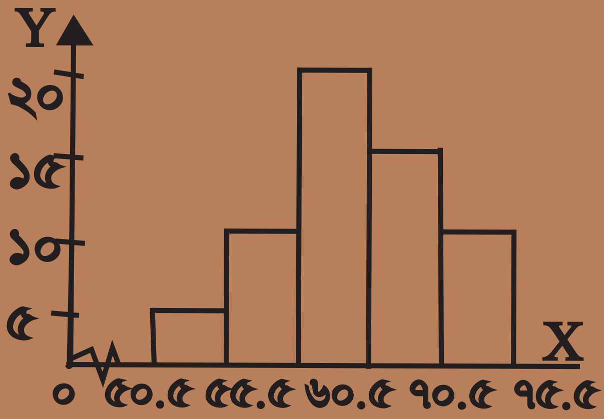
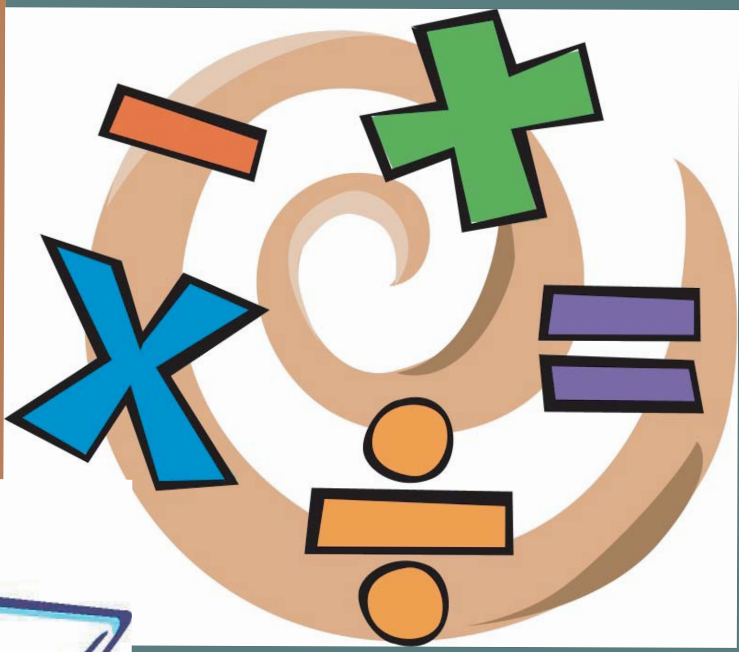
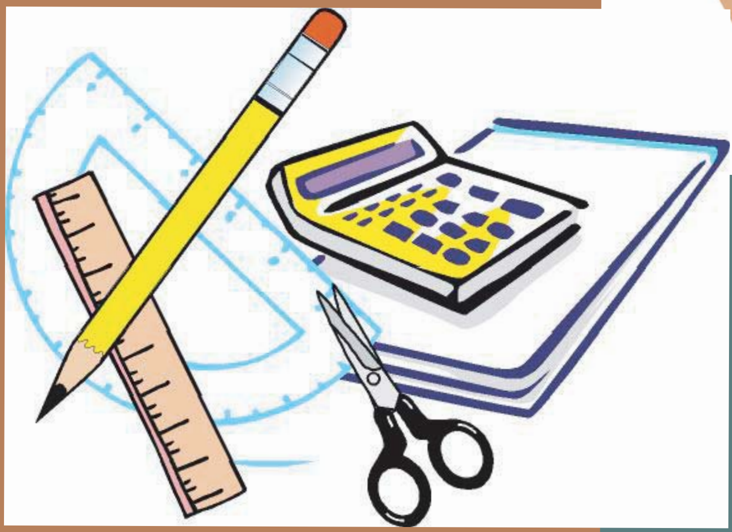
গণিত

নবম-দশম শ্রেণি







জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড, ঢাকা

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড কর্তৃক ২০১৩ শিক্ষাবর্ষ থেকে নবম-দশম শ্রেণির পাঠ্যপুস্তকরূপে নির্ধারিত

গণিত

নবম–দশম শ্রেণি

রচনায়

সালেহ মতিন

ড. অমল হালদার

ড. অমূল্য চন্দ্র মণ্ডল

শেখ কুতুবউদ্দিন

হামিদা বানু বেগম

এ. কে. এম শহীদুল্লাহ্

মোঃ শাহজাহান সিরাজ

সম্পাদনায়

ড. মোঃ আবদুল মতিন

ড. মোঃ আব্দুস ছামাদ

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড, ঢাকা

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড

৬৯-৭০, মতিঝিল বাণিজ্যিক এলাকা, ঢাকা
কর্তৃক প্রকাশিত

পরীক্ষামূলক সংস্করণ

প্রথম প্রকাশ : অক্টোবর- ২০১২

পাঠ্যপুস্তক প্রণয়নে সমন্বয়ক

মোঃ নাসির উদ্দিন

কম্পিউটার কম্পোজ

লেজার স্ক্যান লিমিটেড

প্রচ্ছদ

সুদর্শন বাছার
সুজাউল আবেদীন

চিত্রাঙ্কন

তোহ্ফা এন্টারপ্রাইজ

ডিজাইন

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড

সরকার কর্তৃক বিনামূল্যে বিতরণের জন্য

মুদ্রণ :

প্রসঙ্গ–কথা

শিক্ষা জাতীয় জীবনের সর্বতোমুখী উন্নয়নের পূর্বশর্ত। আর দ্রুত পরিবর্তনশীল বিশ্বের চ্যালেঞ্জ মোকাবেলা করে বাংলাদেশকে উন্নয়ন ও সমৃদ্ধির দিকে নিয়ে যাওয়ার জন্য প্রয়োজন সুশিক্ষিত জনশক্তি। ভাষা আন্দোলন ও মুক্তিযুদ্ধের চেতনায় দেশ গড়ার জন্য শিক্ষার্থীর অন্তর্নিহিত মেধা ও সম্ভাবনার পরিপূর্ণ বিকাশে সাহায্য করা মাধ্যমিক শিক্ষার অন্যতম লক্ষ্য। এছাড়া প্রাথমিক স্তরে অর্জিত শিক্ষার মৌলিক জ্ঞান ও দক্ষতা সম্প্রসারিত ও সুসংহত করার মাধ্যমে উচ্চতর শিক্ষার যোগ্য করে তোলাও এ স্তরের শিক্ষার উদ্দেশ্য। জ্ঞানার্জনের এই প্রক্রিয়ার ভিতর দিয়ে শিক্ষার্থীকে দেশের অর্থনৈতিক, সামাজিক, সাংস্কৃতিক ও পরিবেশগত পটভূমির প্রেক্ষিতে দক্ষ ও যোগ্য নাগরিক করে তোলাও মাধ্যমিক শিক্ষার অন্যতম বিবেচ্য বিষয়।

জাতীয় শিক্ষানীতি–২০১০ এর লক্ষ্য ও উদ্দেশ্যকে সামনে রেখে পরিমার্জিত হয়েছে মাধ্যমিক স্তরের শিক্ষাক্রম। পরিমার্জিত এই শিক্ষাক্রমে জাতীয় আদর্শ, লক্ষ্য, উদ্দেশ্য ও সমকালীন চাহিদার প্রতিফলন ঘটানো হয়েছে, সেই সাথে শিক্ষার্থীদের বয়স, মেধা ও গ্রহণ ক্ষমতা অনুযায়ী শিখনফল নির্ধারণ করা হয়েছে। এছাড়া শিক্ষার্থীর নৈতিক ও মানবিক মূল্যবোধ থেকে শুরু করে ইতিহাস ও ঐতিহ্য চেতনা, মহান মুক্তিযুদ্ধের চেতনা, শিল্প–সাহিত্য–সংস্কৃতিবোধ, দেশপ্রেমবোধ, প্রকৃতি-চেতনা এবং ধর্ম-বর্ণ-গোত্র ও নারী-পুরুষ নির্বিশেষে সবার প্রতি সমমর্যাদাবোধ জাগ্রত করার চেষ্টা করা হয়েছে। একটি বিজ্ঞানমনস্ক জাতি গঠনের জন্য জীবনের প্রতিটি ক্ষেত্রে বিজ্ঞানের স্বতঃস্ফূর্ত প্রয়োগ ও ডিজিটাল বাংলাদেশের রূপকল্প–২০২১ এর লক্ষ্য বাস্তবায়নে শিক্ষার্থীদের সক্ষম করে তোলার চেষ্টা করা হয়েছে।

নতুন এই শিক্ষাক্রমের আলোকে প্রণীত হয়েছে মাধ্যমিক স্তরের প্রায় সকল পাঠ্যপুস্তক। উক্ত পাঠ্যপুস্তক প্রণয়নে শিক্ষার্থীদের সামর্থ্য, প্রবণতা ও পূর্ব অভিজ্ঞতাকে গুরুত্বের সঙ্গে বিবেচনা করা হয়েছে। পাঠ্যপুস্তকগুলোর বিষয় নির্বাচন ও উপস্থাপনের ক্ষেত্রে শিক্ষার্থীর সৃজনশীল প্রতিভার বিকাশ সাধনের দিকে বিশেষভাবে গুরুত্ব দেওয়া হয়েছে। প্রতিটি অধ্যায়ের শুরুতে শিখনফল যুক্ত করে শিক্ষার্থীর অর্জিতব্য জ্ঞানের ইঙ্গিত প্রদান করা হয়েছে এবং বিচিত্র কাজ ও নমুনা প্রশ্নাদি সংযোজন করে মূল্যায়নকে সৃজনশীল করা হয়েছে।

একবিংশ শতকের এই যুগে জ্ঞানবিজ্ঞানের বিকাশে গণিতের ভূমিকা অতীব গুরুত্বপূর্ণ। শুধু তাই নয়, ব্যক্তিগত জীবন থেকে শুরু করে পারিবারিক ও সামাজিক জীবনে গণিতের প্রয়োগ অনেক বেড়েছে। এই সব বিষয় বিবেচনায় রেখে মাধ্যমিক পর্যায়ে নতুন গাণিতিক বিষয় শিক্ষার্থী উপযোগী ও আনন্দদায়ক করে তোলার জন্য গণিতকে সহজ ও সুন্দরভাবে উপস্থাপন করা হয়েছে এবং বেশ কিছু নতুন গাণিতিক বিষয় অন্তর্ভুক্ত করা হয়েছে।

একবিংশ শতকের অঙ্গীকার ও প্রত্যয়কে সামনে রেখে পরিমার্জিত শিক্ষাক্রমের আলোকে পাঠ্যপুস্তকটি রচিত হয়েছে। কাজেই পাঠ্যপুস্তকটির আরও সমৃদ্ধিসাধনের জন্য যেকোনো গঠনমূলক ও যুক্তিসঙ্গত পরামর্শ গুরুত্বের সঙ্গে বিবেচিত হবে। পাঠ্যপুস্তক প্রণয়নের বিপুল কর্মযজ্ঞের মধ্যে অতি স্বল্প সময়ে পুস্তকটি রচিত হয়েছে। ফলে কিছু ভুলত্রুটি থেকে যেতে পারে। পরবর্তী সংস্করণগুলোতে পাঠ্যপুস্তকটিকে আরও সুন্দর, শোভন ও ত্রুটিমুক্ত করার চেষ্টা অব্যাহত থাকবে। বানানের ক্ষেত্রে অনুসৃত হয়েছে বাংলা একাডেমী কর্তৃক প্রণীত বানানরীতি।

পাঠ্যপুস্তকটি রচনা, সম্পাদনা, চিত্রাঙ্কন, নমুনা প্রশ্নাদি প্রণয়ন ও প্রকাশনার কাজে যারা আন্তরিকভাবে মেধা ও শ্রম দিয়েছেন তাঁদের ধন্যবাদজ্ঞাপন করছি। পাঠ্যপুস্তকটি শিক্ষার্থীদের আনন্দিত পাঠ ও প্রত্যাশিত দক্ষতা অর্জন নিশ্চিত করবে বলে আশা করি।

প্রফেসর মোঃ মোস্তফা কামালউদ্দিন
চেয়ারম্যান
জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড, ঢাকা

সূচিপত্র

c_g Aa¨vq
ev¯e msL¨v
(Real Number)

cwigvY‡K cZxK Z_v msL¨v AvKv‡i cKvk Kivi c×wZ †_‡KB MwY‡Zi DrcwË| msL¨vi BwZnvm gvbe mf¨Zvi BwZnv‡mi gZB cvPxb| wMK `vkwbK Gwi÷U‡ji g‡Z, cvPxb wgk‡ii c‡iwnZ m¤c`v‡qi MwYZ Abkxj‡bi gva¨‡g MwY‡Zi AvbôwbK Awf‡lK N‡U| ZvB msL¨wfwËK MwY‡Zi mwó hxïwL‡÷i R‡b¥i cvq `B nvRvi eQi c‡e| Gici bvbv RwZ I mf¨Zvi nvZ N‡i Aabv msL¨v I msL¨vixwZ GKwU mveRbxb i/c avi Y K‡i‡Q|

¯vfweK msL¨v MYbvi c‡qvR‡b cvPxb fviZe‡li MwYZwe`MY mec_g kb¨ I `kwfwËK ¯vbxqgvb c×wZi cPjb K‡ib, hv msL¨v eYbvq GKwU gvBjdjK wnmv‡e we‡ewPZ| fviZxq I Pxbv MwYZwe`MY kb¨, FYvZ¥K, ev¯e, cY I fMvs‡ki avi Yvi we¯wZ NUvb hv ga¨h‡M Avi exq MwYZwe`iv wfwË wn‡m‡e MnY K‡ib| `kwgK fMvs‡ki mvnv‡h¨ msL¨v cKv‡ki KwZZ ga¨cv‡P¨i gmwjg MwYZwe`‡`i e‡j g‡b Kiv nq| Avevi ZvivB GKv`k kZvãx‡Z mec_g exRMwYZxq wØNvZ mgxKi‡Yi mgvavb wn‡m‡e eMgj AvKv‡i Agj` msL¨vi ceZb K‡ib| BwZnvmwe`‡`i aviYv wL÷ce 50 A‡ãi KvQvKwQ wMK `vkwbKivI R¨vwgwZK A¼‡bi c‡qvR‡b Agj` msL¨v, we‡kl K‡i `B-Gi eMg‡ji c‡qvRbxqZv Abfe K‡iwQ‡jb| Ebwesk kZvãx‡Z BD‡ivcxq MwYZwe`iv ev¯e msL¨vi cYvjxe× K‡i cYZv `vb K‡ib| ^`bw›`b c‡qvR‡b ev¯e msL¨v m¤‡Ü wk¶v_x‡`i m¯úó Ávb _vKv c‡qvRb| G Aa¨v‡q ev¯e msL¨v wel‡q mvgwMK Av‡jvPbv Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv–

- ev¯e msL¨vi †kwYweb¨vm Ki‡Z cvi‡e|
- ev¯e msL¨v‡K `kwg‡K cKvk K‡i Avmb gvb wbYq Ki‡Z cvi‡e|
- `kwgK fMvs‡ki †kwYweb¨vm e¨vL¨v Ki‡Z cvi‡e|
- AveË `kwgK fMvsk e¨vL¨v Ki‡Z cvi‡e Ges fMvsk‡K AveË `kwg‡K cKvk Ki‡Z cvi‡e|
- AveË `kwgK fMvsk‡K mvaviY fMvs‡k i/cvši Ki‡Z cvi‡e|
- Amxg AbveË `kwgK fMvsk e¨vL¨v Ki‡Z cvi‡e|
- m`k I wem`k `kwgK fMvsk e¨vL¨v Ki‡Z cvi‡e|
- AveË `kwgK fMvs‡ki †hvM, we‡qvM, ¸Y I fvM Ki‡Z cvi‡e Ges GZ`msµvš wewfb mgm¨vi mgvavb Ki‡Z cvi‡e|

¯vfweK msL¨v (**Natural Number**)

$1, 2, 3, 4.........$ BZ¨vw` msL¨v¸‡jv‡K ¯vfweK msL¨v ev abvZ¥K ALÊ msL¨v e‡j| $2, 3, 5, 7.........$ BZ¨vw` †gŠwjK msL¨v Ges $4, 6, 8, 9,.........$ BZ¨vw` †hŠwMK msL¨v|

cYmsL¨v (**Integers**)

kb¨mn mKj abvZ¥K I FYvZ¥K ALÛ msL¨vmgn‡K cYmsL¨v ejv nq| A_vr $....... -3, -2, -1, 0, 1, 2, 3.........$ BZ¨vw` cYmsL¨v|

fMvsk msL¨v (**Fractional Number**)

p, q ci¯úi mn‡gŠwjK, $q \neq 0$ Ges $q \neq 1$ n‡j, $\frac{p}{q}$ AvKv‡ii msL¨v‡K fMvsk msL¨v e‡j| †hgb : $\frac{1}{2}, \frac{3}{2}, \frac{-5}{3}$ BZ¨vw` fMvsk msL¨v|

$p < q$ n‡j fMvsk‡K cKZ fMvsk Ges $p > q$ n‡j fMvsk‡K AcKZ fMvsk ejv nq| †hgb : $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, ...$ BZ¨vw` cKZ fMvsk Ges $\frac{3}{2}, \frac{4}{3}, \frac{5}{3}, \frac{5}{4}, ....$ BZ¨vw` AcKZ fMvsk|

gj` msL¨v (**Rational Number**)

p I q cYmsL¨v Ges $q \neq 0$ n‡j, $\frac{p}{q}$ AvKv‡ii msL¨v‡K gj` msL¨v ejv nq| †hgb : $\frac{3}{1} = 3, \frac{11}{2} = 5.5, \frac{5}{3} = 1.666...$ BZ¨vw` gj` msL¨v| gj` msL¨v‡K `ywU cYmsL¨vi AbcvZ wnmv‡e cKvk Kiv hvq| mZivs mKj cYmsL¨v Ges mKj fMvsk msL¨v n‡e gj` msL¨v|

Agj` msL¨v (**Irrational Number**)

†h msL¨v‡K $\frac{p}{q}$ AvKv‡i cKvk Kiv hvq bv, †hLv‡b p, q cYmsL¨v Ges $q \neq 0$, †m msL¨v‡K Agj` msL¨v ejv nq| cYeM bq Gi/c †h‡Kv‡bv ¯vfweK msL¨vi eMgj GKwU Agj` msL¨v| †hgb : $\sqrt{2} = 1.414213.....,\ \sqrt{3} = 1.732.....,\ \frac{\sqrt{5}}{2} = 1.58113....$ BZ¨vw` Agj` msL¨v| Agj` msL¨v‡K `ywU cYmsL¨vi AbcvZ wnmv‡e cKvk Kiv hvq bv|

`kwgK fMvsk msL¨v :

gj` msL¨v I Agj` msL¨v‡K `kwg‡K cKvk Kiv n‡j G‡K `kwgK fMvsk ejv nq| †hgb, $3 = 3 \cdot 0, \frac{5}{2} = 2 \cdot 5, \frac{10}{3} = 3 \cdot 3333......., \sqrt{3} = 1 \cdot 732.........$ BZ¨vw` `kwgK fMvsk msL¨v| `kwgK we›`i ci A¼ msL¨v mgxg n‡j, G‡`i‡K mmxg `kwgK fMvsk Ges A¼ msL¨v Amxg n‡j, G‡`i‡K Amxg `kwgK fMwsk ejv nq| †hgb, $0\cdot 52,\ 3\cdot 4152$ BZ¨w` mmxg `kwgK fMwsk Ges $1\cdot 333.......,\ \ 2\cdot 123512367...........$ BZ¨w` Amxg `kwgK fMwsk msL¨v| Avevi, Amxg `kwgK fMwsk msL¨v¸‡jvi g‡a¨ `kwgK we›`i ci A¼¸‡jv cbivewË n‡j, G‡`i‡K Amxg AveË `kwgK fMwsk Ges A¼¸‡jv cbivewË bv n‡j G‡`i Amxg AbveË `kwgK fMwsk msL¨v ejv nq| †hgb, $1\cdot 2323........,$ $5\cdot\dot{6}5\dot{4}$ BZ¨w` Amxg AveË `kwgK fMwsk Ges $0\cdot 523050056........,\ 2\cdot 12340314........$ BZ¨w` AbveË `kwgK fMwsk|

ev¯e msL¨v (**Real Number**)

mKj gj` msL¨v Ges Agj` msL¨v‡K ev¯e msL¨v ejv nq| †hgb :

$0, \pm 1, \pm 2, \pm 3, ...$

$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{4}{3}, ...$

$\sqrt{2},\ \sqrt{3},\ \sqrt{5},\ \sqrt{6}......$

$1\cdot 23,\ 0\cdot 415,\ 1\cdot 3333.......,\ 0\cdot\dot{6}\dot{2},\ 4\cdot 120345061..........$ BZ¨w` ev¯e msL¨v|

abvZ¥K msL¨v (**Positive Number**)

kb¨ A‡c¶v eo mKj ev¯e msL¨v‡K abvZ¥K msL¨v ejv nq|

†hgb, $1,\ 2,\ \frac{1}{2},\ \frac{3}{2},\ \sqrt{2},\ 0\cdot 415,\ 0\cdot\dot{6}\dot{2},\ 4\cdot 120345061..............$ BZ¨w` abvZ¥K msL¨v|

FYvZ¥K msL¨v (**Negative Number**)

kb¨ A‡c¶v †QvU mKj ev¯e msL¨v‡K FYvZ¥K msL¨v ejv nq|

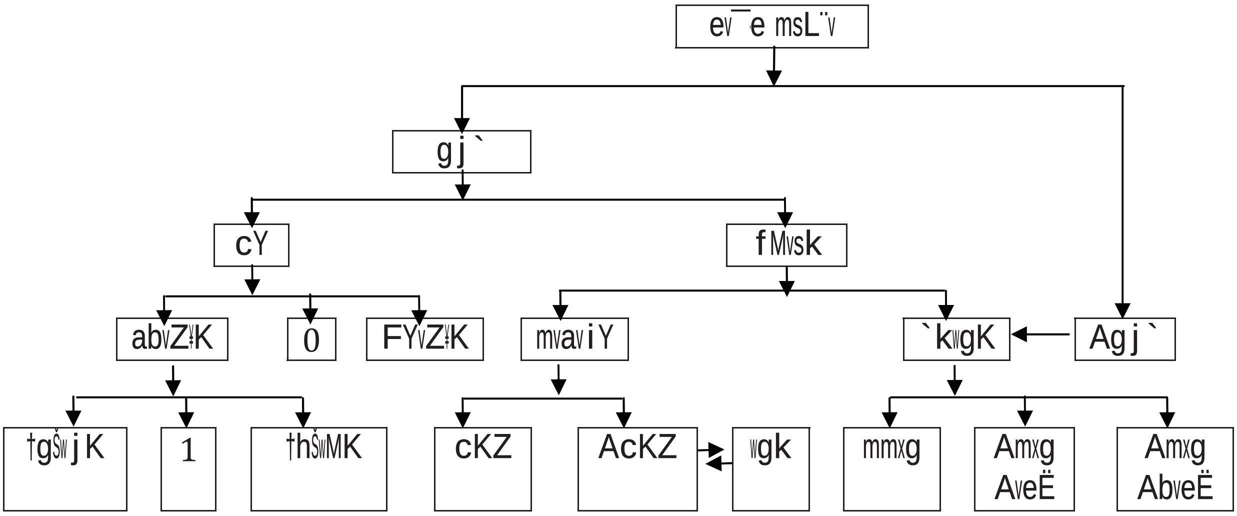
†hgb, $-1,\ -2,\ -\frac{1}{2},\ -\frac{3}{2},\ -\sqrt{2},\ -0\cdot 415,\ -0\cdot\dot{6}\dot{2},\ -4\cdot 120345061..............$ BZ¨w` FYvZ¥K msL¨v|

AFYvZ¥K msL¨v (**Non negative Number**)

kb¨mn mKj abvZ¥K msL¨v‡K AFYvZ¥K msL¨v ejv nq|

†hgb, $0,\ 3,\ \frac{1}{2},\ \ 0\cdot 612,\ 1\cdot\dot{3},\ 2\cdot 120345..............$ BZ¨w` AFYvZ¥K msL¨v|

ev¯Íe msL¨vi †kÖYxweb¨vm



KvR :

$\frac{3}{4}$, 5, 7, $\sqrt{13}$, 0, 1, $\frac{9}{7}$, 12, $2\frac{4}{5}$, 1 1234......, .3$\dot{2}$$\dot{3}$ msL¨v¸‡jv‡K ev¯Íe msL¨vi †kÖYxweb¨v‡m Ae¯'vb †`LvI|

**D`vniY 1|** $\sqrt{3}$ Ges 4 Gi g‡a¨ `yBwU Agj` msL¨v wbY©q Ki|

mgvavb : GLv‡b, $\sqrt{3}$ 1.7320508......

g‡b Kwi, a 2.030033000333.....

Ges b 2.505500555.......

¯úóZ : a I b DfqB `yBwU ev¯Íe msL¨v Ges DfqB $\sqrt{3}$ A‡cÿv eo Ges 4 A‡cÿv †QvU|

A_©vr $\sqrt{3}$ 2.03003300333......... 4

Ges $\sqrt{3}$ 2.505500555................ 4

Avevi, a I b †K fMvsk AvKv‡i cKvk Kiv hvq bv|

a I b `yBwU wb‡Yq Agj` msL¨v|

ev¯Íe msL¨vi Dci †hvM I ¸Yb cÖwµqvi †gŠwjK ˆewkó¨ :

1. a, b ev¯Íe msL¨v n‡j, i a b ev¯Íe msL¨v Ges ii ab ev¯Íe msL¨v
2. a, b ev¯Íe msL¨v n‡j, i a b b a Ges ii ab ba
3. a, b, c ev¯Íe msL¨v n‡j, i a b c a b c Ges ii ab c a bc
4. a ev¯Íe msL¨v n‡j, ev¯Íe msL¨vq †Kej `yBwU msL¨v 0 I 1 we`¨gvb †hLv‡b i 0 1 ii a 0 a iii $a.1$ $1.a$ a

5. a ev¯e msL¨v n‡j, $(i)\ a+(-a)=0$ $(ii)\ a\neq 0$ n‡j, $a.\dfrac{1}{a}=1$
6. $a,\ b,\ c$ ev¯e msL¨v n‡j, $a(b+c)=ab+ac$
7. $a,\ b$ ev¯e msL¨v n‡j, $a<b$ A_ev $a=b$ A_ev $a>b$
8. $a,\ b,\ c$ ev¯e msL¨v Ges $a<b$ n‡j, $a+c<b+c$
9. $a,\ b,\ c$ ev¯e msL¨v Ges $a<b$ n‡j, (i) $ac<bc$ hLb $c>0$ $(ii)\ ac>bc$ n‡j, $c<0$

cÖgvY Ki : $\sqrt{2}$ GKwU Agj` msL¨v|
Avgiv Rwb,

$1<2<4$
$\therefore\ \sqrt{1}<\sqrt{2}<\sqrt{4}$
ev, $1<\sqrt{2}<2$

cgvY : $1^2=1,\ \left(\sqrt{2}\right)^2=2,\ 2^2=4$

mZivs $\sqrt{2}$ Gi gvb 1 A‡c¶v eo Ges 2 A‡c¶v †QvU|

AZGe $\sqrt{2}$ cYmsL¨v bq|

$\therefore\ \sqrt{2}$ gj` msL¨v A_ev Agj` msL¨v| hw` $\sqrt{2}$ gj` msL¨v nq Z‡e

awi, $\sqrt{2}=\dfrac{p}{q}$; †hLv‡b p I q ¯vfvweK msL¨v I ci¯úi mn‡gŠwjK Ges $q>1$

ev, $2=\dfrac{p^2}{q^2}$; eM K‡i

ev, $2q=\dfrac{p^2}{q}$; Dfq c¶‡K q Øviv ¸Y K‡i|

¯úóZ : $2q$ cY msL¨v wKš‘ $\dfrac{p^2}{q}$, cYmsL¨v bq, KviY p I q ¯vfvweK msL¨v I Giv ci¯úi mn‡gŠwjK Ges $q>1$

$\therefore\ 2q$ Ges $\dfrac{p^2}{q}$ mgvb n‡Z cv‡i bv, A_vr $2q\neq\dfrac{p^2}{q}$

$\therefore\ \sqrt{2}$ Gi gvb $\dfrac{p}{q}$ AvKv‡ii †Kv‡bv msL¨v n‡Z cv‡i bv, A_vr $\sqrt{2}\neq\dfrac{p}{q}$

$\therefore\ \sqrt{2}$ GKwU Agj` msL¨v|

D`vniY 2| cgvY Ki †h, †Kv‡bv PviwU µwgK ¯vfvweK msL¨vi ¸Yd‡ji mv‡_ 1 †hvM Ki‡j †hvMdj GKwU cYeM msL¨v n‡e|

mgvavb : g‡b Kwi, PviwU µwgK ¯vfvweK msL¨v h_vµ‡g $x,\ x+1,\ x+2,\ x+3$

µwgK msL¨v PviwUi ¸Yd‡ji mv‡_ 1 †hvM Ki‡j cvIqv hvq,

$x\,(x+1)(x+2)(x+3)+1 = x(x+3)\,(x+1)\,(x+2)+1$
$=(x^2+3x)\left(x^2+3x+2\right)+1$

$=a(a+2)+1;\ \left[x^2+3x=a\right]$

$=a(a+2)+1;$

$=a^2+2a+1 \quad =(a+1)^2 \quad =\left(x^2+3x+1\right)^2$; hv GKwU cYeM msL¨v|

∴ †h‡Kv‡bv Pvi wU µwgK ¯vfvweK msL¨vi ¸Yd‡ji mv‡_ 1 †hvM Ki‡j †hvMdj GKwU cYeM msL¨v n‡e|

KvR : cgvY Ki †h, $\sqrt{3}$ GKwU Agj` msL¨v|

`kwgK fMvs‡ki †kwYweb¨vm

c‡Z¨K ev¯e msL¨v‡K `kwgK fMvs‡k cKvk Kiv hvq| †hgb : $2=2\cdot 0$, $\frac{2}{5}=0.4$, $\frac{1}{3}=0.333....$ BZ¨vw`| `kwgK fMvsk wZb cKvi: mmxg `kwgK, AveË `kwgK Ges Amxg `kwgK fMvsk|

mmxg `kwgK fMvsk : mmxg `kwg‡K `kwgK wP‡ýi Wvbw`‡K mmxg msL¨K A¼ _v‡K| †hgb : 0.12, 1.023, 7.832, 54.67, ...BZ¨vw` mmxg `kwgK fMvsk|

AveË `kwgK fMvsk : AveË `kwg‡K `kwgK wP‡ýi Wvbw`‡Ki A¼¸‡jv ev Askwe‡kl evievi _v‡K| †hgb, 3.333....., 2.454545......, 5.12765765 BZ¨vw` AveË `kwgK fMvsk|

Amxg `kwgK fMvsk : Amxg `kwgK fMvs‡k `kwgK wP‡ýi Wvbw`‡Ki A¼ KL‡bv †kl nq bv, A_vr `kwgK wP‡ýi Wvbw`‡Ki A¼¸‡jv mmxg n‡e bv ev Askwe‡kl evievi Avm‡e bv| †hgb : 1.4142135......, 2.8284271....... BZ¨vw` Amxg `kwgK fMvsk|

mmxg `kwgK I AveË `kwgK fMvsk gj` msL¨v Ges Amxg `kwgK fMvsk Agj` msL¨v| †Kv‡bv Agj` msL¨vi gvb hZ `kwgK ¯vb chš— B"Qv wbYq Kiv hvq| †Kv‡bv fMvs‡ki je I ni‡K ¯vfvweK msL¨vq cKvk Ki‡Z cvi‡j, H fMvskwU gj` msL¨v|

KvR :
1.723, 5.2333........, 0.0025, 2.1356124......., 0.0105105........ Ges
0.450123....... fMvsk¸‡jv‡K KviYmn †kwYweb¨vm Ki|

AveË `kwgK fMvsk

$\frac{23}{6}$ fMvskwU‡K `kwg‡K cKvk Kwi|

$\frac{23}{6}$ = 6) 23 (3.833
18
50
48
20
18
20
18

j¶ Kwi, fMvs‡ki je‡K ni w`‡q fvM K‡i `kwgK fMvs‡k cwiYZ Kivi mgq fv‡Mi cwµqv †kl nq bvB| †`Lv hvq †h, fvMd‡j GKB msL¨v 3 evievi Av‡m| GLv‡b, 3.8333..... GKwU AveË `kwgK fMvsk|

†h mKj `kwgK fMvs‡k `kwgK we›`i Wv‡b GKwU A¼ µgvš‡q evievi ev GKwaK A¼ chvqµ‡g evievi Av‡m, G‡`i AveË `kwgK fMvsk ejv nq| AveË ev †cŠbtcwbK `kwgK fMvs‡k †h Ask evievi A_vr cbtcb nq, G‡K AveË Ask e‡j|

AveË `kwgK fMvs‡k GKwU A¼ AveË n‡j, †m A‡¼i Dci †cŠbtcwbK we›` Ges GKwaK A¼ AveË n‡j, †KejgvÎ c_g I †kl A‡¼i Dci †cŠbtcwbK we›` †`Iqv nq| †hgb 2.555....... †K †jLv nq $2.\dot{5}$ Øviv Ges 3.124124124........ †K †jLv nq, $3.\dot{1}2\dot{4}$ Øviv|

`kwgK fMvs‡k `kwgK we›`i ci AveËvsk Qvov Ab¨ †Kv‡bv A¼ bv _vK‡j, G‡K wei× †cŠbtcwbK e‡j Ges †cŠbtcwbK `kwgK fMvs‡k `kwgK we›`i ci AveËvsk Qvov GK ev GKwaK A¼ _vK‡j, G‡K wgk †cŠbtcwbK e‡j| †hgb, $1.\dot{3}$ wei× †cŠbtcwbK fMvsk Ges $4.235\dot{1}\dot{2}$ wgk †cŠbtcwbK fMvsk|

fMvs‡ki n‡i 2, 5 Qvov Ab¨ †Kv‡bv †gŠwjK ¸bbxqK (Drcv`K) _vK‡j, †mB ni Øviv je‡K fvM Ki‡j, KL‡bv wbt‡k‡l wefvR¨ n‡e bv| †h‡nZz chvqµ‡g fv‡M †k‡li A¼¸‡jv 1, 2, ..., 9 Qvov Ab¨ wKQz n‡Z cv‡i bv, †m‡nZz GK chv‡q fvM‡kl¸‡jv evievi GKB msL¨v n‡Z _vK‡e| AveËvs‡ki msL¨v memgq n‡i †h msL¨v _v‡K, Gi †P‡q †QvU nq|

D`vniY 3| $\frac{3}{11}$ †K `kwgK fMvs‡k cKvk Ki|

mgvavb :

11) 30 (0.2727
22
80
77
30
22
80
77
3

wb‡Yq `kwgK fMvsk = 0.2727 ....= $0.\dot{2}\dot{7}$

D`vniY 4| $\frac{95}{37}$ †K `kwgK fMvs‡k cKvk Ki|

mgvavb :

37) 95 (2.56756
74
210
185
250
222
280
259
210
185
250
222
28

wb‡Yq `kwgK fMvsk = 2.56756...... = $2.\dot{5}6\dot{7}$

AveË `kwgK‡K mvgvb¨ fMvs‡k cwieZb

AveË `kwg‡Ki gvb wbYq :

D`vniY 5| $0.\dot{3}$ †K mvgvb¨ fMvs‡k cKvk Ki|

mgavb : $0.\dot{3} = 0.3333........$

$$0.\dot{3}\times 10 = 0.333.......\times 10 = 3.333......$$

Ges $0.\dot{3}\times 1 \;\; = 0.333.......\times 1 \;\; = 0.333......$

we‡qvM K‡i, $0.\dot{3}\times 10 - 0.\dot{3}\times 1 \qquad = 3$

ev, $0.\dot{3}\times(10-1) = 3$ ev, $0.\dot{3}\times 9 = 3$

AZGe, $0.\dot{3} = \frac{3}{9} = \frac{1}{3}$

wb‡Yq fMvsk $\frac{1}{3}$

D`vniY 6| $0.\dot{2}\dot{4}$ †K mvgvb¨ fMvs‡k cKvk Ki|

mgvavb : $0.\dot{2}\dot{4} = 0.24242424........$

mZivs $0.\dot{2}\dot{4}\times 100 = 0.242424.......\times 100 = 24.2424......$

Ges $0.\dot{2}\dot{4}\times 1 \qquad = 0.242424.......\times 1 \qquad = 0.242424......$

we‡qvM K‡i, $0.\dot{2}\dot{4}(100-1) = 24$

ev, $0.\dot{2}\dot{4}\times 99 \qquad = 24$ ev, $0.\dot{2}\dot{4} = \frac{24}{99} = \frac{8}{33}$

wb‡Yq fMvsk $\frac{8}{33}$

D`vniY 7| $5.1\dot{3}4\dot{5}$ †K mvgvb¨ fMvs‡k cKvk Ki|

mgvavb : $5.1\dot{3}4\dot{5} = 5.1345345345.........$

mZivs $5.1\dot{3}4\dot{5}\times 10000 = 5.1345345......\times 10000 \;\; = 51345.345......$

Ges $5.1\dot{3}4\dot{5}\times 10 \qquad = 5.1345345.......\times 10 \quad = 51.345......$

we‡qvM K‡i, $5.1\dot{3}4\dot{5}\times 9990 \qquad\qquad = 51345 - 51$

AZGe, $5.1\dot{3}4\dot{5} = \frac{51345-51}{9990} = \frac{51294}{9990} = \frac{8549}{1665} = 5\frac{224}{1665}$

wb‡Yq fMvsk $5\frac{224}{1665}$

D`vniY 8| $42.34\dot{7}\dot{8}$ †K mvgvb¨ fMvs‡k cKvk Ki|

mgvavb : $42.34\dot{7}\dot{8} = 42.347878........$

mZivs, $42.34\dot{7}\dot{8} \times 10000 = 42.347878.........\times 10000 = 42348.7878$

Ges $42.34\dot{7}\dot{8} \times 100 = 42.347878........ \times 100 = 4234.7878$

we‡qvM K‡i, $42.34\dot{7}\dot{8} \times 9900 = 423478 - 4234$

AZGe, $42.34\dot{7}\dot{8} = \frac{423478-4234}{9900} = \frac{419244}{9900} = \frac{34937}{825} = 42\frac{287}{825}$

wb‡Yq fMvsk $42\frac{287}{825}$

e¨vL¨v : D`vniY 5, 6, 7 Ges 8 †_‡K †`Lv hvq †h,

- AveË `kwg‡K `kwgK we›`i ci †h KqwU A¼ Av‡Q, †m KqwU kb¨ 1 Gi Wv‡b emw‡q c_‡g AveË `kwgK‡K ¸Y Kiv n‡q‡Q|
- AveË `kwg‡K `kwgK we›`i ci †h KqwU AbveË A¼ Av‡Q, †m KqwU kb¨ 1 Gi Wv‡b emw‡q AveË `kwgK‡K ¸Y Kiv n‡q‡Q|
- c_g ¸Ydj †_‡K wØZxq ¸Ydj we‡qvM Kiv n‡q‡Q| c_g ¸Ydj †_‡K wØZxq ¸Ydj we‡qvM Kivq Wvbc‡¶ cY msL¨v cvIqv †M‡Q| GLv‡b j¶Yxq †h, AveË `kwgK fMvs‡ki `kwgK I †cŠbtcwbK we›` DwV‡q cvß msL¨v †_‡K AbveË As‡ki msL¨v we‡qvM Kiv n‡q‡Q|
- evgc‡¶ AveË `kwg‡K hZ¸‡jv AveË A¼ wQj ZZ¸‡jv 9 wj‡L Ges Zv‡`i Wv‡b `kwgK we›`i ci hZ¸‡jv AbveË A¼ wQj ZZ¸‡jv kb¨ emw‡q Dc‡i cvß we‡qvMdj‡K fvM Kiv n‡q‡Q|
- AveË `kwg‡K fMvs‡k cwiYZ Kivq fMvskwUi ni n‡jv hZ¸‡jv AveË A¼ ZZ¸‡jv 9 Ges 9 ¸‡jvi Wv‡b `kwgK we›`i ci hZ¸‡jv AbveË A¼ ZZ¸‡jv kb¨| Avi je n‡jv AveË `kwg‡Ki `kwgK we›` I †cŠbtcwbK we›` DwV‡q †h msL¨v cvIqv †M‡Q, †m msL¨v †_‡K AveËvsk ev` w`‡q evwK A¼ Øviv MwVZ msL¨v we‡qvM K‡i we‡qvMdj|

gše¨ : AveË `kwgK‡K me mgq fMvs‡k cwiYZ Kiv hvq| mKj AveË `kwgK gj` msL¨v|

D`vniY : 9| $5.23\dot{4}5\dot{7}$

mgvavb : $5.23\dot{4}5\dot{7} = 5.23457457457.........$

mZivs $5.23\dot{4}5\dot{7} \times 100000 = 523457.457457$

Ges $5.23\dot{4}5\dot{7} \times 100 = 523.457457$

we‡qvM K‡i, $5.23\dot{4}5\dot{7} \times 99900 = 522934$

AZGe, $5.23\dot{4}5\dot{7} = \frac{522934}{99900} = \frac{261467}{49950}$

wb‡Yq fMvsk $\frac{261467}{49950}$

e¨vL¨v : `kwgK As‡k cvPwU A¼ i‡q‡Q e‡j GLv‡b AveË `kwgK‡K c_‡g 100000 (GK Gi Wv‡b cvPwU kb¨) Øviv ¸Y Kiv n‡q‡Q| AveË As‡ki ev‡g `kwgK As‡k `BwU A¼ i‡q‡Q e‡j AveË `kwgK‡K 100 (GK Gi Wv‡b `BwU kb¨) Øviv ¸Y Kiv n‡q‡Q| c_g ¸Ydj †_‡K wØZxq ¸Ydj we‡qvM Kiv n‡q‡Q| GB we‡qvMd‡ji GKw`‡K cYmsL¨v Ab¨w`‡K c`Ë AveË `kwg‡Ki gv‡bi $(100000-100) = 99900$ ¸Y| Dfq c¶‡K 99900w`‡q fvM K‡i wb‡Yq fMvsk cvIqv †Mj|

> KvR :
>
> $0.\dot{4}\dot{1}$ Ges $3.04\dot{6}2\dot{3}$ †K fMvs‡k iƒcvšÍi Ki|

AveË `kwgK‡K mvgvb¨ fMvs‡k iƒcvšÍ‡ii wbqg

wb‡Yq fMvs‡ki je = c`Ë `kwgK fMvs‡ki `kwgK we›`y ev`w`‡q cvß msL¨v Ges AbveË Ask Øviv MwVZ msL¨vi we‡qvMdj|

wb‡Yq fMvs‡ki ni = `kwgK we›`i c‡i AveË As‡k hZ¸‡jv A¼ Av‡Q ZZ¸‡jv bq (9) Ges AbveË As‡k hZ¸‡jv A¼ Av‡Q ZZ¸‡jv kb¨ (0) Øviv MwVZ msL¨v|

GLv‡b, G wbqg miivmwi c‡qvM K‡i K‡qKwU AveË `kwg‡K mvgvb¨ fMvs‡k cwiYZ Kiv n‡jv|

D`vniY 10| $45.2\dot{3}4\dot{6}$ †K mvgvb¨ fMvs‡k cKvk Ki|

mgvavb : $45.2\dot{3}4\dot{6} = \frac{452346-452}{9990} = \frac{451894}{9990} = \frac{225947}{4995} = 45\frac{1172}{4995}$

wb‡Yq fMvsk $45\frac{1172}{4995}$

D`vniY 11| $32.\dot{5}6\dot{7}$ †K mvgvb¨ fMvs‡k cKvk Ki|

mgvavb : $32.\dot{5}6\dot{7} = \frac{32567-32}{999} = \frac{32535}{999} = \frac{3615}{111} = \frac{1205}{37} = 32\frac{21}{37}$

wb‡Yq fMvsk $32\frac{21}{37}$.

KvR :

$0.0\dot{1}\dot{2}$ Ges $3.31\dot{2}\dot{4}$ †K fMvs‡k iƒcvši Ki|

m`k AveË `kwgK I wem`k AveË `kwgK

AveË `kwgK¸‡jv‡Z AbveË As‡ki msL¨v mgvb n‡j Ges AveË As‡ki A¼ msL¨vI mgvb n‡j, Zv‡`i m`k AveË `kwgK e‡j| GQvov Ab¨ AveË `kwgK¸‡jv‡K wem`k AveË `kwgK e‡j| †hgb: $12.\dot{4}\dot{5}$ I $6.\dot{3}\dot{2}$; $9.45\dot{3}$ I $125.89\dot{7}$ m`k AveË `kwgK| Avevi, $0.3\dot{4}5\dot{6}$ I $7.45\dot{7}8\dot{9}$; $6.43\dot{5}\dot{7}$ I $2.89\dot{3}4\dot{5}$ wem`k AveË `kwgK|

wem`k AveË `kwgK¸‡jv‡K m`k AveË `kwg‡K cwieZ©‡bi wbqg

†Kv‡bv AveË `kwg‡Ki AveË As‡ki A¼¸‡jv‡K evievi wjL‡j `kwg‡Ki gv‡bi †Kv‡bv cwieZ©b nq bv| †hgb, $6.45\dot{3}\dot{7} = 6.45\dot{3}73\dot{7} = 6.453\dot{7}\dot{3} = 6.4537\dot{3}\dot{7}$ | GLv‡b c‡Z¨KwU AveË `kwgK $6.45373737.........$ GKwU Amxg `kwgK| c‡Z¨KwU AveË `kwgK‡K mvgvb¨ fMvs‡k cwieZ©b Ki‡j †`Lv hv‡e c‡Z¨KwU mgvb|

$$6.45\dot{3}\dot{7} = \frac{64537-645}{9900} = \frac{63892}{9900}$$

$$6.45\dot{3}73\dot{7} = \frac{6453737-645}{999900} = \frac{6453092}{999900} = \frac{63892}{9900}$$

$$6.4537\dot{3}\dot{7} = \frac{6453737-64537}{990000} = \frac{6389200}{990000} = \frac{63892}{9900}$$

m`k AveË `kwg‡K cwiYZ Ki‡Z n‡j msL¨v¸‡jvi g‡a¨ †h msL¨wUi AbveË As‡ki A¼ msL¨v †ewk, c‡Z¨KwU AbveË Ask ZZ A‡¼i Ki‡Z n‡e Ges wewfbœ msL¨vq AveË As‡ki A¼ msL¨v¸‡jvi j.mv.¸ hZ, c‡Z¨KwU `kwg‡Ki AveË Ask ZZ A‡¼i Ki‡Z n‡e|

D`vniY 12| $5.\dot{6}$, $7.3\dot{4}\dot{5}$ I $10.78\dot{4}2\dot{3}$ †K m`k AveË `kwg‡K cwiYZ Ki|

mgvavb : $5.\dot{6}, 7.3\dot{4}\dot{5}$ I $10.78\dot{4}2\dot{3}$ AveË `kwg‡K AbveË As‡ki A¼ msL¨v h_vµ‡g $0, 1$ I $2$ | GLv‡b AbveË A¼ msL¨v $10.78\dot{4}2\dot{3}$ `kwg‡K me‡P‡q †ewk Ges G msL¨v 2 | ZvB m`k AveË `kwgK Ki‡Z n‡j c‡Z¨KwU `kwg‡Ki AbveË As‡ki A¼ msL¨v $2$ n‡e| $5.\dot{6}$, $7.3\dot{4}\dot{5}$ I $10.78\dot{4}2\dot{3}$ AveË `kwg‡K AveË As‡ki msL¨v h_vµ‡g $1, 2$ I 3 | $1, 2$ I 3 Gi j.mv.¸ n‡jv 6 | ZvB m`k AveË `kwgK Ki‡Z n‡j c‡Z¨KwU `kwg‡Ki AveË As‡ki A¼ msL¨v 6 n‡e|

mZivs $5.\dot{6} = 5.66\dot{6}66666\dot{6}$, $7.3\dot{4}\dot{5} = 7.34\dot{5}45454\dot{4}$ I $10.78\dot{4}2\dot{3} = 10.78\dot{4}23423\dot{3}$

wb‡Yq m`k AveË `kwgKmgn h_vµ‡g $5.66\dot{6}66666\dot{6}, 7.34\dot{5}45454\dot{4}, 10.78\dot{4}23423\dot{3}$

D`vniY 13| $1.7643$, $3.\dot{2}\dot{4}$ I $2.78\dot{3}4\dot{6}$ †K m`k AveË `kwg‡K cwieZb Ki|

mgvavb : 1.7643 G AbveË Ask ej‡Z `kwgK we›`i c‡ii 4wU A¼, GLv‡b AveË Ask †bB| $3.\dot{2}\dot{4}$ G AbveË As‡ki A¼ msL¨v 0 Ges AveË As‡ki A¼ msL¨v 2, $2.78\dot{3}4\dot{6}$ G AbveË As‡ki A¼ msL¨v 2 Ges AveË As‡ki msL¨v 3| GLv‡b AbveË As‡ki A¼ msL¨v me‡P‡q †ewk n‡jv 4 Ges AveË As‡ki A¼ msL¨v 2 I 3 Gi j.mv.¸ n‡jv 6| c‡Z¨KwU `kwg‡Ki AbveË As‡ki A¼ msL¨v n‡e 4 Ges AveË As‡ki A¼ msL¨v n‡e 6|

$\therefore 1.7643 = 1.7643\dot{0}0000\dot{0}$, $3.\dot{2}\dot{4} = 3.2424\dot{2}4242\dot{4}$ I $2.78\dot{3}4\dot{6} = 2.7834\dot{6}3463\dot{4}$

wb‡Yq AveË `kwgKmgn: $1.7643\dot{0}0000\dot{0}$, $3.2424\dot{2}4942\dot{4}$, $2.7834\dot{6}3463\dot{4}$

gše¨ : mmxg `kwgK fMvsk¸‡jv‡K m`k `kwg‡K cwiYZ Kivi Rb¨ `kwgK we›`i meWv‡bi A‡¼i ci c‡qvRbxq msL¨K kb¨ ewm‡q c‡Z¨KwU `kwg‡Ki `kwgK we›`i c‡ii AbveË A¼ msL¨v mgvb Kiv n‡q‡Q| Avi AveË `kwg‡K c‡Z¨KwU `kwg‡Ki `kwgK we›`i c‡ii AbveZ A¼ msL¨v mgvb Ges AveË A¼ msL¨v mgvb Kiv n‡q‡Q AveË A¼¸‡jv e¨envi K‡i| AbveË As‡ki ci †h‡Kv‡bv A¼ †_‡K ïi“ K‡i AveË Ask †bIqv hvq|

KvR :
3.467, $2.01\dot{2}4\dot{3}$ Ges $7.52\dot{5}\dot{6}$ †K m`k AveË `kwg‡K cwieZb Ki|

AveË `kwg‡Ki †hvM I we‡qvM

AveË `kwg‡Ki †hvM ev we‡qvM Ki‡Z n‡j AveË `kwgK¸‡jv‡K m`k AveË `kwg‡K cwieZb Ki‡Z n‡e| Gici mmxg `kwg‡Ki wbqg †hvM ev we‡qvM Ki‡Z n‡e| mmxg `kwgK I AveË `kwgK¸‡jvi g‡a¨ †hvM ev we‡qvM Ki‡Z n‡j AveË `kwgK¸‡jv‡K m`k Kivi mgq c‡Z¨KwU AveË `kwg‡Ki AbveË As‡ki A¼ msL¨v n‡e mmxg `kwg‡Ki `kwgK we›`i c‡ii A¼ msL¨v I Ab¨vb¨ AveË `kwg‡Ki AbveË As‡ki A¼ msL¨vi g‡a¨ me‡P‡q eo †h msL¨v †m msL¨vi mgvb| Avi AveË As‡ki A¼ msL¨v n‡e h_vwbq‡g cvß j.mv.¸ Gi mgvb Ges mmxg `kwg‡Ki †¶‡Î AveË As‡ki Rb¨ c‡qvRbxq msL¨K kb¨ emv‡Z n‡e| Gici †hvM ev we‡qvM mmxg `kwg‡Ki wbq‡g Ki‡Z n‡e| Gfv‡e cvß †hvMdj ev we‡qvMdj cKZ †hvMdj ev we‡qvMdj n‡e bv| cKZ †hvMdj ev we‡qvMdj †ei Ki‡Z n‡j †`L‡Z n‡e †h m`kKZ `kwgK¸‡jv †hvM ev we‡qvM Ki‡j c‡Z¨KwU m`kKZ `kwgK¸‡jvi AveË As‡ki meev‡gi A¼¸‡jvi †hvM ev we‡qv‡M nv‡Z †h msL¨wU _v‡K, Zv cvß †hvMdj ev we‡qvMd‡ji AveË As‡ki meWv‡bi A‡¼i mv‡_ †hvM ev A¼ †_‡K we‡qvM Ki‡j cKZ †hvMdj ev we‡qvMdj cvIqv hv‡e| GwUB wb‡Yq †hvMdj ev we‡qvMdj n‡e|

gše¨ : (K) AveË `kwgKwewkó msL¨vi †hvMdj ev we‡qvM I AveË `kwgK nq| GB †hvMdj ev we‡qvMd‡j AbveË Ask AveË `kwgK¸‡jvi g‡a¨ mev‡c¶v AbveË Ask wewkó AveË `kwgKwUi AbveË A¼ msL¨vi mgvb n‡e Ges AveË Ask AveË `kwgK msL¨v¸‡jvi AveË A¼ msL¨vi j.mv.¸ Gi mgvb msL¨K AveË A¼ n‡e| mmxg `kwgK _vK‡j c‡Z¨KwU AveË `kwg‡Ki AbveË As‡ki A¼ msL¨v n‡e mmxg `kwg‡Ki `kwgK we›`i c‡ii A¼ msL¨v I Ab¨vb¨ AveË `kwg‡Ki AbveË As‡ki A¼ msL¨vi g‡a¨ me‡P‡q eo †h msL¨v †h msL¨vi mgvb|

(L) AveË `kwgK fMvsk¸‡jv‡K mvgvb¨ fMvs‡k cwieZb K‡i fMvs‡ki wbq‡g †hvMdj ev we‡qvMdj †ei Kivi ci †hvMdj ev we‡qvMdj‡K Avevi `kwg‡K cwieZb K‡i I †hvM ev we‡qvM Kiv hvq| Z‡e G c×wZ‡Z †hvM ev we‡qvM Ki‡j †ewk mgq jvM‡e|

D`vniY 14| $3\cdot\dot{8}\dot{9}$, $2\cdot1\dot{7}\dot{8}$ I $5\cdot89\dot{7}9\dot{8}$ †hvM Ki|

mgvavb : GLv‡b AbveË As‡ki A¼ msL¨v n‡e 2 Ges AveË As‡ki A¼ n‡e 2, 2 I 3 Gi j.mv.¸ 6| c_‡g wZbwU AveË `kwgK‡K m`k Kiv n‡q‡Q|

$$
\begin{array}{lcr}
3.\dot{8}\dot{9} & = & 3\cdot89\dot{8}98989\dot{9} \\
2\cdot1\dot{7}\dot{8} & = & 2.17\dot{8}7878\dot{7} \\
5\cdot89\dot{7}9\dot{8} & = & 5.89\dot{7}98798\dot{8} \\
\hline
 & & 11\cdot97576574 \\
 & & +2 \\
\hline
 & & 11\cdot97\dot{5}7657\dot{6}
\end{array}
$$

[$8+8+7+2=25$, GLv‡b 2 n‡jv nv‡Zi 2| 25 Gi 2 †hvM n‡q‡Q|]

wb‡Yq †hvMdj $11\cdot97\dot{5}7657\dot{6}$ ev $11\cdot97\dot{5}7\dot{6}$

gše¨ : GB †hvMd‡j 575675 AveË Ask| wKš 576 †K AveË Ask Ki‡j gv‡bi †Kv‡bv cwieZb nq bv|

`óe¨ : meWv‡b 2 †hv‡Mi aviYv †evSvevi Rb¨ G †hvMwU Ab¨ wbq‡g Kiv n‡jv:

$$
\begin{array}{lcl}
3.\dot{8}\dot{9} & = & 3\cdot89\dot{8}98989\dot{9}\,|\,89 \\
2.1\dot{7}\dot{8} & = & 2.17\dot{8}7878\dot{7}\,|\,87 \\
5\cdot89\dot{7}9\dot{8} & = & 5\cdot89\dot{7}98798\dot{8}\,|\,79 \\
\hline
 & & 11.97\dot{5}7657\dot{6}\,|\,55
\end{array}
$$

GLv‡b AveË Ask †kl nIqvi ci Avil 2 A¼ chš— msL¨v‡K evov‡bv n‡q‡Q| AwZwi³ A¼¸‡jv‡K GKUv Lvov †iLv Øviv Avjv`v K‡i †`Iqv n‡q‡Q| Gici †hvM Kiv n‡q‡Q| Lvov †iLvi Wv‡bi A‡¼i †hvMdj †_‡K nv‡Zi 2 G‡m Lvov †iLvi ev‡gi A‡¼i mv‡_ †hvM n‡q‡Q| Lvov †iLvi Wv‡bi A¼wU Avi †cšbtcwbK we›` kb¨ nIqvi A¼wU GKB| ZvB `BwU †hvMdjB GK|

D`vniY 15| $8\cdot9\dot{4}7\dot{8}$, $2\cdot346$ I $4\cdot\dot{7}\dot{1}$ †hvM Ki|

mgvavb : `kwgK¸‡jv‡K m`k Ki‡Z n‡j AbveË Ask 3 A‡¼i Ges AveË Ask n‡e 3 I 2 Gi j.mv.¸ 6 A‡¼i|

$$\begin{array}{lr}
8\cdot9\dot{4}7\dot{8} & = 8\cdot9478\dot{4}7847\dot{7} \\
2\cdot346 & 2\cdot346\dot{0}00000\dot{0} \\
4\cdot\dot{7}\dot{1} & = 4\cdot7171\dot{7}1717\dot{1} \\
\hline
 & 16\cdot011019564 \\
 & +1 \\
\hline
 & 16\cdot011\dot{0}19565\dot{5}
\end{array}$$

[$8+0+1+1=10$, GLv‡b wØZxq 1 n‡jv nv‡Zi 1| 10 Gi 1 †hvM n‡q‡Q|]

wb‡Yq †hvMdj $16\cdot011\dot{0}19565\dot{5}$

KvR : †hvM Ki : 1| $2\cdot0\dot{9}\dot{7}$ I $5\cdot12\dot{7}6\dot{8}$ 2| $1\cdot34\dot{5}$, $0\cdot31\dot{5}\dot{7}\dot{6}$ I $8\cdot056\dot{7}\dot{8}$

D`vniY 16| $8\cdot2\dot{4}\dot{3}$ †_‡K $5\cdot24\dot{6}7\dot{3}$ we‡qvM Ki|

mgvavb : GLv‡b AbveË As‡ki A¼ msL¨v n‡e 2 Ges AveË As‡ki A¼ msL¨v n‡e 2 I 3 Gi j.mv.¸ 6 | GLb `kwgK msL¨v `BwU‡K m`k K‡i we‡qvM Kiv n‡jv|

$$\begin{array}{lr}
8\cdot2\dot{4}\dot{3} & = 8\cdot24\dot{3}4343\dot{4} \\
5\cdot24\dot{6}7\dot{3} & = 5\cdot24\dot{6}7367\dot{3} \\
\hline
 & 2\cdot99669761 \\
 & -1 \\
\hline
 & 2\cdot99\dot{6}69760
\end{array}$$

[3 †_‡K 6 we‡qvM Ki‡j nv‡Z 1 wb‡Z n‡e|]

wb‡Yq we‡qvMdj $2\cdot99\dot{6}69760$|

gše¨ : †cŠbtcwbK we›` †hLv‡b ïi“ †mLv‡b we‡qvRb msL¨v we‡qvR¨ msL¨v †_‡K †QvU n‡j me mgq meWv‡bi A¼ †_‡K 1 we‡qvM Ki‡Z n‡e|

`óe¨ : meWv‡bi A¼ †_‡K 1 †Kb we‡qvM Kiv nq Zv †evSvevi Rb¨ wb‡P Ab¨fv‡e we‡qvM K‡i †`Lv‡bv n‡jv :

$$\begin{array}{lr}
8\cdot2\dot{4}\dot{3} & = 8\cdot24\dot{3}4343\dot{4}\,|\,34 \\
5\cdot24\dot{6}7\dot{3} & = 5\cdot24\dot{6}7367\dot{3}\,|\,67 \\
\hline
 & 2\cdot99\dot{6}6976\dot{0}\,|\,67
\end{array}$$

wb‡Yq we‡qvMdj $2\cdot99\dot{6}6976\dot{0}\,|\,67$ GLv‡b `BwU we‡qvMdjB GK|

D`vniY 17| $24\cdot45\dot{6}4\dot{5}$ †_‡K $16\cdot\dot{4}3\dot{7}$ we‡qvM Ki|

mgvavb :

$$\begin{array}{lr}
24\cdot45\dot{6}4\dot{5} & = 24\cdot45\dot{6}4\dot{5} \\
16\cdot\dot{4}3\dot{7} & =16\cdot43\dot{7}4\dot{3} \\
\hline
\end{array}$$

$$\begin{array}{r} 8\cdot 01902 \\ -1 \\ \hline 8\cdot 01\dot{9}0\dot{1} \end{array}$$

[6 †_‡K 7 we‡qvM Ki‡j nv‡Z 1 wb‡Z n‡e|]

wb‡Yq we‡qvMdj $8.01\dot{9}0\dot{1}$

`óe¨ :

$$\begin{array}{lr} 24\cdot 45\dot{6}4\dot{5} & = 24\cdot 45\dot{6}4\dot{5}|64 \\ 16\cdot \dot{4}3\dot{7} & = 16\cdot 43\dot{7}4\dot{3}|74 \\ \hline & 8\cdot 01\dot{9}0\dot{1}|90 \end{array}$$

KvR :

we‡qvM Ki :

1| $13\cdot 12\dot{7}8\dot{4}$ †_‡K $10\cdot 418$ 2| $23\cdot 03\dot{9}\dot{4}$ †_‡K $9\cdot 12\dot{6}4\dot{5}$

AveË `kwg‡Ki ¸Y I fvM

AveË `kwgK¸‡jv‡K fMœvs‡k cwiYZ K‡i ¸Y ev fv‡Mi KvR mgvav K‡i cvß fMœvskwU‡K `kwg‡K cKvk Ki‡jB AveË `kwgK¸‡jvi ¸Ydj ev fvMdj n‡e| mmxg `kwgK I AveË `kwg‡Ki g‡a¨ ¸Y ev fvM Ki‡Z n‡j G wbq‡gB Ki‡Z n‡e| Z‡e fv‡Mi †¶‡Î fvR¨ I fvRK `BwUB AveË `kwgK n‡j, Dfq‡K m`k AveË `kwgK K‡i wb‡j fv‡Mi KvR mnR nq|

D`vniY 18| $4\cdot\dot{3}$ †K $5\cdot\dot{7}$ Øviv ¸Y Ki|

mgvavb : $4\cdot\dot{3} = \dfrac{43-4}{9} = \dfrac{39}{9} = \dfrac{13}{3}$

$$5\cdot\dot{7} = \frac{57-5}{9} = \frac{52}{9}$$

$$\therefore\ 4\cdot\dot{3} \times 5\cdot\dot{7} = \frac{13}{3}\times\frac{52}{9} = \frac{676}{27} = 25\cdot\dot{0}3\dot{7}$$

wb‡Yq ¸Ydj $25\cdot\dot{0}3\dot{7}$

D`vniY 19| $0\cdot 2\dot{8}$ †K $42\cdot\dot{1}\dot{8}$ Øviv ¸Y Ki|

mgvavb : $0\cdot 2\dot{8} = \dfrac{28-2}{90} = \dfrac{26}{90} = \dfrac{13}{45}$

$$42\cdot\dot{1}\dot{8} = \frac{4218-42}{99} = \frac{4176}{99} = \frac{464}{11}$$

$$= \frac{13}{45}\times\frac{464}{11} = \frac{6032}{495} = 12\cdot\dot{1}8\dot{5}$$

wb‡Yq ¸Ydj $12\cdot\dot{1}8\dot{5}$

D`vniY 20| $2\cdot5\times4\cdot3\dot{5}\times1\cdot2\dot{3}\dot{4}=$ KZ ?

mgvavb : $2\cdot5=\frac{25}{10}=\frac{5}{2}$

$$4\cdot3\dot{5}=\frac{435-43}{90}=\frac{392}{90}$$

$$1\cdot2\dot{3}\dot{4}=\frac{1234-12}{990}=\frac{1222}{990}=\frac{611}{495}$$

$$\therefore\ \frac{5}{2}\times\frac{392}{90}\times\frac{611}{495}=\frac{196\times611}{8910}=\frac{119756}{8910}=13\cdot44062...$$

wb‡Yq ¸Ydj $13\cdot44062$

> KvR :
>
> 1| $1\cdot1\dot{3}$ †K $2\cdot6$ Øviv ¸Y Ki| 2| $0\cdot\dot{2}\times1\cdot\dot{1}\dot{2}\times0\cdot0\dot{8}\dot{1}=$ KZ ?

D`vniY 21| $7\cdot\dot{3}\dot{2}$ †K $0\cdot2\dot{7}$ Øviv fvM Ki|

mgvavb : $7\cdot\dot{3}\dot{2}=\frac{732-7}{99}=\frac{725}{99}$

$$0\cdot2\dot{7}=\frac{27-2}{90}=\frac{25}{90}=\frac{5}{18}$$

$$\therefore\ 7\cdot\dot{3}\dot{2}\div0\cdot2\dot{7}=\frac{725}{99}\div\frac{5}{18}=\frac{725}{99}\times\frac{18}{5}=\frac{290}{11}=26.\dot{3}\dot{6}$$

wb‡Yq fvMdj $26\cdot\dot{3}\dot{6}$

D`vniY 22| $2\cdot\dot{2}71\dot{8}$ †K $1\cdot9\dot{1}\dot{2}$ Øviv fvM Ki|

mgvavb : $2\cdot\dot{2}71\dot{8}=\frac{22718-2}{9999}=\frac{22176}{9999}$

$$1\cdot9\dot{1}\dot{2}=\frac{1912-19}{990}=\frac{1893}{990}$$

$$\therefore\ 2\cdot\dot{2}71\dot{8}\div1\cdot9\dot{1}\dot{2}=\frac{22716}{9999}\div\frac{1893}{990}=\frac{22716}{9999}\times\frac{990}{1893}=\frac{120}{101}=1\cdot\dot{1}88\dot{1}$$

wb‡Yq fvMdj $1\cdot\dot{1}88\dot{1}$

D`vniY 23| $9\cdot45$ †K $2\cdot8\dot{6}\dot{3}$ Øviv fvM Ki|

mgvavb : $9\cdot45\div2\cdot8\dot{6}\dot{3}=\frac{945}{100}\div\frac{2863-28}{990}=\frac{945}{100}\times\frac{990}{2835}$

$$=\frac{189\times99}{2\times2835}=\frac{33}{10}=3\cdot3$$

wb‡Yq fvMdj $3\cdot3$

gšeª : AveË `kwg‡Ki ¸Ydj Ges fvMdj AveË `kwgK n‡Z I cv‡i, bvI n‡Z cv‡i|

KvR :

1| $0\cdot\dot{6}$ †K $0\cdot\dot{9}$ Øviv fvM Ki| 2| $0\cdot7\dot{3}\dot{2}$ †K $0\cdot0\dot{2}\dot{7}$ Øviv fvM Ki|

Amxg `kwgK

A‡bK `kwgK fMvsk Av‡Q hv‡`i `kwgK we›`yi Wv‡bi A‡¼i †kl †bB, Avevi GK ev GKvwaK A¼ evievi cybivq Av‡m bv, Gme `kwgK fMvsk Amxg `kwgK fMvsk| †hgb, $5\cdot134248513942307$......... GKwU Amxg `kwgK msL¨v| 2 Gi eMgj GKwU Amxg `kwgK| GLb, 2 G eMgj †ei Kwi|

1	2	1·4142135........
	1	
24	100	
	96	
281	400	
	281	
2824	11900	
	11296	
28282	60400	
	56564	
282841	383600	
	282841	
2828423	10075900	
	8485269	
28284265	159063100	
	141421325	
	17641775	

Gfv‡e cÖwµqv AbšÍKvj Pj‡Z _vK‡e|

$\therefore\ \sqrt{2}=1\cdot4142135$...... GKwU Amxg `kwgK msL¨v|

wbw`©ó `kwgK ¯'vb chšÍ gvb Ges wbw`©ó `kwgK ¯'vb chšÍ Avmbœ gvb

Amxg `kwg‡Ki gvb †Kv‡bv wbw`©ó `kwgK ¯'vb chšÍ gvb †ei Kiv Ges †Kv‡bv wbw`©ó `kwgK ¯'vb chšÍ Avmbœ gvb †ei Kiv GKB A_© bq|

†hgb, $5\cdot4325893$...... `kwgKwUi ÒPvi `kwgK ¯'vb chšÍ gvbÓ n‡e $5\cdot4325$, wKšÍ $5\cdot4325893$.... `kwgKwUi ÒPvi `kwgK ¯'vb chšÍ Avmbœ gvbÓ n‡e $5\cdot4326$| GLv‡b Ò`B `kwgK ¯'vb chšÍ gvbÓ Ges Ò`B `kwgK ¯'vb chšÍ Avmbœ gvbÓ GKB hv $5\cdot43$| mmxg `kwgK Gfv‡e Avmbœ gvb †ei Kiv hvq|

gšÍe¨ : hZ `kwgK ¯'vb chšÍ gvb †ei Ki‡Z ejv n‡e, ZZ `kwgK ¯'vb chšÍ †h me msL¨v _vK‡e ûeû †m msL¨v¸‡jv wjL‡Z n‡e gvÎ| Avi hZ `kwgK ¯'vb chšÍAvmbœ gvb †ei Ki‡Z ejv n‡e, Gi cieZx© ¯'vbwU‡Z 5, 6, 7, 8 ev 9 nq, Z‡e †kl ¯'vbwUi msL¨vi mv‡_ 1 †hvM Ki‡Z n‡e| wKšÍ hw` 1, 2, 3 ev 4 nq, Z‡e †kl ¯'vbwUi msL¨v †hgb wQj †ZgbB _vK‡e, G‡¶‡Î Ò`kwgK ¯'vb chšÍ gvbÓ Ges Ò`kwgK ¯'vb chšÍ Avmbœ gvbÓ GKB| hZ `kwgK ¯'vb chšÍ †ei Ki‡Z ejv n‡e, `kwg‡Ki ci Gi †P‡q 1 ¯'vb †ewk chšÍ `kwgK msL¨v †ei Ki‡Z n‡e|

D`vniY 24| 13 Gi eMgj †ei Ki Ges wZb `kwgK ¯vb chš Avmb gvb †jL|

mgvavb : 3) 13 (3·605551........

```
        9
    ---------
66   | 400
     | 396
     -----------
7205   | 40000
       | 36025
       -----------
72105    | 3697500
         | 3605525
         -------------
7211101    | 9197500
           | 7211101
           -----------
             1986399
```

∴ wb‡Yq eMgj 3·605551........

∴ wb‡Yq wZb `kwgK ¯vb chš Avmb gvb 3·606

D`vniY 25| 4·4623845....... `kwgKwUi 1, 2, 3, 4 I 5 `kwgK ¯vb chš gvb I Avmb gvb †ei Ki|

mgvavb : 4·4623845 msL¨wUi GK `kwgK ¯vb chš gvb 4·4

Ges GK `kwgK ¯vb chš Avmb gvb 4·5

`B `kwgK ¯vb chš gvb 4·46

Ges `B `kwgK ¯vb chš Avmb gvb 4·46

wZb `kwgK ¯vb chš gvb 4·462

Ges wZb `kwgK ¯vb chš 4·462

Pvi `kwgK ¯vb chš 4·4623

Ges Pvi `kwgK ¯vb chš Avmb 4·4624

cvP `kwgK ¯vb chš gvb 4·46238

Ges cvP `kwgK ¯vb chš Avmb 4·46238 |

KvR : 29 Gi eMgj wbYq Ki Ges eMgj‡K `B `kwgK ¯vb chš gvb Ges `B `kwgK ¯vb chš Avmb gvb †jL|

Abkxjbx 1

1| cgvY Ki †h, (K) $\sqrt{5}$ (L) $\sqrt{7}$ (M) $\sqrt{10}$ c‡Z¨‡K Agj` msL¨v|

2| (K) 0.31 Ges 0.12 Gi g‡a¨ `BwU Agj` msL¨v wbYq Ki|

(L) $\frac{1}{\sqrt{2}}$ Ges $\sqrt{2}$ Gi g‡a¨ GKwU gj` Ges GKwU Agj` msL¨v wbYq Ki|

3| (K) cgvY Ki †h, †h‡Kv‡bv we‡Rvo cY msL¨vi eM GKwU we‡Rvo msL¨v|
(L) cgvY Ki †h, `BwU µwgK †Rvo msL¨vi ¸Ydj 8 (AvU) Øviv wefvR¨|

4| AveË `kwgK fMvs‡k cKvk Ki : (K) $\frac{1}{6}$ (L) $\frac{7}{11}$ (M) $3\frac{2}{9}$ (N) $3\frac{8}{15}$

5| mvgvb¨ fMvs‡k cKvk Ki : (K) $0\cdot\dot{2}$ (L) $0\cdot\dot{3}\dot{5}$ (M) $0\cdot1\dot{3}$ (N) $3\cdot7\dot{8}$ (O) $6\cdot2\dot{3}0\dot{9}$

6| m`k AveË `kwgK fMvs‡k cKvk Ki :

(K) $2\cdot\dot{3},\ 5\cdot2\dot{3}\dot{5}$ (L) $7\cdot2\dot{6},\ 4\cdot23\dot{7}$ (M) $5\cdot\dot{7},\ 8\cdot\dot{3}\dot{4},\ 6\cdot\dot{2}4\dot{5}$ (N) $12\cdot32,\ 2\cdot1\dot{9},\ 4\cdot32\dot{5}\dot{6}$

7| †hvM Ki : (K) $0\cdot4\dot{5}+0\cdot13\dot{4}$ (L) $2\cdot0\dot{5}+8\cdot0\dot{4}+7\cdot018$ (M) $0\cdot00\dot{6}+0\cdot\dot{9}\dot{2}+0\cdot0\dot{1}3\dot{4}$

8| we‡qvM Ki :

(K) $3\cdot\dot{4}-2\cdot1\dot{3}$ (L) $5\cdot\dot{1}\dot{2}-3\cdot4\dot{5}$ (M) $8\cdot49-5\cdot3\dot{5}\dot{6}$ (N) $19\cdot34\dot{5}-13\cdot2\dot{3}4\dot{9}$

9| ¸Y Ki : (K) $0\cdot\dot{3}\times0\cdot\dot{6}$ (L) $2\cdot\dot{4}\times0\cdot\dot{8}\dot{1}$ (M) $0\cdot6\dot{2}\times0\cdot\dot{3}$ (N) $42\cdot\dot{1}\dot{8}\times0\cdot2\dot{8}$

10| fvM Ki : (K) $0\cdot\dot{3}\div0\cdot\dot{6}$ (L) $0\cdot3\dot{5}\div1\cdot\dot{7}$ (M) $2\cdot3\dot{7}\div0\cdot4\dot{5}$ (N) $1\cdot\dot{1}8\dot{5}\div0\cdot\dot{2}\dot{4}$

11| eMgj wbYq Ki (wZb `kwgK ¯vb chš) Ges `B `kwgK ¯vb chš eMgj ¸‡jvi Avmb gvb †jL :

(K) 12 (L) $0\cdot\dot{2}\dot{5}$ (M) $1\cdot3\dot{4}$ (N) $5\cdot1\dot{3}0\dot{2}$

12| wb‡Pi †Kvb msL¨v¸‡jv gj` Ges †Kvb msL¨v¸‡jv Agj` †jL :

(K) $0\cdot\dot{4}$ (L) $\sqrt{9}$ (M) $\sqrt{11}$ (N) $\frac{\sqrt{6}}{3}$ (O) $\frac{\sqrt{8}}{\sqrt{7}}$ (P) $\frac{\sqrt{27}}{\sqrt{48}}$ (Q) $\frac{\frac{2}{3}}{\frac{3}{7}}$ (R) $5\cdot\dot{6}3\dot{9}$

13| mij Ki :

(K) $(0\cdot\dot{3}\times0\cdot8\dot{3})\div(0\cdot5\times0\cdot\dot{1})+0\cdot3\dot{5}\div0\cdot0\dot{8}$

(L) $[(6.27\times0.5)\div\{(0.5\times0.75)\times8.36\}]\div\{(0.25\times0.1)\times(0.75\times21.3)\times0.5\}$

14| $\sqrt{5}$ I 4 `BwU ev¯e msL¨v|

K. †KvbwU gj` I †KvbwU Agj` wb‡`k Ki|

L. $\sqrt{5}$ I 4 G‡`i g‡a¨ `BwU Agj` msL¨v wbYq Ki|

M. cgvY Ki †h, $\sqrt{5}$ GKwU Agj` msL¨v|

wØZxq Aa¨vq

†mU I dvskb

(Set and Function)

†mU kãwU Avgv‡`i mcwiwPZ †hgb : wWbvi †mU, ¯vfweK msL¨vi †mU, gj` msL¨vi †mU BZ¨w`| AvaywbK nwZqvi wn‡m‡e †m‡Ui e¨envi e¨vcK| Rvgvb MwYZwe` RR K¨v›Ui (1844-1918) †mU m¤ú‡K c_g avibY e¨vL¨v K‡ib| wZwb Amxg †m‡Ui avibY c`vb K‡i MwYZ kv‡¯ Av‡jvob mwó K‡ib Ges Zvi †m‡Ui avibY †mU ZË (*Set Theory*) bv‡g cwiwPZ| GB Aa¨v‡q †m‡Ui avibY †_‡K MwYwZK I wP‡ýi gva¨‡g mgm¨v mgvavb Ges dvskb m¤ú‡K mg¨K Ávb ARb Kiv cavb j¶|

Aa¨vq †k‡l wk¶v_xiv –

- †mU I Dc‡m‡Ui avibY e¨vL¨v K‡i cZx‡Ki mvnv‡h¨ cKvk Ki‡Z cvi‡e|
- †mU cKv‡ki c×wZ eYbv Ki‡Z cvi‡e|
- Amxg †mU e¨vL¨v Ki‡Z cvi‡e Ges mmxg I Amxg †m‡Ui cv_K¨ wbiƒcY Ki‡Z cvi‡e|
- †m‡Ui ms‡hvM I †Q` e¨vL¨v Ges hvPvB Ki‡Z cvi‡e|
- kw³ †mU e¨vL¨v Ki‡Z cvi‡e Ges `B I wZb m`m¨wewkó †m‡Ui kw³ †mU MVb Ki‡Z cvi‡e|
- µg‡Rvo I Kv‡Umxq ¸YR e¨vL¨v Ki‡Z cvi‡e|
- D`vniY I †fbwP‡Îi mvnv‡h¨ †mU cwµqvi mnR wewa¸‡jv cgvY Ki‡Z cvi‡e Ges wewa¸‡jv c‡qvM K‡i wewfb mgm¨v mgvavb Ki‡Z cvi‡e|
- Ašq I dvskb e¨vL¨v Ki‡Z I MVb Ki‡Z cvi‡e|
- †Wv‡gb I †iÄ Kx e¨vL¨v Ki‡Z cvi‡e|
- dvsk‡bi †Wv‡gb I †iÄ wbYq Ki‡Z cvi‡e|
- dvsk‡bi †jLwPÎ A¼b Ki‡Z cvi‡e|

†mU (Set)

ev¯e ev wPšv RM‡Zi m-msÁwqZ e¯i mgv‡ek ev msMn‡K †mU e‡j| †hgb, evsjv, Bs‡iwR I MwYZ wel‡q wZbwU cvV¨eB‡qi †mU| c_g `kwU we‡Rvo ¯vfweK msL¨vi †mU, cYmsL¨vi †mU, ev¯e msL¨vi †mU BZ¨w`|

†mU‡K mvaviYZ Bs‡iwR eYgvjvi eo nv‡Zi A¶i $A, B, C, ...X, Y, Z$ Øviv cKvk Kiv nq|

†hgb, $2, 4, 6$ msL¨v wZbwUi †mU $A = \{2, 4, 6\}$

†m‡Ui c‡Z¨K e¯ ev m`m¨‡K †m‡Ui Dcv`vb (*element*) ejv nq| †hgb, $B = \{a, b\}$ n‡j, B †m‡Ui Dcv`vb $a$ Ges $b$; Dcv`vb cKv‡ki wPý '$\in$' .

∴ $a \in B$ Ges cov nq a, B Gi m`m¨ (a *belongs to* B)

$b \in B$ Ges cov nq b, B Gi m`m¨ (b *belongs to* B)

Dc‡ii B †m‡U c Dcv`vb †bB|

∴ $c \notin B$ Ges cov nq c, B Gi m`m¨ bq (c *does not belong to* B) .

†mU cKv‡ki c×wZ (***Method of describing Sets***) :

†mU‡K `B c×wZ‡Z cKvk Kiv nq| h_v : (1) ZvwjKv c×wZ (*Roster Method* ev *Tabular Method*) Ges (2) †mU MVb c×wZ (*Set Builder Method*)

(1) ZvwjKv c×wZ : G c×wZ‡Z †m‡Ui mKj Dcv`vb mwbw`©ó fv‡e D‡jL K‡i wØZxq eÜbx { } Gi g‡a¨ Ave× Kiv nq Ges GKwaK Dcv`vb _vK‡j ÔKgvÕ e¨envi K‡i Dcv`vb¸‡jv‡K Avjv`v Kiv nq|

†hgb, $A = \{a, b\}$, $B = \{2, 4, 6\}$, $C =$ {wbjq, wZkv, ïåv} BZ¨vw`|

(2) †mU MVb c×wZ : G c×wZ‡Z †m‡Ui mKj Dcv`vb mwbw`©ó fv‡e D‡jL bv K‡i Dcv`vb wbavi‡Yi Rb¨ mvaviY a‡gi D‡jL _v‡K| †hgb : $A = \{x : x$ ¯vfvweK we‡Rvo msL¨v$\}$, $B = \{x :\ x$ beg †kwYi c_g cvPRb wk¶v_x$\}$ BZ¨vw`|

GLv‡b, ':' Øviv ÔGi/c †hbÕ ev ms‡¶‡c Ô†hbÕ (*such that*) †evSvq| †h‡nZ G c×wZ‡Z †m‡Ui Dcv`vb wbavi‡Yi Rb¨ kZ ev wbqg (*Rule*) †`Iqv _v‡K, G Rb¨ G c×wZ‡K *Rule Method* I ejv nq|

D`vniY 1| $A = \{7, 14, 21, 28\}$ †mUwU‡K †mU MVb c×wZ‡Z cKvk Ki|

mgvavb : A †m‡Ui Dcv`vbmgn $7, 14, 21, 28$

GLv‡b, c‡Z¨KwU Dcv`vb 7 Øviv wefvR¨, A_vr 7 Gi ¸wYZK Ges 28 Gi eo bq|

∴ $A = \{x : x, 7$ Gi ¸wYZK Ges $x \le 28\}$.

D`vniY 2| $B = \{x : x, 28$ Gi ¸YbxqK$\}$ †mUwU‡K ZvwjKv c×wZ‡Z cKvk Ki|

mgvavb : GLv‡b, $28 = 1 \times 28$

$= 2 \times 14$

$= 4 \times 7$

∴ 28 Gi ¸YbxqKmgn $1, 2, 4, 7, 14, 28$

wb‡Yq †mU $B = \{1, 2, 4, 7, 14, 28\}$

D`vniY 3| $C = \{x : x$ abvZ¥K cYmsL¨v Ges $x^2 < 18\}$ †mUwU‡K ZvwjKv c×wZ‡Z cKvk Ki|

mgvavb : abvZ¥K cYmsL¨vmgn $1, 2, 3, 4, 5,$ ....

GLv‡b, $x = 1$ n‡j, $x^2 = 1^2 = 1$

$x = 2$ n‡j, $x^2 = 2^2 = 4$

$x = 3$ n‡j, $x^2 = 3^2 = 9$

$x = 4$ n‡j, $x^2 = 4^2 = 16$

$x = 5$ n‡j, $x^2 = 5^2 = 25$; hv 18 Gi †P‡q eo

$\therefore$ kZvbmv‡i MnY‡hvM¨ abvZ¥K cYmsL¨vmgn 1, 2, 3, 4

$\therefore$ wb‡Yq †mU $C = \{1, 2, 3, 4\}$.

KvR : 1| $C = \{-9, -6, -3, 3, 6, 9\}$ †mUwU‡K †mU MVb c×wZ‡Z cKvk Ki|
2| $Q = \{y : y$ cY msL¨v Ges $y^3 \le 27\}$ †mUwU‡K Zvwj Kv c×wZ‡Z cKvk Ki|

mmxg †mU ***(Finite Set)*** : †h †m‡Ui Dcv`vb msL¨v MYbv K‡i wbavriY Kiv hvq, G‡K mgxg †mU e‡j\| †hgb, $D = \{x, y, z\}$, $E = \{3, 6, 9, ........, 60\}$, $F = \{x : x$ †gŠwjK msL¨v Ges $30 < x < 70\}$ BZ¨vw` mmxg †mU\| GLv‡b, D †m‡U 3wU Dcv`vb, $E$ †m‡U 20wU Dcv`vb Ges F †m‡U 9wU Dcv`vb Av‡Q\|

Amxg †mU ***(Infinite Set)*** : †h †m‡Ui Dcv`vb msL¨v MYbv K‡i wbavriY Kiv hvq bv, G‡K Amxg †mU e‡j\| †hgb, $A = \{x : x$ we‡Rvo ¯vfvweK msL¨v$\}$, ¯vfvweK msL¨vi †mU $N = \{1, 2, 3, 4, ........\}$, cYmsL¨vi †mU $Z = \{.......-3, -2, -1, 0, 1, 2, 3.......\}$, gj` msL¨vi †mU $Q = \left\{\frac{P}{q} : p\right.$ I q cY msL¨v Ges $q \neq 0\}$, ev¯e msL¨vi †mU R BZ¨vw` Amxg †mU\|

D`vniY 4\| †`LvI †h, mKj ¯vfvweK msL¨vi †mU GKwU Amxg †mU\|

mgvavb : ¯vfvweK msL¨vi †mU $N = \{1, 2, 3, 4, 5, 6, 7, 8, ...\}$

N †mU †_‡K we‡Rvo ¯vfvweK msL¨vmgn wb‡q MwVZ †mU $A = \{1, 3, 5, 7, ...\}$

†Rvo ó ó ó ó ó $B = \{2, 4, 6, 8, ...\}$

3 Gi ¸wYZKmg‡ni †mU $C = \{3, 6, 9, 12, ...\}$ BZ¨vw`\|

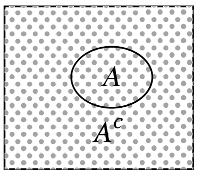
GLv‡b, N †mU †_‡K MwVZ A, B, C †mUmg‡n Dcv`vb msL¨v MYbv K‡i wbavriY Kiv hvq bv\| d‡j A, B, C Amxg †mU\|

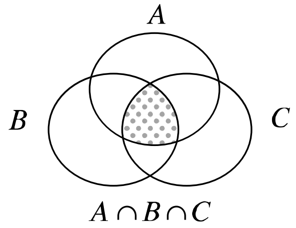
$\therefore$ N GKwU Amxg †mU\|

KvR : wb‡Pi †mU¸‡jv †_‡K mmxg †mU I Amxg †mU †jL :
1| $\{3, 5, 7\}$ 2| $\{1, 2, 2^2, ...2^{10}\}$ 3| $\{3, 3^2, 3^3, ...\}$ 4| $\{x : x$ cYmsLv Ges $x < 4\}$
5| $\left\{\frac{p}{q} : p\right.$ I q ci¯úi mn‡gŠwjK Ges $q > 1\}$ 6| $\{y : y \in N$ Ges $y^2 < 100 < y^3\}$.

dvKv †mU ***(Empty Set)*** : †h †m‡Ui †Kv‡bv Dcv`vb †bB G‡K dvKv †mU e‡j| dvKv †mU‡K { } e $\Phi$ Øviv cKvk Kiv nq| †hgb : nwjµm ¯‹‡ji wZbRb Qv‡Î i †mU, $\{x \in N : 10 < x < 11\}$, $\{x \in N : x$ †gŠwjK msL¨v Ges $23 < x < 29\}$ BZ¨vw`|

†fbwPÎ (***Venn-Diagram***) : Rb †fb (1834-1883) †m‡Ui Kvhwewa wP‡Îi mvnv‡h¨ ceZb K‡ib| G‡Z we‡ePbvaxb †mU¸‡jv‡K mgZ‡j Aew¯Z wewfb AvKv‡ii R¨vwgwZK wPÎ †hgb AvqZvKvi †¶Î, eËvKvi †¶Î Ges wÎfRvKvi †¶Î e¨envi Kiv nq| Rb †f‡bi bvgvbmv‡i wPÎ¸‡jv †fb wPÎ bv‡g cwiwPZ|





Dc‡mU (*Subset*) : $A = \{a, b\}$ GKwU †mU| A †m‡Ui Dcv`vb †_‡K $\{a, b\}$, $\{a\}$, $\{b\}$ †mU¸‡jv MVb Kiv hvq| Avevi, †Kv‡bv Dcv`vb bv wb‡q Φ †mU MVb Ki hvq|

GLv‡b, MwVZ $\{a, b\}$, $\{a\}$, $\{b\}$, Φ c‡Z¨KwU A †m‡Ui Dc‡mU|
mZivs †Kv‡bv †mU †_‡K hZ¸‡jv †mU MVb Kiv hvq, G‡`i c‡Z¨KwU †mU‡K H †m‡Ui Dc‡mU ejv nq| Dc‡m‡Ui wPý $\subset$| hw` B †mU A Gi Dc‡mU nq Z‡e $B \subset A$ cov nq| B, A Gi Dc‡mU A_ev B *is a Subset of A.* Dc‡ii Dc‡mU¸‡jvi g‡a¨ $\{a, b\}$ †mU A Gi mgvb|

$\therefore$ c‡Z¨KwU †mU wb‡Ri Dc‡mU|

Avevi, †h‡Kv‡bv – 3 †mU †_‡K Φ †mU MVb Kiv hvq|

$\therefore$ Φ †h‡Kv‡bv †m‡Ui Dc‡mU|

$P = \{1, 2, 3\}$ Gi $Q = \{1, 2, 3\}$ Ges $R = \{1, 3\}$ `BwU Dc‡mU| Avevi, $P = Q$

$\therefore$ $Q \subseteq P$ Ges $R \subset P$.

cKZ Dc‡mU ***(Proper Subset)*** :

†Kv‡bv †mU †_‡K MwVZ Dc‡m‡Ui g‡a¨ †h Dc‡mU¸‡jvi Dcv`vb msL¨v c`Ë †m‡Ui Dcv`vb msL¨v A‡c¶v Kg G‡`i‡K cKZ Dc‡mU e‡j| †hgb, $A = \{3, 4, 5, 6\}$ Ges $B = \{3, 5\}$ `BwU †mU| GLv‡b, $B$ Gi me Dcv`vb A †m‡U we``gvb $\therefore$ $B \subset A$

Avevi, B †m‡Ui Dcv`vb msL¨v $A$ †m‡Ui Dcv`vb msL¨vi †P‡q Kg|

$\therefore$ B, A Gi GKwU cKZ Dc‡mU Ges $B \subseteq A$ wj‡L cKvk Kiv nq|

D`vniY 5| $P = \{x, y, z\}$ Gi Dc‡mU¸‡jv †jL Ges Dc‡mU¸‡jv †_‡K cKZ Dc‡mU evQvB Ki|

mgvavb : †`Iqv Av‡Q, $P=\{x, y, z\}$

P Gi Dc‡mUmgn $\{x, y, z\}$, $\{x, y\}$, $\{x, z\}$, $\{y, z\}$, $\{x\}$, $\{y\}$, $\{z\}$, Φ.

P Gi cKZ Dc‡mUmgn $\{x, y\}$, $\{x, z\}$, $\{y, z\}$, $\{x\}$, $\{y\}$, $\{z\}$.

†m‡Ui mgZv ***(Equivalent Set)*** :

`B ev Z‡ZwaK †m‡Ui Dcv`vb GKB n‡j, G‡`i‡K †m‡Ui mgZv ejv nq| †hgb : $A=\{3, 5, 7\}$ Ges $B=\{5, 3, 7\}$ `BwU mgvb †mU Ges $A=B$ wPý Øviv †jLv nq|

Avevi, $A=\{3, 5, 7\}$, $B=\{5, 3, 3, 7\}$ Ges $C=\{7, 7, 3, 5, 5\}$ n‡j A, B I C †mU wZbwU mgZv †evSvq| A_vr, $A=B=C$

j¶Yxq, †m‡Ui Dcv`vb¸‡jvi µg e`j‡j ev †Kv‡bv Dcv`vb cbiveË Ki‡j †m‡Ui †Kv‡bv cwieZb nq bv|

†m‡Ui Aši ***(Difference of Set)*** : g‡b Kwi, $A=\{1, 2, 3, 4, 5\}$ Ges $B=\{3, 5\}$| †mU A †_‡K †mU B Gi Dcv`vb¸‡jv ev` w`‡j †h †mUwU nq Zv $\{1, 2, 4\}$ Ges †jLv nq $A\setminus B$ ev $A-B=$ $\{1, 2, 3, 4, 5\}-\{3, 5\}$ = $\{1, 2, 4\}$

mZivs, †Kv‡bv †mU †_‡K Ab¨ GKwU †mU ev` w`‡j †h †mU MwVZ nq Zv‡K ev` †mU e‡j|

D`vniY 6| $P=\{x : x, 12$ Gi ¸YbxqKmgn$\}$ Ges $Q=\{x : x, 3$ Gi ¸wYZK Ges $x\leq 12\}$ n‡j $P-Q$ wbYq Ki|

mgvavb : †`Iqv Av‡Q, $P=\{x : x, 12$ Gi ¸YbxqKmgn$\}$

GLv‡b, 12 Gi ¸YbxqKmgn $1, 2, 3, 4, 6, 12$

$\therefore\ P=\{1, 2, 3, 4, 6, 12\}$

Avevi, $Q=\{x : x, 3$ Gi ¸wYZK Ges $x\leq 12\}$

GLv‡b, 12 chš 3 Gi ¸wYZKmgn $3, 6, 9, 12$

$\therefore\ Q=\{3, 6, 9, 12\}$

$\therefore\ P-Q=\{1, 2, 3, 4, 6, 12)-\{3, 6, 9, 12\}=\{1, 2, 4\}$

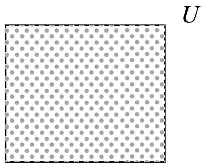
wb‡Yq †mU : $\{1, 2, 4\}$

mwweK †mU ***(Universal Set)*** :

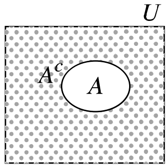
Av‡jvPbv mswkó mKj †mU GKwU wbw`ó †m‡Ui Dc‡mU| †hgb : $A=\{x, y\}$ †mUwU $B=\{x, y, z\}$ Gi GKwU Dc‡mU| GLv‡b, B †mU‡K A †m‡Ui mv‡c‡¶ mwweK †mU e‡j|

mZivs Av‡jvPbv mswkó mKj †mU hw` GKwU wbw`ó †m‡Ui Dc‡mU nq Z‡e H wbw`ó †mU‡K Gi Dc‡mU¸‡jvi mv‡c‡¶ mwweK †mU e‡j|

mwweK †mU‡K mvaviYZ U Øviv cKvk Kiv nq| Z‡e Ab¨ cZx‡Ki mvnv‡h¨I mwweK †mU cKvk Kiv hvq| †hgb : mKj †Rvo ¯^vfvweK msL¨vi †mU $C=\{2, 4, 6........\}$ Ges mKj ¯^vfvweK msL¨vi †mU $N=\{1, 2, 3, 4,...........\}$ n‡j, C †m‡Ui mv‡c‡¶ mwweK †mU n‡e N.



ciK †mU *(Complement of a Set)* :



U mwweK †mU Ges A †mUwU U Gi Dc‡mU| A †m‡Ui ewnfZ mKj Dcv`vb wb‡q MwVZ †mU‡K A †m‡Ui ciK †mU e‡j| A Gi ciK †mU‡K A^c ev A' Øviv cKvk Kiv nq| MwYwZKfv‡e $A^c=U \setminus A$.

g‡b Kwi, P I Q `BwU †mU Ges $Q$ †m‡Ui †hme Dcv`vb P †m‡Ui Dcv`vb bq, H Dcv`vb¸‡jvi †mU‡K P Gi †cwY‡Z Q Gi ciK †mU ejv nq Ges †jLv nq $Q^c=P \setminus Q$.

D`vniY 7| $U=\{1, 2, 3, 4, 6, 7\}$, $A=\{2, 4, 6, 7\}$ Ges $B=\{1, 3, 5\}$ n‡j A^c I B^c wbYq Ki|

mgvavb : $A^c=U \setminus A=\{1, 2, 3, 4, 6, 7\} \setminus \{2, 4, 6, 7\}=\{1, 3, 5\}$

Ges $B^c=U \setminus B=\{1, 2, 3, 4, 6, 7\} \setminus \{1, 3, 5\}=\{2, 4, 6, 7\}$

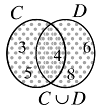
wb‡Yq †mU $A^c=\{1, 3, 5\}$ Ges $B^c=\{2, 4, 6, 7\}$

ms‡hvM †mU *(Union of Sets)* :

`B ev Z‡ZwaK †m‡Ui mKj Dcv`vb wb‡q MwVZ †mU‡K ms‡hvM †mU ejv nq| g‡b Kwi, A I B `BwU †mU| A I B †m‡Ui ms‡hvM‡K $A \cup B$ Øviv cKvk Kiv nq Ges cov nq A ms‡hvM B A_ev A *Union* B| †mU MVb c×wZ‡Z $A \cup B=\{x : x \in A$ A_ev $x \in B\}$.

D`vniY 8| $C=\{3, 4, 5\}$ Ges $D=\{4, 6, 8\}$ n‡j, $C \cup D$ wbYq Ki|

mgvavb : †`Iqv Av‡Q, $C=\{3, 4, 5\}$ Ges $D=\{4, 6, 8\}$

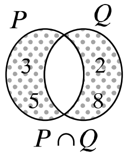


$\therefore$ $C \cup D=\{3, 4, 5\} \cup \{4, 6, 8\}=\{3, 4, 5, 6, 8\}$

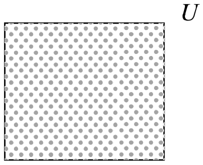
†Q` †mU *(Intersection of Sets):*

`B ev Z‡ZwaK †m‡Ui mvaviY Dcv`vb wb‡q MwVZ †mU‡K †Q` †mU e‡j| g‡b Kwi, $A$ I $B$ `BwU †mU| A I B Gi †Q` †mU‡K $A \cap B$ Øviv cKvk Kiv nq Ges cov nq $A$ †Q` B ev A *intersection* B| †mU MVb c×wZ‡Z $A \cap B=\{x : x \in A$ Ges $x \in B\}$.

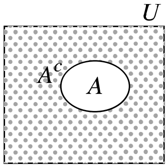
D`vniY 9| $P=\{x \in N : 2 < x \leq 6\}$ Ges $Q=\{x \in N : x$ †Rvo msL¨v Ges $x \leq 8\}$ n‡j, $P \cap Q$ wbYq Ki|



mgvavb : †`Iqv Av‡Q, $P=\{x \in N : 2 < x \leq 6\}$= {3, 4, 5, 6} mwweK †mU‡K mvaviYZ U Øviv cKvk Kiv nq| Z‡e Ab¨ cZx‡Ki mvnv‡h¨I mwweK †mU cKvk Kiv hvq| †hgb : mKj †Rvo ¯vfweK msL¨vi †mU $C=\{2, 4, 6........\}$ Ges mKj ¯vfweK msL¨vi †mU $N=\{1, 2, 3, 4,...........\}$ n‡j, C †m‡Ui mv‡c‡¶ mwweK †mU n‡e N.



ciK †mU *(Complement of a Set)* :



U mwweK †mU Ges A †mUwU U Gi Dc‡mU| A †m‡Ui ewnfZ mKj Dcv`vb wb‡q MwVZ †mU‡K A †m‡Ui ciK †mU e‡j| A Gi ciK †mU‡K A^c ev A' Øviv cKvk Kiv nq| MvwYwZKfv‡e $A^c=U \setminus A$.

g‡b Kwi, P I Q `BwU †mU Ges $Q$ †m‡Ui †hme Dcv`vb P †m‡Ui Dcv`vb bq, H Dcv`vb¸‡jvi †mU‡K P Gi †cwi‡¶‡Z Q Gi ciK †mU ejv nq Ges †jLv nq $Q^c=P \setminus Q$.

D`vniY 7| $U=\{1, 2, 3, 4, 6, 7\}$, $A=\{2, 4, 6, 7\}$ Ges $B=\{1, 3, 5\}$ n‡j A^c I B^c wbYq Ki|

mgvavb : $A^c=U \setminus A=\{1, 2, 3, 4, 6, 7\} \setminus \{2, 4, 6, 7\}=\{1, 3, 5\}$

Ges $B^c=U \setminus B=\{1, 2, 3, 4, 6, 7\} \setminus \{1, 3, 5\}=\{2, 4, 6, 7\}$

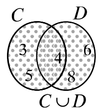
wb‡Yq †mU $A^c=\{1, 3, 5\}$ Ges $B^c=\{2, 4, 6, 7\}$

ms‡hvM †mU *(Union of Sets)* :

`B ev Z‡ZwaK †m‡Ui mKj Dcv`vb wb‡q MwVZ †mU‡K ms‡hvM †mU ejv nq| g‡b Kwi, A I B `BwU †mU| A I B †m‡Ui ms‡hvM‡K $A \cup B$ Øviv cKvk Kiv nq Ges cov nq A ms‡hvM B A_ev A *Union* B| †mU MVb c×wZ‡Z $A \cup B=\{x : x \in A$ A_ev $x \in B\}$.

D`vniY 8| $C=\{3, 4, 5\}$ Ges $D=\{4, 6, 8\}$ n‡j, $C \cup D$ wbYq Ki|

mgvavb : †`Iqv Av‡Q, $C=\{3, 4, 5\}$ Ges $D=\{4, 6, 8\}$

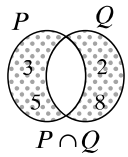


$\therefore \ C \cup D=\{3, 4, 5\} \cup \{4, 6, 8\}=\{3, 4, 5, 6, 8\}$

†Q` †mU *(Intersection of Sets):*

`B ev Z‡ZwaK †m‡Ui mvaviY Dcv`vb wb‡q MwVZ †mU‡K †Q` †mU e‡j| g‡b Kwi, $A$ I $B$ `BwU †mU| A I B Gi †Q` †mU‡K $A \cap B$ Øviv cKvk Kiv nq Ges cov nq $A$ †Q` B ev A *intersection* B| †mU MVb c×wZ‡Z $A \cap B=\{x : x \in A$ Ges $x \in B\}$.

D`vniY 9| $P=\{x \in N : 2 < x \le 6\}$ Ges $Q=\{x \in N : x$ †Rvo msL¨v Ges $x \le 8\}$ n‡j, $P \cap Q$ wbYq Ki|



mgvavb : †`Iqv Av‡Q, $P=\{x \in N : 2 < x \le 6\}=\{3, 4, 5, 6\}$

D`vniY 11| $(2x+y, 3) = (6, x-y)$ n‡j, (x, y) wbYq Ki|

mgvavb : †`Iqv Av‡Q $(2x+y, 3) = (6, x-y)$

µg‡Rv‡oi kZg‡Z, $2x+y=6........(1)$

Ges $x-y=3.........(2)$

mgxKiY (1) I (2) †hvM K‡i cvB, $3x=9$ ev $x=3$

mgxKiY (1) G x Gi gvb emw‡q cvB, $6+y=6$ ev $y=0$

$\therefore\ (x, y)=(3, 0)$.

Kv‡Zmxq ¸YR ***(Cartesian Product) :***

Iqvsm Zvi ewoi GKwU Kvgivi wfZ‡ii †`Iqv‡j mv`v ev bxj is Ges evB‡ii †`Iqv‡j jvj ev nj` ev meR is Gi c‡jc †`Iqvi wm×vš— wb‡jb| wfZ‡ii †`Iqv‡ji is Gi †mU $A=$ {mv`v, bxj} Ges evB‡ii †`Iqv‡j is Gi †mU $B=$ {jvj, nj` I meR}| Iqvsm Zvi Kvgivi is c‡jc (mv`v, jvj), (mv`v, nj`), (mv`v, meR), (bxj, jvj), (bxj, nj`), (bxj, meR) µg‡Rvo AvKv‡i w`‡Z cv‡ib|

D³ µg‡Rv‡oi †mU‡K †jLv nq

$A\times B=$ {(mv`v, jvj), (mv`v, nj`), (mv`v, meR), (bxj, jvj), (bxj, nj`),(bxj, meR)}

GwUB Kv‡Zmxq ¸YR †mU|

†mU MVb c×wZ‡Z, $A\times B=\{(x, y); x\in A$ Ges $y\in B\}$

$A\times B$ †K cov nq A µm B ev $A\, cross\, B$.

D`vniY 12| $P=\{1, 2, 3\}$, $Q=\{3, 4\}$ Ges $R=P\cap Q$ n‡j, $P\times R$ Ges $R\times Q$ wbYq Ki|

mgvavb : †`Iqv Av‡Q, $P=\{1, 2, 3\}$, $Q=\{3, 4\}$

Ges $R=P\cap Q=\{1, 2, 3\}\cap\{3, 4\}=\{3\}$

$\therefore\ P\times R=\{1, 2, 3\}\times\{3\}=\{(1, 3), (2, 3), (3, 3)\}$

Ges $R\times Q=\{3\}\times\{3, 4\}=\{(3, 3), (3, 4)\}$

KvR : 1| $\left(\frac{x}{2}+\frac{y}{3}, 1\right)=\left(1, \frac{x}{3}+\frac{y}{2}\right)$ n‡j, (x, y) wbYq Ki|

2| $P=\{1, 2, 3\}$, $Q=\{3, 4\}$ Ges $R=\{x, y\}$ n‡j, $(P\cap Q)\times R$ Ges $(P\cap Q)\times Q$ wbYq Ki|

D`vniY 13| †h mKj ¯vfvweK msL¨v Øviv 311 Ges 419 †K fvM Ki‡j cwZ †¶‡Î 23 Aewkó _v‡K G‡`i †mU wbYq Ki|

mgvavb : †h ¯vfvweK msL¨v Øviv 311 Ges 419 †K fvM Ki‡j cwZ‡¶‡Î 23 Aewkó _v‡K, †m msL¨v n‡e 23 A‡c¶v eo Ges $311-23=288$ Ges $419-23=396$ Gi mvaviY ¸YbxqK|

g‡b Kwi, 23 A‡c¶v eo 288 Gi ¸YbxqKmg‡ni †mU A Ges 396 Gi ¸YbxqKmg‡ni †mU B

GLv‡b, $288 = 1\times 288 = 2\times 144 = 3\times 96 = 4\times 72 = 6\times 48 = 8\times 36 = 9\times 32 = 12\times 24 = 16\times 18$

$\therefore\ A = \{24, 32, 36, 48, 72, 96, 144, 288\}$

Avevi, $396 = 1\times 396 = 2\times 198 = 3\times 132 = 4\times 99 = 6\times 66 = 9\times 44 = 11\times 36 = 12\times 33 = 18\times 22$

$\therefore\ B = \{33, 36, 44, 66, 99, 132, 198, 396\}$

$\therefore\ A\cap B = \{24, 32, 36, 48, 72, 96, 144, 288\} \cap \{33, 36, 44, 66, 99, 132, 198, 396\} = \{36\}$

wb‡Yq †mU $\{36\}$

D`vniY 14| $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 6, 7\}$, $B = \{2, 3, 5, 6\}$ Ges $C = \{4, 5, 6, 7\}$ n‡j, †`LvI †h, (i) $(A\cup B)' = A'\cap B'$ Ges (ii) $(A\cap B)\cup C = (A\cup C)\cap(B\cup C)$

mgvavb : (i)

wP‡Î GKwU AvqZ‡¶Î Øviv U Ges ci¯úi‡Q`x `BwU eË‡¶Î Øviv h_vµ‡g A, B †mU‡K wb‡`k Kiv n‡jv|

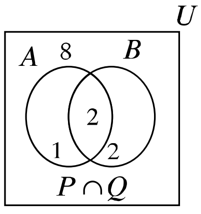
†mU	Dcv`vb
$A\cup B$	1, 2, 3, 5, 6, 7
$(A\cup B)'$	4, 8
A'	3, 4, 5, 8
B'	1, 4, 7, 8
$A'\cap B'$	4, 8

$\therefore\ (A\cup B)' = A'\cap B'$

mgvavb : (ii) wP‡Î GKwU AvqZ‡¶Î Øviv U Ges ci¯úi‡Q`x wZbwU eË‡¶Î Øviv h_vµ‡g $A, B, C$ †mU‡K wb‡`k Kiv n‡jv|

j¶ Kwi ,

†mU	Dcv`vb
$A\cap B$	2, 6
$(A\cap B)\cup C$	2, 4, 5, 6, 7
$A\cup C$	1, 2, 4, 5, 6, 7
$B\cup C$	2, 3, 4, 5, 6, 7
$(A\cup C)\cap(B\cup C)$	2, 4, 5, 6, 7



$\therefore\ (A\cap B)\cup C = (A\cup C)\cap(B\cup C)$

D`vniY 15| 100 Rb wk¶v_xi g‡a¨ †Kv‡bv cix¶vq 92 Rb evsjvq 80 Rb MwY‡Z Ges 70 Rb Dfq wel‡q cvm K‡i‡Q| †fbwP‡Îi mvnv‡h¨ Z_¨¸‡jv cKvk Ki Ges KZRb wk¶v_x Dfq wel‡q †dj K‡i‡Q, Zv wbYq Ki|

mgvavb : †fbwP‡Î AvqZvKvi †¶ÎwU 100 Rb wk¶v_xi †mU U Ges evsjvq I MwY‡Z cvm wk¶v_x‡`i †mU h_vµ‡g $B$ I $M$ Øviv wb‡`k K‡i| d‡j †fbwPÎwU PvixwU wb‡ñ` †m‡U wef³ n‡q‡Q, hv‡`i‡K P, Q, R, F Øviv wPwýZ Kiv n‡jv|

GLv‡b, Dfq wel‡q cvm wk¶v_x‡`i †mU $Q = B \cap M$, hvi m`m¨ msL¨v 70

$P =$ ïa evsjvq cvm wk¶v_x‡`i †mU, hvi m`m¨ msL¨v = $92 - 70 = 18$

$R =$ ïa MwY‡Z cvm wk¶v_x‡`i †mU, hvi m`m¨ msL¨v = $80 - 70 = 10$

$P \cup Q \cup R = B \cup M$, GK Ges Dfq wel‡q cvm wk¶v_x‡`i †mU, hvi m`m¨ msL¨v = $18 + 10 + 70 = 98$

$F =$ Dfq wel‡q †dj Kiv wk¶v_x‡`i †mU, hvi m`m¨ msL¨v = $100 - 98 = 2$

$\therefore$ Dfq wel‡q †dj K‡i‡Q 2 Rb wk¶v_x|

Abkxjbx 2·1

1| wb‡Pi †mU¸‡jv‡K ZvwjKv c×wZ‡Z cKvk Ki :

(K) $\{x \in N : x^2 > 9$ Ges $x^3 < 130\}$

(L) $\{x \in Z : x^2 > 5$ Ges $x^3 \leq 36\}$

(M) $\{x \in N : x, 36$ Gi ¸YbxqK Ges 6 Gi ¸wYZK$\}$

(N) $\{x \in N : x^3 < 25$ Ges $x^4 < 264\}$

2| wb‡Pi †mU¸‡jv‡K †mU MVb c×wZ‡Z cKvk Ki :

(K) $\{3, 5, 7, 9, 11\}$ (L) $\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

(M) $\{4, 8, 12, 16, 20, 24, 28, 32, 36, 40\}$ (N) $\{\pm 4, \pm 5, \pm 6\}$

3| $A = \{2, 3, 4\}$, $B = \{1, 2, a\}$ Ges $C = \{2, a, b\}$ n‡j, wb‡Pi †mU¸‡jv wbYq Ki :

(K) $B \setminus C$ (L) $A \cup B$ (M) $A \cap C$ (N) $A \cup (B \cap C)$ (O) $A \cap (B \cup C)$

4| $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ Ges $C = \{3, 4, 5, 6, 7\}$ n‡j, wbgœwjwLZ †¶‡Î mZ¨Zv hvPvB Ki :

(*i*) $(A \cup B)' = A' \cap B'$ (*ii*) $(B \cap C)' = B' \cup C'$

(*iii*) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ (*iv*) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

5| $Q = \{x, y\}$ Ges $R = \{m, n, \ell\}$ n‡j, $P(Q)$ Ges $P(R)$ wbYq Ki|

6| $A=\{a,b\}$, $B=\{a,b,c\}$ Ges $C=A\cup B$ n‡j, †`LvI †h, $P(C)$ Gi Dcv`vb msL¨v 2^n, †hLv‡b n n‡"Q C Gi Dcv`vb msL¨v|

7| (K) $(x-1, y+2)=(y-2, 2x+1)$ n‡j, x Ges y Gi gvb wbYq Ki|

(L) $(ax-cy, a^2-c^2)=(0, ay-cx)$ n‡j, (x, y) Gi gvb wbYq Ki|

(M) $(6x-y, 13)=(1, 3x+2y)$ n‡j, (x, y) wbYq Ki|

8| (K) $P=\{a\}$, $Q=\{b,c\}$ n‡j, $P\times Q$ Ges $Q\times P$ wbYq Ki|

(L) $A=\{3,4,5\}$, $B=\{4,5,6\}$ Ges $C=\{x,y\}$ n‡j, $(A\cap B)\times C$ wbYq Ki|

(M) $P=\{3,5,7\}$, $Q=\{5,7\}$ Ges $R=P\setminus Q$ n‡j, $(P\cup Q)\times R$ wbYq Ki|

9| A I B h_vµ‡g 35 Ges 45 Gi mKj ¸YbxqK‡Ki †mU n‡j, $A\cup B$ I $A\cap B$ wbYq Ki|

10| †h mKj ¯^vfvweK msL¨v Øviv 346 Ges 556 †K fvM Ki‡j cÖZ¨‡¶‡Î 31 Aewkó _v‡K, G‡`i †mU wbYq Ki|

11| †Kv‡bv †kÖwYi 30 Rb wk¶v_©xi g‡a¨ 20 Rb dUej Ges 15 Rb wµ‡KU †Lj‡Z cQ›` K‡i| `ywU †h †Kv‡bv GKwU †Lj‡Z cQ›` K‡i Ggb wk¶v_©xi msL¨v 10 ; KZRb wk¶v_©x `ywU †Ljv‡ZB cQ›` K‡i bv Zv †fbwP‡Îi mvnv‡h¨ wbYq Ki|

12| 100 Rb wk¶v_©xi g‡a¨ †Kv‡bv cix¶vq 65% wk¶v_©x evsjvq, 48% wk¶v_©x evsjv I Bs‡iwR Dfq wel‡q cvm Ges 15% wk¶v_©x Dfq wel‡q †dj K‡i‡Q|

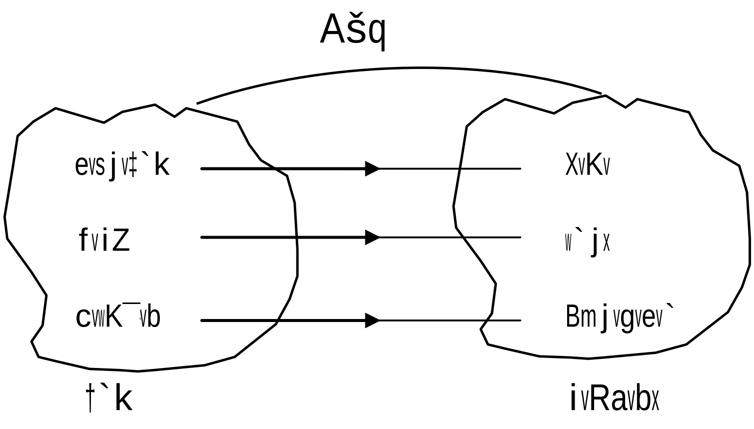
(K) msw¶ß weeiYmn Ic‡ii Z_¨¸‡jv †fbwP‡Î cÖKvk Ki|

(L) ïa evsjvq I Bs‡iwR‡Z cvm K‡i‡Q Zv‡`i msL¨v wbYq Ki|

(M) Dfq wel‡q cvm Ges Dfq wel‡q †dj msL¨vØ‡qi †gŠwjK ¸YbxqKmg‡ni †mU `yBwUi ms‡hvM †mU wbYq Ki|

Aš^q *(Relation)*

Avgiv Rvwb, evsjv‡`‡ki ivRavbx XvKv, fvi‡Zi ivRavbx w`jx Ges cvwK¯Ív‡bi ivRavbx Bmjvgvev`| GLv‡b †`‡ki mv‡_ ivRavbxi GKwU Aš^q ev m¤úK Av‡Q| G m¤úK n‡"Q †`k-ivRavbx Aš^q| D³ m¤úK‡K †mU AvKv‡i wbgœi‡c †`Lv‡bv hvq :



A_vr †`k-ivRavbxi Ašq दिल्ली), (cwK¯—vb, Bmjvgvev`)}|

hw` $A$ I $B$ `BwU †mU nq Z‡e †mUØ‡qi Kv‡Zmxq ¸YR $A \times B$ †m‡Ui AšMZ µg‡Rvo¸‡jvi Akb¨ Dc‡mU R †K A †mU n‡Z B †m‡Ui GKwU Ašq ev m¤úK ejv nq|

GLv‡b, R †mU $A \times B$ †m‡Ui GKwU Dc‡mU A_vr, $R \subseteq A \times B$

D`vniY 15| g‡b Kwi, $A = \{3, 5\}$ Ges $B = \{2, 4\}$

$\therefore \ A \times B = \{3, 5\} \times \{2, 4\} = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$

$\therefore \ R = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$

hw` $x > y$ kZ nq Z‡e, $R = \{(3, 2), (5, 2), (5, 4)\}$

Ges hw` $x < y$ kZ nq Z‡e, $R = \{3, 4\}$

hLb A †m‡Ui GKwU Dcv`vb $x$ I $B$ †m‡Ui GKwU Dcv`vb y Ges $(x, y) \in R$ nq, Z‡e †jLv nq $x\,R\,y$ Ges cov nq x, y Gi mv‡_ AwšZ (x *is related to* y) A_vr Dcv`vb $x$, Dcv`vb y Gi mv‡_ R m¤úKh³|

Avevi, A †mU n‡Z A †m‡Ui GKwU Ašq A_vr $R \subseteq A \times A$ n‡j, R †K A Gi Ašq ejv nq|

mZivs A Ges B `BwU †m‡Ui Dcv`vb¸‡jvi g‡a¨ m¤úK †`Iqv _vK‡j $x \in A$ Gi m‡½ m¤úwKZ $y \in B$ wb‡q †h me µg‡Rvo $(x, y)$ cvIqv hvq, G‡`i Akb¨ Dc‡mU n‡"Q GKwU Ašq|

D`vniY 16| hw` $P = \{2, 3, 4\}$, $Q = \{4, 6\}$ Ges P I Q Gi Dcv`vb¸‡jvi g‡a¨ $y = 2x$ m¤úK we‡ePbvq _v‡K Z‡e Ašq wbYq Ki|

mgvavb : †`Iqv Av‡Q, $P = \{2, 3, 4\}$ Ges $Q = \{4, 6\}$

ckvbmv‡i, $R = \{(x, y) : x \in P, \ y \in Q$ Ges $y = 2x\}$

GLv‡b, $P \times Q = \{2, 3, 4\} \times \{4, 6\} = \{(2, 4), (2, 6), (3, 4), (3, 6), (4, 4), (4, 6)\}$

$\therefore \ R = \{(2, 4), (3, 6)\}$

wb‡Yq Ašq $\{(2, 4), (3, 6)\}$

D`vniY 17| hw` $A = \{1, 2, 3\}$, $B = \{0, 2, 4\}$ Ges C I D Gi Dcv`vb¸‡jvi g‡a¨ $x = y - 1$ m¤úK we‡ePbvq _v‡K, Z‡e Ašq eYbv Ki|

mgvavb : †`Iqv Av‡Q, $A = \{1, 2, 3\}$, $B = \{0, 2, 4\}$

ckvbmv‡i, Ašq $R = \{(x, y) : x \in A, \ y \in B$ Ges $x = y - 1\}$

GLv‡b, $A \times B = \{1, 2, 3\} \times \{0, 2, 4\}$

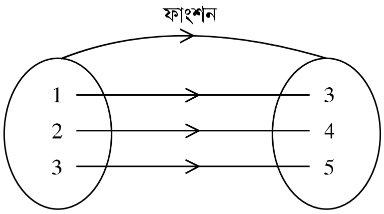
$= \{(1, 0), (1, 2), (1, 4), (2, 0), (2, 2), (2, 4), (3, 0), (3, 2), (3, 4)\}$

$\therefore \ R = \{(1, 2), (3, 4)\}$

KvR : hw` $C=\{2,5,6\}$, $D=\{4,5\}$ Ges $C$ I $D$ Gi Dcv`vb¸‡jvi g‡a¨ $x \le y$ m¤úK we‡ePbvq _v‡K Z‡e Ašq wbYq Ki|

dvskb *(Function)* :

wb‡Pi A I B †m‡Ui Ašq j¶ Ki :



GLv‡b, hLb $y=x+2$, ZLb $x=1$ n‡j, $y=3$

$x=2$ n‡j, $y=4$

$x=3$ n‡j, $y=5$

A_vr x Gi GK-GKwU gv‡bi Rb¨ y Gi gvÎ GKwU gvb cvIqv hvq Ges x I y-Gi g‡a¨ m¤úK ˆZwi nq $y=x+2$ Øviv| mZivs `BwU PjK $x$ Ges $y$ Ggbfv‡e m¤úKh³ †hb $x$ Gi †h‡Kv‡bv GKwU gv‡bi Rb¨ $y$ Gi GKwU gvÎ gvb cvIqv hvq, Z‡e $y$ †K $x$ Gi dvskb ejv nq| $x$ Gi dvskb‡K mvaviYZ $y$, $f(x)$, $g(x)$, $F(x)$ BZ¨wh` Øviv cKvk Kiv nq|

g‡b Kwi, $y=x^2-2x+3$ GKU dvskb| GLv‡b, x Gi †h †Kv‡bv GKwU gv‡bi Rb¨ y Gi GKU gvÎ gvb cvIqv hv‡e| GLv‡b, x Ges y DfqB PjK Z‡e, x Gi gv‡bi Dci y Gi gvb wbfikxj| Kv‡RB x n‡”Q ¯vaxb PjK Ges y n‡”Q Aaxb PjK|

D`vniY 18| $f(x)=x^2-4x+3$ n‡j, $f(-1)$ wbYq Ki|

mgvavb : †`Iqv Av‡Q, $f(x)=x^2-4x+3$

$\therefore\ f(-1)=(-1)^2-4(-1)+3=1+4+3=8$

D`vniY 19| hw` $g(x)=x^3+ax^2-3x-6$ nq, Z‡e a Gi †Kvb gv‡bi Rb¨ $g(-2)=0$ n‡e ?

mgvavb : †`Iqv Av‡Q, $g(x)=x^3+ax^2-3x-6$

$\therefore\ g(-2)=(-2)^3+a(-2)^2-3(-2)-6$

$=-8+4a+6-6$

$=-8+4a=4a-8$

wKš' $g(-2)=0$

$\therefore\ 4a-8=0$

ev $4a=8$

ev $a=2$

$\therefore\ a=2$ n‡j, $g(-2)=0$ n‡e|

†Wv‡gb ***(Domain)*** I †iÄ ***(Range)***

†Kv‡bv Aš‡qi µg‡Rvo¸‡jvi c_g Dcv`vbmg‡ni †mU‡K Gi †Wv‡gb Ges wØZxq Dcv`vbmg‡ni †mU‡K Gi †iÄ ejv nq|

g‡b Kwi, A †mU †_‡K B †m‡U R GKwU Aš‡q A_©vr $R \subseteq A \times B$. RG Ašf³ µg‡Rvo¸‡jvi c_g Dcv`vb †mU n‡e $R$ Gi †Wv‡gb Ges wØZxq Dcv`vbmg‡ni †mU n‡e R Gi †iÄ| R Gi †Wv‡gb‡K †Wvg R Ges †iÄ‡K †iÄ R wj‡L cKvk Kiv nq|

D`vniY 20| Aš‡q $S = \{(2,1), (2,2), (3,2), (4,5)\}$ Aš‡qwUi †Wv‡gb I †iÄ wbY©q Ki|

mgvavb : †`Iqv Av‡Q, $S = \{(2,1), (2,2), (3,2), (4,5)\}$

S Aš‡q µg‡Rvo¸‡jvi c_g Dcv`vbmgn $2, 2, 3, 4$ Ges wØZxq Dcv`vbmgn $1, 2, 2, 5$.

