Sample Exam Questions
ME274 – Basic Mechanics II

Attached are copies of exam questions from past semesters in ME 274. These problems cover material from throughout the course (both midterm exams and final exam).

These problems are provided to you as a study guide for your exams. The level of difficulty may or may not be reflective of the questions that you will encounter in your exams this semester.
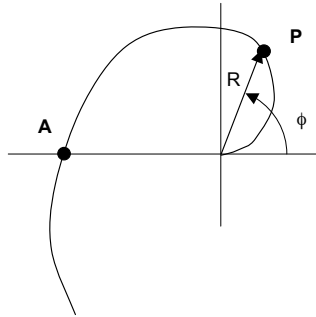
Problem 1

Given: A particle P travels on a path described by the *polar equation*:

$$R = 2\phi \qquad \text{(R in feet and } \phi \text{ in radians)}$$

with a *constant* speed of $v = 3\,ft/sec$.

Find: For the position of P at point A shown below:

  a) determine the values of $\dot{R}$ and $\dot{\phi}$.

  b) make a sketch of path unit vectors $\bar{u}_t$ and $\bar{u}_n$, and the polar unit vectors $\bar{u}_R$ and $\bar{u}_\phi$ in the figure below.

  c) determine the values of $\ddot{R}$ and $\ddot{\phi}$.

  d) determine the radius of curvature of the path.

Problem 2

At the instant shown below, particle P has velocity and acceleration vectors, $\bar{v}_P$ and $\bar{a}_P$ that point directly down (vertical) and directly to the right (horizontal), respectively. Furthermore at this instant, R = 4 meters, $\phi = 36.87°$ and $\left|\bar{a}_P\right| = 2\,m/sec^2$. The speed of P is known to be constant of 10 m/sec for all time. For the instant shown:

  a) show the polar unit vectors $\bar{e}_R$ and $\bar{e}_\phi$ in the figure below.

  b) find the values of $\dot{R}$ and $\dot{\phi}$.

  c) find the values of $\ddot{R}$ and $\ddot{\phi}$.

  d) determine the value of the radius of curvature of the path for point P.
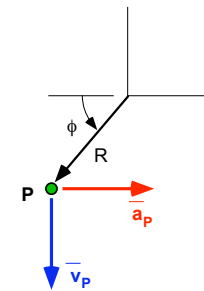
Problem 3

Given: Particle P travels on a path given in terms of its polar coordinates:

$$R = \frac{1}{2}\phi^2 \quad (\text{feet})$$

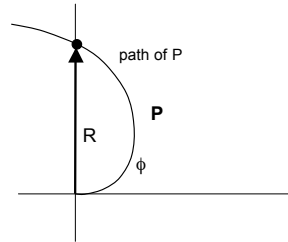where ϕ is in radians and $\dot{\phi} = 4\,rad/\sec = constant.$

Find: For $\phi = \pi/2,$
   a) show the *polar* unit vectors $\bar{u}_R$ and $\bar{u}_\phi$ in the figure below at $\phi = \pi/2$.
   b) find the velocity and acceleration in terms of their polar coordinates.
   c) show the *path* unit vectors $\bar{u}_t$ and $\bar{u}_n$ in the figure below at $\phi = \pi/2$.
   d) determine the *speed* v_P and the *rate of change of speed* of P.

Answers for d):
$\left| v_P \right| = 8.00\,m/\sec$

$\dot{v}_P = 28.1\,m/\sec^2$

path of P

P

R

ϕ

Problem 4

Given: A cord is being unwrapped from a spool with the cord having a CONSTANT speed of $v_{cord} = 5\,ft/\sec$ as it leaves the spool. The cord is then wrapped over a fixed peg O. At the free end of the cord is a particle P. The angular rotation $\dot{\phi}$ for the free end of the cord is related to the radial distance R from O to P by:

$$R^2 \dot{\phi} = constant = -10\,ft^2/\sec$$

where R is in feet and $\dot{\phi}$ is in rad/sec. At the position shown below, $R = 2$ feet and $\phi = \frac{\pi}{3}\,rads$.

Find: For the position of P shown in the figure:
   a) Sketch the polar unit vectors $\bar{e}_R$ and $\bar{e}_\phi$ on the figure.
   b) Determine the velocity $\bar{v}_P$ and acceleration $\bar{a}_P$ of the particle P. Write your answers as vectors.
   c) Sketch $\bar{v}_P$ and $\bar{a}_P$ on the figure below. Include the path description unit vectors $\bar{e}_t$ and $\bar{e}_n$ in your sketch.
   d) Based on your sketch in c) above, is the rate of change of speed of P positive, negative or zero? Provide an explanation with your answer.

Answers to part b):

$$\bar{v}_P = \left(5\bar{e}_R - 5\bar{e}_\phi\right) ft/\sec$$

$$\bar{a}_P = \left(-12.5\,\bar{e}_R\right) ft/\sec^2$$

P

R

ϕ

O (fixed peg)

v_{cord}

spool
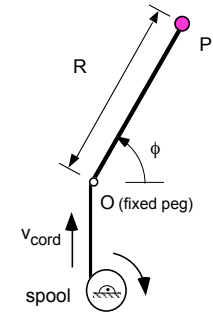
Problem 5

Given: Point P is traveling on a curved path, as shown below. The tangential component of acceleration of P, a_t, and the radius of curvature of the path ρ of P are known to be the following functions of the position s on the path:

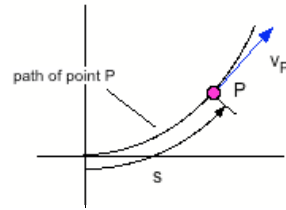$$a_t = \frac{dv}{dt} = 3s^2$$

$$\rho = 0.2s^3 + 1$$

where s is in meters and time is in seconds. When s = 0, the speed of P is 15 meters/sec.

Find: At the position of P when s = 4 meters,

a) determine the acceleration vector of point P, $\bar{a}_P$. Write your answer as a vector.

b) make a sketch $\bar{a}_P$ on the figure below. Include the path description unit vectors $\bar{u}_t$ and $\bar{u}_n$ in your sketch.

