

GOAL:

Use Definitions, Postulates, Properties and Theorems to prove other shortcuts for measuring.

GEOMETRIC POSTULATES:

Ruler Postulate

Segment Addition Postulate

Protractor Postulate

Angle Addition Postulate

ALGEBRAIC PROPERTIES

CONGRUENCE PROPERTIES

Basic rules for measuring

Distance = $|a - b|$

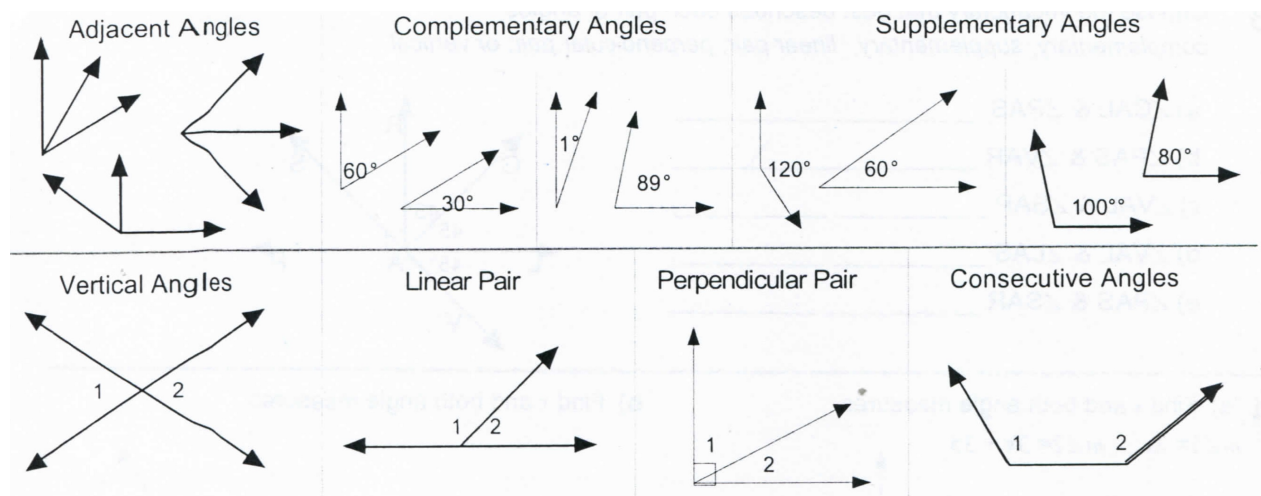
If B is between A and C then $AB + BC = AC$

Angle Measure = $|a - b|$

If B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$

Addition, Subtraction, Multiplication, Division, Reflexive, Symmetric, Transitive.

Reflexive, Symmetric, Transitive



DEFINITIONS

Straight Angle Definition:

Linear Pair Definition:

Perpendicular Pair

Supplementary Angles Definition:

Complementary Angles Definition:

Vertical Angles

USED IN PROOFS

An angle is a straight angle iff its measure is 180

Two angles form a linear pair iff non-shared sides form a straight angle.

Two angles form a perpendicular pair iff non-shared sides form a right angle.

The measure of two angles adds up to 180 iff the angles are supplementary.

The measure of two angles adds up to 90 iff the angles are complementary.

Two angles are vertical angles iff their sides form two pairs of opposite rays.

THEOREMS:

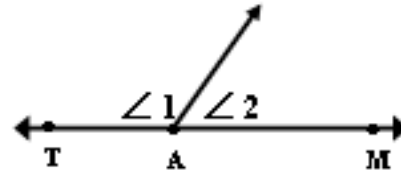
Have to be proven before being used

TO PROVE THEOREMS

1. Start with the given information.
2. Use a Definition to explain the given information.
3. Reason back from what you would like to prove using Definitions, Properties, Postulates and Theorems that have already been proven.
4. Conclude with what needs to be proven.

LINEAR PAIR THEOREM:

If two angles form a linear pair, then **the angles are supplementary**.



