Using the Van Hiele Model In the Classroom

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Teaching geometry may be one of the most difficult tasks a math teacher may encounter. Geometry is where most students are first exposed to using axiomatic systems to prove theorems. This is where geometry differs from previous mathematics courses such as arithmetic and algebra where new methods are introduced but from there, students are only made to go through the algorithms in order to solve questions. In geometry, students are introduced to mathematical proofs, which are sequences of true statements that one can use to achieve one result from either other results or axioms (given definitions). This transition proves to be difficult for most students, but the transition from concrete thinking to geometry's abstract way of thinking can be made easier by following a model proposed by Pierre van Hiele and Dina van Hele between 1957 and 1986 (Halat, 2006). The purpose of this paper is to: discuss how students learn, introduce the Van Hiele model, and how the implementation of the Van Hiele model is important for students and teachers.

How do Students Learn

According to the behavioral psychologist Albert Bandura, learning occurs through modeling, where students either do or do not imitate ones behavior, depending on the outcome of the behavior. This led Bandura to conduct an analysis of behavioral learning, in which he found learning involves four phases: attention, retention, reproduction, and motivation (Slavin, 2012).

Attention. In order for learning to take place, a student must be interested in the material that is being modeled for them. This is why teachers must present interesting material throughout their lessons. (Slavin, 2012)

Retention. This is the period in time where teachers present students with material that they want them to understand and be able to 'retain'. According to Bandura, this presentation process is through modeling, in which the teacher will go through a sequence of steps in order solve a problem that can be replicated by the students. (Slavin, 2012)

Reproduction. Reproduction is the phase in which students take material that they learned during the lesson and reproduce it on their own (through worksheets, homework, etc.). This stage is essential for in order for teachers to be able to judge whether or not students understand the material presented to them. (Slavin, 2012)

Motivation. The final phase, motivation, is what ensures that students will retain the information presented to them. If students are not motivated to learn the material they simply will not learn the material. But if students believe that if they can reproduce what they learned they will be rewarded, then they will be motivated to learn the material. (Slavin, 2012)

Zone of Proximal Development. Bandura's model is easy to apply in a typical classroom, but it does have one major downfall, even if students are paying attention, trying to retain the material, are able to reproduce the material, and are motivated to learn the material, they may not fully grasp the material. This is because the material presented may not be within the students' within the zone of proximal development. To teach in a student's zone of proximal development is to present material at or slightly above what the student is currently able to do, in other words it is teaching a child how to do something they are capable of doing even though they may not currently be able to do it (Slavin, 2012). This is important for teachers to understand, because if students are taught at levels either below or too far above their zone of proximal development, no learning can take place.

A similar concept exists is mathematical education (mainly in geometry), that focuses on the idea that students develop through different levels of understanding. For example, consider a rectangle, younger students (preschool-2nd grade) may recognize the rectangle as a door shape, where middle school students may call it a rectangle, and high school students may call it a parallelogram with four right angles and unequal side lengths. The concept that students develop a geometric understanding through stages was brought to light by Dina van Hiele-Geldof and Pierre van Hiele in 1957 though their most recent work was published in 1987 by Pierre van Hiele (Van Hiele P. M., 1986).

The Van Hiele Model

The Van Hiele model attempts to explain students' ability to learn and understand geometry depending on their Van Hiele Level (Halat, 2006). The Van Hiele model has 5 stages: visual, analysis, inferences related to experience/abstraction, inference resolutions, and the advanced period (ERDOGAN, AKKAYA, & CELEBI AKKAYA, 2009). There has also been a proposal of a stage 0, the pre-recognition period. (Clements & Battista, 1992)

Level 0: Pre-Recognition. In this stage, students are able to recognize and distinguish between shapes, such as a circle and a triangle, but may not be able to distinguish between a heptagon and an octagon. (Clements & Battista, 1992)

Level 1: Visual. Here students are able to differentiate shapes by name or appearance but they are not able to differentiate them based on their properties (Erdogan, Akkaya, & Celebi Akkaya, 2009; Halat, 2008). An Example of what a student might say is "That looks like a wheel" or "A rectangle looks like a door" (Discovering Geometry: The Van Hiele Levels, 2011).