Answer to a): $\bar{a}_P = \left(48\bar{e}_t + 25.6\bar{e}_n\right) m/\sec^2$

path of point P

P

V_P

s

Problem 6

Given: Blocks A and B are connected by a cable that has a length of L = 10 meters with the cable being pulled over a pulley at C. Block A is constrained to move along a guide in such a way that the its acceleration is a function of the position s_A as:

$$a_A = 0.3\,s_A{}^2 \ \ (\text{meters/sec}^2)$$

with the speed of A being zero when $s_A = 0$. Block B is constrained to move along a surface that is perpendicular to the guide for A. Assume that the cable does not stretch or go slack during the motion of the system. Also assume that the pulley C is small compared to the other dimensions of the problem.
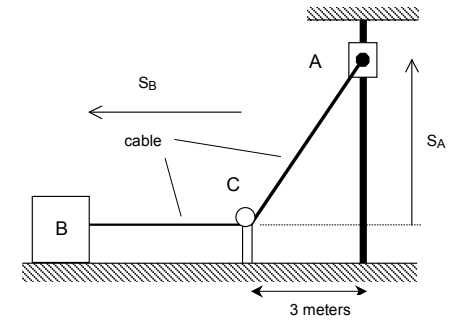
Find: When $s_A = 4$ meters,

a) find the speed of block A.

b) find the speed of block B.

Answers:

$v_A = 3.58 \ m/\sec$

$v_B = 2.86 \ m/\sec$


A

s_B

cable

C

B

s_A

3 meters

Problem 7

Given: A cylinder with an outer radius of R = 3 ft *rolls without slipping* on a horizontal surface. The center of the disk O has an acceleration that is known in terms of the position x of:

$$a_O = \frac{3}{2}x^2 \quad (\text{x in feet and } a_O \text{ in ft/sec}^2)$$

At position 1 (x = 0), point O has a velocity of 1 ft/sec to the right.

Find: At position 2, where x = 2 ft,

    a) determine the *velocity* of point O.

    b) determine the *angular velocity and angular acceleration* of the cylinder. Write your answers as vectors.

    c) determine the *velocity and acceleration of point A* (at this position A is on the same horizontal line as O). Write your answers as vectors.

    d) *sketch* the velocity vector for A (from c) above) in the Position 2 figure below.

Position 1   **Position 2**
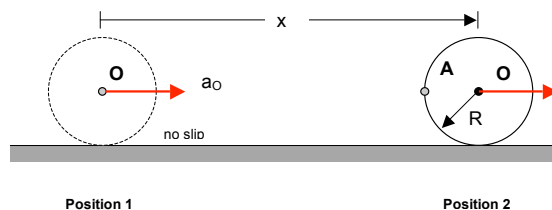
Answers to part c):

$$\underline{v}_A = \left(3\underline{i} + 3\underline{j}\right) ft/sec$$

$$\underline{a}_A = \left(9\underline{i} + 6\underline{j}\right) ft/sec^2$$

Problem 8

Given: A mechanism is made up of two links AB and BC pinned together at point B, and with link AB pinned to ground at point A. Link BC is pinned to a block that is constrained to move on a horizontal plane. At the instant shown, AB is vertical, and point C is moving to the right with a speed of 10 m/sec and slowing down at a rate of 3 m/sec^2.

Find: At the instant shown,

    a) determine the *angular velocity* of links AB and BC. Write your answers as vectors.

    b) determine the *angular acceleration* of links AB and BC. Write your answers as vectors.
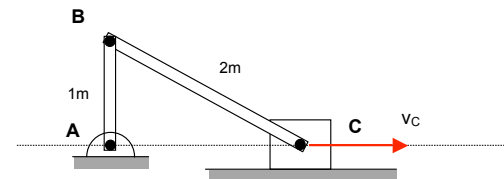
Answers to part b):

$$\underline{\alpha}_{BC} = \left(57.7\ \underline{k}\right) rad/sec^2$$

$$\underline{\alpha}_{AB} = \left(60.7\ \underline{k}\right) rad/sec^2$$

9   10
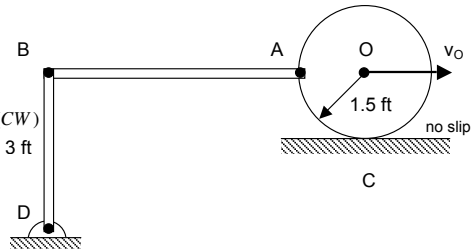
Problem 9

Given: A disk having a radius of r = 1.5 ft is rolling without slipping on a rough horizontal surface to the right with its center O moving at a CONSTANT speed of $v_O = 20 \, ft/sec$. A rigid bar AB having a length of 4 ft is attached to point A on the circumference of the disk. The other end of AB is attached a second rigid bar, BD (having a length of 3 ft), at pin B with point D pinned to ground. At the position shown, bar AB has a horizontal orientation, bar BD has a vertical orientation and point A is on the same horizontal line as point O.

Find: At the position shown,

   a) find the angular velocities of bars AB and BD.

   b) find the angular accelerations of bars AB and BD.

   c) show (or describe in words) the location of the instant center for link AB.

Answers for b):

$$\alpha_{BD} = -366.7/3 = 122.2 \; rad/sec^2 \quad (CW)$$
$$\alpha_{AB} = 33.3 \, rad/sec^2 \quad (CCW)$$

Problem 10

Given: A mechanism is made up of links AB and BC and a wheel pinned to BC at the wheel's center C. The wheel *rolls without slipping* on a horizontal surface. Link AB rotates counterclockwise with a *constant* rate of 3 rad/sec. At the instant shown, link AB is vertical.

Find: At the instant shown,

   a) determine the *angular velocities* of link BC and the wheel.

   b) determine the *angular accelerations* of link BC and the wheel.

   c) determine the *velocity and acceleration of point E* on the perimeter of the wheel (at the instant shown, E is on the same horizontal line as C).

Write your answers as vectors.
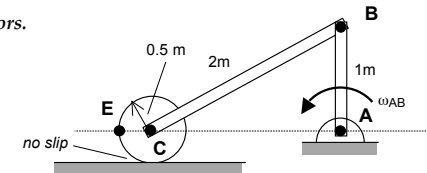
Problem 11

Given: A disk having a radius of r = 2 ft is rolling without slipping on a rough horizontal surface to the right with its center O moving at a CONSTANT speed of $v_O = 20\,ft/sec$. A rigid bar having a length of L = 4 ft is attached to point A on the circumference of the disk. The other end is attached to a block at pin B with the block constrained to moving on a vertical guide. At the position shown, point A is on the same horizontal line as point O, and θ = 53.13°.