$\therefore$ †Wvg $S = \{2, 3, 4\}$ Ges †iÄ $S = \{1, 2, 5\}$

D`vniY 21| $A = \{0, 1, 2, 3\}$ Ges $R = \{(x, y) : x \in A, y \in A$ Ges $y = x + 1\}$ n‡j, R †K ZvwjKv c×wZ‡Z cKvk Ki Ges †Wvg R I †iÄ R wbY©q Ki|

mgvavb : †`Iqv Av‡Q, $A = \{0, 1, 2, 3\}$ Ges $R = \{(x, y) : x \in A, y \in A$ Ges $y = x + 1\}$

R Gi ewY©Z kZ© †_‡K cvB, $y = x + 1$

GLb, c‡Z¨K $x \in A$ Gi Rb¨ $y = x + 1$ Gi gvb wbY©q Kwi|

x	0	1	2	3
y	1	2	3	4

†h‡nZz $4 \notin A$, Kv‡RB $(3, 4) \notin R$

$\therefore$ $R = \{(0,1), (1,2), (2,3)\}$

†Wvg $R = \{0, 1, 2\}$ Ges †iÄ $R = \{1, 2, 3\}$

KvR : 1| $S = \{(-3,8), (-2,3), (-1,0), (0,-1), (1,0), (2,3)\}$ n‡j, S Gi †Wv‡gb I †iÄ wbY©q Ki|

2| $S = \{x, y) : x \in A, y \in A$ Ges $y - x = 1\}$, †hLv‡b $A = \{-3, -2, -1, 0\}$ | †Wvg S I †iÄ S wbY©q Ki|

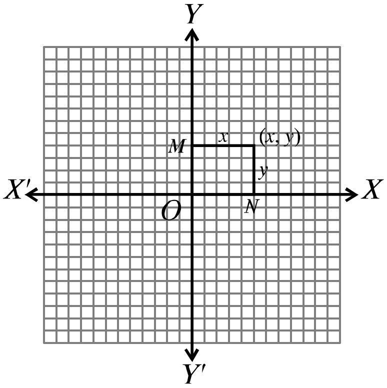
dvsk‡bi †jLwPÎ ***(Graphs)***

dvsk‡bi wPÎiƒc‡K †jLwPÎ ejv nq| dvsk‡bi aviYv m¯úó Kivi †¶‡Î †jLwP‡Îi ¸iæZ Acwimxg| divwm `vkwbK I MwYZwe` †i‡b †`KvZ (Rene Descartes : 1596–1650) mec_g exRMwYZ I R¨vwgwZi g‡a¨ m¤úK© ¯vc‡b AMÖYx fwgKv cvjb K‡ib| wZwb †Kv‡bv mgZ‡j ci¯úi j¤fv‡e †Q`x `BwU dvsk‡bi mvnv‡h¨

we›`i Ae¯'vb mwbw`©óf‡e wbY©‡qi gva¨‡g mgZ‡j Ae¯'vb wbY©q Kiv nq| ZvwË¡K avivi ceZb K‡ib| wZwb ci¯úi j¤fv‡e †Q`x `BwU ‡iLvi mvnv‡h¨ wKQy we›`yi Ae¯'vb wbY©q Kivi avibv †`b| mgZ‡j ci¯úi j¤fv‡e †Q`x `BwU mij‡iLv `BwU‡K A¶‡iLv wn‡m‡e AvL¨vwqZ K‡ib Ges A¶Ø‡qi †Q` we›`y‡K gyjwe›`y ej‡jb|

†Kv‡bv mgZ‡j ci¯úi j¤fv‡e †Q`x `BwU mij‡iLv XOX' Ges YOY' AvKv n‡jv| mgZ‡j Ae¯'vbiZ †h‡Kv‡bv we›`i Ae¯'vb GB †iLvØ‡qi gva¨‡g m¤úYi/‡c Rvbv m¤e| GB †iLvØ‡qi c‡Z¨K‡U‡K A¶ (*axis*) ejv nq| Abf‚wgK †iLv $XOX'$†K $x$-A¶, Dj¤ †iLv $YOY'$ †K $y$-A¶ Ges A¶Ø‡qi †Q`we›` $O$ †K gjwe›` (Origin) ejv nq|

`BwU A‡¶i mgZ‡j Ae¯'Z †Kv‡bv we›` †_‡K A¶Ø‡qi j¤ `i‡Zi h_vh_ wPýh³ msL¨v‡K H we›`i ¯'vbv¼ ejv nq| g‡b Kwi, A¶Ø‡qi mgZ‡j Ae¯'Z P †h †Kv‡bv we›`| $P$ †_‡K $XOX'$ Ges $YOY'$ Gi Dci h_vµ‡g $PN$ I $PM$ j¤ Uvwb| d‡j, $PM = ON$ hv $YOY'$ n‡Z $P$ we›`i j¤ `iZ Ges $PN = OM$ hv $XOX'$n‡Z $P$ we›`i j¤ `iZ| hw` $PM = x$ Ges $PN = y$ nq, Z‡e P we›`i ¯'vbv¼ (x, y)| GLv‡b, x †K fR (*abscissa*) ev x ¯'vbv¼ Ges y †K †KwU (*Ordinate*) ev y ¯'vbv¼ ejv nq| D‡jwLZ ¯'vbv¼‡K Kv‡Zmxq ¯'vbv¼ ejv nq|



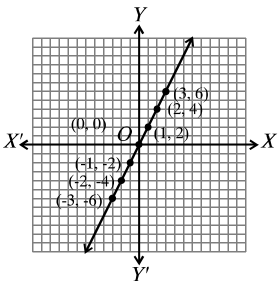
Kv‡Zmxq ¯'vbv‡¼ mn‡RB dvsk‡bi R¨wgwZK wPÎ †`Lv‡bv hvq| GRb¨ mvaviYZ x A¶ eivei ¯'vaxb Pj‡Ki gvb I y A¶ eivei Aaxb Pj‡Ki gvb emv‡bv nq|

$y = f(x)$ dvsk‡bi †jLwPÎ A¼‡bi Rb¨ †Wv‡gb †_‡K ¯'vaxb Pj‡Ki K‡qKwU gv‡bi Rb¨ Aaxb Pj‡Ki Abi/c gvb¸‡jv †ei K‡i µg‡Rvo ˆZwi Kwi| AZtci µg‡Rvo¸‡jv $x - y$ Z‡j ¯'vcb Kwi| cvß we›`¸‡jv g³ n‡¯Z †iLv †U‡b h³ Kwi, hv $y = f(x)$ dvsk‡bi †jLwPÎ|

D`vniY 22| $y = 2x$ dvsk‡bi †jLwPÎ A¼b Ki| †hLv‡b, $-3 \le x \le 3$

mgvavb : $-3 \le x \le 3$ †Wv‡g‡bi x-Gi K‡qKwU gv‡bi Rb¨ y Gi K‡qKwU gvb wbY©q K‡i ZvwjKv ˆZwi Kwi|

x	–3	–2	–1	0	1	2	3
y	–6	–4	–2	0	2	4	6



QK KvM‡R cwZ ¶`ª e‡M©i evûi K‡K GKK a‡i, ZvwjKvq we›`¸‡jv wPwýZ Kwi I g³ n‡¯Í †hvM Kwi|

D`vniY 23| hw` $f(x)=\dfrac{3x+1}{3x-1}$ nq, Z‡e $\dfrac{f\left(\frac{1}{x}\right)+1}{f\left(\frac{1}{x}\right)-1}$ Gi gvb wbYq Ki|

mgvavb : †`Iqv Av‡Q, $f(x)=\dfrac{3x+1}{3x-1}$

$\therefore\ f\left(\dfrac{1}{x}\right)=\dfrac{3\cdot\frac{1}{x}+1}{3\cdot\frac{1}{x}-1}=\dfrac{\frac{3}{x}+1}{\frac{3}{x}-1}=\dfrac{3+x}{3-x}$ [je I ni‡K x Øviv ¸Y K‡i]

ev , $\dfrac{f\left(\frac{1}{x}\right)+1}{f\left(\frac{1}{x}\right)-1}$ = $\dfrac{3+x+3-x}{3+x-3+x}$ [†hvRb-we‡qvRb K‡i]

= $\dfrac{6}{2x}=\dfrac{3}{x}$ wb‡Yq gvb $\dfrac{3}{x}$

D`vniY 24| hw` $f(y)=\dfrac{y^3-3y^2+1}{y(1-y)}$ nq, Z‡e †`LvI †h, $f\left(\dfrac{1}{y}\right)=f(1-y)$

mgvavb : †`Iqv Av‡Q, $f(y)=\dfrac{y^3-3y^2+1}{y(1-y)}$

$\therefore\ f\left(\dfrac{1}{y}\right)=\dfrac{\left(\frac{1}{y}\right)^3-3\left(\frac{1}{y}\right)^2+1}{\frac{1}{y}\left(1-\frac{1}{y}\right)}=\dfrac{\frac{1-3y+y^3}{y^3}}{\frac{y-1}{y^2}}$

= $\dfrac{1-3y+y^3}{y^3}\times\dfrac{y^2}{y-1}=\dfrac{1-3y+y^3}{y(y-1)}$

Avevi, $f(1-y)=\dfrac{(1-y)^3-3(1-y)^2+1}{(1-y)\{1-(1-y)\}}$

= $\dfrac{1-3y+3y^2-y^3-3(1-2y+y^2)+1}{(1-y)(1-1+y)}$

= $\dfrac{1-3y+3y^2-y^3-3+6y-3y^2+1}{y(1-y)}$

= $\dfrac{-1+3y-y^3}{y(1-y)}=\dfrac{-(1-3y+y^3)}{-y(y-1)}$

= $\dfrac{1-3y+y^3}{y(y-1)}$

$\therefore\ f\left(\dfrac{1}{y}\right)=f(1-y)$.

Abkxjbx 2·2

1| 8 Gi ¸YbxqK †mU †KvbwU ?

(K) $\{8, 16, 24, ...\}$ (L) $\{1, 2, 3, 4, 8\}$ (M) $\{2, 4, 8\}$ (N) $\{1, 2\}$

2| †mU C n‡Z †mU B G GKwU m¤úK R n‡j wb‡Pi †KvbwU mwVK ?

(K) $R \subset C$ (L) $R \subset B$ (M) $R \subseteq C \times B$ (N) $C \times B \subseteq R$

3| $A = \{6, 7, 8, 9, 10, 11, 12, 13\}$ n‡j, wb‡Pi ck¸‡jvi DËi `vI :

(i) A †m‡Ui MVb c×wZ †KvbwU ?

4| hw` $A = \{3, 4\}$, $B = \{2, 4\}$ nq, Z‡e $A$ I $B$ Gi Dcv`vb¸‡jvi g‡a¨ $x > y$ m¤úK wb‡Y©q K‡i wi‡jkbwU wbYq Ki|

5| hw` $C = \{2, 5\}$, $D = \{4, 6\}$ Ges $C$ I $D$ Gi Dcv`vb¸‡jvi g‡a¨ $x + 1 < y$ m¤úKwU we‡ePbvq _v‡K Z‡e wi‡jkbwU wbYq Ki|

6| $f(x) = x^4 + 5x - 3$ n‡j, $f(-1)$, $f(2)$ Ges $f\left(\frac{1}{2}\right)$ Gi gvb wbYq Ki|

7| hw` $f(y) = y^3 + ky^3 - 4y - 8$ nq, Z‡e k Gi †Kvb gv‡bi Rb¨ $f(-2) = 0$ n‡e ?

8| $f(x) = x^3 - 6x^2 + 11x - 6$ n‡j, x Gi †Kvb gv‡bi Rb¨ $f(x) = 0$ n‡e ?

9| hw` $f(x) = \dfrac{2x+1}{2x-1}$ nq, Z‡e $\dfrac{f\left(\frac{1}{x^2}\right)+1}{f\left(\frac{1}{x^2}\right)-1}$ Gi gvb wbYq Ki|

10| $g(x) = \dfrac{1+x^2+x^4}{x^2}$ n‡j, †`LvI †h, $g\left(\frac{1}{x^2}\right) = g(x^2)$

11| wb‡Pi Ašq¸‡jv †_‡K †Wv‡gb Ges †iÄ wbYq Ki :

(K) $R = \{(2, 1), (2, 2), (2, 3)\}$ (L) $S = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

(M) $F = \left\{\left(\frac{1}{2}, 0\right), (1, 1), (1, -1), \left(\frac{5}{2}, 2\right), \left(\frac{5}{2}, -2\right)\right\}$

12| wb‡Pi Ašq¸‡jv‡K ZvwjKv c×wZ‡Z cKvk Ki Ges †Wv‡gb I †iÄ wbYq Ki :

(K) $R = \{(x, y) : x \in A, \ y \in A$ Ges $x + y = 1\}$, †hLv‡b $A = \{-2, -1, 0, 1, 2\}$

(L) $F = \{(x, y) : x \in C, \ y \in C$ Ges $x = 2y\}$, †hLv‡b $C = \{-1, 0, 1, 1, 3\}$

13| QK KvM‡R $(-3, 2)$, $(0, -5)$, $\left(\frac{1}{2}, -\frac{5}{6}\right)$ we›`¸‡jv ¯vcb Ki|

14| QK KvM‡R $(1, 2)$, $(-1, 1)$, $(11, 7)$ we›` wZbwU ¯vcb K‡i †`LvI †h, we›` wZbwU GKB mij‡iLvq Aew¯Z|

(K) $\{x \in N:\ 6 < x < 13\}$ (L) $\{x \in N:\ 6 \le x < 13\}$

(M) $\{x \in N:\ 6 \le x \le 13\}$ (N) $\{x \in N:\ 6 < x \le 13\}$

(ii) †gŠwjK msL¨v¸‡jvi †mU †KvbwU ?

(K) $\{6, 8, 10, 12\}$ (L) $\{7,\ 9, 11, 13\}$ (M) $\{7,\ 11,\ 13\}$ (N) $A = \{9,\ 12\}$

(iii) 3 Gi ¸wYZK¸‡jvi †mU †KvbwU ?

(K) $\{6,\ 9\}$ (L) $\{6,\ 11\}$ (M) $\{9,\ 12\}$ (N) $\{6,\ 9,\ 12\}$

(iv) enËg †Rvo msL¨vi ¸Ybxq‡Ki †mU †KvbwU ?

(K) $\{1,\ 13\}$ (L) $\{1,\ 2,\ 3,\ 6\}$ (M) $\{1,\ 3,\ 9\}$ (N) $\{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$

15. mvweK †mU U ={ x:x $\in$ N Ges x we‡Rvo msL¨v}

A = {x $\in$ N : $2 \le x \le 7$

B = { x $\in$ N : 3 < x <6}

C = { x $\in$ N : $x^2 > 5$ Ges $x^3 < 130$}

K. A †mU‡K ZvwjKv c×wZ‡Z cKvk Ki|

L. A' Ges C – B wbYq Ki|

M. B×C Ges P(A $\cap$ C) wbYq Ki|

ZZxq Aa¨vq
exRMwYwZK ivwk
(Algebraical Expressions)

exRMwY‡Z A‡bK mgm¨v mgvav‡b exRMwYwZK mÎ e¨eüZ nq| Avevi A‡bK exRMwYwZK ivwk we‡kølY K‡i Drcv`‡Ki gva¨‡g Dc¯vcb Kiv n‡q _v‡K| ZvB G Aa¨v‡q exRMwYwZK m‡Îi mvnv‡h¨ mgm¨v mgvavb Ges ivwk‡K Drcv`‡K we‡kølY welqK welqe¯ wk¶v_x Dc‡hvMx K‡i Dc¯vcb Kiv n‡q‡Q| AwaKš bvbwea MwYwZK mgm¨v exRMwYwZK m‡Îi mvnv‡h¨ Drcv`‡K we‡kølY K‡i I mgvavb Kiv hvq| c‡ei †kwY‡Z exRMwYwZK mÎvewj I G‡`i mv‡_ m¤ú³ Abywm×vš¸‡jv m¤‡Ü we¯wiZ Av‡jvPbv Kiv n‡q‡Q| G Aa¨v‡q H¸‡jv cbi"‡jøL Kiv n‡jv Ges D`vni‡Yi gva¨‡g G‡`i KwZcq c‡qvM †`Lv‡bv n‡jv| GQvovI G Aa¨v‡q eM I N‡bi m¤cmviY, fvM‡kl Dccv`¨ c‡qvM K‡i Drcv`‡K we‡kølY Ges ev¯e mgm¨v mgvav‡b exRMwYwZK m‡Îi MVb I c‡qvM m¤ú‡K we¯wiZ Av‡jvPbv Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv –

- exRMwYwZK mÎ c‡qvM K‡i eM I N‡bi m¤cmviY Ki‡Z cvi‡e|
- fvM‡kl Dccv`¨ Kx e¨vL¨v Ki‡Z cvi‡e Ges Zv c‡qvM K‡i Drcv`‡K we‡kølY Ki‡Z cvi‡e|
- ev¯e mgm¨v mgvav‡bi Rb¨ exRMwYwZK mÎ MVb Ki‡Z cvi‡e Ges mÎ c‡qvM K‡i mgm¨v mgvavb Ki‡Z cvi‡e|

3·1 exRMwYwZK ivwk

cÖwµqv wPý Ges msL¨vwb‡`kK A¶i cZxK Gi A_‡evaK web¨vm‡K exRMwYwZK ivwk ejv nq| †hgb, $2a+3b-4c$ GKwU exRMwYwZK ivwk| exRMwYwZK ivwk‡Z $a, b, c, p, q, r, m, n, x, y, z, .....$ BZ¨vw` eYgvjvi gva¨‡g wewfbœ Z_¨ cKvk Kiv nq| exRMwYwZK ivwk msewjZ wewfbœ mgm¨v mgvav‡b GB mg¯ eYgvjv‡K e¨envi Kiv nq| cvwUMwY‡Z ïa abvZ¥K msL¨v e¨eüZ nq, Ab¨w`‡K exRMwY‡Z kb¨mn abvZ¥K I FYvZ¥K mKj msL¨v e¨envi Kiv nq| exRMwYZ‡K cvwUMwY‡Zi mevqbKZ iƒc ejv nq| exRMwYwZK ivwk‡Z e¨eüZ msL¨v¸‡jv aeK *(constant)*, G‡`i gvb wbw`ó|

exRMwYwZK ivwk‡Z e¨eüZ A¶i cZxK¸‡jv PjK *(variables)*, G‡`i gvb wbw`ó bq, Giv wewfbœ gvb avi‡Y Ki‡Z cv‡i|

3·2 exRMwYwZK mÎvewj

exRMwYwZK cZxK Øviv cKvwkZ †h‡Kv‡bv mvaviY wbqg ev wm×vš‡K exRMwYwZK mÎ ejv nq| mßg I Aóg †kwY‡Z exRMwYwZK mÎvewj I GZ`msµvš Abywm×vš¸‡jv m¤‡Ü Av‡jvPbv Kiv n‡q‡Q| G Aa¨v‡q H¸‡jv cbi"‡jøL K‡i KwZcq c‡qvM †`Lv‡bv n‡jv|

mÎ 1| $(a+b)^2 = a^2 + 2ab + b^2$

mÎ 2| $(a-b)^2 = a^2 - 2ab + b^2$

gšše¨ : mÎ 1 I mÎ 2 n‡Z †`Lv hvq †h, $a^2 + b^2$ Gi mv‡_ $2ab$ A_ev $-2ab$ †hvM Ki‡j GKwU cYeM, A_vr $(a+b)^2$ A_ev $(a-b)^2$ cvIqv hvq| mÎ 1 G b Gi ¯'‡j $-b$ emv‡j mÎ 2 cvIqv hvq :

$$\{a+(-b)\}^2 = a^2 + 2a(-b) + (-b)^2$$

A_vr, $(a-b)^2 = a^2 - 2ab + b^2$.

Abywm×vš— 1| $a^2 + b^2 = (a+b)^2 - 2ab$

Abywm×vš— 2| $a^2 + b^2 = (a-b)^2 + 2ab$

Abywm×vš— 3| $(a+b)^2 = (a-b)^2 + 4ab$

cgvY : $(a+b)^2 = a^2 + 2ab + b^2$

$$= a^2 - 2ab + b^2 + 4ab$$

$$= (a-b)^2 + 4ab$$

Abywm×vš— 4| $(a-b)^2 = (a+b)^2 - 4ab$

cgvY : $(a-b)^2 = a^2 - 2ab + b^2$

$$= a^2 + 2ab + b^2 - 4ab$$

$$= (a+b)^2 - 4ab$$

Abywm×vš— 5| $a^2 + b^2 = \dfrac{(a+b)^2 + (a-b)^2}{2}$

cgvY : mÎ 1 I mÎ 2 n‡Z,

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

†hvM K‡i, $2a^2 + 2b^2 = (a+b)^2 + (a-b)^2$

ev, $2(a^2 + b^2) = (a+b)^2 + (a-b)^2$

mZivs, $(a^2 + b^2) = \dfrac{(a+b)^2 + (a-b)^2}{2}$

Abywm×vš— 6| $ab = \left(\dfrac{a+b}{2}\right)^2 - \left(\dfrac{a-b}{2}\right)^2$

cgvY : mÎ 1 I mÎ 2 n‡Z,

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

we‡hvM K‡i, $4ab = (a+b)^2 - (a-b)^2$

$(iii)\ (a-b-c)^2 = \{a+(-b)+(-c)\}^2$

$= a^2+(-b)^2+(-c)^2+2a(-b)+2(-b)(-c)+2a(-c)$

$= a^2+b^2+c^2-2ab+2bc-2ac$

D`vniY 1| $(4x+5y)$ Gi eM KZ ?

mgvavb : $(4x+5y)^2 = (4x)^2+2\times(4x)\times(5y)+(5y)^2$

$= 16x^2+40xy+25y^2$

D`vniY 2| $(3a-7b)$ Gi eM KZ ?

mgvavb : $(3a-7b)^2 = (3a)^2-2\times(3a)\times(7b)+(7b)^2$

$= 9a^2-42ab+49b^2$

D`vniY 3| e‡Mi m~Î c‡qvM K‡i 996 Gi eM wbYq Ki|

mgvavb : $(996)^2 = (1000-4)^2$

$= (1000)^2-2\times1000\times4+(4)^2$

$= 1000000-8000+16 = 1000016-8000$

$= 992016$

D`vniY 4| $a+b+c+d$ Gi eM KZ ?

mgvavb : $(a+b+c+d)^2 = \{(a+b)+(c+d)\}^2$

$= (a+b)^2+2(a+b)(c+d)+(c+d)^2$

$= a^2+2ab+b^2+2(ac+ad+bc+bd)+c^2+2cd+d^2$

$= a^2+2ab+b^2+2ac+2ad+2bc+2bd+c^2+2cd+d^2$

$= a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd$

KvR : m‡Îi mvnv‡h¨ eM wbYq Ki :

1| $3xy+2ax$ 2| $4x-3y$ 3| $x-5y+2z$

D`vniY 5| mij Ki : $(5x+7y+3z)^2+2(7x-7y-3z)(5x+7y+3z)+(7x-7y-3z)^2$

mgvavb : awi, $5x+7y+3z=a$ Ges $7x-7y-3z=b$

$\therefore$ c`Ë ivwk $= a^2+2.b.a+b^2$

$= a^2+2ab+b^2$

$= (a+b)^2$

$= \{(5x+7y+3z)+(7x-7y-3z)\}^2$ [a I b Gi gvb ewm‡q]

$= (5x+7y+3z+7x-7y-3z)^2$

$= (12x)^2$

$= 144x^2$

$(iii)\ (a-b-c)^2=\{a+(-b)+(-c)\}^2$

$= a^2+(-b)^2+(-c)^2+2a(-b)+2(-b)(-c)+2a(-c)$

$= a^2+b^2+c^2-2ab+2bc-2ac$

D`vniY 1| $(4x+5y)$ Gi eM KZ ?

mgvavb : $(4x+5y)^2=(4x)^2+2\times(4x)\times(5y)+(5y)^2$

$= 16x^2+40xy+25y^2$

D`vniY 2| $(3a-7b)$ Gi eM KZ ?

mgvavb : $(3a-7b)^2=(3a)^2-2\times(3a)\times(7b)+(7b)^2$

$= 9a^2-42ab+49b^2$

D`vniY 3| e‡Mi mÎ c‡qvM K‡i 996 Gi eM wbYq Ki|

mgvavb : $(996)^2=(1000-4)^2$

$= (1000)^2-2\times1000\times4+(4)^2$

$= 1000000-8000+16=1000016-8000$

$= 992016$

D`vniY 4| $a+b+c+d$ Gi eM KZ ?

mgvavb : $(a+b+c+d)^2=\{(a+b)+(c+d)\}^2$

$= (a+b)^2+2(a+b)(c+d)+(c+d)^2$

$= a^2+2ab+b^2+2(ac+ad+bc+bd)+c^2+2cd+d^2$

$= a^2+2ab+b^2+2ac+2ad+2bc+2bd+c^2+2cd+d^2$

$= a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd$

KvR : m‡Îi mvnv‡h¨ eM wbYq Ki :

1| $3xy+2ax$ 2| $4x-3y$ 3| $x-5y+2z$

D`vniY 5| mij Ki : $(5x+7y+3z)^2+2(7x-7y-3z)(5x+7y+3z)+(7x-7y-3z)^2$

mgvavb : awi, $5x+7y+3z=a$ Ges $7x-7y-3z=b$

$\therefore$ c`Ë ivwk = $a^2+2.b.a+b^2$

$= a^2+2ab+b^2$

$= (a+b)^2$

$= \{(5x+7y+3z)+(7x-7y-3z)\}^2$ [a I b Gi gvb ewm‡q]

$=(5x+7y+3z+7x-7y-3z)^2$

$= (12x)^2$

$= 144x^2$

D`vniY 6| $x - y = 2$ Ges $xy = 24$ n‡j, $x + y$ Gi gvb KZ ?

mgvavb : $(x+y)^2 = (x-y)^2 + 4xy = (2)^2 + 4 \times 24 = 4 + 96 = 100$

$\therefore \quad x + y = \pm\sqrt{100} = \pm 10$

D`vniY 7| hw` $a^4 + a^2b^2 + b^4 = 3$ Ges $a^2 + ab + b^2 = 3$ nq, Z‡e $a^2 + b^2$ Gi gvb KZ ?

mgvavb : $a^4 + a^2b^2 + b^4 = (a^2)^2 + 2a^2b^2 + (b^2)^2 - a^2b^2$

$= (a^2 + b^2)^2 - (ab)^2$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

$= (a^2 + ab + b^2)(a^2 - ab + b^2)$

$\therefore \quad 3 = 3(a^2 - ab + b^2)$ [gvb ewm‡q]

ev, $a^2 - ab + b^2 = \frac{3}{3} = 1$

GLb, $a^2 + ab + b^2 = 3$ Ges $a^2 - ab + b^2 = 1$ †hvM K‡i cvB, $2(a^2 + b^2) = 4$

ev, $a^2 + b^2 = \frac{4}{2} = 2$

$\therefore \; a^2 + b^2 = 2$

D`vniY 8| cgvY Ki †h, $(a+b)^4 - (a-b)^4 = 8ab(a^2 + b^2)$

mgvavb : $(a+b)^4 - (a-b)^4 = \{(a+b)^2\}^2 - \{(a-b)^2\}^2$

$= \{(a+b)^2 + (a-b)^2\}\{(a+b)^2 - (a-b)^2\}$

$= 2(a^2 + b^2) \times 4ab$ [$\because (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$ Ges $(a+b)^2 - (a-b)^2 = 4ab$]

$= 8ab(a^2 + b^2)$

$\therefore \; (a+b)^4 - (a-b)^4 = 8ab(a^2 + b^2)$

D`vniY 9| $a + b + c = 15$ Ges $a^2 + b^2 + c^2 = 83$ n‡j, $ab + bc + ac$ Gi gvb KZ ?

mgvavb : GLv‡b, $2(ab + bc + ac)$

$= (a+b+c)^2 - (a^2 + b^2 + c^2)$

$= (15)^2 - 83$

$= 225 - 83$

$= 142$

$\therefore \; ab + bc + ac = \frac{142}{2} = 71$

weKí c×wZ :

Avgiv Rvwb,

$(a+b+c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ac)$

ev, $(15)^2 = 83 + 2(ab + bc + ac)$

ev, $225 - 83 = 2(ab + bc + ac)$

ev, $2(ab + bc + ac) = 142$

$\therefore \; ab + bc + ac = \frac{142}{2} = 71$

D`vniY 10| $a+b+c=2$ Ges $ab+bc+ac=1$ n‡j, $(a+b)^2+(b+c)^2+(c+a)^2$ Gi gvb KZ ?

mgvavb : $(a+b)^2+(b+c)^2+(c+a)^2$

$= a^2+2ab+b^2+b^2+2bc+c^2+c^2+2ca+a^2$

$= (a^2+b^2+c^2+2ab+2bc+2ca)+(a^2+b^2+c^2)$

$= (a+b+c)^2+\{(a+b+c)^2-2(ab+bc+ac)\}$

$= (2)^2+(2)^2-2\times1$

$= 4+4-2=8-2=6$

D`vniY 11| $(2x+3y)(4x-5y)$ †K `BwU e‡Mi we‡qvMdj iƒ‡c cKvk Ki|

mgvavb : awi, $2x+3y=a$ Ges $4x-5y=b$

$\therefore$ c`Ë ivwk = $ab=\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2$

$= \left(\frac{2x+3y+4x-5y}{2}\right)^2-\left(\frac{2x+3y-4x+5y}{2}\right)^2$ [a I b Gi gvb ewm‡q]

$= \left(\frac{6x-2y}{2}\right)^2-\left(\frac{8y-2x}{2}\right)^2$

$= \left(\frac{2(3x-y)}{2}\right)^2-\left(\frac{2(4y-x)}{2}\right)^2$

$= (3x-y)^2-(4y-x)^2$

$\therefore\ (2x+3y)(4x-5y)=(3x-y)^2-(4y-x)^2$

KvR : 1| mij Ki : $(4x+3y)^2+2(4x+3y)(4x-3y)+(4x-3y)^2$

2| $x+y+z=12$ Ges $x^2+y^2+z^2=50$ n‡j, $(x-y)^2+(y-z)^2+(z-x)^2$ Gi gvb wbYq Ki|

Abkxjbx 3·1

1| m‡Î i mvnv‡h¨ eM wbYq Ki :

(K) $2a+3b$ (L) $2ab+3bc$ (M) $x^2+\frac{2}{y^2}$ (N) $a+\frac{1}{a}$ (O) $4y-5x$ (P) $ab-c$

(Q) $5x^2-y$ (R) $x+2y+4z$ (S) $3p+4q-5r$ (T) $3b-5c-2a$ (U) $ax-by-cz$

(V) $a-b+c-d$ (W) $2a+3x-2y-5z$ (X) 101 (Y) 997 (Z) 1007

2| mij Ki :

(K) $(2a+7)^2+2(2a+7)(2a-7)+(2a-7)^2$

(L) $(3x+2y)^2+2(3x+2y)(3x-2y)+(3x-2y)^2$

(M) $(7p+3r-5x)^2-2(7p+3r-5x)(8p-4r-5x)+(8p-4r-5x)^2$

(N) $(2m+3n-p)^2+(2m-3n+p)^2-2(2m+3n-p)(2m-3n+p)$

(O) $6\cdot35\times6\cdot35+2\times6\cdot35\times3\cdot65+3\cdot65\times3\cdot65$

(P) $5874\times5874+3774\times3774-7548\times5874$

(Q) $\dfrac{7529\times7529-7519\times7519}{7529+7519}$

(R) $\dfrac{2345\times2345-759\times759}{2345-759}$

3| $a-b=4$ Ges $ab=60$ n‡j, $a+b$ Gi gvb KZ ?

4| $a+b=7$ Ges $ab=12$ n‡j, $a-b$ Gi gvb KZ ?

5| $a+b=9m$ Ges $ab=18m^2$ n‡j, $a-b$ Gi gvb KZ ?

6| $x-y=2$ Ges $xy=63$ n‡j, x^2+y^2 Gi gvb KZ ?

7| $x-\dfrac{1}{x}=4$ n‡j, cgvY Ki †h, $x^4+\dfrac{1}{x^4}=322$.

8| $2x+\dfrac{2}{x}=3$ n‡j, $x^2+\dfrac{1}{x^2}$ Gi gvb KZ ?

9| $a+\dfrac{1}{a}=2$ n‡j, †`LvI †h, $a^2+\dfrac{1}{a^2}=a^4+\dfrac{1}{a^4}$.

10| $a+b=\sqrt{7}$ Ges $a-b=\sqrt{5}$ n‡j, cgvY Ki †h, $8ab(a^2+b^2)=24$

11| $a+b+c=9$ Ges $ab+bc+ca=31$ n‡j, $a^2+b^2+c^2$ Gi gvb wbYq Ki|

12| $a^2+b^2+c^2=9$ Ges $ab+bc+ca=8$ n‡j, $(a+b+c)^2$ Gi gvb KZ ?

13| $a+b+c=6$ Ges $a^2+b^2+c^2=14$ n‡j, $(a-b)^2+(b-c)^2+(c-a)^2$ Gi gvb wbYq Ki|

14| $x+y+z=10$ Ges $xy+yz+zx=31$ n‡j, $(x+y)^2+(y+z)^2+(z+x)^2$ Gi gvb KZ ?

15| $x=3, y=4$ Ges $z=5$ n‡j, $9x^2+16y^2+4z^2-24xy-16yz+12zx$ Gi gvb wbYq Ki|

16| cgvY Ki †h, $\left\{\left(\dfrac{x+y}{2}\right)^2-\left(\dfrac{x-y}{2}\right)^2\right\}^2=\left(\dfrac{x^2+y^2}{2}\right)^2-\left(\dfrac{x^2-y^2}{2}\right)^2$

17| $(a+2b)(3a+2c)$ †K `BwU e‡Mi we‡qvMdj iƒ‡c cKvk Ki|

18| $(x+7)(x-9)$ †K `BwU e‡Mi we‡qvMdj iƒ‡c cKvk Ki|

19| $x^2+10x+24$ †K `BwU e‡Mi we‡qvMdj iƒ‡c cKvk Ki|

20| $a^4+a^2b^2+b^4=8$ Ges $a^2+ab+b^2=4$ n‡j, (i) a^2+b^2, (ii) ab -Gi gvb wbYq Ki|

3·3 Nb msewjZ mÎvewj

mÎ 6| $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$= a^3+b^3+3ab(a+b)$

cgvY : $(a+b)^3 = (a+b)(a+b)^2$

$= (a+b)(a^2+2ab+b^2)$

$= a(a^2+2ab+b^2)+b(a^2+2ab+b^2)$

$= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$

$= a^3+3a^2b+3ab^2+b^3$

$= a^3+b^3+3ab(a+b)$

Abwm×vš 9| $a^3+b^3 = (a+b)^3-3ab(a+b)$

mÎ 7| $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$= a^3-b^3-3ab(a-b)$

cgvY : $(a-b)^3 = (a-b)(a-b)^2$

$= (a-b)(a^2-2ab+b^2)$

$= a(a^2-2ab+b^2)-b(a^2-2ab+b^2)$

$= a^3-2a^2b+ab^2-a^2b+2ab^2-b^3$

$= a^3-3a^2b+3ab^2-b^3$

$= a^3-b^3-3ab(a-b)$

j¶ Kwi : mÎ 6 G b Gi ¯‡j $-b$ emv‡j mÎ 7 cvIqv hvq :

$$\{a+(-b)\}^3 = a^3+(-b)^3+3a(-b)\{a+(-b)\}$$

A_vr, $(a-b)^3 = a^3-b^3-3ab(a-b)$

Abwm×vš 10| $a^3-b^3 = (a-b)^3+3ab(a-b)$

mÎ 8| $a^3+b^3 = (a+b)(a^2-ab+b^2)$

cgvY : $a^3+b^3 = (a+b)^3-3ab(a+b)$

$= (a+b)\{(a+b)^2-3ab\}$

$= (a+b)(a^2+2ab+b^2-3ab)$

$= (a+b)(a^2-ab+b^2)$

mÎ 9| $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

cgvY : $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

$= (a-b)\{(a-b)^2 + 3ab\}$

$= (a-b)(a^2 - 2ab + b^2 + 3ab)$

$= (a-b)(a^2 + ab + b^2)$

D`vniY 12| $2x + 3y$ Gi Nb wbYq Ki|

mgvavb : $(2x+3y)^3 = (2x)^3 + 3(2x)^2 \cdot 3y + 3 \cdot 2x(3y)^2 + (3y)^3$

$= 8x^3 + 3 \cdot 4x^2 \cdot 3y + 3 \cdot 2x \cdot 9y^2 + 27y^3$

$= 8x^3 + 36x^2y + 54xy^2 + 27y^3$

D`vniY 13| $2x - y$ Gi Nb wbYq Ki|

mgvavb : $(2x-y)^3 = (2x)^3 - 3 \cdot (2x)^2 y + 3 \cdot 2x \cdot y^2 - y^3$

$= 8x^3 - 3 \cdot 4x^2 y + 6xy^2 - y^3$

$= 8x^3 - 12x^2 y + 6xy^2 - y^3$

KvR : m‡Îi mvnv‡h¨ Nb wbYq Ki :
1| $3x+2y$ 2| $3x-4y$ 3| 397

D`vniY 14| $x = 37$ n‡j, $8x^3 + 72x^2 + 216x + 216$ Gi gvb KZ ?

mgvavb : $8x^3 + 72x^2 + 216x + 216$

$= (2x)^3 + 3 \cdot (2x)^2 \cdot 6 + 3 \cdot 2x.(6)^2 + (6)^3$

$= (2x+6)^3$

$= (2 \times 37 + 6)^3$ [gvb ewm‡q]

$= (74+6)^3$

$= (80)^3$

$= 512000$

D`vniY 15| hw` $x - y = 8$ Ges $xy = 5$ nq, Z‡e $x^3 - y^3 + 8(x+y)^2$ Gi gvb KZ ?

mgvavb : $x^3 - y^3 + 8(x+y)^2$

$= (x-y)^3 + 3xy(x-y) + 8\{(x-y)^2 + 4xy\}$

$= (8)^3 + 3 \times 5 \times 8 + 8(8^2 + 4 \times 5)$ [gvb ewm‡q]

$= 8^3 + 15 \times 8 + 8(64 + 20)$

$= 8^3 + 15 \times 8 + 8 \times 84$

$= 8(8^2 + 15 + 84)$

$= 8(64 + 15 + 84)$

$= 8 \times 163$

$= 1304$

D`vniY 16| $a^2 - \sqrt{3}a + 1 = 0$ n‡j, $a^3 + \frac{1}{a^3}$ Gi gvb KZ ?

mgvavb : †`Iqv Av‡Q, $a^2 - \sqrt{3}a + 1 = 0$

ev, $a^2 + 1 = \sqrt{3}a$ ev, $\frac{a^2 + 1}{a} = \sqrt{3}$

ev, $\frac{a^2}{a} + \frac{1}{a} = \sqrt{3}$ ev, $a + \frac{1}{a} = \sqrt{3}$

$\therefore$ c`Ë iwk = $a^3 + \frac{1}{a^3}$

$= \left(a + \frac{1}{a}\right)^3 - 3a \cdot \frac{1}{a}\left(a + \frac{1}{a}\right)$

$= \left(\sqrt{3}\right)^3 - 3\left(\sqrt{3}\right)$ $[\because a + \frac{1}{a} = \sqrt{3}]$

$= 3\sqrt{3} - 3\sqrt{3}$

$= 0$

D`vniY 17| mij Ki : $(a-b)(a^2 + ab + b^2) + (b-c)(b^2 + bc + c^2) + (c-a)(c^2 + ca + a^2)$

mgvavb : $(a-b)(a^2 + ab + b^2) + (b-c)(b^2 + bc + c^2) + (c-a)(c^2 + ca + a^2)$

$= a^3 - b^3 + b^3 - c^3 + c^3 - a^3$

$= 0$

D`vniY 18| hw` $a = \sqrt{3} + \sqrt{2}$ nq, Z‡e cgvY Ki †h, $a^3 + \frac{1}{a^3} = 18\sqrt{3}$.

mgvavb : †`Iqv Av‡Q, $a = \sqrt{3} + \sqrt{2}$

$\therefore \frac{1}{a} = \frac{1}{\sqrt{3} + \sqrt{2}}$

$= \frac{\sqrt{3} - \sqrt{2}}{\left(\sqrt{3} + \sqrt{2}\right)\left(\sqrt{3} - \sqrt{2}\right)}$ [je I ni‡K $\left(\sqrt{3} - \sqrt{2}\right)$Øviv ¸Y K‡i]

$= \frac{\sqrt{3} - \sqrt{2}}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$ $= \frac{\sqrt{3} - \sqrt{2}}{3 - 2}$

$= \sqrt{3} - \sqrt{2}$

$\therefore\ a+\frac{1}{a}=\left(\sqrt{3}+\sqrt{2}\right)+\left(\sqrt{3}-\sqrt{2}\right)$

$=\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}=2\sqrt{3}$

GLb, $a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)^3-3\cdot a\cdot\frac{1}{a}\left(a+\frac{1}{a}\right)$

$=\left(2\sqrt{3}\right)^3-3\left(2\sqrt{3}\right)$ $[\because a+\frac{1}{a}=2\sqrt{3}]$

$=2^3\cdot\left(\sqrt{3}\right)^3-3\times 2\sqrt{3}$

$=8\cdot 3\sqrt{3}-6\sqrt{3}$

$=24\sqrt{3}-6\sqrt{3}$

$=18\sqrt{3}$ (cgvwYZ)

KvR : 1| $x=-2$ n‡j, $27x^3-54x^2+36x-8$ Gi gvb KZ ?

2| $a+b=5$ Ges $ab=6$ n‡j, $a^3+b^3+4(a-b)^2$ Gi gvb wbYq Ki|

3| $x=\sqrt{5}+\sqrt{3}$ n‡j, $x^3+\frac{1}{x^3}$ Gi gvb wbYq Ki|

Abkxjbx 3·2

1| m‡Îi mvnv‡h¨ Nb wbYq Ki :

(K) $2x+5$ (L) $2x^2+3y^2$ (M) $4a-5x^2$ (N) $7m^2-2n$ (O) 403 (P) 998

(Q) $2a-b-3c$ (R) $2x+3y+z$

2| mij Ki :

(K) $(4a-3b)^3-3(4a-3b)^2(2a-3b)+3(4a-3b)(2a-3b)^2-(2a-3b)^3$

(L) $(2x+y)^3+3(2x+y)^2(2x-y)+3(2x+y)(2x-y)^2+(2x-y)^3$

(M) $(7x+3b)^3-(5x+3b)^3-6x(7x+3b)(5x+3b)$

(N) $(x-15)^3+(16-x)^3+3(x-15)(16-x)$

(O) $(a+b+c)^3-(a-b-c)^3-6(b+c)\{a^2-(b+c)^2\}$

(P) $(m+n)^6-(m-n)^6-12mn(m^2-n^2)^2$

(Q) $(x+y)(x^2-xy+y^2)+(y+z)(y^2-yz+z^2)+(z+x)(z^2-zx+x^2)$

(R) $(2x+3y-4z)^3+(2x-3y+4z)^3+12x\{4x^2-(3y-4z)^2\}$

3| $a-b=5$ Ges $ab=36$ n‡j, a^3-b^3 Gi gvb KZ ?

4| hw` $a^3-b^3=513$ Ges $a-b=3$ nq, Z‡e ab Gi gvb KZ ?

5| $x=19$ Ges $y=-12$ n‡j, $8x^3+36x^2y+54xy^2+27y^3$ Gi gvb wbYq Ki|

6| hw` $a=15$ nq, Z‡e $8a^3+60a^2+150a+130$ Gi gvb KZ ?

7| $a=7$ Ges $b=-5$ n‡j, $(3a-5b)^3+(4b-2a)^3+3(a-b)(3a-5b)(4b-2a)$ Gi gvb KZ

8| hw` $a+b=m, a^2+b^2=n$ Ges $a^3+b^3=p^3$ nq, Z‡e †`LvI †h, $m^3+2p^3=3mn$.

9| hw` $x+y=1$ nq, Z‡e, †`LvI †h, $x^3+y^3-xy=(x-y)^2$

10| $a+b=3$ Ges $ab=2$ n‡j, (K) a^2-ab+b^2 Ges (L) a^3+b^3 Gi gvb wbYq Ki|

11| $a-b=5$ Ges $ab=36$ n‡j, (K) a^2+ab+b^2 Ges (L) a^3-b^3 Gi gvb wbYq Ki|

12| $m+\frac{1}{m}=a$ n‡j, $m^3+\frac{1}{m^3}$ Gi gvb wbYq Ki|

13| $x-\frac{1}{x}=p$ n‡j, $x^3-\frac{1}{x^3}$ Gi gvb wbYq Ki|

14| hw` $a-\frac{1}{a}=1$ nq, Z‡e †`LvI †h, $a^3-\frac{1}{a^3}=4$.

15| hw` $a+b+c=0$ nq, Z‡e †`LvI †h,

(K) $a^3+b^3+c^3=3abc$ (L) $\frac{(b+c)^2}{3bc}+\frac{(c+a)^2}{3ca}+\frac{(a+b)^2}{3ab}=1$

16| $p-q=r$ n‡j, †`LvI †h, $p^3-q^3-r^3=3pqr$

17| $2x-\frac{2}{x}=3$ n‡j, †`LvI †h, $8\left(x^3-\frac{1}{x^3}\right)=63$

18| $a=\sqrt{6}+\sqrt{5}$ n‡j, $\frac{a^6-1}{a^3}$ Gi gvb wbYq Ki|

19| $x^3+\frac{1}{x^3}=18\sqrt{3}$ n‡j, cgvY Ki †h, $x=\sqrt{3}+\sqrt{2}$

20| $a^4-a^2+1=0$ n‡j, cgvY Ki †h, $a^3+\frac{1}{a^3}=0$

3·4 Drcv`‡K বিশ্লেষণ

†Kv‡bv iwk `B ev Z‡ZwaK iwki ¸Yd‡ji mgvb n‡j, †k‡lv³ iwk¸‡jvi c‡Z¨KwU‡K c_‡gv³ iwki Drcv`K ev ¸YbxqK ejv nq|

†Kv‡bv exRMwYwZK iwki m¤¢ve¨ Drcv`K¸‡jv wbYq Kivi ci iwkwU‡K jä Drcv`K¸‡jvi ¸Ydj iƒ‡c cKvk Kiv‡K Drcv`‡K we‡kølY ejv nq|

evRMwYwZK iwk¸‡jv GK ev GKwaK c`wewkó n‡Z cv‡i| †mRb¨ D³ iwki Drcv`K¸‡jvI GK ev GKwaK c`wewkó n‡Z cv‡i|

Drcv`K wbY‡qi KwZcq †KŠkj :

(K) †Kv‡bv eûc`xi c‡Z¨K c‡` mvaviY Drcv`K _vK‡j Zv c_‡g †ei K‡i wb‡Z nq| †hgb :

$(i)\ \ 3a^2b+6ab^2+12a^2b^2=3ab(a+2b+4ab)$

$(ii)\ \ 2ab(x-y)+2bc(x-y)+3ca(x-y)=(x-y)(2ab+2bc+3ca)$

(L) GKwU iwk‡K cY eM AvKv‡i cKvk K‡i :

D`vniY 1| $4x^2+12x+9$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $4x^2+12x+9=(2x)^2+2\times 2x\times 3+(3)^2$

$=(2x+3)^2=(2x+3)(2x+3)$

D`vniY 2| $9x^2-30xy+25y^2$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $9x^2-30xy+25y^2$

$=(3x)^2-2\times 3x\times 5y+(5y)^2$

$=(3x-5y)^2=(3x-5y)(3x-5y)$

(M) GKwU iwk‡K `BwU e‡Mi Ašii¦‡c cKvk K‡i Ges $a^2-b^2=(a+b)(a-b)$ mÎ c‡qvM K‡i :

D`vniY 3| $a^2-1+2b-b^2$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $a^2-1+2b-b^2=a^2-(b^2-2b+1)$

$=a^2-(b-1)^2=\{a+(b-1)\}\{a-(b-1)\}$

$=(a+b-1)(a-b+1)$

D`vniY 4| $a^4+64b^4$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $a^4+64b^4=(a^2)^2+(8b^2)^2$

$=(a^2)^2+2\times a^2\times 8b^2+(8b^2)^2-16a^2b^2$

$=(a^2+8b^2)^2-(4ab)^2$

$=(a^2+8b^2+4ab)(a^2+8b^2-4ab)$

$=(a^2+4ab+8b^2)(a^2-4ab+8b^2)$

KvR : Drcv`‡K we‡kølY Ki :

1| $abx^2+acx^3+adx^4$ 2| $xa^2-144xb^2$ 3| $x^2-2xy-4y-4$

(N) $x^2+(a+b)x+ab=(x+a)(x+b)$ mÎwU e¨envi K‡i :

D`vniY 5| $x^2+12x+35$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $x^2+12x+35=x^2+(5+7)x+5\times 7$

= $(x+5)(x+7)$

G c×wZ‡Z x^2+px+q AvKv‡ii eûc`xi Drcv`K wbYq Kiv m¤¢e nq hw` `BwU cYmsL¨v a I b wbYq Kiv hvq †hb, $a+b=p$ Ges $ab=q$ nq| GRb¨ q Gi `BwU ¯wPý Drcv`K wb‡Z nq hv‡`i exRMwYwZK mgwó p nq| $q>0$ n‡j, a I b GKB wPýh³ n‡e Ges $q<0$ n‡j, a I b wecixZ wPýh³ n‡e|

D`vniY 6| $x^2-5x+6$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $x^2-5x+6=x^2+(-2-3)x+(-2)(-3)$

= $(x-2)(x-3)$

D`vniY 7| $x^2-2x-35$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $x^2-2x-35$

= $x^2+(-7+5)x+(-7)(+5)$

= $(x-7)(x+5)$

D`vniY 8| $x^2+x-20$ †K Drcv`‡K we‡kølY Ki|

mgvavb : x^2+x-20

= $x^2+(5-4)x+(5)(-4)$

= $(x+5)(x-4)$

(O) ax^2+bx+c AvKv‡ii eûc`xi ga¨c` wef³KiY c×wZ‡Z :

$ax^2+bx+c=(rx+p)(sx+q)$ n‡e

hw` $ax^2+bx+c=rsx^2+x(rq+sp)x+pq$

A_vr, $a=rs, b=rq+sp$ Ges $c=pq$ nq|

mZivs, $ac=rspq=(rq)(sp)$ Ges $b=rq+sp$

AZGe, ax^2+bx+c AvKv‡ii eûc`xi Drcv`K wbYq Ki‡Z n‡j ac, A_vr, x^2 Gi mnM Ges x ewRZ c‡`i ¸Yd‡K Ggb `BwU Drcv`‡K cKvk Ki‡Z n‡e, hv‡`i exRMwYwZK mgwó x Gi mnM b Gi mgvb nq|

D`vniY 9| $12x^2+35x+18$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $12x^2+35x+18$

GLv‡b, $12 \times 18 = 216 = 27 \times 8$ Ges $27 + 8 = 35$

$\therefore\ 12x^2 + 35x + 18 = 12x^2 + 27x + 8x + 18$

$= 3x(4x+9) + 2(4x+9)$

$= (4x+9)(3x+2)$

D`vniY 10| $3x^2 - x - 14$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $3x^2 - x - 14 = 3x^2 - 7x + 6x - 14$

$= x(3x-7) + 2(3x-7)$

$= (3x-7)(x+2)$

KvR : Drcv`‡K we‡kølY Ki :
1| $x^2 + x - 56$ 2| $16x^3 - 46x^2 + 15x$ 3| $12x^2 + 17x + 6$

(P) GKwU ivwk‡K cY Nb AvKv‡i cKvk K‡i :

D`vniY 11| $8x^3 + 36x^2y + 54xy^2 + 27y^3$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $8x^3 + 36x^2y + 54xy^2 + 27y^3$

$= (2x)^3 + 3 \times (2x)^2 \times 3y + 3 \times 2x \times (3y)^2 + (3y)^3$

$= (2x+3y)^3 = (2x+3y)(2x+3y)(2x+3y)$

(Q) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ Ges $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ mÎ `BwU e¨envi K‡i:

D`vniY 12| Drcv`‡K we‡kølY Ki : $(i)\ 8a^3 + 27b^3$ $(ii)\ a^6 - 64$

mgvavb : $(i)\ 8a^3 + 27b^3 = (2a)^3 + (3b)^3$

$= (2a+3b)\{(2a)^2 - 2a \times 3b + (3b)^2\}$

$= (2a+3b)(4a^2 - 6ab + 9b^2)$

$(ii)\ a^6 - 64 = (a^2)^3 - (4)^3$

$= (a^2 - 4)\{(a^2)^2 + a^2 \times 4 + (4)^2\}$

$= (a^2 - 4)(a^4 + 4a^2 + 16)$

wKš‘ $a^2 - 4 = a^2 - 2^2 = (a+2)(a-2)$

Ges $a^4 + 4a^2 + 16 = (a^2)^2 + (4)^2 + 4a^2$

$= (a^2+4)^2 - 2(a^2)(4) + 4a^2$

$= (a^2+4)^2 - 4a^2$

$= (a^2+4)^2 - (2a)^2$

$= (a^2 + 4 + 2a)(a^2 + 4 - 2a)$

$= (a^2 + 2a + 4)(a^2 - 2a + 4)$

$\therefore a^6 - 64$

$= (a+2)(a-2)(a^2 + 2a + 4)(a^2 - 2a + 4)$

weKí wbqg :

$a^6 - 64 = (a^3)^2 - (8)^2$

$= (a^3 + 8)(a^3 - 8)$

$= (a^3 + 2^3)(a^3 - 2^3)$

$= (a+2)\ (a^2 - 2a + 4) \times (a-2)\ (a^2 + 2a + 4)$

$= (a+2)(a-2)(a^2 + 2a + 4)(a^2 - 2a + 4)$

KvR : Drcv`‡K we‡kølY Ki :

1| $2x^4+16x$ 2| $8-a^3+3a^2b-3ab^2+b^3$ 3| $(a+b)^3+(a-b)^3$

(R) fMvskmnMh³ iwki Drcv`K :

fMvskh³ iwki Drcv`K¸‡jv‡K wewfbfv‡e cKvk Kiv hvq|

†hgb, $a^3+\frac{1}{27}=a^3+\frac{1}{3^3}=\left(a+\frac{1}{3}\right)\left(a^2-\frac{a}{3}+\frac{1}{9}\right)$

Avevi, $a^3+\frac{1}{27}=\frac{1}{27}(27a^3+1)=\frac{1}{27}\{(3a)^3+(1)^3\}$

$=\frac{1}{27}(3a+1)(9a^2-3a+1)$

GLv‡b, wØZxq mgvav‡b PjK-msewjZ Drcv`K¸‡jv cYmsLïv mnMwewkó| GB dj‡K c_g mgvav‡bi g‡Zv cKvk Kiv hvq :

$\frac{1}{27}(3a+1)(9a^2-3a+1)$

$=\frac{1}{3}(3a+1)\times\frac{1}{9}(9a^2-3a+1)$

$=\left(a+\frac{1}{3}\right)\left(a^2-\frac{a}{3}+\frac{1}{9}\right)$

D`vniY 13| $x^3+6x^2y+11xy^2+6y^3$ †K Drcv`‡K we‡kølY Ki|

mgvavb : $x^3+6x^2y+11xy^2+6y^3$

$=\{x^3+3\cdot x^2\cdot 2y+3\cdot x(2y)^2+(2y)^3\}-xy^2-2y^3$

$=(x+2y)^3-y^2(x+2y)$

$=(x+2y)\{(x+2y)^2-y^2\}$

$=(x+2y)(x+2y+y)(x+2y-y)$

$=(x+2y)(x+3y)(x+y)$

$=(x+y)(x+2y)(x+3y)$

KvR : Drcv`‡K we‡kølY Ki :

1| $\frac{1}{2}x^2+\frac{7}{6}x+\frac{1}{3}$ 2| $a^3+\frac{1}{8}$ 3| $16x^2-25y^2-8xz+10yz$

Abkxjbx 3·3

Drcv`‡K we‡kølY Ki (1 – 43) :

1| $a^2+ab+ac+bc$

2| $ab+a-b-1$

3| $(x-y)(x+y)+(x-y)(y+z)+(x-y)(z+x)$

4| $ab(x-y)-bc(x-y)$

5| $9x^2+24x+16$

6| a^4-27a^2+1

7| $x^4-6x^2y^2+y^4$

8| $(a^2-b^2)(x^2-y^2)+4abxy$

9| $4a^2-12ab+9b^2-4c^2$

10| $9x^4-45a^2x^2+36a^4$

11| $a^2+6a+8-y^2+2y$

12| $16x^2-25y^2-8xz+10yz$

13| $2b^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4$

14| $x^2+13x+36$

15| x^4+x^2-20

16| $a^2-30a+216$

17| $x^6y^6-x^3y^3-6$

18| a^8-a^4-2

19| $a^2b^2-8ab-105$

20| $x^2-37x-650$

21| $4x^4-25x^2+36$

22| $12x^2-38x+20$

23| $9x^2y^2-5xy^2-14y^2$

24| $4x^4-27x^2-81$

25| $ax^2+(a^2+1)x+a$

26| $3(a^2+2a)^2-22(a^2+2a)+40$

27| $14(x+z)^2-29(x+z)(x+1)-15(x+1)^2$

28| $(4a-3b)^2-2(4a-3b)(a+2b)-35(a+2b)^2$

29| $(a-1)x^2+a^2xy+(a+1)y^2$

30| $24x^4-3x$

31| $(a^2+b^2)^3+8a^3b^3$

32| x^3+3x^2+3x+2

33| $a^3-6a^2+12a-9$

34| $a^3-9b^3+(a+b)^3$

35| $8x^3+12x^2+6x-63$

36| $8a^3+\frac{b^3}{27}$

37| $a^3-\frac{1}{8}$

38| $\frac{a^6}{27}-b^6$

39| $4a^2+\frac{1}{4a^2}-2+4a-\frac{1}{a}$

40| $(3a+1)^3-(2a-3)^3$

41| $(x+5)(x-9)-15$

42| $(x+2)(x+3)(x+4)(x+5)-48$

43| $(x-1)(x-3)(x-5)(x-7)-64$

44| †`LvI †h, $x^3+9x^2+26x+24=(x+2)(x+3)(x+4)$

45| †`LvI †h, $(x+1)(x+2)(3x-1)(3x-4)=(3x^2+2x-1)(3x^2+2x-8)$

3·5 fvM‡kl Dccv`¨ *(Remainder Theorem)*

Avgiv wb‡Pi D`vniYwU j¶ Kwi :

$6x^2-7x+5$ †K $x-1$ Øviv fvM Ki‡j fvMdj I fvM‡kl KZ ?

$6x^2-7x+5$ †K $x-1$ Øviv mvaviYfv‡e fvM Ki‡j cvB,

$$
\begin{array}{r|l}
x-1 & 6x^2-7x+5 \ (\ 6x-1 \\
 & \underline{6x^2-6x} \\
 & \quad -x+5 \\
 & \quad \underline{-x+1} \\
 & \qquad\quad 4
\end{array}
$$

GLv‡b, $x-1$ fvRK, $6x^2-7x+5$ fvR¨, $6x-1$ fvMdj Ges 4 fvM‡kl|

Avgiv Rwb, fvR¨ = fvRK × fvMdj + fvM‡kl

GLb hw` Avgiv fvR¨‡K $f(x)$, fvMdj‡K $h(x)$, fvM‡kl‡K r I fvRK‡K $(x-a)$ Øviv mwPZ Kwi, Zvn‡j Dc‡ii m~Î †_‡K cvB,

$f(x)=(x-a)\cdot h(x)+r$, GB m~ÎwU a Gi mKj gv‡bi Rb¨ mZ¨|

Dfqc‡¶ $x=a$ ewm‡q cvB,

$f(a)=(a-a)\cdot h(a)+r=0\cdot h(a)+r=r$

mZivs, $r=f(a)$

AZGe, $f(x)$ †K $(x-a)$ Øviv fvM Ki‡j fvM‡kl nq $f(a)$. GB m~Î fvM‡kl Dccv`¨ *(Remainder theorem)* bv‡g cwiwPZ| A_vr, abvZ¥K gvÎvi †Kv‡bv eûc`x $f(x)$ †K $(x-a)$ AvKv‡ii eûc`x Øviv fvM Ki‡j fvM‡kl KZ n‡e Zv fvM bv K‡i †ei Kivi mÎB n‡jv fvM‡kl Dccv`¨| fvRK eûc`x $(x-a)$ Gi gvÎv $1$, fvRK hw` fv‡R¨i Drcv`K nq, Zvn‡j fvM‡kl n‡e kb¨| Avi hw` Drcv`K bv nq, Zvn‡j fvM‡kl _vK‡e Ges Zv n‡e Akb¨ †Kv‡bv msL¨v|

cwZÁv : hw` $f(x)$ Gi gvÎv abvZ¥K nq Ges $a\neq 0$ nq, Z‡e $f(x)$ †K $(ax+b)$ Øviv fvM Ki‡j fvM‡kl nq $f\left(-\dfrac{b}{a}\right)$.

cgvY : fvRK $ax+b$, $(a\neq 0)$ Gi gvÎv 1,

mZivs Avgiv wjL‡Z cwi,

$$f(x)=(ax+b)\cdot h(x)+r=a\left(x+\frac{b}{a}\right)\cdot h(x)+r$$

$$\therefore \quad f(x)=\left(x+\frac{b}{a}\right)\cdot a\cdot h(x)+r$$

†`Lv hv‡"Q †h, $f(x)$ †K $\left(x+\dfrac{b}{a}\right)$ Øviv fvM Ki‡j fvMdj nq, $a\cdot h(x)$ Ges fvM‡kl nq r.

GLv‡b, fvRK = $x-\left(-\frac{b}{a}\right)$

mZivs fvM‡kl Dccv`¨ Abhvqx, $r = f\left(-\frac{b}{a}\right)$

AZGe, $f(x)$ †K $(ax+b)$ Øviv fvM Ki‡j fvM‡kl nq $f\left(-\frac{b}{a}\right)$.

Abywm×vš— : $(x-a)$, $f(x)$ Gi Drcv`K n‡e, hw` Ges †Kej hw` $f(a)=0$ nq|

cgvY : awi, $f(a)=0$

AZGe, fvM‡kl Dccv`¨ Abhvqx, $f(x)$ †K $(x-a)$ Øviv fvM Ki‡j fvM‡kl kb¨ n‡e| A\_vr, $(x-a)$, $f(x)$ Gi GKwU Drcv`K n‡e|

wecixZµ‡g, awi, $(x-a)$, $f(x)$ Gi GKwU Drcv`K|

AZGe, $f(x)=(x-a)\cdot h(x)$, †hLv‡b $h(x)$ eûc`x|

Dfqc‡¶ $x=a$ ewm‡q cvB,

$f(a)=(a-a)\cdot h(a)=0$

$\therefore\ f(a)=0$.

mZivs, †Kv‡bv eûc`x $f(x), (x-a)$ Øviv wefvR¨ n‡e hw` Ges †Kej hw` $f(a)=0$ nq| GB mÎ Drcv`K Dccv`¨ *(Factor theorem)* bv‡g cwiwPZ|

Abywm×vš— : $ax+b,\ a\neq 0$ n‡j, iwkwU †Kv‡bv eûc`x $f(x)$ Gi Drcv`K n‡e, hw` Ges †Kej hw` $f\left(-\frac{b}{a}\right)=0$ nq|

cgvY : $a\neq 0, ax+b = a\left(x+\frac{b}{a}\right)$, $f(x)$ Gi Drcv`K n‡e, hw` Ges †Kej hw` $\left(x+\frac{b}{a}\right) = x-\left(-\frac{b}{a}\right)$, $f(x)$ Gi GKwU Drcv`K nq| A_vr, hw` Ges †Kej hw` $f\left(-\frac{b}{a}\right)=0$ nq| fvM‡kl Dccv‡`¨i mvnv‡h¨ Drcv`K wbY‡qi GB c×wZ‡K kb¨vqb c×wZ I *(Vanishing method)* e‡j|

D`vniY 1| $x^3-x-6$ †K Drcv`‡K we‡kølY Ki|

mgvavb : GLv‡b, $f(x)=x^3-x-6$ GKwU eûc`x| Gi aec` -6 Gi Drcv`K¸‡jv n‡”Q $\pm 1, \pm 2, \pm 3$ Ges ± 6.

GLb, $x=1,-1$ ewm‡q †`wL, $f(x)$ Gi gvb kb¨ nq bv|

wKš‘ $x=2$ ewm‡q †`wL, $f(x)$ Gi gvb kb¨ nq|

A_vr, $f(2)=2^3-2-6=8-2-6=0$.

mZivs, $x-2$, $f(x)$ eûc`xwUi GKwU Drcv`K|

$\therefore \quad f(x) = x^3 - x - 6$

$= x^3 - 2x^2 + 2x^2 - 4x + 3x - 6$

$= x^2(x-2) + 2x(x-2) + 3(x-2)$

$= (x-2)(x^2+2x+3)$

D`vniY 2| $x^3 - 3xy^2 + 2y^3$ †K Drcv`‡K wefw³ Ki|

mgvavb : GLv‡b, x †K PjK Ges y †K a¨eK wn‡m‡e we‡ePbv Kwi|

c`Ë ivwk‡K $x$-Gi eûc`x we‡ePbv K‡i

awi, $f(x) = x^3 - 3xy^2 + 2y^3$

Zvn‡j, $f(y) = y^3 - 3y \cdot y^2 + 2y^3 = 3y^3 - 3y^3 = 0$

$\therefore (x-y)$, $f(x)$ Gi GKwU Drcv`K|

GLb, $x^3 - 3xy^2 + 2y^3$

$= x^3 - x^2y + x^2y - xy^2 - 2xy^2 + 2y^3$

$= x^2(x-y) + xy(x-y) - 2y^2(x-y)$

$= (x-y)(x^2 + xy - 2y^2)$

$= (x-y)(x^2 + 2xy - xy - 2y^2)$

$= (x-y)\{x(x+2y) - y(x+2y)\}$

$= (x-y)(x+2y)(x-y)$

$= (x-y)^2(x+2y)$

Avevi awi,

$g(x) = x^2 + xy - 2y^2$

$\therefore \quad g(y) = y^2 + y^2 - 2y^2 = 0$

$\therefore (x-y)$, $g(x)$ Gi GKwU Drcv`K

$\therefore \quad x^2 + xy - 2y^2$

$= x^2 - xy + 2xy - 2y^2$

$= x(x-y) + 2y(x-y)$

$= (x-y)(x+2y)$

$\therefore \quad x^3 - 3xy^2 + 2y^3 = (x-y)^2(x+2y)$

D`vniY 3| $54x^4 + 27x^3a - 16x - 8a$ †K Drcv`‡K wefw³ Ki|

mgvavb : awi, $f(x) = 54x^4 + 27x^3a - 16x - 8a$

Zvn‡j, $f\left(-\frac{1}{2}a\right) = 54\left(-\frac{1}{2}a\right)^4 + 27a\left(-\frac{1}{2}a\right)^3 - 16\left(-\frac{1}{2}a\right) - 8a$

$= \frac{27}{8}a^4 - \frac{27}{8}a^4 + 8a - 8a = 0$

$\therefore \quad x - \left(-\frac{1}{2}a\right) = x + \frac{a}{2}$ A_©vr, $2x+a$, $f(x)$ Gi GKwU Drcv`K|

GLb, $54x^4 + 27x^3a - 16x - 8a = 27x^3(2x+a) - 8(2x+a) = (2x+a)(27x^3 - 8)$

$= (2x+a)\{(3x)^3 - (2)^3\} = (2x+a)(3x-2)(9x^2+6x+4)$

KvR : Drcv`‡K we‡kølY Ki :

1| $x^3 - 21x - 20$ 2| $2x^3 - 3x^2 + 3x - 1$ 3| $x^3 + 6x^2 + 11x + 6$

Abkxjbx 3·4

Drcv`‡K we‡kølY Ki :

1| $6x^2 - 7x + 1$
2| $3a^3 + 2a + 5$
3| $x^3 - 7xy^2 - 6y^3$
4| $x^2 - 5x - 6$
5| $2x^2 - x - 3$
6| $3x^2 - 7x - 6$
7| $x^3 + 2x^2 - 5x - 6$
8| $x^3 + 4x^2 + x - 6$
9| $a^3 + 3a + 36$
10| $a^4 - 4a + 3$
11| $a^3 - a^2 - 10a - 8$
12| $x^3 - 3x^2 + 4x - 4$
13| $a^3 - 7a^2b + 7ab^2 - b^3$
14| $x^3 - x - 24$
15| $x^3 + 6x^2y + 11xy^2 + 6y^3$
16| $2x^4 - 3x^3 - 3x - 2$
17| $4x^4 + 12x^3 + 7x^2 - 3x - 2$
18| $x^6 - x^5 + x^4 - x^3 + x^2 - x$
19| $4x^3 - 5x^2 + 5x - 1$
20| $18x^3 + 15x^2 - x - 2$

3·6 ev¯e mgm¨v mgvav‡b exRMwYwZK m~Î MVb I c‡qvM

^`bw›`b Kv‡R wewfbœ mg‡q wewfbœ fv‡e Avgiv ev¯e mgm¨vi m¤§yLxb nB| GB mgm¨v¸‡jv fvlvMZfv‡e ewYZ nq| G Ab‡"Q‡` Avgiv fvlvMZfv‡e ewYZ ev¯e cwi‡e‡ki wewfbœ mgm¨v mgvavbK‡í exRMwYwZK m~Î MVb Ges Zv c‡qvM Kivi c×wZ wb‡q Av‡jvPbv Kie| GB Av‡jvPbvi d‡j wk¶v_©xiv GKw`‡K †hgb ev¯e cwi‡e‡k MwY‡Zi c‡qvM m¤ú‡K aviYv cv‡e, Ab¨w`‡K wb‡R‡`i cwicv‡k©i Ae¯'v MwY‡Zi m¤ú³Zv eS‡Z †c‡i MwYZ wk¶vi cwZ AvMnx n‡e|

mgm¨v mgvav‡bi c×wZ :

(K) c_‡gB mZKZvi mv‡_ mgm¨wU ch©‡e¶Y K‡i Ges g‡bv‡hvM mnKv‡i c‡o †Kvb¸‡jv AÁvZ Ges Kx wbY©q Ki‡Z n‡e Zv wPwýZ Ki‡Z n‡e|

(L) AÁvZ iwk¸‡jvi GKwU‡K †h‡Kv‡bv PjK (awi x) Øviv mwPZ Ki‡Z n‡e| AZtci mgm¨wU fv‡jvfv‡e Abaveb K‡i Ab¨vb¨ AÁvZ iwk¸‡jv‡KI GKB PjK x Gi gva¨‡g cKvk Ki‡Z n‡e|

(M) mgm¨v‡K ¶z`ª ¶z`ª As‡k wef³ K‡i exRMwYwZK iwk Øviv cKvk Ki‡Z n‡e|

(N) c`Ë kZ© e¨envi K‡i ¶z`ª ¶z`ª Ask¸‡jv‡K GK‡Î GKwU mgxKi‡Y cKvk Ki‡Z n‡e|

(O) mgxKiYwU mgvavb K‡i AÁvZ iwk x Gi gvb wbY©q Ki‡Z n‡e|

ev¯e mgm¨v mgvav‡b wewfbœ m~Î e¨envi Kiv nq| m~Î¸‡jv wb‡P D‡jøL Kiv n‡jv :

(1) †`q ev cvc¨ wbY©q :

†`q ev cvc¨, $A = qn$ UvKv

†hLv‡b, q = RbcÖwZ †`q ev cvc¨ UvKvi cwigvY

n = †jv‡Ki msL¨v

(2) mgq I KvR wbY©q :

K‡qKRb †jvK GKwU KvR m¤úbœ Ki‡j,

Kv‡Ri cwigvY, $W = qnx$

†hLv‡b, q = cÖ‡Z¨‡K GKK mg‡q Kv‡Ri †h Ask m¤úbœ K‡i,

n = KvR m¤úv`bKvixi msL¨v

x = Kv‡Ri †gvU mgq

$W = n$ R‡b x mg‡q Kv‡Ri †h Ask m¤úbœ K‡i

(3) mgq I `~iZ¡ wbY©q :

wbw`©ó mg‡q `~iZ¡, $d = vt$.

†hLv‡b, v = cÖwZ NÈvq MwZ‡eM

t = †gvU mgq

(4) bj I †PŠev”Pv wbY©q :

wbw`©ó mg‡q †PŠev”Pvq cvwbi cwigvY, $Q(t) = Q_o \pm qt$

†hLv‡b, Q_o = b‡j i gL L‡j †`I qvi mgq †PŠev”Pvq Rgv cvwbi cwigvY|

q = cÖwZ GKK mg‡q bj w`‡q †h cvwb c‡ek K‡i A_ev †ei nq|

t = AwZµvšÍ mgq|

$Q(t) = t$ mg‡q †PŠev”Pvq cvwbi cwigvY (cvwb c‡ek nIqvi k‡Z ô+ô wPý Ges cvwb †ei nIqvi k‡Z ô–ô wPý e¨envi Ki‡Z n‡e)|

5| kZKiv Ask wbY©q :

$p = br$.

†hLv‡b, b = †gvU iwk

r = kZKiv fMvsk = $\frac{s}{100} = s\%$

p = kZKiv Ask = b Gi $s\%$

6| jvf-¶wZ wbY©q :

$S = C(I \pm r)$

jv‡fi †¶‡Î, $S = C(I + r)$

¶wZi †¶‡Î, $S = C(I - r)$

†hLv‡b, S (UvKv) = weµqgj¨
C (UvKv) = µqgj¨
I = jvf ev gbvdv
r = jvf ev ¶wZi nvi

(7) wewb‡qvM-gbvdv welqK :
mij gbvdvi †¶‡Î,
$I = Pnr$ UvKv
$A = P + I = P + Pnr = P(1+nr)$ UvKv,
Pµewׄ gbvdvi †¶‡Î,

$A = P(1+r)^n$

†hLv‡b, I = n mgq c‡i gbvdv
n = wbw`ó mgq
P = gjab
r = GKK mg‡q GKK gja‡bi gbvdv
A = n mgq c‡i gbvdvmn gjab|

D`vniY 1| ewlK µxov Abôvb Kivi Rb¨ †Kv‡bv GK mwgwZi m`m¨iv 45,000 UvKvi ev‡RU Ki‡jb Ges wm×vš— wb‡jb †h, c‡Z¨K m`m¨B mgvb Pv`v w`‡eb| wKš' 5 Rb m`m¨ Pv`v w`‡Z Am¤§wZ Rvbv‡jb| Gi d‡j c‡Z¨K m`‡m¨i gv_wcQ 15 UvKv Pv`v ew× †cj| H mwgwZ‡Z KZRb m`m¨ wQ‡jb ? mgvavb : g‡b Kwi, mwgwZi m`m¨ msL¨v x Ges RbcwZ †`q Pv`vi cwigvY q UvKv| Zvn‡j.
†gvU Pv`v, $A = qx$ UvKv

cKZc‡¶ m`m¨ msL¨v wQj $(x-5)$ Rb Ges Pv`v n‡jv $(q+15)$ UvKv|
Zvn‡j, †gvU Pv`v n‡jv $(x-5)(q+15)$

ckvbmv‡i, $qx = (x-5)(q+15) \ldots\ldots\ldots.(i)$
Ges $qx = 45{,}000 \ldots\ldots\ldots.(ii)$

mgxKiY (i) †_‡K cvB,
$$qx = (x-5)(q+15)$$
ev, $qx = qx - 5q + 15x - 75$
ev, $5q = 15x - 75 = 5(3x-15)$
$\therefore \quad q = 3x - 15 \ldots\ldots\ldots..(iii)$

mgxKiY (ii) G q Gi gvb ewm‡q cvB,
$$(3x-15) \times x = 45000$$

ev, $3x^2 - 15x = 45000$

ev, $x^2 - 5x = 15000$ [Dfqc¶‡K 3 Øviv fvM K‡i]

ev, $x^2 - 5x - 15000 = 0$

ev, $x^2 - 125x + 120x - 15000 = 0$

ev, $x(x-125) + 120(x-125) = 0$

ev, $(x-125)(x+120) = 0$

mZivs, $(x-125) = 0$ A_ev $(x+120) = 0$

ev, $x = 125$ ev, $x = -120$

†h‡nZ m`m¨ msL¨v FYvZ¥K n‡Z cv‡i bv, ZvB x Gi gvb -120 MnY‡hvM¨ bq|

$\therefore x = 125$

mZivs, mwgwZi m`m¨ msL¨v 125|

D`vniY 2| iwdK GKwU KvR 10 w`‡b Ki‡Z cv‡i| kwdK H KvR 15 w`‡b Ki‡Z cv‡i| Zviv GK‡Î KZ w`‡b KvRwU †kl Ki‡Z cvi‡e?

mgvavb : g‡b Kwi, Zviv GK‡Î d w`‡b KvRwU †kl Ki‡Z cvi‡e|

bvg	KvR m¤úbœ Kivi w`b	1 w`‡b cv‡i KvR‡Ri Ask	d w`‡b K‡i
iwdK	10	$\frac{1}{10}$	$\frac{d}{10}$
kwdK	15	$\frac{1}{15}$	$\frac{d}{15}$

ckvbmv‡i, $\frac{d}{10} + \frac{d}{15} = 1$

ev, $d\left(\frac{1}{10} + \frac{1}{15}\right) = 1$

ev, $d\left(\frac{3+2}{30}\right) = 1$

ev, $\frac{5d}{30} = 1$

ev, $d = \frac{30}{5} = 6$

mZivs, Zviv GK‡Î 6 w`‡b KvRwU †kl Ki‡Z cvi‡e|

D`vniY 3| GKRb gwS †mv‡Zi cwZK‡j t_1 NÈvq x wK.wg. †h‡Z cv‡i| †mv‡Zi AbK‡j H c_ †h‡Z Zvi t_2 NÈv jv‡M| †mv‡Zi †eM I †bŠKvi †eM KZ ?

mgvavb : awi, †mv‡Zi †eM NÈvq v wK.wg. Ges w¯i cwb‡Z †bŠKvi †eM NÈvq u wK.wg.|
Zvn‡j, †mv‡Zi AbK‡j †bŠKvi KvhKix †eM NÈvq $(u+v)$ wK.wg. Ges †mv‡Zi cwZK‡j †bŠKvi KvhKix †eM NÈvq $(u-v)$ wK.wg.|

ckvbmv‡i, $u+v=\dfrac{x}{t_2}$.......(i) [†h‡nZz, †eM = $\dfrac{\text{AwZµvšÍ `iZ}}{\text{mgq}}$]

Ges $u-v=\dfrac{x}{t_1}$.......(ii)

mgxKiY (i) I (ii) †hvM K‡i cvB,

$$2u=\frac{x}{t_1}+\frac{x}{t_2}=x\left(\frac{1}{t_1}+\frac{1}{t_2}\right)$$

ev, $u=\dfrac{x}{2}\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right)$

mgxKiY (i) †_‡K (ii) we‡qvM K‡i cvB,

$$2v=x\left(\frac{1}{t_2}-\frac{1}{t_1}\right)$$

ev, $v=\dfrac{x}{2}\left(\dfrac{1}{t_2}-\dfrac{1}{t_1}\right)$

mZivs, †mv‡Zi †eM NÈvq $\dfrac{x}{2}\left(\dfrac{1}{t_2}-\dfrac{1}{t_1}\right)$ wK.wg.

Ges †bŠKvi †eM NÈvq $\dfrac{x}{2}\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right)$ wK.wg.|

D`vniY 4| GKwU bj 12 wgwb‡U GKwU Lwj †PŠev”Pv cY Ki‡Z cv‡i| Aci GKwU bj cwZ wgwb‡U 14 wjUvi cwb †ei K‡i †`q| †PŠev”PvwU Lwj _vKv Ae¯vq `BwU bj GKm‡½ L‡j †`Iqv nq Ges †PŠev”PvwU 96 wgwb‡U cY nq| †PŠev”PvwU‡Z KZ wjUvi cwb a‡i ?

mgvavb : g‡b Kwi, c_g bj Øviv cwZ wgwb‡U x wjUvi cwb c‡ek K‡i Ges †PŠev”PvwU‡Z †gvU y wjUvi cwb a‡i|

ckvbmv‡i, c_g bj Øviv 12 wgwb‡U Lwj †PŠev”PvwU cY nq

$$\therefore\quad y=12x\text{.......}(i)$$

Avevi, `BwU bj Øviv 96 wgwb‡U Lwj †PŠev”Pv cY nq

$\therefore\ \ y = 96x - 96\times 14 \,.........(ii)$

mgxKiY (i) †_‡K cvB, $x = \dfrac{y}{12}$

x Gi gvb mgxKiY (ii) G ewm‡q cvB,

$$y = 96\times\frac{y}{12} - 96\times 14$$

ev, $y = 8y - 96\times 14$ ev, $7y = 96\times 14$

ev, $y = \dfrac{96\times 14}{7} = 192$

mZivs, †Pšev”PwU‡Z †gvU 192 wjUvi cwb a‡i|

KvR :

1| eb‡fvR‡b hvIqvi Rb¨ GKwU evm 2400 UvKvq fvov Kiv n‡jv Ges wm×vš— MnxZ n‡jv †h, c‡Z¨K hvÎx mgvb fvov w`‡e| 10 Rb hvÎx Abcw¯Z _vKvq gv_wcQ fvov 8 UvKv ew× †cj| ev‡m KZRb hvÎx wM‡qwQj Ges c‡Z¨‡K KZ UvKv K‡i fvov w`‡qwQj ?

2| K I L GK‡Î GKwU KvR p w`‡b Ki‡Z cv‡i| K GKv KvRwU $q$ w`‡b Ki‡Z cv‡i| L GKvKx KZ w`‡b H KvRwU Ki‡Z cvi‡e ?

3| GK e¨w³ †mv‡Zi cwZK‚‡j `vo †e‡q NÈvq 2 wK.wg. †e‡M †h‡Z cv‡i| †mv‡Zi †eM NÈvq 3 wK.wg. n‡j, †mv‡Zi AbK‚‡j 32 wK.wg. †h‡Z Zvi KZ mgq jvM‡e ?

D`vniY 5| GKwU eB‡qi gj¨ 24·00 UvKv| GB gj¨ cKZ g‡j¨i 80%| evwK gj¨ miKvi fZ©wK w`‡q _v‡Kb| miKvi cwZ eB‡q KZ UvKv fZ©wK †`b ?

mgvavb : evRvi gj¨ = cKZ g‡j¨i 80%

Avgiv Rvwb, $p = br$

GLv‡b, $p = 24$ UvKv Ges $r = 80\% = \dfrac{80}{100}$

$$\therefore\ \ 24 = b\times\frac{80}{100}$$

ev, $b = \dfrac{\overset{6}{\cancel{24}}\times\overset{5}{\cancel{100}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{80}}}$ $\therefore\ \ b = 30$

mZivs eB‡qi cKZ gj¨ 30 UvKv|

$\therefore$ fZ©wK gj¨ = $(30-24)$ UvKv

= 6 UvKv

mZivs fZ©wK gj¨ 6 UvKv|

D`vniY 6| UvKvq n msL¨K Kgjv weµq Kivq $r\%$ ¶wZ nq| $s\%$ jvf Ki‡Z n‡j, UvKvq KqwU Kgjv weµq Ki‡Z n‡e ?

mgvavb : µqgj¨ 100 UvKv n‡j, $r\%$ ¶wZ‡Z weµqgj¨ $(100-r)$ UvKv|

Zvn‡j, hLb weµqgj¨ $(100-r)$ UvKv, ZLb µqgj¨ 100 UvKv

∴ hLb weµqgj¨ 1 UvKv, ZLb µqgj¨ $\frac{100}{100-r}$ UvKv|

Avevi, µqgj¨ 100 UvKv n‡j, $s\%$ jv‡f weµqgj¨ $(100+s)$ UvKv|

∴ µqgj¨ $\frac{100}{100-r}$ UvKv n‡j, $s\%$ jv‡f weµqgj¨ $\left(\frac{100+s}{100}\times\frac{100}{100-r}\right)$ UvKv

$= \frac{100+s}{100-r}$ UvKv|

mZivs, $\frac{100+s}{100-r}$ UvKvq weµq Ki‡Z n‡e n msL¨K Kgjv

∴ 1 UvKvq weµq Ki‡Z n‡e $n\times\left(\frac{100-r}{100+s}\right)$ msL¨K Kgjv

mZivs, UvKvq $\frac{n(100-r)}{100+s}$ msL¨K Kgjv weµq Ki‡Z n‡e|

D`vniY 7| kZKiv ewlK 7 UvKv nvi gbvdvq 650 UvKvi 6 eQ‡ii gbvdv KZ ?

mgvavb : Avgiv Rvwb, $I = Pnr$.

GLv‡b, $P = 650$ UvKv, $n = 6$, $s = 7$

∴ $r = \frac{s}{100} = \frac{7}{100}$

∴ $I = 650\times 6\times\frac{7}{100} = 273$

mZivs, gbvdv 273 UvKv|

D`vniY 8| ewlK kZKiv 6 UvKv nvi Pµew× gbvdvq 15000 UvKvi 3 eQ‡ii mew×gj I Pµew× gbvdv wbYq Ki|

mgvavb : Avgiv Rvwb, $C = P(1+r)^n$ [†hLv‡b C Pµew×i †¶‡Î mew×gj]

†`Iqv Av‡Q, $P = 15000$ UvKv, $r = 6\% = \frac{6}{100}$, $n = 3$ eQi

∴ $C = 15000\left(1+\frac{6}{100}\right)^3 = 15000\left(1+\frac{3}{50}\right)^3$

$$= 15000\left(\frac{53}{50}\right)^3$$

$$= 15000\times\frac{53}{50}\times\frac{53}{50}\times\frac{53}{50}$$

$$= \frac{\overset{3}{\cancel{15}}\times 53\times 53\times 53}{\underset{25}{\cancel{125}}} = \frac{3\times 148877}{25}$$

$$= \frac{446631}{25} = 17865\cdot 24$$

∴ mew×gj = $17865\cdot 24$ UvKv

∴ Pµew× gbvdv = $(17865\cdot 24 - 15000)$ UvKv

= $2865\cdot 24$ UvKv|

KvR : 1| UvKvq 10 wU †je weµq Kivq $n\%$ ¶wZ nq| $z\%$ jvf Ki‡Z n‡j, UvKvq KqwU †je weµq Ki‡Z n‡e ?

2| ewlK kZKiv $6\frac{1}{2}$ nvi mij gbvdvq 750 UvKvi 4 eQ‡ii mew×gj KZ UvKv n‡e ?

3| ewlK 4 UvKv nvi Pµew× gbvdvq 2000 UvKvi 3 eQ‡ii mew×gj wbYq Ki|

Abkxjbx 3·5

1| $x^2 - 7x + 6$ Gi Drcv`‡K we‡kwlZ i|c wb‡Pi †KvbwU ?

(K) $(x-2)(x-3)$ (L) $(x-1)(x+8)$

(M) $(x-1)(x-6)$ (N) $(x+1)(x+6)$

2| $f(x) = x^2 - 4x + 4$ n‡j, $f(2)$ Gi gvb wb‡Pi †KvbwU ?

(K) 4 (L) 2

(M) 1 (N) 0

3| $x + y = x - y$ n‡j, y Gi gvb wb‡Pi †KvbwU ?

(K) -1 (L) 0

(M) 1 (N) 2

4| $\frac{x^2+3x^3}{x+3x^2}$ Gi jwNô iƒc wb‡Pi †KvbwU ?

(K) x^2 (L) x

(M) 1 (N) 0

5| $\frac{1-x^2}{1-x}$ Gi jwNô iƒc wb‡Pi †KvbwU ?

(K) 1 (L) x

(M) $(1-x)$ (N) $(1+x)$

6| $\frac{1}{2}\{(a+b)^2-(a-b)^2\}$ Gi gvb wb‡Pi †KvbwU ?

(K) $2(a^2+b^2)$ (L) a^2+b^2

(M) $2ab$ (N) $4ab$

7| $x+\frac{2}{x}=3$ n‡j, $x^3+\frac{8}{x^3}$ Gi gvb KZ ?

(K) 1 (L) 8

(M) 9 (N) 16

8| p^4+p^2+1 Gi Drcv`‡K we‡kølwqZ iƒc wb‡Pi †KvbwU ?

(K) $(p^2-p+1)(p^2+p-1)$ (L) $(p^2-p-1)(p^2+p+1)$

(M) $(p^2+p+1)(p^2+p+1)$ (N) $(p^2+p+1)(p^2-p+1)$

9| x^2-5x+4 Gi Drcv`K KZ ?

(K) $(x-1),(x-4)$ (L) $(x+1),(x-4)$

(M) $(x+2),(x-2)$ (N) $(x-5)(x-1)$

10| $(x-7)(x-5)$ Gi gvb KZ ?

(K) $x^2+12x+35$ (L) $x^2+12x-35$

(M) $x^2-12x+35$ (N) $x^2-12x-35$

11| $\frac{2\cdot9\times2\cdot9-1\cdot1\times1\cdot1}{2\cdot9-1\cdot1}$ Gi gvb KZ ?

(K) $1\cdot8$ (L) $1\cdot9$

(M) 2 (N) 4

12| hw` $x=2-\sqrt{3}$ nq, Z‡e x^2 Gi gvb KZ ?

(K) 1 (L) $7-4\sqrt{3}$

(M) $2+\sqrt{3}$ (N) $\frac{1}{2-\sqrt{3}}$

13| $f(x)=x^2-5x+6$ Ges $f(x)=0$ n‡j, x = KZ ?

(K) $2, 3$　　(L) $-5, 1$

(M) $-2, 3$　　(N) $1, -5$

14|

	x	$+6$
x	x^2	$+6x$
-5	$-5x$	-30

Dc‡ii wP‡Îi me‡gvU †¶Îdj wb‡Pi †KvbwU ?

(K) $x^2-5x+30$　　(L) x^2+x-30

(M) $x^2+6x-30$　　(N) x^2-x+30

15| K †h KvR x w`‡b m¤úb Ki‡Z cv‡i, L †m KvR $3x$ w`‡b m¤úb Ki‡Z cv‡i| GKB mg‡q K, L Gi KZ ¸Y KvR K‡i ?

(K) 2 ¸Y　　(L) $2\frac{1}{2}$ ¸Y

(M) 3 ¸Y　　(N) 4 ¸Y

16| $a+b=-c$ n‡j, $a^2+2ab+b^2$ Gi gvb c Gi gva¨‡g cKvk Ki‡j wb‡Pi †KvbwU n‡e ?

(K) $-c^2$　　(L) c^2

(M) bc　　(N) ca

17| $x+y=3, xy=2$ n‡j, x^3+y^3 Gi gvb KZ ?

(K) 9　　(L) 18

(M) 19　　(N) 27

18| $8x^3+27y^3$ Gi Drcv`‡K we‡kølY i⁄c †KvbwU ?

(K) $(2x-3y)(4x^2+6xy+9y^2)$　　(L) $(2x+3y)(4x^2-6xy+9y^2)$

(M) $(2x-3y)(4x^2-9y^2)$　　(N) $(2x+3y)(4x^2+9y^2)$

19| $9x^2+16y^2$ Gi mv‡_ KZ †hvM Ki‡j †hvMdj cYeM iwk n‡e ?

(K) $6xy$　　(L) $12xy$

(M) $24xy$　　(N) $144xy$

20| $x-y=4$ n‡j, wb‡Pi †Kvb Dw³wU mwVK ?

(K) $x^3-y^3-4xy=64$　　(L) $x^3-y^3-12xy=12$

(M) $x^3-y^3-3xy=64$　　(N) $x^3-y^3-12xy=64$

21| hw` $x^4 - x^2 + 1 = 0$ nq, Z‡e

(1) $x^2 + \frac{1}{x^2}$ = KZ ?

(K) 4 (L) 2

(M) 1 (N) 0

(2) $\left(x + \frac{1}{x}\right)^2$ Gi gvb KZ ?

(K) 4 (L) 3

(M) 2 (N) 1

(3) $x^3 + \frac{1}{x^3}$ = KZ ?

(K) 3 (L) 2

(M) 1 (N) 0

22| K GKwU KvR p w`‡b K‡i Ges L $2p$ w`‡b K‡i| Zviv GKwU KvR Avi¤¢ K‡i Ges K‡qKw`b ci K KvRwU Amgvß †i‡L P‡j †Mj| evwK KvRUK L $r$ w`‡b †kl K‡i| KvRwU KZ w`‡b †kl n‡qwQj ?

23| ^`wbK 8 NÈv cwikg K‡i 50 Rb †jvK GKwU KvR 12 w`‡b Ki‡Z cv‡i| ^`wbK KZ NÈv cwikg K‡i 60 R‡b 16 w`‡b H KvRwU Ki‡Z cvi‡e ?

24| wgZv GKwU KvR x w`‡b Ki‡Z cv‡i| wiZv †m KvR $y$ w`‡b Ki‡Z cv‡i| Zviv GK‡Î KZ w`‡b KvRwU †kl Ki‡Z cvi‡e ?

25| eb‡fvR‡b hvIqvi Rb¨ 57000 UvKvq GKwU evm fvov Kiv n‡jv Ges kZ n‡jv †h, c‡Z¨K hvÎx mgvb fvov enb Ki‡e| 5 Rb hvÎx bv hvIqvq gv_wcQ fvov 3 UvKv e„w× †cj| ev‡m KZRb hvÎx wM‡qwQj ?

26| GKRb gvwS †mv‡Zi cwZK‡j p NÈvq d wK.wg. †h‡Z cv‡i| †mv‡Zi AbK‡j H c_ †h‡Z Zvi q NÈv j‡M| †mv‡Zi †eM I †bŠKvi †eM KZ ?

27| GKRb gvwSi `vo †e‡q 15 wK.wg. †h‡Z Ges †mLvb †_‡K wd‡i Avm‡Z 4 NÈv mgq j‡M| †m †mv‡Zi AbK‡j hZ¶‡Y 5 wK.wg. hvq, †mv‡Zi cwZK‡j ZZ¶‡Y 3 wK.wg. hvq| `v‡oi †eM I †mv‡Zi †eM wbYq Ki|

28| GKwU †PŠev”Pvq `BwU bj msh³ Av‡Q| c_g bj Øviv †PŠev”PwU $t_1$ wgwb‡U cY nq Ges wØZxq bj Øviv $t_2$ wgwb‡U Lwj nq| bj `BwU GK‡Î L‡j w`‡j Lwj †PŠev”PwU KZ¶‡Y cY n‡e ? (GLv‡b $t_1 > t_2$)

29| GKwU bj Øviv 12 wgwb‡U GKwU †PŠev”Pv cY nq| Aci GKwU bj Øviv 1 wgwb‡U Zv †_‡K 15 wjUvi cwb †ei K‡i †`q| †PŠev”PwU Lwj _vKv Ae¯vq `BwU bj GKm‡½ L‡j †`Iqv nq Ges †PŠev”PwU 48 wgwb‡U cY nq| †PŠev”PwU‡Z KZ wjUvi cwb a‡i ?

30| GKwU Kjg 11 UvKvq weµq Ki‡j 10% jvf nq| KjgwUi µqgj¨ KZ ?

31| GKwU LvZv 36 UvKvq weµq Kivq hZ ¶wZ n‡jv, 72 UvKvq weµq Ki‡j Zvi wØ¸Y jvf n‡Zv, LvZvwUi µqgj¨ KZ ?

32| K, L I M Gi g‡a¨ 260 UvKv Gi|‡c fvM K‡i `vI †hb K Gi As‡ki 2 ¸Y, L Gi As‡ki 3 ¸Y Ges M Gi As‡ki 4 ¸Y ci¯úi mgvb nq|

33| GKwU `e¨ $x$% ¶wZ‡Z weµq Ki‡j †h gj¨ cvIqv hvq, $3x$% jv‡f weµq Ki‡j Zvi †P‡q $18x$ UvKv †ewk cvIqv hvq| `e¨wUi µqgj¨ KZ wQj ?

34| 300 UvKvi 4 eQ‡ii mij gbvdv I 400 UvKvi 5 eQ‡ii mij gbvdv GK‡Î 148 UvKv n‡j, kZKiv gbvdvi nvi KZ ?

35| 4% nvi gbvdvq †Kv‡bv UvKvi 2 eQ‡ii gbvdv I Pµew× gbvdvi cv_K¨ 1 UvKv n‡j, gjab KZ ?

36| †Kv‡bv Avmj 3 eQ‡i mij gbvdvmn 460 UvKv Ges 5 eQ‡i mij gbvdvmn 600 UvKv n‡j, kZKiv gbvdvi nvi KZ ?

37| kZKiv ewlK 5 UvKv nvi mij gbvdvq KZ UvKv 13 eQ‡i mew×gj 985 UvKv n‡e ?

38| kZKiv ewlK 5 UvKv nvi gbvdvq KZ UvKv 12 eQ‡i mew×gj 1248 UvKv n‡e ?

39| 5% nvi gbvdvq 8000 UvKvi 3 eQ‡ii mij gbvdv I Pµew× gbvdvi cv_K¨ wbYq Ki|

40| wgwói Dci gj¨ ms‡hvRb Ki (VAT) x%| GKRb we‡µZv f¨vUmn P UvKvi wgwó weµq Ki‡j Zv‡K KZ f¨vU w`‡Z n‡e ? $x = 15,\ P = 2300$ n‡j, f¨v‡Ui cwigvY KZ ?

41. †Kv‡bv msL¨v I H msL¨vi ¸YvZ¥K wecixZ msL¨vi mgwó 3.

K. msL¨wU‡K x P‡K cKvk K‡i Dc‡ii Z_¨‡K GKwU mgxKi‡Yi gva¨‡g cKvk Ki|

L. $x^3 - \frac{1}{x^3}$ Gi gvb wbYq Ki|

M. cgvY Ki $x^5 - \frac{1}{x^5} = 123$

42. †Kv‡bv mwgwZi m`m¨MY c‡Z¨‡KB m`m¨msL¨vi 100 ¸Y Pv`v †`Iqvi wm×vš— wb‡jb| wKš‘ 7 Rb m`m¨ Pv`v bv †`Iqvq c‡Z¨‡Ki Pv`vi cwigvY c‡ei †P‡q 500 UvKv †e‡o †Mj|

K. mwgwZi m`m¨msL¨v $x$ Ges †gvU Pv`vi cwigvY A n‡j, G‡`i g‡a¨ m¤úK wbYq Ki|

L. mwgwZi m`m¨ msL¨v I †gvU Pv`vi cwigvY wbYq Ki|

M. †gvU Pv`vi $\frac{1}{4}$ Ask 5% nv‡i Ges Aewkó UvKv 4% nv‡i 2 eQ‡ii Rb¨ mij gbvdvq wewb‡qvM Kiv n‡jv| †gvU gbvdv wbYq Ki|

PZ_ Aa¨vq

mPK I jMwi`g

(Exponents and Logarithms)

A‡bK eo ev A‡bK †QvU msL¨v ev iwk‡K mP‡Ki mvnv‡h¨ AwZ mn‡R wj‡L cKvk Kiv hvq| d‡j wnmve MYbv I MwYwZK mgm¨v mgvavb mnRZi nq| mP‡Ki gva¨‡gB msL¨vi ˆeÁwbK ev Av`k iƒc cKvk Kiv nq| ZvB c‡Z¨K wk¶v_xi mP‡Ki aviYv I Gi c‡qvM m¤ú‡K Ávb _vKv Avek¨K|

mPK †_‡KB jMwi`‡gi mwó| Avi GB jMwi`‡gi mvnv‡h¨ msL¨v ev iwki ¸Y, fvM I mPK m¤úwKZ MYbvi KvR mnR n‡q‡Q| eZgv‡b K¨vjK‡jUi I Kw¤úDUvi Gi e¨envi cPj‡bi ce chš ˆeÁwbK wn‡me MYbvq jMwi`‡gi e¨envi wQj GKgvÎ Dcvq| Z‡e GLbI G¸‡jvi weKí wnmv‡e jMwi`‡gi e¨envi ¸i“ZcY| G Aa¨v‡q mPK I jMwi`g m¤ú‡K we¯wiZ Av‡jvPbv Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv –

- gj` mPK e¨vL¨v Ki‡Z cvi‡e|
- abvZ¥K cY-mvswL¨K mPK, kb¨ I FYvZ¥K cY-mvswL¨K mPK e¨vL¨v I c‡qvM Ki‡Z cvi‡e|
- mP‡Ki wbqgvewj eYbv I Zv c‡qvM K‡i mgm¨vi mgvavb Ki‡Z cvi‡e|
- nZg gj I gj` fMvsk mPK e¨vL¨v Ki‡Z cvi‡e Ges nZg gj‡K mPK AvKv‡i cKvk Ki‡Z cvi‡e|
- jMwi`g e¨vL¨v Ki‡Z cvi‡e|
- jMwi`‡gi mÎvewj cgvY I c‡qvM Ki‡Z cvi‡e|
- mvaviY jMwi`g I ¯vfvweK jMwi`g e¨vL¨v Ki‡Z cvi‡e|
- msL¨vi ˆeÁwbK iƒc e¨vL¨v Ki‡Z cvi‡e|
- mvaviY jMwi`‡gi cYK I AskK e¨vL¨v Ki‡Z cvi‡e|
- K¨vjK‡jU‡ii mvnv‡h¨ mvaviY I ¯vfvweK jMwi`g wbYq Ki‡Z cvi‡e|

4·1 mPK (*Exponents or Indices*) :

Avgiv lô †kYx‡Z mP‡Ki aviYv †c‡qwQ Ges mßg †kYx‡Z ¸‡Yi I fv‡Mi mPK wbqg m¤ú‡K †R‡bwQ| mPK I wfwË msewjZ iwk‡K mPKxq iwk ejv nq|

KvR : Lvwj Ni ciY Ki :			
GKB msL¨v ev ivwki µwgK ¸Y	mPKxq iwk	wfwË	NvZ ev mPK
$2\times2\times2$	2^3	2	3
$3\times3\times3\times3$		3	
$a\times a\times a$	a^3		
$b\times b\times b\times b\times b$			5

$\boldsymbol{a}$ †h‡Kv‡bv ev¯—e msLv n‡j, $\boldsymbol{n}$ msL¨K $\boldsymbol{a}$ Gi µwgK ¸Y, A_vr, $\boldsymbol{a}\times\boldsymbol{a}\times\boldsymbol{a}\times.....\times\boldsymbol{a}$ †K a^n AvKv‡i †jLv nq, †hLv‡b $\boldsymbol{n}$ abvZ¥K cYmsL¨v|

$\boldsymbol{a}\times\boldsymbol{a}\times\boldsymbol{a}\times.....\times\boldsymbol{a}$ ($\boldsymbol{n}$ msL¨K evi $\boldsymbol{a}$) = a^n.

GLv‡b, $n\rightarrow$ mPK ev NvZ
$a\rightarrow$ wfwË

Aevi, wecixZµ‡g $a^n = a\times a\times a\times........\times a$ (n msL¨K evi a)

mPK ka abvZ¥K cYmsL¨vB bq, FYvZ¥K cYmsL¨v ev abvZ¥K fMvsk ev FYvZ¥K fMvskI n‡Z cv‡i|
A_vr, wfwË $\boldsymbol{a}\in\boldsymbol{R}$ (ev¯—e msL¨vi †mU) Ges mPK $\boldsymbol{n}\in\boldsymbol{Q}$ (gj` msL¨vi †mU) Gi Rb¨ $\boldsymbol{a^n}$ msÁwqZ| Z‡e we‡kl †¶‡Î, $\boldsymbol{n}\in\boldsymbol{N}$ (¯vfvweK msL¨vi †mU) aiv nq| ZvQvov Agj` mPKI n‡Z cv‡i| Z‡e Zv gva¨wgK ¯—‡ii cvV¨m~wP ewnfZ e‡j GLv‡b †m m¤ú‡K Av‡jvPbv Kiv nq wb|

4·2 mP‡Ki mÎvewj

awi, $\boldsymbol{a}\in\boldsymbol{R};\ \boldsymbol{m},\boldsymbol{n}\in\boldsymbol{N}$.

mÎ 1| $a^m\times a^n = a^{m+n}$

mÎ 2| $\dfrac{a^m}{a^n} = \begin{cases} a^{m-n} \text{ hLb } m>n \\ \dfrac{1}{a^{n-m}}, \text{hLb } n>m \end{cases}$

wb‡Pi Q‡Ki Lvwj Ni ciY Ki :

a^m, a^n $a\neq 0$	$m>n$ $m=5,\ n=3$	$n>m$ $m=3,\ n=5$
$a^m\times a^n$	$a^5\times a^3 = (a\times a\times a\times a\times a)\times(a\times a\times a)$ $= a\times a\times a\times a\times a\times a\times a\times a$ $= a^8 = a^{5+3}$	$a^3\times a^5 =$
$\dfrac{a^m}{a^n}$	$\dfrac{a^5}{a^3} =$	$\dfrac{a^3}{a^5} = \dfrac{a\times a\times a}{a\times a\times a\times a\times a}$ $= \dfrac{1}{a^2} = \dfrac{1}{a^{5-3}}$

$\therefore\ a^m\times a^n = a^{m+n}$

Ges $\dfrac{a^m}{a^n} = \begin{cases} a^{m-n} \text{ hLb } m>n \\ \dfrac{1}{a^{n-m}}, \text{hLb } n>m \end{cases}$

mÎ 3| $(ab)^n = a^n \times b^n$

j¶ Kwi, $(5\times2)^3 = (5\times2)\times(5\times2)\times(5\times2)$ $[\because a^3 = a\times a\times a;\ a = 5\times2]$

$= 5\times2\times5\times2\times5\times2$

$= (5\times5\times5)\times(2\times2\times2)$

$= 5^3\times2^3$

mvaviYfv‡e, $(ab)^n = ab\times ab\times ab\times ....\times ab$ [n msL¨K ab Gi µwgK ¸Y]

$= (a\times a\times a\times ....\times a)\times(b\times b\times b\times ....\times b)$

$= a^n b^n$

mÎ 4| $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, (b \neq 0)$

j¶ Kwi, $\left(\dfrac{5}{2}\right)^3 = \dfrac{5}{2}\times\dfrac{5}{2}\times\dfrac{5}{2} = \dfrac{5\times5\times5}{2\times2\times2} = \dfrac{5^3}{2^3}$

mvaviYfv‡e, $\left(\dfrac{a}{b}\right)^n = \dfrac{a}{b}\times\dfrac{a}{b}\times\dfrac{a}{b}\times ....\times\dfrac{a}{b}$ [n msL¨K $\dfrac{a}{b}$ Gi µwgK ¸Y]

$= \dfrac{a\times a\times a\times ......\times a}{b\times b\times b\times ......\times b} = \dfrac{a^n}{b^n}$

mÎ 5| $a^0 = 1, (a \neq 0)$

Avgiv cvB, $\dfrac{a^n}{a^n} = a^{n-n} = a^0$

Avevi, $\dfrac{a^n}{a^n} = \dfrac{a\times a\times a\times .....\times a}{a\times a\times a\times .....\times a}$ [je I ni Dfq‡¶‡Î n msL¨K a Gi ¸Y]

$= 1$

$\therefore\ a^0 = 1.$

mÎ 6| $a^{-n} = \dfrac{1}{a^n}, (a \neq 0)$

Avgiv cvB, $a^{-n} = \dfrac{a^{-n}\times a^n}{1\times a^n}$ [je I ni‡K a^n Øviv ¸Y K‡i]

$= \dfrac{a^{-n+n}}{a^n} = \dfrac{a^o}{a^n} = \dfrac{1}{a^n}$

$\therefore\ a^{-n} = \dfrac{1}{a^n}$

gše¨ : $\frac{1}{a^n} = \frac{a^o}{a^n} = a^{o-n} = a^{-n}$

mÎ 7| $(a^m)^n = a^{mn}$

$(a^m)^n = a^m \times a^m \times a^m \times \ldots\ldots\ldots \times a^m$ [n msL¨K a^m Gi µwgK ¸Y]

$= a^{m+m+m+\ldots\ldots\ldots+m}$ [Nv‡Z n msL¨K mP‡Ki †hvMdj]

$= a^{n\times m} = a^{mn}$

$\therefore \ (a^m)^n = a^{mn}$

D`vniY 1| gvb wbYq Ki : (K) $\frac{5^2}{5^3}$ (L) $\left(\frac{2}{3}\right)^5 \times \left(\frac{2}{3}\right)^{-5}$

mgvavb : (K) $\frac{5^2}{5^3} = 5^{2-3} = 5^{-1} = \frac{1}{5^1} = \frac{1}{5}$

(L) $\left(\frac{2}{3}\right)^5 \times \left(\frac{2}{3}\right)^{-5} = \left(\frac{2}{3}\right)^{5-5} = \left(\frac{2}{3}\right)^o = 1.$

D`vniY 2| mij Ki : (K) $\frac{5^4 \times 8 \times 16}{2^5 \times 125}$ (L) $\frac{3\cdot 2^n - 4\cdot 2^{n-2}}{2^n - 2^{n-1}}$

mgvavb : (K) $\frac{5^4 \times 8 \times 16}{2^5 \times 125} = \frac{5^4 \times 2^3 \times 2^4}{2^5 \times 5^3} = \frac{5^4 \times 2^{3+4}}{5^3 \times 2^5} = \frac{5^4}{5^3} \times \frac{2^7}{2^5} = 5^{4-3} \times 2^{7-5}$

$= 5^1 \times 2^2 = 5 \times 4 = 20$

(L) $\frac{3\cdot 2^n - 4\cdot 2^{n-2}}{2^n - 2^{n-1}} = \frac{3\cdot 2^n - 2^2\cdot 2^{n-2}}{2^n - 2^n \cdot 2^{-1}} = \frac{3\cdot 2^n - 2^{2+n-2}}{2^n - 2^n\cdot\frac{1}{2}}$

$= \frac{3\cdot 2^n - 2^n}{\left(1-\frac{1}{2}\right)\cdot 2^n} = \frac{(3-1)\cdot 2^n}{\frac{1}{2}\cdot 2^n} = \frac{2\cdot 2^n}{\frac{1}{2}\cdot 2^n} = 2\cdot 2 = 4.$

D`vniY 3| †`LvI †h, $(a^p)^{q-r}\cdot(a^q)^{r-p}(a^r)^{p-q} = 1$

mgvavb : $(a^p)^{q-r}\cdot(a^q)^{r-p}\cdot(a^r)^{p-q}$

$= a^{p(q-r)}\cdot a^{q(r-p)}\cdot a^{r(p-q)}$, $[\because (a^m)^n = a^{mn}]$

$= a^{pq-pr}\cdot a^{qr-pq}\cdot a^{pr-qr}$

$= a^{pq-pr+qr-pq+pr-qr}$

$= a^0 = 1.$

KvR : Lwj Ni ciY Ki :

$(i)\ \ 3\times3\times3\times3=3^{\square}$ $(ii)\ 5^{\square}\times5^{3}=5^{5}$ $(iii)\ a^{2}\times a^{\square}=a^{-3}$ $(iv)\ \frac{4}{4^{\square}}=1$ $(v)\ (-5)^{0}=\square$

4·3 n Zg gj

j¶ Kwi, $5^{\frac{1}{2}}\times5^{\frac{1}{2}}=\left(5^{\frac{1}{2}}\right)^{2}$

Avevi, $5^{\frac{1}{2}}\times5^{\frac{1}{2}}=5^{\frac{1}{2}+\frac{1}{2}}=5^{2\times\frac{1}{2}}=5$

$\therefore\ \left(5^{\frac{1}{2}}\right)^{2}=5$

$5^{\frac{1}{2}}$ Gi eM (wØZxq NvZ) = 5 Ges 5 Gi eMgj (wØZxq gj) = $5^{\frac{1}{2}}$

$5^{\frac{1}{2}}$ †K eMg‡ji wPý $\sqrt{\ }$ Gi gva¨‡g $\sqrt{5}$ AvKv‡i †jLv nq|

Avevi, j¶ Kwi, $5^{\frac{1}{3}}\times5^{\frac{1}{3}}\times5^{\frac{1}{3}}=\left(5^{\frac{1}{3}}\right)^{3}$

Avevi, $5^{\frac{1}{3}}\times5^{\frac{1}{3}}\times5^{\frac{1}{3}}=5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}=5^{3\times\frac{1}{3}}=5$

$\therefore\ \left(5^{\frac{1}{3}}\right)^{3}=5.$

$5^{\frac{1}{3}}$ Gi Nb (ZZxq NvZ) $=5$ Ges 5 Gi Nbgj (ZZxq gj) $=5^{\frac{1}{3}}$

$5^{\frac{1}{3}}$ †K Nbg‡ji wPý $\sqrt[3]{\ }$ Gi gva¨‡g $\sqrt[3]{5}$ AvKv‡i †jLv nq|

n Zg g‡ji †¶‡Î,

$a^{\frac{1}{n}}\times a^{\frac{1}{n}}\times a^{\frac{1}{n}}\times.......\times a^{\frac{1}{n}}$ [n msL¨K $a^{\frac{1}{n}}$ Gi µwgK ¸Y]

$=\left(a^{\frac{1}{n}}\right)^{n}.$

Avevi, $a^{\frac{1}{n}}\times a^{\frac{1}{n}}\times a^{\frac{1}{n}}\times.......\times a^{\frac{1}{n}}$

$=a^{\frac{1}{n}+\frac{1}{n}+\frac{1}{n}+.........+\frac{1}{n}}$ [mP‡K n msL¨K $\frac{1}{n}$ Gi †hvM]

$=a^{n\times\frac{1}{n}}=a$

$\therefore\ \left(a^{\frac{1}{n}}\right)^{n}=a.$

$a^{\frac{1}{n}}$ Gi n Zg NvZ $= a$ Ges a Gi n Zg gj $= a^{\frac{1}{n}}$

A_vr, $a^{\frac{1}{n}}$ Gi n Zg NvZ $= \left(a^{\frac{1}{n}}\right)^n = a$ Ges a Gi n Zg gj $(a)^{\frac{1}{n}} = a^{\frac{1}{n}} = \sqrt[n]{a}$ | a Gi n Zg gj‡K $\sqrt[n]{a}$ AvKv‡i †jLv nq|

D`vniY 4| mij Ki : (K) $7^{\frac{3}{4}} \cdot 7^{\frac{1}{2}}$ (L) $(16)^{\frac{3}{4}} \div (16)^{\frac{1}{2}}$ (M) $\left(10^{\frac{2}{3}}\right)^{\frac{3}{4}}$

mgvavb : (K) $7^{\frac{3}{4}} \cdot 7^{\frac{1}{2}} = 7^{\frac{3}{4}+\frac{1}{2}} = 7^{\frac{5}{4}}$

(L) $(16)^{\frac{3}{4}} \div (16)^{\frac{1}{2}} = \dfrac{(16)^{\frac{3}{4}}}{(16)^{\frac{1}{2}}} = (16)^{\frac{3}{4}-\frac{1}{2}} = (16)^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2.$

(M) $\left(10^{\frac{2}{3}}\right)^{\frac{3}{4}} = 10^{\frac{2}{3}\times\frac{3}{4}} = 10^{\frac{1}{2}} = \sqrt{10}.$

D`vniY 5| mij Ki : (K) $(12)^{-\frac{1}{2}} \times \sqrt[3]{54}$ (L) $(-3)^3 \times \left(-\dfrac{1}{2}\right)^2$

mgvavb : (K) $(12)^{-\frac{1}{2}} \times \sqrt[3]{54} = \dfrac{1}{(12)^{\frac{1}{2}}} \times (54)^{\frac{1}{3}}$

$$= \frac{1}{(2^2 \times 3)^{\frac{1}{2}}} \times (3^3 \times 2)^{\frac{1}{3}} = \frac{1}{(2^2)^{\frac{1}{2}} \cdot \times 3^{\frac{1}{2}}} \times (3^3)^{\frac{1}{3}} \cdot 2^{\frac{1}{3}}$$

$$= \frac{1}{2 \cdot 3^{\frac{1}{2}}} \times 3 \cdot 2^{\frac{1}{3}} = \frac{2^{\frac{1}{3}}}{2^1} \times \frac{3^1}{3^{\frac{1}{2}}} = \frac{3^{1-\frac{1}{2}}}{2^{1-\frac{1}{3}}} = \frac{3^{\frac{1}{2}}}{2^{\frac{2}{3}}} = \frac{3^{\frac{1}{2}}}{4^{\frac{1}{3}}} = \frac{\sqrt{3}}{\sqrt[3]{4}}.$$

(L) $(-3)^3 \times \left(-\dfrac{1}{2}\right)^2$

$= (-3)(-3)(-3) \times \left(-\dfrac{1}{2}\right)\left(-\dfrac{1}{2}\right)$

$= -27 \times \dfrac{1}{4}$

$= -\dfrac{27}{4}$

KvR : mij Ki : (i) $\dfrac{2^4 \cdot 2^2}{32}$ (ii) $\left(\dfrac{2}{3}\right)^5 \times \left(\dfrac{2}{3}\right)^{-5}$ (iii) $8^{\frac{3}{4}} \div 8^{\frac{1}{2}}$

j¶Yxq :

1. $a > 0, a \neq 1$ k‡Z $a^x = a^y$ n‡j, $x = y$

2. $a > 0, b > 0, x \neq 0$ k‡Z $a^x = b^x$ n‡j, $a = b$

D`vniY 6| mgvavb Ki $4^{x+1} = 32$

mgvavb : $4^{x+1} = 32$

ev $(2^2)^{x+1} = 32$, ev $2^{2x+2} = 2^5$

$\therefore 2x + 2 = 5$, [$a^x = a^y$ n‡j, $x = y$]

ev $2x = 5 - 2$, ev $2x = 3$

$\therefore x = \frac{3}{2}$

Abkxjbx 4·1

mij Ki (1 – 10) :

1| $\frac{3^3 \cdot 3^5}{3^6}$ 2| $\frac{5^3 \cdot 8}{2^4 \cdot 125}$ 3| $\frac{7^3 \times 7^{-3}}{3 \times 3^{-4}}$ 4| $\frac{\sqrt[3]{7^2} \cdot \sqrt[3]{7}}{\sqrt{7}}$ 5| $(2^{-1} + 5^{-1})^{-1}$

6| $(2a^{-1} + 3b^{-1})^{-1}$ 7| $\left(\frac{a^2 b^{-1}}{a^{-2} b}\right)^2$ 8| $\sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x}, (x > 0, y > 0, z > 0)$

9| $\frac{2^{n+4} - 4 \cdot 2^{n+1}}{2^{n+2} \div 2}$ 10| $\frac{3^{m+1}}{(2^m)^{m-1}} \div \frac{9^{m+1}}{(3^{m-1})^{m+1}}$

cgvY Ki (11 – 18) :

11| $\frac{4^n - 1}{2^n - 1} = 2^n + 1$ 12| $\frac{2^{p+1} \cdot 3^{2p-q} \cdot 5^{p+q} \cdot 6^p}{6^q \cdot 10^{p+2} \cdot 15^q} = \frac{1}{50}$

13| $\left(\frac{a^\ell}{a^m}\right)^n \cdot \left(\frac{a^m}{a^n}\right)^\ell \cdot \left(\frac{a^n}{a^\ell}\right)^m = 1$ 14| $\frac{a^{p+q}}{a^{2r}} \times \frac{a^{q+r}}{a^{2p}} \times \frac{a^{r+p}}{a^{2q}} = 1$

15| $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$ 16| $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

17| $\left(\frac{x^p}{x^q}\right)^{p+q-r} \times \left(\frac{x^q}{x^r}\right)^{q+r-p} \times \left(\frac{x^r}{x^p}\right)^{r+p-q} = 1$

18| hw` $a^x = b,\ b^y = c$ Ges $c^z = a$ nq, Z‡e †`LvI †h, $xyz = 1$

mgvavb Ki (19 – 22) :

19| $4^x = 8$ 20| $2^{2x+1} = 128$ 21| $\left(\sqrt{3}\right)^{x+1} = \left(\sqrt[3]{3}\right)^{2x-1}$ 22| $2^x + 2^{1-x} = 3$

4·4 jMwi`g *(Logarithm)*

mPKxq iwki gvb †ei Ki‡Z jMwi`g e¨envi Kiv nq| jMwi`g‡K ms‡¶‡c jM (Log) †jLv nq| eo eo msL¨v ev iwki ¸Ydj, fvMdj BZ¨w` log Gi mvnv‡h¨ mn‡R wbYq Kiv hvq|

Avgiv Rwb, $2^3=8$; GB MwYwZK Dw³wU‡K j‡Mi gva¨‡g †jLv nq $\log_2 8=3$. Aevi, wecixZµ‡g, $\log_2 8=3$ n‡j, mP‡Ki gva¨‡g †jLv hv‡e $2^3=8$; A_vr, $2^3=8$ n‡j $\log_2 8=3$ Ges wecixZµ‡g, $\log_2 8=3$ n‡j $2^3=8$. GKBfv‡e, $2^{-3}=\frac{1}{2^3}=\frac{1}{8}$ †K j‡Mi gva¨‡g †jLv hvq, $\log_2\frac{1}{8}=-3$.

