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RST Digital Controls

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Fulvio Boattini
CERN TE\EPC

Bibliography

- *“Digital Control Systems”: Ioan D. Landau; Gianluca Zito*
- *“Computer Controlled Systems. Theory and Design”: Karl J. Astrom; Bjorn Wittenmark*
- *“Advanced PID Control”: Karl J. Astrom; Tore Hagglund;*
- *“Elementi di automatica”: Paolo Bolzern*

SUMMARY

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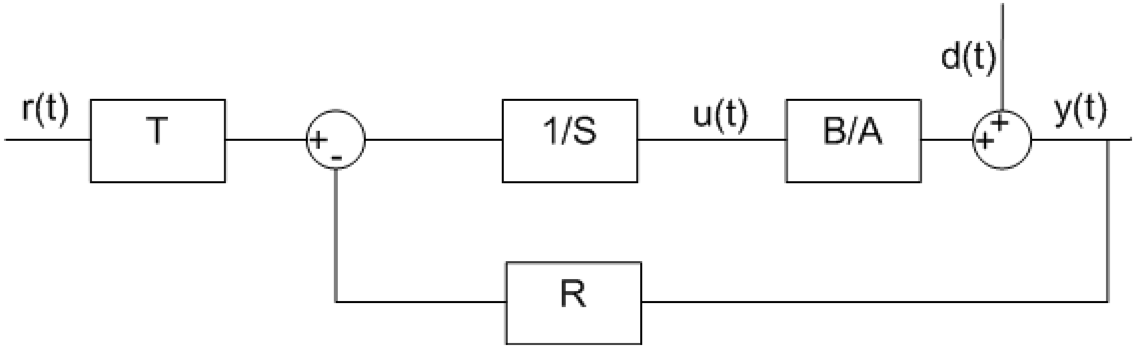
- ***RST Digital control: structure and calculation***
- ***RST equivalent for PID controllers***
- ***RS for regulation, T for tracking***
- ***Systems with delays***
- ***RST at work with POPS***
    - ***Vout Controller***
    - ***Imag Controller***
    - ***Bfield Controller***
- ***Conclusions***

RST Digital control: structure and calculation

RST calculation: control structure



$$R = r_0 + r_1 \cdot z^{-1} + r_2 \cdot z^{-2} \ldots + r_n \cdot z^{-n}$$

$$S = s_0 + s_1 \cdot z^{-1} + s_2 \cdot z^{-2} \ldots + s_n \cdot z^{-n}$$

$$T = t_0 + t_1 \cdot z^{-1} + t_2 \cdot z^{-2} \ldots + t_n \cdot z^{-n}$$

A combination of FFW and FBK actions
that can be tuned separately

$$S \cdot u = T \cdot r - R \cdot y$$

$$Hff = \frac{T}{S}$$

$$Hfb = \frac{R}{S}$$

REGULATION

$$\frac{y}{d} = \frac{A \cdot S}{A \cdot S + B \cdot R}$$

$$A(z^{-1}) \cdot S(z^{-1}) + B(z^{-1}) \cdot R(z^{-1}) = P_{des}(z^{-1})$$

TRACKING

$$\frac{y}{r} = \frac{B \cdot T}{A \cdot S + B \cdot R}$$

$$T = \frac{\dfrac{P_{des}(z^{-1})}{B(1)}}{\dfrac{P_{des}(1)}{B(1)}}$$

RST calculation: Diophantine Equation

Getting the desired polynomial

$$\frac{\omega^2}{s^2+2\cdot\zeta\cdot\omega\cdot s+\omega^2} \qquad \begin{array}{c}\omega = 2\cdot\pi\cdot 100\\ \zeta = 0.3\end{array} \quad \xrightarrow{\text{Sample Ts=100us}} \quad \frac{0.001949\ \ z^{-1} + 0.001924\ \cdot z^{-2}}{1 - 1.959\ \ z^{-1} + 0.963\ \cdot z^{-2}}$$

$$P_{des} = 1 - 1.959\ \ z^{-1} + 0.963\ \cdot z^{-2}$$

Calculating R and S: Diophantine Equation

$$A(z^{-1})\cdot S(z^{-1}) + B(z^{-1})\cdot R(z^{-1}) = P_{des}(z^{-1}) \Rightarrow M\cdot x = p$$

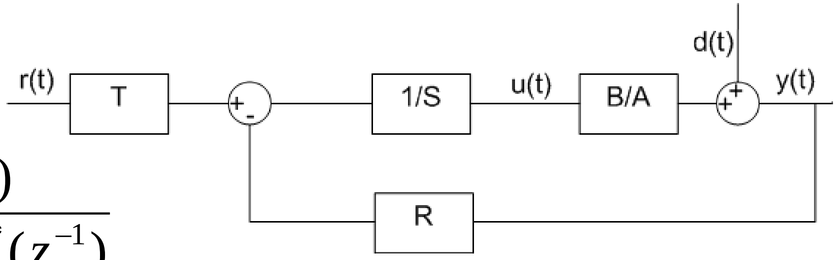
Matrix form:

$$x^T = [1, s_1, \ldots, s_{nS}, r_0, r_1, \ldots, r_{nR}]$$

$$p^T = [1, p_1, \ldots, p_{nP}, 0, \ldots 0]$$

$$M = \left.\left[\begin{array}{cccccccc}
\multicolumn{4}{c}{\overbrace{\hphantom{1\quad 0\quad \ldots\quad 0}}^{nB+d}} & \multicolumn{4}{c}{\overbrace{\hphantom{0\quad \ldots\quad \ldots\quad 0}}^{nA}} \\
1 & 0 & \ldots & 0 & 0 & \ldots & \ldots & 0 \\
a_1 & & & 0 & b_1 & & & 0 \\
a_2 & & & 1 & b_2 & & & 0 \\
 & & & a_1 & & & & b_1 \\
a_{nA} & & & a_2 & b_{nB} & & & b_2 \\
0 & & & & 0 & & & \\
0 & \ldots & 0 & a_{nA} & 0 & 0 & 0 & b_{nB}
\end{array}\right]\right\} nA+nB+d$$

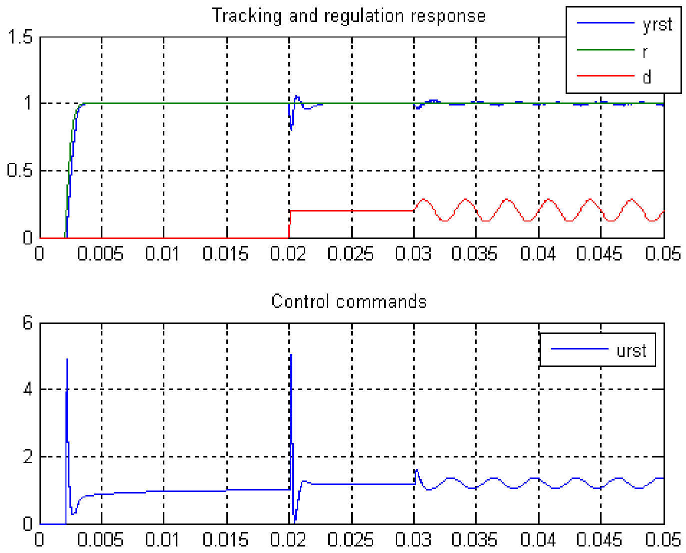
RST calculation: fixed polynomials



Controller TF is:

$$Hfb = \frac{R(z^{-1})}{S(z^{-1})} \xrightarrow{\text{Add integrator}} \frac{R(z^{-1})}{[1-z^{-1}]\cdot S^{*}(z^{-1})}$$

$$\frac{y}{d} = \frac{A\cdot S}{A\cdot S+B\cdot R} \xrightarrow{\text{Add 2 zeros more on S}} \frac{A\cdot[1-z^{-1}]\cdot[1+\alpha_1\cdot z^{-1}+\alpha_2\cdot z^{-2}]\cdot S^{*}}{A\cdot S+B\cdot R}$$



Integrator active on step reference and step disturbance.
Attenuation of a 300Hz disturbance

Calculating R and S: Diophantine Equation

$$A(z^{-1})\cdot Hs(z^{-1})\cdot S(z^{-1})+B(z^{-1})\cdot Hr(z^{-1})\cdot R(z^{-1}) = P_{des}(z^{-1})$$

$$Hs(z^{-1}) = \text{fixed part of S}$$

$$Hr(z^{-1}) = \text{fixed part of R}$$

RST equivalent for PID controllers

RST equivalent of PID controller: continuous PID design

Consider a II order system:

$$\frac{B}{A} = \frac{\omega^2}{s^2 + 2\cdot\zeta\cdot\omega\cdot s + \omega^2} \qquad \omega = 2\cdot\pi\cdot 100 \qquad \zeta = 0.3$$