Find: At the position shown,

    a) determine the <u>velocity</u> and <u>acceleration</u> of point B. Express your answers as vectors.

    b) show the location of the instant center of bar AB in the figure below.
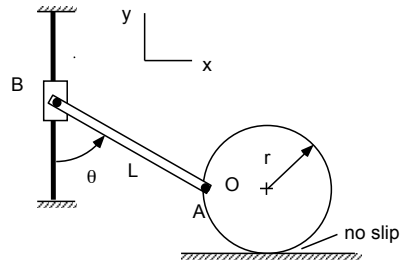
Answers for a):

$$\overline{v}_B = \left(-6.66\,\overline{j}\right)ft/sec$$

$$\overline{a}_B = \left(-729.4\,\overline{j}\right)ft/sec^2$$

Problem 12

In the mechanism below, link AB is rotating CCW at a <u>constant</u> rate of 3 rad/sec, and point D is moving to the right with a <u>constant</u> rate of 5 ft/sec. Point D is the center of the wheel shown, with the wheel rolling without slipping on a horizontal surface. At the instant shown, links AB and CD are vertical, and link BC is horizontal. At this instant, find

    a) <u>angular velocity</u> of links BC and CD.

    b) <u>angular acceleration</u> of links BC and CD.

    c) determine the <u>location of the instant center</u> for link CD.

Express your answers in a) and b) as vectors. Clearly indicate the unit vectors in the figures that you use in your analysis.
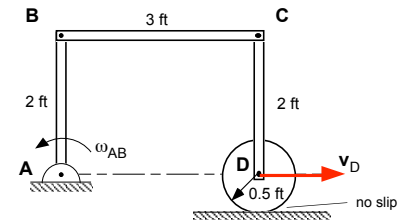
Answers for a) and b):

$$\overline{\omega}_{CD} = 5.5\,\overline{k}\;\;rad/sec$$

$$\overline{\omega}_{BC} = \overline{0}$$

$$\overline{\alpha}_{CD} = \overline{0}$$

$$\overline{\alpha}_{BC} = -14.2\,\overline{k}\,rad/sec^2$$
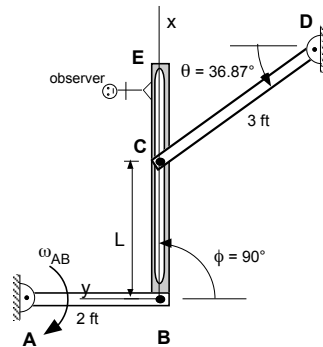
Problem 13

In the mechanism shown below, rigid links AB and EB are pinned together at point B. A third rigid link is pinned to ground at point D, and has pin C sliding within a slot cut into link EB. At the instant shown in Figure (b), link AB is horizontal and rotating CW at a rate of 4 rad/sec, L = 2.5 ft, $\phi = 90°$ and $\dot{\phi} = 6\,rad/\sec$.

At this instant, find the <u>velocity</u> of pin C. Express you answer as a vector.

<u>Hint</u>: Use link EB as a moving reference frame (observer shown in figure) with an xyz coordinate attached to this link with its origin at B and oriented as shown below.
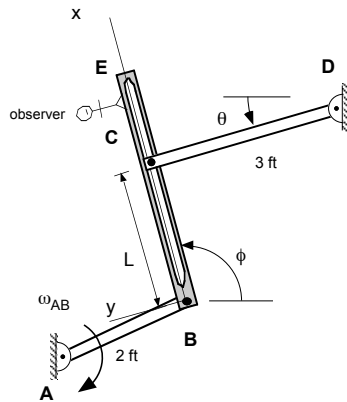
Figure(a)   **Figure(b)**

Answer: $\bar{v}_C = \left(20\vec{i} + 15\vec{j}\right) ft/\sec$

Problem 14

During a dynamic test of an automobile tire, the support arm AO for the wheel is given an oscillatory motion in the fixed vertical plane of $\theta(t) = 0.1 \sin 10t$, and the wheel has a <u>constant</u> rotation rate *relative to the support arm* of $\omega_1 = 2$ rad/sec. A pebble P is stuck in the tread of the tire at the outer radius of the tire of R = 14 inches. The length of the support arm is L = 24 inches.

An xyz coordinate system is <u>attached to the wheel</u> with its origin at O (center of wheel) with the pebble P being on the y-axis for all time. A <u>fixed</u> XYZ coordinate system is defined in such a way that the xyz axes are aligned with the XYZ axes at time t = 0.
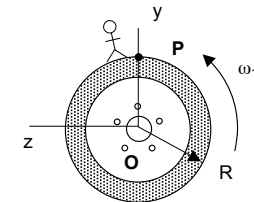
At t = 0, determine

i)   the angular velocity and angular acceleration of the wheel.

ii) the acceleration of the pebble P.

Use an observer that is fixed to the xyz coordinate system.
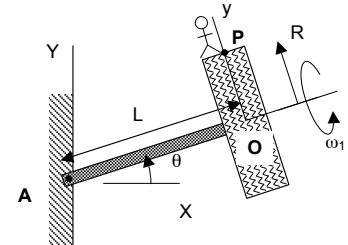

Side View of Tire   **End View of Tire**

Answer to ii): $\underline{a}_P = \left(-24\,\underline{i} - 70\,\underline{j}\right) in/\sec^2$
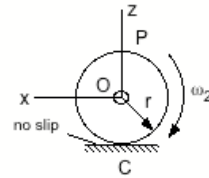
Problem 15

Arm OA, having a length of R = 2 meters, rotates about a fixed vertical axis at a <u>constant</u> rate of $\omega_1 = 4\,rad/\sec$. A thin disk with its center at O and a radius of r = 1 meter rotates with respect to OA with a rate of ω_2 and rolls without slipping on a horizontal surface.

An xyz coordinate system is <u>attached to the disk</u> with its origin at O. A <u>fixed</u> XYZ coordinate system is defined in such a way that Z is aligned with the rotation axis of OA. At the instant shown below in the figures, the xyz and XYZ axes are aligned with respect to each other.

i)   Determine the rate ω_2 with which the disk rotates with respect to arm OA. (Consider the no-slip condition at point C on the disk.)

ii) Determine the angular velocity and angular acceleration of the disk. Write your answers as vectors using xyz components.

iii) Write down the acceleration of point O as a vector.

iv) Determine the acceleration of point P on the disk (at the instant shown, point P is immediately above point O on the circumference of the disk).
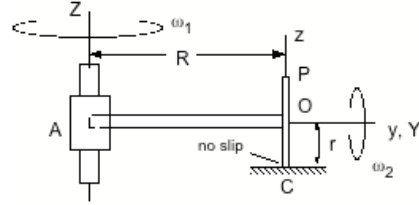

Side View

End View
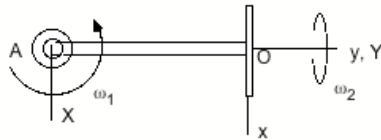
Top View

Answer: $\bar{a}_P = \left(-96\,\bar{j} + 64\,\bar{k}\right) m/\sec^2$

Problem 16

Given: A particle P travels in a tube with $\dot{R} = 6\ ft/\sec = constant$. The tube is being raised at a *constant* rate of $\dot{\theta} = 3\ rad/\sec$. In addition, the tube is attached to a vertical shaft, which is rotating about the *fixed* Y axis with a *constant* rate of $\omega = 4\ rad/\sec$. An observer is attached to the tube with the xyz axes also attached to the tube with its origin at point O.