$a^x=N,(a>0,a\neq1)$ n‡j, $x=\log_a N$ †K

N Gi a wfwËK jM ejv nq|

j¶Yxq : x abvZ¥K ev FYvZ¥K hvB †nvK bv †Kb, a^x me`v abvZ¥K| ZvB ïa abvZ¥K msL¨viB j‡Mi gvb Av‡Q hv ev¯e| kb¨ ev FYvZ¥K msL¨vi j‡Mi ev¯e gvb †bB|

KvR 1 : j‡Mi gva¨‡g cKvk Ki :		KvR 2 : dvKv RvqMv ciY Ki :	
$(i)\ 10^2=100$		mP‡Ki gva¨‡g	j‡Mi gva¨‡g
$(ii)\ 3^{-2}=\frac{1}{9}$		$10^0=1$	$\log_{10}1=0$
$(iii)\ 2^{-\frac{1}{2}}=\frac{1}{\sqrt{2}}$		$e^0=.....$ $a^0=1$	$\log_e 1=......$ ...... = ...
$(iv)\ \sqrt[4]{2}=4$		$10^1=10$	$\log_{10}10=1$
		$e^1=...$	.... = ...
		.... = ...	$\log_a a=1$

jMwi`‡gi mÎvewj

awi, $a>0,a\neq1;b>0,b\neq1$ Ges $M>0,N>0$.

mÎ 1| (K) $\log_a 1=0,(a>0,a\neq1)$

(L) $\log_a a=1,(a>0,a\neq1)$

cgvY (K) mP‡Ki mÎ n‡Z Rwb, $a^0=1$

$\therefore$ j‡Mi msÁv n‡Z cvB, $\log_a 1=0$ (cgwYZ)

(L) mP‡Ki mÎ n‡Z Rwb, $a^1=a$

$\therefore$ j‡Mi msÁv n‡Z cvB, $\log_a a=1$ (cgwYZ)|

mÎ 2| $\log_a(MN)=\log_a M+\log_a N$

cgvY : awi, $\log_a M=x,\log_a N=y$;

$$\therefore\ M=a^x,N=a^y$$

GLb, $MN = a^x \cdot a^y = a^{x+y}$

$\therefore \log_a(MN) = x + y$, ev $\log_a(MN) = \log_a M + \log_a N$ [x, y Gi gvb ewm‡q]

$\therefore \log_a(MN) = \log_a M + \log_a N$. (cgvYZ)

` óe¨-1| $\log_a(MNP.....) = \log_a M + \log_a N + \log_a P + ...$

` óe¨-2| $\log_a(M \pm N) \neq \log_a M \pm \log_a N$

mÎ 3| $\log_a \frac{M}{N} = \log_a M - \log_a N$

cgvY : awi, $\log_a M = x, \log_a N = y$;

$\therefore M = a^x, N = a^y$

GLb, $\frac{M}{N} = \frac{a^x}{a^y} = a^{x-y}$

$\therefore \log_a\left(\frac{M}{N}\right) = x - y$

$\therefore \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$ (cgvYZ)|

mÎ 4| $\log_a M^r = r \log_a M$.

cgvY : awi, $\log_a M = x$; $\therefore M = a^x$

ev $(M)^r = (a^x)^r$; ev $M^r = a^{rx}$

$\therefore \log_a M^r = rx$; ev $\log_a M^r = r \log_a M$

$\therefore (\log_a M^r = r \log_a M$.cgvYZ)

` óe¨ : $(\log_a M)^r \neq r \log_a M$

mÎ 5| $\log_a M = \log_b M \times \log_a b$, (wfwË cwieZb)

cgvY : awi, $\log_a M = x, \log_b M = y$

$\therefore a^x = M, b^y = M$

$\therefore a^x = b^y$, ev $(a^x)^{\frac{1}{y}} = (b^y)^{\frac{1}{y}}$

ev $b = a^{\frac{x}{y}}$

$\therefore \quad \therefore \frac{x}{y} = \log_a b$

ev, $x = y \log_a b$, ev $\log_a M = \log_b M \times \log_a b$ (cgvYZ)

Abywm×vš : $log_a b = \frac{1}{log_b a}$, A_ev $\log_b a = \frac{1}{\log_a b}$

cgvY : Avgiv Rvwb, $\log_a M = \log_b M \times \log_a b$ [m~Î 5]

$M = a$ ewm‡q cvB, $\log_a a = \log_b a \times \log_a b$

ev, $1 = log_b a \times log_a b$; $\therefore log_b a = \frac{1}{log_a b}$, A_ev $log_a b = \frac{1}{log_b a}$ (cgwYZ)|

ev, $1 = \log_b a \times \log_a b$; $\therefore \log_b a = \frac{1}{\log_a b}$, A_ev $\log_a b = \frac{1}{\log_b a}$ (cgwYZ)|

D`vniY 7| gvb wbYq Ki : (K) $\log_{10} 100$ (L) $\log_3 \left(\frac{1}{9}\right)$ (M) $\log_{\sqrt{3}} 81$

mgvavb :

(K) $\log_{10} 100 = \log_{10} 10^2 = 2\log_{10} 10 [\because \log_{10} M^r = r\log_{10} M]$

$= 2 \times 1 \ [\because \log_a a = 1] = 2$

(L) $\log_3 \left(\frac{1}{9}\right) = \log_3 \left(\frac{1}{3^2}\right) = \log_3 3^{-2} = -2\log_3 3 \ [\because \log_a M^r = r\log_a M]$

$= -2 \times 1 [\because \log_a a = 1] = -2$

(M) $\log_{\sqrt{3}} 81 = \log_{\sqrt{3}} 3^4 = \log_{\sqrt{3}} \{(\sqrt{3})^2\}^4 = \log_{\sqrt{3}} (\sqrt{3})^8$

$= 8\log_{\sqrt{3}} \sqrt{3} \ [\because \log_a M^r = r\log_a M]$

$= 8 \times 1, [\because \log_a a = 1]$

= 8

D`vniY 8| (K) $5\sqrt{5}$ Gi 5 wfwËK jM KZ ?

(L) 400 Gi jM 4; wfwË KZ ?

mgvavb : (K) $5\sqrt{5}$ Gi 5 wfwËK jM

$= \log_5 5\sqrt{5} = \log_5 (5 \times 5^{\frac{1}{2}}) = \log_5 5^{\frac{3}{2}}$

$\frac{3}{2}\log_5 5, [\because \log_a M^r = r\log_a M]$

$= \frac{3}{2} \times 1, [\because \log_a a = 1]$

$= \frac{3}{2}$

(L) awi, wfwË a

$\therefore$ ckg‡Z, $\log_a 400 = 4$

$\therefore a^4 = 400$

ev, $a^4 = (20)^2 = \{(2\sqrt{5})^2\}^2 = (2\sqrt{5})^4$

ev, $a^4 = (2\sqrt{5})^4$

$\therefore a = 2\sqrt{5}$ $[\because a^x = b^x$ n‡j, $a = b]$

$\therefore$ wfwË $2\sqrt{5}$

D`vniY 9| x Gi gvb wbYq Ki :

(K) $\log_{10} x = -2$ (L) $\log_x 324 = 4$

mgvavb :

(K) $\log_{10} x = -2$

$\therefore x = 10^{-2} = \dfrac{1}{10^2}$

ev $x = \dfrac{1}{100} = 0 \cdot 01$

$\therefore x = 0 \cdot 01$

(L) $\log_x 324 = 4$

$\therefore x^4 = 324 = 3 \times 3 \times 3 \times 3 \times 2 \times 2$

$= 3^4 \times 2^2 = 3^4 \times (\sqrt{2})^4$

ev $x^4 = (3\sqrt{2})^4$

$\therefore x = 3\sqrt{2}.$

D`vniY 10| cgvY Ki †h, $3\log_{10} 2 + \log_{10} 5 = \log_{10} 40$

mgvavb : evgc¶ = $3\log_{10} 2 + \log_{10} 5$

$= \log_{10} 2^3 + \log_{10} 5, [\because \log_a M^r = r\log_a M]$

$= \log_{10} 8 + \log_{10} 5$

$= \log_{10}(8 \times 5), [\because \log_n(MN) = \log_a M + \log_a N]$

$= \log_{10} 40$

$= \log_{10} 2^3 + \log_{10} 5, [\because \log_a M^r = r\log_a M]$

$= \log_{10} 8 + \log_{10} 5$

$= \log_{10}(8 \times 5), [\because \log_n(MN) = \log_a M + \log_a N]$

$= \log_{10} 40$

= Wvbc¶

D`vniY 11| mij Ki : $\dfrac{\log_{10}\sqrt{27} + \log_{10} 8 - \log_{10}\sqrt{1000}}{\log_{10} 1 \cdot 2}$

mgvavb : $\dfrac{\log_{10}\sqrt{27} + \log_{10} 8 - \log_{10}\sqrt{1000}}{\log_{10} 1 \cdot 2}$

$$= \frac{\log_{10}(3^3)^{\frac{1}{2}} + \log_{10} 2^3 - \log_{10}(10^3)^{\frac{1}{2}}}{\log_{10}\frac{12}{10}}$$

$$= \frac{\log_{10} 3^{\frac{3}{2}} + \log_{10} 2^3 - \log_{10} 10^{\frac{3}{2}}}{\log_{10} 12 - \log_{10} 10}$$

$$= \frac{\frac{3}{2}\log_{10}(3 + 3\log_{10} 2 - \log_{10} 10}{\log_{10}(3\times 2^2) - \log_{10} 10}$$

$$\frac{\frac{3}{2}(\log_{10} 3 + 2\log_{10} 2 - 1)}{(\log_{10} 3 + 2\log_{10} 2 - 1)}, [\because \log_{10} 10 = 1]$$

$$= \frac{3}{2}.$$

Abkxjbx 4.2

1| gvb wbYq Ki : (K) $\log_3 81$ (L) $\log_5 \sqrt[3]{5}$ (M) $\log_4 2$ (N) $\log_{2\sqrt{5}} 400$
(O) $\log_5\left(\sqrt[3]{5}.\sqrt{5}\right)$

2| x Gi gvb wbYq Ki : (K) $\log_5 x = 3$ (L) $\log_x 25 = 2$ (M) $\log_x \frac{1}{16} = -2$

3| †`LvI †h,
(K) $5\log_{10} 5 - \log_{10} 25 = \log_{10} 125$
(L) $\log_{10}\frac{50}{147} = \log_{10} 2 + 2\log_{10} 5 - \log_{10} 3 - 2\log_{10} 7$
(M) $3\log_{10} 2 + 2\log_{10} 3 + \log_{10} 5 = \log_{10} 360$

4| mij Ki :
(K) $7\log_{10}\frac{10}{9} - 2\log\frac{25}{24} + 3\log\frac{81}{80}$
(L) $\log_7\left(\sqrt[5]{7}.\sqrt{7}\right) - \log_3\sqrt[3]{3} + \log_4 2$
(M) $\log_e\frac{a^3b^3}{c^3} + \log_e\frac{b^3c^3}{d^3} + \log_e\frac{c^3d^3}{a^3} - 3\log_e b^2c$

4.5 msL¨vi ˆeÁvwbK ev Av`k© iƒc

mP‡Ki mvnv‡h¨ Avgiv A‡bK eo ev A‡bK †QvU msL¨v‡K †QvU I mnR AvKv‡i cKvk Ki‡Z cvwi| †hgb, Av‡jvi MwZ = 300000 wK.wg./†m. = 300000000 wgUvi/†m.

= 3×100000000 wg./†m. = 3×10^8 wg./†m.

Avevi, GKwU nvB‡Wv‡Rb cigvYi e¨vmva

= $0\cdot 0000000037$ †m. wg.

= $\frac{37}{10000000000}$ †m.wg.= 37×10^{-10} †m.wg.

= $3\cdot7\times10\times10^{-10}$ †m.wg.= $3\cdot7\times10^{-9}$ †m.wg.|

mweavi Rb¨ A‡bK eo ev A‡bK †QvU msL¨v‡K $a\times10^{n}$ AvKv‡i cKvk Kiv nq, †hLv‡b, $1\leq a<10$ Ges $n\in Z$. †Kv‡bv msL¨vi $a\times10^{n}$ i/c‡K ejv nq msL¨wUi ˆeÁvwbK ev Av`k i/c|

KvR : wb‡Pi msL¨v¸‡jv‡K ˆeÁvwbK AvKv‡i cKvk Ki :

(K) 15000 (L) $0\cdot000512$

4·6 jMvwi`g c×wZ

jMvwi`g c×wZ `B ai‡bi :

(K) ¯vfvweK jMvwi`g (Natural logarithm) :

¯‹Uj¨v‡Ûi MwYZwe` Rb †bwcqvi (*John Napier* :1550 – 1617) 1614 mv‡j $e$ †K wfwË a‡i c_g jMvwi`g m¤úwKZ eB cKvk K‡ib| e GKwU Agj` msL¨v, $e=2\cdot71828.........$| Zvi GB jMvwi`g‡K †bwcwiqvb jMvwi`g ev $e$ wfwËK jMvwi`g ev ¯vfvweK jMvwi`gI ejv nq| $\log_e x$ †K $\ln x$ AvKv‡iI †jLv nq|

(L) mvaviY jMvwi`g (Common Logarithm) :

Bsj¨v‡Ûi MwYZwe` †nbwi weMm (*Henry Briggs* :1561 – 1630) 1624 mv‡j 10†K wfwË a‡i jMvwi`‡gi †Uwej (jM †Uwej ev jM mviwY) ˆZwi K‡ib| Zvi GB jMvwi`g‡K weMm jMvwi`g ev 10 wfwËK jMvwi`g ev e¨enwiK jMvwi`gI ejv nq|

`óe¨ : jMvwi`‡gi wfwËi D‡jL bv _vK‡j iwki (exRMwYZxq) †¶‡Î e †K Ges msL¨vi †¶‡Î 10 †K wfwË wn‡m‡e aiv nq| jM mviwY‡Z wfwË 10 ai‡Z nq|

4·7 mvaviY jMvwi`‡gi cYK I AskK

(K) cYK (Characteristics) :

awi, GKwU msL¨v N †K ˆeÁvwbK AvKv‡i cKvk K‡i cvB,

$N=a\times10^{n}$, †hLv‡b $N>0, 1\leq a<10$ Ges $n\in Z$ |

Dfqc‡¶ 10 wfwË‡Z jM wb‡q cvB,

$\log_{10}N=\log_{10}(a\times10^{n})$

$\therefore \log_{10}a+\log_{10}10^{n}=\log_{10}a+n\log_{10}10$

= $\log_{10}N=n+\log_{10}a, [\because \log_{10}10=1]$

wfwË 10 Dn¨ †i‡L cvB,

$\log N=n+\log a$

n†K ejv nq $\log N$ Gi cYK|

j¶ Kwi : QK 1

N	N Gi $a\times10^n$ AvKvi	mPK	`kwg‡Ki ev‡gi As‡ki A¼msL¨v	cYK
6237	$6\cdot237\times10^3$	3	4	$4-1=3$
$623\cdot7$	$6\cdot237\times10^2$	2	3	$3-1=2$
$62\cdot37$	$6\cdot237\times10^1$	1	2	$2-1=1$
$6\cdot237$	$6\cdot237\times10^0$	0	1	$1-1=0$
$0\cdot6237$	$6\cdot237\times10^{-1}$	-1	0	$0-1=-1$

j¶ Kwi : QK 2

N	N Gi $a\times10^n$ AvKvi	mPK	`kwg‡Ki ev‡gi As‡ki A¼msL¨v	cYK
$0\cdot6237$	$6\cdot237\times10^{-1}$	-1	0	$-(0+1)=-1$
$0\cdot06237$	$6\cdot237\times10^{-2}$	-2	1	$-(1+1)=-2$
$0\cdot006237$	$6\cdot237\times10^{-3}$	-3	2	$-(2+1)=-3$

QK 1 †_‡K j¶ Kwi :

c`Ë msL¨vi cY As‡k hZ¸‡jv A¼ _vK‡e, msL¨wUi jMwi`‡gi cYK n‡e †mB A¼msL¨vi †P‡q 1 Kg Ges Zv n‡e abvZ¥K|

QK-2 †_‡K j¶ Kwi :

c`Ë msL¨vi cY Ask bv _vK‡j `kwgK we›` I Gi c‡ii c_g mv_K A‡¼i gv‡S hZ¸‡jv 0 (kb¨) _vK‡e, msL¨wUi jMwi`‡gi cYK n‡e k‡b¨i msL¨vi †P‡q 1 †ewk Ges Zv n‡e FYvZ¥K|

`óe¨ 1| cYK abvZ¥K ev FbvZ¥K n‡Z cv‡i, wKš AskK me`v abvZ¥K|

`óe¨ 2| †Kv‡bv cYK FbvZ¥K n‡j, cYKwUi ev‡g ô–ô wPý bv w`‡q cYKwUi Dc‡i ô–ô (evi wPý) w`‡q †jLv nq| †hgb, cYK $-3$ †K †jLv n‡e $\bar{3}$ w`‡q| Zv bv n‡j AskKmn j‡Mi m¤úY AskwU FYvZ¥K eSv‡e|

D`vniY 12| wb‡Pi msL¨v¸‡jvi j‡Mi cYK wbYq Ki :

(i) 5570 (ii) $45\cdot70$ (iii) $0\cdot4305$ (iv) $0\cdot000435$

mgvavb : (i) $5570=5\cdot570\times1000=5\cdot570\times10^3$

$\therefore$ msL¨wUi j‡Mi cYK 3.

Ab¨fv‡e, 5570 msL¨wU‡Z A‡¼i msL¨v 4 wU|

$\therefore$ msL¨wUi j‡Mi cYK $=4-1=3$

$\therefore$ msL¨wUi j‡Mi cYK 3.

(*ii*) $45\cdot 70 = 4\cdot 570\times 10^{1}$

∴ msL¨wUi cYK 1.

Ab¨fv‡e, msL¨wUi `kwg‡Ki ev‡g, A_vr cY As‡k 2 wU A¼ Av‡Q|

∴ msL¨wUi j‡Mi cYK $= 2-1 = 1$

∴ $45\cdot 70$ msL¨wUi j‡Mi cYK 1

(*iii*) $0\cdot 4305 = 4\cdot 305\times 10^{-1}$

∴ msL¨wUi cYK -1

Ab¨fv‡e, msL¨wUi `kwgK we›`i Av‡M, A_vr cY As‡k †Kv‡bv mv_K A¼ †bB, ev kb¨wU A¼ Av‡Q|

∴ msL¨wUi cYK $= 0-1 = -1 = \bar{1}$

Ab¨fv‡e, $0\cdot 4305$ msL¨vi `kwgK we›` I Gi cieZx 1g mv_K A¼ 4 Gi gv‡S †Kv‡bv 0 (kb¨) †bB, A_vr kb¨wU 0 Av‡Q|

∴ msL¨wUi cYK $= -(0+1) = -1 = \bar{1}$

∴ $0\cdot 4305$ msL¨wUi j‡Mi cYK $\bar{1}$

(*iv*) $0\cdot 000435 = 4\cdot 35\times 10^{-4}$

∴ msL¨wUi j‡Mi cYK -4 ev $\bar{4}$

Ab¨fv‡e, msL¨wUi `kwgK we›` I Gi cieZx 1g mv_K A¼ 4 Gi gv‡S 3 wU 0 (kb¨) Av‡Q|

∴ msL¨wUi j‡Mi cYK $= -(3+1) = -4 = \bar{4}$

∴ $0\cdot 000435$ Gi j‡Mi cYK $\bar{4}$

(L) AskK (*Mantissa*) :

†Kv‡bv msL¨vi mvaviY j‡Mi AskK 1 A‡c¶v †QvU GKwU AFYvZ¥K msL¨v| GwU gjZ: Agj` msL¨v| Z‡e GKwU wbw`ó `kwgK ¯vb chš— As‡Ki gvb †ei Kiv nq|

†Kv‡bv msL¨vi j‡Mi AskK jM Zwj‡Kv †_‡K †ei Kiv hvq| Avevi Zv K¨vjK‡jU‡ii mvnv‡h¨I †ei Kiv hvq| Avgiv wØZxq c×wZ‡Z, A_vr K¨vjK‡jU‡ii mvnv‡h¨ msL¨vi j‡Mi AskK †ei Ki‡ev|

K¨vjK‡jU‡ii mvnv‡h¨ msL¨vi mvaviY jM wbYq :

D`vniY 13| log 2717 Gi cYK I AskK wbYq Ki :

mgvavb : K¨vjK‡jUi e¨envi Kwi :

| AC | | log | | 2717 | | = | | $3\cdot 43408$

∴ log 2717 Gi cYK 3 Ges AskK $\cdot 43408$

D`vniY 14| $\log\ 43\cdot517$ Gi cYK I AskK †ei Ki|

mgvavb : K¨vjK‡jUi e¨envi Kwi :

AC	log	43·517	=	1·63866

$\therefore\ \log 43\cdot517$ Gi cYK 1 Ges AskK $\cdot 63866$

D`vniY 15| $0\cdot00836$ Gi j‡Mi cYK I AskK KZ ?

mgvavb : K¨vjK‡jUi e¨envi Kwi :

AC	log	0·00836	=	$-3\cdot92221=\bar{3}\cdot92221$

$\therefore\ \log 0\cdot00836$ Gi cYK -3 ev $\bar{3}$ Ges AskK $\cdot92221$

D`vniY 16| $\log_e 10$ wbYq Ki :

mgvavb : $\log_e 10 = \dfrac{1}{\log_{10} e}$

$= \dfrac{1}{\log_{10} 2\cdot71828} = \dfrac{1}{0.43429}$ [K¨vjK‡jUi e¨envi K‡i]

$= 2.30259$ (cvq)|

weKí : K¨vjK‡jUi e¨envi Kwi :

AC	ln	10	=	2.30259 (cvq)

KvR : K¨vjK‡jUi e¨envi K‡i wbgwjwLZ msL¨v¸‡jvi 10 wfwËK I e wfwËK jM wbYq Ki :
(i) 2550 (ii) 52·143 (iii) 0·4145 (iv) 0·0742

Abkxjbx 4·3

1| †Kvb k‡Z $a^0=1$?

K. $a=0$ L. $a\neq0$ M. $a>0$ N. $a\neq1$

2| $\sqrt[3]{5}\cdot\sqrt[3]{5}$ Gi gvb wb‡Pi †KvbwU ?

K. $\sqrt[6]{5}$ L. $\left(\sqrt[3]{5}\right)^3$ M. $\left(\sqrt{5}\right)^3$ N. $\sqrt[3]{25}$

3| mwVK †Kvb k‡Z $\ell og_a\ a=1$?

K. $a>0$ L. $a\neq1$ M. $a>0, a\neq1$ N. $a\neq0, a>1$

4| $log_x\ 4=2$ n‡j, x Gi gvb KZ ?

K. 2 L. ±2 M. 4 N. 10

5| GKwU msL¨v‡K $a\times10^n$ AvKv‡i †jLvi Rb¨ kZ †KvbwU ?

K. $1<a<10$ L. $1\le a\le10$ M. $1\le a<10$ N. $1<a\le10$

6| wb‡Pi Z_¨¸‡jv j¶ Ki :

$i.\ log_a(m)^p = plog_a m$

$ii.\ 2^4 = 16$ Ges $log_2 16 = 4$ mgv_K

$iii.\ log_a(m+n) = log_a m + log_a n$

Ic‡ii †Kvb Z_¨¸‡jv mwVK ?

K. i I ii L. ii I iii M. i I iii N. i, ii I iii

7| $0\cdot0035$ Gi mvaviY j‡Mi cYK KZ ?

K. 3 L. 1 M. $\bar{2}$ N. $\bar{3}$

8| $0\cdot0225$ msL¨wU we‡ePbv K‡i wb‡Pi ck¸‡jvi DËi `vI :

(1) msL¨wUi a^n AvKvi wb‡Pi †KvbwU ?

K. $(2\cdot5)^2$ L. $(\cdot015)^2$ M. $(1\cdot5)^2$ N. $(\cdot15)^2$

(2) msL¨wUi ˆeÁvwbK AvKvi wb‡Pi †KvbwU ?

K. 225×10^{-4} L. $22\cdot5\times10^{-3}$ M. $2\cdot25\times10^{-2}$ N. $\cdot225\times10^{-1}$

(3) msL¨wUi mvaviY j‡Mi cYK KZ ?

K. $\bar{2}$ L. $\bar{1}$ M. 0 N. 2

9| ˆeÁvwbK i/‡c cKvk Ki :

(K) 6530 (L) $60\cdot831$ (M) $0\cdot000245$ (N) 37500000 (O) $0\cdot00000014$

10| mvaviY `kwgK i/‡c cKvk Ki :

(K) 10^5 (L) 10^{-5} (M) $2\cdot53\times10^4$ (N) $9\cdot813\times10^{-3}$ (O) $3\cdot12\times10^{-5}$

11| wb‡Pi msL¨v¸‡jvi mvaviY j‡Mi cYK †ei Ki (K¨vjK‡jUi e¨envi bv K‡i) :

(K) 4820 (L) $72\cdot245$ (M) $1\cdot734$ (N) $0\cdot045$ (O) $0\cdot000036$

12| K¨vjK‡jUi e¨envi K‡i wb‡Pi msL¨v¸‡jvi mvaviY j‡Mi cYK I AskK wbYq Ki :

(K) 27 (L) $63\cdot147$ (M) $1\cdot405$ (N) $0\cdot0456$ (O) $0\cdot000673$

13| ¸Yd‡ji/fvMd‡ji mvaviY jM (Avmb cvP `kwgK ¯vb chš) wbYq Ki :

(K) $5\cdot34\times8\cdot7$ (L) $0\cdot79\times0\cdot56$ (M) $22\cdot2642\div3\cdot42$ (N) $0\cdot19926\div32\cdot4$

14| hw` $log2 = 0\cdot30103, log3 = 0\cdot47712$ Ges $log7 = 0\cdot84510$ nq, Z‡e wb‡Pi iwk¸‡jvi gvb wbYq Ki :

(K) $log\ 9$ (L) $log\ 28$ (M) $log\ 42$

15| †`Iqv Av‡Q, $x = 1000$ Ges $y = 0\cdot0625$

K. x †K $a^n b^n$ AvKv‡i cKvk Ki, †hLv‡b a I b †gŠwjK msL¨v|

L. x I y Gi ¸Ydj‡K ˆeÁvwbK AvKv‡i cKvk Ki|

M. xy Gi mvaviY j‡Mi cYK I AskK wbYq Ki|

cÂg Aa¨vq
GK PjKwewkó mgxKiY
(Equations in One Variable)

Avgiv c‡ei †kwY‡Z PjK I mgxKiY Kx Zv †R‡bwQ Ges G‡`i e¨envi wk‡LwQ| GK PjKwewkó mij mgxKi‡Yi mgvavb wk‡LwQ Ges ev¯ewfwËK mgm¨vi mij mgxKiY MVb K‡i Zv mgvavb Kiv m¤ú‡K mg¨K Ávb jvf K‡iwQ| G Aa¨v‡q GK PjKwewkó GKNvZ I wØNvZ mgxKiY Ges A‡f` m¤ú‡K Av‡jvPbv Kiv n‡q‡Q Ges ev¯ewfwËK mgm¨vi mgvav‡b G‡`i e¨envi †`Lv‡bv n‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv –

- Pj‡Ki aviYv e¨vL¨v Ki‡Z cvi‡e|
- mgxKiY I A‡f‡`i cv_K¨ e¨vL¨v Ki‡Z cvi‡e|
- GKNvZ mgxKi‡Yi mgvavb Ki‡Z cvi‡e|
- ev¯ewfwËK mgm¨vi GKNvZ mgxKiY MVb K‡i mgvavb Ki‡Z cvi‡e|
- wØNvZ mgxKi‡Yi mgvavb Ki‡Z cvi‡e I mgvavb †mU wbYq Ki‡Z cvi‡e|
- ev¯ewfwËK mgm¨vi wØNvZ mgxKiY MVb K‡i mgvavb Ki‡Z cvi‡e|

5·1 PjK

Avgiv Rwb, $x+3=5$ GKwU mgxKiY| GwU mgvavb Ki‡Z n‡j Avgiv AÁvZ iwk x Gi gvb †ei Kwi| GLv‡b AÁvZ iwk x GKwU PjK| Avevi, $x+a=5$ mgxKiYwU mgvavb Ki‡Z n‡j, Avgiv x Gi gvb wbYq Kwi, a Gi gvb bq| GLv‡b x †K PjK I a †K a“eK wn‡m‡e aiv nq| G‡¶‡Î x Gi gvb a Gi gva¨‡g cvIqv hv‡e| Z‡e a Gi gvb wbYq Ki‡Z n‡j, Avgiv wjL‡ev $a=5-x$; A_vr a Gi gvb x Gi gva¨‡g cvIqv hv‡e| GLv‡b a PjK I x a“eK wn‡m‡e we‡ewPZ| Z‡e we‡kl †Kv‡bv wb‡`kbv bv _vK‡j cPwjZ ixwZ Abhvqx $x$ †K PjK wn‡m‡e aiv nq| mvaviYZ Bs‡iwR eYgvjvi †QvU nv‡Zi †k‡li w`‡Ki A¶i x, y, z †K PjK wn‡m‡e Ges c_g w`‡Ki A¶i a, b, c †K a“eK wn‡m‡e e¨envi Kiv nq|

†h mgxKi‡Y GKwU gvÎ AÁvZ iwk _v‡K, Zv‡K GK PjKwewkó mgxKiY ev mij mgxKiY ejv nq| †hgb, $x+3=5$ mgxKi‡Y x GKwU gvÎ PjK, ZvB GwU mij mgxKiY ev GK PjKwewkó mgxKiY|

Avgiv †mU m¤ú‡K Rwb| hw` GKwU †mU $S=\{x : x\in R, 1\le x\le 10\}$ nq, Z‡e $x$-Gi gvb $1$ †_‡K $10$ chš— †h‡Kv‡bv ev¯e msL¨v n‡Z cv‡i| GLv‡b $x$ GKwU PjK| Kv‡RB Avgiv ej‡Z cwi †h, hLb †Kv‡bv A¶i cZxK †Kv‡bv †m‡Ui Dcv`vb †evSvq ZLb Zv‡K PjK e‡j|

mgxKi‡Yi NvZ: †Kv‡bv mgxKi‡Yi Pj‡Ki m‡ev”P NvZ‡K mgxKiYwUi NvZ e‡j| $x+1=5,\ 2x-1=x+5,$ $y+7=2y-3$ mgxKiY¸‡jvi c‡Z¨KwUi NvZ 1; G¸‡jv GK PjKwewkó GKNvZ mgxKiY|

Avevi, $x^2+5x+6=0$, $y^2-y=12$, $4x^2-2x=3-6x$ mgxKiY¸‡jvi c‡Z¨KwUi NvZ 2 ; G¸‡jv GK PjKwewkó wØNvZ mgxKiY| $2x^3-x^2-4x+4=0$ mgxKiYwU GK PjKwewkó wÎNvZ mgxKiY|

5·2 mgxKiY I A‡f`

mgxKiY : mgxKi‡Y mgvb wP‡ýi `Bc‡¶ `BwU eûc`x _v‡K, A_ev GKc‡¶ (cavbZ Wvbc‡¶) kb¨ _vK‡Z cv‡i| `B c‡¶i eûc`xi Pj‡Ki m‡ev”P NvZ mgvb bvI n‡Z cv‡i| mgxKiY mgvavb K‡i Pj‡Ki m‡ev”P Nv‡Zi mgvb msL¨K gvb cvIqv hv‡e| GB gvb ev gvb¸‡jv‡K ejv nq mgxKiYwUi gj| GB gj ev gj¸‡jv Øviv mgxKiYwU wm× n‡e| GKwaK g‡ji †¶‡Î G¸‡jv mgvb ev Amgvb n‡Z cv‡i| †hgb, $x^2-5x+6=0$ mgxKiYwUi gj $2, 3$| Avevi, $(x-3)^2=0$ mgxKi‡Y x Gi gvb 3 n‡jI Gi gj $3, 3$|

A‡f` : mgvb wP‡ýi `Bc‡¶ mgvb NvZwewkó `BwU eûc`x _v‡K| Pj‡Ki m‡ev”P Nv‡Zi msL¨vi †P‡qI AwaK msL¨K gv‡bi Rb¨ A‡f`wU wm× n‡e| mgvb wP‡ýi Dfq c‡¶i g‡a¨ †Kv‡bv †f` †bB e‡jB A‡f`| †hgb, $(x+1)^2-(x-1)^2=4x$ GKwU A‡f`; GwU x Gi mKj gv‡bi Rb¨ wm× n‡e| ZvB GB mgxKiYwU GKwU A‡f`| c‡Z¨K exRMwYZxq mÎ GKwU A‡f`| †hgb, $(a+b)^2=a^2+2ab+b^2$, $(a-b)^2=a^2-2ab+b^2$, $a^2-b^2=(a+b)(a-b)$, $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ BZ¨vw` A‡f`|

mKj mgxKiY A‡f` bq| A‡f‡` mgvb (=) wP‡ýi cwie‡Z '≡' wPý e¨eüZ nq| Z‡e mKj A‡f`B mgxKiY e‡j A‡f‡`i †¶‡Î I mvaviYZ mgvb wPý e¨envi Kiv nq|

mgxKiY I A‡f‡`i cv_K¨ wb‡P †`Iqv n‡jv :

mgxKiY	A‡f`
1| mgvb wP‡ýi `B c‡¶ `BwU eûc`x _vK‡Z cv‡i A_ev GK c‡¶ kb¨ _vK‡Z cv‡i|	1| `B c‡¶ `BwU eûc`x _v‡K|
2| Dfq c‡¶i eûc`xi gvÎv Amgvb n‡Z cv‡i|	2| Dfq c‡¶ eûc`xi gvÎv mgvb _v‡K|
3| Pj‡Ki GK ev GKwaK gv‡bi Rb¨ mgZwU mZ¨ nq|	3| Pj‡Ki gj †m‡Ui mKj gv‡bi Rb¨ mvaviYZ mgZwU mZ¨ nq|
4| Pj‡Ki gv‡bi msL¨v mewaK gvÎvi mgvb n‡Z cv‡i|	4| Pj‡Ki AmsL¨ gv‡bi Rb¨ mgZwU mZ¨|
5| mKj mgxKiY mÎ bq|	5| mKj exRMwYZxq mÎB A‡f`|

KvR : 1| wb‡Pi mgxKiY¸‡jvi †KvbwUi NvZ KZ I gj KqwU ?

$(i)\ 3x+1=5$ $\qquad (ii)\ \dfrac{2y}{5}-\dfrac{y-1}{3}=\dfrac{3y}{2}$

2| wZbwU A‡f` †jL|

5·3 GKNvZ mgxKi‡Yi mgvavb

mgxKiY mgvav‡bi †¶‡Î K‡qKwU wbqg c‡qvM Ki‡Z nq| GB wbqg¸‡jv Rvbv _vK‡j mgxKi‡Yi mgvavb wbYq mnRZi nq| wbqg¸‡jv n‡jv :

1| mgxKi‡Yi Dfqc‡¶ GKB msL¨v ev ivwk †hvM Ki‡j c¶Øq mgvb _v‡K|

2| mgxKi‡Yi Dfqc¶ †_‡K GKB msL¨v ev ivwk we‡qvM Ki‡j c¶Øq mgvb _v‡K|

3| mgxKi‡Yi Dfqc¶‡K GKB msL¨v ev ivwk Øviv ¸Y Ki‡j c¶Øq mgvb _v‡K|

4| mgxKi‡Yi Dfqc¶‡K Akb¨ GKB msL¨v ev ivwk Øviv fvM Ki‡j c¶Øq mgvb _v‡K|

Dc‡ii ag¸‡jv‡K exRMwYZxq ivwki gva¨‡g cKvk Kiv hvq :

hw` $x=a$ Ges $c \neq 0$ nq Zvn‡j,

$$(i)\ x+c=a+c \quad (ii)\ x-c=a-c \quad (iii)\ xc=ac \quad (iv)\ \frac{x}{c}=\frac{a}{c}$$

GQvov hw` a, b I c wZbwU ivwk nq Z‡e, $a=b+c$ n‡j, $a-b=c$ n‡e Ges $a+c=b$ n‡j, $a=b-c$ n‡e|

GB wbqgwU c¶všÍi wewa wn‡m‡e cwiwPZ Ges GB wewa c‡qvM K‡i wewfbœ mgxKiY mgvavb Kiv nq|

†Kv‡bv mgxKi‡Yi c`¸‡jv fMvsk AvKv‡i _vK‡j, je¸‡jv‡Z Pj‡Ki NvZ 1 Ges ni¸‡jv a"eK n‡j, †m¸‡jv GKNvZ mgxKiY|

D`vniY 1| mgvavb Ki : $\frac{5x}{7}-\frac{4}{5}=\frac{x}{5}-\frac{2}{7}$

mgvavb : $\frac{5x}{7}-\frac{4}{5}=\frac{x}{5}-\frac{2}{7}$ ev, $\frac{5x}{7}-\frac{x}{5}=\frac{4}{5}-\frac{2}{7}$ [c¶všÍi K‡i]

ev, $\frac{25x-7x}{35}=\frac{28-10}{35}$ ev, $\frac{18x}{35}=\frac{18}{35}$

ev, $18x=18$

ev, $x=1$

$\therefore$ mgvavb $x=1$.

GLb, Avgiv Ggb mgxKi‡Yi mgvavb Ki‡ev hv wØNvZ mgxKi‡Yi AvKv‡i _v‡K| G mKj mgxKiY mijxKi‡Yi gva¨‡g mgZj mgxKi‡Y iƒcvšÍi K‡i $ax=b$ AvKv‡ii GKNvZ mgxKi‡Y cwiYZ Kiv nq| Avevi, n‡i PjK _vK‡j I mijxKiY K‡i GKNvZ mgxKi‡Y iƒcvšÍi Kiv nq|

D`vniY 2| mgvavb Ki : $(y-1)(y+2)=(y+4)(y-2)$

mgvavb : $(y-1)(y+2)=(y+4)(y-2)$

ev, $y^2-y+2y-2=y^2+4y-2y-8$

ev, $y-2=2y-8$

ev, $y-2y=-8+2$ [c¶všÍi K‡i]

ev, $-y=-6$

ev, $y=6$

$\therefore$ mgvavb $y=6$

D`vniY 3| mgvavb Ki I mgvavb †mU †jL : $\frac{6x+1}{15}-\frac{2x-4}{7x-1}=\frac{2x-1}{5}$

mgvavb : $\frac{6x+1}{15}-\frac{2x-4}{7x-1}=\frac{2x-1}{5}$

ev, $\frac{6x+1}{15}-\frac{2x-1}{5}=\frac{2x-4}{7x-1}$ [c¶všÍi K‡i]

ev, $\frac{6x+1-6x+3}{15}=\frac{2x-4}{7x-1}$ ev, $\frac{4}{15}=\frac{2x-4}{7x-1}$

ev, $15(2x-4)=4(7x-1)$ [Avo¸Yb K‡i]

ev, $30x-60=28x-4$

ev, $30x-28x=60-4$ [c¶všÍi K‡i]

ev, $2x=56$, ev, $x=28$

$\therefore$ mgvavb $x=28$

Ges mgvavb †mU $S=\{28\}$

D`vniY 4| mgvavb Ki : $\frac{1}{x-3}+\frac{1}{x-4}=\frac{1}{x-2}+\frac{1}{x-5}$

mgvavb : $\frac{1}{x-3}+\frac{1}{x-4}=\frac{1}{x-2}+\frac{1}{x-5}$

ev, $\frac{x-4+x-3}{(x-3)(x-4)}=\frac{x-5+x-2}{(x-2)(x-5)}$ ev, $\frac{2x-7}{x^2-7x+12}=\frac{2x-7}{x^2-7x+10}$

`B c‡¶i fMvsk `BwUi gvb mgvb| Avevi, `B c‡¶i je mgvb, wKš‘ ni Amgvb| G‡¶‡Î GKgvÎ j‡ei gvb kb¨ n‡jB `B c¶ mgvb n‡e|

$\therefore$ $2x-7=0$ ev, $2x=7$ ev, $x=\frac{7}{2}$

$\therefore$ $x=\frac{7}{2}$

D`vniY 5| mgvavb †mU wbYq Ki : $\sqrt{2x-3}+5=2$

mgvavb : $\sqrt{2x-3}+5=2$

ev, $\sqrt{2x-3}=2-5$ [c¶všÍi K‡i]

ev, $\left(\sqrt{2x-3}\right)^2=(-3)^2$ [Dfqc¶‡K eM© K‡i]

ev, $2x-3=9$

ev, $2x=12$

ev, $x=6$

c`Ë mgxKi‡Y eMg‡ji wPý _vKvi Kvi‡Y ïw× cix¶v c‡qvRb|

weKí wbqg :

$\sqrt{2x-3}+5=2$

ev, $\sqrt{2x-3}=2-5$

ev, $\sqrt{2x-3}=-3$

†Kv‡bv ev¯e iwki eMgj FYvZ¥K n‡Z cv‡i bv|

$\therefore$ mgxKiYwUi †Kv‡bv mgvavb †bB|

$\therefore$ mgvavb †mU : $S=\{\ \}$ ev, Φ

c`Ë mgxKiYwU‡Z $x=6$ ewm‡q cvB,

$\sqrt{2\times 6-3}+5=2$ ev, $\sqrt{9}+5=2$

ev, $3+5=2$

ev, $8=2$, hv Am¤¢e|

∴ mgxKiYwUi †Kv‡bv mgvavb †bB|

∴ mgvavb †mU : $S=\{\ \}$ ev, Φ

KvR : 1| $(\sqrt{5}+1)x+4=4\sqrt{5}$ n‡j, †`LvI †h, $x=6-2\sqrt{5}$

2| mgvavb Ki I mgvavb †mU †jL : $\sqrt{4x-3}+5=2$

5·4 GKNvZ mgxKi‡Yi e¨envi

ev¯e Rxe‡b wewfb ai‡bi mgm¨vi mgvavb Ki‡Z nq| GB mgm¨v mgvav‡bi AwaKvsk †¶‡Î MwYwZK Ávb, `¶Zv I hw³i c‡qvRb nq| ev¯e †¶‡Î MwYwZK Ávb I `¶Zvi c‡qv‡M GKw`‡K †hgb mgm¨vi mô mgvavb nq, Ab¨w`‡K †Zgwb cvZ¨wnK Rxe‡b MwY‡Zi gva¨‡g mgm¨vi mgvavb cvIqv hvq weavq, wk¶v_xiv MwY‡Zi cwZ AvKó nq| GLv‡b cvZ¨wnK Rxe‡bi wewfb mgm¨v‡K mgxKi‡Yi gva¨‡g cKvk K‡i Zvi mgvavb Kiv n‡e|

ev¯ewfwËK mgm¨v mgvav‡b AÁvZ msL¨v wbY‡qi Rb¨ Gi cwie‡Z PjK a‡i wb‡q mgm¨vq c`Ë kZvbmv‡i mgxKiY MVb Kiv nq| Zvici mgxKiYwU mgvavb Ki‡jB PjKwUi gvb, A_vr AÁvZ msL¨wU cvIqv hvq|

D`vniY 6| `B A¼wewkó †Kv‡bv msL¨vi GKK ¯vbxq A¼wU `kK ¯vbxq A¼ A‡c¶v $2$ †ewk| A¼Øq ¯vb wewbgq Ki‡j †h msL¨v cvIqv hv‡e Zv c`Ë msL¨vi wØ¸Y A‡c¶v 6 Kg n‡e| msL¨wU wbYq Ki|

mgvavb : g‡b Kwi, `kK ¯vbxq A¼wU x; AZGe, GKK ¯vbxq A¼wU n‡e $x+2$.

∴ msL¨wU $10x+(x+2)$ ev, $11x+2$.

A¼Øq ¯vb wewbgq Ki‡j cwiewZZ msL¨wU n‡e $10(x+2)+x$ ev, $11x+20$

ckg‡Z, $11x+20=2(11x+2)-6$

ev, $11x+20=22x+4-6$

ev, $22x-11x=20+6-4$ [c¶vši K‡i]

ev, $11x=22$

ev, $x=2$

∴ msL¨wU $11x+2=11\times 2+2=24$

∴ c`Ë msL¨wU 24.

D`vniY 7| GKwU †kwYi cwZ‡e‡Â 4 Rb K‡i Qv‡Î emv‡j 3wU †eÂ Lwj _v‡K| Avevi, cwZ‡e‡Â 3 Rb K‡i Qv‡Î emv‡j 6 Rb Qv‡Î‡K `vwo‡q _vK‡Z nq| H †kwYi Qv‡Î msL¨v KZ ?

mgvavb : g‡b Kwi, †kwYwUi Qv‡Î msL¨v x.

†h‡nZ cwZ‡e‡Â 4 Rb K‡i emv‡j 3wU †eÂ Lwj _v‡K, †m‡nZ H †kwYi †e‡Âi msL¨v $=\frac{x}{4}+3$

Avevi, †h‡nZ cwZ‡e‡Â 3 Rb K‡i emv‡j 6 Rb‡K `vwo‡q _vK‡Z nq, †m‡nZ H †kwYi †e‡Âi msL¨v

$=\frac{x-6}{3}$

†h‡nZ †e‡Âi msL¨v GKB _vK‡e,

mZivs, $\frac{x}{4}+3=\frac{x-6}{3}$ ev, $\frac{x+12}{4}=\frac{x-6}{3}$

ev, $4x-24=3x+36$, ev, $4x-3x=36+24$

ev, $x=60$

$\therefore$ H †kwYi Qv‡Î msL¨v 60.

D`vniY 8| Kwei mv‡ne Zvi 56000 UvKvi wKQ UvKv evwlK 12% gbvdvq I evwK UvKv evwlK 10% gbvdvq wewb‡qvM Ki‡jb| GK eQi ci wZwb †gvU 6400 UvKv gbvdv †c‡jb| wZwb 12% gbvdvq KZ UvKv wewb‡qvM K‡i‡Qb ?

mgvavb : g‡b Kwi, Kwei mv‡ne 12% gbvdvq x UvKv wewb‡qvM K‡i‡Qb|

$\therefore$ wZwb 10% gbvdvq wewb‡qvM K‡i‡Qb $(56000-x)$ UvKv|

GLb, x UvKvi 1 eQ‡ii gbvdv $x\times\frac{12}{100}$ UvKv, ev, $\frac{12x}{100}$ UvKv|

Avevi, $(56000-x)$ UvKvi 1 eQ‡ii gbvdv $(56000-x)\times\frac{10}{100}$ UvKv, ev, $\frac{10(56000-x)}{100}$ UvKv|

ckg‡Z, $\frac{12x}{100}+\frac{10(56000-x)}{100}=6400$

ev, $12x+560000-10x=640000$

ev, $2x=640000-560000$

ev, $2x=80000$

ev, $x=40000$

$\therefore$ Kwei mv‡ne 12% gbvdvq 40000 UvKv wewb‡qvM K‡i‡Qb|

KvR : mgxKiY MVb K‡i mgvavb Ki :

1| $\frac{3}{5}$ fMvskwUi je I n‡ii mv‡_ †Kvb GKB msL¨v †hvM Ki‡j fMvskwU $\frac{4}{5}$ n‡e ?

2| `BwU µwgK ¯^vfvweK msL¨vi e‡Mi Aš—i 151 n‡j, msL¨v `BwU wbYq Ki|

3| 120 wU GK UvKvi g`v I `B UvKvi g`vq †gvU 180 UvKv n‡j, †Kvb cKv‡ii g`vi msL¨v KqwU ?

Abkxjbx 5·1

mgvavb Ki (1-10) :

1| $3(5x-3)=2(x+2)$ 2| $\frac{ay}{b}-\frac{by}{a}=a^2-b^2$ 3| $(z+1)(z-2)=(z-4)(z+2)$

4| $\frac{7x}{3}+\frac{3}{5}=\frac{2x}{5}-\frac{4}{3}$ 5| $\frac{4}{2x+1}+\frac{9}{3x+2}=\frac{25}{5x+4}$ 6| $\frac{1}{x+1}+\frac{1}{x+4}=\frac{1}{x+2}+\frac{1}{x+3}$

7| $\frac{a}{x-a}+\frac{b}{x-b}=\frac{a+b}{x-a-b}$ 8| $\frac{x-a}{b}+\frac{x-b}{a}+\frac{x-3a-3b}{a+b}=0$ 9| $\frac{x-a}{a^2-b^2}=\frac{x-b}{b^2-a^2}$

10| $(3+\sqrt{3})z+2=5+3\sqrt{3}.$

mgvavb †mU wbYq Ki (11-20) :

11| $2x(x+3)=2x^2+12$ 12| $2x+\sqrt{2}=3x-4-3\sqrt{2}$ 13| $\frac{x+a}{x-b}=\frac{x+a}{x+c}$

14| $\frac{z-2}{z-1}=2-\frac{1}{z-1}$ 15| $\frac{1}{x}+\frac{1}{x+1}=\frac{2}{x-1}$ 16| $\frac{m}{m-x}+\frac{n}{n-x}=\frac{m+n}{m+n-x}$

17| $\frac{1}{x+2}+\frac{1}{x+5}=\frac{1}{x+4}+\frac{1}{x+3}$ 18| $\frac{2t-6}{9}+\frac{15-2t}{12-5t}=\frac{4t-15}{18}$

19| $\frac{x+2b^2+c^2}{a+b}+\frac{x+2c^2+a^2}{b+c}+\frac{x+2a^2+b^2}{c+a}=0$

mgxKiY MVb K‡i mgvavb Ki (21-30) :

20| GKwU msL¨v Aci GKwU msL¨vi $\frac{2}{5}$ ¸Y| msL¨v `BwUi mgwó 98 n‡j, msL¨v `BwU wbYq Ki|

21| GKwU cKZ fMœvs‡ki je I n‡ii Aš—i 1 ; je †_‡K 2 we‡qvM I n‡ii mv‡_ 2 †hvM Ki‡j †h fMœvsk cvIqv hv‡e Zv $\frac{1}{6}$ Gi mgvb| fMœvskwU wbYq Ki|

22| `B A¼wewkó GKwU msL¨vi A¼Ø‡qi mgwó 9 ; A¼ `BwU ¯'vb wewbgq Ki‡j †h msL¨v cvIqv hv‡e Zv c`Ë msL¨v n‡Z 45 Kg n‡e| msL¨wU KZ ?

23| `B A¼wewkó GKwU msL¨vi `kK ¯'vbxq A¼ GKK ¯'vbxq A‡¼i wØ¸Y| †`LvI †h, msL¨wU A¼Ø‡qi mgwói mvZ¸Y|

24| GKRb ¶`ª e¨emvqx 5600 UvKv wewb‡qvM K‡i GK eQi ci wKQ UvKvi Dci 5% Ges Aewkó UvKvi Dci 4% jvf Ki‡jb| wZwb KZ UvKvi Dci 5% jvf Ki‡jb ?

25| GKwU j‡Â hvÎx msL¨v 47 ; gv_wcQ †Kwe‡bi fvov †W‡Ki fvovi wØ¸Y| †W‡Ki fvov gv_wcQ 30 UvKv Ges †gvU fvov cwiß 1680 UvKv n‡j, †Kwe‡bi hvÎx msL¨v KZ ?

26| 120 wU cwPk cqmvi g`v I cÂvk cqmvi g`vq †gvU 35 UvKv n‡j, †Kvb cKv‡ii g`vi msL¨v KqwU ?

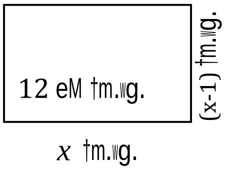
27| GKwU Mwo NÈvq 60 wK.wg. †e‡M wKQ c_ Ges NÈvq 40 wK.wg. †e‡M Aewkó c_ AwZµg Ki‡jv| MwowU †gvU 5 NÈvq 240 wK.wg. c_ AwZµg Ki‡j, NÈvq 60 wK.wg. †e‡M KZ`i wM‡q‡Q ?

5·5 GK PjKwewkó wØNvZ mgxKiY

$ax^2+bx+c=0$ [†hLv‡b, a,b,c a"eK Ges $a\neq 0$] AvKv‡ii mgxKiY‡K GK PjKwewkó wØNvZ mgxKiY ejv nq| wØNvZ mgxKi‡Yi evgc¶ GKwU wØgwÎK eûc`x| mgxKi‡Yi Wvbc¶ kb¨ aiv nq|

12 eM †m.wg. †¶Îdjwewkó GKwU AvqZvKvi‡¶‡Îi ˆ`N¨ x †m.wg. I c¯ $(x-1)$ †m.wg.|

$\therefore$ AvqZvKvi‡¶ÎwUi †¶Îdj = $x(x-1)$ eM †m.wg.



ckg‡Z, $x(x-1)=12$, ev $x^2-x-12=0$

mgxKiYwU‡Z GKwU PjK x Ges x Gi m‡ev"P NvZ 2|

Gi/c mgxKiY n‡jv wØNvZ mgxKiY|

†h mgxKi‡Y Pj‡Ki m‡ev"P NvZ 2, Zv‡K wØNvZ mgxKiY e‡j|

Avgiv Aóg †kwY‡Z x^2+px+q Ges ax^2+bx+c AvKv‡ii GK PjKwewkó wØNvZ iwki Drcv`‡K we‡klY K‡iwQ| GLv‡b Avgiv $x^2+px+q=0$ Ges $ax^2+bx+c=0$ AvKv‡ii wØNvZ mgxKi‡Yi evgc¶‡K Drcv`‡K we‡klY K‡i Pj‡Ki gvb wbY‡qi gva¨‡g Gi/c mgxKiY mgvavb Ki‡ev|

Drcv`‡K we‡kqY c×wZ‡Z ev¯e msL¨vi GKwU ¸iZcY ag c‡qvM Kiv nq| agwU wbgi/c :

hw` `BwU iwki ¸Ydj kb¨ nq, Z‡e iwkØ‡qi †h‡Kv‡bvwU A_ev Dfq iwk kb¨ n‡e| A_vr, `BwU iwk a I b Gi ¸Ydj $ab=0$ n‡j, $a=0$ ev, $b=0$, A_ev $a=0$ Ges $b=0$ n‡e|

D`vniY 9| mgvavb Ki : $(x+2)(x-3)=0$

mgvavb : $(x+2)(x-3)=0$

$\therefore$ $x+2=0$, A_ev $x-3=0$

$x+2=0$ n‡j, $x=-2$

Avevi, $x-3=0$ n‡j, $x=3$

$\therefore$ mgvavb $x=-2$ A_ev 3

D`vniY 10| mgvavb †mU wbYq Ki : $y^2=\sqrt{3}y$

mgvavb : $y^2=\sqrt{3}y$

ev, $y^2-\sqrt{3}y=0$ [c¶vši K‡i Wvbc¶ kb¨ Kiv n‡q‡Q]

ev, $y(y-\sqrt{3})=0$

$\therefore$ $y=0$, A_ev $y-\sqrt{3}=0$

Avevi, $y-\sqrt{3}=0$ n‡j, $y=\sqrt{3}$

$\therefore$ mgvavb †mU $\{0,\sqrt{3}\}$

D`vniY 11| mgvavb Ki I mgvavb †mU †jL : $x-4=\frac{x-4}{x}$

mgvavb : $x-4=\frac{x-4}{x}$

ev, $x(x-4)=x-4$ [Avo¸Yb K‡i]

ev, $x(x-4)-(x-4)=0$ [c¶vši K‡i]

ev, $(x-4)(x-1)=0$

$\therefore$ $x-4=0$, A_ev $x-1=0$

$x-4=0$ n‡j, $x=4$

Avevi, $x-1=0$ n‡j, $x=1$

$\therefore$ mgvavb $x=1$ A_ev 4

Ges mgvavb †mU $\{1,\ 4\}$

D`vniY 12| mgvavb Ki : $\left(\frac{x+a}{x-a}\right)^2-5\left(\frac{x+a}{x-a}\right)+6=0$

mgvavb : $\left(\frac{x+a}{x-a}\right)^2-5\left(\frac{x+a}{x-a}\right)+6=0..........(1)$

awi, $\frac{x+a}{x-a}=y$

$\therefore$ (1) n‡Z cvB, $y^2-5y+6=0$

ev, $y^2-2y-3y+6=0$

ev, $y(y-2)-3(y-2)=0$

ev, $(y-2)(y-3)=0$

$\therefore$ $y-2=0$ n‡j, $y=2$

A_ev $y-3=0$ n‡j, $y=3$

GLb, $y=2$ n‡j,

$\frac{x+a}{x-a}=\frac{2}{1}$ [y Gi gvb ewm‡q] ev, $\frac{x+a+x-a}{x+a-x+a}=\frac{2+1}{2-1}$ [†hvRb-we‡qvRb K‡i]

ev, $\frac{2x}{2a}=\frac{3}{1}$

ev, $x=3a$

Avevi, $y=3$ n‡j, $\frac{x+a}{x-a}=\frac{3}{1}$

ev, $\frac{x+a+x-a}{x+a-x+a}=\frac{3+1}{3-1}$ [†hvRb-we‡qvRb K‡i]

ev, $\frac{2x}{2a}=\frac{4}{2}$

ev, $\frac{x}{a}=\frac{2}{1}$

ev, $x=2a$

$\therefore$ mgvavb $x=2a$ A_ev, $3a$

KvR :

1| $x^2-1=0$ mgxKiYwU‡K $ax^2+bx+c=0$ mgxKi‡Yi mv‡_ Zjbv K‡i a,b,c Gi gvb †jL|

2| $(x-1)^2=0$ mgxKiYwUi NvZ KZ ? Gi gj KqwU I Kx Kx ?

5·6 wØNvZ mgxKi‡Yi e¨envi

Avgv‡`i ˆ`bw›`b Rxe‡bi A‡bK mgm¨v mij mgxKiY I wØNvZ mgxKi‡Y iƒcvšÍi K‡i mn‡R mgvavb Kiv hvq| GLv‡b, ev¯ÍewfwËK mgm¨vq c`Ë kZ †_‡K wØNvZ mgxKiY MVb K‡i mgvavb Kivi †KŠkj †`Lv‡bv n‡jv|

D`vniY 13| GKwU cKZ fMvs‡ki ni, je A‡c¶v 4 †ewk| fMvskwU eM Ki‡j †h fMvsk cvIqv hv‡e Zvi ni, je A‡c¶v 40 †ewk n‡e| fMvskwU wbYq Ki|

mgvavb : awi, fMvskwU $\frac{x}{x+4}$

fMvskwUi eM = $\left(\frac{x}{x+4}\right)^2=\frac{x^2}{(x+4)^2}=\frac{x^2}{x^2+8x+16}$

GLv‡b, je = x^2 Ges ni = $x^2+8x+16$.

ckg‡Z, $x^2+8x+16=x^2+40$

ev, $8x+16=40$

ev, $8x = 40 - 16$

ev, $8x = 24$

ev, $x = 3$

$\therefore\ x + 4 = 3 + 4 = 7$

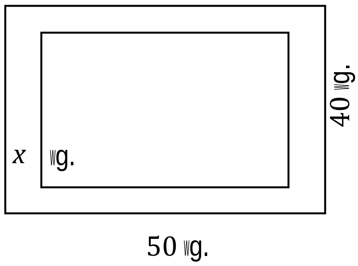
$\therefore\ \dfrac{x}{x+4} = \dfrac{3}{3+4} = \dfrac{3}{7}$

$\therefore$ fMvskwU $\dfrac{3}{7}$

D`vniY 14| 50 wgUvi ˆ`N¨ Ges 40 wgUvi c¯'wewkó GKwU AvqZvKvi evMv‡bi wfZ‡ii Pviw`‡K mgvb PIov GKwU iv¯Ív Av‡Q| iv¯Ív ev‡` evMv‡bi †¶Îdj 1200 eMwgUvi n‡j, iv¯ÍvwU KZ wgUvi PIov ?

mgvavb : g‡b Kwi, iv¯ÍvwU x wgUvi PIov|

iv¯Ív ev‡` evMvbwUi ˆ`N¨ $(50-2x)$ wgUvi Ges c¯' $(40-2x)$ wgUvi|



$\therefore$ iv¯Ív ev‡` evMvbwUi †¶Îdj = $(50-2x) \times (40-2x)$ eMwgUvi|

ckg‡Z, $(50-2x)(40-2x) = 1200$

ev, $2000 - 80x - 100x + 4x^2 = 1200$

ev, $4x^2 - 180x + 800 = 0$

ev, $x^2 - 45x + 200 = 0$ [4 w`‡q fvM K‡i]

ev, $x^2 - 5x - 40x + 200 = 0$

ev, $x(x-5) - 40(x-5) = 0$

ev, $(x-5)(x-40) = 0$

$\therefore\ x - 5 = 0$, A_ev $x - 40 = 0$

$x - 5 = 0$ n‡j, $x = 5$

$x - 40 = 0$ n‡j, $x = 40$

wKš' iv¯ÍvwUi PIov evMvbwUi c¯' 40 wgUvi †_‡K Kg n‡e|

$\therefore\ x \neq 40$; $\therefore\ x = 5$

$\therefore$ iv¯ÍvwU 5 wgUvi PIov|

D`vniY 15| kwnK 240 UvKvq KZK¸‡jv Kjg wKbj| †m hw` H UvKvq GKwU Kjg †ewk †c‡Zv Z‡e cwZwU Kj‡gi `vg M‡o 1 UvKv Kg co‡Zv| †m KZ¸‡jv Kjg wKbj ?

mgvavb : g‡b Kwi, kwnK 240 UvKvq †gvU x wU Kjg wK‡bwQj| G‡Z cwZwU Kj‡gi `vg c‡o $\dfrac{240}{x}$ UvKv| †m hw` 240 UvKvq $(x+1)$ wU Kjg †c‡Zv Z‡e cwZwU Kj‡gi `vg co‡Zv $\dfrac{240}{x+1}$ UvKv|

ckg‡Z, $\frac{240}{x+1}=\frac{240}{x}-1$, ev, $\frac{240}{x+1}=\frac{240-x}{x}$

ev, $240x=(x+1)(240-x)$ [Avo¸Yb K‡i]

ev, $240x=240x+240-x^2-x$

ev, $x^2+x-240=0$ [c¶vši K‡i]

ev, $x^2+16x-15x-240=0$

ev, $x(x+16)-15(x+16)=0$

ev, $(x+16)(x-15)=0$

$\therefore x+16=0$, A_ev $x-15=0$

$x+16=0$ n‡j, $x=-16$

$x-15=0$ n‡j, $x=15$

wKš Kj‡gi msL¨v x FYvZ¥K n‡Z cv‡i bv|

$\therefore\ x\neq-16$; $\therefore\ x=15$

$\therefore$ kwnK 15wU Kjg wK‡bwQj|

KvR : mgxKiY MVb K‡i mgvavb Ki :

1| GKwU ¯vfweK msL¨vi e‡Mi mv‡_ H msL¨wU †hvM Ki‡j †hvMdj wVK cieZx ¯vfweK msL¨vi bq¸‡Yi mgvb n‡e| msL¨wU KZ ?

2| 10 †m.wg. e¨vmvaweko GKwU e‡Ëi †K›` n‡Z GKwU R¨v Gi Dci Aw¼Z j‡¤i ``N¨ eËwUi Aa-R¨v A‡c¶v 2 †m.wg. Kg| AvbgwbK wPÎ A¼b K‡i R¨wUi ``N¨ wbYq Ki|

D`vniY 16| GKwU we`¨vj‡qi beg †kwYi GKwU cix¶vq x Rb Qv‡Îi MwY‡Z cvß †gvU b¤i 1950; GKB cix¶vq Ab¨ GKRb bZb Qv‡Îi MwY‡Z cvß b¤i 34 †hvM Kivq cvß b¤‡ii Mo 1 K‡g †Mj|

K. c_Kfv‡e x Rb Qv‡Îi Ges bZb QvÎmn mK‡ji cvß b¤‡ii Mo x Gi gva¨‡g †jL|

L. c`Ë kZvbmv‡i mgxKiY MVb K‡i †`LvI †h, $x^2+35x-1950=0$

M. x Gi gvb †ei K‡i `B‡¶‡Î b¤‡ii Mo KZ Zv wbYq Ki|

mgvavb : K. x Rb Qv‡Îi cvß b¤‡ii Mo = $\frac{1950}{x}$

bZb Qv‡Îi b¤imn $(x+1)$ Rb Qv‡Îi cvß b¤‡ii Mo $\frac{1950+34}{x+1}=\frac{1984}{x+1}$

L. ckg‡Z, $\frac{1950}{x}=\frac{1984}{x+1}+1$

ev, $\frac{1950}{x}-\frac{1984}{x+1}=1$ [c¶vši K‡i] ev, $\dfrac{1950x+1950-1984x}{x(x+1)}=1$

ev, $x^2+x=1950x-1984x+1950$ [Avo¸Yb K‡i]

ev, $x^2+x=1950-34x$

$\therefore$ $x^2+35x-1950=0$ [†`Lv‡bv n‡jv]

M. $x^2+35x-1950=0$

ev, $x^2+65x-30x-1950=0$

ev, $x(x+65)-30(x+65)=0$

ev, $(x+65)(x-30)=0$

$\therefore x+65=0$, A_ev $x-30=0$

$x+65=0$ n‡j, $x=-65$

Avevi, $x-30=0$ n‡j, $x=30$

†h‡nZ Qv‡Îi msL¨v x FYvZ¥K n‡Z cv‡i bv,

mZivs, $x\neq-65$

$\therefore$ $x=30$

$\therefore$ c_g †¶‡Î, Mo = $\dfrac{1950}{30}=65$

Ges wØZxq †¶‡Î, Mo = $\dfrac{1984}{31}=64$.

Abkxjbx 5·2

1| x †K PjK a‡i $a^2x+b=0$ mgxKiYwUi NvZ wb‡Pi †KvbwU ?

K. 3 L. 2 M. 1 N. 0

2| wb‡Pi †KvbwU A‡f` ?

K. $(x+1)^2+(x-1)^2=4x$ L. $(x+1)^2+(x-1)^2=2(x^2+1)$

M. $(a+b)^2-(a-b)^2=2ab$ N. $(a-b)^2=a^2+2ab+b^2$

3| $(x-4)^2=0$ mgxKi‡Yi gj KqwU ?

K. 1 wU L. 2 wU M. 3 wU N. 4 wU

4| $x^2-x-12=0$ mgxKi‡Yi gjØq wb‡Pi †KvbwU ?

K. $3,\ 4$ L. $3,\ -4$ M. $-3,\ 4$ N. $-3,\ -4$

5| $3x^2 - x + 5 = 0$ mgxKi‡Y x Gi mnM KZ ?

K. 3 L. 2 M. 1 N. -1

6| wb‡Pi mgxKiY¸‡jv j¶ Ki :

i. $2x+3=9$ *ii.* $\frac{x}{2}-2=-1$ *iii.* $2x+1=5$

Dc‡ii †Kvb mgxKiY¸‡jv ci¯úi mgZj ?

K. *i* I *ii* L. *ii* I *iii* M. *i* I *iii* N. *i*, *ii* I *iii*

7| $x^2-(a+b)x+ab=0$ mgxKi‡Yi mgvavb †mU wb‡Pi †KvbwU ?

K. $\{a,\ b\}$ L. $\{a,\ -b\}$ M. $\{-a,\ b\}$ N. $\{-a,\ -b\}$

8| `B A¼wewkó GKwU msL¨vi `kK ¯vbxq A¼ GKK ¯vbxq A‡¼i wØ¸Y| GB Z‡_¨i Av‡jv‡K wb‡Pi ck¸‡jvi DËi `vI ?

(1) GKK ¯vbxq A¼ x n‡j, msL¨wU KZ ?

K. $2x$ L. $3x$ M. $12x$ N. $21x$

(2) A¼Øq ¯vb wewbgq Ki‡j msL¨wU KZ n‡e ?

K. $3x$ L. $4x$ M. $12x$ N. $21x$

(3) $x=2$ n‡j, gj msL¨vi mv‡_ ¯vb wewbgqKZ msL¨vi cv_K¨ KZ ?

K. 18 L. 20 M. 34 N. 36

mgvavb Ki (9–18) :

9| $(x+2)(x-\sqrt{3})=0$

10| $(\sqrt{2}x+3)(\sqrt{3}x-2)=0$

11| $y(y-5)=6$

12| $(y+5)(y-5)=24$

13| $2(z^2-9)+9z=0$

14| $\frac{3}{2z+1}+\frac{4}{5z-1}=2$

15| $\frac{4}{\sqrt{10x-4}}+\sqrt{10x-4}=5$

16| $\frac{x-2}{x+2}+\frac{6(x-2)}{x-6}=1$

17| $\frac{x}{a}+\frac{a}{x}=\frac{x}{b}+\frac{b}{x}$

18| $\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

mgvavb †mU wbYq Ki (19–25):

19| $\frac{3}{x}+\frac{4}{x+1}=2$

20| $\frac{x+7}{x+1}+\frac{2x+6}{2x+1}=5$

21| $\frac{1}{x}+\frac{1}{a}+\frac{1}{b}=\frac{1}{x+a+b}$

22| $\frac{ax+b}{a+bx}=\frac{cx+d}{c+dx}$

23| $x+\frac{1}{x}=2$

24| $2x^2-4ax=0$

25| $\frac{(x+1)^3-(x-1)^3}{(x+1)^2-(x-1)^2}=2$

mgxKiY MVb K‡i mgvavb Ki (26–31) :

26| `B A¼wewkó †Kv‡bv msL¨vi A¼Ø‡qi mgwó 15 Ges G‡`i ¸Ydj 56 ; msL¨wU wbYq Ki|

27| GKwU AvqZvKvi N‡ii †g‡Si †¶Îdj 192 eMwgUvi| †g‡Si ˆ`N¨ 4 wgUvi Kgv‡j I c¯ 4 wgUvi evov‡j †¶Îdj AcwiewZZ _v‡K| †g‡Si ˆ`N¨ I c¯ wbYq Ki|

28| GKwU mg‡KvYx wÎf‡Ri AwZf‡Ri ˆ`N¨ 15 †m.wg. I Aci evûØ‡qi ˆ`‡N¨i Aši 3 †m.wg.| H evûØ‡qi ˆ`N¨ wbYq Ki|

29| GKwU wÎf‡Ri fwg Zvi D”PZvi wØ¸Y A‡c¶v 6 †m.wg. †ewk| wÎfR †¶ÎwUi †¶Îdj 810 eM †m.wg. n‡j, Gi D”PZv KZ ?

30| GKwU †kwY‡Z hZRb QvÎ-QvÎx c‡o c‡Z¨‡K Zvi mncvVxi msL¨vi mgvb UvKv Pv`v †`Iqvq †gvU 420 UvKv Pv`v DVj| H †kwYi QvÎ-QvÎxi msL¨v KZ Ges c‡Z¨‡K KZ UvKv K‡i Pv`v w`j ?

31| GKwU †kwY‡Z hZRb QvÎ-QvÎx c‡o, c‡Z¨‡K ZZ cqmvi †P‡q Avi I 30 cqmv †ewk K‡i Pv`v †`Iqv‡Z †gvU 70 UvKv DVj| H †kwYi QvÎ-QvÎxi msL¨v KZ ?

32| `B A¼wewkó GKwU msL¨vi A¼Ø‡qi mgwó 7 ; A¼Øq ¯vb wewbgq Ki‡j †h msL¨v cvIqv hvq Zv c`Ë msL¨v †_‡K 9 †ewk|

K. PjK x Gi gva¨‡g c`Ë msL¨wU I ¯vb wewbgqKZ msL¨wU †jL|

L. msL¨wU wbYq Ki|

M. c`Ë msL¨wUi A¼Øq hw` †mw›UwgUv‡i †Kv‡bv AvqZ‡¶‡Îi ˆ`N¨ I c¯ wb‡`k K‡i Z‡e H AvqZ‡¶ÎwUi K‡Yi ˆ`N¨ wbYq Ki| KYwU‡K †Kv‡bv e‡Mi evû a‡i eM‡¶ÎwUi K‡Yi ˆ`N¨ wbYq Ki|

33| GKwU mg‡KvYx wÎf‡Ri fwg I D”PZv h_vµ‡g $(x-1)$ †m.wg. I x †m.wg. Ges GKwU e‡Mi evûi ˆ`N¨ wÎfRwUi D”PZvi mgvb| Avevi, GKwU AvqZ‡¶‡Îi evûi ˆ`N¨ $(x+3)$ †m.wg. I c¯ x †m.wg.|

K. GKwUgvÎ wP‡Îi gva¨‡g Z_¨¸‡jv †`LvI|

L. wÎfR‡¶ÎwUi †¶Îdj 10 eM †m.wg. n‡j, Gi D”PZv KZ ?

M. wÎfR‡¶Î, eM‡¶Î I AvqZ‡¶‡Îi †¶Îd‡ji avivewnK AbcvZ †ei Ki|

Iô Aa¨vq
†iLv, †KvY I wÎfR

R¨vwgwZ ev '*Geometry*' MwYZ kv‡¯¿i GKwU cvPxb kvLv| '*Geometry*' kãwU MxK *Geo* - fwg (earth) I *metrein* - cwigvc (measure) k‡ãi mgš‡q ˆZwi| ZvB ÒR¨vwgwZÓ k‡ãi A_ Òfwg cwigvcÓ| KwlwfwËK mf¨Zvi h‡M fwg cwigv‡ci c‡qvR‡bB R¨vwgwZi mwó n‡qwQj| Z‡e R¨vwgwZ AvRKvj †Kej fwg cwigv‡ci Rb¨B e¨eüZ nq bv, eis eû RwUj MwwYwZK mgm¨v mgvav‡b R¨vwgwZK Ávb GLb Acwinvh| cvPxb mf¨Zvi wb`kb¸‡jv‡Z R¨vwgwZ PPvi cgvY cvIqv hvq| HwZnvwmK‡`i g‡Z cvPxb wgk‡i AvbgwbK Pvi nvRvi eQi Av‡MB fwg Rwi‡ci Kv‡R R¨vwgwZK aviYv e¨envi Kiv n‡Zv| cvPxb wgki, e¨wejb, fviZ, Pxb I BbKv mf¨Zvi wewfb e¨enwiK Kv‡R R¨vwgwZi c‡qv‡Mi wb`kb i‡q‡Q| cvK-fviZ Dcgnv‡`‡k wmÜ DcZ¨Kvi mf¨Zvq R¨vwgwZi eûj e¨envi wQj| niàv I g‡n‡Äv`v‡ivi Lb‡b mcwiKwíZ bMixi Aw¯‡Zi cgvY †g‡j| kn‡ii iv¯v¸‡jv wQj mgvšivj Ges fMf¯ wb®vmb e¨e¯v wQj DbZ| ZvQvov Niewoi AvKvi †`‡L †evSv hvq †h, kn‡ii Awaevmxiv fwg cwigv‡cI `¶ wQ‡jb| ˆew`K h‡M †ew` ˆZwi‡Z wbw`ó R¨vwgwZK AvKvi I †¶Îdj †g‡b Pjv n‡Zv| G¸‡jv cavbZ wÎfR, PZfR I UwcwRqvg AvKv‡ii mgš‡q MwVZ n‡Zv|

Z‡e cvPxb MxK mf¨Zvi h‡MB R¨vwgwZK cYvjxe× i|cwU m¯úófv‡e j¶ Kiv hvq| MxK MwYZwe` †\_wjm‡K c\_g R¨vwgwZK cgv‡Yi KwZZ †`qv nq| wZwb hw³gjK cgvY †`b †h, e¨vm Øviv eË mgwØLwÊZ nq| †_wj‡mi wkl¨ wc_v‡Mvivm R¨vwgwZK Z‡Ëi we¯wZ NUvb| AvbgwbK wLóce 300 A‡ã MxK cwÊZ BDwKW R¨vwgwZi BZ¯Z wew¶ß mÎ¸‡jv‡K wewae×fv‡e mweb¨¯ K‡i Zvi weL¨vZ Mš ÒBwj‡g›UmÓ iPbv K‡ib| †Z‡iv L‡Ê m¤úY Kv‡jvËxY GB ÒBwj‡g›UmÓ MšwUB AvawbK R¨vwgwZi wfwË¯i|c| GB Aa¨v‡q BDwK‡Wi Abmi‡Y hw³gjK R¨vwgwZ Av‡jvPbv Kiv n‡e|

Aa¨vq †k‡l wk¶v_xiv Ñ

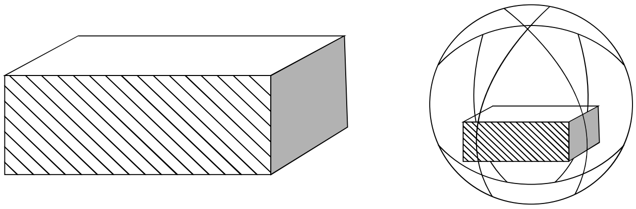
- mgZjxq R¨vwgwZi †gŠwjK ¯xKvh¸‡jv eYbv Ki‡Z cvi‡e|
- wÎfR msµvš Dccv`¨¸‡jv cgvY Ki‡Z cvi‡e|
- wÎfR msµvš Dccv`¨ I Abwm×vš¸‡jv c‡qvM K‡i mgm¨v mgvavb Ki‡Z cvi‡e|

6·1 ¯vb, Zj, †iLv I we›`i aviYv

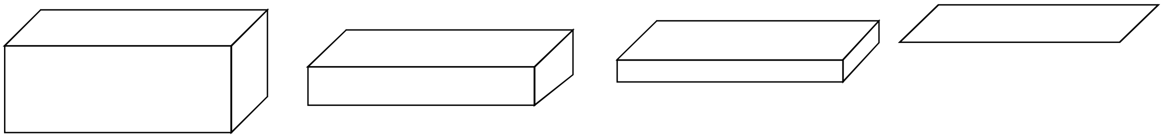
Avgv‡`i Pvicv‡k we¯Z RMZ (*Space*) mxgvnxb| Gi wewfb Ask R‡o i‡q‡Q †QvU eo bvbv iKg e¯| †QvU eo e¯ ej‡Z evjKYv, Avjwcb, †cwÝj, KvMR, eB, †Pqvi, †Uwej, BU, cv_i, ewoNi, cvnvo, cw_ex, Mn-b¶Î meB †evSvb nq| wewfb e¯ ¯v‡bi †h Ask R‡o _v‡K †m ¯vbUKi AvKvi, AvKwZ, Ae¯vb, ˆewkó¨ cfwZ †_‡KB R¨vwgwZK aviYvi D™¢e|

†Kv‡bv Nbe¯ (*Solid*) †h ¯vb AwaKvi K‡i _v‡K, Zv wZb w`‡K we¯Z| G wZb w`‡Ki we¯v‡iB e¯wUi wZbwU gvÎv (`N¨, c¯ I D"PZv) wb‡`k K‡i| †mRb¨ c‡Z¨K Nbe¯B wÎgwÎK (*Three dimensional*) |

†hgb, GKwU BU ev ev‡·i wZbwU gvÎv (^`N¨, c¯ I D”PZv) Av‡Q| GKwU †Mvj‡Ki wZbwU gvÎv Av‡Q| Gi wZb gvÎvi wfbZv ¯úó †evSv bv †M‡j I G‡K ^`N¨-c¯-D”PZv wewkó L‡Ê wef³ Kiv hvq|



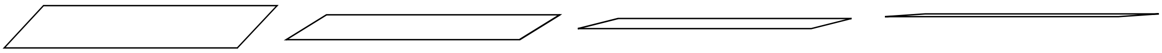
Nbe¯i DcwifvM Zj (*Surface*) wb‡`k K‡i A_vr, c‡Z¨K Nbe¯ GK ev GKvwaK Zj Øviv mxgvex× _v‡K| †hgb, GKwU ev‡·i QqwU cô QqwU mgZ‡ji cwZi/c| †Mvj‡Ki DcwifvM I GKwU Zj| Z‡e ev‡·i côZj I †Mvj‡Ki cô Zj wfb cKv‡ii| c_gwU mgZj (*Plane*), wØZxqwU eµZj (*Curved Surface*)|



Zj wØgvwÎK (*Two – dimensional*) : Gi ïa ^`N¨ I c¯ Av‡Q, †Kv‡bv D”PZv bvB| GKwU ev‡·i `BwU gvÎv wVK †i‡L ZZxq gvÎv µgk nvm K‡i k‡b¨ cwiYZ Ki‡j, ev·wUi côwe‡kl gvÎ Aewkó _v‡K| Gfv‡e Nbe¯ †_‡K Z‡ji avibvq Avmv hvq|

`BwU Zj ci¯úi‡K †Q` Ki‡j GKwU †iLv (*line*) Drcb nq| †hgb, ev‡·i `BwU côZj ev‡·i GKav‡i GKwU †iLvq wgwjZ nq| GB †iLv GKwU mij‡iLv (*straight line*)| GKwU †je‡K GKwU cvZjv Qwi w`‡q KvU‡j, Qwii mgZj †hLv‡b †jei eµZj‡K †Q` K‡i †mLv‡b GKwU eµ‡iLv (*curved line*) Drcb nq|

†iLv GKgvwÎK (*one – dimensional* : Gi ïa ^`N¨ Av‡Q, c¯ I D”PZv †bB| ev‡·i GKwU cô-Z‡ji c¯ µgk nvm †c‡q m¤úY kb¨ n‡j, H Z‡ji GKwU †iLv gvÎ Aewkó _v‡K| Gfv‡e Z‡ji avibv †_‡K †iLvi avibvq Avmv hvq|



`BwU †iLv ci¯úi †Q` Ki‡j we›`i DrcwË nq| A_vr, `BwU †iLvi †Q`¯vb we›` (*point*) Øviv wbw`ó nq| ev‡·i `BwU avi †hgb, ev‡·i GK †KvYvq GKwU we›`‡Z wgwjZ nq|

we›`i ^`N¨, c¯ I D”PZv bvB, ïa Ae¯vb Av‡Q| GKwU †iLvi ^`N¨ µgk nvm †c‡j Ae‡k‡l GKwU we›`‡Z chewmZ nq| we›`‡K kb¨ gvÎvi mËv (*entity*) e‡j MY¨ Kiv nq|

6·2 BDwK‡Wi ¯xKvh

Dc‡i Zj, †iLv I we›` m¤ú‡K †h avibv †`Iqv n‡jv, Zv Zj, †iLv I we›`i msÁv bq- eYbv gvÎ| GB eYbvq gvÎv ej‡Z ^`N¨, c¯, D”PZv BZ¨vw` avibv e¨envi Kiv n‡q‡Q, †h¸‡jv msÁvwqZ bq| BDwKW Zvi ÔBwj‡g›UmÕ M‡š'i c_g L‡Êi ïi"‡ZB we›`, †iLv I Z‡ji †h ÔmsÁvÕ D‡jL K‡i‡Qb Zv-I Avawbk `wó fw½ Abmv‡i Am¤úY| BDwKW c`Ë K‡qKwU eYbv wbgi/c :

(1) hvi †Kv‡bv Ask bvB, ZvB we›`|

(2) †iLvi cvš— we›` †bB|

(3) hvi †Kej ˆ`N©¨ Av‡Q, wKš‘ c—¯' I D"PZv bvB, ZvB †iLv|

(4) †h †iLvi Dcwiw¯'Z we›`¸‡jv GKB eivei _v‡K, ZvB mij‡iLv|

(5) hvi †Kej ˆ`N©¨ I c—¯' Av‡Q, ZvB Zj|

(6) Z‡ji cvš— n‡jv †iLv|

(7) †h Z‡ji mij‡iLv¸‡jv Zvi Ici mgfv‡e _v‡K, ZvB mgZj|

j¶ Ki‡j †`Lv hvq †h, GB eYbvq Ask, ˆ`N©¨, c—¯', mgfv‡e BZ¨vw` kã¸‡jv AmsÁvwqZfv‡e MnY Kiv n‡q‡Q| a‡i †bqv n‡q‡Q †h, G¸‡jv m¤ú‡K Avgv‡`i cv_wgK avibv i‡q‡Q| Gme avibvi Dci wfwË K‡i we›`, mij‡iLv I mgZ‡ji avibv †`Iqv n‡q‡Q| ev¯weK c‡¶, †h‡Kv‡bv MwYwZK Av‡jvPbvq GK ev GKwaK cv_wgK avibv ¯xKvi K‡i wb‡Z nq| BDwKW G¸‡jv‡K ¯Ztwm× (*Axioms*) e‡j AvL¨vwqZ K‡ib| BDwKW c`Ë K‡qKwU ¯Ztwm× :

1| †hmKj e¯' GKB e¯'i mgvb, †m¸‡jv ci¯úi mgvb|

2| mgvb mgvb e¯'i mv‡_ mgvb e¯' †hvM Kiv n‡j †hvMdj mgvb|

3| mgvb mgvb e¯' ‡_‡K mgvb e¯' we‡qvM Kiv n‡j we‡qvMdj mgvb|

4| hv ci¯ú‡ii mv‡_ wg‡j hvq, Zv ci¯úi mgvb|

5| cY Zvi Ask †P‡q eo|

AvawbK R¨vwgwZ‡Z we›`, mij‡iLv I mgZj‡K cv_wgK avibv wn‡m‡e MnY K‡i Zv‡`i wKQ ˆewkó¨‡K ¯xKvi K‡i †bIqv nq| GB ¯xKZ ˆewkó¨¸‡jv‡K R¨vwgwZK ¯xKvh (*postulate*) ejv nq| ev¯e avibvi m‡½ m½wZ †i‡LB GB ¯xKvhmgn wbavibY Kiv n‡q‡Q| BDwKW c`Ë cvPwU ¯xKvh n‡jv :

¯xKvh 1| GKwU we›` †_‡K Ab¨ GKwU we›` chš— GKwU mij‡iLv AvKv hvq|

¯xKvh 2| LwÊZ †iLv‡K h‡_"Q fv‡e evov‡bv hvq|

¯xKvh 3| †h‡Kv‡bv †K›`ª I †h‡Kv‡bv e¨vmva wb‡q eË AvKv hvq|

¯xKvh 4| mKj mg‡KvY ci¯úi mgvb|

¯xKvh 5| GKwU mij‡iLv `BwU mij‡iLv‡K †Q` Ki‡j Ges †Q`‡Ki GKB cv‡ki Aš—¯' †KvYØ‡qi mgwó `B mg‡Kv‡Yi †P‡q Kg n‡j, †iLv `BwU‡K h‡_"Q fv‡e ewaZ Ki‡j †hw`‡K †Kv‡Yi mgwó `B mg‡Kv‡Yi †P‡q Kg, †mw`‡K wgwjZ nq|

BDwKW msÁv, ¯Ztwm× I ¯xKvh¸‡jvi mvnv‡h¨ hw³gjK bZb cwZÁv cgvY K‡ib| wZwb msÁv, ¯Ztwm×, ¯xKvh I cgwYZ cwZÁvi mvnv‡h¨ Avevi bZb GKwU cwZÁvI cgvY K‡ib| BDwKW Zvi ÔBwj‡g›UmÕ M‡š' †gvU 465wU k•Ljve× cwZÁvi cgvY w`‡q‡Qb hv AvawbK hw³gjK R¨vwgwZi wfwË|

j¶ Kwi †h, BDwK‡Wi c_g ¯xKv‡h wKQ Am¤úYZv i‡q‡Q| `BwU wfb we›` w`‡q †h GKwU Abb¨ mij‡iLv A¼b Kiv hvq Zv D‡cw¶Z n‡q‡Q| cÂg ¯xKvh Ab¨ PviwU ¯xKv‡hi †P‡q RwUj| Ab¨w`‡K, c_g †_‡K

PZ_ ¯xKvh¸‡jv G‡Zv mnR †h G¸‡jv Ô¯úóB mZ¨Õ e‡j cZxqgvb nq| wKš G¸‡jv cgvY Kiv hvq bv| mZivs, Dw³ ¸‡jv ÔcgvYwenxb mZ¨Õ ev ¯xKvh e‡j †g‡b †bqv nq| cÂg ¯xKvhwU mgvšivj mij‡iLvi mv‡_ RwoZ weavq cieZx‡Z Av‡jvPbv Kiv n‡e|

6·3 mgZj R¨wgwZ

c‡eB we›`, mij‡iLv I mgZj R¨wgwZi wZbwU cv_wgK avi Yv D‡jL Kiv n‡q‡Q| G‡`i h_vh_ msÁv †`Iqv m¤e bv n‡j I G‡`i m¤ú‡K Avgv‡`i ev¯e AwfÁZvcmZ avi Yv n‡q‡Q| wegZ R¨wgwZK avi Yv wn‡m‡e ¯vb‡K we›`mg‡ni †mU aiv nq Ges mij‡iLv I mgZj‡K GB mwweK †m‡Ui Dc‡mU we‡ePbv Kiv nq| A_vr,

¯xKvh 1| RMZ (*Space*) mKj we›`i †mU Ges mgZj I mij‡iLv GB †m‡Ui Dc‡mU|

GB ¯xKvh †_‡K Avgiv j¶ Kwi †h, c‡Z¨K mgZj I c‡Z¨K mij‡iLv GK GKwU †mU, hvi Dcv`vb n‡"Q we›`| R¨wgwZK eYbvq mvavi YZ †mU cZx‡Ki e¨envi cwinvi Kiv nq| †hgb, †Kv‡bv we›` GKwU mij‡iLvi (ev mgZ‡ji) Ašf³ n‡j we›`wU H mij‡iLvq (ev mgZ‡j) Aew¯Z A_ev, mij‡iLwU (ev mgZjwU) H we›` w`‡q hvq| GKBfv‡e, GKwU mij‡iLv GKwU mgZ‡ji Dc‡mU n‡j mij‡iLwU H mgZ‡j Aew¯Z, A_ev, mgZjwU H mij‡iLv w`‡q hvq G iKg evK¨ Øviv Zv eYbv Kiv nq|

mij‡iLv I mgZ‡ji ˆewkó¨ wn‡m‡e ¯xKvi K‡i †bIqv nq †h,

¯xKvh 2| `BwU wfb we›`i Rb¨ GKwU I †Kej GKwU mij‡iLv Av‡Q, hv‡Z Dfq we›` Aew¯Z|

¯xKvh 3| GKB mij‡iLvq Aew¯Z bq Ggb wZbwU wfb we›`i Rb¨ GKwU I †Kej GKwU mgZj Av‡Q, hv‡Z we›` wZbwU Aew¯Z|

¯xKvh 4| †Kv‡bv mgZ‡ji `BwU wfb we›` w`‡q hvq Ggb mij‡iLv H mgZ‡j Aew¯Z|

¯xKvh 5| (K) RM‡Z (*Space*) GKwaK mgZj we`¨gvb|

(L) c‡Z¨K mgZ‡j GKwaK mij‡iLv Aew¯Z|

(M) c‡Z¨K mij‡iLvi we›`mgn Ges ev¯e msL¨vmgn‡K Ggbfv‡e m¤úwKZ Kiv hvq †hb, †iLwUi c‡Z¨K we›`i m‡½ GKwU Abb¨ ev¯e msL¨v mswkó nq Ges c‡Z¨K ev¯e msL¨vi m‡½ †iLwUi GKwU Abb¨ we›` mswkó nq|

gše¨ : ¯xKvh 1 †_‡K ¯xKvh 5 †K AvcZb ¯xKvh ejv nq|

R¨wgwZ‡Z `i‡Zi avi Yv I GKwU cv_wgK avi Yv| G Rb¨ ¯xKvi K‡i †bIqv nq †h,

¯xKvh 6| (K) P I Q we›`hMj GKwU Abb¨ ev¯e msL¨v wbw`ó K‡i _v‡K| msL¨wU‡K P we›` †_‡K $Q$ we›`i `iZ ejv nq Ges PQ Øviv mwPZ Kiv nq|

(L) P I Q wfb we›` n‡j PQ msL¨wU abvZ¥K| Ab¨_vq, $PQ = 0$|

(M) P †_‡K Q Gi `iZ Ges $Q$ †_‡K $P$ Gi `iZ GKB| A_vr $PQ = QP$|

$PQ = QP$ nIqv‡Z GB `iZ‡K mvaviYZ $P$ we›` I Q we›`i ga¨eZx `iZ ejv nq| e¨enwiKfv‡e, GB `iZ ce wbawiZ GK‡Ki mvnv‡h¨ cwigvc Kiv nq|

¯xKvh 5 (M) Abhvqx c‡Z¨K mij‡iLvq Aew¯Z we›`mg‡ni †mU I ev¯e mgm¨vi †m‡Ui g‡a¨ GK-GK wgj ¯vcb Kiv hvq| G cm‡½ ¯xKvi K‡i †bIqv nq †h,

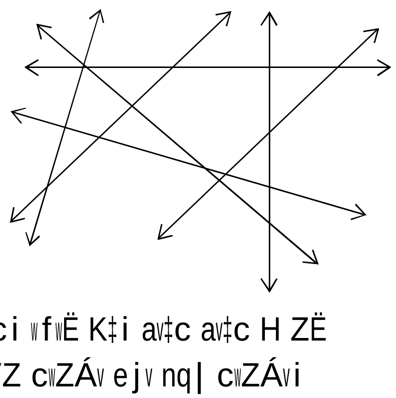
¯xKvh 7| †Kv‡bv mij‡iLvq Aew¯Z we›`mg‡ni †mU Ges ev¯e msL¨vi †m‡Ui g‡a¨ Ggbfv‡e GK-GK wgj ¯vcb Kiv hvq, †hb †iLwUi †h‡Kv‡bv we›` P, Q Gi Rb¨ $PQ = |a - b|$ nq, †hLv‡b wgjKi‡Yi d‡j P I Q Gi m‡½ h_vµ‡g a I b ev¯e msL¨v mswkøó nq|

GB ¯xKv‡h ewYZ wgjKiY Kiv n‡j, †iLwU GKwU msL¨v‡iLvq cwiYZ n‡q‡Q ejv nq| msL¨v‡iLvq P we›`i m‡½ $a$ msL¨wU mswkøó n‡j $P$ †K $a$ Gi †jLwe›` Ges a †K P Gi ¯vbv¼ ejv nq| †Kv‡bv mij‡iLv‡K msL¨v‡iLvq cwiYZ Kivi Rb¨ c_‡g †iLwUi GKwU we›`i ¯vbv¼ $0$ Ges Aci GKwU we›`i ¯vbv¼ 1 a‡i †bIqv nq| G‡Z †iLwU‡Z GKwU GKK `iZ Ges GKwU abvZ¥K w`K wbw`ó nq| G Rb¨ ¯xKvi K‡i †bIqv nq †h,

¯xKvh 8| †h‡Kv‡bv mij‡iLv AB †K Ggbfv‡e msL¨v‡iLvq cwiYZ Kiv hvq †h, A Gi ¯vbv¼ O Ges B Gi ¯vbv¼ abvZ¥K nq|

gše¨ : ¯xKvh 6 †K `iZ ¯xKvh, ¯xKvh 7 †K i/jvi ¯xKvh Ges ¯xKvh 8 †K i/jvi ¯vcb ¯xKvh ejv nq| R¨wgwZK eYbv‡K ¯úó Kivi Rb¨ wPÎ e¨envi Kiv nq| KvM‡Ri Ici †cwÝj ev Kj‡gi m² †dvUv w`‡q we›`i cwZi/c AvKv nq| †mvRv i/jvi eivei `vM †U‡b mij‡iLvi cwZi/c AvKv nq| mij‡iLvi wP‡Î `B w`‡K ZxiwPý w`‡q †evSv‡bv nq †h, †iLwU Dfq w`‡K mxgvnxbfv‡e we¯Z| ¯xKvh 2 Abhvqx `BwU wfbœ we›` A I B GKwU Abb¨ mij‡iLv wbw`ó K‡i hv‡Z we›` `BwU Aew¯Z nq| GB †iLv‡K $AB$ †iLv ev $BA$ †iLv ejv nq| ¯xKvh 5 (M) Abhvqx Gi/c c‡Z¨K mij‡iLv AmsL¨ we›` aviY K‡i|

¯xKvh (5) (K) Abhvqx GKwaK mgZj we`¨gvb| Gi/c c‡Z¨K mgZ‡j AmsL¨ mij‡iLv i‡q‡Q| R¨wgwZi †h kvLvq GKB mgZ‡j Aew¯Z we›`, †iLv Ges Zv‡`i m‡½ m¤úwKZ wewfbœ R¨wgwZK mËv m¤ú‡K Av‡jvPbv Kiv nq, Zv‡K mgZj R¨wgwZ (*Plane Geometry*) ejv nq| G c¯‡K mgZj R¨wgwZB Avgv‡`i gj we‡eP¨ welq| mZivs, we‡kl †Kv‡bv D‡jL bv _vK‡j eS‡Z n‡e †h, Av‡jvP¨ mKj we›`, †iLv BZ¨w` GKB mgZ‡j Aew¯Z| Gi/c GKwU wbw`ó mgZjB Av‡jvPbvi mwweK †mU|



MwwYwZK Dw³i cgvY

†h‡Kv‡bv MwwYwZK Z‡Ë KwZcq cv_wgK aviYv, msÁv Ges ¯xKv‡hi Dci wfwË K‡i av‡c av‡c H ZË m¤úwKZ wewfbœ Dw³ †hŠw³Kfv‡e cgvY Kiv nq| Gi/c Dw³‡K mvaviYZ cwZÁv ejv nq| cwZÁvi †hŠw³KZv cgv‡Yi Rb¨ hw³we`¨vi wKQ wbqg c‡qvM Kiv nq| ‡hgb,

(K) Av‡ivn c×wZ (Mathematical Induction)

(L) Ae‡ivn c×wZ (Mathematical Deduction)

(M) we‡iva c×wZ BZ¨vw` |

we‡iva c×wZ *(Proof by contradiction)*

`vkwbK Gwi÷Uj hw³gjK cgv‡Yi G c×wZwUi mPbv K‡ib| G c×wZi wfwË n‡jv:

Ñ GKB ¸Y‡K GKB mgq ¯xKvi I A¯xKvi Kiv hvq bv|

Ñ GKB wRwb‡li `BwU ci¯úiwe‡ivax ¸Y _vK‡Z cv‡i bv|

Ñ hv ci¯úiwe‡ivax Zv AwPš¨bxq|

Ñ †Kv‡bv e¯ GK mg‡q †h ¸‡Yi AwaKvix nq, †mB e¯ †mB GKB mg‡q †mB ¸‡Yi AbwaKvix n‡Z cv‡i bv|

6·4 R¨wgwZK cgvY

R¨wgwZ‡Z KZK¸‡jv cwZÁv‡K we‡kl ¸i“Z w`‡q Dccv`¨ wn‡m‡e MnY Kiv nq Ges Ab¨vb¨ cwZÁv cgv‡Y µg Abhvqx G‡`i e¨envi Kiv nq| R¨wgwZK cgv‡Y wewfb Z_¨ wP‡Îi mvnv‡h¨ eYbv Kiv nq| Z‡e cgvY Aek¨B hw³wbfi n‡Z n‡e|

R¨wgwZK cwZÁvi eYbvq mvaviY wbePb (*general enunciation*) A_ev we‡kl wbePb (*particular enunciation*) e¨envi Kiv nq| mvaviY wbePb n‡”Q wPÎwbi‡c¶ eYbv Avi we‡kl wbePb n‡”Q wPÎwbfi eYbv| †Kv‡bv cwZÁvi mvaviY wbePb †`Iqv _vK‡j cwZÁvi welqe¯ we‡kl wbeP‡bi gva¨‡g wbw`ó Kiv nq| G Rb¨ c‡qvRbxq wPÎ A¼b Ki‡Z nq| R¨wgwZK Dccv‡`¨i cgv‡Y mvaviYZ wb‡gv³ avc¸‡jv _v‡K :

(1) mvaviY wbePb

(2) wPÎ I we‡kl wbePb

(3) c‡qvRbxq A¼‡bi eYbv Ges

(4) cgv‡Yi †hŠw³K avc¸‡jvi eYbv|

hw` †Kv‡bv cwZÁv mivmwi fv‡e GKwU Dcvcv‡`¨i wm×vš— †_‡K cgwYZ nq, Z‡e Zv‡K A‡bK mgq H Dccv‡`¨i Abwm×vš— (*Corollary*) wn‡m‡e D‡jL Kiv hvq| wewfb cwZÁv cgvY Kiv QvovI R¨wgwZ‡Z wewfb wPÎ A¼b Kivi c¯vebv we‡ePbv Kiv nq| G¸‡jv‡K m¤úv`¨ ejv nq| m¤úv`¨ welqK wPÎ A¼b K‡i wPÎv¼‡bi eYbv I †hŠw³KZv D‡jL Ki‡Z nq|

Abkxjbx 6·1

1| ¯vb, Zj, †iLv Ges we›`i aviYv `vI|

2| BDwK‡Wi cvPwU ¯xKvh eYbv Ki|

3| cvPwU AvcZb ¯'xKvh eYbv Ki|

4| `eZ ¯'xKvhwU eYbv Ki|

5| i"jvi ¯'xKvhwU eYbv Ki|

6| msL¨v‡iLv eYbv Ki|

7| i"jvi ¯'vcb ¯'xKvhwU eYbv Ki|

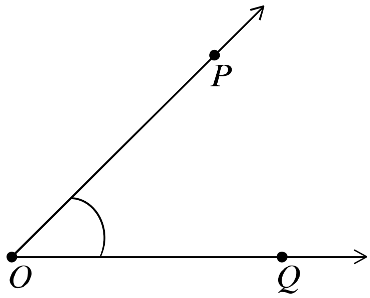
8| ci¯úi‡Q`x mij‡iLv I mgvšivj mij‡iLvi msÁv `vI|

†iLv, iwk¥, †iLvsk

mgZjxq R¨vwgwZi ¯'xKvh Abhvqx mgZ‡j mij‡iLv we`¨gvb hvi cwZwU we›` mgZ‡j Aew¯'Z| g‡b Kwi, mgZ‡j AB GKwU mij‡iLv Ges †iLwUi Dci Aew¯'Z GKwU we›` $C$| $C$ we›`‡K A I B we›`i AšeZx ejv nq hw` A, C I B GKB mij‡iLvi wfb wfb we›` nq Ges $AC+CB=AB$ nq| $A$, $C$ I $B$ we›` wZbwU‡K mg‡iL we›`I ejv nq| $A$ I $B$ Ges G‡`i AšeZx mKj we›`i †mU‡K $A$ I $B$ we›`i ms‡hvRK †iLvsk ev ms‡¶‡c AB †iLvsk ejv nq| A I B we›`i AšeZx c‡Z¨K we›`‡K †iLvs‡ki Ašt¯' we›` ejv nq|

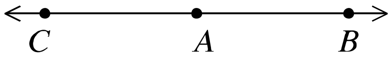
†KvY

mgZ‡j `BwU iwk¥i cvšwe›` GKB n‡j †KvY %Zwi nq| iwk¥ `BwU‡K †Kv‡Yi evû Ges Zv‡`i mvaviY we›`‡K kxlwe›` e‡j| wP‡Î, OP I OQ iwk¥Øq Zv‡`i mvaviY cvšwe›` O†Z $\angle POQ$ Drcb K‡i‡Q| O we›`wU $\angle POQ$ Gi kxlwe›`| OP Gi †h cv‡k Q Av‡Q †mB cv‡k Ges OQ Gi †h cv‡k P Av‡Q †mB cv‡k Aew¯'Z mKj we›`i †mU‡K $\angle POQ$ Gi Af¨šÍi ejv nq| †KvYwUi Af¨š‡i A_ev †Kv‡bv evû‡Z Aew¯'Z bq Ggb mKj we›`i †mU‡K Gi ewnfvM ejv nq|



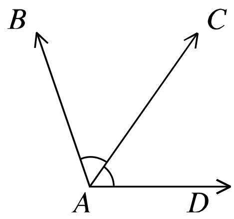
mij †KvY

`BwU ci¯úi wecixZ iwk¥ Zv‡`i mvaviY cvšwe›`‡Z †h †KvY Drcb K‡i, Zv‡K mij †KvY e‡j| cv‡ki wP‡Î, $AB$ iwk¥i cvßwe›` A †_‡K AB Gi wecixZ w`‡K $AC$ iwk¥ AvKv n‡q‡Q| $AC$ I $AB$ iwk¥Øq Zv‡`i mvaviY cvšwe›` $A$ †Z $\angle BAC$ Drcb K‡i‡Q| $\angle BAC$ †K mij †KvY e‡j| mij †Kv‡Yi cwigvc `B mg‡KvY ev 180^0|



mwbwnZ †KvY

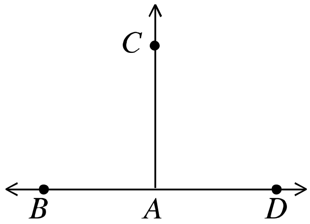
hw` mgZ‡j `BwU †Kv‡Yi GKB kxl©we›`y nq I Zv‡`i GKwU mvaviY iwk¥ _v‡K Ges †KvYØq mvaviY iwk¥i wecixZ cv‡k Ae¯’vb K‡i, Z‡e H †KvYØq‡K mwbwnZ †KvY e‡j|



cv‡ki wP‡Î, A we›`yU $\angle BAC$ I $\angle CAD$ Gi kxl©we›`y|
A we›`y $\angle BAC$ I $\angle CAD$ DrcbKvix iwk¥¸‡jvi g‡a¨ $AC$ mvaviY iwk¥ | †KvY `BwU mvaviY iwk¥ AC Gi wecixZ cv‡k Aew¯’Z| $\angle BAC$ Ges $\angle CAD$ ci¯úi mwbwnZ †KvY|

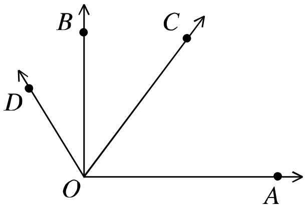
j¤, mg‡KvY

GKwU mij‡Kv‡Yi mgwØLÊK‡K j¤ Ges mswkó mwbwnZ †Kv‡Yi c‡Z¨KwU‡K mg‡KvY e‡j| cv‡ki wP‡Î, $\angle BAD$ mij‡KvY A we›`‡Z $AC$ iwk¥ Øviv d‡j $\angle BAC$ I $\angle CAD$ mwbwnZ †KvY `BwUi c‡Z¨‡K mg‡KvY Ges BD I AC evûØq ci¯ú‡ii Dci j¤|



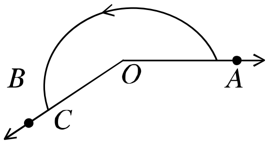
m²‡KvY I ¯’j‡KvY

GK mg‡KvY †_‡K †QvU †KvY‡K m²‡KvY Ges GK mg‡KvY †_‡K eo wKš‘ `B mg‡KvY †_‡K †QvU †KvY‡K ¯’j‡KvY ejv nq| wP‡Î $\angle AOC$ m²‡KvY Ges $\angle AOD$ ¯’j‡KvY| GLv‡b $\angle AOB$ GK mg‡KvY|



cex †KvY

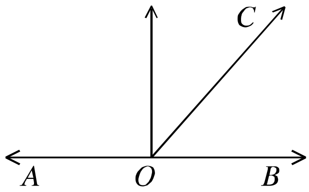
`B mg‡KvY †_‡K eo wKš‘ Pvi mg‡KvY †_‡K †QvU †KvY‡K cex‡KvY ejv nq| wP‡Î wPwýZ $\angle AOC$ cex‡KvY|

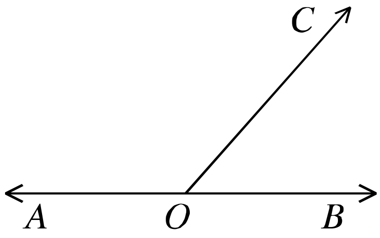


ciK †KvY

`BwU †Kv‡Yi cwigv‡ci †hvMdj 1 mg‡KvY n‡j †KvY `BwUi GKwU AciwUi ciK †KvY|

cv‡ki wP‡Î, $\angle AOB$ GKwU mg‡KvY| OC iwk¥ †KvYwUi evûØ‡qi Af¨š‡i Aew¯’Z| Gi d‡j $\angle AOC$ Ges $\angle COB$ GB `BwU †KvY Drcb n‡jv| †KvY `BwUi cwigv‡ci †hvMdj $\angle AOB$ Gi cwigv‡ci mgvb, A_©vr 1 mg‡KvY| $\angle AOC$ Ges $\angle COB$ ci¯úi ciK †KvY|

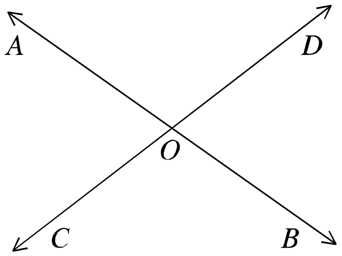




m¤úiK †KvY

`BwU †Kv‡Yi cwigv‡ci †hvMdj 2 mg‡KvY n‡j †KvY `BwU ci¯ci m¤úiK †KvY|

AB GKwU mij‡iLvi O Ašt¯ GKwU we›`| $OC$ GKwU iwk¥ hv $OA$ iwk¥ I $OB$ iwk¥ †\_‡K wfbœ| Gi d‡j $\angle AOC$ Ges $\angle COB$ GB `BwU †KvY Drcbœ n‡jv| †KvY `BwUi cwigv‡ci †hvMdj $\angle AOB$ †Kv‡Yi cwigv‡ci mgvb, A_vr 2 mg‡KvY, †Kbbv $\angle AOB$ GKwU mij‡KvY| $\angle AOC$ Ges $\angle COB$ ci¯úi m¤úiK †KvY|

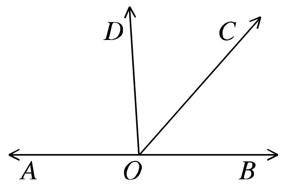


wecZxc †KvY

†Kv‡bv †Kv‡Yi evûØ‡qi wecixZ iwk¥Øq †h †KvY ˆZwi K‡i Zv H †Kv‡Yi wecZxc †KvY|

wP‡Î OA I OB ci¯úi wecixZ iwk¥| Avevi OC I OD ci¯úi wecixZ iwk¥| $\angle BOD$ I $\angle AOC$ ci¯úi wecZxc †KvY| Avevi $\angle BOC$ I $\angle DOA$ GKwU AciwUi wecZxc †KvY| `BwU mij‡iLv †Kv‡bv we›`‡Z ci¯úi‡K †Q` Ki‡j, †Q` we›`‡Z `B †Rvov wecZxc †KvY Drcbœ nq|

Dccv`¨ 1



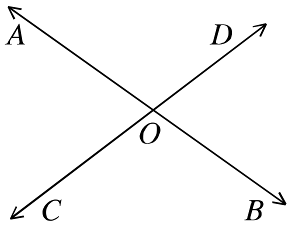
GKwU mij‡iLvi GKwU we›`‡Z Aci GKwU iwk¥ wgwjZ n‡j, †h `BwU mwbœwnZ †KvY Drcbœ nq Zv‡`i mgwó `B mg‡KvY|

g‡b Kwi, AB mij‡iLwUi O we›`‡Z $OC$ iwk¥i cvšwe›` O wgwjZ n‡q‡Q| d‡j $\angle AOC$ I $\angle COB$ `BwU mwbœwnZ †KvY Drcbœ nj| AB †iLvi Dci DO j¤ AwwK|

mwbœwnZ †KvYØ‡qi mgwó $= \angle AOC + \angle COB = \angle AOD + \angle DOC + \angle COB$

$= \angle AOD + \angle DOB$ = 2 mg‡KvY |

Dccv`¨ 2



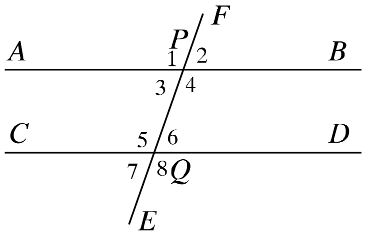
`BwU mij‡iLv ci¯úi †Q` Ki‡j, Drcbœ wecZxc †KvY¸‡jv ci¯úi mgvb|

g‡b Kwi, AB I CD †iLvØq ci¯úi O we›`‡Z †Q` K‡i‡Q| d‡j O we›`‡Z $\angle AOC$, $\angle COB$, $\angle BOD$, $\angle AOD$ †KvY Drcbœ n‡q‡Q|

$\angle AOC$ = wecZxc $\angle BOD$ Ges $\angle COB$ = wecZxc $\angle AOD$ |

6·4 mgvšivj mij‡iLv

GKvši †KvY, Abi‚c †KvY, †Q`‡Ki GKB cvk¦© Ašt¯’ †KvY



Dc‡ii wP‡Î, AB I CD `ywU mij‡iLv Ges $EF$ mij‡iLv G‡`i‡K P I Q we›`y‡Z †Q` K‡i‡Q| EF mij‡iLv AB I CD mij‡iLvØ‡qi †Q`K| †Q`KwU AB I CD mij‡iLv `ywUi mv‡_ $\angle 1, \angle 2$, $\angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$ †gvU AvUwU †KvY ˆZwi K‡i‡Q| G †KvY¸‡jvi g‡a¨

(K) $\angle 1$ Ges $\angle 5, \angle 2$ Ges $\angle 6$, $\angle 3$ Ges $\angle 7, \angle 4$ Ges $\angle 8$ ci¯úi Abi“c †KvY|

(L) $\angle 3$ Ges $\angle 6, \angle 4$ Ges $\angle 5$ n‡jv ci¯úi GKvši †KvY

(M) $\angle 4, \angle 6$ Wvbcv‡ki Ašt¯’ †KvY|

(N) $\angle 3, \angle 5$ evgcv‡ki Ašt¯’ †KvY|

mgZ‡j `ywU mij‡iLv ci¯úi‡K †Q` Ki‡Z cv‡i A_ev Zviv mgvšivj| mij‡iLvØq ci¯úi‡Q`x nq, hw` Dfq‡iLvq Aew¯’Z GKwU mvaviY we›` _v‡K| Ab¨_vq mij‡iLv `ywU mgvšivj| j¶Yxq †h, `ywU wfbœ mij‡iLvi mewaK GKwU mvaviY we›` _vK‡Z cv‡i|

GKB mgZ‡j Aew¯’Z `ywU mij‡iLvi mgvšivjZv wb‡gœvY³ wZbfv‡e msÁvwqZ Kiv hvq:

(K) mij‡iLv `ywU KLbI ci¯úi‡K †Q` K‡i bv (`yB w`‡K Amxg chš— ewa©Z Kiv n‡jI)|

(L) GKwU mij‡iLvi cÖwZwU we›` Aci wU ‡_‡K mgvb ¶z`ªZg `ivZ¡ Ae¯’vb K‡i|

(M) mij‡iLv `ywU‡K Aci GKwU mij‡iLv †Q` Ki‡j hw` GKvši †KvY ev Abic †KvY¸‡jv mgvb nq|

msÁv (K) Abmv‡i GKB mgZ‡j Aew¯’Z `ywU mij‡iLv G‡K Aci‡K †Q` bv Ki‡j †m¸‡jv mgvšivj| `ywU mgvšivj mij‡iLv †_‡K †h‡Kv‡bv `ywU †iLvsk wb‡j, †iLvsk `ywUI ci¯úi mgvšivj nq|

msÁv (L) Abmv‡i `ywU mgvšivj mij‡iLvi GKwUi †h‡Kv‡bv we›` †_‡K AciwUi j¤^-`iZ¡ me©`v mgvb| j¤^-`iZ¡ ej‡Z Zv‡`i GKwUi †h‡Kv‡bv we›` n‡Z AciwUi Dci Aw¼Z j‡¤^i ˆ`N©¨‡KB †evSvq| Avevi wecixZfv‡e, `ywU mij‡iLvi GKwUi †h‡Kv‡bv `ywU we›` †_‡K AciwUi j¤^-`iZ¡ ci¯úi mgvb n‡jI †iLvØq mgvšivj| GB j¤^-`iZ¡‡K `ywU mgvšivj †iLvØ‡qi `iZ¡ ejv nq|

msÁv (M) BDwK¬‡Wi cÂg ¯xKv‡hi mgZj¨| R¨wgwZK cgvY I A¼‡bi Rb¨ G msÁvwU AwaKZi Dc‡hvMx|

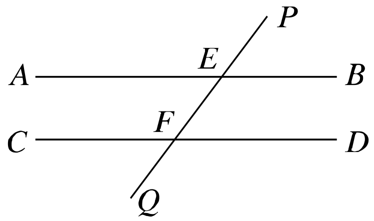
j¶ Kwi, †Kv‡bv wbw`ó mij‡iLvi Dci Aew¯’Z bq Gic we›`i ga¨ w`‡q H mij‡iLvi mgvšivj K‡i GKwU gvÎ mij‡iLv AvKv hvq|

Dccv`¨ 3

`BwU mgvšÍivj mij‡iLvi GKwU †Q`K Øviv Drcb

(K) c‡Z¨K GKvšÍi †KvY †Rvov mgvb n‡e|

(L) †Q`‡Ki GKB cv‡ki Ašt¯ †KvY `BwU ci¯úi m¤úiK|



wP‡Î, $AB \parallel CD$ Ges PQ †Q`K Zv‡`i h_vµ‡g E I F we›`‡Z †Q` K‡i‡Q|

mZivs, (K) $\angle PEB$ = Abic $\angle EFD$ [msÁvbmv‡i]

(L) $\angle AEF$ = GKvšÍi $\angle EFD$

(M) $\angle BEF + \angle EFD$ = `B mg‡KvY|

> KvR :
> 1| mgvšÍivj mij‡iLvi weKí msÁvi mvnv‡h¨ mgvšÍivj mij‡iLv msµvšÍ Dccv`¨¸‡jv cgvY Ki|

Dccv`¨ 4

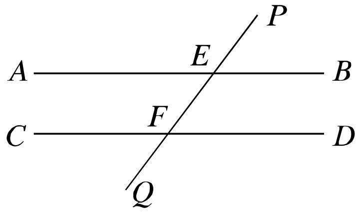
`BwU mij‡iLv Aci GKwU mij‡iLv‡K †Q` Ki‡j hw`

(K) Abic †KvY¸‡jv ci¯úi mgvb nq, A_ev

(L) GKvšÍi †KvY¸‡jv ci¯úi mgvb nq, A_ev

(M) †Q`‡Ki GKB cv‡ki Ašt¯ †KvYØ‡qi †hvMdj `B mg‡Kv‡Yi mgvb nq,

Z‡e H mij‡iLv `BwU ci¯úi mgvšÍivj|



wP‡Î, AB I CD †iLvØq‡K PQ †iLv h_vµ‡g E I F we›`‡Z †Q` K‡i‡Q Ges

(K) $\angle PEB$ = Abic $\angle EFD$

A_ev, (L) $\angle AEF$ = GKvšÍi $\angle EFD$

A_ev, (M) $\angle BEF + \angle EFD$ = `B mg‡KvY|

mZivs, AB I CD †iLv `BwU ci¯úi mgvšÍivj|

Abywm×všÍ 1| †hme mij‡iLv GKB mij‡iLvi mgvšÍivj †m¸‡jv ci¯úi mgvšÍivj|

Abykxjbx 6·2

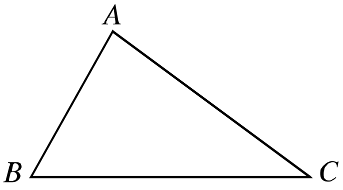
1| †Kv‡Yi Af¨šÍi I ewnfv‡Mi msÁv `vI|

2| hw` GKB mij‡iLv¯ wZbwU wfb we›` nq, Z‡e wP‡Îi Drcb †KvY¸‡jvi bvgKiY Ki|

3| mwbwnZ †Kv‡Yi msÁv `vI Ges Gi evû¸‡jv wPwýZ Ki|

4| wPÎmn msÁv `vI: wecZxc †KvY, ciK †KvY, m¤úiK †KvY, mg‡KvY, m²‡KvY Ges ¯j‡KvY|

wÎfR

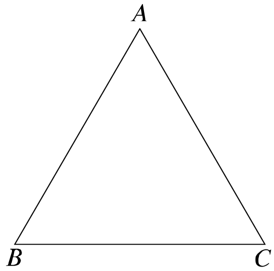


wZbwU †iLvsk Øviv Ave× wPÎ GKwU wÎfR| †iLvsk¸‡jv‡K wÎf‡Ri evû e‡j| †h‡Kv‡bv `BwU evûi mvaviY we›`‡K kxlwe›` ejv nq| wÎf‡Ri †h‡Kv‡bv `BwU evû kxlwe›`‡Z †KvY Drcb K‡i| wÎf‡Ri wZbwU evû I wZbwU †KvY i‡q‡Q| evû‡f‡` wÎfR wZb cÖKvi : mgevû, mgwØevû I welgevû| Avevi †KvY‡f‡` I wÎfR wZb cÖKvi : m²‡KvYx, ¯’j‡KvYx I mg‡KvYx|

wÎf‡Ri evû wZbwUi ˆ`‡N©¨i mgwó‡K cwimxgv e‡j| wÎf‡Ri evû¸‡jv Øviv mxgve×‡¶Î‡K wÎfR‡¶Î e‡j| wÎf‡Ri †h‡Kv‡bv kxlwe›` n‡Z wecixZ evûi ga¨we›` chš— A¼xZ †iLvsk‡K ga¨gv e‡j| Avevi, †h‡Kv‡bv kxlwe›` n‡Z wecixZ evû Gi j¤ `iZB wÎf‡Ri D”PZv|

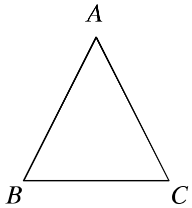
cv‡ki wP‡Î ABC GKwU wÎfR| A,B,C Gi wZbwU kxlwe›`| AB, BC, CA Gi wZbwU evû Ges Gi wZbwU †KvY $\angle BAC, \angle ABC, \angle BCA$ AB, BC, CA evûi cwigv‡ci †hvMdj wÎfRwUi cwimxgv|

mgevû wÎfR



†h wÎf‡Ri wZbwU evû mgvb Zv mgevû wÎfR| cv‡ki wP‡Î ABC wÎf‡Ri $AB=BC=CA$| A_©vr evû wZbwUi ˆ`N©¨ mgvb| ABC wÎfRwU GKwU mgevû wÎfR|

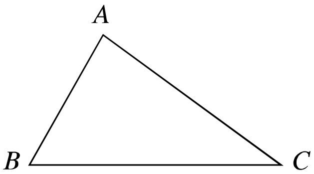
mgwØevû wÎfR



†h wÎf‡Ri `BwU evû mgvb Zv mgwØevû wÎfR|

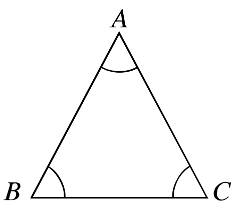
cv‡ki wP‡Î ABC wÎf‡Ri $AB = AC \neq BC$| A_©vr `BwU evûi ˆ`N©¨ mgvb, hv‡`i †Kv‡bvwUB ZZxq evûi mgvb bq| ABC wÎfRwU mgwØevû|

wel gevû wÎfR



†h wÎf‡Ri wZbwU evûB ci¯úi Amgvb Zv wel gevû wÎfR| cv‡ki wP‡Î ABC wÎf‡Ri AB, BC, CA evû¸‡jvi ˆ`N©¨ ci¯úi Amgvb| ABC wÎfRwU wel gevû|

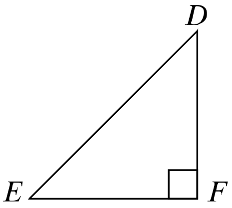
m²‡KvYx wÎfR



†h wÎf‡Ri c‡Z¨KwU †KvY m²‡KvY, Zv m²‡KvYx wÎfR| ABC wÎf‡R $\angle BAC, \angle ABC, \angle BCA$ †KvY wZbwUi c‡Z¨‡K m²‡KvY| A_©vr c‡Z¨KwU †Kv‡Yi cwigvY 90^0 A‡c¶v Kg| ΔABC GKwU m²‡KvYx wÎfR|

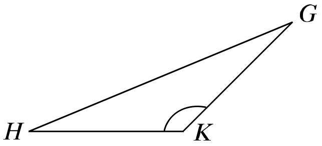
mg‡KvYx wÎfR

†h wÎf‡Ri GKwU †KvY mg‡KvY, Zv mg‡KvYx wÎfR| DEF wÎf‡R $\angle DFE$ mg‡KvY, Aci †KvY `BwU $\angle DEF$ I $\angle EDF$ c‡Z¨‡K m²‡KvY| ΔDEF GKwU mg‡KvYx wÎfR|



¯’j‡KvYx wÎfR

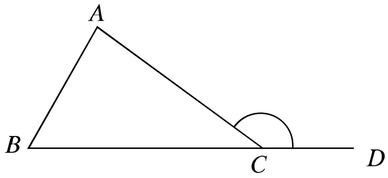
†h wÎf‡Ri GKwU †KvY ¯’j‡KvY, Zv ¯’j‡KvYx wÎfR| GHK wÎf‡R $\angle GKH$ GKwU ¯’j‡KvY, Aci †KvY `BwU $\angle GHK$ I $\angle HGK$ c‡Z¨‡K m²‡KvY| ΔGHK GKwU ¯’j‡KvYx wÎfR|



9·3 wÎf‡Ri ewnt¯’ I Ašt¯’ †KvY

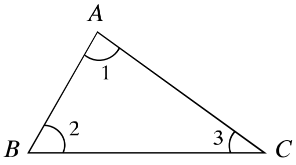
†Kv‡bv wÎf‡Ri GKwU evû ewaZ Ki‡j †h †KvY Drcb nq Zv wÎfRwUi GKwU ewnt¯’ †KvY| GB †Kv‡Yi mwbwnZ †KvYwU Qvov wÎf‡Ri Aci `BwU †KvY‡K GB ewnt¯’ †Kv‡Yi wecixZ Ašt¯’ †KvY e‡j|

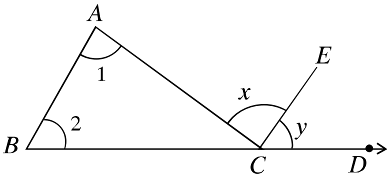
cv‡ki wP‡Î, ΔABC Gi BC evû‡K D chšÍ ewaZ Kiv n‡q‡Q| $\angle ACD$ wÎfRwUi GKwU ewnt¯’ †KvY| $\angle ABD$, $\angle BAC$ I $\angle ACB$ wÎfRwUi wZbwU Ašt¯’ †KvY| $\angle ACB$ †K $\angle ACD$ Gi †cwÖ¶‡Z mwbwnZ Ašt¯’ †KvY ejv nq| $\angle ABC$ I $\angle BAC$ Gi c‡Z¨K‡K $\angle ACD$ Gi wecixZ Ašt¯’ †KvY ejv nq|



Dccv`¨ 5

wÎf‡Ri wZb †Kv‡Yi mgwó `B mg‡Kv‡Yi mgvb|





g‡b Kwi, ABC GKwU wÎfR| wÎfRwUi $\angle BAC + \angle ABC + \angle ACB =$ `B mg‡KvY|

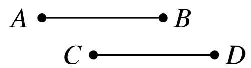
Abywm×všÍ 1| wÎf‡Ri GKwU evû‡K ewaZ Ki‡j †h ewnt¯’ †KvY Drcb nq, Zv Gi wecixZ Ašt¯’ †KvYØ‡qi mgwói mgvb|

Abywm×všÍ 2| wÎf‡Ri GKwU evû‡K ewaZ Ki‡j †h ewnt¯’ †KvY Drcb nq, Zv Gi Ašt¯’ wecixZ †KvY `BwUi c‡Z¨KwU A‡c¶v enËi|

Abywm×všÍ 3| mg‡KvYx wÎf‡Ri m²‡KvYØq ci¯úi ciK|

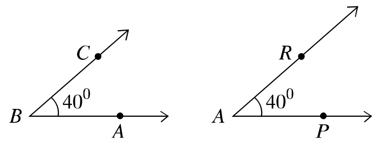
KvR : 1| cgvY Ki †h, wÎf‡Ri GKwU evû‡K ewaZ Ki‡j †h ewnt¯’ †KvY Drcb nq, Zv Gi Ašt¯’ wecixZ †KvY `BwUi c‡Z¨KwU A‡c¶v enËi|

evû I †Kv‡Yi memgZv :



`BwU †iLvs‡ki ˆ`N©¨ mgvb n‡j †iLvsk `BwU memg| Avevi wecixZfv‡e, `BwU †iLvsk memg n‡j Zv‡`i ˆ`N©¨ mgvb|

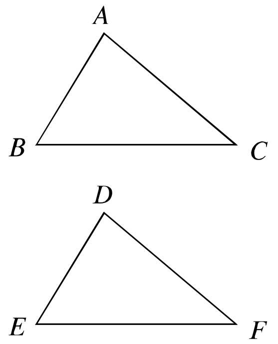
`BwU †Kv‡Yi cwigvc mgvb n‡j †KvY `BwU memg| Avevi wecixZfv‡e, `BwU †KvY memg n‡j Zv‡`i cwigvcI mgvb|



(i) (ii)

wÎf‡Ri memgZv

GKwU wÎfR‡K Aci GKwU wÎf‡Ri Dci ¯’vcb Ki‡j hw` wÎfR `BwU me‡Zvfv‡e wg‡j hvq, Z‡e wÎfR `BwU memg nq| memg wÎf‡Ri Abic evû I Abic †KvY¸‡jv mgvb|

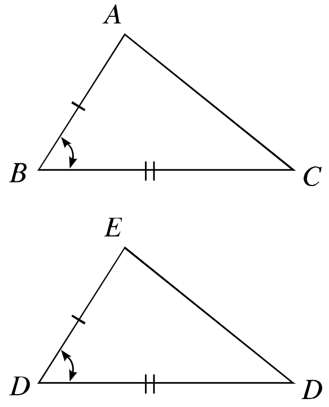


cv‡ki wP‡Î ΔABC I ΔDEF memg| ΔABC I ΔDEF memg n‡j Ges A, B, C kxl h_vµ‡g D, E, F kx‡li Dci cwZZ n‡j $AB = DE$, $AC = DF$, $BC = EF$ Ges $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ n‡e| ΔABC I ΔDEF memg †evSv‡Z $\Delta ABC \cong \Delta DEF$ †jLv nq|

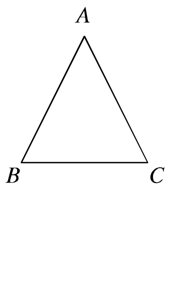
Dccv`¨ 6 ( evû-†KvY-evû Dccv`¨)

hw` `BwU wÎf‡Ri GKwUi `B evû h_vµ‡g AciwUi `B evûi mgvb nq Ges evû `BwUi Ašf©³ †KvY `BwU ci¯úi mgvb nq, Z‡e wÎfR `BwU memg|

g‡b Kwi, ΔABC I ΔDEF G $AB = DE$, $AC = DF$ Ges Ašf©³ $\angle BAC$ = Ašf©³ $\angle EDF$. Zvn‡j, $\Delta ABC \cong \Delta DEF$.



