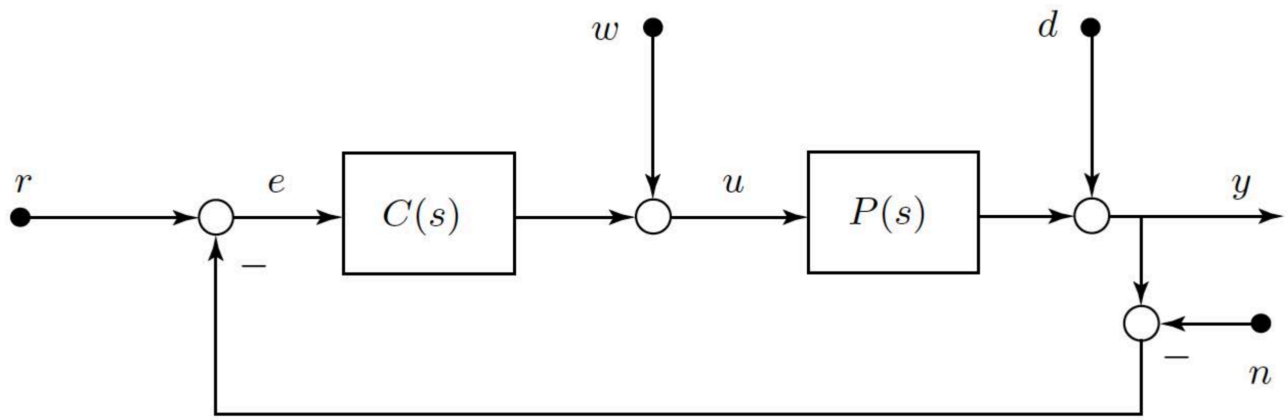
RTI Vorlesung 11

29.11.2019

Definitions



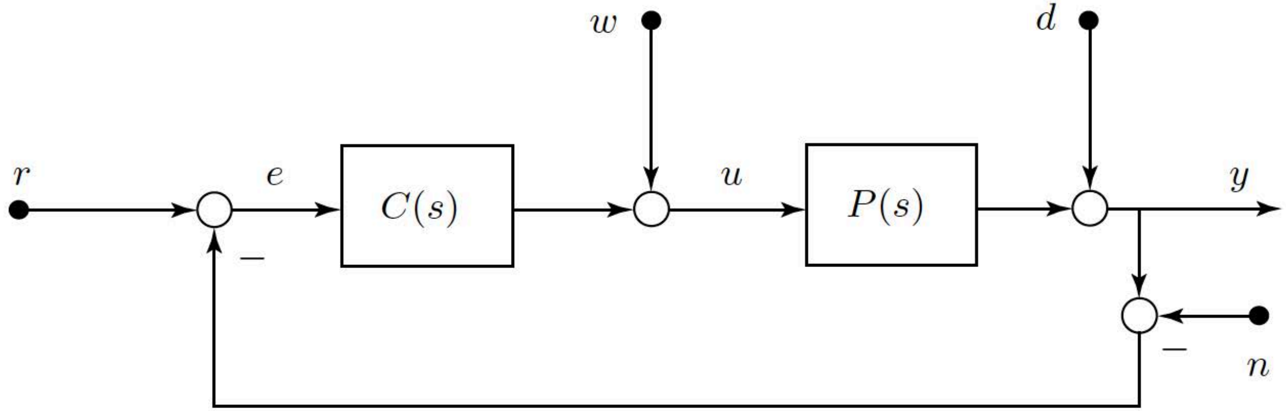
loop gain $L(s) = P(s) \cdot C(s)$

return diference $1 + L(s)$

sensitivity $S(s) = \dfrac{1}{1 + L(s)}$

complementary sensitivity $T(s) = \dfrac{L(s)}{1 + L(s)}$

Closed-Loop System



$$Y(s) = S(s) \cdot \big(D(s) + P(s) \cdot W(s)\big) + T(s) \cdot \big(R(s) + N(s)\big)$$

$$|L(s)| \quad \gg \quad 1 \;\Rightarrow\; S(s) \approx \frac{1}{L(s)} \text{ and } T(s) \approx 1$$

$$|L(s)| \quad \ll \quad 1 \;\Rightarrow\; T(s) \approx L(s) \text{ and } S(s) \approx 1$$

$$T(s) + S(s) = \frac{L(s)}{1 + L(s)} + \frac{1}{1 + L(s)} = 1, \quad \forall\, s \in \mathbb{C}$$

Nyquist Theorem

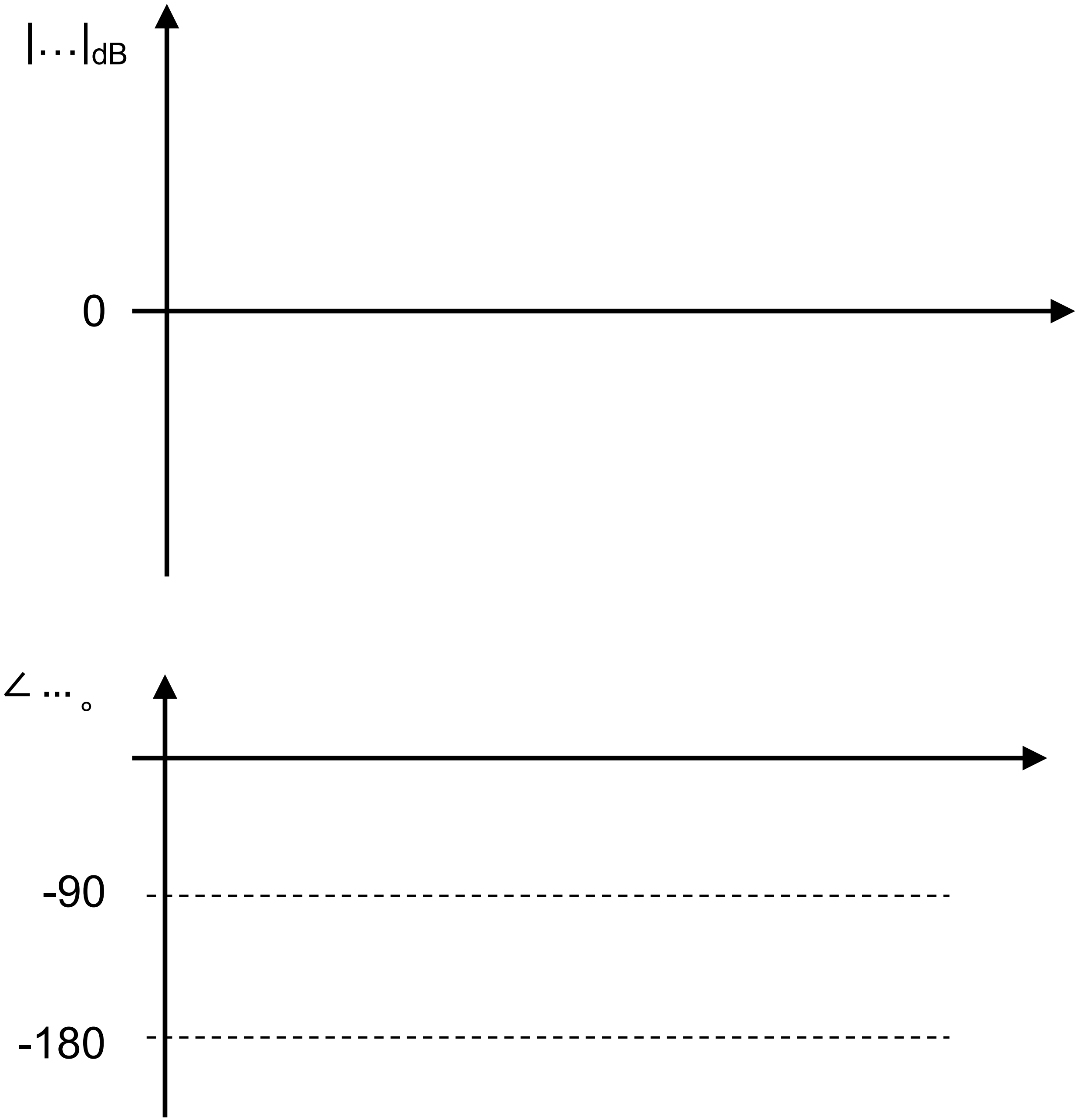
$L(s)$: $n_0 =$ number of poles with real part = 0

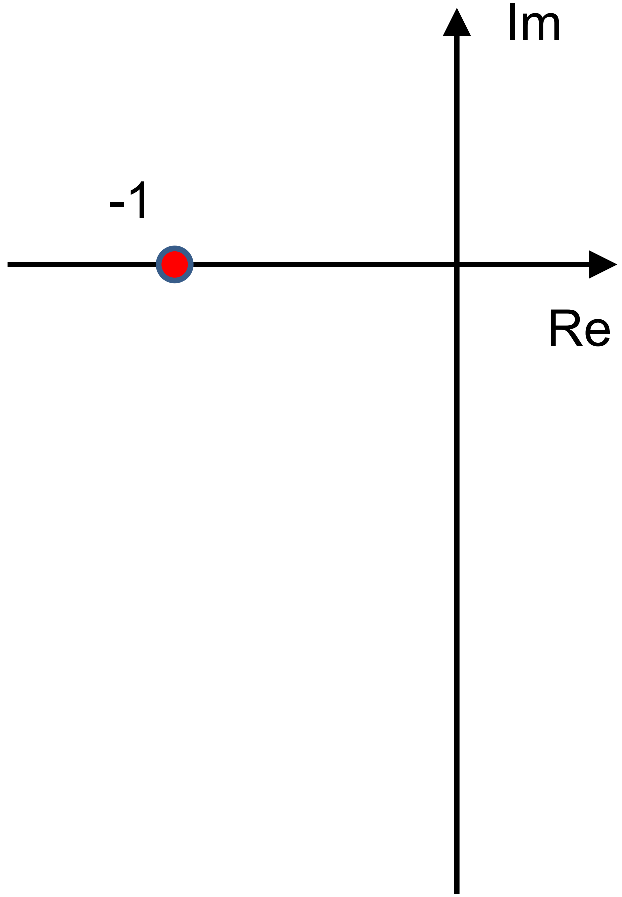
$n_+ =$ number of poles with real part > 0