Find: You are asked to find the acceleration of particle P when R = 5 feet and $\theta = 36.87°$. In doing so, answer the following questions.

a) Find the *angular velocity vector of the observer*.

b) Find the *angular acceleration vector of the observer*.

c) What is the *velocity of point P as seen by the observer*?

d) What is the *acceleration of point P as seen by the observer*?

e) Using the results from a)-d) above, find the *acceleration of point P*.
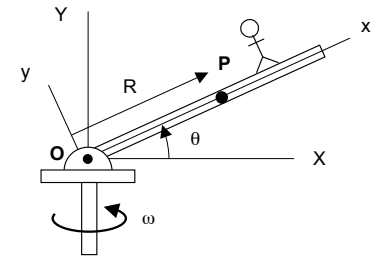
Answer to part e):
$\underline{a}_P = \left(-96.2\,\underline{i} + 74.4\,\underline{j} + 33.6\,\underline{k}\right) ft/\sec^2$

Problem 17

Given: A turntable rotates about a fixed, vertical axis with a *constant* rate of $\omega_1 = 4\,rad/\sec$. Arm OA is hinged on the turntable at point O (with O being located on the rotation axis of the turntable), and is being raised at a *constant* rate of $\dot{\theta} = 3\,rad/\sec$. Block P is sliding outward on arm OA at a constant rate of $v = 5\,m/\sec$ relative to the arm. A moving coordinate system xyz is attached to arm OA, with the x-axis aligned with the motion of P on the arm, as shown in the figure to the right. The XYZ axes shown in figure are fixed, with the Y-axis aligned with the rotation axis of the turntable. The xyz and XYZ axes are aligned with each other when $\theta = 0$.

Find: When R = 1.5 meters and $\theta = 36.87°$,

    a) determine the *angular velocity* and *angular acceleration* of arm OA.
    b) determine the *acceleration* of block P.

Note: Express your answers in terms of their xyz components.
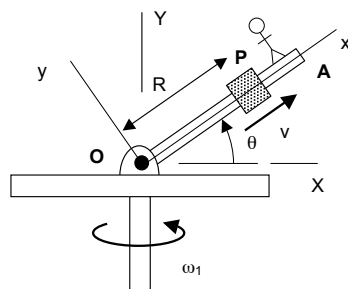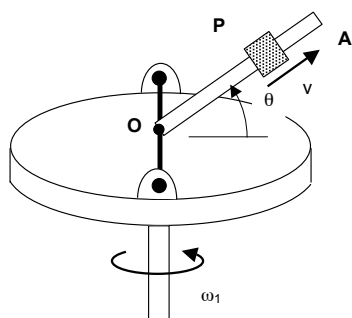
Side View

Answer for b):
$$\bar{a}_P = \left(-28.86\,\bar{i} + 41.51\,\bar{j} - 10.4\,\bar{k}\right)m/\sec^2$$

19

Problem 18

Given: Collar P having a mass of 20 kg is fit over the angled portion of a bent rod. The rod is rotating with a *constant* rate of ω about a vertical axis, as shown in the figure. The static and kinetic coefficients of friction between the collar and the rod are $\mu_s = 0.5$ and $\mu_k = 0.2$, respectively.

Find: Determine the *maximum rotation rate* ω such that the collar <u>does not slide</u> on the rod at the position shown. You will need to provide an accurate free body diagram for the collar and an appropriate set of coordinate axes in order to receive full credit for your solution.

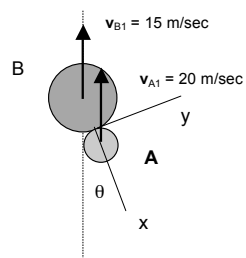Answer: $\omega = 5.20\,rad/\sec$

20
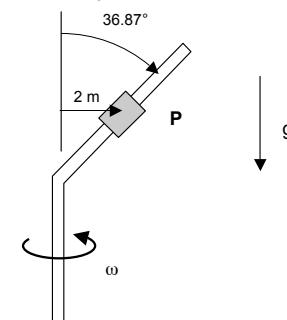
Problem 19

Given: Disks A and B (having masses of 2 and 4 kg, respectively) are initially traveling in the same direction on a horizontal surface with speeds of 20 m/sec and 15 m/sec, respectively. The radius of A is half that of B, and A is initially traveling along a line such that its left edge is lined up with the center of B, as shown in the figure below. The coefficient of restitution of the impact of A with B is 0.7.

Find: Determine the velocity vectors for A and B after impact. You will need to show appropriate FBD's and coordinate axes in order to receive full credit for your solution.

Answers:

$$v_{A2} = \left(-13.5\,\underline{i} + 6.68\,\underline{j}\right)\,m/\sec$$

$$v_{B2} = \left(-16.8\,\underline{i} + 5\,\underline{j}\right)\,m/\sec$$

v_{B1} = 15 m/sec

B

v_{A1} = 20 m/sec

y

A

θ

x

Problem 20

A particle B having a mass of m is released from rest on a smooth incline with an angle θ from a height of h. The particle slides down the incline impacting a smooth horizontal surface, with the impact having a coefficient of restitution of e.

a) Determine the angle φ that defines the tangent to the path of B after impacting the horizontal surface. Leave your answer in terms of e and θ. Is the angle φ larger or smaller than the angle θ? Explain.

b) Find the horizontal distance d traveled by B after impact before striking the ground for the second time. Leave your answer in terms of e, h and θ.

c) If the value of the coefficient of restitution e is increased, does the value of d found in a) increase or decrease?

smooth

B
1

y, n

v_3

mg

DATUM

h

θ

φ

x

2,3

d

4
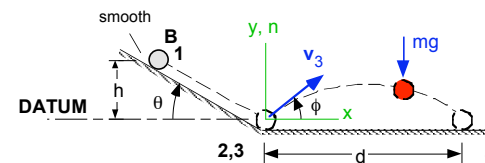
Answer for b):

$$d = 4\,e\,h\cos\theta\sin\theta$$

Problem 21

Given: A system is made up of two identical homogeneous disks (with each disk having a mass of M = 20 kg and outer radius of R = 0.5 m) and an inextensible cable. One end of the cable is attached to point O of disk 1 and the other end of the cable is wrapped around the outer radius of disk 2. Disk 1 sits on a SMOOTH horizontal surface, whereas disk 2 is on a rough incline and does NOT slip on the incline. The system is released from rest.

