SAMPLE CONTENT

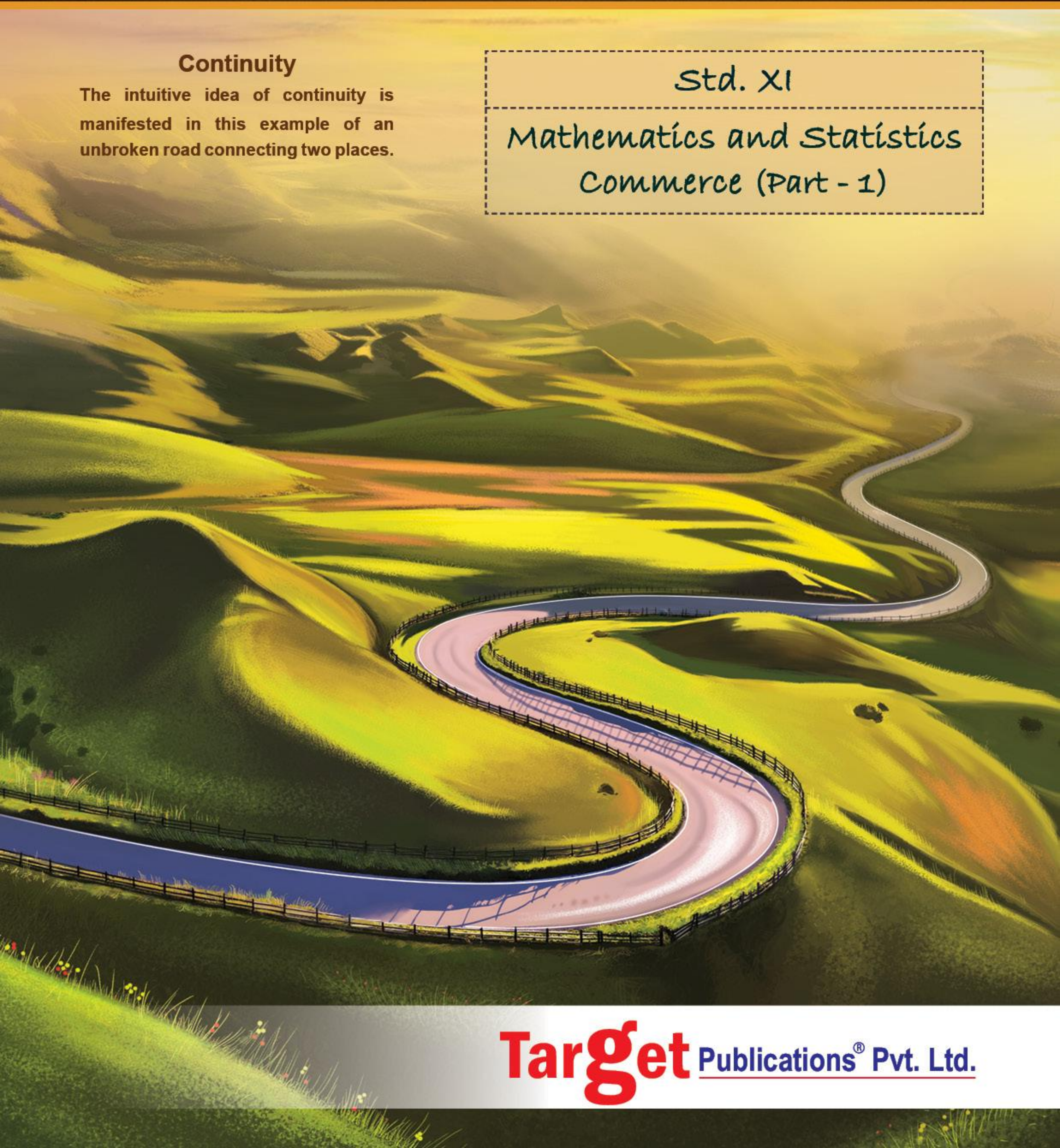
SMART NOTES

Continuity

The intuitive idea of continuity is manifested in this example of an unbroken road connecting two places.

Std. XI

Mathematics and Statistics Commerce (Part - 1)



Target Publications® Pvt. Ltd.

SMART NOTES

MATHEMATICS & STATISTICS – COMMERCE (PART - I)

Std. XI Commerce

MAHARASHTRA STATE BOARD

(As per the new textbook published by Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune. w.e.f. Academic Year 2019-20)

Salient Features:

- Written as per the new textbook
- Exhaustive coverage of entire syllabus
- Topic-wise distribution of textual questions and practice problems at the start of every chapter.
- Precise theory for every topic
- Covers answers to all exercises and miscellaneous exercises given in the textbook.
- All derivations and theorems covered
- Includes additional problems for practice and MCQs
- Illustrative examples for selective problems
- Recap of important formulae at the end of the book
- Activity Based Questions covered in every chapter
- Smart Check to enable easy rechecking of solutions

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PREFACE

"The only way to learn Mathematics is to do Mathematics" – Paul Halmos

"Mathematics & Statistics – Commerce (Part – I): Std. XI" forms a part of **'Target Perfect Notes'** prepared as per the **New Textbook**. It is a complete and thorough guide critically analysed and extensively drafted to boost the students' confidence.

The book provides **answers to all textbook questions** included in exercises as well as miscellaneous exercises. Apart from these questions, we have provided **ample questions for additional practice** to students based on every exercise of the textbook. Only the final answer has been provided for such additional practice questions. At the start of the chapter, we have provided a **table to birfucate the textbook questions and additional practice questions** as per the different type of problems/concepts in the chapter. This will help in systematic study of the entire chapter.

Precise theory has been provided at the required places for better understanding of concepts. Further, all **derivations and theorems have been covered** wherever required. A **recap of all important formulae** has been provided at the end of the book for quick revision. We have also included **activity based questions** in every chapter. We all know that there are certain sums that can be solved by multiple methods. Besides, there are also other ways to check your answer in Maths. **'Smart Check'** has been included to help you understand how you can check the correctness of your answer.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you. Pls write to us on: mail@targetpublications.org

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

From,
Publisher

Edition: First

INDEX

“**Note**: Solved examples from textbook are indicated by “+”.
Smart check is indicated by ✓ symbol.”

2 Functions

Type of Problems	Exercise	Q. Nos.
Identify the function Domain and range	2.1	Q. 1, 2
	Practice Problems (Based on Exercise 2.1)	Q. 1, 2
	Miscellaneous Exercise 2	Q. 1
	Practice Problems (Based on Miscellaneous Exercise 2)	Q. 1
Value of a function	2.1	Q. 3 to Q. 7
	Practice Problems (Based on Exercise 2.1)	Q. 3 to Q. 11
	Miscellaneous Exercise 2	Q. 3 to Q. 7
	Practice Problems (Based on Miscellaneous Exercise 2)	Q. 3 to Q. 7
Algebra of functions	2.1	Q. 8
	Practice Problems (Based on Exercise 2.1)	Q. 12
Inverse functions	Practice Problems (Based on Exercise 2.1)	Q. 15, 16
	Miscellaneous Exercise 2	Q. 2
	Practice Problems (Based on Miscellaneous Exercise 2)	Q. 2
Composite function	2.1	Q. 9
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	Practice Problems (Based on Miscellaneous Exercise 2)	Q. 8, 9

Syllabus

- Function, Domain, Co-domain, Range.
- Representation of a function.
- Types of functions – One-one, Onto.
- Evaluation of a function.
- Some fundamental functions and their graphs.
- Some special functions.

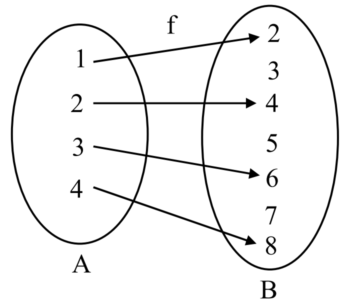
Let's Study

Function

Definition:
A function from set A to the set B is a relation which assigns every element of a set A, a unique element of set B and is denoted by $f: A \rightarrow B$. If f is a function from A to B and $(x, y) \in f$, then we write it as $y = f(x)$

y is called the image of x under f. y is the value of the function at x.

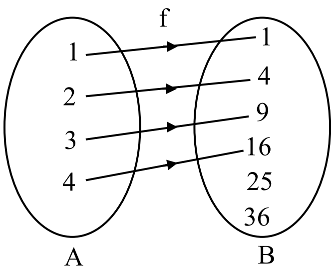
Example:
Let A = {1, 2, 3, 4}, B = {2, 3, 4, 5, 6, 7, 8}



Let a relation from A to B be given as "twice of" then we observe that every element x of set A is related to one and only one element of set B. Hence this relation is a function from set A to set B. In this case f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8 are the values of function $f(x) = 2x$ at x = 1, 2, 3, 4 respectively.

Range of function:
If f is a function from set A to set B, then the set of all values of the function f is called the range of the function f.
Thus the range set of the function f: A $\rightarrow$ B is
$\{f(x) / x \in A\}$
Note that the range set is a subset of co-domain. This subset may be proper or improper.

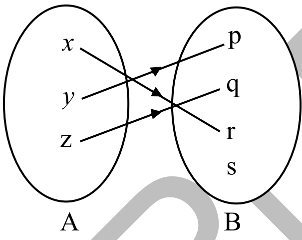
Let f: A $\rightarrow$ B be a function where
A = {1, 2, 3, 4}, B = {1, 4, 9, 16, 25, 36}. Then, A, B are domain and co-domain respectively and set {1, 4, 9, 16} is the range of the function f.