Pole Placement for continuous PID

$$PID = Kp\cdot\left(1 + \frac{1}{s\cdot Ti} + s\cdot Td\right) \qquad Ki = \frac{Kp}{Ti} \qquad Kd = Kp\cdot Td$$

$$HclPIDc = \frac{PID\cdot Hsy}{1 + PID\cdot Hsy} = HclPIDc.Den = 1 + PID\cdot Hsy =$$

$$s^3 + \left(2\cdot\zeta\cdot\omega + Kd\cdot\omega^2\right)\cdot s^2 + \left(Kp\cdot\omega^2 + \omega^2\right)\cdot s + Ki\cdot\omega^2 = \left(s + \alpha_0\cdot\omega_0\right)\cdot\left(s^2 + 2\cdot\zeta_0\cdot\omega_0\cdot s + \omega_0^{\ 2}\right)$$

RST equivalent of PID controller: continuous PID design

Consider a II order system:

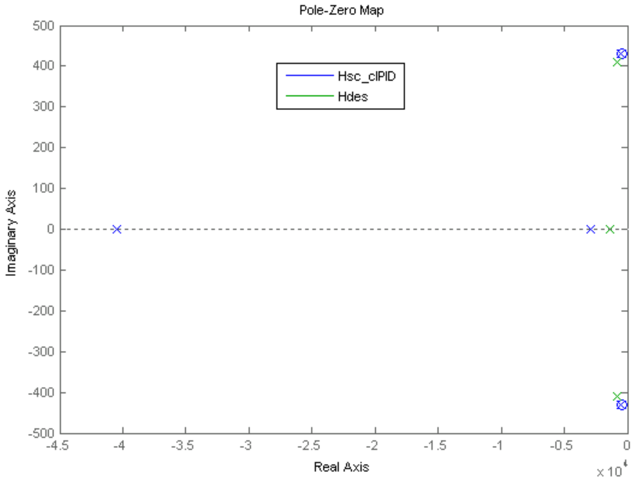
$$\frac{B}{A} = \frac{\omega^2}{s^2 + 2\cdot\zeta\cdot\omega\cdot s + \omega^2} \qquad \omega = 2\cdot\pi\cdot 100 \qquad \zeta = 0.3$$

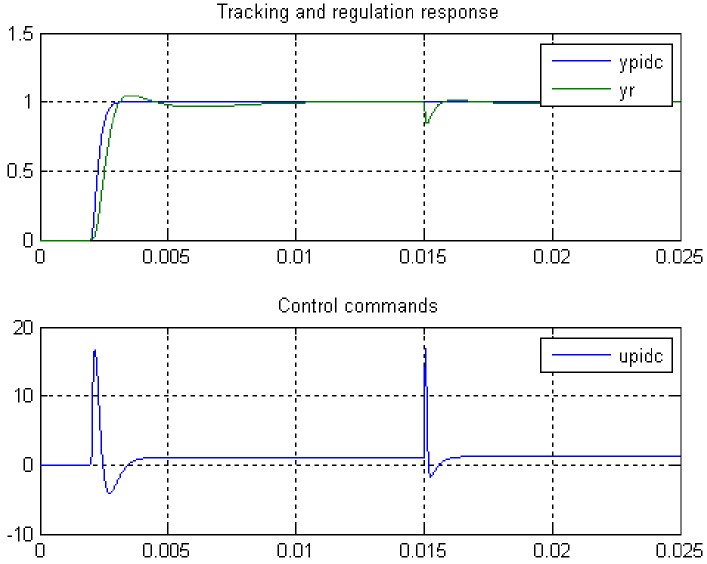
$$Kp = (1 + 2\cdot\alpha_0\cdot\zeta_0)\cdot\frac{\omega_0^2}{\omega^2} - 1$$

$$Ki = \alpha_0\cdot\frac{\omega_0^3}{\omega^2}$$

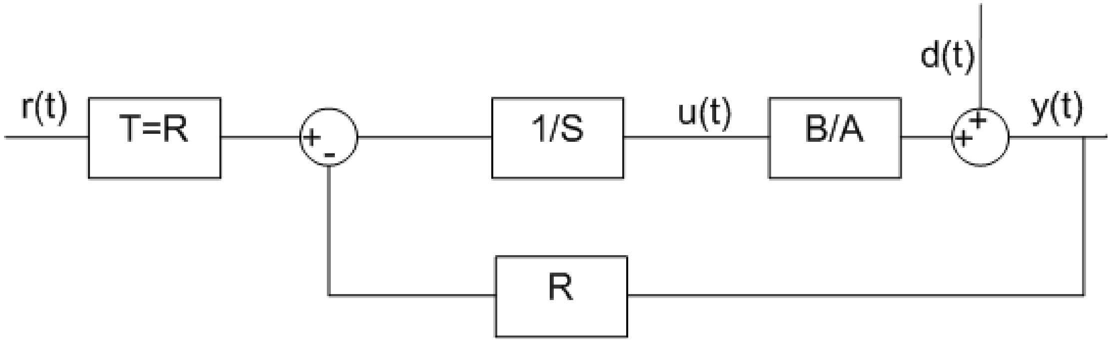
$$Kd = \frac{(\alpha_0 + 2\cdot\zeta_0)\cdot\omega_0 - 2\cdot\zeta\cdot\omega}{\omega^2}$$

$$PIDf = Kp\cdot\left(1 + \frac{1}{s\cdot Ti} + \frac{s\cdot Td}{1 + s\cdot {}^{Td}\!/_{N}}\right) \qquad Ki = \frac{Kp}{Ti} \qquad Kd = Kp\cdot Td$$





RST equivalent of PID: s to z substitution

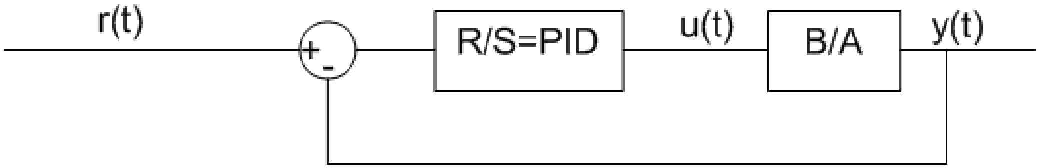


All control actions on error

$$T(z^{-1}) = R(z^{-1})$$

Proportional on error;
Int+deriv on output

$$T(z^{-1}) = R(1)$$



$$\frac{R(z^{-1})}{S(z^{-1})} = \frac{r_0 + r_1 \cdot z^{-1} + r_2 \cdot z^{-2}}{s_0 + s_1 \cdot z^{-1} + s_2 \cdot z^{-2}} = PIDd(z^{-1})$$

Choose R and S coeffs such that the 2 TF are equal

$$PIDd = Kp \cdot \left[1 + \frac{Ts}{Ti} \cdot \frac{\alpha + (1-\alpha) \cdot z^{-1}}{1 - z^{-1}} + \frac{N \cdot Td \cdot (1 - z^{-1})}{(Td + N \cdot Ts \cdot \alpha) + (N \cdot Ts \cdot (1-\alpha) - Td) \cdot z^{-1}} \right] = \frac{R}{S} = \frac{r0 + r1 \cdot z^{-1} + r2 \cdot z^{-2}}{s0 + s1 \cdot z^{-1} + s2 \cdot z^{-2}}$$