Find:

    a) Draw a free body diagram of each disk individually.
    b) Determine the *acceleration* of the center A of disk 2 immediately after release.
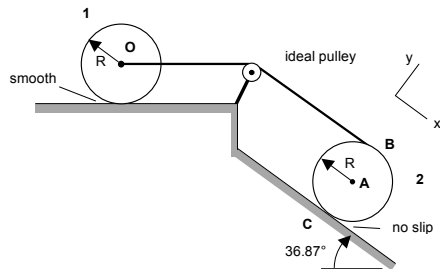
Answer: $\bar{a}_A = \left(1.07\vec{i}\right) m/\sec^2$

Problem 22

Pellet P having a mass of m = 20 kg is pushed through a barrel by means of compressed air such that the force on the pellet by the compressed air is a constant F = 100 newtons. The barrel is rotating CCW within a vertical plane with a constant rate of 5 rad/sec. The kinetic coefficient of friction between the pellet and the barrel is μ_k = 0.3. When θ = 53.13°, R is increasing at a rate of 4 m/sec.

When θ = 53.13°, find

    a) the acceleration of P.
    b) the normal force acting on P by the barrel.

Include an accurate free body diagram of the pellet and a sketch of the coordinate axes used in determining your solution.

Answer to a): $\underline{a}_P = \left(-16.6\,\underline{e}_r + 40.0\,\underline{e}_\theta\right) m/\sec^2$


Vertical plane
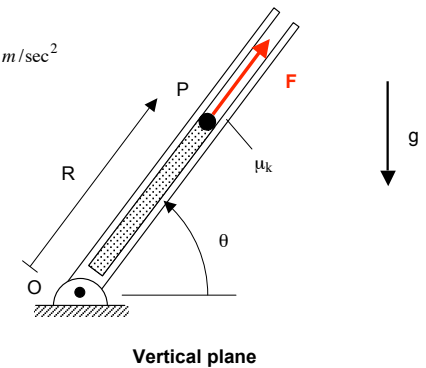
Problem 23

Wedge B having a mass of $m_B = 4$ kg is initially traveling in a horizontal plane to the left with speed of 20 m/sec. The wedge strikes a stationary sphere A, where A has a mass of $m_A = 8$ kg. The coefficient of restitution of the impact of B with A is e = 0.5. Assume all surfaces to be smooth, and treat both bodies as particles.

Find the velocities of A and B immediately after impact. Write your answers as vectors.

Please include an appropriate free body diagram for each equation used. Also include a sketch of the coordinates used in your equations.
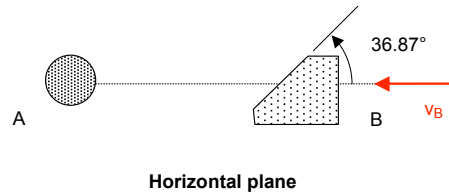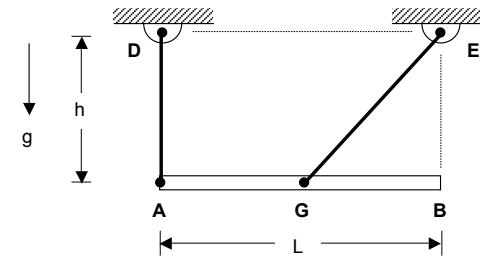

Horizontal plane

Answers:

$v_{A2} = 6 \ m/\sec$

$v_{B2} = 16 \ m/\sec$

Problem 24

A homogeneous bar AB with a mass of M and length L is supported by cables AD and EG, where G is the center of mass of the bar. At the position shown below, the bar is horizontal, point A is directly below D, and end B is directly below E.

If the bar is released from rest in the position shown, determine the <u>angular acceleration</u> of the bar immediately after release. In your calculations, use M = 20 kg, L = 4 meters and h = 1.5 meters. Ignore the mass of cables AD and EG.

You need to provide an accurate free body diagram of the bar and to indicate the coordinate system used in order to receive full credit for this problem.


Answer: $\alpha = 2.58 \ rad/\sec^2 \ (CW)$

Particle A, having a mass of m = 4 kg, initially moves to the left on a smooth horizontal plane with a speed of $v_{A1} = 20 \, m/\text{sec}$. Particle B, also having a mass of m = 4 kg, is attached to an elastic cord whose unstretched length is 3 meters, whose stiffness is K = 100 newtons/meter and is attached to a fixed pin at O. Initially B is at rest at a distance of R = 2 meters from O with $\phi = 36.87°$ with the cord being slack. The impact of A with B has a coefficient of restitution of e = 0.6.

a) Draw a free body diagram (FBD) of A and B together during impact.

b) Determine the speed of B immediately after being struck by A.

c) Draw an FBD of B after impact.

d) When the cord has stretched, such that B is at a distance of 5 meters from O, find $\dot{R}$ and $\dot{\phi}$ for the motion of B.
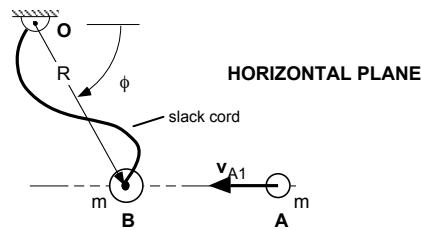
HORIZONTAL PLANE

Answer to d):

$$\dot{\phi}_3 = 0.768 \, rad/\text{sec}$$
$$\dot{R}_3 = 11.9 \, m/\text{sec}$$

A particle P having a mass of m = 10 kg is constrained to move in a horizontal plane. A cable this attached to P is pulled over an ideal pulley with a constant force F = 800 newtons at the other end of the cable. The cable has a total length of L = 8 meters. When d = 2 meters, $\theta = 53.13°$ and P is moving to the left with a speed of $v_P = 15 \, m/\text{sec}$.

