## **Rotational Motion Problems Solutions**

**12.1. Model:** A spinning skater, whose arms are outstretched, is a rigid rotating body. **Visualize:** 



**Solve:** The speed  $v = r\omega$ , where r = 140 cm/2 = 0.70 m. Also,  $180 \text{ rpm} = (180)2\pi/60 \text{ rad/s} = 6\pi \text{ rad/s}$ . Thus,  $v = (0.70 \text{ m})(6\pi \text{ rad/s}) = 13.2 \text{ m/s}$ .

Assess: A speed of 13.2 m/s  $\approx$  26 mph for the hands is a little high, but reasonable.

**12.7.** Visualize: Please refer to Figure EX12.7. The coordinates of the three masses  $m_A$ ,  $m_B$ , and  $m_C$  are (0 cm, 10 cm), (10 cm, 10 cm), and (10 cm, 0 cm), respectively.

Solve: The coordinates of the center of mass are

$$x_{\rm cm} = \frac{m_{\rm A}x_{\rm A} + m_{\rm B}x_{\rm B} + m_{\rm C}x_{\rm C}}{m_{\rm A} + m_{\rm B} + m_{\rm C}} = \frac{(200 \text{ g})(0 \text{ cm}) + (300 \text{ g})(10 \text{ cm}) + (100 \text{ g})(10 \text{ cm})}{(200 \text{ g} + 300 \text{ g} + 100 \text{ g})} = 6.7 \text{ cm}$$
  
$$y_{\rm cm} = \frac{m_{\rm A}y_{\rm A} + m_{\rm B}y_{\rm B} + m_{\rm C}y_{\rm C}}{m_{\rm A} + m_{\rm B} + m_{\rm C}} = \frac{(200 \text{ g})(10 \text{ cm}) + (300 \text{ g})(10 \text{ cm}) + (100 \text{ g})(0 \text{ cm})}{(200 \text{ g} + 300 \text{ g} + 100 \text{ g})} = 8.3 \text{ cm}$$

**12.17.** Model: The door is a slab of uniform density.

Solve: (a) The hinges are at the edge of the door, so from Table 12.2,

$$I = \frac{1}{3} (25 \text{ kg}) (0.91 \text{ m})^2 = 6.9 \text{ kg m}^2$$

(b) The distance from the axis through the center of mass along the height of the door is

 $d = \left(\frac{0.91 \text{ m}}{2} - 0.15 \text{ m}\right) = 0.305 \text{ m}.$  Using the parallel–axis theorem,

$$I = I_{\rm cm} + Md^2 = \frac{1}{12} (25 \text{ kg}) (0.91 \text{ m})^2 + (25 \text{ kg}) (0.305 \text{ cm})^2 = 4.1 \text{ kg m}^2$$

Assess: The moment of inertia is less for a parallel axis through a point closer to the center of mass.

12.19. Visualize:



**Solve:** Torque by a force is defined as  $\tau = Fr \sin \phi$  where  $\phi$  is measured counterclockwise from the  $\vec{r}$  vector to the  $\vec{F}$  vector. The net torque on the pulley about the axle is the torque due to the 30 N force plus the torque due to the 20 N force: (30 N) $r_1 \sin \phi_1 + (20 \text{ N})r_2 \sin \phi_2 = (30 \text{ N})(0.02 \text{ m}) \sin (-90^\circ) + (20 \text{ N})(0.02 \text{ m}) \sin (90^\circ)$ 

$$= (-0.60 \text{ N m}) + (0.40 \text{ N m}) = -0.20 \text{ N m}$$

Assess: A negative torque causes a clockwise acceleration of the pulley.

**12.23. Model:** The beam is a solid rigid body. **Visualize:** 



The steel beam experiences a torque due to the gravitational force on the construction worker  $(\vec{F}_G)_C$  and the gravitational force on the beam  $(\vec{F}_G)_B$ . The normal force exerts no torque since the net torque is calculated about the point where the beam is bolted into place.

