SAU PhD Maths Questions Papers

*SAU : South Asian Uinversity

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Department of Mathematics

PhD (Applied Mathematics)

The Department of Mathematics started its PhD programme in July 2013. A wide range of the following research areas are offered in which the enrolled students can pursue their PhD work:

Numerical Analysis

Differential Equations and Boundary Value Problems

Fourier Analysis

Analysis, Function Spaces

Integral Operators and weighted Norm Inequalities

Graph Theory, Discrete Mathematics

Finite Elements Methods

Parallel Computations

Statistical Approximation, Stochastic Processes

Mathematical Biology, Non-linear Dynamical Systems

Optimization, Swarm Intelligence

Admission to PhD is through a common entrance test held in all the SAARC countries followed by an interview.

South Asian University PhD Admission Syllabus and Sample Questions

Sample Question Paper for PhD Applied Mathematics

Details of Syllabus

Format of the Entrance Test Paper

- The duration of the Entrance Test will be 2 hours and the question paper will consist of 50 multiple choice questions.

- **Analysis:** Real functions; limit, continuity, differentiability; sequences; series; uniform convergence; functions of complex variables; analytic functions, complex integration; singularities, power and Laurent series; metric spaces; stereographic projection; topology, compactness, connectedness; normed linear spaces, inner product spaces; dual spaces, linear operators; Lebesgue measure and integration; convergence theorems.

- **Algebra:** Basic theory of matrices and determinants; eigen values and eigen vectors; Groups and their elementary properties; subgroups, normal subgroups, cyclic groups, permutation groups; Lagrange's theorem; quotient groups, homomorphism of groups; Cauchy Theorem and p-groups; the structure of groups; Sylow's theorems and their applications; rings, integral domains and fields; ring homomorphism and ideals; polynomial rings and irreducibility criteria; vector space, vector subspace, linear independence of vectors, basis and dimensions of a vector space, inner product spaces, orthonormal basis; Gram-Schmidt process, linear transformations.

- **Differential Equations:** First order ordinary differential equations (ODEs); solution of first order initial value problems; singular solution of first order ODEs; system of linear first order ODEs; method of solution of dx/P=dy/Q=dz/R; orthogonal trajectory; solution of Pfaffian differential equations in three variables; linear second order ODEs; Sturm-Liouville problems; Laplace transformation of ODEs; series solutions; Cauchy problem for first order partial differential equations (PDEs); method of characteristics; second order linear PDEs in two variables and their classification; separation of variables; solution of Laplace, wave and diffusion equations; Fourier transform and Laplace transform of PDEs.

- **Numerical Analysis:** Numerical solution of algebraic and transcendental equations; direct and iterative methods for system of linear equations; matrix eigenvalue problems; interpolation and approximations; numerical differentiation and integration; composite numerical integration; double numerical integration; numerical solution for initial value problems; finite difference and finite element methods for boundary value problems.

- **Probability and Statistics:** Axiomatic approach of probability; random variables; expectation, moments generating functions, density and distribution functions; conditional expectation.

- **Linear Programming:** Linear programming problem and its formulation; graphical method, simplex method; artificial starting solution; sensitivity analysis; duality and post-optimality analysis.

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QUESTION PAPER SERIES CODE
A

Centre Name : ______________________

Roll No. : ______________________

Name of Candidate : ______________________

S A U

Entrance Test for M.Phil./Ph.D. (Applied Mathematics), 2014

[PROGRAMME CODE : PAM]

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Centre Name in the space provided for the purpose on the top of this Question Paper and in the OMR/Answer Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (Objective-type) has 30 questions of **1** mark each. All questions are compulsory.

(iv) Part—B (Objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR/Answer Sheet in the space provided.**

(vii) Part—A and Part—B (Multiple Choice) questions should be answered on the OMR/Answer Sheet.

(viii) Answers written by the candidates inside the Question Paper will **NOT** be evaluated.

(ix) Calculators and Log Tables may be used. Mobile Phones are **NOT** allowed.

(x) Pages at the end have been provided for Rough Work.

(xi) **Return the Question Paper and the OMR/Answer Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR/ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS IN THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Example :

   Question Paper Series Code

   Write Question Paper Series Code A or B and darken appropriate circle.

   ☐ A or B

   ●

   Ⓑ

   Programme Code

   Write Programme Code out of 14 codes given and darken appropriate circle.

   Write Programme Code ☐

MEC	○	MAM	○	PCS	○
MSO	○	MLS	○	PBT	○
MIR	○	PEC	○	PAM	●
MCS	○	PSO	○	PLS	○
MBT	○	PIR	○		

2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.
3. Please darken the whole Circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

   Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	⊗ ⓑ ⓒ ⓓ	⊗ ⓑ ⓒ ⊘	⊙ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. There will be <u>no negative marking</u> in evaluation.
10. Write your six digits Roll Number in small boxes provided for the purpose; and also darken appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    Example :

ROLL NUMBER

1	3	5	7	2	0
●	①	①	①	①	①
②	②	②	②	●	②
③	●	③	③	③	③
④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●

PART—A

1. Let $\{f_n\}$ be a sequence of functions where $f_n : [0, 1] \to R$ is defined by $f_n(x) = x^n$ for $n = 1, 2, \ldots$. Then the sequence $\{f_n\}$ is

(a) convergent but not uniformly convergent on $[0, 1]$

(b) uniformly convergent on $[0, 1]$

(c) not convergent on $[0, 1]$

(d) convergent to a continuous function on $[0, 1]$

2. Let $a_1, a_2, \ldots, a_n, \ldots$ be a sequence of positive real numbers such that

$$\sum_{n=1}^{\infty} a_n^2$$

converges. Which of the following necessarily holds?

(a) $\sum_{n=1}^{\infty} a_n$ converges

(b) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges

(c) $\sum_{n=1}^{\infty} n\, a_n$ converges

(d) None of the above

3. Let f be a real valued continuous function defined on $[0, 1]$ such that

$$\int_0^1 f(x)\,dx = 0$$

Then

(a) $f \equiv 0$

(b) $\int_0^x f(t)dt = 0$ for all x

(c) there is a subinterval of $[0, 1]$ on which f is non-negative

(d) there is always a proper subinterval $[\alpha, \beta] \subset [0, 1]$ on which $f(x) = 0$

4. Let $f(x) = e^{-x^2}$ and $g(x) = \frac{1}{1+x^2}$. Which of the following is true?

(a) $f(x) \geq g(x)$ for all $x \geq 0$

(b) $f(x) \leq g(x)$ for all $x \geq 0$

(c) $f(x) - g(x)$ changes sign finitely many times as x varies over $[0, \infty)$

(d) $f(x) - g(x)$ changes sign infinitely many times as x varies over $[0, \infty)$

5. The improper integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}dx$$

exists if and only if

(a) $m > 0, n > 0$

(b) $m > 0, n < 0$

(c) $m < 0, n > 0$

(d) $m < 0, n < 0$

6. Let f be an increasing function and g be a decreasing function on an interval such that $f \circ g$ exists. Then $f \circ g$ is

(a) increasing

(b) decreasing

(c) monotone

(d) neither increasing nor decreasing

7. A group with at least two elements but with no proper non-trivial subgroups must be

(a) finite and of odd order

(b) finite and of prime order

(c) finite and of even order

(d) finite with no restriction over the order

8. $\mathbb{Z}_3 \times \mathbb{Z}_4$ is of order

(a) 3

(b) 4

(c) 12

(d) None of the above

9. Let G be an Abelian group of order 72. Number of subgroups of G of order 4 is

(a) 1

(b) 9

(c) 4

(d) Not a fixed number

10. If H and N are subgroups of a group G, and N is normal in G, then $H \cap N$ is always normal in

(a) G

(b) H

(c) N

(d) Not a normal group

11. A group is simple if

(a) it is non-trivial and has no proper non-trivial normal subgroups

(b) it is non-trivial and has proper non-trivial normal subgroups

(c) it is non-trivial and has no proper non-trivial subgroups

(d) it is non-trivial and has proper non-trivial subgroups

12. Which of the following is true?

(a) Every field is also a ring

(b) Every ring has a multiplicative identity

(c) Every ring with unity has at least two units

(d) Every ring with unity has at most two units

13. The order of the matrix ring $M_2(Z_2)$ is

(a) 4

(b) 2

(c) 16

(d) 8

14. The necessary and sufficient condition for the differential equation $f(x, y)dx + g(x, y)dy = 0$ to be exact is given by

(a) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

(b) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$

(c) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial x}$

(d) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial y}$

15. The necessary condition for the partial differential equation

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} = 0$$

where $u = u(x, y)$ and A, B, C are functions of x and y, to be elliptic is given by