GIVEN: $\angle 1$ and $\angle 2$ form a linear pair

PROVE: $\angle 1$ and $\angle 2$ are supplementary

STATEMENT

1. $\angle 1$ and $\angle 2$ form a linear pair.
2. $\angle TAM$ is a straight angle.
3. $m\angle TAM = 180^\circ$
4. $m\angle 1 + m\angle 2 = m\angle TAM$
5. $m\angle 1 + m\angle 2 = 180^\circ$
6. $\angle 1$ and $\angle 2$ are supplementary angles

REASONS

Given

Linear Pair Definition

(Two angles form a linear pair iff their non-shared sides form a straight angle)

Straight Angle Definition

(An angle is a straight angle iff its measure is 180)

Angle Addition Postulate

Substitution

Supplementary Angle Definition

(The measure of two angles is 180 iff the angles are supplementary)

TO PROVE THEOREMS

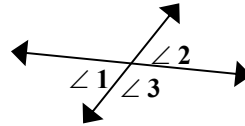
1. Start with the given information.
2. Use a Definition to explain the given information.
3. Reason back from what you would like to prove using Definitions, Properties, Postulates and Theorems that have already been proven.
4. Conclude with what needs to be proven

LINEAR PAIR THEOREM:

If two angles form a linear pair, then the angles are supplementary.

VERTICAL ANGLES THEOREM:

If two angles are vertical angles, then the angles have equal measures.



GIVEN: $\angle 1$ and $\angle 2$ are vertical angles

PROVE: $\angle 1$ and $\angle 2$ have equal measures

STATEMENT

1. $\angle 1$ and $\angle 2$ are vertical angles
2. $\angle 1$ and $\angle 3$ form a linear pair
 $\angle 2$ and $\angle 3$ form a linear pair
3. $\angle 1$ and $\angle 3$ are supplementary
 $\angle 2$ and $\angle 3$ are supplementary
4. $m\angle 1 + m\angle 3 = 180^\circ$
 $m\angle 2 + m\angle 3 = 180^\circ$
5. $m\angle 1 = 180^\circ - m\angle 3$
 $m\angle 2 = 180^\circ - m\angle 3$
6. $m\angle 1 = m\angle 2$

REASONS

Given

Linear Pair Definition

(Two angles form a linear pair iff their non-shared sides form a straight angle)

Linear Pair Theorem

(If two angles form a linear pair then they are supplementary)

Supplementary Angle Definition

(The measure of two angles is 180 iff the angles are supplementary)

Subtraction Prop. of Equality

Substitution

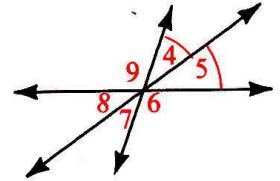
LINEAR PAIR THEOREM:

If two angles form a linear pair, then the angles are supplementary

VERTICAL ANGLES THEOREM:

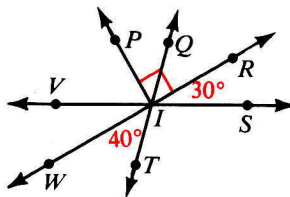
If two angles are vertical angles, they have equal measures

Class Discussion



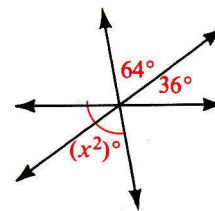
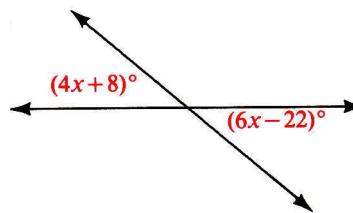
1. In the diagram, $\angle 4 \cong \angle 5$. Name two other angles congruent to $\angle 5$

2.



3. Find the value of X

- a. $m\angle QIR = \underline{\quad?}$
- b. $m\angle PIQ = \underline{\quad?}$
- c. $m\angle VIT = \underline{\quad?}$
- d. $m\angle VIQ = \underline{\quad?}$
- e. $m\angle SIT = \underline{\quad?}$



Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles, Same Side Interior Angles, Same Side Exterior Angles

Angles formed by a transversal line and two coplanar lines.