Dccv`¨ 7



hw` †Kv‡bv wÎf‡Ri `BwU evû ci¯úi mgvb nq, Z‡e G‡`i wecixZ †KvY `BwUI ci¯úi mgvb n‡e|

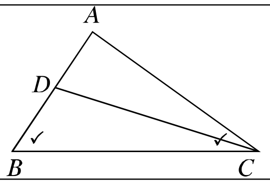
g‡b Kwi, ABC wÎf‡R $AB = AC$ | Zvn‡j, $\angle ABC = \angle ACB$ |

Dccv`¨ 8

hw` †Kv‡bv wÎf‡Ri `BwU †KvY ci¯úi mgvb nq, Z‡e G‡`i wecixZ evû `BwUI ci¯úi mgvb n‡e|

we‡kl wbePb: g‡b Kwi, ABC wÎf‡R $\angle ABC = \angle ACB$ | cgvY Ki‡Z n‡e †h, $AB = AC$ |

cgvY:

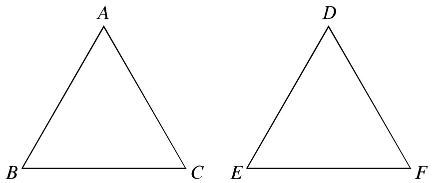


avc	h_v_Zv
(1) hw` $AB = AC$ Ges G‡`i †KvbwUB AB Gi mgvb bv nq, Z‡e (i) $AB > AC$ A_ev (ii) $AB < AC$ n‡e|	[mgwØevû wÎf‡Ri f~wg msjMœ †KvYØq mgvb]
g‡b Kwi, (i) $AB > AC$. AB †_‡K AC Gi mgvb AD †K‡U wbB| GLb, ADC wÎfRwU mgwØevû| mZivs $\angle ADC = \angle ACD$ ΔDBC Gi ewnt¯’ †KvY $\angle ADC > \angle ABC$	[ewnt¯’ †KvY Aš—¯’ wecixZ †KvY `BwU c‡Z¨KwU A‡c¶v enËi]
$\therefore \angle ACD > \angle ABC$ mZivs, $\angle ACB > \angle ABC$ wKš‘ Zv c`Ë kZ©we‡ivax|	
(2) Abiƒcfv‡e, (ii) $AB < AC$ n‡j †`Lv‡bv hvq †h $\angle ABC > \angle ACB$. wKš‘ ZvI c`Ë kZ©we‡ivax|	
(3) mZivs, $AB > AC$ A_ev $AB < AC$ n‡Z cv‡i bv| $\therefore AB = AC$ (cÖgvwYZ)	

Dccv`¨ 9 (evû-evû-evû Dccv`¨)

hw` GKwU wÎf‡Ri wZb evû Aci GKwU wÎf‡Ri wZb evûi mgvb nq, Z‡e wÎfR `BwU memg n‡e|

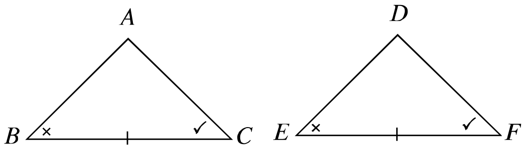
g‡b Kwi, ΔABC Ges ΔDEF G $AB = DE$, $AC = DF$ Ges $BC = EF$. Zvn‡j, $\Delta ABC \cong \Delta DEF$.



Dccv`¨ 10 ( †KvY-evû-†KvY Dccv`¨)

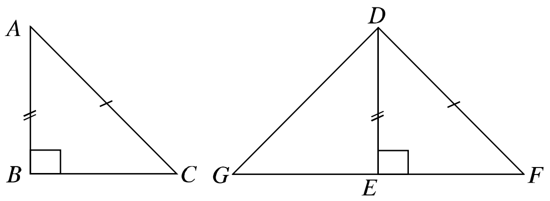
hw` GKwU wÎf‡Ri `BwU †KvY I Zv‡`i msjMœ evû h_vµ‡g Aci GKwU wÎf‡Ri `BwU †KvY I Zv‡`i msjMœ evûi mgvb nq, Z‡e wÎfR `BwU memg n‡e|

g‡b Kwi, ΔABC I ΔDEF -G $\angle B = \angle E$, $\angle C = \angle F$ Ges †KvYØ‡qi msjM BC evû = Abic EF evû| Z‡e wÎfR `BwU memg, A_vr $\Delta ABC \cong \Delta DEF$.



Dccv`¨ 11 (AwZfR-evû Dccv`¨)

`BwU mg‡KvYx wÎf‡Ri AwZfRØq mgvb n‡j Ges GKwUi GK evû AciwUi Aci GK evûi mgvb n‡j, wÎfRØq memg|



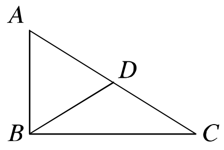
ABC I DEF mg‡KvYx wÎfRØ‡q AwZfR AC =AwZfR DF Ges $AB = DE$ .Zvn‡j, $\Delta ABC \cong \Delta DEF$.

wÎf‡Ri evû I †Kv‡Yi g‡a¨ m¤úK i‡q‡Q| G m¤úK wb‡Pi Dccv`¨ 11 I Dccv`¨ 12 Gi cwZcv`¨ welq|

Dccv`¨ 12

†Kv‡bv wÎf‡Ri GKwU evû Aci GKwU evû A‡c¶v enËi n‡j, enËi evûi wecixZ †KvY ¶`Zi evûi wecixZ †KvY A‡c¶v enËi|

g‡b Kwi, ΔABC -G $AC > AB$. mZivs $\angle ABC > \angle ACB$.



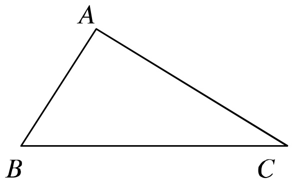
Dccv`¨ 13

†Kv‡bv wÎf‡Ri GKwU †KvY Aci GKwU †KvY A‡c¶v enËi n‡j, enËi †Kv‡Yi wecixZ evû ¶`Zi †Kv‡Yi wecixZ evû A‡c¶v enËi|

we‡kl wbePb: g‡b Kwi, ΔABC Gi $\angle ABC > \angle ACB$

cgvY Ki‡Z n‡e †h, $AC > AB$

cgvY:



avc	h_v_Zv
(1) hw` AC evû AB evû A‡c¶v enËi bv nq,	[mgwØevû wÎf‡Ri mgvb evûØ‡qi wecixZ †KvYØq mgvb]

Z‡e (i) $AC = AB$ A_ev (ii) $AC < AB$ n‡e|

(i) hw` $AC = AB$ nq, $\angle ABC = \angle ACB$

wKš kZvbhvqx $\angle ABC > \angle ACB$

Zv c`Ë kZwe‡ivax|

(ii) Avevi, hw` $AC < AB$ nq, Z‡e $\angle ABC < \angle ACB$ n‡e|

wKš ZvI c`Ë kZwe‡ivax|

(2) mZivs, AC evû AB Gi mgvb ev AB †_‡K ¶`Zi n‡Z cv‡i bv| $\therefore AC > AB$ (cgwYZ)|

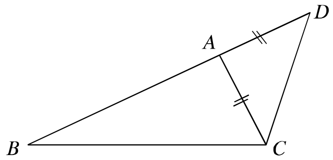
[¶`Zi evûi wecixZ †KvY ¶`Zi]

wÎf‡Ri †h‡Kv‡bv `B evûi ˆ`‡N¨i mgwói ev Aš‡ii mv‡_ ZZxq evûi ˆ`‡N¨i m¤úK i‡q‡Q|

Dccv`¨ 14

wÎf‡Ri †h‡Kv‡bv `B evûi ˆ`‡N¨i mgwó Gi ZZxq evûi ˆ`N¨ A‡c¶v enËi|

g‡b Kwi, ABC GKwU wÎfR| awi, BC wÎfRwUi enËg evû| Zvn‡j, $AB + AC > BC$ |

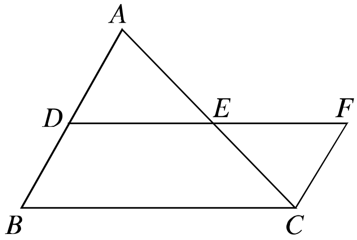


Abwm×vš 1| wÎf‡Ri †h‡Kv‡bv `B evûi ˆ`‡N¨i Aši Gi ZZxq evûi ˆ`N¨ A‡c¶v ¶`Zi|
g‡b Kwi, ABC GKwU wÎfR| ΔABC Gi †h‡Kv‡bv `B evûi ˆ`‡N¨i Aši Gi ZZxq evûi ˆ`N¨ A‡c¶v ¶`Zi| ‡hgb, $AB - AC < BC$ |

Dccv`¨ 15

wÎf‡Ri †h‡Kv‡bv `B evûi ga¨we›`i ms‡hvRK †iLvsk ZZxq evûi mgvšivj Ges ˆ`‡N¨ Zvi A‡aK|

g‡b Kwi, ABC GKwU wÎfR| D I E h_vµ‡g wÎfRwUi AB I AC evûi ga¨we›`| Zvn‡j, cgvY Ki‡Z n‡e †h $DE \parallel BC$ Ges $DE = \frac{1}{2}BC$.



A¼b: D I E †hvM K‡i ewaZ Kwi †hb $EF = DE$ nq|

cgvY:

avc	h_v_Zv
(1) ΔADE I ΔCEF Gi g‡a¨ $AE = EC$,	[†` Iqv Av‡Q]
$DE = EF$	[A¼bvbmv‡i]
$\angle AED = \angle CEF$	[wecZxc †KvY]

$\Delta ADE \cong \Delta CEF$ [evû-‡KvY-evû Dccv`¨]

$\therefore \angle ADE = \angle EFC$ Ges $\angle DAE = \angle ECF$. [GKvšÍi †KvY]

$\therefore$ $DF \parallel BC$ ev $DE \parallel BC$.

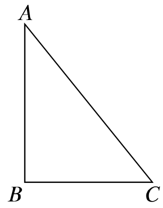
(2) Avevi, $DF = BC$ ev $DE + EF = BC$

ev $DE + DE = BC$ ev $2DE = BC$ ev $DE = \frac{1}{2} BC$

Dccv`¨ 16 (wc_v‡Mvi‡mi Dccv`¨)

mg‡KvYx wÎf‡Ri AwZf‡Ri Ici AwZ eM‡¶‡Îi †¶Îdj Aci `B evûi Ici AwZ eM‡¶‡Î©‡qi †¶Îd‡ji mgwói mgvb|

g‡b Kwi, ABC mg‡KvYx wÎf‡Ri $\angle ABC$ mg‡KvY Ges AC AwZfR| Zvn‡j, $AC^2 = AB^2 + BC^2$.



Abkxjbx 6·3

1| wb‡P wZbwU evûi ˆ`N¨ †`Iqv n‡jv| †Kvb †¶‡Î wÎfR A¼b m¤¢e?

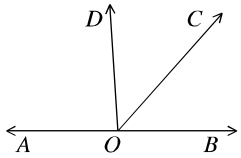
K. 5 †m. wg., 6 †m. wg. I 7 †m. wg. L. 3 †m. wg., 4 †m. wg. I 7 †m. wg.
M. 5 †m. wg., 7 †m. wg. I 14 †m. wg. N. 2 †m. wg., 4 †m. wg. I 8 †m. wg.

2| wb‡Pi Z_¨¸‡jv j¶ Ki :

i †h wÎf‡Ri wZbwU †KvY mg‡KvY Zv‡K mg‡KvYx wÎfR e‡j
ii †h wÎf‡Ri wZbwU †KvY m²‡KvY Zv‡K m²‡KvYx wÎfR e‡j|
iii †h wÎf‡Ri wZbwU evû mgvb Zv‡K mgevû wÎfR e‡j
wb‡Pi †KvbwU mwVK ?

K. i I ii L. i I iii
M. ii I iii N. i, ii I iii

3| c`Ë wPÎ Abhvqx 3 I 4 bs c‡ki DËi `vI|



GKmg‡Kv‡Yi mgvb †KvY †KvYwU?

K. $\angle$BOC L. $\angle$BOD
M. $\angle$COD N. $\angle$AOD

4| $\angle$BOC Gi ciK †Kvb †KvYwU?

K. $\angle$AOC L. $\angle$BOD
M. $\angle$COD N. $\angle$AOD

5| cgvY Ki †h, mgevû wÎf‡Ri evû¸‡jvi ga¨we›`mgn †hvM Ki‡j †h wÎfR Drcb nq, Zv mgevû n‡e|

6| cgvY Ki †h, mgevû wÎf‡Ri ga¨gv wZbwU ci¯úi mgvb|

7| cgvY Ki †h, wÎf‡Ri †h‡Kv‡bv `BwU evnt¯ †Kv‡Yi mgwó `B mg‡KvY A‡c¶v enËi|

8| ΔABC Gi Af¨š‡i D GKwU we›`| cgvY Ki †h, $AB+AC>BD+DC.$

9| ΔABC Gi BC evûi ga¨we›` D n‡j, cgvY Ki †h, $AB+AC>2AD.$

10| cgvY Ki †h, wÎf‡Ri ga¨gvÎ‡qi mgwó Zvi cwimxgv A‡c¶v ¶`Zi|

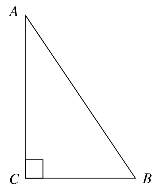
11| ABC mgwØevû wÎf‡R, BA evû‡K D chš Gi/cfv‡e ewaZ Kiv nj, †hb $BA=AD$ nq| cgvY Ki †h, $\angle BCD$ GKwU mg‡KvY|

12| ΔABC Gi $\angle B$ I $\angle C$ Gi mgwØLÊKØq O we›`‡Z wgwjZ nq|

cgvY Ki †h, $\angle BOC=90^\circ+\frac{1}{2}\angle A.$

13| ΔABC Gi AB I AC evû‡K ewaZ Ki‡j B I C we›`‡Z †h evnt‡KvY `BwU Drcb nq, Zv‡`i mgwØLÊK `BwU O we›`‡Z wgwjZ n‡j,

cgvY Ki †h, $\angle BOC=90^\circ-\frac{1}{2}\angle A.$



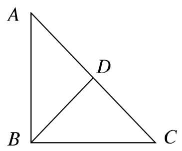
14| wP‡Î, †`Iqv Av‡Q, $\angle C$ = GK mg‡KvY

Ges $\angle B=2\angle A$

cgvY Ki †h, $AB=2BC$.

15| cgvY Ki †h, wÎf‡Ri GKwU evû ewaZ Ki‡j †h evnt¯ †KvY Drcb nq, Zv wecixZ Ašt¯ †KvYØ‡qi mgwói mgvb|

16| cgvY Ki †h, wÎf‡Ri †h‡Kv‡bv `B evûi Aši Zvi ZZxq evû A‡c¶v ¶`Zi|



17| wP‡Î, ABC wÎf‡Ri $\angle B=$ GK mg‡KvY

Ges D, AwZfR AC Gi ga¨we›`|

cgvY Ki †h, $BD=\frac{1}{2}AC.$

18| ΔABC G $AB>AC$ Ges $\angle A$ Gi mgwØLÊK AD, BC evû‡K D we›`‡Z †Q` K‡i|

cgvY Ki †h, $\angle ADB$ ¯'j‡KvY|

19| cgvY Ki †h, †Kv‡bv †iLvs‡ki j¤wØLÊ‡Ki Dcwi¯'Z †h‡Kv‡bv we›` D³ †iLvs‡ki cvš we›`Øq n‡Z mg`ieZx|

20. ABC GKwU mg‡KvYx wÎfR hvi $\angle$A= GK mg‡KvY| BC evûi ga¨we›` D.

K. c`Ë Z_¨ Abhvqx ABC wÎfRwU A¼b Ki|

L. †`LvI †h, AB+AC> 2AD

M. cgvY Ki †h, AD=$\frac{1}{2}$BC

mßg Aa¨vq

e¨enwiK R¨wgwZ

c‡ei †kwY‡Z R¨wgwZi wewfbœ Dccv`¨ cgv‡Y I Abkxjbx‡Z wPÎ A¼‡bi c‡qvRb wQj| †m me wPÎ m²fv‡e A¼‡bi c‡qvRb wQj bv| wKš KL‡bv KL‡bv R¨wgwZK wPÎ m²fv‡e A¼‡bi c‡qvRb nq| †hgb, GKRb ¯cwZ hLb †Kv‡bv ewoi bKmv K‡ib wKsev c‡KŠkjx hLb h‡ši wewfbœ As‡ki wPÎ Av‡Kb| G ai‡bi R¨wgwZK A¼‡b ïa †¯‹j I †cwÝj K¤úv‡mi mvnvh¨ †bIqv nq| B‡Zvc‡e †¯‹j I †cwÝj K¤úv‡mi mvnv‡h¨ wÎfR I PZfR AvK‡Z wk‡LwQ| G Aa¨v‡q we‡kl ai‡bi wÎfR I PZfR A¼‡bi Av‡jvPbv Kiv n‡e|

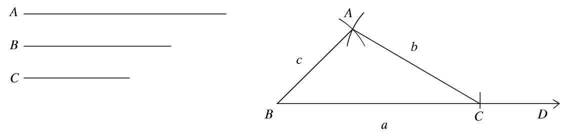
Aa¨vq †k‡l wk¶v_xiv

- wP‡Îi mvnv‡h¨ wÎfR I PZfR e¨vL¨v Ki‡Z cvi‡e|
- c`Ë DcvË e¨envi K‡i wÎfR A¼b Ki‡Z cvi‡e|
- c`Ë DcvË e¨envi K‡i mvgvšwiK A¼b Ki‡Z cvi‡e|

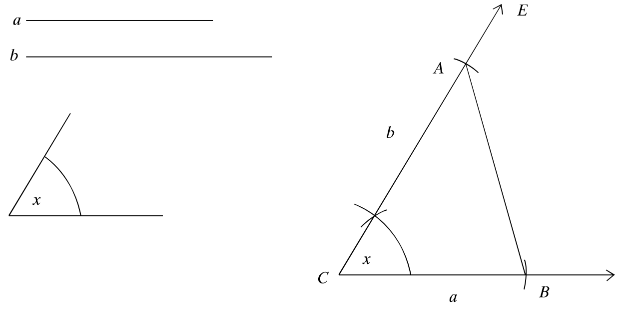
7·1 wÎfR A¼b

c‡Z¨K wÎf‡Ri wZbwU evû I wZbwU †KvY i‡q‡Q| Z‡e †Kv‡bv wÎf‡Ri AvKvi I AvKwZ wbw`ó Kivi Rb¨ me¸‡jv evû I †Kv‡Yi c‡qvRb nq bv| †hgb, wÎf‡Ri wZb †Kv‡Yi mgwó `B mg‡KvY e‡j Gi †h‡Kv‡bv `BwU †Kv‡Yi gvb †`Iqv _vK‡j ZZxq †KvYwUi gvb †ei Kiv hvq| Avevi, wÎf‡Ri memgZv msµvš Dccv`¨¸‡jv †_‡K †`Lv hvq †h, †Kv‡bv wÎf‡Ri wZbwU evû I wZbwU †KvY A_vr QqwUi g‡a¨ †Kejgvθ wbgwjwLZ wZbwU Aci GK wÎf‡Ri Abiƒc wZbwUi As‡ki mgvb n‡jB wÎfR `BwU memg nq| A_vr, G wZbwU As‡ki Øviv wbw`ó AvKv‡ii Abb¨ wÎfR AvKv hvq| mßg †kwY‡Z Avgiv wbgewYZ DcvË †_‡K wÎfR AvK‡Z wk‡LwQ|

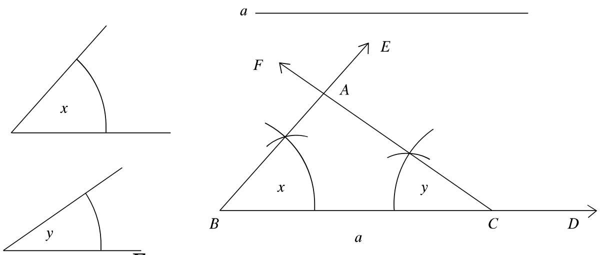
(1) wZbwU evû



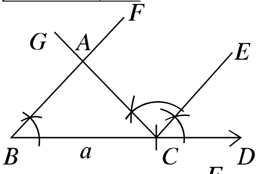
(2) `BwU evû I Zv‡`i Ašf³ †KvY



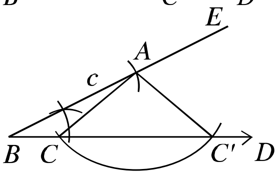
(3) `BwU †KvY I Zv‡`i msjM evû



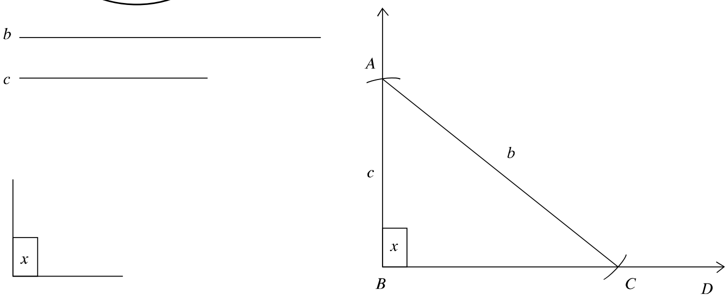
(4) `BwU †KvY I GKwUi wecixZ evû



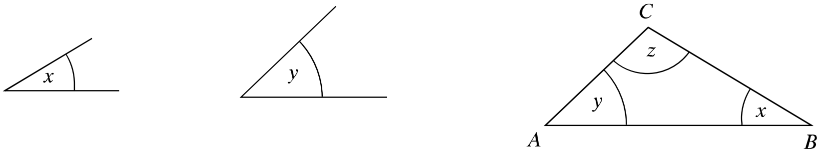
(5) `BwU evû I Zv‡`i GKwUi wecixZ †KvY

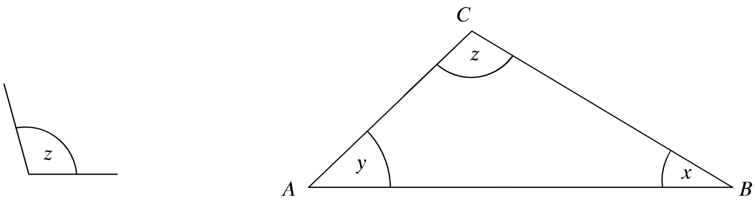


(6) mg‡KvYx wÎf‡Ri AwZfR I Aci GKwU evû



j¶Yxq †h, Dc‡ii c‡Z¨K †¶‡Î wÎf‡Ri wZbwU Ask wbw`ó Kiv n‡q‡Q| wKš †h‡Kv‡bv wZbwU Ask wbw`ó Ki‡jB wÎfRwU wbw`ó nq bv| †hgb, wÎf‡Ri wZbwU †KvY †`Iqv _vK‡j wewfb AvKv‡ii AmsL¨ wÎfR AvKv hvq (hv‡`i m`k wÎfR ejv hvq)|



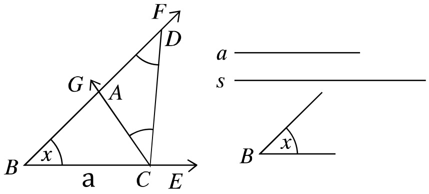


A‡bK mgq wÎfR AvKvi Rb¨ Ggb wZbwU DcvË †`Iqv _v‡K, hv‡`i mvnv‡h¨ wewfb A¼‡bi gva¨‡g wÎfRwU wbaviY Kiv hvq| Gi¦c K‡qKwU m¤úv`¨ wb‡P eYbv Kiv n‡jv|

m¤úv`¨ 1

wÎf‡Ri fwg, fwg msjMœ GKwU †KvY I Aci `B evûi mgwó †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|

g‡b Kwi, †Kv‡bv wÎf‡Ri fwg a, fwg msjMœ GKwU †KvY $\angle x$ Ges Aci `B evûi mgwó $s$ †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|



A¼b :

(1) †h‡Kv‡bv GKwU iwk¥ BE †_‡K fwg a Gi mgvb K‡i BC †iLvsk †K‡U wbB| BC †iLvs‡ki B we›`‡Z $\angle x$ Gi mgvb $\angle CBF$ AwK|

(2) BF iwk¥ †_‡K s Gi mgvb BD Ask KwU|

(3) C, D †hvM Kwi| C we›`‡Z $DC$ †iLvs‡ki †h cv‡k $B$ we›` Av‡Q †mB cv‡k $\angle BDC$ Gi mgvb $\angle DCG$ AwK|

(4) CG iwk¥ BD †K A we›`‡Z †Q` K‡i|

Zvn‡j, ΔABC B DwÏó wÎfR|

cgvY : ΔACD G $\angle ADC = \angle ACD$ [A¼b Abmv‡i]

$\therefore \quad AC = AD.$

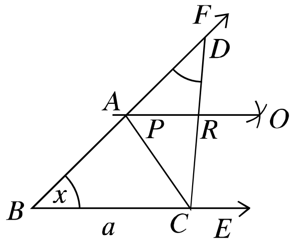
GLb, ΔABC G $\angle ABC = \angle x,\ BC = a,$ [A¼b Abmv‡i]

Ges $BA + AC = BA + AD = BD = s$ | AZGe, ΔABC B wb‡Yq wÎfR|

weKí c×wZ

g‡b Kwi, †Kv‡bv wÎf‡Ri fwg a, fwg msjMœ GKwU †KvY $\angle x$ Ges Aci `B evûi mgwó $s$ †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|

A¼b :



(1) †h‡Kv‡bv GKwU iwk¥ BE †_‡K fwg a Gi mgvb K‡i BC †iLvsk †K‡U wbB| BC †iLvs‡ki B we›`‡Z $\angle x$ Gi mgvb $\angle CBF$ AwK|

(2) BF iwk¥ †_‡K s Gi mgvb BD Ask KwU|

(3) C, D †hvM Kwi| CD Gi j¤ wØLÊK PQ AwK|

(4) PQ iwk¥ BD iwk¥‡K A we›`‡Z †Q` K‡i| A, C †hvM Kwi|

Zvn‡j, ΔABC B DwÏó wÎfR|

cgvY : ΔACR Ges ΔADR G $CR = DR$ $AR = AR$ Ges Ašf³ $\angle ARC = \angle ARD$ [mg‡KvY]

$\Delta ACR \cong \Delta ADR$. $\therefore$ $AC = AD$

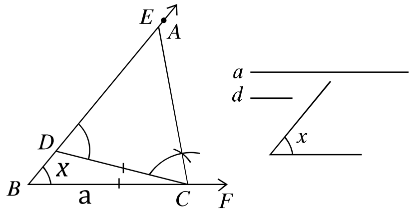
GLb, ΔABC G $\angle ABC = \angle x$, $BC = a$, [A¼b Abmv‡i]

Ges $BA + AC = BA + AD = BD = s$. AZGe, ΔABC B wb‡Yq wÎfR|

m¤úv`¨ 2

wÎf‡Ri fwg, fwg msjMœ GKwU m²‡KvY I Aci `B evûi Aši †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|

g‡b Kwi, †Kv‡bv wÎf‡Ri fwg a fwg msjM m²‡KvY $\angle x$. Ges Aci `B evûi Aši $d$ †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|



A¼b :

(1) †h‡Kv‡bv GKwU iwkœ BF †_‡K fwg a Gi mgvb K‡i BC †iLvsk †K‡U wbB| BC †iLvs‡ki B we›`‡Z $\angle x$ Gi mgvb $\angle CBE$ AvwK|

(2) BE iwkœ †_‡K d Gi mgvb BD Ask †K‡U wbB|

(3) C, D †hvM Kwi| DC †iLvs‡ki †h cv‡k E we›` Av‡Q †mB cv‡k $C$ we›`‡Z $\angle EDC$ Gi mgvb $\angle DCA$ AvwK| CA iwkœ BE iwkœ‡K A we›`‡Z †Q` K‡i| Zvn‡j, ΔABC B DwÏó wÎfR|

cgvY : A¼b Abmv‡i, ΔACD G $\angle ADC = \angle ACD$

$\therefore$ $AC = AD$.

mZivs `B evûi Aši, $AB - AC = AB - AD = BD = d$.

GLb, ΔABC G $BC = a, AB - AC = d$ Ges $\angle ABC = \angle x$. mZivs, ΔABC B wb‡Yq wÎfR|

we‡kl `óe¨ :

KvR :

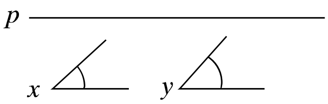
1| c`Ë †KvY m²‡KvY bv n‡j, Dc‡ii c×wZ‡Z A¼b Kiv m¤¢e bq| †Kb ? G †¶‡Î wÎfRwU AvKvi †Kv‡bv Dcvq †ei Ki|

2| wÎf‡Ri fwg, fwg msjMœ GKwU m²‡KvY I Aci `B evûi Aši †`Iqv Av‡Q| wfbœ c×wZ‡Z wÎfRwU A¼b Ki|

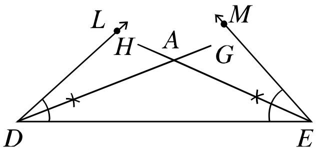
m¤úv`¨ 3

wÎf‡Ri fwg msjM `BwU †KvY I cwimxgv †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|

g‡b Kwi, GKwU wÎf‡Ri cwimxgv p Ges fwg msjM `BwU †KvY $\angle x$ I $\angle y$ †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|

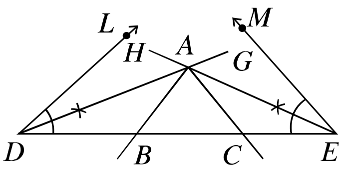


A¼b :



(1) †h‡Kv‡bv GKwU iwk¥ DF †_‡K cwimxgv p Gi mgvb K‡i DE Ask †K‡U wbB| D I E we›`‡Z DE †iLvs‡ki GKB cv‡k $\angle x$ Gi mgvb $\angle EDL$ Ges $\angle y$ Gi mgvb $\angle DEM$ AwwK|

(2) †KvY `BwUi wØLÊK DG I EH AwwK|



(3) g‡b Kwi, DG I EH iwk¥Øq ci¯úi‡K A we›`‡Z †Q` K‡i| A we›`‡Z $\angle ADE$ Gi mgvb $\angle DAB$ Ges $\angle AED$ Gi mgvb $\angle EAC$ AwwK|

(4) AB Ges AC iwk¥Øq DE †iLvsk‡K h_vµ‡g B I C we›`‡Z †Q` K‡i|

Zvn‡j, $\triangle ABC$ B DwÏó wÎfR|

cgvY : $\triangle ADB$ G $\angle ADB = \angle DAB$ [A¼b Abmv‡i], $\therefore \; AB = DB.$

Avevi, $\triangle ACE$ G $\angle AEC = \angle EAC$; $\therefore \; CA = CE.$

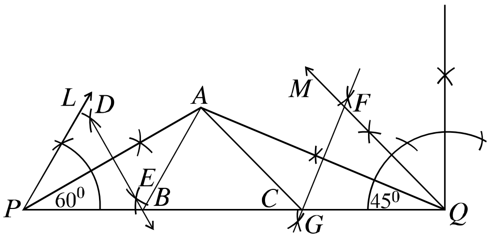
mZivs $\triangle ABC$ G $AB + BC + CA = DB + BC + CE = DE = p.$

$$\angle ABC = \angle ADB + \angle DAB = \frac{1}{2}\angle x + \frac{1}{2}\angle x = \angle x$$

Ges $\angle ACB = \angle AEC + \angle EAC = \frac{1}{2}\angle y + \frac{1}{2}\angle y = \angle y.$ mZivs $\triangle ABC$ B wb‡Yq wÎfR|

KvR : wÎf‡Ri fwg msjM `BwU m²‡KvY I cwimxgv †`Iqv Av‡Q| weKí c×wZ‡Z wÎfRwU A¼b Ki|

D`vniY 1| GKwU wÎfR ABC AvK, hvi $\angle B = 60°, \angle C = 45°$ Ges cwimxgv $AB + BC + CA = 11$ †m.wg.|



A¼b : wb‡Pi avcmgn AbmiY Kwi:

(1) †iLvsk $PQ = 11$ †m.wg. AwK|

(2) PQ †iLvs‡ki GKB cv‡k P Ges Q we›`‡Z h_vµ‡g $\angle QPL = 60^\circ$ I $\angle PQM = 45^\circ$ †KvY AwK|

(3) †KvY `BwUi wØLÊK $PG$ I $QH$ AwwK| g‡b Kwi, $PG$ I $GH$ iwk¥Øq ci¯úi‡K $A$ we›`‡Z †Q` K‡i|

(4) PA, QA †iLvs‡ki j¤ mgwØLÊK AwwK hv PQ †iLvsk‡K h_vµ‡g B I C we›`‡Z †Q` K‡i|

(5) A,B Ges A,C †hvM Kwi|

Zvn‡j, ΔABC B DwÏó wÎfR|

KvR : mg‡KvYx wÎf‡Ri mg‡KvY msjM GKwU evû Ges AwZfR I Aci evûi Aši †`Iqv Av‡Q| wÎfRwU AvK|

Abkxjbx 7·1

1| wb‡g c`Ë DcvË wb‡q wÎfR A¼b Ki :

K. wZbwU evûi ˆ`N¨ h_vµ‡g 3 †m.wg., 3·5 †m.wg., 2·8 †m.wg.|

L. `BwU evûi ˆ`N¨ 4 †m.wg., 3 †m.wg. Ges Ašf©³ †KvY 60°|

M. `BwU †KvY 60° I 45° Ges G‡`i msjM evûi ˆ`N¨ 5 †m.wg.|

N. `BwU †KvY 60° I 45° Ges 45° †Kv‡Yi wecixZ evûi ˆ`N¨ 5 †m.wg.|

O. `BwU evûi ˆ`N¨ h_vµ‡g 4.5 †m.wg. I 3.5 †m.wg. Ges wØZxq evûi wecixZ †KvY 30°|

P. mg‡KvYx wÎf‡Ri AwZfR I GKwU evûi ˆ`N¨ h_vµ‡g 6 †m.wg. I 4 †m.wg.|

2| wb‡g c`Ë DcvË wb‡q wÎfR A¼b Ki :

K. fwg 3·5 †m.wg., fwg msjM GKwU †KvY 60° I Aci `B evûi mgwó 8 †m.wg |

L. fwg 4 †m.wg., fwg msjM GKwU †KvY 50° I Aci `B evûi mgwó 7.5 †m.wg |

M. fwg 4 †m.wg., fwg msjM GKwU †KvY 50° I Aci `B evûi Aši 1.5 †m.wg |

N. fwg 5 †m.wg., fwg msjM GKwU †KvY 45° I Aci `B evûi Aši 1 †m.wg |

O. fwg msjM †KvY `BwU h_vµ‡g 60° I 45° I cwimxgv 12 †m.wg.|

O. fwg msjM †KvY `BwU h_vµ‡g 30° I 45° I cwimxgv 10 †m.wg.|

3| GKwU wÎf‡Ri fwg msjM `BwU †KvY Ges kxl© †_‡K fwgi Dci Aw¼Z j‡¤i ˆ`N¨ †`Iqv Av‡Q| wÎfRwU AvK|

4| mg‡KvYx wÎf‡Ri AwZfR I Aci `B evûi mgwó †`Iqv Av‡Q| wÎfRwU AvK|

5| wÎf‡Ri fwg msjM GKwU †KvY, D”PZv I Aci `B evûi mgwó †`Iqv Av‡Q| wÎfRwU AvK|

6| mgevû wÎf‡Ri cwimxgv †`Iqv Av‡Q| wÎfRwU AvK|

7| wÎf‡Ri fwg, fwg msjM GKwU ¯’j‡KvY I Aci `B evûi Aši †`Iqv Av‡Q| wÎfRwU AvK|

7·2 PZfR A¼b

Avgiv †`‡LwQ †h, wÎf‡Ri wZbwU DcvË †`Iqv _vK‡j A‡bK †¶‡ÎB wÎfRwU wbw`ó fv‡e AvKv m¤¢e| wKš PZf‡Ri PviwU evû †`Iqv _vK‡jB GKwU wbw`ó PZfR AvKv hvq bv| wbw`ó PZfR AvKvi Rb¨ cvPwU ¯Zš DcvË c‡qvRb nq| wb‡g ewYZ cvPwU DcvË Rvbv _vK‡j, wbw`ó PZfR AvKv hvq|

(1) PviwU evû I GKwU †KvY

(2) PviwU evû I GKwU KY

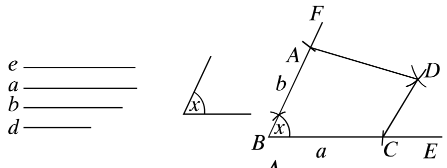
(3) wZbwU evû I `BwU KY

(4) wZbwU evû I Zv‡`i Aš—f³ `BwU †KvY

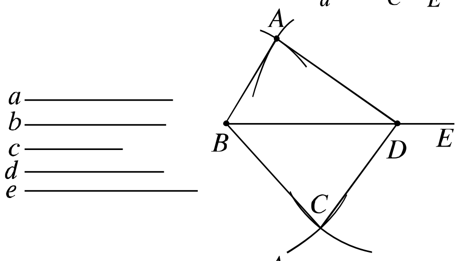
(5) `BwU evû I wZbwU †KvY|

Aóg †kÖwY‡Z D‡jøwLZ DcvË w`‡q PZfR A¼b wel‡q Av‡jvPbv Kiv n‡q‡Q| A¼‡bi †KŠkj j¶ K‡i †`Lv hvq wKQ †¶‡Î mivmwi PZfR AvKv nq| Avevi wKQ †¶‡Î wÎfR A¼‡bi gva¨‡g PZfR AvKv nq| †h‡nZ KY PZfR‡K `BwU wÎf‡R wef³ K‡i, †m‡nZ DcvË wnmv‡e GKwU ev `BwU KY c`Ë n‡j wÎfR A¼‡bi gva¨‡g PZfR AvKv m¤¢e nq|

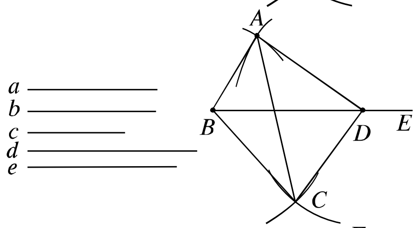
(1) PviwU evû I GKwU †KvY



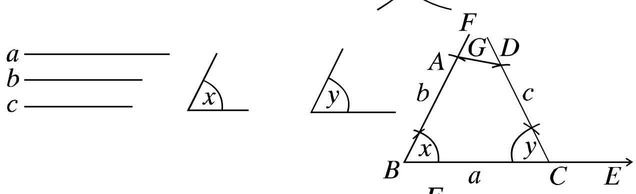
(2) PviwU evû I GKwU KY



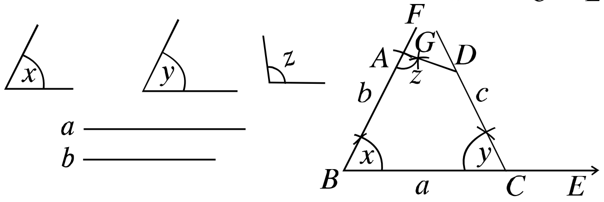
(3) wZbwU evû I `BwU KY



(4) wZbwU evû I Zv‡`i Aš—f³ `BwU †KvY



(5) `BwU evû I wZbwU †KvY|

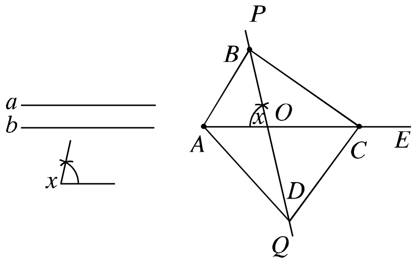


we‡kl ai‡bi PZfR A¼‡bi Rb¨ A‡bK mgq Ggb DcvË †`Iqv _v‡K hv †_‡K wb‡`©ó PZfR AvKvi Rb¨ c‡qvRbxq cvPwU ¯Zš DcvË cvIqv hvq| Zvn‡j H Dcv‡Ëi mvnv‡h¨ PZfRwU AvKv hvq| †hgb, mvgvšwi‡Ki `BwU msjM evû I Zv‡`i Aš—f©³ †KvYwU †`Iqv _vK‡j mvgvšwiKwU AvKv hvq| GLv‡b wZbwU gvÎ DcvË †`Iqv Av‡Q| Avevi e‡Mi gvÎ GKwU evû †`Iqv _vK‡jB eMwU AvKv hvq| KviY, Zv‡Z cvPwU DcvË, h_v e‡Mi Pvi mgvb evû I GK †KvY (mg‡KvY) wb‡`©ó nq|

m¤úv`¨ 4

mvgvšwi‡Ki `BwU KY I Zv‡`i Aš—f©³ GKwU †KvY †`Iqv Av‡Q| mvgvšwiKwU AvK‡Z n‡e|

g‡b Kwi, mvgvšwi‡Ki KY `BwU $a$ I $b$ Ges KYØ‡qi Aš—f©³ GKwU †KvY $\angle x$ †`Iqv Av‡Q| mvgvšwiKwU AvK‡Z n‡e|



A¼b : †h‡Kv‡bv iwk¥ AM †_‡K a Gi mgvb AC †iLvsk wbB| AC Gi ga¨we›` $O$ wbYq Kwi| $O$ we›`‡Z $\angle x$ Gi mgvb $\angle AOP$ AwK| OP Gi wecixZ iwk¥ OQ A¼b Kwi| OP I OQ iwk¥Øq †_‡K $\frac{1}{2}b$ Gi mgvb h_vµ‡g OB I OD †iLvsk¸q wbB| A, B; A, D; C, B I C, D †hvM Kwi|

Zvn‡j, $ABCD$ B DwÏó mvgvšwiK|

cgvY : ΔAOB I ΔCOD G $OA = OC = \frac{1}{2}a,\ OB = OD = \frac{1}{2}b$ [A¼bvbmv‡i]

Ges Aš—f©³ $\angle AOB$ =Aš—f©³ $\angle COD$ [wecZxc †KvY]|

AZGe, $\Delta AOB \cong \Delta COD$

mZivs, $AB = CD$

Ges $\angle ABO = \angle CDO$; wKš †KvY `BwU GKvši †KvY|

$\therefore$ AB I CD mgvb I mgvšivj|

Abi|cfv‡e, AD I BC mgvb I mgvšivj|

mZivs, $ABCD$ GKwU mvgvšwiK hvi KYØq $AC = AO + OC = \frac{1}{2}a + \frac{1}{2}a = a$

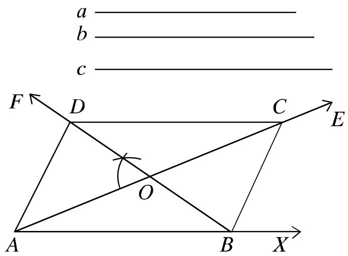
I $BD = BO + OD = \frac{1}{2}b + \frac{1}{2}b = b$ Ges KY `BwUi Aš—f©³ $\angle AOB = \angle x$

AZGe, $ABCD$ B wb‡Yq mvgvšwiK|

m¤úv`¨ 5

mvgvšÍwi‡Ki `yBwU KY I GKwU evû †`Iqv Av‡Q| mvgvšÍwiKwU AvK‡Z n‡e|

g‡b Kwi mvgvšÍwi‡Ki `yBwU KY $a$ I $b$ Ges GKwU evû $c$ †`Iqv Av‡Q| mvgvšÍwiKwU AvK‡Z n‡e|



A¼b : a I b KYØq‡K mgvb `yBfv‡M wef³ Kwi| †h‡Kv‡bv iwk¥ $AX$ †_‡K $c$ Gi mgvb $AB$ wbB| $A$ I $B$ †K †K›`ª K‡i h_vµ‡g $\frac{a}{2}$ I $\frac{b}{2}$ Gi mgvb e¨vmva© wb‡q AB Gi GKB cv‡k `yBwU eËPvc AuwK| g‡b Kwi, eËPvc `yBwU ci¯úi‡K O we›`‡Z †Q` K‡i| A, O I O, B †hvM Kwi| AO †K AE eivei Ges BO †K BF eivei ewaZ Kwi| OE †_‡K $\frac{a}{2} = OC$ Ges OF †_‡K $\frac{b}{2} = OD$ wbB| A, D; D, C I B, C †hvM Kwi|

Zvn‡j, $ABCD$ B DwÏó mvgvšÍwiK|

cÖgvY : ΔAOB I ΔCOD G $OA = OC = \frac{a}{2}; OB = OD = \frac{b}{2},$ [A¼bvbmv‡i]

Ges Ašÿf³ $\angle AOB$ = Ašÿf³ $\angle COD$ [wecÖZxc †KvY]

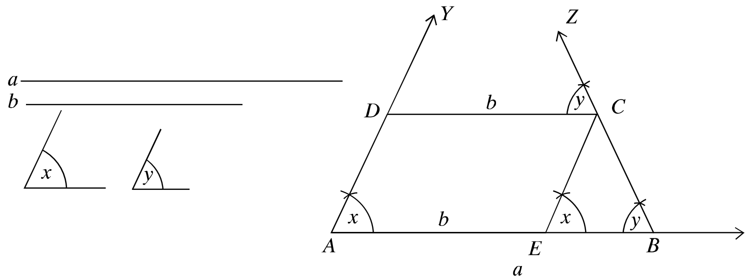
$\therefore \ \Delta AOB \cong \Delta COD$.

$\therefore \ AB = CD$ Ges $\angle ABO = \angle ODC$; wKš‘ †KvY `yBwU GKvšÍi †KvY|

$\therefore \ AB$ I CD mgvb I mgvšÍivj|

Abiƒcfv‡e, AD I BC mgvb I mgvšÍivj| AZGe, $ABCD$ B wb‡Y©q mvgvšÍwiK|

D`vniY 1| UªvwcwRqv‡gi `yBwU mgvšÍivj evû Ges G‡`i g‡a¨ enËi evû msjMœ `yBwU †KvY †`Iqv Av‡Q| UªvwcwRqvgwU AvK|



g‡b Kwi, UªvwcwRqv‡gi mgvšÍivj evûØq a Ges b, †hLv‡b $a > b$ Ges enËi evû a msjMœ †KvYØq $\angle x$ I $\angle y$| UªvwcwRqvgwU AvK‡Z n‡e|

A¼b : †h‡Kv‡bv iwk¥ AX †_‡K $AB = a$ wbB| B †iLvs‡ki A we›`‡Z $\angle x$ Gi mgvb $\angle BAY$ Ges $B$ we›`‡Z $\angle y$ Gi mgvb $\angle ABZ$ AuwK|

Gevi AB †iLvsk †_‡K $AE = b$ †K‡U wbB| E we›`‡Z $BC || AY$ AuwK hv $BZ$ iwk¥‡Z $C$ we›`‡Z †Q` K‡i| Gevi $CD || BA$ AuwK| $CD$ †iLvsk $AY$ iwk¥‡K $D$ we›`‡Z †Q` K‡i| Zvn‡j, $ABCD$ B DwÏó UªvcwRqvg| I

cgvY : A¼bvbmv‡i, $AB || CD$ Ges $AD || EC$ mZivs $ABCD$ GKwU mvgvšÍwiK Ges $CD = AE = b$. GLb, PZfR $ABCD$ I $AB = a$, $CD = b$, $AB || CD$ Ges $\angle BAD = \angle x$, $\angle ABC = \angle y$ (A¼b Abmv‡i) AZGe, $ABCD$ B wb‡Yq UªvcwRqvg|

KvR : i¤‡mi cwimxgv I GKwU †KvY †`Iqv Av‡Q| i¤mwU AvK|

Abkxjbx 7·2

1| mg‡KvYx wÎf‡Ri Aci `BwU †Kv‡Yi cwigvY †`Iqv _vK‡j wb‡gi †Kvb †¶‡Î wÎfR A¼b Kiv m¤e|

K. 63^0 I 36^0 L. 30^0 I 70^0

M. 40^0 I 50^0 N. 80^0 I 20^0

2| i AvqZ GKwU mvgvšÍwiK

ii eM GKwU AvqZ

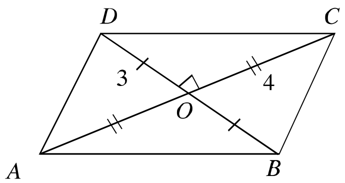
iii i¤m GKwU eM

Ic‡ii Z‡_¨i Av‡jv‡K wb‡gi †KvbwU mwVK ?

K. i I ii   L. i I iii

M. ii I iii   N. i, ii I iii

c`Ë wP‡Îi Av‡jv‡K 3 I 4 bs c‡ki DËi `vI



3| ΔAOB Gi †¶Îdj KZ?

K. 6 eM GKK   L. 7 eM GKK

M. 12 eM GKK   N. 14 eM GKK

4| PZfRwUi cwimxgv

K. 12 GKK   L. 14 GKK

M. 20 GKK   N. 28 GKK

5| wb‡g c`Ë DcvË wb‡q PZfR A¼b Ki :

K. PviwU evûi ˆ`N¨ 3 †m.wg., 3·5 †m.wg., 2.5 †m.wg. I 3 †m,wg. Ges GKwU †KvY 45°|

L. PviwU evûi ˆ`N¨ 3.5 †m.wg, 4 †m.wg., 2·5 †m.wg. I 3.5 †m.wg. Ges GKwU KY 5 †m.wg.|

M. wZbwU evûi ˆ`N¨ 3·2 †m.wg., 3 †m.wg., 3·5 †m.wg. Ges `BwU KY 2·8 †m.wg. I 4.5 †m.wg.|

N. wZbwU evûi ˆ`N¨ 3 †m.wg., 3·5 †m.wg., 4 †m.wg. Ges `BwU †KvY 60° I 45°|

6| wb‡g c`Ë DcvË wb‡q mvgvšÍwiK A¼b Ki :

K. `BwU K‡Yi ˆ`N©¨ 4 †m.wg., 6·5 †m.wg. Ges G‡`i AšÍf©³ †KvY 45°|

L. `BwU K‡Yi ˆ`N©¨ 5 †m.wg., 6·5 †m.wg. Ges G‡`i AšÍf©³ †KvY 30°|

M. GKwU evûi ˆ`N©¨ 4 †m.wg. Ges `BwU K‡Yi ˆ`N©¨ 5 †m.wg., 6·5 †m.wg.|

N. GKwU evûi ˆ`N©¨ 5 †m.wg. Ges `BwU K‡Yi ˆ`N©¨ 4.5 †m.wg., 6 †m.wg.|

7| $ABCD$ PZf‡Ri AB I BC evû Ges $\angle B, \angle C$ I $\angle D$ †KvY †`Iqv Av‡Q| PZfRwU AvK|

8| PZf‡Ri KY `BwUi †Q`we›` Øviv KY `BwUi PviwU LwÊZ Ask Ges Zv‡`i AšÍf©³ GKwU †KvY h_vµ‡g $OA = 4$ †m.wg., $OB = 5$ †m.wg., $OC = 3·5$ †m.wg., $OD = 4·5$ †m.wg. I $\angle AOB = 80°$. PZfRwU AvK|

9| i¤‡mi GKwU evûi ˆ`N©¨ 3·5 †m.wg. I GKwU †KvY 45° ; i¤mwU AvK|

10| i¤‡mi GKwU evû Ges GKwU K‡Yi ˆ`N©¨ †`Iqv Av‡Q| i¤mwU AvK|

11| `BwU K‡Yi ˆ`N©¨ †`Iqv Av‡Q| i¤mwU AvK|

12| eM‡¶‡Îi cwimxgv †`Iqv Av‡Q| eM‡¶ÎwU AvK|

13| RKx I Rvdi j mv‡n‡ei emZ ewo GKB mxgv‡iLvi g‡a¨ Aew¯Z Ges ewoi †¶Îdj mgvb| Z‡e RKxi mv‡n‡ei ewoi AvKwZ AvqZvKvi Ges Rvdvi j mv‡n‡ei ewo mvgvšÍwiK AvKwZi|

K. fwgi ˆ`N©¨ 10 GKK Ges D"PZv 8 GKK a‡i Zv‡`i ewoi mxgv‡iLv A¼b Ki|

L. †`LvI †h, RKx mv‡n‡ei ewoi mxgv‡iLv Rvdvi j mv‡n‡ei ewoi mxgv‡iLv A‡c¶v †QvU|

M. RKx mv‡n‡ei ewoi ˆ`N©¨ I c‡¯i AbycvZ 4:3 Ges †¶Îdj 300 eM GKK n‡j, Zv‡`i ewoi †¶ÎdjØ‡qi AbycvZ wbYq Ki|

14| GKwU mg‡KvYx wÎf‡Ri AwZfR 7 †m.wg I GK evûi ˆ`N©¨ 4 †m.wg, $\angle A = 85^0$, $\angle B = 80^0$ Ges $\angle C = 95^0$

Ic‡ii Z‡_¨i Av‡jv‡K wb‡Pi cÖk¸‡jvi DËi `vI :

K. wÎfRwUi Aci evûi ˆ`N©¨ wbYq Ki|

L. wÎfRwU A¼b Ki|

M. wÎfRwUi cwimxgvi mgvb cwimxgv wewkó GKwU eM A¼b Ki|

15| ABCD PZf‡Ri AB = 4 †m.wg. BC = 5 †m.wg

Ic‡ii Z‡_¨i Av‡jv‡K wb‡Pi cÖk¸‡jvi DËi `vI

K. GKwU i¤m A¼b K‡i Dnvi bvg `vI|

L. c`Ë Z_¨ Abhvqx ABCD PZfRwU A¼b Ki|

M. c`Ë PZf‡Ri cwimxgvi mgvb cwimxgv wewkó GKwU mgevû wÎfR A¼b Ki|

Aóg Aa¨vq
eË

Avgiv †R‡bwQ †h, eË GKwU mgZj¸q R¨vwgwZK wPÎ hvi we›`¸‡jv †Kv‡bv wbw`ó we›` †_‡K mg`i‡Z Aew¯Z| eË m¤úwKZ wewfbaviYv †hgb †K›`, e¨vm, e¨vmva, R¨v BZ¨vw` wel‡q Av‡jvPbv Kiv n‡q‡Q| G Aa¨v‡q mgZ‡j †Kv‡bv e‡Ëi Pvc I ¯úkK m¤úwKZ cwZÁvi Av‡jvPbv Kiv n‡e|

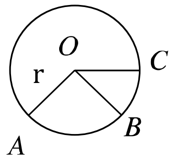
Aa¨vq †k‡l wk¶v_xiv

- eËPvc, †K›`¯ †KvY, eË¯ †KvY, e‡Ë AšwjwLZ PZfR e¨vL¨v Ki‡Z cvi‡e|
- eË msµvšÍ Dccv`¨ cgvY Ki‡Z cvi‡e|
- eË m¤úwKZ m¤úv`¨ eYbv Ki‡Z cvi‡e|

8·1 eË

eË GKwU mgZj¸q R¨vwgwZK wPÎ hvi we›`¸‡jv †Kv‡bv wbw`ó we›` †_‡K mg`i‡Z Aew¯Z| wbw`ó we›`wU e‡Ëi †K›`| wbw`ó we›` †_‡K mg`iZ eRvq †i‡L †Kv‡bv we›` †h Ave× c_ wPwÎZ K‡i ZvB eË| †K›` n‡Z eË¯ †Kv‡bv we›`i `iZ‡K e¨vmva e‡j|

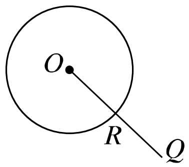
g‡b Kwi, O mgZ‡ji †Kv‡bv wbw`ó we›` Ges r wbw`ó cwigvc| mgZj¯ †h mKj we›` O †_‡K r `i‡Z Aew¯Z, Zv‡`i †mU eË, hvi †K›` $O$ I e¨vmva $r$. wP‡Î $O$ e‡Ëi †K›`, A, B I C eË¯ we›`| OA, OB I OC Gi c‡Z¨KwU eËwUi e¨vmva|



mgZj¯ KwZcq we›`‡K mgeË we›` ejv nq hw` we›`¸‡jv w`‡q GKwU eË hvq A_vr, Ggb GKwU eË _v‡K hv‡Z we›`¸‡jv Aew¯Z nq| Dc‡ii wP‡Î A, B I C mgeË we›`|

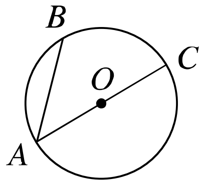
e‡Ëi Af¨šÍi IewnfvM

hw` †Kv‡bv e‡Ëi †K›` O Ges e¨vmva r nq Z‡e O †_‡K mgZ‡ji †h mKj we›`i `iZ r †_‡K Kg Zv‡`i †mU‡K eËwUi Af¨šÍi Ges $O$ †_‡K mgZ‡ji †h mKj we›`i `iZ $r$ †_‡K †ewk Zv‡`i †mU‡K eËwUi ewnfvM ejv nh| e‡Ëi Af¨šÍi¯ `BwU we›`i ms‡hvRK †iLvsk m¤úYfv‡e e‡Ëi Af¨š‡iB _v‡K|



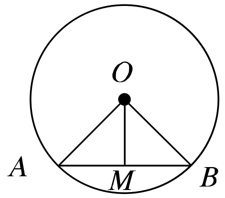
†Kv‡bv e‡Ëi Af¨šÍi¯ GKwU we›` I ewnt¯ GKwU we›`i ms‡hvRK †iLvsk eËwU‡K GKwU I †Kej GKwU we›`‡Z †Q` K‡i| wP‡Î, P e‡Ëi Af¨šÍi¯ GKwU we›` Ges $Q$ e‡Ëi ewnt¯ GKwU we›`| PQ †iLvsk eËwU‡K †Kej R we›`‡Z †Q` K‡i|

e‡Ëi R¨v I e¨vm



e‡Ëi `BwU wfb we›`i ms‡hvRK †iLvsk eËwUi GKwU R¨v| e‡Ëi †Kv‡bv R¨v hw` †K›` w`‡q hvq Z‡e R¨vwU‡K e‡Ëi e¨vm ejv nq| A_vr e‡Ëi †K›`Mvgx †h‡Kv‡bv R¨v n‡jv e¨vm| wP‡Î, AB I AC eËwUi `BwU R¨v Ges eËwUi †K›` O| G‡`i g‡a¨ $AC$ R¨vwU e¨vm; KviY R¨vwU eËwUi †K›`Mvgx| OA I OC e‡Ëi `BwU e¨vmva| mZivs, e‡Ëi †K›` c‡Z¨K e¨v‡mi ga¨we›`| AZGe c‡Z¨K e¨v‡mi ˆ`N¨ $2r$, †hLv‡b r eËwUi e¨vmva|

Dccv`¨ 1| e‡Ëi †K›` I e¨vm wfb †Kv‡bv R¨v Gi ga¨we›`i ms‡hvRK †iLvsk H R¨v Gi Ici j¤|



g‡b Kwi, O †K›`wewkó $ABC$ e‡Ë e¨vm bq Ggb GKwU R¨v $AB$ Ges GB R¨v Gi ga¨ we›` M| O, M †hvM Kwi|

cgvY Ki‡Z n‡e †h, OM †iLvsk AB R¨v Gi Dci j¤|

A¼b : O, A Ges O, B †hvM Kwi|

cgvY :

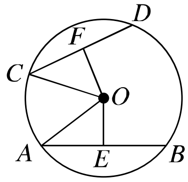
avcmgn	h_v_Zv
(1) ΔOAM Ges ΔOBM G	
$AM = BM$	[M, AB Gi ga¨we›`]
$OA = OB$	[Df‡q GKB e‡Ëi e¨vmva]
Ges $OM = OM$	[mvaviY evû]
mZivs, $\Delta OAM \cong \Delta OBM$	[evû-evû-evû Dccv`¨]
$\therefore \angle OMA = \angle OMB$	
(2) †h‡nZ †KvYØq ˆiwLK hMj †KvY Ges Zv‡`i cwigvc mgvb, mZivs, $\angle OMA = \angle OMB$ = 1 mg‡KvY| AZGe, $OM \perp AB$. (cgwYZ)	

Abywm×vš— 1| e‡Ëi †h‡Kv‡bv R¨v Gi j¤-wØLÊK †K›`Mvgx|

Abywm×vš— 2| †h‡Kv‡bv mij‡iLv GKwU e‡Ë‡K `B‡qi AwaK we›`‡Z †Q` Ki‡Z cv‡i bv|

> KvR :
>
> 1| Dccv`¨ 1 Gi wecixZ Dccv`¨wU wbgic: e‡Ëi †K›` †_‡K e¨vm wfb Ab¨ †Kv‡bv R¨v Gi Ici Aw¼Z j¤ H R¨v‡K mgwØLwÊZ K‡i Ñ cgvY Ki|

Dccv`¨ 2| e‡Ëi mKj mgvb R¨v †K›` †_‡K mg`ieZx|



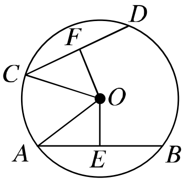
g‡b Kwi, O e‡Ëi †K›` Ges $AB$ I $CD$ e‡Ëi `BwU mgvb R¨v| cgvY Ki‡Z n‡e †h, O †_‡K AB Ges CD R¨vØq mg`ieZx|

A¼b : O †_‡K AB Ges CD R¨v Gi Dci h_vµ‡g OE Ges OF j¤ AwwK| O, A Ges O, C †hvM Kwi|

cgvY :

avc	h_v_Zv
(1) $OE \perp AB$ I $OF \perp CD.$ mZivs, $AE = BE$ Ges $CF = DF.$ $\therefore AE = \frac{1}{2}AB$ Ges $CF = \frac{1}{2}CD.$	[†K›` †_‡K e¨vm wfb †h‡Kv‡bv R¨v Gi Dci Aw¼Z j¤ R¨v‡K mgwØLwÊZ K‡i]
(2) wKš $AB = CD$ $\therefore$ $AE = CF.$	[Kíbv]
(3) GLb ΔOAE Ges ΔOCF mg‡KvYx wÎfRØ‡qi g‡a¨ AwZfR $OA =$ AwZfR OC Ges	[Df‡q GKB e‡Ëi e¨vmva]
$AE = CF.$	[avc 2]
$\therefore$ $\Delta OAE \cong \Delta OCF$	[mg‡KvYx wÎf‡Ri AwZfR-evû mgmZv Dccv`¨]
$\therefore$ $OE = OF.$	
(4) wKš OE Ges OF †K›` $O$ †_‡K h\_vµ‡g $AB$ R¨v Ges $CD$ R¨v Gi `iZ| mZivs, AB Ges CD R¨vØq e‡Ëi †K›` †_‡K mg`ieZx|	

Dccv`¨ 3| e‡Ëi †K›` †_‡K mg`ieZx mKj R¨v ci¯úi mgvb|



g‡b Kwi, O e‡Ëi †K›` Ges $AB$ I $CD$ `BwU R¨v| O †_‡K AB I CD Gi Dci h_vµ‡g OE I OF j¤| Zvn‡j OE I OF †K›` †\_‡K h\_vµ‡g $AB$ I $CD$ R¨v‡qi `iZ wb‡`k K‡i| $OE = OF$ n‡j cgvY Ki‡Z n‡e †h, $AB = CD.$

A¼b : O, A Ges O, C †hvM Kwi|

cgvY :

avc	h_v_Zv
(1) †h‡nZ $OE \perp AB$ Ges $OF \perp CD$. mZivs, $\angle OEA = \angle OFC =$ GK mg‡KvY|	[mg‡KvY]
(2) GLb, ΔOAE Ges ΔOCF mg‡KvYx wÎfRØ‡qi	
g‡a¨ AwZfR $OA =$ AwZfR OC Ges $OE = OF$ [Kíbv] $\therefore \Delta OAE \cong \Delta OCF$ $\therefore AE = CF$.	[Df‡q GKB e‡Ëi e¨vmva] [mg‡KvYx wÎf‡Ri AwZfR-evû memgZv Dccv`¨
(3) $AE = \frac{1}{2} AB$ Ges $CF = \frac{1}{2} CD$	[†K›` †_‡K e¨vm wfbœ †h‡Kv‡bv R¨v Gi Dci Aw¼Z j¤ R¨v‡K mgwØLwÊZ K‡i]
(4) mZivs $\frac{1}{2} AB = \frac{1}{2} CD$ A_vr., $AB = CD$.	

Abwm×všÍ 1\| e‡Ëi e¨vmB enËg R¨v\|

Abkxjbx 8·1

1\| cvgY Ki †h, †Kv‡bv e‡Ëi `BwU R¨v ci¯úi‡K mgwØLwÊZ Ki‡j Zv‡`i †Q`we›` eËwUi †K›` n‡e\|

2\| cgvY Ki †h, `BwU mgvšÍivj R¨v Gi ga¨we›`i ms‡hvRK mij‡iLv †K›`Mvgx Ges R¨vØ‡qi Ici j¤\|

3\| †Kv‡bv e‡Ëi AB I AC R¨v `BwU $A$ we›`Mvgx e¨vmv‡ai mv‡_ mgvb †KvY Drcb K‡i\| cgvY Ki †h, $AB = AC$.

4\| wP‡Î O e‡Ëi †K›` Ges R¨v $AB =$ R¨v AC.

cgvY Ki †h, $\angle BAO = \angle CAO$.

5\| †Kv‡bv eË GKwU mg‡KvYx wÎf‡Ri kxlwe›`¸‡jv w`‡q hvq\| †`LvI †h, eËwUi †K›` AwZf‡Ri ga¨we›`\|

6\| `BwU mg‡Kw›`K e‡Ëi GKwUi AB R¨v Aci eË‡K C I D we›`‡Z †Q` K‡i\|

cgvY Ki †h, $AC = BD$.

7\| e‡Ëi `BwU mgvb R¨v ci¯úi‡K †Q` Ki‡j †`LvI †h, Zv‡`i GKwUi AskØq AciwUi Ask؇qi mgvb\|

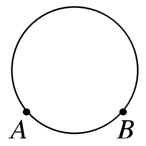
8\| cgvY Ki †h, e‡Ëi mgvb R¨v Gi ga¨we›`¸‡jv mgeË\|

9\| †`LvI †h, e¨v‡mi `B cvšÍ †_‡K Zvi wecixZ w`‡K `BwU mgvb R¨v A¼b Ki‡j Zviv mgvšÍivj nq\|

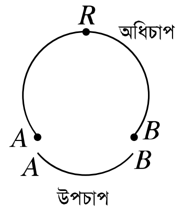
10\| †`LvI †h, e¨v‡mi `B cvšÍ †_‡K Zvi wecixZ w`‡K `BwU mgvšÍivj R¨v AvK‡j Zviv mgvb nq\|

11\| †`LvI †h, e‡Ëi `BwU R¨v Gi g‡a¨ enËi R¨v-wU ¶`Zi R¨v A‡c¶v †K‡›`i wbKUZi\|

8·2 eËPvc



e‡Ëi †h‡Kv‡bv `BwU we›`i g‡a¨i cwiwai Ask‡K Pvc e‡j| wP‡Î A I B `BwU we›`i gv‡S e‡Ëi Ask¸‡jv j¶ Kwi| †`Lv hvq, `BwU As‡ki GKwU Ask †QvU, Ab¨wU ZjbvgjKfv‡e eo| †QvU AskwU‡K DcPvc I eowU‡K AwaPvc ejv nq| A I B GB Pv‡ci cvšwe›` Ges Pv‡ci Ab¨ mKj we›` Zvi Ašt¯ we›`| Pv‡ci Ašt¯ we›`¸wU GKwU we›` $C$ wbw`ó K‡i PvcwU‡K ACB Pvc e‡j AwfwnZ Kiv nq Ges ACB cZxK Øviv cKvk Kiv nq| Avevi KL‡bv DcPvcwU AB cZxK Øviv cKvk Kiv nq| e‡Ëi `BwU we›` A I B eËwU‡K `BwU Pv‡c wef³ K‡i| Dfq Pv‡ci cvšwe›` A I B Ges cvšwe›` Qvov Pvc `BwUi Ab¨ †Kv‡bv mvaviY we›` †bB|



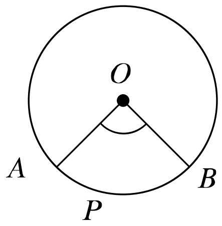
†KvY KZK LwÊZ Pvc

GKwU †KvY †Kv‡bv e‡Ë GKwU Pvc LwÊZ ev wQb K‡i ejv nq hw`

(1) PvcwUi c‡Z¨K cvšwe›` †KvYwUi evû‡Z Aew¯Z nq,

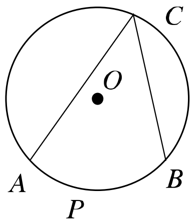
(2) †KvYwUi c‡Z¨K evû‡Z PvcwUi AšZ GKwU cvšwe›`, Aew¯Z nq Ges

(3) PvcwUi Ašt¯ c‡Z¨KwU we›` †KvYwUi Af¨š‡i _v‡K| wP‡Î c`wkZ †KvYwU O †Kw›`K e‡Ë APB Pvc LwÊZ K‡i|



eË¯ †KvY

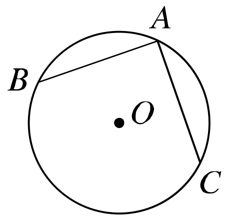
GKwU †Kv‡Yi kxlwe›` †Kv‡bv e‡Ëi GKwU we›` n‡j Ges †KvYwUi c‡Z¨K evû‡Z kxlwe›` Qvov I e‡Ëi GKwU we›` _vK‡j †KvYwU‡K GKwU eË¯ †KvY ev e‡Ë AšwjwLZ †KvY ejv nq| wP‡Î †KvY¸‡jv eË¯ †KvY| c‡Z¨K eË¯ †KvY e‡Ë GKwU Pvc LwÊZ K‡i| GB Pvc DcPvc, AaeË A_ev AwaPvc n‡Z cv‡i|



GKwU eË¯ †KvY e‡Ë †h Pvc LwÊZ K‡i, †KvYwU †mB Pv‡ci Ici `Êvqgvb Ges LwÊZ Pv‡ci AbeÜx Pv‡c AšwjwLZ ejv nq| cv‡ki wP‡Î eË¯ †KvYwU $APB$ Pv‡ci Ici `Êvqgvb Ges ACB Pv‡c AšwjwLZ|

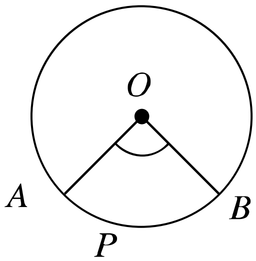
j¶Yxq †h, APB I ACB G‡K Ac‡ii AbeÜx Pvc|

gše¨ : e‡Ëi †Kv‡bv Pv‡c Aš÷jwLZ GKwU †KvY n‡"Q †mB †KvY hvi kxlwe›` H Pv‡ci GKwU Aštˉwe›` Ges hvi GK GKwU evû H Pv‡ci GK GKwU cvšÍwe›` w`‡q hvq| e‡Ëi †Kv‡bv Pv‡c `Êvqgvb GKwU eˉ †KvY n‡"Q H Pv‡ci AbeÜx Pv‡c Aš÷jwLZ GKwU †KvY|

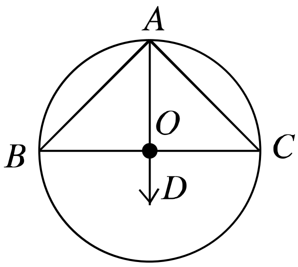


†K›`ª †KvY

GKwU †Kv‡Yi kxlwe›` †Kv‡bv e‡Ëi †K‡›` Aew¯'Z n‡j, †KvYwU‡K H e‡Ëi GKwU †K›`ª †KvY ejv nq Ges †KvYwU e‡Ë †h Pvc LwÊZ K‡i †mB Pv‡ci Ici Zv `Êvqgvb ejv nq| cv‡ki wP‡Î $\angle AOB$ †KvYwU GKwU †K›`ª †KvY Ges Zv $APB$ Pv‡ci Ici `Êvqgvb|



c‡Z¨K †K›`ª †KvY e‡Ë GKwU DcPvc LwÊZ K‡i| wP‡Î $APB$ GKwU DcPvc| e‡Ëi †Kv‡bv DcPv‡ci Ici `Êvqgvb †K›`ª †KvY ej‡Z Gic †KvY‡KB †evSvq hvi kxlwe›` e‡Ëi †K‡›` Aew¯'Z Ges hvi evûØq H Pv‡ci cvšÍwe›` `BwU w`‡q hvq|



Aae‡Ëi Ici `Êvqgvb †K›`ª †KvY we‡ePbvi Rb¨ Ic‡i DwjøwLZ eYbv A_en bq| Aae‡Ëi †¶‡Î †K›`ª †KvY $\angle BOC$ mij‡KvY Ges eˉ †KvY $\angle BAC$ mg‡KvY|

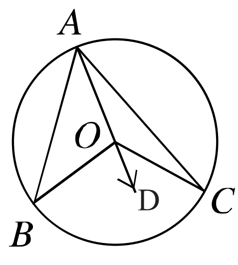
Dccv`¨ 4

e‡Ëi GKB Pv‡ci Ici `Êvqgvb †K›`ª †KvY eˉ †Kv‡Yi wØ¸Y|

g‡b Kwi, O †K›`wewkó $ABC$ GKwU eË Ges Zvi GKB DcPvc $BC$ Gi Ici `Êvqgvb eˉ $\angle BAC$ Ges †K›`ª $\angle BOC$ |

cgvY Ki‡Z n‡e †h, $\angle BOC = 2\angle BAC$

A¼b: g‡b Kwi, AC †iLvsk †K›`Mvgx bq| G †¶‡Î $A$ we›` w`‡q †K›`Mvgx †iLvsk AD AwK|



cgvY

avc	h_v_Zv
(1) ΔAOB Gi ewnt¯ †KvY $\angle BOD = \angle BAO + \angle ABO$	[ewnt¯ †KvY Ašt¯ wecixZ
(2) ΔAOB G $OA = OB$	†KvYØ‡qi mgwói mgvb]
AZGe, $\angle BAO = \angle ABO$	[GKB e‡Ëi e¨vmva]
(3) avc (1) I (2) †_‡K $\angle BOD = 2\angle BAO$.	[mgw`evû wÎf‡Ri fwg msjM †KvY
(4) GKBfv‡e ΔAOC †_‡K $\angle COD = 2\angle CAO$	`BwU mgvb]
(5) avc (3) I (4) †_‡K	
$\angle BOD + \angle COD = 2\angle BAO + 2\angle CAO$	
A_vr $\angle BOC = 2\angle BAC$. [cgwYZ]	[†hvM K‡i]

Ab¨fv‡e ejv hvq, e‡Ëi GKB Pv‡ci Ici `Êvqgvb eË¯ †KvY †K›`¯ †Kv‡Yi A‡aK|

KvR : O †K›`ª wewkó ABC e‡Ëi AC †K›`ªMvgx n‡j Dccv`¨ 8 cgvY Ki|

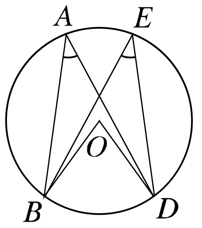
Dccv`¨ 5

e‡Ëi GKB Pv‡ci Dci `Êvqgvb eË¯ †KvY¸‡jv ci¯úi mgvb|

g‡b Kwi, O e‡Ëi †K›` Ges e‡Ëi $BCD$ Pv‡ci Ici `Êvqgvb $\angle BAD$ I $\angle BED$ `BwU eË¯ †KvY|

cgvY Ki‡Z n‡e †h, $\angle BAD = \angle BED$

A¼b : O, B Ges O, D †hvM Kwi|



cgvY :

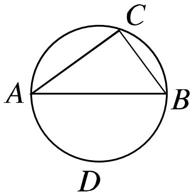
avc	h_v_Zv
(1) GLv‡b BCD Pv‡ci Ici `Êvqgvb †K›`¯ †KvY $\angle BOD$	
mZivs, $\angle BOD = 2\angle BAD$ Ges $\angle BOD = 2\angle BED$	[GKB Pv‡ci Ici `Êvqgvb †K›`¯
$\therefore \quad 2\angle BAD = 2\angle BED$	†KvY eË¯ †Kv‡Yi wØ¸Y]
ev $\angle BAD = \angle BED$	

Dccv`¨ 6

AaeË¯ †KvY GK mg‡KvY

g‡b Kwi, O †K›`ªwewkó e‡Ë AB GKwU e¨vm Ges $\angle ACB$ GKwU AaeË¯ †KvY|

cgvY Ki‡Z n‡e †h, $\angle ACB =$ GK mg‡KvY|



A¼b : AB Gi †h cv‡k C we›` Aew¯Z, Zvi wecixZ cv‡k
e‡Ëi Dci GKwU we›` D wbB|
cgvY :

avc	h_v_Zv
(1) ADB Pv‡ci Ici `Êvqgvb eË¯ $\angle ACB = \frac{1}{2}$ (‡K›`¯ mij †KvY $\angle AOB$) (2) wKš mij‡KvY $\angle AOB$ `B mg‡KvY|   $\therefore \angle ACB = \frac{1}{2}$ (`B mg‡KvY) = GK mg‡KvY|	[GKB Pv‡ci Ici `Êvqgvb eË¯ †KvY †K›`¯ †Kv‡Yi A‡aK]

Abwm×vš 1| mg‡KvYx wÎf‡Ri AwZfR‡K e¨vm a‡i eË A¼b Ki‡j Zv mg‡KŠwYK kxIwe›` w`‡q hv‡e|
Abwm×vš 2| †Kv‡bv e‡Ëi AwaPv‡c AšwjwLZ †KvY m²‡KvY|

KvR :
1| cgvY Ki †h, †Kv‡bv e‡Ëi DcPv‡c AšwjwLZ †KvY ¯j‡KvY|

Abkxjbx 8·2

1| O †K›`wewkó †Kv‡bv e‡Ë $ABCD$ GKwU AšwjwLZ PZfR| $AB$, $CD$ KYØq .. we›`‡Z †Q` Ki‡j cgvY Ki †h, $\angle AOB + \angle COD = 2 \angle AEB$.

2| $ABCD$ e‡Ë AB I CD R¨v `BwU ci¯úi $E$ we›`‡Z †Q` K‡i‡Q| †`LvI †h, ΔAED I ΔBED m`k‡KvYx|

3| O †K›`wewkó e‡Ë $\angle ADB + \angle BDC =$ GK mg‡KvY| cgvY Ki †h, A I B Ges C GK mij‡iLvq Aew¯Z|

4| AB I CD `BwU R¨v e‡Ëi Af¨š‡i $E$ we›`‡Z †Q` K‡i‡Q| cgvY Ki †h, $AB$ I $CD$ PvcØq †K‡›` †h `BwU †KvY Drcb K‡i, Zv‡`i mgwó $\angle AEC$ Gi wØ¸Y|

5| †`LvI †h, eË¯ UwcwRqv‡gi wZhK evûØq ci¯úi mgvb|

6| AB I CD †Kv‡bv e‡Ëi `BwU R¨v Ges $P$ I $Q$ h_vµ‡g Zv‡`i Øviv wQb DcPvc `BwUi ga¨we›`| PQ R¨v AB I CD R¨v‡K h_vµ‡g D I E we›`‡Z †Q` K‡i| †`LvI †h, $AD = AE$.

8·3 eË¯ PZfR

eËxq PZfR ev e‡Ë AšwjwLZ PZfR n‡jv Ggb PZfR hvi PviwU kxIwe›` e‡Ëi Dci Aew¯Z| G mKj PZf‡Ri GKwU we‡kl ag i‡q‡Q| weIqwU Abave‡bi Rb¨ wb‡Pi KvRwU Kwi|

KvR :

wewfb AvKv‡ii K‡qKwU eËxq PZfR $ABCD$ AvK| K‡qKwU wewfb e¨vmv‡ai eË A¼b K‡i cwZwUi Dci PviwU K‡i we›` wb‡q PZfR¸‡jv mn‡RB AvKv hvq| PZf‡Ri †KvY¸‡jv †g‡c wb‡Pi mviwYwU ciY Ki|

µwgK bs	$\angle A$	$\angle B$	$\angle C$	$\angle D$	$\angle A+\angle C$	$\angle B+\angle D$
1						
2						
3						
4						
5						

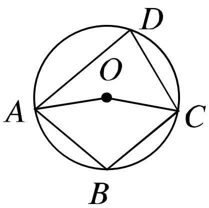
mviwY †_‡K Kx †evSv hvq ?

eË msµvš Dccv`¨

Dccv`¨ 7

e‡Ë AšwjLZ PZf‡Ri †h‡Kv‡bv `BwU wecixZ †Kv‡Yi mgwó `B mg‡KvY|

g‡b Kwi, O †K›`wewkó GKwU eË $ABCD$ PZfRwU AšwjLZ n‡q‡Q|



cgvY Ki‡Z n‡e †h, $\angle ABC + \angle ADC$ = `B mg‡KvY|

Ges $\angle BAD + \angle BCD$ = `B mg‡KvY|

A¼b : O, A Ges O, C †hvM Kwi|

cgvY :

avc	h_v_Zv
(1) GKB Pvc ADC Gi Dci `Êvqgvb †K›`¯	GKB Pv‡ci Dci `Êvqgvb †K›`¯
$\angle AOC = 2$ (eË¯ $\angle ABC$)	†KvY eË¯ †Kv‡Yi wØ¸Y|
A_vr, $\angle AOC = 2\angle ABC$	GKB Pv‡ci Dci `Êvqgvb †K›`¯
(2) Avevi, GKB Pvc ABC Gi Dci `Êvqgvb †K›`¯	†KvY eË¯ †Kv‡Yi wØ¸Y|
ce× †KvY $\angle AOC = 2$ (eË¯ $\angle ADC$)	
A_vr ce× †KvY $\angle AOC = 2\ \angle ADC$	
$\therefore\ \angle AOC +$ ce× †KvY $\angle AOC = 2(\angle ABC + \angle ADC)$	
wKš $\angle AOC$ + ce× †KvY $\angle AOC$ = Pvi mg‡KvY	

$\therefore\ 2(\angle ABC + \angle ADC$ = Pvi mg‡KvY

$\therefore\ \angle ABC + \angle ADC$ = `B mg‡KvY|

GKBfv‡e, cgvY Kiv hvq †h, $\angle BAD + \angle BCD$ = `B mg‡KvY|

Abywm×vš— 1| e‡Ë AšÍwjwLZ PZzf©‡Ri GKwU evûewaZ Ki‡j †h ewnt¯ †KvY Drcbœ nq Zv wecixZ Ašt¯ †Kv‡Yi mgvb|

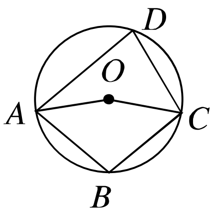
Abywm×vš— 2| e‡Ë AšÍwjwLZ mvgvšÍwiK GKwU AvqZ‡¶Î|

Dccv`¨ 8

†Kv‡bv PZzf©‡Ri `BwU wecixZ †KvY m¤úiK n‡j Zvi kxl©we›`yPviwU mgeË nq|

g‡b Kwi, $ABCD$ PZzf©‡R $\angle ABC + \angle ADC$ = `B mg‡KvY|

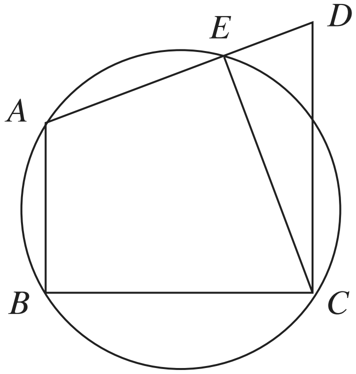
cgvY Ki‡Z n‡e †h, A, B, C, D we›`y PviwU mgeË|



A¼b : †h‡nZz A, B, C we›`y wZbwU mg‡iL bq, mZivs we›`y wZbwU w`‡q hvq Gi¦c GKwU I †Kej GKwU eË Av‡Q| g‡b Kwi, eËwU $AD$ †iLvsk‡K $E$ we›`y‡Z †Q` K‡i| A, E †hvM Kwi|

cgvY :

avc	h_v_Zv
A¼b Abmv‡i $ABCE$ eË¯ PZzfR|	e‡Ë AšÍwjwLZ PZzf©‡Ri `BwU wecixZ †Kv‡Yi mgwó `B mg‡KvY|
mZivs $\angle ABC + \angle AEC$ = `B mg‡KvY wKš‘ $\angle ABC + \angle ADC$ = `B mg‡KvY [‡`Iqv Av‡Q] $\therefore\ \angle AEC = \angle ADC$	
wKš‘ Zv Am¤¢e| KviY ΔCED Gi ewnt¯ $\angle AEC >$ wecixZ Ašt¯ $\angle ADC$	ewnt¯ †KvY wecixZ Ašt¯ †h‡Kv‡bv †Kv‡Yi †P‡q eo|
mZivs E Ges D we›`yØq wfbœ n‡Z cv‡i bv| $E$ we›`y AvekÿB D we›`yi mv‡_ wg‡j hv‡e| AZGe, $A, B, C, D$ we›`y PviwU mgeË|	



Abkxjbx 8.3

1| ΔABC G $\angle B$ I $\angle C$ Gi mgwØLÛKØq P we›`‡Z Ges ewnwØLÊKØq $Q$ we›`‡Z wgwjZ n‡j, cgvY Ki †h, B,P,C,Q we›` PvionU mgeË|

2| cgvY Ki †h, eË¯ PZf‡Ri †h‡Kv‡bv †Kv‡Yi mgwØLÊK I Zvi wecixZ †Kv‡Yi ewnwØLÊK e‡Ëi Ic‡i †Q` K‡i|

3| $ABCD$ GKwU eË| $\angle CAB$ I $\angle CBA$ Gi mgwØLÊK `BwU $P$ we›`‡Z Ges $\angle DBA$ I $\angle DAB$ †KvYØ‡qi mgwØLÊK `BwU $Q$ we›`‡Z wgwjZ n‡j, cgvY Ki †h, A,Q,P,B we›` PvionU mgeË|

4| O †K›`wewkó e‡Ëi $AB$ I $CD$ R¨v `BwU e‡Ëi Af¨š‡i Aew¯Z †Kv‡bv we›`‡Z mg‡Kv‡Y wgwjZ n‡q‡Q| cgvY Ki †h, $\angle AOD + \angle BOC$ = `B mg‡KvY|

5| mgvb mgvb fwgi Ici Aew¯Z †h †Kv‡bv `BwU wÎf‡Ri wkit‡KvYØq m¤úiK n‡j, cgvY Ki †h, Zv‡`i cwieËØq mgvb n‡e|

6| $ABCD$ PZf‡Ri wecixZ †KvYØq ci¯úi m¤úiK| AC †iLv hw` $\angle BAD$ Gi mgwØLÊK nq, Z‡e cgvY Ki †h, $BC = CD$ |

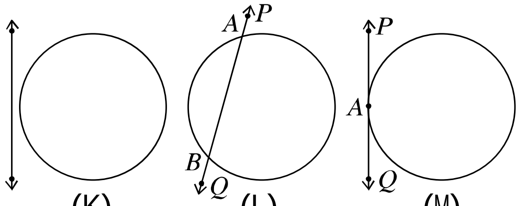
8.4 e‡Ëi †Q`K I ¯úkK

mgZ‡j GKwU eË I GKwU mij‡iLvi cvi¯úwiK Ae¯vb we‡ePbv Kwi| G‡¶‡Î wb‡Pi wP‡Îi c`Ë wZbwU m¤vebv i‡q‡Q:

(K) eË I mij‡iLvi †Kv‡bv mvaviY we›` †bB,

(L) mij‡iLwU eË‡K `BwU we›`‡Z †Q` K‡i‡Q,

(M) mij‡iLwU eË‡K GKwU we›`‡Z ¯úk K‡i‡Q|



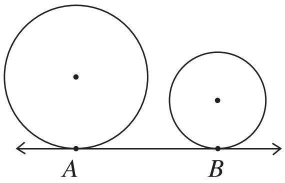
(K) (L) (M)

mgZ‡j GKwU eË I GKwU mij‡iLvi mewaK `BwU †Q`we›` _vK‡Z cv‡i| mgZj¯ GKwU eË I GKwU mij‡iLvi hw` `BwU †Q`we›` _v‡K Z‡e †iLwU‡K eËwUi GKwU †Q`K ejv nq Ges hw` GKwU I †Kej GKwU mvaviY we›` _v‡K Z‡e †iLwU‡K eËwUi GKwU ¯úkK ejv nq| †k‡lv³ †¶‡Î, mvaviY we›`wU‡K H ¯úk‡Ki ¯úkwe›` ejv nq| Dc‡ii wP‡Î GKwU eË I GKwU mij‡iLvi cvi¯úwiK Ae¯vb ‡`Lv‡bv n‡q‡Q| wPÎ-K G eË I $PQ$ mij‡iLvi ‡Kv‡bv mvaviY we›` †bB, wPÎ-L G PQ mij‡iLwU eË‡K A I B `BwU we›`‡Z †Q` K‡i‡Q Ges wPÎ-M G $PQ$ mij‡iLwU eË‡K $A$ we›`‡Z ¯úk K‡i‡Q| PQ eËwUi ¯úkK I A GB ¯úk‡Ki ¯úkwe›`|

gšev : e‡Ëi c‡Z¨K †Q`‡Ki †Q`we›`Ø‡qi AšeZx mKj we›` eËwUi Af¨š‡i _v‡K|

mvaviY ¯úkK

GKwU mij‡iLv hw` `BwU e‡Ëi ¯úkK nq, Z‡e Zv‡K eË `BwUi GKwU mvaviY ¯úkK ejv nq|cv‡ki wPÎ¸‡jv‡Z $AB$ Dfq e‡Ëi mvaviY ¯úkK| wPÎ-K I wPÎ-L G ¯úkwe›` GKB| wPÎ-M I wPÎ-N G ¯úkwe›` wfb wfb|

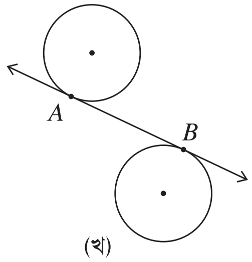


(ক)

`BwU e‡Ëi †Kv‡bv mvaviY ¯úk‡Ki ¯úkwe›` `BwU wfb n‡j ¯úkKwU‡K

(K) mij mvaviY ¯úkK ejv nq hw` eË `BwUi †K›`Øq ¯úk‡Ki GKB cv‡k _v‡K Ges

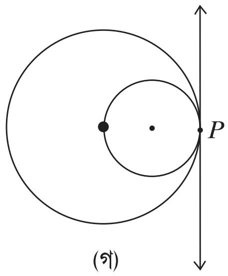
(L) wZhK mvaviY ¯úkK ejv nq hw` eË `BwUi †K›`Øq ¯úk‡Ki wecixZ cv‡k _v‡K|



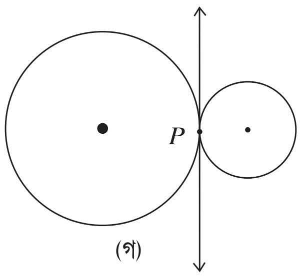
(খ)

wPÎ-M G ¯úkKwU mij mvaviY ¯úkK Ges wPÎ-N G ¯úkKwU wZhK mvaviY ¯úkK|

`BwU e‡Ëi mvaviY ¯úkK hw` eË `BwU‡K GKB we›`‡Z ¯úk K‡i Z‡e H we›`‡Z eË `BwU ci¯úi‡K ¯úk K‡i ejv nq| Gi|c †¶‡Î, eË `BwUi Ašt¯úk n‡q‡Q ejv nq hw` †K›`Øq ¯úk‡Ki GKB cv‡k _v‡KGes ewnt¯úk n‡q‡Q ejv nq hw` †K›`Øq ¯úk‡Ki wecixZ cv‡k _v‡K| wPÎ-K G eË `BwUi Ašt¯úk Ges wPÎ-L G ewnt¯úk n‡q‡Q|



(গ)

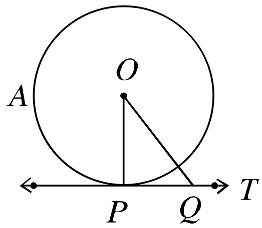


(গ)

Dccv`¨ 9

e‡Ëi †h‡Kv‡bv we›`‡Z AwuZ ¯úkK ¯úkKwe›`Mvgx e¨vmv‡ai Ici j¤|

g‡b Kwi, O †K›`ewkó GKwU e‡Ëi Ici¯ $P$ we›`‡Z PT GKwU ¯úkK Ges OP ¯úk we›`Mvgx e¨vmva| cgvY Ki‡Z n‡e †h, $PT \perp OP$.



A¼b : PT ¯úk‡Ki Ici †h‡Kv‡bv GKwU we›` Q wbB Ges O, Q †hvM Kwi|

cgvY : †h‡nZ e‡Ëi P we›`‡Z $PT$ GKwU ¯úkK, mZivs H $P$ we›` e¨ZxZ PT Gi Ici¯ Ab¨ mKj we›` e‡Ëi evB‡i _vK‡e| mZivs $Q$ we›`wU e‡Ëi evB‡i Aew¯Z|

$\therefore$ OQ e‡Ëi e¨vmva OP Gi †P‡q eo, A_vr, $OQ > OP$ Ges Zv ¯úk we›` $P$ e¨ZxZ $PT$ Gi Ici¯ $Q$ we›`i mKj Ae¯v‡bi Rb¨ mZ¨|

$\therefore$ †K›` $O$ †_‡K $PT$ ¯úk‡Ki Ici $OP$ nj ¶`Zg `iZ| mZivs $PT \perp OP$.