Level 2: Analysis. Analysis is where students begin to experiment with shapes and prove features and rules about shapes through activities (ERDOGAN, AKKAYA, & CELEBI AKKAYA, 2009). For example, a student at this stage might say that a square has four equal sides and four right angles, but may not assert that a square is not a parallelogram (Discovering Geometry: The Van Hiele Levels, 2011). This is because students at this stage are unable to relate properties of shapes that come from two different classes (Halat, 2008).

Level 3: Inferences Related to Experience/Abstraction. At this level, students are able to group shapes by category and order shapes. Though still informal, they are able to discuss properties of shapes and group them according to their properties. For example, students at this stage may be able to see that a square is a rectangle and a rectangle is a parallelogram (Halat, 2008). They are also able to understand implication, if then, statements (ERDOGAN, AKKAYA, & CELEBI AKKAYA, 2009).

Level 4: Inference Resolutions. Here students are able to prove (reason rigorously) theorems deductively and are able to understand that it is possible to come to the same conclusions, even if the ways of getting there are different (ERDOGAN, AKKAYA, & CELEBI AKKAYA, 2009). In other words, students at this stage are able to deduce results from definitions, axioms, and other conclusions, but are still unable to see that these results are arbitrary. It is also at this stage that students can see the need for proofs, as opposed to just thinking that the explanations are ‘obvious’ (De Villiers, 2004).

Level 5: Advanced Period. In the final stage, students are able see that it is possible to draw different conclusions when using different axiomatic systems. (ERDOGAN, AKKAYA, & CELEBI AKKAYA, 2009). Students at this level are able to deduce, prove, understand, and compare the similarities and differences in the results from different systems, this is the highest level of mathematical understanding (Halat, 2008; Discovering Geometry: The Van Hiele Levels, 2011).

Advancing Through the Van Hiele Stages. Along with the five stages of the Van Hiele model, the transition from one stage to the next requires one to go through five phases: information, bound orientation, explicitation, free orientation, and integration (Halat, 2006). The outline of the phases is: first the teacher acquires an approximation of the information that the students already know, then the teacher takes the students through sample practice activities where they are exposed to new material, then the teacher formally introduces the material, then the teacher allows the students explore and practice what was just taught to them, finally the students are asked to summarize what they learned and why it is important. (ERDOGAN, AKKAYA, & CELEBI AKKAYA, 2009). It is also important to note that, according to the Van Hiele model, students who are taught at levels above their own level either hinders or prevents growth in geometric understanding (Unal, Jakubowski, & Corey, 2009). This is the central reason for failure in geometry at the high school level (De Villiers, 2004).

The Van Hiele Model in the Classroom

A strong understanding of geometry is an important aspect of understanding advanced mathematics and other sciences. Geometry is not isolated to a geometry classroom as it occurs frequently in algebra, trigonometry, calculus, physics, chemistry, and engineering as well as more advanced mathematical courses such as number theory, analysis, and topology.

According to Halat (2008), the majority of high school students are in the first or second levels, where according to NCTM (2000) students should be at the second level by 8th grade, and the third or fourth level by 12th grade. This means that students are not performing at appropriate standards according to the NCTM (Halat, 2008). The cause of this discrepancy comes on the part of the teacher 1. Not being at the proper level, or 2. The teacher not using the correct teaching methods(Halat, 2008; Unal, Jakubowski, & Corey, 2009; Villiers, 2004).

Students and the Van Hiele Model. The Van Hiele model focuses on developing high level skills, namely creative thinking. Students who are strong creative thinkers are able to search for and produce original products (ERDOGAN, AKKAYA, & CELEBI AKKAYA, 2009). According to a study done by Erdogan, Akkaya, and Celebi Akkaya on 55 sixth grade students in Bolu, Turkey, it has been found that using traditional teaching methods on students is ineffective in developing creative thinking skills as opposed to teaching according to the Van Hiele Model. Students taught according to the Van Hiele model felt the need to research, try and explore. In other words, teaching in correspondence to the Van Hiele levels enables students to learn with one another and learn while doing, which advances their creative thinking levels (ERDOGAN, AKKAYA, & CELEBI AKKAYA, 2009). An example of a way to allow students to be creative when it comes to geometry is given by De Villiers (2004), in which he suggests letting students use *geometers sketchpad*, a computer program that allows children to create and distort geometric shapes and allows them to visualize which properties remain the same and which change. This is the creative process that lets them come up with theorems and properties of certain shapes given certain properties (De Villiers, 2004).