α = 0 forward Euler

α = 1 backward Euler

α = 0.5 Tustin

RST equivalent of PID: pole placement in z

Choose desired poles

$$\frac{\omega^2}{s^2 + 2\cdot\zeta\cdot\omega\cdot s + \omega^2} \qquad \begin{matrix}\omega = 2\cdot\pi\cdot 150 \\ \zeta = 0.8\end{matrix} \xrightarrow{\text{Sample Ts=100us}} \frac{0.001949\ \text{z}^{-1} + 0.001924\ \cdot\text{z}^{-2}}{1 - 1.959\ \text{z}^{-1} + 0.963\ \cdot\text{z}^{-2}}$$

$$P_{des} = 1 - 1.959\ \text{z}^{-1} + 0.963\ \cdot\text{z}^{-2}$$

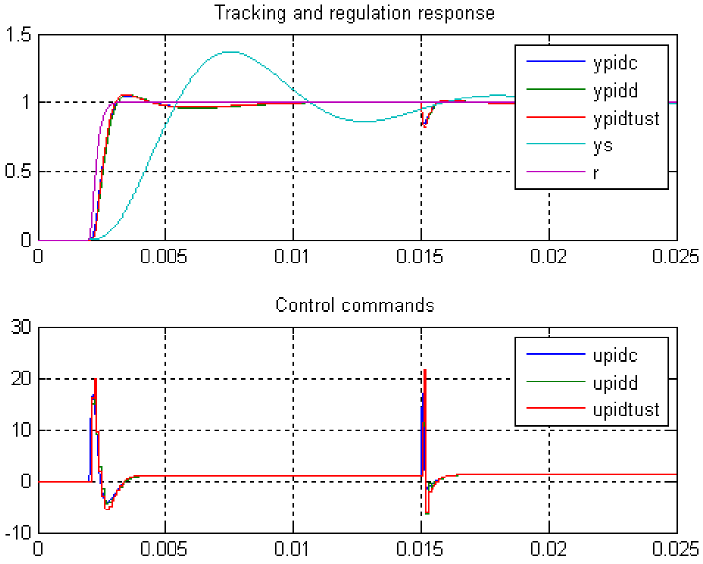
Choose fixed parts for R and S

$$Hs = 1 - z^{-1} \qquad Hr = 1$$

Calculating R and S: Diophantine Equation

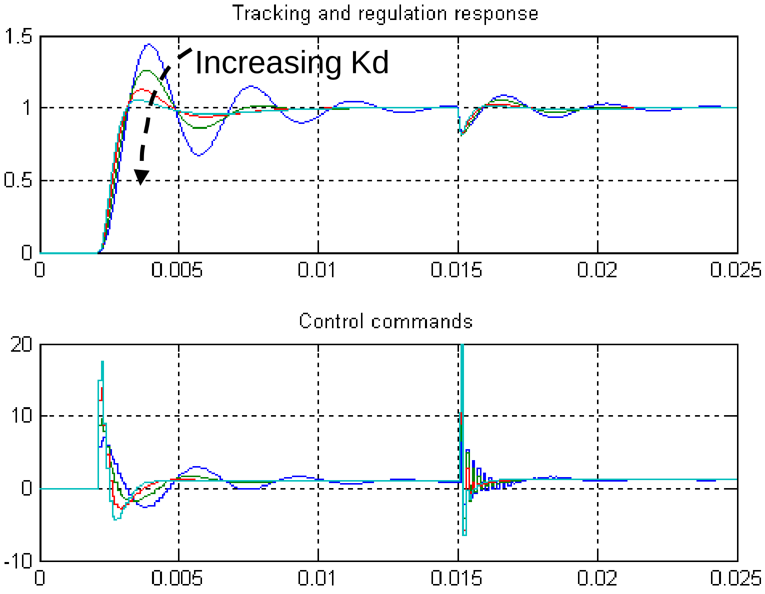
$$A(z^{-1})\cdot Hs(z^{-1})\cdot S^*(z^{-1}) + B(z^{-1})\cdot Hr(z^{-1})\cdot R^*(z^{-1}) = P_{des}(z^{-1})$$

RST equivalent of PID: pole placement in z



The 3 regulators behave very similarly

Manual Tuning with Ki, Kd and Kp is still possible



RS for regulation, T for Tracking

RS for regulation, T for tracking

REGULATION

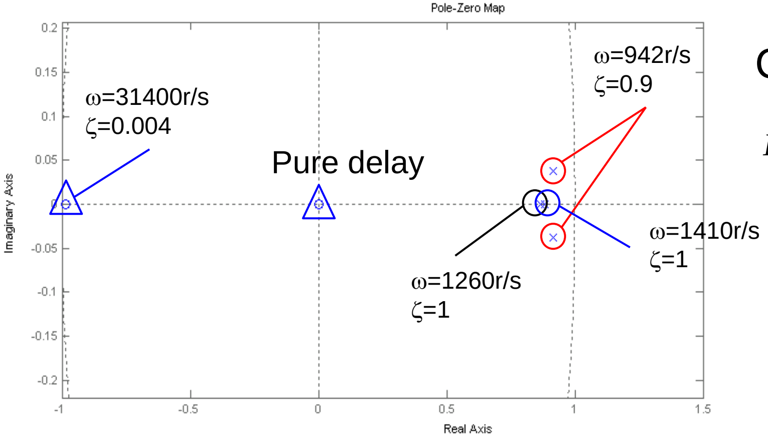
$$Pdes = (s+\alpha_0\cdot\omega_0)\cdot(s^2+2\cdot\zeta_0\cdot\omega_0\cdot s+\omega_0{}^2) \qquad \alpha_0=1.5 \quad \omega_0=940r/s \quad \zeta_0=0.9$$

$Hs = [1 -1]$ integrator

Diophantine Equation : $A\cdot Hs\cdot S^* + B\cdot R = Pdes\cdot Paux$

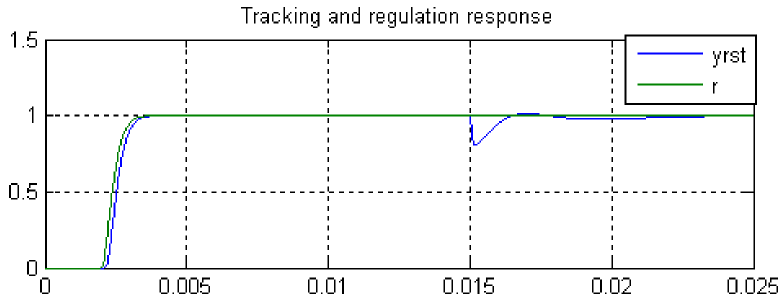
After the D. Eq solved we get the following open loop TF:

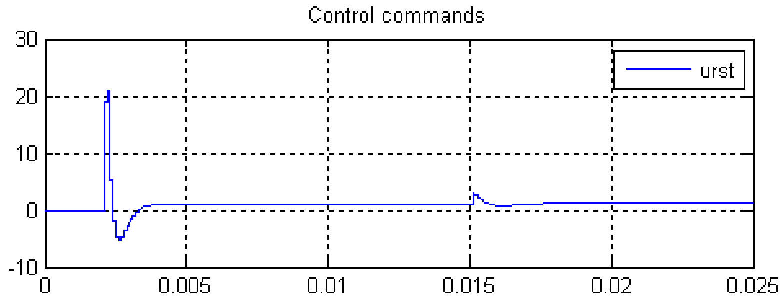
$$Hol = \frac{B\cdot R}{A\cdot S} = \frac{0.022626\ \ z\text{^-}1\,(1+0.9875z\text{^-}1\ \ )\,(1-1.929z\text{^-}1\ \ +0.9322z\text{^-}2\ \ )}{(1-z\text{^-}1)\ \ (1-0.6494z\text{^-}1\ \ )\,(1-1.959z\text{^-}1\ \ +0.963z\text{^-}2)}$$



Closed Loop TF without T

$$Hcl = \frac{B}{A\cdot S+B\cdot R} = \frac{0.0019487\ \ \ z\text{^-}1\,(1+0.9875z\text{^-}1\ \ )}{(1-0.8819z\text{^-}1\ \ )\,(1-0.8682z\text{^-}1\ \ )\,(1-1.836z\text{^-}1\ \ +0.844z\text{^-}2)}$$





TRACKING

$$T = \frac{Pdes\cdot Paux}{B(1)}$$

$$Hcl = \frac{T\cdot B}{A\cdot S+B\cdot R} = \frac{0.50314z\text{^-}1(1+0.9875z\text{^-}1)}{1}$$