Abykxjbx 1| e‡Ëi †Kv‡bv we›`y‡Z GKwUgvÎ ¯úkK A¼b Kiv hvq|

Abykxjbx 2| ¯úk we›`y‡Z ¯úk‡Ki Ici Aw¼Z j¤ †K›`ª‡Mvgx|

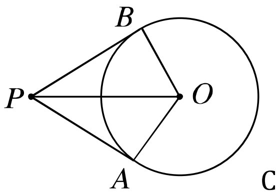
Abykxjbx 3| e‡Ëi †Kv‡bv we›` w`‡q H we›`y‡Mvgx e¨vmv‡ai Ici Aw¼Z j¤ D³ we›`y‡Z eËwUi ¯úkK nq|

Dccv`¨ 10

e‡Ëiewnt¯ †Kv‡bv we›` †_‡K e‡Ë `BwU ¯úkK Uvb‡j, H we›` †_‡K ¯úk we›`Øq‡qi `iZ mgvb|

g‡b Kwi, O †K›`ªwewkó $ABC$ e‡Ëi $P$ GKwU ewn:¯ we›` Ges PA I PB iwkØq e‡Ëi A I B we›`y‡Z `BwU ¯úkK| cgvY Ki‡Z n‡e †h, $PA = PB$

A¼b : O, A; O, B Ges O, P †hvM Kwi|



cgvY :

avc	h_v_Zv
(1) †h‡nZ PA ¯úkK Ges OA ¯úkwe›`y‡Mvgx e¨vmva, †m‡nZ $PA \perp OA$. $\therefore$ $\angle PAO$ = GK mg‡KvY| Abiƒ‡c $\angle PBO$ = GK mg‡KvY| $\therefore$ ΔPAO Ges ΔPBO DfqB mg‡KvYx wÎfR|	[¯úkK ¯úkKwe›`y‡Mvgx e¨vmv‡ai Ici j¤]
(2) GLb, ΔPAO I ΔPBO mg‡KvYx wÎfRØ‡q AwZfR PO = AwZfR PO Ges $OA = OB$ $\therefore$ $\Delta PAO \cong \Delta PBO$. $\therefore$ $PA = PB$	[GKB e‡Ëi e¨vmva] [mg‡KvYx wÎf‡Ri AwZfR- evû memgZv]

gše¨ :

1. `BwU eË ci¯úi‡K ewnt¯úk Ki‡j, ¯úkwe›` Qvov c‡Z¨K e‡Ëi Ab¨ mKj we›` Aci e‡Ëi evB‡i _vK‡e|
2. `BwU eË ci¯úi‡K Ašt¯úk Ki‡j, ¯úkwe›` Qvov †QvU e‡Ëi Ab¨ mKj we›` eo eËwUi Af¨š‡i _vK‡e|

Dccv`¨ 11

`BwU eË ci¯úi‡K ewnt¯úk Ki‡j, Zv‡`i †K›`Øq I ¯úk we›` mg‡iL|

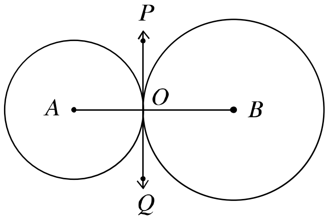
g‡b Kwi, A Ges B †K›`ªwewkó `BwU eË ci¯úi O we›`y‡Z ewnt¯úk K‡i| cgvY Ki‡Z n‡e †h, $A, O$ Ges $B$ we›` wZbwU mg‡iL|

A¼b : †h‡nZ eËØq ci¯úi O we›`y‡Z ¯úk K‡i‡Q, mZivs $O$ we›`y‡Z Zv‡`i GKwU mvaviY ¯úkK _vK‡e| GLb $O$ we›`y‡Z mvaviY ¯úkK POQ A¼b Kwi Ges O, A I O, B †hvM Kwi|

cgvY

A †K›`ªwewkó e‡Ë $OA$ ¯úk© we›`yMvgx e¨vmva© Ges POQ ¯úkK|

mZivs $\angle POA$ = GK mg‡KvY| Z`c $\angle POB$ = GK mg‡KvY|



$\angle POA + \angle POB$ = GK mg‡KvY + GK mg‡KvY = `B mg‡KvY|

ev, $\angle AOB$ = `B mg‡KvY

A_©vr, $\angle AOB$ GKwU mij‡KvY| ∴ A, O Ges B we›`Îq mg‡iL|

Abywm×vš— 1| `BwU eË ci¯úi‡K ewnt¯úk Ki‡j, †K›`Ø‡qi `iZ¡ eËØ‡qi e¨vmv‡a©i mgwói mgvb|

Abywm×vš— 2| `BwU eË ci¯úi‡K Ašt¯úk Ki‡j, †K›`Ø‡qi `iZ¡ eËØ‡qi e¨vmv‡a©i Aš—i mgvb|

KvR : 1| cgvY Ki †h, `BwU eË ci¯úi Ašt¯úk Ki‡j, Zv‡`i †K›`Øq I ¯úkwe›` mg‡iL n‡e|

Abkxjbx 8·4

1| O †K›`ªwewkó GKwU e‡Ëi ewnt¯ †Kv‡bv we›` P †_‡K e‡Ë `BwU ¯úkK UvbV nj| cgvY Ki †h, OP mij‡iLv ¯úk-R¨v Gi j¤^wØLÊK|

2| †`Iqv Av‡Q, $O$ e‡Ëi †K›` Ges PA I PB ¯úkKØq eË‡K h_vµ‡g A I B we›`‡Z ¯úk K‡i‡Q| cgvY Ki †h, PO, $\angle APB$ †K mgwØLwÊZ K‡i|

3| cgvY Ki †h, `BwU eË GK‡Kw›`K n‡j Ges enËi eËwUi †Kv‡bv R¨v ¶`Zi eËwU‡K ¯úk Ki‡j D³ R¨v ¯úkwe›`‡Z mgwØLwÊZ nq|

4| AB †Kv‡bv e‡Ëi e¨vm Ges BC e¨vmv‡a©i mgvb GKwU R¨v| hw` $A$ I $C$ we›`‡Z Aw¼Z ¯úkKØq ci¯úi D we›`‡Z wgwjZ nq, Z‡e cgvY Ki †h, ACD GKwU mgevû wÎfR|

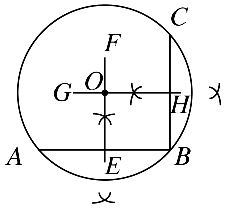
5| cgvY Ki †h, †Kv‡bv e‡Ëi cwiwjwLZ PZf‡Ri †h‡Kv‡bv `BwU wecixZ evû †K‡›` †h `BwU †KvY avi‡Y K‡i, Zviv ci¯úi m¤úiK|

8·5 eË m¤úKxq m¤úv`¨

m¤úv`¨ 1

GKwU eË ev eËPvc †`Iqv Av‡Q, †K›` wbYq Ki‡Z n‡e|

GKwU eË wPÎ-1 ev eËPvc wPÎ-2 †`Iqv Av‡Q, eËwUi ev eËPvcwUi †K›` wbYq Ki‡Z n‡e|



A¼b : c`Ë e‡Ë ev eËPv‡c wZbwU we›` A, B I C wbB| A, B Ges B, C †hvM Kwi| AB I BC R¨v `BwUi j¤^mgwØLÊK h_vµ‡g $EF$ I $GH$ †iLvsk `BwU Uwb| g‡b Kwi, Zviv ci¯úi O we›`‡Z †Q` K‡i| mZivs, O we›`B e‡Ëi ev eËPv‡ci †K›`|

cgvY : EF †iLvsk AB R¨v Gi Ges GH †iLvsk BC R¨v Gi j¤mgwØLÊK| wKš‘ EF I GH Df‡q †K›`Mvgx Ges $O$ Zv‡`i mvaviY †Q` we›`| mZivs O we›`B e‡Ëi ev eËPv‡ci †K›`|

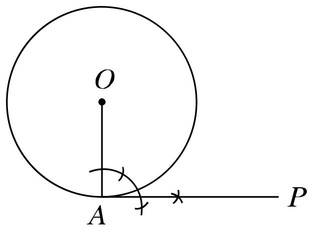
e‡Ëi ¯úkK A¼b

Avgiv †R‡bwQ †h, e‡Ëi wfZ‡i Aew¯Z †Kv‡bv we›` †_‡K e‡Ëi ¯úkK AvKv hvq bv| we›`wU hw` e‡Ëi Ici _v‡K Zvn‡j D³ we›`‡Z e‡Ëi GKwUgvÎ ¯úkK A¼b Kiv hvq| ¯úkKwU ewYZ we›`‡Z Aw¼Z e¨vmv‡ai Dci j¤ nq| mZivs, eËw¯Z †Kv‡bv we›`‡Z e‡Ëi ¯úkK A¼b Ki‡Z n‡j ewYZ we›`‡Z e¨vmva A¼b K‡i e¨vmv‡ai Dci j¤ AvK‡Z n‡e| Avevi we›`wU e‡Ëi evB‡i Aew¯Z n‡j Zv †_‡K e‡Ë `BwU ¯úkK AvKv hv‡e|

m¤úv`¨ 2

e‡Ëi †Kv‡bv we›`‡Z GKwU ¯úkK AvK‡Z n‡e|

g‡b Kwi, O †K›`wewkó e‡Ë $A$ GKwU we›`| A we›`‡Z eËwU‡Z GKwU ¯úkK AvK‡Z n‡e|



A¼b :

(1) O, A †hvM Kwi| A we›`‡Z OA Gi Dci AP j¤ AwK| Zvn‡j AP wbYq ¯úkK|

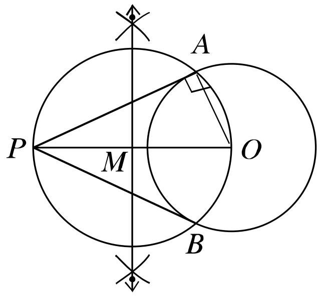
cgvY : OA †iLvsk A we›`Mvgx e¨vmva Ges AP Zvi Ici j¤| mZivs, AP †iLvB wb‡Yq ¯úkK|

we‡kl `óe¨ : e‡Ëi †Kv‡bv we›`‡Z GKwUgvÎ ¯úkK AvKv nq|

m¤úv`¨ 3

e‡Ëi ewnt¯ †Kv‡bv we›` †_‡K eËwUi ¯úkK AvK‡Z n‡e|

g‡b Kwi, O †K›`wewkó e‡Ëi $P$ GKwU ewnt¯ we›`| P we›` †_‡K H e‡Ë ¯úkK AvK‡Z n‡e|



A¼b :

(1) P, O †hvM Kwi| PO †iLvs‡ki ga¨we›` M wbYq Kwi|

(2) GLb M †K †K›` K‡i $MO$ Gi mgvb e¨vmva wb‡q GKwU eË AwK| g‡b Kwi, bZb Aw¼Z eËwU c`Ë e‡K A I B we›`‡Z †Q` K‡i|

(3) A, P Ges B, P †hvM Kwi|

Zvn‡j, AP , BP Df‡qB wb‡Yq ¯úkK|

cgvY : A, O Ges B, O †hvM Kwi| APB e‡Ë PO e¨vm|

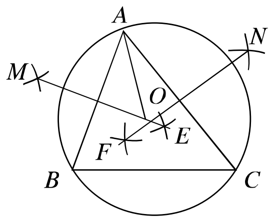
$\therefore \quad \angle PAO$ = GK mg‡KvY [AaeË¯ †KvY mg‡KvY]

mZivs, OA †iLvsk AP †iLvs‡ki Ici j¤| AZGe, O †Kw›`K e‡Ëi $A$ we›`‡Z AP †iLvsk GKwU ¯úkK| Abi/cfv‡e, BP †iLvskI GKwU ¯úkK|

we‡kl `óe¨ : e‡Ëi ewnt¯ †Kv‡bv we›` †_‡K H e‡Ë `BwU I †Kej `BwU ¯úkK AvKv hvq|

m¤úv`¨ 4

†Kv‡bv wbw`ó wÎf‡Ri cwieË AvK‡Z n‡e|



g‡b Kwi, ABC GKwU wÎfR| Gi cwieË AvK‡Z n‡e| A_vr, Ggb GKwU eË AvK‡Z n‡e, hv wÎf‡Ri wZbwU kxlwe›` $A, B$ I $C$ we›` w`‡q hvq|

A¼b :

(1) AB I AC †iLvs‡ki j¤ mgwØLÊK h_vµ‡g EM I FN †iLvsk AwK| g‡b Kwi, Zviv ci¯úi‡K O we›`‡Z †Q` K‡i|

(2) A, O †hvM Kwi| O †K †K›` K‡i OA Gi mgvb e¨vmva wb‡q GKwU eË AwK|

Zvn‡j, eËwU A, B I C we›`Mvgx n‡e Ges GB eËwUB ΔABC Gi wb‡Yq cwieË|

cgvY : B, O Ges C, O †hvM Kwi| O we›`wU AB Gi j¤mgwØLÊK EM Gi Ici Aew¯Z|

$\therefore \quad OA = OB$, GKBfv‡e, $OA = OC$

$\therefore \quad OA = OB = OC$

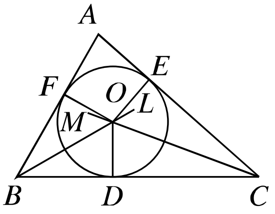
mZivs O †K †K›` K‡i $OA$ Gi mgvb e¨vmva wb‡q A¼Z eËwU $A, B$ I $C$ we›` wZbwU w`‡q hv‡e| mZivs GB eËwUB ΔABC Gi cwieË|

> KvR : Ic‡ii wP‡Î GKwU m²‡KvYx wÎf‡Ri cwieË AvKv n‡q‡Q| ¯j‡KvYx Ges mg‡KvYx wÎf‡Ri cwieË A¼b Ki|

j¶Yxq †h, m²‡KvYx wÎf‡Ri †¶‡Î cwi‡K›` wÎf‡Ri Af¨š‡i, ¯j‡KvYx wÎf‡Ri †¶‡Î cwi‡K›` wÎf‡Ri ewnfv‡M Ges mg‡KvYx wÎf‡Ri †¶‡Î cwi‡K›` AwZf‡Ri Ici Aew¯Z|

m¤úv`¨ 5

†Kv‡bv wbw`ó wÎf‡Ri AšeË AvK‡Z n‡e|



g‡b Kwi, ΔABC GKwU wÎfR| Gi AšeË AvK‡Z n‡e| A_vr, ΔABC Gi wfZ‡i Ggb GKwU eË AvK‡Z n‡e, hv BC, CA I AB evû wZbwUi c‡Z¨KwU‡K ¯úk K‡i|

A¼b : $\angle ABC$ I $\angle ACB$ Gi mgwØLÊK h_vµ‡g BL I CM AwK| g‡b Kwi, Zviv O we›`‡Z †Q` K‡i| O †_‡K BC Gi Ici OD j¤ AwK Ges g‡b Kwi, Zv BC †K D we›`‡Z †Q` K‡i| O †K †K›`` K‡i OD Gi mgvb e¨vmva wb‡q GKwU eË AwK| Zvn‡j, GB eËwUB wb‡Yq AšeË|

cgvY : O †_‡K AC I AB Gi Ici h_vµ‡g OE I OF j¤ Uwb| g‡b Kwi, j¤Øq evûØq‡K h_vµ‡g E I F we›`‡Z †Q` K‡i|

O we›` $\angle ABC$ Gi wØLÊ‡Ki Ici Aew¯Z|

$\therefore \quad OF = OD$

Abi/cfv‡e, O we›` $\angle ABC$ Gi wØLÊ‡Ki Ici Aew¯Z e‡j $OF = OD$

$\therefore \quad OD = OE = OF$

mZivs O †K †K›`` K‡i OD Gi mgvb e¨vmva wb‡q eË AvK‡j Zv D, E Ges F we›` w`‡q hv‡e|

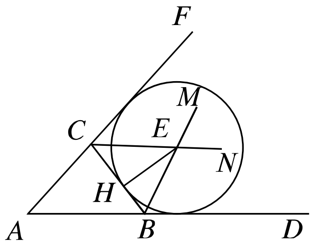
Avevi, OD, OE I OF Gi cvšwe›`‡Z h_vµ‡g BC, AC I AB j¤|

mZivs eËwU ΔABC Gi wfZ‡i †_‡K Gi evû wZbwU‡K h_vµ‡g D, E I F we›`‡Z ¯úk K‡i|

AZGe, DEF eËwUB ΔABC Gi AšeË n‡e|

m¤úv`¨ 6

†Kv‡bv wbw`ó wÎf‡Ri ewneË AvK‡Z n‡e|



g‡b Kwi, ABC GKwU wÎfR| Gi ewneË AvK‡Z n‡e| A_vr, Ggb GKwU eË AvK‡Z n‡e, hv wÎf‡Ri GKwU evû‡K Ges Aci `B evûi ewaZvsk‡K ¯úk K‡i|

A¼b : AB I AC evûØq‡K h_vµg D I F chš ewaZ Kwi| $\angle DBC$ I $\angle FCB$ Gi mgwØLÊK BM Ges CN AwK| g‡b Kwi, E Zv‡`i †Q` we›`| $E$ †_‡K $BC$ Gi Ici $EH$ j¤ AwK Ges g‡b Kwi Zv $BC$ †K $H$ we›`‡Z †Q` K‡i| E †K †K›`` K‡i EH Gi mgvb e¨vmva wb‡q GKwU eË AwK|

Zvn‡j, GB eËwUB wb‡Yq ewneË|

cgvY : E †_‡K BD I CF †iLvs‡ki Ici h_vµ‡g EG I EL j¤ Uwb| g‡b Kwi, j¤Øq, †iLvskØq‡K h_vµ‡g G I L we›`‡Z †Q` K‡i|

E we›`wU $\angle DBC$ Gi wØLÊ‡Ki Ici Aew¯'Z

$\therefore \quad EH = EG$

Abi/cfv‡e, E we›`wU $\angle FCB$ Gi wØLÊ‡Ki Ici Aew¯'Z e‡j $EH = EL$

$\therefore \quad EH = EG = EL$

mZivs E †K †K›`ª K‡i $EL$ Gi mgvb e¨vmva© wb‡q Aw¼Z eË $H, G$ Ges $L$ we›` w`‡q hv‡e|

Avevi, EH, EG I EL Gi cv`we›`‡Z h_vµ‡g BC, BD I CF †iLvsk wZbwU j¤|

mZivs eËwU †iLvsk wZbwU‡K h_vµ‡g H, G I L we›`‡Z ¯úk K‡i|

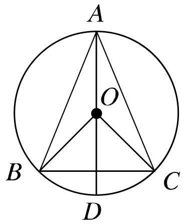
AZGe, HGL eËwUB ΔABC Gi ewneË n‡e|

gšÍe¨ : †Kv‡bv wÎf‡Ri wZbwU ewneË AvKv hvq|

KvR : 1| wÎf‡Ri Aci `BwU ewneË AvK|

Abkxjbx 8·5

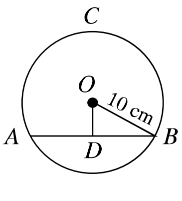
1. wb‡Pi Z_¨¸‡jv j¶ Ki :
   i e‡Ëi ¯úkK ¯úk we›`Mvgx e¨vmv‡ai Ici j¤
   ii AaeË¯' †KvY GK mg‡KvY
   iii e‡Ëi mKj mgvb R¨v †K›`ª †_‡K mg`ieZx©
   wb‡Pi †KvbwU mwVK ?
   K. i I ii   L. i I iii
   M. ii I iii   N. i, ii I iii



Ic‡ii wPÎ Abhvqx 2 I 3 bs c‡ki DËi `vI:

2. $\angle$BOD Gi cwigvY n‡e-
   K. $\frac{1}{2}\angle$BAC   L. $\frac{1}{2}\angle$BAD
   গ. 2 $\angle$BAC   ঘ. 2 $\angle$BAD

3. eËwU ABC wÎf‡Ri-
   K. AšeË
   L. cwieË
   M. ewnteË
   N. DceË
4. †Kv‡bv e‡Ëi AwaPv‡c AšwjwLZ †KvY-
   K. m²‡KvY
   L. mg‡KvY
   M. ¯j †KvY
   N. ciK‡KvY
5. †Kv‡bv e‡Ë Ggb GKwU ¯úkK AvK †hb Zv wbw`ó mij‡iLvi mgvšivj nq|
6. †Kv‡bv e‡Ë Ggb GKwU ¯úkK AvK †hb Zv wbw`ó mij‡iLvi Dci j¤ nq|
7. †Kv‡bv e‡Ë Ggb `BwU ¯úkK AvK †hb Zv‡`i Ašf³ †KvY 60° nq|
8. 3 †m.wg., 4 †m.wg. I 4.5 †m.wg. evûwewkó GKwU wÎf‡Ri cwieË AvK Ges GB e‡Ëi e¨vmva wbYq Ki|
9. 5 †m.wg evûwewkó GKwU mgevû wÎfR ABC Gi AC evû‡K ¯úk Kwi‡q GKwU ewneË AvK|
10. GKwU e‡Mi AšeË I cwieË AvK|
11. cgvY Ki †h, mgwØevû wÎf‡Ri mgvb evûØq‡K e¨vm a‡i `BwU eË A¼b Ki‡j, Zviv f‚wgi ga¨we›`‡K ci¯úi †Q` K‡i|
12. cgvY Ki †h, mg‡KvYx wÎf‡Ri AwZf‡Ri ga¨we›` I wecixZ kx‡li ms‡hvRK †iLvsk AwZf‡Ri A‡aK|
13. ABC GKwU wÎfR| AB †K e¨vm wb‡q Aw¼Z eË hw` $BC$ evû‡K $D$ we›`‡K †Q` K‡i, Z‡e cgvY Ki †h, $AC$ evû‡K e¨vm wb‡q Aw¼Z eËI $D$ we›` w`‡q hv‡e|
14. AB I CD GKB e‡Ë `BwU mgvšivj R¨v| cgvY Ki †h, Pvc AC = Pvc BD .
15. O †K›`wewkó †Kv‡bv e‡Ëi $AB$ I $CD$ R¨v `BwU e‡Ëi Af¨ši¯ E we›`‡Z †Q` Ki‡j cgvY Ki †h, $\angle AEC = \frac{1}{2}(\angle BOD + \angle AOC)$.
16. `BwU mgvb e¨vmwewkó e‡Ëi mvaviY R¨v $AB$| $B$ we›` w`‡q Aw¼Z †Kv‡bv mij‡iLv hw` eË `BwUi mv‡_ $P$ I $Q$ we›`‡Z wgwjZ nq, Z‡e cgvY Ki †h, ΔOAQ mgwØevû|
17. O †K›`wewkó ABC e‡Ë R¨v AB = x †m.wg. OD ⊥ AB
   cv‡ki wPÎ Abhvqx wb‡Pi ck¸‡jvi DËi `vI:



   K. eËwUi †¶Îdj wbYq Ki|
   L. †`LvI †h, D, AB Gi ga¨we›`|
   M. OD = ($\frac{x}{2}$ -2) †m. wg. n‡j x Gi gvb wbYq Ki|
18. GKwU wÎf‡Ri wZbwU evûi ˆ`N¨ h_vµ‡g 4 †m. wg. 5 †m. wg. I 6 †m. wg.
   Ic‡ii Z_¨ Abhvqx wb‡gi ck¸‡jvi DËi `vI:
   K. wÎfRwU A¼b Ki
   L. wÎfRwUi cwieË A¼b Ki|
   M. wÎf‡Ri cwie‡Ëi ewn‡i †h †Kvb GKwU wbw`ó we›` †_‡K e‡Ëi `BwU ¯úkK A¼b K‡i †`LvI †h ¯úkKØ‡qi `iZ mgvb nq|

beg Aa¨vq
wÎ‡KvYwgwZK AbcvZ

(Trigonometric Ratios)

Avgiv cwZwbqZ wÎfR, we‡kl K‡i mg‡KvYx wÎf‡Ri e¨envi K‡i _wK| Avgv‡`i Pvwiw`‡Ki cwi‡e‡k bvbv D`vniY †`Lv hvq †hLv‡b Kíbvq mg‡KvYx wÎfR MVb Kiv hvq| †mB cvPxb h‡M gvbl R¨wgwZi mvnv‡h¨ b`xi Zx‡i `wo‡q b`xi c¯ wbYq Kivi †KŠkj wk‡LwQj| Mv‡Q bv D‡V I Mv‡Qi Qvqvi m‡½ jwVi Zjbv K‡i wbLZfv‡e Mv‡Qi D”PZv gvc‡Z wk‡LwQj| GB MwYwZK †KŠkj †kLv‡bvi Rb¨ mwó n‡q‡Q wÎ‡KvYwgwZ bv‡g MwY‡Zi GK we‡kl kvLv| Trigonometry kãwU wMK kã tri(A_ wZb) gon(A_ avi) metron(A_ cwigvc) Øviv MwVZ| wÎ‡KvYwgwZ‡Z wÎf‡Ri evû I †Kv‡Yi g‡a¨ m¤úK we‡kølY Kiv nq| wgki I e¨wejbxq mf¨Zvq wÎ‡KvYwgwZ e¨env‡ii wb`kb i‡q‡Q| wgkixqiv fwg Rwic I c‡KŠkj Kv‡R Gi eûj e¨envi KiZ e‡j aviYv Kiv nq| Gi mvnv‡h¨ †R¨wZwe©`MY c_w_ex †_‡K `ieZx Mn-b¶‡Îi `iZ wbYq Ki‡Zb| Aabv wÎ‡KvYwgwZi e¨envi MwY‡Zi mKj kvLvq| wÎfR msµvš— mgm¨vi mgvavb, †bwf‡Mkb BZ¨w` †¶‡Î wÎ‡KvYwgwZi e¨vcK e¨envi n‡q _v‡K| MwY‡Zi ¸iæZcY †R¨wZwe©Ávb kvLvmn K¨vjKyj‡m Gi eûj e¨envi i‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv–

- m²‡Kv‡Yi wÎ‡KvYwgwZK AbcvZ eYbv Ki‡Z cvi‡e|
- m²‡Kv‡Yi wÎ‡KvYwgwZK AbcvZ¸‡jvi g‡a¨ cvi¯úwiK m¤úK wbYq Ki‡Z cvi‡e|
- m²‡Kv‡Yi wÎ‡KvYwgwZK AbcvZ¸‡jvi aeZv hvPvB K‡i cgvY I MwYwZK mgm¨v mgvavb Ki‡Z cvi‡e|
- R¨wgwZK c×wZ‡Z $30^\circ, 45^\circ, 60^\circ$ †Kv‡Yi wÎ‡KvYwgwZK Abcv‡Zi gvb wbYq I c‡qvM Ki‡Z cvi‡e|
- 0° I 90° †Kv‡Yi A_cY wÎ‡KvYwgwZK AbcvZ¸‡jvi gvb wbYq K‡i c‡qvM Ki‡Z cvi‡e|
- wÎ‡KvYwgwZK A‡f`vewj cgvY Ki‡Z cvi‡e|
- wÎ‡KvYwgwZK A‡f`vewji c‡qvM Ki‡Z cvi‡e|

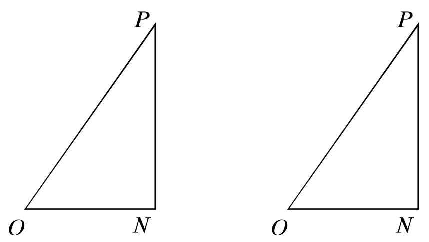
9·1 mg‡KvYx wÎf‡Ri evû¸‡jvi bvgKiY

Avgiv Rwb, mg‡KvYx wÎf‡Ri evû¸‡jv AwZfR, fwg I DbwZ bv‡g AwfwnZ nq| wÎf‡Ri AbfwgK Ae¯v‡bi Rb¨ G bvgmgn mv_K| Avevi mg‡KvYx wÎf‡Ri m²‡KvYØ‡qi GKwUi mv‡c¶ Ae¯v‡bi †cw¶‡Z I evû¸‡jvi bvgKiY Kiv nq| h_v:

K. ÔAwZfRÕ, mg‡KvYx wÎf‡Ri enËg evû hv mg‡Kv‡Yi wecixZ evû

L. ÔwecixZ evûÕ, hv n‡jv c`Ë †Kv‡Yi mivmwi wecixZ w`‡Ki evû

M. ÔmwbwnZ evûÕ, hv c`Ë †KvY mwóKvix GKwU †iLvsk|



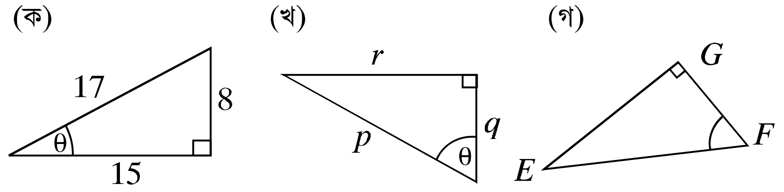
$\angle PON$ †Kv‡Yi Rb¨ AwZfR OP, mwbœwnZ evû ON, wecixZ evû PN	$\angle OPN$ †Kv‡Yi Rb¨ AwZfR OP, mwbœwnZ evû PN, wecixZ evû ON

R¨vwgwZK wP‡Îi kxl©we›`y wPwýZ Kivi Rb¨ eo nv‡Zi eY© I evû wb‡`©k Ki‡Z †QvU nv‡Zi eY© e¨envi Kiv nq| †KvY wb‡`©‡ki Rb¨ cÖvqkB wMÖK eY© e¨eüZ nq| wMÖK eY©gvjvi QqwU eûj e¨eüZ eY© n‡jv :

alpha α	beta β	gamma γ	theta θ	phi ϕ	omega ω
(Avjdv)	(wewUv)	(Mvgv)	(w_Uv)	(cvB)	(I‡gMv)

cÖvPxb wMÖ‡mi weL¨vZ me MwYZwe`‡`i nvZ a‡iB R¨vwgwZ I wÎ‡KvYwgwZ‡Z wMÖK eY¸‡jv e¨envi n‡q Avm‡Q|

D`vniY 1| θ †Kv‡Yi Rb¨ AwZfR, mwbœwnZ evû I wecixZ evû wPwýZ Ki|



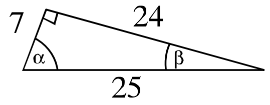
mgvavb :

(K) AwZfR 17 GKK
wecixZ evû 8 GKK
mwbœwnZ evû 15 GKK

(L) AwZfR p
wecixZ evû r
mwbœwnZ evû q

(M) AwZfR EF
wecixZ evû EG
mwbœwnZ evû FG

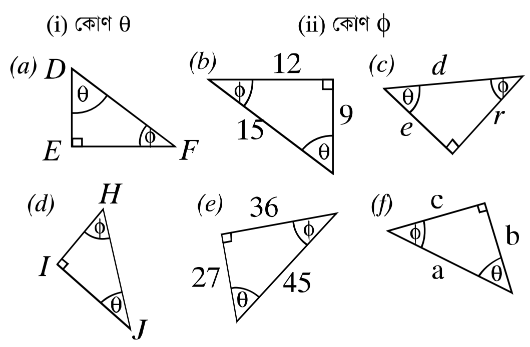
D`vniY 2| $\alpha$ I $\beta$ †Kv‡Yi Rb¨ AwZfR, mwbœwnZ evû I wecixZ evûi ˆ`N¨© wbY©q Ki|



(K) α †Kv‡Yi Rb¨
AwZfR 25 GKK
wecixZ evû 24 GKK
mwbœwnZ evû 7 GKK

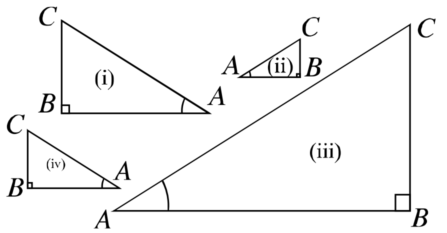
(L) β †Kv‡Yi Rb¨
AwZfR 25 GKK
wecixZ evû 7 GKK
mwbœwnZ evû 24 GKK

KvR :
θ I ϕ †Kv‡Yi Rb¨ AwZfR, mwbœwnZ evû I wecixZ evû wb‡`©k Ki|



9·2 m`k mg‡KvYx wÎf‡Ri evû¸‡jvi AbcvZmg‡ni aeZv

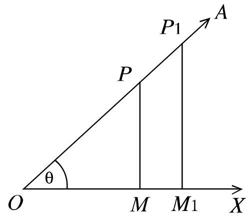
KvR : wb‡Pi PviwU m`k mg‡KvYx wÎf‡Ri evû¸‡jvi ˆ`N¨ †g‡c mviwYwU ciY Ki| wÎf‡Ri AbcvZ¸‡jv m¤ú‡K Kx j¶ Ki ?



evûi ˆ`N¨			AbcvZ (†Kv‡Yi mv‡c‡¶)		
BC	AB	AC	BC/AC	AB/AC	BC/AB

g‡b Kwi, $\angle XOA$ GKwU m²‡KvY| OA evû‡Z †h‡Kv‡bv GKwU we›`y $P$ wbB| $P$ †_‡K $OX$ evû chš‘ $PM$ j¤ Uvwb| d‡j GKwU mg‡KvYx wÎfR $POM$ MwVZ n‡jv| GB $\Delta POM$ Gi $PM, OM$ I $OP$ evû¸‡jvi †h wZbwU AbcvZ cvIqv hvq Zv‡`i gvb OA evû‡Z wbewPZ P we›`yi Ae¯'v‡bi Ici wbfi K‡i bv|

$\angle XOA$ †Kv‡Yi OA evû‡Z †h‡Kv‡bv we›`y $P$ I $P_1$ †_‡K $OX$ evû chš‘ h_vµ‡g $PM$ I $P_1M_1$ j¤ A¼b Ki‡j $\Delta POM$ I $\Delta P_1OM_1$ `BwU m`k mg‡KvYx wÎfR MwVZ nq|



GLb, ΔPOM I ΔP_1OM_1 m`k nIqvq,

$$\frac{PM}{P_1M_1} = \frac{OP}{OP_1} \quad \text{ev,} \quad \frac{PM}{OP} = \frac{P_1M_1}{OP_1} \; \ldots\ldots (i)$$

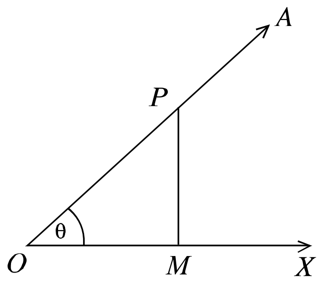
$$\frac{OM}{OM_1} = \frac{OP}{OP_1} \quad \text{ev,} \quad \frac{OM}{OP} = \frac{OM_1}{OP_1} \quad .... (ii)$$

$$\frac{PM}{P_1M_1} = \frac{OM}{OM_1} \quad \text{ev,} \quad \frac{PM}{OM} = \frac{P_1M_1}{OM_1} \quad .... (iii)$$

A_vr, AbcvZmg‡ni c‡Z¨KwU a"eK| GB AbcvZmgn‡K wÎ‡KvYwgwZK AbcvZ e‡j|

9·3 m² ‡Kv‡Yi wÎ‡KvYwgwZK AbcvZ

g‡b Kwi, $\angle XOA$ GKwU m²‡KvY| OA evû‡Z †h‡Kv‡bv GKwU we›`y $P$ wbB| $P$ †\_‡K $OX$ evû chš— $PM$ j¤ Uwb| d‡j GKwU mg‡KvYx wÎfR $POM$ MwVZ n‡jv| GB $\Delta POM$ Gi $PM, OM$ I $OP$ evû¸‡jvi †h QqwU AbcvZ cvIqv hvq Zv‡`i $\angle XOA$ Gi wÎ‡KvYwgwZK AbcvZ ejv nq Ges Zv‡`i c‡Z¨KwU‡K GK GKwU mwbw`ó bv‡g bvgKiY Kiv nq|



$\angle XOA$ mv‡c‡¶ mg‡KvYx wÎfR POM Gi PM wecixZ evû, OM mwbwnZ evû, OP AwZfR| GLb $\angle XOA = \theta$ ai‡j, θ †Kv‡Yi †h QqwU wÎ‡KvYwgwZK AbcvZ cvIqv hvq Zv wb‡g eYbv Kiv n‡jv|

wPÎ †_‡K,

$$\sin\theta = \frac{PM}{OP} = \frac{\text{wecixZ evû}}{\text{AwZfR}} \quad [\ \theta \text{ †Kv‡Yi mvBb } (\sin e)\ ]$$

$$\cos\theta = \frac{OM}{OP} = \frac{\text{mwbwnZ evû}}{\text{AwZfR}} \quad [\theta \text{ †Kv‡Yi †KvmvBb } \cos ine\ ]$$

$$\tan\theta = \frac{PM}{OM} = \frac{\text{wecixZ evû}}{\text{mwbwnZ evû}} \quad [\theta \text{ †Kv‡Yi U¨vb‡R›U } \tan gent\,]$$

Ges G‡`i wecixZ AbcvZ

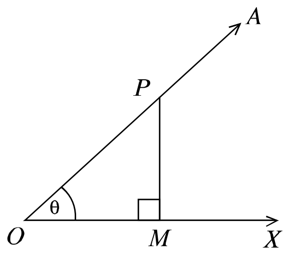
$$\cos ec\theta = \frac{1}{\sin\theta} \quad [\theta \text{ †Kv‡Yi †Kv‡mK¨v›U } \text{cosec}ant\,]$$

$$\sec\theta = \frac{1}{\cos\theta} \quad [\theta \text{ †Kv‡Yi †mK¨v›U } \sec ant\,]$$

$$\cot\theta = \frac{1}{\tan\theta} \quad [\theta \text{ †Kv‡Yi †KvU¨vb‡R›U } \cot angent\,]$$

j¶ Kwi, $\sin\theta$ cZxKwU θ †Kv‡Yi mvBb-Gi AbcvZ‡K †evSvq; sin I θ Gi ¸Ydj‡K bq| θ ev‡` sin Avjv`v †Kv‡bv A_ enb K‡i bv| wÎ‡KvYwgwZK Ab¨vb¨ AbcvZ¸‡jvi †¶‡Î I welqwU c‡hvR¨|

9·4 wÎ‡KvYwgwZK AbcvZ¸‡jvi m¤úK



g‡b Kwi, $\angle XOA = \theta$ GKwU m² ‡KvY|

cv‡ki wPÎ mv‡c‡¶, msÁvbhvqx,

$$\sin\theta = \frac{PM}{OP}, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{OP}{PM}$$

$$\cos\theta = \frac{OM}{OP}, \quad \sec\theta = \frac{1}{\cos\theta} = \frac{OP}{OM}$$

$$\tan\theta = \frac{PM}{OM}, \quad \cot\theta = \frac{1}{\tan\theta} = \frac{OM}{PM}$$

Avevi, $\tan\theta = \dfrac{PM}{OM} = \dfrac{\frac{PM}{OP}}{\frac{OM}{OP}}$ [je I ni‡K OP Øviv fvM K‡i]

$$= \frac{\sin\theta}{\cos\theta}$$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Ges GKBfv‡e,

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

9·5 wÎ‡KvYwgwZK A‡f`vewj

(i) $(\sin\theta)^2 + (\cos\theta)^2 = \left(\dfrac{PM}{OP}\right)^2 + \left(\dfrac{OM}{OP}\right)^2$

$= \dfrac{PM^2}{OP^2} + \dfrac{OM^2}{OP^2} = \dfrac{PM^2 + OM^2}{OP^2} = \dfrac{OP^2}{OP^2}$ [wc_v‡Mvivm mÎ]

$= 1$

ev, $(\sin\theta)^2 + (\cos\theta)^2 = 1$

$$\therefore \sin^2\theta + \cos^2\theta = 1$$

gšev : cYmsL¨v mPK n Gi Rb¨ $(\sin\theta)^n$ †K $(\sin^n\theta), (\cos\theta)^n$ †K $\cos^n\theta$ BZ¨w` †jLv nq|

(ii) $\sec^2\theta = (\sec\theta)^2 = \left(\dfrac{OP}{OM}\right)^2$

$= \dfrac{OP^2}{OM^2} = \dfrac{PM^2 + OM^2}{OM^2}$ [OP mg‡KvYx ΔPOM Gi AwZfR e‡j]

$$= \frac{PM^2}{OM^2} + \frac{OM^2}{OM^2}$$

$$= 1 + \left(\frac{PM}{OM}\right)^2 = 1 + (\tan\theta)^2 = 1 + \tan^2\theta$$

$\therefore\ \sec^2\theta = 1 + \tan^2\theta$

ev, $\boxed{\sec^2\theta - \tan^2\theta = 1}$

ev, $\boxed{\tan^2\theta = \sec^2\theta - 1}$

(iii) $\operatorname{cosec}^2\theta = (\operatorname{cosec}\theta)^2 = \left(\frac{OP}{PM}\right)^2$

$$= \frac{OP^2}{PM^2} = \frac{PM^2 + OM^2}{PM^2}$$ [OP mg‡KvYx ΔPOM Gi AwZfR e‡j]

$$= \frac{PM^2}{PM^2} + \frac{OM^2}{PM^2} = 1 + \left(\frac{OM}{PM}\right)^2$$

$$= 1 + (\cot\theta)^2 = 1 + \cot^2\theta$$

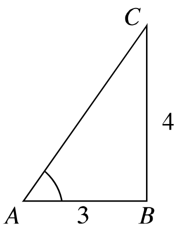
$\therefore$ $\boxed{\operatorname{cosec}^2\theta - \cot^2\theta = 1}$ Ges $\boxed{\cot^2\theta = \operatorname{cosec}^2\theta - 1}$

KvR t

1| wb‡Pi wÎ‡KvYwgwZK mÎ¸‡jv mn‡R g‡b ivLvi Rb¨ ZvwjKv ˆZwi Ki|

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\tan\theta = \frac{1}{\cot\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\sin^2\theta + \cos^2\theta = 1$ $\sec^2\theta = 1 + \tan^2\theta$ $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

D`vniY 1| $\tan A = \frac{4}{3}$ n‡j, A †Kv‡Yi Ab¨vb¨ wÎ‡KvYwgwZK AbcvZmgn wbYq Ki|



mgvavb : †`Iqv Av‡Q, $\tan A = \frac{4}{3}$.

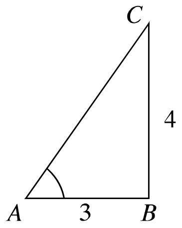
AZGe, A †Kv‡Yi wecixZ evû = 4, mwbwnZ evû = 3

AwZfR = $\sqrt{4^2 + 3^2} = \sqrt{25} = 5$

mZivs, $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$, $\cot A = \frac{3}{4}$

$cosec\, A = \frac{5}{4}$, $\sec A = \frac{5}{3}$.

D`vniY 2| ABC mg‡KvYx wÎf‡Ri $\angle B$ †KvYwU mg‡KvY| $\tan A = 1$ n‡j $2\sin A \cos A = 1$ Gi mZ¨Zv hvPvB Ki|



mgvavb : †`Iqv Av‡Q, $\tan A = \frac{4}{3}$.

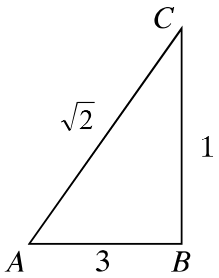
AZGe, A †Kv‡Yi wecixZ evû = 4, mwbwnZ evû = 3

AwZfR = $\sqrt{4^2 + 3^2} = \sqrt{25} = 5$

mZivs, $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$, $\cot A = \frac{3}{4}$

$cosec\, A = \frac{5}{4}$, $\sec A = \frac{5}{3}$.

D`vniY 2| ABC mg‡KvYx wÎf‡Ri $\angle B$ †KvYwU mg‡KvY| $\tan A = 1$ n‡j $2\sin A \cos A = 1$ Gi mZ¨Zv hvPvB Ki|



mgvavb : †`Iqv Av‡Q, $\tan A = 1$.

AZGe, wecixZ evû = mwbwnZ evû = 1

AwZfR = $\sqrt{1^2 + 1^2} = \sqrt{2}$

mZivs, $\sin A = \frac{1}{\sqrt{2}}$, $\cos A = \frac{1}{\sqrt{2}}$.

GLb evgc¶ = $2\sin A \cos A = 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 2 \cdot \frac{1}{2} = 1$

= Wvbc¶ |

$\therefore 2\sin A \cos A = 1$ evK¨wU mZ¨|

KvR :

1| ABC mg‡KvYx wÎf‡Ri $\angle C$ mg‡KvY, $AB = 29$ †m.wg., $BC = 21$ †m.wg. Ges $\angle ABC = \theta$ n‡j, $\cos^2\theta - \sin^2\theta$ Gi gvb †ei Ki|

D`vniY 3| cgvY Ki †h, $\tan\theta + \cot\theta = \sec\theta . cosec\theta$.

mgvavb :

evgc¶ = $\tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta}$$

$$= \frac{1}{\sin\theta \cdot \cos\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}$$

$= cosec\theta \cdot \sec\theta$

$= \sec\theta \cdot cosec\theta$ = Wvbc¶ (cgwYZ)|

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$$

$$= \frac{1}{\cos^2\theta \sin^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\cos^2\theta} \cdot \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta \cdot \operatorname{cosec}^2\theta$$

= Wvbc¶ (cgwYZ)|

D`vniY 5| cgvY Ki †h, $\frac{1}{1+\sin^2\theta} + \frac{1}{1+\operatorname{cosec}^2\theta} = 1$

mgvavb : evgc¶ $= \frac{1}{1+\sin^2\theta} + \frac{1}{1+\operatorname{cosec}^2\theta}$

$$= \frac{1}{1+\sin^2\theta} + \frac{1}{1+\frac{1}{\sin^2\theta}}$$

$$= \frac{1}{1+\sin^2\theta} + \frac{\sin^2\theta}{1+\sin^2\theta}$$

$$= \frac{1+\sin^2\theta}{1+\sin^2\theta}$$

= 1 = Wvbc¶ (cgwYZ)|

D`vniY 6| cgvY Ki : $\frac{1}{2-\sin^2 A} + \frac{1}{2+\tan^2 A} = 1$

mgvavb : evgc¶ $= \frac{1}{2-\sin^2 A} + \frac{1}{2+\tan^2 A}$

$$= \frac{1}{2-\sin^2 A} + \frac{1}{2+\frac{\sin^2 A}{\cos^2 A}}$$

$$= \frac{1}{2-\sin^2 A} + \frac{\cos^2 A}{2\cos^2 A + \sin^2 A}$$

$$= \frac{1}{2-\sin^2 A} + \frac{\cos^2 A}{2(1-\sin^2 A) + \sin^2 A}$$

$$= \frac{1}{2-\sin^2 A} + \frac{\cos^2 A}{2 - 2\sin^2 A + \sin^2 A}$$

$$= \frac{1}{2-\sin^2 A} + \frac{1-\sin^2 A}{2-\sin^2 A}$$

$$= \frac{2-\sin^2 A}{2-\sin^2 A}$$

= 1 = Wvbc¶ (cgwYZ)

D`vniY 7| cgvY Ki : $\frac{\tan A}{\sec A+1}-\frac{\sec A-1}{\tan A}=0$

mgvavb : evgc¶ = $\frac{\tan A}{\sec A+1}-\frac{\sec A-1}{\tan A}$

$= \frac{\tan^2 A-(\sec^2 A-1)}{(\sec A+1)\tan A}$ $[\because \sec^2 A-1=\tan^2 A]$

$= \frac{\tan^2 A-\tan^2 A}{(\sec A+1)\tan A}$

$= \frac{0}{(\sec A+1)\tan A}$

$= 0$ = Wvbc¶ (cgvYZ)

D`vniY 8| cgvY Ki : $\sqrt{\frac{1-\sin A}{1+\sin A}}=\sec A-\tan A$

mgvavb : evgc¶ = $\sqrt{\frac{1-\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1-\sin A)(1-\sin A)}{(1+\sin A)(1-\sin A)}}$ [je I ni‡K $\sqrt{(1-sin A)}$ Øviv ¸Y K‡i]

$= \sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}}$

$= \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$

$= \frac{1-\sin A}{\cos A}$

$= \frac{1}{\cos A}$ Ñ $\frac{\sin A}{\cos A}$

$= \sec A-\tan A$

= Wvbc¶ (cgvYZ)

D`vniY 9| $\tan A+\sin A=a$ Ges $\tan A-\sin A=b$ n‡j, cgvY Ki †h, $a^2-b^2=4\sqrt{ab}$.

mgvavb : GLv‡b c`Ë, $\tan A+\sin A=a$ Ges $\tan A-\sin A=b$

evgc¶ $= a^2-b^2$

$= (\tan A+\sin A)^2-(\tan A-\sin A)^2$

$= 4\tan A\sin A$ $[\because (a+b)^2-(a-b)^2=4ab]$

$= 4\sqrt{\tan^2 A\sin^2 A}$

$= 4\sqrt{\tan^2 A\,(1-\cos^2 A)}$

$= 4\sqrt{\tan^2 A - \tan^2 A \cdot \cos^2 A}$

$= 4\sqrt{\tan^2 A - \sin^2 A}$

$= 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$

$= 4\sqrt{ab}$

= Wvbc¶ (cgwYZ)

KvR : 1| $\cot^4 A - \cot^2 A = 1$ n‡j, cgvY Ki †h, $\cos^4\theta + \cos^2 A = 1$
2| $\sin^2 A + \sin^4 A = 1$ n‡j, cgvY Ki †h, $\tan^4 A + \tan^2 A = 1$

D`vniY 10| $\sec A + \tan A = \frac{5}{2}$ n‡j, $\sec A - \tan A$ Gi gvb wbYq Ki|

mgvavb : GLv‡b c`Ë, $\sec A + \tan A = \frac{5}{2}$.............(i)

Avgiv Rwb, $\sec^2 A = 1 + \tan^2 A$

ev, $\sec^2 A - \tan^2 A = 1$

ev, $(\sec A + \tan A)(\sec A - \tan A) = 1$

ev, $\frac{5}{2}(\sec A - \tan A) = 1$ [(i) n‡Z]

$\therefore \quad \sec A - \tan A = \frac{2}{5}$

Abkxjbx 9·1

1| wb‡Pi MwYwZK Dw³¸‡jvi mZ¨-wg_¨v hvPvB Ki| †Zvgvi DË‡ii c‡¶ hw³ `vI|

K. $\tan A$ Gi gvb me`v 1 Gi †P‡q Kg

L. $\cot A$ n‡jv cot I A Gi ¸Ydj

M. A Gi †Kvb gv‡bi Rb¨ $\sec A = \frac{12}{5}$

N. cos n‡jv cotangent Gi msw¶ß i/c

2| $\sin A = \frac{3}{4}$ n‡j, A †Kv‡Yi Ab¨vb¨ wÎ‡KvYwgwZK AbcvZmgn wbYq Ki|

3| †`Iqv Av‡Q, $15\cot A = 8$, $\sin A$ I $\sec A$ Gi gvb †ei Ki|

$= 4\sqrt{\tan^2 A\,(1-\cos^2 A)}$

$= 4\sqrt{\tan^2 A - \tan^2 A \cdot \cos^2 A}$

$= 4\sqrt{\tan^2 A - \sin^2 A}$

$= 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$

$= 4\sqrt{ab}$

= Wvbc¶ (cgwYZ)

KvR : 1| $\cot^4 A - \cot^2 A = 1$ n‡j, cgvY Ki †h, $\cos^4\theta + \cos^2 A = 1$ 2| $\sin^2 A + \sin^4 A = 1$ n‡j, cgvY Ki †h, $\tan^4 A + \tan^2 A = 1$

D`vniY 10| $\sec A + \tan A = \frac{5}{2}$ n‡j, $\sec A - \tan A$ Gi gvb wbYq Ki|

mgvavb : GLv‡b c`Ë, $\sec A + \tan A = \frac{5}{2}$.............(i)

Avgiv Rvwb, $\sec^2 A = 1 + \tan^2 A$

ev, $\sec^2 A - \tan^2 A = 1$

ev, $(\sec A + \tan A)(\sec A - \tan A) = 1$

ev, $\frac{5}{2}(\sec A - \tan A) = 1$ [(i) n‡Z]

$\therefore \quad \sec A - \tan A = \frac{2}{5}$

Abkxjbx 9·1

1| wb‡Pi MwYwZK Dw³¸‡jvi mZ¨-wg_¨v hvPvB Ki| †Zvgvi DË‡ii c‡¶ hyw³ `vI|

K. $\tan A$ Gi gvb me©`v 1 Gi †P‡q Kg

L. $\cot A$ n‡jv cot I A Gi ¸Ydj

M. A Gi †Kvb gv‡bi Rb¨ $\sec A = \frac{12}{5}$

N. cos n‡jv cotangent Gi msw¶ß iƒc

2| $\sin A = \frac{3}{4}$ n‡j, A †Kv‡Yi Ab¨vb¨ wÎ‡KvYwgwZK AbycvZmgn wbYq Ki|

3| †`Iqv Av‡Q, $15\cot A = 8$, $\sin A$ I $\sec A$ Gi gvb †ei Ki|

4| ABC mg‡KvYx wÎf‡Ri $\angle C$ mg‡KvY, AB = 13 †m.wg., BC = 12 †m.wg. Ges $\angle ABC = \theta$ n‡j, $\sin\theta$, $\cos\theta$ I $\tan\theta$ Gi gvb †ei Ki|

5| ABC mg‡KvYx wÎf‡Ri $\angle B$ †KvYwU mg‡KvY| $\tan A = \sqrt{3}$ n‡j, $\sqrt{3}\sin A\cos A = 4$ Gi mZ¨Zv hvPvB Ki|

cgvY Ki (6 Ñ 20) :

6| (*i*) $\frac{1}{\sec^2 A} + \frac{1}{\mathrm{cosec}^2 A} = 1$; (*ii*) $\frac{1}{\cos^2 A} - \frac{1}{\cot^2 A} = 1$; (*iii*) $\frac{1}{\sin^2 A} - \frac{1}{\tan^2 A} = 1$;

7| (*i*) $\frac{\sin A}{\mathrm{cosec} A} + \frac{\cos A}{\sec A} = 1$; (*ii*) $\frac{\sec A}{\cos A} - \frac{\tan A}{\cot A} = 1$.

(*iii*) $\frac{1}{1+\sin^2 A} + \frac{1}{1+\mathrm{cosec}^2 A} = 1$

8| (*i*) $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \sec A\,\mathrm{cosec} A + 1$; (*ii*) $\frac{1}{1+\tan^2 A} + \frac{1}{1+\cot^2 A} = 1$

9| $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A.$

10| $\tan A\sqrt{1-\sin^2 A} = \sin A.$

11| $\frac{\sec A + \tan A}{\mathrm{cosec} A + \cot A} = \frac{\mathrm{cosec} A - \cot A}{\sec A - \tan A}$

12| $\frac{\mathrm{cosec} A}{\mathrm{cosec} A - 1} + \frac{\mathrm{cosec} A}{\mathrm{cosec} A + 1} = 2\sec^2 A.$

13| $\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = 2\sec^2 A.$

14| $\frac{1}{\mathrm{cosec} A - 1} - \frac{1}{\mathrm{cosec} A + 1} = 2\tan^2 A.$

15| $\frac{\sin A}{1-\cos A} + \frac{1-\cos A}{\sin A} = 2\,\mathrm{cosec}\,A.$

16| $\frac{\tan A}{\sec A + 1} - \frac{\sec A - 1}{\tan A} = 0$

17| $(\tan\theta + \sec\theta)^2 = \frac{1+\sin\theta}{1-\sin\theta}$

18| $\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \,.\, \tan B.$

19| $\sqrt{\frac{1-\sin A}{1+\sin A}} = \sec A - \tan A.$

20| $\sqrt{\frac{\sec A + 1}{\sec A - 1}} = \cot A + \mathrm{cosec}\,A.$

21| $\cos A + \sin A = \sqrt{2}\cos A$ n‡j, Z‡e cgvY Ki †h, $\cos A - \sin A = \sqrt{2}\sin A$

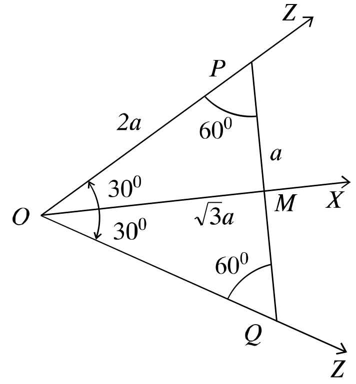
22| hw` $\tan A = \frac{1}{\sqrt{3}}$ nq, Z‡e $\frac{\mathrm{cosec}^2 A - \sec^2 A}{\mathrm{cosec}^2 A + \sec^2 A}$ Gi gvb wbYq Ki|

23| $\mathrm{cosec} A - \cot A = \frac{4}{3}$ n‡j, $\mathrm{cosec} A + \cot A$ Gi gvb KZ ?

24| $\cot A = \frac{b}{a}$ n‡j, $\frac{a\sin A - b\cos A}{a\sin A + b\cos A}$ Gi gvb wbYq Ki|

9·6 30°, 45° I 60° †Kv‡Yi wÎ‡KvYwgwZK AbcvZ

R¨wgwZK Dcv‡q 30°, 45° I 60° cwigv‡ci †KvY AvK‡Z wk‡LwQ| G mKj †Kv‡Yi wÎ‡KvYwgwZK Abcv‡Zi cKZ gvb R¨wgwZK c×wZ‡Z wbYq Kiv hvq|



30° I 60° †Kv‡Yi wÎ‡KvYwgwZK AbcvZ

g‡b Kwi, $\angle XOZ = 30^\circ$ Ges OZ evû‡Z P GKwU we›`| $PM \perp OX$ AwK Ges PM †K Q chš— ewaZ Kwi †hb $MQ = PM$ nq| O, Q †hvM K‡i Z chš— ewaZ Kwi

GLb ΔPOM I ΔQOM Gi g‡a¨ $PM = QM$,

OM mvaviY evû Ges Aš—f³ $\angle PMO$ = Aš—f³ $\angle QMO = 90^\circ$

$\therefore \ \Delta POM \cong \Delta QOM$

AZGe, $\angle QOM = \angle POM = 30^\circ$

Ges $\angle OQM = \angle OPM = 60^\circ$

Avevi, $\angle POQ = \angle POM + \angle QOM = 30^\circ + 30^\circ = 60^\circ$

$\therefore \ \Delta OPQ$ GKwU mgevû wÎfR|

hw` $OP = 2a$ nq, Z‡e $PM = \frac{1}{2}PQ = \frac{1}{2}OP = a$ [†h‡nZ ΔOPQ GKwU mgevû wÎfR]

mg‡KvYx ΔOPM n‡Z cvB,

$OM = \sqrt{OP^2 - PM^2} = \sqrt{4a^2 - a^2} = \sqrt{3}a$.

wÎ‡KvYwgwZK AbcvZmgn †ei Kwi :

$\therefore \ \sin 30^\circ = \frac{PM}{OP} = \frac{a}{2a} = \frac{1}{2}$, $\cos\ 30^\circ = \frac{OM}{OP} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{PM}{OM} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$.

$\text{cosec}\ 30^\circ = \frac{OP}{PM} = \frac{2a}{a} = 2$, $\sec\ 30^\circ = \frac{OP}{OM} = \frac{2a}{\sqrt{3}a} = \frac{2}{\sqrt{3}}$

$\cot\ 30^\circ = \frac{OM}{PM} = \frac{\sqrt{3}a}{a} = \sqrt{3}$.

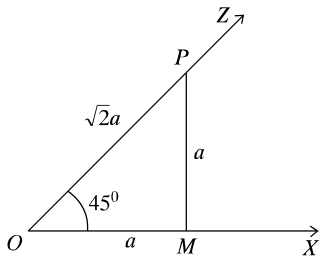
GKBfv‡e,

$$\sin\ 60^\circ = \frac{OM}{OP} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2},\ \cos\ 60^\circ = \frac{PM}{OP} = \frac{a}{2a} = \frac{1}{2},\ \tan\ 60^\circ = \frac{OM}{PM} = \frac{\sqrt{3}a}{a} = \sqrt{3}$$

$$\operatorname{cosec}\ 60^\circ = \frac{OP}{OM} = \frac{2a}{\sqrt{3}a} = \frac{2}{\sqrt{3}},\ \sec\ 60^\circ = \frac{OP}{PM} = \frac{2a}{a} = 2,\ \cot\ 60^\circ = \frac{PM}{OM} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

45° †Kv‡Yi wÎ‡KvYwgwZK AbycvZ

g‡b Kwi, $\angle XOZ = 45^\circ$ Ges P, OZ Gi Dci¯ GKwU we›`| $PM \perp OX$ AwK|



ΔOPM mg‡KvYx wÎfzR $\angle POM = 45^\circ$

mZivs, $\angle OPM = 45^\circ$

AZGe, $PM = OM = a$ (g‡b Kwi)

GLb, $OP^2 = OM^2 + PM^2 = a^2 + a^2 = 2a^2$

ev, $OP = \sqrt{2}\,a$

wÎ‡KvYwgwZK Abycv‡Zi msÁv †_‡K Avgiv cvB,

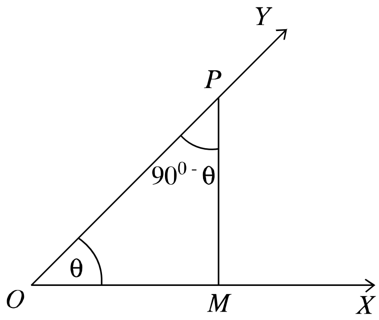
$$\sin 45^\circ = \frac{PM}{OP} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}},\ \cos\ 45^\circ = \frac{OM}{OP} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}},\ \tan\ 45^\circ = \frac{PM}{OM} = \frac{a}{a} = 1$$

$$\operatorname{cosec}\ 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2},\ \sec\ 45^\circ = \frac{1}{\cos 45^\circ} = \sqrt{2},\ \cot\ 45^\circ = \frac{1}{\tan 45^\circ} = 1$$

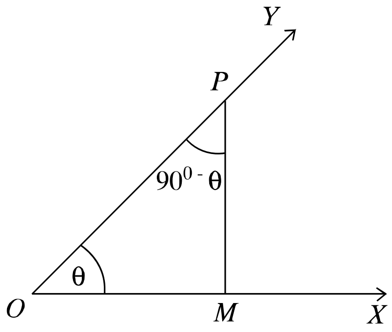
9·7 ciK †Kv‡Yi wÎ‡KvYwgwZK AbycvZ

Avgiv Rwb †h, `BwU m²‡Kv‡Yi cwigv‡ci mgwó $90^\circ$ n‡j, Zv‡`i GKwU‡K AciwUi ciK †KvY ejv nq| †hgb, 30° I 60° Ges 15° I 75° ci¯úi ciK †KvY|

mvaviYfv‡e, θ †KvY I $(90^\circ - \theta)$ †KvY ci¯ú‡ii ciK †KvY|



ciK †Kv‡Yi wÎ‡KvYwgwZK AbycvZ



g‡b Kwi, $\angle XOY = \theta$ Ges P GB †Kv‡Yi OY evûi Dci GKwU we›`| $PM \perp OX$ AwK|

†h‡nZz wÎf‡Ri wZb †Kv‡Yi mgwó `B mg‡KvY,

AZGe, POM mg‡KvYx wÎf‡R $\angle PMO = 90^\circ$

Ges $\angle OPM + \angle POM$ = GK mg‡KvY = 90°

$\therefore \angle OPM = 90^\circ - \angle POM = 90^\circ - \theta$

[†h‡nZz $\angle POM = \angle XOY = \theta$]

$$\therefore \quad \sin(90^\circ - \theta) = \frac{OM}{OP} = \cos\angle POM = \cos\theta$$

$$\cos(90^\circ - \theta) = \frac{PM}{OP} = \sin\angle POM = \sin\theta$$

$$\tan(90^\circ - \theta) = \frac{OM}{PM} = \cot\angle POM = \cot\theta$$

$$\cot(90^\circ - \theta) = \frac{PM}{OM} = \tan\angle POM = \tan\theta$$

$$\sec(90^\circ - \theta) = \frac{OP}{PM} = \operatorname{cosec}\angle POM = \operatorname{cosec}\theta$$

$$\operatorname{cosec}(90^\circ - \theta) = \frac{OP}{OM} = \sec\angle POM = \sec\theta .$$

Dc‡ii mÎ¸‡jv wbgwjwLZfv‡e K_vq cKvk Kiv hvq :

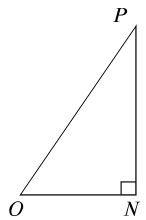
ciK †Kv‡Yi *sine* = †Kv‡Yi *cosine*;

ciK †Kv‡Yi *cosine* = †Kv‡Yi *sine*;

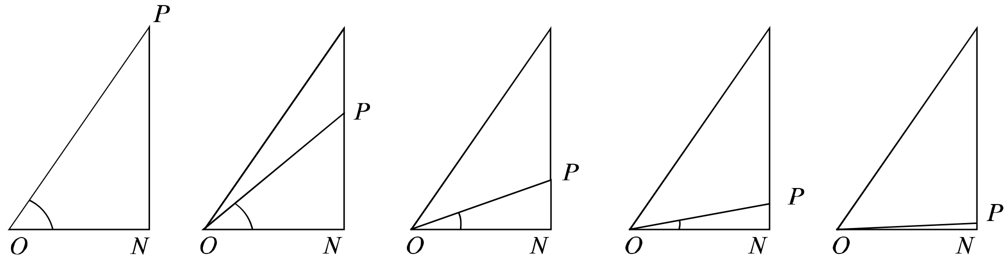
ciK †Kv‡Yi *tangent* = †Kv‡Yi *cotangent*, BZ¨wv`|

> KvR : $\sec(90^\circ - \theta) = \frac{5}{3}$ n‡j, $\operatorname{cosec}\theta - \cot\theta$ Gi gvb wbYq Ki|

9·8 0° I 90° †Kv‡Yi wÎ‡KvYwgwZK AbycvZ



Avgiv mg‡KvYx wÎf‡Ri m¶‡KvY θ Gi Rb¨ wÎ‡KvYwgwZK AbycvZ¸‡jv wbYq Ki‡Z wk‡LwQ| Gevi †`wL, †KvYwU µgkt †QvU Kiv n‡j wÎ‡KvYwgwZi AbycvZ¸‡jv Kxi/c nq| $\theta$ †KvYwU hZB †QvU n‡Z _v‡K, wecixZ evû $PN$ Gi %`N¨ ZZB †QvU nq| P we›`wU $N$ we›`i wbKUZi nq Ges Ae‡k‡l θ †KvYwU hLb 0° Gi Le Kv‡Q Aew¯Z nq, OP cvq ON Gi mv‡_ wg‡j hvq|



hLb θ †KvYwU 0° Gi Le wbK‡U Av‡m PN †iLvs‡ki ˆ`N¨ k‡b¨i †KvVvq †b‡g Av‡m Ges G‡¶‡Î

$\sin\theta = \frac{PN}{OP}$ Gi gvb cvq kb¨| GKB mgq, θ †KvYwU 0° Gi Le Kv‡Q G‡j OP Gi ˆ`N¨ cvq $ON$ Gi ˆ`‡N¨i mgvb nq Ges $\cos\theta = \frac{ON}{OP}$ Gi gvb cvq 1.

wÎ‡KvYwgwZ‡Z Av‡jvPbvi myweav‡_© 0° †Kv‡Yi AeZviYv Kiv nq Ges cwgZ Ae¯v‡b 0° †Kv‡Yi cvšÍxq evû I Aw` evû GKB iwk¥ aiv nq| mZivs c‡ei Av‡jvPbvi m‡½ mvgÄm¨ †i‡L ejv nq †h, $\cos 0^\circ = 1$, $\sin 0^\circ = 0$.

θ m²‡KvY n‡j Avgiv †`‡LwQ

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta},$$

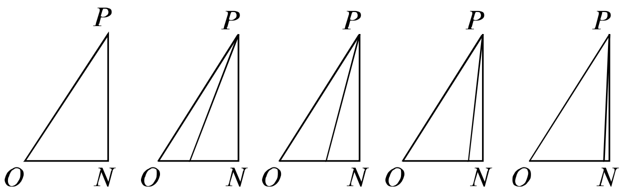
$$\sec\theta = \frac{1}{\cos\theta}, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta},$$

0° †Kv‡Yi Rb¨ m¤ve¨ †¶‡Î G m¤úK¸‡jv hv‡Z eRvq _v‡K †m w`‡K j¶ †i‡L msÁvwqZ Kiv nq|

$$\tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = \frac{0}{1} = 0$$

$$\sec 0^\circ = \frac{1}{\cos 0^\circ} = \frac{1}{1} = 1.$$

0 Øviv fvM Kiv hvq bv weavq $\operatorname{cosec} 0^\circ$ I $\cot 0^\circ$ msÁvwqZ Kiv hvq bv|



Avevi, hLb θ ‡KvYwU 90° Gi Le Kv‡Q, AwZfR OP cvq PN Gi mgvb| mZivs, $\sin\theta$ Gi gvb cvq 1| Ab¨w`‡K, θ ‡KvYwU cvq 90° Gi mgvb n‡j ON k‡b¨i KvQvKwQ; $\cos\theta$ Gi gvb cvq 0. mZivs, c‡e ewYZ m‡Îi m‡½ mvgÄm¨ †i‡L ejv nq †h, $\cos 90^\circ = 0$, $\sin 90^\circ = 1$.

$$\cot 90^\circ = \frac{\cos 90^\circ}{\sin 90^\circ} = \frac{0}{1} = 0$$

$$\text{cosec}\,90^\circ = \frac{1}{\sin 90^\circ} = \frac{1}{1} = 1$$

c‡ei b¨vq 0 Øviv fvM Kiv hvq bv weavq $\tan 90^\circ$ I $\sec 90^\circ$ msÁvwqZ Kiv hvq bv|

`óe¨ : e¨env‡ii myweav‡_ $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$ I $90^\circ$ †KvY¸‡jvi wÎ‡KvYwgwZK AbcvZ¸‡jvi gvb wb‡Pi Q‡K †`Lv‡bv n‡jv :

AbcvZ \ †KvY	0°	30°	45°	60°	90°
sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	AmsÁvwqZ
cotangent	AmsÁvwqZ	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
secant	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	AmsÁvwqZ
cosecant	AmsÁvwqZ	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

j¶ Kwi : wbawiZ K‡qKwU †Kv‡Yi Rb¨ wÎ‡KvYwgwZK gvbmgn g‡b ivLvi mnR Dcvq|

(i) 0, 1, 2, 3 Ges 4 msL¨v¸‡jvi c‡Z¨KwU‡K 4 Øviv fvM K‡i fvMd‡ji eMgj wb‡j h_vµ‡g $\sin 0^\circ$, $\sin 30^\circ$, $\sin 45^\circ$, $\sin 60^\circ$ Ges $\sin 90^\circ$ Gi gvb cvIqv hvq|

(ii) 4, 3, 2, 1 Ges 0 msL¨v¸‡jvi c‡Z¨KwU‡K 4 Øviv fvM K‡i fvMdj¸‡jvi eMgj wb‡j h_vµ‡g $\cos 0^\circ$, $\cos 30^\circ$, $\cos 45^\circ$, $\cos 60^\circ$ Ges $\cos 90^\circ$ Gi gvb cvIqv hvq|

(iii) 0, 1, 3 Ges 9 msL¨v¸‡jvi c‡Z¨KwU‡K 3 Øviv fvM K‡i fvMdj¸‡jvi eMgj wb‡j h_vµ‡g $\tan 0^\circ$, $\tan 30^\circ$, $\tan 45^\circ$ Ges $\tan 60^\circ$ Gi gvb cvIqv hvq| (D‡jL¨ †h, $\tan 90^\circ$ msÁvwqZ bq)|

(iv) 9, 3, 1 Ges 0 msL¨v¸‡jvi c‡Z¨KwU‡K 3 Øviv fvM K‡i fvMdj¸‡jvi eMgj wb‡j h_vµ‡g $\cot 45^\circ$, $\cot 60^\circ$, $\cot 90^\circ$ Gi gvb cvIqv hvq| (D‡jL¨ †h, $\cot 0^\circ$ msÁvwqZ bq)|

D`vniY 1| gvb wbYq Ki :

(K) $\dfrac{1-sin^2 45^\circ}{1+sin^2 45^\circ}+tan^2 45^\circ$

(L) $cot\,90^\circ \cdot tan\,0^\circ \cdot sec\,30^\circ \cdot cosec\,60^\circ$

(M) $sin\,60^\circ \cdot cos\,30^\circ + cos\,60^\circ \cdot sin\,30^\circ$

(N) $\dfrac{1-\tan^2 60^\circ,}{1+\tan^2 60^\circ}+\sin^2 60^\circ$

mgvavb :

(K) c`Ë ivwk $= \dfrac{1-sin^2 45^\circ}{1+sin^2 45^\circ}+tan^2 45^\circ$

$$= \frac{1-\left(\frac{1}{\sqrt{2}}\right)^2}{1+\left(\frac{1}{\sqrt{2}}\right)^2}+(1)^2 \quad [\because sin45^\circ = \frac{1}{\sqrt{2}} \text{ I } tan45^\circ = 1]$$

$$= \frac{1-\frac{1}{2}}{1+\frac{1}{2}}+1 = \frac{\frac{1}{2}}{\frac{3}{2}}+1 = \frac{1}{3}+1 = \frac{4}{3}$$

(L) c`Ë ivwk $= cot\,90^\circ \cdot tan\,0^\circ \cdot sec\,30^\circ \cdot cosec\,60^\circ$

$$= 0\cdot 0\cdot \frac{2}{\sqrt{3}}\cdot\frac{2}{\sqrt{3}} = 0$$

$$[\because cot\,90^\circ = 0,\ tan\,0^\circ = 0,\ sec\,30^\circ = \frac{2}{\sqrt{3}},\ cosec\,60^\circ = \frac{2}{\sqrt{3}}]$$

(M) c`Ë ivwk $= sin\,60^\circ \cdot cos\,30^\circ + cos\,60^\circ \cdot sin\,30^\circ$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{1}{2}\cdot\frac{1}{2}$$

$$[\because \sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2},\ \sin 30^\circ = \cos 60^\circ = \frac{1}{2}]$$

$$= \frac{3}{4}+\frac{1}{4} = \frac{4}{4} = 1$$

(N) c`Ë ivwk $= \dfrac{1-\tan^2 60^\circ}{1+\tan^2 60^\circ}+\sin^2 60^\circ$

$$= \frac{1-\left(\sqrt{3}\right)^2}{1+\left(\sqrt{3}\right)^2}+\left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1-3}{1+3}+\frac{3}{4} = \frac{-2}{4}+\frac{3}{4}$$

$$= \frac{-2+3}{4} = \frac{1}{4}$$

D`vniY 2|

(K) $\sqrt{2}\cos(A-B)=1,\ 2\sin(A+B)=\sqrt{3}$ Ges A, B m²‡KvY n‡j, A I B Gi gvb wbYq Ki|

(L) $\frac{\cos A-\sin A}{\cos A+\sin A}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$ n‡j, A Gi gvb wbYq Ki|

(M) cgvY Ki †h, $\cos 2A=\frac{1-\tan^2 A}{1+\tan^2 A}$, hw` $A=45^\circ$ nq|

(N) mgvavb Ki : $2\cos^2\theta+3\sin\theta-3=0$, †hLv‡b θ m²‡KvY|

mgvavb : (K) $\sqrt{2}cos(A-B)=1$

ev, $cos(A-B)=\frac{1}{\sqrt{2}}$

ev, $cos(A-B)=cos45^\circ$ $[\because cos45^\circ=\frac{1}{\sqrt{2}}]$

$\therefore\quad A-B=45^\circ$..................$(i)$

Ges $2sin(A+B)=\sqrt{3}$

ev, $sin(A+B)=\frac{\sqrt{3}}{2}$

ev, $sin(A+B)=sin\,60^\circ$ $[\because sin\,60^\circ=\frac{\sqrt{3}}{2}]$

$\therefore\quad A+B=60^\circ$....................$(ii)$

(i) I (ii) bs †hvM K‡i cvB,

$2A=105^\circ$

$\therefore\quad A=\frac{105^\circ}{2}=52\frac{1}{2}^\circ$

Avevi, (ii) n‡Z (i) we‡qvM K‡i cvB,

$2B=15^\circ$

ev, $B=\frac{15^\circ}{2}$

$\therefore\quad B=7\frac{1}{2}^\circ$

wb‡Yq $A=52\frac{1}{2}^\circ$ I $B=7\frac{1}{2}^\circ$

dgv-22, MwYZ-9g-10g

(L) $\dfrac{cosA - sinA}{cosA + sinA} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$

ev, $\dfrac{\cos A - \sin A + \cos A + \sin A}{\cos A - \sin A - \cos A - \sin A} = \dfrac{1-\sqrt{3}+1+\sqrt{3}}{1-\sqrt{3}-1-\sqrt{3}}$

ev, $\dfrac{2cosA}{-2sinA} = \dfrac{2}{-2\sqrt{3}}$

ev, $\dfrac{cosA}{sinA} = \dfrac{1}{\sqrt{3}}$

ev, $cotA = \cot 60^\circ$

$\therefore$ $A = 60^\circ$

(M) †`Iqv Av‡Q, $A = 45^\circ$

cgvY Ki‡Z n‡e, $cos2A = \dfrac{1-tan^2A}{1+tan^2A}$

evgc¶ = $cos2A$

= $cos(2\times 45^\circ) = cos\,90^\circ = 0$

Wvbc¶ = $\dfrac{1-tan^2A}{1+tan^2A}$

= $\dfrac{1-tan^2 45^\circ}{1+tan^2 45^\circ} = \dfrac{1-(1)^2}{1+(1)^2}$

= $\dfrac{0}{2} = 0$

$\therefore$ evgc¶ = Wvbc¶ (cgwYZ)

(N) c`Ë mgxKiY $2cos^2\theta + 3sin\theta - 3 = 0$

ev, $2(1-\sin^2\theta) - 3(1-\sin\theta) = 0$

ev, $2(1+sin\theta)(1-sin\theta) - 3(1-sin\theta) = 0$

ev, $(1-sin\theta)\{2(1+sin\theta)-3\} = 0$

ev, $(1-sin\theta)\{2sin\theta - 1\} = 0$

ev, $1 - sin\theta = 0$ A_ev, $2\sin\theta - 1 = 0$

$\therefore$ $sin\theta = 1$ ev, $2sin\theta = 1$

ev, $sin\theta = sin\,90^\circ$ ev, $sin\theta = \dfrac{1}{2}$

$\therefore$ $\theta = 90^\circ$ ev, $sin\theta = sin\,30^\circ$

ev, $\theta = 30^\circ$

†h‡nZ θ m²‡KvY, †m‡nZ, $\theta = 30^\circ$.

Abkxjbx 9·2

1| $\cos\theta = \frac{1}{2}$ n‡j $\cot\theta$ Gi gvb †KvbwU?

K. $\frac{1}{\sqrt{3}}$ L. 1

M. $\sqrt{3}$ N. 2

2| (i) $\sin^2\theta = 1 - \cos^2\theta$

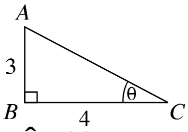
(ii) $\sec^2\theta = 1 + \tan^2\theta$

(iii) $\cot^2\theta = 1 - \tan^2\theta$

cv‡ki Z‡_¨i Av‡jv‡K wb‡gi †KvbwU mwVK ?

K. i I ii L. i I iii

M. ii I iii N. i, ii I iii



wPÎ Abhvqx 3 I 4 bs c‡ki DËi `vI|

3| $\sin\theta$ Gi gvb †KvbwU ?

K. $\frac{3}{4}$ L. $\frac{4}{3}$

M. $\frac{3}{5}$ N. $\frac{4}{5}$

4| $\cot\theta$ Gi gvb †KvbwU ?

K. $\frac{3}{4}$ L. $\frac{3}{5}$

M. $\frac{4}{5}$ N. $\frac{4}{3}$

gvb wbYq Ki (5-8)

5| $\frac{1-\cot^2 60^\circ}{1+\cot^2 60^\circ}$

6| $\tan 45^\circ \cdot \sin^2 60^\circ \cdot \tan 30^\circ \cdot \tan 60^\circ.$

7| $\frac{1-\cos^2 60^\circ}{1+\cos^2 60^\circ} + \sec^2 60^\circ$

8| $\cos 45^\circ \cdot \cot^2 60^\circ \cdot \operatorname{cosec}^2 30^\circ$

†`LvI †h, (9-11)

9| $\cos^2 30^\circ - \sin^2 30^\circ = \cos 60^\circ.$

10| $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = \sin 90^\circ$

11| $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \cos 30^\circ$

12| $\sin 3A = \cos 3A.$ hw` $A = 15^\circ$ nq|

13| $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$ hw` $A = 45^\circ$ nq|

14| $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ hw` $A = 30^\circ$ nq|

15| $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$ hw` $A = 60^\circ$ nq|

16| $2\cos(A+B) = 1 = 2\sin(A-B)$ Ges A, B m²‡KvY n‡j †`LvI †h, $A = 45^\circ$, $B = 15^\circ$ |

17| $\cos(A-B) = 1,\ 2\sin(A+B) = \sqrt{3}$ Ges A, B m²‡KvY n‡j, A I B Gi gvb wbYq Ki|

18| mgvavb Ki : $\dfrac{\cos A - \sin A}{\cos A + \sin A} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

19| A I B m²‡KvY Ges $\cot(A+B) = 1,\ \cot(A-B) = \sqrt{3}$ n‡j, A I B Gi gvb wbYq Ki|

20| †`LvI †h, $\cos 3A = 4\cos^3 A - 3\cos A$ hw` $A = 30^\circ$ nq|

21| mgvavb Ki : $\sin\theta + \cos\theta = 1$, hLb $0^\circ \le \theta \le 90^\circ$

22| mgvavb Ki : $\cos^2\theta - \sin^2\theta = 2 - 5\cos\theta$ hLb θ m²‡KvY|

23| mgvavb Ki : $2\sin^2\theta + 3\cos\theta - 3 = 0$, θ m²‡KvY|

24| mgvavb Ki : $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0.$

25| gvb wbYq Ki : $3\cot^2 60^\circ + \dfrac{1}{4}\text{cosec}^2 30^\circ + 5\sin^2 45^\circ - 4\text{co}^2 60^\circ$

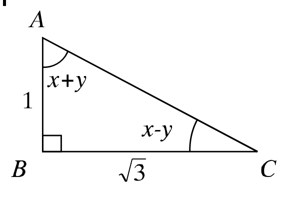
26| ΔABC Gi $\angle B = 90^0$, AB = 5 cm, BC = 12 cm.

K. AC Gi ˆ`N¨ wbYq Ki|

L. $\angle C = \theta$ n‡j $\sin\theta + \cos\theta$ Gi gvb wbYq Ki|

M. †`LvI †h, $\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\,\text{cosec}^2\theta$

27|



K. AC Gi cwigvY KZ?

L. tanA + tanC Gi gvb wbYq Ki|

M. x I y Gi gvb wbYq Ki|

`kg Aa¨vq

`iZ I D”PZv

AwZ cvPxb Kvj †_‡KB `ieZx †Kv‡bv e¯i `iZ I D”PZv wbYq Ki‡Z wÎ‡KvYwgwZK Abcv‡Zi c‡qvM Kiv nq| eZgvb h‡M wÎ‡KvYwgwZK Abcv‡Zi e¨envi †e‡o hvIqvq Gi ¸i“Z Acwimxg| †h me cvnvo, ceZ, UvIqvi, Mv‡Qi D”PZv Ges b`-b`xi c¯ mn‡R gvcv hvq bv †m me †¶‡Î D”PZv I c¯ wÎ‡KvYwgwZi mvnv‡h¨ wbYq Kiv hvq| G‡¶‡Î m²‡Kv‡Yi wÎ‡KvYwgwZK Abcv‡Zi gvb †R‡b ivLv c‡qvRb|

Aa¨vq †k‡l wk¶v_xivÑ

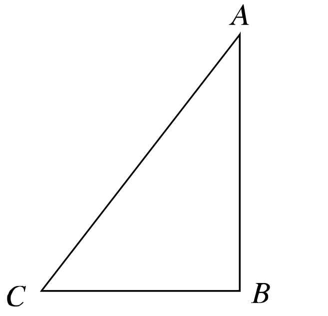
- f-†iLv, EaŸ‡iLv, Dj¤^Zj, DbwZ †KvY I AebwZ †KvY e¨vL¨v Ki‡Z cvi‡e|
- wÎ‡KvYwgwZi mvnv‡h¨ `iZ I D”PZv welqK MwYwZK mgm¨v mgvavb Ki‡Z cvi‡e|
- wÎ‡KvYwgwZi mvnv‡h¨ nv‡Z-Kj‡g `iZ I D”PZv welqK wewfbœ cwigvc Ki‡Z cvi‡e|

f-†iLv, EaŸ‡iLv Ges Dj¤^Zj :

f-†iLv n‡”Q fwg Z‡j Aew¯Z †h‡Kv‡bv mij‡iLv| f-†iLv‡K kqb‡iLvI ejv nq| EaŸ‡iLv n‡”Q fwg Z‡ji Dci j¤ †h‡Kv‡bv mij‡iLv| G‡K Dj¤^ †iLvI e‡j|

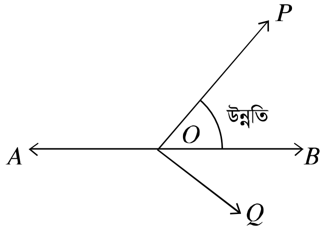
fwg Z‡ji Dci j¤fv‡e Aew¯Z ci¯úi‡”Q`x f-†iLv I EaŸ‡iLv GKwU Zj wbw`ó K‡i| G Zj‡K Dj¤^ Zj e‡j|

wP‡Î : fwg Z‡ji †Kv‡bv ¯vb C †_‡K CB `i‡Z $AB$ D”PZv wewkó GKwU MvQ Lvov Ae¯vq `Ûvqgvb| GLv‡b CB †iLv n‡”Q f-†iLv, BA †iLv n‡”Q EaŸ‡iLv Ges ABC ZjwU fwgi Dci j¤ hv Dj¤^ Zj|

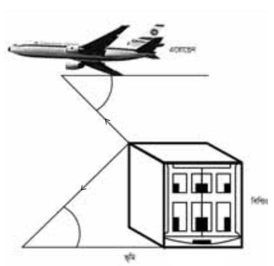


DbwZ †KvY I AebwZ †KvY :

wPÎwU j¶ Kwi, fwgi mgvšivj AB GKwU mij‡iLv| O, B, P we›`¸‡jv GKB Dj¤^ Z‡j Aew¯Z| $AB$ mij‡iLvi Dc‡ii $P$ we›`wU AB †iLvi mv‡_ $\angle POB$ Drcb K‡i| GLv‡b, O we›`‡Z $P$ we›`i DbwZ †KvY n‡”Q $\angle POB$|



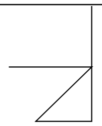
mZivs, fZ‡ji Dc‡ii †Kvb we›` fwgi mgvšivj †iLvi mv‡_ †h †KvY Drcb K‡i Zv‡K DbwZ †KvY ejv nq|



Avevi, O, A, Q we›`¸‡jv GKB Dj¤^ Z‡j Aew¯'Z Ges Q we›` f~-†iLvi mgvšÍivj AB †iLvi wb‡Pi w`‡K Aew¯'Z| GLv‡b, $O$ we›`‡Z Q we›`i AebwZ †KvY n‡"Q $\angle QOA$ mZivs fZ‡ji mgvšÍivj †iLvi wb‡Pi †Kv‡bv we›` f~-†iLvi mv‡_ †h †KvY Drcb K‡i Zv‡K AebwZ †KvY ejv nq|

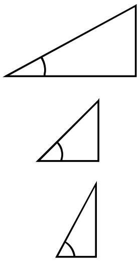
KvR :

wPÎwU wPwýZ Ki Ges f~-†iLv EaŸ©‡iLv, Dj¤^Zj, DbwZ †KvY I AebwZ †KvY wb‡`k Ki|



←fwg→

we‡kl `óe¨ : G Aa¨v‡q mgm¨v mgvav‡bi †¶‡Î AvbgwbK mwVK wPÎ Avek¨K| wPÎ A¼‡bi mgq wb‡Pi †KŠkj Aej¤^b Kiv `iKvi|



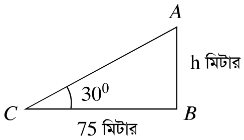
(1) 30° †KvY A¼‡bi †¶‡Î fwg > j¤^ n‡e|

(2) 45° †KvY A¼‡bi †¶‡Î fwg = j¤^ n‡e|

(3) 60° †KvY A¼‡bi †¶‡Î fwg < j¤^ n‡e|

D`vniY 1| GKwU UvIqv‡ii cv`‡`k †\_‡K 75 wgUvi `‡i fZ‡j¯' †Kv‡bv we›`‡Z UvIqv‡ii kx‡li DbwZ 30° n‡j, UvIqv‡ii D"PZv wbYq Ki|

mgvavb : g‡b Kwi, UvIqv‡ii D"PZv $AB = h$ wgUvi UvIqv‡ii cv`‡`k †_‡K $BC = 75$ wgUvi `‡i fZ‡j¯' $C$ we›`‡Z UvIqv‡ii kxl A we›`i DbwZ $\angle ACB = 30^\circ$



75 wgUvi

mg‡KvYx ΔABC †_‡K cvB, $tan\angle ACB = \dfrac{AB}{BC}$

ev, $tan30^\circ = \dfrac{h}{75}$ ev, $\dfrac{1}{\sqrt{3}} = \dfrac{h}{75}$ $\left[\because tan30^\circ = \dfrac{1}{\sqrt{3}}\right]$ ev, $\sqrt{3}h = 75$ ev, $h = \dfrac{75}{\sqrt{3}}$

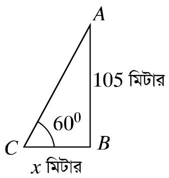
ev, $h = \dfrac{75\sqrt{3}}{3}$ [ni Ges je‡K $\sqrt{3}$ Øviv ¸Y K‡i] ev, $h = 25\sqrt{3}$

$\therefore$ $h = 43.301$ (cvq)|

wb‡Yq UvIqv‡ii D"PZv 43.301 wgUvi (cvq)|

D`vniY 2| GKwU Mv‡Qi D"PZv $105$ wgUvi| MvQwUi kx‡li DbwZ fwgi †Kv‡bv we›`‡Z DbwZ †KvY 60° n‡j, MvQwUi †Mvov †_‡K fj¯' we›`wUi `iZ wbYq Ki|

mgvavb : g‡b Kwi, Mv‡Qi †Mvov †_‡K fZj¯' we›`wUi `iZ $BC = x$ wgUvi, Mv‡Qi D"PZv $AB = 105$ wgUvi Ges C we›`‡Z MvQwUi kxl we›`i DbwZ $\angle ACB = 60^\circ$



ΔABC †_‡K cvB,

$tan\angle ACB = \frac{AB}{BC}$ ev, $tan60^0 = \frac{105}{x}$ $\left[\because \tan 60^0 = \sqrt{3}\right]$

ev, $\sqrt{3} = \frac{105}{x}$ ev, $\sqrt{3}x = 105$ ev, $x = \frac{105}{\sqrt{3}}$ ev, $x = \frac{105\sqrt{3}}{3}$ ev, $x = 35\sqrt{3}$

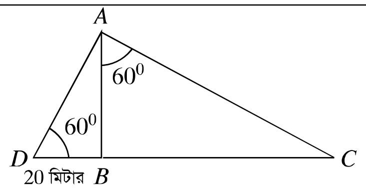
$\therefore$ $x = 60.622$ wgUvi (cvq)

$\therefore$ MvQwUi †Mvov †_‡K fZj¯ we›`wU `iZ 60.622 wgUvi (cvq)|

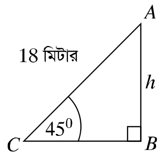
KvR :

wP‡Î AB GKwU MvQ| wP‡Î c`Ë ZË †_‡K –

1. MvQwUi D"PZv wbYq Ki|
2. MvQwUi cv`‡`k †_‡K fZj¯ C we›`i `iZ wbYq Ki|



D`vniY 3| 18 wgUvi j¤v GKwU gB GKwU †`Iqv‡ji Qv` eivei †Vm w`‡q fwgi m‡½ 45° †KvY Drcb K‡i| †`IqvjwUi D"PZv wbYq Ki|



mgvavb : g‡b Kwi, †`IqvjwUi D"PZv $AB = h$ wgUvi, gBwUi ˆ`N¨ $AC = 18$ wgUvi Ges fwgi m‡½ $\angle ACB = 45^\circ$ Drcb K‡i|

ΔABC †_‡K cvB, $\sin\angle ACB = \frac{AB}{AC}$

ev, $\sin 45^\circ = \frac{h}{18}$

ev, $\frac{1}{\sqrt{2}} = \frac{h}{18}$ $\left[\because \sin 45^\circ = \frac{1}{\sqrt{2}}\right]$ ev, $\sqrt{2}h = 18$ ev, $h = \frac{18}{\sqrt{2}}$

ev, $\sqrt{2}h = 18$ ev, $h = \frac{18}{\sqrt{2}}$

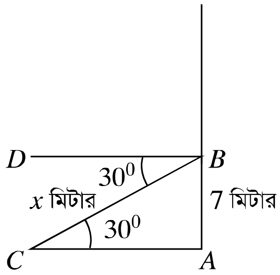
ev, $h = \frac{18\sqrt{2}}{2}$ [ni Ges je‡K $\sqrt{2}$ Øviv ¸Y K‡i] ev, $h = 9\sqrt{2}$

$\therefore$ $h = 12.728$ (cvq)

mZivs †`IqvjwUi D"PZv 12.728 wgUvi (cvq)|

D`vniY 4| S‡o GKwU MvQ †n‡j co‡jv| Mv‡Qi †Mvov †_‡K 7 wgUvi D"PZvq GKwU jwV †Vm w`‡q MvQwU‡K †mvRv Kiv n‡jv| gwU‡Z jwVwUi ¯úk we›`i AebwZ †KvY 30° n‡j, jwVi ˆ`N¨ wbYq Ki|

mgvavb : g‡b Kwi, Mv‡Qi †Mvov †_‡K $AB = 7$ wgUvi D”PZvq jwVwU †Vm w`‡q Av‡Q Ges AebwZ $\angle DBC = 30^\circ$ |



$\therefore \angle ACB = \angle DBC = 30^\circ$ [GKvši †KvY e‡j]

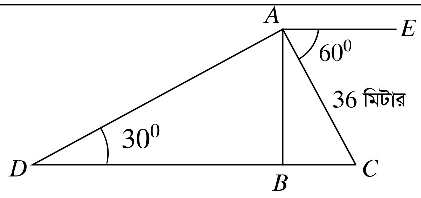
ΔABC †_‡K cvB,

$Sin\angle ACB = \dfrac{AB}{BC}$ ev, $Sin 30^\circ = \dfrac{7}{BC}$

ev, $\dfrac{1}{2} = \dfrac{7}{BC}$ $\left[\because Sin 30^\circ = \dfrac{1}{2}\right]$

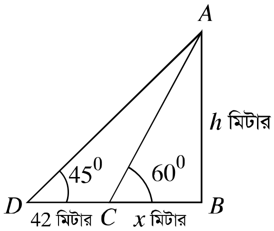
$\therefore BC = 14$

$\therefore$ jwVwUi ˆ`N¨ 14 wgUvi|

> KvR : wP‡Î AebwZ $\angle CAE = 60^\circ$, DbwZ $\angle ADB = 30^\circ$
> $AC = 36$ wgUvi Ges B, C, D GKB mij‡iLvq Aew¯Z n‡j,
> AB, AD Ges CD evûi ˆ`N¨ wbYq Ki|
>
> 
>

D`vniY 5| fZj¯ †Kv‡bv ¯'v‡b GKwU `vjv‡bi Qv‡`i GKwU we›`i DbwZ †KvY 60°| H ¯'vb †_‡K 42 wgUvi wcwQ‡q †M‡j `vjv‡bi H we›`i DbwZ †KvY 45° nq| `vjv‡bi D”PZv wbYq Ki|

mgvavb : g‡b Kwi, `vjv‡bi D”PZv $AB = h$ wgUvi, kx‡li DbwZ $\angle ACB = 60^\circ$ Ges C ¯'vb †_‡K $CD = 42$ wgUvi wcwQ‡q †M‡j DbwZ $\angle ADB = 45^\circ$ nq|



awi, $BC = x$ wgUvi|

$\therefore BD = BC + CD = (x + 42)$ wgUvi|

ΔABC †_‡K cvB,

$tan 60^0 = \dfrac{AB}{BC}$ ev, $\sqrt{3} = \dfrac{h}{x}$ $\left[\because tan 60^0 = \sqrt{3}\right]$

$\therefore x = \dfrac{h}{\sqrt{3}}$ ....(i)

Avevi, ΔABD †_‡K cvB, $\tan 45^\circ = \dfrac{AB}{BD}$

ev, $1 = \dfrac{h}{x + 42}$ $\left[\because tan 45^0 = 1\right]$ ev, $h = x + 42$

ev, $h = \dfrac{h}{\sqrt{3}} + 42$; (i) bs mgxKi‡Yi mvnv‡h¨|

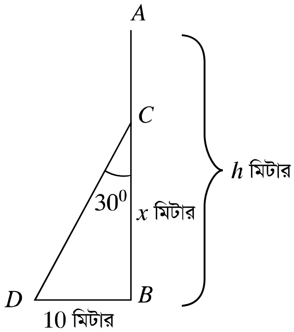
ev, $\sqrt{3}h = h + 42\sqrt{3}$ ev, $\sqrt{3}h - h = 42\sqrt{3}$ ev, $(\sqrt{3}-1)h = 42\sqrt{3}$ ev, $h = \frac{42\sqrt{3}}{\sqrt{3}-1}$

$\therefore h = 99.373$ wgUvi (cÖvq)

`vjvbwUi D"PZv 99.373 wgUvi (cÖvq)|

D`vniY 6| GKwU LwU Ggb fv‡e †f‡O †Mj †h, Zvi fvOv Ask `Êvqgvb As‡ki mv‡_ 30° †KvY Drcb K‡i LwUi †Mvov †_‡K 10 wgUvi `‡i gwU¯úk K‡i| LwUi m¤úY© ˆ`N©¨ wbY©q Ki|

mgvavb : g‡b Kwi, LwUi m¤úY© ˆ`N©¨ $AB = h$ wgUvi| LwUwU $BC = x$ wgUvi D"PZvq †f‡O wM‡q wew"Qbœ bv n‡q fvOv Ask `Êvqgvb As‡ki mv‡_ $\angle BCD = 30^\circ$ Drcb K‡i †Mvov †_‡K $BD = 10$ wgUvi `‡i gwU¯úk K‡i|



GLv‡b, $CD = AC = AB - BC = (h - x)$ wgUvi

ΔBCD †_‡K cvB,

$\tan 30^\circ = \frac{BD}{BC}$ ev, $\frac{1}{\sqrt{3}} = \frac{10}{x}$ $\therefore x = 10\sqrt{3}$

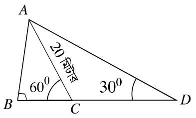
Avevi, $\sin 30^\circ = \frac{BD}{CD}$ ev, $\frac{1}{2} = \frac{10}{h-x}$

ev, $h - x = 20$ ev, $h = 20 + x$ ev, $h = 20 + 10\sqrt{3}$; [x-Gi gvbewm‡q]

$\therefore h = 37.321$ (cÖvq) $\therefore$ LwUi ˆ`N©¨ 37.321 wgUvi (cÖvq)|

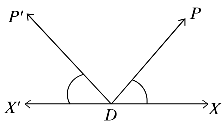
KvR :
`BwU gvBj †cv‡ói ga¨eZx© †Kv‡bv ¯'v‡bi Dc‡i GKwU †ejb Do‡Q| †ej‡bi ¯'vb n‡Z gvBj †cv÷ `BwUi AebwZ †KvY h_vµ‡g 30° I 60° n‡j, †ejbwUi D"PZv wgUv‡i wbY©q Ki|

Abkxjbx 10



1| K. $\angle CAD$ Gi cwigvb wbY©q Ki|
   L. AB I BC Gi ˆ`N©¨ wbY©q Ki|
   M. A I D Gi `iZ¡ wbY©q Ki|

2| `wU wK‡jvwgUvi †cv÷ A I B Gi ga¨eZx© †Kvb ¯'v‡bi Dci O we›`‡Z GKwU †nwjKÞvi n‡Z H wK‡jvwgUvi †cv÷Ø‡qi AebwZ †Kvb h_vµ‡g 60^0 Ges 30^0
   K. msw¶ß eY©bvmn AvbycwZK wPÎ A¼b Ki|
   L. †nwjKÞviwU gwU †_‡K KZ DP‡Z Aew¯'Z?
   M. A we›` †\_‡K †nwjKÞv‡ii mivmwi `iZ¡ wbY©q Ki|



3| Ic‡ii wP‡Î O we›`‡Z P we›`i DbwZ †KvY †KvbwU?
   K. $\angle QOB$ L. $\angle POA$ M. $\angle QOA$ N. $\angle POB$

4| i f~-†iLv n‡"Q fwg Z‡j Aew¯'Z †h‡Kv‡bv mij‡iLv|
   ii DaŸ©‡iLv n‡"Q fwg Z‡ji Ici j¤ †h‡Kv‡bv mij †iLv|
   iii fwg Z‡ji Dci j¤fv‡e Aew¯'Z ci¯úi‡"Q`x f~-†iLv I DaŸ©‡iLv GKwU Zj wbw`©ó K‡i| G Zj‡K Dj¤ Zj e‡j|

Ic‡ii evK¨¸‡jvi g‡a¨ †KvbwU mwVK?

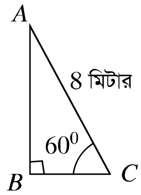
K. i I ii

L. i I iii

M. ii I iii

N. i, ii I iii

cv‡ki wPÎ Abhvqx 5-6 ck `BwUi DËi `vI|



5| BC Gi ˆ`N¨ n‡e Ñ

K. $\frac{4}{\sqrt{3}}m$

L. 4m

M. $4\sqrt{2}$ m

N. $4\sqrt{3}\,m$

6| AB Gi ˆ`N¨ n‡eÑ

K. $\frac{4}{\sqrt{3}}m$

L. 4m

M. $4\sqrt{2}$ m

N. $4\sqrt{3}\,m$

7| GKwU wgbv‡ii cv`‡`k †_‡K wKQ `‡i GKwU ¯v‡b wgbviwUi kx‡li DbwZ $30^\circ$ Ges wgbviwUi D”PZv 26 wgUvi n‡j, wgbvi †_‡K H ¯vbwUi `iZ wbYq Ki|

8| GKwU Mv‡Qi cv`‡`k †_‡K 20 wgUvi `‡i fZ‡ji †Kv‡bv we›`‡Z Mv‡Qi Povi DbwZ †KvY 60° n‡j, MvQwUi D”PZv wbYq Ki|

9| 18wgUvi ˆ`N¨ GKwU gB fwgi mv‡_ $45^\circ$ †KvY Drcb K‡i †`Iqv‡ji Qv` ¯úk K‡i| †`IqvjwUi D”PZv wbYq Ki|

10| GKwU N‡ii Qv‡`i †Kv‡bv we›`‡Z H we›` †_‡K 20 wgUvi `‡ii fZj¯ GKwU we›`i AebwZ †KvY 30° n‡j, NiwUi D”PZv wbYq Ki|

11| fZ‡j †Kv‡bv ¯v‡b GKwU ¯‡¤i kx‡li DbwZ 60°| H ¯vb †_‡K 25 wgUvi wcwQ‡q †M‡j ¯¤wUi DbwZ †KvY 30° nq| ¯¤wUi D”PZv wbYq Ki|

12| †Kv‡bv ¯vb †_‡K GKwU wgbv‡ii w`‡K 60 wgUvi GwM‡q Avm‡j wgbv‡ii kxl we›`i DbwZ 45°†_‡K 60° nq| wgbviwUi D”PZv wbYq Ki|

13| GKwU b`xi Zx‡i †Kv‡bv GK ¯v‡b `wo‡q GKRb †jvK †`Lj †h, wVK †mvRv‡mwR Aci Zx‡i Aew¯Z GKwU UvIqv‡ii DbwZ †KvY $60^\circ$| H ¯vb †_‡K 32 wgUvi wcwQ‡q †M‡j DbwZ †KvY $30^\circ$ nq| UvIqv‡ii D”PZv Ges b`xi we¯vi wbYq Ki|

14| 64 wgUvi j¤v GKwU LwU †f‡O wM‡q m¤úY wew”Qb bv n‡q fwgi mv‡_ 60° Drcb K‡i| LwUwUi fvOv As‡ki ˆ`N¨ wbYq Ki|

15| GKwU MvQ S‡o Ggbfv‡e †f‡O †Mj †h, fvOv Ask `Êvqgvb As‡ki mv‡_ $30^\circ$ †KvY K‡i Mv‡Qi †Mvov †_‡K 12 wgUvi `‡i gwU ¯úk K‡i| MvQwU m¤úY ˆ`N¨ wbYq Ki|

16| GKwU b`xi GK Zx‡i †Kv‡bv ¯v‡b `wo‡q GKRb †jvK †`L‡jv †h, wVK †mvRv‡mwR Aci Zx‡i Aew¯Z 150 wgUvi j¤v GKwU Mv‡Qi kx‡li DbwZ †KvY $30^\circ$| †jvKwU GKwU †bŠKv‡hv‡M MvQwU‡K j¶¨ K‡i hvÎv ïi“ Ki‡jv| wKš‘ cwbi †mv‡Zi Kvi‡Y †jvKwU MvQ †_‡K 10 wgUvi `‡i Zx‡i †cŠQj|

(K) Dc‡iv³ eYbwU wP‡Îi gva¨‡g †`LvI|

(L) b`xi we¯vi wbYq Ki|

(M) †jvKwUi hvÎv ¯vb †_‡K Mše¨ ¯v‡bi `iZ wbYq Ki|

GKv`k Aa¨vq

exRMwYZxq AbcvZ I mgvbcvZ

(Algebraic Ratio and Proportion)

AbcvZ I mgvbcv‡Zi aviYv _vKv Avgv‡`i Rb¨ LeB ¸iZcY| mßg †kÖwY‡Z cvwUMwYZxq AbcvZ I mgvbcvZ wek`fv‡e Av‡jvPbv Kiv n‡q‡Q| G Aa¨v‡q Avgiv exRMwYZxq AbcvZ I mgvbcvZ m¤ú‡K Av‡jvPbv Ki‡ev| Avgiv cwZwbqZB wbgvY mvgMx I wewfb cKvi Lv`¨ mvgMx ^Zwi‡Z, †fvM¨cY¨ Drcv`‡b, Rwg‡Z mvi c‡qv‡M, †Kv‡bv I wKQi AvKvi-AvqZb `wób›`b Ki‡Z Ges ^`bw›`b Kvhµ‡gi Avi I A‡bK †¶‡Î AbcvZ I mgvbcv‡Zi aviYv c‡qvM K‡i _wK| Bnv e¨envi K‡i ^`bw›`b Rxe‡b A‡bK mgm¨vi mgvavb Kiv hvq|

Aa¨vq †k‡l wk¶v_xiv–

- exRMwYZxq AbcvZ I mgvbcvZ e¨vL¨v Ki‡Z cvi‡e|
- mgvbcvZ msµvš wewfb i|cvši wewa c‡qvM Ki‡Z cvi‡e|
- avivewnK AbcvZ eYbv Ki‡Z cvi‡e|
- ev¯e mgm¨v mgvav‡b AbcvZ, mgvbcvZ I avivewnK AbcvZ e¨envi Ki‡Z cvi‡e|

11·1 AbcvZ

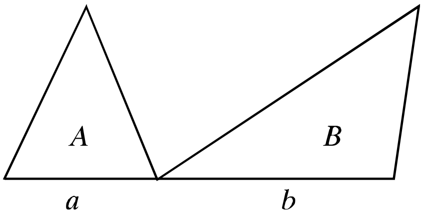
GKB GK‡K mgRvZxq `BwU iwki cwigv‡Yi GKwU AciwUi KZ ¸Y ev KZ Ask Zv GKwU fMvsk Øviv cKvk Kiv hvq| GB fMvskwU‡K iwk `BwUi AbcvZ e‡j|

`BwU iwk $p$ I $q$ Gi AbcvZ†K $p:q=\frac{p}{q}$ wjLv nq| $p$ I $q$ iwk `BwU mgRvZxq I GKB GK‡K n‡Z n‡e| Abcv‡Z p †K ce iwk Ges q †K DËi iwk ejv nq|

A‡bK mgq Avbgwb‡K cwigvc Ki‡ZI Avgiv AbcvZ e¨envi Kwi| †hgb, mKvj 8 Uvq iv¯vq †h msL¨K Mvox _v‡K, 10 Uvq Zvi wØ¸Y Mvox _v‡K| G †¶‡Î AbcvZ wbY‡q Mvoxi cKZ msL¨v Rvbvi c‡qvRb nq bv| Avevi A‡bK mgq Avgiv e‡j _wK, †Zvgvi N‡ii AvqZb Avgvi N‡ii AvqZ‡bi wZb¸Y n‡e| GLv‡bI N‡ii mwVK AvqZb Rvbvi c‡qvRb nq bv| ev¯e Rxe‡b GiKg A‡bK †¶‡Î Avgiv Abcv‡Zi avibv e¨envi K‡i _wK|

11·2 mgvbcvZ

hw` PviwU iwk Gi|c nq †h, c_g I wØZxq iwki AbcvZ ZZxq I PZ_ iwki Abcv‡Zi mgvb nq, Z‡e H PviwU iwk wb‡q GKwU mgvbcvZ Drcb nq| $a, b, c, d$ Gi|c PviwU iwk n‡j Avgiv wjwL $a:b=c:d$| mgvbcv‡Zi PviwU iwkB GKRvZxq n‡Z nq bv| c‡Z¨K Abcv‡Zi iwk `BwU GK RvZxq n‡jB P‡j|



Dc‡ii wP‡Î, `BwU wÎf‡Ri fwg h_vµ‡g $a$ I $b$ Ges Zv‡`i c‡Z¨‡Ki D”PZv h GKK| wÎfRØ‡qi †¶Îdj A I B eMGKK n‡j Avgiv wjL‡Z cvwi

$$\frac{A}{B}=\frac{\frac{1}{2}ah}{\frac{1}{2}bh}=\frac{a}{b}$$ ev, $A:B=a:b$

A_vr, †¶Îdj‡qi AbcvZ fwgØ‡qi Abcv‡Zi mgvb|

µwgK mgvbcvZx

a, b, c µwgK mgvbcvZx ej‡Z †evSvq $a : b = b : c$.

a, b, c µwgK mgvbcvZx n‡e hw` Ges †Kej hw` $b^2 = ac$ nq| µwgK mgvbcv‡Zi †¶‡Î me¸‡jv iwk GK RvZxq n‡Z n‡e| G‡¶‡Î c †K a I b Gi ZZxq mgvbcvZx Ges b †K a I c Gi ga¨mgvbcvZx ejv nq|

D`vniY 1| $A$ I $B$ wbw`ó c_ AwZµg K‡i h_vµ‡g t_1 Ges t_2 wgwb‡U| A I B Gi Mo MwZ‡e‡Mi AbcvZ wbYq Ki|

mgvavb : g‡b Kwi, A I B Gi Mo MwZ‡eM cwZ wgwb‡U h_vµ‡g v_1 wgUvi I v_2 wgUvi| Zvn‡j,

t_1 wgwb‡U A AwZµg K‡i v_1t_1 wgUvi Ges t_2 wgwb‡U B AwZµg K‡i v_2t_2 wgUvi |

ckvbmv‡i, $v_1t_1 = v_2t_2$, $\therefore \frac{v_1}{v_2}=\frac{t_2}{t_1}$

GLv‡b MwZ‡e‡Mi AbcvZ mg‡qi e¨¯Í Abcv‡Zi mgvb|

KvR : 1| 3.5 : 5.6 ‡K 1 : a Ges b : 1 AvKv‡i cKvk Ki|

2| $x : y = 5 : 6$ n‡j $3x : 5y$ = KZ ?

11·3 Abcv‡Zi i“cvši

GLv‡b Abcv‡Zi iwk¸‡jv abvZ¥K msL¨v|

(1) $a:b=c:d$ n‡j, $b:a=d:c$ [e¨¯ÍKiY (*Invertendo*)]

cgvY : †`Iqv Av‡Q,

$$\frac{a}{b}=\frac{c}{d}$$

$\therefore ad=bc$ [Dfqc¶‡K bd Øviv ¸Y K‡i] ev, $\frac{ad}{ac}=\frac{bc}{ac}$ [Dfq c¶‡K ac Øviv fvM K‡i †hLv‡b a, c Gi †KvbwUB kb¨ bq]

ev, $\frac{d}{c}=\frac{b}{a}$

A_vr, $b:a=d:c$

(2) $a:b=c:d$ n‡j, $a:c=b:d$ [GKvšÍiKiY ($alternendo$)]

cgvY : †`Iqv Av‡Q,

$$\frac{a}{b}=\frac{c}{d}$$

$\therefore ad=bc$ [Dfqc¶‡K bd Øviv ¸Y K‡i]

ev, $\frac{ad}{cd}=\frac{bc}{cd}$ [Dfq c¶‡K cd Øviv fvM K‡i †hLv‡b c, d Gi †KvbwUB kb¨ bq]

ev, $\frac{a}{c}=\frac{b}{d}$

A_vr, $a:c=b:d$

(3) $a:b=c:d$ n‡j, $\frac{a+b}{b}=\frac{c+d}{d}$ [†hvRb ($componendo$)]

cgvY : †`Iqv Av‡Q,

$$\frac{a}{b}=\frac{c}{d}$$

$$\therefore \frac{a}{b}+1=\frac{c}{d}+1$$ [Dfqc‡¶ 1†hvM K‡i]

A_vr, $\frac{a+b}{b}=\frac{c+d}{d}$

(4) $a:b=c:d$ n‡j, $\frac{a-b}{b}=\frac{c-d}{d}$ [we‡qvRb ($dividendo$)]

cgvY : †`Iqv Av‡Q,

$$\frac{a}{b}=\frac{c}{d}$$

$$\therefore \frac{a}{b}-1=\frac{c}{d}-1$$ [Dfqc¶ ‡_‡K 1 we‡qvM K‡i]

A_vr, $\frac{a-b}{b}=\frac{c-d}{d}$

(5) $a:b=c:d$ n‡j, $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ [†hvRb-we‡qvRb ($componendo-dividendo$)]

cgvY : $a:b=c:d$

†hvRb K‡i cvB,

$$\frac{a+b}{b}=\frac{c+d}{d}..............(i)$$

Avevi we‡qvRb K‡i cvB,

$$\frac{a-b}{b}=\frac{c-d}{d}$$

ev, $\frac{b}{a-b}=\frac{d}{c-d}$ [e¨¯ÍKiY K‡i] ....(ii)

mZivs, $\frac{a+b}{b}\times\frac{b}{a-b}=\frac{c+d}{d}\times\frac{d}{c-d}$ [(i) I (ii) ¸Y K‡i]

A_vr, $\frac{a+b}{a-b}=\frac{c+d}{c-d}$. [GLv‡b $a\neq b$ Ges $c\neq d$]

(6) $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}$ n‡j, c‡Z¨KwU AbcvZ $=\frac{a+c+e+g}{b+d+f+h}$.

cgvY : g‡b Kwi, $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=k.$

$$\therefore a=bk,\quad c=dk,\quad e=fk,\quad g=hk$$

$$\therefore \frac{a+c+e+g}{b+d+f+h}=\frac{bk+dk+fk+hk}{b+d+f+h}=\frac{k(b+d+f+h)}{b+d+f+h}=k.$$

wKš k c`Ë mgvbcv‡Zi c‡Z¨KwU Abcv‡Zi mgvb|

$$\therefore \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=\frac{a+c+e+g}{b+d+f+h}.$$

KvR : 1| gvZv I Kb¨vi eZgvb eq‡mi mgwó s eQi| Zv‡`i eq‡mi AbcvZ $t$ eQi c‡e wQj $r:p$, $x$ eQi c‡i Zv‡`i eq‡mi AbcvZ KZ n‡e ?

2| GKwU j¨v¤ú‡cv÷ †_‡K p wgUvi `‡i `vov‡bv r wgUvi D”PZv wewkó GK e¨w³i Qvqvi ˆ`N¨ $s$ wgUvi| j¨v¤ú‡cv‡ói D”PZv $h$ wgUvi n‡j, H e¨w³ j¨v¤ú‡cv÷ †_‡K KZ `‡i `vov‡bv wQ‡jb?

D`vniY 2| wcZv I c‡Îi eZgvb eq‡mi AbcvZ 7 : 2 Ges 5 eQi c‡i Zv‡`i eq‡mi AbcvZ 8 : 3 n‡e| Zv‡`i eZgvb eqm KZ ?

mgvavb : g‡b Kwi, wcZvi eZgvb eqm a eQi Ges c‡Îi eZgvb eqm b eQi|

c‡ki c_g I wØZxq kZvbmv‡i h_vµ‡g cvB,

$$\frac{a}{b}=\frac{7}{2}...........(i)$$

$$\frac{a+5}{b+5}=\frac{8}{3}...........(ii)$$

mgxKiY (i) ‡_‡K cvB,

$$a = \frac{7b}{2} \ldots\ldots\ldots (iii)$$

mgxKiY (ii) †_‡K cvB,

$$3(a+5) = 8(b+5)$$

ev, $3a + 15 = 8b + 40$

ev, $3a - 8b = 25$

ev, $3 \times \frac{7b}{2} - 8b = 25$ [(iii) e¨envi K‡i]

ev, $\frac{21b - 16b}{2} = 25$

ev, $5b = 50$

$\therefore\ b = 10$

mgxKiY (iii) G $b = 10$ ewm‡q cvB, $a = 35$

$\therefore$ wcZvi eZgvb eqm 35 eQi Ges c‡Îi eZgvb eqm 10 eQi|

D`vniY 3| hw` $a : b = b : c$ nq, Z‡e cgvY Ki †h, $\left(\frac{a+b}{b+c}\right)^2 = \frac{a^2+b^2}{b^2+c^2}$.

mgvavb : †`Iqv Av‡Q, $a : b = b : c$

$$\therefore\ b^2 = ac$$

GLb, $\left(\frac{a+b}{b+c}\right)^2 = \frac{(a+b)^2}{(b+c)^2}$

$$= \frac{a^2 + 2ab + b^2}{b^2 + 2bc + c^2}$$

$$= \frac{a^2 + 2ab + ac}{ac + 2bc + c^2}$$

$$= \frac{a(a+2b+c)}{c(a+2b+c)} = \frac{a}{c}$$

Ges $\frac{a^2+b^2}{b^2+c^2} = \frac{a^2+ac}{ac+c^2}$

$$= \frac{a(a+c)}{c(a+c)}$$

$$= \frac{a}{c}$$

$$\therefore\ \left(\frac{a+b}{b+c}\right)^2 = \frac{a^2+b^2}{b^2+c^2}$$

D`vniY 4| $\dfrac{a}{b}=\dfrac{c}{d}$ n‡j, †`Lvl †h, $\dfrac{a^2+b^2}{a^2-b^2}=\dfrac{ac+bd}{ac-bd}$.

mgvavb : g‡b Kwi, $\dfrac{a}{b}=\dfrac{c}{d}=k$; $\therefore\ a=bk$ Ges $c=dk$

GLb, $\dfrac{a^2+b^2}{a^2-b^2}=\dfrac{(bk)^2+b^2}{(bk)^2-b^2}=\dfrac{b^2(k^2+1)}{b^2(k^2-1)}=\dfrac{k^2+1}{k^2-1}$

Ges $\dfrac{ac+bd}{ac-bd}=\dfrac{bk\cdot dk+bd}{bk\cdot dk-bd}=\dfrac{bd(k^2+1)}{bd(k^2-1)}=\dfrac{k^2+1}{k^2-1}$

$$\therefore\quad \frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}.$$

D`vniY 5| mgvavb Ki : $\dfrac{1-ax}{1+ax}\sqrt{\dfrac{1+bx}{1-bx}}=1,\quad 0<b<2a<2b.$

mgvavb : †`Iqv Av‡Q, $\dfrac{1-ax}{1+ax}\sqrt{\dfrac{1+bx}{1-bx}}=1$

$$\therefore\quad \sqrt{\frac{1+bx}{1-bx}}=\frac{1+ax}{1-ax}$$

ev, $\dfrac{1+bx}{1-bx}=\dfrac{(1+ax)^2}{(1-ax)^2}$ [Dfq c¶‡K eM© K‡i]

ev, $\dfrac{1+bx}{1-bx}=\dfrac{1+2ax+a^2x^2}{1-2ax+a^2x^2}$

ev, $\dfrac{1+bx+1-bx}{1+bx-1+bx}=\dfrac{1+2ax+a^2x^2+1-2ax+a^2x^2}{1+2ax+a^2x^2-1+2ax-a^2x^2}$ [†hvRb-we‡qvRb K‡i]

ev, $\dfrac{2}{2bx}=\dfrac{2(1+a^2x^2)}{4ax}$

ev, $\dfrac{1}{bx}=\dfrac{1+a^2x^2}{2ax}$

ev, $2ax=bx(1+a^2x^2)$

ev, $x\{2a-b(1+a^2x^2)\}=0$

$\therefore$ nq $x=0$ A_ev $2a-b(a+a^2x^2)=0$

ev, $b(1+a^2x^2)=2a$

ev, $1+a^2x^2=\frac{2a}{b}$

ev, $a^2x^2=\frac{2a}{b}-1$

ev, $x^2=\frac{1}{a^2}\left(\frac{2a}{b}-1\right)$

$\therefore\ x=\pm\frac{1}{a}\sqrt{\frac{2a}{b}-1}$

$\therefore$ wb‡Yq mgvavb $x=0$, $x=\pm\frac{1}{a}\sqrt{\frac{2a}{b}-1}$.

D`vniY 6| mgvavb Ki : $\frac{6}{x}=\frac{1}{a}+\frac{1}{b}$ n‡j †`LvI †h, $\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=2,\ a\neq b.$

mgvavb : †`Iqv Av‡Q, $\frac{6}{x}=\frac{1}{a}+\frac{1}{b}$

$\therefore 6ab=(a+b)x$ [Dfq c¶‡K abx Øviv ¸Y K‡i]

A_vr, $x=\frac{6ab}{(a+b)}$

ev, $\frac{x}{3a}=\frac{2b}{a+b}$

$\therefore\ \frac{x+3a}{x-3a}=\frac{2b+a+b}{2b-a-b}$ [†hvRb-we‡qvRb K‡i]

ev, $\frac{x+3a}{x-3a}=\frac{a+3b}{b-a}$

Avevi, $\frac{x}{3b}=\frac{2a}{a+b}$

ev, $\frac{x+3b}{x-3b}=\frac{2a+a+b}{2a-a-b}$ [†hvRb-we‡qvRb K‡i]

$\therefore\ \frac{x+3b}{x-3b}=\frac{3a+b}{a-b}$

GLb, $\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{a+3b}{b-a}+\frac{3a+b}{a-b}$

$=\frac{a+3b}{b-a}-\frac{3a+b}{b-a}=\frac{a+3b-3a-b}{b-a}=\frac{2(b-a)}{b-a}=2.$

$\therefore\ \frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=2.$

D`vniY 7| $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=p$ n‡j, cgvY Ki †h, $p^2-\frac{2p}{x}+1=0.$

mgvavb : †`Iqv Av‡Q, $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=p$

$\therefore \frac{\sqrt{1+x}+\sqrt{1-x}+\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}-\sqrt{1+x}+\sqrt{1-x}}=\frac{p+1}{p-1}$ [†hvRb-we‡qvRb K‡i]

ev, $\frac{2\sqrt{1+x}}{2\sqrt{1-x}}=\frac{p+1}{p-1}$ ev, $\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{p+1}{p-1}$

ev, $\frac{1+x}{1-x}=\frac{(p+1)^2}{(p-1)^2}=\frac{p^2+2p+1}{p^2-2p+1}$ [Dfq c¶‡K eM K‡i]

ev, $\frac{1+x+1-x}{1+x-1+x}=\frac{p^2+2p+1+p^2-2p+1}{p^2+2p+1-p^2+2p-1}$ [†hvRb-we‡qvRb K‡i]

ev, $\frac{1}{x}=\frac{p^2+1}{2p}$ ev, $p^2+1=\frac{2p}{x}$

$\therefore \quad p^2-\frac{2p}{x}+1=0.$

D`vniY 8| $\frac{a^3+b^3}{a-b+c}=a(a+b)$ n‡j cgvY Ki †h, a, b, c µwgK mgvbycvZx|

mgvavb : †`Iqv Av‡Q, $\frac{a^3+b^3}{a-b+c}=a(a+b)$

ev, $\frac{a^3+b^3}{a-b+c}=a(a+b)$

ev, $\frac{(a+b)(a^2-ab+b^2)}{a-b+c}=a(a+b)$

ev, $\frac{a^2-ab+b^2}{a-b+c}=a$ [Dfqc¶‡K $(a+b)$ Øviv fvM K‡i]

ev, $a^2-ab+b^2=a^2-ab+ac$

$\therefore \quad b^2=ac$

$\therefore$ a, b, c µwgK mgvbycvZx|

D`vniY 9| hw` $\frac{a+b}{b+c}=\frac{c+d}{d+a}$ nq, Z‡e cgvY Ki †h, $c=a$ A_ev $a+b+c+d=0$.

mgvavb : †`Iqv Av‡Q, $\frac{a+b}{b+c}=\frac{c+d}{d+a}$

ev, $\frac{a+b}{b+c}-1=\frac{c+d}{d+a}-1$ [Dfqc¶ †_‡K 1 we‡qvM K‡i] ev, $\dfrac{a+b-b-c}{b+c}=\dfrac{c+d-d-a}{d+a}$

ev, $\dfrac{a-c}{b+c}=\dfrac{c-a}{d+a}$

ev, $\dfrac{a-c}{b+c}+\dfrac{a-c}{d+a}=0$

ev, $(a-c)\left(\dfrac{1}{b+c}+\dfrac{1}{d+a}\right)=0$

ev, $(a-c)\dfrac{(d+a+b+c)}{(b+c)(d+a)}=0$

ev, $(a-c)(d+a+b+c)=0$

$\therefore$ nq $a-c=0$ A_vr $a=c$

A_ev, $a+b+c+d=0.$

D`vniY 10| hw` $\dfrac{x}{y+z}=\dfrac{y}{z+x}=\dfrac{z}{x+y}$ Ges x, y, z mK‡j ci¯úi mgvb bv nq, Z‡e cgvY Ki †h, cÖZÖU Abcv‡Zi gvb -1 A_ev $\dfrac{1}{2}$ Gi mgvb n‡e|

mgvavb : g‡b Kwi,

$$\frac{x}{y+z}=\frac{y}{z+x}=\frac{z}{x+y}=k$$

$$\therefore x=k(y+z)................(i)$$

$$y=k(z+x)...................(ii)$$

$$z=k(x+y)...................(iii)$$

mgxKiY (i) †_‡K (ii) we‡qvM K‡i cvB,

$x-y=k(y-x)$ ev, $k(y-x)=-(y-x)$

$\therefore\ k=-1$

Avevi, mgxKiY (i), (ii) I (iii) †hvM K‡i cvB,

$$x+y+z=k(y+z+z+x+x+y)=2k(x+y+z)$$

$$\therefore k=\frac{1}{2}\frac{(x+y+z)}{(x+y+z)}=\frac{1}{2}$$

$\therefore$ cÖZÖU Abcv‡Zi gvb -1 A_ev $\dfrac{1}{2}$.

D`vniY 11| hw` $ax = by = cz$ nq, Z‡e †`LvI †h, $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{bc}{a^2}+\frac{ca}{b^2}+\frac{ab}{c^2}$.

mgvavb : g‡b Kwi,

$$ax = by = cz = k$$

$$\therefore\ x=\frac{k}{a},\quad y=\frac{k}{b},\quad z=\frac{k}{c}$$

GLb, $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{k^2}{a^2}\times\frac{bc}{k^2}+\frac{k^2}{b^2}\times\frac{ca}{k^2}+\frac{k^2}{c^2}\times\frac{ab}{k^2}=\frac{bc}{a^2}+\frac{ca}{b^2}+\frac{ab}{c^2}$

A_vr, $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{bc}{a^2}+\frac{ca}{b^2}+\frac{ab}{c^2}$.

Abkxjbx 11·1

1| `BwU eM‡¶‡Îi evûi ˆ`N¨ h_vµ‡g a wgUvi Ges b wgUvi n‡j, Zv‡`i †¶Îd‡ji AbycvZ KZ ?

2| GKwU eË‡¶‡Îi †¶Îdj GKwU eM‡¶‡Îi †¶Îd‡ji mgvb n‡j, Zv‡`i cwimxgvi AbycvZ wbYq Ki|

3| `BwU msL¨vi AbycvZ $3:4$ Ges Zv‡`i j.mv.¸. 180; msL¨v `BwU wbYq Ki|

4| GKw`b †Zvgv‡`i Kv‡m †`Lv †Mj Abcw¯Z I Dcw¯Z QvÎ msL¨vi AbycvZ $1:4$, Abcw¯Z QvÎ msL¨v‡K †gvU QvÎ msL¨vi kZKivq cKvk Ki|

5| GKwU `e¨ µq K‡i 28% ¶wZ‡Z weµq Kiv n‡jv| weµqgj¨ I µqg‡j¨i AbycvZ wbYq Ki|

6| wcZv I c‡Îi eZgvb eq‡mi mgwó 70 eQi| Zv‡`i eq‡mi AbycvZ 7 eQi c‡e wQj 5:2| 5 eQi c‡i Zv‡`i eq‡mi AbycvZ KZ n‡e ?

7| hw` $a:b=b:c$ nq, Z‡e cgvY Ki †h,

(i) $\frac{a}{c}=\frac{a^2+b^2}{b^2+c^2}$ (ii) $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

(iii) $\frac{abc(a+b+c)^3}{(ab+bc+ca)^3}=1$ (iv) $a-2b+c=\frac{(a-b)^2}{a}=\frac{(b-c)^2}{c}$

8| mgvavb Ki : (i) $\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}=\frac{1}{3}$ (ii) $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}=b$

(iii) $\frac{a+x-\sqrt{a^2-x^2}}{a+x+\sqrt{a^2-x^2}}=\frac{b}{x}$, $2a>b>0$ Ges $x\neq 0$.

(iv) $\frac{\sqrt{x-1}+\sqrt{x-6}}{\sqrt{x-1}-\sqrt{x-6}}=5$ (v) $\frac{\sqrt{ax+b}+\sqrt{ax-b}}{\sqrt{ax+b}-\sqrt{ax-b}}=c$

$(vi)\quad 81\left(\frac{1-x}{1+x}\right)^3=\frac{1+x}{1-x}$

9| $\frac{a}{b}=\frac{c}{d}$ n‡j, †`LvI th, $(i)\ \frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{c^2+cd+d^2}{c^2-cd+d^2}$ $\quad (ii)\ \frac{ac+bd}{ac-bd}=\frac{c^2+d^2}{c^2-d^2}$

10| $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$ n‡j, †`LvI th,

$(i)\ \frac{a^3+b^3}{b^3+c^3}=\frac{b^3+c^3}{c^3+d^3}$

$(ii)\ (a^2+b^2+c^2)(b^2+c^2+d^2)=(ab+bc+cd)^2$

11| $x=\frac{4ab}{a+b}$ n‡j, †`LvI th, $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}=2,\quad a\neq b.$

12| $x=\frac{\sqrt[3]{m+1}+\sqrt[3]{m-1}}{\sqrt[3]{m+1}-\sqrt[3]{m-1}}$ n‡j, cgvY Ki th, $x^3-3mx^2+3x-m=0$

13| $x=\frac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$ n‡j, †`LvI th, $3bx^2-4ax+3b=0.$

14| $\frac{a^2+b^2}{b^2+c^2}=\frac{(a+b)^2}{(a+c)^2}$ n‡j, cgvY Ki th, a, b, c µwgK mgvbycvZx|

15| $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$ n‡j, cgvY Ki th, $\frac{a}{y+z-x}=\frac{b}{z+x-y}=\frac{c}{x+y-z}.$

16| $\frac{bz-cy}{a}=\frac{cx-az}{b}=\frac{ay-bx}{c}$ n‡j, cgvY Ki th, $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}.$

17| $\frac{a+b-c}{a+b}=\frac{b+c-a}{b+c}=\frac{c+a-b}{c+a}$ Ges $a+b+c\neq 0$ n‡j, cgvY Ki th, $a=b=c.$

19| $\frac{x}{xa+yb+zc}=\frac{y}{ya+zb+xc}=\frac{z}{za+xb+yc}$ Ges $x+y+z\neq 0$ n‡j, †`LvI th,

cÖwZwU AbycvZ $=\frac{1}{a+b+c}.$

20| hw` $(a+b+c)p=(b+c-a)q=(c+a-b)r=(a+b-c)s$ nq, Z‡e cgvY Ki th,
$\frac{1}{q}+\frac{1}{r}+\frac{1}{s}=\frac{1}{p}.$

21| hw` $lx=my=nz$ nq, Z‡e †`LvI th, $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{mn}{l^2}+\frac{nl}{m^2}+\frac{lm}{n^2}.$

23| hw` $\frac{p}{q}=\frac{a^2}{b^2}$ Ges $\frac{a}{b}=\frac{\sqrt{a+q}}{\sqrt{a-q}}$ nq, Z‡e †`LvI th, $\frac{p+q}{a}=\frac{p-q}{q}.$

11·4 avivewnK AbcvZ

g‡b Ki, iwbi Avq 1000 UvKv, mwbi Avq 1500 UvKv Ges mwgi Avq 2500 UvKv|

GLv‡b, iwbi Avq : mwbi Avq = 1000 : 1500 = 2 : 3; mwbi Avq : mwgi Avq = 1500 : 2500 = 3 : 5. mZivs iwbi Avq : mwbi Avq : mwgi Avq = 2 : 3 : 5.

`BwU AbcvZ hw` K : L Ges L : M AvKv‡ii nq, Zvn‡j Zv‡`i‡K mvaviYZ K : L : M AvKv‡i †jLv hvq| G‡K avivewnK AbcvZ ejv nq| †h†Kv‡bv `BwU ev Z‡ZwaK AbcvZ‡K GB AvKv‡i cKvk Kiv hvq| GLv‡b j¶Yxq †h, `BwU AbcvZ‡K K : L : M AvKv‡i cKvk Ki‡Z n‡j c_g AbcvZwUi DËi iwk, wØZxq AbcvZwUi ce iwki mgvb n‡Z n‡e| †hgb, 2 : 3 Ges 4 : 3 AbcvZ `BwU K : L : M AvKv‡i cKvk Ki‡Z n‡j c_g AbcvZwUi DËi iwkwU‡K wØZxq AbcvZwUi ce iwki mgvb Ki‡Z n‡e| A_vr H `BwU iwk‡K Zv‡`i j.mv.¸. Gi mgvb Ki‡Z n‡e|

GLb, $2:3=\frac{2}{3}=\frac{2\times 4}{3\times 4}=\frac{8}{12}=8:12$ Avevi, $4:3=\frac{4}{3}=\frac{4\times 3}{3\times 3}=\frac{12}{9}=12:9$

AZGe 2 : 3 Ges 4 : 3 AbcvZ `BwU K : L : M AvKv‡i n‡e 8 : 12 : 9.

j¶ Kwi †h, Dc‡ii D`vni‡Y mwgi Avq hw` 1125 UvKv nq, Zvn‡j Zv‡`i Av‡qi AbcvZ I 8 : 12 : 9 AvKv‡i †jLv hv‡e|

D`vniY 12| K, L I M GK RvZxq iwk Ges K : L = 3 : 4, L : M = 6 : 7 n‡j, K : L : M KZ ?

mgvavb: $\frac{K}{L}=\frac{3}{4}=\frac{3\times 3}{4\times 3}=\frac{9}{12}$ Ges $\frac{L}{M}=\frac{6}{7}=\frac{6\times 2}{7\times 2}=\frac{12}{14}$ [GLv‡b 4 I 6 Gi j. mv. ¸. 12]

$\therefore$ K : L : M = 9 : 12 : 14.