$n_c =$ number of encirclements of $L(j\omega)$ of -1, when ω goes from -∞ to +∞, counting positive counter clockwise

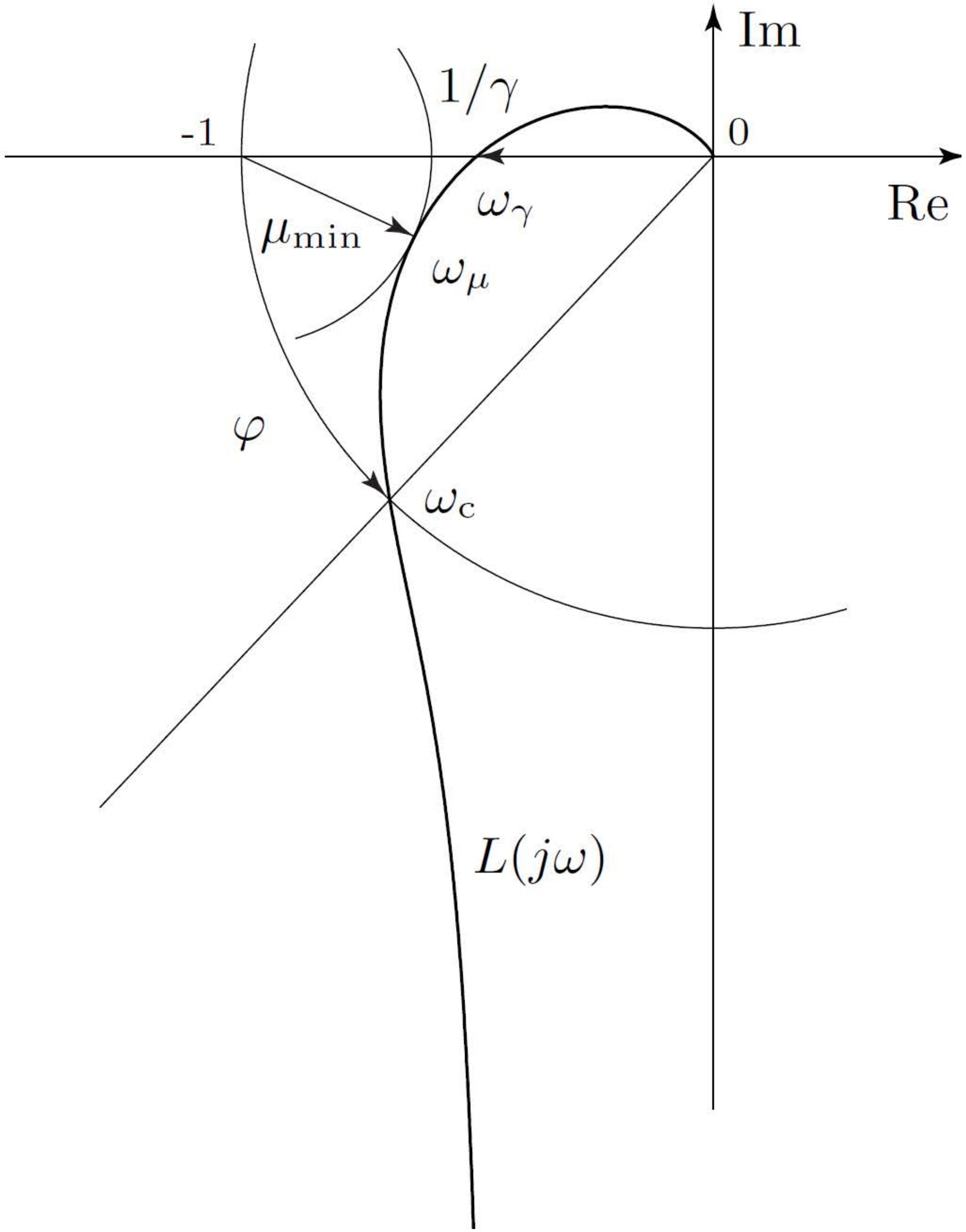
$S(s)$, $T(s)$, … asymptotically stable iff $n_c = n_+ + n_0/2$ and no unstable or non-minimumphase poles and zeros of $L(s)$ are cancelled.

Typical Design





Robustness measures

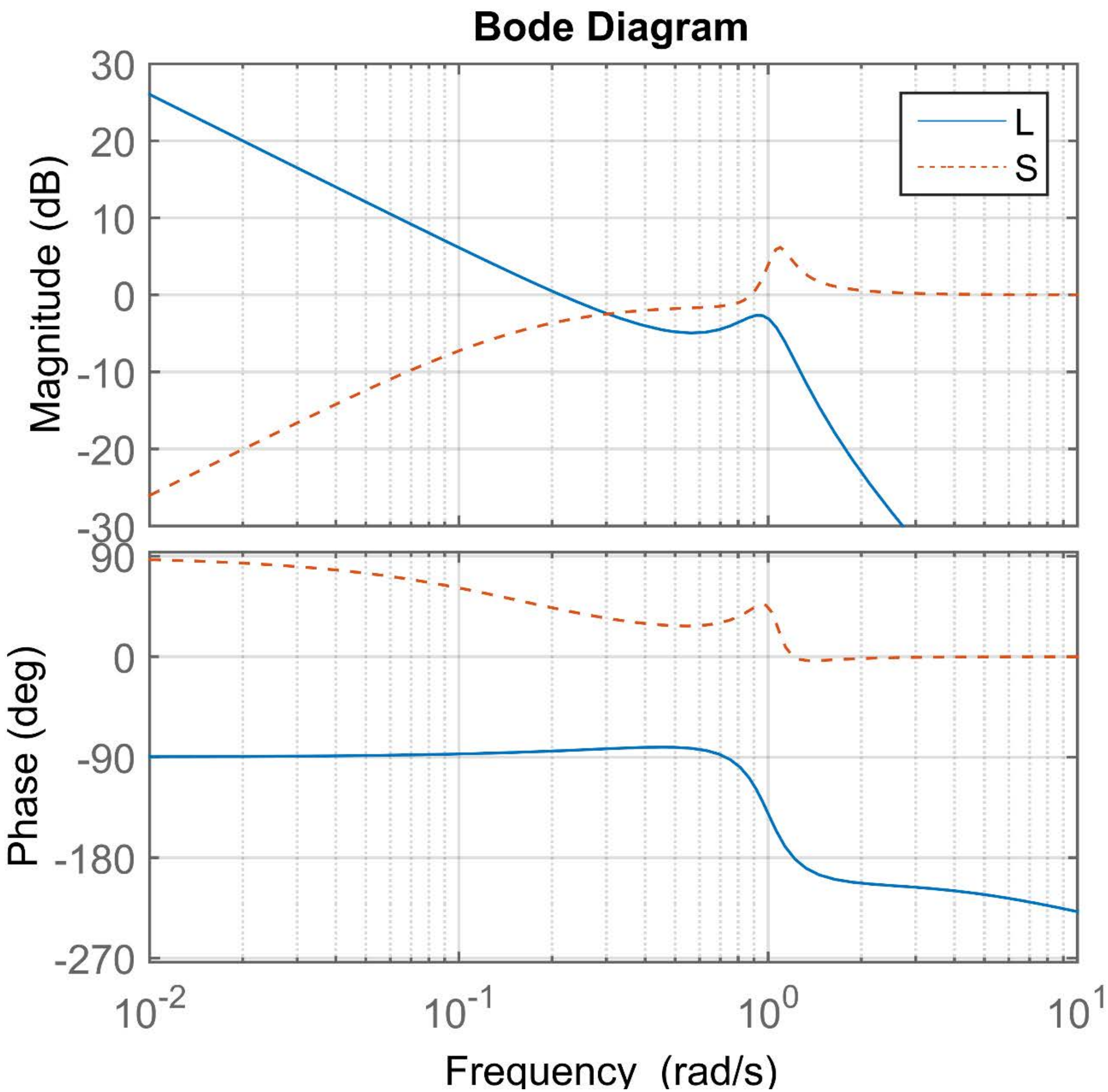


The *phase margin* φ is the distance from -180° where $L(s)$ enters the unit circle in the Nyquist diagram (magnitude 1).

The *gain margin* γ is the the inverse of the magnitude at -180°

The *minimum return difference* $\mu_{\min}$ is the minimum distance from -1.

