Chapter 9

Rotation of Rigid Bodies

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University Physics, Thirteenth Edition
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Goals for Chapter 9

- To describe rotation in terms of angular coordinate, angular velocity, and angular acceleration
- To analyze rotation with constant angular acceleration
- To relate rotation to the linear velocity and linear acceleration of a point on a body
- To understand moment of inertia and how it relates to rotational kinetic energy
- To calculate moment of inertia

Introduction

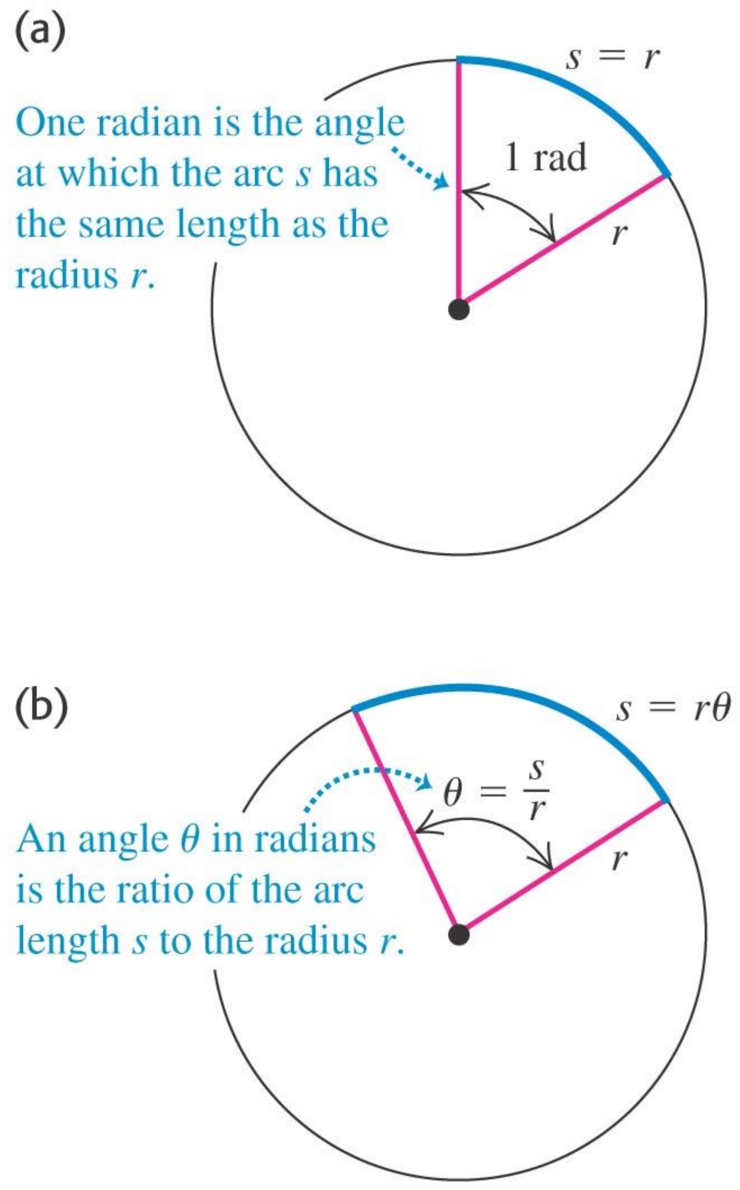
- A wind turbine, a CD, a ceiling fan, and a Ferris wheel all involve rotating rigid objects.
- Real-world rotations can be very complicated because of stretching and twisting of the rotating body. But for now we'll assume that the rotating body is perfectly rigid.

Units of angles

- Position of particle is expressed in angle θ.
- An angle in radians is $\theta = s/r$, as shown in the figure.
- One complete revolution is $360° = 2\pi$ radians.

1 revolution = 2π radians.

- $1\text{rev} = 2\pi$ rad



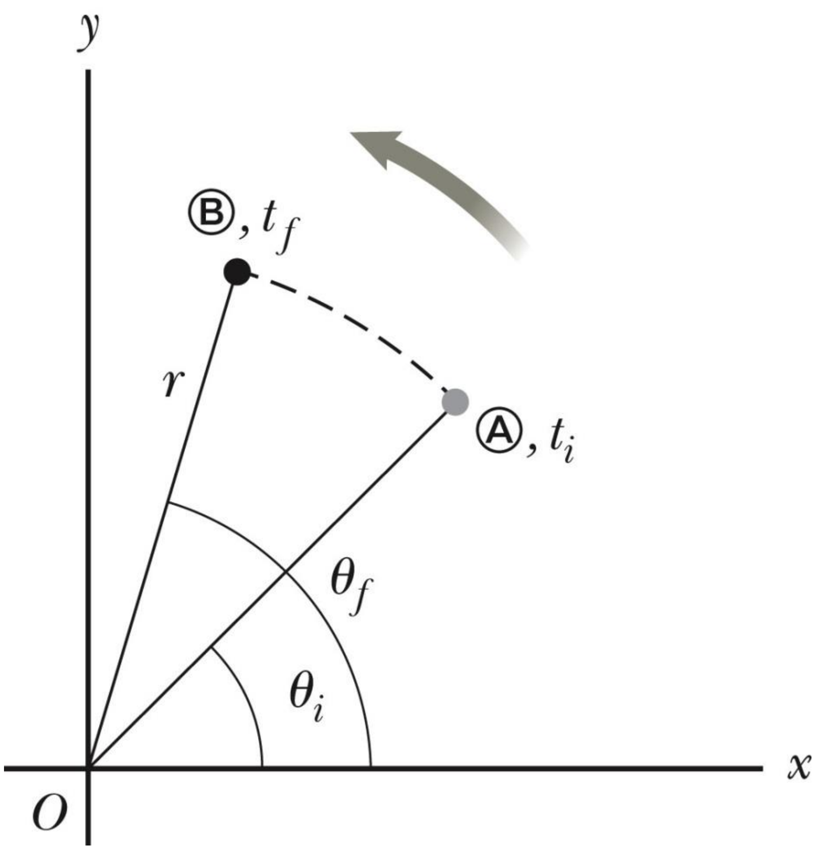
Angular Displacement

The *angular displacement* is defined as the angle the object rotates through during some time interval.

$$\Delta\theta = \theta_f - \theta_i$$

This is the angle that the reference line of length *r* sweeps out.

Unit: radians



Average Angular Speed

The *average* angular speed, ω_{avg}, of a rotating rigid object is the ratio of the angular displacement to the time interval.