D`vniY 13| GKwU wÎf‡Ri wZbwU †Kv‡Yi AbcvZ 3 : 4 : 5; †KvY wZbwU wbY©q Ki|

mgvavb : wÎf‡Ri wZb †Kv‡Yi mgwó = 180º

g‡b Kwi, c`Ë AbcvZ Abmv‡i †KvY wZbwU h_vµ‡g 3x, 4x Ges 5x.

cklbmv‡i, 3x + 4x + 5x = 180º ev, 12x = 180º ev, x = 15º

AZGe, †KvY wZbwU n‡jv 3x = 3 × 15º = 45º

4x = 4 × 15º = 60º

Ges 5x = 5 × 15º = 75º

D`vniY 14| hw` †Kv‡bv eM©‡¶‡Îi c‡Z¨K evûi cwigvY 10% ew× cvq, Z‡e Zvi †¶Îdj kZKiv KZ ew× cv‡e ?

mgvavb : g‡b Kwi, eM©‡¶‡Îi c‡Z¨K evûi ˆ`N¨ a wgUvi|

$\therefore$ eM©‡¶ÎwUi †¶Îdj a^2 eMwgUvi|

10% ew× †c‡j c‡Z¨K evûi ˆ`N¨ nq $(a+a$ Gi 10%) wgUvi ev $1{\cdot}10a$ wgUvi|

G‡¶‡Î, eM©‡¶ÎwUi †¶Îdj $(1{\cdot}10a)^2$ eMwgUvi ev $1\cdot 21a^2$ eMwgUvi

†¶Îdj e„w× cvq $(1\cdot 21a^2 - a^2) = 0.21a^2$ eMFGUvi

$\therefore$ †¶Îdj kZKiv e„w× cv‡e $\dfrac{0\cdot 21a^2}{a^2}\times 100\% = 21\%$

KvR 1| †Zvgv‡`i †kÖYx‡Z 35 Rb QvÎ I 25 Rb QvÎx Av‡Q| eb‡fvR‡b wLPzwi LvIqvi Rb¨ c‡Z¨K QvÎ I QvÎxi c`Ë Pvj I Wv‡ji AbycvZ h_vµ‡g 3 : 1 Ges 5 : 2 n‡j, ‡gvU Pvj I †gvU Wv‡ji AbycvZ †ei Ki|

11·5 mgvbycvwZK fvM

†Kv‡bv iwk‡K wbw`©ó Abycv‡Z fvM Kiv‡K mgvbycvwZK fvM ejv nq| S †K $a:b:c:d$ Abyc‡v‡i fvM Ki‡Z n‡j, S †K †gvU $(a + b + c + d)$ fvM K‡i h_vµ‡g a, b, c I d fvM wb‡Z nq|

AZGe

1g Ask $= S$ Gi $\dfrac{a}{a+b+c+d} = \dfrac{Sa}{a+b+c+d}$

2q Ask $= S$ Gi $\dfrac{b}{a+b+c+d} = \dfrac{Sb}{a+b+c+d}$

3q Ask $= S$ Gi $\dfrac{c}{a+b+c+d} = \dfrac{Sc}{a+b+c+d}$

4_ Ask $= S$ Gi $\dfrac{d}{a+b+c+d} = \dfrac{Sd}{a+b+c+d}$

Gfv‡e †h‡Kv‡bv iwk‡K †h‡Kv‡bv wbw`©ó Abycv‡Z fvM Kiv hvq|

D`vniY 15| wZb e¨w³i g‡a¨ 5100 UvKv Gi|‡c fvM K‡i `vI †hb, 1g e¨w³i Ask : 2q e¨w³i Ask : 3q e¨w³i Ask $= \dfrac{1}{2} : \dfrac{1}{3} : \dfrac{1}{9}$ nq|

mgvavb : GLv‡b $\dfrac{1}{2}:\dfrac{1}{3}:\dfrac{1}{9} = \left(\dfrac{1}{2}\times 18\right):\left(\dfrac{1}{3}\times 18\right):\left(\dfrac{1}{9}\times 18\right)$ [2, 3 I 9 Gi j.mv.¸. 18]

$= 9:6:2$

Abyc‡v‡Zi iwk¸‡jvi †hvMdj = 9 + 6 + 2 = 17.

1g e¨w³i Ask $= 5100\times\dfrac{9}{17}$ UvKv = 2700 UvKv

2q e¨w³i Ask $= 5100\times\dfrac{6}{17}$ UvKv = 1800 UvKv

3q e¨w³i Ask $= 5100\times\dfrac{2}{17}$ UvKv = 600 UvKv

AZGe wZb e¨w³ h_vK‡g 2700 UvKv , 1800 UvKv Ges 600 UvKv cv‡eb|

Abkxjbx 11.2

1| a, b, c µwgK mgvbycvZx n‡j wb‡Pi †KvbwU mwVK?

K. $a^2 = bc$ L. $b^2 = ac$

M. $ab = bc$ N. $a = b = c$

2| Awid I AwK‡ei eq‡mi AbycvZ 5:3; Awi‡di eqm 20 eQi n‡j, KZ eQi ci Zv‡`i eq‡mi AbycvZ 7:5 n‡e?

K. 5 eQi L. 6 eQi

M. 8 eQi N. 10 eQi

3| wb‡Pi Z_¨¸‡jv j¶¨ Ki:

i mgvbycv‡Zi PviwU iwkB GKRvZxq nIqvi c‡qvRb nq bv|

ii `BwU wÎfR †¶‡Îi †¶Îd‡ji AbycvZ Zv‡`i fwgØ‡qi Abycv‡Zi mgvb|

iii $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h}$ n‡j G‡`i cÖwZwU Abycv‡Zi gvb $\frac{a+c+e+g}{b+b+f+h}$

Dc‡ii Z_¨¸‡jvi wfwË‡Z wb‡Pi †KvbwU mwVK ?

K. i I ii L. ii I iii

M. i I iii N. i, ii I iii

ΔABC Gi †KvY¸‡jvi AbycvZ 2:3:5 Ges ABCD PZf©‡Ri †KvY PviwUi AbycvZ 3:4:5:6 Dc‡ii Z‡_¨i wfwË‡Z 4 I 5 bs c‡ki DËi `vI|

4| GKwU e‡Mi evûi ^`N¨ wØ¸Y n‡j Dnvi †¶Îdj KZ¸Y e„w× cv‡e|

K. 2 ¸Y L. 4 ¸Y

M. 8 ¸Y N. 6 ¸Y

5| x : y = 7 : 5, y : z = 5 : 7 n‡j x : z = KZ ?

K. 35 : 49 L. 35 : 35

M. 25 : 49 N. 49 : 25

6| GKwU Kv‡Vi cj ˆZwii cv°wjZ e¨q 90,000 UvKv | wKš LiP †ewk n‡q‡Q 21,600 UvKv| LiP kZKiv KZ e„w× †c‡q‡Q ?

7| av‡b Pvj I Z‡li AbycvZ 7 : 3 n‡j, G‡Z kZKiv Kx cwigvY Pvj Av‡Q ?

8| 1 Nb †m. wg. Kv‡Vi IRb 7 †WwmMÖvg| Kv‡Vi IRb mgAvqZb cvwbi IR‡bi kZKiv KZ fvM ?

9| K, L ,M ,N Gi g‡a¨ 300 UvKv Ggbfv‡e fvM K‡i `vI †hb, K Gi Ask : L Gi Ask = 2 : 3, L Gi Ask : M Gi Ask = 1 :2 Ges M Gi Ask : N Gi Ask = 3 : 2 nq|

10| wZbRb †R‡j 690 wU gvQ a‡i‡Q| Zv‡`i As‡ki AbycvZ $\frac{2}{3}, \frac{4}{5}$ Ges $\frac{5}{6}$ n‡j, †K KqwU gvQ †cj?

11| GKwU wÎf‡Ri cwimxgv 45 †m. wg.| evû¸‡jvi ‰`‡N¨i AbycvZ 3: 5: 7 n‡j , c‡Z¨K evûi cwigvY wbYq Ki|

12| 1011 UvKv‡K $\frac{3}{4}:\frac{4}{5}:\frac{6}{7}$ Abycv‡Z wef³ Ki|

13| `BwU msL¨vi AbycvZ 5 : 7 Ges Zv‡`i M. mv. ¸. 4 n‡j, msL¨v `BwUi j. mv. ¸. KZ ?

14| wµ‡KU †Ljvq mwwKe, gkwdKi I gvkivdx 171 ivb Ki‡jv| mwwKe I gkwdK‡ii Ges gkwdKi I gvkivdxi iv‡bi AbycvZ 3 : 2 n‡j †K KZ ivb K‡i‡Q ?

15| GKwU Awd‡m 2 Rb KgKZv, 7 Rb KiwYK Ges 3 Rb wcIb Av‡Q| GKRb wcIb 1 UvKv †c‡j GKRb KiwYK cvq 2 UvKv, GKRb KgKZv cvq 4 UvKv| Zv‡`i mK‡ji †gvU †eZb 150,000 UvKv n‡j, †K KZ †eZb cvq ?

16| GKwU mwgwZi ‡bZv wbevPb `BRb cwZØ›`xi g‡a¨ ‡Wvbvì mv‡ne 4 : 3 †fv‡U Rqjvf Ki‡jb| hw` †gvU m`m¨ msL¨v 581 nq Ges 91 Rb m`m¨ †fvU bv w`‡q _v‡Kb, Z‡e †Wvbvì mv‡n‡ei cwZØ›`x KZ †fv‡Ui e¨eav‡b civwRZ n‡q‡Qb ?

17| hw` †Kv‡bv eM‡¶‡Îi evûi cwigvY 20% e„w× cvq, Z‡e Zvi †¶Îdj kZKiv KZ e„w× cv‡e ?

18| GKwU AvqZ‡¶‡Îi ˆ`N¨ 10% e„w× Ges c¯ 10% nvm ‡c‡j AvqZ‡¶‡Îi ‡¶Îdj kZKiv KZ e„w× ev nvm cv‡e ?

19| GKwU gv‡Vi Rwg‡Z †m‡Pi m‡hvM Avmvi Av‡Mi I c‡ii dj‡bi AbycvZ 4 : 7. H gv‡V †h Rwg‡Z Av‡M 304 KB›Uvj avb dj‡Zv, †mP cvIqvi c‡i Zvi djb KZ n‡e ?

20| avb I avb †_‡K Drcb Pv‡ji AbycvZ 3 : 2 Ges Mg I Mg †_‡K Drcb mwRi AbycvZ 4 : 3 n‡j, mgvb cwigv‡Yi avb I Mg †_‡K Drcb Pvj I mwRi AbycvZ †ei Ki |

21| GKwU Rwgi †¶Îdj 432 eMwgUvi| H Rwgi ˆ`N¨ I c‡¯i m‡½ Aci GKwU Rwgi ˆ`N¨ I c‡¯i AbycvZ h_vµ‡g 3 : 4 Ges 2 : 5 n‡j, Aci Rwgi †¶Îdj KZ ?

22| †Rwg I wmwg GKB e¨vsK †_‡K GKB w`‡b 10% mij gbvdvq Avjv`v Avjv`v cwigvY A_ FY †bq| †Rwg 2 eQi ci gbvdv-Avm‡j hZ UvKv †kva K‡i 3 eQi ci wmwg gbvdv-Avm‡j ZZ UvKv †kva K‡i| Zv‡`i F‡Yi AbycvZ wbYq Ki|

23| GKwU wÎf‡Ri evû¸‡jvi AbycvZ 5:12:13 Ges cwimxgv 30 †m.wg.
K. wÎfRwU A¼b Ki Ges †KvY †f‡` wÎfRwU Kx ai‡bi Zv wjL?
L. enËi evû‡K ˆ`N¨ Ges ¶`Zi evû‡K c¯ a‡i Aw¼Z AvqZ‡¶‡Îi K‡Yi mgvb evû wewkóe‡Mi †¶Îdj wbYq Ki|
M. D³ AvqZ‡¶‡Îi ˆ`N¨ 10% Ges c¯ 20% e„w× †c‡j †¶Îdj kZKiv KZ e„w× cv‡e?

24| GKw`b †Kv‡bv Kv‡m Abcw¯Z I Dcw¯Z wk¶v_xi AbycvZ 1:4|
K. Abcw¯Z wk¶v_x‡`i‡K †gvU wk¶v_xi kZKivq cKvk Ki|
L. 10 Rb wk¶v_x †ewk Dcw¯Z n‡j Abcw¯Z I Dcw¯Z wk¶v_xi AbycvZ n‡Zv 1:9. †gvU wk¶v_xi msL¨v KZ ?
M. †gvU wk¶v_xi g‡a¨ QvÎ msL¨v QvÎx msL¨vi wØ¸Y A‡c¶v 20 Rb Kg| QvÎ I QvÎxmsL¨vi AbycvZ wbYq Ki|

Øv`k Aa¨vq

`B PjKwewkó mij mnmgxKiY

(Simple Simultaneous Equations in Two Variables)

MwYwZK mgm¨v mgvav‡bi Rb¨ exRMwY‡Zi me‡P‡q ¸i“ZcY welq n‡jv mgxKiY| lô I mßg †kwY‡Z Avgiv mij mgxKi‡Yi aviYv †c‡qwQ Ges Kxfv‡e GK PjKwewkó mij mgxKiY mgvavb Ki‡Z nq Zv †R‡bwQ| Aóg †kwY‡Z mij mgxKiY cwZ¯vcb I Acbqb c×wZ‡Z Ges †jLwP‡Îi mvnv‡h¨ mgvavb K‡iwQ| Kxfv‡e ev¯ewfwËK mgm¨vi mij mnmgxKiY MVb K‡i mgvavb Kiv nq ZvI wk‡LwQ| G Aa¨v‡q mij mnmgxKi‡Yi aviYv m¤cmviY Kiv n‡q‡Q I mgvav‡bi Av‡iv bZb c×wZ m¤ú‡K Av‡jvPbv Kiv n‡q‡Q| G QvovI G Aa¨v‡q †jLwP‡Îi mvnv‡h¨ mgvavb I ev¯ewfwËK mgm¨vi mnmgxKiY MVb I mgvavb m¤ú‡K we¯wiZ Av‡jvPbv Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_©xiv –

- `B PjKwewkó mij mnmgxKi‡Yi m½wZ hvPvB Ki‡Z cvi‡e|
- `B PjKwewkó `BwU mgxKi‡Yi ci¯úi wbfikxjZv hvPvB Ki‡Z cvi‡e|
- mgvav‡bi Avo¸Yb c×wZ e¨vL¨v Ki‡Z cvi‡e|
- ev¯ewfwËK MwYwZK mgm¨vi mnmgxKiY MVb K‡i mgvavb Ki‡Z cvi‡e|
- †jLwP‡Îi mvnv‡h¨ `B PjKwewkó mij mnmgxKiY mgvavb Ki‡Z cvi‡e|

12·1 mij mnmgxKiY

mij mnmgxKiY ej‡Z `B PjKwewkó `BwU mij mgxKiY‡K †evSvq hLb Zv‡`i GK‡Î Dc¯vcb Kiv nq Ges PjK `BwU GKB ˆewk‡ó¨i nq| Avevi Gi¦c `BwU mgxKiY‡K GK‡Î mij mgxKiY‡RvUI e‡j| Aóg †kwY‡Z Avgiv Gi¦c mgxKiY‡Rv‡Ui mgvavb K‡iwQ I ev¯ewfwËK mgm¨vi mnmgxKiY MVb K‡i mgvavb Ki‡Z wk‡LwQ| G Aa¨v‡q G m¤ú‡K Av‡iv we¯wiZ Av‡jvPbv Kiv n‡q‡Q|

c_‡g Avgiv $2x+y=12$ mgxKiYwU we‡ePbv Kwi| GwU GKwU `B PjKwewkó mij mgxKiY| mgxKiYwU‡Z evgc‡¶ $x$ I $y$ Gi Ggb gvb cvIqv hv‡e wK hv‡`i c_gwU wØ¸‡Yi mv‡_ wØZxqwUi †hvMdj Wvbc‡¶i 12 Gi mgvb nq, A_©vr H gvb `BwU Øviv mgxKiYwU wm× nq ?

GLb, $2x+y=12$ mgxKiYwU †_‡K wb‡Pi QKwU ciY Kwi :

x Gi gvb	y Gi gvb	evgc¶ $(2x+y)$ Gi gvb	Wvbc¶
-2	16	$-4+16=12$	12
0	12	$0+12=12$	12
3	6	$6+6=12$	12
5	2	$10+2=12$	12
....	.....	..... $=12$	12

mgxKiYwUi AmsL¨ mgvavb Av‡Q| Zvi g‡a¨ PviwU mgvavb $(-2,16)$, $(0,12)$, $(3,6)$ I $(5,2)$|

Avevi, Ab¨ GKwU mgxKiY $x-y=3$ wb‡q wb‡Pi QKwU ciY Kwi :

x Gi gvb	y Gi gvb	evgc¶ $(x-y)$ Gi gvb	Wvbc¶
-2	-5	$-2+5=3$	3
0	-3	$0+3=3$	3
3	0	$3-0=3$	3
5	2	$5-2=3$	3
....	.....	..... $=3$	3

mgxKiYwUi AmsL¨ mgvavb Av‡Q| Zvi g‡a¨ PviwU mgvavb :
$(-2,-5)$, $(0,-3)$, $(3,0)$ I $(5,2)$

hw` Av‡jvP¨ mgxKiY `BwU‡K GK‡Î †RvU wn‡m‡e aiv nq, Z‡e GKgvÎ $(5,2)$ Øviv Dfq mgxKiY hMcr wm× nq| Avi Ab¨ †Kv‡bv gvb Øviv Dfq mgxKiY hMcr wm× n‡e bv|

AZGe, mgxKiY‡RvU $2x+y=12$ Ges $x-y=3$ Gi mgvavb : $(x,y)=(5,2)$

KvR : $x-2y+1=0$ I $2x+y-3=0$ mgxKiYØ‡qi c‡Z¨KwUi cvPwU K‡i mgvavb †jL †hb Zb¥‡a¨ mvaviY mgvavbwUI _v‡K|

12·2 `B PjKwewkó mij mnmgxKi‡Yi mgvavb †hvM¨Zv

(K) c‡ei Av‡jwPZ mgxKiY‡RvU $\left.\begin{array}{r}2x+y=12\\ x-y=3\end{array}\right\}$ Gi Abb¨ (GKwU gvÎ) mgvavb cvIqv †M‡Q|

Gi¦c mgxKiY‡RvU‡K m½wZcY ev mvgÄm¨cY $(Consistent)$ ejv nq| mgxKiY `BwUi x I y Gi mnM Zjbv K‡i (mn‡Mi AbcvZ wb‡q) cvB, $\frac{2}{1}\neq\frac{1}{-1}$, mgxKiY‡RvUwUi GKwU mgxKiY‡K Ab¨wUi gva¨‡g cKvk Kiv hvq bv| G Rb¨ Gi¦c mgxKiY‡K ci¯úi Awbfikxj $(Independent)$ mgxKiY‡RvU ejv nq| m½wZcY I ci¯úi Awbfikxj mgxKiY‡RvUi †¶‡Î AbcvZ¸‡jv mgvb bq|
G‡¶‡Î a"eKc` Zjbv Kivi c‡qvRb nq bv|

(L) GLb Avgiv $\left.\begin{array}{l}2x-y=6\\ 4x-2y=12\end{array}\right\}$ mgxKiY‡RvUwU we‡ePbv Kwi| GB `BwU mgxKiY mgvavb Kiv hv‡e wK ?

GLv‡b, 1g mgxKiYwUi Dfqc¶‡K 2 Øviv ¸Y K‡i 2q mgxKiYwU cvIqv hv‡e| Avevi, 2q mgxKi‡Yi Dfqc¶‡K 2 Øviv fvM K‡i 1g mgxKiYwU cvIqv hv‡e| A_vr, mgxKiY `BwU ci¯úi wbfikxj|

Avgiv Rwb, 1g mgxKiYwUi AmsL¨ mgvavb Av‡Q| Kv‡RB, 2q mgxKiYwUiI H GKB AmsL¨ mgvavb Av‡Q| Giƒc mgxKiY‡RvU‡K m½wZcY I ci¯úi wbfikxj (*dependent*) mgxKiY‡RvU e‡j| Giƒc mgxKiY‡Rv‡Ui AmsL¨ mgvavb Av‡Q|

GLv‡b, mgxKiY `BwUi $x$ I $y$ Gi mnM Ges a“eK c` Zjbv K‡i cvB, $\frac{2}{4}=\frac{-1}{-2}=\frac{6}{12}\left(=\frac{1}{2}\right)$

A_vr, m½wZcY I ci¯úi wbfikxj mgxKiY‡Rv‡Ui †¶‡Î AbcvZ¸‡jv mgvb nq|

(M) Gev‡i Avgiv $\left.\begin{aligned}2x+y&=12\\4x+2y&=5\end{aligned}\right\}$ mgxKiY‡RvUwU mgvavb Kivi †Póv Kwi|

GLv‡b, 1g mgxKiYwUi Dfqc¶‡K 2 Øviv ¸Y K‡i cvB, $4x+2y=24$

2q mgxKiYwU $4x+2y=\ \ 5$

we‡qvM K‡i cvB, $0=19$, hv Am¤¢e|

Kv‡RB ej‡Z cwi, G ai‡bi mgxKiY‡RvU mgvavb Kiv m¤¢e bq| Giƒc mgxKiY‡RvU Am½wZcY (*inconsistent*) I ci¯úi Awbfikxj| Giƒc mgxKiY‡Rv‡Ui †Kv‡bv mgvavb †bB|

GLv‡b mgxKiY `BwUi $x$ I $y$ Gi mnM Ges a“eK c` Zjbv K‡i cvB, $\frac{2}{4}=\frac{1}{2}\neq\frac{12}{5}$.

A_vr, Am½wZcY I ci¯úi Awbfikxj mgxKiY‡Rv‡Ui †¶‡Î PjK¸‡jvi mnM¸‡jvi AbcvZ a“eK¸‡jvi AbcvZ mgvb bq|

mvaviYfv‡e, $\left.\begin{aligned}a_1x+b_1y&=c_1\\a_2x+b_2y&=c_2\end{aligned}\right\}$ mgxKiY‡RvUwU wb‡q wb‡Pi Q‡K gva¨‡g `BwU mij mgxKi‡Yi mgvavb †hvM¨Zvi kZ D‡jL Kiv n‡jv :

	mgxKiY‡RvU	mnM I aeK c` Zjbv	m½wZcY/Am½wZcY	ci¯úi wbfikxj/ Awbfikxj	mgvavb Av‡Q (KqwU)/†bB
(i)	$a_1x+b_1y=c_1$ $a_2x+b_2y=c_2$	$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$	m½wZcY	Awbfikxj	Av‡Q (GKwUgvÎ)
(ii)	$a_1x+b_1y=c_1$ $a_2x+b_2y=c_2$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$	m½wZcY	wbfikxj	Av‡Q (AmsL¨)
(iii)	$a_1x+b_1y=c_1$ $a_2x+b_2y=c_2$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$	Am½wZcY	Awbfikxj	†bB

GLb, hw` †Kv‡bv mgxKiY‡Rv‡U Dfq mgxKi‡Y a"eK c` bv _v‡K, A_©vr, $c_1 = c_2 = 0$ nq, Z‡e Q‡Ki

(i) Abhvqx $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ n‡j, mgxKiY‡RvU me`v m½wZcY I ci¯úi Awbfik©j| †m‡¶‡Î GKwUgvÎ (Abb¨) mgvavb _vK‡e|

(ii) I (iii) †_‡K $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ n‡j, mgxKiY‡RvU m½wZcY I ci¯úi wbfik©j| †m‡¶‡Î AmsL¨ mgvavb _vK‡e|

D`vniY : wb‡Pi mgxKiY‡RvU¸‡jv m½wZcY/Am½wZcY, wbfik©j/Awbfik©j wK bv e¨vL¨v Ki Ges G‡`i mgvav‡bi msL¨v wb‡`k Ki|

(K) $x + 3y = 1$
$2x + 6y = 2$

(L) $2x - 5y = 3$
$x + 3y = 1$

(M) $3x - 5y = 7$
$6x - 106y = 15$

mgvavb :

(K) c`Ë mgxKiY‡RvU : $\left.\begin{aligned} x + 3y &= 1 \\ 2x + 6y &= 2 \end{aligned}\right\}$

x Gi mnMØ‡qi AbycvZ $\frac{1}{2}$

y ,, ,, ,, $\frac{3}{6}$ ev $\frac{1}{2}$

a"eK c`Ø‡qi AbycvZ $\frac{1}{2}$

$\therefore \quad \frac{1}{2} = \frac{3}{6} = \frac{1}{2}$

AZGe, mgxKiY‡RvUwU m½wZcY I ci¯úi wbfik©j| mgxKiY‡RvUwUi AmsL¨ mgvavb Av‡Q|

(L) c`Ë mgxKiY‡RvU : $\left.\begin{aligned} 2x - 5y &= 3 \\ x + 3y &= 1 \end{aligned}\right\}$

x Gi mnMØ‡qi AbycvZ $\frac{2}{1}$

y ,, ,, ,, $\frac{-5}{3}$

Avgiv cvB, $\frac{2}{1} \neq \frac{-5}{3}$

$\therefore$ mgxKiY‡RvUwU m½wZcY I ci¯úi Awbfik©j| mgxKiY‡RvUwUi GKwUgvÎ (Abb¨) mgvavb Av‡Q|

(M) c`Ë mgxKiY‡RvU : $3x - 5y = 7$

$6x - 10y = 15$

x Gi mnMØ‡qi AbcvZ $\frac{3}{6}$ ev $\frac{1}{2}$

y ,, ,, ,, $\frac{-5}{-10}$ ev $\frac{1}{2}$

a“eK c`Ø‡qi AbcvZ $\frac{7}{15}$

Avgiv cvB, $\frac{3}{6} = \frac{-5}{-10} \neq \frac{7}{15}$

∴ mgxKiY‡RvUwU Am½wZcY I ci¯úi Awbfikxj| mgxKiY‡RvUwUi †Kv‡bv mgvavb †bB|

KvR : $x - 2y + 1 = 0, 2x + y - 3 = 0$ mgxKiY‡RvUwU m½wZcY wK bv, ci¯úi wbfikxj wK bv hvPvB Ki Ges mgxKiY‡RvUwUi KqwU mgvavb _vK‡Z cv‡i Zv wb‡`k Ki|

Abkxjbx 12·1

wb‡Pi mij mnmgxKiY¸‡jv m½wZcY/Am½wZcY, ci¯úi wbfikxj/Awbfikxj wK bv hw³mn D‡jL Ki Ges G¸‡jvi mgvav‡bi msL¨v wb‡`k Ki :

1| $x - y = 4$
$x + y = 10$

2| $2x + y = 3$
$4x + 2y = 6$

3| $x - y - 4 = 0$
$3x - 3y - 10 = 0$

4| $3x + 2y = 0$
$6x + 4y = 0$

5| $3x + 2y = 0$
$9x - 6y = 0$

6| $5x - 2y - 16 = 0$
$3x - \frac{6}{5}y = 2$

7| $-\frac{1}{2}x + y = -1$
$x - 2y = 2$

8| $-\frac{1}{2}x - y = 0$
$x - 2y = 0$

9| $-\frac{1}{2}x + y = -1$
$x + y = 5$

10| $ax - cy = 0$
$cx - ay = c^2 - a^2.$

12·3 mij mnmgxKi‡Yi mgvavb

Avgiv ïa m½wZcY I ci¯úi Awbfikxj mij mnmgxKi‡Yi mgvavb m¤ú‡K Av‡jvPbv Ki‡ev| Gi¨c mgxKiY‡Rv‡Ui GKwUgvÎ (Abb¨) mgvavb Av‡Q|

GLv‡b, mgvav‡bi PviwU c×wZi D‡jL Kiv n‡jv :

(1) cwZ¯vcb c×wZ (2) Acbqb c×wZ (3) Avo¸Yb c×wZ I (4) ˆjwLK c×wZ|

Avgiv Aóg †kYx‡Z cwZ¯vcb I Acbqb c×wZ‡Z mgvavb Kx fv‡e Ki‡Z nq †R‡bwQ| G `B c×wZi GKwU K‡i D`vniY †`Iqv n‡jv :

D`vniY 1| cwZ¯vcb c×wZ‡Z mgvavb Ki :

$$2x + y = 8$$
$$3x - 2y = 5$$

mgvavb : c`Ë mgxKiYØq $2x + y = 8$.............(1)
$3x - 2y = 5$...........(2)

mgxKiY (1) n‡Z cvB, $y = 8 - 2x$.........(3)

mgxKiY (2) G y Gi gvb $8 - 2x$ ewm‡q cvB,

$3x - 2(8 - 2x) = 5$	x Gi gvb mgxKiY (3) G ewm‡q cvB,
ev $3x - 16 + 4x = 5$	$y = 8 - 2 \times 3$
ev $3x + 4x = 5 + 16$	$= 8 - 6$
ev $7x = 21$	$= 2$
ev $x = 3$	

$\therefore$ mgvavb $(x, y) = (3,2)$

cwZ¯vcb c×wZ‡Z mgvavb : mweavgZ GKwU mgxKiY †_‡K GKwU Pj‡Ki gvb Aci Pj‡Ki gva¨‡g cKvk K‡i cvß gvb Aci mgxKi‡Y emv‡j GK PjKwewkó mgxKiY cvIqv hvq| AZtci mgxKiYwU mgvavb K‡i PjKwUi gvb cvIqv hvq| GB gvb c`Ë mgxKi‡Yi †h †Kv‡bwU‡Z emv‡bv †h‡Z cv‡i| Z‡e †hLv‡b GKwU PjK‡K Aci Pj‡Ki gva¨‡g cKvk Kiv n‡q‡Q †mLv‡b emv‡j mgvavb mnR nq| GLvb †_‡K Aci Pj‡Ki gvb cvIqv hvq|

D`vniY 2| Acbqb c×wZ‡Z mgvavb Ki : $2x + y = 8$
$3x - 2y = 5$

[`óe¨ : cwZ¯vcb I Acbqb c×wZi cv_K¨ †evSv‡ZB D`vniY 1 Gi mgxKiYØqB D`vniY 2 G †bqv n‡jv]

mgvavb : c`Ë mgxKiYØq $2x + y = 8$.............(1)
$3x - 2y = 5$...........(2)

mgxKiY (1) Gi Dfqc¶‡K 2 Øviv ¸Y K‡i, $4x + 2y = 16$.............(3)

mgxKiY (2) $3x - 2y = 5$...........(2)

mgxKiY (2) I (3) †hvM K‡i cvB,

$$7x = 21$$
ev, $x = 3$

x Gi gvb mgxKiY (1) G ewm‡q cvB,
$$2 \times 3 + y = 8$$
ev, $y = 8 - 6$
ev, $y = 2$

$\therefore$ mgvavb $(x, y) = (3,2)$

Acbqb c×wZ‡Z mgvavb : mwVKfv‡e mgvavb Ki‡Z n‡j GKwU mgxKiY‡K ev Dfq mgxKiY‡K Gi/c msL¨v w`‡q ¸Y Ki‡Z n‡e †hb ¸Y‡bi ci Dfq mgxKi‡Yi †h‡Kv‡bv GKwU Pj‡Ki mn‡Mi ciggvb mgvb nq| Gici c‡qvRbgZ mgxKiY `BwU‡K †hvM ev we‡qvM Ki‡j mnM mgvbKZ PjKwU Acbx Z ev Acmwi Z nq| Zvici mgxKiYwU mgvavb Ki‡j we`¨gvb PjKwUi gvb cvIqv hvq| H gvb mwVKfvgZ c`Ë mgxKiYØ‡qi †h‡Kv‡bvwU‡Z emv‡j Aci PjKwUi gvb cvIqv hvq|

(3) Avo¸Yb c×wZ :

Avo¸Yb c×wZ‡K eR¸Yb c×wZI e‡j|

wb‡Pi mgxKiY `BwU we‡ePbv Kwi :

$$a_1x + b_1y + c_1 = 0 \text{..........}(1)$$
$$a_2x + b_2y + c_2 = 0 \text{..........}(2)$$

mgxKiY (1) †K b_2 w`‡q I mgxKiY (2) †K $b_1$ w`‡q ¸Y K‡i cvB,

$$a_1b_2x + b_1b_2y + b_2c_1 = 0 \text{.........}(3)$$
$$a_2b_1x + b_1b_2y + b_1c_2 = 0 \text{.........}(4)$$

mgxKiY (3) †_‡K mgxKiY (4) we‡qvM K‡i cvB,

$$(a_1b_2 - a_2b_1)x + b_2c_1 - b_1c_2 = 0$$
ev, $(a_1b_2 - a_2b_1)x = b_1c_2 - b_2c_1$

ev, $\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{1}{a_1b_2 - a_2b_1}$ ....(5)

Avevi, mgxKiY (1) †K a_2 w`‡q I mgxKiY (2) †K $a_1$ w`‡q ¸Y K‡i cvB,

$$a_1a_2x + a_2b_1y + c_1a_2 = 0 \text{.........}(6)$$
$$a_1a_2x + a_1b_2y + c_2a_1 = 0 \text{.........}(7)$$

mgxKiY (6) †_‡K mgxKiY (7) we‡qvM K‡i cvB,

$$(a_2b_1 - a_1b_2)y + c_1a_2 - c_2a_1 = 0$$
ev, $-(a_1b_2 - a_2b_1)y = -(c_1a_2 - c_2a_1)$

ev, $\dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1}$ ....(8)

(5) I (8) †_‡K cvB,

$$\boxed{\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}}$$

x I y Gi Gi¦c m¤úK †_‡K G‡`i gvb wbY‡qi †KŠkj‡K Avo¸Yb c×wZ e‡j|

x I y Gi DwjwLZ m¤úK †_‡K cvB,

$$\frac{x}{b_1c_2-b_2c_1}=\frac{1}{a_1b_2-a_2b_1}\text{ , ev } x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$$

Avevi, $\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$, ev $y=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$

∴ c`Ë mgxKiYØ‡qi mgvavb : $(x,y)=\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)$

j¶ Kwi :

mgxKiY	x I y Gi g‡a¨ m¤úK	g‡b ivLvi wPÎ
$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$	$\begin{array}{c\|cccc} & x & & y & 1 \\ a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \end{array}$

`óe¨ : c`Ë Dfq mgxKi‡Yi a"eK c` Wvbc‡¶ †i‡L I Avo¸Yb c×wZ c‡qvM Kiv hvq| Z‡e †m‡¶‡Î wP‡ýi wKQz cwieZb n‡e| wKš— mgvavb GKB cvIqv hv‡e|

KvR : $\left.\begin{array}{l}4x-y-7=0\\3x+y\quad\;=0\end{array}\right\}$ mgxKiY‡RvU‡K

$\left.\begin{array}{l}a_1x+b_1y+c_1=0\\a_2x+b_2y+c_2=0\end{array}\right\}$ mgxKiY‡Rv‡Ui AvKv‡i cKvk Ki‡j

a_1,b_1,c_1,a_2,b_2,c_2 Gi gvb †ei Ki|

D`vniY 3| Avo¸Yb c×wZ‡Z mgvavb Ki : $6x-y=1$

$3x+2y=13$

mgvavb : c¶vš—i cwµqvq c`Ë mgxKiYØ‡qi Wvbc¶ 0 (kb¨) K‡i cvB,

$6x-y-1=0$

$3x+2y-13=0$

mgxKiYØq‡K h_vµ‡g $a_1x+b_1y_2+c_1=0$

Ges $a_2x+b_2y+c_2=0$

Gi mv‡_ Zjbv K‡i cvB, $a_1=6,b_1=-1,c_1=-1$

$a_2=3,b_2=2,c_2=-13$

Avo¸Yb c×wZ‡Z cvB,

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

e¨vL¨v

$$\begin{array}{c|cccc} & x & & y & 1 \\ a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \end{array}$$

$$\downarrow$$

$$\begin{array}{c|cccc} & x & & y & 1 \\ 6 & -1 & -1 & 6 & -1 \\ 3 & 2 & -13 & 3 & 2 \end{array}$$

ev $\frac{x}{(-1)\times(-13)-2\times(-1)}=\frac{y}{(-1)\times 3-(-13)\times 6}=\frac{1}{6\times 2-3\times(-1)}$

ev $\frac{x}{13+2}=\frac{y}{-3+78}=\frac{1}{12+3}$

ev $\frac{x}{15}=\frac{y}{75}=\frac{1}{15}$

$\therefore$ $\frac{x}{15}=\frac{1}{15}$ ev $x=\frac{15}{15}=1$

Avevi, $\frac{y}{75}=\frac{1}{15}$ ev $y=\frac{75}{15}=5$

$\therefore$ mgvavb $(x, y)=(1,5)$

D`vniY 4| Avo¸Yb c×wZ‡Z mgvavb Ki : $3x-4y=0$
$2x-3y=-1$

mgvavb : c`Ë mgxKiYØq

$$\left.\begin{array}{l}3x-4y=0\\2x-3y=-1\end{array}\right\}$$

ev, $$\left.\begin{array}{l}3x-4y+0=0\\2x-3y+1=0\end{array}\right\}$$

Avo¸Yb c×wZ‡Z cvB,

$$\frac{x}{-4\times 1-(-3)\times 0}=\frac{y}{0\times 2-1\times 3}=\frac{1}{3\times(-3)-2\times(-4)}$$

$$\begin{array}{c|cccc} & x & y & 1 & \\ 3 & -4 & 0 & 3 & -4 \\ 2 & -3 & 1 & 2 & -3 \end{array}$$

ev $\frac{x}{-4+0}=\frac{y}{0-3}=\frac{1}{-9+8}$

ev $\frac{x}{-4}=\frac{y}{-3}=\frac{1}{-1}$

ev $\frac{x}{4}=\frac{y}{3}=\frac{1}{1}$

$\therefore$ $\frac{x}{4}=\frac{1}{1}$ ev, $x=4$

Avevi, $\frac{y}{3}=\frac{1}{1}$ ev, $y=3$

$\therefore$ mgvavb $(x, y)=(4,3)$

D`vniY 5| Avo¸Yb c×wZ‡Z mgvavb Ki : $\frac{x}{2}+\frac{y}{3}=8$

$$\frac{5x}{4}-3y=-3$$

mgvavb : c`Ë mgxKiYØq‡K $ax+by+c=0$ AvKv‡i mvwR‡q cvB,

$\frac{x}{2}+\frac{y}{3}=8$

ev $\frac{3x+2y}{6}=8$

ev $3x+2y-48=0$

Avevi, $\frac{5x}{4}-3y=-3$

ev $\frac{5x-12y}{4}=-3$

ev $5x-12y+12=0$

$\therefore$ mgxKiYØq $3x+2y-48=0$

$5x-12y+12=0$

Avo¸Yb c×wZ‡Z cvB,

$$\frac{x}{2\times12-(-12)\times(-48)}=\frac{y}{(-48)\times5-12\times3}=\frac{1}{3\times(-12)-5\times2}$$

	x		y	1
3	2	−48	3	2
5	−12	12	5	−12

ev $\frac{x}{24-576}=\frac{y}{-240-36}=\frac{1}{-36-10}$

ev $\frac{x}{-552}=\frac{y}{-276}=\frac{1}{-46}$

ev $\frac{x}{552}=\frac{y}{276}=\frac{1}{46}$

$\therefore$ $\frac{x}{552}=\frac{1}{46}$ ev, $x=\frac{552}{46}=12$

Avevi, $\frac{y}{276}=\frac{1}{46}$ ev. $y=\frac{276}{46}=6$

$\therefore$ mgvavb : $(x,y)=(12,6)$

mgvav‡bi ïw× cix¶v : cvß x I y Gi gvb c`Ë mgxKi‡Y ewm‡q cvB,

1g mgxKi‡Y, evgc¶ = $\frac{x}{2}+\frac{y}{3}=\frac{12}{2}+\frac{6}{3}=6+2$

$=8=$ Wvbc¶

2q mgxKi‡Y, evgc¶ = $\frac{5x}{4}-3y=\frac{5\times12}{4}-3\times6$

$=15-18=-3$ = Wvbc¶|

$\therefore$ mgvavb ï× n‡q‡Q|

D`vniY 6| Avo¸Yb c×wZ‡Z mgvavb Ki : $ax-by=ab=bx-ay$.

mgvavb : c`Ë mgxKiYØq,

$$\left.\begin{aligned} ax-by&=ab \\ bx-ay&=ab \end{aligned}\right\} \text{ ev, } \left.\begin{aligned} ax-by-ab&=0 \\ bx-ay-ab&=0 \end{aligned}\right\}$$

$$\therefore \frac{x}{(-b)\times(-ab)-(-a)(-ab)}=\frac{y}{(-ab)\times b-(-ab)\times a}=\frac{1}{a\times(-a)-b\times(-b)}$$

$$\begin{array}{c|cccc} & x & & y & 1 \\ a & -b & -ab & a & -b \\ b & -a & -ab & b & -a \end{array}$$

ev $\dfrac{x}{ab^2-a^2b}=\dfrac{y}{-ab^2+a^2b}=\dfrac{1}{-a^2+b^2}$

ev $\dfrac{x}{-ab(a-b)}=\dfrac{y}{ab(a-b)}=\dfrac{1}{-(a+b)(a-b)}$

ev $\dfrac{x}{ab(a-b)}=\dfrac{y}{-ab(a-b)}=\dfrac{1}{(a+b)(a-b)}$

$\therefore \dfrac{x}{ab(a-b)}=\dfrac{1}{(a+b)(a-b)}$, ev $x=\dfrac{ab(a-b)}{(a+b)(a-b)}=\dfrac{ab}{a+b}$

Avevi, $\dfrac{y}{-ab(a-b)}=\dfrac{1}{(a+b)(a-b)}$, ev $y=\dfrac{-ab(a-b)}{(a+b)(a-b)}=\dfrac{-ab}{a+b}$

$$\therefore (x,y)=\left(\frac{ab}{a+b},\frac{-ab}{a+b}\right)$$

Abkxjbx 12·2

cwZ¯vcb c×wZ‡Z mgvavb Ki (1 Ñ 3) :

1| $7x-3y=31$, $9x-5y=41$

2| $\dfrac{x}{2}+\dfrac{y}{3}=1$, $\dfrac{x}{3}+\dfrac{y}{2}=1$

3| $\dfrac{x}{a}+\dfrac{y}{b}=2$, $ax+by=a^2+b^2$

Acbqb c×wZ‡Z mgvavb Ki (4 Ñ 6) :

4| $7x-3y=31$, $9x-5y=41$

5| $7x-8y=-9$, $5x-4y=-3$

6| $ax+by=c$, $a^2x+b^2y=c^2$

Avo¸Yb c`awZ‡Z mgvavb Ki (7 Ñ 15) :

7| $2x+3y+5=0$, $4x+7y+6=0$

8| $3x-5y+9=0$, $5x-3y-1=0$

9| $x+2y=7$, $2x-3y=0$

10| $4x+3y=-12$ 11| $-7x+8y=9$ 12| $3x-y-7=0=2x+y-3$

$2x=5$ $5x-4y=-3$

13| $ax+by=a^2+b^2$ 14| $y(3+x)=x(6+y)$

$2bx-ay=ab$ $3(3+x)=5(y-1)$

15| $(x+7)(y-3)+7=(y+3)(x-1)+5$

$5x-11y+35=0$

12·4 ˆjwLK c×wZ‡Z mgvavb

`B PjKwewkó GKwU mij mgxKi‡Y we`¨gvb PjK x I y Gi m¤úK‡K wP‡Î i mvnv‡h¨ cKvk Kiv hvq| GB wPÎ‡K H m¤ú‡Ki †jLwPÎ e‡j| G RvZxq mgxKi‡Yi †jLwP‡Î AmsL¨ we›` _v‡K| Gi/c K‡qKwU we›` ¯’vcb K‡i G‡`i ci¯úi msh³ Ki‡jB †jLwPÎ cvIqv hvq|

mij mnmgxKi‡Yi c‡Z¨KwUi AmsL¨ mgvavb i‡q‡Q| c‡Z¨KwU mgxKi‡Yi †jL GKwU mij‡iLv| mij‡iLwUi c‡Z¨KwU we›`i ¯’vbvsK mgxKiYwU‡K wm× K‡i| †Kv‡bv †jL wbw`ó Ki‡Z `B ev Z‡ZwaK we›` †bqv Avek¨K|

GLb Avgiv wb‡Pi mgxKiY‡RvUwU mgvavb Kiv i †Póv Kie : $2x+y=3$..........(1)

$4x+2y=6$........(2)

mgxKiY (1) †_‡K cvB, $y=3-2x$.

mgxKiYwU‡Z x Gi K‡qKwU gvb wb‡q y Gi Abi/c gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

x	-1	0	3
y	5	3	-3

$\therefore$ mgxKiYwUi †j‡Li Dci wZbwU we›` $(-1,5),(0,3)$ I $(3,-3)$|

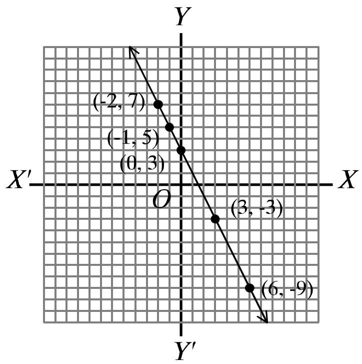
Avevi, mgxKiY (2) †_‡K cvB, $2y=6-4x$ ev, $y=\frac{6-4x}{2}$

x	-2	0	6
y	7	3	-9

mgxKiYwU‡Z x Gi K‡qKwU gvb wb‡q y Gi Abi/‡c gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

$\therefore$ mgxKiYwUi †j‡Li Dci wZbwU we›` $(-2,7),(0,3)$ I $(6,-9)$|

g‡b Kwi, QK Kv‡M‡R XOX' I YOY' h_vµ‡g x-A¶ I y-A¶ Ges O gjwe›`|



QK Kv‡Ri Dfq A¶ eivei ¶`Zg eM‡¶‡Îi cwZevûi ˆˆN¨‡K GKK awi| GLb mgxKiY (1) n‡Z cvß $(-1,5),(0,3)$ I $(3,-3)$ we›`M‡jv ¯’vcb Kwi I Zv‡`i ci¯úi msh³ Kwi| †jLwU GKwU mij‡iLv|

Avevi, mgxKiY (2) n‡Z cvß $(-2,7),(0,3)$ I $(6,-9)$ we›`¸‡jv ¯'vcb Kwi I Zv‡`i ci¯úi msh³ Kwi| G‡¶‡Î I †jLwU GKwU mij‡iLv| Z‡e j¶ Kwi, mij‡iLv `BwU ci¯ú‡ii Dci mgvcwZZ n‡q GKwU mij‡iLvq cwiYZ n‡q‡Q| Avevi, mgxKiY (2) Gi Dfqc¶‡K 2 Øviv fvM Ki‡j mgxKiY (1) cvIqv hvq| G Kvi‡Y mgxKiYØ‡qi †jL ci¯úi mgvcwZZ n‡q‡Q|

GLv‡b, $\left.\begin{array}{l}2x+y=3..........(1)\\4x+2y=6........(2)\end{array}\right\}$ mgxKiY‡RvUwU m½wZcY I ci¯úi wbfikxj| Gi|c mgxKiY‡Rv‡Ui AmsL¨ mgvavb Av‡Q Ges mgxKiY‡RvUwUi †jL GKwU mij‡iLv|

Gevi Avgiv wb‡Pi mgxKiY‡RvUwU mgvavb Kivi †Póv Kie : $2x-y=4..........(1)$

$$4x-2y=12......(2)$$

mgxKiY (1) †_‡K cvB, $y=2x-4$.

mgxKiYwU‡Z x Gi K‡qKwU gvb wb‡q y Gi Abi|c gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

x	-1	0	4
y	-6	-4	4

$\therefore$ mgxKiYwUi †j‡Li Dci wZbwU we›` $(-1,-6),(0,-4),(4,4)$ |

Avevi, mgxKiY (2) †_‡K cvB,

$4x-2y=12$, ev $2x-y=6$ [Dfqc¶‡K 2 Øviv fvM K‡i]

ev $y=2x-6$

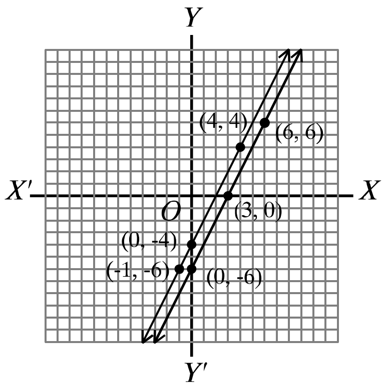
mgxKiYwU‡Z x Gi K‡qKwU gvb wb‡q y Gi Abi|c gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

x	0	3	6
y	-6	0	6

$\therefore$ mgxKiYwUi †j‡Li Dci wZbwU we›` $(0,-6),(3,0),(6,6)$ |

g‡b Kwi, QK KvM‡R XOX' I YOY' h_vµ‡g x-A¶ I y-A¶ Ges O gjwe›`|

QK KvM‡Ri Dfq A¶ eivei ¶`Zg eM©‡¶‡Îi cwZevûi ˆ``N¨‡K GKK a‡i mgxKiY (1) n‡Z cvß $(-1,-6),(0,-4)$ I $(4,4)$ we›`M‡jv ¯'vcb Kwi I Zv‡`i ci¯úi msh³ Kwi| †jLwU GKwU mij‡iLv|



Avevi, mgxKiY (2) n‡Z cvß $(0,-6),(3,0),(6,6)$ we›`¸‡jv ¯'vcb Kwi I G‡`i ci¯úi msh³ Kwi| G‡¶‡Î I †jLwU GKwU mij‡iLv|

wP‡Î j¶ Kwi, c`Ë mgxKiYØ‡qi c_Kfv‡e c‡Z¨KwUi AmsL¨ mgvavb _vK‡j I †RvU wn‡m‡e Zv‡`i mvaviY mgvavb †bB| Avi I j¶ Kwi †h, c`Ë mgxKiY `BwUi †jLwPÎ `BwU ci¯úi mgvšivj mij‡iLv| A_vr, †iLv `BwU KL‡bv G‡K Aci‡K †Q` Ki‡e bv| AZGe, G‡`i †Kv‡bv mvaviY †Q` we›` cvIqv hv‡e bv| G †¶‡Î Avgiv ewj †h, Gi|c mgxKiY‡Rv‡Ui †Kv‡bv mgvavb †bB| Avgiv Rwb, Gi|c mgxKiY‡RvU Am½wZcY I ci¯úi Awbfikxj|

Avgiv GLb †jLwP‡Îi mvnv‡h¨ m½wZcY I ci¯úi Awbfikxj mgxKiY‡RvU mgvavb Ki‡ev|

`B PjKwewkó `BwU m½wZcY I ci¯úi Awbfikxj mij mgxKi‡Yi †jL GKwU we›`‡Z †Q` K‡i| H †Q` we›`i ¯vbvsK Øviv Dfq mgxKiY wm× n‡e| †Q`we›`wUi ¯vbvsKB n‡e mgxKiYØ‡qi mgvavb|

D`vniY 7| mgvavb Ki I mgvavb †jLwP‡Î †`LvI : $2x+y=8$

$3x-2y=5$

mgvavb : : c`Ë mgxKiYØq $2x+y-8=0$...........(1)

$3x-2y-5=0$.............(2)

Avo¸Yb c×wZ‡Z cvB,

$$\frac{x}{1\times(-5)-(-2)\times(-8)}=\frac{y}{(-8)\times3-(-5)\times2}=\frac{1}{2(-2)-3\times1}$$

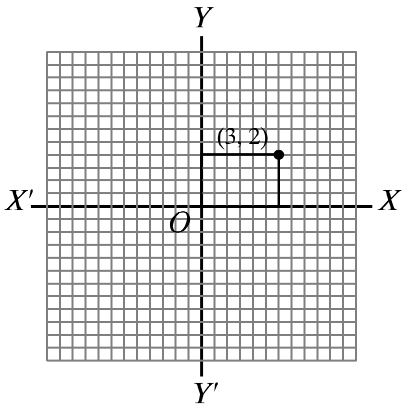
ev $\frac{x}{-5-16}=\frac{y}{-24+10}=\frac{1}{-4-3}$

ev $\frac{x}{-21}=\frac{y}{-14}=\frac{1}{-7}$

ev $\frac{x}{21}=\frac{y}{14}=\frac{1}{7}$

$\therefore \frac{x}{21}=\frac{1}{7}$, ev $x=\frac{21}{7}=3$

Avevi, $\frac{y}{14}=\frac{1}{7}$, ev $y=\frac{14}{7}=2$



$\therefore$ mgvavb : $(x,y)=(3,2)$

g‡b Kwi, XOX' I YOY' h_vµ‡g x-A¶ I y-A¶ Ges O gjwe›`|

QK KvM‡Ri Dfq A¶ eivei ¶`Zg e‡Mi cwZ `B evûi ˆ`N¨‡K GKK a‡i (3,2) we›`wU ¯vcb Kwi|

D`vniY 8| †jLwP‡Îi mvnv‡h¨ mgvavb Ki :

$3x-y=3$

$5x+y=21$

mgvavb : c`Ë mgxKiYØq $3x-y=3$..........(1)

$5x+y=21$........(2)

mgxKiY (1) †_‡K cvB, $3x - y = 3$, ev $y = 3x - 3$

mgxKiYwU‡Z x Gi K‡qKwU gvb wb‡q y Gi Abiƒc gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

x	-1	0	3
y	-6	-3	6

∴ mgxKiYwUi †j‡Li Dci wZbwU we›` $(-1,-6),(0,-3),(3,6)$

Avevi, mgxKiY (2) †_‡K cvB, $5x + y = 21$, ev $y = 21 - 5x$

mgxKiYwU‡Z x Gi K‡qKwU gvb wb‡q y Gi Abiƒc gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

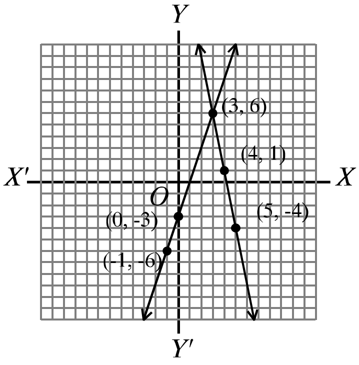
x	3	4	5
y	6	1	-4

∴ mgxKiYwUi †j‡Li Dci wZbwU we›` $(3,6),(4,1),(5,-4)$|

g‡b Kwi, XOX' I YOY' h_vµ‡g x-A¶ I y-A¶ Ges O gjwe›`|

QK KvM‡Ri Dfq A¶ eivei ¶`Zg e‡Mi cwZ evûi ˆ`N¨‡K GKK awi|

GLb QK KvM‡R mgxKiY (1) n‡Z cvß $(-1,-6),(0,-3),(3,6)$ we›`M‡jv ¯vcb Kwi I Zv‡`i ci¯úi msh³ Kwi| †jLwU GKwU mij‡iLv|



GKBfv‡e, mgxKiY (2) n‡Z cvß $(3,6),(4,1),(5,-4)$ we›`¸‡jv ¯vcb Kwi I Zv‡`i ci¯úi msh³ Kwi| G‡¶‡Î I †jLwU GKwU mij‡iLv|

g‡b Kwi, mij‡iLØq ci¯úi P we›`‡Z †Q` K‡i‡Q| wPÎ †_‡K †`Lv hvq, $P$ we›`i ¯vbvsK $(3,6)$

∴ mgvavb : $(x, y) = (3,6)$

D`vniY 9| ˆjwLK c×wZ‡Z mgvavb Ki :

$$2x + 5y = -14$$
$$4x - 5y = 17$$

mgvavb : : c`Ë mgxKiYØq

$$2x + 5y = -14 \text{...........}(1)$$
$$4x - 5y = 17 \text{........}(2)$$

mgxKiY (1) †_‡K cvB, $5y = -14 - 2x$, ev $y = \frac{-2x - 14}{5}$

mgxKiYwU‡Z x Gi mweavgZ K‡qKwU gvb wb‡q y Gi Abiƒc gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

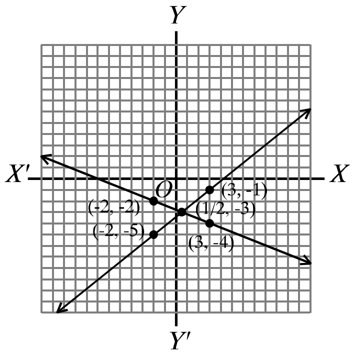
x	3	$\frac{1}{2}$	-2
y	-4	-3	-2

∴ mgxKiYwUi †j‡Li Dci wZbwU we›` $(3,-4),\left(\frac{1}{2},-3),(-2,-2)\right)$|

x	3	$\frac{1}{2}$	-2
y	-1	-3	-5

Avevi, mgxKiY (2) †_‡K cvB, $5y = 4x - 17$, ev $y = \frac{4x-17}{5}$

mgxKiYwU‡Z x Gi mwewavgZ K‡qKwU gvb wb‡q y Gi Abi~c gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :



∴ mgxKiYwUi †j‡Li Dci wZbwU we›` $(3,1), \left(\frac{1}{2}, -3\right), (-2, -5)$

g‡b Kwi, XOX' I YOY' h_vµ‡g x-A¶ I y-A¶ Ges O gjwe›`| QK KvM‡Ri Dfq A¶ eivei ¶`Zg e‡Mi cwZ `B evûi ˆ`N©¨‡K GKK awi|

GLb, QK KvM‡R mgxKiY (1) †_‡K cvß $(3,-4), \left(\frac{1}{2}, -3\right)$ I $(-2,-2)$ we›`¸‡jv ¯'vcb K‡i Zv‡`i cici msh³ Kwi| †jLwU GKwU mij‡iLv|

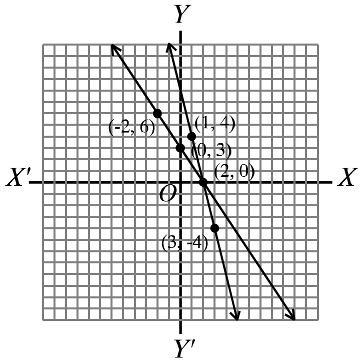
GKBfv‡e, mgxKiY (2) †_‡K cvß $(3,-1), \left(\frac{1}{2}, -3\right), (-2,-5)$ we›`¸‡jv ¯'vcb K‡i Zv‡`i cici msh³ Kwi| †jLwU GKwU mij‡iLv|

g‡b Kwi, mij‡iLvØq ci¯úi P we›`‡Z †Q` K‡i‡Q| wP‡Î †`Lv hvq, $P$ we›`i ¯'vbvsK $\left(\frac{1}{2}, -3\right)$

∴ mgvavb : $(x, y) = \left(\frac{1}{2}, -3\right)$

D`vniY 10| †j‡Li mvnv‡h¨ mgvavb Ki : $3 - \frac{3}{2}x = 8 - 4x$

mgvavb : c`Ë mgxKiY $3 - \frac{3}{2}x = 8 - 4x$



awi, $y = 3 - \frac{3}{2}x = 8 - 4x$

∴ $y = 3 - \frac{3}{2}x$..........(1)

Ges $y = 8 - 4x$..........(2)

GLb, mgxKiY (1) G x Gi K‡qKwU gvb wb‡q y Gi Abi~c gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

x	− 2	0	2
y	6	3	0

mgxKiYwUi †j‡Li Dci wZbwU we›` $(-2,6), (0,3), (2,0)$

Avevi, mgxKiY (2) G x-Gi K‡qKwU gvb wb‡q y-Gi Abi~c gvb †ei Kwi I cv‡ki QKwU ˆZwi Kwi :

x	1	2	3
y	4	0	− 4

∴ mgxKiYwUi †j‡Li Dci wZbwU we›` $(1,4), (2,0), (3,-4)$

g‡b Kwi, XOX' I YOY' h_vµ‡g x-A¶ I y-A¶ Ges O gjwe›`| QK KvM‡Ri Dfq A¶ eivei ¶`Zg e‡Mi cwZ evûi ˆ`N©¨‡K GKK awi|

GLb, QK KvM‡R mgxKiY (1) †_‡K cvß $(-2,6),(0,3),(2,0)$ we›`¸‡jv ¯vcb Kwi I we›`¸‡jv cici msh³ Kwi| Zvn‡j, †jLwU n‡e GKwU mij‡iLv| GKBfv‡e, mgxKiY (2) †_‡K cvß $(1,4),(2,0),(3,-4)$ we›`¸‡jv ¯vcb K‡i G¸‡jv cici msh³ Kwi| Zvn‡j, †jLwU n‡e GKwU mij‡iLv| g‡b Kwi, mij‡iLvØq ci¯úi $P$ we›`‡Z †Q` K‡i| wP‡Î †`Lv hvq, †Q`we›`wUi ¯vbvsK $(2,0)$ |

∴ mgvavb : $x = 2$, ev mgvavb : 2

KvR : $2x - y - 3 = 0$ mgxKi‡Yi †j‡Li Dci Q‡Ki gva¨‡g PviwU we›` wbYq Ki| AZ:ci QK KvM‡R wbw`ó ^`‡N¨i GKK wb‡q we›`¸‡jv ¯vcb Ki I Zv‡`i ci¯úi msh³ Ki| †jLwU wK mij‡iLv n‡q‡Q ?

Abkxjbx 12·3

†jLwP‡Îi mvnv‡h¨ mgvavb Ki :

1| $3x + 4y = 14$, $4x - 3y = 2$

2| $2x - y = 1$, $5x + y = 13$

3| $2x + 5y = 1$, $x + 3y = 2$

4| $3x - 2y = 2$, $5x - 3y = 5$

5| $\frac{x}{2} + \frac{y}{3} = 2$, $2x + 3y = 13$

6| $3x + y = 6$, $5x + 3y = 12$

7| $3x + 2y = 4$, $3x - 4y = 1$

8| $\frac{x}{2} + \frac{y}{3} = 3$, $x + \frac{y}{6} = 3$

9| $3x + 2 = x - 2$

10| $3x - 7 = 3 - 2x$

12·5 ev¯ewfwËK mgm¨vi mnmgxKiY MVb I mgvavb

^`bw›`b Rxe‡b Ggb wKQ MwYwZK mgm¨v Av‡Q hv mgxKiY MV‡bi gva¨‡g mgvavb Kiv mnRZi nq| G Rb¨ mgm¨vi kZ ev kZvewj †_‡K `BwU AÁvZ iwki Rb¨ `BwU MwYwZK cZxK, cavbZ PjK x, y aiv nq| AÁvZ iwk `BwUi gvb wbY‡qi Rb¨ `BwU mgxKiY MVb Ki‡Z nq| MwVZ mgxKiYØq mgvavb Ki‡jB AÁvZ iwk `BwUi gvb cvIqv hvq|

D`vniY 11| `B A¼wewkó †Kv‡bv msL¨vi A¼Ø‡qi mgwói mv‡_ 5 †hvM Ki‡j †hvMdj n‡e msL¨wUi `kK ¯vbxq A‡¼i wZb¸Y| Avi msL¨wUi A¼Øq ¯vb wewbgq Ki‡j †h msL¨v cvIqv hv‡e, Zv gj msL¨wU †_‡K 9 Kg n‡e| msL¨wU wbYq Ki|

mgvavb : g‡b Kwi, wb‡Yq msL¨wUi `kK ¯vbxq A¼ x Ges GKK ¯vbxq A¼ y| AZGe, msL¨wU $10x + y$.

$\therefore$ 1g kZvbmv‡i $x+y+5=3x$.........(1)

Ges 2q kZvbmv‡i, $10y+x=(10x+y)-9$.......(2)

mgxKiY (1) †_‡K cvB, $y=3x-x-5$, ev $y=2x-5$........(3)

Avevi, mgxKiY (2) †_‡K cvB,

$10y-y+x-10x+9=0$

ev $9y-9x+9=0$

ev $y-x+1=0$

ev $2x-5-x+1=0$ [(3) n‡Z y-Gi gvb ewm‡q]

ev $x=4$

(3) G x Gi gvb ewm‡q cvB,

$y=2\times4-5$

$=8-5$

$=3$

$\therefore$ wb‡Yq msL¨wU n‡e

$10x+y=10\times4+3$

$=40+3$

$=43$

$\therefore$ msL¨wU 43

D`vniY 12| AvU eQi c‡e wcZvi eqm c‡Îi eq‡mi AvU¸Y wQj| `k eQi ci wcZvi eqm c‡Îi eq‡mi wØ¸Y n‡e| eZgv‡b Kvi eqm KZ ?

mgvavb : g‡b Kwi, eZgv‡b wcZvi eqm x eQi I c‡Îi eqm y eQi|

$\therefore$ 1g kZvbmv‡i $x-8=8(y-8)$........(1)

Ges 2q kZvbmv‡i, $x+10=2(y+10)$.......(2)

(1) n‡Z cvB, $x-8=8y-64$

ev $x=8y-64+8$

ev $x=8y-56$.............(3)

(2) n‡Z cvB, $x+10=2y+20$

ev $8y-56+10=2y+20$ [(3) n‡Z x Gi gvb ewm‡q]

ev $8y-2y=20+56-10$

ev $6y=66$

ev $y=11$

$\therefore$ (3) n‡Z cvB, $x=8\times11-56=88-56=32$

$\therefore$ eZgv‡b wcZvi eqm 32 eQi I c‡Îi eqm 11 eQi|

D`vniY 13| GKwU AvqZvKvi evMv‡bi c‡¯'i wØ¸Y, ˆ`N¨ A‡c¶v 10 wgUvi †ewk Ges evMvbwUi cwimxgv 100 wgUvi|

K. evMvbwUi ˆ`N¨ x wg I c¯' y wg. a‡i mgxKiY‡RvU MVb Ki|

L. evMvbwUi ˆ`N¨ I c¯' wbYq Ki|

M. evMvbwUi mxgvbvi evB‡i Pvwi`‡K 2 wgUvi PIov iv¯Ív Av‡Q| iv¯ÍwU BU w`‡q ˆZwi Ki‡Z cwZeMwgUv‡i $110 \cdot 00$ UvKv wn‡m‡e †gvU KZ LiP n‡e ?

mgvavb : K. AvqZvKvi evMvbwUi ˆ`N©¨ x wgUvi I cÖ¯' y wgUvi|

$\therefore$ 1g kZvbmv‡i $2y = x + 10 ....(1)$

Ges 2q kZvbmv‡i, $2(x + y) = 100 ....(2)$

L. mgxKiY (1) n‡Z cvB, $2y = x + 10 ....(1)$

mgxKiY (2) n‡Z cvB, $2x + 2y = 100 ....(2)$

ev $2x + x + 10 = 100$ [(1) n‡Z $2y$ Gi gvb ewm‡q]

ev $3x = 90$ ev $x = 30$

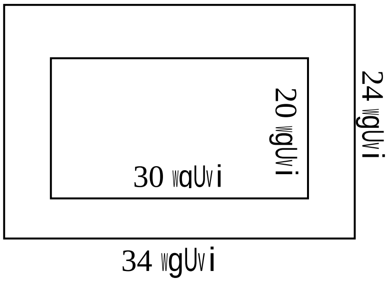
$\therefore$ (1) n‡Z cvB, $2y = 30 + 10$ [x Gi gvb ewm‡q]

ev, $2y = 40$ ev, $y = 20$

$\therefore$ evMvbwUi ˆ`N©¨ 30 wgUvi I cÖ¯' 20 wgUvi|

M. iv¯Ívi evB‡ii ˆ`N©¨ $(30 + 4)$ wg.= 34 wg

Ges cÖ¯' = $(20 + 4)$ wg. = 24 wg.



$\therefore$ iv¯Ívi †¶Îdj = iv¯Ívmn evMv‡bi †¶Îdj – evMv‡bi †¶Îdj

$= (34 \times 24 - 30 \times 20)$ eMwgUvi|

$= (816 - 600)$ eMwgUvi

$= 216$ eMwgUvi|

$\therefore$ BU w`‡q iv¯Ív ˆZwi Kivi LiP

$= 216 \times 110$ UvKv

$= 23760$ UvKv|

KvR : ABC wÎf‡R $\angle B = 2x$ wWwM, $\angle C = x$ wWwM, $\angle A = y$ wWwM Ges $\angle A = \angle B + \angle C$ n‡j, x I y Gi gvb wbYq Ki|	

Abkxjbx 12·4

1| wb‡Pi †Kvb k‡Z $ax + by + c = 0$ I $px + qy + r = 0$ mgxKiY‡RvUwU m½wZcY I ci¯úi Awbf©ikxj n‡e ?

K. $\frac{a}{p} \neq \frac{b}{q}$ L. $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ M. $\frac{a}{p} = \frac{b}{q} \neq \frac{c}{r}$ N. $\frac{a}{p} = \frac{b}{q}$

2| $x + y = 4, x - y = 2$ n‡j (x, y) Gi gvb wb‡Pi †KvbwU ?

K. $(2, 4)$ L. $(4, 2)$ M. (3.1) N. $(1, 3)$

3| $x + y = 6$ I $2x = 4$ n‡j, y gvb KZ ?

K. 2 L. 4 M. 6 N. 8

4| wb‡Pi †KvbwUi Rb¨ cv‡ki QKwU mwVK ?

x	0	2	4
y	-4	0	4

K. $y = x - 4$ L. $y = 8 - x$ M. $y = 4 - 2x$ N. $y = 2x - 4$

5| $2x - y = 8$ Ges $x - 2y = 4$ n‡j, $x + y$ = KZ ?

K. 0 L. 4 M. 8 N. 12

6| wb‡Pi Z_¨¸‡jv j¶ Ki :

$i.$ $2x - y = 0$ I $x - 2y = 0$ mgxKiYØq ci¯úi wbfikxj|

$ii.$ $x - 2y + 3 = 0$ mgxKi‡Yi †jLwPÎ $(-3, 0)$ we›`Mvgx|

$iii.$ $3x + 4y = 1$ mgxKi‡Yi †jLwPÎ GKwU mij‡iLv|

Dc‡ii Z‡_¨i wfwË‡Z wb‡Pi †KvbwU mwVK ?

K. i I ii L. ii I iii M. i I iii N. i, ii I iii

7| AvqZvKvi GKwU N‡ii †g‡Si ˆ`N¨, c¯' A‡c¶v 2 wgUvi †ewk Ges †g‡Si cwimxgv 20 wgUvi| wb‡Pi ck¸‡jvi DËi `vI :

(1) NiwUi †g‡Si ˆ`N¨ KZ wgUvi ?

K. 10 L. 8 M. 6 N. 4

(2) NiwUi †g‡Si †¶Îdj KZ eMwgUvi ?

K. 24 L. 32 M. 48 N. 80

(3) NiwUi †g‡S †gvRvBK Ki‡Z cwZ eMwgUv‡i 900 UvKv wn‡m‡e †gvU KZ LiP n‡e ?

K. 72000 L. 43200 M. 28800 N. 21600

mnmgxKiY MVb K‡i mgvavb Ki (8 Ñ 17) :

8| †Kv‡bv fMvs‡ki je I n‡ii c‡Z¨KwUi mv‡_ 1 †hvM Ki‡j fMvskwU $\frac{4}{5}$ n‡e| Avevi, je I n‡ii c‡Z¨KwU †_‡K 5 we‡qvM Ki‡j fMvskwU $\frac{1}{2}$ n‡e| fMvskwU wbYq Ki|

9| †Kv‡bv fMvs‡ki je †_‡K 1 we‡qvM I n‡ii mv‡_ 2 †hvM Ki‡j fMvskwU $\frac{1}{2}$ nq| Avi je †_‡K 7 we‡qvM Ges ni †_‡K 2 we‡qvM Ki‡j fMvskwU $\frac{1}{3}$ nq| fMvskwU wbYq Ki|

10| `B A¼wewkó GKwU msL¨vi GKK ¯vbxq A¼ `kK ¯vbxq A‡¼i wZb¸Y A‡c¶v 1 †ewk| wKš A¼Øq ¯vb wewbgq Ki‡j †h msL¨v cvIqv hvq, Zv A¼Ø‡qi mgwói AvU¸‡Yi mgvb| msL¨wU KZ ?

11| `B A¼wewkó GKwU msL¨vi A¼Ø‡qi Aši 4; msL¨wUi A¼Øq ¯vb wewbgq Ki‡j †h msL¨v cvIqv hvq, Zvi I gj msL¨wUi †hvMdj 110 ; msL¨wU wbYq Ki|

12| gvZvi eZgvb eqm Zvi `B Kb¨vi eq‡mi mgwói Pvi¸Y| 5 eQi ci gvZvi eqm H `B Kb¨vi eq‡mi mgwói wØ¸Y n‡e| gvZvi eZgvb eqm KZ ?

13| GKwU AvqZ‡¶‡Îi ˆ`N¨ 5 wgUvi Kg I c¯ 3 wgUvi †ewk n‡j †¶Îdj 9 eMwgUvi Kg n‡e| Avevi ˆ`N¨ 3 wgUvi †ewk I c¯ 2 wgUvi †ewk n‡j †¶Îdj 67 eMwgUvi †ewk n‡e| †¶ÎwUi ˆ`N¨ I c¯ wbYq Ki|

14| GKwU †bŠKv `vo †e‡q †mv‡Zi AbK‡j NÈvq 15 wK.wg. hvq Ges †mv‡Zi cwZK‡j hvq NÈvq 5 wK.wg.| †bŠKvi I †mv‡Zi †eM wbYq Ki|

15| GKRb Mv‡g›Um kwgK gwmK †eZ‡b PvKwi K‡ib| cwZeQi †k‡l GKwU wbw`ó †eZbew× cvb| Zvi gwmK †eZb 4 eQi ci 4500 UvKv I 8 eQi ci 5000 UvKv nq| Zvi PvKwi ïi“i †eZb I ewlK †eZb ew×i cwigvY wbYq Ki|

16| GKwU mij mgxKiY‡RvU

$$x + y = 10$$
$$3x - 2y = 0$$

K. †`LvI †h, mgxKiY‡RvUwU m½wZcY| Gi KqwU mgvavb Av‡Q ?

L. mgxKiY‡RvUwU mgvavb K‡i (x, y) wbYq Ki|

M. mgxKiYØq Øviv wb‡`wkZ mij‡iLØq x-A‡¶i mv‡_ †h wÎfR MVb K‡i Zvi †¶Îdj wbYq Ki|

17| †Kv‡bv fMvs‡ki j‡ei mv‡_ 7 †hvM Ki‡j fMvskwUi gvb cYmsL¨v 2 nq| Avevi ni n‡Z 2 we‡qvM Ki‡j fMvskwUi gvb cYmsL¨v 1 nq|

K. fMvskwU $\frac{x}{y}$ a‡i mgxKiY‡RvU MVb Ki|

L. mgxKiY‡RvUwU Avo¸Yb c×wZ‡Z mgvavb K‡i (x, y) wbYq Ki| fMvskwU KZ ?

M. mgxKiY‡RvUwUi †jL A¼b K‡i (x, y) Gi cvß gv‡bi mZ¨Zv hvPvB Ki|

Î‡qv`k Aa¨vq

mmxg aviv

Finite Series

cvZ¨wnK Rxe‡b ÔµgÕ eûj cÖPwjZ GKwU kã| †hgb- †`vKv‡bi Zv‡K †fvM¨cY¨ mvRv‡Z, bvUK I Abôv‡bi NUbvejx mvRv‡Z, ¸`vgN‡i m›`i fv‡e `e¨vw` ivL‡Z µ‡gi aviYv e¨enƒZ nq| Avevi A‡bK KvR mn‡R Ges `wób›`bfv‡e m¤úv`b Ki‡Z Avgiv eo n‡Z †QvU, wkï n‡Z e×, nvjKv n‡Z fvix BZ¨vw` ai‡bi µg e¨envi Kwi| GB µ‡gi aviYv n‡ZB wewfbœ cKvi MwYwZK avivi D™¢e n‡q‡Q| GB Aa¨v‡q Abµg I avivi g‡a¨ m¤úK I GZ`msµvšÍ wewfbœ Dc¯’vcb Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_©xiv-

- Abµg I avivi eY©bv Ki‡Z I Zv‡`i cv_©K¨ wbiƒcY Ki‡Z cvi‡e|
- mgvšÍi avivi e¨vL¨v Ki‡Z cvi‡e|
- mgvšÍi avivi wbw`©óZg c` I wbw`©ó msL¨K c‡`i mgwó wbY©‡qi m~Î MVb Ki‡Z cvi‡e Ges m~Î c‡qvM K‡i MwYwZK mgm¨v mgvavb Ki‡Z cvi‡e|
- ¯^vfvweK msL¨vi e‡Mi I N‡bi mgwó wbY©q Ki‡Z cvi‡e|
- avivi wewfbœ m~Î c‡qvM K‡i MwYwZK mgm¨vi mgvavb Ki‡Z cvi‡e|
- ¸‡YvËi avivi wbw`©óZg c` I wbw`©ó msL¨K c‡`i mgwó wbY©‡qi m~Î MVb Ki‡Z cvi‡e Ges m~Î c‡qvM K‡i MwYwZK mgm¨vi mgvavb Ki‡Z cvi‡e|

Abµg

wb‡Pi m¤úKwU j¶ Kwi :

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & \cdots\cdots & n & \cdots\cdots \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow & \\ 2 & 4 & 6 & 8 & 10 & \cdots\cdots & 2n & \cdots\cdots \end{array}$$

GLv‡b c‡Z¨K ¯^vfvweK msL¨v n Zvi wظY msL¨v $2n$ Gi mv‡_ m¤úwK©Z| A_©vr ¯^vfvweK msL¨vi †mU $N=\{1,2,3,.......\}$ †_‡K GKwU wbw`©ó wbq‡g †hvM‡evaK †Rvo msL¨vi †mU $\{2,4,6,8,.......\}$ cvIqv hvq| GB mvRv‡bv †RvomsL¨vi †mUwU GKwU Abµg| mZivs, KZK¸‡jv ivwk GKwU we‡kl wbq‡g µgvš^‡q Ggbfv‡e mvRv‡bv nq ‡h c‡Z¨K ivwk Zvi c‡ei c` I c‡ii c‡`i mv‡_ Kxfv‡e m¤úwK©Z Zv Rvbv hvq| Gfv‡e mvRv‡bv ivwk¸‡jvi †mU‡K Abµg (Sequence) ejv nq|

Dc‡ii m¤úKwU‡K dvskb e‡j Ges $f(n)=2n$ wjLv nq| GB Abµ‡gi mvaviY c` $2n$. †h‡Kv‡bv Abµ‡gi c`msL¨v Amxg| AbµgwU mvaviY c‡`i mvnv‡h¨ wjLvi c×wZ n‡jv $<2n>, n=1,2,3,.........$ ev, $<2n>_{n=1}^{+\infty}$ ev, $<2n>$.

Abµ‡gi c_g iwk‡K c_g c\`, wØZxq iwk‡K wØZxq c\`, ZZxq iwk‡K ZZxq c\` BZ¨vw\` ej¨ nq| 1, 3, 5, 7,… … Abµ‡gi c_g c\` =1, wØZxq c\` = 3, BZ¨vw\`|

wb‡P Abµ‡gi PviwU D\`vniY †\`Iqv n‡jv :

$1, \quad 2, \quad 3, \ldots\ldots\ldots\ldots, n, \ldots\ldots\ldots\ldots$

$1, \quad 3, \quad 5, \ldots\ldots\ldots\ldots, (2n-1), \ldots\ldots\ldots$

$1, \quad 4, \quad 9, \ldots\ldots\ldots\ldots, n^2, \ldots\ldots\ldots\ldots$

$\frac{1}{2}, \quad \frac{2}{3}, \quad \frac{3}{4}, \ldots\ldots\ldots\ldots, \frac{n}{n+1}, \ldots\ldots\ldots\ldots$

KvR : 1| wb‡P Qq wU Abµ‡gi mvaviY c\` †\`Iqv Av‡Q| Abµg¸wj †jL :

$(i)\ \frac{1}{n} \quad (ii)\ \frac{n-1}{n+1} \quad (iii)\ \frac{1}{2^n} \quad (iv)\ \frac{1}{2^{n-1}} \quad (v)\ (-1)^{n+1}\frac{n}{n+1} \quad (vi)\ (-1)^{n-1}\frac{n}{2n+1}$.

2| †Zvgiv c‡Z¨‡K GKwU K‡i Abµ‡gi mvaviY c\` wj‡L Abµg wU †jL|

aviv

†Kv‡bv Abµ‡gi c\`¸‡jv cici Ô+Õ wPý Øviv h³ Ki‡j GKwU aviv (Series) cvIqv hvq| †hgb, $1+3+5+7+\ldots\ldots$ GKwU aviv| aviwUi cici \`BwU c‡\`i cv\_K¨ mgvb| Avevi $2+4+8+16+\ldots\ldots$ GKwU aviv| Gi cici \`BwU c‡\`i AbycvZ mgvb| mZivs, †h‡Kv‡bv avivi cici \`BwU c‡\`i g‡a¨ m¤úK‡Ki Dci wbfi K‡i aviwUi ˆewkó¨| aviv¸‡jvi g‡a¨ ¸i"ZcY \`BwU aviv n‡jv mgvšÍi aviv I ¸‡YvËi avi|

mgvšÍi avi

†Kv‡bv avivi †h‡Kv‡bv c\` I Zvi ceeZx c‡\`i cv_K¨ me mgq mgvb n‡j, †mB aviwU‡K mgvšÍi avi e‡j|

D\`vniY : $1+3+5+7+9+11$ GKwU avi|

GB aviwUi c_g c\` 1, wØZxq c\` 3, ZZxq c\` 5, BZ¨vw\`|

GLv‡b, wØZxq c\` – c\_g c\` = $3-1=2$, ZZxq c\` – wØZxq c\` = $5-3=2$,

PZ_ c\` – ZZxq c\` = $7-5=2$, cÂg c\` – PZ\_ c\` = $9-7=2$,

lô c\` – cÂg c\` $=11-9=2$

mZivs, aviwU GKwU mgvšÍi avi|

GB avivq cvß \`BwU c‡\`i we‡qvMdj‡K mvaviY AšÍi ej¨ nq| DwjøwLZ avivi mvaviY AšÍi 2. aviwUi c\` msL¨v wbw\`ó| G Rb¨ GwU GKwU mmxg ev mvšÍavi (Finite Series)| D‡jøL¨, mgvšÍi avivi c\`msL¨v wbw\`ó bv n‡j Zv‡K Amxg ev AbšÍavi (Infinite Series) e‡j| †hgb, $1+4+7+10+\ldots\ldots$ GKwU Amxg avi| mgvšÍi avivq mvaviYZ c_g c\`‡K $a$ Øviv Ges mvaviY AšÍi‡K $d$ Øviv cKvk Kiv nq| Zvn‡j msÁvbmv‡i, c\_g c\` a n‡j, wØZxq c\` $a+d$, ZZxq c\` $a+2d$, BZ¨vw\`| mZivs, aviwU n‡e,

$a+(a+d)+(a+2d)+\ldots\ldots$

mgvšÍi avivi mvaviY c` wbYq

g‡b Kwi, ‡h‡Kv‡bv mgvšÍi avivi c_g c` $= a$ I mvaviY AšÍi $= d$; Zvn‡j aviwUi

c_g c` $= a \quad = a + (1-1)\,d$

wØZxq c` $= a + d \quad = a + (2-1)\,d$

ZZxq c` $= a + 2d \quad = a + (3-1)\,d$

PZ_ c` $= a + 3d \quad = a + (4-1)\,d$

....

....

$\therefore$ nZg c` = $a + (n-1)d$

GB nZg c`‡KB mgvšÍi avivi mvaviY c` ejv nq| †Kv‡bv mgvšÍi avivi c_g c` $a$, mvaviY AšÍi $d$ Rvbv _vK‡j $n$Zg c‡` $n = 1, 2, 3, 4, \ldots\ldots$ emm‡q chvqµ‡g aviwUi c‡Z¨KwU c` wbYq Kiv hvq|

g‡b Kwi, GKwU mgvšÍi avivi c_g c` 3 Ges mvaviY AšÍi 2| Zvn‡j aviwUi

wØZxq c` $= 3 + 2 = 5$, ZZxq c` $= 3 + 2 \times 2 = 7$, PZ_ c` $= 3 + 3 \times 2 = 9$, BZ¨vw`|

AZGe, aviwUi nZg c` $= 3 + (n-1) \times 2 = 2n + 1$.

KvR : †Kv‡bv mgvšÍi avivi c_g c` 5 Ges mvaviY AšÍi 7 n‡j, aviwUi c_g QqwU c`, 22Zg c`, $r$ Zg Ges $(2p+1)$Zg c` wbYq Ki|

D`vniY 1| $5 + 8 + 11 + 14 + \cdots\cdots$ aviwUi †Kvb c` 383 ?

mgvavb : aviwUi c_g c` $a = 5$, mvaviY AšÍi $d = 8 - 5 = 11 - 8 = 3$

$\therefore$ Bnv GKwU mgvšÍi aviv|

g‡b Kwi, aviwUi nZg c` = 383

Avgiv Rvwb, nZg c` = $a + (n-1)d$.

$\therefore \quad a + (n-1)d = 383$

ev, $5 + (n-1)3 = 383$

ev, $5 + 3n - 3 = 383$

ev, $3n = 383 - 5 + 3$

ev, $3n = 381$

ev, $n = \dfrac{381}{3}$

$\therefore \quad n = 127$

$\therefore$ c`Ë avivi 127Zg c` = 383.

mgvšÍi avivi n msL¨K c‡`i mgwó

g‡b Kwi, †h‡Kv‡bv mgvšÍi avivi c_g c` $a$, †kl c` p, mvaviY AšÍi d, c` msL¨v $n$ Ges avivwUi $n$ msL¨K c‡`i mgwó S_n.

avivwU‡K c_g c` n‡Z Ges wecixZK‡g †kl c` n‡Z wj‡L cvIqv hvq

$$S_n = a + (a+d) + (a+2d) + \cdots \cdots + (p-2d) + (p-d) + p \qquad (i)$$

Ges $$S_n = p + (p-d) + (p-2d) + \cdots \cdots + (a+2d) + (a+d) + a \qquad (ii)$$

†hvM K‡i, $2S_n = (a+p) + (a+p) + (a+p) + \ldots\ldots + (a+p) + (a+p) + (a+p)$

ev, $2S_n = n(a+p)$ [$\because$ avivwUi c` msL¨v n]

$$\therefore \quad S_n = \frac{n}{2}(a+p) \qquad (iii)$$

Avevi, nZg c` $= p = a + (n-1)d$. p Gi gvb (iii) G ewm‡q cvB,

$$S_n = \frac{n}{2}[a + \{a + (n-1)d\}]$$

A_vr, $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

†Kv‡bv mgvšÍi avivi c_g c` $a$, †kl c` p Ges c` msL¨v $n$ Rvbv _vK‡j, $(iii)$ bs m‡Îi mvnv‡h¨ avivwUi mgwó wbY©q Kiv hvq| wKš‘ c_g c` a, mvaviY AšÍi d, c` msL¨v n Rvbv _vK‡j, (iv) bs m‡Îi mvnv‡h¨ avivwUi mgwó wbY©q Kiv hvq|

c_g n msL¨K ¯^vfvweK msL¨vi mgwó wbY©q

g‡b Kwi, n msL¨K ¯^vfvweK msL¨vi mgwó S_n

A_vr, $S_n = 1 + 2 + 3 + \cdots\cdots\cdots + (n-1) + n \qquad (i)$

avivwU‡K c_g c` n‡Z Ges wecixZK‡g †kl c` n‡Z wj‡L cvIqv hvq

$$S_n = 1 + 2 + 3 + \cdots\cdots\cdots + (n-2) + (n-1) + n \qquad (i)$$

Ges $$S_n = n + (n-1) + (n-2) + \cdots\cdots + 3 \;\; + 2 \;\; + 1 \qquad (ii)$$

†hvM K‡i, $2S_n = (n+1) + (n+1) + (n+1) + \cdots\cdots\cdots + (n+1)$ [n msL¨K c`]

ev, $2S_n = n(n+1)$

$$\therefore \; S_n = \frac{n(n+1)}{2} \qquad (iii)$$

D`vniY 2| c_g 50 wU ¯^vfvweK msL¨vi ‡hvMdj wbY©q Ki|

mgvavb : Avgiv (iii) bs mÎ e¨envi K‡i cvB,

$$S_{50} = \frac{50(50+1)}{2} = 25 \times 51 = 1275$$

$\therefore$ c_g 50 wU ¯^vfvweK msL¨vi ‡hvMdj 1275.

D`vniY 3| $1+2+3+4+\cdots\cdots\cdots+99=$ KZ ?

mgvavb : avivwUi c_g c` $a=1$, mvaviY Aš—i $d=2-1=1$ Ges †kl c` $p=99$.

$\therefore$ Bnv GKwU mgvšÍi avivi|

g‡b Kwi, avivwUi nZg c` = 99

Avgiv Rvwb, mgvšÍi avivi nZg c` = $a+(n-1)d$

$\therefore \quad a+(n-1)d=99$

ev, $\quad 1+(n-1)1=99$

ev, $\quad 1+n-1=99$

$\therefore \quad n=99$

weKí c×wZ:

†h‡nZ

$$S_n=\frac{n}{2}(a+p),$$

$$\therefore S_{99}=\frac{99}{2}(1+99)$$

$$=\frac{99\times100}{2}=4950$$

(iv) bs mÎ n‡Z, mgvšÍi avivi c_g n-msL¨K c‡`i mgwó-

$$S_n=\frac{n}{2}\{2a+(n-1)d\}.$$

mZivs, avivwUi 99wU c‡`i mgwó $S_{99}=\frac{99}{2}\{2\times1+(99-1)\times1\}=\frac{99}{2}(2+98)$

$$=\frac{99\times100}{2}=99\times50=4950$$

D`vniY 4| $7+12+17+\cdots\cdots\cdots$ avivwUi 30wU c‡`i mgwó KZ ?

mgvavb : avivwU c_g c` $a=7$, mvaviY Aš—i $d=12-7=5$

$\therefore$ Bnv GKwU mgvšÍi avivi| GL‡b c` msL¨v $n=30$.

Avgiv Rvwb, mgvšÍi avivi c_g n-msL¨K c‡`i mgwó,

$$S_n=\frac{n}{2}\{2a+(n-1)d\}.$$

Zvn‡j, 30wU c‡`i mgwó $S_{30}=\frac{30}{2}\{2.7+(30-1)5\}=15(14+29\times5)$

$$=15(14+145)=15\times159$$

$$=2385$$

D`vniY 5| K Zvi †eZb †_‡K c_g gv‡m 1200 UvKv mÂq K‡ib Ges cieZx gvm¸‡jvi cÖwZgv‡m Gi ceeZx gv‡mi Zjbvq 100 UvKv †ewk mÂq K‡ib|

(i) wZwb nZg gv‡m KZ UvKv mÂq K‡ib ?

(ii) Dc‡iv³ mgm¨wU‡K n msL¨K c` chš— avivq cKvk Ki|

(iii) wZwb c_g n msL¨K gv‡m KZ UvKv mÂq K‡ib ?

(iv) GK eQ‡i wZwb KZ UvKv mÂq K‡ib ?

mgvavb : (i) c_g gv‡m mÂq K‡ib 1200 UvKv

wØZxq gv‡m mÂq K‡ib $(1200+100)$ UvKv $=1300$ UvKv

ZZxq gv‡m mÂq K‡ib $(1300+100)$ UvKv $=1400$ UvKv

PZ_ gv‡m mÂq K‡ib $(1400+100)$ UvKv $=1500$ UvKv

mZivs, GwU GKwU mgvšÍi aviv, hvi c_g c` $a=1200,$ mvaviY Ašii $d=1300-1200=100.$

avivwUi nZg c` $=a+(n-1)d$

$=1200+(n-1)100=1200+100n-100$

$=100n+1100$

AZGe, wZwb nZg gv‡m mÂq K‡ib $(100n+1100)$ UvKv|

(ii) G‡¶‡Î n msL¨K c` chšÍavivwU n‡e $1200+1300+1400+\cdots\cdots\cdots+(100n+1100)$

(iii) wZwb c_g n msL¨K gv‡m mÂq K‡ib-

$\frac{n}{2}\{2a+(n-1)d\}$ UvKv $=\frac{n}{2}\{2\times1200+(n-1)100\}$ UvKv

$=\frac{n}{2}(2400+100n-100)$ UvKv $=\frac{n}{2}\times2(1150+50n)$ UvKv

$=n(50n+1150)$ UvKv|

(iv) Avgiv Rvwb, GK eQi = 12 gvm| G‡¶‡Î, $n=12.$

AZGe, [Dc‡ii (iii) n‡Z] K GK eQ‡i mÂq K‡ib $12(50\times12+1150)$ UvKv

$=12(600+1150)$ UvKv $=12\times1750$ UvKv = 21000 UvKv|

Abkxjbx 13·1

1| $2-5-12-19-\cdots\cdots\cdots$ avivwUi mvaviY Ašii Ges 12Zg c` wbYq Ki|

2| $8+11+14+17+\cdots\cdots\cdots$ avivwUi †Kvb c` 392 ?

3| $4+7+10+13+\cdots\cdots\cdots$ avivwUi †Kvb c` 301?

4| †Kv‡bv mgvšÍi avivi pZg c` $p^2$ Ges $q$Zg c` q^2 n‡j, avivwUi $(p+q)$Zg c` KZ ?

5| †Kv‡bv mgvšÍi avivi mZg c` $n$ I $n$Zg c` m n‡j, $(m+n)$Zg c` KZ ?

6| $1+3+5+7+\cdots\cdots\cdots$ avivwUi n c‡`i mgwó KZ ?

7| $8+16+24+\cdots\cdots\cdots$ avivwUi c_g 9wU c‡`i mgwó KZ ?

8| $5+11+17+23+\cdots\cdots\cdots+59=$ KZ ?

9| $29+25+21+\cdots\cdots\cdots-23=$ KZ ?

10| †Kv‡bv mgvšÍi avivi 12Zg c` 77 n‡j, Gi c_g 23wU c‡`i mgwó KZ ?

11| GKwU mgvšÍi avivi 16Zg c` $-20$ n‡j, Gi c_g 31wU c‡`i mgwó KZ ?

12| $9+7+5+\cdots\cdots\cdots$ avivwUi c_g n msL¨K c‡`i †hvMdj -144 n‡j, n Gi gvb wbYq Ki|

13| $2+4+6+8+\cdots\cdots\cdots$ avivwUi c_g n msL¨K c‡`i mgwó 2550 n‡j, n Gi gvb wbYq Ki|

14| †Kv‡bv avivi c_g n msL¨K c‡`i mgwó $n(n+1)$ n‡j, avivwU wbYq Ki|

15| †Kv‡bv avivi c_g n msL¨K c‡`i mgwó $n(n+1)$ n‡j, avivwUi 10wU c‡`i mgwó KZ ?

16| GKwU mgvšÍi avivi c_g 12 c‡`i mgwó 144 Ges c_g 20 c‡`i mgwó 560 n‡j, Gi c_g 6 c‡`i mgwó wbYq Ki|

17| †Kv‡bv mgvšÍi avivi c_g m c‡`i mgwó $n$ Ges c_g $n$ c‡`i mgwó m n‡j, Gi c_g $(m+n)$ c‡`I mgwó wbYq Ki|

18| †Kv‡bv mgvšÍi avivq p Zg, q Zg I r Zg c` h_vµ‡g $a, b, c$ n‡j, †`LvI †h,

$$a(q-r)+b(r-p)+c(p-q)=0.$$

19| †`LvI †h, $1+3+5+7+\cdots\cdots\cdots+125=169+171+173+\cdots\cdots\cdots+209.$

20| GK e¨w³ 2500 UvKvi GKwU FY wKQmsL¨K wKw¯‡Z cwi‡kva Ki‡Z ivRx nb| c‡Z¨K wKw¯ c‡ei wKw¯ †_‡K 2 UvKv †ewk| hw` c_g wKw¯ 1 UvKv nq, Z‡e KZ¸‡jv wKw¯‡Z H e¨w³ Zvi FY †kva Ki‡Z cvi‡eb ?

c_g n msL¨K ¯vfvweK msL¨vi e‡Mi mgwó wbYq

g‡b Kwi, c_g n msL¨K ¯vfvweK msL¨vi e‡Mi mgwó S_n.

A_vr, $S_n = 1^2+2^2+3^2+\cdots\cdots\cdots+n^2$

Avgiv Rvwb,

$$r^3-3r^2+3r-1=(r-1)^3$$

ev, $r^3-(r-1)^3=3r^2-3r+1$

Dc‡ii A‡f`wU‡Z, $r=1, 2, 3, \ldots\ldots, n$ evmv‡q cvB,

$$1^3-0^3=3.1^2-3.1+1$$
$$2^3-1^3=3.2^2-3.2+1$$
$$3^3-2^3=3.3^2-3.3+1$$
$$\cdots\quad\cdots\quad\cdots\quad\cdots$$
$$\cdots\quad\cdots\quad\cdots\quad\cdots$$
$$n^3-(n-1)^3=3.n^2-3.n+1$$

†hvM K‡i cvB,

$$n^3-0^3=3(1^2+2^2+3^2+\cdots\cdots\cdots+n^2)-3(1+2+3+\cdots\cdots\cdots+n)+(1+1+1+\cdots\cdots\cdots+1)$$

ev, $n^3=3S_n-3.\dfrac{n(n+1)}{2}+n$ $\left[\because 1+2+3+\cdots\cdots\cdots+n=\dfrac{n(n+1)}{2}\right]$

ev, $3S_n=n^3+\dfrac{3n(n+1)}{2}-n$

$$= \frac{2n^3 + 3n^2 + 3n - 2n}{2} = \frac{2n^3 + 3n^2 + n}{2} = \frac{n(2n^2 + 3n + 1)}{2}$$

$$= \frac{n(2n^2 + 2n + n + 1)}{2} = \frac{n\{2n(n+1) + 1(n+1)\}}{2}$$

ev, $3S_n = \frac{n(n+1)(2n+1)}{2}$

$\therefore \quad S_n = \frac{n(n+1)(2n+1)}{6}$

c_g *n* msL¨K ¯vfvweK msL¨vi N‡bi mgwó wbYq

g‡b Kwi, c_g n msL¨K ¯vfvweK msL¨vi N‡bi mgwó S_n.

A_©vr, $S_n = 1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3$

Avgiv Rvwb, $(r+1)^2 - (r-1)^2 = (r^2 + 2r + 1) - (r^2 - 2r + 1) = 4r.$

ev, $(r+1)^2 r^2 - r^2 (r-1)^2 = 4r.r^2 = 4r^3$ [Dfqc¶‡K r^2 Øviv ¸Y K‡i]

Dc‡ii A‡f`wU‡Z, $r = 1, 2, 3, \ldots\ldots, n$ emv‡q cvB,

$2^2.1^2 - 1^2.0^2 = 4.1^3$

$3^2.2^2 - 2^2.1^2 = 4.2^3$

$4^2.3^2 - 3^2.2^2 = 4.3^3$

$\cdots \quad \cdots \quad \cdots \quad \cdots$

$\cdots \quad \cdots \quad \cdots \quad \cdots$

$(n+1)^2 n^2 - n^2 (n-1)^2 = 4n^3$

†hvM K‡i, $(n+1)^2.n^2 - 1^2.0^2 = 4(1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3)$

ev, $(n+1)^2.n^2 = 4S_n$

ev, $S_n = \frac{n^2(n+1)^2}{4}$

$\therefore \quad S_n = \left\{\frac{n(n+1)}{2}\right\}^2$

c‡qvRbxq m‚Î

$$1 + 2 + 3 + \cdots\cdots\cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots\cdots\cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

we‡kl `óe¨: $1^3+2^3+3^3+\cdots\cdots\cdots+n^3=(1+2+3+\cdots\cdots\cdots+n)^2$.

KvR : 1| c_g n msL¨K ¯^vfvweK †Rvo msL¨vi mgwó wbYq Ki|
2| c_g n msL¨K ¯^vfvweK we†Rvo msL¨vi e‡Mi mgwó wbYq Ki|

¸‡YvËi aviv

†Kv‡bv avivi †h‡Kv‡bv c` I Gi ceeZx c‡`i AbcvZ me mgq mgvb n‡j A_vr, †h‡Kv‡bv c`‡K Gi ceeZx c` Øviv fvM K‡i fvMdj me`v mgvb cvIqv †M‡j, †m aviwU‡K ¸‡YvËi aviv e‡j Ges fvMdj‡K mvaviY AbcvZ e‡j| †hgb, $2+4+8+16+32$ aviwUi c_g c` 2, wØZxq c` 4, ZZxq c` 8, PZ_ c` 16, cÂg c` 32. GLv‡b,

wØZxq c‡`i mv‡_ c_g c‡`i AbcvZ = $\frac{4}{2}=2$, ZZxq c‡`i mv‡_ wØZxq c‡`i AbcvZ = $\frac{8}{4}=2$

PZ_ c‡`i mv‡_ ZZxq c‡`i AbcvZ = $\frac{16}{8}=2$, cÂg c‡`i mv‡_ PZ_ c‡`i AbcvZ = $\frac{32}{16}=2$.

mZivs, aviwU GKwU ¸‡YvËi aviv| GB avivq †h‡Kv‡bv c` I Gi ceeZx c‡`i AbcvZ me`v mgvb| DwjøwLZ avivq mvaviY AbcvZ 2| aviwUi c` msL¨v wbw`ó| G Rb¨ GwU GKwU ¸‡YvËi mmxg aviv|

†fŠZ I Rxe weÁv‡bi wewfbœ †¶‡Î, e¨vsK I exgv BZ¨vw` cwZôv‡b Ges wewfbœ cKvi chw³ we`¨vq ¸‡YvËi avivi e¨vcK c‡qvM Av‡Q|

¸‡YvËi avivi c` msL¨v wbw`ó bv _vK‡j G‡K Abš— ¸‡YvËi aviv e‡j|

¸‡YvËi avivi c_g c`‡K mvaviYZ $a$ Øviv Ges mvaviY AbcvZ‡K $r$ Øviv cKvk Kiv nq| Zvn‡j msÁvbmv‡i, c_g c` a n‡j, wØZxq c` $ar$, ZZxq c` ar^2, BZ¨vw`| mZivs, aviwU n‡e,

$$a+ar+ar^2+ar^3+\cdots\cdots\cdots$$

KvR : wbgwjwLZ †¶‡Î ¸‡YvËi aviv¸‡jv †jL :
(i) c_g c` 4, mvaviY AbcvZ 10 ($ii$) c_g c` 9, mvaviY AbcvZ $\frac{1}{3}$ (iii) c_g c` 7, mvaviY AbcvZ $\frac{1}{10}$
(iv) c_g c` 3, mvaviY AbcvZ 1 ($v$) c_g c` 1, mvaviY AbcvZ $-\frac{1}{2}$ (vi) c_g c` 3, mvaviY AbcvZ -1

¸‡YvËi avivi mvaviY c`

g‡b Kwi, †h‡Kv‡bv ¸‡YvËi avivi c_g c` a, mvaviY AbcvZ r, Zvn‡j aviwUi

c_g c` $=a=ar^{1-1}$ wØZxq c` $=ar=ar^{2-1}$
ZZxq c` $=ar^2=ar^{3-1}$ PZ_ c` $=ar^3=ar^{4-1}$
... ...
... ...
nZg c` = ar^{n-1}

GB nZg c`‡KB ¸‡YvËi avivi mvaviY c` ejv nq| †Kv‡bv ¸‡YvËi avivi c_g c` $a$ I mvaviY AbcvZ $r$ Rvbv _vK‡j $n$Zg c‡` chvqµ‡g $r = 1, 2, 3, \ldots\ldots$ BZ¨w` ewm‡q avivwUi †h‡Kv‡bv c` wbYq Kiv hvq|

D`vniY 6| $2 + 4 + 8 + 16 + \cdots\cdots$ avivwUi $10$Zg c` KZ ?

mgvavb : avivwUi c_g c` $a = 2$, mvaiY AbcvZ $r = \frac{4}{2} = 2.$

$\therefore$ c`Ë avivwU GKwU ¸‡YvËi aviv|

Avgiv Rwb, ¸‡YvËi avivi nZg c` = ar^{n-1}

$\therefore$ avivwUi 10Zg c` = $2 \times 2^{10-1}$

= $2 \times 2^{9} = 1024$

D`vinY 7| $128 + 64 + 32 + \cdots\ \cdots\ \cdots$ avivwUi mvaviY c` KZ ?

mgvavb : c`Ë avivwUi c_g c` $a = 128$, mvaviY AbcvZ $r = \frac{64}{128} = \frac{1}{2}.$

$\therefore$ Bnv GKwU ¸‡YvËi aviv|

Avgiv Rwb, ¸‡YvËi avivi mvaviY c` = ar^{n-1}

mZivs, avivwUi mvaviY c` $= 128 \times \left(\frac{1}{2}\right)^{n-1} = \frac{2^{7}}{2^{n-1}} = \frac{1}{2^{n-1-7}} = \frac{1}{2^{n-8}}.$

D`vniY 8| GKwU ¸‡YvËi avivi c_g I wØZxq c` h_vµ‡g 27 Ges 9 n‡j, avivwUi cÂg c` Ges `kg c` wbYq Ki|

mgvavb : c`Ë avivwUi c_g c` $a = 27$, wØZxq c` $= 9$

Zvn‡j mvaviY AbcvZ $r = \frac{9}{27} = \frac{1}{3}.$

$\therefore$ cÂg c` = $ar^{5-1} = 27 \times \left(\frac{1}{3}\right)^{4} = \frac{27 \times 1}{27 \times 3} = \frac{1}{3}$

Ges `kg c` = $ar^{10-1} = 27 \times \left(\frac{1}{3}\right)^{9} = \frac{3^{3}}{3^{3} \times 3^{6}} = \frac{1}{3^{6}} = \frac{1}{729}.$

¸‡YvËi avivi mgwó wbYq

g‡b Kwi, ¸‡YvËi avivi c_g c` $a$, mvaviY AbcvZ $r$ Ges c` msL¨v n. hw` $n$ msL¨K c‡`i mgwó S_n nq, Zvn‡j

$$S_n = a + ar + ar^{2} + \cdots\cdots\cdots + ar^{n-2} + ar^{n-1} \qquad (i)$$

Ges $$r.S_n = \qquad ar + ar^{2} + ar^{3} + \cdots\cdots\cdots + ar^{n-1} + ar^{n} \quad [\,(i)\text{ ‡K } r \text{ Øviv ¸Y K‡i}\,] \qquad (ii)$$

we‡qvM K‡i, $S_n - rS_n = a - ar^{n}$

ev, $S_n(1 - r) = a(1 - r^{n})$

$\therefore \quad S_n = \dfrac{a(1-r^n)}{1-r}$, hLb $r<1$

Avevi (ii) †_‡K (i) we‡qvM K‡i cvB,

$rS_n - S_n = ar^n - a$ ev, $S_n(r-1) = a(r^n-1)$

A_vr, $S_n = \dfrac{a(r^n-1)}{(r-1)}$, hLb $r>1$.

j¶Yxq : mvaviY AbcvZ $r=1$ n‡j c‡Z¨K c` $=a$

mZivs, G‡¶‡Î $S_n = a+a+a+\cdots\ \cdots\ n$ c` chš—

$= an.$

KvR : K Zvi †Q‡j‡K ¯‹z‡j †bqv-Avbvi Rb¨ GK e¨w³‡K 1jv Gwcÿj †_‡K GK gv‡mi Rb¨ wb‡qvM Ki‡jb| Zvi cvwikÖwgK wVK Kiv n‡jv- c_g w`b GK cqmv, wØZxq w`b c_g w`‡bi wØ¸Y A\_vr `B cqmv, ZZxq w`b wØZxq w`‡bi wØ¸Y A_vr Pvi cqmv| GB wbq‡g cvwikÖwgK w`‡j mvßvwnK QywUi w`bmn GK gvm ci H e¨w³ KZ UvKv cv‡eb ?

D`vniY 9\| $12+24+48+\cdots\ \cdots\ \cdots+768$ avivwUi mgwó KZ ?

mgvavb : c`Ë avivwUi c\_g c` $a=12$, mvaviY AbcvZ $r=\dfrac{24}{12}=2>1$.

$\therefore$ avivwU GKwU ¸‡YvËi avivÑ\|

g‡b Kwi, avivwUi n Zg c` $=768$

Avgiv Rvwb, n Zg c` = ar^{n-1}

$\therefore \quad ar^{n-1} = 768$

ev, $\quad 12\times 2^{n-1} = 768$

ev, $\quad 2^{n-1} = \dfrac{768}{12} = 64$

ev, $\quad 2^{n-1} = 2^6$

ev, $\quad n-1 = 6$

$\therefore \quad n = 7.$

mZivs, avivwUi mgwó $= \dfrac{a(r^n-1)}{(r-1)}$, hLb $r>1$

$$= \frac{12(2^7-1)}{2-1} = 12\times(128-1) = 12\times 127 = 1524.$$

D`vniY 10\| $1+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\cdots\ \cdots\ \cdots$ avivwUi c\_g AvUwU c‡`i mgwó wbYq Ki\|

mgvavb : c`Ë avivwUi c\_g c` $a=1$, mvaviY AbcvZ $r=\dfrac{\frac{1}{2}}{1}=\dfrac{1}{2}<1$

$\therefore$ Bnv GKwU ¸‡YvËi aviv\|

GLv‡b c` msL¨v $n=8$.

Avgiv Rvwb, ¸‡YvËi avivi n-msL¨K c‡`i mgwó

$$S_n = \frac{a(1-r^n)}{1-r}, \qquad \text{hLb } r<1.$$

mZivs, avivwUi 8wU c‡`i mgwó $S_8 = \dfrac{1\times\left\{1-\left(\frac{1}{2}\right)^8\right\}}{1-\frac{1}{2}} = \dfrac{1-\frac{1}{256}}{\frac{1}{2}} = 2\left(\dfrac{256-1}{256}\right) = \dfrac{255}{128} = 1\dfrac{127}{128}$

Abkxjbx 13·2

1. a, b, c I d mgvšÍi avivi PviwU µwgK c` n‡j wb‡Pi †KvbwU mwVK?

   K. b = $\dfrac{c+d}{2}$ L. a = $\dfrac{b+c}{2}$

   M. c = $\dfrac{b+d}{2}$ N. d = $\dfrac{a+c}{2}$

2. i a+(a+d)+(d+2d).................... avivwUi c_g n msL¨K c‡`i mgwó = $\dfrac{n}{2}$ {2a+(n-1)d}

   ii 1+2+3............+n = $\dfrac{n(n+1)(2n+1)}{6}$

   iii 1+3+5+..............+(2n-1) = n^2

   Dc‡ii evK¨¸‡jvi †KvbwU mwVK ?

   K. i I ii L. i I iii

   M. ii I iii N. i, ii I iii

   wb‡Pi avivwUi wfwË‡Z 3 I 4 b¤i c‡ki DËi `vI :

   log 2+ log 4 + log 8 +.......................

3. avivwUi mvaviY Aš—i †KvbwU?

   K. 2 L. 4

   M. log 2 N. 2 log 2

4. avivwUi 7g c` KZ?

   K. log 32 L. log 64

   M. log 128 N. log 256

5| $64+32+16+8+\cdots\cdots\cdots$ avivwUi Aóg c` wbYq Ki|

6| $3+9+27+\cdots\ \cdots\ \cdots$ avivwUi c_g †PŠÏwU c‡`i mgwó wbYq Ki|

7| $128+64+32+\cdots\cdots\cdots$ avivwUi †Kvb c` $\dfrac{1}{2}$?

8| GKwU ¸‡YvËi avivi cÂg c` $\dfrac{2\sqrt{3}}{9}$ Ges `kg c` $\dfrac{8\sqrt{2}}{81}$ n‡j, avivwUi ZZxq c` wbYq Ki|

9| $\frac{1}{\sqrt{2}}, -1, \sqrt{2}, \cdots \cdots \cdots$ avivwUi †Kvb c` $8\sqrt{2}$?

10| $5+x+y+135$ ¸‡YvËi avivf³ n‡j, x Ges y Gi gvb wbYq Ki|

11| $3+x+y+z+243$ ¸‡YvËi avivf³ n‡j, x, y Ges z Gi gvb wbYq Ki|

12| $2-4+8-16+\cdots \cdots \cdots$ avivwUi c_g mvZwU c‡`i mgwó KZ ?

13| $1-1+1-1+\cdots \cdots \cdots$ avivwUi $(2n+1)$ msL¨K c‡`i mgwó wbYq Ki|

14| $\log 2+\log 4+\log 8+\cdots \cdots \cdots$ avivwUi c_g `kwU c‡`i mgwó KZ ?

15| $\log 2+\log 16+\log 512+\cdots \cdots \cdots$ avivwUi c_g evivwU c‡`i mgwó wbYq Ki|

16| $2+4+8+16+\cdots \cdots \cdots$ avivwUi n-msL¨K c‡`i mgwó 254 n‡j, n-Gi gvb KZ ?

17| $2-2+2-2+\cdots \cdots \cdots$ avivwUi $(2n+2)$ msL¨K c‡`i mgwó KZ ?

18| c_g n msL¨K ¯^vfvweK msL¨vi N‡bi mgwó 441 n‡j, n Gi gvb wbYq Ki Ges H msL¨v¸‡jvi mgwó wbYq Ki|

19| c_g n msL¨K ¯^vfvweK msL¨vi N‡bi mgwó 225 n‡j, n Gi gvb KZ ? H msL¨v¸‡jvi e‡Mi mgwó KZ ?

20| †`LvI †h, $1^3+2^3+3^3+\cdots \cdots \cdots+10^3 = (1+2+3+\cdots \cdots \cdots+10)^2$.

21| $\frac{1^3+2^3+3^3+\cdots \cdots \cdots+n^3}{1+2+3+\cdots \cdots \cdots+n} = 210$ n‡j n-Gi gvb KZ ?

22| 1 wgUvi ˆ`N¨wewkó GKwU †j͛n `Û‡K 10wU UKivq wef³ Kiv n‡jv hv‡Z UKiv¸‡jvi ˆ`N¨ ¸‡YvËi aviv MVb K‡i| hw` enËg UKiwU ¶z`Zg UKivi 10 ¸Y nq, Z‡e ¶z`Zg UKiwUi ˆ`‡N¨i gvb Avmbœ wgwjwgUv‡i wbYq Ki|

23| GKwU ¸‡YvËi avivi 1g c` $a$, mvaviY AbycvZ $r$, avivwUi 4_ c` -2 Ges 9g c` $8\sqrt{2}$

K. Dc‡iv³ Z_¨¸‡jv‡K `BwU mgxKi‡Yi gva¨‡g cKvk Ki|

L. avivwUi 12 Zg c` wbYq Ki|

M. avivwU wbYq K‡i c_g 7wU c‡`i mgwó wbYq Ki|

24| †Kvb avivi n Zg c` 2n-4

K. avivwU wbYq Ki|

L. avivwUi 10Zg c` Ges c_g 20wU c‡`i mgwó wbYq Ki|

M. cÖß avivwUi c_g c`‡K c_g c` Ges mvaviY Aš—i‡K mvaviY AbycvZ a‡i GKwU bZb aviv ˆZwi Ki Ges mÎ c‡qvM K‡i avivwUi c_g 8 c‡`i mgwó wbYq Ki|

PZ`k Aa¨vq

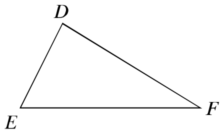
AbcvZ, m`kZv I cwZmgZv

`BwU iwki ZjbvKivi Rb¨ Zv‡`i AbcvZ we‡ePbv Kiv nq| AbcvZ wbY©‡qi Rb¨ iwk `BwU GKB GK‡K cwigvc Ki‡Z nq| G m¤ú‡K eÿRMwY‡Z we¯—wiZ Av‡jvPbv Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_©xiv Ñ

- R¨wgwZK AbcvZ m¤ú‡K e¨vL¨v Ki‡Z cvi‡e|
- †iLvs‡ki Aš—wefw³ e¨vL¨v Ki‡Z cvi‡e|
- AbcvZ m¤úwKZ Dccv`¨¸‡jv hvPvB I cgvY Ki‡Z cvi‡e|
- m`kZvi AbcvZ msµvš— Dccv`¨¸‡jv hvPvB I cgvY Ki‡Z cvi‡e|
- cwZmgZvi avibv e¨vL¨v Ki‡Z cvi‡e|
- nv‡Z-Kj‡g ev¯—e DcKi‡Yi mvnv‡h¨ †iLv I NYb cwZmgZv hvPvB Ki‡Z cvi‡e|

14·1 AbcvZ I mgvbcv‡Zi ag



(i) a t $b = x$ t y Ges c t $d = x$ t y n‡j, a t $b = c$ t d

(ii) a t $b = b$ t a n‡j, $a = b$

(iii) a t $b = x$ t y n‡j, b t $a = y$ t x (e¨¯—KiY)

(iv) a t $b = x$ t y n‡j, a t $x = b$ t y (GKvš—iKiY)

(v) a t $b = c$ t d n‡j, $ad = bc$ (Avo¸Yb)

(vi) a t $b = x$ t y n‡j, $a+b$ t $b = x+y$ t y (†hvRb)

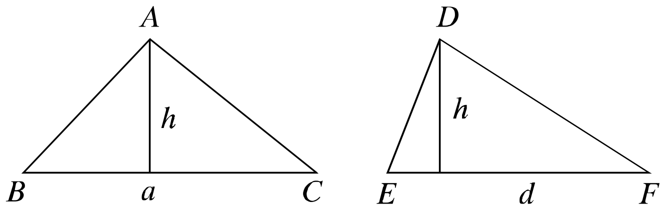
Ges $a-b$ t $b = x-y$ t y (we‡qvRb)

(vii) $\frac{a}{b} = \frac{c}{d}$ n‡j, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (†hvRb I we‡qvRb)

R¨wgwZK mgvbcvZ

Avgiv wÎfR‡¶‡Îi †¶Îdj wbY©q Ki‡Z wk‡LwQ| G †_‡K `BwU c‡qvRbxq AbcvZ‡Zi avibv ˆZwi Kiv hvq|

(1) `BwU wÎfR‡¶‡Îi D"PZv mgvb n‡j, Zv‡`i †¶Îdj I fwg mgvbcvwZK|



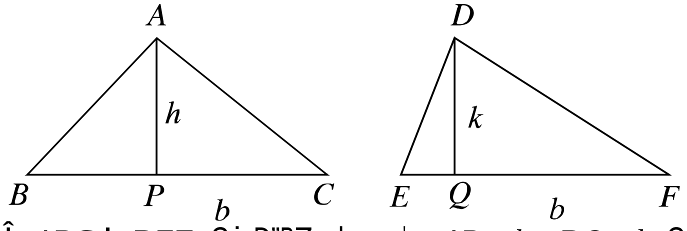
g‡b Kwi, wÎfR‡¶Î ABC I DEF Gi fwg h_vµ‡g $BC = a$, $EF = d$ Ges Dfq †¶‡Îi D"PZv h|

mZivs, wÎfR‡¶Î ABC Gi †¶Îdj = $\frac{1}{2}a \times h$, wÎfR‡¶Î DEF Gi †¶Îdj = $\frac{1}{2}d \times h$

AZGe, wÎfR‡¶Î ABC Gi †¶Îdj t wÎfR‡¶Î DEF Gi †¶Îdj = $\frac{1}{2}a \times h$ t $\frac{1}{2}d \times h$

$= a$ t $d = BC$ t EF |

(2) `BwU wÎfR‡¶‡Îi fwg mgvb n‡j, Zv‡`i †¶Îdj I D”PZv mgvbcvwZK|



g‡b Kwi wÎfR‡¶Î ABC I DEF Gi D”PZv h_vµ‡g $AP=h$, $DQ=k$ Ges Dfq‡¶‡Îi fwg b|

mZivs, wÎfR‡¶Î ABC Gi †¶Îdj = $\frac{1}{2}b\times h$, wÎfR‡¶Î DEF Gi †¶Îdj = $\frac{1}{2}b\times k$

AZGe, wÎfR‡¶Î ABC Gi †¶Îdj t wÎfR‡¶Î DEF Gi †¶Îdj = $\frac{1}{2}b\times h$ t $\frac{1}{2}b\times k$

= h t k = AP t DQ |

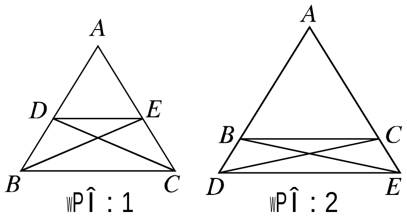
Dccv`¨ 1

wÎf‡Ri †h‡Kv‡bv evûi mgvšÍivj mij‡iLv H wÎf‡Ri Aci evûØq‡K ev Zv‡`i ewaZvskØq‡K mgvb Abcv‡Z wef³ K‡i|

we‡kl wbePb : ABC wÎf‡Ri BC evûi mgvšÍivj DE †iLvsk AB I AC evûØq‡K A_ev Zv‡`i ewaZvskØq‡K h_vµ‡g $D$ I $E$ we›`‡Z †Q` K‡i‡Q|

cgvY Ki‡Z n‡e †h, AD t $DB = AE$ t EC.

A¼b : B, E Ges C, D †hvM Kwi|



wPÎ : 1 wPÎ : 2

cgvY :

avc	h_v_Zv
(1) ΔADE Ges ΔBDE GKB D”PZwewkó $\therefore \frac{\Delta ADE}{\Delta BDE} = \frac{AD}{DB}$	[GKB D”PZwewkó wÎfRmg‡ni †¶Îdj fwgi mgvbcvwZK]
(2)Avevi, ΔADE Ges ΔDEC GKB D”PZwewkó $\therefore \frac{\Delta ADE}{\Delta DEC} = \frac{AE}{EC}$	[GKB D”PZwewkó wÎfRmg‡ni †¶Îdj fwgi mgvbcvwZK]
(3) wKš‘ $\Delta BDE = \Delta DEC$ $\therefore \frac{\Delta ADE}{\Delta BDE} = \frac{\Delta ADE}{\Delta DEC}$	[GKB fwg DE I GKB mgvšÍivj hM‡ji g‡a¨ Aew¯'Z]
(4) AZGe, $\frac{AD}{DB} = \frac{AE}{EC}$ A_vr, AD t $DB = AE$ t EC.	