When d = 5 meters, find:

a) $\dot{\theta}$.

b) the speed of point A .
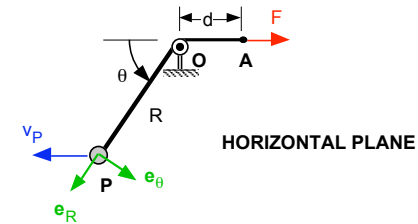

HORIZONTAL PLANE

Answer:
$$v_{A2} = 11.4 \, m/\text{sec}$$

Problem 27

Given: Particle A of mass m is moving in a <u>horizontal plane</u> with initial velocity V_{A1}, with V_{A1} being parallel to line BC. After impact with the *fixed*, rigid surface, particle A rebounds with a velocity V_{A2}, with V_{A2} being parallel to line CD. Line BC is perpendicular to line CD. Let e be the coefficient of restitution for the impact of particle A with the surface. Consider the contacting surface to be smooth.
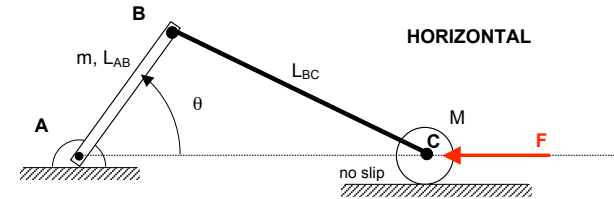
Find:

a) Determine the angle θ needed to ensure that V_{A2} is parallel to line CD. Write your answer in terms of e only.

b) If e = 0.4, m = 3kg, and V_{A1} = 5m/s, find the average force applied to mass A during impact if the impact occurs over a period of time of 0.002 sec.

HORIZONTAL PLANE
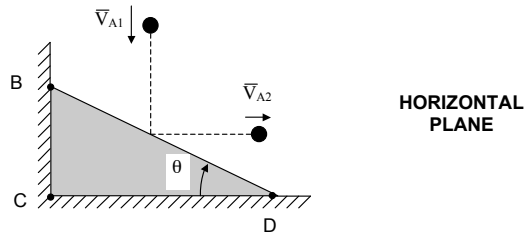
Answer for a): $\theta = \tan^{-1}\left(e^{1/2}\right)$

Problem 28

Given: A mechanism is made up of a crank AB, coupler BC and a wheel attached to BC at the wheel's centroid C. AB is a thin, homogeneous link having a mass m = 10 kg, and a length of L_{AB} = 1.5 meters. BC is a link having *negligible mass* and a length of L_{BC} = 2.5 meters. The wheel is a homogeneous disk having a mass of M = 16 kg and outer radius R = 0.5 meters, and *rolls without slipping* on a rough surface. A constant force F = 200 N acts to the left at C. Consider the motion of all points on the mechanism move in a HORIZONTAL plane.
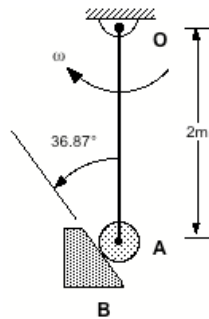

HORIZONTAL

Find: Find the angular acceleration of crank AB when θ = 90° if the system is instantaneously at rest when θ = 90°. Write your answer as a vector.

<u>Answer</u>: $\underline{\alpha}_{AB} = \left(4.88 \,\underline{k}\right) rad/\sec^2$

Problem 29

Particle A having a mass of 30 kg is attached to bar OA that has a length of 2 meters and has negligible mass. Bar OA is pinned to ground in such a way that particle A moves in a HORIZONTAL PLANE. Bar OA is given an initial angular velocity of $\omega = 5$ rad/sec. At the position shown below, particle A strikes a second particle B that has a mass of 20 kg. Assume that the contact surface between A and B is smooth and that the impact of A with B has a coefficient of restitution of 0.7.

i)   Show the forces acting on a system made up of particles A and B in the free body diagram of the system given below.

ii) Based on the forces indicated on your free body diagram, identify a direction (or directions) along which linear momentum for A + B is conserved during impact.

iii) Determine the velocities of particles A and B after impact. Write your answers as vectors.

HORIZONTAL PLANE
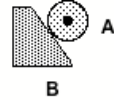
**FBD of System A + B**

Answers for iii):

$$\bar{v}_{B2} = -\left(7.62\,\bar{i} + 5.71\bar{j}\right) m/\sec$$

$$\bar{v}_{A2} = \left(-4.92\bar{i}\right) m/\sec$$

Problem 30

A thin, homogeneous bar having a mass of 80 kg and length of 3 meters, is constrained to move within a vertical plane. Ends A and B are constrained to slide along smooth horizontal and vertical walls, respectively. A cable, attached to end A of the bar, supports a block C that has a mass of 20 kg. The system is released from rest with the bar oriented as shown in the figure below.

Determine the angular acceleration of the bar immediately after the system is released. Express your answer as a vector.
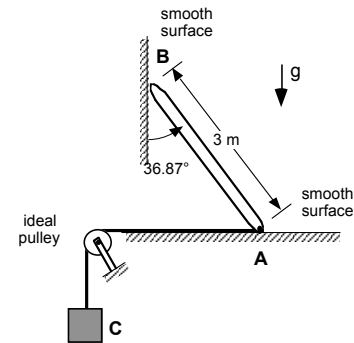
Answer:

$$\bar{\alpha} = \left(0.663\,rad/\sec^2\right)\bar{k}$$

Problem 31

An inhomogeneous wheel, having a mass of m and centroidal radius of gyration k_G, is moving in such a way that it rolls without slipping and the geometric center of the wheel C is moving to the right with a speed of v_O at the instant shown below. At the instant shown, the centroid G is to the left of O at the same elevation as O.

For this instant, determine the normal force acting on the wheel due to the contact of the wheel with the ground.

Use the following parameter values in your calculations: m = 80 kg, r = 2 meters, $k_G = 1.5\,meters$ and $v_O = 5\,m/\sec$.

HINT: In solving this problem, please be reminded that the center of mass G is NOT at the geometric center of the wheel.
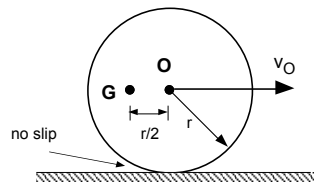
Answer:

$N = 538.3\ newtons$

Problem 32

Particles A and B, having weights of 20 lb and 10 lb, respectively, are attached to an L-shaped bar (*with the bar having negligible mass*). The L-shaped bar is pinned to ground at pin O in such a way that A and B move in a vertical plane. An elastic band (having a stiffness of k = 800 lb/ft and an unstretched length of 3.5 ft) is attached between particle B and point C where C is at the same elevation as pin O. At the position shown in Figure 1, OB is vertical, OA is horizontal and the bar is at rest. At the position shown in Figure 2, OB is horizontal and OA is vertical.

Find the angular velocity of the L-shaped bar at the position shown in Figure 2.