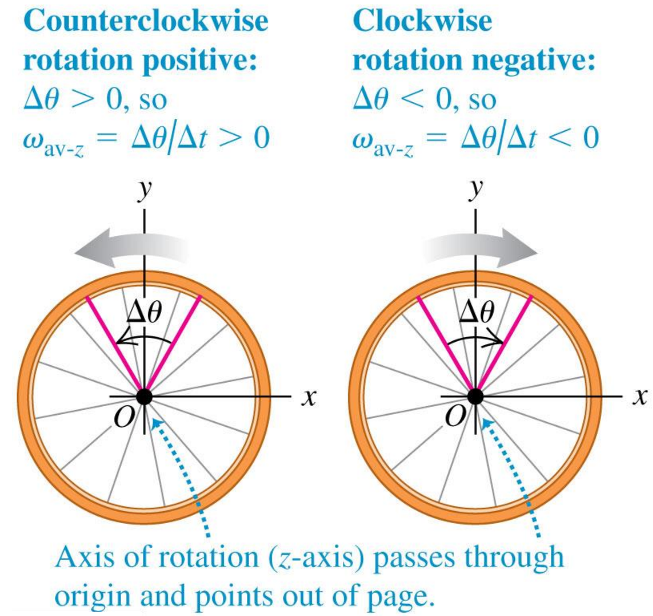
$$\omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

The *instantaneous* angular speed is defined as the limit of the average speed as the time interval approaches zero.

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

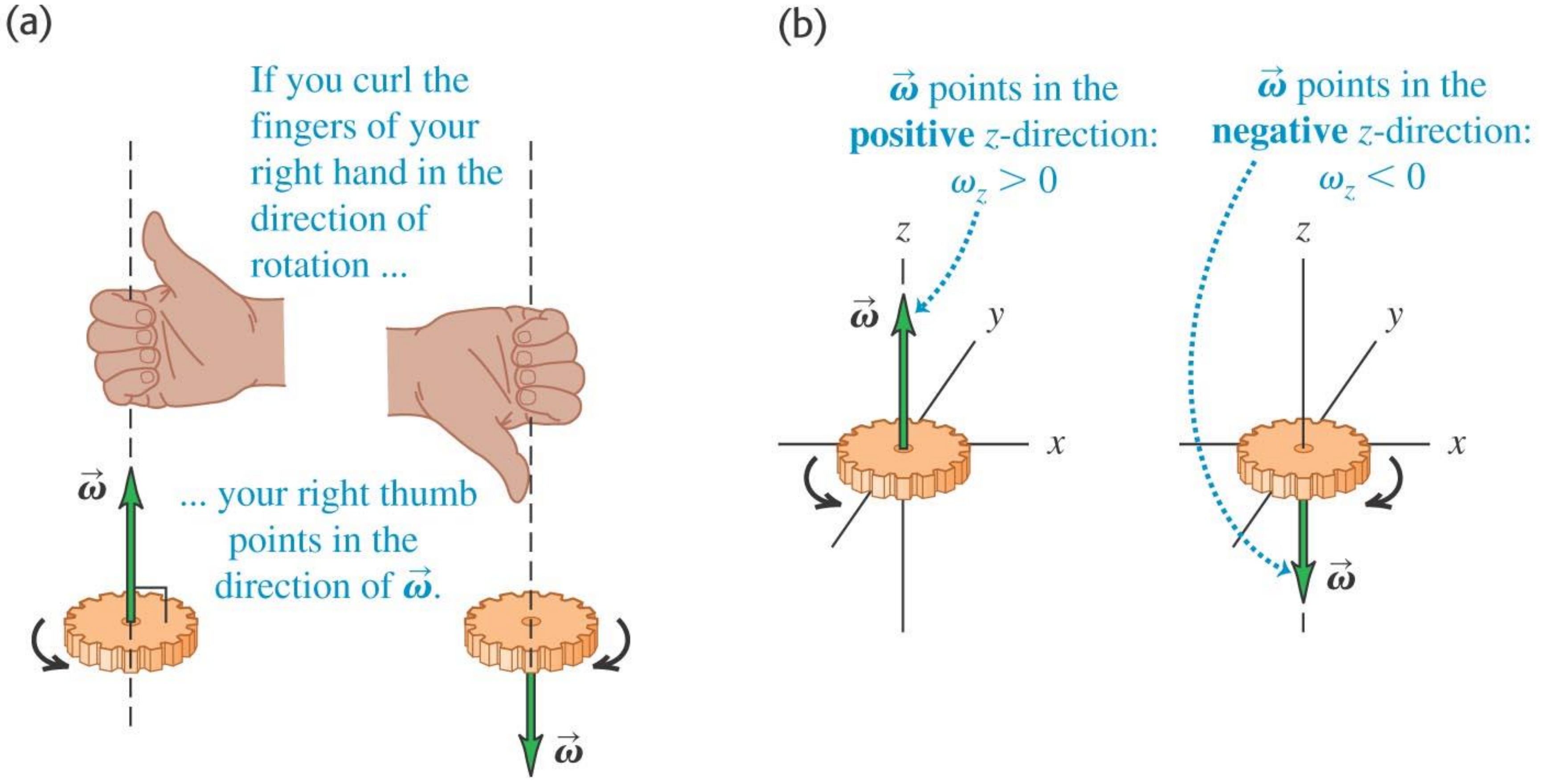
Units of angular speed are radians/sec.

- rad/s or s^{-1} .
- A counterclockwise rotation is positive;
- A clockwise rotation is negative.



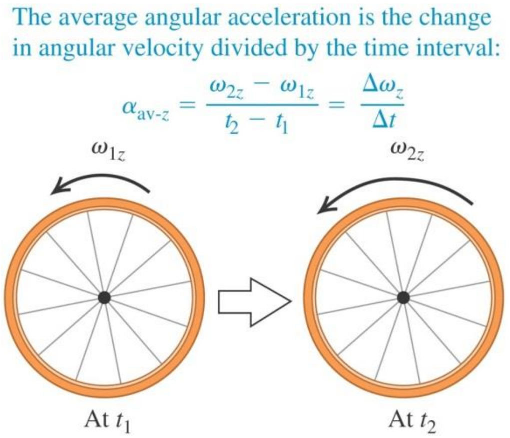
Angular velocity is a vector

- Angular velocity is defined as a vector whose direction is given by the right-hand rule.



Angular Acceleration

The average angular acceleration, α_{avg}, of an object is the ratio of the change in the angular velocity to the time it takes for the object to undergo the change.