**Solve:** The net torque on the steel beam about point O is the sum of the torque due to  $(\vec{F}_{G})_{C}$  and the torque due to  $(\vec{F}_{G})_{B}$ . The gravitational force on the beam acts at the center of mass.

m

$$\tau = ((F_G)_C)(4.0 \text{ m})\sin(-90^\circ) + ((F_G)_B)(2.0 \text{ m})\sin(-90^\circ)$$
  
= -(70 kg)(9.80 m/s<sup>2</sup>)(4.0 m) - (500 kg)(9.80 m/s<sup>2</sup>)(2.0 m) = -12.5 kN

The negative torque means these forces would cause the beam to rotate clockwise. The magnitude of the torque is 12.5 kN m.

**12.27. Model:** Two balls connected by a rigid, massless rod are a rigid body rotating about an axis through the center of mass. Assume that the size of the balls is small compared to 1 m. **Visualize:** 



We placed the origin of the coordinate system on the 1.0 kg ball. **Solve:** The center of mass and the moment of inertia are

$$x_{\rm cm} = \frac{(1.0 \text{ kg})(0 \text{ m}) + (2.0 \text{ kg})(1.0 \text{ m})}{(1.0 \text{ kg} + 2.0 \text{ kg})} = 0.667 \text{ m} \text{ and } y_{\rm cm} = 0 \text{ m}$$
$$I_{\rm about \, cm} = \sum m_i r_i^2 = (1.0 \text{ kg})(0.667 \text{ m})^2 + (2.0 \text{ kg})(0.333 \text{ m})^2 = 0.667 \text{ kg m}^2$$

We have  $\omega_f = 0$  rad/s,  $t_f - t_i = 5.0$  s, and  $\omega_i = -20$  rpm  $= -20(2\pi \text{ rad}/60 \text{ s}) = -\frac{2}{3}\pi \text{ rad/s}$ , so  $\omega_f = \omega_i + \alpha(t_f - t_i)$  becomes

0 rad/s = 
$$\left(-\frac{2\pi}{3}$$
 rad/s $\right) + \alpha(5.0 \text{ s}) \Rightarrow \alpha = \frac{2\pi}{15}$  rad/s<sup>2</sup>

Having found I and  $\alpha$ , we can now find the torque  $\tau$  that will bring the balls to a halt in 5.0 s:

$$\tau = I_{\text{about cm}} \alpha = \left(\frac{2}{3} \text{kg m}^2\right) \left(\frac{2\pi}{15} \text{ rad/s}^2\right) = \frac{4\pi}{45} \text{ N m} = 0.28 \text{ N m}$$

The magnitude of the torque is 0.28 N m, applied in the counterclockwise direction.

**12.31.** Model: The rod is in rotational equilibrium, which means that  $\tau_{net} = 0$ . Visualize:



As the gravitational force on the rod and the hanging mass pull down (the rotation of the rod is exaggerated in the figure), the rod touches the pin at two points. The piece of the pin at the very end pushes down on the rod; the right end of the pin pushes up on the rod. To understand this, hold a pen or pencil between your thumb and forefinger, with your thumb on top (pushing down) and your forefinger underneath (pushing up).

**Solve:** Calculate the torque about the left end of the rod. The downward force exerted by the pin acts through this point, so it exerts no torque. To prevent rotation, the pin's normal force  $\vec{n}_{pin}$  exerts a positive torque (ccw about the left end) to balance the negative torques (cw) of the gravitational force on the mass and rod. The gravitational force on the rod acts at the center of mass, so

$$\tau_{\text{net}} = 0 \text{ N m} = \tau_{\text{pin}} - (0.40 \text{ m})(2.0 \text{ kg})(9.8 \text{ m/s}^2) - (0.80 \text{ m})(0.50 \text{ kg})(9.8 \text{ m/s}^2)$$
$$\Rightarrow \tau_{\text{pin}} = 11.8 \text{ N m}$$