T polynomial compensate most of the system dynamic

Systems with delays

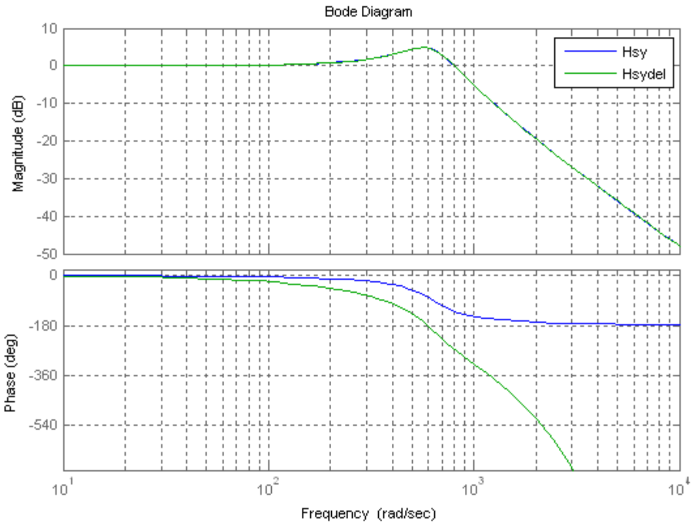
Systems with delays

II order system with pure delay

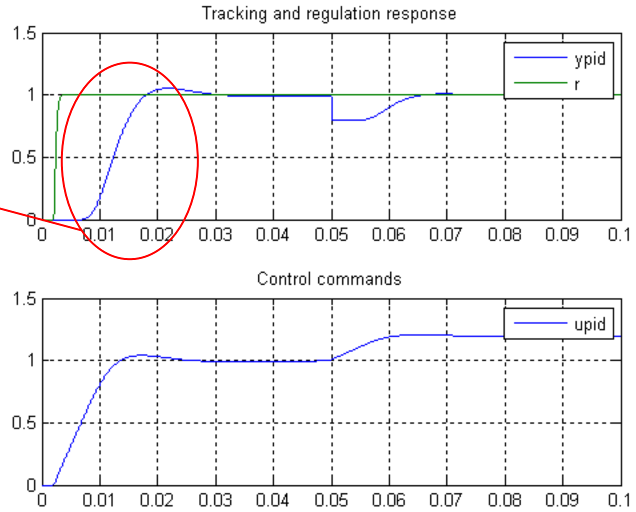
$$\frac{B}{A}=\frac{\omega^2\cdot e^{-s\cdot\tau}}{s^2+2\cdot\zeta\cdot\omega\cdot s+\omega^2}$$

$$\omega=2\cdot\pi\cdot 100r/s$$
$$\zeta=0.3$$
$$\tau=3ms$$

$$\xrightarrow{\text{Sample Ts=1ms}} \frac{0.1692\ z^{-4}+0.149\ z^{-5}}{1-1.368\ z^{-1}+0.6859\ z^{-2}}$$

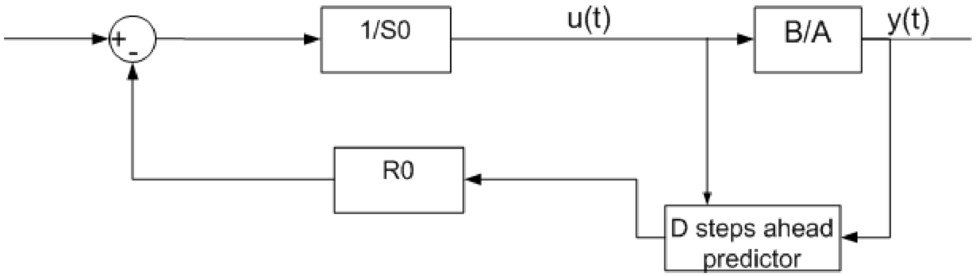


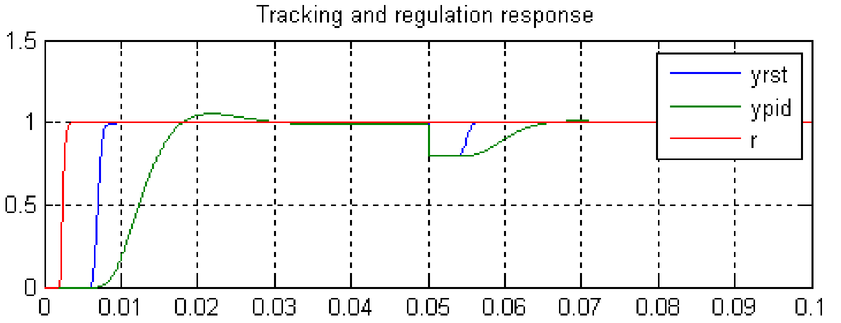
Continuous time PID is much slower than before

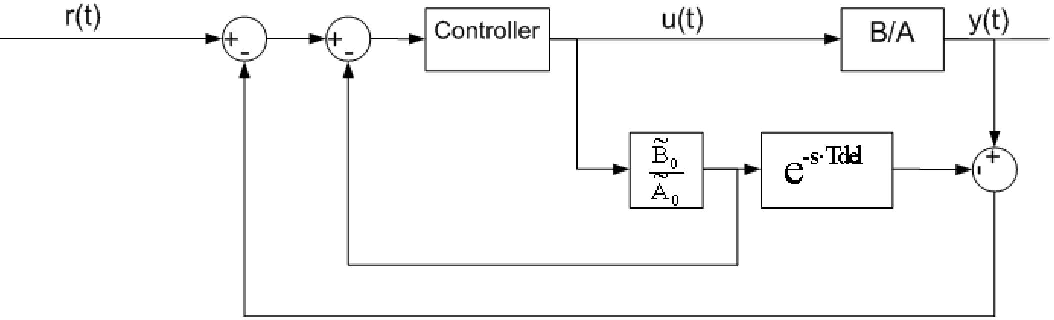


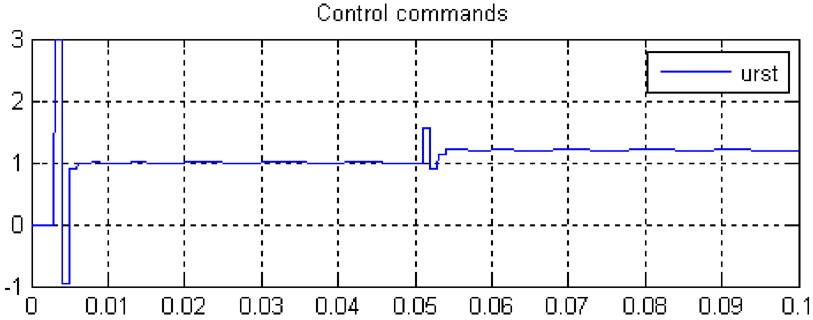
Systems with delays

Predictive controls







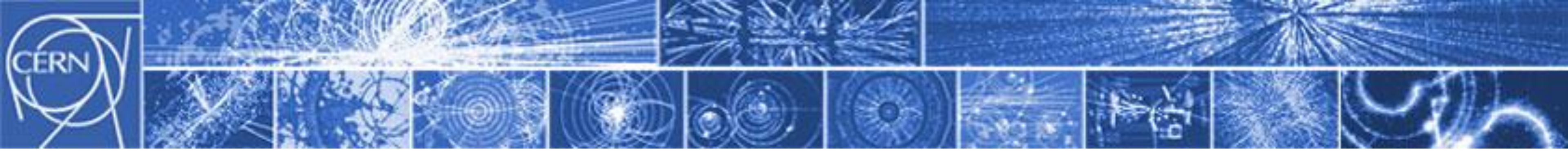


Diophantine Equation

$$A(z^{-1}) \cdot S(z^{-1}) + z^{-d} \cdot B_0(z^{-1}) \cdot R(z^{-1}) = A(z^{-1}) \cdot P_{aux}(z^{-1})$$

