

# S.E. Exam Review: Prestressed Concrete

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## Applicable Specifications

- Buildings:
  - ACI 318-14
    - Reorganized by member type (e.g. beam, column, slab) rather than stress (e.g. flexural, shear).
    - Reinforced concrete and prestressed concrete are now mixed together in the code.
    - Common material is now in “toolbox” chapters.
    - There is no separate chapter on prestressed any more.
  - PCI Design Handbook 7th Ed (covers pretensioned) based on ACI 318-08/11
    - 8<sup>th</sup> Edition came out in 2018; based on ACI 318-14
    - NCEES no longer names a PCI Handbook

## This Webinar

- Due to time limitations, only pretensioned under gravity loads can be covered.
- It is assumed that the participant knows how to calculate the critical loads and load combinations per ASCE 7.
- We will analyze a building beam.
  - ACI 318 – 14
  - PCI Design Handbook, 7th Ed.
- For prestressed concrete, many building concepts are applicable to bridges.
- PCI has information (including FREE information) on bridge design. Check the website [pci.org](http://pci.org) for information.

## Exam Areas Covered

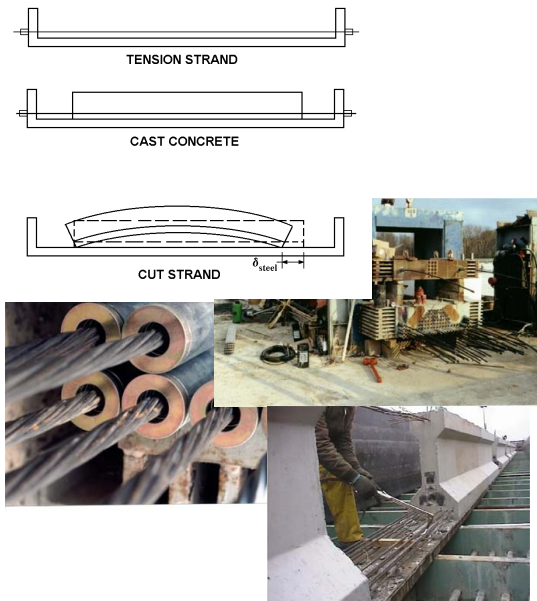
Note: This module only covers gravity loads. Lateral design of prestressed structures is a very specialized area which is beyond the scope of a 2 hour webinar.

Day	Area	Approx. % on Exam
Friday Morning	Analysis of Structures – Load	10%
	Analysis of Structures – Methods	20%
	Design and Details of Structures – General Considerations	7.5%
	Design and Details of Structures – Concrete	12.5%
Friday Afternoon	Concrete Building Structures	25%
*Lateral loads (Saturday exam) are not covered in this module.		

## Steps in Prestress Design

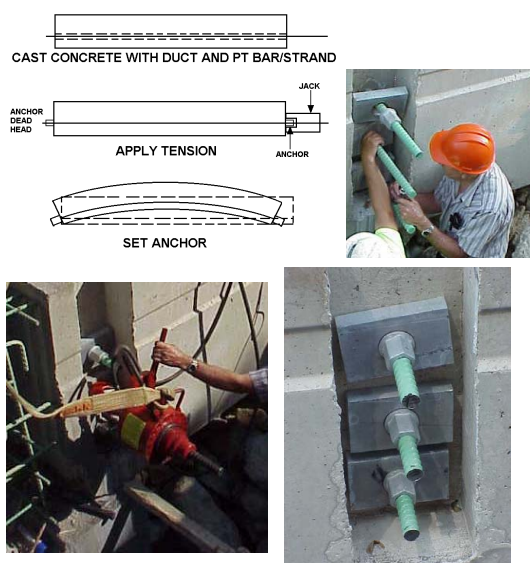
- Determine loads/moments
- Estimate cross section/reinforcing
- Determine loss of prestressing force
- Check service load stresses
- Check nominal bending strength ( $M_n$ )
- Check release stresses
- Check shear
- Miscellaneous checks

## Types of Prestressed Concrete – Pretensioned



- Pretensioned
  - 7 wire strand is anchored to end supports.
  - Concrete is cast around the strand.
  - When the concrete reaches strength, the strand is cut releasing the prestressing.
  - Force is transferred through bond.

## Types of Prestressed Concrete



- Post-tensioned
  - The concrete (precast or cast-in-place) is cast with a duct.
  - Strand or bar is run through the duct.
  - When the concrete reaches strength, the bar/strand is tensioned and anchored. Force is transferred by the anchor.
  - The strand/bar may be grouted (bonded) or left unbonded.

## Post-tensioning vs. Pretensioning

- At transfer of prestressing force:
  - Different allowable stresses for compression in the concrete and tension in the steel
  - In post tensioned, there are limits on bearing stresses in the concrete due to anchorage devices
- At service loads
  - No difference between methods
- At ultimate load
  - A different formula is used for the steel stress,  $f_{ps}$ , for if unbonded steel is used in post-tensioned structures.
  - No difference between methods, otherwise
- Prestressing force losses
  - There are some differences for each method
- Concrete Strength:
  - The term  $f'_{ci}$  = concrete strength when prestressing is applied
  - The term  $f'_c$  = design concrete strength (usually 28 or 56 d)

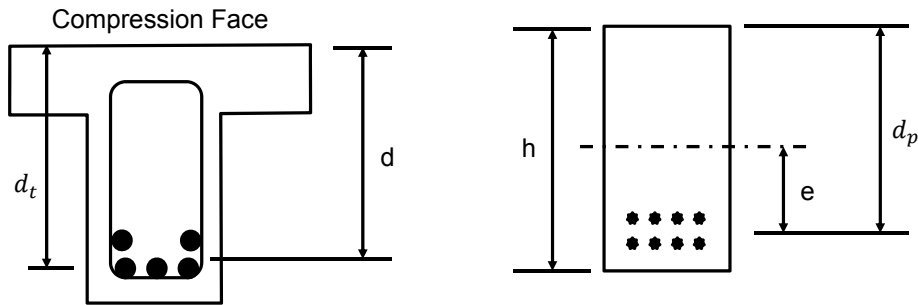
## Prestressing Strand

- ASTM A416
- 7 wire – helically wrapped.
- Strength: GR 250 = 250 ksi or GR 270 = 270 ksi
- Stress Relieved (less common) or Low Relaxation (more common)
- Common Diameters: 3/8", 1/2", 5/8" special, 0.6".
- $E_p = 28,500$  ksi due to helical wrap
- Areas found in tables.



Tables with areas of strands can be found in the PCI Design Handbook, prestressed concrete text books or on the web from manufacturers.

## Important ACI Definitions



$h$  = overall height of the section.

$d$  = distance from compression face to centroid of **all** tensile steel. May be limited  $\geq 0.8h$ .

$d_p$  = distance from compression face to centroid of prestressing steel **only**. May or may not be limited to  $\geq 0.8h$ .

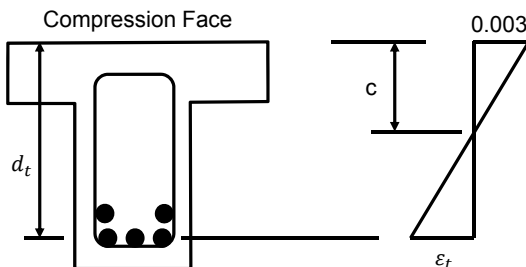
$d_t$  = distance from compression face to centroid of extreme tensile steel (mild or prestressed).

$e$  = distance from centroid of gross section to centroid of prestressing steel, only. Called eccentricity.

## Definition of Section Types (ACI 21.2.2)

Strain in the extreme tensile steel, $\epsilon_t$	Type of Section	$\phi$
$\epsilon_t \leq f_y/E_s$	Compression Controlled $c/d_t \geq 3/5^*$	0.65 Tied Conf. 0.75 Spiral Conf.
$0.005 > \epsilon_t > f_y/E_s$	Transition $3/8 < c/d_t < 3/5^*$	Interpolated based on $\epsilon_t$
$\epsilon_t > 0.005$	Tension Controlled $c/d_t \geq 3/8$	0.9

\*For Grade 60 steel and prestressing steel, where  $f_y/E_s$  may be taken as 0.002.



The same definition is used for both building and bridge for prestressing steel, which is the same as for Grade 60 mild reinforcing.

Note: ACI states a flexural member ( $P < 0.1A_g f'_c$ ) must have  $\epsilon_t > 0.004$ ; this does not apply to prestressed members. See 9.3.3.1.

## Definition of Development and Transfer Lengths

- $\ell_d$  = development length;
  - Length the strand needs to be bonded to develop the full nominal moment,  $M_n$ .
  - The stress in the strand at  $M_n$  is called  $f_{ps}$ .
- $\ell_{tr}$  = transfer length;
  - Length the strand needs to be bonded to develop the service load stresses.
  - Service load stress in the strand is called  $f_{se}$ .

The equations, which will be covered in detail later, are:

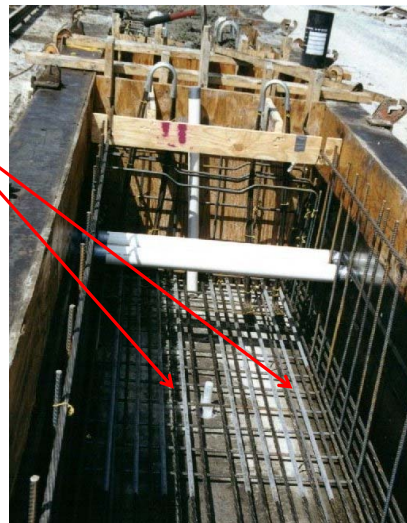
$$\ell_d = \frac{f_{se}}{3,000} d_b + \frac{f_{ps} - f_{se}}{1,000} d_b$$
$$\ell_{tr} = \frac{f_{se}}{3,000} d_b$$

Units are psi for  $f_{ps}$  and  $f_{se}$ ;  $d_b$  is nominal strand diameter.

## Debonding

To control stresses at the end of a girder, strand is sometimes “debonded”. Reasons for this are explained later.

Note the Teflon sleeves placed around some of the strand. This is “debonding”. These strand will not bond over the coated length. This strand behaves as though it begins at the end of the sleeve.





## Requirements for End Conditions

Strength reduction factors ( $\phi$ ) at the ends of pretensioned members (21.2.3).

$\ell_d$  = development length;  $\ell_{tr}$  = transfer length

$\ell_{db}$  = debonded length

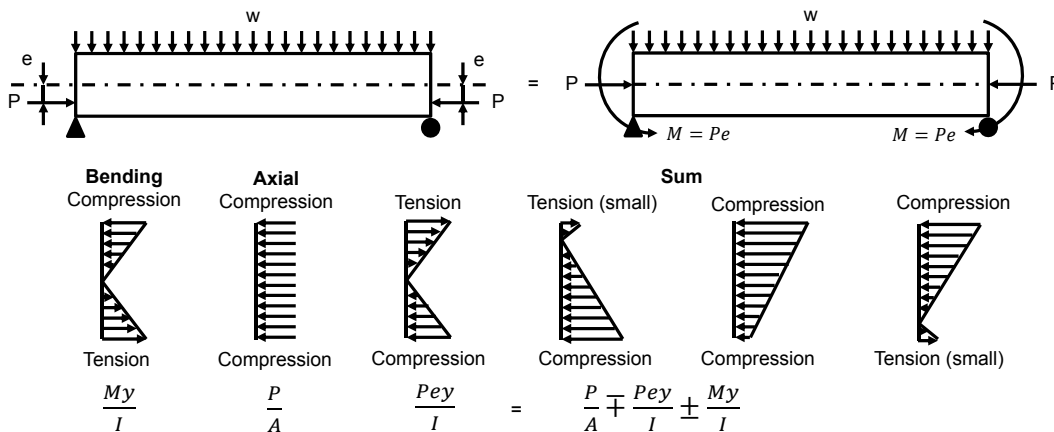
Condition at end of member	Stress in concrete under service load <sup>1</sup>	Distance from end of member to section under consideration	$\phi$	
All strands bonded	Not applicable	$\leq \ell_{tr}$	0.75	(a)
		$\ell_{tr}$ to $\ell_d$	Linear Interpolation from 0.75 to 0.90 <sup>2</sup>	(b)
One or more strands debonded	No tension calculated	$\leq (\ell_{db} + \ell_{tr})$	0.75	(c)
		$(\ell_{db} + \ell_{tr})$ to $(\ell_{db} + \ell_d)$	Linear Interpolation from 0.75 to 0.90 <sup>2</sup>	(d)
	Tension Calculated	$(\ell_{db} + \ell_{tr})$	0.75	(e)
		$(\ell_{db} + \ell_{tr})$ to $(\ell_{db} + 2\ell_d)$	Linear Interpolation from 0.75 to 0.90 <sup>2</sup>	(f)

<sup>1</sup>Stresses calculated using gross cross sectional properties in extreme fiber of precompressed tension zone under service loads after allowance for all prestress losses at section under service conditions.