Representation of functions

i. Arrow diagram:

In this diagram, we use arrows. Arrow starts from an element of domain and point out it's image under f.



ii. Ordered pair:

Let A, B be two sets and A × B be the cartesian product. Then a subset f of A × B is a function if for every a $\in$ A, there is b in B such that (a, b) $\in$ f.

Example: If A = {a, b, c, d}, B = {1, 2, 3, 4, 5}, then f = {(a, 1), (b, 2), (c, 3), (d, 4)} is a function.

iii. Tabular form:

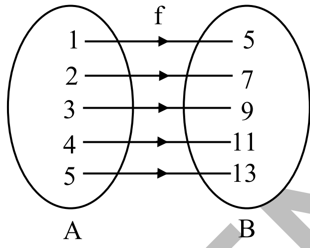
If the sets A and B are finite, then a function f: A$\rightarrow$B can be exhibited with the help of a table of corresponding elements.

A function f = {(1, 7), (2, 9), (3, 11), (4, 13)} can be represented in tabular form as follows:

x	1	2	3	4
f(x)	7	9	11	13

iv. By Formula:

Let A = {1, 2, 3, 4, 5}, B = {5, 7, 9, 11, 13} and f: A $\rightarrow$ B be a function represented by arrow diagram.

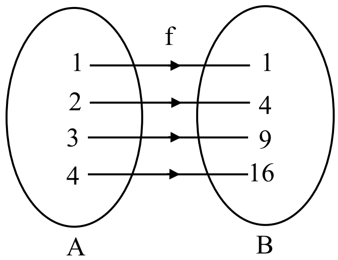


In this case we observe that, if we take an element x of the set A, then the element of the set B related to x is obtained by adding 3 to twice of x. Applying this rule we get in general $f(x) = 2x + 3$, for all $x \in A$.
This is the formula which exhibits the function f.
If we denote the value of f at x by y, then we get $y = 2x + 3$, for all $x \in A$.

Types of functions

1. One-one function:

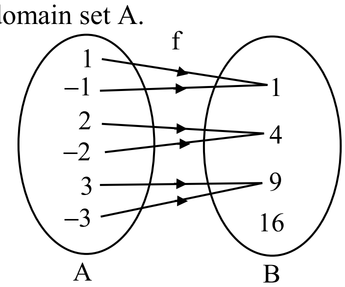
Let f: A $\rightarrow$ B be a function such that $f(x_1) = f(x_2)$, $x_1, x_2 \in A$
Function f is one-one if $x_1 = x_2$.



In this case, f: A $\rightarrow$ B is a one-one function.

2. Many-one function:

Let f: A $\rightarrow$ B be a function such that $f(x_1) = f(x_2)$, $x_1, x_2 \in A$. Function f is many one if $x_1 \neq x_2$. If function f : A $\rightarrow$ B is many one then two or more elements in a set A have the same image in set B.
The function f: A $\rightarrow$ B represented by the following arrow diagram is such that the co-domain B contains 1, 4 and 9 each of which is the image of two distinct elements of the domain set A.



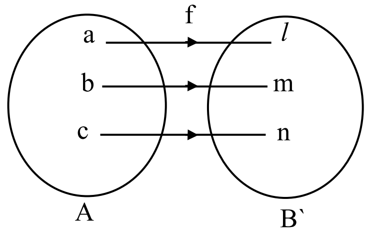
Clearly f: A $\rightarrow$ B is many-one function.

3. **Onto function:**

If the function f: A → B is such that every element in B is the image of some element in A, then f is said to be an onto function.

In case of onto function, the range of function f is same as its co-domain B.

Consider the function f: A → B represented by the following arrow diagram:



In this case, range is equal to co-domain

= {*l*, m, n}

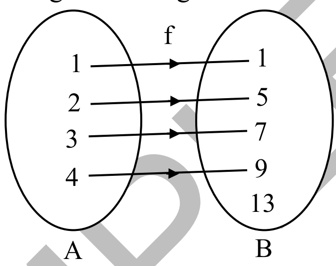
Hence f: A → B is an onto function.

4. **Into function:**

If a function f: A → B is not onto, then f is an into function. For an into function f, there exists at least one element in B which is not the image of any element in A.

In case of into function the range of function f is a proper subset of its co-domain.

Let the function f: A → B be represented by the following arrow diagram.



Range of function f = {1, 5, 7, 9}, which is a proper subset of co-domain {1, 5, 7, 9, 13}

Hence, f: A → B is an into function.

Graph of a function

The Pictorial representation of a function can be done by a graph.

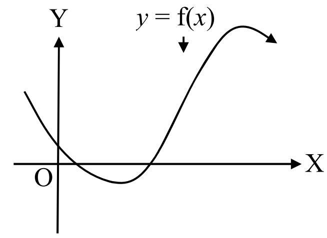
Let f: A → B be a function, given by $y = f(x)$, $x \in A$, $y \in B$

The varying quantity x is called the independent variable and y is known as the dependent variable.

To draw a graph we need two axes.

A horizontal axis is kept for the independent variable and a vertical axis is kept for the dependent variable.

Example:

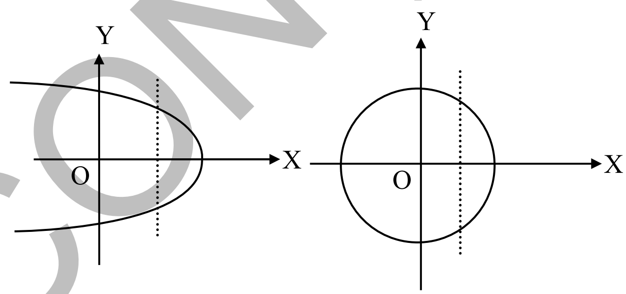


Every graph need not represent a function. To identify if the graph represents a function we perform the following test

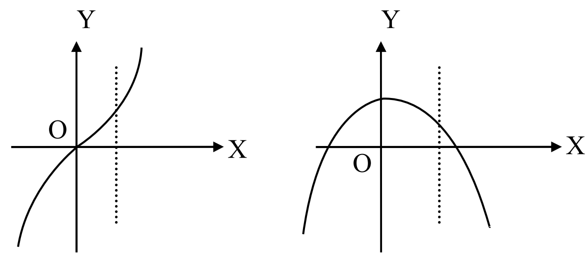
Vertical line test:

We can test a graph of a relation for it to be a function.

A vertical line would cut the graph of a relation, that is not a function, at least at two points.
A vertical line would cut the graph of a relation, that is a function, at atmost one point.



(Graphs of non-function relation)

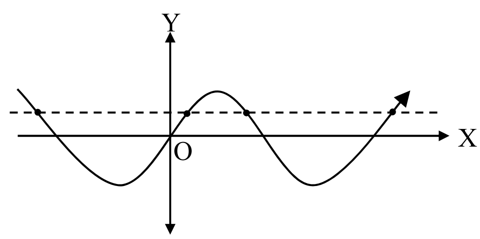


(Graphs of function relation)

With the help of graph of a given function we may check if given function is one-one function (injective), by performing following test.

Horizontal line test:

Draw the graph of $y = f(x)$ and if a horizontal line meets the graph of $y = f(x)$ at more than one point, the function f is one-one function.



(Graph of an injective function)

Evaluation of function at a given value of x can be done by replacing x by the given value.

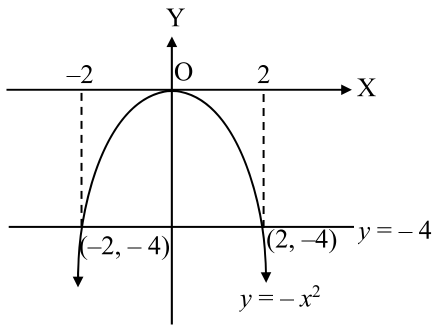
Evaluation of x when $f(x)$ is given, can be done with the help of graph of $y = f(x)$ too.

Let $f(x) = b$

Draw $y = f(x)$ and line $y = b$.

The abscissa of a point of intersection is a solution.

Example: $f(x) = -x^2$ and find the value of x such that $f(x) = -4$.



The line $y = -4$ intersects $y = -x^2$ at $(2, -4)$ and $(-2, -4)$.

$\therefore$ The required values of $x = -2, 2$.

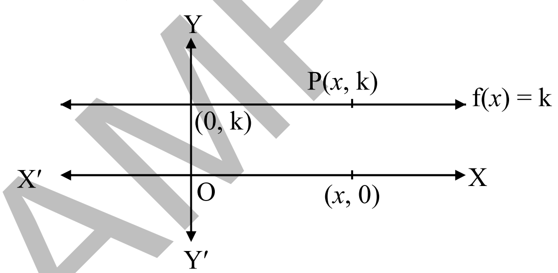
Value of a function:

$f(a)$ is called the value of a function $f(x)$ at $x = a$.

Some fundamental functions and their graphs

1. Constant function:

A function f defined by $f(x) = k$, for all $x \in R$, where k is a constant, is called a constant function. The graph of a constant function is a line parallel to the X-axis, intersecting Y-axis at $(0, k)$.

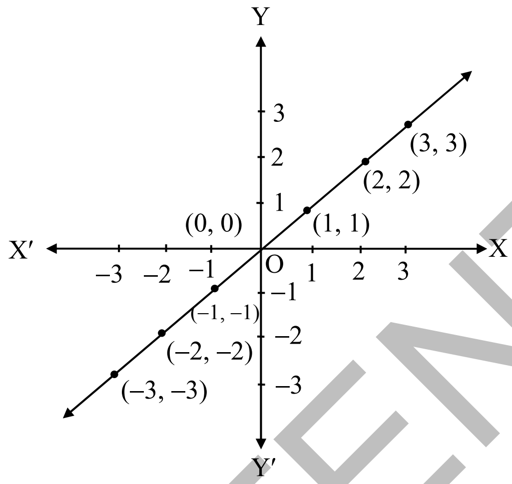


For example, function f defined as $f(x) = 5$, is a constant function.