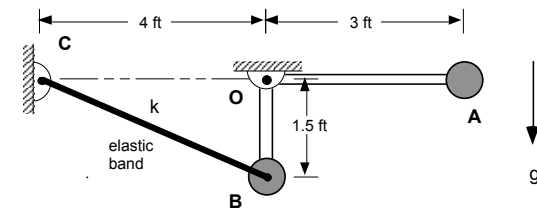HINT: The elastic band can carry a load under tension but not under compression.
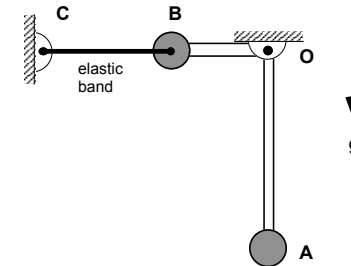

Figure 1

Figure 2

Answer:

$\omega_2 = 9.50\ rad/\sec$

Problem 33

A thin homogeneous bar of length L = 1.5 meters and mass m = 30 kg is pinned to ground at point O. A homogeneous disk with a mass of M = 100 kg and radius R = 0.6 meters is PINNED to end A of the bar. The disk rolls without slipping on the inside of a circular surface. The system is *released from rest* with θ = 90°.

Find the <u>angular velocity</u> of the bar when θ = 0.

You need to include an appropriate FBD for the system used as well as an indication of the DATUM line(s) used in your analysis.
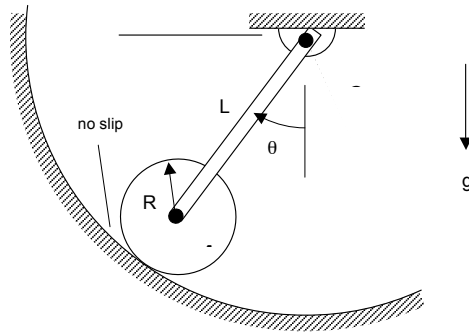

Answer: ω = 3.07 *rad*/sec

Problem 34

Given: A homogeneous disk (having a mass of M = 10 kg and outer radius of R = 1 meter) is welded to a thin, homogeneous bar having a mass of m = 20 kg and length L = 4 meters with the end of the bar located at the center O of the disk. The disk is able to roll without slipping on a horizontal surface. The system is released from rest at position 1 with the bar being horizontal at this position. On release, the system rolls to the right. At position 2 shown below the bar is vertical.
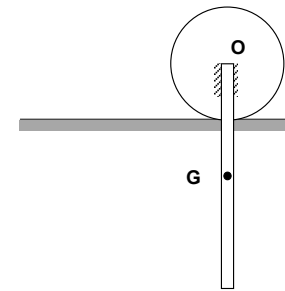
Find:

a) Draw a free body diagram (FBD) of the system of the bar and disk together.

b) Classify all of the forces in your FBD in a) as to whether they do work or will be included in potential energy.

c) Determine the velocity of point G when the system is at position 2.
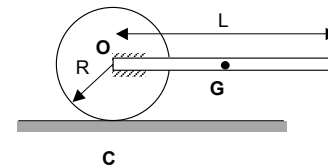
Position 1   **Position 2**

Answer: $\bar{v}_G = -3.57\,\bar{i}$ m/sec

35   36

Problem 35

Given: A thin homogeneous bar having a mass of M = 40 kg and length L = 2 meters is at rest on a smooth horizontal surface, as shown in the top view below. A constant force of F = 500 newtons acts as shown at end A of the bar over a short interval of time Δt = 0.4 seconds.

Find: Determine velocity of the end point B after the application of the force F. Write your answer as a vector.

Answer: $\bar{v}_{B2} = \left(-8\vec{i} - 3\vec{i}\right) m/\sec$
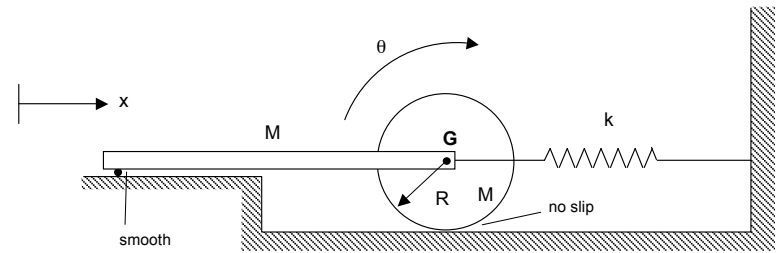
53.13°

F

y

x

A

G

L

B

top view
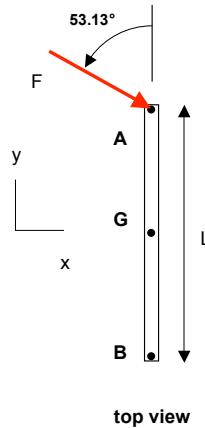
Problem 36

A homogeneous disk having a mass of M and outer radius of R rolls *without slipping* on a horizontal surface. A block having a mass of M is pinned to the disk at the disk's center of mass G and is supported on its left hand side by a smooth horizontal surface, as shown below. A spring having a stiffness of k is connected between G and a wall.

Let x be a coordinate that describes the position of the block (measured *positive to the right*), and θ be the angle of rotation of the disk (measured *positive clockwise*), as shown in the figure. The springs are unstretched when x = θ = 0.

1. Derive the <u>equation of motion</u> for vibrations (in terms of M, k, x, and $\ddot{x}$) for this system.
2. Find the natural frequency of this system. Use M = 20 kg and k = 20,000 N/m.

θ

x

M

G

k

R   M

no slip

smooth

Answer for b): natural frequency = 20 rad/sec

Problem 37

Given: A system is made up of three components: a homogeneous disk A having a mass of m and outer radius R; a homogeneous wheel B having a mass of 3m and outer radius R; and a block having a mass of 2m. Disk A is pinned to ground at its center, and wheel B rolls *without slipping* on a rough horizontal surface. The block moves horizontally while in contact with the disk and wheel at points D and E, respectively. Assume that the *block does not slip* at either D or E. A spring of stiffness k is connected between the block and ground. Let θ represent the rotation of disk A measured positively CCW (as shown) with θ = 0 when both springs are unstretched.
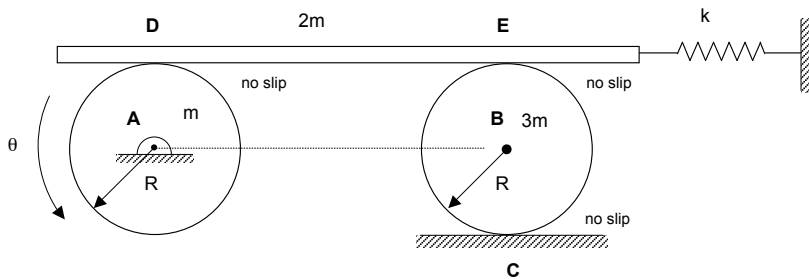

Find: For this system:

a) Draw individual free body diagrams for the disk, wheel and block below.

b) Determine the differential equation of motion (EOM) for the system corresponding to the coordinate θ. Your EOM should be in terms of, at most, m, k, R, θ and $\ddot{\theta}$.

c) Based on your EOM in b), what is the natural frequency for free response of this system? Use R = 0.5 meters, m = 10 kg and k = 7000 N/m.