(a) $B^2 - AC = 0$

(b) $B^2 - 4AC > 0$

(c) $B^2 - 4AC = 0$

(d) $B^2 - 4AC < 0$

16. The bisection method of finding roots of non-linear equation falls under the category of —— methods.

(a) bracketing

(b) open

(c) random

(d) graphical

17. In —— method, a system is reduced to an equivalent diagonal form using elementary transformations.

(a) Jacobi

(b) Gauss elimination

(c) Gauss-Jordan

(d) Gauss-Seidel

18. The highest order of polynomial integrand for which Simpson's $1/3$ rule of integration is exact, is

(a) first

(b) second

(c) third

(d) fourth

19. Given $n+1$ data pairs, a unique polynomial of degree —— passes through the $n+1$ data points.

(a) $n+1$

(b) n

(c) n or less

(d) $n+1$ or less

20. The truncation error in quadratic interpolation in an equidistant table is bounded by

(a) $\frac{h^2}{9\sqrt{3}} \max |f'''(\xi)|$

(b) $\frac{h^2}{\sqrt{3}} \max |f'''(\xi)|$

(c) $\frac{h^2}{9} \max |f'''(\xi)|$

(d) $\frac{h^2}{\sqrt{2}} \max |f'''(\xi)|$

21. How will the mass of the object affect the way it speeds up or slows down?

(a) No effect

(b) More massive objects are easier to accelerate and harder to decelerate

(c) More massive objects are harder to accelerate and harder to decelerate

(d) More massive objects are easier to accelerate and easier to decelerate

22. When an object is moving faster through a fluid, then

(a) the force of friction is greater

(b) the force of friction is less

(c) the force of friction is unaffected

(d) None of the above

23. The amount by which an objective function coefficient can change before a different set of values for the decision variables becomes optimal is the

(a) optimal solution

(b) dual solution

(c) range of optimality

(d) range of feasibility

24. A basic solution is called degenerate if

(a) the value of at least one of the basic variables is non-zero

(b) the value of all the basic variables is zero

(c) the value of at least one of the basic variables is zero

(d) the value of all the basic variables is non-zero

25. If in a simplex table, the relative cost $z_j - c_j$ is zero for a non-basic variable, then there exists an alternate optimal solution, provided

(a) it is a starting simplex table

(b) it is an optimal simplex table

(c) it can be any simplex table

(d) None of the above

26. Let S be a non-empty closed convex set. Then

(a) S has finite number of vertices

(b) S has infinite number of vertices

(c) S may have finite or infinite number of vertices

(d) None of the above

27. The constraints of an LPP including non-negativity restrictions are

(a) either closed half spaces or hyperplanes

(b) closed half spaces

(c) hyperplanes only

(d) None of the above

28. Let X is any non-negative integer-valued random variable with its generating function

$$P(z) = \sum_{j=0}^{\infty} z^j P\{X = j\}$$

Now if it is given that X is Poisson with mean λ, then $P\{X \text{ is even}\}$ is given by

(a) $\dfrac{1+e^{-2\lambda}}{2}$

(b) $\dfrac{1+e^{-\lambda}}{2}$

(c) $\dfrac{1-e^{-2\lambda}}{2}$

(d) $\dfrac{1-e^{-\lambda}}{2}$

29. In an election, candidate A receives n votes and candidate B receives m votes, where $n > m$. Assuming that all orderings are equally likely, the probability that A is always ahead in the count of votes is

(a) $\frac{n-m}{2^{mn}}$

(b) $\frac{n-m}{2^{m+n}}$

(c) $\frac{2^{n-m}}{2^{m+n}}$

(d) $\frac{n-m}{n+m}$

30. The continuous random variable X is uniformly distributed with mean 1 and variance 3. Then $P(X < 0)$ is

(a) $\frac{1}{6}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

PART—B

31. The matrix

$$A=\begin{pmatrix}1 & 0\\ 2 & 4\end{pmatrix}$$

is given. Then eigenvalues of $4A^{-1}+3A+2I$ are

(a) 6, 15

(b) 9, 12

(c) 9, 15

(d) 7, 15

32. Which of the following system of vectors is linearly independent?

(a) $X_1=(1,-1,1)$, $X_2=(2,1,1)$, $X_3=(3,0,2)$

(b) $X_1=(3,1,-4)$, $X_2=(2,2,-3)$, $X_3=(0,-4,1)$

(c) $X_1=(1,6,4)$, $X_2=(0,2,3)$, $X_3=(0,1,2)$

(d) None of the above

33. Among the following given statements which one is true?

(a) Any plane w in R^3 is a subspace of R^3

(b) Set of all real valued discontinuous function forms a subspace of V (Vector space of real valued function with real domain)

(c) Set of all continuous real valued functions f defined on the interval $[0, 1]$ forms a subspace of V (Vector space of real valued function with real domain)

(d) All of the above

34. If S_1 and S_2 be any two subsets of the vector space V, then which of the following statements is not correct?

(a) Span (S_1) = Span (S_2) if and only if $S_1 \subset$ Span (S_2) or $S_2 \subset$ Span (S_1)

(b) Span $(S_1 \cap S_2) \subseteq$ Span $(S_1) \cap$ Span (S_2)

(c) Span $(S_1) \cup$ Span $(S_2) \subseteq$ Span $(S_1 \cup S_2)$

(d) $S_1 \subseteq S_2$, then Span $(S_1) \cup$ Span (S_2) = Span $(S_1 \cup S_2)$

35. Let $L: R^3 \to R^3$ be a rotation about z-axis through an angle of $\frac{\pi}{3}$. Then its matrix A with respect to standard basis

(a) is diagonalizable

(b) is not diagonalizable

(c) A has eigenvalue 1 with algebraic multiplicity 2

(d) A has eigenvalue 2 with algebraic multiplicity 1

36. For a complex variable z, the function $f(z) = |z|^2$ is

(a) differentiable nowhere

(b) differentiable everywhere

(c) differentiable only at $z = 0$

(d) differentiable everywhere except at $z = 0$

37. Let C be the circle centered at origin and with radius 1. Then the value of the integral $\int_C \frac{e^z}{z^3} dz$ is

(a) $e^{\pi i}$

(b) πi

(c) $-\pi i$

(d) $e^{-\pi i}$

38. Under the stereographic projection, the point $z = \infty$ of the extended complex plane has the following corresponding point on the unit sphere.

(a) (0, 0, 0)

(b) (0, 0, 1)

(c) (0, 1, 0)

(d) (1, 0, 0)

39. For the function $f(z) = \frac{z - \sin z}{z^3}$, the point $z = 0$ is

(a) a removable singularity

(b) a pole

(c) an essential singularity

(d) None of the above

40. If C is the circle $|z| = \frac{1}{2}$, then $\int_C \frac{z^2 - z + 1}{z - 1} dz$ equals

(a) 1

(b) −1

(c) 0

(d) $\frac{1}{2}$

41. Which of the following statements is true?

(a) Cauchy-Riemann equations are necessary conditions for a function to be differentiable

(b) Cauchy-Riemann equations are sufficient conditions for a function to be differentiable

(c) Cauchy-Riemann equations are both necessary as well as sufficient conditions for a function to be differentiable

(d) Cauchy-Riemann equations are neither necessary nor sufficient conditions for a function to be differentiable

42. Let $f : [0, 1] \to R$ be a function such that $|f|$ is Riemann integrable over [0, 1]. Which of the following is necessarily true?

(a) f is Riemann integrable over [0, 1]

(b) f is Lebesgue integrable over [0, 1]

(c) f is both Riemann as well as Lebesgue integrable over [0, 1]

(d) f is neither Riemann nor Lebesgue integrable over [0, 1]

43. In an incomplete metric space

(a) no Cauchy sequence is convergent

(b) no convergent sequence has a convergent subsequence

(c) there is a Cauchy sequence which has no convergent subsequence

(d) there is a non-convergent sequence which has a convergent subsequence

44. Number of fixed points of the mapping $T: (0, 1) \to (0, 1)$ defined by $Tx = \frac{1}{x}$ is

(a) 0

(b) 1

(c) 2

(d) infinite

45. Which of the following statements is false?

(a) Every inner product space defines a norm

(b) Every inner product space defines a metric

(c) Every norm defines a metric

(d) Every metric defines an inner product space

46. The dual space of ℓ^3 is

(a) $\ell^{3/2}$

(b) $\ell^{2/3}$

(c) $\ell^{1/3}$

(d) $\ell^{1/2}$

47. Let H be a Hilbert space and $T : H \to H$ be a bijective bounded linear operator such that $T^* = T^{-1}$. Then

(a) T is self-adjoint

(b) T is unitary

(c) T is normal

(d) None of the above

48. Which of the following statements is false?

(a) Every countable set is measurable

(b) Cantor set is uncountable

(c) Cantor set is measurable

(d) Every measurable set is countable

49. If R is a ring with unity and N is an ideal of R containing a unit, then

(a) $N \subset R$

(b) $N \supset R$

(c) $N = R$

(d) None of the above

50. A Sylow 3-subgroup of a group of order 12 has order

(a) 4

(b) 9

(c) 12

(d) 3

51. Let p be a prime. A p-group is a group with the property that

(a) every element has order p

(b) at least one element has order p

(c) no element has order p

(d) one and only one element has order p

52. If $f(x) = x + 1$ and $g(x) = x + 1$, then in $\mathbb{Z}_2[x]$, $f(x) + g(x) =$

(a) $2x + 2$

(b) $2x$

(c) 2

(d) 0

53. Let E be the finite extension of degree n over a finite field F. If F has q elements, then E has

(a) q^n elements

(b) q elements

(c) nq elements

(d) Cannot say

54. Which of the following can be the order of a finite field?

(a) 4096

(b) 3127

(c) 36

(d) 60

55. The eigenvalues of the Sturm-Liouville problems are

(a) imaginary

(b) real

(c) both real and imaginary

(d) not defined

56. The solution of the total differential equation $xdy - ydx - 2x^2zdz = 0$ is given by

(a) $\frac{y}{x} - z^2 = c$

(b) $\frac{y}{x} + z^2 = c$

(c) $\frac{y}{x} - z = c$

(d) $\frac{y}{x} + z = c$

57. If $J_n(x)$ defines the Bessel's function of the first kind and of order n, then which of the following is true for $n > 2$?