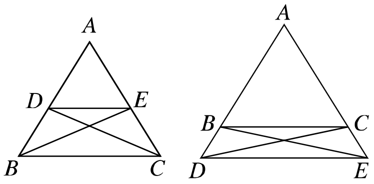
Abywm×vš— 1| ABC wÎf‡Ri BC evûi mgvšÍivj †Kv‡bv †iLv hw` $AB$ I $AC$ evû‡K h_vµ‡g $D$ I $E$ we›`‡Z †Q` K‡i, Z‡e $\frac{AB}{AD} = \frac{AC}{AE}$ Ges $\frac{AB}{BD} = \frac{AC}{CE}$ n‡e|

Abywm×vš— 2| wÎf‡Ri †Kv‡bv evûi ga¨we›`y w`‡q Aci GK evûi mgvšÍivj †iLv ZZxq evû‡K mgwØLwÊZ K‡i|

Dccv`¨ 1 Gi wecixZ cÖZÁvI mZ¨| A_vr †Kv‡bv mij‡iLv GKwU wÎf‡Ri `B evû‡K A_ev Zv‡`i ewaZvsk؇K mgvb Abcv‡Z wef³ Ki‡j D³ mij‡iLv wÎfR‡Ui ZZxq evûi mgvšÍivj n‡e| wb‡P cÖZÁwU cÖgvY Kiv n‡jv|

Dccv`¨ 2

†Kv‡bv mij‡iLv GKwU wÎf‡Ri `B evû‡K A_ev Zv‡`i ewaZvsk؇K mgvb Abcv‡Z wef³ Ki‡j D³ mij‡iLv wÎfRwUi ZZxq evûi mgvšÍivj|



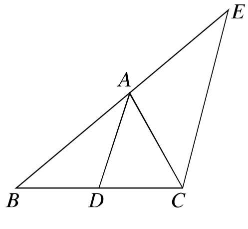
we‡kl wbePb : DE †iLvsk ABC wÎf‡Ri AB I AC evûØq‡K A_ev Zv‡`i ewaZvsk؇K mgvb Abcv‡Z wef³ K‡i‡Q|
A_vr, AD t $DB = AE$ t EC
cgvY Ki‡Z n‡e †h, DE Ges BC mgvšÍivj|
A¼b : B, E Ges C, D †hvM Kwi|
cgvY :

avc	h_v_Zv
(1) $\frac{\Delta ADE}{\Delta BDE} = \frac{AD}{DB}$	[wÎfR `BwU GKB D”PZwewkó]
Ges $\frac{\Delta ADE}{\Delta DEC} = \frac{AE}{EC}$	[wÎfR `BwU GKB D”PZwewkó]
(2) wKš $\frac{AD}{DB} = \frac{AE}{EC}$	[¯xKvi]
(3) AZGe, $\frac{\Delta ADE}{\Delta BDE} = \frac{\Delta ADE}{\Delta BDE}$	[(1) Ges (2) †_‡K]

$\therefore \quad \Delta BDE = \Delta DEC$
(4) wKš ΔBDE Ges ΔDEC GKB fwg DE Gi GKB cv‡k Aew¯Z| mZivs Zv‡`i GKB mgvšÍivj hM‡ji g‡a¨ Aew¯Z|
$\therefore$ BC I DE mgvšÍivj|

Dccv`¨ 3

wÎf‡Ri †h‡Kv‡bv †Kv‡Yi AšÍØLÊK wecixZ evû‡K D³ †KvY msjMœ evûØ‡qi Abcv‡Z Ašwef³ K‡i|



we‡kl wbePb : g‡b Kwi, AD †iLvsk ΔABC Gi Ašt¯ $\angle A$ †K mgwØLwÊZ K‡i BC evû‡K D we›`‡Z †Q` K‡i| cgvY Ki‡Z n‡e †h, BD t $DC = BA$ t AC
A¼b : DA †iLvs‡ki mgvšÍivj K‡i C we›` w`‡q CE †iLvsk A¼b Kwi, †hb Zv ewaZ BA evû‡K E we›`‡Z †Q` K‡i|
cgvY :

avc	h_v_Zv
(1) †h‡nZ DA II CE Ges BC I AC Zv‡`i †Q`K	[A¼b]
$\angle AEC = \angle BAD$	[Abiƒc †KvY]
Ges $\angle ACE = \angle CAD$	[GKvšÍi †KvY]

(2) wKš $\angle BAD = \angle CAD$ — [¯xKvi]

$\therefore \angle AEC = \angle ACE$; $\therefore AC = AE$ — [Dccv`¨ 1]

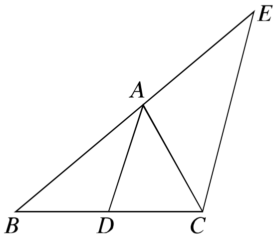
(3) Avevi, †h‡nZ DA II CE, $\therefore \dfrac{BD}{DC} = \dfrac{BA}{AE}$ — [avc (2)]

(4) wKš $AE = AC$

$\therefore \dfrac{BD}{DC} = \dfrac{BA}{AC}$

Dccv`¨ 4

wÎf‡Ri †h‡Kv‡bv evû Aci `B evûi Abcv‡Z Ašwef³ n‡j, wefvM we›` †_‡K wecixZ kxl chš— Aw¼Z †iLvsk D³ kxl‡Kv‡Yi mgwØLÊK n‡e|



we‡kl wbePb : g‡b Kwi, ABC wÎf‡Ri A we›` †_‡K Aw¼Z $AD$ mij‡iLvsk $BC$ evû‡K $D$ we›`‡Z Gi/‡c Ašt¯fv‡e wef³ K‡i‡Q †h, BD t $DC = BA$ t AC

cgvY Ki‡Z n‡e †h, AD †iLvsk $\angle BAC$ Gi mgwØLÊK A_vr, $\angle BAD = \angle CAD$.

A¼b : DA †iLvs‡ki mgvšivj K‡i C we›` w`‡q Gi/c CE †iLvsk A¼b Kwi †hb Zv BA evûi ewaZvsk‡K E we›`‡Z †Q` K‡i|

cgvY :

avc	h_v_Zv
(1) ΔBCE Gi DA II CE	[A¼b]
$\therefore BA$ t $AE = BD$ t DC	[Dccv`¨ 1]
(2) wKš BD t $DC = BA$ t AC	[¯xKvi]
$\therefore BA$ t $AE = BA$ t AC	[avc 1 I avc 2 †_‡K]
$\therefore AE = AC$	
AZGe $\angle ACE = \angle AEC$	[mgwØevû wÎf‡Ri fwg msjM †KvY `BwU mgvb]
(3) wKš $\angle AEC = \angle BAD$	[Abi/c †KvY]
Ges $\angle ACE = \angle CAD$	[GKvšii †KvY]
AZGe, $\angle BAD = \angle CAD$	[avc 2 †_‡K]
A_vr AD †iLvsk $\angle BAC$ Gi mgwØLÊK|	

Abkxjbx 14·1

1| †Kv‡bv wÎf‡Ri fwg msjM †KvYØ‡qi mgwØLÊKØq wecixZ evû `BwU‡K $X$ I $Y$ we›`‡Z †Q` K‡i| XY fwgi mgvšivj n‡j cgvY Ki †h, wÎfRwU mgwØevû|

2| cgvY Ki †h, KZK¸‡jv ci¯úi mgvšivj mij‡iLv‡K `BwU mij‡iLv †Q` Ki‡j Abi/c Ask¸‡jv mgvbcvwZK n‡e|

3| cgvY Ki †h, UªvcwRqv‡gi KYØq Zv‡`i †Q`we›`‡Z GKB Abcv‡Z wef³ nq|

4| cgvY Ki †h, UªvcwRqv‡gi wZhK evûØ‡qi ga¨we›`i ms‡hvRK †iLvsk mgvšivj evûØ‡qi mgvšivj|

5| ABC wÎf‡Ri AD I BE ga¨gvØq ci¯úi G we›`‡Z †Q` K‡i‡Q| G we›`i ga¨ w`‡q Aw¼Z DE Gi mgvšivj †iLvsk AC †K F we›`‡Z †Q` K‡i| cgvY Ki †h, $AC = 6EF$.

6| ΔABC Gi BC evû¯ †h‡Kv‡bv we›`  $X$ Ges $AX$ †iLv¯ $O$ GKwU we›`| cgvY Ki †h, ΔAOB t $\Delta AOC = BX$ t XC

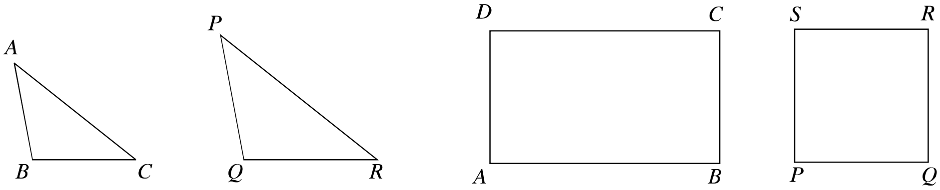
7| ΔABC Gi $\angle A$ Gi mgwØLÊK BC †K D we›`‡Z †Q` K‡i| BC Gi mgvšivj †Kv‡bv †iLvsk AB I AC †K h_vµ‡g E I F we›`‡Z †Q` K‡i|
cgvY Ki †h, BD t $DC = BE$ t CF

8| ABC I DEF m`k‡KvYx wÎfRØ‡qi D"PZv AM I DN n‡j cgvY Ki †h, AM t $DN = AB$ t DE.

14·2 m`kZv (*Similarity*)

mßg †kwY‡Z wÎf‡Ri memgZv I m`kZv wb‡q Av‡jvPbv Kiv n‡q‡Q| mvaviYfv‡e, memgZv m`kZvi we‡kl iƒc| `BwU wPÎ memg n‡j †m¸‡jv m`k; Z‡e wPÎ `BwU m`k n‡j ‡m¸‡jv memg bvI n‡Z cv‡i|

m`‡KvYx eûfR : mgvb msL¨K evûwewkó `BwU eûf‡Ri GKwUi †KvY¸‡jv hw` avivewnKfv‡e AciwUi †KvY¸‡jvi mgvb nq, Z‡e eûfR `BwU‡K m`‡KvYx (*equiangular*) ejv nq|



m`k eûfR : mgvb msL¨K evûwewkó `BwU eûf‡Ri GKwUi kxl©we›`¸‡jv‡K hw` avivewnKfv‡e AciwUi kxl©we›`¸‡jvi m‡½ Ggbfv‡e wgj Kiv hvq †h, eûfR `BwUi (1) Abiƒc †KvY¸‡jv mgvb nq Ges (2) Abiƒc evû¸‡jvi AbycvZ¸‡jv mgvb nq, Z‡e eûfR `BwU‡K m`k (*Similar*) eûfR ejv nq|

Dc‡ii wP‡Î Avgiv j¶ Kwi †h, $ABCD$ AvqZ I $PQRS$ eM© m`‡KvYx| KviY, Dfq wP‡Î evûi msL¨v 4 Ges Avq‡Zi †KvY¸‡jv avivewnKfv‡e eM©wUi †KvY¸‡jvi mgvb (me¸‡jv †KvY mg‡KvY)| wKš' wPÎ¸‡jvi Abiƒc †KvY¸‡jv mgvb n‡jI Abiƒc evû¸‡jvi AbycvZ mgvb bq| d‡j †m¸‡jv m`k bq | wÎf‡Ri †¶‡Î Aek¨ GiKg nq bv| `BwU wÎf‡Ri kxl©we›`¸‡jvi †KvY wgjKi‡Yi d‡j m`kZvi msÁvq D‡jøwLZ kZ© `BwUi GKwU mZ¨ n‡j AciwUI mZ¨ nq Ges wÎfR `BwU m`k nq| A_vr, m`k wÎfR me`v m`‡KvYx Ges m`‡KvYx wÎfR me`v m`k|

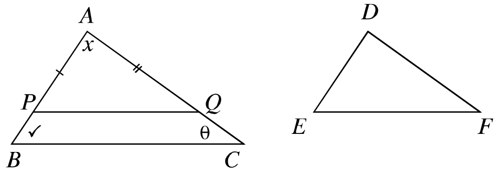
`BwU wÎfR m`‡KvYx n‡j Ges G‡`i †Kv‡bv GK †Rvov Abiƒc evû mgvb n‡j wÎfRØq mgmg nq| `BwU m`‡KvYx wÎf‡Ri Abiƒc evû¸‡jvi AbycvZ a"eK| wb‡P G msµvš Dccv‡`¨i cgvY †`Iqv n‡jv|

Dccv`¨ 5

`BwU wÎfR m`‡KvYx n‡j Zv‡`i Abiƒc evû¸‡jv mgvbcvwZK|

we‡kl wbePb : g‡b Kwi, ABC I DEF wÎfRØ‡qi $\angle A = \angle D$, $\angle B = \angle E$ Ges $\angle C = \angle F$

cgvY Ki‡Z n‡e †h, $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$



A¼b : ABC I DEF wÎfRØ‡qi c‡Z¨K Abiƒc evûhM©j Amgvb we‡ePbv Kwi| AB evû‡Z P we›` Ges $AC$ evû‡Z $Q$ we›` wbB †hb $AP = DE$ Ges $AQ = DF$ nq| P I Q †hvM K‡i A¼b m¤úb Kwi|

cgvY

avc	h_v_Zv
(1) ΔAPQ I ΔDEF Gi $AP = DE, \quad AQ = DF$, $\angle A = \angle D$	
AZGe, $\Delta APQ \cong \Delta DEF$	[evû-‡KvY-evûi mgmgZv]
mZivs, $\angle APQ = \angle DEF = \angle ABC$ Ges $\angle AQP = \angle DFE = \angle ACB.$	
A_vr, PQ †iLvsk I BC evû‡K AB evû I AC †iLv †Q` Kivq Abi‚c †KvYhMj mgvb n‡q‡Q|	
mZivs, $PQ \parallel BC$; $\therefore \frac{AB}{AP} = \frac{AC}{AQ}$ ev, $\frac{AB}{DE} = \frac{AC}{DF}$.	[Dccv`¨ 1]
(2) GKBfv‡e BA evû I BC evû †_‡K h_vµ‡g ED †iLvsk I EF †iLvs‡ki mgvb †iLvsk †K‡U wb‡q †`Lv‡bv hvq †h, $\frac{BA}{ED} = \frac{BC}{EF}$	[Dccv`¨ 1]
A_vr $\frac{AB}{DE} = \frac{BC}{EF}$; $\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.	

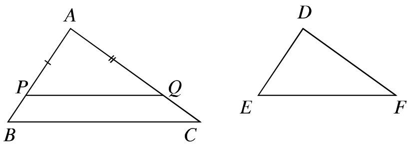
Dccv`¨ 5 Gi wecixZ cwZÁwU I mZ¨\|

Dccv`¨ 6

`BwU wÎfy‡Ri evû¸‡jv mgvbcvwZK n‡j Abi‚c evûi wecixZ †KvY¸‡jv ci¯úi mgvb\|

we‡kl wbePb : g‡b Kwi,

ΔABC I ΔDEF Gi $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.



cgvY Ki‡Z n‡e †h, . $\angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$.

A¼b:

ΔABC I ΔDEF Gi c‡Z¨K Abi‚c evûhMj Amgvb we‡ePbv Kwi\| AB evû‡Z P we›` Ges $AC$ evû‡Z $Q$ we›` wbB †hb $AP = DE$ Ges $AQ = DF$ nq\| P I Q †hvM K‡i A¼b m¤úb Kwi\|

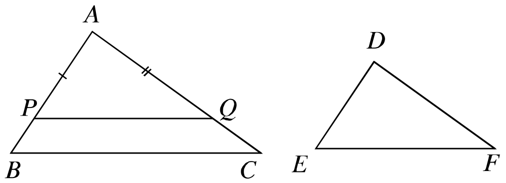
cgvY :

avc	h_v_Zv
(1) †h‡nZ $\frac{AB}{DE} = \frac{AC}{DF}$, mZivs $\frac{AB}{AP} = \frac{AC}{AQ}$.	
mZivs, $PQ \parallel BC$	[Dccv`¨ 2]
$\therefore \angle ABC = \angle APQ$ Ges $\angle ACB = \angle AQP$	[AB †Q`K Øviv Drcb Abi‚c †KvY] [$AC$ †Q`K Øviv Drcb Abi‚c †KvY]
$\therefore \Delta ABC$ I ΔAPQ m`k‡KvYx|	
mZivs $\frac{AB}{AP} = \frac{BC}{PQ}$ ev, $\frac{AB}{DE} = \frac{BC}{PQ}$.	[Dccv`¨ 5]

$\therefore \frac{BC}{EF} = \frac{BC}{PQ}$ [Kíbvbmv‡i] ; $\therefore \frac{AB}{DE} = \frac{BC}{EF}$ $\therefore EF = PQ$ mZivs, ΔAPQ I ΔDEF memg| $\therefore \angle PAQ = \angle EDF, \angle APQ = \angle DEF, \angle AQP = \angle DFE$ $\therefore \angle APQ = \angle ABC$ Ges $\angle AQP = \angle ACB$ $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F.$	[evû-evû-evû Dccv`¨]

Dccv`¨ 7

`BwU wÎf‡Ri GKwUi GK †KvY AciwUi GK †Kv‡Yi mgvb n‡j Ges mgvb mgvb †KvY ms‡j· evû¸‡jv mgvbcvwZK n‡j wÎfRØq m`k|



we‡kl wbePb : g‡b Kwi, ΔABC Ges ΔDEF Ggb †h,

$\angle A = \angle D$ Ges $\frac{AB}{DE} = \frac{AC}{DF}$

cgvY Ki‡Z n‡e †h, ΔABC Ges ΔDEF m`k|

A¼b :

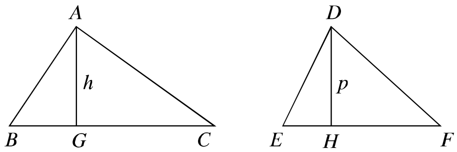
ΔABC I ΔDEF Gi c‡Z¨K Abi|c evûhM‡j Amgvb we‡ePbv Kwi| AB evû‡Z P we›` Ges $AC$ evû‡Z $Q$ we›` wbB †hb $AP = DE$ Ges $AQ = DF$ nq| P I Q †hvM K‡i A¼b m¤úb Kwi|

cgvY :

avc	h_v_Zv
ΔAPQ I ΔDEF Gi $AP = DE,\ AQ = DF$ Ges Ašf ³ $\angle A =$ Ašf ³ $\angle D$, $\therefore \Delta ABC \cong \Delta DEF$	[evû-‡KvY-evû Dccv`¨]
$\therefore \angle A = \angle D, \angle APQ = \angle E, \angle AQP = \angle F.$	
Avevi, †h‡nZ $\frac{AB}{DE} = \frac{AC}{DF}$, mZivs $\frac{AB}{AP} = \frac{AC}{AQ}.$	[Dccv`¨ 2]
$\therefore PQ \parallel BC$	
mZivs $\angle ABC = \angle APQ$ Ges $\angle ACB = \angle AQP$	
$\therefore \angle A = \angle D, \angle B = \angle E$ Ges $\angle C = \angle F$	
A_vr, ΔABC I ΔDEF m`k‡KvYx|	
mZivs ΔABC I ΔDEF m`k|	

Dccv`¨ 8

`BwU m`k wÎfR‡¶‡Îi †¶Îdj؇qi AbcvZ Zv‡`i †h‡Kv‡bv `B Abi|c evûi Dci AwZ eM‡¶‡Îi †¶Îdj؇qi Abcv‡Zi mgvb|



we‡kl wbePb : g‡b Kwi, ABC I DEF wÎfRØq m`k Ges Zv‡`i `BwU Abi|c evû BC I EF .

cgvY Ki‡Z n‡e †h, ΔABC t $\Delta DEF = BC^2$ t EF^2

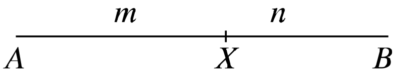
A¼b : BC I EF Gi Ici h_vµ‡g AG I DH j¤ AwsK| g‡b Kwi, $AG = h$, $DH = p$.

cgvY :

avc	h_v_Zv
(1) $\Delta ABC = \frac{1}{2} BC.h$ Ges $\Delta DEF = \frac{1}{2} EF.p$ $\therefore \frac{\Delta ABC}{\Delta DEF} = \frac{\frac{1}{2}BC.h}{\frac{1}{2}EF.p} = \frac{h.BC}{p.EF} = \frac{h}{p} \times \frac{BC}{EF}$ (2) ABG Ges DEH wÎfR؇qi $\angle B = \angle E$, $\angle AGB = \angle DHE$ (= GK mg‡KvY)| $\therefore \angle BAG = \angle EDH$ ΔABG I ΔDEH m\`k‡KvYx, ZvB m\`k| (3) $\frac{h}{p} = \frac{AB}{DE} = \frac{BC}{EF}$ [KviY ΔABC I ΔDEF m\`k] $\therefore \frac{\Delta ABC}{\Delta DEF} = \frac{h}{p} \times \frac{BC}{EF} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2}$	

14·3| wbw`ó Abcv‡Z †iLvs‡ki wefw³KiY

mgZ‡j `BwU wfbœ we›`ž A I B Ges m I n †h‡Kv‡bv ¯vfvweK msL¨v n‡j Avgiv ¯xKvi K‡i wbB †h, AB †iLvq Ggb Abb¨ we›`ž $X$ Av‡Q †h, $X$ we›`wU A I B we›`ži AšeZx Ges AX t $XB = m$ t n.

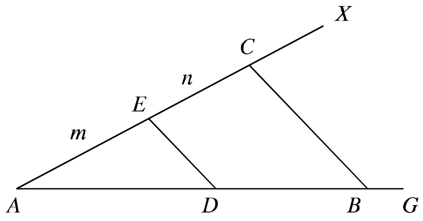


Ic‡ii wP‡Î, AB †iLvsk X we›`‡Z m t n Abcv‡Z Ašwef³ n‡q‡Q| Zvn‡j, AX t $XB = m$ t n.

m¤úv`¨ 1

†Kv‡bv †iLvsk‡K GKwU wbw`ó Abcv‡Z Ašwef³ Ki‡Z n‡e|

g‡b Kwi, AB †iLvsk‡K m t n Abcv‡Z Ašwef³ Ki‡Z n‡e|



A¼‡bi weeiY : A we›`‡Z †h‡Kv‡bv †KvY $\angle BAX$ A¼b Kwi Ges $AX$ iwk¥ †_‡K cici $AE = m$ Ges $EC = n$ Ask †K‡U wbB| $B, C$ †hvM Kwi| $E$ we›`y w`‡q $CB$ Gi mgvšivj $ED$ †iLvsk A¼b Kwi hv $AB$ †K $D$ we›`‡Z †Q` K‡i| Zvn‡j $AB$ †iLvsk $D$ we›`‡Z m t n Abcv‡Z Ašwef³ n‡jv|

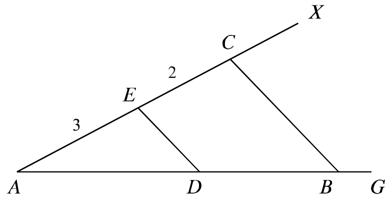
cgvY : †h‡nZz DE †iLvsk ABC wÎf‡Ri GK evû BC Gi mgvšivj,

$\therefore AD$ t $DB = AE$ t $EC = m$ t n

KvR : 1| weKí c×wZ‡Z †Kv‡bv †iLvsk‡K wbw`ó Abcv‡Z Ašwef³ Ki|

D`vniY 1| 7 †m.wg. ^`‡N¨i GKwU †iLvsk‡K 3t2 Abcv‡Z AšÍwe©f³ Ki|

mgvavb : †h‡Kv‡bv GKwU iwk¥ AG AwaK Ges AG †_‡K 7 †m.wg. mgvb †iLvsk AB wbB| A we›`y‡Z †h‡Kv‡bv †KvY $\angle BAX$ A¼b Kwi| $AX$ iwk¥ †_‡K $AE = 3$ †m.wg. †K‡U wbB Ges $EX$ †_‡K $EC = 2$ †m.wg. †K‡U wbB| $B, C$ †hvM Kwi| $E$ we›`y‡Z $\angle ACB$ Gi mgvb $\angle AED$ A¼b Kwi hvi ED †iLv AB †K D we›`y‡Z †Q` K‡i| Zvn‡j AB †iLvsk D we›`y‡Z 3 t 2 Abcv‡Z AšÍwe©f³ n‡jv|



D`vniY 2| GKwU wbw`©ó wÎf‡Ri m`„k GKwU wÎfR A¼b Ki hvi evû¸‡jv gj wÎf‡Ri evû¸‡jvi $\frac{3}{5}$ ¸Y|

Abkxjbx 14·2

1| wb‡Pi Z_¨¸‡jv j¶¨ Ki:

i `BwU iwkki Zjbv Kivi Rb¨ Zv‡`i Abcv‡Z we‡ePbv Kiv nq

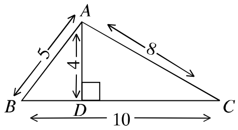
ii Abcv‡Z wbY©‡qi Rb¨ iwk `BwU GKB GK‡K cwigvc Ki‡Z nq

iii Abcv‡Z wbY©‡qi †¶‡Î iwk `wU GKB RvZxq n‡Z nq

wb‡Pi †KvbwU mwVK ?

K. i I ii L. ii I iii

M. i I iii N. i, ii I iii



Dc‡ii wP‡Îi Z_¨vbmv‡i (2 I 3) bs c‡ki DËi `vI:

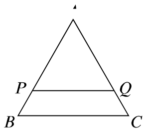
2| ΔABC Gi D"PZv I fwgi Abcv‡Z KZ?

K. $\frac{1}{2}$ L. $\frac{4}{5}$ M. $\frac{2}{5}$ N. $\frac{5}{4}$

3| ΔABD Gi †¶Îdj KZ eM GKK?

K. 6 L. 20 M. 40 N. 50

4| ΔABC- G PQ11 BC n‡j wb‡Pi †KvbwU mwVK?



K. AP : PB = AQ : QC L. AB : PQ = AC : PQ

M. AB : AC = PQ : BC N. PQ : BC = BP : BQ

5| GKwU e‡Mi m‡ev©"P (†gvU) KZwU cwZmvg¨ †iLv Av‡Q?

K. 10wU L. 8wU

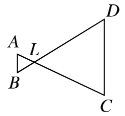
M. 6wU N. 4wU

6| cgvY Ki †h, `BwU wÎf‡Ri c‡Z¨KwU hw` Aci ZZxq GKwU wÎf‡Ri m`„k nq, Z‡e Zviv ci¯úi m`„k|

7| cgvY Ki †h, `BwU mg‡KvYx wÎf‡Ri GKwUi GKwU m²‡KvY AciwUi GKwU m²‡Kv‡Yi mgvb n‡j, wÎfR `BwU m`k n‡e|

8| cgvY Ki †h, mg‡KvYx wÎf‡Ri mg‡KŠwYK kx©l †_‡K AwZf‡Ri Dci j¤ AvK‡j †h `BwU mg‡KvYx wÎfR Drcb nq, Zviv ci¯úi m`k Ges c‡Z¨‡K gj wÎf‡Ri m`k|

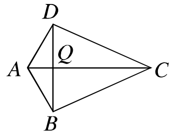
9| cv‡ki wP‡Î, $\angle B = \angle D$ Ges $CD = 4AB.$
cgvY Ki †h, $BD = 5BL.$



10| $ABCD$ mvgvšwi‡Ki A kx©l w`‡q Aw¼Z GKwU †iLvsk $BC$ evû‡K $M$ we›`‡Z Ges DC evû ewaZvsk‡K N we›`‡Z †Q` K‡i| cgvY Ki †h, $BM \times DN$ GKwU a"eK|

11| cv‡ki wP‡Î $BD \perp AC$ Ges

$DQ = BA = 2AQ = \dfrac{1}{2}QC.\ BD = 5BL.$

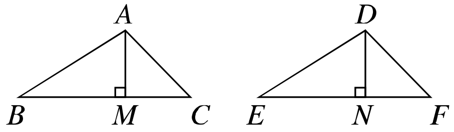


cgvY Ki †h, $DA \perp DC.$

12| ΔABC I ΔDEF Gi $\angle A = \angle D$.
cgvY Ki †h, ΔABC t ΔDEF $= AB.AC$ t $DE.DF$.

13| ΔABC Gi $\angle A$ Gi mgwØLÊK AD, BC †K D we›`‡Z †Q` K‡i‡Q| DA Gi mgvšivj CE †iLvsk ewaZ BA evû‡K E we›`‡Z †Q` K‡i‡Q|
K. Z_¨ Abmv‡i wPÎwU A¼b Ki|
L. cgvY Ki †h, BD t DC = BA t AC
M. BC Gi mgvšivj †Kv‡bv †iLvsk AB I AC †K h_vµ‡g P I Q we›`‡Z †Q` Ki‡j, cgvY Ki †h, BD t DC = BP t CQ

14| wP‡Î ABC Ges DEF `BwU m`k wÎfR|
K. wÎfR `BwUi Abi/c evû I Abi/c †KvY¸‡jvi bvg wjL|
L. cgvY Ki †h,

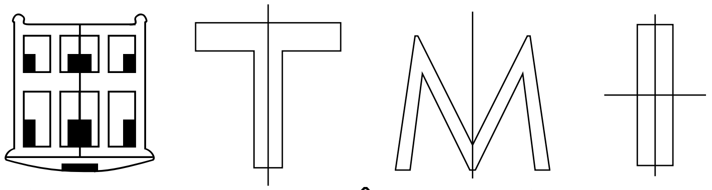


$$\frac{\Delta ABC}{\Delta DEF} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$

M. hw` $BC = 3$ †m.wg., $EF = 8$ †m.wg., $\angle B == 60^\circ, \dfrac{BC}{AB} = \dfrac{3}{2}$ Ges $\Delta ABC = 3$ eM †m.wg. nq, Z‡e ΔDEF A¼b Ki Ges Gi †¶Îdj wbYq Ki|

14·4 cwZmgZv

cwZmgZv GKwU c‡qvRbxq R¨wgwZK aviYv hv cKwZ‡Z we`¨gvb Ges hv Avgv‡`i KgKv‡Ê cwZwbqZ e¨envi K‡i _wK| cwZmgZvi aviYv‡K wkíx, KwiMi, wWRvBbvi, mZviiv cwZwbqZ e¨envi K‡i _v‡Kb| Mv‡Qi cvZv, dj, †gŠPvK, Niewo, †Uwej, †Pqvi mewKQi g‡a¨ cwZmgZv we`¨gvb| hw` †Kv‡bv mij‡iLv eivei †Kv‡bv wPÎ fvR Ki‡j Zvi Ask `BwU m¤úYfv‡e wg‡j hvq †m‡¶‡Î mij‡iLwU‡K cwZmvg¨ †iLv ejv nq|

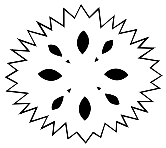


Dc‡ii wPÎ¸‡jvi cwZwUi cwZmvg¨ †iLv i‡q‡Q| †k‡li wPÎwUi GKwaK cwZmvg¨ †iLv i‡q‡Q|

KvR :

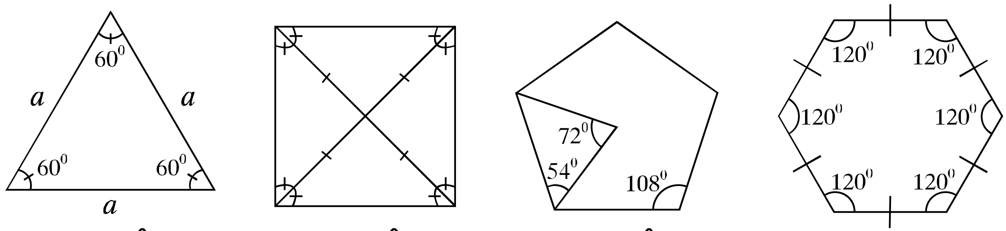
1| mvg KvMR †K‡U cv‡ki wP‡Îi wWRvBb ˆZwi Ki‡Q| wP‡Î cwZmg †iLvmgn wPwýZ Ki| Gi KqwU cwZmg †iLv i‡q‡Q ?

2| Bs‡iwR eYgvjvi †h mKj e‡Yi cwZmvg¨ †iLv i‡q‡Q †m¸‡jv wj‡L cwZmvg¨ †iLv wPwýZ Ki|



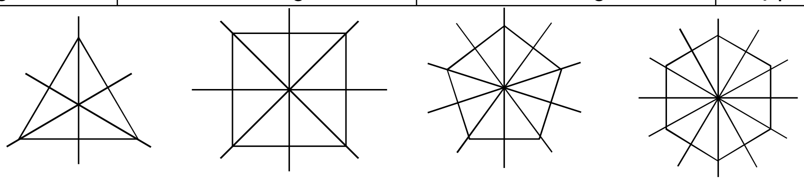
14·5 mlg eûf‡Ri cwZmvg¨ †iLv

eûfR KZK¸‡jv †iLvsk Øviv Avea× wPÎ| eûf‡Ri †iLvsk¸‡jvi ˆ`N¨ mgvb I †KvY¸‡jv mgvb n‡j Zv‡K mlg eûfR ejv nq| wÎfR n‡jv me‡P‡q Kg msL¨K †iLvsk w`‡q MwVZ eûfR| mgevû wÎfR n‡jv wZb eûwewkó mlg eûfR| mgevû wÎf‡Ri evû I †KvY¸‡jv mgvb| Pvi eûwewkó mlg eûfR n‡jv eM‡¶Î| eM‡¶‡Îi evû I †KvY¸‡jv mgvb| Abi~cfv‡e, mlg cÂfR I mlg lof‡Ri evû I †KvY¸‡jv mgvb|

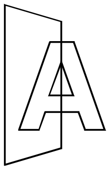


mgevû wÎfR   eM‡¶Î   mlg cÂfR   mlg lofR

c‡Z¨K mlg eûfR GKwU cwZmg wPÎ| mZivs Zv‡`i cwZmvg¨ †iLvi m¤ú‡K Rvbv Avek¨K| mlg eûf‡Ri A‡bK evûi cvkvcvwk GKwaK cwZmvg¨ †iLv i‡q‡Q|

wZbwU cwZmvg¨ †iLv	PviwU cwZmvg¨ †iLv	cvPwU cwZmvg¨ †iLv	QqwU cwZmvg¨ †iLv
			
mgevû wÎfR	eM‡¶Î	mlg cÂfR	mlg lofR

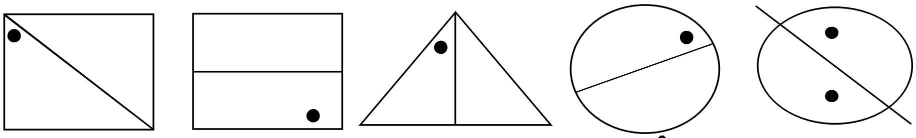
cwZmgZvi aviYvi mv‡_ Avqbvi cwZdj‡bi m¤úK i‡q‡Q| †Kv‡bv R¨wgwZK wP‡Îi cwZmvg¨ †iLv ZLbB _v‡K, hLb Zvi Aavsk‡ki cwZ”Qwe evwK Aavsk‡ki mv‡_ wg‡j hvq| GRb¨ cwZmvg¨ †iLv wbY‡q KvíwbK Avqbvi Ae¯’vb †iLvi mvnvh¨ ‡bqv nq| †iLv cwZmgZv‡K cwZdjb cwZmgZvI ejv nq|



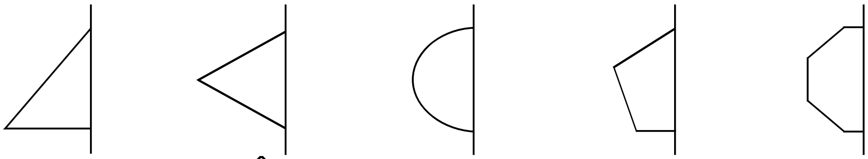
Abkxjbx 14·3

1| wb‡Pi wPÎmg‡ni †KvbwUi cwZmvg¨ †iLv i‡q‡Q?
(K) ewoi wPÎ (L) gmwR‡`i wPÎ (M) gw›`‡ii wPÎ (M) MxRvi wPÎ, (M) c¨v‡MvWvi wPÎ (N) cvjv‡g›U fe‡bi wPÎ, (O) g‡Lv‡ki wPÎ (P) ZvRgn‡ji wPÎ

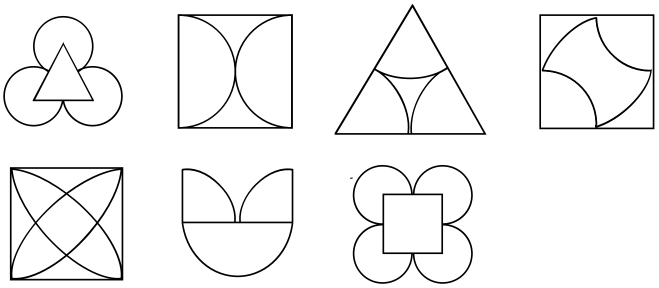
2| cwZmvg¨ †iLv †`Iqv Av‡Q, Ab¨ dUwK c`kb Ki :



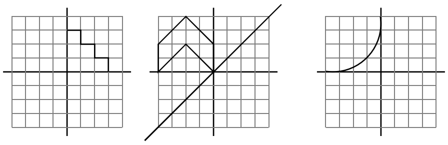
3| cwZmvg¨ †iLv †`Iqv Av‡Q (W¨vkh³ †iLv), R¨vwgwZK wPÎ m¤úY Ki Ges kbv³ Ki|



4| wb‡Pi R¨vwgwZK wP‡Î cwZmvg¨ †iLv wb‡`k Ki:



5| wb‡Pi Am¤úY R¨vwgwZK wPÎ m¤úY Ki †hb Avqbv †iLv mv‡c‡¶ cwZmg nq :



6| wb‡Pi R¨vwgwZK wP‡Îi cwZmvg¨ †iLvi msL¨v wbYq Ki:
(K) mgwØevû wÎfR (L) welgevû wÎfR (M) eM‡¶Î (N) i¤m
(O) mIg lofR (P) cÂfR (Q) eË

7| Bs‡iwR eYgvjvi †h mKj e‡Yi
(K) AbfwgK Avqbv (L) Dj¤ Avqbv
(M) AbfwgK I Dj¤ Dfq Avqbv
mv‡c‡¶ cwZdjb cwZmgZv i‡q‡Q †m¸‡jv AvK|

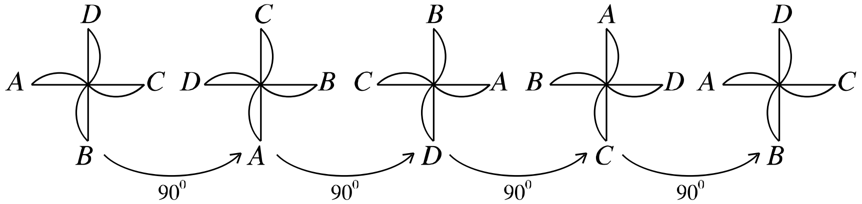
7| cwZmgZv †bB Ggb wZbwU wPÎ A¼b Ki|

14·6 NYb cwZmgZv

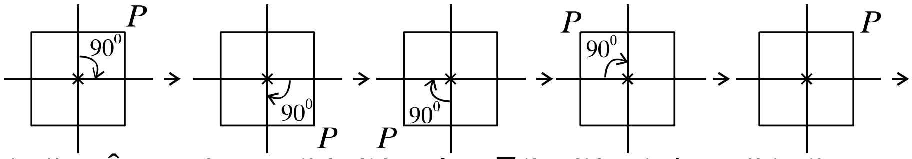
†Kv‡bv wbw`ó we›`i mv‡c‡¶ NY‡bi d‡j e¯'i AvKwZ I AvKv‡ii cwieZb nq bv| Z‡e e¯'i wewfb As‡ki Ae¯'v‡bi cwieZb nq| NY‡bi d‡j e¯'i bZb Ae¯'v‡b e¯'i AvKwZ I AvKvi Aw` Ae¯'‡bi b¨vq GKB n‡j Avgiv ewj e¯'wUi NYb cwZmgZv i‡q‡Q| †hgb, mvB‡K‡ji PvKv, wmwjs d¨vb, eM BZ¨vw`| GKwU wmwjs d¨v‡bi cvLv¸‡jvi NY‡bi d‡j GKwaKevi gj Ae¯'v‡bi mv‡_ wg‡j hvq| cvLv¸‡jv Nwoi KvUvi w`‡K I Nwoi KvUvi wecixZ w`‡K I Ni‡Z cv‡i| mvB‡K‡ji PvKv Nwoi KvUvi w`‡K I Ni‡Z cv‡i, Avevi wecixZ w`‡K I Ni‡Z cv‡i| Nwoi KvUvi w`‡K NYb‡K abvZ¥K w`K wnmv‡e aiv nq|

†h we›`i mv‡c‡¶ e¯‘wU †Nv‡i Zv n‡jv NYb †K›`ª| NY‡bi mgq †h cwigvY †Kv‡Y †Nv‡i Zv n‡jv NYb †KvY| GKevi cY NY‡bi †Kv‡Yi cwigvY 360°, AaŸ NY‡bi †Kv‡Yi cwigvY 180°|

wP‡Î Pvi cvLwewkó d¨v‡bi 90° K‡i NY‡bi d‡j wewfb Ae¯'vb ‡`Lv‡bv n‡q‡Q| j¶ Kwi, GKevi cY NY‡b wVK PviwU Ae¯'v‡b (90°, 180°, 270° I 360° ‡Kv‡Y NY‡bi d‡j) d¨vbwU †`L‡Z ûeû GKB iKg| GRb¨ ejv nq d¨vbwUi NYb cwZmgZvi gvÎv 4|



NYb cwZmgZvi Ab¨ GKwU D`vniY †bqv hvq| GKwU  e‡Mi KY `BwUi †Q`we›`‡K NYb †K›`ª awi| NYb †K‡›`ªi mv‡c‡¶ eM©wUi GK-PZz_vsk NY‡bi d‡j †h‡Kv‡bv †KŠwYK wK›`i Ae¯'vb wØZxq wP‡Îi b¨vq n‡e| Gfv‡e Pvievi GK-PZz_vsk NY‡bi d‡j eM©wU Aw` Ae¯'v‡b wd‡i Av‡m| ejv nq, e‡Mi 4 gvÎvi NYb cwZmgZv i‡q‡Q|



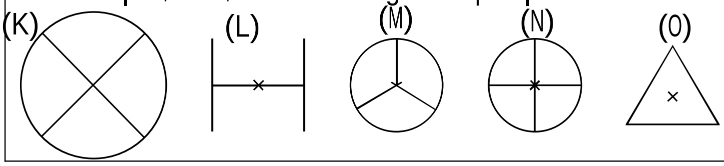
j¶ Kwi, †h‡Kv‡bv wPÎ GKevi cY NY‡bi d‡j Aw` Ae¯'v‡b wd‡i Av‡m| ZvB †h‡Kv‡bv R¨wgwZK wP‡Îi 1 gvÎvi NYb cwZmgZv i‡q‡Q|

NYb cwZmgZv wbY‡qi †¶‡Î wb‡Pi welq¸‡jv j¶ ivL‡Z n‡e:

(K) NYb †K›`ª (L) NYb †KvY (M) NY‡bi w`K (N) NYb cwZmgZvi gvÎv|

KvR : 1| †Zvgvi Pvicv‡ki cwi‡ek †_‡K 5wU mgZjxq e¯'i D`vniY `vI hv‡`i NYb cwZmgZv i‡q‡Q|

2| wb‡Pi wP‡Îi NYb cwZmgZv wbYq Ki|



14·7 †iLv cwZmgZv I NYb cwZmgZv

Avgiv †`‡LwQ ‡h wKQ R¨wgwZK wP‡Îi ïa †iLv cwZmgZv i‡q‡Q, wKQi ïa NYb cwZmgZv i‡q‡Q| Avevi †Kv‡bv †Kv‡bv wP‡Îi †iLv cwZmgZv I NYb cwZmgZv DfqB we`¨gvb| †hgb, e‡Mi ‡hgb PviwU cwZmvg¨ †iLv i‡q‡Q, †Zgwb 4 gvÎvi NYb cwZmgZv i‡q‡Q|

eË GKwU Av`k cwZmg wPÎ| eË‡K Gi †K‡›`ªi mv‡c‡¶ †h‡Kv‡bv †Kv‡Y I †h‡Kv‡bv w`‡K Niv‡j Gi Ae¯'v‡bi cwieZb j¶ Kiv hvq bv| AZGe, e‡Ëi NYb cwZmgZvi gvÎv Amxg| GKB mgq e‡Ëi †K›`Mvgx †h‡Kv‡bv †iLv Gi cwZmvg¨ †iLv| mZivs, e‡Ëi AmsL¨ cwZmvg¨ †iLv i‡q‡Q|

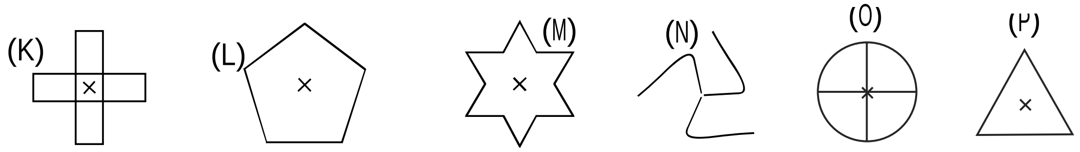
KvR :

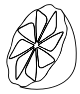
1| Bs‡iwR eYgvjvi K‡qKwU e‡Yi †iLv cwZmgZv I NYb cwZmgZv wbavi Y Ki Ges wb‡Pi mviwYwU ciY Ki: (GKwU K‡i †`Lv‡bv n‡jv)

eY	†iLv cÖZmgZv	cÖZmvg¨ †iLvi msL¨v	NYb cÖZmgZv	NYb cÖZmgZvi gvÎv
Z	†bB	0	n¨v	2
H				
O				
E				
C				

Abkxjbx 14·4

1| wb‡Pi wP‡Îi NYb cÖZmgZv wbYq Ki :



2| GKwU †je AvovAwo †K‡U wP‡Îi b¨vq AvKvi cvIqv †Mj| mgZjxq wPÎwUi NYb cÖZmgZv wbYq Ki|	

3| kb¨¯’vb ciY Ki :

wPÎ	NYb †K›`ª	NYb cÖZmgZvi gvÎv	NYb cÖZmgZvi ‡KvY
eM			
AvqZ			
i¤m			
mgevû wÎfR			
AaeË			
mIg cÂfR			

4| †h mKj PZf‡Ri †iLv cÖZmgZv I 1 Gi AwaK gvÎvi NYb cÖZmgZv i‡q‡Q, Zv‡`i ZvwjKv Ki|

5| 1 Gi AwaK gvÎvi NYb cÖZmgZv i‡q‡Q Gi/c wP‡Îi NYb †KvY 18° n‡Z cv‡i Kx ? †Zvgvi DË‡ii c‡¶ hw³ `vI|

cÂ`k Aa¨vq

†¶Îdj m¤úwKZ Dccv`¨ I m¤úv`¨

(Area Related Theorems and Constructions)

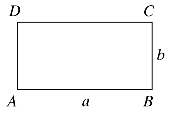
Avgiv Rwb mxgve× mgZj †¶‡Îi AvKwZ wewfbœ iKg n‡Z cv‡i| mgZj †¶Î hw` PviwU evûØviv mxgve× nq, Z‡e Zv‡K Avgiv PZfR e‡j _wK| GB PZf‡Ri Avevi †kwY wefvM Av‡Q Ges AvKwZ I ˆewk‡ó¨i Dci wfwË K‡i Zv‡`i bvgKiY I Kiv n‡q‡Q| GB mKj mgZj †¶‡Îi evB‡i A‡bK †¶Î Av‡Q hv‡`i evû Pv‡ii AwaK| Av‡jwPZ G mKj †¶ÎB eûfR‡¶Î| c‡Z¨K mxgve× mgZj‡¶‡Îi wbw`ó cwigvc Av‡Q hv‡K †¶Îdj e‡j AewnZ Kiv nq| GB mKj †¶Îdj cwigv‡ci Rb¨ mvaviYZ GK GKK evûwewkó eM‡¶‡Îi †¶Îdj e¨envi Kiv nq Ges Zv‡`i †¶Îdj‡K eM GKK wn‡m‡e †jLv nq| †hgb, evsjv‡`‡ki †¶Îdj 144 (cvq) nvRvi eM wK‡jwgUvi| Avgv‡`i ˆ`bw›`b Rxe‡bi c‡qvRb †gUv‡Z eûfR †¶‡Îi †¶Îdj Rvbv‡Z I cwigvc Ki‡Z nq| ZvB G ¯‡ii wk¶v_x‡`i eûfR †¶‡Îi †¶Îdj m¤‡Ü mg¨K Ávb c`vb Kiv AZxe ¸i"ZcY| GLv‡b eûfR †¶‡Îi †¶Îd‡ji aviYv Ges GZ`msµvš— KwZcq Dccv`¨ I m¤úv`¨ welqK welqe¯ Dc¯vcb Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv –

- eûfR †¶‡Îi †¶Îd‡ji aviYv e¨vL¨v Ki‡Z cvi‡e|
- †¶Îdj msµvš— Dccv`¨ hvPvB I cgvY Ki‡Z cvi‡e|
- c`Ë DcvË e¨envi K‡i eûfR †¶Î A¼b I A¼‡bi h_vhZv hvPvB Ki‡Z cvi‡e|
- wÎfR‡¶‡Îi †¶Îd‡ji mgvb PZfR‡¶Î A¼b Ki‡Z cvi‡e|
- PZfR‡¶‡Îi †¶Îd‡ji mgvb wÎfR‡¶Î A¼b Ki‡Z cvi‡e|

15·1 mgZj †¶‡Îi †¶Îdj

c‡Z¨K mxgve× mgZj †¶‡Îi wbw`ó †¶Îdj i‡q‡Q| GB †¶Îdj cwigv‡ci Rb¨ mvaviYZ GK GKK evûwewkó eM‡¶‡Îi †¶Îdj‡K eM GKK wn‡m‡e MnY Kiv nq| †hgb, †h eM‡¶‡Îi GK evûi ˆ`N¨ GK †mw›UwgUvi Zvi †¶Îdj n‡e GK eM‡mw›UwgUvi|



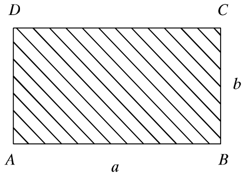
Avgiv Rwb,

(K) $ABCD$ AvqZ‡¶‡Îi

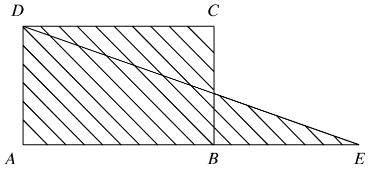
ˆ`N¨ $AB = a$ GKK (h_v, wgUvi)

c¯ $BC = b$ GKK (h_v, wgUvi) n‡j,

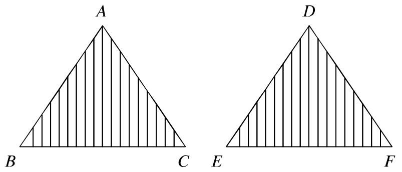
$ABCD$ AvqZ‡¶‡Îi †¶Îdj = ab eM GKK (h_v, eMwgUvi)|



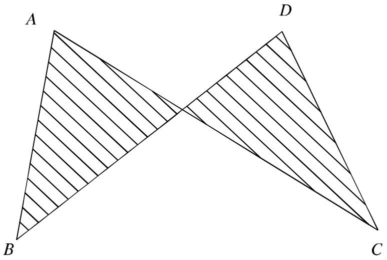
(L) *ABCD* eM©‡¶‡Î i evûi ˆ`N¨ = a GKK (h_v, wgUvi) n‡j, *ABCD* eM©‡¶‡Î i †¶Îdj = a^2 eM GKK (h_v, eMwgUvi)|



`BwU †¶‡Î i †¶Îdj mgvb n‡j Zv‡`i g‡a¨ ˆ`N©¨ wPý e¨envi Kiv nq| †hgb, *ABCD* AvqZ‡¶‡Î i †¶Îdj = *AED* wÎ fR‡¶‡Î i †¶Îdj|



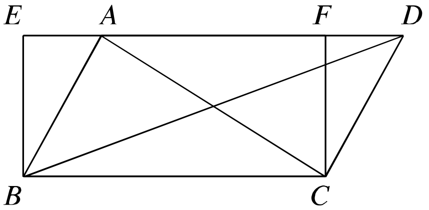
D‡jøL¨ †h, ΔABC I ΔDEF memg n‡j, $\Delta ABC \cong \Delta DEF$ †jLv nq| G‡¶‡Î Aek¨B ΔABC Gi †¶Îdj = ΔDEF Gi †¶Îdj|



wKš `BwU wÎ fR‡¶‡Î i †¶Îdj mgvb n‡jB wÎ fR `BwU memg nq bv| †hgb, wP‡Î ΔABC Gi †¶Îdj = ΔDBC Gi †¶Îdj| wKš ΔABC I ΔDBC memg bq|

Dccv`¨ 15·1

GKB fwgi Dci Ges GKB mgvšÍ iv‡j †iLvhM‡j i g‡a¨ Aew¯Z mKj wÎ fR‡¶‡Î i †¶Îdj mgvb|



g‡b Kwi, *ABC* I *DBC* wÎ fR‡¶ÎØq GKB fwg *BC* Gi Dci Ges GKB mgvšÍ iv‡j †iLvhMj *BC* I *AD* Gi g‡a¨ Aew¯Z| cgvY Ki‡Z n‡e †h, ∆ †¶Î *ABC* Gi †¶Îdj = ∆ †¶Î *DBC* Gi †¶Îdj|

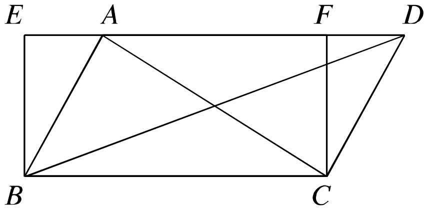
A¼b : *BC* †iLvs‡ki *B* I *C* we›`‡Z h\_vµ‡g *BE* I *CF* j¤ A¼b Kwi| Giv *AD* †iLvi ewaZ Ask‡K *E* we›`‡Z Ges *AD* †iLv‡K *F* we›`‡Z †Q` K‡i| d‡j *EBCF* GKwU AvqZ‡¶Î ˆZwi nq|

cgvY : *EBCF* GKwU AvqZ‡¶Î, GLb ∆ †¶Î *ABC* Ges AvqZ‡¶Î *EBCF* GKB fwg *BC* Gi Dci Ges *BC* I *ED* mgvšÍ iv‡j †iLvs‡ki g‡a¨ Aew¯Z| mZivs ∆ †¶Î *ABC* = $\frac{1}{2}$ (AvqZ‡¶Î *EBCF*) Abi|cfv‡e, ∆ †¶Î *DBC* †¶‡Î i †¶Îdj = $\frac{1}{2}$ (AvqZ‡¶Î *EBCF*)

∴ ∆ †¶Î *ABC* †¶Îdj = ∆ †¶Î *DBC* -Gi †¶Îdj (cgwYZ)|

Dccv`¨ 1

GKB fwgi Dci Ges GKB mgvšÍivj †iLvhM‡j i g‡a¨ Aew¯’Z mKj wÎfR‡¶‡Îi †¶Îdj mgvb|



g‡b Kwi, ABC I DBC wÎfR‡¶ÎØq GKB fwg BC Gi Dci Ges GKB mgvšÍivj †iLvhMj BC I AD Gi g‡a¨ Aew¯’Z| cgvY Ki‡Z n‡e †h, Δ †¶Î ABC Gi †¶Îdj = Δ †¶Î DBC Gi †¶Îdj|

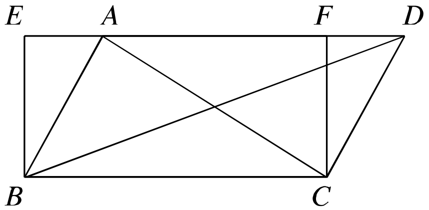
A¼b : BC †iLvs‡ki B I C we›`‡Z h_vµ‡g $BE$ I $CF$ j¤ A¼b Kwi| Giv $AD$ †iLvi ewaZ Ask‡K $E$ we›`‡Z Ges AD †iLv‡K F we›`‡Z †Q` K‡i| d‡j $EBCF$ GKwU AvqZ‡¶Î ˆZwi nq|

cgvY : $EBCF$ GKwU AvqZ‡¶Î, GLb Δ †¶Î ABC Ges AvqZ‡¶Î $EBCF$ GKB fwg BC Gi Dci Ges BC I ED mgvšÍivj †iLvs‡ki g‡a¨ Aew¯’Z| mZivs Δ †¶Î ABC = $\frac{1}{2}$ (AvqZ‡¶Î $EBCF$) Abiƒcfv‡e, Δ †¶Î DBC †¶‡Îi †¶Îdj = $\frac{1}{2}$ (AvqZ‡¶Î $EBCF$)

$\therefore$ Δ †¶Î ABC †¶Îdj = Δ †¶Î DBC-Gi †¶Îdj (cgwYZ)|

Dccv`¨ 2

GKB fwgi Dci Ges GKB mgvšÍivj †iLvhM‡j i g‡a¨ Aew¯’Z mvgvšÍwiK‡¶Îmg‡ni †¶Îdj mgvb|



wP‡Î, $ABCD$ I $EFGH$ mvgvšÍwiK‡¶Î `BwU AB I EF fwg AB Gi Dci Ges F GKB mgvšÍivj †iLvhMj AF I DG Gi g‡a¨ Aew¯’Z|

cgvY Ki‡Z n‡e †h, mvgvšÍwiK $ABCD$ Gi †¶Îdj = mvgvšÍwiK‡¶Î $EFGH$.

$EFGH$ Gi fwg EF mgvb nq| GLb AC I EG †hvM Kwi| C I G we›` †_‡K fwg AF I Gi ewaZ †iLvs‡ki Dci CL I GK j¤ Uwb|

cgvY : ΔABC Gi †¶Îdj = $\frac{1}{2} AB \times GL$ Ges

ΔEFG Gi †¶Îdj $\frac{1}{2} EF \times GK$.

$\because$ $AB = EF$ Ges $CL = GK$, (A¼bmv‡i)

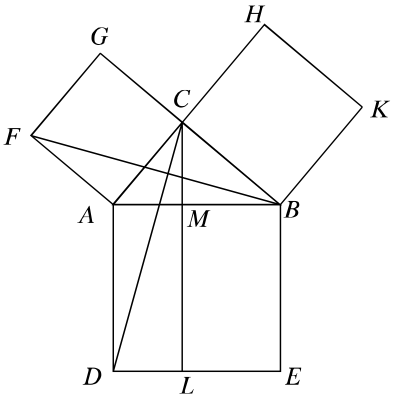
AZGe, ΔABC Gi †¶Îdj = . ΔEFG Gi †¶Îdj

$\Rightarrow \frac{1}{2}$ mvgvšwiK $ABCD$ Gi †¶Îdj = $\frac{1}{2}$ mvgvšwiK $EFGH$ Gi †¶Îdj

$\therefore$ mvgvšwiK $ABCD$ Gi †¶Îdj = mvgvšwiK $EFGH$ (cgwYZ|

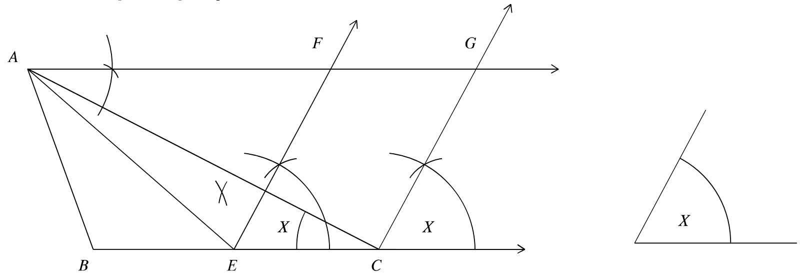
Dccv`¨ 3 (cx_v‡Mviv‡mi Dccv`¨)

mg‡KvYx wÎf‡Ri AwZf‡Ri Dci AwW¼Z eM‡¶‡Îi †¶Îdj Aci `B evûi Dci AwW¼Z eM‡¶‡Î؇qi †¶Îd‡ji mgwói mgvb|

we‡kl wbePb : g‡b Kwi, ABC mg‡KvYx wÎf‡Ri $\angle ACB$ mg‡KvY Ges AB AwZfR| cgvY Ki‡Z n‡e †h, $AB^2 = BC^2 + AC^2$. A¼b : AB, AC Ges BC evûi Dci h_vµ‡g $ABED$, $ACGF$ Ges $BCHK$ eM‡¶Î A¼b Kwi| C we›` w`‡q AD ev BE †iLvi mgvšivj CL †iLv AwK| g‡bKwi, Zv AB †K M we›`‡Z Ges $DE$ †K $L$ we›`‡Z †Q` K‡i| C I D Ges B I F †hvM Kwi| cgvY :	
avc	h_v_Zv
(1) ΔCAD I ΔFAB G $CA = AF$, $AD = AB$ Ges Ašf³ $\angle CAD = \angle CAB + \angle BAD$ $= \angle CAB + \angle CAF$ = Ašf³ $\angle BAF$ AZGe, $\Delta CAD \cong \Delta FAB$	[$\angle BAD = \angle CAF = 1$ mg‡KvY] [evû-†KvY-evû Dccv`¨]
(2) wÎfR‡¶Î CAD Ges AvqZ‡¶Î $ADLM$ GKB fwg AD Gi Dci Ges AD I CL mgvšivj †iLv؇qi g‡a¨ Aew¯Z| mZivs, AvqZ‡¶Î $ADLM = 2$ (wÎfR‡¶Î CAD)	 [Dccv`¨ 1]
(3) wÎfR‡¶Î BAF Ges eM‡¶Î $ACGF$ GKB fwg AF Gi Dci Ges AF I BG mgvšivj †iLv؇qi g‡a¨ Aew¯Z| mZivs, eM‡¶Î $ACGF = 2$ (wÎfR‡¶Î FAB) $= 2$ (wÎfR‡¶Î CAD)	 [Dccv`¨ 1]
(4) AvqZ‡¶Î $ADLM$ = eM‡¶Î $ACGF$	
(5) Abiƒcfv‡e C, E I A, K †hvM K‡i cgvY Kiv hvq †h, AvqZ‡¶Î $BELM$ = eM‡¶Î $BCHK$	
(6) AvqZ‡¶Î ($ADLM + BELM$) = eM‡¶Î $ACGF$ + eM‡¶Î $BCHK$ ev, eM‡¶Î $ABED$ = eM‡¶Î $ACGF$ + eM‡¶Î $BCHK$ A_©vr, $AB^2 = BC^2 + AC^2$ [cgwYZ]	[(2) Ges (3) †_‡K] [(4) Ges (5) †_‡K]

m¤úv`¨ 1

Ggb GKwU mvgvšwiK AvK‡Z n‡e, hvi GKwU †KvY GKwU wbw`ó †Kv‡Yi mgvb Ges hv Øviv mxgvex× †¶Î GKwU wÎfR‡¶‡Îi †¶Îd‡ji mgvb|



g‡b Kwi, ABC GKwU wbw`ó wÎfR‡¶Î Ges $\angle x$ GKwU wbw`ó †KvY| Gi/c mvgvšwiK AvK‡Z n‡e, hvi GKwU †KvY $\angle x$ Gi mgvb Ges hv Øviv mxgvex× †¶‡Îi †¶Îdj Δ †¶Î ABC Gi †¶Îd‡ji mgvb|

A¼b : BC evû‡K E we›`‡Z mgwØLwÊ Kwi| $EC$ †iLvs‡ki $E$ we›`‡Z $\angle x$ Gi mgvb $\angle CEF$ AwwK| A we›` w`‡q BC evûi mgvšivj AG iwk¥ Uwb Ges g‡b Kwi Zv EF iwk¥‡K F we›`‡Z †Q` K‡i| C we›` w`‡q EF †iLvs‡ki mgvšivj CG iwk¥ Uwb Ges g‡b Kwi Zv AG iwk¥‡K G we›`‡Z †Q` K‡i| Zvn‡j, $ECGF$ B DwÏó mvgvšwiK|

cgvY : A, E †hvM Kwi|

GLb, Δ †¶Î ABE Gi †¶Îdj = Δ †¶Î AEC Gi †¶Îdj [†h‡nZ fwg BE = fwg EC Ges Df‡qi GKB D"PZv]

$\therefore$ Δ †¶Î ABC Gi †¶Îdj = 2 (Δ †¶Î AEC Gi †¶Îdj)

Avevi, mvgvšwiK †¶Î $ECGF$ Gi †¶Îdj 2(Δ †¶Î AEC Gi †¶Îdj) [†h‡nZ, Df‡q GKB fwg EC Gi Dci Aew¯Z Ges $EC \parallel AG$

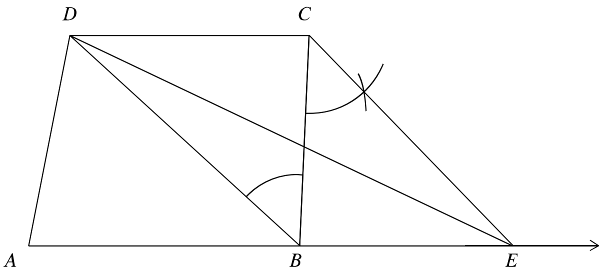
$\therefore$ mvgvšwiK †¶Î $ECGF$ Gi †¶Îdj = Δ †¶Î ABC Gi †¶Îdj

Avevi, . $\angle CEF$ = $\angle x$ [†h‡nZ $EF \parallel CG$, A¼b Abmv‡i]

$\therefore$ mvgvšwiK $ECGF$ B wb‡Yq mvgvšwiK|

m¤úv`¨ 2

Ggb GKwU wÎfR AvK‡Z n‡e hv Øviv mxgvex× †¶‡Îi †¶Îdj GKwU wbw`ó PZfR‡¶‡Îi †¶Îd‡ji mgvb|



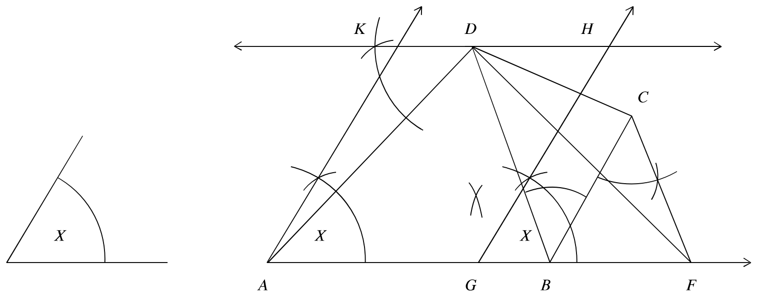
g‡b Kwi, $ABCD$ GKwU PZfR‡¶Î| Gi/c GKwU wÎfR AvK‡Z n‡e hv Øviv mxgvex× †¶‡Îi †¶Îdj $ABCD$ PZfR‡¶‡Îi †¶Îd‡ji mgvb|

A¼b : D, B †hvM Kwi| C we›` w`‡q $CE \parallel DB$ Uwb| g‡b Kwi, Zv AB evûi ewaZvsk‡K E we›`‡Z †Q` K‡i| D, E †hvM Kwi|
Zvn‡j, ΔDAE B DwÏó wÎfR|

cgvY : BD fwgi Dci ΔBDC I ΔBDE Aew¯Z Ges $DB \parallel CE$ [A¼b Abmv‡i]
∴ Δ †¶Î BDC Gi †¶Îdj = Δ †¶Î BDE Gi †¶Îdj
∴ Δ †¶Î BDC Gi †¶Îdj + Δ †¶Î ABD Gi †¶Îdj = Δ †¶Î BDE Gi †¶Îdj + Δ †¶Î ABD Gi †¶Îdj|
∴ Δ PZfR‡¶Î $ABCD$ Gi †¶Îdj = Δ †¶Î ADE Gi †¶Îdj|
AZGe, ΔADE B wb‡Yq wÎfR|

we‡kl `óe¨ : Dc‡ii c×wZi mvnv‡h¨ wbw`ó PZfR‡¶‡Îi †¶Îd‡ji mgvb †¶Îdj wewkó AmsL¨ wÎfR‡¶Î AvKv hv‡e|

m¤úv`¨ 3
Ggb GKwU mvgvšwiK AvK‡Z n‡e hvi GKwU †KvY †`Iqv Av‡Q Ges Zv Øviv mxgve× †¶Î GKwU wbw`ó PZfR‡¶‡Îi †¶Îd‡ji mgvb|



g‡b Kwi, $ABCD$ GKwU wbw`ó PZfR‡¶Î Ges $\angle x$ GKwU wbw`ó †KvY| Gi/c GKwU mvgvšwiK AvK‡Z n‡e hvi GKwU †KvY c`Ë $\angle x$ Gi mgvb Ges mxgve× †¶‡Îi †¶Îdj $ABCD$ †¶‡Îi †¶Îd‡ji mgvb|
A¼b : B, D †hvM Kwi| C we›` w`‡q $CF \parallel DB$ Uwb Ges g‡b Kwi, CE, AB evûi ewaZvsk‡K F we›`‡Z †Q` K‡i| AF †iLvs‡ki ga¨we›` $G$ wbYq Kwi| $AG$ †iLvs‡ki $A$ we›`‡Z $\angle x$ Gi mgvb $\angle GAK$ AwK Ges G we›` w`‡q $GH \parallel AK$ Uwb| D we›` w`‡q $KDH \parallel AG$ Uwb Ges g‡b Kwi, Zv AK I GH †K h_vµ‡g K I H we›`‡Z †Q` K‡i|
Zvn‡j, $AGHK$ B DwÏó mvgvšwiK|
cgvY : D, F †hvM Kwi| $AGHK$ GKwU mvgvšwiK [A¼b Abmv‡i]
†hLv‡b, $\angle GAK = \angle x$ Avevi, Δ †¶Î DAF Gi †¶Îdj = PZfR‡¶Î $ABCD$ Gi †¶Îdj Ges mvgvšwiK †¶Î $AGHK$ Gi †¶Îdj = wÎfR‡¶Î DAF Gi †¶Îdj|
AZGe, $AGHK$ B wb‡Yq mvgvšwiK|

Abkxjbx 15

1| wÎf‡Ri wZbwU evûi ˆ`N©¨ †`Iqv Av‡Q; wb‡Pi †Kvb †¶‡Î mg‡KvYx wÎfR A¼b m¤¢e bq?

K. 3 cm, 4 cm, 5 cm   L. 6 cm, 8 cm, 10 cm

M. 5 cm, 7 cm, 9 cm   N. 5 cm, 12 cm, 13 cm

2| wb‡Pi Z_¨¸‡jv j¶ Ki:

i c‡Z¨K mxgve× mgZj †¶‡Îi wbw`©ó †¶Îdj i‡q‡Q

ii `BwU wÎfR †¶‡Îi †¶Îdj mgvb n‡jB wÎfR `BwU memg

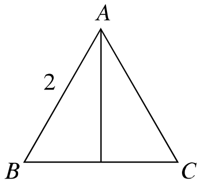
iii `BwU wÎfR memg n‡j Zv‡`i †¶Îdj mgvb

wb‡Pi †KvbwU mwVK ?

K. i I ii   L. i I iii

M. ii I iii   N. i, ii I iii

wb‡Pi wP‡Î, ΔABC mgevû, $AD \perp BC$ Ges $AB=2$ Z‡_¨i wfwË‡Z (3 I 4) bs c‡ki DËi `vI :



3| BD = KZ ?

K. 1 L. $\sqrt{2}$

M. 2 N. 4

4| wÎfRwUi D”PZv KZ ?

K. $\frac{4}{\sqrt{3}}$ e. GKK   L. $\sqrt{3}$ e. GKK

M. $\frac{2}{\sqrt{3}}$ e. GKK   N. $2\sqrt{3}$ e. GKK

5| cgvY Ki †h, mvgvšwi‡Ki KYØq mvgvšwiK‡¶ÎwU‡K PviwU mgvb wÎfR‡¶‡Î wef³ K‡i|

6| cgvY Ki †h, †Kv‡bv eM©‡¶Î Zvi K‡Yi Dci Aw¼Z eM©‡¶‡Îi A‡aK|

7| cgvY Ki †h, wÎf‡Ri †h‡Kv‡bv ga¨gv wÎfR‡¶ÎwU‡K mgvb †¶Îdj wewkó `BwU wÎfR‡¶‡Î wef³ K‡i|

8| GKwU mvgvšwiK‡¶‡Îi Ges mgvb †¶Îdjwewkó GKwU AvqZ‡¶Î GKB fwgi Dci Ges Gi GKB cv‡k Aew¯'Z| †`LvI †h, mvgvšwiK‡¶ÎwUi cwimxgv AvqZ‡¶ÎwUi cwimxgv A‡c¶v enËi|

9| ΔABC Gi AB I AC evû؇qi ga¨we›`y h_vµ‡g X I Y.

cgvY Ki †h, Δ †¶Î AXY Gi †¶Îdj = $\frac{1}{4}$ (Δ †¶Î ABC GiK †¶Îdj)|

10| wP‡Î, $ABCD$ GKwU UªvcwRqvg| Gi AB I CD evû `BwU mgvšivj| UªvcwRqvg‡¶Î $ABCD$ Gi †¶Îdj wbYq Ki|

11| mvgvšwiK $ABCD$ Gi Af¨š‡i P †h‡Kv‡bv GKwU we›`| cgvY Ki †h, ∆ †¶Î PAB Gi †¶Îdj + ∆ †¶Î PCD Gi †¶Îdj = $\frac{1}{2}$ (mvgvšwiK‡¶Î $ABCD$ Gi †¶Îdj)

12| ΔABC G BC fwgi mgvšivj †h‡Kv‡bv mij‡iLv AB I AC evû‡K h_vµ‡g D I F we›`‡Z †Q` K‡i| cgvY Ki †h, ∆ †¶Î DBC = ∆ †¶Î EBC Ges ∆ †¶Î DBF = ∆ †¶Î CDE .

13| ABC wÎf‡Ri $\angle A$ = GK mg‡KvY| D, AC Gi Dci¯ GKwU we›`|

cgvY Ki †h, $BC^2 + AD^2 = BD^2 + AC^2$.

14| ABC GKwU mgevû wÎfR Ges AD, BC Gi Ici j¤|

‡`LvI †h, $4AD^2 = 3AB^2$.

15| ABC GKwU mgwØevû mg‡KvYx wÎfR| BC Gi AwZfR Ges P, BC Gi Ici †h‡Kv‡bv we›`|

cgvY Ki †h, $PB^2 + PC^2 = 2PA^2$.

16| ΔABC Gi $\angle C$ ¯’j‡KvY ; AD, BC Gi

Ici j¤| †`LvI †h,

$AB^2 = AC^2 + BC^2 + 2BC.CD.$

17| ΔABC Gi $\angle C$ m²‡KvY ; AD, BC Gi

Ici j¤| †`LvI †h,

$AB^2 = AC^2 + BC^2 - 2BC.CD.$

18| ΔABC Gi AD GKwU ga¨gv| †`LvI †h,

$AB^2 + AC^2 = 2(BD^2 + AD^2)$

lô`k Aa¨vq

cwiwgwZ

(Mensuration)

e¨enwiK c‡qvR‡b, †iLvi ˆ`N¨, Z‡ji †¶Îdj, Nbe¯‘i AvqZb BZ¨vw` cwigvc Kiv nq| G iKg †h‡Kv‡bv ivwk cwigv‡ci †¶‡Î GKB RvZxq wbw`©ó cwigv‡Yi GKwU ivwk‡K GKK wnmv‡e MnY Kiv nq| cwigvcKZ ivwk Ges G iƒc wbaviZ GK‡Ki AbcvZB ivwkwUi cwigvc wbaviY K‡i|

A_vr cwigvc = $\frac{\text{cwigvcKZ ivwk}}{\text{GKK ivwk}}$|

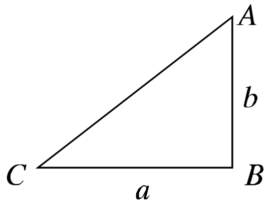
wbaviZ GKK m¤ú‡K c‡Z¨K cwigvc GKwU msL¨v hv cwigvcKZ ivwkwUi GKK ivwki KZ¸Y Zv wb‡`k K‡i| †hgb, †eÂwU 5 wgUvi j¤v| GLv‡b wgUvi GKwU wbw`©ó ˆ`N¨ hv‡K GKK wnmv‡e aiv n‡q‡Q Ges hvi Zjbvq †eÂwU 5 ¸Y j¤v|

Aa¨vq †k‡l wk¶v_xiv –

- wÎfR‡¶Î I PZfR‡¶‡Îi †¶Îd‡ji m¤Î c‡qvM K‡i eûfR‡¶‡Îi †¶Îdj wbYq Ges GZ`m¤úwKZ mgm¨v mgvavb Ki‡Z cvi‡e|
- e‡Ëi cwiwa I eËvs‡ki ˆ`N¨ wbYq Ki‡Z cvi‡e|
- e‡Ëi †¶Îdj wbYq Ki‡Z cvi‡e|
- eË‡¶Î I Zvi Askwe‡k‡li †¶Îdj wbYq K‡i GZ`m¤úwKZ mgm¨v mgvavb Ki‡Z cvi‡e|
- AvqZKvi Nbe¯, NbK I †ej‡bi †¶Îdj cwigvc Ki‡Z cvi‡e Ges G m¤úwKZ mgm¨v mgvavb Ki‡Z cvi‡e|
- mlg I †hŠwMK Nbe¯i cô‡Zi †¶Îdj cwigvc Ki‡Z cvi‡e|

16·1 wÎfR‡¶‡Îi †¶Îdj

c‡ei †kwY‡Z Avgiv †R‡bwQ, wÎfR‡¶‡Îi †¶Îdj = $\frac{1}{2}\times$ fwg $\times$ D"PZv



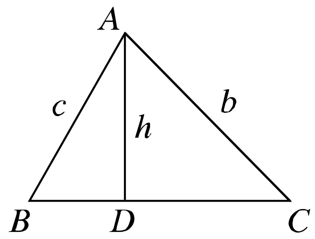
(1) mg‡KvYx wÎfR : g‡b Kwi, ABC mg‡KvYx wÎf‡Ri mg‡KvY msjMœ evûØq h_vµ‡g $BC = a$ Ges $AB = b$| BC †K fwg Ges AB †K D"PZv we‡ePbv Ki‡j,

ΔABC Gi †¶Îdj $= \frac{1}{2} \times$ fwg $\times$ D”PZv

$= \frac{1}{2}ab$

(2) wÎfR‡¶‡Îi `B evû I Zv‡`i Aš—f©³ †KvY †`Iqv Av‡Q| g‡b Kwi, ABC wÎf‡Ri evûØq $BC = a$, $CA = b$, $AB = c$ | A †_‡K BC evûi Dci AD j¤ AvwK|

awi, D”PZv $AD = h$ |



†KvY C we‡ePbv Ki‡j cvB, $\frac{AD}{CA} = \sin C$

ev, $\frac{h}{b} = \sin C$ ev, $h = b\sin C$

Δ †¶Î ABC Gi †¶Îdj $= \frac{1}{2}BC \times AD$

$= \frac{1}{2}a \times b\sin C$

$= \frac{1}{2}ab\sin C$

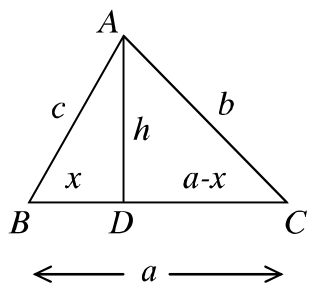
Abiƒcfv‡e Δ †¶Î ABC Gi †¶Îdj $= \frac{1}{2}bc\sin A$

$= \frac{1}{2}ca\sin B$

(3) wÎf‡Ri wZbevû †`Iqv Av‡Q| g‡b Kwi, ΔABC Gi $BC = a$, $CA = b$ Ges $AB = c$ |

$\therefore$ Gi cwimxgv $2s = a + b + c$

$AD \perp BC$ AvwK|



awi, $BD = x$ Zvn‡j, $CD = a - x$

ΔABD Ges ΔACD mg‡KvYx

$\therefore AD^2 = AB^2 - BD^2$ Ges $AD^2 = AC^2 - CD^2$

$\therefore\ AB^2 - BD^2 = AC^2 - CD^2$

ev, $c^2 - x^2 = b^2 - (a - x)^2$

ev, $c^2 - x^2 = b^2 - a^2 + 2ax - x^2$

ev, $2ax = c^2 + a^2 - b^2$

$\therefore\ x = \frac{c^2 + a^2 - b^2}{2a}$

Avevi, $AD^2 = c^2 - x^2$

$$= c^2 - \left(\frac{c^2+a^2-b^2}{2a}\right)^2$$

$$= \left(c + \frac{c^2+a^2-b^2}{2a}\right)\left(c - \frac{c^2+a^2-b^2}{2a}\right)$$

$$= \frac{2ac+c^2+a^2-b^2}{2a} \cdot \frac{2ac-c^2-a^2+b^2}{2a}$$

$$= \frac{\{(c+a)^2-b^2\}\{b^2-(c-a)^2\}}{4a^2}$$

$$= \frac{(a+b+c)(a+b+c-2b)(a+b+c-2a)(a+b+c-2c)}{4a^2}$$

$$= \frac{2s(2s-2b)(2s-2a)(2s-2c)}{4a^2}$$

$$= \frac{4s(s-a)(s-b)(s-c)}{a^2}$$

$\therefore\ AD = \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}$

∆ †¶Î ABC Gi †¶Îdj $= \frac{1}{2} BC \cdot AD$

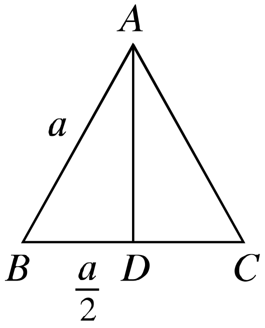
$$= \frac{1}{2} \cdot a \cdot \frac{2}{a}\sqrt{s(s-a)(s-b)(b-c)}$$

$$= \sqrt{s(s-a)(s-b)(b-c)}$$

(4) mgevû wÎfR :

g‡b Kwi, ABC mgevû wÎf‡Ri c‡Z¨K evûi ˆ`N¨ a

$AD \perp BC$ AwK| $\therefore\ BD = CD = \frac{a}{2}$



ΔABD mg‡KvYx

$\therefore\ BD^2 + AD^2 = AB^2$

ev, $AD^2 = AB^2 - BD^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$

$\therefore\ AD = \frac{\sqrt{3}a}{2}$

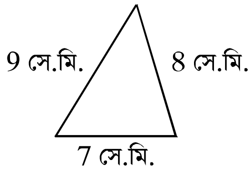
∆ †¶Î ABC Gi †¶Îdj $= \frac{1}{2} \cdot BC \cdot AD$

$$= \frac{1}{2} \cdot a \cdot \frac{\sqrt{3}a}{2}$$ ev, $\frac{\sqrt{3}}{4}a^2$

D`vniY 3| GKwU wÎf‡Ri wZbwU evûi ˆ`N©¨ h_vµ‡g 7 †m.wg., 8 †m.wg. I 9 †m.wg.| Gi †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, wÎfRwUi evû¸‡jvi ˆ`N©¨ h_vµ‡g $a = 7$ †m.wg., $b = 8$ †m.wg. Ges $c = 9$ †m.wg.

$\therefore$ Aacwimxgv $s = \frac{a+b+c}{2} = \frac{7+8+9}{2}$ †m.wg. = 12 †m.wg.



$\therefore$ Gi †¶Îdj $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-7)(12-8)(12-9)}$ eM †m.wg.

$= \sqrt{12\times5\times6\times7}$ eM †m.wg. = $50\cdot2$ eM †m.wg.

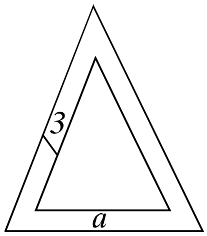
$\therefore$ wÎfRwU †¶Îdj $50\cdot2$ eM †m.wg. (cÖvq)|

D`vniY 4| GKwU mgevû wÎf‡Ri c‡Z¨K evûi ˆ`N©¨ 1 wgUvi evov‡j †¶Îdj $3\sqrt{3}$ eMwgUvi †e‡o hvq| wÎfRwUi evûi ˆ`N©¨ wbYq Ki|

mgvavb : g‡b Kwi, mgevû wÎf‡Ri c‡Z¨K evûi ˆ`N©¨ a wgUvi|

$\therefore$ Gi †¶Îdj $= \frac{\sqrt{3}}{4}a^2$ eMwgUvi|

wÎfRwUi c‡Z¨K evûi ˆ`N©¨ 1 wgUvi evov‡j wÎfRwUi †¶Îdj $= \frac{\sqrt{3}}{4}(a+1)^2$ eMwgUvi|



cÖkœvbymv‡i, $\frac{\sqrt{3}}{4}(a+1)^2 - \frac{\sqrt{3}}{4}a^2 = 3\sqrt{3}$

ev, $(a+1)^2 - a^2 = 12$; [$\frac{\sqrt{3}}{4}$ Øviv fvM K‡i]

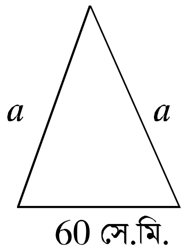
ev, $a^2 + 2a + 1 - a^2 = 12$ ev, $2a = 11$ ev, $a = 5\cdot5$

wb‡Yq evûi ˆ`N©¨ $5\cdot5$ wgUvi|

D`vniY 5| GKwU mgwØevû wÎf‡Ri fwgi ˆ`N©¨ 60 †m.wg.| Gi †¶Îdj 1200 eM †m.wg. n‡j, mgvb mgvb evûi ˆ`N©¨ wbYq Ki|

mgvavb : g‡b Kwi, mgwØevû wÎf‡Ri fwg $b = 60$ †m.wg. Ges mgvb mgvb evûi ˆ`N©¨ a |

$\therefore$ Gi †¶Îdj $= \frac{b}{4}\sqrt{4a^2 - b^2}$



cÖkœvbymv‡i, $\frac{b}{4}\sqrt{4a^2 - b^2} = 1200$

ev, $\frac{60}{4}\sqrt{4a^2 - (60)^2} = 1200$

ev, $15\sqrt{4a^2 - 3600} = 1200$

ev, $\sqrt{4a^2 - 3600} = 80$

ev, $4a^2 - 3600 = 6400$; eM K‡i

ev, $4a^2 = 10000$

ev, $a^2 = 2500$

$\therefore\ a = 50$

$\therefore$ wÎfRwUi mgvb evûi ˆ`N¨ 50 †m.wg.|

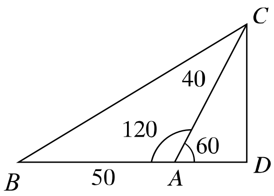
D`vniY 6| GKwU wbw`ó ¯’vb †_‡K `BwU iv¯Ív $120^o$ †Kv‡Y P‡j †M‡Q| `BRb †jvK H wbw`ó ¯’vb †_‡K h_vµ‡g NÈvq 10 wK‡jvwgUvi I NÈvq 8 wK‡jvwgUvi †e‡M wecixZ w`‡K iIbv n‡jv| 5 NÈv c‡i Zv‡`i g‡a¨ miVmwi `iZ wbYq Ki|

mgvavb : g‡b Kwi, A ¯’vb †_‡K `BRb †jvK h_vµ‡g NÈvq 10 wK‡jvwgUvi I NÈvq 8 wK‡jvwgUvi †e‡M iIbv n‡q 5 NÈv ci $B$ I $C$ ¯’v‡b †cŠQj| Zvn‡j, 5 NÈv ci Zv‡`i g‡a¨ miVmwi `iZ n‡e BC.

C †_‡K BA Gi ewaZvs‡ki Dci CD j¤ UwB|

$\therefore\ AB = 5\times 10$ wK‡jvwgUvi = 50 wK‡jvwgUvi, $AC = 5\times 8$ wK‡jvwgUvi = 40 wK‡jvwgUvi

Ges $\angle BAC = 120^o$



$\therefore\ \angle DAC = 180^o - 120^o = 60^o$

ACD mg‡KvYx

$\therefore\ \dfrac{CD}{AC} = \sin 60^o$ ev, $CD = AC\sin 60^o = 40\times\dfrac{\sqrt{3}}{2} = 20\sqrt{3}$

Ges $\dfrac{AD}{AC} = \cos 60^o$ ev, $AD = AC\cos 60^o = 40\times\dfrac{1}{2} = 20$

Avevi, ΔBCD mg‡KvYx †_‡K cvB,

$BC^2 = BD^2 + CD^2 = (BA + AD)^2 + CD^2$

$= (50+20)^2 + (20\sqrt{3})^2 = 4900 + 1200 = 6100$

$\therefore\ BC = 78\cdot 1$ (cÖvq)

wb‡Yq `iZ $78\cdot 1$ wK‡jvwgUvi (cÖvq)

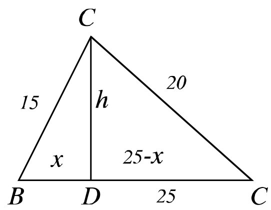
D`vniY 7| GKwU wÎf‡Ri evû¸‡jvi ˆ`N¨ h_vµ‡g 25 GKK, 20 GKK I 15 GKK| enËi evûi wecixZ kxl©we›`y †_‡K Aw¼Z j¤ wÎfRwU‡K †h `BwU wÎf‡R wef³ K‡i Zv‡`i †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, ABC wÎf‡Ri $BC = 25$ GKK, $AC = 20$ GKK, $AB = 15$ GKK|

A kxl©we›`y †_‡K BC evûi Dci Aw¼Z j¤ AD wÎfR‡¶ÎwU‡K ΔABD I ΔACD †¶‡Î wef³ K‡i|

awi, $BD = x$ Ges $AD = h$

$\therefore\ CD = BC - BD = 25 - x$



ΔABD mg‡KvYx -G

$BD^2 + AD^2 = AB^2$ ev, $x^2 + h^2 = (15)^2$

$\therefore\ x^2 + h^2 = 225..........(i)$

Ges ΔACD mg‡KvYx

$CD^2 + AD^2 = AC^2$ ev, $(25 - x)^2 + h^2 = (20)^2$

ev, $625 - 50x + x^2 + h^2 = 400$

ev, $625 - 50x + 225 = 400$; mgxKiY (i) Gi mvnv‡h¨

ev, $50x = 450 \quad \therefore\ x = 9$

mgxKiY (i) G x Gi gvb emv‡q cvB,

$81 + h^2 = 225$ ev, $h^2 = 144 \quad \therefore\ h = 12$

Δ †¶Î ABD Gi †¶Îdj = $\frac{1}{2} BD \cdot AD = \frac{1}{2} \times 9 \times 12$ eMGKK = 36 eMGKK

Ges Δ †¶Î ACD Gi †¶Îdj = $\frac{1}{2} BD \cdot AD = \frac{1}{2}(25 - 9) \times 12$ eMGKK

= $\frac{1}{2} \times 16 \times 12$ eMGKK = 96 eMGKK

wb‡Yq †¶Îdj 36 eMGKK Ges 96 eMGKK|

Abkxjbx 16·1

1| GKwU mg‡KvYx wÎf‡Ri AwZfR 25 wgUvi| Gi GKwU evû AciwUi $\frac{3}{4}$ Ask n‡j, evû `BwUi ˆ`N¨ wbYq Ki|

2| 20 wgUvi j¤v GKwU †`Iqv‡ji mv‡_ Lvovfv‡e Av‡Q| gBwUi †Mvov †`Iqvj †_‡K KZ `‡i miv‡j Ic‡ii cvš— 4 wgUvi wb‡P bvg‡e|

3| GKU mgwØevû wÎf‡Ri cwimxgv 16 wgUvi| Gi mgvb mgvb evûi ˆ`N¨ fwgi $\frac{5}{6}$ Ask n‡j, wÎfRwUi †¶Îdj wbYq Ki|

4| GKwU wÎf‡Ri `BwU evûi ˆ`N¨ 25 †m.wg., 27 †m.wg. Ges cwimxgv 84 †m.wg.| wÎfRwUi †¶Îdj wbYq Ki|

5| GKwU mgevû wÎf‡Ri c‡Z¨K evûi ˆ`N¨ 2 wgUvi evov‡j Gi †¶Îdj $6\sqrt{3}$ eMwgUvi †e‡o hvq| wÎfRwUi evûi ˆ`N¨ wbYq Ki|

6| GKwU wÎf‡Ri `B evûi ˆ`N¨ h_vµ‡g 26 wgUvi, 28 wgUvi Ges †¶Îdj 182 eMwgUvi n‡j, evûØ‡qi Ašf³ †KvY wbYq Ki|

7| GKwU mg‡KvYx wÎf‡Ri j¤ fwgi $\frac{11}{12}$ Ask †_‡K 6 †m.wg. Kg Ges AwZfR fwgi $\frac{4}{3}$ Ask †_‡K 3 †m.wg. Kg| wÎfRwUi fwgi ˆ`N¨ wbYq Ki|

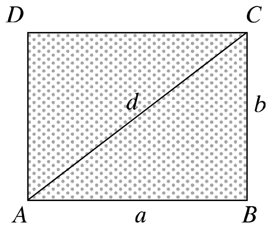
8| GKwU mgwØevû wÎf‡Ri mgvb mgvb evûi ˆ`N¨ 10 wgUvi Ges †¶Îdj 48 eMwgUvi n‡j, fwgi ˆ`N¨ wbYq Ki|

9| GKwU wbw`ó ¯’vb †_‡K `BwU iv¯Ív ci¯úi 135° †KvY K‡i `Bw`‡K P‡j †M‡Q| `BRb †jvK H wbw`ó ¯’vb †_‡K h_vµ‡g NÈvq 7 wK‡jwgUvi I NÈvq 5 wK‡jwgUvi †e‡M wecixZ g‡L iIbv n‡jv| 4 NÈv ci Zv‡`i g‡a¨ mivmwi `iZ wbYq Ki|

10| GKwU mgevû wÎf‡Ri Af¨šÍi¯’ GKwU we›`y †_‡K wZbwUi Ici AwâZ j‡¤^i ˆ`N¨ h_vµ‡g 6 †m.wg., 7 †m.wg. I 8 †m.wg.| wÎfRwUi evûi ˆ`N¨ Ges †¶Îdj wbYq Ki|

16·2 PZzfR‡¶‡Îi †¶Îdj

(1) AvqZ‡¶‡Îi †¶Îdj



g‡b Kwi, $ABCD$ AvqZ‡¶‡Îi ˆ`N¨ $AB = a$

c¯’ $BC = b$ Ges KY $AC = d$

Avgiv Rvwb, AvqZ‡¶‡Îi KY AvqZ‡¶ÎwU‡K

mgvb `BwU wÎfR‡¶‡Î wef³ K‡i|

$\therefore$ AvqZ‡¶Î $ABCD$ Gi †¶Îdj $= 2\times\Delta$ †¶Î ABC Gi †¶Îdj

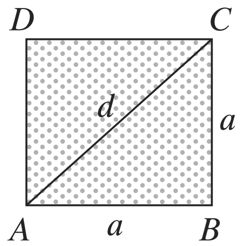
$$= 2\times\frac{1}{2}a\cdot b = ab$$

AvqZ‡¶ÎwUi cwimxgv $s = 2(a+b)$

Ges ΔABC mg‡KvYx

$AC^2 = AB^2 + BC^2$ ev, $d^2 = a^2 + b^2$ $\therefore$ $d = \sqrt{a^2+b^2}$

(2) eM‡¶‡Îi †¶Îdj



g‡b Kwi, $ABCD$ eM‡¶‡Îi cwZ evûi ˆ`N¨ a Ges KY d

AC KY eM‡¶ÎwU‡K mgvb `BwU wÎfR‡¶‡Î wef³ K‡i|

$\therefore$ eM‡¶Î $ABCD$ Gi †¶Îdj $= 2\times\Delta$ †¶Î ABC Gi †¶Îdj

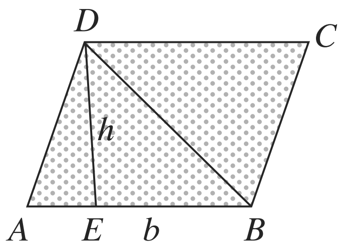
$$= 2\times\frac{1}{2}a\cdot a = a^2$$

j¶ Kwi. eM‡¶‡Îi cwimxgv $s = 4a$

Ges KY $d = \sqrt{a^2+a^2} = \sqrt{2a^2} = \sqrt{2}a$

(3) mvgvšÍiK‡¶‡Îi †¶Îdj

(K) fwg I D”PZv †`Iqv Av‡Q|



g‡b Kwi, $ABCD$ mvgvšÍiK‡¶‡Îi fwg $AB = b$

Ges D”PZv $DE = h$

BD KY mvgvšÍiK‡¶ÎwU‡K mgvb

`BwU wÎfR‡¶‡Î wef³ K‡i|

$\therefore$ mvgvšÍwiK‡¶Î $ABCD$ Gi †¶Îdj $= 2\times\Delta$ †¶Î ABD Gi †¶Îdj

$$= 2\times\frac{1}{2}b\cdot h$$

$$= bh$$

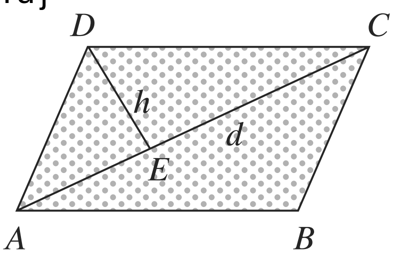
(L) GKwU K‡Yi ^`N¨ Ges H K‡Yi wecixZ †KŠwYK we›` †_‡K D³ K‡Yi Ici Aw¼Z j‡¤i ^`N¨ †`Iqv Av‡Q|

g‡b Kwi, $ABCD$ mvgvšÍwiK‡¶‡Îi KY $AC = d$ Ges Gi wecixZ †KŠwYK we›` $D$ †_‡K $AC$ Gi Dci Aw¼Z j¤ $DE = h$| KY $AC$ mvgvšÍwiK‡¶ÎwU‡K mgvb `BwU wÎfR‡¶‡Î wef³ K‡i|

$\therefore$ mvgvšÍwiK‡¶Î $ABCD$ Gi †¶Îdj $= 2\times\Delta$ †¶Î ACD Gi †¶Îdj

$$= 2\times\frac{1}{2}d\cdot h$$

$$= dh$$



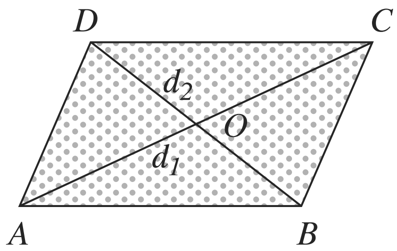
(4) i¤‡mi †¶Îdj

i¤‡mi `BwU KY †`Iqv Av‡Q|

g‡b Kwi, $ABCD$ i¤‡mi KY $AC = d_1$, KY $BD = d$ Ges KYØq ci¯úi O we›`‡Z †Q` K‡i|

KY AC i¤m‡¶ÎwU‡K mgvb `BwU wÎfR‡¶‡Î wef³ K‡i|

Avgiv Rwb, i¤‡mi KYØq ci¯úi‡K mg‡Kv‡Y mgwØLwÊZ K‡i



$\therefore$ ΔACD Gi D"PZv $= \frac{d_2}{2}$

$\therefore$ i¤m $ABCD$ Gi †¶Îdj $= 2\times\Delta$ †¶Î ACD Gi †¶Îdj

$$= 2\times\frac{1}{2}d_1\times\frac{d_2}{2}$$

$$= \frac{1}{2}d_1d_2$$

(5) UwcwRqvg‡¶‡Îi †¶Îdj

UwcwRqvg‡¶‡Îi mgvšÍivj `BwU evû Ges G‡`i ga¨eZx© j¤ `iZ¡ †`Iqv Av‡Q|

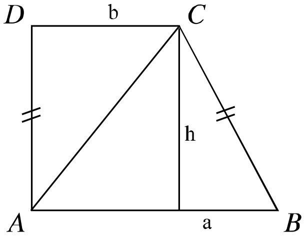
g‡b Kwi, $ABCD$ UwcwRqvg‡¶‡Îi mgvšÍivj evûØ‡qi ^`N¨ h_vµ‡g $AB = a$ GKK, $CD = b$ GKK Ges G‡`i ga¨eZx© `iZ¡ $CE = AF = h$ | AC KY UwcwRqvg $ABCD$ †¶ÎwU‡K ΔABC I ΔACD †¶‡Î wef³ K‡i|

UwcwRqvg‡¶Î $ABCD$ Gi †¶Îdj

$$= \Delta \text{ †¶Î } ABC \text{ Gi †¶Îdj} + \Delta \text{ †¶Î } ACD \text{ Gi †¶Îdj}$$

$$= \frac{1}{2}AB\times CE + \frac{1}{2}CD\times AF$$

$$= \left(\frac{1}{2}ah + \frac{1}{2}bh\right) = \frac{1}{2}h(a+b)$$



D`vniY 1| GKwU AvqZvKvi N‡ii ˆ`N©¨ c‡¯’i $\frac{3}{2}$ ¸Y| Gi †¶Îdj 384 eMwgUvi n‡j, cwimxgv I K‡Yi ˆ`N©¨ wbYq Ki|

mgvavb : g‡b Kwi, AvqZvKvi N‡ii c¯’ x wgUvi|

$\therefore$ N‡ii ˆ`N©¨ $\frac{3x}{2}$ wgUvi

Ges †¶Îdj $\frac{3x}{2} \times x$ ev, $\frac{3x^2}{2}$ eMwgUvi|

cÖkœvbymv‡i, $\frac{3x^2}{2} = 384$ ev, $3x^2 = 768$ ev, $x^2 = 256$ $\therefore$ $x = 16$ wgUvi

$\therefore$ AvqZvKvi NiwUi ˆ`N©¨ = $\frac{3}{2} \times 16$ wgUvi = 24 wgUvi

Ges c¯’ = 16 wgUvi|

$\therefore$ Gi cwimxgv = $2(24+16)$ wgUvi = 80 wgUvi

Ges K‡Yi ˆ`N©¨ = $\sqrt{(24)^2 + (16)^2}$ wgUvi = $\sqrt{832}$ wgUvi = $28 \cdot 84$ wgUvi (cÖvq)

wb‡Yq cwimxgv 80 wgUvi Ges K‡Yi ˆ`N©¨ $28 \cdot 84$ wgUvi (cÖvq)|

D`vniY 2| GKwU AvqZ‡¶‡Îi †¶Îdj 2000 eMwgUvi| hw` Gi ˆ`N©¨ 10 wgUvi Kg nZ Zvn‡j GwU GKwU eM‡¶Î nZ| AvqZ‡¶ÎwUi ˆ`N©¨ I c¯’ wbYq Ki|

mgvavb : g‡b Kwi, AvqZ‡¶ÎwUi ˆ`N©¨ x wgUvi Ges c¯’ y wgUvi|

$\therefore$ AvqZ‡¶ÎwUi †¶Îdj = xy eMwgUvi|

cÖkœvbymv‡i, $xy = 2000................(1)$

Ges $x - 10 = y.............(2)$

mgxKiY (2) †_‡K cvB, $y = x - 10.............(3)$

mgxKiY (1) G $y = x = 10$ ewm‡q cvB

$x(x-10) = 2000$ ev, $x^2 - 10x - 2000 = 0$

ev, $x^2 - 50x + 40x - 2000 = 0$ ev, $(x-50)(x+40) = 0$

$\therefore$ $x - 50 = 0$ A_ev $x + 40 = 0$

ev, $x = 50$ A_ev $x = -40$

wKš‘ ˆ`N©¨ FYvZ¥K n‡Z cv‡i bv|

$\therefore$ $x = 50$

GLb, mgxKiY (3) G x Gi gvb ewm‡q cvB,

$y = 50 - 10 = 40$

$\therefore$ AvqZ‡¶ÎwUi ˆ`N©¨ 50 wgUvi Ges c¯’ 40 wgUvi|

D`vniY 3| eMvKvi GKwU gv‡Vi wfZ‡i Pvwi`‡K 4 wgUvi PIov GKwU iv¯v Av‡Q| hw` iv¯vi †¶Îdj 1 †n±i nq, Z‡e iv¯v ev‡` gv‡Vi wfZ‡ii †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, eMvKvi gv‡Vi ˆ`N¨ x wgUvi|

$\therefore$ Gi †¶Îdj x^2 eMwgUvi|

gv‡Vi wfZ‡i Pvwi`‡K 4 wgUvi PIov GKwU iv¯v Av‡Q|

$\therefore$ iv¯v ev‡` eMvKvi gv‡Vi ˆ`N¨ = $(x-2\times 4)$ ev $(x-8)$ wgUvi|

$\therefore$ iv¯v ev‡` eMvKvi gv‡Vi †¶Îdj = $(x-8)^2$ eMwgUvi

mZivs iv¯vi †¶Îdj = $\{x^2-(x-8)^2\}$ eMwgUvi

Avgiv Rwb, 1 †n±i = 10000 eMwgUvi

ckvbmv‡i, $x^2-(x+8)^2=10000$

ev, $x^2-x^2+16x-64=10000$

ev, $16x=10064$

$\therefore\ x=629$

iv¯vev‡` eMvKvi gv‡Vi †¶Îdj = $(629-8)^2$ eMwgUvi

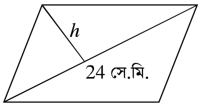
= 385641 eMwgUvi

= $38\cdot 56$ †n±i (cvq)

wb‡Yq †¶Îdj $38\cdot 56$ †n±i (cvq)|

D`vniY 4| GKwU mvgvšwiK‡¶‡Îi †¶Îdj 120 eM †m.wg. Ges GKwU KY 24 †m.wg.| KYwUi wecixZ †KŠwYK we›` †_‡K D³ K‡Yi Ici Aw¼Z j‡¤i ˆ`N¨ wbYq Ki|

mgvavb : g‡b Kwi, mvgvšwiK‡¶‡Îi GKwU KY $d=24$ †m.wg. Ges Gi wecixZ †KŠwYK we›` †_‡K K‡Yi Ici Aw¼Z j‡¤i ˆ`N¨ h †m.wg.|



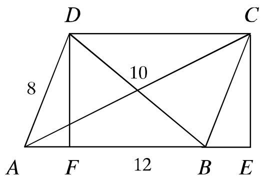
$\therefore$ mvgvšwiK‡¶ÎwUi †¶Îdj = dh eM †m.wg.

ckvbmv‡i, $dh=120$ ev, $h=\frac{120}{d}=\frac{120}{24}=5$

wb‡Yq K‡Yi ˆ`N¨ 5 †m.wg.|

D`vniY 5| GKwU mvgvšwi‡Ki evûi ˆ`N¨ 12 wgUvi I 8 wgUvi Ges ¶`Zg KYwU 10 wgUvi n‡j, Aci KYwUi ˆ`N¨ wbYq Ki|

mgvavb : g‡b Kwi, $ABCD$ mvgvšwi‡Ki $AB=a=12$ wgUvi, $AD=c=8$ wgUvi Ges KY $BD=b=10$ wgUvi| D I C †_‡K AB Gi Dci Ges AB Gi ewaZvs‡ki Dci DF I CE j¤ Uwb| A, C I B, D †hvM Kwi|



ΔABD Gi Aa cwimxgv $s = \dfrac{12+10+8}{2}$ wgUvi = 15 wgUvi

$\therefore$ Δ †¶Î ABD Gi †¶Îdj = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{15(15-12)(15-10)(15-8)}$ eMwgUvi

= $\sqrt{1575}$ eMwgUvi

= $39\cdot 68$ eMwgUvi (cvq)|

Avevi, Δ †¶Î ABD Gi †¶Îdj = $\dfrac{1}{2}AB\times DF$

ev, $39\cdot 68 = \dfrac{1}{2}\times 12\times DF$ ev, $6DF = 39\cdot 68$ $\therefore$ $DF = 6\cdot 61$

GLb, ΔBCE mg‡KvYx

$\therefore$ $BE^2 = BC^2 - CE^2 = AD^2 - DF^2 = 8^2 - (6\cdot 61)^2 = 20\cdot 31$

$\therefore$ $BE = 4\cdot 5$

AZGe, $AE = AB + BE = 12 + 4\cdot 5 = 16\cdot 5$

ΔBCE mg‡KvYx †_‡K cvB,

$AC^2 = AE^2 - CE^2 = (16\cdot 5)^2 - (6\cdot 61)^2 = 315\cdot 94$

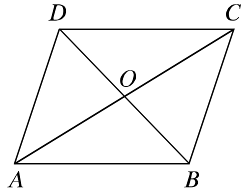
$\therefore$ $AC = 17\cdot 77$ (cvq)

wb‡Yq K‡Yi ˆ`N¨ $17\cdot 77$ wgUvi (cvq)

D`vniY 6| GKwU i¤¢‡mi GKwU KY 10 wgUvi Ges †¶Îdj 120 eMwgUvi n‡j, Aci KY Ges cwimxgv wbYq Ki|

mgvavb : g‡b Kwi, $ABCD$ i¤¢‡mi KY $BD = d_1 = 10$ wgUvi

Ges Aci KY d_2 wgUvi



$\therefore$ i¤¢mwUi †¶Îdj = $\dfrac{1}{2}d_1d_2$ eMwgUvi

cKvbmv‡i, $\dfrac{1}{2}d_1d_2 = 120$ ev, $d_2 = \dfrac{120\times 2}{10} = \dfrac{120\times 2}{10} = 24$

Avgiv Rwb, i¤¢‡mi KYØq ci¯úi‡K mg‡Kv‡Y mgwØLwÊZ K‡i|

$\therefore$ $OD = OB = \dfrac{10}{2}$ wgUvi = 5 wgUvi Ges $OA = OC = \dfrac{24}{2}$ wgUvi = 12 wgUvi

Ges ΔAOD mg‡KvYx -G

$\therefore$ $AD^2 = OA^2 + OD^2 = 5^2 + (12)^2 = 169$ $\therefore$ $AD = 13$

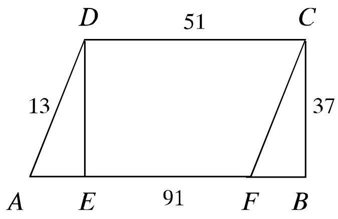
$\therefore$ i¤¢‡mi cwZevûi ˆ`N¨ 13 wgUvi|

$\therefore$ i¤¢‡mi cwimxgv = 4×13 wgUvi = 52 wgUvi|

wb‡Yq K‡Yi ˆ`N¨ 24 wgUvi Ges cwimxgv 52 wgUvi|

D`vniY 7| GKwU UwcwRqv‡gi mgvšÍivj evû؇qi ˆ`N©¨ h_vµ‡g 91 †m.wg. I 51 †m.wg. Ges Aci evû `BwUi ˆ`N©¨ q_vµ‡g 37 †m.wg. I 13 †m.wg.| UwcwRqvgwUi †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, $ABCD$ UwcwRqv‡gi $AB = 91$ †m.wg., $CD = 51$ †m.wg.| D I C †_‡K AB Gi Dci h_vµ‡g DE I CF j¤^ UwbwU|



$\therefore$ $CDEF$ GKwU AvqZ‡¶Î|

$\therefore$ $EF = CD = 51$ †m.wg.|

awi, $AE = x$ Ges $DE = CF = h$

$\therefore$ $BF = AB - AF = 91 - (AE + EF) = 91 - (x + 51) = 40 - x$

ΔADE mg‡KvYx †_‡K cvB,

$AE^2 + DE^2 = AD^2$ ev, $x^2 + h^2 = (13)^2$ ev, $x^2 + h^2 = 169.........(i)$

Avevi, mg‡KvYx Gi †¶‡Î ΔBCF

$BF^2 + CF^2 = BC^2$ ev, $(40 - x)^2 + h^2 = (37)^2$

ev, $1600 - 80x + x^2 + h^2 = 1369$

ev, $1600 - 80x + 169 = 1396$; (1) bs Gi mvnv‡h¨

ev, $1600 + 169 - 1396 = 80x$; mgxKiY (1) Gi gvb ewm‡q cvB,

ev, $80x = 400 \quad \therefore \quad x = 5$

mgxKiY (1) G x Gi gvb ewm‡q cvB,

$5^2 + h^2 = 163$ ev, $h^2 = 169 - 25 = 144 \quad \therefore h = 12$

UwcwRqvg = $ABCD$ Gi †¶Îdj $\frac{1}{2}(AB + CD) \cdot h$

= $\frac{1}{2}(91 + 51) \times 12$ eM †m.wg.

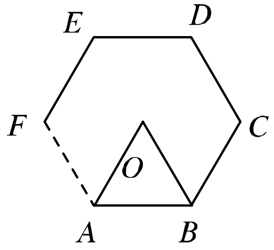
= 852 eM †m.wg.

wb‡Yq †¶Îdj 852 eM †m.wg.|

16·3 mlg eûf‡Ri †¶Îdj :

mlg eûf‡Ri evû¸‡jvi ˆ`N©¨ mgvb| Avevi †KvY¸‡jv mgvb| $n$ msL¨K evûwewkó mlg eûf‡Ri †K›`ª I kxl© we›`¸‡jv †hvM Ki‡j n msL¨K mgwØevû wÎfR Drcb K‡i|

mZivs eûf‡Ri †¶Îdj = n $\times$ GKwU wÎfR †¶‡Îi‡¶Îdj|



$ABCDEF$ .... GKwU mlgevû eûfR, hvi †K›`ª 0.

n msL¨K evû Ges cÖwZ evûi ˆ`N©¨ a.

O, A ; O, B †hvM Kwi|

awi, ΔAOB Gi D”PZv $OA = h$ Ges $\angle OAB = \theta$

mlg eûf‡Ri cÖwZwU kx‡l© Drcb †Kv‡Yi cwigvY = 2θ

$\therefore$ n msL¨K mlg eûf‡Ri kxl© †Kv‡Yi mgwó = $2\theta \cdot n$

mlg eûf‡Ri †K‡›`ª Drcb †Kv‡Yi cwigvY = 4 mg‡KvY

$\therefore$ n msL¨K wÎf‡Ri †Kv‡Yi mgwó $2\theta \cdot (n+4)$ mg‡KvY

ΔOAB Gi wZb‡Kv‡Yi mgwó = 2 mg‡KvY

$\therefore$ Gi|c n msL¨K wÎf‡Ri †Kv‡Yi mgwó $n \cdot 2$ mg‡KvY

$\therefore$ $2\theta \cdot (n+4)$ mg‡KvY = $n \cdot 2$ mg‡KvY

ev, $2\theta \cdot n = (2n-4)$ mg‡KvY

ev, $\theta = \dfrac{2n-4}{2n}$ mg‡KvY

ev, $\theta = \left(1-\dfrac{2}{n}\right)$ mg‡KvY

ev, $\theta = \left(1-\dfrac{2}{n}\right)\times 90^\circ$

$$\therefore \quad \theta = 90^\circ - \frac{180^\circ}{n}$$

GLb, $\tan\theta = \dfrac{h}{\frac{a}{2}} = \dfrac{2h}{a} \quad \therefore h = \dfrac{a}{2}\tan\theta$

ΔOAB Gi †¶Îdj $= \dfrac{1}{2}\, a\, h$

$= \dfrac{1}{2}\, a \times \dfrac{a}{2}\tan\theta$

$= \dfrac{a^2}{4}\tan\left(90^\circ - \dfrac{180^\circ}{n}\right)$

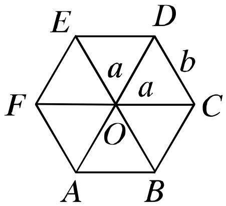
$= \dfrac{a^2}{4}\cot\left(\dfrac{180^\circ}{n}\right)$

$\therefore$ n msL¨K evûwewkó mlg eûf‡Ri †¶Îdj = $\dfrac{a^2}{4}\cot\left(\dfrac{180^\circ}{n}\right)$

D`vniY 8| GKwU mlg cÂf‡Ri cwZevûi ˆ`N¨ 4 †m.wg. n‡j, Gi †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, mlg cÂf‡Ri evûi ˆ`N¨ $a = 4$ †m.wg.

Ges evûi msL¨v $n = 5$



Avgiv Rvwb, mlg eûf‡Ri †¶Îdj = $\dfrac{a^2}{4}\cot\dfrac{180^\circ}{n}$

$\therefore$ mlg cÂf‡Ri †¶Îdj $= \dfrac{4^2}{4}\cot\dfrac{180^\circ}{5}$ eM© †m.wg.

= $4\times\cot 36^\circ$ eM †m.wg.

= $4\times 1\cdot 376$ eM †m.wg. (K¨vjK‡jU‡ii mvnv‡h¨)

= $5\cdot 506$ eM †m.wg. (cÖvq)

wb‡Yq †¶Îdj $5\cdot 506$ eM †m.wg. (cÖvq)

D`vniY 9| GKwU mlg lofy‡Ri †K›` †_‡K †KŠwYK we›`yi `iZ¡ 4 wgUvi n‡j, Gi †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, *ABCDEF* GKwU mlg lofyR| Gi †K›` *O* †_‡K kxl©we›`y¸‡jv †hvM Kiv n‡jv| d‡j 6wU mgvb †¶Îwewkó wÎfyR Drcb nq|

$\therefore\ \angle COD=\dfrac{360^\circ}{6}=60^\circ$

g‡b Kwi, †K›` *O* †_‡K kxl©we›`y¸‡jvi `iZ¡ *a* wgUvi

$\therefore\ \Delta$ †¶‡Î *COD* Gi †¶Îdj = $\dfrac{1}{2}a\cdot a\sin 60^\circ=\dfrac{1}{2}a^2\cdot\dfrac{\sqrt{3}}{2}=\dfrac{\sqrt{3}}{4}a^2$

= $\dfrac{\sqrt{3}}{4}\times 4^2$ eMwgUvi = $4\sqrt{3}$ eMwgUvi

mlg lofy‡Ri †¶‡Îi †¶Îdj

= $6\times\Delta$ †¶Î *COD* Gi †¶Îdj

= $6\times 4\sqrt{3}$ eMwgUvi

= $24\sqrt{3}$ eMwgUvi|

Abkxjbx 16·2

1| GKwU AvqZvKvi †¶‡Îi ˆ`N©¨ we¯Ív‡ii wØ¸Y| Gi †¶Îdj 512 eMwgUvi n‡j, cwimxgv wbYq Ki|

2| GKwU Rwgi ˆ`N©¨ 80 wgUvi Ges cÖ¯' 60 wgUvi| H Rwgi gv‡S GKwU cKzi Lbb Kiv n‡jv| hw` cKz‡ii c‡Z¨K cv‡oi we¯Ívi 4 wgUvi nq, Z‡e cKz‡ii cv‡oi †¶Îdj wbYq Ki|

3| GKwU evMv‡bi ˆ`N©¨ 40 wgUvi Ges cÖ¯' 30 wgUvi| evMv‡bi wfZ‡i mgvb cvowewkó GKwU cKzi Av‡Q| cKz‡ii †¶Îdj evMv‡bi †¶Îd‡ji $\dfrac{1}{2}$ Ask n‡j, cKz‡ii ˆ`N©¨ I cÖ¯' wbYq Ki|

4| GKwU eMvKvi gv‡Vi evB‡i Pviw`‡K 5 wgUvi PIov GKwU iv¯Ív Av‡Q| iv¯Ívi †¶Îdj 500 eMwgUvi n‡j, evMv‡bi †¶Îdj wbYq Ki|

5| GKwU eM©‡¶‡Îi cwimxgv GKwU AvqZ‡¶‡Îi cwimxgvi mgvb| AvqZ‡¶ÎwUi ˆ`N©¨ c‡¯'i wZb¸Y Ges †¶Îdj 768 eMwgUvi| cwZwU 40 †m.wg. eMvKvi cv_i w`‡q eM©‡¶ÎwU evay‡Z †gvU KZwU cv_i jvM‡e|

6| GKwU AvqZvKvi †¶‡Îi †¶Îdj 160 eMwgUvi| hw` Gi ˆ`N¨ 6 wgUvi Kg nq, Z‡e †¶ÎwU eMvKvi nq| AvqZvKvi †¶‡Îi ˆ`N¨ I c¯ wbYq Ki|

7| GKwU mvgvšwi‡Ki fwg D”PZvi $\frac{3}{4}$ Ask Ges †¶Îdj 363 eMBwÂ n‡j, †¶ÎwUi fwg I D”PZv wbYq Ki|

8| GKwU mvgvšwiK‡¶‡Îi †¶Îdj GKwU eM‡¶‡Îi mgvb| mvgvšwi‡Ki fwg 125 wgUvi Ges D”PZv 5 wgUvi n‡j, eM‡¶‡Îi K‡Yi ˆ`N¨ wbYq Ki|

9| GKwU mvgvšwi‡Ki evûi ˆ`N¨ 30 †m.wg. Ges 26 †m.wg.| Gi ¶`Zg KYwU 28 †m.wg. n‡j, Aci K‡Yi ˆ`N¨ wbYq Ki|

10| GKwU i¤‡mi cwimxgv 180 †m.wg. Ges ¶`Zg KYwU 54 †m.wg.| Gi Aci KY Ges †¶Îdj wbYq Ki|

11| GKwU UwcwRqv‡gi mgvšivj evû `BwUi ˆ`‡N¨i Aši 8 †m.wg. Ges Zv‡`i j¤ `iZ 24 †m.wg.| UwcwRqvg `BwUi mgvšivj evûi ˆ`N¨ wbYq Ki|

12| GKwU UwcwRqv‡gi mgvšivj evûØ‡qi ˆ`N¨ h_vµ‡g 31 †m.wg. I 11 †mw›UwgUvi Ges Aci evû `BwUi ˆ`N¨ h_vµ‡g 10 †m.wg. I 12 †m.wg.| Gi †¶Îdj wbYq Ki|

13| GKwU mIg Aóf‡Ri †K›` †_‡K †KŠwYK we›`i `iZ 1·5 wgUvi n‡j, Gi †¶Îdj wbYq Ki|

14| AvqZvKvi GKwU d‡ji evMv‡bi ˆ`N¨ 150 wgUvi Ges c¯ 100 wgUvi| evMvbwU‡K cwiPhv Kivi Rb¨ wVK gvS w`‡q 3 wgUvi PIov ˆ`N¨ I c¯ eivei iv¯v Av‡Q|

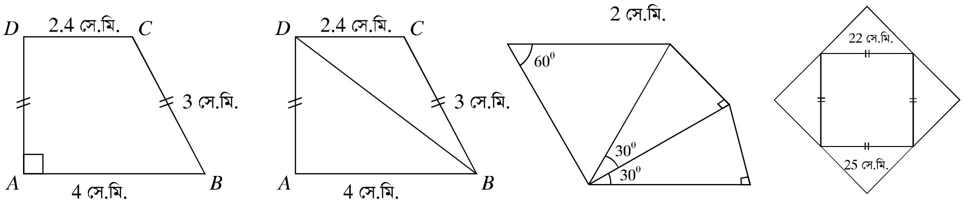
(K) Dc‡ii Z_¨wU wP‡Îi mvnv‡h¨ msw¶ß eYbv `vI|

(L) iv¯vi †¶Îdj wbYq Ki|

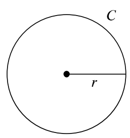
(M) iv¯wU cvKv Ki‡Z 25 †m.wg. ˆ`N¨ Ges 12·5 c¯ wewkó KqwU B‡Ui c‡qvRb n‡e|

15| eûfR wP‡Î Z_¨ Abmv‡i Gi †¶Îdj wbYq Ki|

16| wb‡Pi wP‡Îi Z_¨ †_‡K Gi †¶Îdj wbYq Ki|



6·4 eË msµvš cwigvc



(1) e‡Ëi cwiwa

e‡Ëi ˆ`N¨‡K Zvi cwiwa ejv nq| g‡b Kwi, †Kv‡bv e‡Ëi e¨vmva $r$ n‡j, Gi cwiwa $c = 2\pi r$ †hLv‡b $\pi = 3 \cdot 14159265..........$ GKwU Agj` msL¨v| π Gi Avmj gvb wnmv‡e 3.1416 e¨envi Kiv hvq|

mZivs †Kv‡bv e‡Ëi e¨vmva Rvbv _vK‡j π Gi Avmb gvb e¨envi K‡i e‡Ëi cwiwai Avmb gv‡b wbYq Kiv hvq|

D`vniY 1| GKwU e‡Ëi e¨vm 26 †m.wg. n‡j, Gi †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, e‡Ëi e¨vmva r

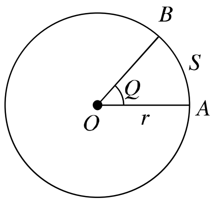
$\therefore$ e‡Ëi e¨vm = $2r$ Ges cwiwa = $2\pi\, r$

ckvbmv‡i, $2r = 26$ ev, $r = \dfrac{26}{2}$ $\quad\therefore\ r = 13$

$\therefore$ e‡Ëi cwiwa = $2\pi\, r$ = $2\times 3{\cdot}1616\times 13$ †m.wg. = $3{\cdot}1616\times 13$ $81{\cdot}64$ †m.wg.(cvq)

wb‡Yq e‡Ëi cwiwa $81{\cdot}64$ †m.wg. (cvq)|

(2) eËvs‡ki ˆ`N¨



g‡b Kwi, O †K›`ªwewkó¨ e‡Ëi e¨vmva $r$ Ges $AB = s$ eËPvc †K‡›` θ^o †KvY Drcb K‡i|

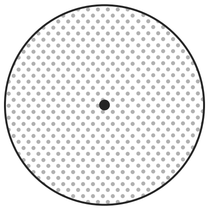
$\therefore$ e‡Ëi cwiwa = $2\pi\, r$

e‡Ëi †K‡›` †gvU Drcb †KvY = $360^o$ Ges Pvc $s$ Øviv †K‡›` Drcb †Kv‡Yi wWMÖx cwigvY θ°

Avgiv Rwb, e‡Ëi †Kv‡bv Pvc Øviv Drcb †K›`¯ †KvY H eËPv‡ci mgvbcvwZK|

$\therefore\ \dfrac{\theta}{360^o} = \dfrac{s}{2\pi\, r}$ ev, $s = \dfrac{\pi\, r\theta}{180}$

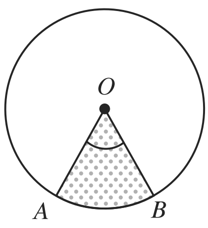
(3) eË‡¶Î I eËKjv †¶Îdj :



†Kv‡bv eË I Gi Af¨šÍ‡i ms‡hv‡M MwVZ mgZ‡ji Dc‡mUwU‡K GKwU eË‡¶Î ejv nq Ges eËwU‡K Gi/c eË‡¶‡Îi mxgv‡iLv ejv nq|

eËKjv : GKwU Pvc I Pv‡ci cvšÍwe›` msjMœ e¨vmva Øviv †ewóZ †¶Î‡K eËKjv ejv nq|

O †K›`ªwewkó e‡Ëi cwiwai Ici $A$ I $B$ `BwU we›` n‡j $\angle AOB$ Gi Af¨šÍi OA I OB e¨vmva Ges AB Pv‡ci ms‡hv‡M MwVZ GKwU eËKjv|



c‡ei †kÖYx‡Z Avgiv wk‡L G‡mwQ †h, e‡Ëi e¨vmva r n‡j, e‡Ëi †¶Îdj = πr^2

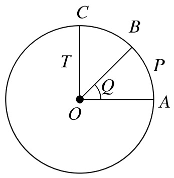
Avgiv Rwb, e‡Ëi †Kv‡bv Pvc Øviv Drcb †K›`¯ †KvY H eËPv‡ci mgvbcvwZK|

mZivs G chv‡q Avgiv ¯xKvi K‡i wb‡Z cvwi †h, GKB e‡Ëi `BwU eËvsk †¶Î Ges Giv †h Pvc `BwUi Dci `Êvqgvb G‡`i cwigvc mgvbcvwZK|

g‡b Kwi, O †K›`ªwewkó e‡Ëi e¨vmva r

AOB eËKjv †¶ÎwU APB Pv‡ci Dci `Êvqgvb, hvi wWMÖx cwigvc θ | OA Dci OC j¤^ Uwb|

$\therefore$ $\dfrac{\text{eËKjv } AOB \text{ Gi †¶Îdj}}{\text{eËKjv } AOC \text{ Gi †¶Îdj}} = \dfrac{\angle AOB \text{ Gi cwigvc}}{\angle ADC \text{ Gi cwigvc}}$



ev, $\dfrac{\text{eËKjv } AOB \text{ Gi †¶Îdj}}{\text{eËKjv } AOC \text{ Gi †¶Îdj}} = \dfrac{\theta}{90^o}$; $[\angle AOC = 90^o]$

ev, eËKjv AOB Gi †¶Îdj = $\dfrac{\theta}{90^o} \times$ eËKjv ADC Gi †¶Îdj

$= \dfrac{\theta}{90^o} \times \dfrac{1}{4} \times$ eË‡¶‡Îi †¶Îdj

$= \dfrac{\theta}{90^o} \times \dfrac{1}{4} \times \pi r^2$

$= \dfrac{\theta}{360^o} \times \pi r^2$

mZivs, eËKjvi †¶Îdj = $\dfrac{\theta}{360^o} \times \pi r^2$

D`vniY 2| GKwU e‡Ëi e¨vmva 8 †m.wg. Ges GKwU eËPvc †K‡›`ª 56^o Drcb Ki‡j, eËPv‡ci ˆ`N¨ Ges eËKjvi †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, e‡Ëi e¨vmva $r = 8$ †m.wg., eËPv‡ci ˆ`N¨ $s$ Ges eËPvc Øviv †K‡›`ª Drcb †KvY $\theta = 560^o$|

Avgiv Rvwb, $s = \dfrac{\pi r\theta}{180^o} = \dfrac{3\cdot1416 \times 8 \times 56}{180}$ †m.wg. = $7\cdot82$ †m.wg. (cvq)

Ges eËvs‡ki †¶Îdj = $\dfrac{\theta}{360^o} \times \pi r^2$

$= \dfrac{56}{350} \times 3\cdot1416 \times 8^2$ eM †m.wg.