$$\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

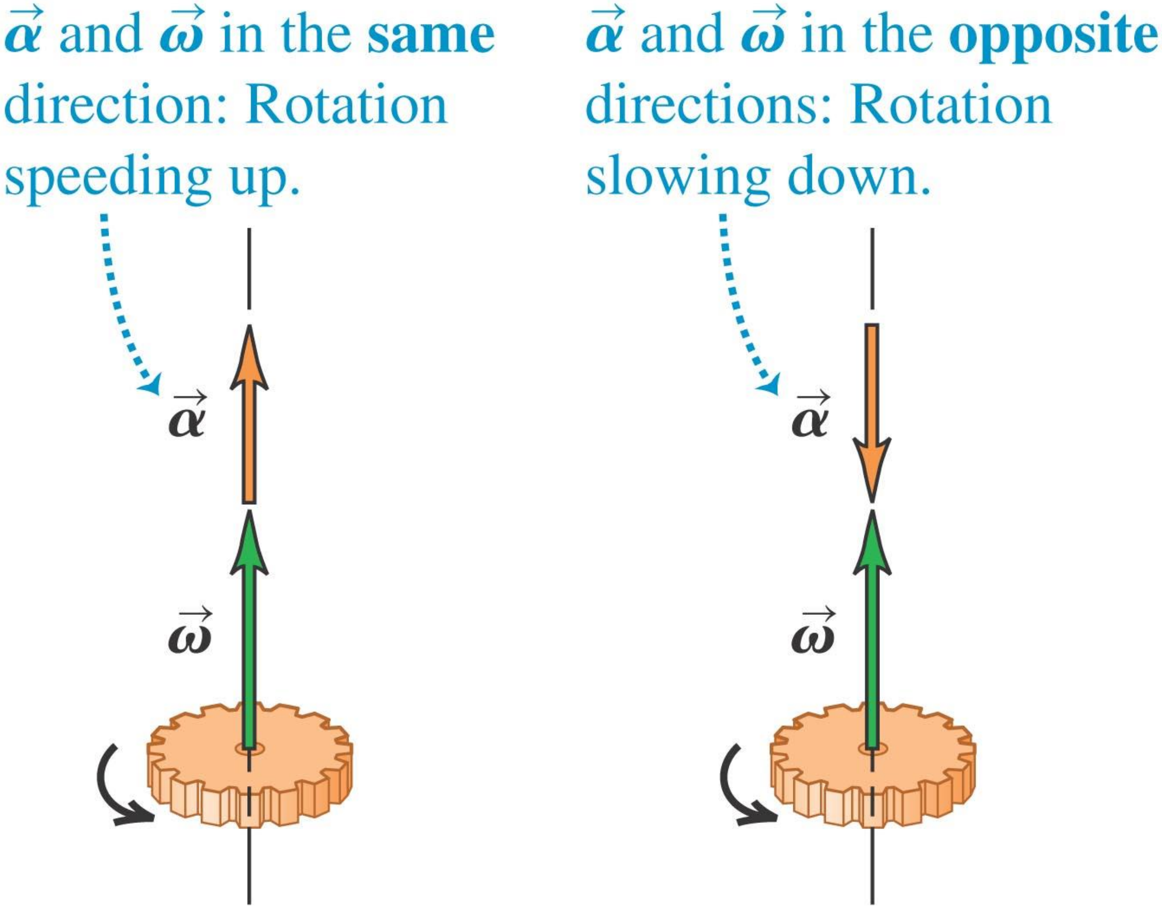
The instantaneous angular acceleration is defined as the limit of the average angular acceleration as the time goes to 0.

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Units of angular acceleration are rad/s² or s^{-2} since radians have no dimensions.

Angular acceleration as a vector

- For a fixed rotation axis, the angular acceleration and angular velocity vectors both lie along that axis.



Rotation with constant angular acceleration

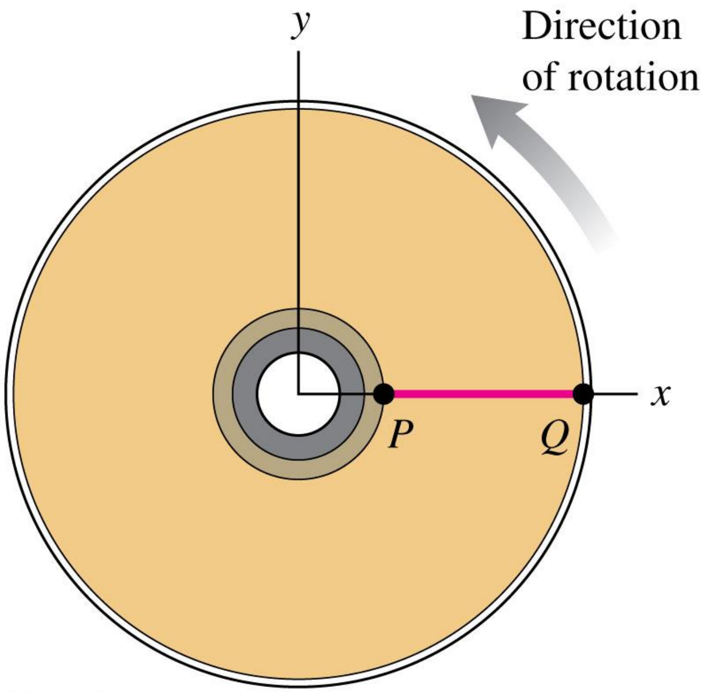
For constant acceleration, the rotational formulas have the same form as the straight-line formulas, as shown in Table 9.1 below.

Table 9.1 Comparison of Linear and Angular Motion with Constant Acceleration

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	$\theta = \theta_0 + \omega_{0z} t + \frac{1}{2} \alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z (\theta - \theta_0)$
$x - x_0 = \frac{1}{2} (v_x + v_{0x}) t$	$\theta - \theta_0 = \frac{1}{2} (\omega_z + \omega_{0z}) t$

Rotation of a Blu-ray disc

A Blu-ray disc is coming to rest after being played. The disc's angular velocity at t= 0 is 27.5 rad/s and its angular acceleration is -10 rad/s^2. Through how many revolutions does the disc rotates before coming to rest .



Relationship Between Angular and Linear Quantities

The linear velocity is always tangent to the circular path.

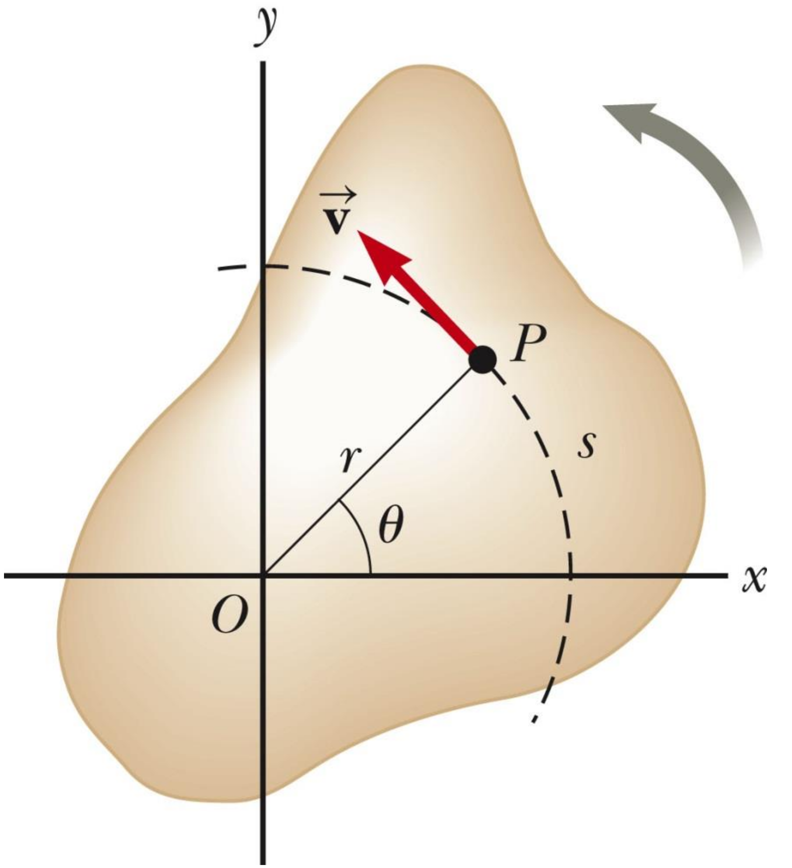
- Called the tangential velocity

The magnitude is defined by the tangential speed .