(a) $\frac{d}{dx}(x^n J_n(x)) = x^n$

(b) $\frac{d}{dx}(x^n J_n(x)) = J_{n-1}(x)$

(c) $\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$

(d) $\frac{d}{dx}(x^n J_n(x)) = 0$

58. The generating function for the Legendre polynomial is given by

(a) $(1-2xz)^{-1/2}$

(b) $(1-z^2)^{-1/2}$

(c) $(1-2x+z)^{-1/2}$

(d) $(1-2xz+z^2)^{-1/2}$

59. The orthogonal trajectories of the system of curves

$$\left(\frac{dy}{dx}\right)^2 = \frac{a}{x}$$

is

(a) $9a(y+c)^2 = 4x^3$

(b) $9a(y+c)^2 = x$

(c) $9a(y+c)^2 = 2x^2$

(d) $9a(y+c) = 4$

60. If $y = e^{ax}$ is a solution of the linear second-order ordinary differential equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = 0$$

then which of the following holds true?

(a) $a^2 + p(x)a + q(x) = 0$

(b) $a^2 - p(x)a + q(x) = 0$

(c) $a^2 + p(x)a - q(x) = 0$

(d) $a^2 - p(x)a - q(x) = 0$

61. The solution of the initial value problem

$$\frac{d^2y}{dx^2} - 1 = 0, \; y(0) = 1, \left(\frac{dy}{dx}\right)_{x=0} = 2$$

is given by

(a) $y(x) = \frac{x^2}{2} - 2x + 1$

(b) $y(x) = \frac{x^2}{2} + 2x - 1$

(c) $y(x) = \frac{x^2}{2} + 2x + 1$

(d) $y(x) = \frac{x^4}{2} + 4x + 1$

62. The general solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x \partial y} = x + y$$

is of the form

(a) $\frac{1}{2}xy(x+y) + F(x) + G(y)$

(b) $\frac{1}{2}xy(x-y) + F(x) + G(y)$

(c) $\frac{1}{2}xy(x-y) + F(x)G(y)$

(d) $\frac{1}{2}xy(x+y) + F(x)G(y)$

63. Which of the following equations is parabolic?

(a) $f_{xy} - f_x = 0$

(b) $f_{xx} + 2f_{xy} + f_{yy} = 0$

(c) $f_{xx} + 2f_{xy} + 4f_{yy} = 0$

(d) None of the above

64. When the given equation cannot be reduced to any of the standard form, then to solve the differential equation, we apply

(a) Lagrange's method

(b) Charpit's method

(c) Monge's method

(d) Picard's iteration method

65. The complete integral of the equation $p^2x + q^2y = z$ is

(a) $\sqrt{(1-a)z} = \sqrt{ax} - \sqrt{y} + b$

(b) $(1+a)z = ax + y + b$

(c) $\sqrt{(1+a)z} = \sqrt{ax} - \sqrt{y} + b$

(d) $\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b$

66. Applying Charpit's method, solution of the equation $(p+q)(px+qy) = 1$ is

(a) $z + b = \frac{2}{\sqrt{1+c}}(cx + y)^{1/2}$

(b) $z + b = \frac{2}{\sqrt{1+c}}(cy + x)^{1/2}$

(c) $z + b = \frac{2}{\sqrt{1+c}}(cx + y)$

(d) $z + b = \frac{2}{\sqrt{1+c}}(cy + x)$

67. Which of the following is Lagrange's linear partial differential equation?

(a) $Rr + Ss + Tt = V$

(b) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

(c) $Pdx + Qdy = R$

(d) $Pp + Qq = R$

68. LU decomposition of a square matrix

(a) is unique

(b) is not unique

(c) does not exist

(d) may be unique or not unique

69. The principle of least action

(a) is a variational principle

(b) when applied to the action of a mechanical system, can be used to obtain equation of motion of the system

(c) being applied in the theory of relativity, quantum mechanics and quantum field theory

(d) All of the above

70. The Euler-Lagrange equation is

(a) $\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$

(b) $\frac{\partial L}{\partial q} - \left(\frac{\partial L}{\partial \dot{q}}\right) = 0$

(c) $\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial q}\right) = 0$

(d) $\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 1$

★ ★ ★

14

QUESTION PAPER SERIES CODE
B

Centre Name : ____________________

Roll No. : ____________________

Name of Candidate : ____________________

S A U

Entrance Test for M.Phil./Ph.D. (Applied Mathematics), 2015

[PROGRAMME CODE : PAM]

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Centre Name in the space provided for the purpose on the top of this Question Paper and in the OMR/Answer Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (Objective-type) has 30 questions of **1** mark each. All questions are compulsory.

(iv) Part—B (Objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(v) **One-fourth (1/4) of marks assigned to any question in Part—A and Part—B will be deducted for wrong answers.**

(vi) Symbols have their usual meanings.

(vii) **Please darken the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR/Answer Sheet in the space provided.**

(viii) Part—A and Part—B (Multiple Choice) questions should be answered on the OMR/Answer Sheet.

(ix) Answers written by the candidates inside the Question Paper will **NOT** be evaluated.

(x) Calculators and Log Tables may be used. Mobile Phones are **NOT** allowed.

(xi) Pages at the end have been provided for Rough Work.

(xii) **Return the Question Paper and the OMR/Answer Sheet** to the Invigilator at the end of the Entrance Test.

(xiii) **DO NOT FOLD THE OMR/ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS IN THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

Example :

Question Paper Series Code

Write Question Paper Series Code A or B and darken appropriate circle.

☐ A or B

Ⓐ

●

Programme Code

Write Programme Code out of 14 codes given and darken appropriate circle.

Write Programme Code ☐

MEC	○	MAM	○	PCS	○
MSO	○	MLS	○	PBT	○
MIR	○	PEC	○	PAM	●
MCS	○	PSO	○	PLS	○
MBT	○	PIR	○		

2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.
3. Please darken the whole Circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	~~ⓐ~~ ⓑ ⓒ ⓓ	~~ⓐ~~ ⓑ ⓒ ~~ⓓ~~	⊙ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **One-fourth (1/4) of marks assigned to any question will be deducted for wrong answers in multiple choice questions.**
10. Write your six-digit Roll Number in small boxes provided for the purpose; and also darken appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

Example :

ROLL NUMBER

1	3	5	7	2	0
●	①	①	①	①	①
②	②	②	②	●	②
③	●	③	③	③	③
④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●

PART—A

1. The general solution to the PDE $\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\,\partial y}+\frac{\partial^2 u}{\partial y^2}=0$, is

(a) $u(x, y)=f(y+x)+g(2y+x)$

(b) $u(x, y)=f(y+3x)+g(y-3x)$

(c) $u(x, y)=f(y+x)+g(y-x)+F(y+ix)+G(y-ix)$

(d) $u(x, y)=f(y-x)+xg(y-x)$

2. If $U_k(x, y)$, $k=1(1)n$ are solutions of the homogeneous linear partial differential equations $f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right)=0$, then which of the following is also a solution?

(a) $\sum_{k=1}^{n} a_k U_k$

(b) $\sum_{k=1}^{n} a_k U_k^2$

(c) $\sum_{k=1}^{n} a_k U_k^3$

(d) $\sum_{k=1}^{n} a_k U_k^4$

3. The one-dimensional diffusion equation $\frac{\partial^2 U}{\partial x^2}=\frac{1}{k}\frac{\partial U}{\partial t}$, where $U(x, t)\to 0$, as $t\to\infty$, admits the solution

(a) $u(x, t)=\sum_{n=0}^{\infty} C_n \sin(nx+\varepsilon_n)e^{-n^2kt}$

(b) $u(x, t)=\sum_{n=0}^{\infty} C_n \cos(nx+\varepsilon_n)e^{-n^2kt}$

(c) $u(x, t)=\sum_{n=0}^{\infty} C_n \exp(nx+\varepsilon_n)e^{-n^2kt}$

(d) $u(x, t)=\sum_{n=0}^{\infty} C_n \cosh(nx+\varepsilon_n)e^{-n^2kt}$

4. Let y be the solution of the initial value problem $\frac{d^2y}{dx^2}+y=\cos(2x)$, $y(0)=1$, $y'(0)=-2$. Let the Laplace transform of y be $F(s)$. Then the value of $F(1)$ is

(a) 10/3

(b) 2/5

(c) −2/5

(d) 3/10

5. Which of the following represents two-dimensional Laplacian operator in polar coordinate?

(a) $\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}$

(b) $\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$

(c) $\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$

(d) $\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$

6. The numerical value obtained by applying the seven-point trapezoidal rule to the integral $\int_0^6 \frac{1}{1+x^2}\,dx$ is