**12.35.** Solve: (a) According to Equation 12.35, the speed of the center of mass of the tire is

$$v_{\rm cm} = R\omega = 20 \text{ m/s} \Rightarrow \omega = \frac{v_{\rm cm}}{R} = \frac{20 \text{ m/s}}{0.30 \text{ m}} = 66.67 \text{ rad/s} = (66.7) \left(\frac{60}{2\pi}\right) \text{ rpm} = 6.4 \times 10^2 \text{ rpm}$$

(b) The speed at the top edge of the tire relative to the ground is  $v_{top} = 2v_{cm} = 2(20 \text{ m/s}) = 40 \text{ m/s}.$ 

(c) The speed at the bottom edge of the tire relative to ground is  $v_{\text{bottom}} = 0$  m/s.

**12.43.** Solve: (a)  $\vec{C} \times \vec{D} = 0$  implies that  $\vec{D}$  must also be in the same or opposite direction as the  $\vec{C}$  vector or zero, because  $\hat{i} \times \hat{i} = 0$ . Thus  $\vec{D} = n\hat{i}$ , where *n* could be any real number.

(b) \$\vec{C}\$\times\$\vec{E}\$ = 6\$\hat{k}\$ implies that \$\vec{E}\$ must be along the \$\jet{j}\$ vector, because \$\vec{i}\$\times\$\vec{j}\$ = \$\hat{k}\$. Thus \$\vec{E}\$ = 2\$\jet{j}\$.
(c) \$\vec{C}\$\times\$\vec{F}\$ = -3\$\jet{j}\$ implies that \$\vec{F}\$ must be along the \$\hat{k}\$ vector, because \$\vec{i}\$\times\$\hat{k}\$ = -\jet{j}\$. Thus \$\vec{F}\$ = 1\$\hat{k}\$.

**12.49.** Model: The disk is a rotating rigid body.

**Visualize:** Please refer to Figure EX12.49.

Solve: From Table 12.2, the moment of inertial of the disk about its center is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.0 \text{ kg})(0.020 \text{ m})^2 = 4.0 \times 10^{-4} \text{ kg m}^2$$

The angular velocity  $\omega$  is 600 rpm =  $600 \times 2\pi/60$  rad/s =  $20\pi$  rad/s. Thus,  $L = I\omega = (4.0 \times 10^{-4} \text{ kg m}^2)(20\pi \text{ rad/s}) = 0.025 \text{ kg m}^2/\text{s}$ . If we wrap our right fingers in the direction of the disk's rotation, our thumb will point in the -x direction. Consequently,

$$\vec{L} = -0.025 \ \hat{i} \ \text{kg m}^2/\text{s} = (0.025 \ \text{kg m}^2/\text{s}, \text{ into page})$$

**12.55. Model**: The disk is a rigid rotating body. The axis is perpendicular to the plane of the disk. **Visualize:** 



Solve: (a) From Table 12.2, the moment of inertia of a disk about its center is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.0 \text{ kg})(0.10 \text{ m})^2 = 0.010 \text{ kg m}^2$$

(b) To find the moment of inertia of the disk through the edge, we can make use of the parallel axis theorem:

 $I = I_{\text{center}} + Mh^2 = (0.010 \text{ kg m}^2) + (2.0 \text{ kg})(0.10 \text{ m})^2 = 0.030 \text{ kg m}^2$ 

Assess: The larger moment of inertia about the edge means there is more inertia to rotational motion about the edge than about the center.

**12.63.** Model: The structure is a rigid body. **Visualize:** 



**Solve:** We pick the left end of the beam as our pivot point. We don't need to know the forces  $F_{\rm h}$  and  $F_{\rm v}$  because the pivot point passes through the line of application of  $F_{\rm h}$  and  $F_{\rm v}$  and therefore these forces do not exert a torque. For the beam to stay in equilibrium, the net torque about this point is zero. We can write

$$\tau_{\text{about left end}} = -(F_{\text{G}})_{\text{B}}(3.0 \text{ m}) - (F_{\text{G}})_{\text{W}}(4.0 \text{ m}) + (T \sin 150^{\circ})(6.0 \text{ m}) = 0 \text{ N m}$$

Using  $(F_G)_B = (1450 \text{ kg})(9.8 \text{ m/s}^2)$  and  $(F_G)_W = (80 \text{ kg})(9.8 \text{ m/s}^2)$ , the torque equation can be solved to yield T = 15,300 N. The tension in the cable is slightly more than the cable rating. The worker should be worried.