A **transversal** is a line that intersects two or more coplanar lines in different points. In the next diagram, t is a transversal of h and k . The angles formed have special names.

Interior angles: angles 3, 4, 5, 6 *Exterior angles:* angles 1, 2, 7, 8

Alternate interior angles (alt. int. \sphericalangle) are two nonadjacent interior angles on opposite sides of the transversal.

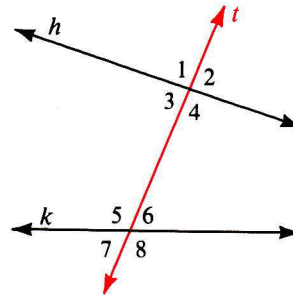
$\sphericalangle 3$ and $\sphericalangle 6$ $\sphericalangle 4$ and $\sphericalangle 5$

Same-side interior angles (s-s. int. \sphericalangle) are two interior angles on the same side of the transversal.

$\sphericalangle 3$ and $\sphericalangle 5$ $\sphericalangle 4$ and $\sphericalangle 6$

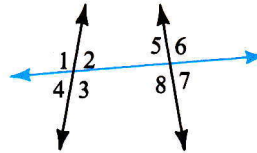
Corresponding angles (corr. \sphericalangle) are two angles in corresponding positions relative to the two lines.

$\sphericalangle 1$ and $\sphericalangle 5$ $\sphericalangle 2$ and $\sphericalangle 6$ $\sphericalangle 3$ and $\sphericalangle 7$ $\sphericalangle 4$ and $\sphericalangle 8$



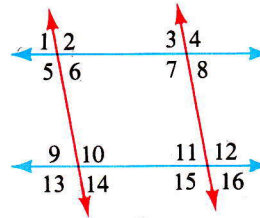
Classroom Exercises

1. The blue line is a transversal.
 - a. Name four pairs of corresponding angles.
 - b. Name two pairs of alternate interior angles.
 - c. Name two pairs of same-side interior angles.



Classify each pair of angles as alternate interior angles, same-side interior angles, corresponding angles, or none of these.

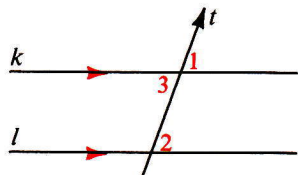
- | | |
|--|--|
| 2. $\sphericalangle 2$ and $\sphericalangle 4$ | 3. $\sphericalangle 6$ and $\sphericalangle 10$ |
| 4. $\sphericalangle 7$ and $\sphericalangle 15$ | 5. $\sphericalangle 7$ and $\sphericalangle 12$ |
| 6. $\sphericalangle 7$ and $\sphericalangle 10$ | 7. $\sphericalangle 14$ and $\sphericalangle 15$ |
| 8. $\sphericalangle 11$ and $\sphericalangle 14$ | 9. $\sphericalangle 1$ and $\sphericalangle 11$ |
10. Name alternate exterior angles.
 11. Name same side exterior angles.



CORRESPONDING ANGLES POSTULATE
IF TWO PARALLEL LINES ARE CUT BY A TRANSVERSAL
THEN CORRESPONDING ANGLES ARE CONGRUENT

Given: Parallel Lines cut by a Transversal
If $k \parallel l$, then $\angle 1 \cong \angle 2$

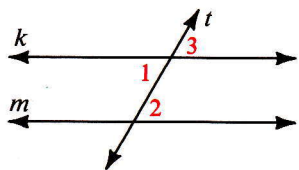
Conclude: Alternate Interior Angles are Congruent
(Corresponding Angles Postulate)



ALTERNATE INTERIOR ANGLE THEOREM
IF TWO PARALLEL LINES ARE CUT BY A TRANSVERSAL
THEN ALTERNATE INTERIOR ANGLES ARE CONGRUENT

Given: Parallel Lines cut by a Transversal
If $k \parallel l$, then $\angle 1 \cong \angle 2$

Conclude: Alternate Interior Angles are Congruent
(Alternate Interior Angle Theorem)