(a) 1·4108

(b) 3·4108

(c) 6·4108

(d) 8·4108

7. Using the Jacobi iteration method with the initial guess $\{x_1^{(0)}=x_2^{(0)}=x_3^{(0)}=0\}$, the second approximation $\{x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\}$ for the solution of the system of equations

$$20x_1+x_2-2x_3=17$$
$$3x_1+20x_2-x_3=-18$$
$$2x_1-3x_2+20x_3=25$$

is

(a) $x_1^{(2)}=9\cdot02,\ x_2^{(2)}=-0\cdot965,\ x_3^{(2)}=1\cdot1515$

(b) $x_1^{(2)}=1\cdot02,\ x_2^{(2)}=-0\cdot965,\ x_3^{(2)}=1\cdot1515$

(c) $x_1^{(2)}=1\cdot02,\ x_2^{(2)}=0\cdot965,\ x_3^{(2)}=1\cdot1515$

(d) $x_1^{(2)}=1\cdot02,\ x_2^{(2)}=-0\cdot965,\ x_3^{(2)}=4\cdot1515$

8. The fourth-order Runge-Kutta method given by

$$y_{k+1} = y_k + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

is used to solve the initial value problem $\frac{dy}{dx} = x + y$, $y(0) = 1$. If $y(0\cdot 2) = 1\cdot 2428$ is obtained by taking the step size $h = 0\cdot 2$, then the value of K_2 is

(a) 8·41

(b) 3·41

(c) 5·24

(d) 0·24

9. Using Euler's method taking the step size as 0·1, the approximate value of y obtained corresponding to $x = 0\cdot 2$ for the initial value problem $\frac{dy}{dx} = x + y$, $y(0) = 1$, is

(a) 0·85

(b) 5·89

(c) 3·87

(d) 1·36

10. Three groups of children contain respectively

3 girls, 1 boy; 2 girls, 2 boys; 1girl, 3 boys

One child is selected at random from each group. The chance that the three selected consist of 1 girl and 2 boys is

(a) 1/4

(b) 9/32

(c) 11/32

(d) 13/32

11. If a random variable X has a Poisson distribution such that $p(1) = p(2)$, then the mean of the distribution is

(a) 2

(b) 3/2

(c) 1

(d) 1/2

12. Addition of a new constraint to an LPP can affect

(a) feasibility condition only

(b) optimality condition only

(c) feasibility and optimality conditions both

(d) neither feasibility nor optimality condition

13. Let in simplex method for an LPP, the variable x_j leave the basis in some iteration. Then

(a) x_j can enter the basis in the next iteration

(b) x_j cannot enter the basis in the next iteration

(c) Both the above cases are possible depending upon the LPP

(d) None of the above

14. Let A be the set of all rational numbers with discrete metric d. Then which of the following is not true in the metric space (A, d)?

(a) Every subset of A is open

(b) Every subset of A is closed

(c) (A, d) is incomplete

(d) (A, d) is disconnected

15. Which one of the following is not true?

(a) The sum of two Lebesgue measurable functions is measurable

(b) The product of two Lebesgue measurable functions is measurable

(c) The composite of two Lebesgue measurable functions is measurable

(d) Maximum of two Lebesgue measurable functions is measurable.

16. If X^{**} denotes the second dual space of a normed linear space X, then which of the following is not true?

(a) $(\mathbb{C}^n)^{**} = \mathbb{C}^n$

(b) $(l_1)^{**} = l_1$

(c) $(l_2)^{**} = l_2$

(d) $(l_3)^{**} = l_3$

(The notations stand their usual meanings.)

17. Let f be defined on the set $\mathbb{C}$ of complex numbers as

$$f(z) = \begin{cases} \dfrac{x^3 y(y - ix)}{x^6 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Then f does not satisfy which of the following?

(a) f is analytic at $z = 0$

(b) f is continuous at $z = 0$

(c) Cauchy-Riemann equations are satisfied at $z = 0$

(d) $\dfrac{f(z) - f(0)}{z} \to 0$ as $z \to 0$ along x-axis or y-axis

18. The value of the integral

$$\frac{1}{2\pi i}\int_C \frac{8z}{(z-1)(z+3)(z-5)^2}\,dz, \text{ where } C = \{z : |z| = 2\}$$

equals

(a) 1/8

(b) 1/4

(c) 1/2

(d) 1/32

19. If $< a_n >$ is a sequence of real numbers defined as

$$a_1 = 1$$
$$a_{n+1} = \sqrt{6 + a_n}, \; n = 1, 2, \dots$$

then

(a) $< a_n >$ converges to 2

(b) $< a_n >$ converges to 3

(c) $< a_n >$ converges to 6

(d) $< a_n >$ is oscillatory and not convergent

20. For a similar matrix A and B

(a) A, B have same characteristic polynomial

(b) A, B may have different characteristic polynomials

(c) A, B have same eigenvalue

(d) A, B have different eigenvalues

21. The homomorphisms from Z_{20} onto Z_{10} are

(a) 4

(b) 20

(c) 8

(d) 10

22. The alternating group A_4 on 4 symbols has a normal subgroup of order

(a) 2

(b) 3

(c) 4

(d) 6

23. The incorrect statement for a non-Abelian group G of order p^3, where p is prime, is

(a) $Z(G)$; the center of the group G has p elements

(b) G'; the commutator subgroup of G has p elements

(c) $Z(G) = G'$

(d) None of the above

24. The elements of order 5 in S_7 are

(a) 120

(b) 21

(c) 504

(d) 24

25. The order of 2-Sylow subgroup A_4 is

(a) 4

(b) 6

(c) 2

(d) None of the above

26. If G is a group of order 91, then which statement is false?

(a) G has one subgroup of order 7

(b) G has two subgroups of order 13

(c) G has subgroups of order 7 and 13

(d) None of the above

27. The $n \times n$ matrix P is idempotent if $P^2 = P$ and orthogonal if $P'P = I$. Which of the following is false?

(a) If P and Q are orthogonal $n \times n$ matrices, then PQ is orthogonal

(b) If P and Q are idempotent $n \times n$ matrices and $PQ = QP = O$, then $P + Q$ is idempotent

(c) If P is idempotent, then $-P$ is idempotent

(d) $P = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$ is orthogonal

28. Which of the following is a parabolic partial differential equation?

(a) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial x}$

(b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(c) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial x}$

(d) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$

29. Let $U(x, t)$ be a function defined for $a \le x \le b$, $t > 0$. Then which of the following is Laplace transformation of $\frac{\partial U}{\partial t}$?

(a) $sL\{U(x, t)\} - U(x, 0)$

(b) $sL\{U(x, t)\} - U(x, \infty)$

(c) $sL\{U(x, t)\}$

(d) $sL\{U(x, t)\} + U(x, 0)$

30. The solution to the linear PDE $\frac{\partial^4 u}{\partial x^4} - \frac{\partial^4 u}{\partial y^4} = 0$, is

(a) $u(x, y) = f(y + x) + g(y + ix)$

(b) $u(x, y) = f(y + 3x) + g(y - 3x)$

(c) $u(x, y) = f(y + x) + g(y - x) + F(y + ix) + G(y - ix)$

(d) $u(x, y) = f(y + x) + g(y + 2ix)$

PART—B

31. The differential equation of the family of trajectories of the family of curve given by $F\left(x, y, \frac{dy}{dx}\right) = 0$, is

(a) $F\left(x, y, -\frac{dy}{dx}\right) = 0$

(b) $F\left(x, y, \frac{dx}{dy}\right) = 0$

(c) $F\left(x, y, \frac{dy}{dx}\right) = 0$

(d) $F\left(x, y, -\frac{dx}{dy}\right) = 0$

32. Which of the following satisfies the initial value problem $\frac{dy}{dx} = (4x + y + 1)^2$, $y(0) = 1$?

(a) $4x - y + 1 = 2\tan(2x + \pi/4)$

(b) $4x + y + 1 = \tan(x + \pi/4)$

(c) $x - y + 1 = 2\tan(2x + \pi/4)$

(d) $4x + y + 1 = 2\tan(2x + \pi/4)$

33. The characteristic of the first-order equation $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x$ is given by the curve