<sup>2</sup>It shall be permitted to use a strength reduction factor of 0.75.

## Basic Equation of Prestressing

The basic equation of prestressing (for both buildings and bridges):



The eccentric prestressing force causes both axial compression and bending opposite to the applied forces. In cases where the section can be considered elastic, the net stress is found from superposition. This equation applies at release of prestressing force and under service load.

## Basic Equation of Prestressing

$$f_c = \frac{P}{A_g} \mp \frac{Pec}{I_g} \pm \frac{Mc}{I_g}$$

$f_c$  = concrete stress (+ = comp; - = tens).

P = total prestressing force after all losses

e = eccentricity

c = distance from neutral axis to extreme fiber. There may be different values of “c” for top and bottom!

M = moment from external forces

$A_g$  = gross area

$I_g$  = gross moment of inertia

For a simply supported beam, the top sign is for the stress at the top of the beam and the bottom sign is for the stress at the bottom of the beam.

## Basic Equation of Prestressing

If the section is made composite by adding a slab later, the loads must be separated based on whether they were applied to non-composite precast section or to the entire composite section. This is not difficult, it just requires bookkeeping. The basic equation becomes:

$$f_c = \frac{P}{A_g} \mp \left( \frac{Pec}{I_g} \right)_{nc} \pm \left( \frac{Mc}{I_g} \right)_{nc} \pm \left( \frac{Mc}{I_g} \right)_{comp}$$

Where “nc” refers to the non-composite section properties and the forces/moments applied to the non-composite section and “comp” refers to the properties and moments applied to the composite section.

## Classes of Prestressed Beams - Buildings

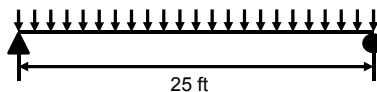
- ACI-318 (24.5.2.1)
  - Class U – Uncracked. Service level **gross section tension cannot exceed**  $7.5(\sqrt{f'_c})$ .
  - Class C – Cracked. Service level **gross section tension exceeds**  $12(\sqrt{f'_c})$ .
  - Class T – Transition. Service level **gross section tension** between  $7.5(\sqrt{f'_c})$  and  $12(\sqrt{f'_c})$ .
- ACI-318 Section 24.5.2.1 and Table R24.5.2.1 (PCI 5.2.1) provide the serviceability design requirements for these classes.

## Classes U and T

- We will analyze a Class U member, as those are the most common.
- Class C members are rare.
- A Class T member is designed and analyzed **exactly** the same as a Class U, with one exception:
  - A Class T member must have special deflection check using a bilinear curve.
  - For Class T deflection, use ACI 24.2.3.9 and 24.2.4.2.
  - See Example 5.2.2.3 of the PCI Design Handbook for information and an example!

## Problem Statement

Load Type	w (kips/ft) <u>unfactored</u>	$M_{max}$ (k-in) at Midspan 25 ft. span c/c bearing
Self Weight	0.25	234.4
DL	0.5	468.8
LL	1.2	1125
TOTAL Service	1.95	1828



The beam is **simply supported**.  $M = wL^2/8$

$L = 25$  ft c/c bearing

$L_{overall} = 27$  ft – out to out.

For prestressed beams, both the span (c/c bearing) and the overall length are needed.

The following would have to be given in the problem statement:

28 day (design) concrete strength:  $f'_c = 5,000$  psi

Concrete strength at release of prestressing force:  $f'_{ci} = 4,000$  psi

Prestressing steel: ASTM A416, 7 wire prestressing strand, Grade 270; Low Relaxation,  $\frac{1}{2}$ "

( $A_p = 0.153$  in<sup>2</sup>/strand)

Strength  $f_{pu} = 270$  ksi,  $f_{py} = 230$  ksi

Initial stress =  $0.75f_{pu}$

Nonprestressed steel: ASTM A615 Grade 60.

## Prestressing Steel

Initial prestressing force in the strand

$$f_{pi} = 0.75f_{pu} = 0.75(270 \text{ ksi}) = 202.5 \text{ ksi}$$

This is the stress in the steel **before** release. This is not standard and would have to be given.

### 20.3.2.5.1 Limits on Prestressing Steel Stress:

$$\begin{aligned} \text{Due to jacking} &\leq 0.94f_{py} = 0.94(230) = 216 \text{ ksi} \\ &\leq 0.8f_{pu} = 216 \text{ ksi} \end{aligned}$$

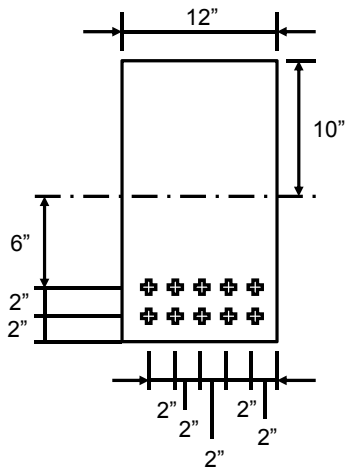
After transfer of prestressing – None for pretensioned.  
(provision removed in 2011)

## Initial Section Selection

This is a partial reproduction of a table on Pg 3-44 of the 7th Edition of the PCI Design Handbook. It covers rectangular beams. The span is 25 feet and the load is 1,950 pounds/ft. Use a 12RB20. The approximate service load with 8 strands is 2,410 pound/ft.

Section	# 1/2" Str	Approximate Service Load pound/ft for a given span (in feet) – Assumed 50% DL + 50% LL										
		16	18	20	22	24	26	28	30	32	34	36
12RB16	5	3,350	2,770	2,210	1,790	1,480	1,230	1,040				
12RB20	8	6,160	4,820	3,860	3,150	2,620	2,200	1,860	1,600	1,380	1,190	1,040

## Proposed Initial Section



Strand patterns are usually 2 inch centers.

From the previous slide, 8 strands hold 2.41 k/ft. The applied load is 1.95 k/ft.

Estimate  $(1.95/2.41)(8) = 6.5$  strand

The table assumes 50% LL + 50% DL. Actual LL is higher than DL.

Also 6 strands has a larger “e” than 8 strands, so round down to 6 as a first estimate.

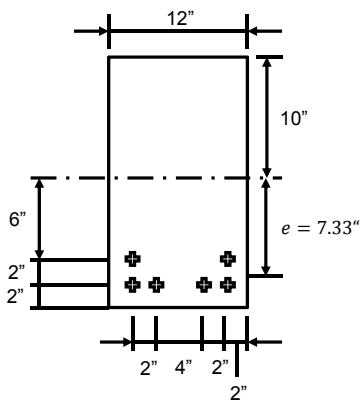
Section Properties:

$$A = 240 \text{ in}^2$$

$$I = 8,000 \text{ in}^4 = bh^3/12$$

$$S = I/c = 800 \text{ in}^3$$

## Proposed Initial Section



Try this strand pattern. Must be symmetrical about the vertical axis or the beam will deflect laterally.

The distance from the bottom of the beam to the centroid of strand,  $y_b$ , is:

$$y_b = \frac{2 \text{ strand}(4 \text{ in}) + 4 \text{ strand}(2 \text{ in})}{6 \text{ strand}} = 2.67 \text{ in}$$

$$e = \frac{h}{2} - y_b = 10 - 2.67 = 7.33 \text{ in}$$

A single strand is  $0.153 \text{ in}^2$ . This is found in a table.

$$A_p = 6(0.153) = 0.918 \text{ in}^2$$

The “initial pull” (force in the steel prior to transfer of prestress):

$$P_i = 0.918 \text{ in}^2(0.75)(270 \text{ ksi}) = 185.9 \text{ kips}$$

## Loss of Prestressing

As soon as the prestressing force is applied, creep, shrinkage and relaxation cause the prestressing force to start to drop. After a long time, this loss of prestressing force stabilizes. These losses are needed for various calculations.

ACI 318 does not say how to calculate the loss of prestressing force, so the procedure in the PCI Design Handbook 7th Ed. (Section 5.7.3 starting on pg. 5-84) is used.

(8th ed. procedure is same as in 7th ed.)

## Loss of Prestressing

(20.3.2.6.1) The losses are Elastic shortening; Creep of the concrete; Shrinkage of the concrete and Relaxation of the steel. These apply to both pre and post tensioned.

Post tensioned also has anchorage set loss and loss due the friction when the tendons are pulled through curved ducts.

Losses are for **service load** calculations. **Unfactored loads** are used.

## Elastic Modulus of Concrete

The elastic moduli are needed (ACI 19.2.2.1 for concrete). Note that there are two; one for release and one for service level stresses.  $E$  is given for 145 pcf concrete.

The prestressing steel value is from the appendix of the PCI Design Handbook. It is less than 29,000,000 psi due to the helical wrap of the prestressing strand.

$$E_{ci} = 33(145)^{1.5}\sqrt{4,000} = 3,600,000 \text{ psi}$$

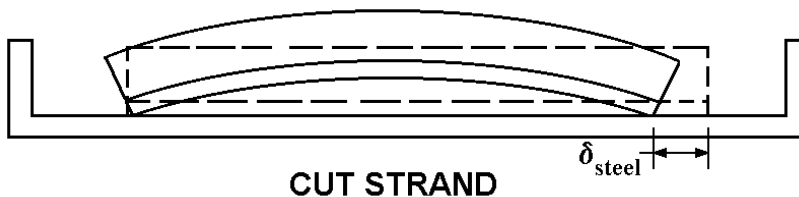
$$E_c = 33(145)^{1.5}\sqrt{5,000} = 4,030,000 \text{ psi}$$

$$E_p = 28,500,000 \text{ psi}$$

## Elastic Shortening

When the steel is cut, it “springs” back, compressing the concrete. Since the steel and concrete are bonded, the steel shortens and loses stress.

This is elastic shortening.





## Elastic Shortening

$$f_{cir} = K_{cir} \left( \frac{P_i}{A_g} + \frac{P_i e^2}{I_g} \right) - \frac{M_g e}{I_g}$$

$$\varepsilon_c = \frac{f_{cir}}{E_{ci}} = \Delta\varepsilon_p$$

$$ES = \Delta f_p = E_p \Delta\varepsilon_p$$

$$ES = K_{ES} \frac{E_p}{E_{ci}} f_{cir}$$

This is the stress in concrete at the centroid of the steel at release.  $K_{cir}$  is an empirical constant = 0.9.

Since concrete and steel are bonded, the change in steel strain is equal to the concrete strain.

Elastic shortening is the change in steel stress.  $K_{ES}$  is an empirical constant = 1.

## Span Length Definitions

Normally, prestressed beams are cast longer than the center of bearing to center of bearing span to allow for the width of bearing pads. **This beam is cast 27 feet long.**

1. When calculating the stress when the prestressing is released (release condition), it is usual to use overall length; 27 feet in this case.
2. When calculating stresses under service or ultimate loads, center of bearing to center of bearing span is used; 25 feet in this case.
3. If overall length is **not** given, the c/c bearing can be used for the release condition; some example problems even use c/c bearing at release.

## Calculation of $f_{cir}$

From the *PCI Handbook*,  $K_{cir} = 0.9$ . The term  $f_{cir}$  is the concrete stress at the centroid of the steel (in My/I use  $y = e$ ). The only load is the beam's own DL.

$P_i = 185.9$  k and  $e = 7.33$  in (found prev.).

$$M_g = \frac{wl^2}{8} = \frac{(0.25 \text{ k/ft})(27 \text{ ft})^2}{8} (12 \text{ in/ft}) = 273.4 \text{ k-in}$$

$$f_{cir} = K_{cir} \left( \frac{P_i}{A_g} + \frac{P_i e^2}{I_g} \right) - \frac{M_g e}{I_g}$$

$$f_{cir} = 0.9 \left( \frac{185.9 \text{ k}}{240 \text{ in}^2} + \frac{185.9 \text{ k}(7.33 \text{ in})^2}{8,000 \text{ in}^4} \right) - \frac{273.4 \text{ k-in}(7.33 \text{ in})}{8,000 \text{ in}^4}$$

$$f_{cir} = 1.57 \text{ ksi}$$

## Elastic Shortening Loss

$K_{es} = 1.0$  is from the *PCI Design Handbook*

$$E_{ci} = 57,000\sqrt{4,000} = 3,600,000 \text{ psi}$$

$$ES = K_{ES} \frac{E_p}{E_{ci}} f_{cir}$$

$$ES = 1.0 \frac{28,500 \text{ ksi}}{3,600 \text{ ksi}} 1.57 \text{ ksi} = 12.5 \text{ ksi}$$

Since this is normal wt. concrete, here is an alternate  $E_c$  equation.

For post-tensioned structures, the ES equation above is valid if all tendons are tightened simultaneously. If they are tightened one at a time, ES above is multiplied by  $(N-1)/2N$ ; where  $N$  is the number of tendons. The following equations for CR, SH and RE work for post tensioning, too.