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$

Since r is not the same for all points on the object, the tangential speed of every point is not the same.

The tangential speed increases as one moves outward from the center of rotation.

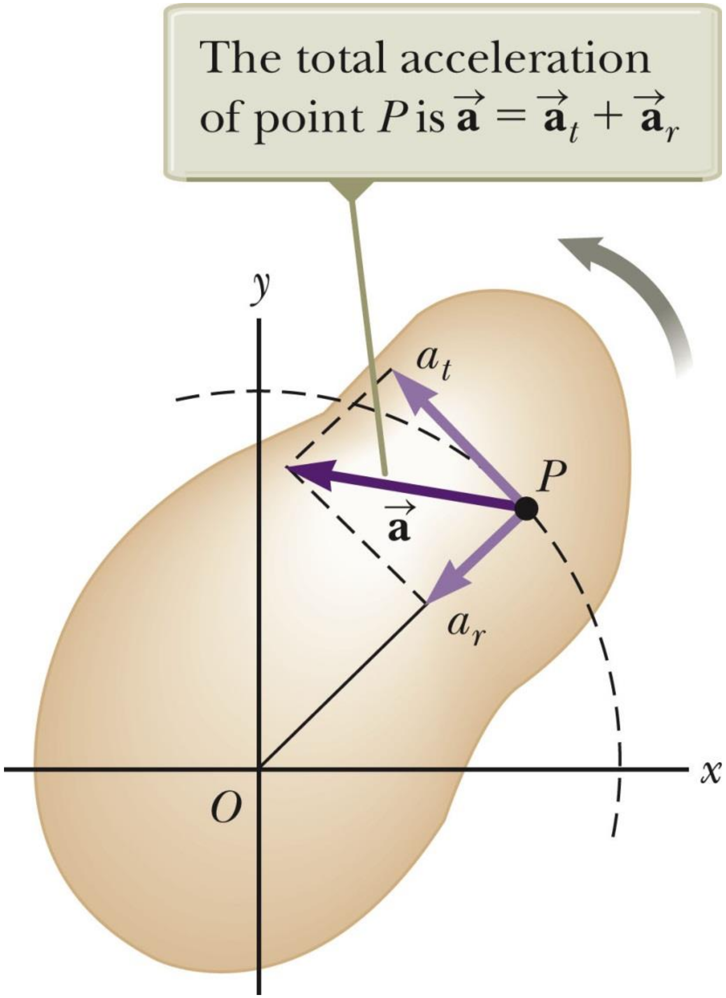


Relationship Between Angular and Linear Quantities - Acceleration

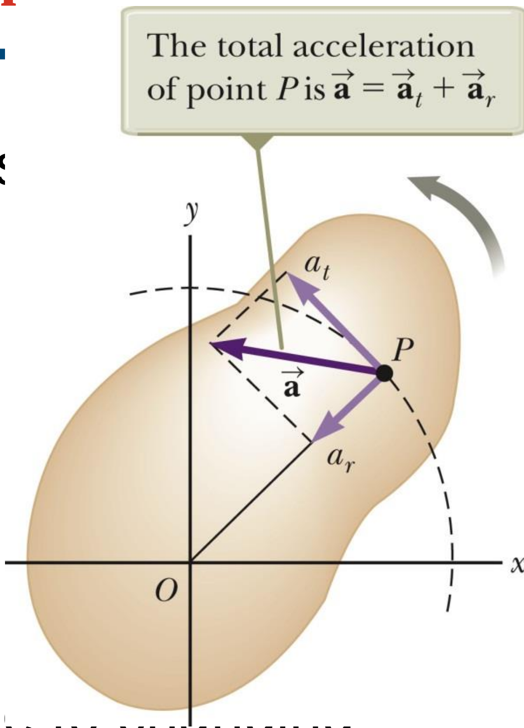
The tangential acceleration is the derivative of the tangential velocity

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$

- Displacement $s = \theta r$
- Speed $v = \omega r$
- Tangential Acceleration $a_t = \alpha r$



Centripetal Acceleration, Resultant Acceleration



An object traveling in a circle, even though it moves with a constant speed, will have an acceleration.

- Therefore, each point on a rotating rigid object will experience a centripetal acceleration.

$$a_C = \frac{v^2}{r} = r\omega^2$$

The tangential component of the acceleration is due to changing speed. The centripetal component of the acceleration is due to changing direction. Total acceleration can be found from these components:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}$$