(a) $y = ce^{-x}$

(b) $y = ce^{x}$

(c) $y = \sin(x)$

(d) $y = \cosh(x)$

34. The solution of the equation $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$ is

(a) $x^2 + 2y^2 + z^2 = f(lx + 5my + nz)$

(b) $x^2 + y^2 z^2 = f(lx + my + nz)$

(c) $x^2 - y^2 - z^2 = f(lx + my + nz)$

(d) $x^2 + y^2 + z^2 = f(lx + my + nz)$

35. Which of the following defines the central difference operators?

(a) $\delta y(x) = y\left(x+\frac{h}{2}\right) y\left(x+\frac{3h}{2}\right)$

(b) $\delta y(x) = y\left(x+\frac{h}{2}\right) y\left(x-\frac{h}{2}\right)$

(c) $\delta y(x) = y\left(x+\frac{h}{2}\right) - y\left(x-\frac{h}{2}\right)$

(d) $\delta y(x) = y\left(x+\frac{h}{2}\right) + y\left(x-\frac{h}{2}\right)$

36. The Newton's scheme of iteration for finding the square root of a positive integer N is

(a) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$

(b) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$

(c) $x_{n+1} = \frac{1}{2}\left(x_{n+1} + \frac{N}{x_n}\right)$

(d) $x_{n+1} = \frac{1}{2}\left(x_{n-1} + \frac{N}{x_n}\right)$

37. A relation between the differences of an unknown function at one or more general values of the argument is known as

(a) partial differential equation

(b) linear integral equation

(c) differential equation

(d) difference equation

38. The Runge-Kutta method of second-order is same as

(a) Euler method

(b) modified Euler method

(c) Taylor method

(d) fourth-order Runge-Kutta method

39. Which of the following is true for the solution of $AX = b$, where

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} ?$$

(a) Jacobi method converges

(b) Jacobi method diverges

(c) Gauss-Seidel method converges

(d) Both Jacobi method and Gauss-Seidel method converge

40. The contents of urns I, II and III are as follows :

Urn I : 1 white, 2 black and 3 red balls
Urn II : 2 white, 1 black and 1 red balls
Urn III : 4 white, 5 black and 3 red balls

One urn is chosen at random and two balls drawn. They happen to be white and red. The probability that they come from urn III is

(a) 55/118

(b) 33/118

(c) 15/59

(d) 13/59

41. Suppose X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} kx, & 0 \le x < 2 \\ 2k, & 2 \le x < 4 \\ -kx + 6k, & 4 \le x < 6 \end{cases}$$

Then the mean value of X is

(a) 4

(b) 3

(c) 2

(d) 1

42. Let

$$f(x,\ y) = \begin{cases} xe^{-x(1+y)}, & x \geq 0,\ y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

be the probability distribution function of the random variables X and Y. Then which one of the following is true?

(a) $E(XY)$ does not exist

(b) $E(Y/X)$ does not exist

(c) $E(Y)$ does not exist

(d) All the three $E(XY)$, $E(Y/X)$, $E(Y)$ exist

43. Let $P(Y = \pm 1) = \frac{1}{2}$ and define the sequence $< X_n >$ by

$$X_n = \begin{cases} Y, & \text{if } n \text{ is odd} \\ -Y, & \text{if } n \text{ is even} \end{cases}$$

Then the sequence $< X_n >$

(a) converges in distribution and not in probability

(b) converges in probability but not in distribution

(c) neither converges in probability nor in distribution

(d) converges both in probability and distribution

44. Consider the following LPP :

Maximize $Z = x_1 + 3x_2$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 2 \\ -x_1 + x_2 &\leq 4 \end{aligned}$$

x_1 is unrestricted and $x_2 \geq 0$.

Then which of the following is the best basic feasible objective function value?

(a) 8

(b) 11

(c) 6

(d) None of the above

45. The optimal value of the objective function for the following LPP

Minimize $Z = 10x_1 + 4x_2 + 5x_3$

subject to

$$5x_1 - 7x_2 + 3x_3 \geq 50$$
$$x_1, x_2, x_3 \geq 0$$

is

(a) 50

(b) 250/3

(c) 270/4

(d) None of the above

46. Let the optimal basic solution to the primal be degenerate. Then the dual problem has

(a) alternative optimal solutions

(b) no solution

(c) an unbounded solution

(d) None of the above

47. In a maximization LPP, the variable corresponding to minimum ratio with solution column leaves the basis. This ensures

(a) largest rise in the objective function

(b) that the next solution will be a BFS

(c) that the next solution will not be unbounded

(d) None of the above

48. In two-phase simplex method, original problem may be maximization or minimization problem but the phase–I problem

(a) is always a minimization problem

(b) is always a maximization problem

(c) may be minimization or maximization problem

(d) None of the above

49. Let $\mathbb{R}$ be the set of real numbers and let $\Im$ be the usual topology on $\mathbb{R}$. Let f be a function defined from $(\mathbb{R}, \Im) \to (\mathbb{R}, \Im)$ as $f(x) = 1$ for all x. Then f satisfies which of the following?

(a) f is continuous and closed but not open

(b) f is continuous and open but not closed

(c) f is open and closed but not continuous

(d) f is continuous, closed and open

50. Which of the following is not true in the context of topological spaces?

(a) Continuous image of a compact set is compact

(b) Continuous image of a bounded set is bounded

(c) Continuous image of a connected set is connected

(d) Continuous image of a sequentially compact set is sequentially compact

51. Let $< f_n >$ be a sequence of functions defined on the set $\mathbb{R}$ of real numbers as :

$$f_n(x) = \begin{cases} nx(1-x^2)^n, & 0 < x < 1, \quad n = 1, 2, 3, \ldots \\ 0, & \text{otherwise}, \quad n = 1, 2, 3, \ldots \end{cases}$$

Then which of the following is not true?

(a) Each f_n is Lebesgue integrable

(b) $f_n(x) \to f(x)$ for all x and f is bounded measurable

(c) f is Lebesgue integrable

(d) $\lim \int f_n = \int f$

52. Let $H = C[0, 1]$ be the space of all complex-valued continuous functions with mapping

$$H \times H \to \mathbb{C} \text{ as } (f, g) = \int_0^1 f(x)\overline{g(x)}\,dx$$

Then which of the following is not true?

(a) H is an inner product space

(b) H is a normed linear space with $\|f\| = (\int |f|^2)^{1/2}$

(c) H is complete

(d) The norm defined in (b) satisfies parallelogram law

53. Suppose f is an entire function and $|f(z)| < a + b|z|^{1/2}$ for all z. Then which one of the following is true?

(a) f is a constant function

(b) f can be a polynomial of degree 1

(c) f can be a polynomial of degree 2

(d) f can be a polynomial of degree 3

54. If

$$f(z) = \begin{cases} \dfrac{1-\cos z}{z^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

then which one of the following is true?

(a) $z = 0$ is a pole of order 1

(b) $z = 0$ is a pole of order 2

(c) $z = 0$ is a removable singularity

(d) $z = 0$ is an essential singularity

55. Let

$$f(x) = \begin{cases} \dfrac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \leq x < 0 \\ \dfrac{2x+1}{x-2}, & 0 \leq x \leq 1 \end{cases}$$

If $f(x)$ is continuous in the interval $[-1, 1]$, then p equals

(a) 1/2

(b) −1/2

(c) 1

(d) −1

56. Let

$$f(x) = \begin{cases} x^3 \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Then f satisfies which of the following?

(a) f is differentiable nowhere

(b) f is differentiable everywhere except at $x = 0$

(c) f is differentiable once at $x = 0$ but not twice differentiable at $x = 0$

(d) f is twice differentiable at $x = 0$ but not thrice differentiable at $x = 0$

57. A real quadratic form $X^T AX$ is positive semidefinite, if

(a) all eigenvalues of $A \geq 0$

(b) all eigenvalues of $A \leq 0$

(c) all eigenvalues of $A = 0$

(d) None of the above

58. The following vectors

$$(1/4,\ 0,\ -1/4),\ (1/3,\ -1/3,\ 0) \text{ and } (0,\ 1/2,\ -1/2)$$

are

(a) linearly independent

(b) linearly dependent

(c) constants

(d) None of the above

59. Let T be a linear operator on a finite dimensional vector space V and c is any scalar. Then c is a characteristic value of T, if

(a) the operator $(T-cI)$ is singular

(b) the operator $(T-cI)$ is non-singular

(c) the operator $(T-cI)$ is identity

(d) the operator $(T-cI)$ is zero

60. If n is the order of element a of group G, then $a^m = e$ (an identity element) if and only if

(a) m/n

(b) n/m

(c) $n = m$

(d) n does not divide m

61. Consider the following statements :

Statement A : All cyclic groups are Abelian.

Statement B : The order of a cyclic group is same as the order of its generator.

Choose the correct option.

(a) A and B are false

(b) A is true, B is false

(c) B is true, A is false

(d) A and B are true

62. Consider the following statements :

Statement A : Every isomorphic image of a cyclic group is cyclic.

Statement B : Every quotient group of a cyclic group is cyclic.

Choose the correct option.