## Creep Loss

Because the prestressing force keeps a constant stress on the concrete, the concrete will creep. Creep shortens the beam and causes a loss of prestressing. Additional DL moments mitigate these stresses, so they are subtracted out. Self weight is already subtracted out of the  $f_{cir}$  term. Again, stress is found at  $y = e$ . The additional dead load is applied after the beam is in place, so the c/c bearing (service load) span is used.

$$M_{dl} = \frac{(0.5 \text{ k/f})(25 \text{ ft})^2}{8} (12 \text{ in/ft}) = 468.8 \text{ k-in}$$

$$f_{c ds} = \frac{M_{dl}e}{I} = \frac{468.8 \text{ k-in}(7.33 \text{ in})}{8,000 \text{ in}^4} = 0.43 \text{ ksi}$$

$$CR = 2.0E_p \left( \frac{f_{cir} - f_{c ds}}{E_c} \right)$$

Elastic strain in concrete from sustained loads. The concrete is assumed 28 days old. 2 is the creep coefficient.

$$CR = 2.0 \frac{28,500 \text{ ksi}}{4,030 \text{ ksi}} (1.57 \text{ ksi} - 0.43 \text{ ksi}) = 16.4 \text{ ksi}$$

## Shrinkage Loss

This shows the loss due to shrinkage of the concrete. The volume to surface area ratio (V/S) is approximated by the area to perimeter ratio. Relative Humidity varies; here it is taken as 70%. There is a map in the *PCI Design Handbook* (pg 3-114) which gives local RH values.

$$SH = 8.2 \times 10^{-6} K_{sh} E_p \left( 1 - 0.06 \frac{V}{S} \right) (100 - RH)$$

$$SH = 8.2 \times 10^{-6} (1) (28,500 \text{ ksi}) (1 - 0.06(3.75)) (100 - 70)$$

$$SH = 5.43 \text{ ksi}$$

## Relaxation Loss and Total Loss

Finally, steel under stress relaxes. The relaxation loss is:

$$RE = [K_{re} - J(ES + CR + SH)]C$$

The values of  $K_{re}$ ,  $J$  and  $C$  are found in the *PCI Design Handbook* (Pg 5-84). For Grade 270 low relaxation steel initially stressed to  $0.75f_{pu}$ ;  $K_{re} = 5,000$  psi,  $J = 0.04$  and  $C = 1.0$ .

$$RE = [5 \text{ ksi} - 0.04(12.5 + 16.4 + 5.4)]1.0$$

$$RE = 3.7 \text{ ksi}$$

$$\text{Total} = 12.5 + 16.4 + 5.4 + 3.7 = 38 \text{ ksi}$$

$$\frac{38 \text{ ksi}}{0.75(270 \text{ ksi})} (100\%) = 19\%$$

## Effective Stress

Thus, the stress in the strand after all losses is:

$$f_{se} = 0.75f_{pu} - \text{loss} = 0.75(270 \text{ ksi}) - 38 \text{ ksi} = 164 \text{ ksi}$$

$$P = (6 \text{ strand})(0.153 \text{ in}^2)(164 \text{ ksi}) = 150.5 \text{ k}$$

This will be the stress in the strand after a long time.

It is used to calculate service load behavior.

Typical loss values are 15-30%, with 20% being average.

## Service Load Allowable Stresses Classes “U” and “T”

- Sections 24.5.2.1; 24.5.4.1 ACI 318 (Use gross section properties)
- Class U tension in the precompressed zone (bottom of the beam in this case):
  - $7.5\sqrt{f'_c} = 7.5(5,000 \text{ psi})^{0.5} = 530 \text{ psi}$
- Compression under all loads (to prevent fatigue):
  - $0.6f'_c = 0.6(5,000 \text{ psi}) = 3,000 \text{ psi}$
- Compression under sustained load only (to control creep deflections):
  - $0.45f'_c = 0.45(5,000 \text{ psi}) = 2,250 \text{ psi}$
- The compression limits also apply to Class “T”. Class “T” tensile limit is  $12\sqrt{f'_c}$ .
- Service stress limits not applicable to Class “C”.

## Check Service Load Stress

For all loads at midspan:

$$f_c = \frac{P}{A} \mp \frac{Pe}{S} \pm \frac{M}{S} \quad S = \frac{I}{c}$$

$$f_c = \frac{150.5 \text{ k}}{240 \text{ in}^2} \mp \frac{150.5 \text{ k}(7.33 \text{ in})}{800 \text{ in}^3} \pm \frac{(234+469+1125)\text{k-in}}{800 \text{ in}^3}$$

$$f_c = 1.53 \text{ ksi C Top}$$

$$f_c = -0.28 \text{ ksi} = 0.28 \text{ ksi T Bottom}$$

For sustained loads at midspan (only top is checked):

$$f_c = \frac{150.5 \text{ k}}{240 \text{ in}^2} - \frac{(150.5 \text{ k})(7.33 \text{ in})}{800 \text{ in}^3} + \frac{(234.4+468.8)\text{k-in}}{800 \text{ in}^3}$$

$$f_c = 0.13 \text{ ksi C top}$$

Remember, this beam is symmetric about the horizontal axis so  $S_{top} = S_{bot}$ . On an exam, your beam may not be symmetric!

## Service Load Stresses

	Stress ksi	Allowable ksi	
Compression under all loads	1.53	3.00	OK
Compression under sustained load	0.13	2.25	OK
Tension	0.28	0.53	OK

**Note:** If the number of strands is reduced to 5, the tensile allowable is exceeded. With 5 strands, the beam could be designed as a Class “T” – as long as the pattern can be made symmetrical about the vertical axis. Class “T” also requires a deflection check Class “U” does not.

## Definition of Transfer Length

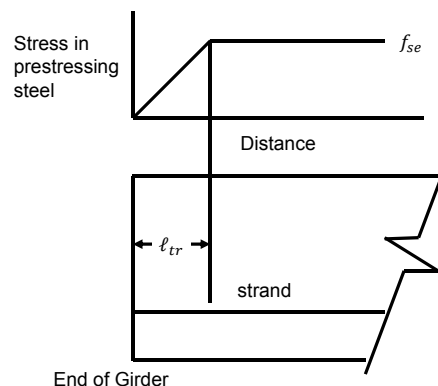
This is a pretensioned beam. Stress from the strand is transferred to the concrete through bond over a distance called the “transfer length”. The transfer length, measured from the end of the beam is:

$$\ell_{tr} = \frac{f_{se}}{3,000} d_b = \frac{164,000 \text{ psi}}{3,000 \text{ psi}} (0.5 \text{ in}) = 27.3 \text{ in}$$

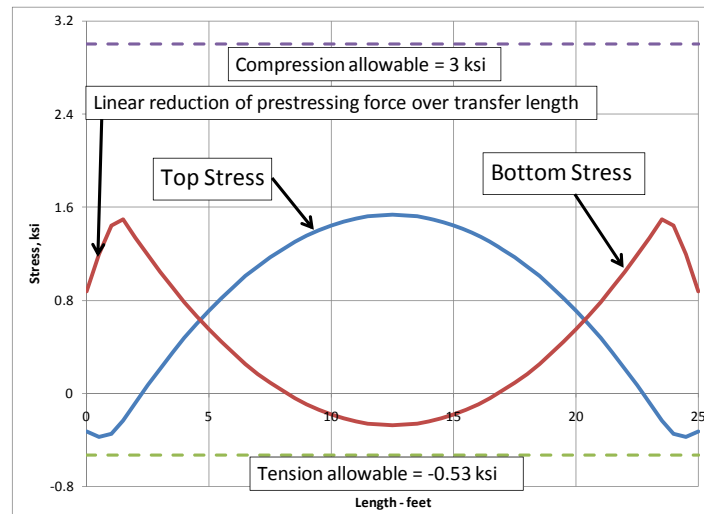
Where  $d_b$  is the diameter of the strand.

It is assumed the prestressing force varies linearly over the transfer length.

See 21.2.3



## Service Load Stresses



This graph shows the stresses from applying all service loads are below the allowable along the entire beam. A similar graph would show top stress under sustained service loads is below the allowable at all points. The stress does not go to 0 at the supports because the beam overhangs the supports by 1 foot.

### Strength Limit State 20.3.2.3.1

Unlike reinforced concrete, the steel stress is not assumed as yield, but must be calculated. This equation is valid if  $f_{se} > 0.5f_{pu}$ :

$$f_{ps} = f_{pu} \left\{ 1 - \frac{\gamma_p}{\beta_1} \left[ \rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p} (\omega - \omega') \right] \right\}$$

Ignore any compression steel if the following conditions are not met:

$$\left[ \rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p} (\omega - \omega') \right] \geq 0.17$$

And

$$d' \leq 0.15d_p$$

## Strength Limit State 20.3.2.3.1

$f_{ps}$  = stress in the prestressing steel at the ultimate condition

$f_{pu}$  = ultimate strength of the prestressing steel

$f_{py}$  = yield strength of the prestressing steel

$f'_c$  = concrete design strength

$\gamma_p$  = prestressing steel constant

= 0.55 for  $f_{py}/f_{pu} \leq 0.8$  (prestressing bar)

= 0.4 for  $f_{py}/f_{pu} \leq 0.85$  (stress relieved strand)

= 0.28 for  $f_{py}/f_{pu} \leq 0.9$  (low relaxation strand)

$\beta_1$  = the stress block constant for concrete

= 0.85 for  $f'_c \leq 4,000$  psi

= 0.65 for  $f'_c \geq 8,000$  psi

= interpolate in between

## Strength Limit State 20.3.2.3.1

$$\rho_p = \frac{A_p}{bd_p}$$

$$\rho = \frac{A_s}{bd}$$

$$\rho' = \frac{A'_s}{bd}$$

Careful,  
it's d, not  
d'!

$$\omega = \frac{\rho f_y}{f'_c}$$

$$\omega' = \frac{p' f_y}{f'_c}$$

$d_p$  = effective depth of the prestressing steel. (Prestressing strand is never considered effective in compression so if there is strand on the compression side, a strain compatibility approach is needed – see PCI Handbook)

$b$  = width of the compression face

$d$  = effective depth to the tensile steel

$d'$  = depth to non-prestressed, compression steel.



### Strength Limit State 20.3.2.3.1

Since there is no compression steel or non-prestressed tensile steel:

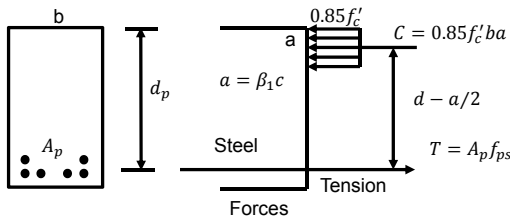
$$d_p = 17.3 \text{ in} \quad \gamma_p = 0.28 \quad \beta_1 = 0.8 \text{ for } f'_c = 5,000 \text{ psi}$$

$$f_{ps} = f_{pu} \left\{ 1 - \frac{\gamma_p}{\beta_1} \left[ \rho_p \frac{f_{pu}}{f'_c} \right] \right\}$$

$$f_{ps} = 270 \text{ ksi} \left\{ 1 - \frac{0.28}{0.80} \left[ \left( \frac{6(0.153 \text{ in}^2)}{(12 \text{ in})(17.3 \text{ in})} \right) \left( \frac{270 \text{ ksi}}{5 \text{ ksi}} \right) \right] \right\}$$

$$f_{ps} = 247.4 \text{ ksi}$$

### Strength Limit State 20.3.2.3.1



$M_n$  is calculated the same way for both prestressed and reinforced concrete (22.3). The only difference is the steel stress! If this were RC,  $f_{ps}$  would be replaced by  $f_y$ .

$$a = \frac{A_p f_{ps}}{0.85 f'_c b} = \frac{0.918 \text{ in}^2 (247.4 \text{ ksi})}{0.85 (5 \text{ ksi}) (12 \text{ in})} = 4.45 \text{ in}$$

$$M_n = A_p f_{ps} \left( d - \frac{a}{2} \right)$$

$$M_n = 0.918 \text{ in}^2 (247.4 \text{ ksi}) \left[ 17.3 \text{ in} - \frac{4.45 \text{ in}}{2} \right] = 3,423 \text{ k-in}$$

$$\Phi M_n = 0.9 (3,423 \text{ k-in}) = 3,080 \text{ k-in}$$

$$\Phi M_n > M_u = 2,644 \text{ k-in} = 1.2(234.4 + 468.8) + 1.6(1,125) \text{ OK}$$

$$M_u = 1.2 M_{DL} + 1.6 M_{LL} \text{ (ACI Section 5.3)}$$

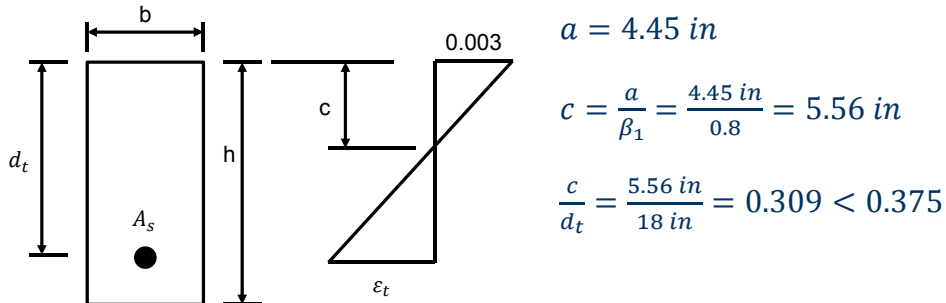
OK if  $\Phi = 0.9$ !