Example

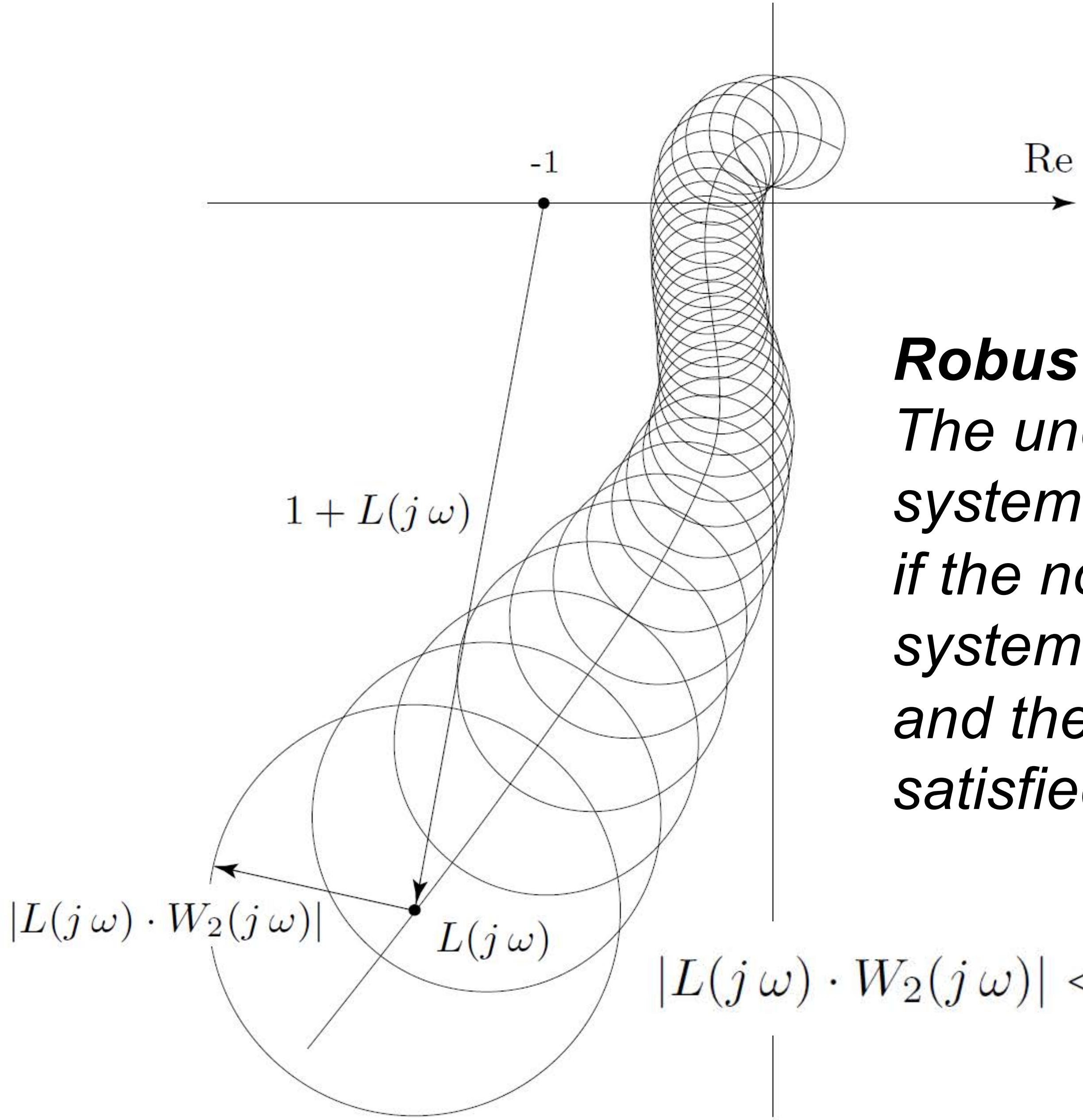


$\varphi =$

$\gamma =$

$\mu_{\min} =$

Robust closed-loop stability



Robust Nyquist Theorem:
The uncertain closed-loop system is asymptotically stable if the nominal closed-loop system is asymptotically stable and the following inequality is satisfied

$$|L(j\,\omega) \cdot W_2(j\,\omega)| < |1 + L(j\,\omega)|, \quad \forall\, \omega \in [0, \infty)$$

20.09. Lektion 1 – Einführung

Modellierung

27.09. Lektion 2 – Modellbildung
4.10. Lektion 3 – Systemdarstellung, Normierung, Linearisierung

Systemanalyse im Zeitbereich

11.10. Lektion 4 – Analyse I, allg. Lösung, Systeme erster Ordnung, Stabilität
18.10. Lektion 5 – Analyse II, Zustandsraum, Steuerbarkeit/Beobachtbarkeit

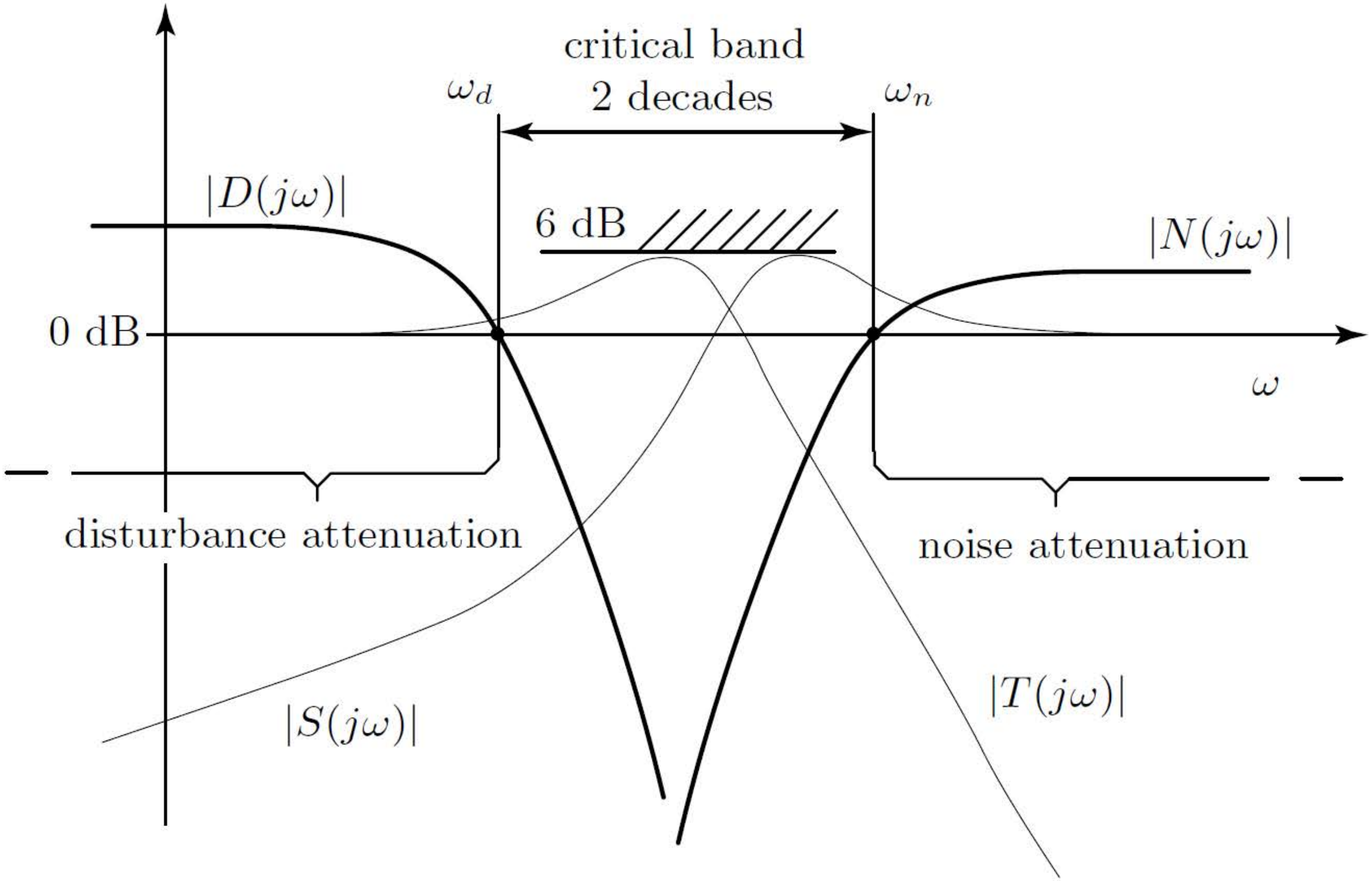
Systemanalyse im Frequenzbereich

25.10. Lektion 6 – Laplace I, Übertragungsfunktionen
1.11. Lektion 7 – Laplace II, Lösung, Pole/Nullstellen, BIBO-Stabilität
8.11. Lektion 8 – Frequenzgänge (RH hält VL)
15.11. Lektion 9 – Systemidentifikation, Modellunsicherheiten
22.11. Lektion 10 – Analyse geschlossener Regelkreise
29.11. Lektion 11 – Randbedingungen

Reglerauslegung

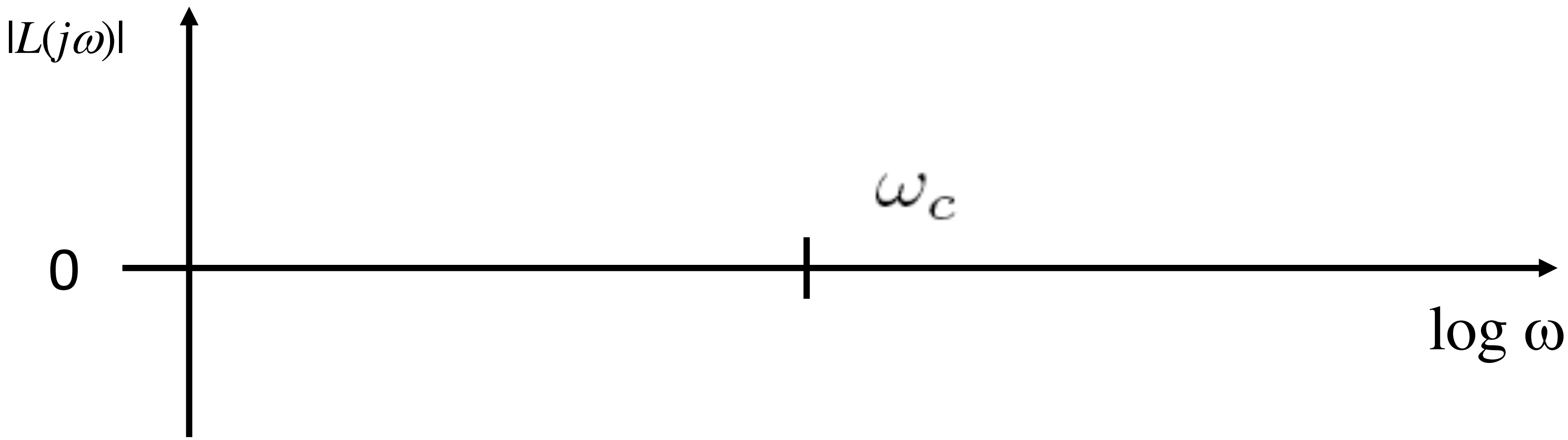
6.12. Lektion 12 – Spezifikationen geregelter Systeme
13.12. Lektion 13 – Reglerentwurf I, PID (RH hält VL)
20.12. Lektion 14 – Reglerentwurf II, „loop shaping“