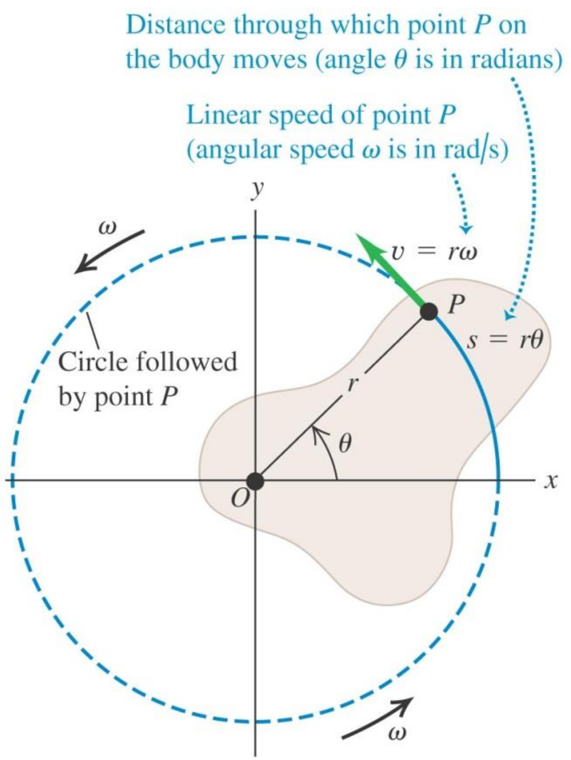
Relating linear and angular kinematics

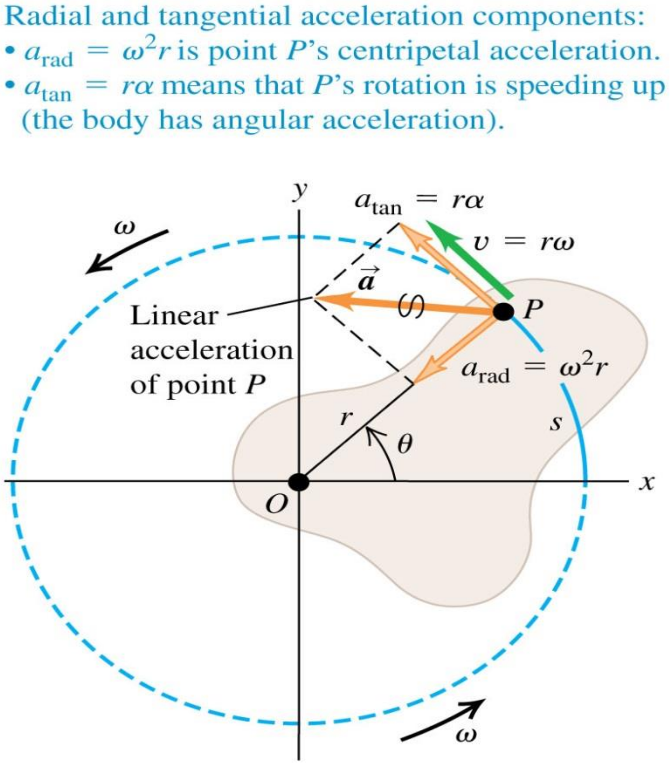
- For a point a distance r from the axis of rotation:

  its linear speed is $v = r\omega$

  its tangential acceleration is $a_{\text{tan}} = r\alpha$

  its centripetal (radial) acceleration is $a_{\text{rad}} = v^2/r = r\omega^2$





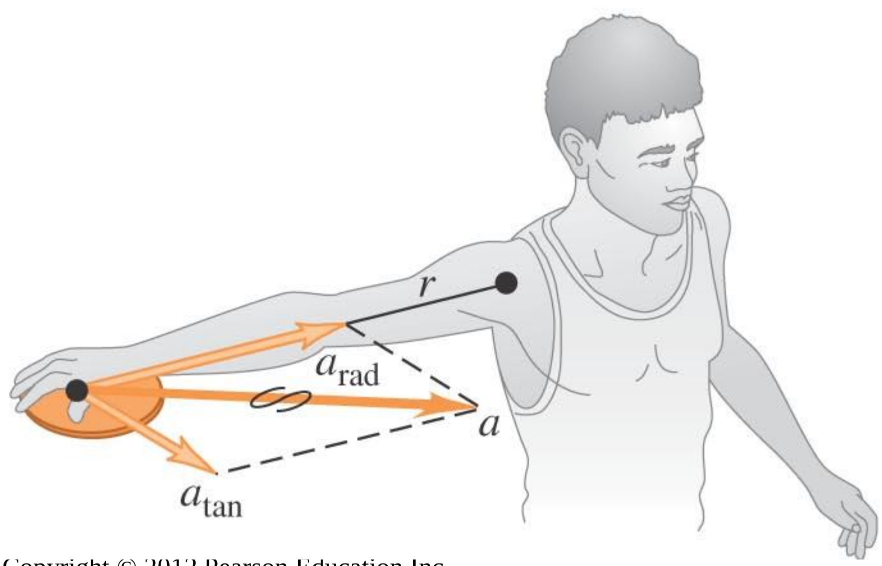
An athlete throwing a discus

An athlete whirls a discus in a circle of radius 80 cm. At certain instant, the athlete is rotating at 10 rad/s and the angular speed is increasing at 50 rad/s². At this instant, find the tangential and centripetal components of the acceleration and its magnitude.

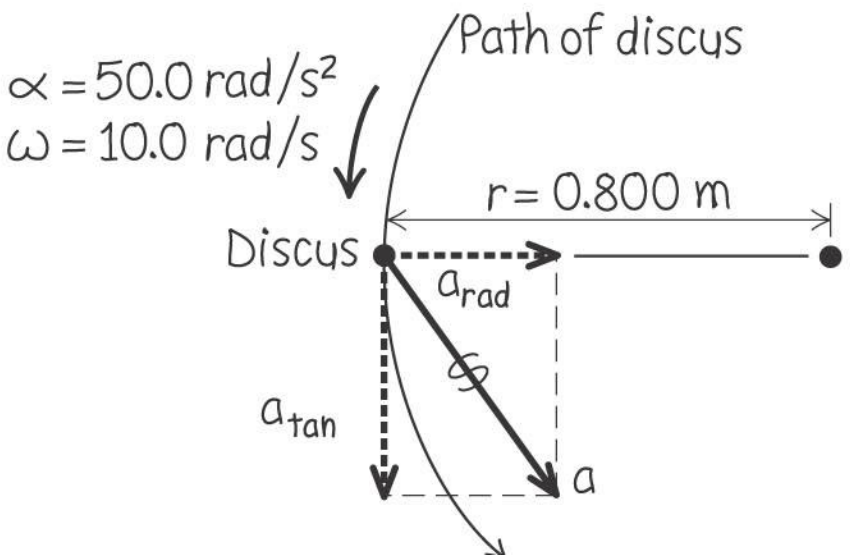
$a_t = \alpha r = (50\ \text{rad/s}^2)\cdot 0.8\ \text{m} = 40\text{m/s}^2$ $a_r = \omega^2 r = (10\text{rad/s})^2(0.8\text{m}) = 80\text{m/s}^2$

$a = \sqrt{(40^2 + 80^2)} = 89.4\text{m/s}^2$

(a)