Class U, T or C is only for service loads. The  $M_n$ ,  $\Phi$  and ductility limit calculations apply to **all** classes!

## Strength Limit State

For prestressing steel, tension control, compression control and transition sections are defined the same as for Gr 60 rebar.

Tension control if  $\varepsilon_t > 0.005$ . It was shown previously that this condition is met if  $c/d_t < 0.375$ .



Or:  $\varepsilon_t = 0.003 \left( \frac{d_t}{c} - 1 \right) = 0.003 \left( \frac{18}{5.56} - 1 \right) = 0.0067 > 0.005$

The section is tension controlled so  $\Phi = 0.9$ .

Remember,  $d_t$  is to the extreme tension steel, not the steel centroid.

## Ductility Limit

Now check the ductility limit (minimum steel). ACI 318 (9.6.2.1) requires the ultimate moment to be **at least** 1.2 times the cracking moment (unless both  $\Phi V_n \geq 2V_u$  **and**  $\Phi M_n \geq 2M_u$ ).

$$\Phi M_n \geq (1.2)M_{cr}$$

$$M_{cr} = S_{nc} \left( \frac{P}{A} + \frac{Pe}{S} + 7.5\sqrt{f'_c} \right)$$

If the section is composite:

$$M_{cr} = S_c \left( 7.5\sqrt{f'_c} + \frac{P}{A_{nc}} + \frac{Pe}{S_{nc}} \right) - M_{dnc} \left( \frac{S_c}{S_{nc}} - 1 \right) \geq S_c (7.5\sqrt{f'_c})$$

Where  $A_{nc}$  is the non-composite area,  $S_{nc}$  is the non-composite section modulus,  $S_c$  is the composite section modulus and  $M_{dnc}$  are the moments applied only to the non-composite section. P is the effective prestressing force at service load after all losses. This applies to **all** Classes.

## Ductility Limit

Taking compression (+) and tension (-)

$$\Phi M_n \geq (1.2)M_{cr}$$

$$-7.5\sqrt{f'_c} = \frac{P}{A} + \frac{Pe}{S} - \frac{M_{cr}}{S}$$

This is the equation on the last slide, just rearranged to look like the basic equation of prestressing! Tension is negative.

$$-0.53 \text{ ksi} = \frac{150.5 \text{ kips}}{240 \text{ in}^2} + \frac{(150.5 \text{ kips})(7.33 \text{ in})}{800 \text{ in}^3} - \frac{M_{cr}}{800 \text{ in}^3}$$

$$M_{cr} = 2,024 \text{ kip in}$$

$$(1.2)M_{cr} = (1.2)(2,024 \text{ kip in}) = 2,429 \text{ kip in}$$

$$\Phi M_n = 3,080 \text{ kip in} > 2,429 \text{ kip in OK}$$

## Check Development Length Section ACI 25.4.8.1

$$\ell_d = \frac{f_{se}}{3,000} d_b + \frac{f_{ps} - f_{se}}{1,000} d_b$$

$$\ell_d = \frac{164,000}{3,000} (0.5 \text{ in}) + \frac{247,400 - 164,000}{1,000} (0.5 \text{ in}) = 69 \text{ in}$$

Units for  $f_{se}$  and  $f_{ps}$  are psi for this equation.

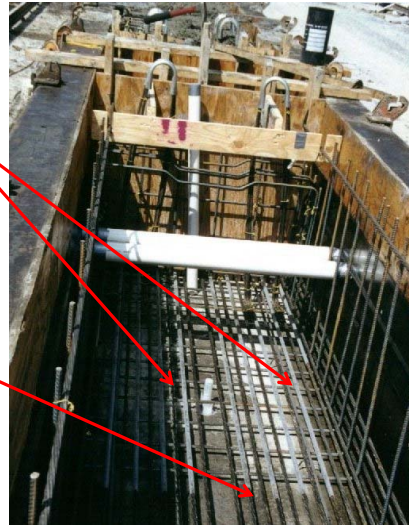
The prestressing strand must be embedded at least 69 inches = 5.8 ft from the point of maximum stress. The point of maximum stress is midspan. Since the beam is 27 ft overall, the distance from the end of the beam to midspan is 13.5 ft, so this requirement is met.

## Check Development Length Section ACI 25.4.8.1

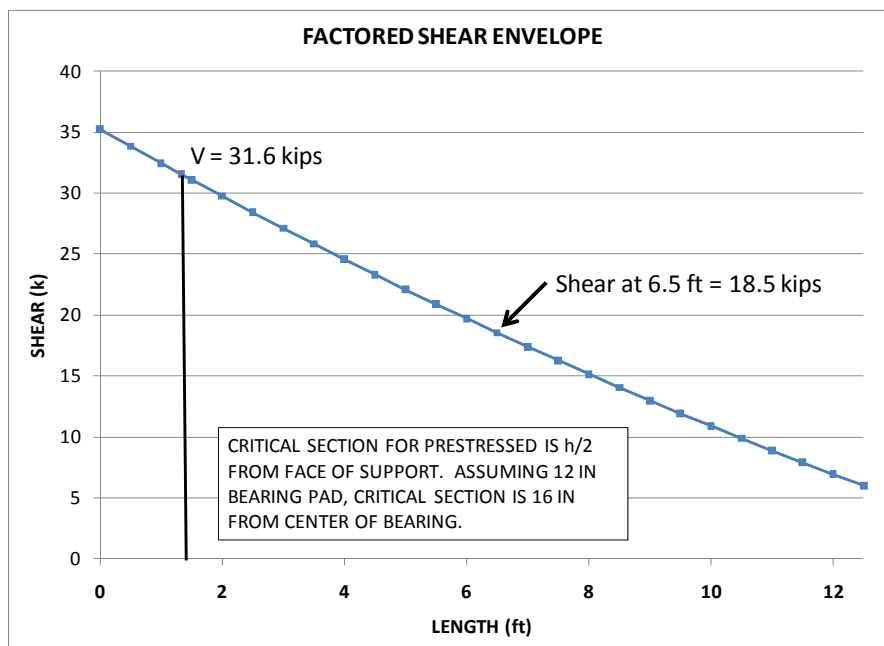
If the strand is “debonded” (a bond breaker is used to lower release stresses), development length is doubled (25.4.8.1).

Note the Teflon sleeves placed around some of the strand. This is “debonding”. These strand will not bond over the coated length. This strand behaves as though it begins at the end of the sleeve.

Development and transfer lengths are calculated from the end of the sleeve.



## Shear



## Shear – Given

- The shear envelope is shown on the previous slide.
- If the section meets ACI 9.4.3.2 – critical section is  $h/2$  from face of support for prestressed.
  - Support reaction in direction of applied shear.
  - No concentrated loads between face of support and critical section.
  - Loads applied to top.
- The term  $h/2$  used because  $d$  can be small in a prestressed beam with harped/draped strand.
- Assume beam sits on 12 in bearing pad, so face of support is 6 in from center of bearing. Critical section is  $6 + 20/2 = 16$  in from center of bearing.

## Shear

- Shear strength for prestressed or non-prestressed concrete is  $V_n = V_c + V_s + V_p$ 
  - $V_c$  = shear strength of concrete
  - $V_s$  = shear strength of steel
  - $V_p$  = Vertical component of harped strand
- $V_s$  is calculated the same as RC
- $V_c$  for prestressed concrete has two methods;
  - $V_{ci}$  and  $V_{ci}$  (22.5.8.3)
  - Simplified  $V_c$  (22.5.8.2)

## Shear

The simplified  $V_c$  method (22.5.8.2) is shown.  $V_c$  is the least of:

$$V_c = \left(0.6\lambda\sqrt{f'_c} + 700\frac{V_u d_p}{M_u}\right) b_w d$$

$$\frac{V_u d_p}{M_u} \leq 1$$

$$V_c = (0.6\lambda\sqrt{f'_c} + 700) b_w d$$

$$V_c = 5\lambda\sqrt{f'_c} b_w d$$

But not less than:  $2\lambda\sqrt{f'_c} b_w d$

In this equation,  $M_u$  is the maximum factored moment and  $V_u$  is the shear associated with  $M_u$  at the section being analyzed. The term  $b_w$  is the web width. The term  $\lambda$  is for lightweight. **Note:** This provision uses both “ $d$ ” and “ $d_p$ ”; that will be discussed in a later slide!

In non-prestressed design, the much simpler:

$$V_c = 2\lambda(\sqrt{f'_c})b_w d$$

is used (22.5.5.1).

## Shear

To use the simplified method:

$$A_{ps}f_{se} \geq 0.4(A_{ps}f_{ps} + A_s f_y)$$

$$(0.918 \text{ in}^2)(164 \text{ ksi}) > 0.4[(0.918 \text{ in}^2)(247.4 \text{ ksi}) + 0]$$

$$150.5 \text{ k} > 90.8 \text{ k OK}$$

ACI 22.5.8.2

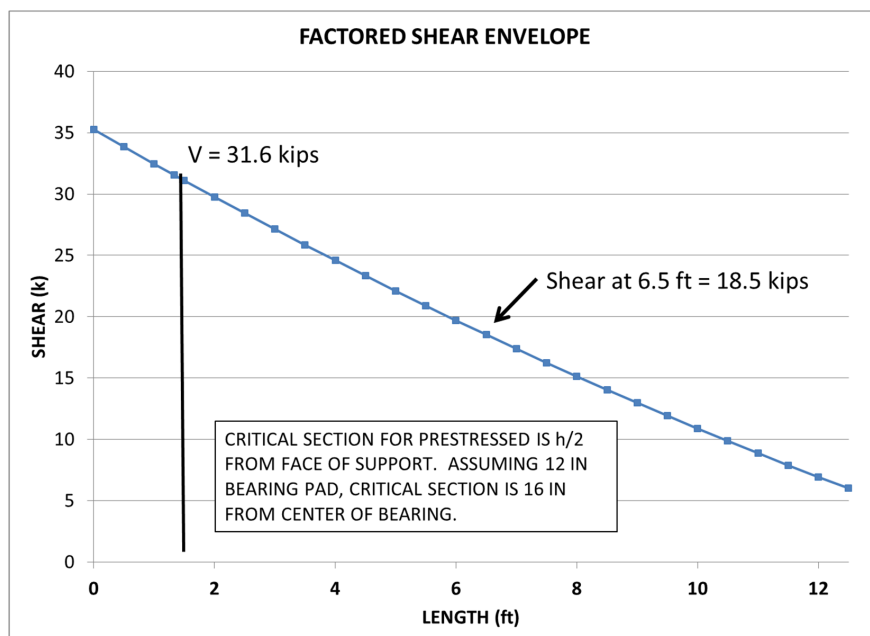
For ALL shear methods:  $\sqrt{f'_c} \leq 100 \text{ psi}$  (22.5.3.1).

## Shear

The beam is divided into intervals, usually, critical section, every 0.1L and any significant points, like a harp point. A shear analysis is performed at each interval.

Time does not permit multiple sections to be analyzed. Rule of thumb: in a prestressed beam, the greatest demand for stirrups is usually at 0.25L. A sample analysis at this section will be performed. This is approximately 6.5 feet from the center of the support.

## Shear Envelope



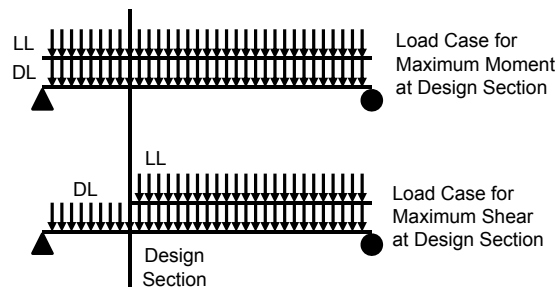
## Shear

From analysis of the beam (using the factored DL of 0.9 klf and factored LL of 1.92 klf) at 6.5 feet from the center of the support:

$V_u = 17$  kips;  $M_u = 2,035$  k – in. This is the maximum **factored** moment and the associated shear.

$V_{u,max} = 18.5$  k This is the maximum factored shear (different load case)

$$b_w = 12 \text{ inches}$$



## Shear

$$d_p = 17.3 \text{ inches}$$

$$d = 17.3 \text{ in} > 0.8 h = 16 \text{ in}$$

**Note:** This provision uses both “ $d$ ” and “ $d_p$ ”.

Section 22.5.2.1 states that “ $d$ ” does not need to be less than  $0.8h$ . The term “ $d$ ” is the distance from the extreme compression fiber to the tensile reinforcement – both prestressed and non-prestressed.