(a) Both A and B are false

(b) Only A is true

(c) Only B is true

(d) Both A and B are true

63. For a group G of order 15, the number of 3-Sylow subgroups of G is

(a) 0

(b) 1

(c) 3

(d) 5

64. Let R is a commutative ring with unity and only ideals are $\{0\}$ and R. Then

(a) R is finite integral domain

(b) R is integral domain

(c) R is division ring

(d) R is a field

65. The homomorphism f from ring R onto ring R' is an isomorphism if and only if kernel of f is

(a) $\{0\}$

(b) R

(c) R'

(d) None of the above

66. Which of the following statements is false?

(a) $F[x]$ is an integral domain

(b) $F[x]$ is a Euclidean ring

(c) $F[x]$ is a principal ideal ring

(d) $F[x]$ is not a group

67. The solution of the linear ordinary differential equation $y'' - 3y' + 2y = e^x$ is

(a) $a_1 e^x + a_2 e^{2x} - xe^x$

(b) $a_1 e^x + a_2 e^{-2x} - xe^x$

(c) $a_1 e^{-x} + a_2 e^{2x} - xe^x$

(d) $a_1 e^x + a_2 e^{2x} + xe^x$

68. The solution of the initial value problem $\frac{dy}{dx} = e^{x+y}$, $y(1) = 1$ at $x = -1$ is

(a) 2

(b) 0

(c) -1

(d) 1

69. The general solution of the differential equation $2yzdx + zxdy - xy(1+z)dz = 0$ is

(a) $x^2 y = cze^z$

(b) $x^2 y^2 = cze^z$

(c) $xy = ce^z$

(d) $x^2 y = cz$

70. Which of the following is true?

(a) All eigenvalues of Sturm-Liouville problem are zero

(b) Eigenvalues of Sturm-Liouville problem are imaginary

(c) All eigenvalues of Sturm-Liouville problem are real

(d) All eigenvalues of Sturm-Liouville problem are unity

★ ★ ★

50005

QUESTION PAPER SERIES CODE
A

Test Centre : ________________

Roll No. : ________________

Name of the Candidate : ________________

S A U

Entrance Test for M.Phil./Ph.D. (Applied Mathematics), 2016

[PROGRAMME CODE : PAM]

Question Paper

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (Objective-type) has 30 questions of **1** mark each. All questions are compulsory. Part—B (Objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(iv) **A wrong answer will lead to deduction of one-fourth of the marks assigned to that questions.**

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(vii) All questions should be answered on the OMR sheet.

(viii) Answers written inside the Question Paper will **NOT** be evaluated.

(ix) **Calculators and Log Tables may be used. Mobile Phones are NOT allowed.**

(x) Three pages at the end of the Question Paper have been provided for Rough Work.

(xi) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR SHEET.**

/14-A

INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Example :

   Question Paper Series Code

   Write Question Paper Series Code A or B in the box and darken appropriate circle.

   ☐ A or B

   ●
   Ⓑ

   Programme Code

   Write Programme Code in the box and darken appropriate circle.

   Write Programme Code ☐

MEC	○	MAM	○	PCS	○
MSO	○	MLS	○	PBT	○
MIR	○	PEC	○	PAM	●
MCS	○	PSO	○	PLS	○
MBT	○	PIR	○		

2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.
3. Please darken the whole Circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

   Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	⊗ ⓑ ⓒ ⓓ	⊗ ⓑ ⓒ ⊗	⊙ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth (¼) of the marks assigned to that question.**
10. Write your six-digit Roll Number in small boxes provided for the purpose; and also darken appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    Example :

    ROLL NUMBER

1	3	5	7	2	0
●	①	①	①	①	①
②	②	②	②	●	②
③	●	③	③	③	③
④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●

PART—A

1. The sequence $< a_n >$ defined by $a_n = \sum_{j=1}^{n} \frac{(-1)^j 2}{n}$

(a) is oscillatory

(b) diverges to $+\infty$

(c) diverges to $-\infty$

(d) converges to 0

2. The value of the integral

$$\int_C \frac{(\sin^2 z + 2)(\cos z + 1)}{z(z+6)} dz$$

where, z is complex and $C = \{z : |z| = 1\}$ equals

(a) $\pi i / 3$

(b) $2\pi i / 3$

(c) $4\pi i / 3$

(d) 0

3. If $\mathbb{R}$ denotes the set of real numbers, $\mathbb{T}$ the cofinite topology on $\mathbb{R}$ and $\mathbb{U}$ the usual topology on $\mathbb{R}$ and if $f : (\mathbb{R}, \mathbb{T}) \to (\mathbb{R}, \mathbb{U})$ and $g : (\mathbb{R}, \mathbb{U}) \to (\mathbb{R}, \mathbb{T})$ be identity maps, then which of the following is true?

(a) f is continuous, g is discontinuous

(b) g is continuous, f is discontinuous

(c) f and g both are continuous

(d) f and g both are discontinuous

4. In which of the following Banach spaces is the parallelogram law satisfied?

(a) l_2

(b) l_1

(c) l_∞

(d) $C[0, 1]$

5. If T is a continuous linear operator on a normed linear space X, then which of the following is ***not*** true?

(a) If $< x_n > \to x$ weakly in X, then $< Tx_n > \to Tx$ weakly in X

(b) If $< x_n > \to x$ strongly in X, then $< Tx_n > \to Tx$ strongly in X

(c) If $< x_n > \to x$ strongly in X, then $< Tx_n > \to Tx$ weakly in X

(d) If $< x_n > \to x$ weakly in X, then $< Tx_n > \to Tx$ strongly in X

6. If f and g be two real-valued functions defined on a closed bounded interval I and if $f = g$ almost everywhere on I, then which one of the following is ***not*** true?

(a) If f is measurable (Lebesgue), then g is also measurable on I

(b) If f is non-measurable, then g is also non-measurable on I

(c) If f is continuous, then g is also continuous on I

(d) If f is integrable, then g is also integrable on I

7. Which one of the following statements is ***not*** true?

(a) Cantor's ternary set has measure (Lebesgue) zero.

(b) The set of irrationals of the form $\sqrt{n}+\sqrt{m}$, (n and m are natural numbers) has measure zero.

(c) The set of algebraic real numbers has measure zero.

(d) The set of transcendental real numbers has measure zero.

8. The dimension of the subspace of $M_{2\times 2}$ spanned by

$$\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix} \text{ and } \begin{pmatrix} 2 & -4 \\ -5 & 7 \end{pmatrix}$$

is

(a) 1

(b) 2

(c) 3

(d) 4

9. If A and B are n-square positive definite matrices, then which of the following is positive definite?

(a) $A+B$

(b) ABA

(c) AB

(d) A^2+I

10. If G is a group of order 91, then which of the following statements is false?

(a) G has one subgroup of order 7

(b) G has two subgroups of order 13

(c) G has subgroups of order 7 and 13

(d) None of the above

11. Let G be a group and let H and K be two subgroups of G. If both H and K have 12 elements, then which of the following numbers cannot be the cardinality of the set

$$HK = \{hk : h \in H, k \in K\}?$$

(a) 72

(b) 60

(c) 48

(d) 36

12. In $U(40)$, the cyclic subgroups of order 4 are

(a) 4

(b) only one

(c) at most equal to order of the group

(d) exactly two

13. If S_n be the symmetric group of n letters, then there exists an onto group homomorphism

(a) from S_5 to S_4

(b) from S_4 to S_2

(c) from S_5 to $\mathbb{Z}/5$

(d) from S_4 to $\mathbb{Z}/4$

14. For a group G, if Aut(G) denotes the group of automorphisms of G, then which of the following statements is true?

(a) Aut $(\mathbb{Z})$ is isomorphic to $\mathbb{Z}_2$

(b) If G is cyclic, then Aut (G) is cyclic

(c) If Aut (G) is trivial, then G is trivial

(d) Aut $(\mathbb{Z})$ is isomorphic to $\mathbb{Z}$

15. The general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 10e^x \cos x$$

is

(a) $e^{-2x}(k_1 \cos 2x + k_2 \sin 2x) - e^x(2\cos x - \sin x)$

(b) $e^{-2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x + \sin x)$

(c) $e^{2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x - \sin x)$

(d) $e^{2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x + \sin x)$

16. The region in which the equation $xu_{xx} + u_{yy} = x^2$ is hyperbolic is

(a) the half plane $x < 0$

(b) the whole plane $\mathbb{R}^2$

(c) the half plane $y > 0$

(d) the half plane $x > 0$

17. The general solution of the differential equation $(6x^2 - e^{-y^2})dx + 2xye^{-y^2}dy = 0$ is

(a) $x(2x^2 + e^{-y^2}) = c$

(b) $x(2x^2 - e^{-y^2}) = c$

(c) $x^2(2x + e^{-y^2}) = c$

(d) $x^2(2x - e^{-y^2}) = c$

18. The solution of the initial value problem

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0, \ y(0) = -3, \ y'(0) = -1$$

is

(a) $y = e^{3x}(2\sin 4x - 3\cos 4x)$

(b) $y = e^{3x}(2\cos 2x + 3\sin 2x)$

(c) $y = e^{-3x}(2\sin 2x - 3\cos 2x)$

(d) $y = e^{-3x}(2\sin 4x + 3\cos 4x)$

19. The general integral of the partial differential equation $z(xp - yq) = y^2 - x^2$ is

(a) $z^2 = x^2 + y^2 + f(xy)$

(b) $z^2 = x^2 - y^2 + f(xy)$

(c) $z^2 = y^2 - x^2 + f(xy)$

(d) $z^2 = -x^2 - y^2 + f(xy)$

20. The general solution of

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$$

is

(a) $c_1 + c_2x + c_3x^2e^{-x}$

(b) $c_1 + c_2x + c_3x^2e^{x}$

(c) $(c_1 + c_2x + c_3x^2)e^{-x}$

(d) $(c_1 + c_2x + c_3x^2)e^{x}$

21. For the equation

$$x^2(x-2)\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} - 3xy = 0$$

consider the following statements :