Noise and Disturbance



$$Y(s) = S(s) \cdot D(s) + T(s) \cdot N(s)$$

Limitations on Crossover Frequency



dominant NMP zero

Test plant: $$P_3(s) = P_0(s) \cdot \frac{s - \zeta^+}{s - \pi^+}$$

BIBO stable and minimum phase plant, but uncertain

dominant unstable pole

Limitations Imposed by Model Uncertainty

$$P(s) = P_0(s)$$

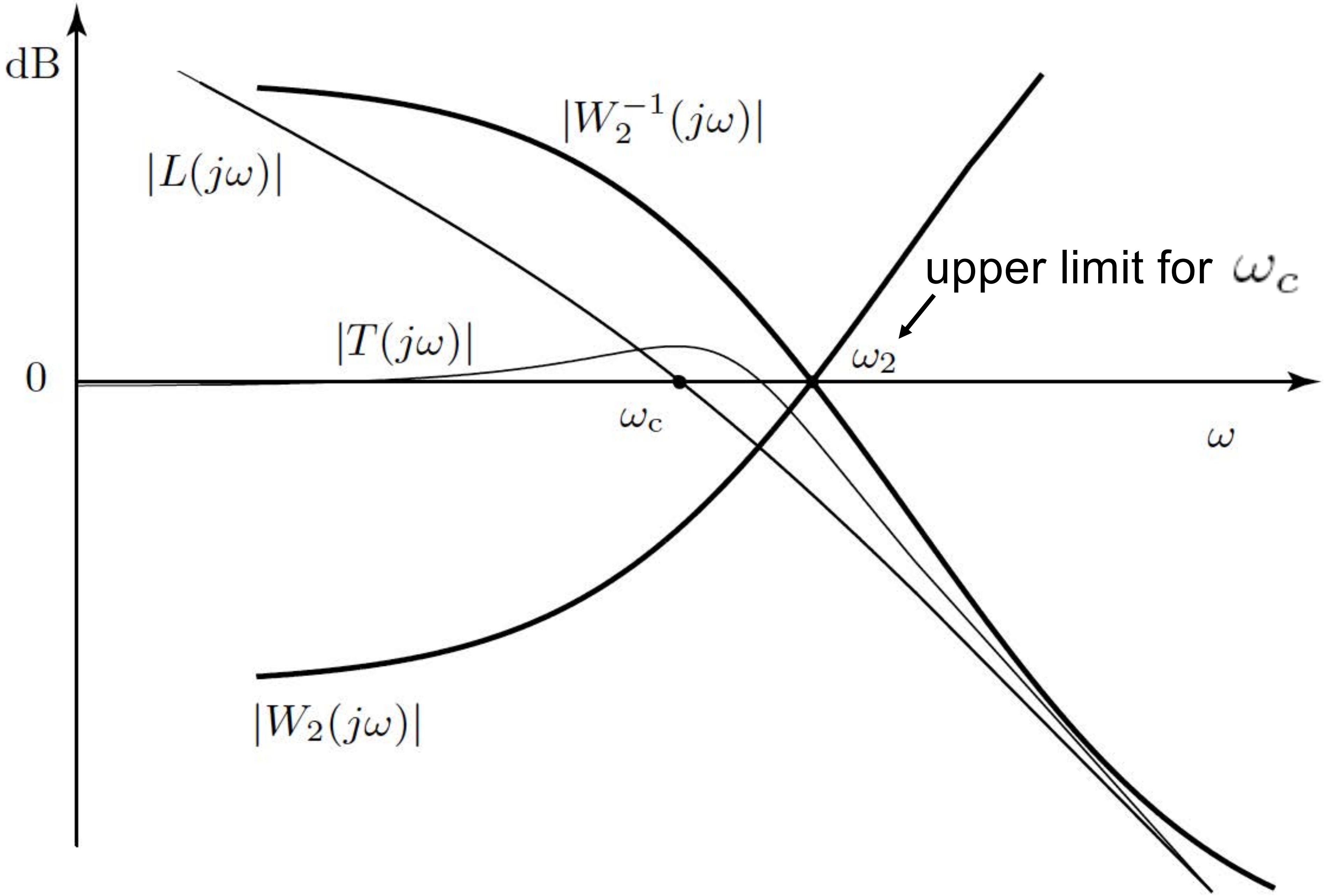
Robust stability Theorem:

$$|W_2(j\,\omega) \cdot L(j\,\omega)| < |1 + L(j\,\omega)|, \quad \forall\ \omega \in [0, \infty]$$

Therefore

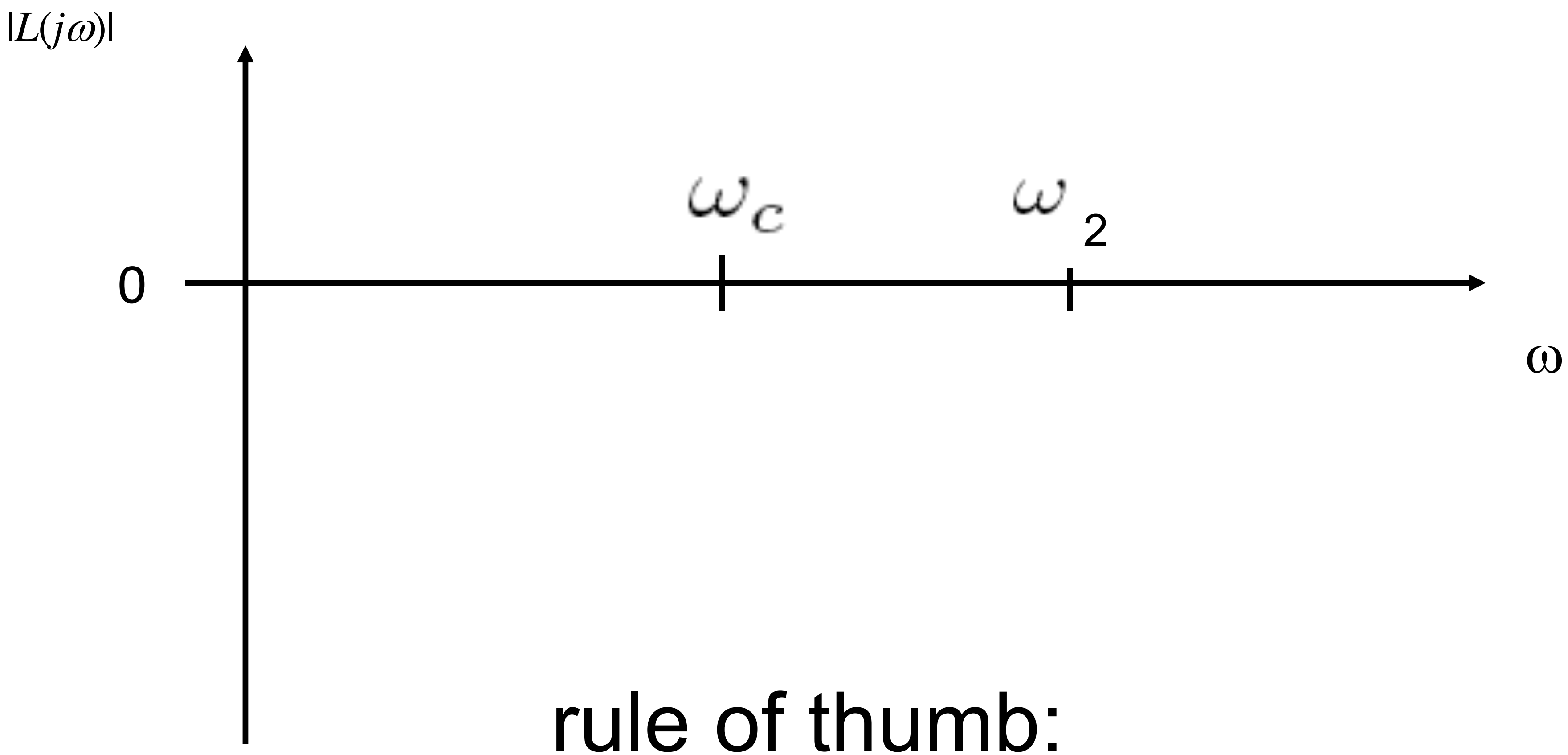
$$|T(j\,\omega)| < |W_2^{-1}(j\,\omega)|, \quad \forall\ \omega \in I\!R_+$$

$$|T(j\,\omega)| \text{ can be small only if } |L(j\,\omega)| \text{ is small}$$



dB
$|W_2^{-1}(j\omega)|$
$|L(j\omega)|$
upper limit for ω_c
$|T(j\omega)|$
ω_2
0
ω_c
ω
$|W_2(j\omega)|$

Generalization



rule of thumb:

Limitations Imposed by Nonminimum Phase Zeros

Assume:

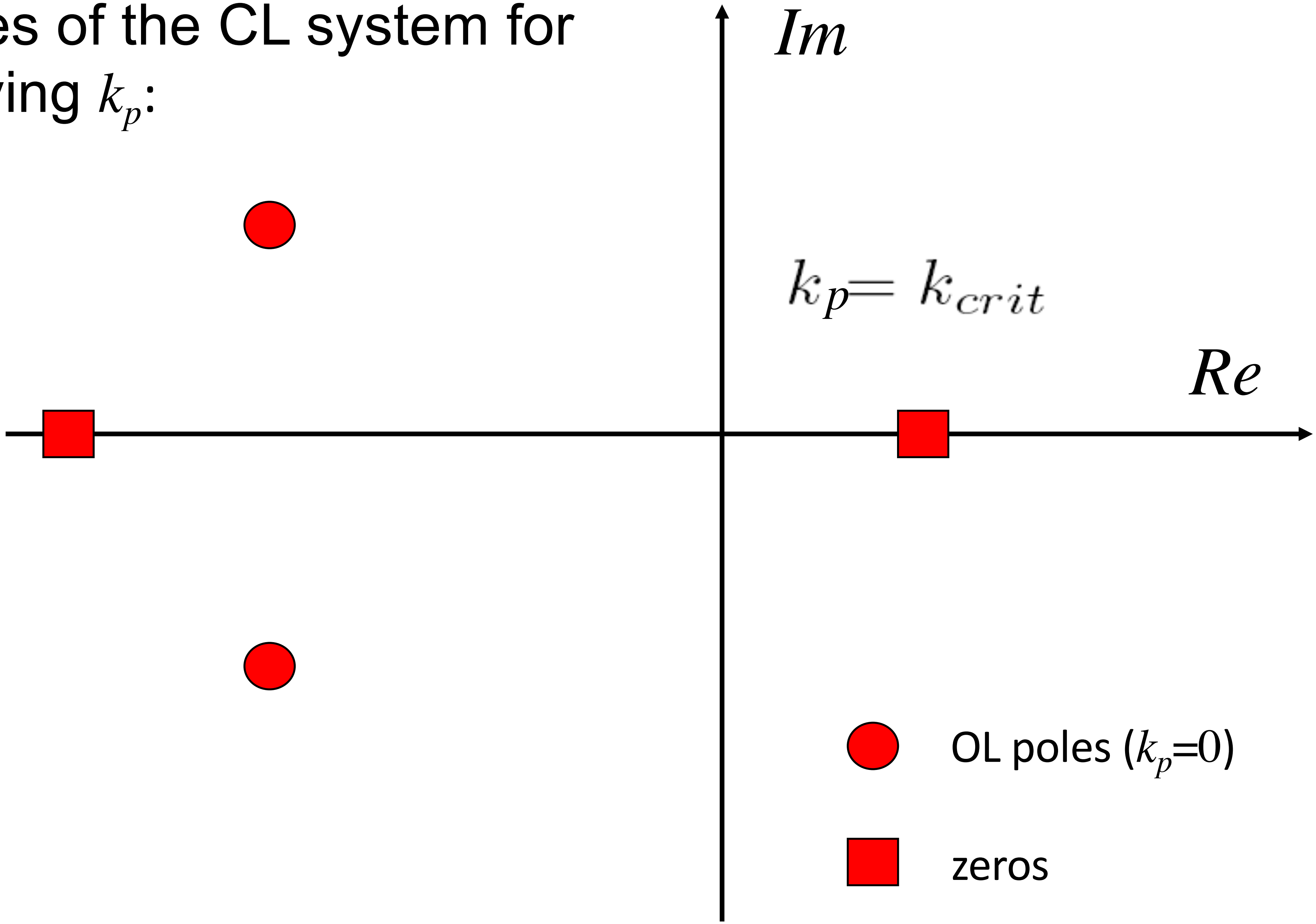
$$P_1(s) = P_0(s) \cdot (s - \zeta^+) = n(s)/d(s)$$