The term  $d_p$  is the distance from the extreme compression fiber to the all the **prestressed** reinforcement. This **may** be taken less than  $0.8h$  for the simplified method.



## Shear

$$V_c = \left( 0.6\lambda\sqrt{f'_c} + 700\frac{V_u d_p}{M_u} \right) b_w d$$

$$V_c = \frac{(0.6(1)\sqrt{5,000 \text{ psi}} + 700\frac{17 \text{ k}(17.3 \text{ in})}{2,035 \text{ k-in}})(12 \text{ in})(17.3 \text{ in})}{1,000}$$

The term  $\lambda = 0.75$  for lightweight, 0.85 for sand lightweight and 1 for normal weight. See 8.4.

$$V_c = 29.8 \text{ k (controls)} \quad \phi V_c = 0.75(29.8 \text{ k}) = 22.4 \text{ k}$$

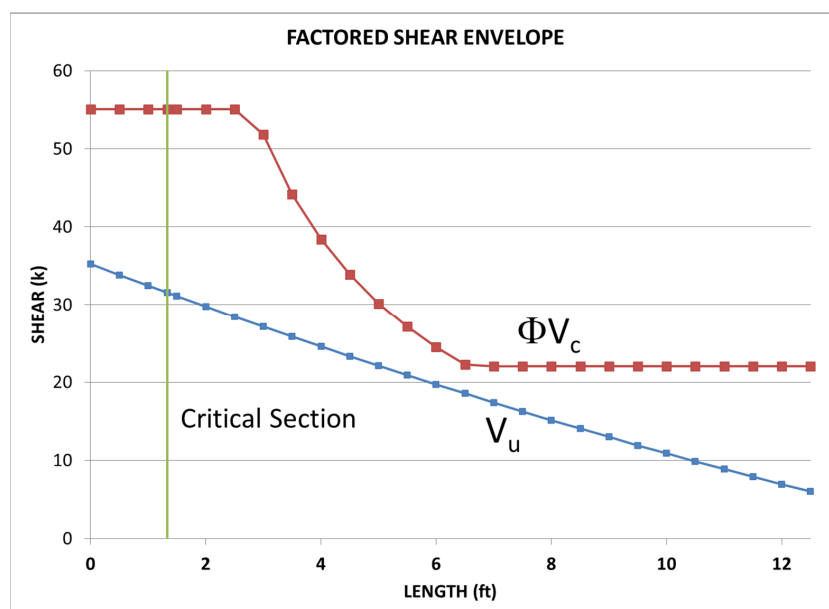
$$\frac{V_u d_p}{M_u} = 0.14 < 1 \text{ OK}$$

$$\frac{(0.6(1)\sqrt{5,000 \text{ psi}} + 700)(12 \text{ in})(17.3 \text{ in})}{1,000} = 154 \text{ k} > 29.8 \text{ OK}$$

$$5\lambda\sqrt{f'_c} b_w d = 73.4 \text{ k} > 29.8 \text{ OK}$$

$$\text{But not less than: } 2\lambda\sqrt{f'_c} b_w d = 29.2 \text{ k} < 29.8 \text{ OK}$$

## Shear Demand vs. Capacity



## Shear

$$\Phi V_c = 22.4 \text{ k} > V_u 18.5 \text{ k} > \frac{1}{2} \Phi V_c = 11.2 \text{ k}$$

Minimum stirrups are needed (9.6.3.1). Maximum spacing is (9.7.6.2.2):

$$\text{For } V_s \leq 4\sqrt{f'_c} b_w d$$

- $s_{max}$  is the lesser of 0.75h or 24 inches

$$\text{For } 4\sqrt{f'_c} b_w d \leq V_s \leq 8\sqrt{f'_c} b_w d$$

- $s_{max}$  is the lesser of 0.375h or 12 inches

$$\text{For } V_s \geq 8\sqrt{f'_c} b_w d$$

- resize the section (22.5.1.2)

$$V_{s,reqd} = 0; 0.75 h = 15 \text{ in} < 24 \text{ in}. \text{ Use 15 inches.}$$

Non prestressed beams use the same force limits, but the maximum spacings are  $d/2$  or  $d/4$  for non-prestressed. The difference is because  $d$  can get small in a prestressed beam with harped strand!

## Shear

Determine the minimum stirrup area required for a prestressed beam (9.6.3.3):

$$A_{v,min} = 0.75\sqrt{f'_c} \frac{b_w s}{f_y} \geq \frac{50b_w s}{f_y}$$

$$= 0.75\sqrt{5,000 \text{ psi}} \frac{(12 \text{ in})(15 \text{ in})}{(60,000 \text{ psi})} = 0.16 \text{ in}^2 \text{ (controls)}$$

$$\frac{50b_w s}{f_y} = \frac{50(12 \text{ in})(15 \text{ in})}{60,000 \text{ psi}} = 0.15 \text{ in}^2$$

$$\text{Or } A_{v,min} = \frac{A_{ps} f_{pu} s}{80 f_y d} \sqrt{\frac{d}{b_w}}$$

$$= \frac{(0.918 \text{ in}^2)(270,000 \text{ psi})(15 \text{ in})}{80(60,000 \text{ psi})(17.3 \text{ in})} \sqrt{\frac{17.3 \text{ in}}{12 \text{ in}}} = 0.05 \text{ in}^2$$

$$A_{v,min} = 0.05 \text{ in}^2 \text{ at 15 inches c/c}$$

This is the same as for a non-prestressed beam.

This applies only to prestressed.

Use the smaller of these two.

## Shear

The smallest stirrup is a #3, so #3 @ 15 inches would be needed.

If stirrups were required for strength (22.5.10.5.3):

$$V_s = \frac{A_v f_y d}{s}$$

$$\Phi = 0.75$$

$$V_u \leq \Phi(V_c + V_s)$$

**Note:** When calculating maximum spacing, minimum stirrup area or required stirrup strength,  $d$  for prestressed concrete does **not** need to be taken less than  $0.8h$ !

## $V_{ci}$ and $V_{cw}$

As an alternate to the simplified method of shear presented previously, the ACI code allows the use of  $V_{ci}$  and  $V_{cw}$  (22.5.8.3).

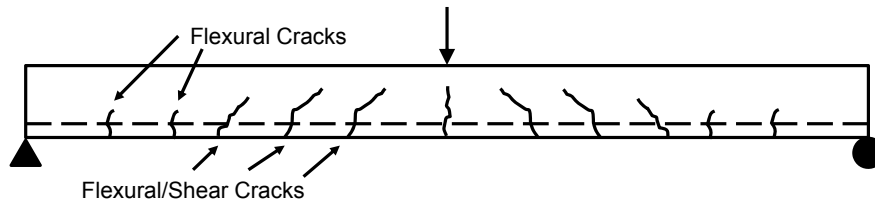
In the shear equation,  $V_c$  is the **lesser** of  $V_{ci}$  and  $V_{cw}$ .

$V_{ci}$  is the shear that causes a flexural crack to become a diagonal crack. This term controls near the midspan where moment is greatest and flexural cracking occurs.

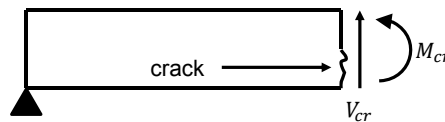
$V_{cw}$  is the shear that causes web cracking. This term tends to control near the end of the beam.

Shear:  $V_{ci}$

$V_{ci}$  is the nominal shear strength provided by concrete when diagonal cracking results due to combined actions of shear and moment.



It's first necessary to determine the shear in the section when the beam cracks, i.e., when the moment at a section reaches  $M_{cr}$ . The shear at the section that exists at the time of cracking is called  $V_{cr}$ .



Shear:  $V_{ci}$

Assuming that shear and moment increase at the same rate:

$$V_{cr} = \frac{V_u M_{cr}}{M_u}$$

Research shows that the increase in shear that causes the flexural crack to become a shear crack is:

$$V_{addl} = 0.6\lambda\sqrt{f'_c}b_wd_p$$

$$\text{So: } V_{ci} = 0.6\lambda\sqrt{f'_c}b_wd_p + \frac{V_u M_{cr}}{M_u}$$

## Shear: $V_{ci}$

For composite sections or sections with point loads the DL does not increase so it is considered separately:

$$V_{ci} = 0.6\lambda\sqrt{f'_c}b_wd_p + V_d + \frac{V_iM_{cre}}{M_{max}} \geq 1.7\lambda\sqrt{f'_c}b_wd_p$$

$$M_{cre} = \frac{I}{y_t} (6\lambda\sqrt{f'_c} - f_{pe} - f_d)$$

## Shear: $V_{ci}$

$\lambda$  = modification factor reflecting the reduced mechanical properties of lightweight concrete, all relative to normal weight concrete of the same compressive strength.

$f'_c$  = concrete compressive strength (psi)

$b_w$  = web width (in.)

$d_p$  = distance from extreme compression fiber to centroid of prestressing steel (in.) need not be taken less than  $0.80h$ , where  $h$  = overall height of member (in.) This is different from the simplified method.

$V_d$  = shear force at section due to **unfactored** dead load (lb.)

$V_i$  = **factored** shear force at section due to externally applied loads occurring simultaneously with  $M_{max}$  (lb.)

$M_{max}$  = maximum **factored** moment at section due to externally applied load (lb-in.)

$M_{cre}$  = the total moment, excluding **unfactored** dead load, required to cause cracking (lb-in.)

$I$  = moment of inertia about centroidal axis (in.)

$y_t$  = distance from centroidal axis of gross section, neglecting reinforcement, to tension face (in.)

$f_{pe}$  = compressive stress in concrete due to effective prestress only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (psi). For simply supported beams, this stress is at the bottom extreme fiber.

$f_d$  = stress due to **unfactored** dead load, at extreme fiber of section where tensile stress is caused by externally applied loads (psi). For simply supported beams, this stress is at the bottom extreme fiber.

Shear:  $V_{ci}$

The commentary allows (R22.5.8.3 1b and c):

$$V_i = V_u - V_d$$

$$M_{max} = M_u - M_d$$

Where:

$V_u$  and  $M_u$  are the factored shear and moment;

$V_d$  and  $M_d$  are the unfactored dead load shear and moment.

Note that  $V_i$  is the shear that occurs simultaneously with  $M_{max}$ .

Shear:  $V_{ci}$

For non-composite beams with **uniform** loads only, the commentary (R22.5.8.3.1d) allows:

$$V_{ci} = 0.6\lambda\sqrt{f'_c}b_wd_p + \frac{V_uM_{ct}}{M_u} \geq 1.7\lambda\sqrt{f'_c}b_wd_p$$

$$M_{ct} = \frac{I}{y_t}(6\lambda\sqrt{f'_c} - f_{pe})$$

Shear:  $V_{ci}$

As an example, consider the point 6.5 feet from the support for straight strands:

$$f_{pe} = \frac{150.5 \text{ k}}{240 \text{ in}^2} + \frac{150.5 \text{ k}(7.33 \text{ in})}{800 \text{ in}^3} = 2 \text{ ksi}$$

Previously found:

$V_u = 17 \text{ kips}$ ;  $M_u = 2,035 \text{ k-in}$  This is the maximum **factored** moment and the associated shear.

$$b_w = 12 \text{ inches}$$

$$d_p = 17.3 \text{ in} \geq 0.8 h = 16 \text{ in}$$

Shear:  $V_{ci}$

Using the formula for a non-composite beam with a uniform load:

$$M_{ct} = \frac{\left(\frac{8,000 \text{ in}^4}{10 \text{ in}}\right)[6(1)\sqrt{5,000 \text{ psi}} + 2,000 \text{ psi}]}{1,000} = 1,940 \text{ k-in}$$

$$V_{ci} = \frac{0.6(1)\sqrt{5,000}(12 \text{ in})(17.3 \text{ in})}{1,000} + \frac{17 \text{ k}(1,940 \text{ k-in})}{2,035 \text{ k-in}} = 25 \text{ k}$$

$$\geq \frac{1.7(1)\sqrt{5,000 \text{ psi}}(12 \text{ in})(17.3 \text{ in})}{1,000} = 25 \text{ k}$$

The simplified method gave  $V_c = 29.8 \text{ k}$

Shear:  $V_{ci}$

Although the beam is non-composite and has no point loads, the other form of the equation can still be used:

$$SW + DL = 0.75 \text{ k/ft}$$

Unfactored DL shear and moment:

$$V_d = (0.75 \text{ k/ft}) \left( \frac{25 \text{ ft}}{2} - 6.5 \text{ ft} \right) = 4.5 \text{ k}$$

$$M_d = \frac{0.75 \text{ k/ft}(6.5 \text{ ft})}{2(12 \text{ in/ft})} (25 \text{ ft} - 6.5 \text{ ft}) = 541 \text{ k-in}$$

$$f_d = \frac{541 \text{ k-in}}{800 \text{ in}^3} = 0.68 \text{ ksi}$$

Shear:  $V_{ci}$

$$V_i = 17 \text{ k} - 4.5 \text{ k} = 12.5 \text{ k}$$

$$M_{max} = 2,035 \text{ k-in} - 541 \text{ k-in} = 1,494 \text{ k-in}$$



Shear:  $V_{ci}$

Using the general formula (which subtracts out DL):

$$M_{cre} = \frac{\left(\frac{8,000 \text{ in}^4}{10 \text{ in}}\right)[6(1)\sqrt{5,000 \text{ psi}} + 2,000 \text{ psi} - 680 \text{ psi}]}{1,000} = 1,395 \text{ k-in}$$

$$V_{ci} = \frac{0.6(1)\sqrt{5,000}(12 \text{ in})(17.3 \text{ in})}{1,000} + 4.5 \text{ k} + \frac{12.5 \text{ k}(1,395 \text{ k-in})}{1,494 \text{ k-in}} = 25 \text{ k}$$

$$\geq \frac{1.7(1)\sqrt{5,000 \text{ psi}}(12 \text{ in})(17.3 \text{ in})}{1,000} = 25 \text{ k}$$

The same answer as before.