2. Identity function:

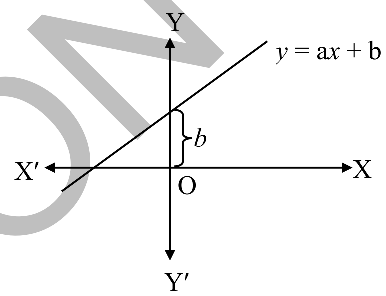
The function f defined as $f(x) = x$, where $x \in R$ is called an identity function. The graph of the identity function is the line $y = x$.

x	–3	–2	–1	0	1	2	3
$f(x)$	–3	–2	–1	0	1	2	3



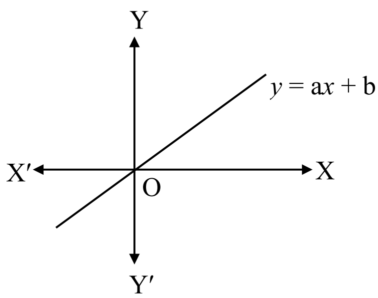
3. Linear function:

i. $y = ax + b$, a, b > 0



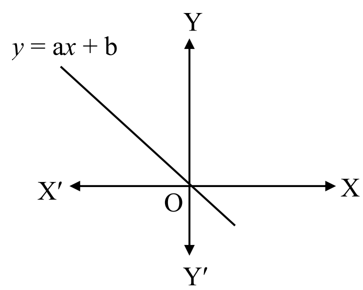
Domain = R, Range = R

ii. $y = ax + b$, b = 0, a > 0



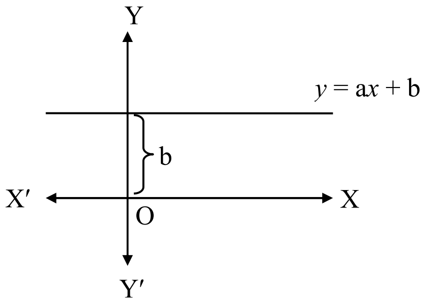
Domain = R, Range = R

iii. $y = ax + b$, a < 0, b = 0



Domain = R, Range = R

iv. $y = ax + b$, a = 0, b > 0



This graph represents a constant function.
Domain = R, Range = {b}

4. Quadratic function:

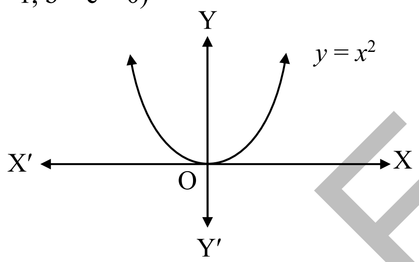
$y = ax^2 + bx + c$, $a \neq 0$

$$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \qquad \ldots\text{(i)}$$

$$\therefore \quad y + \frac{b^2 - 4ac}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

Let $X = x + \frac{b}{2a}$, $\qquad Y = y + \frac{b^2 - 4ac}{4a}$

Then we have $Y = aX^2$. Before drawing graph of $Y = aX^2$, let us first draw the graph of $y = x^2$. (a = 1, b = c = 0)



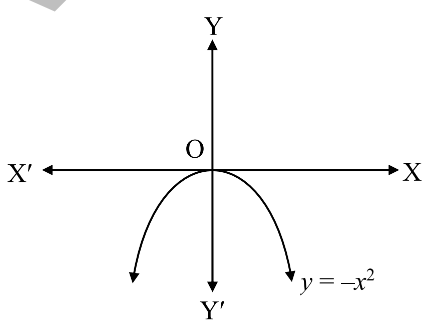
O (the origin) is the vertex.
In case of (i), Vertex = (X = 0, Y = 0)

$$= \left(-\frac{b}{2a}, -\frac{b^2 - 4ac}{4a}\right)$$

The graph of $Y = aX^2$ is similar (in shape) to $y = x^2$, with vertex at (X = 0, Y = 0) and elongated vertically as $Y = ay$.

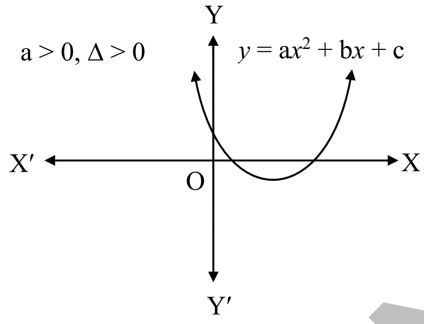
Graph of $y = -x^2$:

If a = – 1, b = c = 0, $y = -x^2$



Let discriminant be $\Delta = b^2 - 4ac$. Then

i.



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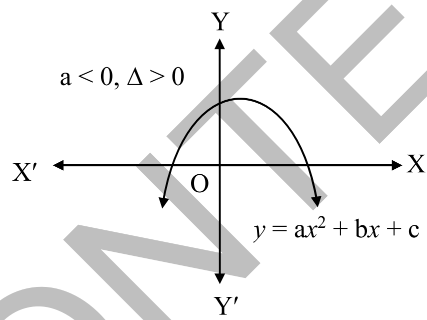

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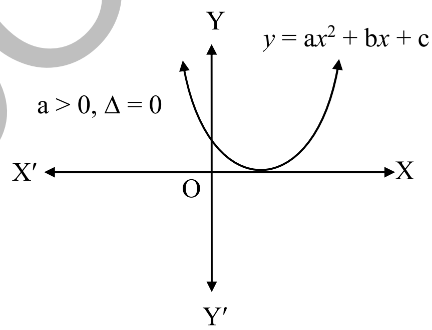

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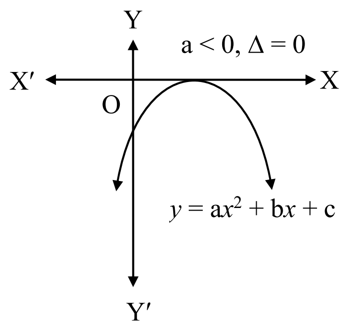

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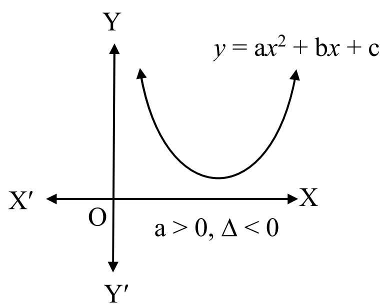

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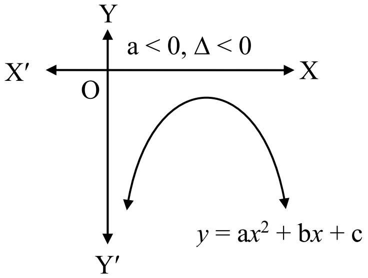
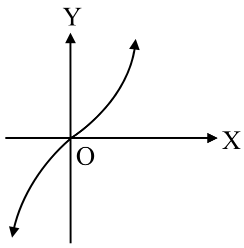


5. Cubic function:

$y = ax^3 + bx^2 + cx + d$

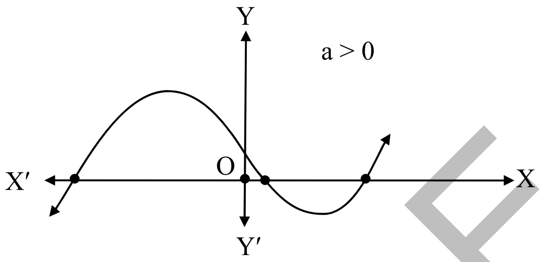
If a = 1, b = c = d = 0, then



Domain = R, Range = R

$y = ax^3 + bx^2 + cx + d$,

i.



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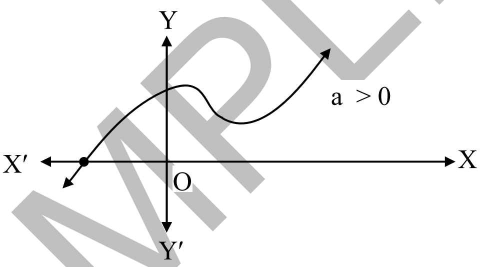
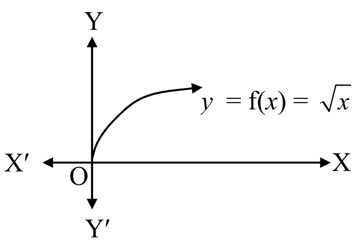


Domain = R, Range = R

6. Square root function:

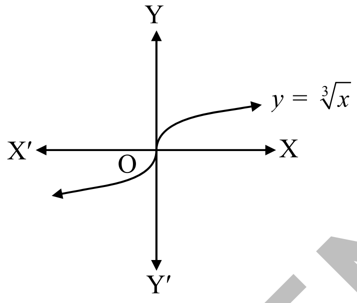
$y = \sqrt{x} = f(x)$



Domain = $[0, \infty)$, Range = $[0, \infty)$

7. Cube root function:

$y = x^{\frac{1}{3}} = f(x)$



Domain = R, Range = R

8. Polynomial function:

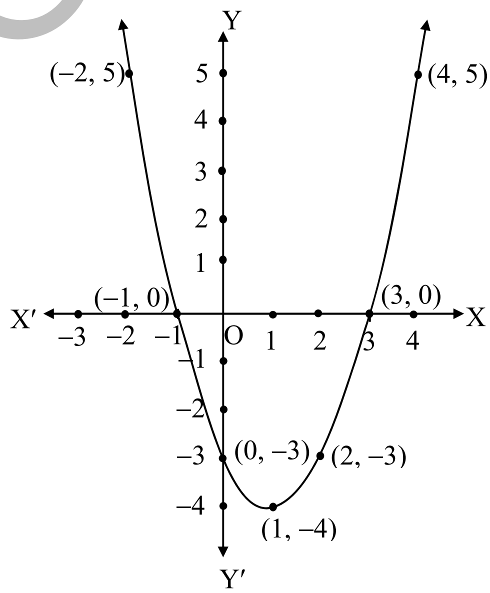
A function of the form

$f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, where n is a non-negative integer and $a_0, a_1, a_2, \ldots, a_n \in R$ is called a polynomial function.