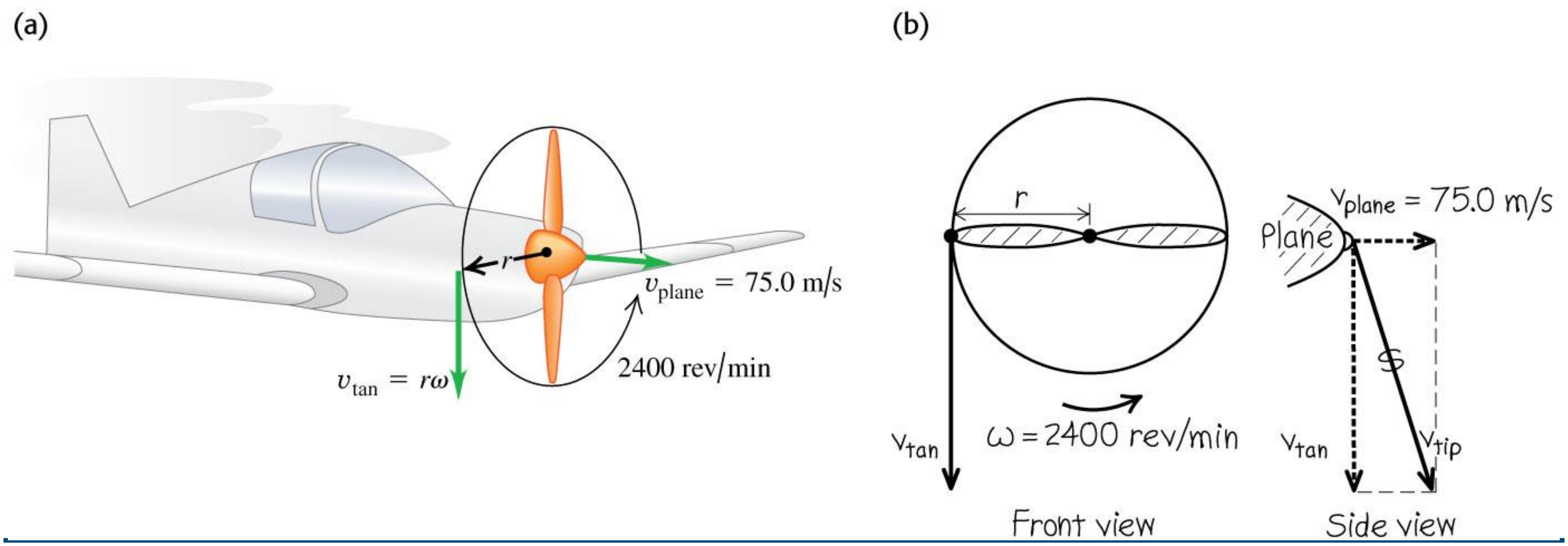


(b)



Designing a propeller

You are designing an airplane propeller that is to turn at 2400 rpm. The forward airspeed of the plane is to be 75 m/s and the speed of the tips of the propeller blades through the air must not exceed 270 m/s. (a) What is the maximum possible propeller radius? (b) With this radius , what is the acceleration of the propeller tip?



Rotational Kinetic Energy,

The total rotational kinetic energy of the system of particles rotating with the same angular velocity about the same axis is the sum of the energies of all its particles.

$$K_i = \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2} m_i (\omega r_i)^2 = \tfrac{1}{2} m_i r_i^2 \omega^2$$

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

$$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$$

I is called the moment of inertia of the system.

The units of rotational kinetic energy are Joules (J).

Moment of Inertia

The definition of moment of inertia of a set of particles is

$$I = \sum_i r_i^2 m_i$$

The dimensions of moment of inertia are ML^2 and its SI units are $kg\cdot m^2$.

- The moment of inertia depends on the mass and how the mass is distributed around the rotational axis.
- For a continuous rigid object, imagine the object to be divided into many small elements, each having a mass of Δm_i.

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

Rotational kinetic energy

- The moment of inertia of a set of particles is

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots = \Sigma m_i r_i^2$$

- The rotational kinetic energy of a set of particles having a moment of inertia I is