**12.69.** Model: The flywheel is a rigid body rotating about its central axis. Visualize:



Solve: (a) The radius of the flywheel is R = 0.75 m and its mass is M = 250 kg. The moment of inertia about the axis of rotation is that of a disk:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(250 \text{ kg})(0.75 \text{ m})^2 = 70.31 \text{ kg m}^2$$

The angular acceleration is calculated as follows:

$$\tau_{\rm net} = I\alpha \Longrightarrow \alpha = \tau_{\rm net}/I = (50 \text{ N m})/(70.31 \text{ kg m}^2) = 0.711 \text{ rad/s}^2$$

Using the kinematic equation for angular velocity gives

$$\omega_1 = \omega_0 + \alpha (t_1 - t_0) = 1200 \text{ rpm} = 40 \ \pi \text{ rad/s} = 0 \text{ rad/s} + 0.711 \text{ rad/s}^2 (t_1 - 0 \text{ s})$$
  
 $\Rightarrow t_1 = 177 \text{ s}$ 

(b) The energy stored in the flywheel is rotational kinetic energy:

$$K_{\rm rot} = \frac{1}{2}I\omega_{\rm l}^2 = \frac{1}{2}(70.31 \text{ kg m}^2)(40\pi \text{ rad/s})^2 = 5.55 \times 10^5 \text{ J}$$

The energy stored is  $5.6 \times 10^5$  J.

(c) Average power delivered = 
$$\frac{\text{energy delivered}}{\text{time interval}} = \frac{(5.55 \times 10^3 \text{ J})/2}{2.0 \text{ s}} = 1.39 \times 10^5 \text{ W} = 139 \text{ kW}$$

(d) Because  $\tau = I\alpha$ ,  $\Rightarrow \tau = I \frac{\Delta\omega}{\Delta t} = I \left( \frac{\omega_{\text{full energy}} - \omega_{\text{half energy}}}{\Delta t} \right)$ .  $\omega_{\text{full energy}} = \omega_1$  (from part (a)) = 40 $\pi$  rad/s.  $\omega_{\text{half energy}}$  can be

obtained as:

$$\frac{1}{2}I\omega_{\text{half energy}}^2 = \frac{1}{2} K_{\text{rot}} \Longrightarrow \omega_{\text{half energy}} = \sqrt{\frac{K_{\text{rot}}}{I}} = \sqrt{\frac{5.55 \times 10^5 \text{ J}}{70.31 \text{ kg m}^2}} = 88.85 \text{ rad/s}$$

Thus

$$\tau = (70.31 \text{ kg m}^2) \left( \frac{40 \ \pi \text{ rad/s} - 88.85 \text{ rad/s}}{2.0 \text{ s}} \right) = 1.30 \text{ kN m}$$

**12.71. Model:** Assume the string does not slip on the pulley. **Visualize:** 



The free-body diagrams for the two blocks and the pulley are shown. The tension in the string exerts an upward force on the block  $m_2$ , but a downward force on the outer edge of the pulley. Similarly the string exerts a force on block  $m_1$  to the right, but a leftward force on the outer edge of the pulley.