Example:

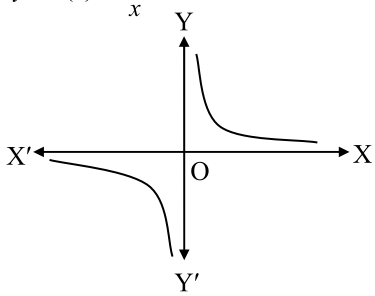
$f(x) = x^2 - 2x - 3$ for $x \in R$

x	-2	-1	0	1	2	3	4
$f(x) = x^2 - 2x - 3$	5	0	-3	-4	-3	0	5



9. Rational function:

$y = f(x) = \frac{1}{x}$

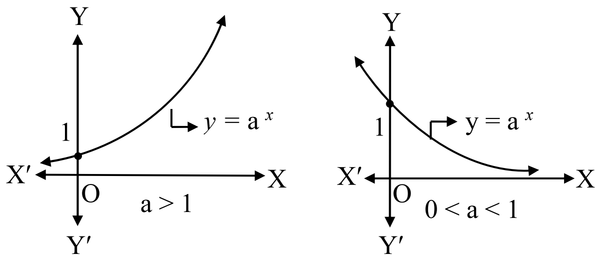


Domain = R – {0}, Range = R – {0}

10. Exponential function:

$y = a^x = f(x), \quad a > 0$

i.



We observe from the graphs

- When $a < 1$, a^x decreases as x increases.
- When $a > 1$, a^x increases as x increases.
- $a^x > 0$ for all x.
- For $a < 1$ and $x < 0$, $a^x > 1$
- For $a > 1$ and $x < 0$, $a^x < 1$
- For $a < 1$ and $x > 0$, $a^x < 1$
- For $a > 1$ and $x > 0$, $a^x > 1$

ii. The following statements are true for any positive a and real x and y

- $a^x a^y = a^{x+y}$
- $\dfrac{a^x}{a^y} = a^{x-y}$
- $a^{-x} = \dfrac{1}{a^x}$
- $(a^x)^y = a^{xy}$
- $a^0 = 1$

Illustration:

Solve $9^x + 6^x = 2(4^x)$

***Solution*:**

$9^x + 6^x = 2(4^x)$

$\therefore \quad (3^2)^x + 3^x 2^x = 2(2^2)^x$

$\therefore \quad (3^x)^2 + 3^x 2^x = 2(2^x)^2$

Let $3^x = a$ and $2^x = b$ $(a, b > 0)$, then

$a^2 + ab - 2b^2 = 0$

$\therefore \quad \underbrace{(a + 2b)}_{>0} (a - b) = 0$ as $a, b > 0$

$\therefore \quad a = b$

$\therefore \quad 3^x = 2^x$

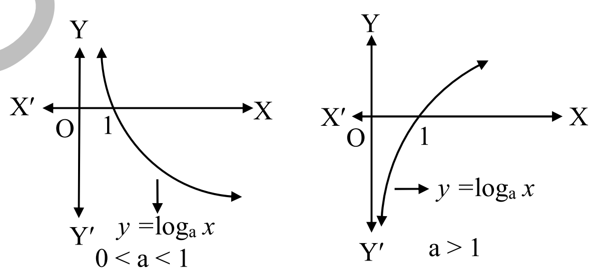
$\therefore \quad x = 0$

11. Logarithmic Function:

$y = \log_a x$ if and only if $x = a^y$

where $a > 0$, $x > 0$ and $a \neq 1$.

'a' is called the base of logarithm.



We observe from the graphs

- When $a < 1$, $\log_a x$ decreases as x increases.
- When $a > 1$, $\log_a x$ increases as x increases.
- $\log_a x$ is defined only for positive values of x.
- For $a < 1$ and $0 < x < 1$, $\log_a x > 0$
- For $a > 1$ and $0 < x < 1$, $\log_a x < 0$
- For $a < 1$ and $x > 1$, $\log_a x < 0$
- For $a > 1$ and $x > 1$, $\log_a x > 0$

Properties

- $\log_a (mn) = \log_a m + \log_a n$
- $\log_a \dfrac{m}{n} = \log_a m - \log_a n$
- $\log_a (m^n) = n \log_a m$
- $\log_n m = \dfrac{\log m}{\log n}$ **(Change of base property)**

  (On R.H.S., any base, but same, can be chosen)
- $\log_a a = 1$
- $\log_a x = \dfrac{1}{\log_x a}$
- $a^{\log_a x} = x$

Illustration:

Solve $\log(x^2 - 6x + 7) = \log(x - 3)$

***Solution*:**

Observe that base of log has not been mentioned explicitly. In such cases we can take any base $c > 0$, $c \neq 1$.

To get feasible region, in which the equation is solvable,

$x^2 - 6x + 7 > 0$ and $x - 3 > 0$

$\therefore \quad (x - 3)^2 > 2$ and $x > 3$

$\therefore \quad \left(x < 3 - \sqrt{2} \text{ or } x > 3 + \sqrt{2}\right)$ and $x > 3$

$\therefore \quad$ Feasible region $= (3 + \sqrt{2}, \infty)$

Any value of $x < 3 + \sqrt{2}$ cannot be a solution.

$\log(x^2 - 6x + 7) = \log(x - 3)$

$\therefore \quad x^2 - 6x + 7 = x - 3$

$\therefore \quad x^2 - 7x + 10 = 0$

$\therefore \quad (x - 5)(x - 2) = 0$

$\therefore \quad x = 5, 2$

$x = 2$ is rejected as it does not belong to the feasible region, as $2 < 3 + \sqrt{2}$

$\therefore \quad x = 5$ is the solution.

Algebra (operations) of functions

Let $X \subset R$, then various operations on functions are defined as follows:

i. Sum of functions:

Let $f: X \to R$ and $g: X \to R$ be the two functions, then $f + g: X \to R$ is defined by $(f + g)(x) = f(x) + g(x)$ for all $x \in X$.

ii. Difference of functions:
Let f: X → R and g: X → R be the two functions, then f − g: X → R is defined by $(f - g)(x) = f(x) - g(x)$ for all $x \in X$.

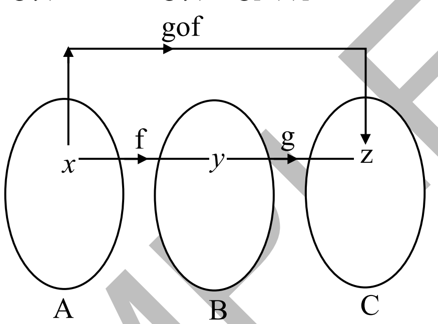
iii. Product of functions:
Let f: X → R and g: X → R be the two functions, then f.g : X → R is defined by $(f.g)(x) = f(x).g(x)$ for all $x \in X$.

iv. Quotient of functions:
Let f: X → R and g: X → R be the two functions.
Let $X_0 = \{x \in X / g(x) = 0\}$.
Then $\frac{f}{g} : X - X_0 \to R$ is defined by
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, for all $x \in X - X_0$

v. Scalar multiplication of a function:
Let f: X → R be a function and k be a scalar, then (kf): X → R is defined by
$(kf)(x) = kf(x)$ for all $x \in X$

Composite function

If f: A → B and g: B → C are two functions, then the composite function of f and g is the function
gof: A → C given by $(gof)(x) = g[f(x)]$, for all $x \in A$
Let $z = g(y)$ then $z = g(y) = g[f(x)] \in C$



This shows that every element x of the set A is related to one and only one element $z = g[f(x)]$ of C. This gives rise to a function from the set A to the set C. This function is called the composite of f and g.
Note that $(fog)(x) \neq (gof)(x)$

Inverse Functions

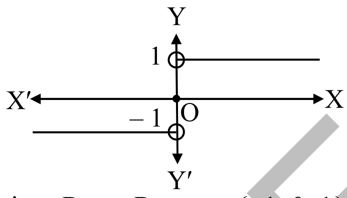
If a function f: A → B is one-one and onto function defined by $y = f(x)$, then the function g: B → A defined by $g(y) = x$ is called the inverse of f and is denoted by f^{-1}.

Thus $f^{-1} : B \to A$ is defined by $x = f^{-1}(y)$
We also write if $y = f(x)$ then $x = f^{-1}(y)$
Note that if the function is not one-one or not onto, then its inverse does not exist.

Some special functions

Signum Function:
It is denoted by sgn.

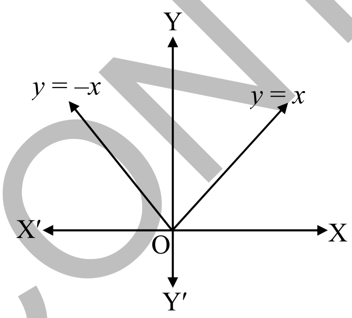
$$\text{sgn}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$



Domain = R, Range = {–1, 0, 1}

Absolute Value Function (Modulus function):

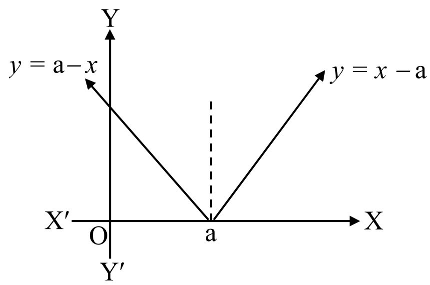
i. $y = f(x) = |x|$



$y = -x$ $x < 0$ and
$= x$, $x \geq 0$
$x = 0$ is the critical point.
The critical point is a point near which a function changes its behaviour.