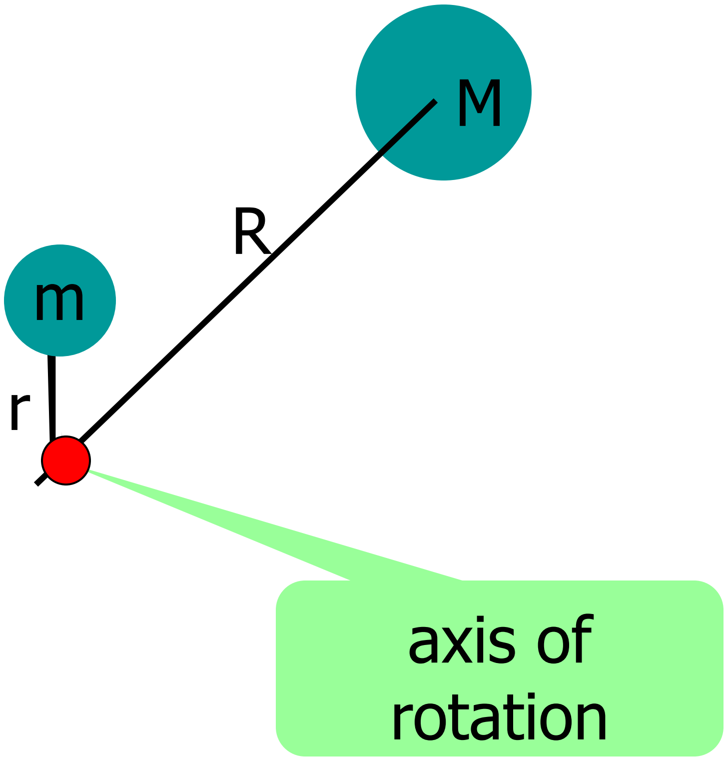
$$K_R = \frac{1}{2} I \omega^2$$

- The moment of inertia of a rigid object is

$$I = \int r^2 dm$$

- The rotational kinetic energy of a rigid body having a moment of inertia I is

$$K_R = \frac{1}{2} I \omega^2$$

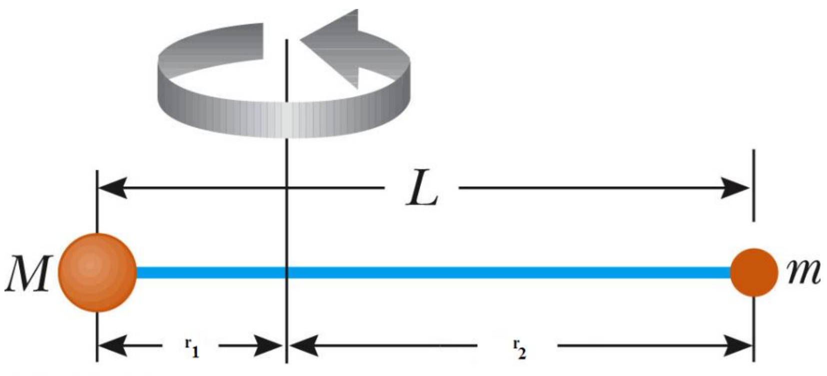


$$I = mr^2 + MR^2$$

Example

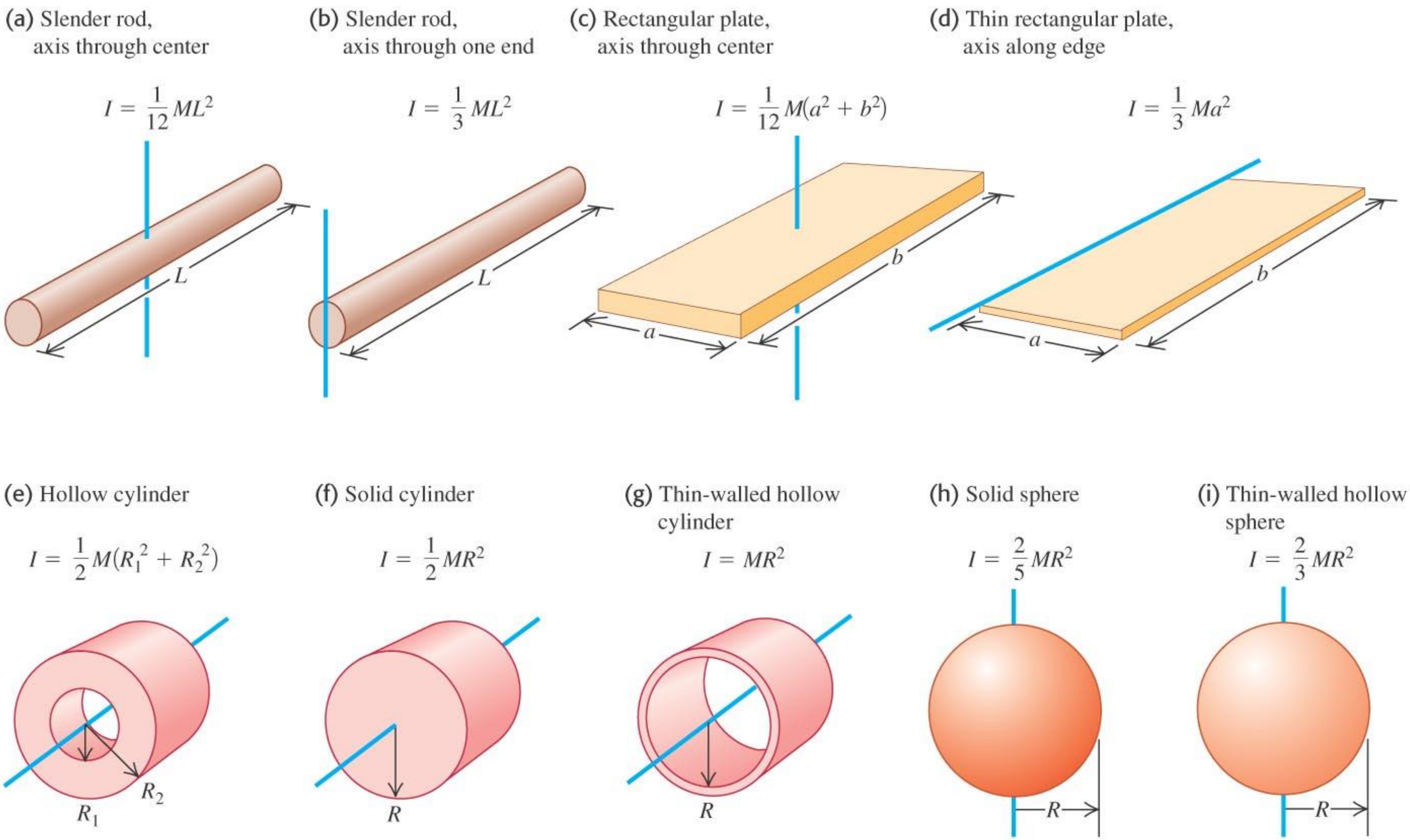
If M=10 kg, m=2 kg, L =2m, r_1 = 0.3 m and mass of the rod can be neglected, what is the moment of inertia about axis of rotation?

$$I = \sum_i r_i^2 m_i \qquad I = Mr_1^2 + mr_2^2$$



Moments of inertia of some common bodies

- Table 9.2 gives the moments of inertia of various bodies.



The parallel-axis theorem

The *parallel-axis theorem* is:

$$I_P = I_{\mathrm{cm}} + Md^2.$$

Gravitational potential energy of an extended body

- The gravitational potential energy of an extended body is the same as if all the mass were concentrated at its center of mass: $U_{grav} = Mgy_{cm}$.
- Total energy of the system

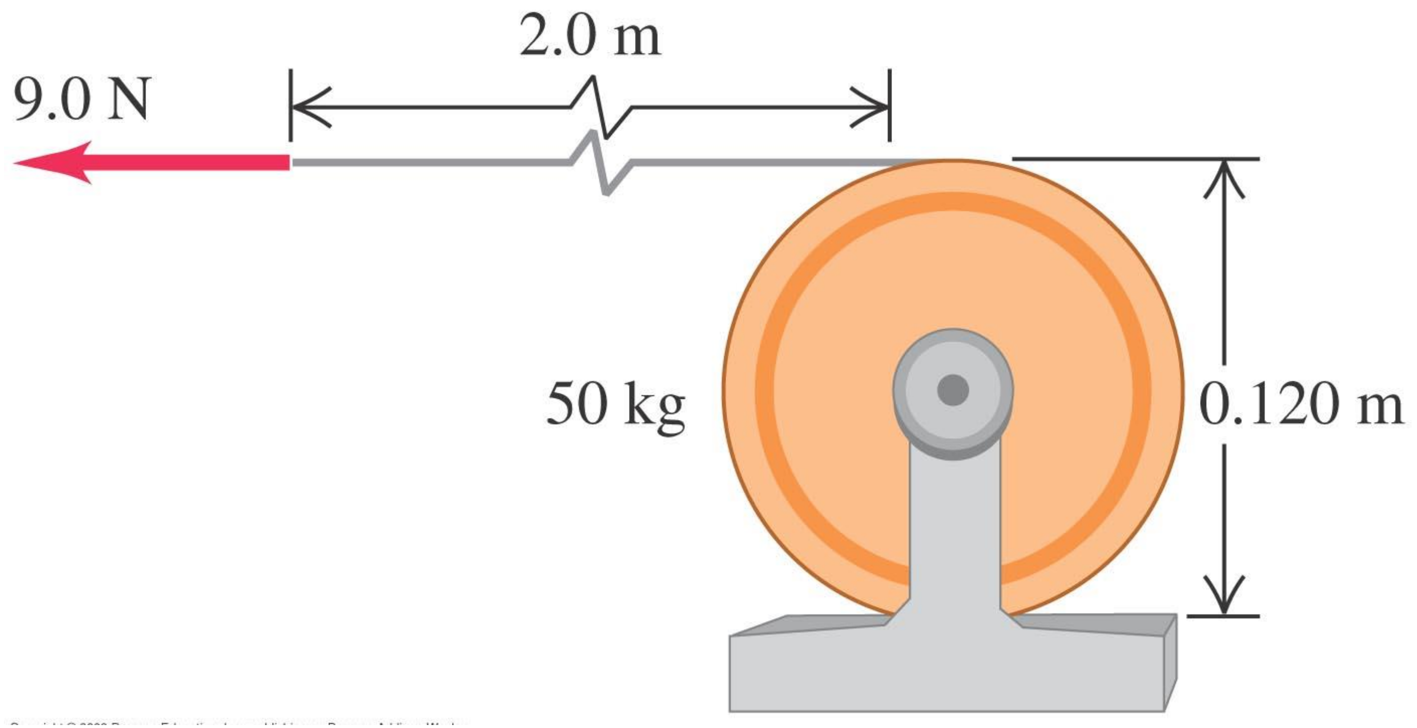
$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy_{cm}$$