$$C(s) = k_p$$

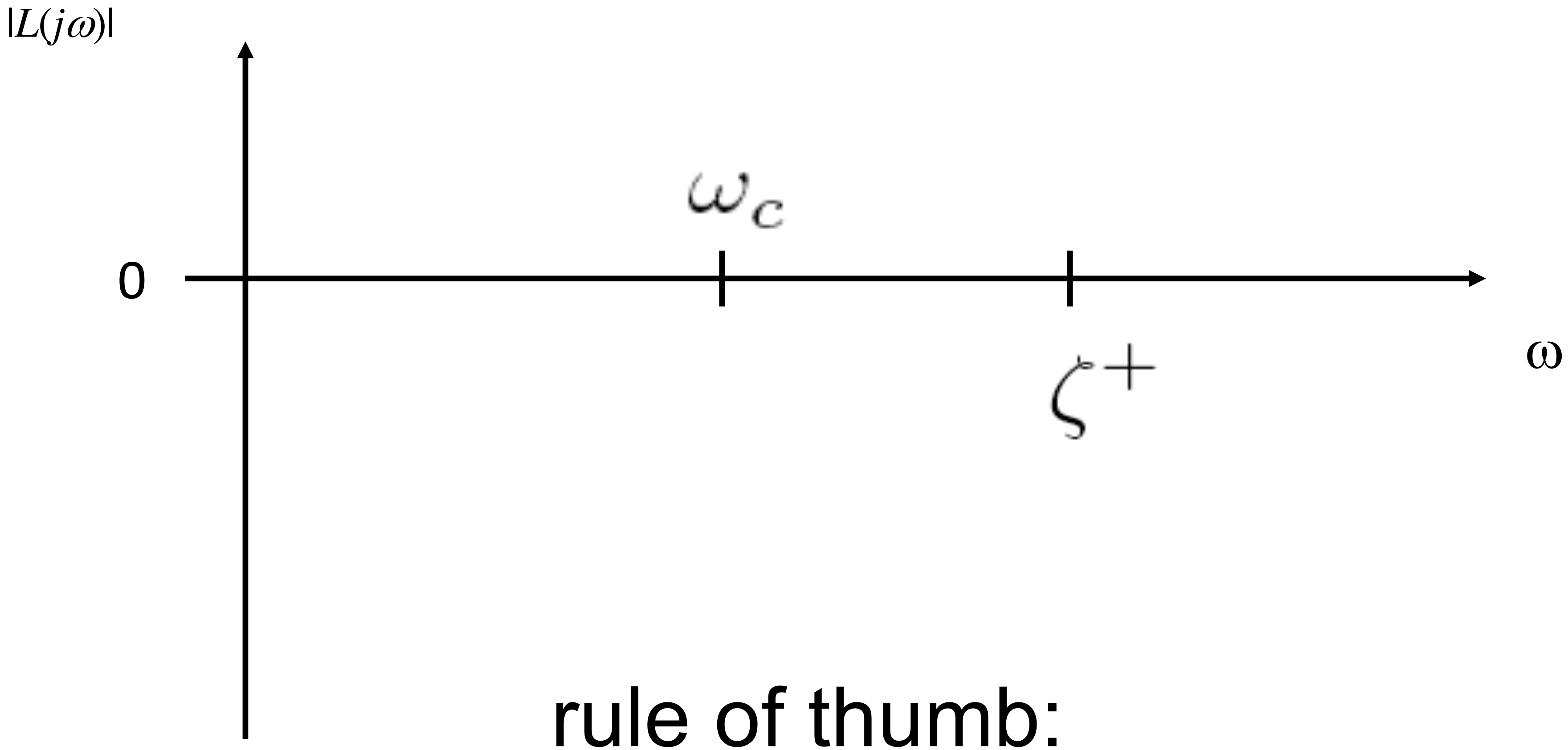
$$T(s) = \frac{k_p n(s)}{\pi(s)}$$

$$\pi(s) = d(s) + k_p \cdot n(s)$$

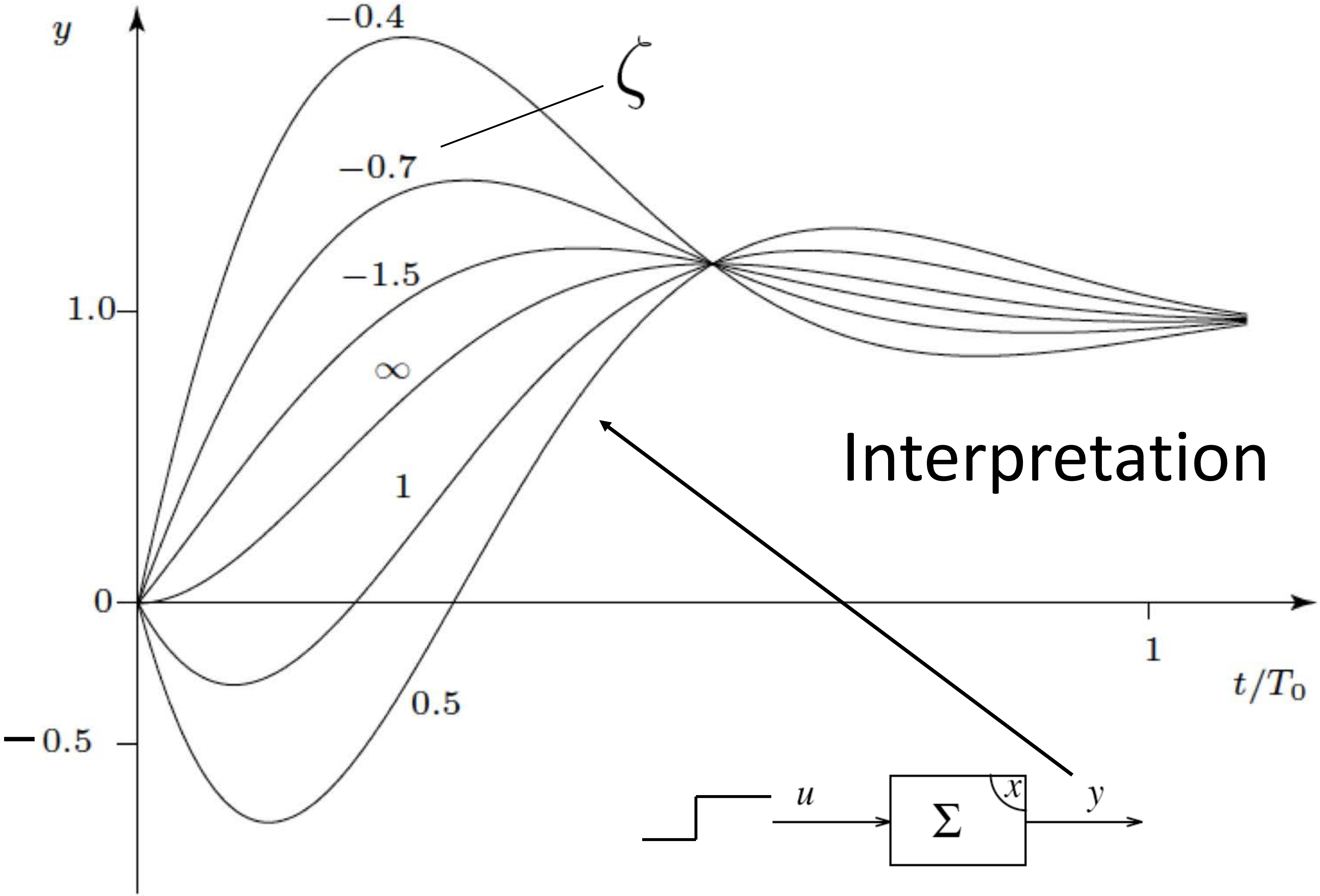
Poles of the CL system for varying k_p:



Generalization

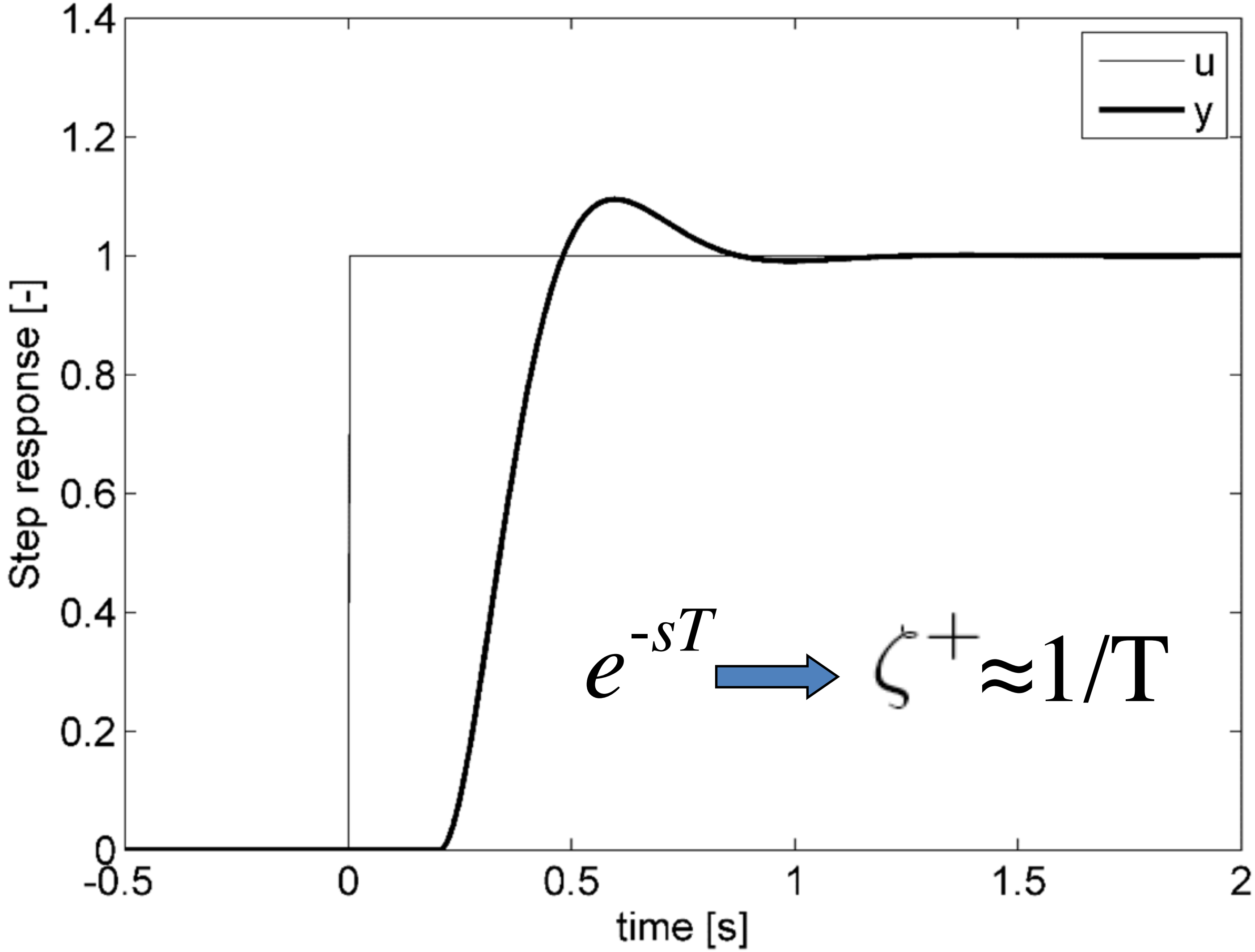


rule of thumb:



Interpretation

Delays are similar to NMP Zeros



e^{-sT} → $\zeta^{+} \approx 1/T$

Limitations Imposed by Unstable Poles

Simplest case:

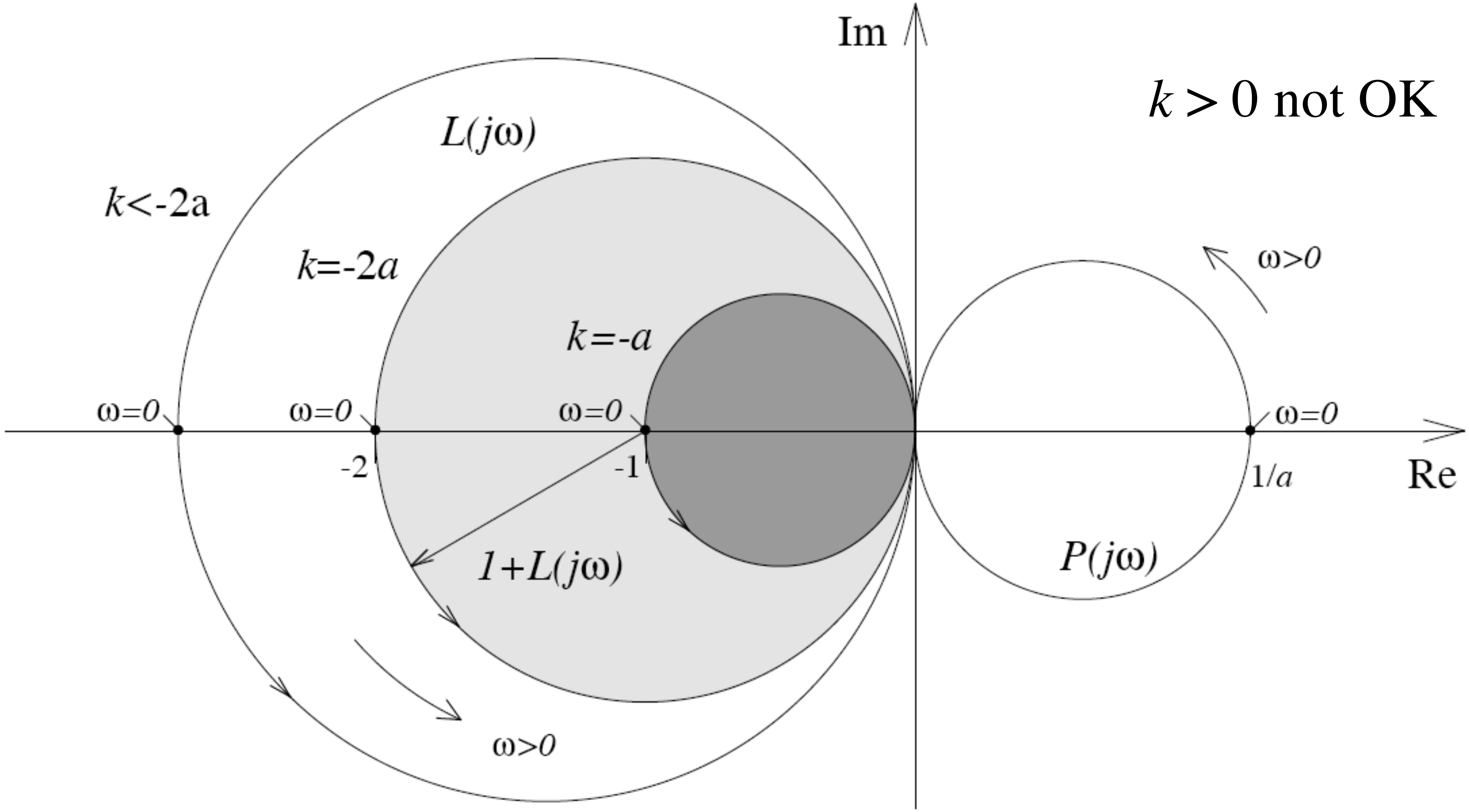
$$P_2(s) = \frac{1}{a-s}, \quad a > 0 \qquad \pi^+ = a$$