Choose fixed parts for R and S

$$Hs = 1 - z^{-1} \qquad Hr = 1$$



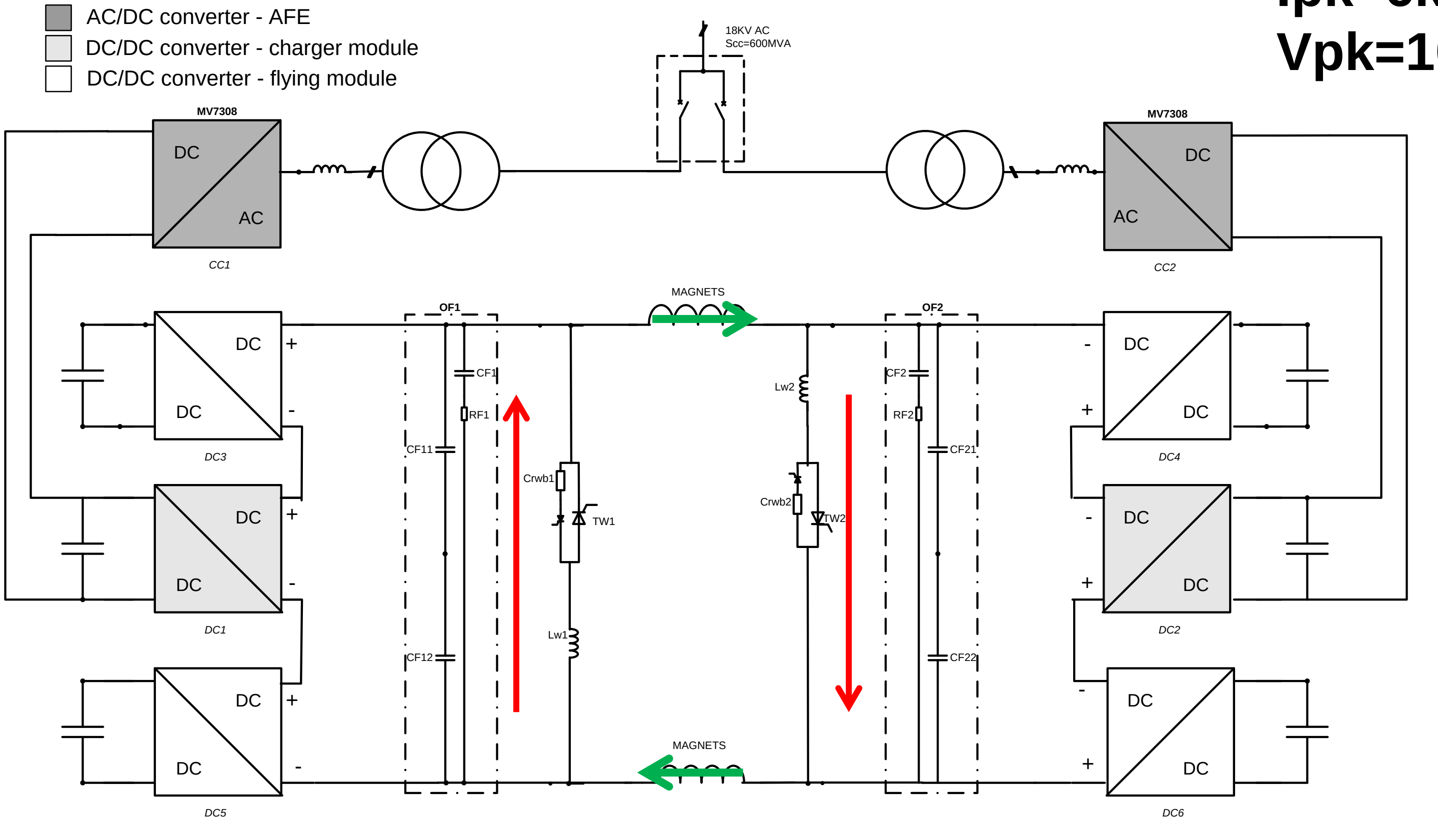
RST at work with POPS

RST at work with POPS

Vout Controller

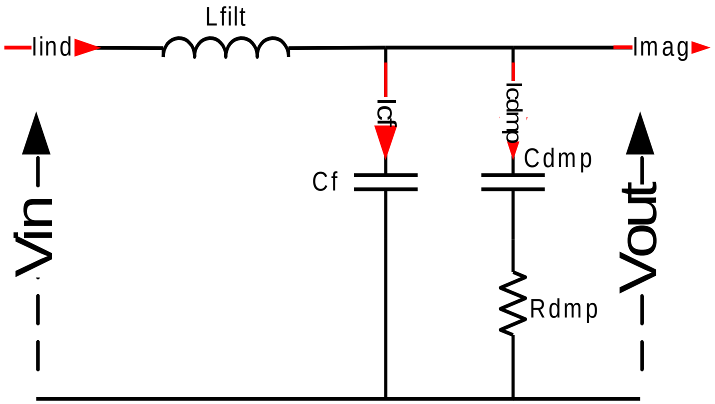
Imag or Bfield control

Ppk=60MW
Ipk=6kA
Vpk=10kV



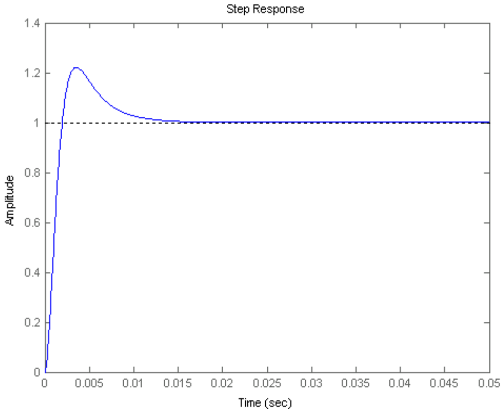
▨ AC/DC converter - AFE
▨ DC/DC converter - charger module
☐ DC/DC converter - flying module

RST at work with POPS: Vout Control



III order output filter

$$H_{flt} = \frac{Vout}{Vin} = \frac{Rd \cdot Cd \cdot S + 1}{Lf \cdot Cf \cdot Rd \cdot Cd \cdot S^3 + Lf \cdot (Cf + Cd) \cdot S^2 + Rd \cdot Cd \cdot S + 1}$$



Decide desired dynamics

$$\frac{\omega^2}{s^2 + 2\cdot\zeta\cdot\omega\cdot s + \omega^2} \qquad \begin{matrix} \omega = 2\cdot\pi\cdot 150 \\ \zeta = 0.85 \end{matrix} \xrightarrow{c2d\ \mathrm{Ts=1ms}} \frac{0.1431\cdot z^{-1} + 0.1043\cdot z^{-2}}{1\text{-}1.142\cdot z^{-1} + 0.3897\cdot z^{-2}}$$

$$P_{des} = 1\text{-}1.142\cdot z^{-1} + 0.3897\cdot z^{-2}$$

Eliminate process well dumped zeros

$$P_{zeros} = z - 0.8053$$

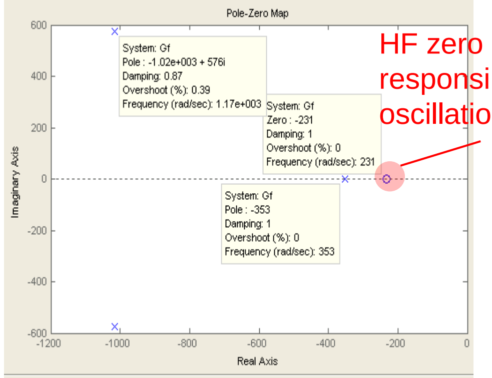
$$\frac{y}{r} = \frac{B\cdot T}{A\cdot S + B\cdot R}$$

Solve Diophantine Equation

$$A(z^{-1})\cdot S(z^{-1}) + B(z^{-1})\cdot R(z^{-1}) = P_{des}(z^{-1})\cdot P_{zeros}(z^{-1})$$

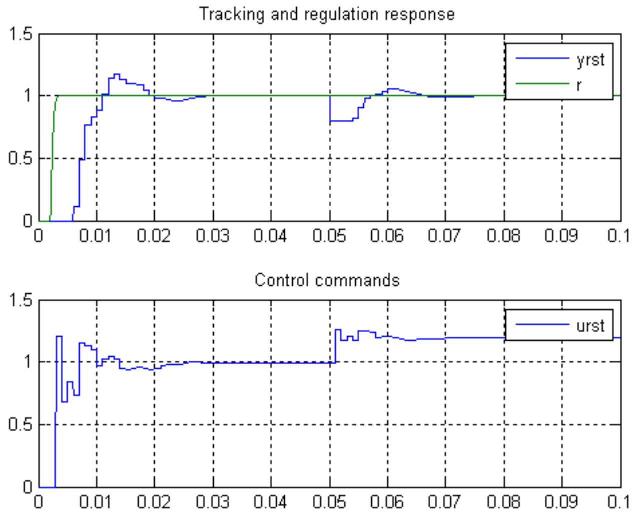
Calculate T to eliminate all dynamics

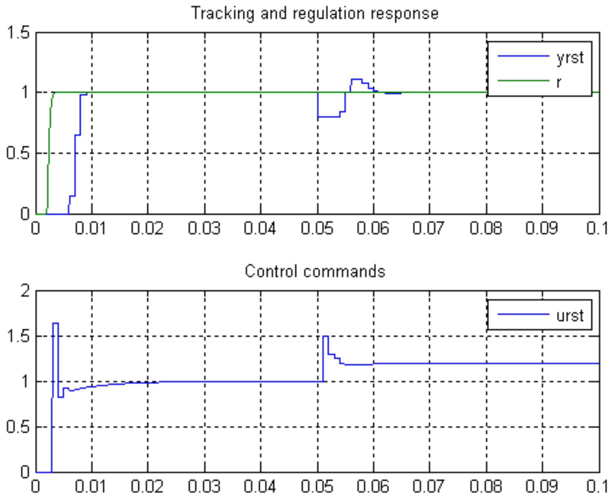
$$T(z^{-1}) = \frac{P_{des}(z^{-1})}{B(1)}$$



HF zero responsible for oscillations

RST at work with POPS: Vout Control

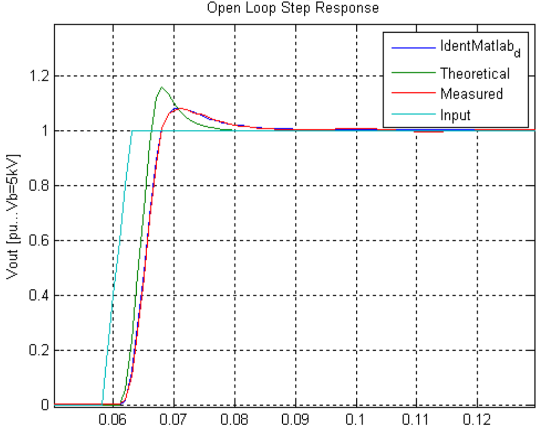




Well… not very nice performance….