**Solve:** (a) Newton's second law for  $m_1$  and  $m_2$  is  $T = m_1a_1$  and  $T - m_2g = m_2a_2$ . Using the constraint  $-a_2 = +a_1 = a$ , we have  $T = m_1a$  and  $-T + m_2g = m_2a$ . Adding these equations, we get  $m_2g = (m_1 + m_2)a$ , or

$$a = \frac{m_2 g}{m_1 + m_2} \Longrightarrow T = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2}$$

(b) When the pulley has mass m, the tensions  $(T_1 \text{ and } T_2)$  in the upper and lower portions of the string are different. Newton's second law for  $m_1$  and the pulley are:

$$T_1 = m_1 a$$
 and  $T_1 R - T_2 R = -I \alpha$ 

We are using the minus sign with  $\alpha$  because the pulley accelerates clockwise. Also,  $a = R\alpha$ . Thus,  $T_1 = m_1 a$  and

$$T_2 - T_1 = \frac{I}{R} \frac{a}{R} = \frac{aI}{R^2}$$

Adding these two equations gives

$$T_2 = a \left( m_1 + \frac{I}{R^2} \right)$$

Newton's second law for  $m_2$  is  $T_2 - m_2 g = m_2 a_2 = -m_2 a$ . Using the above expression for  $T_2$ ,

$$a\left(m_1 + \frac{I}{R^2}\right) + m_2 a = m_2 g \Longrightarrow a = \frac{m_2 g}{m_1 + m_2 + I/R^2}$$

Since  $I = \frac{1}{2}m_{\rm p}R^2$  for a disk about its center,

$$a = \frac{m_2 g}{m_1 + m_2 + \frac{1}{2}m_p}$$

With this value for *a* we can now find  $T_1$  and  $T_2$ :

$$T_{1} = m_{1}a = \frac{m_{1}m_{2}g}{m_{1} + m_{2} + \frac{1}{2}m_{p}} \qquad T_{2} = a(m_{1} + I/R^{2}) = \frac{m_{2}g}{\left(m_{1} + m_{2} + \frac{1}{2}m_{p}\right)} \left(m_{1} + \frac{1}{2}m_{p}\right) = \frac{m_{2}\left(m_{1} + \frac{1}{2}m_{p}\right)g}{m_{1} + m_{2} + \frac{1}{2}m_{p}}$$

Assess: For m = 0 kg, the equations for a,  $T_1$ , and  $T_2$  of part (b) simplify to

$$a = \frac{m_2 g}{m_1 + m_2}$$
 and  $T_1 = \frac{m_1 m_2 g}{m_1 + m_2}$  and  $T_2 = \frac{m_1 m_2 g}{m_1 + m_2}$ 

These agree with the results of part (a).

**12.77. Model:** The hoop is a rigid body rotating about an axle at the edge of the hoop. The gravitational torque on the hoop causes it to rotate, transforming the gravitational potential energy of the hoop's center of mass into rotational kinetic energy.

## Visualize:



We placed the origin of the coordinate system at the hoop's edge on the axle. In the initial position, the center of mass is a distance R above the origin, but it is a distance R below the origin in the final position.

**Solve:** (a) Applying the parallel-axis theorem,  $I_{edge} = I_{cm} + mR^2 = mR^2 + mR^2 = 2mR^2$ . Using this expression in the energy conservation equation  $K_f + U_{gf} = K_i + U_{gi}$  yields:

$$\frac{1}{2}I_{edge}\omega_{1}^{2} + mgy_{1} = \frac{1}{2}I_{edge}\omega_{0}^{2} + mgy_{0} \qquad \frac{1}{2}(2mR^{2})\omega_{1}^{2} - mgR = 0 \text{ J} + mgR \Longrightarrow \omega_{1} = \sqrt{\frac{2g}{R}}$$

(b) The speed of the lowest point on the hoop is

$$v = (\omega_1)(2R) = \sqrt{\frac{2g}{R}}(2R) = \sqrt{8gR}$$

Assess: Note that the speed of the lowest point on the loop involves a distance of 2*R* instead of *R*.