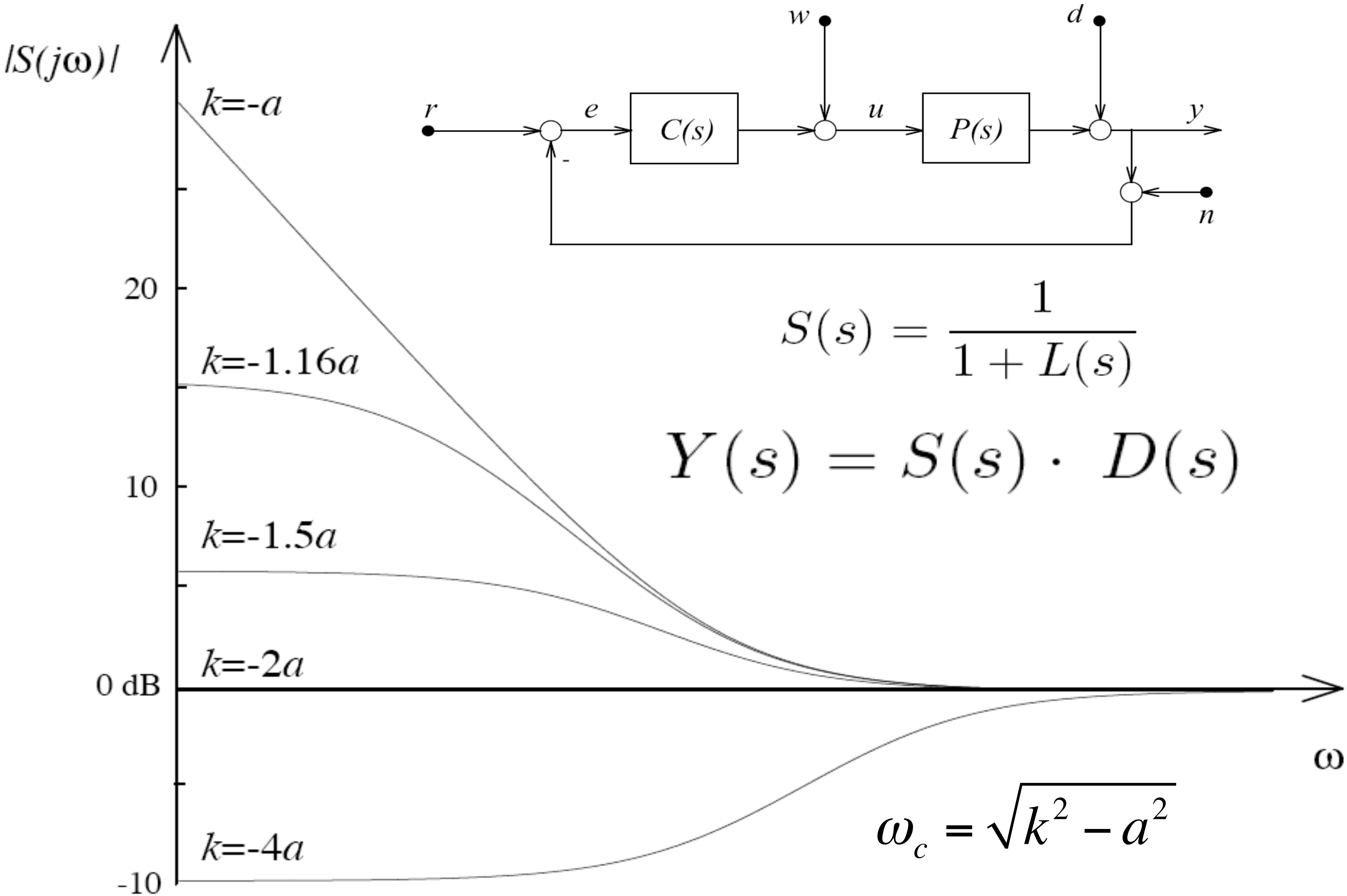
$$C(s) = k$$

$$L(j\omega) = k/(a\text{-}j\omega) = k(a+j\omega)/(a^2+\omega^2)$$

$$n_+ = 1, n_0 = 0$$

k < 0 can be OK



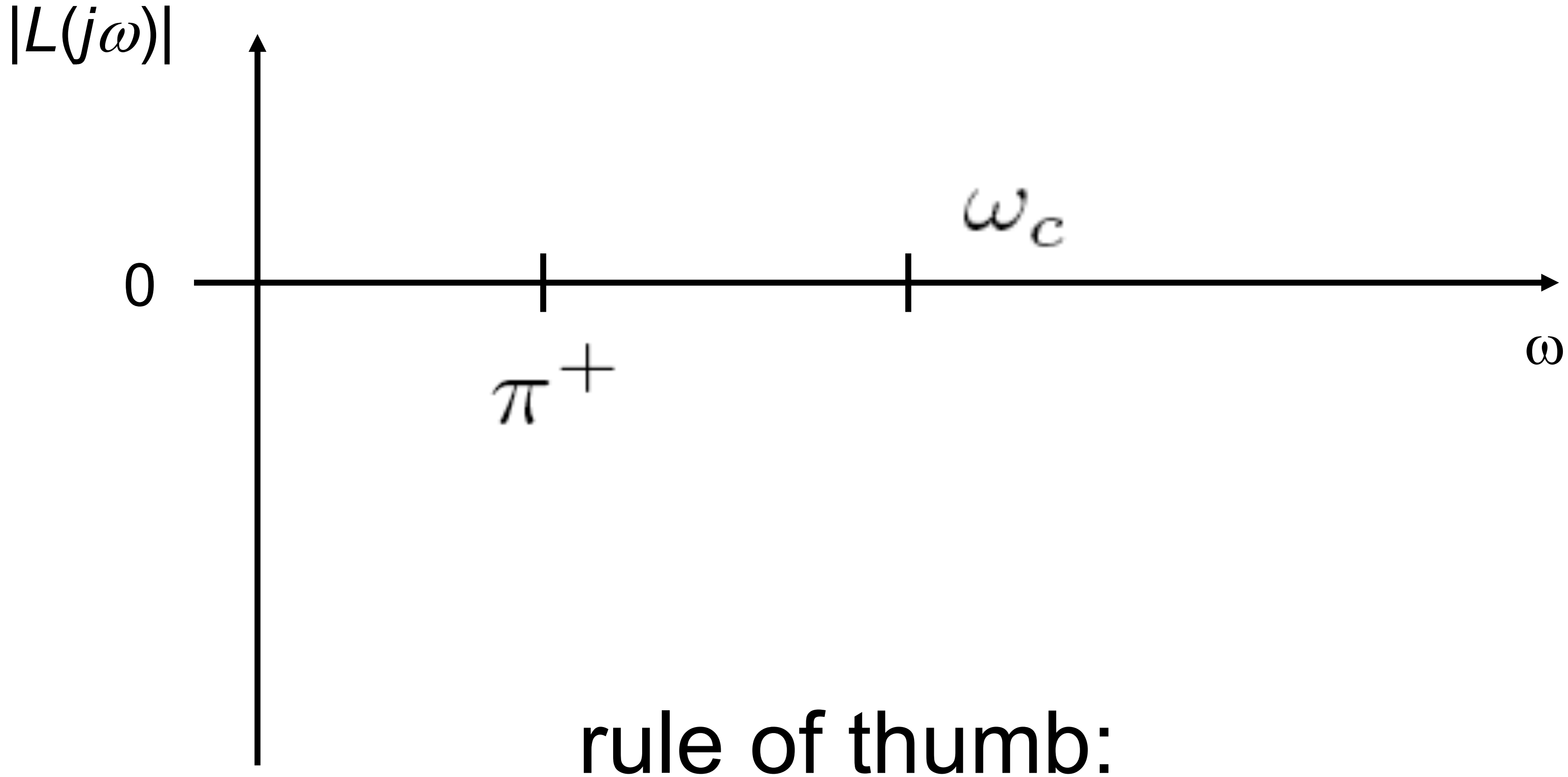


$$S(s) = \frac{1}{1 + L(s)}$$

$$Y(s) = S(s) \cdot \; D(s)$$

$$\omega_c = \sqrt{k^2 - a^2}$$

Generalization



rule of thumb:

Summary of Limitations

ω_c = cross-over frequency of the loop gain

ω_d = specified maximum disturbance rejection frequency

π^+ = dominant („fastest“) unstable pole

ζ^+ = dominant („slowest“) non-minimum phase zero

ω_T = frequency defined by the delay of the loop gain

ω_2 = frequency beyond which the model uncertainty is ≥ 100%

ω_n = specified minimum noise rejection frequency

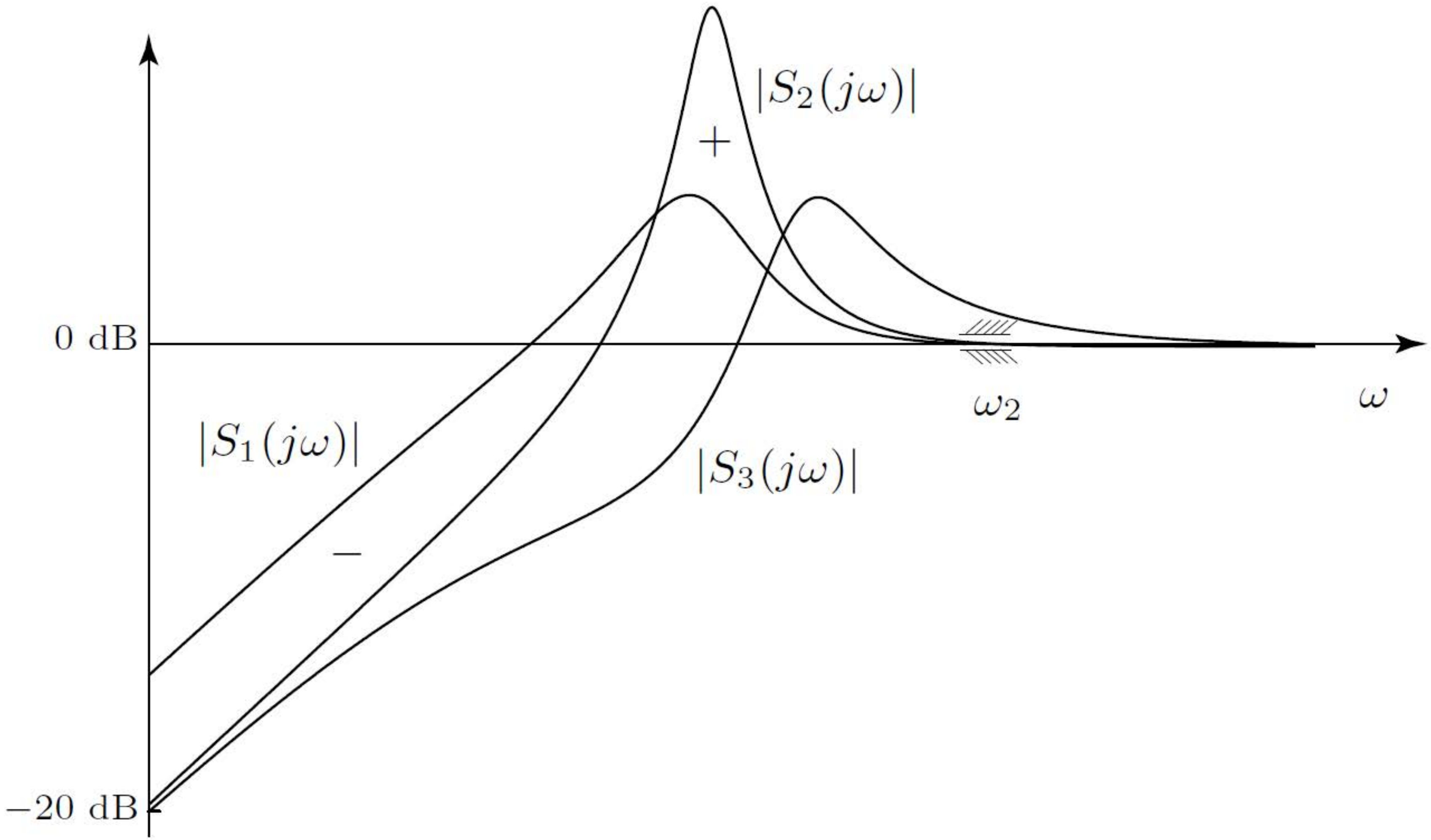
Rule of t thumb:

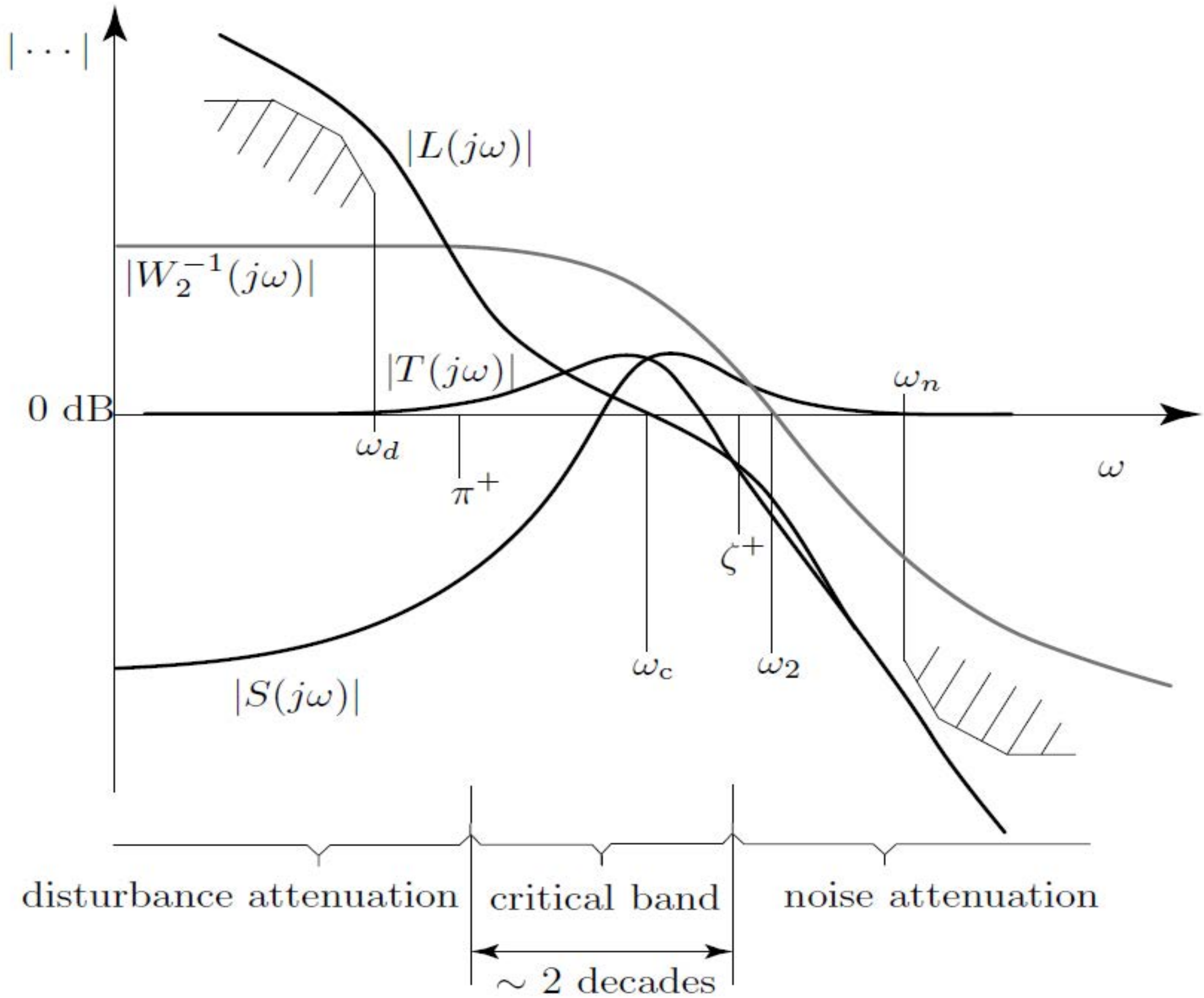
$$\omega_c \in \left[\max(10 \cdot \omega_d, 2 \cdot \pi^+)\,, \min\left(\frac{\zeta^+}{2}, \frac{\omega_T}{2}, \frac{\omega_2}{5}, \frac{\omega_n}{10}\right)\right]$$

There are more constraints (see book).

Example

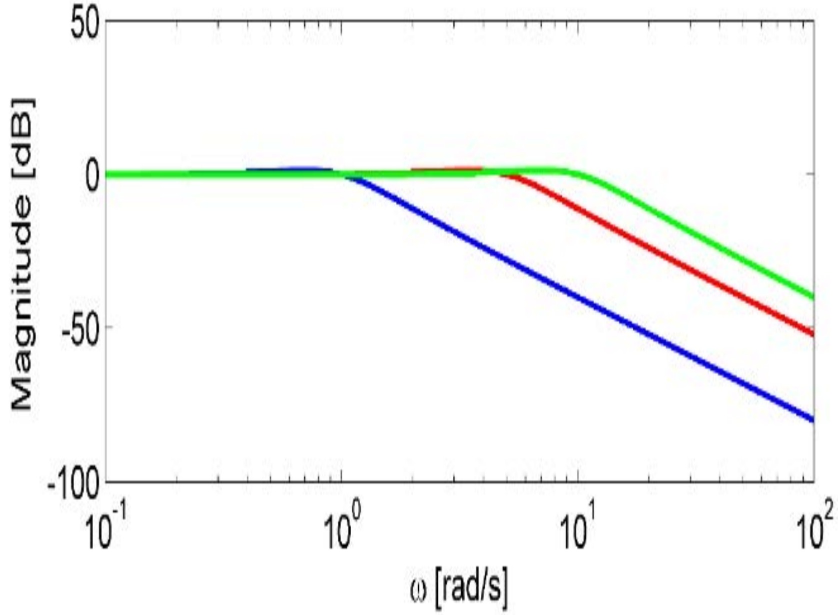
$$L(s) = P(s) \cdot C(s) = \frac{1}{s \cdot (s^2 + s + 1)} \cdot k$$

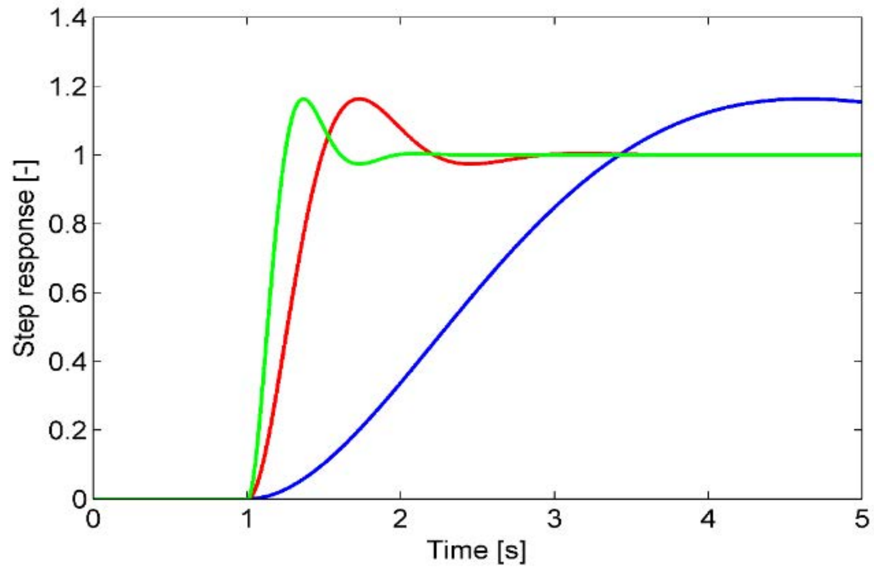



$|\cdots|$
$|L(j\omega)|$
$|W_2^{-1}(j\omega)|$
$|T(j\omega)|$
ω_n
0 dB
ω_d
π^+
ω
ζ^+
ω_c
ω_2
$|S(j\omega)|$
disturbance attenuation
critical band
noise attenuation
$\sim$ 2 decades

Bandwidth and Settling Time

The *bandwidth* ω_b is the frequency, at which the complementary sensitivity $T(s)$ (closed loop!) falls below -3 dB. The bandwith determines the «speed» of the time-domain behavior.





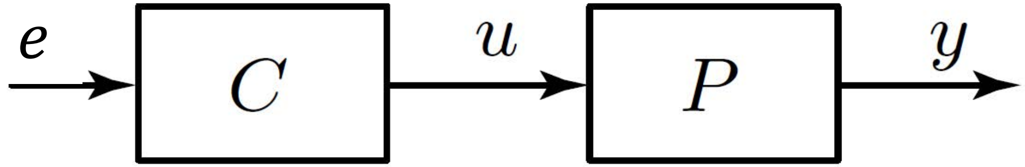
The bandwidth always is close to the crossover frequency. Therefore, the crossover frequency (open loop) is a measure for the speed of the closed-loop system.

Example

Quick Check 9.5.2: Assume that a plant $P(s) = s^{-1}$ is controlled using a controller $C(s) = k_p$.

- Compute the crossover frequency ω_c of the corresponding loop gain $L(s)$.
- What is the bandwidth ω_b of this system?
- What form will the output $y(t)$ of the closed-loop system have when the reference signal is a step function $r(t) = h(t)$?

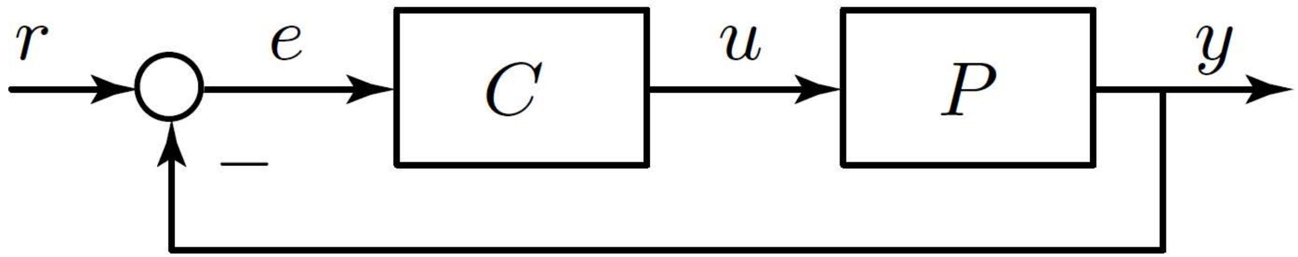
Series Connection in State Space



Plant: $$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = A \cdot x(t) + b \cdot u(t), \quad y(t) = c \cdot x(t)$$

Controller: $$\frac{\mathrm{d}}{\mathrm{d}t}z(t) = F \cdot z(t) + g \cdot e(t), \quad u(t) = h \cdot z(t)$$

Feedback Connection in State Space



$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A & b \cdot h \\ -g \cdot c & F \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} 0 \\ g \end{bmatrix} \cdot r(t)$$

$$y(t) = \begin{bmatrix} c & 0 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$$