There must be something odd !!!

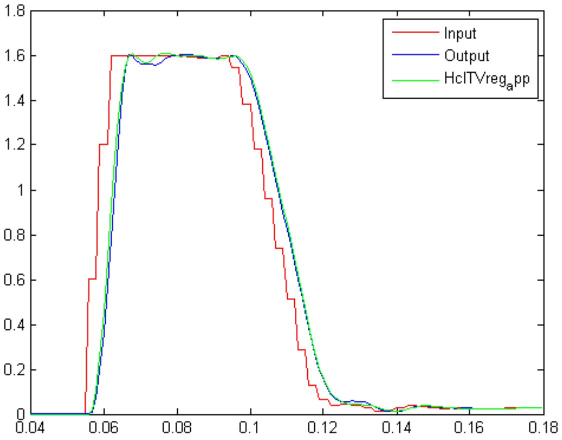
Identification of output filter with a step



Put this back in the RST calculation sheet

$$Hol\_Ident = \frac{0.08904\,z^3 + 0.2251\,z^2 - 0.06442\,z - 0.1636}{z^6 - 1.397\,z^5 + 0.5507\,z^4 - 0.09403\,z^3 + 0.02594\,z^2}$$

RST at work with POPS: Vout Control

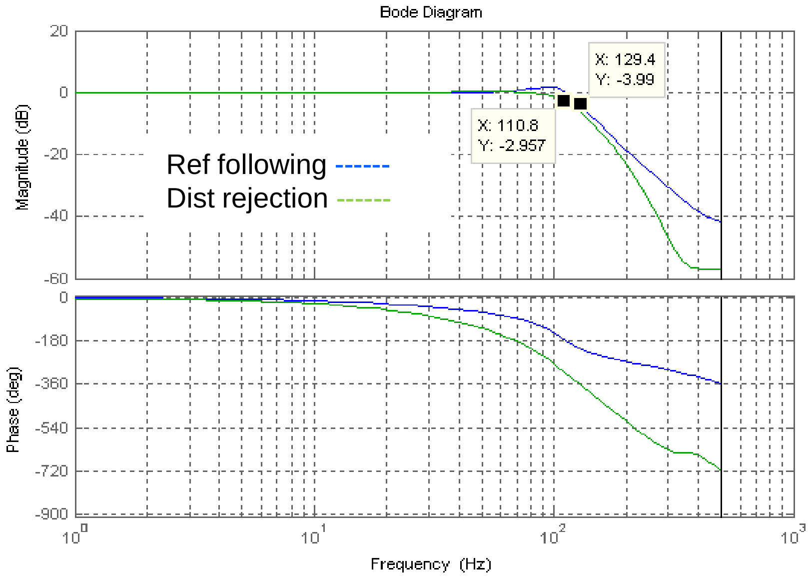


In reality the response is a bit less nice… but still very good.

Performance to date (identified with initial step response):

Ref following: 130Hz

Disturbance rejection: 110Hz

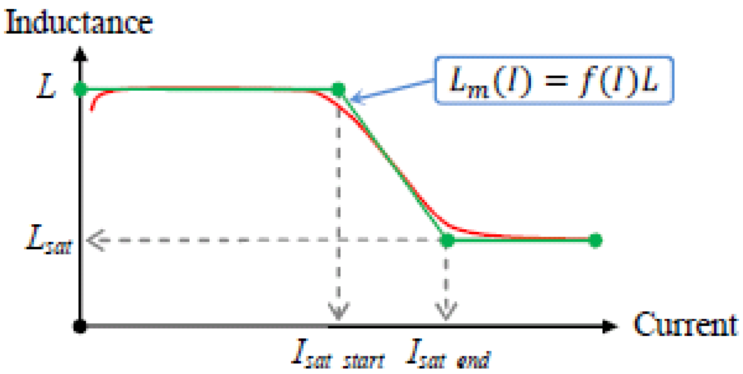


RST at work with POPS: Imag Control

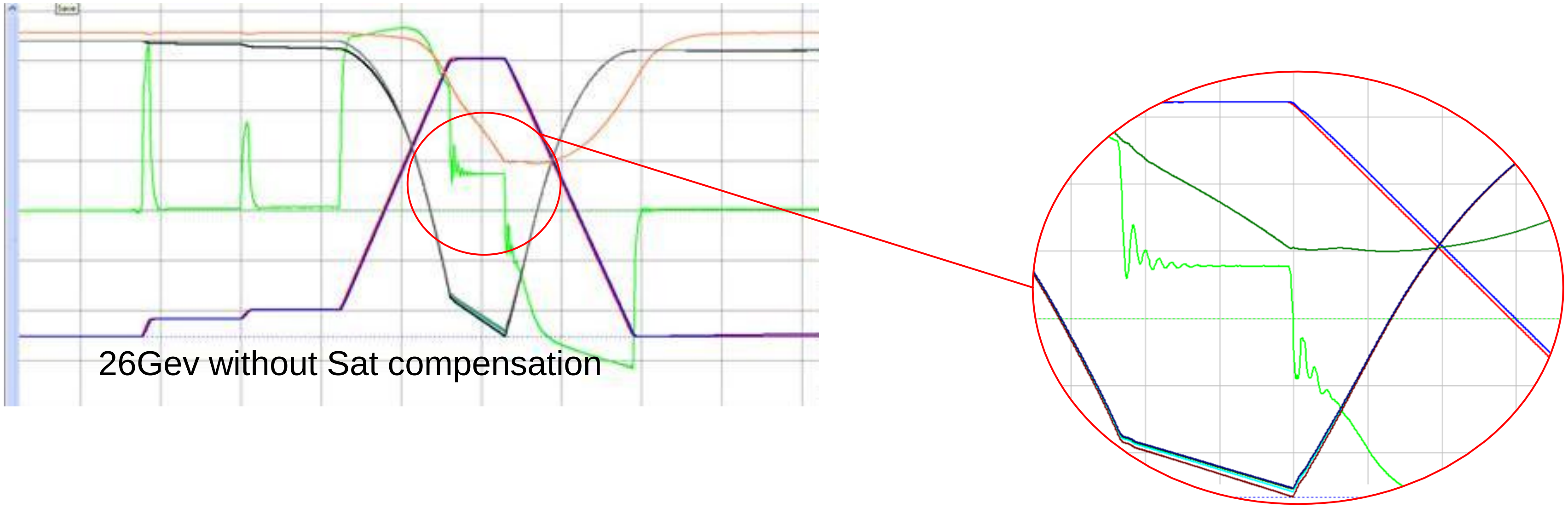
Magnet transfer function for Imag:

$$\frac{I_{mag}}{V_{mag}} = \frac{1}{s \cdot L_{mag} + R_{mag}} \qquad \begin{aligned} L_{mag} &= 0.96H \\ R_{mag} &= 0.32\Omega \end{aligned}$$

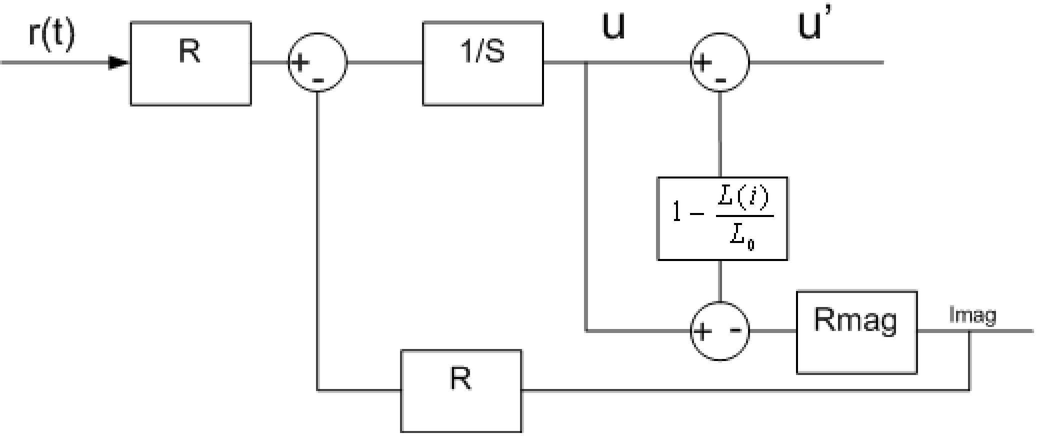
PS magnets deeply saturate:

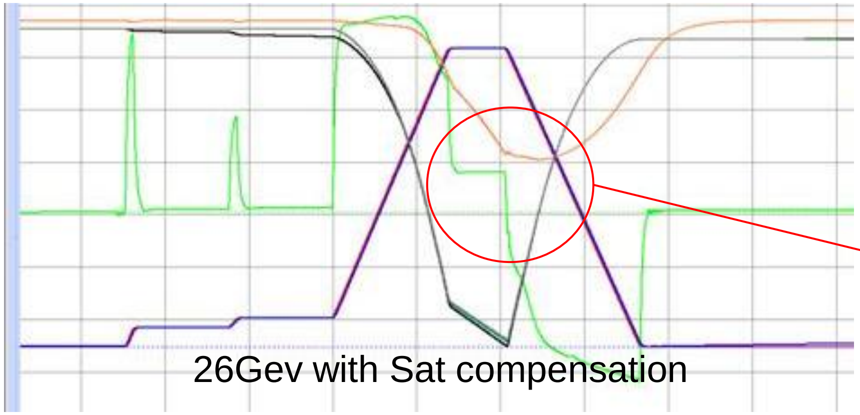


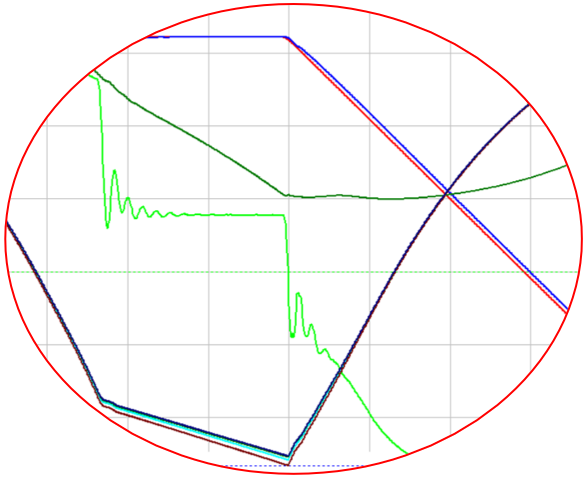
The RST controller was badly oscillating at the flat top because the gain of the system was changed

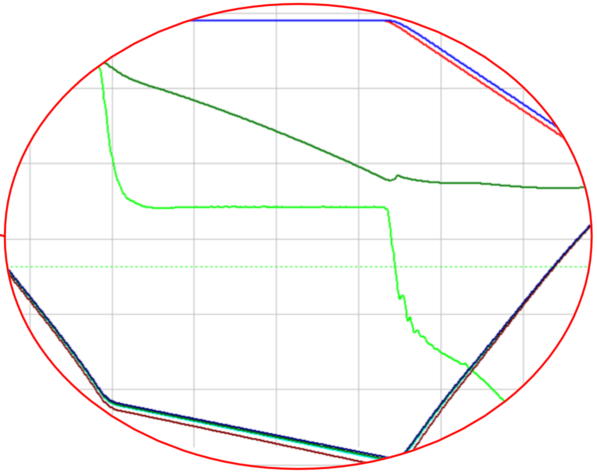

26Gev without Sat compensation

RST at work with POPS: Imag Control




26Gev with Sat compensation





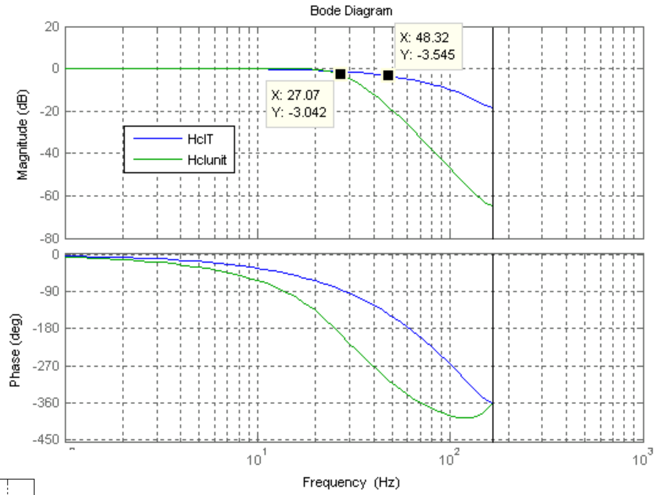
RST at work with POPS: Bfield Control

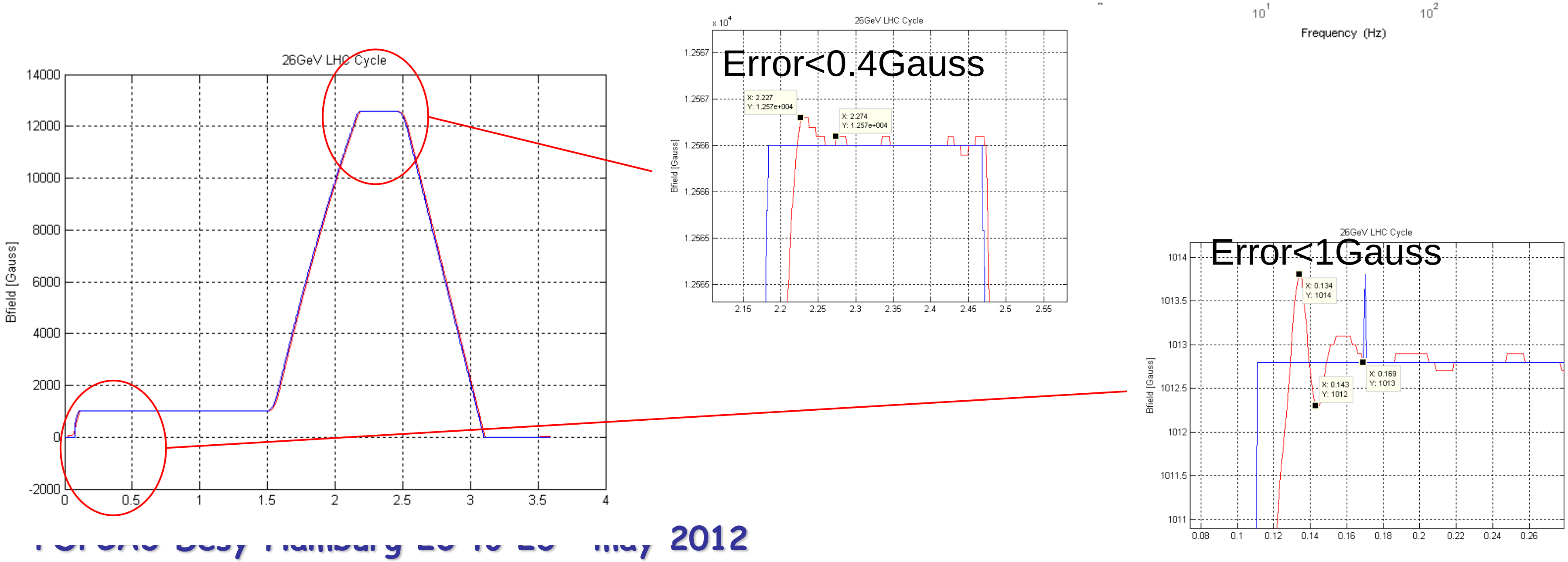
Magnet transfer function for Bfield:

$$\frac{B_{mag}}{V_{mag}} = \frac{1}{s \cdot K_{mag} + \frac{R_{mag}}{L_{mag}} \cdot K_{mag}}$$

$$L_{mag} = 0.96H$$
$$R_{mag} = 0.32\Omega$$
$$K_{mag} = 2.5$$

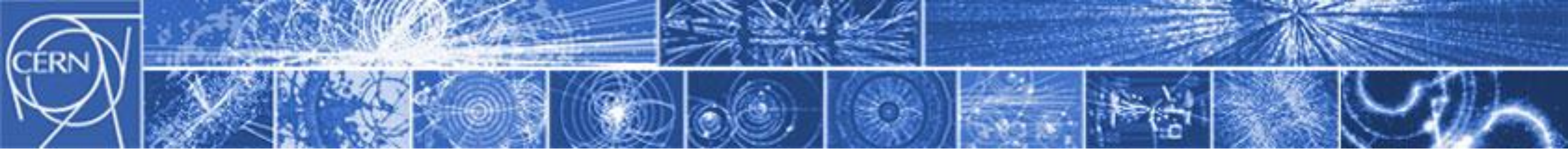
Tsampl=3ms.
Ref following: 48Hz
Disturbance rejection: 27Hz





RST control: Conclusions

- RST structure can be used for “basic” PID controllers and conserve the possibility to manual tune the performances
- It has a 2 DOF structure so that Tracking and Regulation can be tuned independently
- It include “naturally” the possibility to control systems with pure delays acting as a sort of predictor.
- When system to be controlled is complex, identification is necessary to refine the performances (no manual tuning is available).
- A lot more…. But time is over !