Shear:  $V_{cw}$  (22.5.8.3.2)

$V_{cw}$  is the shear required to create a crack in the web. It can be calculated as the shear required to cause the principal tensile stress in the web to be equal to  $4\sqrt{f'_c}$ .

However the code gives the following equation which can be used:

$$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p$$

### Shear: $V_{cw}$ (22.5.8.3.2)

$$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p$$

$\lambda$  = modification factor reflecting the reduced mechanical properties of lightweight concrete, all relative to normal weight concrete of the same compressive strength.

$f'_c$  = concrete compressive strength (psi)

$b_w$  = web width (in.)

$d_p$  = distance from extreme compression fiber to centroid of prestressing steel (in.) need not be taken less than  $0.80h$ , where  $h$  = overall height of member (in.)

$f_{pc}$  = compressive stress (psi) in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange. For non-composite sections,  $f_{pc} = P_{eff}/A$ , where  $P_{eff}$  = effective prestressing force (after allowance for all prestress losses), and  $A$  = cross-sectional area of the member.

$V_p$  = vertical component of effective prestress force (for harped strands) at section (lb). This component will be zero for straight prestressing strands.

### Shear: $V_{cw}$

If the strands are straight,  $f_{pc}$  is constant along the beam except within the transfer length where the value of  $P_{eff}$  must be reduced for the lack of development. Within the transfer length,  $P_{eff}$  is linearly interpolated from 0 at the end of the beam to  $P_{eff}$  at the end of the transfer length.

If strands are straight,  $V_p = 0$ .

$$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p$$

$$V_{cw} = \left( 3.5(1) \frac{\sqrt{5,000 \text{ psi}}}{1,000} + 0.3 \frac{150.5 \text{ k}}{240 \text{ in}^2} \right) (12 \text{ in})(17.3 \text{ in}) + 0$$

$$V_{cw} = 90.4 \text{ k}$$

Shear:  $V_{cw}$

Within the transfer length,  $P_{eff}$  is linearly interpolated from 0 at the end of the beam to  $P_{eff}$  at the end of the transfer length.

Recall the beam 27 feet long and the span is 25 feet so the beam overhangs the support by 1 foot.

The transfer length,  $\ell_{tr}$ , was previously found to be 27.3 inches. At 12 inches from the end of the beam (center of bearing) the strand is not fully developed and  $P_{eff}$  must be adjusted:

$$P_{eff} = \frac{12 \text{ in}}{27.3 \text{ in}} (150.5 \text{ k}) = 66.2 \text{ k}$$

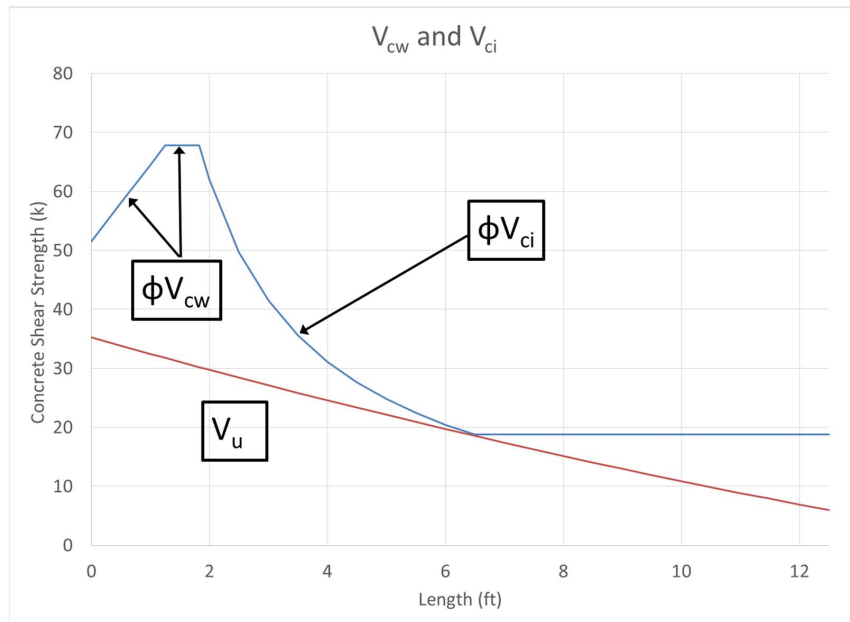
Shear:  $V_{cw}$

Adjusting for lack of development within the transfer length:

$$V_{cw} = \left( 3.5(1) \frac{\sqrt{5,000 \text{ psi}}}{1,000} + 0.3 \frac{66.2 \text{ k}}{240 \text{ in}^2} \right) (12 \text{ in})(17.3 \text{ in}) + 0$$

$$V_{cw} = 68.7 \text{ k}$$

## Shear Demand vs. Capacity



### Shear: $V_{cw}$ and $V_{ci}$

As previously stated,  $V_c$  is taken as the lesser of  $V_{cw}$  and  $V_{ci}$ .

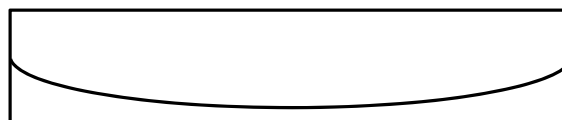
Once  $V_c$  is determined by this method, design for shear and design of the stirrups is the same as the simplified method.

## Release Stresses 24.5.3.1

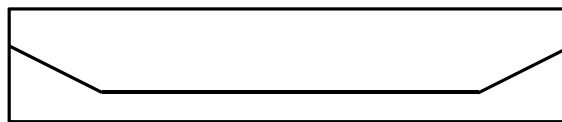
- Allowables:
  - Compression =  $0.6f'_{ci} = 0.6(4,000 \text{ psi}) = 2,400 \text{ psi}$
  - Tension =  $3\sqrt{f'_{ci}} = 3\sqrt{(4,000)} = 190 \text{ psi}$
  - Tension at the end of simply supported beam\* =  $6\sqrt{f'_{ci}} = 6\sqrt{(4,000)} = 380 \text{ psi}$
  - Compression at the end of simply supported beam\* =  $0.7f'_{ci} = 2,800 \text{ psi}$
  - If tension limit is exceeded, mild steel must be added.
  - If the compression limit is exceeded, harp or debond strand.

\* ACI does **not** define what constitutes “the end of a simply supported beam”.

## Deflected Strand Patterns



Draped



Harped

To control end stresses, designers sometimes drape (post-tensioned) or harp (pretensioned) strands. This changes the prestressing moment by changing “e”.

Note that the strand now has a vertical component of force.

This is far more common for bridge rather than buildings.

## Release Stresses

- At release, the overall length of the beam is used (27 feet for this example).
- The only applied load is self weight.
- Initial pull is  $0.75f_{pu} = 202.5$  ksi
- It is assumed the only loss of prestressing force =  $ES^*$ .
  - $P = 6 \text{ strand}(0.153 \text{ in}^2)(202.5 \text{ ksi} - 12.5 \text{ ksi}) = 174.4 \text{ k}$
- The prestressing force is linearly increased over the transfer length from 0 at the end of the beam to  $P$  at the transfer length.

\* Not in ACI. Some designers use 10% for the loss at release. Note the stress values of 202.5 ksi and 174 ksi do not violate steel stress limits of 20.3.2.5.1.

## Release Stresses

Here is a sample calculation of stress at the midspan:

$$M_g = \frac{0.25 \text{ k/ft}(27 \text{ ft})^2}{8} (12 \text{ in/ft}) = 273.4 \text{ k-in}$$

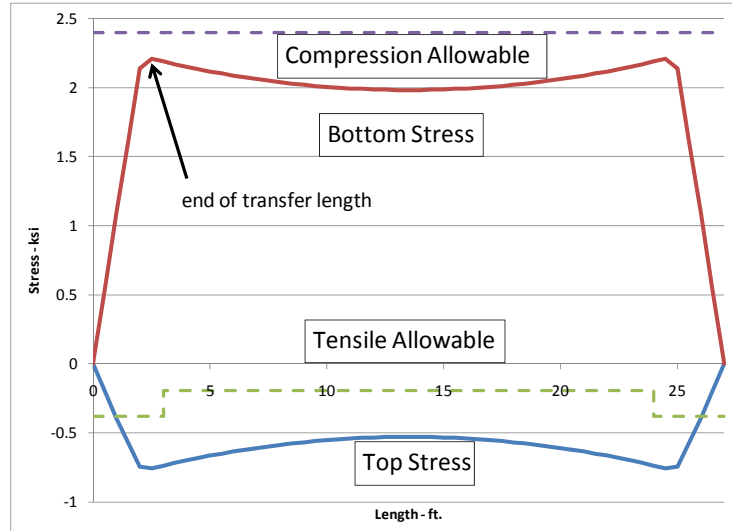
$$f_c = \frac{P}{A} \mp \frac{Pe}{S} \pm \frac{M_g}{S}$$

$$f_c = \frac{174.4 \text{ k}}{240 \text{ in}^2} \mp \frac{174.4 \text{ k}(7.33 \text{ in})}{800 \text{ in}^3} \pm \frac{273.4 \text{ k-in}}{800 \text{ in}^3}$$

$$f_c = -0.529 \text{ ksi} = 0.529 \text{ ksi T Top}$$

$$f_c = 1.98 \text{ ksi} = 1.98 \text{ ksi C Bottom}$$

## Release Stresses

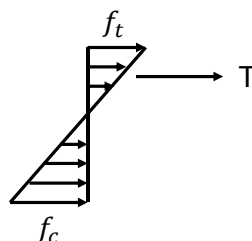


The bottom is OK, but the top is overstressed in tension. The stress goes to 0 at the supports because the full beam length is used. End of transfer length is shown.

## Release Stresses

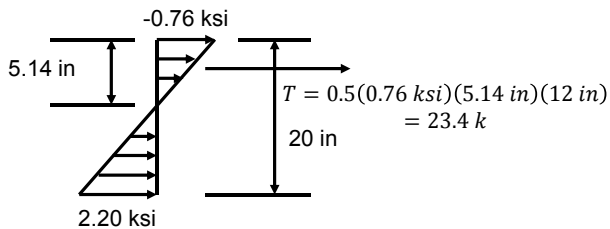
When the top is overstressed in tension, the code requires that mild steel be added to the top of the beam to take the **entire** tensile force in the top of the beam. The **commentary** suggests the steel may be stressed to no more than  $0.6f_y$  or 30 ksi, whichever is less (R24.5.3.2).

Top steel is calculated by remembering that the bending stress distribution is linear. Draw the bending stress distribution and calculate the **tensile force** from the stress diagram.



## Release Stresses

The critical point is the end of the transfer length. The total tensile force is the **volume** under the stress wedge:

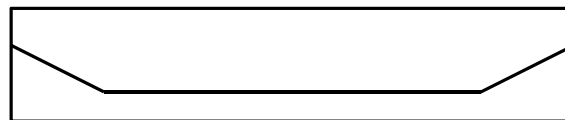


The top and bottom stresses are found from the previously shown graph. The beam is 12" wide.

Top steel is provided to take 23.4 kips of force. The commentary suggests a stress of 30 ksi (if Gr. 60 is used) for crack control.

$23.4 \text{ k} / 30 \text{ ksi} = 0.78 \text{ in}^2$ ;  
use 3 #5 bars or 4 #4.

## Harped Strand



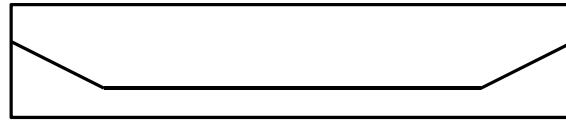
Harped

Harping, or deflecting, strand is a way to control end stresses. In the harped area, the eccentricity,  $e$ , is reduced so the prestressing moment,  $Pe$ , is less.