ALTERNATE EXTERIOR ANGLE THEOREM
IF TWO PARALLEL LINES ARE CUT BY A TRANSVERSAL
THEN ALTERNATE EXTERIOR ANGLES ARE CONGRUENT

Given: Parallel Lines cut by a Transversal

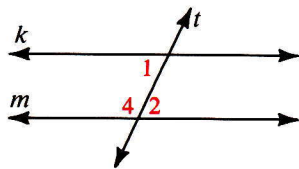
Conclude: Alternate Exterior Angles are Congruent

SAME SIDE INTERIOR ANGLE THEOREM
IF TWO PARALLEL LINES ARE CUT BY A TRANSVERSAL
THEN SAME SIDE INTERIOR ANGLES ARE CONGRUENT

Given: Parallel Lines cut by a Transversal

Conclude: Same Side Interior Angles are
Supplementary
(Same Side Interior Angle Theorem)

If $k \parallel l$, then $\angle 1 \cong \angle 4$



SAME SIDE EXTERIOR ANGLE THEOREM
IF TWO PARALLEL LINES ARE CUT BY A TRANSVERSAL
THEN SAME SIDE EXTERIOR ANGLES ARE CONGRUENT

Given: Parallel Lines cut by a Transversal

Conclude: Same Side Exterior Angles are
Supplementary

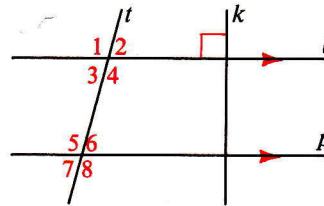
CW#23

If $l \parallel p$ cut by transversal t , which angle pairs are Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles, Same Side Interior Angles, Same Side Exterior Angles?

Which angle pairs are congruent?

Which angle pairs are supplementary?

1. What do the arrowheads in the diagram tell you?
2. a. How are lines k and l related?
b. How are lines k and p related? Why?

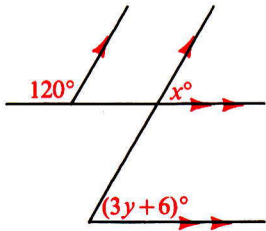


Exs. 1-11

State the postulate or theorem that justifies each statement.

3. $\angle 1 \cong \angle 5$
 4. $\angle 3 \cong \angle 6$
 5. $m\angle 4 + m\angle 6 = 180$
 6. $m\angle 4 = m\angle 8$
 7. $m\angle 4 = m\angle 5$
 8. $\angle 6 \cong \angle 7$
 9. $k \perp p$
 10. $\angle 3$ is supplementary to $\angle 5$.
11. If $m\angle 1 = 130$, what are the measures of the other numbered angles?

12. Find the values of x and y .



Substitution Property of Equality
 Substitute $(m\angle 1)$ for $(180^\circ - m\angle 3)$
 in $m\angle 2 = (180^\circ - m\angle 3)$

HW#23

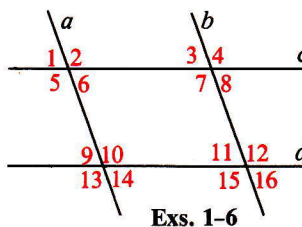
If $c \parallel d$ cut by transversal a , which angle pairs are Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles, Same Side Interior Angles, Same Side Exterior Angles?

Which angle pairs are congruent?

Which angle pairs are supplementary?

Show your work.

- A** 1. If $a \parallel b$, name all angles that must be congruent to $\angle 1$.
 2. If $c \parallel d$, name all angles that must be congruent to $\angle 1$.



Exs. 1-6

Assume that $a \parallel b$ and $c \parallel d$.

3. Name all angles congruent to $\angle 4$.
 4. Name all angles supplementary to $\angle 4$.
 5. If $m\angle 16 = 50$, then $m\angle 14 = ?$ and $m\angle 2 = ?$.
 6. If $m\angle 9 = x$, then $m\angle 12 = ?$ and $m\angle 7 = ?$.

Find the values of x and y .