**12.85. Model:** The mechanical energy of both the hoop (h) and the sphere (s) is conserved. The initial gravitational potential energy is transformed into kinetic energy as the objects roll down the slope. The kinetic energy is a combination of translational and rotational kinetic energy. We also assume no slipping of the hoop or of the sphere. **Visualize:** 



The zero of gravitational potential energy is chosen at the bottom of the slope. **Solve:** (a) The energy conservation equation for the sphere or hoop  $K_{\rm f} + U_{\rm gf} = K_{\rm i} + U_{\rm gi}$  is

$$\frac{1}{2}I(\omega_1)^2 + \frac{1}{2}m(v_1)^2 + mgy_1 = \frac{1}{2}I(\omega_0)^2 + \frac{1}{2}m(v_0)^2 + mgy_0$$

For the sphere, this becomes

$$\frac{1}{2} \left(\frac{2}{5}mR^2\right) \frac{(v_1)_s^2}{R^2} + \frac{1}{2}m(v_1)_s^2 + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + mgh_s$$
$$\Rightarrow \frac{7}{10}(v_1)_s^2 = gh \Rightarrow (v_1)_s = \sqrt{10gh/7} = \sqrt{10(9.8 \text{ m/s}^2)(0.30 \text{ m})/7} = 2.05 \text{ m/s}$$

For the hoop, this becomes

$$\frac{1}{2}(mR^2)\frac{(v_1)_h^2}{R^2} + \frac{1}{2}m(v_1)_h^2 + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + mgh_{\text{hoop}}$$
$$\implies h_{\text{hoop}} = \frac{(v_1)_h^2}{g}$$

For the hoop to have the same velocity as that of the sphere,

$$h_{\text{hoop}} = \frac{(v_1)_s^2}{g} = \frac{(2.05 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 42.9 \text{ cm}$$

The hoop should be released from a height of 43 cm.

(b) As we see in part (a), the speed of a hoop at the bottom depends only on the starting height and not on the mass or radius. So the answer is No.

**12.89.** Model: The toy car is a particle located at the rim of the track. The track is a cylindrical hoop rotating about its center, which is an axis of symmetry. No net torques are present on the track, so the angular momentum of the car and track is conserved.

Visualize:



Solve: The toy car's steady speed of 0.75 m/s relative to the track means that

$$v_{\rm c} - v_{\rm t} = 0.75 \text{ m/s} \Rightarrow v_{\rm c} = v_{\rm t} + 0.75 \text{ m/s},$$

where  $v_t$  is the velocity of a point on the track at the same radius as the car. Conservation of angular momentum implies that

$$\begin{split} L_{t} &= L_{t} \\ 0 &= I_{c}\omega_{c} + I_{t}\omega_{t} = (mr^{2})\omega_{c} + (Mr^{2})\omega_{t} = m\omega_{c} + M\omega_{t} \end{split}$$

The initial and final states refer to before and after the toy car was turned on. Table 12.2 was used for the track. Since  $\omega = \frac{v_c}{v_c}$ ,  $\omega = \frac{v_t}{v_c}$ , we have

$$c = r, \quad w_t = r, \quad w_t = mv_c + Mv_t$$
  

$$\Rightarrow m(v_t + 0.75 \text{ m/s}) + Mv_t = 0$$
  

$$\Rightarrow v_t = -\frac{M}{m+M} (0.75 \text{ m/s}) = -\frac{(0.200 \text{ kg})}{(0.200 \text{ kg} + 1.0 \text{ kg})} (0.75 \text{ m/s}) = -0.125 \text{ m/s}$$

The minus sign indicates that the track is moving in the opposite direction of the car. The angular velocity of the track is

$$\omega_{\rm t} = \frac{v_{\rm t}}{r} = \frac{(0.125 \text{ m/s})}{0.30 \text{ m}} = 0.417 \text{ rad/s clockwise}$$

In rpm,

$$\omega_{\rm t} = (0.417 \text{ rad/s}) \left(\frac{\text{rev}}{2\pi \text{rad}}\right) \left(\frac{60 \text{ s}}{\text{min}}\right)$$
  
= 4.0 rpm

Assess: The speed of the track is less than that of the car because it is more massive.