This reduces top and bottom stresses near the ends of the beam where there is no DL moment to mitigate the prestressing stresses.



## Harped Strand



Harped

This is far more common for bridges rather than buildings.

High top stresses crack the top of beam. In buildings, ACI allows this cracking to be controlled by reinforcing bar.

However, the AASHTO LRFD Specifications do **not** allow the top of the girder to crack, so harping may be used to stop top cracking.

Harping may be used in any beam where top cracking may be problematic, such as a parking structure where salt might get into the cracks.

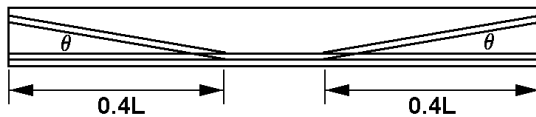
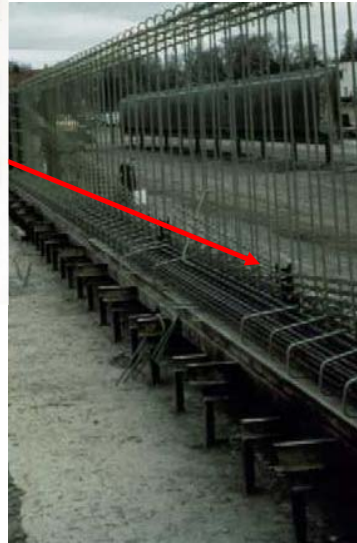
## Harped Strand

- The loss of prestressing force is usually calculated for strand pattern at midspan and is assumed valid for the entire length.
- Harped strand above the midheight of the beam is assumed ineffective for calculating  $f_{ps}$  for finding  $M_n$ .
- The vertical component of the harped strand,  $V_p$ , can be added to  $V_n$ .

## Harped Strand

This is an example of a “hold down” for holding harped strand.

It is sacrificial.

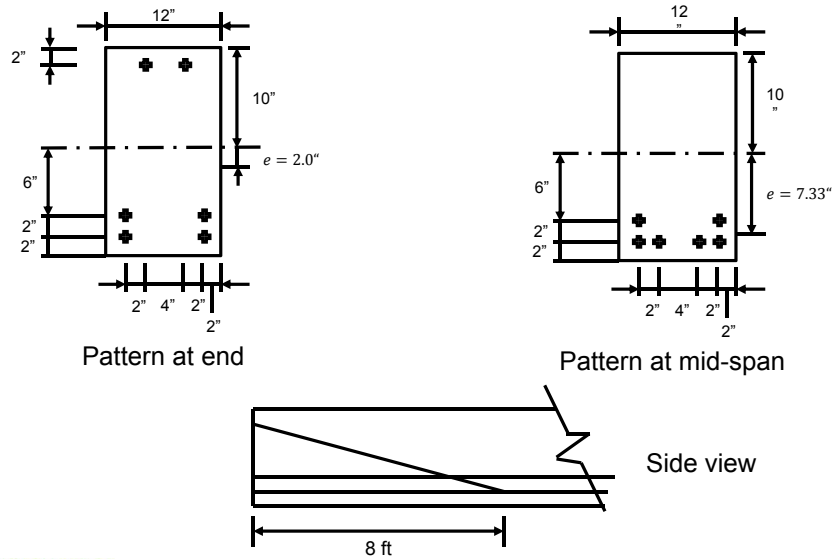


## Harped Strand

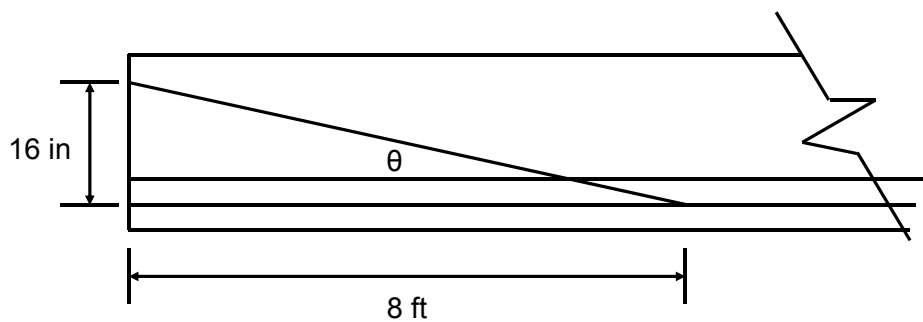
- The hold down exerts a considerable force on the prestressing bed, so not all precasters can harp.
- Some have limitations on their ability to harp due to bed capacity.
- Check with local precasters to determine if they can harp before specifying harping.
- Some sections (e.g. box girders) cannot have harped strand.

## Harped Strand

To illustrate the use of harped strand, harp the two center strands 8 feet from the end.



## Harped Strand



The angle,  $\theta$ , is found from trigonometry:

$$\theta = \tan^{-1}(16 \text{ in}/96 \text{ in}) = 9.46^\circ$$

The position of the harped strand at any point in the cross section can now be found.

## Harped Strand

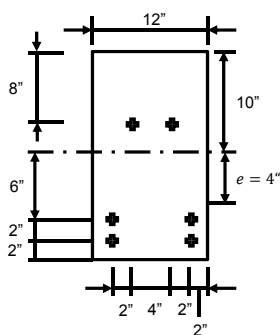
- The harp point is usually  $0.3\ell$  and  $0.5\ell$  from the end.
- The position of the harp point is determined by trial and error.
- The number of harped strands is determined by trial and error.
- See the PCI Design Handbook for more information on harping strand.

## Harped Strand

To illustrate this, consider a section 3 feet from the end of the beam.

The strand position, measured from the top, is found from:

$$2 \text{ in} + 36 \text{ in}[\tan(9.46)] = 8 \text{ inches}$$



Then centroid of the strand from the bottom is:

$$y_b = \frac{2(2 \text{ in}) + 2(4 \text{ in}) + 2(20 \text{ in} - 8 \text{ in})}{6 \text{ strand}} = 6 \text{ in}$$

The eccentricity,  $e$ , is now:

$$10 \text{ in} - 6 \text{ in} = 4 \text{ inches below the centroid.}$$

## Harped Strand

So the calculation at 3 feet from the end is:

$$M_g = \frac{0.25 \text{ k/ft}(3 \text{ ft})}{2} (27 \text{ ft} - 3 \text{ ft})(12 \text{ in/ft}) = 108 \text{ k-in}$$

$$f_c = \frac{P}{A} \mp \frac{Pe}{S} \pm \frac{M_g}{S}$$

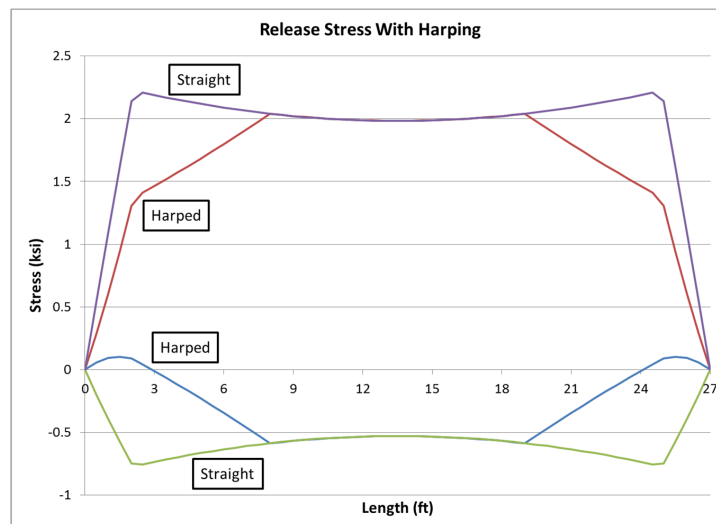
$$f_c = \frac{174.4 \text{ k}}{240 \text{ in}^2} \mp \frac{174.4 \text{ k}(4 \text{ in})}{800 \text{ in}^3} \pm \frac{108 \text{ k-in}}{800 \text{ in}^3}$$

$$f_c = -0.0103 \text{ ksi} = 0.0103 \text{ ksi T Top}$$

$$f_c = 1.46 \text{ ksi} = 1.46 \text{ ksi C Bottom}$$

## Harped Strand

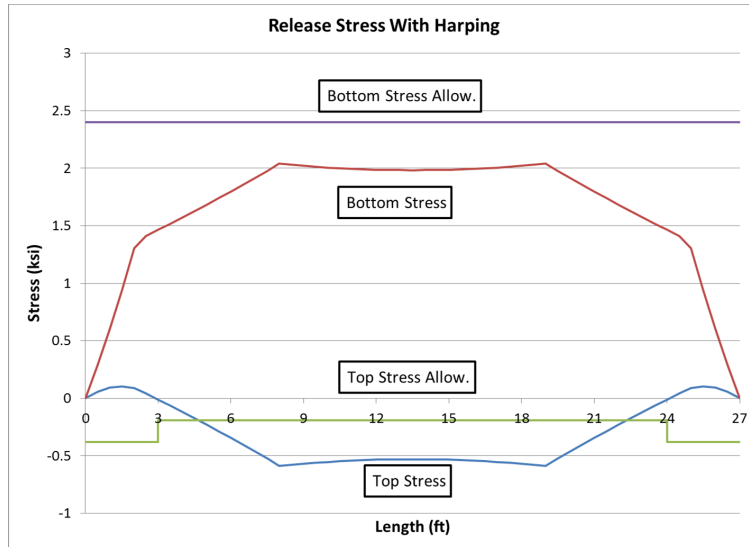
The effect of harping at release can be seen in this graph:



Note how the end stresses are lowered.

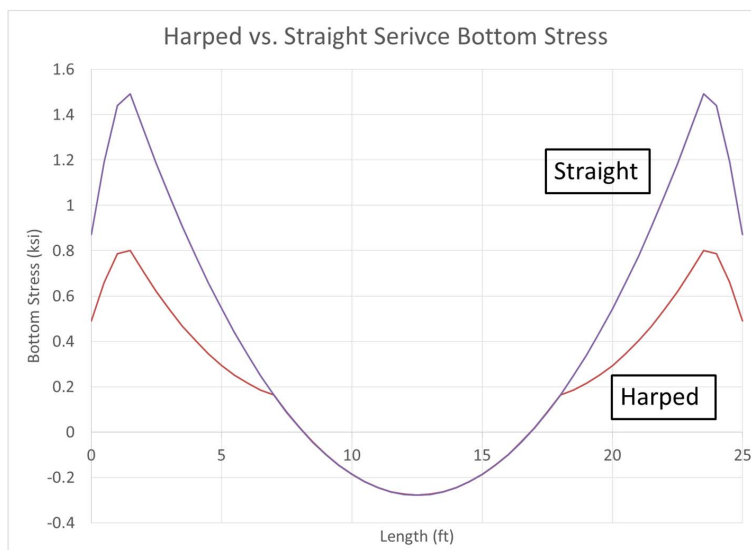
## Harped Strand

However, while harping brings the end stress below the allowable, the top stress at midspan is still too high, so top steel would still be needed.

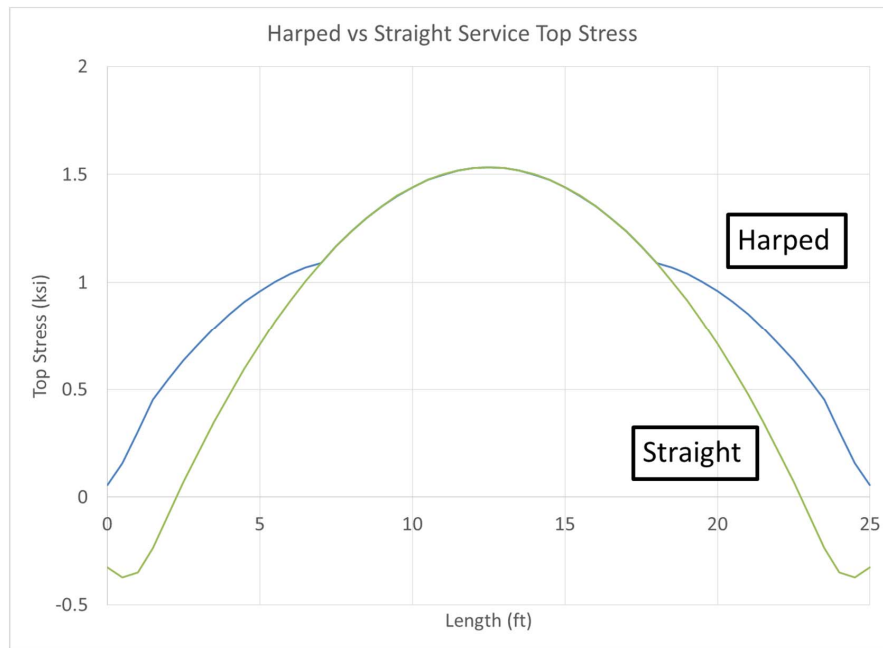


## Harped Strand

Harping is used to limit end stresses at release. However, harping also affects service level stresses which must be checked.