To get the graph, some points like (1, 1), (–1, 1), (2,2), (–2, 2), (0,0) etc. are to be plotted.
x and $-x$ are linear polynomials and **graphs of linear polynomials are lines.**
A drastic change can be seen around the critical point $x = 0$.
Domain = R, Range = $[0, \infty)$

ii. $y = |x - a|$, $a > 0$



$y = a - x$, $x < a$
$y = x - a$, $x \geq a$
$x = a$ is the critical point.
Domain = R, Range = $[0, \infty)$

Illustration:
How many solutions does the equation $|2^x - 1| + |2^x + 1| = 2$ have?
***Solution*:**
$2^x > 0 \Rightarrow 2^x + 1 > 0$
Given equation $|2^x - 1| + |2^x + 1| = 2$

$\therefore \quad |2^x - 1| + 2^x + 1 = 2$

$\therefore \quad |2^x - 1| = 1 - 2^x$

$\therefore \quad 2^x - 1 \le 0 \qquad \ldots[\,|x| = -x \text{ if } x \le 0]$

$\therefore \quad x \le 0$

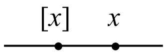
i.e., Solution set = $(-\infty, 0]$

$\therefore$ There are infinitely many solutions.

Greatest Integer Function(Step function):

Greatest integer function extracts the greatest integer contained in a real number x.

It is denoted as $[x]$.



Clearly $[x] \le x$

(Equality occurs only when x is an integer.)

Example: $[9.3] = 9$, $[-7.5] = -8$

Properties:

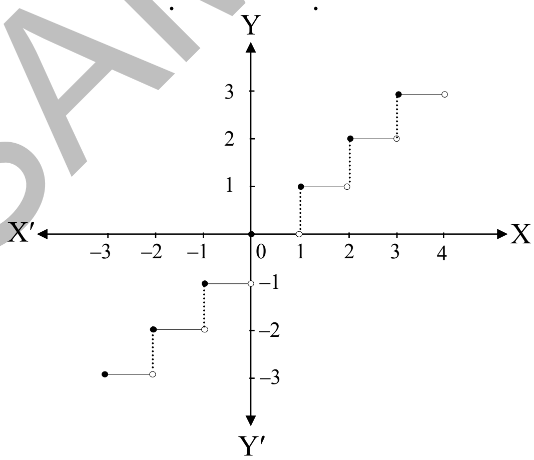
- $x - 1 < [x] \le x$
- $[x + \text{n}] = [x] + \text{n}$, n : an integer
- $[x + y] \ge [x] + [y]$
- $[x] + \left[x + \frac{1}{\text{n}}\right] + \left[x + \frac{2}{\text{n}}\right] + \ldots + \left[x + \frac{\text{n}-1}{\text{n}}\right] = [\text{n}x]$

  For $\text{n} \in \text{N}$ and $x \in \text{R}$.

Graph of $y = [x]$

Greatest integer function is a piecewise defined function. Let us express $[x]$ explicitly.

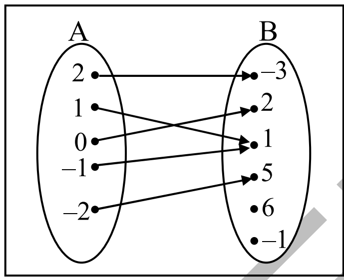
$$
\begin{aligned}
&\vdots && \vdots\\
y = [x] &= -3, && -3 \le x < -2\\
&= -2, && -2 \le x < -1\\
&= -1, && -1 \le x < 0\\
&= 0, && 0 \le x < 1\\
&= 1, && 1 \le x < 2\\
&= 2, && 2 \le x < 3\\
&= 3, && 3 \le x < 4\\
&\vdots && \vdots
\end{aligned}
$$



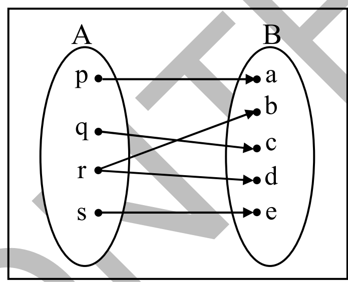
Exercise 2.1

1. Check if the following relations are functions.

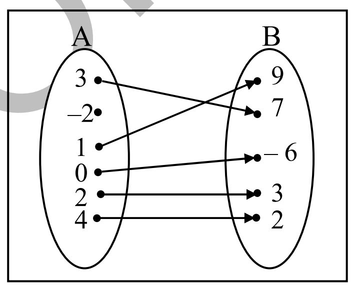
i.



ii.



iii.



Solution:

i. Yes

Reason:

Every element of set A has been assigned a unique element in set B.

ii. No.

Reason:

An element of set A has been assigned more than one element from set B.

iii. No.

Reason:

Not every element of set A has been assigned an image from set B.

2. Which sets of ordered pairs represent functions from A = {1, 2, 3, 4} to B = {–1, 0, 1, 2, 3}? Justify.

i. {(1, 0), (3, 3), (2, –1), (4, 1), (2, 2)}

ii. {(1, 2), (2, –1), (3, 1), (4, 3)}

iii. {(1, 3), (4, 1), (2, 2)}

iv. {(1, 1), (2, 1), (3, 1), (4, 1)}

Solution:

i. {(1, 0), (3, 3), (2, –1), (4, 1), (2, 2)} does not represent a function.
Reason:
(2, –1) and (2, 2) show that element 2∈A has been assigned two images – 1 and 2 from set B.

ii. {(1, 2), (2, –1), (3, 1), (4, 3)} represents a function.
Reason:
Every element of set A has a unique image in set B.

iii. {(1, 3), (4, 1), (2, 2)} does not represent a function.
Reason:
3∈A does not have an image in set B.

iv. {(1, 1), (2, 1), (3, 1), (4, 1)} represents a function
Reason:
Every element of set A has been assigned a unique image in set B.

3. If f (m) = m² – 3m + 1, find

i. **f (0)** **ii.** **f (– 3)**

iii. **f $\left(\frac{1}{2}\right)$** **iv.** **f (x + 1)**

v. **f (– x)**

Solution:

f (m) = $m^2 - 3m + 1$

i. $f(0) = 0^2 - 3(0) + 1 = 1$

ii. $f(-3) = (-3)^2 - 3(-3) + 1$
$= 9 + 9 + 1 = 19$

iii. $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1 = \frac{1}{4} - \frac{3}{2} + 1$
$= \frac{1-6+4}{4} = -\frac{1}{4}$

iv. $f(x+1) = (x+1)^2 - 3(x+1) + 1$
$= x^2 + 2x + 1 - 3x - 3 + 1$
$= x^2 - x - 1$

v. $f(-x) = (-x)^2 - 3(-x) + 1$
$= x^2 + 3x + 1$

4. Find x, if g (x) = 0 where

i. $g(x) = \frac{5x-6}{7}$

ii. $g(x) = \frac{18-2x^2}{7}$

iii. $g(x) = 6x^2 + x - 2$

Solution:

i. $g(x) = \frac{5x-6}{7}$
$g(x) = 0$
$\therefore \frac{5x-6}{7} = 0$
$\therefore 5x - 6 = 0$
$\therefore x = \frac{6}{5}$

Smart Check

If $g\left(\frac{6}{5}\right) = 0$, then our answer is correct.

$g(x) = \frac{5x-6}{7}$

$\therefore g\left(\frac{6}{5}\right) = \frac{5\left(\frac{6}{5}\right) - 6}{7}$

$= \frac{6-6}{7} = 0$

Thus, our answer is correct.

ii. $g(x) = \frac{18-2x^2}{7}$
$g(x) = 0$
$\therefore \frac{18-2x^2}{7} = 0$
$\therefore 18 - 2x^2 = 0$
$\therefore x^2 = \frac{18}{2} = 9$
$\therefore x = \pm 3$

iii. $g(x) = 6x^2 + x - 2$
$g(x) = 0$
$\therefore 6x^2 + x - 2 = 0$
$\therefore 6x^2 + 4x - 3x - 2 = 0$
$\therefore 2x(3x + 2) - 1(3x + 2) = 0$
$\therefore (2x - 1)(3x + 2) = 0$
$\therefore 2x - 1 = 0$ or $3x + 2 = 0$
$\therefore x = \frac{1}{2}$ or $x = -\frac{2}{3}$

5. Find x, if f(x) = g(x) where
f(x) = $x^4 + 2x^2$, g(x) = $11x^2$.

Solution:

$f(x) = x^4 + 2x^2$, $g(x) = 11x^2$
$f(x) = g(x)$
$\therefore x^4 + 2x^2 = 11x^2$
$\therefore x^4 - 9x^2 = 0$
$\therefore x^2(x^2 - 9) = 0$
$\therefore x = 0$ or $x^2 - 9 = 0$
$\therefore x = 0$ or $x^2 = 9$
$\therefore x = 0$ or $x = \pm 3$

6. **If $f(x) = \begin{cases} x^2 + 3, & x \le 2 \\ 5x + 7, & x > 2 \end{cases}$, then find**

i. **f(3)** **ii.** **f(2)** **iii.** **f(0)**

Solution:

$f(x) = x^2 + 3, \quad x \le 2$

$\quad = 5x + 7, \quad x > 2$

i. $f(3) = 5(3) + 7 = 15 + 7 = 22$

ii. $f(2) = 2^2 + 3 = 4 + 3 = 7$

iii. $f(0) = 0^2 + 3 = 3$

---

7. **If $f(x) = \begin{cases} 4x - 2, & x \le -3 \\ 5, & -3 < x < 3 \\ x^2, & x \ge 3 \end{cases}$, then find**

i. **f(–4)** **ii.** **f(–3)**

iii. **f(1)** **iv.** **f(5)**

Solution:

$f(x) = 4x - 2, \quad x \le -3$

$\quad = 5, \quad -3 < x < 3$

$\quad = x^2, \quad x \ge 3$

i. $f(-4) = 4(-4) - 2 = -16 - 2 = -18$

ii. $f(-3) = 4(-3) - 2 = -12 - 2 = -14$

iii. $f(1) = 5$

iv. $f(5) = 5^2 = 25$

---

8. **If $f(x) = 3x + 5$, $g(x) = 6x - 1$, then find**

i. **$(f + g)(x)$**

ii. **$(f - g)(2)$**

iii. **$(f g)(3)$**

iv. **$\left(\frac{f}{g}\right)(x)$ and its domain**

Solution:

$f(x) = 3x + 5, \; g(x) = 6x - 1$

i. $(f + g)\, x = f(x) + g(x)$

$\quad = 3x + 5 + 6x - 1 = 9x + 4$

ii. $(f - g)(2) = f(2) - g(2)$

$\quad = [3(2) + 5] - [6(2) - 1]$

$\quad = 6 + 5 - 12 + 1 = 0$

iii. $(f g)(3) = f(3)\, g(3)$

$\quad = [3(3) + 5]\, [6(3) - 1]$

$\quad = (14)(17) = 238$

iv. $\left(\frac{f}{g}\right) x = \frac{f(x)}{g(x)} = \frac{3x + 5}{6x - 1}, x \ne \frac{1}{6}$

Domain = $R - \left\{\frac{1}{6}\right\}$

---

9. **If $f(x) = 2x^2 + 3$, $g(x) = 5x - 2$, then find**

i. **fog** **ii.** **gof**

iii. **fof** **iv.** **gog**

Solution:

$f(x) = 2x^2 + 3, \; g(x) = 5x - 2$

i. $(fog)(x) = f(g(x)) = f(5x - 2)$

$\quad = 2(5x - 2)^2 + 3$

$\quad = 2(25x^2 - 20x + 4) + 3$

$\quad = 50x^2 - 40x + 8 + 3$

$\quad = 50x^2 - 40x + 11$

ii. $(gof)(x) = g(f(x)) = g(2x^2 + 3)$

$\quad = 5(2x^2 + 3) - 2$

$\quad = 10x^2 + 15 - 2$

$\quad = 10x^2 + 13$

iii. $(fof)(x) = f(f(x)) = f(2x^2 + 3)$

$\quad = 2(2x^2 + 3)^2 + 3$

$\quad = 2(4x^4 + 12x^2 + 9) + 3$

$\quad = 8x^4 + 24x^2 + 18 + 3$

$\quad = 8x^4 + 24x^2 + 21$

iv. $(gog)(x) = g(g(x)) = g(5x - 2)$

$\quad = 5(5x - 2) - 2$

$\quad = 25x - 10 - 2$

$\quad = 25x - 12$

Miscellaneous Exercise – 2

1. **Which of the following relations are functions? If it is a function determine its domain and range.**

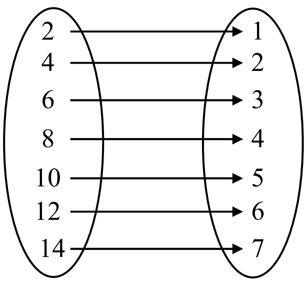
i. **{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5) (12, 6), (14, 7)}**

ii. **{(0, 0), (1, 1), (1, –1), (4, 2), (4, –2), (9, 3), (9, –3), (16, 4), (16, – 4)}**

iii. **{(1, 1), (3, 1), (5, 2)}**

Solution:

i. {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5) (12, 6), (14, 7)}



A (Domain) B (Codomain)

Every element of set A has been assigned a unique element in set B.

∴ Given relation is a function.

Domain = {2, 4, 6, 8, 10, 12, 14},

Range = {1, 2, 3, 4, 5, 6, 7}

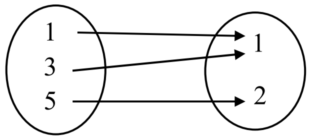
ii. $\{(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)\}$

$\because$ $(1, 1), (1, -1) \in$ the relation

$\therefore$ Given relation is not a function.

As the element 1 of domain has not been assigned a unique element of co-domain.

iii. $\{(1, 1), (3, 1), (5, 2)\}$



A (Domain) B (Codomain)

Every element of set A has been assigned a unique element in set B.

$\therefore$ Given relation is a function.

Domain = $\{1, 3, 5\}$, Range = $\{1, 2\}$

2. A function f: R $\to$ R defined by $f(x) = \dfrac{3x}{5} + 2$, $x \in$ R. Show that f is one-one and onto. Hence, find f^{-1}.

Solution:

f: R $\to$ R defined by $f(x) = \dfrac{3x}{5} + 2$

First we have to prove that f is one-one function for that we have to prove if $f(x_1) = f(x_2)$ then $x_1 = x_2$

Here $f(x) = \dfrac{3x}{5} + 2$

Let $f(x_1) = f(x_2)$

$\therefore$ $\dfrac{3x_1}{5} + 2 = \dfrac{3x_2}{5} + 2$

$\therefore$ $\dfrac{3x_1}{5} = \dfrac{3x_2}{5}$

$\therefore$ $x_1 = x_2$

$\therefore$ f is a one-one function.

Now, we have to prove that f is an onto function.

Let $y \in$ R be such that

$y = f(x)$

$\therefore$ $y = \dfrac{3x}{5} + 2$

$\therefore$ $y - 2 = \dfrac{3x}{5}$

$\therefore$ $x = \dfrac{5(y-2)}{3} \in$ R

$\therefore$ for any $y \in$ co-domain R, there exist an element $x = \dfrac{5(y-2)}{3} \in$ domain R such that $f(x) = y$

$\therefore$ f is an onto function.

$\therefore$ f is one-one onto function.

$\therefore$ f^{-1} exists

$\therefore$ $f^{-1}(y) = \dfrac{5(y-2)}{3}$

$\therefore$ $f^{-1}(x) = \dfrac{5(x-2)}{3}$

3. A function f is defined as follows: $f(x) = 4x + 5$, for $-4 \le x < 0$. Find the values of $f(-1)$, $f(-2)$, $f(0)$, if they exist.

Solution:

$f(x) = 4x + 5, \ -4 \le x < 0$

$f(-1) = 4(-1) + 5 = -4 + 5 = 1$

$f(-2) = 4(-2) + 5 = -8 + 5 = -3$

$x = 0 \notin$ domain of f

$\therefore$ f(0) does not exist.

4. A function f is defined as follows: $f(x) = 5 - x$ for $0 \le x \le 4$. Find the value of x such that $f(x) = 3$.

Solution:

$f(x) = 5 - x$

$f(x) = 3$

$\therefore$ $5 - x = 3$

$\therefore$ $x = 5 - 3 = 2$

5. If $f(x) = 3x^2 - 5x + 7$, find $f(x - 1)$.

Solution:

$f(x) = 3x^2 - 5x + 7$

$\therefore$ $f(x-1) = 3(x-1)^2 - 5(x-1) + 7$

$= 3(x^2 - 2x + 1) - 5(x - 1) + 7$

$= 3x^2 - 6x + 3 - 5x + 5 + 7$

$= 3x^2 - 11x + 15$

6. If $f(x) = 3x + a$ and $f(1) = 7$, find a and $f(4)$.

Solution:

$f(x) = 3x + a$,

$f(1) = 7$

$\therefore$ $3(1) + a = 7$

$\therefore$ $a = 7 - 3 = 4$

$\therefore$ $f(x) = 3x + 4$

$\therefore$ $f(4) = 3(4) + 4 = 12 + 4 = 16$

7. If $f(x) = ax^2 + bx + 2$ and $f(1) = 3$, $f(4) = 42$, find a and b.

Solution:

$f(x) = ax^2 + bx + 2$

$f(1) = 3$

$\therefore$ $a(1)^2 + b(1) + 2 = 3$

$\therefore$ $a + b = 1$ …(i)

$f(4) = 42$

$\therefore$ $a(4)^2 + b(4) + 2 = 42$

$\therefore$ $16a + 4b = 40$

Dividing by 4, we get
$4a + b = 10$ …(ii)
Solving (i) and (ii), we get
$a = 3, b = -2$

8. If $f(x)=\dfrac{2x-1}{5x-2}, x \neq \dfrac{2}{5}$, verify whether (fof) ($x$) = x.

Solution:

$$\text{(fof)}(x) = f(f(x))$$
$$= f\left(\frac{2x-1}{5x-2}\right)$$
$$= \frac{2\left(\frac{2x-1}{5x-2}\right)-1}{5\left(\frac{2x-1}{5x-2}\right)-2}$$
$$= \frac{4x-2-5x+2}{10x-5-10x+4} = \frac{-x}{-1} = x$$

9. If $f(x)=\dfrac{x+3}{4x-5}$, $g(x)=\dfrac{3+5x}{4x-1}$, then verify that (fog) (x) = x.

Solution:

$$f(x)=\frac{x+3}{4x-5},\ g(x)=\frac{3+5x}{4x-1}$$
$$\text{(fog)}(x) = f(g(x))$$
$$= f\left(\frac{3+5x}{4x-1}\right)$$
$$= \frac{\frac{3+5x}{4x-1}+3}{4\left(\frac{3+5x}{4x-1}\right)-5}$$
$$= \frac{3+5x+12x-3}{12+20x-20x+5} = \frac{17x}{17} = x$$

Activities for Practice

1. If $f(x) = \dfrac{x+3}{x-2}$, $g(x) = \dfrac{2x+3}{x-1}$, verify whether fog(x) = gof(x). *(Textbook page no. 32)*

2. $f(x) = 3x^2 - 4x + 2$, $x \in \{0, 1, 2, 3, 4\}$, then represent the function
i. by arrow diagram
ii. as set of ordered pairs
iii. in tabular form
iv. in graphical form *(Textbook page no. 32)*

3. If $f(x) = 5x - 2$, $x > 0$, find $f^{-1}(x)$, $f^{-1}(7)$. For what value of x is $f(x) = 0$. *(Textbook page no. 32)*

4. If $g(a) = \log\left(\dfrac{5+a}{5-a}\right)$ for $0 < a < 5$, find $g\left(\dfrac{50a}{25+a^2}\right)$ by completing the following activity.