P : $x = 0$ is a regular singular point

Q : $x = 2$ is a regular singular point

Then

(a) P is false but Q is true

(b) P is true but Q is false

(c) both P and Q are true

(d) both P and Q are false

22. If Δ and ∇ are the forward and the backward difference operators respectively, then $\nabla - \Delta$ is equal to

(a) $\frac{\Delta}{\nabla}$

(b) $\Delta\nabla$

(c) $-\Delta\nabla$

(d) $\Delta + \nabla$

23. The maximum step size h is such that the error in linear interpolation for the function $y = \sin(x)$ in $[0, \pi]$ is less than 5×10^{-5} is

(a) 0·02

(b) 0·002

(c) 0·04

(d) 0·06

24. If $|\text{error}| < 10^{-4}$, then the number of iterations for finding root of $\cos(x) - e^{-x} = 0$ by bisection method in interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is

(a) at least 10

(b) at least 14

(c) at least 17

(d) None of the above

25. A problem in statistics is given to three students whose chances of solving it are 1/2, 1/3 and 1/4 respectively. The probability that the problem will be solved is

(a) 1/8

(b) 1/24

(c) 3/4

(d) 3/8

26. The density function of a random variable x is given by $f(x) = kx(2-x)$, $0 \le x \le 2$. Its mean is given by

(a) 1

(b) 3/4

(c) 1/2

(d) 1/4

27. The value of the moment generating function $M_0(t)$ of the exponential distribution $f(x) = 8e^{-8x}$, $0 \le x < \infty$ at $t = 3$ is given by

(a) 5/8

(b) 1/2

(c) 8/5

(d) 2

28. The solution set which satisfies the non-negativity constraint is classified as

(a) basic feasible solution

(b) feasible value solution

(c) basic solution

(d) positive-negative value

29. The primal problem has unbounded solution if

(a) dual has bounded solution

(b) dual has unbounded solution

(c) dual has no feasible solution

(d) dual has feasible solution

30. In a maximization LPP, the variable corresponding to the minimum ratio with solution column leaves the basis. This ensures

(a) the largest rise in the objective function

(b) that the next solution will be a BFS

(c) that the next solution will not be unbounded

(d) None of the above

PART—B

31. Let f be defined as $f(x) = \text{Max}\{[x^2], x\}$, $x \in [0, 2]$; where $[y]$ denotes the greatest integer less than or equal to y. Then f satisfies which of the following?

(a) f is continuous at all points in $[0, 2]$ except one point

(b) f is continuous at all points in $[0, 2]$ except two points

(c) f is continuous at all points in $[0, 2]$ except three points

(d) f is continuous at all points in $[0, 2]$

32. Let f be defined as $f(x) = x^3 - 3x + 1$, $x \in [0, 1]$. Then f satisfies which of the following?

(a) $f(x) \neq 0$ for any x in $[0, 1]$

(b) $f(x)$ has exactly one zero in $[0, 1]$

(c) $f(x)$ has exactly two zeros in $[0, 1]$

(d) $f(x)$ has all the three zeros in $[0, 1]$

33. The series

$$1 + \frac{x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \cdots, \; x > 0$$

is

(a) convergent for all $x > 0$

(b) divergent for $x = 1/e^3$

(c) divergent for $x = 1/e^2$

(d) divergent for $x = 1/e$

34. The sequence of functions $< f_n >$ defined on $[0, 1]$ as $f_n(x) = \frac{nx}{1 + n^3 x^2}$, $n = 1, 2, 3, \cdots$ satisfies which of the following?

(a) uniformly convergent over $[0, 1]$

(b) only pointwise but not uniformly convergent over $[0, 1]$

(c) uniformly convergent in $[0, \frac{1}{2}]$ and pointwise only for $\frac{1}{2} < x \leq 1$

(d) pointwise convergent only at $x = 0$

35. The analytic function whose imaginary part is $\frac{x-y}{x^2+y^2}$ is

(a) $\frac{iz}{1+z^2}+c$

(b) $\frac{(1+z)i}{1+z^2}+c$

(c) $\frac{1+i}{z}+c$

(d) $\frac{(e^z-1)i}{1+z}+c$

36. Residue of the function $f(z)=z\cos\left(\frac{1}{z}\right)$ *at* $z=0$ is

(a) $-1/2$

(b) $-1/3$

(c) $-1/4$

(d) 0

37. Which of the following is not true?

(a) The set of rational numbers with the usual relative topology of $\mathbb{R}$ is disconnected

(b) The set of irrational numbers with the usual relative topology of $\mathbb{R}$ is disconnected

(c) The set of real numbers $\mathbb{R}$ with the usual topology is disconnected

(d) The set of real numbers $\mathbb{R}$ with the topology generated by semi-open interval $(a, b]$ is disconnected

38. Which of the following normed linear spaces is not Banach?

(a) The space of all continuous functions on [0, 1] with norm $\|f\|=\int_0^1 |f(x)|\,dx$

(b) The space of all continuous functions on [0, 1] with norm $\|f\|=\sup\{|f(x)|: x\in[0,1]\}$

(c) The space $\mathbf{C}^n=\{(z_1, z_2, \cdots, z_n): z_i \in \mathbf{C}\}$ with norm $\|(z_1, z_2, \cdots, z_n)\|=\sqrt{\sum_{i=1}^{n}|z_i|^2}$

(d) The space of all Lebesgue integral functions on [0, 1] with norm $\|f\|=\int_0^1|f(t)|\,dt$

39. Let T be a bounded linear operator defined on $\mathbb{C}^2$ as $T(1, 0) = (0, 1)$ and $T(0, 1) = (1, 0)$. Then $\sigma(T)$, the spectrum of T equals

(a) $\{0, 1\}$

(b) $\{0, -1\}$

(c) $\{1, -1\}$

(d) $\{1\}$

40. Let $< f_n >$ be a sequence of functions defined on $[0, 1]$ as

$$f_n(x) = \begin{cases} 0 \ , & 0 \le x < \dfrac{1}{2n} \\ 2n \ , & \dfrac{1}{2n} \le x \le \dfrac{1}{n} \\ 0 \ , & \dfrac{1}{n} < x \le 1 \end{cases}$$

Then which of the following does not hold?

(a) $f_n \to f$ almost everywhere on $[0, 1]$

(b) Fatou's lemma holds

(c) dominated convergence theorem cannot be applied

(d) $\int_0^1 \lim_{n\to\infty} f_n(x)\,dx = \lim_{n\to\infty} \int_0^1 f_n(x)\,dx$

41. Let A and B be $n \times n$ real matrices such that $AB = BA = O$ and $A + B$ is invertible. Then which of the following may not be true?

(a) rank (A) = rank (B)

(b) rank (A) + rank (B) = n

(c) nullity (A) + nullity (B) = n

(d) $A - B$ is invertible

42. Let $T : R^3 \to R^3$ such that $T(a, b, c) = (0, a, b)$, for $(a, b, c) \in R^3$. Then $T + I$ is a zero of the polynomial

(a) t

(b) t^2

(c) t^3

(d) None of the above

43. If A and B are square matrices such that $AB = I$, then zero is an eigenvalue of

(a) A but not of B

(b) B but not of A

(c) both A and B

(d) neither A nor B

44. Let $A = (a_{ij})$ be $n \times n$ complex matrix and A^* denote the conjugate transpose of A. Then which of the following statements is false?

(a) If A is invertible, then $\operatorname{tr}(A^* A) \neq 0$, i.e., trace of $A^* A$ is nonzero

(b) If $\operatorname{tr}(A^* A) \neq 0$, then A is invertible

(c) If $|\operatorname{tr}(A^* A)| < n^2$, then $|a_{ij}| < 1$ for some i, j

(d) If $\operatorname{tr}(A^* A) = 0$, then A is zero matrix

45. If $\sigma : \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4, 5\}$ be a permutation (one-to-one and onto function) such that $\sigma^{-1}(j) \leq \sigma(j)$ for all j, $1 \leq j \leq 5$, then which of the following is false?

(a) $\sigma \circ \sigma(j) = j$ for all j, $1 \leq j \leq 5$

(b) $\sigma^{-1}(j) = \sigma(j)$ for all j, $1 \leq j \leq 5$

(c) The set $\{k : \sigma(k) \neq k\}$ has an even number of elements

(d) The set $\{k : \sigma(k) = k\}$ has an odd number of elements

46. Let G be a group. Suppose $|G| = p^2 q$, where p and q are distinct prime numbers satisfying

$$q \neq 1 \bmod p$$

Then which of the following is true?

(a) G has more than one p-Sylow subgroup

(b) G has a normal p-Sylow subgroup

(c) The number of q-Sylow subgroups of G is divisible by pd

(d) G has a unique q-Sylow subgroup

47. If $C([0,\ 1])$ be the ring of all real-valued continuous functions on $[0,\ 1]$, which of the following statements is true?