= $62\cdot55$ eM †m.wg. (cvq)|

D`vniY 3| GKwU e‡Ëi e¨vm I cwiwai cv_K¨ 90 †m.wg. n‡j, e‡Ëi e¨vm wbYq Ki|

mgvavb : g‡b Kwi, e‡Ëi e¨vmva r

$\therefore$ e‡Ëi e¨vm = $2r$ Ges cwiwa = $2\pi r$

ckvbmv‡i, $2\pi r - 2r = 90$

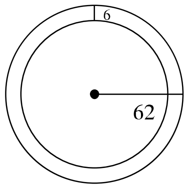
ev, $2r(\pi - 1) = 90$ ev, $r = \dfrac{90}{2(\pi - 1)} = \dfrac{45}{3\cdot1416 - 1} = 21\cdot01$ (cvq)

wb‡Yq e‡Ëi e¨vmva $21\cdot01\pi r$ †m.wg. (cvq)|

D`vniY 4| GKwU eËvKvi gv‡Vi e¨vm 124 wgUvi| gv‡Vi mxgvbv †N‡l 6 wgUvi PIov GKwU iv¯Ív Av‡Q| iv¯Ívi †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, eËvKvi gv‡Vi e¨vmva r Ges iv¯Ívmn eËvKvi gv‡Vi e¨vmva R|

$\therefore\ r=\dfrac{124}{2}$ wgUvi = 62 wgUvi Ges $R=(62+6)$ wgUvi = 68 wgUvi



eËvKvi gv‡Vi †¶Îdj = πr^2

Ges iv¯Ívmn eËvKvi gv‡Vi †¶Îdj = πR^2

$\therefore$ iv¯Ívi †¶Îdj = iv¯Ívmn gv‡Vi †¶Îdj – gv‡Vi †¶Îdj

= $(\pi R^2-\pi r^2)$ = $\pi(R^2-r^2)$

= $3\cdot1416\{(68)^2-(62)^2\}$ eMwgUvi

= $3\cdot1416(4624-3844)$ eMwgUvi

= $3\cdot1416\times780$ eMwgUvi

= $2450\cdot44$ eMwgUvi (cÖvq)

wb‡Yq iv¯Ívi †¶Îdj $2450\cdot44$ eMwgUvi (cÖvq)|

KvR : GKwU e‡Ëi cwiwa 440 wgUvi| H e‡Ë AšÍwjwLZ eM©‡¶‡Îi evûi ˆ`N©¨ wbY©q Ki|

D`vniY 5| GKwU e‡Ëi e¨vmva© 12 †m.wg. Ges eËPv‡ci ˆ`N©¨ 14 †m.wg.| eËPvcwU †K‡›`ª †h †KvY Drcbœ K‡i Zv wbY©q Ki|

mgvavb : g‡b Kwi, e‡Ëi e¨vmva© $r=12$ †m.wg., eËPv‡ci ˆ`N©¨ $s=14$ †m.wg. Ges †K‡›`ª Drcbœ †Kv‡Yi cwigvY θ |

Avgiv Rvwb, $S=\dfrac{\pi r\theta}{180^o}$

ev, $\pi r\theta=180^o\times S$

ev, $\theta=\dfrac{180^o\times S}{\pi r}=\dfrac{180^o\times14}{3\cdot1416\times12}=66\cdot85^o$ (cÖvq)

wb‡Yq †KvY $66\cdot85^o$ (cÖvq)|

D`vniY 6| GKwU PvKvi e¨vm $4\cdot5$ wgUvi| PvKwU 360 wgUvi c_ AwZµg Ki‡Z KZ evi Ni‡e ?

mgvavb : †`Iqv Av‡Q, PvKvi e¨vm $4\cdot5$ wgUvi

$\therefore$ PvKwUi e¨vmva© $r=\dfrac{4\cdot5}{2}$ wgUvi Ges cwiwa = $2\pi r$

g‡b Kwi, PvKwU 360 wgUvi c_ AwZµg Ki‡Z n evi Ni‡e|

cÖkœvbymv‡i, $n\times2\pi r=360$

ev, $n=\dfrac{360}{2\pi r}=\dfrac{360\times2}{2\times3\cdot1416\times4\cdot5}=18$ (cÖvq)

$\therefore$ PvKwU cÖvq 18 evi Ni‡e|

D`vniY 7| 211 wgUvi 20 †m.wg. †h‡Z `BwU PvKv h_vµ‡g 32 Ges 48 evi Ni‡jv| PvKv `BwUi e¨vmv‡ai Aš—i wbYq Ki|

mgvavb : 211 wgUvi 20 †m.wg. = 21120 †m.wg.

g‡b Kwi, PvKv `BwUi e¨vmva h_vµ‡g R I r; †hLv‡b $R > r$.

$\therefore$ PvKv `BwUi cwiwa h_vµ‡g $2\pi R$ I $2\pi r$ Ges e¨vmv‡ai Aš—i $(R-r)$

ckvbmv‡i, $32 \times 2\pi R = 21120$

ev, $R = \dfrac{21120}{32 \times 2\pi} = \dfrac{21120}{32 \times 2 \times 3\cdot1416} = 105\cdot04$ (cvq)

Ges $48 \times 2\pi r = 21120$

ev, $r = \dfrac{21120}{48 \times 2\pi} = \dfrac{21120}{48 \times 2 \times 3\cdot1416} = 70\cdot03$ (cvq)

$\therefore$ $R - r = (105\cdot04 - 70\cdot03)$ †m.wg. = $35\cdot01$ †m.wg. = $\cdot35$ †m.wg. (cvq)

$\therefore$ PvKv `BwUi e¨vmv‡ai Aš—i $\cdot35$ wgUvi (cvq)|

D`vniY 8| GKwU e‡Ëi e¨vmva 14 †m.wg.| GKwU e‡Mi †¶Îdj D³ e‡Ëi †¶Îd‡ji mgvb| eM‡¶‡Îi ˆ`N¨ wbYq Ki|

mgvavb : g‡b Kwi, e‡Ëi e¨vmva $r = 14$ †m.wg. Ges eM‡¶ÎwUi ˆ`N¨ a

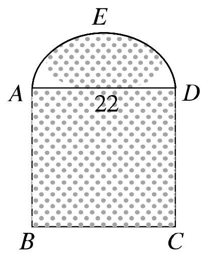
$\therefore$ e‡Ëi †¶Îdj πr^2 Ges eM‡¶ÎwUi †¶Îdj = a^2

ckvbmv‡i, $a^2 = \pi r^2$

ev, $a = \sqrt{\pi}\, r = \sqrt{3\cdot1416} \times 14 = 24\cdot81$ (cvq)

wb‡Yq ˆ`N¨ $24\cdot81$ †m.wg. (cvq)|

D`vniY 9| wP‡Î $ABCD$ GKwU eM‡¶Î hvi cwZevûi ˆ`N¨ 22 wgUvi Ges AED †¶ÎwU GKwU AaeË| m¤úY †¶ÎwUi †¶Îdj wbYq Ki|



mgvavb : g‡b Kwi, $ABCD$ eM‡¶ÎwUi cwZ evûi ˆ`N¨ a

$\therefore$ eM‡¶‡Îi †¶Îdj = a^2

Avevi, AED GKwU AwaeË

$\therefore$ Aae‡Ëi e¨vmva $r = \dfrac{22}{2}$ wgUvi = 11 wgUvi

mZivs, AED Aae‡Ëi †¶Îdj = $\dfrac{1}{2}\pi r^2$

$\therefore$ m¤úY Z‡ji †¶Îdj = $ABCD$ eM‡¶‡Îi †¶Îdj + AED Aae‡Ëi †¶Îdj

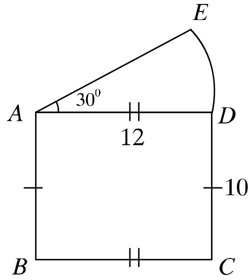
$= \left(a^2 + \dfrac{1}{2}\pi r^2\right)$

$$= \{(22)^2 + \frac{1}{2} \times 3\cdot1416 \times (11)^2 \text{ eMwgUvi } = 674\cdot07 \text{ eMwgUvi (cÖvq)}$$

wb‡Yq †¶Îdj $674\cdot07$ eMwgUvi (cÖvq)|

D`vniY 10| wP‡Î $ABCD$ GKwU AvqZ‡¶Î hvi ˆ`N¨ I c¯'_ h_vµ‡g 12 wgUvi I 10 wgUvi Ges DAE GKwU eÜvsk| eÜvsk DE Gi ˆ`N¨ Ges m¤úY© †¶‡Îi †¶Îdj wbYq Ki|

mgvavb : eÜvs‡ki e¨vm $r = AD = 12$ wgUvi Ges †K‡›`ª Drcb †KvY $\theta = 30^o$



$\therefore$ eÜPvc DE Gi ˆ`N¨ = $\frac{\pi r\theta}{180^o}$

$$= \frac{3\cdot1416 \times 12 \times 30}{180} \text{ wgUvi } = 6\cdot28 \text{ wgUvi (cÖvq)}$$

ADE eÜvs‡ki †¶Îdj = $\frac{\theta}{360^o} \times \pi r^2 = \frac{30}{360} \times 3\cdot1416 \times (12)^2$ eMwgUvi

$$= 37\cdot7 \text{ eMwgUvi (cÖvq)}$$

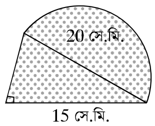
AvqZ‡¶Î $ABCD$ Gi ˆ`N¨ 12 wgUvi Ges c¯' 10 wgUvi|

$\therefore$ AvqZ‡¶ÎwUi †¶Îdj = ˆ`N¨ × c¯' = 12×10 wgUvi = 120 eMwgUvi

$\therefore$ m¤úY© †¶‡Îi †¶Îdj = $(37\cdot7 + 120)$ eMwgUvi = $157\cdot7$ eMwgUvi

wb‡Yq †¶Îdj $157\cdot7$ eMwgUvi (cÖvq)|

KvR : wP‡Î Mvp wPwýZ †¶ÎwUi †¶Îdj wbYq Ki :



Abkxjbx 16·3

1| GKwU eÜPvc †K‡›`ª $30^\circ$ †KvY Drcb K‡i| e‡Ëi e¨vm $126$ †m.wg. n‡j Pv‡ci ˆ`N¨ wbYq Ki|

2| cÖwZ wgwb‡U 66 wgUvi †e‡M $1\frac{1}{2}$ wgwb‡U GKwU †Nvov †Kv‡bv gvV N‡i G‡jv| H gv‡Vi e¨vm wbYq Ki|

3| GKwU eÜvs‡ki †¶Îdj 77 eMwgUvi Ges e‡Ëi e¨vmva© 21 wgUvi| eÜPvcwU †K‡›`ª †h †KvY Drcb K‡i, Zv wbYq Ki|

4| GKwU e‡Ëi e¨vmva© 14 †m.wg. Ges eÜPvc †K‡›`ª 76° †KvY Drcb K‡i| eÜvs‡ki †¶Îdj wbYq Ki|

5| GKwU eËvKvi gvV‡K wN‡i GKwU iv¯v Av‡Q| iv¯wUi wfZ‡ii cwiwa A‡c¶v evB‡ii cwiwa 44 wgUvi †ewk| iv¯wUi PIov wbYq Ki|

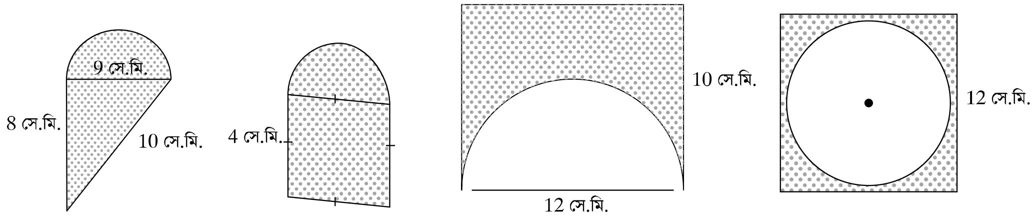
6| GKwU eËvKvi cv‡Ki e¨vm 26 wgUvi| cvKwU‡K †eób K‡i evB‡i 2 wgUvi ck¯ GKwU c_ Av‡Q| c_wUi †¶Îdj wbYq Ki|

7| GKwU Mwoi mvg‡bi PvKvi e¨vm 28 †m.wg. Ges wcQ‡bi PvKvi e¨vm 35 †m.wg.| 88 wgUvi c_ †h‡Z mvg‡bi PvKv wcQ‡bi PvKv A‡c¶v KZ cYmsL¨K evi †ewk Ni‡e ?

8| GKwU e‡Ëi cwiwa 220 wgUvi| H e‡Ë Aš—wjwLZ eM‡¶‡Îi evûi ˆ`N¨ wbYq Ki|

9| GKwU e‡Ëi cwiwa GKwU mgevû wÎf‡Ri cwimxgvi mgvb| G‡`i †¶Î‡dj AbcvZ wbYq Ki|

10| wb‡Pi wP‡Îi Z_¨ Abhvqx Mvp wPwýZ †¶Î¸‡jvi †¶Îdj wbYq Ki :



6·5 AvqZvKvi Nbe¯ :

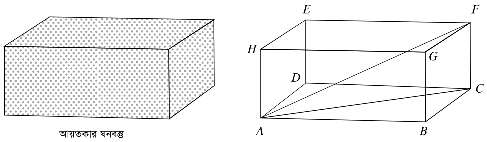
wZb †Rvov mgvšivj AvqZvKvi mgZj ev cô Øviv Ave× Nbe¯‡K AvqZKvi Nbe¯ e‡j|

g‡b Kwi, $ABCDEFGH$ GKwU AvqZKvi Nbe¯| Gi ˆ`N¨ $AB = a$, c¯ $BC = b$, D"PZv $AH = c$

(1) KY wbYq : $ABCDEFGH$ AvqZKvi Nbe¯i KY AF

ΔABC -G $BC \perp AB$ Ges AC AwZf³|

$\therefore \; AC^2 = AB^2 + BC^2 = a^2 + b^2$

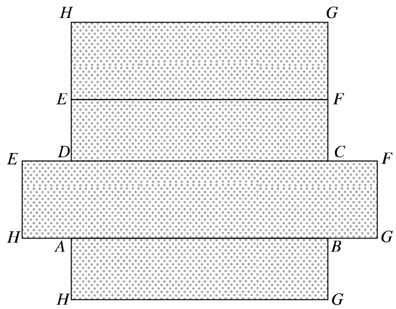


Avevi, ΔACF G $FC \perp AC$ Ges AF AwZfR|

$\therefore \; AF^2 = AC^2 + CF^2 = a^2 + b^2 + c^2$

$\therefore \; AF = \sqrt{a^2 + b^2 + c^2}$

$\therefore$ AvqZvKvi Nbe¯wUi KY = $\sqrt{a^2 + b^2 + c^2}$



(2) mgM Z‡ji †¶Îdj wbYq :

AvqZKvi Nbe¯wUi 6wU Zj

†hLv‡b, wecixZ Zj¸‡jv ci¯úi mgvb|

AvqZvKvi Nb¯wUi mgM Z‡ji †¶Îdj

= 2($ABCD$ Z‡ji †¶Îdj + $ABGH$ Z‡ji †¶Îdj + $BCFG$ Z‡ji †¶Îdj)

$= 2(AB \times AD + AB \times AH + BC \times BG)$

$= 2(ab + ac + bc)$

$= 2(ab + bc + ca)$

(3) AvqZvKvi Nbe¯‘i AvqZb $=$ ˆ`N©¨ $\times$ c¯’ $\times$ D”PZv

$= abc$

D`vniY 1| AvqZvKvi Nbe¯‘i ˆ`N©¨, c¯’ I D”PZv h_vµ‡g, 25 †m.wg., 20 †m.wg. Ges 15 †m.wg.| Gi mgM Z‡ji †¶Îdj, AvqZb Ges K‡Y©i ˆ`N©¨ wbY©q Ki|

mgvavb : g‡b Kwi, AvqZvKvi Nbe¯‘i ˆ`N©¨ $a = 25$ †m.wg., c¯’ $b = 20$ †m.wg. Ges D”PZv $c = 15$ †m.wg.|

$\therefore$ AvqZvKvi Nbe¯‘wUi mgM Z‡ji †¶Îdj $= 2(ab + bc + ca)$

$= 2(25 \times 20 + 20 \times 15 + 15 \times 25)$ eM© †m.wg.

$= 2350$ eM© †m.wg.

AvqZb $= abc$

$= 25 \times 20 \times 15$ Nb †m.wg.

$= 7500$ Nb †m.wg.

Ges K‡Y©i ˆ`N©¨ $= \sqrt{a^2 + b^2 + c^2}$

$= \sqrt{(25)^2 + (20)^2 + (15)^2}$ †m.wg.

$= \sqrt{625 + 400 + 225}$ †m.wg.

$= \sqrt{1250}$ †m.wg.

$= 35 \cdot 353$ †m.wg. (cÖvq)

wb‡Y©q mgM Z‡ji †¶Îdj 2350 eM© †m.wg., AvqZb 2500 Nb †m.wg. Ges K‡Y©i ˆ`N©¨ $35 \cdot 353$ †m.wg. (cÖvq)|

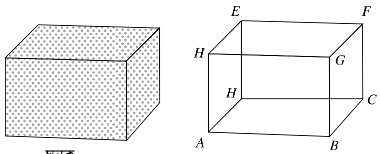
KvR : †Zvgvi MwYZ eB‡qi ˆ`N©¨, c¯’ I D”PZv †g‡c Gi AvqZb, mgM Z‡ji †¶Îdj Ges K‡Y©i ˆ`N©¨ wbY©q Ki|

6·6 NYK :

AvqZKvi Nbe¯‘i ˆ`N©¨, c¯’ I D”PZv mgvb n‡j Zv‡K NbK ejv nq|

g‡b Kwi, $ABCDEFGH$ GKwU NbK|

Gi ˆ`N©¨ = c¯’ = D”PZv = a GKK



ঘনক

(1) NYKwUi K‡Y©i ˆ`N©¨ $= \sqrt{a^2 + b^2 + c^2} = \sqrt{3a^2} = \sqrt{3}a$

(2) NY‡Ki mgM Z‡ji †¶Îdj $= 2(a\cdot a + a\cdot a + a\cdot a)$

$= 2(a^2 + a^2 + a^2) = 6a^2$

(3) NYKwUi AvqZb = $a\cdot a\cdot a = a^3$

D`vniY 2| GKwU NY‡Ki m¤úY c‡ôi †¶Îdj 96 eMwgUvi| Gi K‡Yi ˆ`N©¨ wbYq Ki|

mgvavb : g‡b Kwi, NYKwUi avi a

$\therefore$ Gi m¤úY c‡ôi †¶Îdj = $6a^2$ Ges K‡Yi ˆ`N©¨ = $\sqrt{3}a$

ckvbmv‡i, $6a^2 = 96$ ev, $a^2 = 16$ $\therefore a = 4$

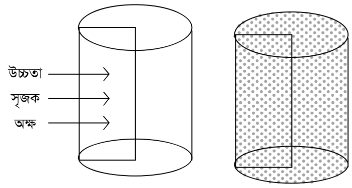
$\therefore$ NYKwUi K‡Yi ˆ`N©¨ = $\sqrt{3}a = \sqrt{3}\times a = 6\cdot928$ wgUvi (cvq)

wb‡Yq K‡Yi ˆ`N©¨ $6\cdot928$ wgUvi (cvq)|

KvR : wZbwU avZe NY‡Ki avi h_vµ‡g 3 †m.wg., 4 †m.wg. I 5 †m.wg.| NYK wZbwU‡K Mwj‡q GKwU bZb NYK ˆZwi Kiv n‡jv| bZb NY‡Ki m¤úY c‡ôi †¶Îdj I K‡Yi ˆ`N©¨ wbYq Ki|

6·7 †ejb :

†Kv‡bv AvqZ‡¶‡Îi †h‡Kv‡bv evû‡K A¶ a‡i AvqZ‡¶ÎwU‡K H evûi PZw`©‡K †Nviv‡j †h Nbe¯‘i mwó nq, Zv‡K mgeËfwgK †ejb ev wmwjÊvi ejv nq| mgeËfwgK †ej‡bi `B cvš‡K eËvKvi Zj, eµZj‡K eµcô ejv nq Ges mgM Zj‡K côZj ejv nq| AvqZ‡¶‡Îi A‡¶i mgvšivj NYvqgvb evûwU‡K †ej‡bi mRK ev Drcv`K †iLv e‡j|



g‡b Kwi, wPÎ K GKwU mgeËfwgK †ejb| hvi fwgi e¨vmva r Ges D”PZv h

(1) fwgi †¶Îdj = πr^2

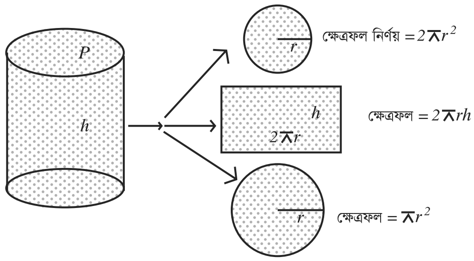
(2) eµc‡ôi †¶Îdj

= fwgi cwiwa × D”PZv

= $2\pi rh$

(3) m¤úYZ‡ji †¶Îdj ev mgMZ‡ji †¶Îdj

ev, côZ‡ji †¶Îdj = $(\pi r^2 + 2\pi rh + \pi r^2) = 2\pi r(r+h)$



(4) AvqZb = fwgi †¶Îdj × D”PZv

= $\pi r^2 h$

D`vniY 3| GKwU mgeËfwgK †ej‡bi D”PZv 10 †m.wg. Ges fwgi e¨vmva 7 †m.wg. n‡j, Gi AvqZb Ges m¤úY c‡ôi †¶Îdj wbYq Ki|

mgvavb : g‡b Kwi, mgeËfwgK †ej‡bi D”PZv $h = 10$ †m.wg. Ges fwgi e¨vmva r

$\therefore$ Gi AvqZb $= \pi r^2 h = 3\cdot1416 \times 7^2 \times 10$

$= 1539\cdot38$ Nb †m.wg. (cvq)

Ges mgMc‡ôi †¶Îdj $= 2\pi r(r+h)$

$= 2\times3\cdot1416\times7(7+10)$ eMwgUvi (cvq)

$= 747\cdot7$ eMwgUvi (cvq)

KvR : GKwU AvqZvKvi KvM‡Ri cvZv gwo‡q GKwU mgeËfwgK wmwjÊvi ˆZwi Ki| Gi côZ‡ji †¶Îdj Ges AvqZb wbYq Ki|

D`vniY 4| XvKbvmn GKwU ev‡·i evB‡ii gvc h_vµ‡g $10$ †m.wg., $9$ †m.wg. I $7$ †m.wg. Ges wfZ‡ii mgM ev·wU †¶Îdj $262$ eM †m.wg.| Gi †`Iqv‡ji ciªZ mgvb n‡j, ev‡·i †ea wbYq Ki|

mgvavb : g‡b Kwi, ev‡·i †ea x

XvKbvmn ev‡·i evB‡ii gvc h_vµ‡g 10 †m.wg., 9 †m.wg. I 7 †m.wg.

$\therefore$ ev‡·i wfZ‡ii gvc h_vµ‡g $a=(10-2x)$, $b=(9-2x)$ I $c=(7-2x)$ †m.wg.

$\therefore$ ev‡·i wfZ‡ii mgM Z‡ji †¶Îdj $= 2(ab+b+ca)$

ckvbmv‡i, $2(ab+b+ca) = 262$

ev, $(10-2x)(9-2x)+(9-2x)(7-2x)+(7-2x)(10-2x)=131$

ev, $90-38x+4x^2+63-32x+4x^2+70-34x+4x^2-131=0$

ev, $12x^2-104x+92=0$

ev, $3x^2-26x+23=0$

ev, $3x^2-3x-23x+23=0$

ev, $3x(x-1)-23(x-1)=0$

ev, $(x-1)(3x-23)=0$

ev, $x-1=0$ A_ev $3x-23=0$

ev, $x=1$ A_ev, $x=\frac{23}{3}=7\cdot67$ (cvq)

wKš ev‡·i †ea Gi ˆ`N¨ ev c¯ ev D”PZvi mgvb A_ev eo n‡Z cv‡i bv|

$\therefore$ $x=1$

wb‡Yq ev‡·i †ea 1 †m.wg.|

D`vniY 5| †Kv‡bv NY‡Ki côZ‡ji K‡Yi ˆ`N© $8\sqrt{2}$ †m.wg. n‡j Gi K‡Yi ˆ`N© I AvqZb wbYq Ki|

mgvavb : g‡b Kwi, NY‡Ki avi a

$\therefore$ NYKwUi côZ‡ji K‡Yi ˆ`N© = $\sqrt{2}a$

K‡Yi ˆ`N© = $\sqrt{3}a$

Ges AvqZb = a^3

ckvbmv‡i, $\sqrt{2}a = 8\sqrt{2}$ $\therefore$ $a = 8$

$\therefore$ NYKwUi K‡Yi ˆ`N© = $\sqrt{3}\times 8$ †m.wg. = $13\cdot856$ †m.wg. (cvq)

Ges AvqZb = 8^3 Nb †m.wg. = 512 Nb †m.wg.|

wb‡Yq K‡Yi ˆ`N© $13\cdot856$ †m.wg. (cvq) Ges AvqZb 512 Nb †m.wg.|

D`vniY 6| †Kv‡bv AvqZ‡¶‡Îi ˆ`N© 12 †m.wg. Ges c¯' 5 †m.wg.| G‡K enËi evûi PZw`©‡K †Nviv‡j †h Nbe¯' Drcb nq Zvi côZ‡ji †¶Îdj Ges AvqZb wbYq Ki|

mgvavb : †`Iqv Av‡Q GKwU AvqZ‡¶‡Îi ˆ`N© 12 †m.wg. Ges c¯' 5 †m.wg.| G‡K enËi evûi PZw`©‡K †Nviv‡j GKwU mgeËfwgK †ejb AvKwZi Nbe¯' Drcb n‡e, hvi D"PZv $h = 12$ †m.wg. Ges fwgi e¨vmva $r = 5$ †m.wg.|

$\therefore$ Drcb NY‡Ki côZ‡ji †¶Îdj = $2\pi r(r+h)$

= $2\times 3\cdot1416\times 5(5+12)$ eM †m.wg.

= $534\cdot071$ eM †m.wg. (cvq)

Ges AvqZb = $\pi r^2 h$

= $3\cdot1416\times 5^2\times 12$ Nb †m.wg.

= $942\cdot48$ Nb †m.wg. (cvq)

wb‡Yq côZ‡ji †¶Îdj $534\cdot071$ eM †m.wg. (cvq) Ges AvqZb $942\cdot48$ Nb †m.wg. (cvq)|

Abkxjbx 16·4

1| GKwU mvgvšwi‡Ki `BwU mwbwnZ evûi ˆ`N© h_vµ‡g 7 †m.wg., 5 †m.wg. n‡j, Gi cwimxgvi A‡aK KZ ?

(K) 12 (L) 20 (M) 24 (N) 28

2| GKwU mgevû wÎf‡Ri evûi ˆ`N© 6 †m.wg. n‡j, Gi †¶Îdj KZ eM †m.wg. ?

(K) $3\sqrt{3}$ (L) $4\sqrt{3}$ (M) $6\sqrt{3}$ (N) $9\sqrt{3}$

3| GKwU UªwcwRqv‡gi D"PZv 8 †m.wg. Ges mgvšivj evûØ‡qi ˆ`N© h_vµ‡g 9 †m.wg. I 7 †m.wg. n‡j, Gi †¶Îdj KZ eM †m.wg. ?

(K) 24 (L) 64 (M) 96 (N) 504

4| wb‡Pi Z_¨¸‡jv j¶ Ki :

(*i*) 4 †m.wg. eMvKvi cv‡_i cwimxgv 16 †m.wg.|

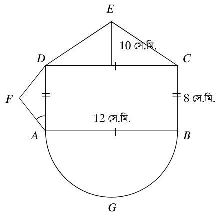
(*ii*) 3 †m.wg. e¨vmv‡ai eËvKvi cv‡Zi †¶Îdj 3π eM †m.wg.|

(*iii*) 5 †m.wg. D”PZv Ges 2 †m.wg. e¨vmv‡ai †ejb AvKwZi e¯i AvqZb 20π Nb †m.wg.|

Dc‡ii Z‡_¨i wfwË‡Z wb‡Pi †KvbwU mwVK ?

(K) *i* I *ii* (L) *i* I *iii* (M) *ii* I *iii* (N) *i*, *ii* I *iii*

wP‡Îi Z_¨ Abmv‡i wb‡Pi ck¸‡jvi DËi `vI :



5| *ABCD* AvqZ‡¶‡Îi K‡Yi ˆ`N¨ KZ ?

(K) 13 (L) 14 (M) 14·4 (cvq) (N) 15

6| *ADF* eËvs‡ki †¶Îdj KZ ?

(K) 16 (L) 32 (M) 64 (N) 128

7| *AGB* Aae‡Ëi cwiwa KZ ?

(K) 18 (L) 18·85 (cvq) (M) 37·7 (cvq) (N) 96

8| GKwU AvqZvKvi Nbe¯i ˆ`N¨, c¯ I D”PZv h_vµ‡g 16 wgUvi, 12 wgUvi I 4·5 wgUvi| Gi cóZ‡ji †¶Îdj, K‡Yi ˆ`N¨ I AvqZb wbYq Ki|

9| GKwU AvqZvKivi Nbe¯i ˆ`N¨, c¯ I D”PZvi Abcv Z 21:16:12 Ges K‡Yi ˆ`N¨ 87 †m.wg. n‡j, Nb e¯wUi Z‡ji †¶Îdj wbYq Ki|

10| GKwU AvqZvKvi Nbe¯ 48 eMwgUvi fwgi Dci `Êvqgvb| Gi D”PZv 3 wgUvi Ges KY 13 wgUvi| AvqZKivi Nbe¯i ˆ`N¨ I c¯ wbYq Ki|

11| GKwU AvqZvKvi Kv‡Vi ev‡·i evB‡ii gvc h_vµ‡g 8 †m.wg., 6 †m.wg., I 4 †m.wg.| Gi wfZ‡ii m¤úY c‡ói †¶Îdj 88 eM †m.wg.| ev·wUi Kv‡Vi ci“Z wbYq Ki|

12| GKwU †`Iqv‡ji ˆ`N¨ 25 wgUvi, D”PZv 6 wgUvi Ges ci“Z 30 †m.wg.| GKwU B‡Ui ˆ`N¨ 10 †m.wg., c¯ 5 †m.wg. Ges D”PZv 3 †m.wg.| †`IqvjwU BU w`‡q ˆZwi Ki‡Z c‡qvRbxq B‡Ui msL¨v wbYq Ki|

13| GKwU NYK AvKwZe¯i côZ‡ji †¶Îdj 2400 eM †m.wg. n‡j, Gi K‡Yi ˆ`N¨ wbYq Ki|

14| 12 †m.wg. D”PZwewkó GKwU †ej‡bi fwgi e¨vmva 5 †m.wg.| Gi côZ‡ji †¶Îdj I AvqZb wbYq Ki|

15| GKwU †ej‡bi eµZ‡ji †¶Îdj 100 eM †m.wg. Ges AvqZb 150 Nb †m.wg.| †ej‡bi D”PZv Ges fwgi e¨vmva wbYq Ki|

16| GKwU mgeËfwgK wmwjÊv‡ii eµZ‡ji †¶Îdj 4400 eM †m.wg.| Gi D”PZv 30 †m.wg. n‡j, mgMZj wbYq Ki|

17| GKwU †jvnvi cvB‡ci wfZ‡ii I evB‡ii e¨vm h_vµ‡g 12 †m.wg. I 14 †m.wg. Ges cvB‡ci D”PZv 5 wgUvi| 1 Nb †m.wg. †jvnvi IRb 7·2 Mvg n‡j, cvB‡ci †jvnvi IRb wbYq Ki|

18| GKwU AvqZvKvi †¶‡Îi ˆ`N¨ 12 wgUvi Ges c¯ 5 wgUvi| AvqZvKvi †¶ÎwU‡K cwi‡ewóZ K‡i GKwU eËvKvi †¶Î Av‡Q †hLv‡b AvqZKvi †¶Î Øviv AbwaKZ As‡k Nvm jvMv‡bv n‡jv|

(K) Dc‡ii Z‡_¨i wfwË‡Z msw¶ß eYbvmn wPÎ AvK|

(L) eËvKvi †¶ÎwUi e¨vm wbYq Ki|

(M) cwZ eMwgUvi Nvm jvMv‡Z 50 UvKv LiP n‡j, †gvU LiP wbYq Ki|

19| ∆ABC I ∆BCD GKB fwg BC Gi Dci Ges GKB mgvšivj hMj BC I AD Gi g‡a¨ Aew¯Z|

K. Dc‡ii eYbv Abmv‡i wPÎwU AvK|

L. cgvY Ki †h, ∆ †¶Î ABC = ∆ †¶Î BCD.

M. ∆ †¶‡Î ABC Gi mgvb †¶Îdj wewkó GKwU mvgvšwiK AvK hvi GKwU †KvY GKwU wbw`ó †Kv‡Yi mgvb| (A¼‡bi wPý I weeiY Avek¨K)|

20| GKwU mvgvšwiK †¶Î ABCD Ges GKwU AvqZ‡¶Î BCEF Df‡qi fwg BC.

K. GKB D”PZv we‡ePbv K‡i mvgvšwiK †¶Î I AvqZ‡¶ÎwUi wPÎ AvK|

L. †`LvI †h, ABCD †¶ÎwUi cwimxgv BCEF †¶ÎwUi cwimxgv A‡c¶v enËi|

M. AvqZ‡¶ÎwUi ˆ`N¨ I c‡¯i AbycvZ 5:3 Ges †¶ÎwUi cwimxgv 48 wgUvi n‡j, mvgvšwiK †¶ÎwUi †¶Îdj wbYq Ki|

mß`k Aa¨vq

cwimsL¨vb

weÁvb I chw³i Dbq‡bi AMhvÎvq Z_¨ Dcv‡Ëi Ae`v‡bi d‡j cw_ex cwiYZ n‡q‡Q wekMv‡g| Z_¨ I Dcv‡Ëi `“Z mÂvjb I we¯—v‡ii Rb¨ m¤e n‡q‡Q wekvq‡bi| ZvB Dbq‡bi avivi Ae¨vnZ ivLv I wekvq‡b AskMnY I Ae`vb ivL‡Z n‡j Z_¨ I DcvË m¤ú‡Ü mg¨K Ávb ARb G¯—‡ii wk¶v_x‡`i Rb¨ Acwinvh| cvmw½Kfv‡e wk¶v_xi Ávb AR‡bi Pwn`v †gUv‡bvi j‡¶¨ lô †kwY †_‡K Z_¨ I Dcv‡Ëi Av‡jvPbv Kiv n‡q‡Q Ges av‡c av‡c †kwYwfwËK welqe¯—i web¨vm Kiv n‡q‡Q| GiB avivewnKZvq G †kwY‡Z wk¶v_xiv µg‡hwRZ MYmsL¨v, MYmsL¨v eûfR, AwRf †iLv, †Kw›`ªq ceYZv cwigv‡c msw¶ß c×wZ‡Z Mo, ga¨K I cPiK BZ¨wn` m¤ú‡Ü Rvb‡e I wkL‡e|

Aa¨vq †k‡l wk¶v_xiv-

- µg‡hwRZ MYmsL¨v, MYmsL¨v eûfR I AwRf †iLv e¨vL¨v Ki‡Z cvi‡e|
- MYmsL¨v eûfR I AwRf †iLvi mvnv‡h¨ DcvË e¨vL¨v Ki‡Z cvi‡e|
- †Kw›`ªq ceYZvi cwigvc c×wZ e¨vL¨v Ki‡Z cvi‡e|
- †Kw›`ªq ceYZv cwigv‡c msw¶ß c×wZi c‡qvRbxqZv e¨vL¨v Ki‡Z cvi‡e|
- msw¶ß c×wZi mvnv‡h¨ Mo, ga¨K I cPiK wbYq Ki‡Z cvi‡e|
- MYmsL¨v eûfR I AwRf †iLv †jLwP‡Îi e¨vL¨v Ki‡Z cvi‡e|

Dcv‡Ëi Dc¯—vcb : Avgiv Rwb ¸YevPK bq Ggb msL¨vmPK Z_¨vewj cwimsL¨v‡bi DcvË| AbmÜvbvaxb DcvË cwimsL¨v‡bi KvPvgvj| G¸‡jv Awe¨¯—fv‡e _v‡K Ges Awe¨¯— DcvË †_‡K mivmwi c‡qvRbxq wm×v‡š— DcbxZ nIqv hvq bv| c‡qvRb nq DcvË¸‡jvi web¨¯— I mviwYf³ Kiv| Avi DcvËmg‡ni mviwYf³ Kiv n‡jv Dcv‡Ëi Dc¯—vcb| Av‡Mi †kwY‡Z Avgiv DcvËmgn Kxfv‡e mviwYf³ K‡i web¨¯— Ki‡Z nq Zv wk‡LwQ| Avgiv Rwb †Kv‡bv Dcv‡Ëi mviwYf³ Ki‡Z n‡j c_‡g Zvi cwimi wbaviY Ki‡Z nq| Gici †kwY e¨eavb I †kwY msL¨v wbaviY K‡i U¨wj wPý e¨envi K‡i MYmsL¨v wb‡ekY mviwY ˆZwi Kiv nq| GLv‡b eSvi myweav‡_ wb‡Pi D`vni‡Yi gva¨‡g MYmsL¨v wb‡ekb mviwY ˆZwi Kivi c×wZi cbiv‡jvPbv Kiv n‡jv|

D`vniY 1| †Kv‡bv GK kxZ †gŠm‡g kxg½‡ji Rvbqwi gv‡mi 31 w`‡bi Zvcgv‡Îv (†mjwmqvm) wb‡P †`Iqv n‡jv| Zvcgv‡Îvi MYmsL¨v wb‡ekb mviwY ˆZwi Ki|

14°, 14°, 14°, 13°, 12°, 13°, 10°, 10°, 11°, 12°, 11°, 10°, 9°, 8°, 9°,

11°, 10°, 10°, 8°, 9°, 7°, 6°, 6°, 6°, 6°, 7°, 8°, 9°, 9°, 8°, 7°|

mgvavb : GLv‡b Zvcgv‡¤¿v wb‡`kK Dcv‡Ëi me‡P‡q †QvU msL¨v 6 Ges eo msL¨v 14|

mZivs Dcv‡Ëi cwimi = (14 – 6) + 1 = 9|

GLb †kwY e¨eavb hw` 3 †bIqv nq Z‡e †kwY msL¨v n‡e $\frac{9}{3}$ ev 3|

†kwY e¨eavb 3 wb‡q wZb †kwY‡Z DcvËmgn web¨vm Ki‡j MYmsL¨v (NUb msL¨vI ejv nq) wb‡ekb mvwiwY n‡e wbgiƒc :

Zvcgv‡¤¿v (†mjwmqvm)	U¨wj wPý	MYmsL¨v ev NUb msL¨v
6° – 8°	𝍸 𝍸 I	11
9° – 11°	𝍸 𝍸 III	13
12° – 14°	𝍸 II	7
		†gvU 31|

KvR : †Zvgv‡`i †kwY‡Z Aa¨vqbiZ mKj wk¶v_©x‡`i `BwU `j MVb Ki| `‡ji m`m¨‡`i IR‡bi (†KwR‡Z) MYmsL¨v wb‡ekY mvwiwY ˆZwi Ki|

µg‡hwRZ MYmsL¨v (*Cumulative Frequency*) :

D`vniY 1 Gi †kwY 3 e¨eavb a‡i †kwYmsL¨v wbavib K‡i MYmsL¨v wb‡ekY mviwY ˆZwi Kiv n‡q‡Q| D‡jøwLZ Dcv‡Ëi †kwY msL¨v 3| c_g †kwYi mxgv n‡jv 6° – 8°| GB †kwYi wbgmxgv 6° Ges D”Pmxgv 8° †m.| GB †kwYi MYmsL¨v 11|

wØZxq †kwYi MYmsL¨v 13| GLb c_g †kwYi MYmsL¨v 11 Gi mv‡_ wØZxq †kwYi MYmsL¨v 13 †hvM K‡i cvB 24| GB 24 n‡e wØZxq †kwYi µg‡hwRZ MYmsL¨v| Avi c_g †kwY w`‡q ïi“ nIqvq GB †kwYi µg‡hwRZ MYmsL¨v n‡e 11| Avevi wØZxq †kwYi µg‡hwRZ MYmsL¨v 24 Gi mv‡_ ZZxq †kwYi MYmsL¨v †hvM Ki‡j 24 + 7 = 31, hv ZZxq †kwYi µg‡hwRZ MYmsL¨v| GBfv‡e µg‡hwRZ MYmsL¨v mviwY ˆZwi Kiv nq| Dc‡ii Av‡jvPbvi †cÖ¶‡Z D`vniY 1 Gi Zvcgv‡¤¿vi µg‡hwRZ MYmsL¨v mviwY wbgiƒc :

Zvcgv‡¤¿v †mw›UwgUv‡i	MYmsL¨v	µg‡hwRZ MYmsL¨v
6° – 8°	11	11
9° – 11°	13	(11 + 13) = 24
12° – 14°	7	(24 + 7) = 31

D`vniY 2| wb‡P 40 Rb wk¶v_©xi ewlK cix¶vq Bs‡iwR‡Z cÖvß b¤^i †`Iqv n‡jv| cÖvß b¤^‡ii µg‡hwRZ MYmsL¨v mviwY ˆZwi Ki|

70, 40, 35, 60, 55, 58, 45, 60, 65, 80, 70, 46, 50, 60, 65, 70, 58, 60, 48, 70, 36, 85, 60, 50, 46, 65, 55, 61, 72, 85, 90, 68, 65, 50, 40, 56, 60, 65, 46, 76|

mgvavb : Dcv‡Ëi cwiwa = (m‡ev”P gvb – mewbggvb) + 1
= (90 – 35) + 1
= 55 + 1
= 56

†kwY e¨eavb hw` 5 aiv nq, Z‡e †kwY msL¨v = $\frac{56}{5}$
= 11·2 ev 12

mZivs †kwY e¨eavb 5 a‡i µg‡hwRZ MYmsL¨v mviwY n‡e wbgiƒc :

cvß b¤i	MYmsL¨v	µg‡hwRZ MYmsL¨v	cvß b¤i	MYmsL¨v	µg‡hwRZ MYmsL¨v
35 – 39	2	2	70 – 74	4	4 + 31 = 35
40 – 44	2	2 + 2 = 4	75 – 79	1	1 + 35 = 36
45 – 49	5	5 + 4 = 9	80 – 84	1	1 + 36 = 37
50 – 54	3	3 + 9 = 12	85 – 89	2	2 + 37 = 39
55 – 59	5	5 + 12 = 17	90 – 94	1	1 + 39 = 40
60 – 64	8	8 + 17 = 25	95 – 99	0	0 + 40 = 40
65 – 69	6	6 + 25 = 31			

PjK : Avgiv Rwb msL¨vmPK Z_¨mgn cwimsL¨v‡bi DcvË| Dcv‡Ë e¨eüZ msL¨vmgn n‡jv PjK| †hgb, D`vniY 1 G Zvcgv‡Îv wb‡`kK msL¨v¸‡jv PjK| Z`ªbiƒc D`vniY 2 G cvß b¤i¸‡jv e¨eüZ Dcv‡Ëi PjK|

wewQb I Awew”Qb PjK : cwimsL¨v‡b e¨eüZ PjK `B cKv‡ii nq| †hgb wewQb PjK I Awew”Qb PjK| †h Pj‡Ki gvb ïagv‡Î cYmsL¨v nq Zv wew”Qb PjK, †hgb D`vniY 2 G e¨eüZ cvß b¤i| Z`ªbiƒc RbmsL¨v wb‡`kK Dcv‡Ë cYmsL¨v e¨eüZ nq| ZvB RbmsL¨vgjK Dcv‡Ëi PjK n‡”Q wew”Qb PjK| Avi †hmKj Pj‡Ki gvb †h‡Kv‡bv ev¯e msL¨v n‡Z cv‡i, †m mKj PjK Awew”Qb PjK| †hgb D`vniY 1-G e¨eüZ Zvcgv‡Îv wb‡`kK Dcv‡Ë †h‡Kv‡bv ev¯e msL¨v n‡Z cv‡i| G Qvov eqm, D”PZv, IRb BZ¨w` msw¶ó Dcv‡Ë †h‡Kv‡bv ev¯e msL¨v e¨envi Kiv hvq| ZvB G¸‡jvi Rb¨ e¨eüZ PjK n‡”Q Awew”Qb PjK| Awew”Qb Pj‡Ki `BwU gv‡bi ga¨eZx †h‡Kv‡bv msL¨vI H Pj‡Ki gvb n‡Z cv‡i| A‡bK mgq †kwY e¨eavb Awew”Qb Kivi c‡qvRb nq| †kwY e¨eavb Awew”Qb Kivi Rb¨ †Kv‡bv †kwYi D”Pmxgv Ges cieZx †kwYi wbgmxgvi ga¨we›` wb‡q †mB †kwYi cKZ D”Pmxgv Ges cieZx †kwYi cKZ wbgmxgv wbavib Kiv nq| †hgb, D`vniY 1 G c_g †kwYi cKZ D”Pmxgv I wbgmxgv h_vµ‡g 8·5° I 5·5° Ges wØZxq †kwYi D”Pmxgv I wbgmxgv 11·5° I 8·5° BZ¨w`|

KvR : †Zvgv‡`i †kwYi wk¶v_x‡`i wb‡q Aba¶ 40 R‡bi `j MVb Ki| `‡ji m`m¨‡`i IRb/D”PZv wb‡q `‡j MYmsL¨v wb‡ekY I µg‡hwRZ MYmsL¨v mviwY ˆZwi Ki|

Dcv‡Ëi †jLwPÎ : Avgiv †`‡LwQ †h, AbmÜvbvaxb msMnxZ DcvË cwimsL¨vb‡bi KvPvgvj| G¸‡jv MYmsL¨v wb‡ekY mviwYf³ ev µg‡hwRZ mviwYf³ Kiv n‡j G‡`i m¤‡Ü mg¨K avibv Kiv I wm×vš— †bIqv mnR nq| GB mviwYf³ DcvËmgn hw` †jLwP‡Îi gva¨‡g Dc¯vcb Kiv nq, Z‡e Zv eSvi Rb¨ †hgb Avil mnR nq †Zgwb wPËvKlK nq| G Rb¨ cwimsL¨v‡bi DcvËmgn mviwYf³ Kiv I †jLwP‡Îi gva¨‡g Dc¯vcb eûj cPwjZ Ges e¨vcK e¨eüZ c×wZ| 8g †kwY chš— wewfb cKvi †jLwP‡Îi g‡a¨ †iLwPÎ I AvqZ‡jL m¤‡Ü we¯wiZ Av‡jvPbv Kiv n‡q‡Q Ges G¸‡jv wKfv‡e AvK‡Z nq Zv †`Lv‡bv n‡q‡Q| GLv‡b Kxfv‡e MYmsL¨v wb‡ekY I µg‡hwRZ MYmsL¨v mviwY †_‡K MYmsL¨v eûfR, cvBwPÎ I AwRf †iLv AvKv nq Zv wb‡q Av‡jvPbv Kiv n‡e|

MYmsL¨v eûfR (*Frequency Polygon*) : 8g †kwY‡Z Avgiv wew”Qb Dcv‡Ëi AvqZ‡jL AvKv wk‡LwQ| GLv‡b Kxfv‡e c_‡g Awew”Qb Dcv‡Ëi AvqZ‡jL G‡K Zvi MYmsL¨v eûfR AvKv nq, Zv D`vni‡Yi gva¨‡g Dc¯vcb Kiv n‡jv|

D`vniY 3| †Kvb ¯‹‡ji 10g †kwYi 60 Rb wk¶v_xi IR‡bi (wK‡jvMvg) MYmsL¨v wb‡ekY n‡jv wbgi¦c :

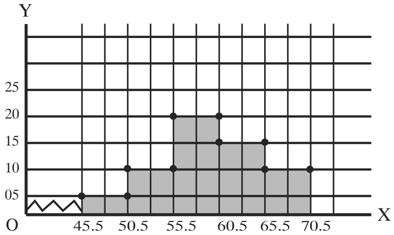
IRb (†KwR)	46 – 50	51 – 55	56 – 60	61 – 65	66 – 70
MYmsL¨v (wk¶v_x‡`i msL¨v)	5	10	20	15	10

(K) MYmsL¨v wb‡ek‡Yi AvqZ‡jL AvK|
(L) AvqZ‡j‡Li MYmsL¨v eûfR AvK|

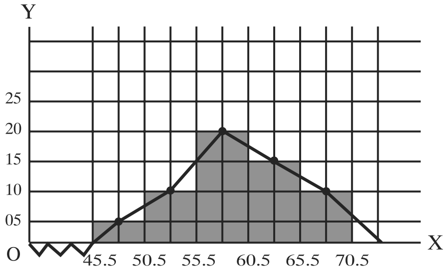
mgvavb : c`Ë mviwY‡Z Dcv‡Ëi †kwY e¨eavb wew”Qb| †kwY e¨eavb Awew”Qb Kiv n‡j c`Ë mviwY n‡e :

†kwY e¨eavb IRb (†KwR)	Awew”Qb †kwYmxgv	†kwY ga¨we›`	MYmsL¨v
46 – 50	45·5 – 50·5	48	5
51 – 55	50·5 – 55·5	53	10
56 – 60	55·5 – 60·5	58	20
61 – 65	60·5 – 65·5	63	15
66 – 70	65·5 – 70·5	68	10

(K) QK KvM‡Ri cwZ Ni‡K GK GKK a‡i x-A¶ eivei †kwYmxgv Ges y-A¶ eivei MYmsL¨v wb‡q AvqZ‡jL AvKv n‡q‡Q| x-A¶ eivei †kwYmxgv 45·5 †_‡K Avi¤ n‡q‡Q| gjwe›` †_‡K 45·5 chš— ceeZx Ni¸‡jv Av‡Q eSv‡Z fvOv wPý e¨envi Kiv n‡q‡Q|



(L) AvqZ‡jL n‡Z MYmsL¨v eûfR AvKvi Rb¨ cvß AvqZ‡j‡Li AvqZmg‡ni fwgi mgvšivj wecixZ evûi ga¨we›`mgn wbavP Kiv n‡q‡Q| wPwýZ ga¨we›`mgn †iLvsk Øviv msh³ K‡i MYmsL¨v eûfR AvKv n‡q‡Q (cv‡ki wP‡Î †`Lv‡bv n‡jv)| MYmsL¨v eûfR m›`i †`Lv‡bvi Rb¨ c_g I †kl Av‡Zi ga¨we›`i ms‡hvM †iLvs‡ki cvš— we›`Øq †kwY e¨eavb wb‡`kK x-A‡¶i mv‡_ msh³ Kiv n‡q‡Q|

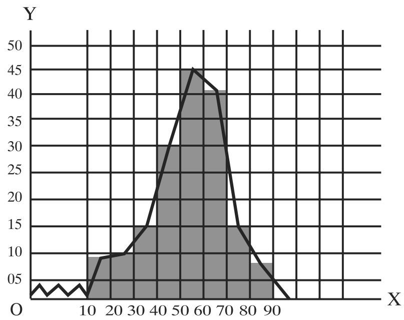


MYmsL¨v eûfR : AwewQb Dcv‡Ëi †kwY e¨eav‡bi wecixZ MYmsL¨v wb‡`kK we›`mgn‡K chvqµ‡g †iLvsk Øviv h³ K‡i †h †jLwPÎ cvIqv hvq, ZvB n‡jv MYmsL¨v eûfR|

D`vniY 4| wb‡Pi MYmsL¨v wb‡ekY mviwYi eûfR A¼b Ki|

†kwY e¨eavb	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90
ga¨we›`	15	25	35	45	55	65	75	85
MYmsL¨v	8	10	15	30	45	41	15	7

mgvavb : x-A¶ eivei QK KvM‡Ri cwZ `B Ni‡K †kwY e¨eav‡bi 5 GKK a‡i Ges $y$-A¶ eivei QK KvM‡Ri `B Ni‡K MYmsL¨vi 5 GKK a‡i c`Ë MYmsL¨v wb‡ek‡Yi AvqZ‡jL AvKv n‡jv| AvqZ‡j‡Li AvqZmg‡ni fwgi wecixZ evûi g‡a¨ we›` hv †kwYi ga¨we›` wPwýZ Kwi| GLb wPwýZ ga¨we›`mgn †iLvsk Øviv msh³ Kwi| c_g †kwYi cvš— we›` I †kl †kwYi cvš— we›`Øq‡K †kwY e¨eavb wb‡`kK x A‡¶i mv‡_ msh³ K‡i MYmsL¨v eûfR A¼b Kiv n‡jv|



KvR : †Zvgv‡`i †kwY‡Z Aa¨qbiZ wk¶v_x‡`i c_g mvgwqK cix¶vq evsjvq cvß b¤i wb‡q MYmsL¨v eûfR AvK|

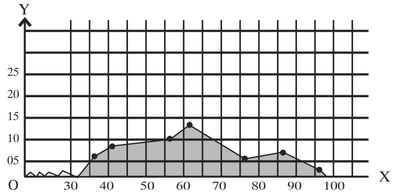
D`vniY 5| 10g †kwYi 50 Rb wk¶v_xi weÁvb wel‡q cvß b¤‡ii MYmsL¨v wb‡ekY mviwY †`Iqv n‡jv| c`Ë Dcv‡Ëi MYmsL¨v eûfR AvK (AvqZ‡jL e¨envi bv K‡i)|

cvß b¤‡ii †kwY e¨eavb	31–40	41–50	51–60	61–70	71–80	81–90	91–100
MYmsL¨v	6	8	10	12	5	7	2

mgvavb : GLv‡b c`Ë DcvË wew”Qb| G‡¶‡Î †kwY e¨eav‡bi ga¨we›` †ei K‡i mivmwi MYmsL¨v eûfR AvKv mweavRbK|

†kwY e¨eavb	31–40	41–50	51–60	61–70	71–80	81–90	91–100
ga¨we›`	$\frac{40+31}{2} = 35\cdot5$	45·5	55·5	65·5	75·5	85·5	95·5
MYmsL¨v	6	8	10	12	5	7	2

x-A¶ eivei QK KvM‡Ri cwZ 2 Ni‡K †kwY e¨eav‡bi ga¨we›`i 10 GKK a‡i Ges $y$-A¶ eivei QK KvM‡Ri 1 Ni‡K MYmsL¨vi 1 GKK a‡i c`Ë Dcv‡Ëi MYmsL¨v eûfR AvKv n‡jv|



KvR : 100 Rb K‡jR Qv‡Îi D”PZvi MYmsL¨v wb‡ekY †_‡K MYmsL¨v eûfR AvK|

D”PZv (†m.wg.)	141–150	151–160	161–170	171–180	181–190

µg‡hwRZ MYmsL¨v †jLwPÎ ev AwRf †iLv : †Kv‡bv Dcv‡Ëi †kwY web¨v‡mi ci †kwY e¨eav‡bi D”Pmxgv x-A¶ eivei Ges †kwYi µg‡hwRZ MYmsL¨v y A¶ eivei ¯’vcb K‡i µg‡hwRZ MYmsL¨vi †jLwPÎ ev AwRf †iLv cvIqv hvq|

D`vniY 6| †Kv‡bv †kwYi 60 wk¶v_©xi 50 b¤‡ii MvwYwZK cix¶vq cvß b¤‡ii MYmsL¨v wb‡ekY mviwY n‡jv :

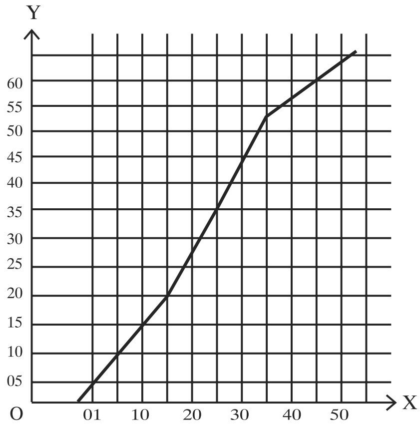
cvß b¤‡ii †kwY e¨eavb	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
MYmsL¨v	8	12	15	18	7

GB MYmsL¨v wb‡ek‡Yi AwRf †iLv AvK|

mgvavb : c`Ë Dcv‡Ëi MYmsL¨v wb‡ek‡Yi µg‡hwRZ MYmsL¨v mviwY n‡jv :

cvß b¤‡ii †kwY e¨eavb	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
MYmsL¨v	8	12	15	18	7
µg‡hwRZ MYmsL¨v	8	8 + 12 = 20	15 + 20 = 35	18 + 35 = 53	7 + 53 = 60

x-A¶ eivei QK KvM‡Ri `B Ni‡K †kÖY e¨eavb D”PmxgvI GKK Ges $y$-A¶ eivei QK KvM‡Ri GK Ni‡K µg‡hwRZ MYmsL¨vi 5 GKK a‡i c`Ë Dcv‡Ëi µg‡hwRZ MYmsL¨vi AwRf †iLv AvKv n‡jv|



KvR : †Kvb GK cix¶vq MwY‡Z †Zvgv‡`i †kÖYi 50 I Z`‡a¶v b¤icvß wk¶v_x‡`i b¤‡ii µg‡hwRZ MYmsL¨v mviwY ˆZwi Ki Ges AwRf †iLv AvK|

†K›`ªxq cÖeYZv : mßg I Aóg †kÖY‡Z †K›`ªxq cÖeYZv I Gi cwigvc mg‡Ü Av‡jvPbv Kiv n‡q‡Q| Avgiv †`‡LwQ †h, AbmÜvbvaxb Awebª¯Í DcvËmgn gv‡bi µgvbmv‡i mvRv‡j, DcvËmgn gvSvgwS †Kv‡bv gv‡bi KvQvKvwQ cwÄfZ nq| Avevi Awebª¯Í DcvËmgn MYmsL¨v wb‡ekY mviwY‡Z Dc¯'vcb Kiv n‡j gvSvgwS GKwU †kÖY‡Z MYmsL¨vi cvPh †`Lv hvq| A_vr, gvSvgwS GKwU †kÖY‡Z MYmsL¨v Le †ewk nq| e¯Z DcvËmg‡ni †K›`ª gv‡bi w`‡K cwÄfZ nIqvi GB cÖeYZvB n‡jv †K›`ªq cÖeYZv| †K›`ªq gvb GKwU msL¨v Ges GB msL¨v DcvËmg‡ni cwZwbwaZ K‡i| GB msL¨v Øviv †K›`ªq cÖeYZv cwigvc Kiv nq| mvaviYZ †K›`ªq cÖeYZvi cwigvc n‡jv : (1) MwYwZK Mo (2) ga¨K (3) cPiK|

MwYwZK Mo : Avgiv Rwb, DcvËmg‡ni gv‡bi mgwó‡K hw` Zvi msL¨v Øviv fvM Kiv nq, Z‡e DcvËmg‡ni Mo gvb cvIqv hvq| Z‡e DcvËmg‡ni msL¨v hw` Le †ewk nq Zvn‡j G c×wZ‡Z Mo wbYq Kiv mgqmv‡c¶, †ek KwVb I fj nIqvi m¤¢vebv _v‡K| G mKj †¶‡Î DcvËmgn †kÖY webv¨v‡mi gva¨‡g mviwYe× K‡i msw¶ß c×wZ‡Z Mo wbYq Kiv nq|

D`vniY 7| wb‡P †Kv‡bv GKwU †kÖYi wk¶v_x‡`i MwY‡Z cÖß b¤‡ii MYmsL¨v wb‡ekb mviwY †`Iqv n‡jv| cÖß b¤‡ii MwYwZK Mo wbYq Ki|

†kÖY e¨wß	25–34	35–44	45–54	55–64	65–74	75–84	85–94
MYmsL¨v	5	10	15	20	30	16	4

mgvavb : GLv‡b †kÖY e¨wß †`Iqv Av‡Q wKšÍ wk¶v_x‡`i e¨w³MZ b¤i KZ Zv Rvbv hvq bv| G †¶‡Î c‡Z¨K †kÖYi †kÖY ga¨gvb wbYq Kivi c‡qvRb nq|

$$†kÖY ga¨gvb = \frac{†kÖY EaŸgvb+†kÖYi wbggvb}{2}$$

hw` †kÖY ga¨gvb $x_i(i=1.....,k)$ nq Z‡e ga¨gvb msewjZ mviwY n‡e wbgœiƒc :

†kÖY e¨wß	†kÖY ga¨gvb (x_i)	MYmsL¨v (f_i)	$(f_i x_i)$
25 – 34	29·5	5	147·5
35 – 44	39·5	10	395·0
45 – 54	49·5	15	742·5
55 – 64	59·5	20	1190·0

65 – 74	69·5	30	2085·0
75 – 84	79·5	16	1272·0
85 – 94	89·5	4	358·0
	†gvU	100	6190·00

wb‡Yq MwYwZK Mo = $\frac{1}{n}\sum_{l=1}^{k} f_i x_i \quad = \frac{1}{100}\times 6190$

$= 61{\cdot}9$

†kÖYxeÜbK…Z DcvË¨i MwYwZK Mo (msw¶ß c×wZ)

†kÖYxeÜbK…Z DcvË MwYwZK Mo wbY©‡qi Rb¨ msw¶ß c×wZ n‡jv mnR|

msw¶ß c×wZ‡Z Mo wbY©‡qi avcmgn Ñ

1| †kÖYmg~‡ni ga¨gvb wbYq Kiv

2| ga¨gvbmgn †_‡K mweavRbK †Kvb gvb‡K AvbgwbK Mo (a) aiv

3| c‡Z¨K †kwYi ga¨gvb †_‡K AvbgwbK Mo we‡qvM K‡i Zv‡K †kwY e¨wß Øviv fvM K‡i avc wePz¨wZ

$u = \dfrac{\text{ga¨gvb Ñ AvbgwbK Mo}}{\text{e¨wß}}$ wbYq Kiv

4| avc wePz¨wZ‡K msw¯ó †kwYi MYmsL¨v Øviv ¸Y Kiv

5| wePz¨wZi Mo wbYq Kiv Ges Gi mv‡_ AvbgwbK Mo †hvM K‡i KvwâLZ Mo wbYq Kiv|

msw¶ß c×wZ : G c×wZ‡Z DcvËmg~‡ni MwYwZK Mo wbY©‡q e¨eüZ m~Î n‡jv :

$\bar{x} = a + \frac{\sum f_i u_i}{n} \times h$ †hLvb, $\bar{x}$ = wb‡Yq Mo, a = AvbgwbK Mo, f_i = i-Zg †kÖYxi MYmsL¨v, $u_i f_i$ = i

Zg †kÖYxi MYmsL¨v avc wePz¨wZ h = †kÖY e¨wß

D`vniY 8| †Kv‡bv `‡e¨i Drcv`‡b wewfbœ chv‡q †h LiPmgn (kZ UvKvq) nq Zv wb‡Pi mviwY‡Z †`Lv‡bv

n‡q‡Q| msw¶ß c×wZ‡Z Mo LiP wbY©q Ki|

Drcv`b LiP (kZ UvKvq)	2–6	6–10	10–14	14–18	18–22	22–26	26–30	30–34
MYmsL¨v	1	9	21	47	52	36	19	3

mgvavb : msw¶ß c×wZ‡Z AbmZ av‡ci Av‡jv‡K Mo wbY©‡qi mviwY n‡e wbgiƒc :

†kÖY e¨wß	ga¨gvb x_i	MYmsL¨v f_i	avc wePz¨wZ $u_i = \frac{x_i - a}{h}$	MYmsL¨v avc wePz¨wZ $f_i u_i$
2 – 6	4	1	– 4	– 4
6 – 10	8	9	– 3	– 27
10 – 14	12	21	– 2	– 42
14 – 18	16	47	– 1	– 47
18 – 22	$20 \leftarrow a$	52	0	0

22 – 26	24	36	1	36
26 – 30	28	19	2	38
30 – 34	32	3	3	9
†gvU		188		– 37

Mo $\bar{x} = a + \frac{\sum f_i u_i}{n} \times h$

$= 20 + \frac{-37}{188} \times 4$

$= 20 - \cdot 79$

$= 19 \cdot 21$

∴ Drcv`‡b AvbgwbK Mo LiP 19 kZ UvKv|

¸i“Z c`Ë Dcv‡Ëi Mo wbYq

A‡bK †¶‡Î AbmÜvbvaxb cwimsL¨v‡bi Pj‡Ki mvswL¨K gvb $x_1, x_2, \ldots\ldots, x_n$ wewfbœ KviY/¸i“Z/fvi Øviv cfvweZ n‡Z cv‡i| G mKj †¶‡Î Dcv‡Ëi gvb $x_1, x_2, \ldots\ldots, x_n$ Gi mv‡_ G‡`i KviY/¸i“Z/fvi $w_1, w_2, \ldots\ldots, w_n$ we‡ePbv K‡i MwYwZK Mo wbYq Ki‡Z nq|

hw` $n$ msL¨K Dcv‡Ëi gvb $x_1, x_2, \ldots\ldots, x_n$ n‡jv Ges G‡`i ¸i“Z hw` $w_1, w_2, \ldots\ldots, w_n$ nq Z‡e G‡`i ¸i“Z c`Ë MwYwZK Mo n‡e

$$\overline{x_w} = \frac{\sum_{i=1}^{n} x_i w_i}{\sum_{i=1}^{n} w_i}$$

D`vniY 9| †Kv‡bv wekwe`¨vj‡qi K‡qKwU wefv‡Mi ¯vZK m¤§vb †kwY‡Z cv‡mi nvi I wk¶v_xi msL¨v wb‡Pi mviwY‡Z Dc¯vcb Kiv n‡jv| D³ wekwe`¨vj‡qi H KqwU wefv‡Mi ¯vZK m¤§vb †kwY‡Z cv‡mi Mo nvi wbYq Ki|

wefv‡Mi bvg	MwYZ	cwimsL¨vb	Bs‡iwR	evsjv	cwYwe`¨v	ivóweÁvb
cv‡ki nvi (kZKivq)	70	80	50	90	60	85
wk¶v_xi msL¨v	80	120	100	225	135	300

mgvavb : GLv‡b cv‡mi nvi I wk¶v_xi msL¨v †`Iqv Av‡Q| cv‡mi nv‡ii fvi n‡jv wk¶v_xi msL¨v| hw` cv‡mi nv‡ii PjK x Ges wk¶v_xi msL¨v PjK w aiv nq, Z‡e ¸i“Z c`Ë MwYwZK Mo wbY‡qi mviwY n‡e wbgiƒc :

wefv‡Mi bvg	x_i	w_i	$x_i \; w_i$
MwYZ	70	80	5600
cwimsL¨vb	80	120	9600
Bs‡iwR	50	100	5000

evsjv	90	225	20250
cwiYwe`¨v	60	135	8100
ivóªweÁvb	85	300	25500
†gvU		960	74050

$$\overline{x_w} = \frac{\sum_{i=1}^{6} x_i w_i}{\sum_{i=2}^{6} w_i} = \frac{74050}{960} = 77 \cdot 14$$

cv‡mi Mo nvi 77·14

KvR : †Zvgv‡`i Dc‡Rjvi K‡qKwU ¯‹‡ji Gm.Gm.wm. cv‡mi nvi I Zv‡`i msL¨v msMn Ki Ges cv‡mi Mo nvi wbYq Ki|

ga¨K

8g †kwY‡Z Avgiv wk‡LwQ †h, †Kv‡bv cwimsL¨v‡bi DcvË¸‡jv gv‡bi µgvbmv‡i mvRv‡j †hmKj DcvË mgvb `Bfv‡M fvM K‡i †mB gvbB n‡e DcvË¸‡jvi ga¨K| Avgiv Avi I †R‡bwQ †h, hw` Dcv‡Ëi msL¨v n nq Ges n hw` we‡Rvo msL¨v nq Z‡e ga¨K n‡e $\frac{n+1}{2}$ Zg c‡`i gvb| Avi n hw` †Rvo msL¨v nq, Z‡e ga¨K n‡e $\frac{n}{2}$ Zg I $\left(\frac{n}{2}+1\right)$ Zg c` `BwUi mvsL¨K gv‡bi Mo| GLv‡b Avgiv mÎ e¨envi bv K‡i Ges e¨envi K‡i Kxfv‡e ga¨K wbYq Kiv nq Zv D`vniY‡Yi gva¨‡g Dc¯’vcb Kiv n‡jv|

D`vniY 10| wb‡Pi 51 Rb wk¶v_©xi D”PZvi (†m.wg.) MYmsL¨v wb‡ekb mviwY †`Iqv n‡jv| ga¨K wbYq Ki|

D”PZv (†m.wg.)	150	155	160	165	170	175
MYmsL¨v	4	6	12	16	8	5

mgvavb : ga¨K wbY‡qi MYmsL¨v mviwY

D”PZv †m.wg.)	150	155	160	165	170	175
MYmsL¨v	4	6	12	16	8	5
µg‡hwRZ MYmsL¨v	4	10	22	38	46	51

GLv‡b n = 51 hv we‡Rvo msL¨v

∴ ga¨K = $\frac{51+1}{2}$ Zg c‡`i gvb

= 26 Zg c‡`i gvb, = 165

wb‡Yq ga¨K 165 †m.wg.|

j¶ Kwi : 23 †_‡K 38 Zg c‡`i gvb 165|

D`vniY 11 : wb‡Pi 60 Rb wk¶v_©xi MwY‡Z cvß b¤‡ii MYmsL¨v wb‡ekb mviwY †`Iqv n‡jv| ga¨K wbYq Ki :

cvß b¤i	40	45	50	55	60	70	80	85	90	95	100
MYmsL¨v	2	4	4	3	7	10	16	6	4	3	1

mgvavb : ga¨K wbY‡qi µg‡hwRZ MYmsL¨v mviwY n‡jv :

cvß b¤i	40	45	50	55	60	70	80	85	90	95	100
MYmsL¨v	2	4	4	3	7	10	16	6	4	3	1
µg‡hwRZ MYmsL¨v	2	6	10	13	20	30	46	52	56	59	60

GLv‡b, n = 60 hv †Rvo msL¨v|

$$\therefore \text{ ga¨K } = \frac{\frac{60}{2}\text{ Zg I } \frac{60}{2}+1 \text{ Zg c` `BwUi gv‡bi mgwó}}{2}$$

$$= \frac{\text{30 Zg I 31 Zg c` `BwUi gv‡bi mgwó}}{2}$$

$$= \frac{70+80}{2} = \frac{150}{2} = 75$$

∴ wb‡Yq ga¨K 75|

KvR : 1| †Zvgv‡`i †kwYi 49 Rb wk¶v_x©i D”PZv (†m.wg.) wb‡q MYmsL¨v mviwY ˆZwi Ki Ges †Kvb mÎ e¨envi bv K‡i ga¨K wbYq Ki|
2| c‡ei mgm¨v †_‡K 9 R‡bi D”PZv ev` w`‡q 40 R‡bi D”PZvi (†m.wg.) ga¨K wbYq Ki|

†kwYweb¨¯Í Dcv‡Ëi ga¨K wbYq

hw` †kwYweb¨¯Í Dcv‡Ëi msL¨v nq $n$, Z‡e †kwYweb¨¯Í Dcv‡Ëi $\frac{n}{2}$ Zg c‡`i gvb n‡”Q ga¨K| Avi $\frac{n}{2}$ Zg c‡`i gvb ev ga¨K wbY‡q e¨eüZ mÎ n‡jv ga¨K = $L+\left(\frac{n}{2}-F_c\right)\times\frac{h}{f_m}$, †hLv‡b L n‡jv †h †kwY‡Z ga¨K Aew¯Z †mB †kwYi wbgœmxgv, n MYmsL¨v, F_c ga¨K †kwYi ceeZx †kwYi †hwRZ MYmsL¨v, f_m ga¨K †kwYi MYmsL¨v Ges h †kwY e¨wß|

D`vniY 12| wb‡Pi MYmsL¨v wb‡ekY mviwY †_‡K ga¨K wbYq Ki :

mgq (†m‡KÊ)	30–35	36–41	42–47	48–53	54–59	60–65
MYmsL¨v	3	10	18	25	8	6

mgvavb : ga¨K wbY‡qi MYmsL¨v wb‡ekY mviwY :

mgq (†m‡K‡Ê) †kwY e¨wß	MYmsL¨v	µg‡hwRZ MYmsL¨v
30 – 35	3	3
36 – 41	10	13
42 – 47	18	31
48 – 53	25	56

60 – 65	6	70
	n = 70	

GLv‡b, $n = 70$ Ges $\frac{n}{2} = \frac{70}{2}$ ev 35|

AZGe, ga¨K n‡jv 35 Zg c‡`i gvb| 35 Zg c‡`i Ae¯'vb n‡e (48-53) †kwY‡Z| AZGe ga¨K †kwY n‡jv (48-53)|

mZivs, $L = 48, F_c = 31, f_m = 25$ Ges $h = 6$|

ga¨K $= L + \left(\frac{n}{2} - F_c\right) \times \frac{h}{fm}$

$= 48 + (35 - 31) \times \frac{6}{25} = 48 + 4 \times \frac{6}{25}$

$= 48 + 0{\cdot}96$

$= 48{\cdot}96$

wb‡Yq ga¨K 48·96

KvR : †Zvgv‡`i †kwYi mKj wk¶v\_x‡K wb‡q 2wU `j MVb Ki| GKwU mgm¨v mgvavb c‡Z¨‡Ki KZ mgq jv‡M (K) Zvi MYmsL¨v wb‡ekY mviwY ˆZwi Ki, (L) mviwY n‡Z ga¨K wbYq Ki|

cPiK

8g †kwY‡Z Avgiv wk‡LwQ †h, †Kv‡bv Dcv‡Ë †h msL¨v mewaK evi Dc¯'wcZ nq, †mB msL¨vB Dcv‡Ëi cPiK| GKwU Dcv‡Ëi GK ev GKwaK cPiK _vK‡Z cv‡i| †Kv‡bv Dcv‡Ë hw` †Kv‡bv msL¨vB GKwaKevi bv _v‡K Z‡e †mB Dcv‡Ëi †Kv‡bv cPiK †bB| GLv‡b Avgiv Kxfv‡e mÎ e¨envi K‡i †kwYweb¨¯— Dcv‡Ëi cPiK wbYq Ki‡Z nq, ZvB Av‡jvPbv Kiv n‡jv|

†kwY web¨¯— Dcv‡Ëi cPiK wbYq

†kwY web¨¯— Dcv‡Ëi cPiK wbY‡qi mÎ n‡jv :

cPiK = $L + \frac{f_1}{f_1 + f_2} \times h$ †hLv‡b L cPiK †kwYi A_vr †h †kwY‡Z cPiK Aew¯'Z Zvi wbggvb,

f_1 = cPiK †kwYi MYmsL¨v–ceeZx †kwYi MYmsL¨v, $f_2 =$ cPiK †kwYi MYmsL¨v-cieZx †kwYi MYmsL¨v

Ges h = †kwY e¨wß|

D`vniY 13| wb‡Pi MYmsL¨v wb‡ekY mviwY †_‡K cPiK wbYq Ki|

†kwY	MYmsL¨v
31 – 40	4
41 – 50	6
51 – 60	8
61 – 70	12

mgvavb :

cPiK = $L + \frac{f_1}{f_1 + f_2} \times h$

GLv‡b MYmsL¨v mewaK evi 12 Av‡Q (71-80) †kwY‡Z|

mZivs, $L = 61$

$f_2 = 12 - 8 = 4$

$f_2 = 12 - 9 = 3$

$d = 10$

71 – 80	9
81 – 90	7
91 – 100	4

$\therefore$ cPiK $= 61 + \dfrac{4}{4+3} \times 10 = 61 + \dfrac{4}{7} \times 10$

$= 61 + \dfrac{40}{7} = 61 + 5\cdot7 = 66\cdot7$ |

wb‡Yq cPiK 66·714

D`vniY 14| wb‡Pi MYmsL¨v wb‡ekY mviwY †_‡K cPiK wbYq Ki :

†kwY	MYmsL¨v
41 – 50	25
51 – 60	20
61 – 70	15
71 – 80	8

mgvavb : GLv‡b MYmsL¨v mewaK evi 25 Av‡Q (41-50) †kwY‡Z| mZivs, cPiK GB †kwY‡Z Av‡Q| Avgiv Rwb,

cPiK = $L + \dfrac{f_1}{f_1 + f_2} \times h$

GLv‡b, $L = 41$ [c_g †kwY‡Z MYmsL¨v †ewk n‡j, ceeZx †kwYi MYmsL¨v kb¨]

$f_1 = 25 - 0$

$f_2 = 25 - 20 = 5$

$\therefore$ cPiK $= 41 + \dfrac{25}{25+5} \times 10$

$= 41 + \dfrac{25}{30} \times 10 = 51 + 8 \cdot 33$ |

$= 49\cdot33$

wb‡Yq cPiK 49·33

†kwY web¨— Dcv‡Ë c_g †kwY cPiK †kwY n‡j, Zvi Av‡Mi †kwYi MYmsL¨v kb¨ ai‡Z nq

D`vniY 15| wb‡Pi MYmsL¨v wb‡ekY mviwYi cPiK wbYq Ki :

mgvavb :

GLv‡b MYmsL¨v mewaK evi 25 Av‡Q (41-50) †kwY‡Z| GB †kwY‡Z cPiK we`¨gvb Avgiv Rwb,

cPiK = $L + \dfrac{f_1}{f_1 + f_2} \times h$

†kwY	MYmsL¨v
10 – 20	4
21 – 30	16
31 – 40	20
41 – 50	25

GLv‡b, $L = 41$
$f_1 = 25 - 20 = 5$
$f_2 = 25 - 0$ [†kl †kwY cPiK †kwY n‡j, cieZx
†kwYi NUb msL¨v kb¨ aiv nq]
$h = 10$

AZGe, cPiK = 41 + $\frac{5}{25} \times 10$
= 41 + 2 = 42

wb‡Yq cPiK 42|

Abkxjbx 17

mwVK DË‡i wUK (√) wPý `vI :

1| wb‡Pi †KvbwU Øviv †kwY e¨wß †evSvq ?
(K) DcvËmg‡ni g‡a¨ enËg I ¶`Zg Dcv‡Ëi e¨eavb
(L) DcvËmg‡ni g‡a¨ c_g I †kl Dcv‡Ëi e¨eavb
(M) c‡Z¨K †kwYi Aš—f© enËg I ¶`Zg msL¨vi cv_K¨
(N) c‡Z¨K †kwYi Aš—f© enËg I ¶`Zg msL¨vi mgwó

2| DcvËmgn mvwiYf³ Kiv n‡j cwZ †kwY‡Z hZ¸‡jv DcvË Aš—f© nq Zvi wb‡`kK wb‡Pi †KvbwU ?
(K) †kwY mxgv (L) †kwYi ga¨we›` (M) †kwY msL¨v (N) †kwYi MYmsL¨v

3| cwimsL¨v‡bi Aweb¨š— DcvËmgn g‡bi µgvbmv‡i mvRv‡j DcvËmgn gvSvgwS †Kv‡bv g‡bi KvQvKwQ cwÄfZ nq| Dcv‡Ëi GB ceYZv‡K ejv nq
(K) cPiK (L) †Kw›`q ceYZv (M) Mo (N) ga¨K

kxZKv‡j evsjv‡`‡ki †Kvb GKwU A‡ji 10 w`‡bi ZvcgvÎvi (†mw›U‡MW) cwimsL¨vb n‡jv 10°, 9°, 8°, 6°, 11°, 12°, 7°, 13°, 14°, 5°| GB cwimsL¨v‡bi †cwі‡Z (4-6) chš— ck¸‡jvi DËi `vI |

4| Dc‡ii msL¨vmPK Dcv‡Ëi cPiK †KvbwU ?
(K) 12° (L) 5° (M) 14° (N) cPiK †bB

5| Dc‡ii msL¨vmPK Dcv‡Ëi Mo ZvcgvÎv †KvbwU ?
(K) 8° (L) 8.5° (M) 9.5° (N) 9°

6| DcvËmg‡ni ga¨K †KvbwU ?
(K) 9.5° (L) 9° (M) 8.5° (N) 8°

7| mvwiYf³ †kwYweb¨¯— Dcv‡Ëi msL¨v n‡jv n, ga¨K †kwYi wbgmxgv L, ga¨K †kwYi ceeZx †kwYi µg‡hwRZ MYmsL¨v F_c, ga¨K †kwYi MYmsL¨v f_m Ges †kwY e¨wß h GB Z‡_¨i Av‡jv‡K wb‡Pi †KvbwU ga¨K wbY‡qi mÎ ?
(K) $L + \left(\frac{n}{2} - F_c\right) \times \frac{h}{f_m}$ (L) $L + \left(\frac{n}{2} - f_m\right) \times \frac{h}{F_m}$

(M) $L-\left(\frac{n}{2}-F_c\right)\times\frac{h}{f_m}$ (N) $L-\left(\frac{n}{2}-f_n\right)\times\frac{h}{F_m}$

wb‡P †Zvgv‡`i ¯‹‡ji 8g †kwYi mgvcbx cix¶vq evsjvq cvß b¤^‡ii MYmsL¨v mviwY †`Iqv n‡jv|
GB mviwY †_‡K (8–17)chš— c‡ki DËi `vI :

†kwY e¨wß	31–40	41–50	51–60	61–80	71–80	81–90	91–100
MYmsL¨v	6	12	16	24	12	8	2
µg‡hwRZ MYmsL¨v	6	18	34	58	70	78	80

8| DcvËmg‡ni KqwU †kwY‡Z wef³ Kiv n‡q‡Q ?
(K) 6 (L) 7 (M) 8 (N) 9

9| mviwY‡Z Dc¯vwcZ Dcv‡Ëi †kwY e¨wß KZ ?
(K) 5 (L) 9 (M) 10 (N) 15

10| 4_ †kwYi ga¨gvb KZ ?
(K) 71.5 (L) 61.5 (M) 70.5 (N) 75.6

11| Dcv‡Ëi ga¨K †kwY †KvbwU ?
(K) 41–50 (L) 51–60 (M) 61–70 (N) 71–80

12| ga¨K †kwYi ceeZx †kwYi †hwRZ MYmsL¨v KZ ?
(K) 18 (L) 34 (M) 58 (N) 70

13| ga¨K †kwYi wbgmxgv KZ ?
(K) 41 (L) 51 (M) 61 (N) 71

14| ga¨K †kwYi MYmsL¨v KZ ?
(K) 16 (L) 24 (M) 34 (N) 58

15| Dc¯wcZ Dcv‡Ëi ga¨K KZ ?
(K) 63 (L) 63.5 (M) 65 (N) 65.5

16| Dc¯wcZ Dcv‡Ëi cPiK KZ ?
(K) 61.4 (L) 61 (M) 70 (N) 70.4

17| †Kvb ¯‹‡ji 10g †kwYi 50 Rb wk¶v_xi IRb (wK‡jvMvg) n‡jv :
45, 50, 55, 51, 56, 57, 56, 60, 58, 60, 61, 60, 62, 60, 63, 64, 60,
61, 63, 66, 67, 61, 70, 70, 68, 60, 63, 61, 50, 55, 57, 56, 63, 60,
62, 56, 67, 70, 69, 70, 69, 68, 70, 60, 56, 58, 61, 63, 64|
(K) †kwY e¨eavb 5 a‡i MYmsL¨v wb‡ekb mviwY ˆZwi Ki|
(L) mviwY †_‡K msw¶ß c×wZ‡Z Mo wbYq Ki|
(M) MYmsL¨v wb‡ekb mviwY‡Z Dc¯wcZ Dcv‡Ëi MYmsL¨v eûfR AvK|

18| 10g †kwYi 50 Rb wk¶v_xi MwYZ wel‡q cvß b¤^‡ii MYmsL¨v wb‡ekb mviwY †`Iqv n‡jv| c`Ë Dcv‡Ëi MYmsL¨v eûfR AvK|

†kwY e¨wß	31–40	41–50	51–60	61–80	71–80	81–90	91–100
MYmsL¨v	6	8	10	12	5	7	2

19| †Kvb †kÖwYi 60 Rb wk¶v_©xi 50 b¤^‡ii mgwqK cix¶vq cÖvß b¤^‡ii MYmsL¨v wb‡ekb mviwY n‡jv :

cÖvß b¤^i	1–10	11–20	21–30	31–40	41–50
MYmsL¨v	7	10	16	18	9

Dcv‡Ëi AwRf †iLv AvK|

20| wb‡P 50 Rb wk¶v_©xi IR‡bi (†KwR) MYmsL¨v wb‡ekb mviwY †`Iqv n‡jv| ga¨K wbYq Ki|

IRb (†KwR)	45	50	55	60	65	70
MYmsL¨v	2	6	8	16	12	6

21| †Zvgv‡`i †kÖwYi 60 Rb wk¶v_©xi IR‡bi (†KwR) MYmsL¨v wb‡ekb mviwY n‡jv :

e¨wß	45-49	50-54	55-59	60-64	65-69	70-74
MYmsL¨v	4	8	10	20	12	6
†hvwRZ dj	4	12	22	42	54	60

(K) Dcv‡Ëi ga¨K wbYq Ki|

(L) Dcv‡Ëi cPziK wbYq Ki|

22| Dcv‡Ëi †¶‡Î cPziK-

(i) †K›`ªxq cebZvi cwigvc :

(ii) me‡P‡q †ekx evi Dc¯wcZ gvb

(iii) me‡¶‡Î Abb¨ bvI n‡Z cv‡i

Dc‡ii Z‡_¨i wfwË‡Z wb‡Pi †KvbwU mwVK?

K) i I ii L) i I iii

M) ii I iii N) i, ii I iii

23| †Kv‡bv we`¨vj‡qi ewlK cix¶vq 9g †kÖwYi 50 Rb wk¶v_©xi MwY‡Z cÖvß b¤^i¸‡jv wbgœiƒc:

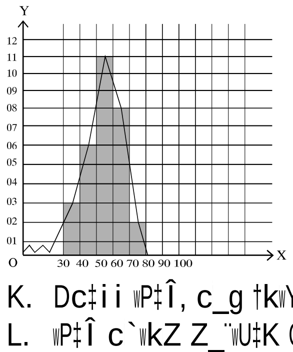
76,	65,	98,	79,	64	68,	56,	73,	83,	57
55,	92,	45,	77,	87	46,	32,	75,	89,	48
97,	88,	65,	73,	93	58,	41,	69,	63,	39
84,	56,	45,	73,	93	62,	67,	69,	65,	53
78,	64,	85,	53,	73	34,	75,	82,	67,	62

K. c`Ë Z_¨wU aiY Kxiƒc? †Kv‡bv wb‡e©I‡Y GKwU †kÖwYi MYmsL¨v Kx wb‡`©k K‡i ?

L. Dch³ †kÖwY e¨wß wb‡q MYmsL¨v wb‡e©lY ˆZwi Ki|

M. msw¶ß c×wZ‡Z cÖvß b¤^‡ii Mo wbYq Ki|

24|



K. Dc‡ii wP‡Î, c_g †kÖwYwUi †kÖwY ga¨gvb I †kl †kÖwYwUi MYmsL¨v KZ?

L. wP‡Î c`wkZ Z_¨wU‡K Q‡K Ki gva¨‡g cKvk Ki|

M. ÔLÕ-As‡k cÖvß QK †_‡K wb‡e©lYwUi ga¨K wbYq Ki|

DËigvjv

Abkxjbx 1

4| (K) $0\cdot1\dot{6}$ (L) $0\cdot\dot{6}\dot{3}$ (M) $3\cdot\dot{2}$ (N) $3\cdot5\dot{3}$

5| (K) $\frac{2}{9}$ (L) $\frac{35}{99}$ (M) $\frac{2}{15}$ (N) $3\frac{71}{90}$ (O) $6\frac{769}{3330}$

6| (K) $2\cdot3\dot{3}\dot{3}$, $5\cdot2\dot{3}\dot{5}$ (L) $7\cdot26\dot{6}$, $4\cdot23\dot{7}$ (M) $5\cdot\dot{7}7777\dot{7}$, $8\cdot\dot{3}4343\dot{4}$, $6\cdot\dot{2}4524\dot{5}$

(N) $12\cdot32\dot{0}\dot{0}$, $2\cdot19\dot{9}\dot{9}$, $4\cdot32\dot{5}\dot{6}$

7| (K) $0\cdot58\dot{9}$ (L) $17\cdot117\dot{9}$ (M) $1\cdot92\dot{6}3\dot{1}$

8| (K) $1\cdot3\dot{1}$ (L) $1\cdot6\dot{6}\dot{5}$ (M) $3\cdot13\dot{3}\dot{4}$ (N) $6\cdot11\dot{0}6\dot{2}$

9| (K) $0\cdot2$ (L) 2 (M) $0\cdot\dot{2}07\dot{4}$ (N) $12\cdot1\dot{8}\dot{5}$

10| (K) $0\cdot5$ (L) $0\cdot2$ (M) $5\cdot\dot{2}195\dot{1}$ (N) $4\cdot\dot{8}$

11| (K) $3\cdot4641$, $3\cdot464$ (L) $0\cdot5025$, $0\cdot503$ (M) $1\cdot1595$, $1\cdot160$ (N) $2\cdot2650$, $2\cdot265$

12| (K) gj` (L) gj` (M) Agj` (N) Agj` (O) Agj` (P) Agj` (Q) gj` (R) gj`

13| (K) 9 (L) 5

Abkxjbx 2·1

1| (K) $\{4, 5\}$ (L) $\{\pm3, \pm4, \pm5, \pm6\}$ (M) $\{6, 12, 18, 36\}$ (N) $\{3, 4\}$

2| (K) $\{x \in N : x$ we‡Rvo msL¨v Ges $1 < x < 13\}$ (L) $\{x \in N : x, 36$ Gi ¸YbxqK$\}$ (M) $\{x \in N : x, 4$ Gi ¸YwbqK Ges $x \leq 40\}$ (N) $\{x \in Z : x^2 \geq 16$ Ges $x^3 \leq 216\}$

3| (K) $\{1\}$ (L) $\{1, 2, 3, 4, a\}$ (M) $\{2\}$ (N) $\{2, 3, 4, a\}$ (O) $\{2\}$

5| $\{\{x, y\}, \{x\}, \{y\}, \Phi\}$, $\{\{m, n, l\}, \{m, n\}, \{m, l\}, \{n, l\}, \{m\}, \{n\}, \{l\}, \Phi\}$

7| (K) $2, 3$ (L) (a, c) (M) $(1, 5)$

8| (K) $\{(a, b), (a, c)\}, \{(b, a), (c, a)\}$ (L) $\{(4, x), (4, y), (5, x), (5, y)\}$ (M) $\{(3, 3), (5, 3), (7, 3)\}$

9| $\{1, 3, 5, 7, 9, 15, 35, 45\}$ Ges $\{1, 5\}$ 10| $\{35, 105\}$ 11| 5 Rb

Abkxjbx 2·2

4| $\{(3, 2), (4, 2)\}$ 5| $\{(2, 4), (2, 6)\}$ 6| $-7, 23, \frac{-7}{16}$ 7| 2 8| 1 A_ev 2 A_ev 3 9| $\frac{4}{x}$

11| (K) $\{2\}, \{1, 2, 3\}$ (L) $\{-2, -1, 0, 1, 3), (2, -1)\}$ (M) $\left\{\frac{1}{2}, 1, \frac{5}{2}\right\}, \{0, 1, -1, 2, -2\}$

12| (K) $\{(-1, 2), (0, 1), (1, 0), (2, -1)\}$, $\{-1, 0, 1, 2\}$, $\{2, 1, 0, -1\}$

(L) $\{(-1, -2), (0, 0), (1, 2)\}$, $\{-1, 0, 1\}$, $\{-2, 0, 2\}$

Abkxjbx 3·1

1| (K) $4a^2+12ab+9b^2$ (L) $4a^2b^2+12ab^2c+9b^2c^2$ (M) $x^4+\frac{4x^2}{y^2}+\frac{4}{y^4}$ (N) $a^2+2+\frac{1}{a^2}$

(O) $16y^2-40xy+25x^2$ (P) $a^2b^2-2abc+c^2$ (Q) $25x^4-10x^2y+y^2$

(R) $x^2+4y^2+16z^2+4xy+16yz+8zx$ (S) $9p^2+16q^2+25r^2+24pq-40qr-30pr$

(T) $9b^2+25c^2+4a^2-30bc+20ca-12ab$ (U) $a^2x^2+b^2y^2+c^2z^2-2abxy+2bcyz-2cazx$

(V) $a^2+b^2+c^2+d^2-2ab+2ac-2ad-2bc+2bd-2cd$

(W) $4a^2+9x^2+4y^2+25z^2+12ax-8ay-20az-12xy-30xz+20yz$ (X) 10201

(Y) 994009 (Z) 10140491

2| (K) $16a^2$ (L) $36x^2$ (M) p^2+49r^2-14rp (N) $36n^2-24pn+4p^2$ (O) 100

(P) 4410000 (Q) 10 (R) 3104

3| ± 16 4| ± 1 5| $\pm 3m$ 6| 130 8| $\frac{1}{4}$ 11| 19 12| 25 13| 6 14| 138

15| 9 17| $(2a+b+c)^2-(b-a-c)^2$ 18| $(x-1)^2-8^2$ 19| $(x+5)^2-1^2$ 20| (i) 3

20| (ii) 1

Abkxjbx 3·2

1| (K) $8x^3+60x^2+150x+125$ (L) $8x^6+36x^4y^2+54x^2y^4+27y^6$

(M) $64a^3-240a^2x^2+300ax^4-125x^6$ (N) $343m^6-294m^4n+84m^2n^2-8n^3$

(O) 65450827 (P) 994011992

(Q) $8a^3-b^3-27c^3-12a^2b-36a^2c+6ab^2+54ac^2-9b^2c-27bc^2+36abc$

(R) $8x^3+27y^3+z^3+36x^2y+12x^2z+54xy^2+27y^2z+6xz^2+9yz^2+36xyz$

2| (K) $8a^3$ (L) $64x^3$ (M) $8x^3$ (N) 1 (O) $8(b+c)^3$ (P) $64m^3n^3$ (Q) $2(x^3+y^3+z^3)$ (R) $64x^3$

3| 665 4| 54 5| 8 6| 42880 7| 1728 10| (K) 3 (L) 9 11| (K) 133 (L) 665

12| a^3-3a 13| p^3+3p 14| $46\sqrt{5}$

Abkxjbx 3·3

1| $(a+b)(a+c)$ 2| $(b+1)(a-1)$

3| $2(x-y)(x+y+z)$ 4| $b(x-y)(a-c)$

5| $(3x+4)^2$

6| $(a^2+5a-1)(a^2-5a-1)$

7| $(x^2+2xy-y^2)(x^2-2xy-y^2)$

8| $(ax+by+ay-by)(ax+bx-ay+bx)$

9| $(2a-3b+2c)(2a-3b-2c)$

10| $9(x+a)(x-a)(x+2a)(x-2a)$

11| $(a+y+2)(a-y+4)$

12| $(4x-5y)(4x+5y-2z)$

13| $(a+b+c)(b+c-a)(c+a-b)(a+b-c)$

14| $(x+4)(x+9)$

15| $(x+2)(x-2)(x^2+5)$

16| $(a-18)(a-12)$

17| $(x^3y^3-3)(x^3y^3+2)$

18| $(a^4-2)(a^4+1)$

19| $(ab+7)(ab-15)$

20| $(x+13)(x-15)$

21| $(x+2)(x-2)(2x+3)(2x-3)$

22| $(2x-5)(6x-4)$

23| $y^2(x+1)(9x-14)$

24| $(x+3)(x-3)(4x^2+9)$

25| $(x+a)(ax+1)$

26| $(a^2+2a-4)(3a^2+6a-10)$

27| $(2z-3x-5)(10x+7z+3)$

28| $-(3a+17b)(9a+7b)$

29| $(x+ay+y)(ax-x+y)$

30| $3x(2x-1)(4x^2+2x+1)$

31| $(a+b)^2(a^4-2a^3b+6a^2b^2-2ab^3+b^4)$

32| $(x+2)(x^2+x+1)$

33| $(a-3)(a^2-3a+3)$

34| $(a-b)(2a^2+5ab+8b^2)$

35| $(2x-3)(4x^2+12x+21)$

36| $\frac{1}{27}(6a+b)(36a^2-6ab+b^2)$

37| $\frac{1}{8}(2a-1)(4a^2+2a+1)$

38| $\left(\frac{a^2}{3}-b^2\right)\left(\frac{a^4}{9}+\frac{a^2b^2}{3}+b^4\right)$

39| $\left(2a-\frac{1}{2a}\right)\left(2a-\frac{1}{2a}+2\right)$

40| $(a+4)(19a^2-13a+7)$

41| $(x+6)(x-10)$

42| $(x^2+7x+4)(x^2+7x-18)$

43| $(x^2-8x+20)(x^2-8x+2)$

Abkxjbx 3·4

1| $(6x-1)(x-1)$

2| $(a+1)(3a^2-3a+5)$

3| $(x+y)(x-3y)(x+2y)$

4| $(x-6)(x+1)$

5| $(2x-3)(x+1)$

6| $(x-3)(3x+2)$

7| $(x-2)(x+1)(x+3)$

8| $(x-1)(x+2)(x+3)$

৯। $(a+3)(a^2-3a+12)$

১০। $(a-1)(a-1)(a^2+2a+3)$

১১। $(a+1)(a-4)(a+2)$

১২। $(x-2)(x^2-x+2)$

১৩। $(a-b)(a^2-6ab+b^2)$

১৪। $(x-3)(x^2+3x+8)$

১৫। $(x+y)(x+3y)(x+2y)$

১৬। $(x-2)(2x+1)(x^2+1)$

১৭। $(2x-1)(x+1)(x+2)(2x+1)$

১৮। $x(x-1)(x^2+x+1)(x^2-x+1)$

১৯। $(4x-1)(x^2-x+1)$

২০। $(2x+1)(3x+2)(3x-1)$

অনুশীলনী ৩·৫

১। (গ) ২। (ঘ) ৩। (খ) ৪। (খ)

৫। (ঘ) ৬। (গ) ৭। (গ) ৮। (ঘ)

৯। (ক) ১০। (গ) ১১। (ঘ) ১২। (খ)

১৩। (ক) ১৪। (খ) ১৫। (গ) ১৬। (খ)

১৭। (ক) ১৮। (খ) ১৯। (গ) ২০। (ঘ)

২১ (১) (ঘ), (২) (খ) ২১ (৩)। (ঘ) ২২। $\frac{2}{3}(p+r)$ দিনে ২৩। 5 ঘণ্টা

২৪। $\frac{xy}{x+y}$ দিনে ২৫। 95 জন

২৬। স্রোতের বেগ ঘণ্টায় $\frac{d}{2}\left(\frac{1}{q}-\frac{1}{p}\right)$ কি.মি. এবং নৌকার বেগ ঘণ্টায় $\frac{d}{2}\left(\frac{1}{p}+\frac{1}{q}\right)$ কি.মি.

২৭। দাঁড়ের বেগ 8 কি.মি./ঘণ্টা এবং স্রোতের বেগ 2 কি.মি./ঘণ্টা

২৮। $\frac{t_1 t_2}{t_2 - t_1}$ মিনিট ২৯। 240 লিটার ৩০। 10 টাকা। ৩১। 48 টাকা ৩২। (ক) 120 টাকা, (খ) 80 টাকা, (গ) 60 টাকা ৩৩। ক্রয়মূল্য 450 টাকা ৩৪। 4% ৩৫। 625 টাকা ৩৬। 5%

৩৭। $522 \cdot 37$ টাকা (প্রায়) ৩৮। 780 টাকা ৩৯। 61 টাকা

৪০। $\frac{px}{100+x}$ টাকা ভ্যাট ; ভ্যাটের পরিমাণ 300 টাকা।

অনুশীলনী ৪·১

১। 9 ২। $\frac{1}{2}$ ৩। $\frac{10}{7}$ ৪। $\frac{ab}{3a+2b}$ ৫। 27 ৬। $\frac{a^2}{b}$ ৭। 343

৮। 1 ৯। 4 ১০। $\frac{1}{9}$ ১৯। $\frac{3}{2}$ ২০। 3 ২১। 5 ২২। 0, 1

অনুশীলনী ৪·২

১। (ক) 4 (খ) $\frac{1}{3}$ (গ) $\frac{1}{2}$ (ঘ) 4 (ঙ) $\frac{5}{6}$

২। (ক) 125 (খ) 5 (গ) 4

৪। (ক) $\log 2$ (খ) $\frac{13}{15}$ (গ) 0

অনুশীলনী ৪·৩

১। খ ২। ঘ ৩। গ ৪। ক ৫। গ ৭। ঘ ৮। (১) ঘ (২) গ (৩) ক

৯। (ক) $6 \cdot 530 \times 10^3$ (খ) $6 \cdot 0831 \times 10^1$ (গ) $2 \cdot 45 \times 10^{-4}$ (ঘ) $3 \cdot 75 \times 10^7$ (ঙ) $1 \cdot 4 \times 10^{-7}$

১০। (ক) 100000 (খ) $0 \cdot 000001$ (গ) 25300 (ঘ) $0 \cdot 009813$ (ঙ) $0 \cdot 0000312$

১১। (ক) 3 (খ) 1 (গ) 0 (ঘ) $\bar{2}$ (ঙ) $\bar{5}$

১২। (ক) পূর্ণক 1, অংশক $\cdot 43136$ (খ) পূর্ণক 1, অংশক $\cdot 80035$ (গ) পূর্ণক 0, অংশক $\cdot 14765$ (ঘ) পূর্ণক $\bar{2}$, অংশক $\cdot 65896$ (ঙ) পূর্ণক $\bar{4}$, অংশক $\cdot 82802$

১৩। (ক) $1\cdot 66706$ (খ) $\bar{1}\cdot 64562$ (গ) $0\cdot 81358$ (ঘ) $\bar{3}\cdot 78888$

১৪। (ক) $0\cdot 95424$ (খ) $1\cdot 44710$ (গ) $1\cdot 62325$

১৫। ক. $2^3 \cdot 5^3$ খ. $6\cdot 25\times 10^1$ গ. পূর্ণক 1, অংশক $\cdot 79588$

অনুশীলনী ৫·১

১। 1 ২। ab ৩। -6 ৪। -1 ৫। $-\frac{3}{5}$ ৬। $-\frac{5}{2}$ ৭। $\frac{a+b}{2}$ ৮। $a+b$

৯। $\frac{a+b}{2}$ ১০। $\sqrt{3}$ ১১। $\{2\}$ ১২। $\{4(1+\sqrt{2})\}$ ১৩। $\{-a\}$ ১৪। Φ

১৫। $\left\{-\frac{1}{3}\right\}$ ১৬। $\left\{\frac{m+n}{2}\right\}$ ১৭। $\left\{-\frac{7}{2}\right\}$ ১৮। $\{6\}$ ১৯। $\left\{-(a^2+b^2+c^2)\right\}$

২০। 28, 70 ২১। $\frac{3}{4}$ ২২। 72 ২৩। ২৪। 18 ২৫। .9

২৬। পঁচিশ পয়সার মুদ্রা 100টি, পঞ্চাশ পয়সার মুদ্রা 20টি।

২৭। 120 কিলোমিটার

অনুশীলনী ৫·২

১। গ ২। খ ৩। খ ৪। গ ৫। ঘ ৬। খ ৭। ক ৮। (১) ঘ (২) গ (৩) ক

৯। $-2, \sqrt{3}$ ১০। $-\frac{3\sqrt{2}}{2}, \frac{2\sqrt{3}}{3}$ ১১। $-1, 6$ ১২। ± 7 ১৩। $-6, \frac{3}{2}$ ১৪। $1, -\frac{3}{20}$

১৫। $\frac{1}{2}, 2$ ১৬। $0, \frac{2}{3}$ ১৭। $\pm\sqrt{ab}$ ১৮। $0,\ a+b$ ১৯। $\left\{3, -\frac{1}{2}\right\}$ ২০। $\left\{-\frac{2}{3}, 2\right\}$

২১। $\{-a, -b\}$ ২২। $\{1, -1\}$ ২৩। $\{1\}$ ২৪। $\{0, 2a\}$ ২৫। $\left\{\frac{1}{3}, 1\right\}$ ২৬। 78 বা 87

২৭। দৈর্ঘ্য 16 মিটার, প্রস্থ 12 মিটার ২৮। 9 সে.মি., 12 সে.মি. ২৯। 27 সে.মি.

৩০। 21 জন, 20 টাকা করে। ৩১। 70 ৩২। ক. $70-9x,\ 9x+7$ খ. 34 গ. 5 সে.মি., $5\sqrt{2}$ সে.মি.

৩৩। খ. 5 সে.মি. গ. 2:5:8

অনুশীলনী-৯.১

২। $\cos A = \frac{\sqrt{7}}{4}$, $\tan A = \frac{3}{\sqrt{7}}$, $\cot A = \frac{\sqrt{7}}{3}$, $\sec A = \frac{4}{\sqrt{7}}$, $\operatorname{cosec} A = \frac{4}{3}$

৩। $\sin A = \frac{15}{17}$, $\cos A = \frac{8}{17}$

৪। $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

২২। $\frac{1}{2}$, ২৩। $\frac{3}{4}$, ২৪। $\frac{4}{3}$, ২৫। $\frac{a^2-b^2}{a^2+b^2}$,

অনুশীলনী-৯.২

৫। $\frac{1}{2}$ ৬। $\frac{3}{4\sqrt{2}}$ ৭। $\frac{23}{5}$ ৮। $\frac{2\sqrt{2}}{3}$ ১৭। $A = 30^0,\ B = 30^0$ ১৮। $A = 30^0$

১৯। $A = 37\frac{1}{2}^0$, $B = 7\frac{1}{2}^0$ ২১। $\theta = 90^0$ ২২। $\theta = 60^0$ ২৩। $\theta = 60^0$ ২৪। $\theta = 45^0$

২৫। 3

অনুশীলনী ১০

১-৬ নিজে কর।

৭। 45.033 মিটার (প্রায়) ৮। 34.641 মিটার (প্রায়) ৯। 12.728 মিটার (প্রায়) ১০। 10 মিটার

১১। 21.651 মিটার (প্রায়) ১২। 141.962 মিটার (প্রায়) ১৩। 83.138 মিটার (প্রায়) এবং 48 মিটার

১৪। 34.298 মিটার (প্রায়) ১৫। 44.785 মিটার (প্রায়) ১৬। (খ) 259.808 মিটার

অনুশীলনী ১১·১

১। $a^2 : b^2$, ২। $\sqrt{\pi} : 2$, ৩। 45, 60, ৪। 20%, ৫। 18 : 25, ৬। 13 : 7, ৮। (i) $\frac{3}{4}$, (ii) $\frac{2ab}{b^2+1}$, (iii) $x = \pm\sqrt{2ab - b^2}$, (iv) 10, (v) $\frac{b}{2a}\left(c + \frac{1}{c}\right)$, (vi) $\frac{1}{2}$, 2, 22. 3

অনুশীলনী ১১·২

১। খ ২। গ ৩। গ ৪। খ ৫। খ

৬। 24%, ৭। 70%, ৮। 70%, ৯। ক 40 টাকা, খ 60 টাকা, গ 120 টাকা, ঘ 80 টাকা, ১০। 200, 240, 250, ১১। 9 সে. মি., 15 সে. মি., 21 সে. মি., ১২। 315 টাকা, 336 টাকা, 360 টাকা, ১৩। 140, ১৪। 81 রান, 54 রান, 36 রান, ১৫। কর্মকর্তা 24000 টাকা, করণিক 12000 টাকা, পিওন 6000 টাকা, ১৬। 70, ১৭। 44%, ১৮। 1% হ্রাস পাবে, ১৯। 532 কুইন্টাল, ২০। 8 : 9, ২১। 1440 বর্গমিটার, ২২। 13 : 12.

অনুশীলনী-১২.১

১। সঙ্গতিপূর্ণ, অনির্ভরশীল, একটিমাত্র সমাধান ২। সঙ্গতিপূর্ণ, নির্ভরশীল, অসংখ্য সমাধান ৩। অসঙ্গতিপূর্ণ, অনির্ভরশীল, সমাধান নেই ৪। সঙ্গতিপূর্ণ, নির্ভরশীল, অসংখ্য সমাধান ৫। সঙ্গতিপূর্ণ, অনির্ভরশীল, একটিমাত্র সমাধান ৬। অসঙ্গতিপূর্ণ, অনির্ভরশীল, সমাধান নেই ৭। সঙ্গতিপূর্ণ, নির্ভরশীল, অসংখ্য সমাধান ৮। সঙ্গতিপূর্ণ, নির্ভরশীল, অসংখ্য সমাধান৯। সঙ্গতিপূর্ণ, অনির্ভরশীল, একটিমাত্র সমাধান ১০। সঙ্গতিপূর্ণ, অনির্ভরশীল, একটিমাত্র সমাধান

অনুশীলনী-১২.২

১। (4, -1) ২। $(\frac{6}{5}, \frac{6}{5})$ ৩। (a, b) ৪। (4, -1) ৫। (1, 2) ৬। $\left(\frac{a(b-c)}{a(b-a)}, \frac{c(c-a)}{b(b-a)}\right)$
৭। $(-\frac{17}{2}, 4)$ ৮। (2, 3) ৯। (3, 2) ১০। $(\frac{5}{2}, -\frac{22}{3})$ ১১। (1, 2) ১২। (2, -1) ১৩। (a, b)

১৪। (2, 4) ১৫। (4, 5)

অনুশীলনী-১২.৩

১। (2, 2) ২। (2, 3) ৩। (-7, 3) ৪। (4, 5) ৫। (2, 3) ৬। (1.5, 1.5) ৭। $(1, \frac{1}{2})$ ৮। (2, 6)

৯। -2 ১০। 2

অনুশীলনী-১২.৪

১। ক ২। গ ৩। খ ৪। ঘ ৫। খ ৬। খ ৭(১)। গ ৭(২)। ঘ ৭ (৩) ঘ ৮। $\frac{7}{9}$ ৯। $\frac{15}{26}$ ১০। 27

১১। 37 বা 73 ১২। 30 বছর ১৩। দৈর্ঘ্য 17 মিটার, প্রস্থ 9 মিটার ১৪। নৌকার বেগ ঘণ্টায় 10 কি. মি., স্রোতের বেগ ঘণ্টায় 5 কি. মি.। ১৫। চাকরি শুরুর বেতন 4000 টাকা, বার্ষিক বেতনবৃদ্ধি 25 টাকা।

১৬। ক. একটি খ. (4, 6) গ. 30 বর্গ একক ১৭। ক. $\frac{x+7}{y} = 2, \frac{x}{y\text{-}2} = 1$, খ. (3, 5), $\frac{3}{5}$

অনুশীলনী ১৩·১

১। -7 এবং -75, ২। 129 তম, ৩। 100 তম, ৪। $p^2 + pq + q^2$, ৫। 0, ৬। n^2, ৭। 360, ৮। 320, ৯। 42, ১০। 1771, ১১। 620, ১২। 18, ১৩। 50, ১৪। 2+4+6+........, ১৫। 110, ১৬। 0, ১৭। $-(m+n)$, ২০। 50টি।

অনুশীলনী ১৩·২

১। গ ২। খ ৩। গ ৪। গ

৫। $\frac{1}{2}$, ৬। $\frac{3}{2}(3^{14} - 1)$, ৭। 9ম পদ, ৮। $\frac{1}{\sqrt{3}}$, ৯। 9ম পদ, ১০। $x = 15,\ y = 45$,

১১। $x = 9,\ y = 27,\ z = 81$, ১২। 86, ১৩। 1, ১৪। $55\log 2$, ১৫। $650\log 2$, ১৬। $n = 7$, ১৭। 0, ১৮। $n = 6,\ S = 21$, ১৯। $n = 5,\ S = 55$, ২১। 20, ২২। 24.47 মি. মি. (প্রায়)

অনুশীলনী ১৬·১

১। 20 মিটার, 15 মিটার ২। 12 মিটার ৩। 12 বর্গমিটার ৪। 327·26 বর্গ সে.মি. (প্রায়) ৫। 5 মিটার

৬। 30^o ৭। 36 বা 12 সে.মি. ৮। 12 বা 16 মিটার ৯। 44·44 কিলোমিটার (প্রায়)

১০। 24·249 সে.মি. (প্রায়), 254·611 বর্গ সে.মি. (প্রায়)

অনুশীলনী ১৬·২

১। 96 মিটার ২। 1056 বর্গমিটার ৩। 30 মিটার ও 20 মিটার ৪। 400 মিটার

৫। 6400 টি ৬। 16 মিটার ও 10 মিটার ৭। 16·5 মিটার ও 22 মিটার ৮। 35·35 মিটার (প্রায়)

৯। 48·66 সে.মি. (প্রায়) ১০। 72 সে.মি., 1944 বর্গ সে.মি. ১১। 17 সে.মি. ও 9 সে.মি.

১২। 95·75 বর্গ সে.মি. (প্রায়) ১৩। 6·36 বর্গমিটার (প্রায়)।

অনুশীলনী ১৬.৩

১। 32·987 সে.মি. (প্রায়) ২। 31·513 মিটার (প্রায়)৩। 20·008 (প্রায়)। ৪। 128·282 বর্গ সে.মি. (প্রায়) ৫। 7·003 মিটার (প্রায়) ৬। 175·93 মিটার (প্রায়)৭। 20 বার ৮। 49·517 মিটার (প্রায়) ৯। $3\sqrt{3}:\pi$

অনুশীলনী ১৬·৪

৮। 636 বর্গমিটার, 20·5 মিটার, 864 ঘনমিটার ৯। 14040 বর্গ সে.মি. ১০। 12 মিটর, 4 মিটার ১১। 1 সে.মি. ১২। 300000 টি ১৩। 34·641 সে.মি. (প্রায়) ১৪। 534·071 বর্গসে.মি.(প্রায়), 942·48 ঘন সে.মি. (প্রায়) ১৫। 5·305 বর্গ সে.মি., 3 সে.মি. ১৬। 6111·8 বর্গ সে.মি. ১৭। 147·027 কিলোগ্রাম (প্রায়)

অনুশীলনী ১৭

১। (গ) ২। (খ) ৩। (খ) ৪। (ঘ) ৫। (গ) ৬। (ক) ৭। (ক) ৮। (খ) ৯। (গ) ১০। (গ) ১১। (গ) ১২। (গ) ১৩। (গ) ১৪। (খ) ১৫। (খ) ১৬। (ক) ২০। মধ্যক ৬০ ২১। (ক) ৬২ কেজি, (খ) ৬২.৮ কেজি

২০১৩
শিক্ষাবর্ষ
৯,১০-গণিত

সমৃদ্ধ বাংলাদেশ গড়ে তোলার জন্য যোগ্যতা অর্জন কর

– মাননীয় প্রধানমন্ত্রী শেখ হাসিনা

জ্ঞান মানুষের অন্তরকে আলোকিত করে

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড
ঢাকা বাংলাদেশ

২০১০ শিক্ষাবর্ষ থেকে সরকার কর্তৃক বিনামূল্যে বিতরণের জন্য

মুদ্রণে :