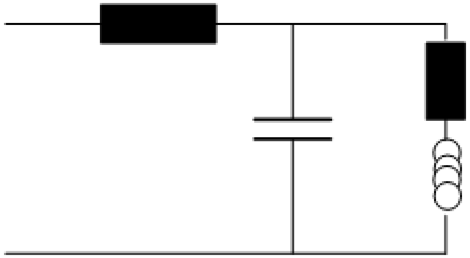
Thanks for the attention

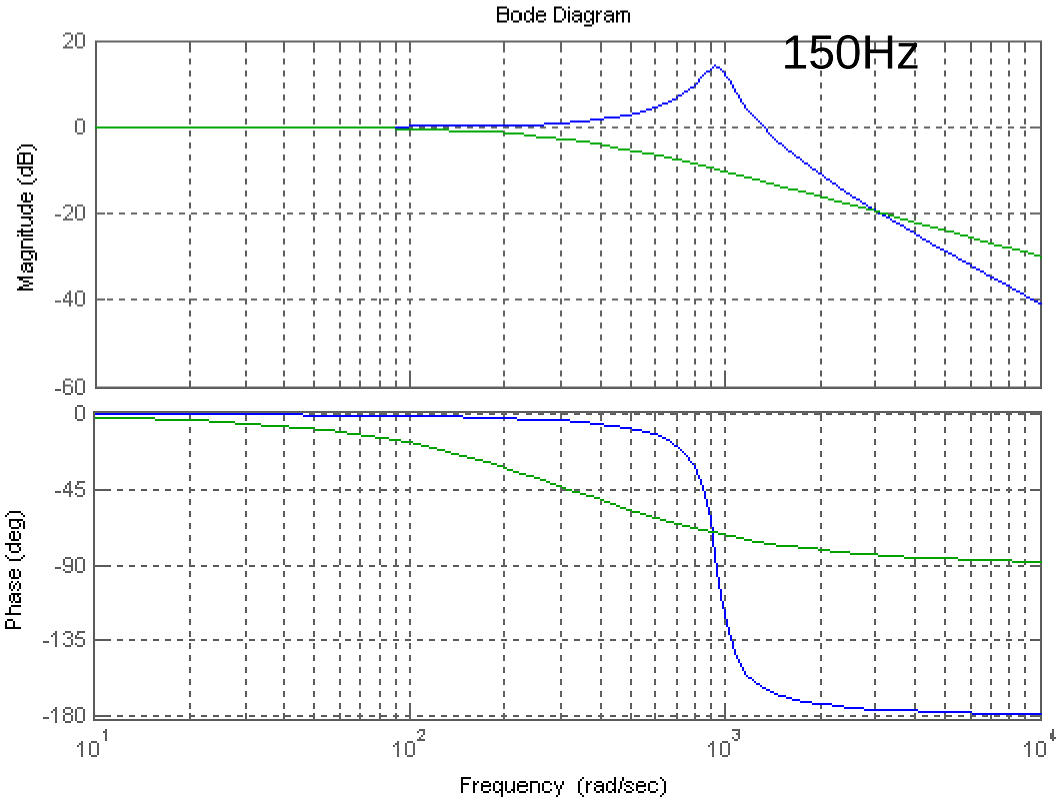
Questions?

Towards more complex systems

(test it before !!!)

Unstable filter+magnet+delay





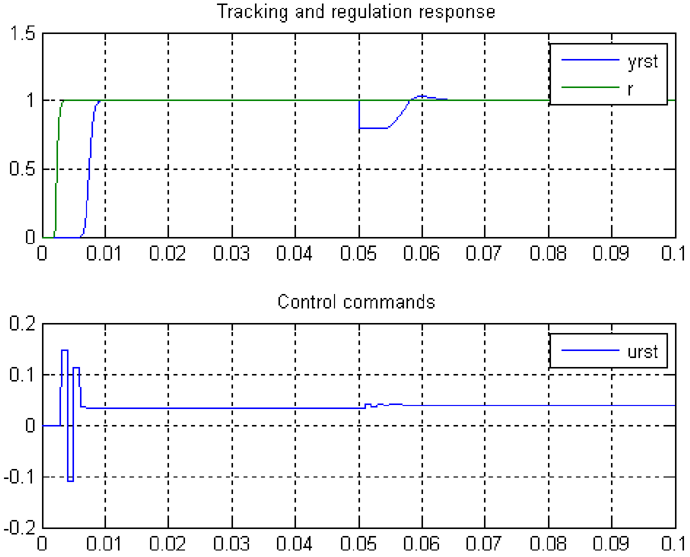
150Hz

Ts=1ms

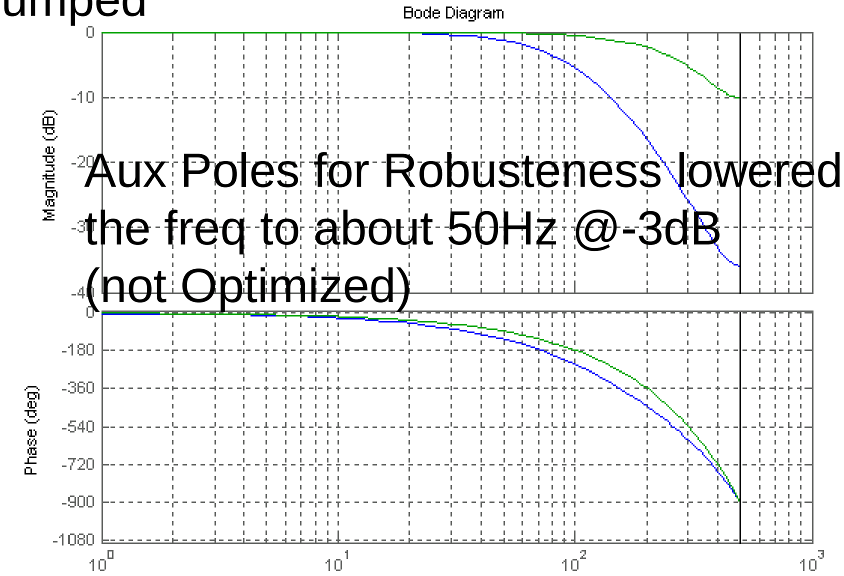
$$\frac{1.2487 (z+3.125) (z+0.2484)}{z^3 (z-0.7261) (z^2 - 1.077z + 0.8282)}$$

Unstable filter+magnet+delay

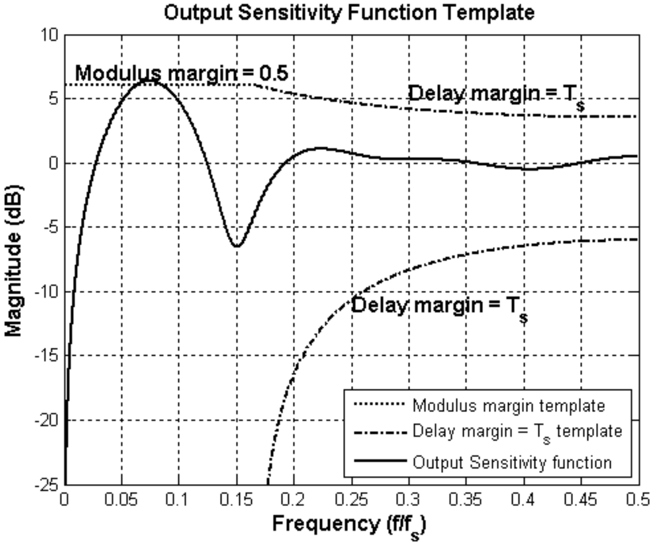
Choose Pdes as 2nd order system 100Hz well dumped


Tracking and regulation response

Control commands


Bode Diagram

Aux Poles for Robusteness lowered the freq to about 50Hz @-3dB (not Optimized)


Output Sensitivity Function Template