## Harped Strand



## Shear – Harped Strand

In the simplified  $V_c$  method shown.  $V_c$  is lesser of:

$$V_c = \left( 0.6\lambda\sqrt{f'_c} + 700 \frac{V_u d_p}{M_u} \right) b_w d$$

$$\frac{V_u d_p}{M_u} < 1$$

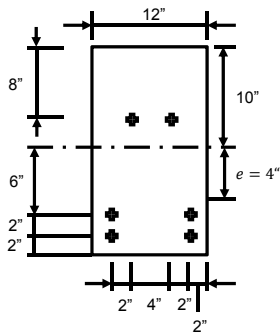
$$V_c = (0.6\lambda\sqrt{f'_c} + 700) b_w d$$

$$V_c = 5\sqrt{f'_c} b_w d$$

$$2\sqrt{f'_c} b_w d \leq V_c$$

The effect of harped strand is that  $d_p$  and  $d$  change.

## Shear – Harped Strand



As an example, calculate  $V_c$  at 2 foot from the support. Since the beam overhangs the support by 1 foot, this point is 3 feet from the end of beam.

This is the same point considered at release.

Note that now  $d_p = 14$  inches; but  $d = 16$  inches since  $d$  is not taken less than  $0.8 h = 0.8(20 \text{ in}) = 16 \text{ in}$ .

## Shear

$$V_c = \left( 0.6(1)\sqrt{5,000 \text{ psi}} + 700 \frac{29.6 \text{ k}(14 \text{ in})}{778 \text{ k-in}} \right) (12 \text{ in})(16 \text{ in})$$

$$V_c = 79.7 \text{ k}$$

$$\frac{V_u d_p}{M_u} = 0.53 < 1 \text{ OK}$$

At this point:

$$M_u = 778 \text{ k-in}$$

$$V_u = 29.8 \text{ k}$$

$$V_u \text{ concur. } w/M_u = 29.6 \text{ k}$$

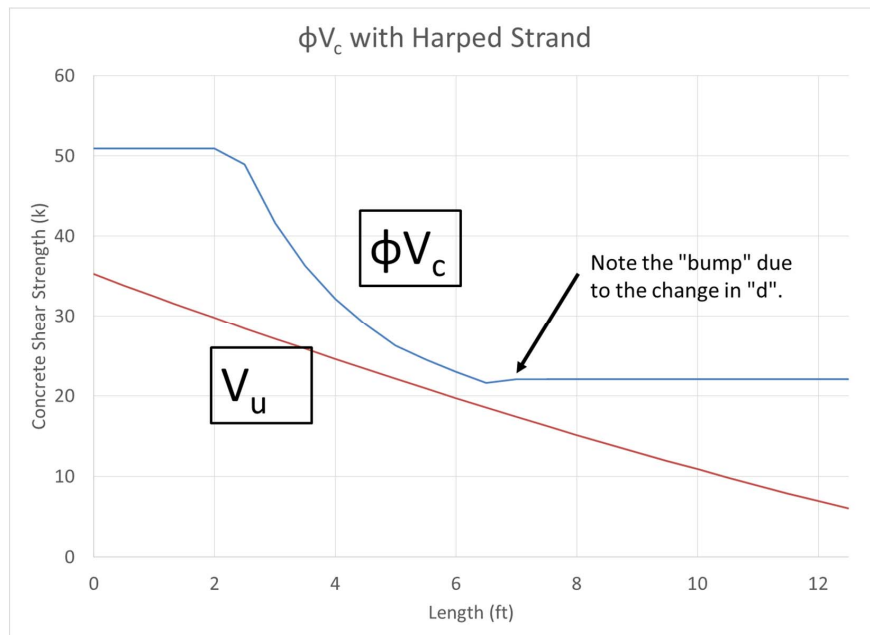
$$V_c = \frac{(0.6(1)\sqrt{5,000 \text{ psi}} + 700)(12 \text{ in})(17.3 \text{ in})}{1,000} = 154 \text{ k} > 79.7 \text{ OK}$$

$$V_c = 5\sqrt{f'_c} b_w d = 67.9 \text{ k} < 79.7 \text{ k use } V_c = 0.75(67.9 \text{ k}) = 50.9 \text{ k}$$

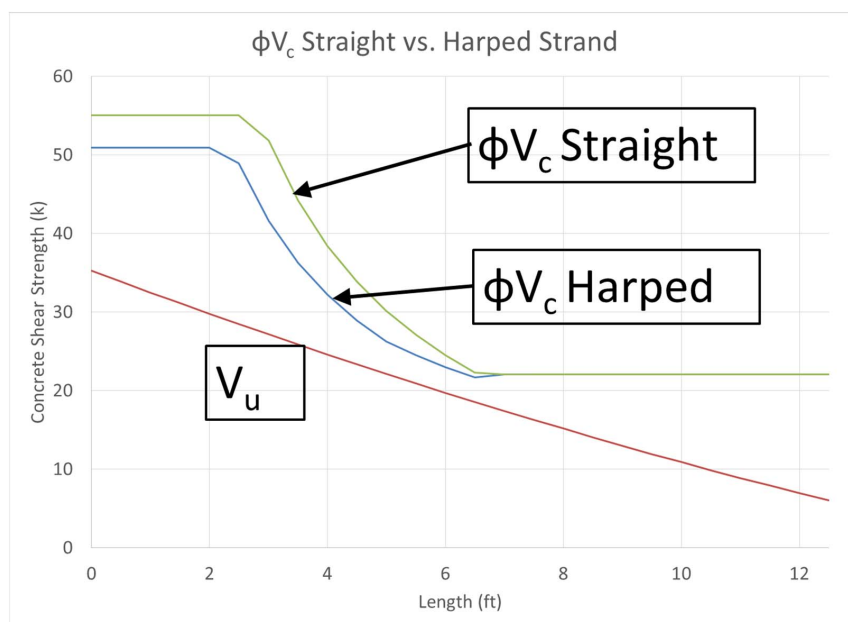
$$2\sqrt{f'_c} b_w d = 29.4 \text{ k} < 67.9 \text{ k OK}$$



## Shear Capacity vs Demand; Harped Strand



## Shear Capacity vs Demand; Compared



## Harped Strand – $V_{ci}$ and $V_{cw}$

The effect of harping strand is that  $d$  and  $d_p$  become smaller so the shear strength in the **simplified** equation is lower.

Additional stirrups will usually take care of this.

As an alternative, using  $V_{ci}$  and  $V_{cw}$  may sometimes provide higher capacities (and sometimes not).

## Harped Strand – $V_{ci}$ and $V_{cw}$

- Harped strand affects  $V_{ci}$  as follows:
  - The term  $d_p$  changes.
  - Stress  $f_{pe}$  changes.
- Harped strand affects  $V_{cw}$  as follows:
  - The term  $d_p$  changes.
  - Stress  $f_{pe}$  changes.
  - There is now a  $V_p$  term. In the example:

$$V_p = (2 \text{ strand})(0.153 \text{ in}^3)(164 \text{ ksi}) \sin 9.46 = 8.2 \text{ k}$$

## Bursting Stirrups

At the end of the girder, it is possible cracking will occur due to release of prestressing force.

As a result, bursting stirrups are needed.

Section 5.2.4 in the 7th edition of PCI Design Handbook provides the following methodology for computing the amount of additional stirrups. (Note: ACI does not provide specific equations.)

## Bursting Stirrups

PCI provides:

$$A_{vt} = \frac{0.021P_o h}{f_s \ell_{tr}}$$

$A_{vt}$  = area of stirrups that must be distributed along a length of  $h/5$  from the end of the beam

$P_o$  = prestress force at transfer

$h$  = depth of the beam

$f_s$  = allowable stress in the stirrups (taken as 30 ksi for Gr. 60 steel)

$\ell_{tr}$  = transfer length

## Bursting Stirrups

Using numbers previously found:

$$A_{vt} = \frac{0.021P_o h}{f_s \ell_{tr}} = \frac{0.021(0.75)(270 \text{ ksi})(6)(0.153 \text{ in}^2)(20 \text{ in})}{30 \text{ ksi}(27.3 \text{ in})} = 0.0953$$

Thus a single #3 stirrup ( $A_{vt} = 0.22 \text{ in}^2$ ) is needed within  $h/5=4$  in of the end.

## Deflections/Camber

Camber is the upward deflection of the beam due to prestressing.

In general camber is not a design consideration except that excessive camber might be a sign of something wrong with the beam.

Also, excessive differential camber between beams may make alignment in the structure difficult.

## Deflections/Camber

Initially, camber is caused by the effects of prestressing forces.

Over time, the prestressing forces cause creep in the beam and the beam both shortens and cambers up. Shrinkage also causes shortening and may increase or decrease camber depending on the shape and shrinkage of the beam.

This additional camber due to creep and shrinkage is called “Growth in Storage.”

## Camber/Deflection

<b>Table 5.8.2 Suggested simple span multipliers to be used as a guide in estimating long-term cambers and deflections for typical prestressed components</b>		
	Without composite topping	With composite topping
At erection:		
1. Deflection (downward) component – apply to the elastic deflection due to the component weight at release of prestress	1.85	1.85
2. Camber (upward) component – apply to the elastic camber due to prestress at the time of release of prestress	1.80	1.80
Final:		
3. Deflection (downward) – apply to the elastic camber due to prestress at the time of release of prestress	2.70	2.40
4. Camber (upward) component – apply to the elastic camber due to prestress at the time of release of prestress	2.45	2.20
5. Deflection (downward) – apply to elastic deflection due to superimposed dead load only	3.00	3.00
6. Deflection (downward) – apply to elastic deflection caused by the composite topping	-	2.30

To get long term cambers, multiply the short term camber/deflection by these values (Table 5.8.2, PCI Design Handbook, 7th ed. These values are used for bridges AASHTO C5.7.3.6.2).

## Deflections/Camber

Consider the case of the straight strand.

Basically, the deflection due to the prestressing force is found from the equation for a beam with a moment at each end:

$$\delta_{ps} = \frac{Pe\ell^2}{8E_{ci}I}$$

At release:  $P = 174.4$  k (found previously);  $e = 7.33$  in

$$\delta_{ps} = \frac{(174.4 \text{ k})(7.33 \text{ in})[(25 \text{ ft})12 \text{ in/ft}]^2}{8(3,600 \text{ ksi})(8,000 \text{ in}^4)} = 0.5 \text{ in } \uparrow$$

## Deflections/Camber

Deflection due to self weight:

$$\delta_{sw} = \frac{5w\ell^4}{384E_{ci}I}$$

At release:  $w = 0.25$  k/ft = 0.0208 k/in

$$\delta_{sw} = \frac{5(0.0208 \text{ k/in})[(25 \text{ ft})12 \text{ in/ft}]^4}{384(3,600 \text{ ksi})(8,000 \text{ in}^4)} = 0.08 \text{ in } \downarrow$$

## Deflections/Camber

The initial camber is:

$$\delta_{init} = 0.50 \text{ in} - 0.08 \text{ in} = 0.42 \text{ in} \uparrow$$

At erection:

$$\delta_{erect} = 1.80(0.50 \text{ in}) - 1.85(0.08 \text{ in}) = 0.75 \text{ in} \uparrow$$

## Deflections/Camber

Deflection due to additional dead load:

$$\delta_{DL} = \frac{5w\ell^4}{384E_cI}$$

At release:  $w = 0.5 \text{ k/ft} = 0.0417 \text{ k/in}$

$$\delta_{DL} = \frac{5(0.0417 \text{ k/in})[(25 \text{ ft})12 \text{ in/ft}]^4}{384(4,030 \text{ ksi})(8,000 \text{ in}^4)} = 0.14 \text{ in} \downarrow$$

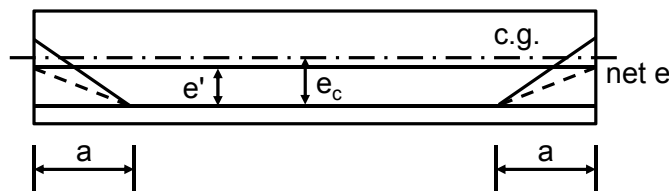
## Deflections/Camber

Long term camber:

$$\begin{aligned}\delta_{LT} &= 2.45(0.5 \text{ in}) - 2.7(0.08 \text{ in}) - 3.0(0.14 \text{ in}) \\ &= 0.59 \text{ in } \uparrow\end{aligned}$$

## Deflections/Camber

For other cases, such as harped strand, the PCI Design Handbook and PCI Bridge Design Handbook, have other formulae. For example, for a beam with both straight and harped strand, camber is:



$$\Delta_p = \frac{P_i}{E_c I_g} \left( \frac{e_c L^2}{8} - \frac{e' a^2}{6} \right)$$

See the appendices of the PCI Design Handbook and PCI Bridge Design Handbook for more formulae.



## Questions

Thank you!

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