Answer to c): *natural frequency = 13.9 rad/sec*
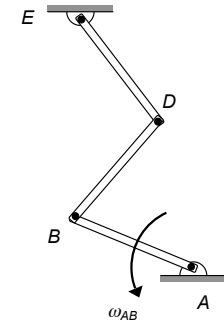
Problem 38 - Part (a)

Vertical shaft OA rotates about a fixed axis with a *constant* rate of Ω = 8 *rad/sec*. Arm AB is pinned to OA and is being raised at a *constant* rate of $\dot{\theta} = 10 \, rad/sec$. An observer and the xyz axes are attached to AB. The XYZ axes are stationary.

What is the *angular acceleration* of arm AB when θ = 90°? Write your answer as a vector.
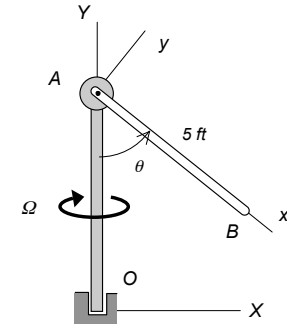
Problem 38 - Part (b)

Consider the mechanism shown below that is made up of links AB, BD and DE. Link AB is rotating counterclockwise as shown.

- Show the location of the instant center for link BD.
- What is the *direction of rotation* of link BD (clockwise or counterclockwise)? Provide a written justification for your answer.

An automobile A travels along a highway with a speed of v_A. A police officer, at a distance of r at point O, accurately measures $\dot{r}$ (the time derivative of r) with a hand-held radar device. What is the size of $|\dot{r}|$ relative to v_A:

    A. $|\dot{r}| > v_A$ *(officer overestimates the speed of the automobile)*

    B. $|\dot{r}| = v_A$ *(officer exactly measures the speed of the automobile)*

    C. $|\dot{r}| < v_A$ *(officer underestimates the speed of the automobile)*

Provide a written justification for your answer.
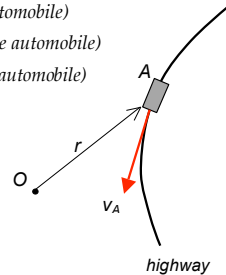
Problem 38 - Part (d)

A wheel rolls without slipping with the center of the wheel O having a constant speed of v_O. In the sketch below right, make sketches that indicate the *direction of the acceleration vectors* for points A, B and E on the wheel.

Problem 38 - Part (e)

The rigid body shown below has a *counterclockwise angular velocity* ω and a *clockwise angular acceleration* α. The direction of the acceleration of point A, $\underline{a}_A$, is shown in the figure, where $\underline{a}_A$ is perpendicular to line AB. In the figure below, make a sketch showing the direction of the acceleration vector for point B, $\underline{a}_B$.

(The sketch of $\underline{a}_B$ does not need to be accurate; simply show its direction relative to the two dashed lines in the figure (the two dashed lines are perpendicular to and parallel to $\underline{a}_A$.)
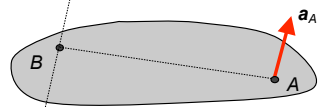
Problem 39 - Part (a)

Particle A strikes a stationary particle B with a speed of $v_{A1} = 10$ *ft/sec* in the direction shown below left. After impact, A has a speed of $v_{A2} = 6$ *ft/sec* (*direction of the velocity of A after impact is not known*) and B has a speed of $v_{B2} = 4$ ft/sec. All motion of the particles is in a horizontal plane, and the contact surface between A and B is to be assumed to be smooth.

Determine a numerical value for the *coefficient of restitution* for the impact of A with B.

BEFORE impact   **AFTER impact**
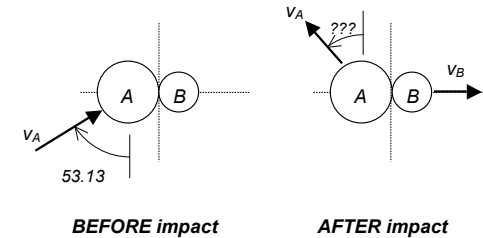
Problem 39 - Part (b)

Bar AB is released from rest with point A in contact with a smooth, horizontal surface. Describe in the words or with a sketch the path of the center of mass G of the bar after the bar is released.

You need to provide an accurate free body diagram of the bar in order to receive full credit for this problem.
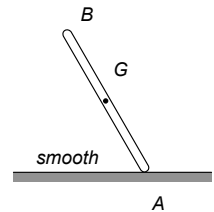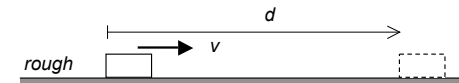
Problem 39 - Part c)

A block on a rough horizontal surface is given an initial velocity to the right of v = 3 m/sec. The block comes to rest after it has slid a distance of d = 2 meters.

If the same block is instead given an initial speed of v = 6 m/sec, how far will it slide to the right before stopping?

    D. d = 2 meters

    E. d = 4 meters

    F. d = 6 meters

    G. d = 8 meters

    H. d = 9 meters

    I. A numerical value for the kinetic coefficient of friction is needed in order to answer this question.
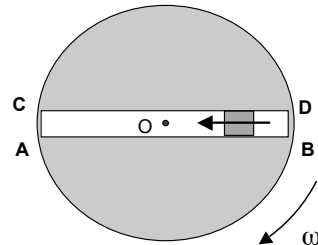
Problem 39 - Part d)

A disk rotates <u>clockwise</u> at a *constant* rate of ω about a vertical shaft that passes through point O on the disk. A block of mass m is moving radially <u>inward</u> toward the shaft O. On which surface of the slot does the block make contact?

   A. Surface AB.

   B. Surface CD.

   C. Neither surface since there is zero contact force of the slot on the block.

   D. More information is needed about the sliding speed and disk rotation in order to answer this question.

You must provide a written justification for your answer in order to receive full credit for this problem.

Problem 39 - Part e)

You are on a cart that is initially at rest on a frictionless track. You throw a ball at a partition that is rigidly mounted on the cart. If the ball bounces back as shown in the figure, then

   A. the cart moves to the right.

   B. the cart remains stationary.

   C. the cart moves to the left.

You must provide a written justification for your answer in order to receive full credit for this problem.

HINT: Consider the linear-impulse momentum equation on the system made up of you, the cart and the ball.