Teachers and the Van Hiele Model. As stated earlier, students who are taught at levels above their Van Hiele Level are not able to properly develop their geometric understanding, a similar occurrence happens when teachers have not yet achieved high enough Van Hiele levels. This creates a problem because, according to Stipek (as cited in Halat, 2008), the amount of knowledge of the teacher has a significant impact on the overall success of students (Halat, 2008). Also, according to Crowley (1987) and Fuys (1988) (as cited in Halat, 2008), the knowledge of the teacher is the single most important factor in determining whether or not a student will progress from one Van Hiele level to the next (Halat, 2008).

A study done by Halat (2008), found that most middle school mathematics teachers are on level three and the majority of high school teachers reason at level four. This is important for teachers to take in to consideration when teaching middle to high school aged children, and take note that their levels of geometric understanding may be different than that of their students. But their levels must also be high enough to help their students meet the expectations set by other teachers and NCTM (Halat, 2008). It has also been shown by a study done by Halat (2006) that when teachers teach according to the Van Hiele model, it is more likely to reduce the gender gap between male and female students (Halat, 2006).

Van Hiele Levels and Understanding. A study done by Unal, Jakubowski, and Corey (2009), on 28 pre-service mathematics teachers, showed that when subjected to a geometry class based in the middle Van Hiele levels, those with the highest levels of understanding develop the most, and those with the least understanding hardly develop (Unal, Jakubowski, & Corey, 2009). This is reason for there to be a stronger focus on developing geometric understanding at younger ages, and to continue to encourage growth throughout middle school and high school, so that students are able to achieve the proper Van Hiele level by the 12th grade.

Conclusion

The importance of geometric understanding should not be underestimated as it is an important part in not only geometry classrooms but is also important in many other math classes and applied sciences. Therefore an understanding of the levels and progressions of the Van Hiele levels is important on behalf of both the teacher and students.

References

Clements, D. H., & Battista, M. T. (1992). Geometry and Spatial Reasoing. *Handbook of Research on Mathematics Teaching and Learning* .

De Villiers, M. (2004). Using dynamic geometry to expand mathematics teachers' understanding of proof. *International Journal of Mathematical Education in Science and Technology* , pp. 703-724.

Discovering Geometry: The Van Hiele Levels. (2011). Retrieved November 1, 2011, from Key Curriculum Press: http://www.keypress.com/x6772.xml

ERDOGAN, T., AKKAYA, R., & CELEBI AKKAYA, S. (2009). The Effect of the Van Hiele Model Based Instruction on the Creative Thinking Levels of 6th Grade Primary School Students. *Educational Sciences: Theory & Practice* , 181-194.

Halat, E. (2008). In-Service Middle and High School Mathematics Teachers: Geometric Reasoning Stages and Gender. *The Mathematics Educator* , pp. 8-14.

Halat, E. (2006). Sex-Related Differences In The Acquisition Of The Van Hiele Levels And Motivation In Learning Geometry. *Asia Pacific Education Review* , pp. 173-183.

Mason, M. (n.d.). *The van Hiele Levels of Geometric Understanding*. Retrieved November 1, 2011, from The University of Georgia: http://jwilson.coe.uga.edu/EMAT8990/GEOMETRY/Mason,%20Marguerite.%20The%20van%20Hiele%20Levels%20of%20Geometric%20Understanding.%202002.pdf

Slavin, R. E. (2012). *Educational Psychology.* Boston: Pearson Education, Inc.

Unal, H., Jakubowski, E., & Corey, D. (2009, December 15). Differences in learning geometry among high and low spatial ability pre-service mathematics teachers *International Journal Of Mathematical Education in Science and Technology* , pp. 997-1012.

Van Hiele, P. M. (1986). *Structure and Insight: A theory of mathematics education.* Orland, Flordia: Academic Press.