Solution:

$$g(a) = \log\left(\frac{5+a}{5-a}\right)$$
$$\therefore\ g\left(\frac{50a}{25+a^2}\right) = \log\left(\frac{5+\frac{50a}{25+a^2}}{5-\frac{50a}{25+a^2}}\right)$$
$$= \log\left(\frac{5(25+a^2)+50a}{5(25+a^2)-50a}\right)$$
$$= \log\left(\frac{25+a^2+\boxed{}}{25+a^2-\boxed{}}\right)$$
$$= \log\left(\frac{5+a}{5-a}\right)^2$$
$$= \boxed{}\ \log\left(\frac{5+a}{5-a}\right)$$
$$= 2\ \boxed{}$$

5. If $f(x, y) = 3x^2 - y$, then to evaluate $f(-1, f(2, 3))$ complete the following activity.

Solution:

$f(x, y) = 3x^2 - y$
$\therefore$ $f(2, 3) = 3\ \boxed{} - \boxed{}$
$\therefore$ $f(-1, f(2, 3)) = 3(-1)^2 - \boxed{}$
$= \boxed{}$

6. If $f : R \rightarrow R$ and $g : R \rightarrow R$ be defined as $f(x) = 3x - 7$ and $g(x) = 2 + x$ and $(f^{-1} \text{ o } g^{-1})\, x = 3$, then to find x, complete the following activity.

Solution:

$f(x) = 3x - 7$, $g(x) = 2 + x$
$(g \text{ o } f)(x) = g(f(x)) = 2 + (3x - 7)$
$= \boxed{}$
Let $(g \text{ o } f)(x) = y$
$\therefore$ $x = (g \text{ o } f)^{-1}(y) = \dfrac{\boxed{}}{3}$
But $(g \text{ o } f)^{-1} = f^{-1} \text{ o } g^{-1}$
$\therefore$ $(f^{-1} \text{ o } g^{-1})(x) = \boxed{}$
Given $(f^{-1} \text{ o } g^{-1})x = 3$
$\therefore$ $x = \boxed{}$

Answers

1. $f(x) = \dfrac{x+3}{x-2}$, $g(x) = \dfrac{2x+3}{x-1}$

$fog(x) = f[g(x)]$

$= f\left(\dfrac{2x+3}{x-1}\right)$

$= \dfrac{\dfrac{2x+3}{x-1}+3}{\dfrac{2x+3}{x-1}-2}$

$= \dfrac{2x+3+3x-3}{2x+3-2x+2}$

$= \dfrac{5x}{5}$

$= x$ …(i)

$gof(x) = g[f(x)]$

$= g\left(\dfrac{x+3}{x-2}\right)$

$= \dfrac{2\left(\dfrac{x+3}{x-2}\right)+3}{\dfrac{x+3}{x-2}-1}$

$= \dfrac{2x+6+3x-6}{x+3-x+2}$

$= \dfrac{5x}{5}$

$= x$ …(ii)

From (i) and (ii), we get

$fog(x) = gof(x)$

2. $f(x) = 3x^2 - 4x + 2$

$f(0) = 3(0)^2 - 4(0) + 2 = 2$

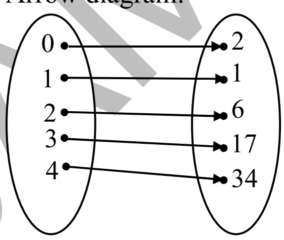
$f(1) = 3(1)^2 - 4(1) + 2 = 3 - 4 + 2 = 1$

$f(2) = 3(2)^2 - 4(2) + 2 = 12 - 8 + 2 = 6$

$f(3) = 3(3)^2 - 4(3) + 2 = 27 - 12 + 2 = 17$

$f(4) = 3(4)^2 - 4(4) + 2 = 48 - 16 + 2 = 34$

i. Arrow diagram:



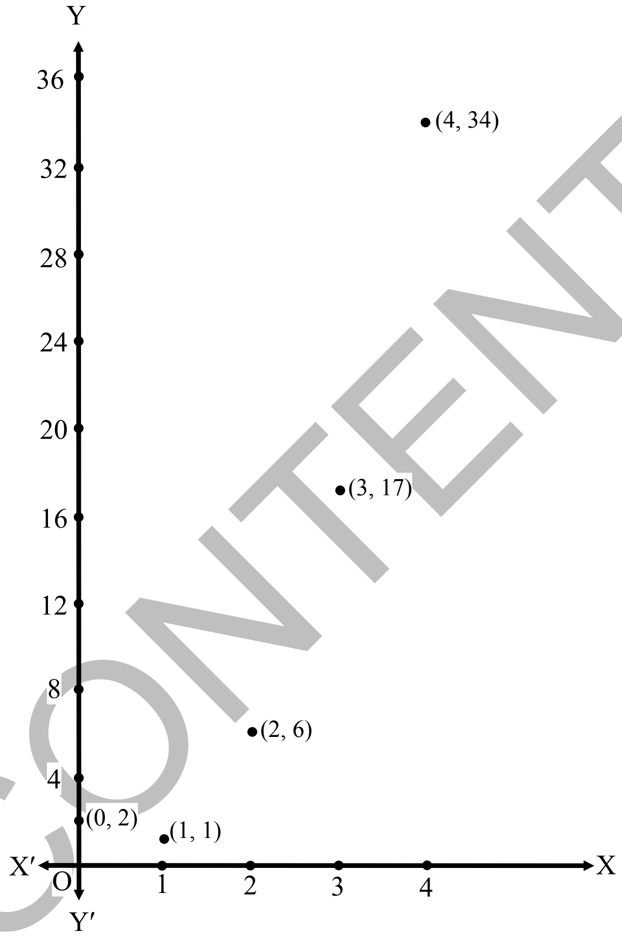
ii. Set of ordered pairs:

= {(0,2), (1, 1), (2, 6), (3, 17), (4, 34)}

iii. Tabular form:

x	0	1	2	3	4
y	2	1	6	17	34

iv. Graphical form:



3. $f(x) = 5x - 2, x > 0$

Let $y = f(x) = 5x - 2$

$\therefore$ $5x = y + 2$

$\therefore$ $x = \dfrac{y+2}{5}$

$\therefore$ $f^{-1}(y) = \dfrac{y+2}{5}$

Replacing y by x, we get

$f^{-1}(x) = \dfrac{x+2}{5}$

$\therefore$ $f^{-1}(7) = \dfrac{7+2}{5} = \dfrac{9}{5}$

$f(x) = 0$

$\therefore$ $5x - 2 = 0$

$\therefore$ $x = \dfrac{2}{5}$

4. i. 10 a ii. 10 a
iii. 2 iv. g(a)

5. i. 4 ii. 3
iii. 9 iv. – 6

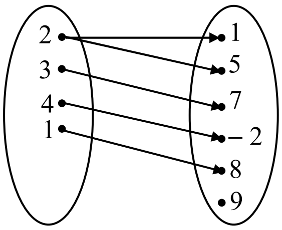
6. i. $3x - 5$ ii. $y + 5$
iii. $\dfrac{x+5}{3}$ iv. 4

Additional Problems for Practice

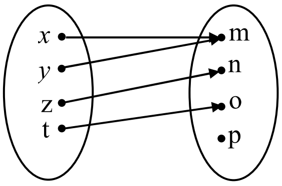
Based on Exercise 2.1

1. Check if the following relations are functions.

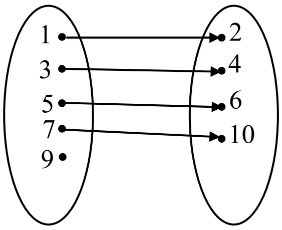
i.



ii.



iii.



2. Which of the following relations are functions? Justify your answer.

i. {(2, 1), (3, 1), (5, 2)}

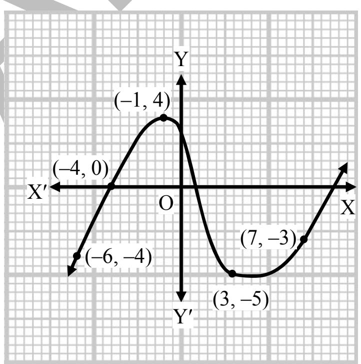
ii. {(2, 3), (3, 2), (2, 5), (5, 2)}

+3. Evaluate

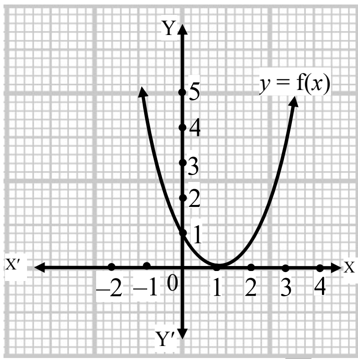
$f(x) = 2x^2 – 3x + 4$ at $x = 7$ and $x = – 2t$.