- Work- energy theorem

$$E_i + W_{nc} = E_f$$

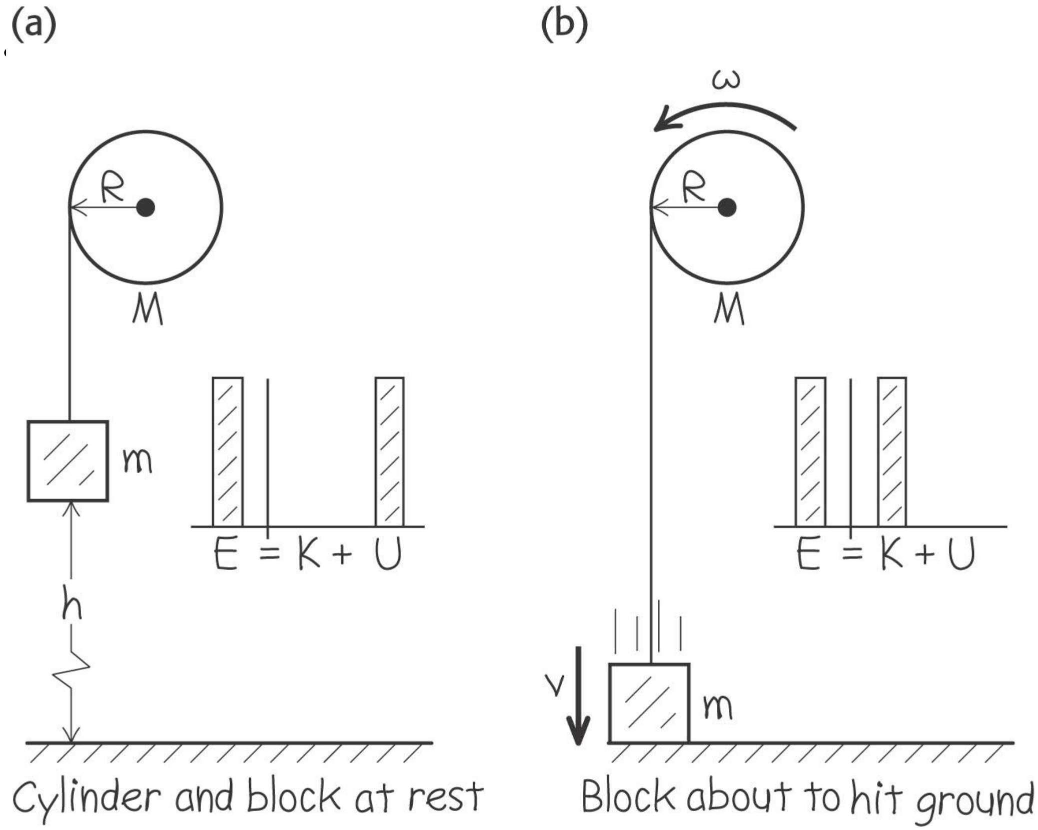
An unwinding cable

We wrap a light cable around a solid cylinder of mass of 50 kg and diameter 0.12 m, which rotates about a horizontal axis.. We pull the free end of the cable with constant 9-N force for a distance 2.0m; it turns the cylinder as it unwinds. The cylinder is initially at rest. Find its final angular speed.

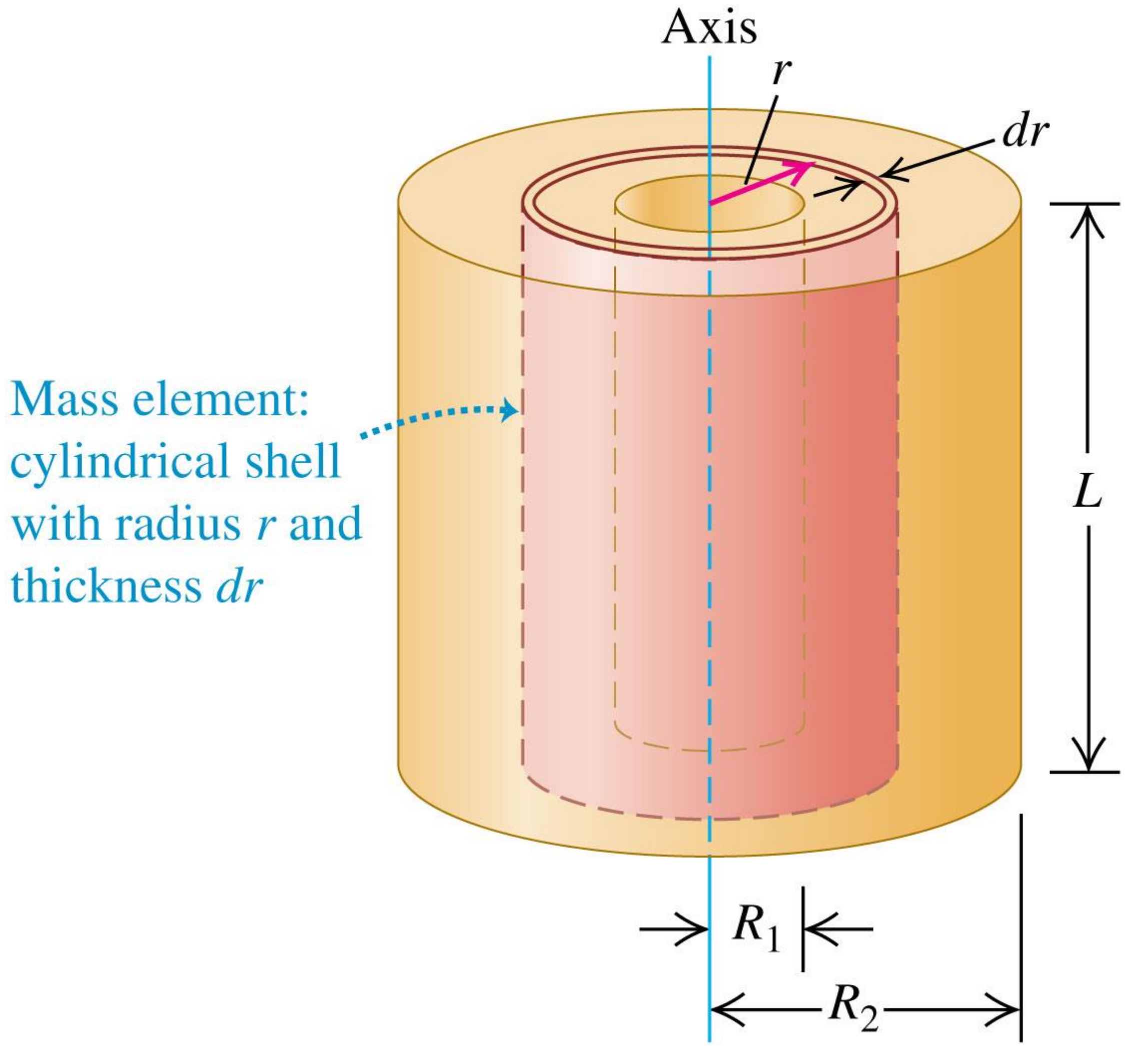


More on an unwinding cable

A block m attached to pulley in form of cylinder of mass M and radius R is released from rest at a distance h above the floor. Find the expressions for the speed of the falling block and an angular velocity of the cylinder as the block strikes the floor.



Moment of inertia of a hollow or solid cylinder



Moment of inertia of a uniform solid sphere

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