4. If $f(x) = (x + 5)(3x – 1)$, then find f(1), f (–2).

+5. Using the graph of $y = g(x)$, find g(– 4) and g(3).



+6. From the graph below find x for which $f(x) = 4$



+7. If $f(x) = 3x^2 – x$ and f (m) = 4, then find m.

8. Find x, if $f(x) = 0$ where

i. $f(x) = \dfrac{4x-3}{7}$

ii. $f(x) = 3x^2 – 11x – 4$

9. Find the set of values of x for which $f(x) = g(x)$, where $f(x) = 2x^2 – 1$ and $g(x) = 1 – 3x$.

+10. From the equation $4x + 7y = 1$ express

i. y as a function of x

ii. x as a function of y

11. i. If $f(x) = \dfrac{x^3+1}{x^2+1}$, find f(–3), f(–1).

ii. If $f(x) = (x – 1)(2x + 1)$, find f(1), f(2), f(–3).

+12. If $f(x) = x^2 + 2$ and $g(x) = 5x – 8$, then find

i. (f + g) (1) ii. (f – g) (–2)

iii. (f.g) (3m) iv. $\dfrac{f}{g}$ (0)

+13. If $f(x) = \dfrac{2}{x+5}$ and $g(x) = x^2 – 1$, then find

i. (fog) (x) ii. (gof) (3)

+14. If $f(x) = x^2$, $g(x) = x + 5$, and $h(x) = \dfrac{1}{x}$, $x \neq 0$, find (gofoh) (x).

+15. i. If f is one-one onto function with $f(x) = 9 – 5x$, find $f^{-1}(–1)$.

ii. Determine whether the function $f(x)=\dfrac{2x+1}{x-3}$ has inverse, if it exists find it.

+16. Verify that $f(x) = \dfrac{x-5}{8}$ and $g(x) = 8x + 5$ are inverse functions of each other.

Based on Miscellaneous Exercise – 2

1. Which of the following relations are functions? If it is a function, determine its domain and range.
   i. {(1, 2), (2, 3), (3, 4), (4, 5)}
   ii. {(1, 2), (1, 4), (2, 4), (3, 6)}

2. A function f : R → R defined by $f(x) = 5 + \frac{x}{6}$, $x \in R$. Show that f is one-one and onto. Hence find f^{-1}.

3. A function f is defined as follows:
   $f(x) = 3x + 7$, for $-3 \le x \le 1$.
   Find the values of f(–2), f(1), f(2), if they exist.

4. A function f is defined as follows:
   $f(x) = 3 + x$ for $-2 < x < 2$. Find the value of x such that $f(x) = 4$.

5. If $f(x) = 2x^3 - 3x + 11$, find $f(x + 1)$.

6. If $f(x) = 2x + a$ and $f(2) = 9$, find a and f(3).

7. If $f(x) = ax^2 + bx + 5$ and $f(1) = 12$, $f(2) = 21$, find a and b.

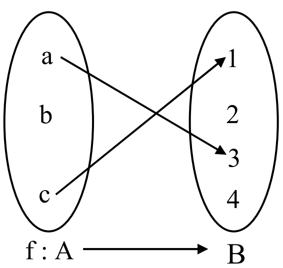
8. If $f(x) = \frac{3x+1}{5x-3}, x \ne \frac{3}{5}$, then show that $(f \circ f)(x) = x$.

9. If $f(x) = \frac{x+2}{3x-7}$ and $g(x) = \frac{2+7x}{3x-1}$, then show that $(f \circ g)(x) = x$.

Multiple Choice Questions

1. Let A = { 1,2,3,4} and B = {1,6,8,11,15}. Which of the following are functions from A to B?
   i. f(1) = 1, f(2) = 6, f(3) = 8, f(4) = 8
   ii. f(1) = 1, f(2) = 6, f(3) = 15
   iii. f(1) = 6, f(2) = 6, f(3) = 6, f(4) = 6
   (A) (ii) & (iii) (B) (i) & (ii)
   (C) (ii) (D) (i) &(iii)

2. The diagram given below shows that



   (A) f is a function from A to B
   (B) f is a one-one function from A to B
   (C) f is a bijection from A to B
   (D) f is not a function.

3. Let A = {1, 2, 3} and B = {2, 3, 4}, then which of the following is a function from A to B?
   (A) {(1, 2), (1, 3), (2, 3), (3, 3)}
   (B) {(0, 3), (2, 4)}
   (C) {(1, 3), (2, 3), (3, 3)}
   (D) {(1, 2), (2, 3), (3, 4), (3, 2)}

4. The function f: R → R defined by
   f(x) = 1; if $x > 0$
   = 0; if $x = 0$
   = – 1; if $x < 0$ is a
   (A) rational function
   (B) modulus function
   (C) signum function
   (D) sine function

5. If f: R → R is defined by $f(x) = x^2 - 3x + 2$, then the value of f[f(5)] is
   (A) 111 (B) 110
   (C) 109 (D) 101

6. The domain of the function $\frac{1}{(2x-3)(x+1)}$ is
   (A) R – {– 1} (B) $R - \left\{\frac{3}{2}\right\}$
   (C) $R - \{-1, \frac{3}{2}\}$ (D) R

7. If $f(x) = 3x - 5$, then $f^{-1}(x)$ is
   (A) $\frac{1}{3x-5}$ (B) $\frac{x+5}{3}$
   (C) $\frac{y+3}{5}$ (D) does not exist

8. If $f(x) = \frac{3x+2}{4x-3}$ for $x \ne \frac{3}{4}$, then fof (x) is
   (A) $17x$ (B) $3x$
   (C) $4x$ (D) x

9. If $f(x) = x^2 + 5x + 7$, then the value of x for which $f(x) = f(x + 1)$ is
   (A) 3 (B) –6
   (C) –3 (D) 6

10. If $f(x) = x^2 - 6x + 9, 0 \le x \le 4$, then f(3) =
    (A) 4 (B) 1
    (C) 0 (D) does not exist

11. If $f(x) = x^2 + \frac{1}{x}, x \ne 0$, then the value of f(–1) is
    (A) $\frac{9}{2}$ (B) 1 (C) 0 (D) 2

12. If $f\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$, $x \neq 0$, then $f(x) =$

(A) $x - \frac{1}{x}$ (B) $x^3 + 3x$

(C) $x^3 - 3$ (D) $x^3 - 3x$

13. If $f(x) = \frac{2^x + 2^{-x}}{2}$, then $f(x + y) \cdot f(x - y) =$

(A) $\frac{1}{2}$ [f (2x) + f (2y)]

(B) $\frac{1}{4}$ [f (2x) + f (2y)]

(C) $\frac{1}{2}$ [f (2x) − f (2y)]

(D) $\frac{1}{4}$ [f (x) − f (2y)]

14. If f : R → R is defined by $f(x) = \frac{x}{x^2+1}$, find f (f (2)).

(A) $\frac{29}{10}$ (B) $\frac{1}{29}$

(C) 29 (D) $\frac{10}{29}$

15. If $f(x) = \frac{3^x - 3^{-x}}{3^x + 3^{-x}}$, then inverse of f is

(A) $\log_3(2 - x)$ (B) $\frac{1}{2}\log_3(10x - 1)$

(C) $\frac{1}{2}\log_3\left(\frac{1+x}{1-x}\right)$ (D) $\frac{1}{4}\log_3\frac{2x}{2-x}$

Answers to Additional Practice Problems

Based on Exercise 2.1

1. i. Not a function ii. It is a function
   iii. Not a function
2. i. It is a function ii. Not a function
3. $f(7) = 81$, $f(-2t) = 8t^2 + 6t + 4$
4. $f(1) = 12$, $f(-2) = -21$
5. $g(-4) = 0$, $g(3) = -5$
6. $x = -1$ and $x = 3$
7. $m = \frac{4}{3}$ or $m = -1$
8. i. $\frac{3}{4}$
   ii. $x = 4$ or $x = -\frac{1}{3}$
9. $\left\{-2, \frac{1}{2}\right\}$
10. i. $\frac{1-4x}{7}$ ii. $\frac{1-7y}{4}$
11. i. $f(-3) = -2.6$, $f(-1) = 0$
    ii. $f(1) = 0$, $f(2) = 5$, $f(-3) = 20$
12. i. 0
    ii. 24
    iii. $135m^3 - 72m^2 + 30m - 16$
    iv. $-\frac{1}{4}$
13. i. $\frac{2}{x^2+4}$ ii. $-\frac{15}{16}$
14. $\frac{1}{x^2} + 5$
15. i. 2 ii. $f^{-1}(x) = \frac{3x+1}{x-2}$

Based on Miscellaneous Exercise – 2

1. i. It is a function
   Domain = {1, 2, 3, 4}
   Range = {2, 3, 4, 5}
   ii. Not a function
2. $f^{-1}(x) = 6(x - 5)$
3. $f(-2) = 1$, $f(1) = 10$, f(2) does not exist.
4. 1
5. $2x^3 + 6x^2 + 3x + 10$
6. a = 5, f(3) = 11
7. a = 1, b = 6

Answers to Multiple Choice Questions

1.	(D)	2.	(D)	3.	(C)	4.	(C)
5.	(B)	6.	(C)	7.	(B)	8.	(D)
9.	(C)	10.	(C)	11.	(C)	12.	(B)
13.	(A)	14.	(D)	15.	(C)		