(a) $C([0,\ 1])$ is an integral domain

(b) The set of all functions vanishing at 0 is a maximal ideal

(c) The set of all functions vanishing at both 0 and 1 is a prime ideal

(d) If $f \in C([0,\ 1])$ is such that $(f(x))^n = 0$ for all $x \in [0, 1]$ for some $n > 1$, then $f(x) = 0$ for all $x \in [0,\ 1]$

48. Which of the following polynomials is irreducible over the indicated rings?

(a) $x^5 - 3x^4 + 2x^3 - 5x + 8$ over $\mathbb{R}$

(b) $x^4 + x^2 + 1$ over $\mathbb{Z}/2\mathbb{Z}$

(c) $x^3 + 3x^2 - 6x + 3$ over $\mathbb{Z}$

(d) None of the above

49. Let PID, ED, UFD denote the set of all principal ideal domains, Euclidean domains, unique factorization domains, respectively. Then

(a) $\text{UFD} \subset \text{ED} \subset \text{PID}$

(b) $\text{PID} \subset \text{ED} \subset \text{UFD}$

(c) $\text{ED} \subset \text{PID} \subset \text{UFD}$

(d) $\text{PID} \subset \text{UFD} \subset \text{ED}$

50. The solution of the Cauchy problem for the first-order partial differential equation

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z, \text{ on } D = \{(x,\ y,\ z) : x^2 + y^2 \neq 0, z > 0\}$$

with the initial conditions $x^2 + y^2 = 1, z = 1$ is

(a) $z = x^2 + y^2$

(b) $z = (x^2 + y^2)^2$

(c) $z = (x^2 + y^2)^{\frac{1}{2}}$

(d) $z = (2 - (x^2 + y^2))^{\frac{1}{2}}$

51. The orthogonal trajectories of the system of parabolas $y^2 = 4a(x+a)$ belong to

(a) the system of circles $x^2 + y^2 = a^2$

(b) the system of hyperbolas $xy = a^2$

(c) the system of parabolas $y^2 = 4a(x+a)$

(d) None of the above

52. Applying Charpit's method, the solution of the equation $px + qy = pq$ is

(a) $az = \frac{1}{2}(ax + y) + b$

(b) $az = \frac{1}{2}(ay + x) + b$

(c) $az = \frac{1}{2}(ay + x)^{\frac{1}{2}} + b$

(d) $az = \frac{1}{2}(ax + y)^2 + b$

53. The general solution of the second-order partial differential equation

$$u_{xx} + u_{xy} - 2u_{yy} = (y+1)e^x$$

is given by

(a) $\phi_1(y + x) + \phi_2(y - 2x) + ye^x$

(b) $\phi_1(y + x) + \phi_2(y - 2x) + xe^{-y}$

(c) $\phi_1(y - x) + \phi_2(y + 2x) + ye^{-x}$

(d) $\phi_1(y - x) + \phi_2(y + 2x) + xe^y$

54. The solution of the total differential equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ is given by

(a) $xy + \frac{z^3}{x} = c$

(b) $xz + \frac{y^3}{x} = c$

(c) $yz + \frac{z^3}{y} = c$

(d) $xz + \frac{x^3}{y} = c$

55. The partial differential equation $\frac{\partial^2 u}{\partial y^2} - y\frac{\partial^2 u}{\partial x^2} = 0$ has

(a) no real characteristics for $y > 0$

(b) two families of real characteristic curves for $y < 0$

(c) branches of quadratic curves as characteristics for $y \neq 0$

(d) vertical lines as a family of characteristic curves for $y = 0$

56. The general integral of the partial differential equation

$$(x^2 - y^2 - z^2)\frac{\partial z}{\partial x} + 2xy\frac{\partial z}{\partial y} = 2xz$$

is

(a) $f\left(\frac{y}{z}, \frac{x^2 + y^2 + z^2}{x}\right) = 0$

(b) $f\left(\frac{y}{z}, \frac{x^2 + y^2 + z^2}{z}\right) = 0$

(c) $f\left(\frac{xy}{z}, \frac{y^2}{x^2 + z^2}\right) = 0$

(d) $f\left(\frac{y}{zx}; \frac{x^2}{y^2 + z^2}\right) = 0$

57. If $J_n(x)$ defines the Bessel's function of the first kind and of order n, then which of the following is true?

(a) $\frac{d}{dx}(x^{-n}J_n(x)) = x^{-n}$

(b) $\frac{d}{dx}(x^{-n}J_n(x)) = J_{n+1}(x)$

(c) $\frac{d}{dx}(x^{-n}J_n(x)) = -x^{-n}J_{n+1}(x)$

(d) $\frac{d}{dx}(x^{-n}J_n(x)) = 0$

58. The partial differential equation for the family of surfaces $z = ce^{\omega t}\sin(\omega x)$, where c and ω are arbitrary constants is

(a) $z_{xx} - z_{tt} = 0$

(b) $z_{xx} + z_{tt} = 0$

(c) $z_{xt} + z_{tt} = 0$

(d) $z_{xt} + z_{xx} = 0$

59. The interpolating polynomial for the function satisfying the data

$$f(-1) = -2,\ f(0) = -1,\ f(2) = 7,\ f'(-1) = -3,\ f'(0) = 4,\ f'(2) = 36$$

is

(a) $p(x) = x^5 + 4x^3 + 4x - 1$

(b) $p(x) = x^5 - 4x^3 + 4x - 1$

(c) $p(x) = x^5 - 4x^3 - 4x - 1$

(d) None of the above

60. The second-order Runge-Kutta method with step-size 0·5 applied to the equation

$$\frac{dy}{dx} = 1 + \frac{y}{x},\ y(1) = 1$$

gives the approximate solution

(a) $y(2\cdot 0) = 1\cdot 39$

(b) $y(2\cdot 0) = 2\cdot 39$

(c) $y(2\cdot 0) = 3\cdot 39$

(d) $y(2\cdot 0) = 4\cdot 39$

61. The coefficients in the three-step multistep method

$$y_{i+1} - y_i = h[a_0 f(x_i, y_i) + a_1 f(x_{i-1}, y_{i-1})]$$

for the equation $y'(x) = f(x, y)$ are

(a) $a_0 = 2, a_1 = 3$

(b) $a_0 = 3/2, a_1 = -1/2$

(c) $a_0 = 2/3, a_1 = 3/2$

(d) $a_0 = 1, a_1 = 1$

62. The least square polynomial approximation of degree one for $f(x) = x^3$ on the interval $[0, 1]$ with weight function 1 is

(a) $\frac{9x-2}{5}$

(b) $\frac{2-9x}{5}$

(c) $\frac{2-9x}{10}$

(d) $\frac{9x-2}{10}$

63. A man parks his car among 27 cars in a row not at either end. On his return, he finds that 12 places are still occupied. The probability, that both neighbouring places are empty, is

(a) 12/25

(b) 21/65

(c) 6/13

(d) 13/25

64. A random variable x can assume any positive integral value n with a probability proportional to $\frac{1}{3^n}$. Then $E(x)$ equals

(a) 3/2

(b) 4/3

(c) 5/4

(d) 1

65. A and B alternately throw a pair of dice. The one who throws 9 first wins. The chances of their winning are in the ratio

(a) 6 : 5

(b) 7 : 6

(c) 8 : 7

(d) 9 : 8

66. The frequency distribution of a measurable characteristic varying between 0 and 2 is

$$f(x) = \begin{cases} x^3 & , \ 0 \le x \le 1 \\ (2-x)^3 & , \ 1 \le x \le 2 \end{cases}$$

The mean deviation about the mean is

(a) 1/3

(b) 1/4

(c) 1/5

(d) 1/6

67. Addition of a new constraint to an LPP can affect

(a) feasibility condition only

(b) optimality condition only

(c) feasibility and optimality conditions both

(d) neither feasibility nor optimality conditions

68. If the primal constraint is originally in equation form, then the corresponding dual variable is necessarily

(a) non-negative

(b) positive

(c) unrestricted

(d) None of the above

69. Pick the wrong statement.

(a) The dual of the dual is primal.

(b) An equation in a constraint of a primal problem implies the associated variable in the dual problem to be unrestricted in sign.

(c) If a primal variable is non-negative, the corresponding dual constraint is an equation.

(d) The objective function coefficients in the primal problem become right-hand side of constraints of the dual.

70. Change in availability vector and addition of new constraint, simultaneously to an LPP

(a) may disturb feasibility

(b) may disturb optimality

(c) may disturb both feasibility and optimality

(d) None of the above

Test Centre : ____________________

Roll No. : ____________________

Name of the Candidate : ____________________

S A U

Entrance Test for Ph.D. (Applied Mathematics) 2017

[PROGRAMME CODE : 50005]

Question Paper Series Code : A

QUESTION PAPER

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (objective-type) has 30 questions of **1** mark each. All questions are compulsory. Part—B (objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(iv) **A wrong answer will lead to the deduction of one-fourth (¼) of the marks assigned to that question.**

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(vii) All questions should be answered on the OMR sheet.

(viii) Answers written inside the Question Paper will **NOT** be evaluated.

(ix) **Calculators and Log Tables may be used. Mobile Phones are NOT allowed.**

(x) Pages at the end of the Question Paper have been provided for rough work.

(xi) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR SHEET.**

/14-A

INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Question Paper Series Code

   Write Question Paper Series Code **A** or **B**
   in the box and darken the appropriate circle.

   ☐ **A** or **B**
   ●
   Ⓑ

2. Use only Blue/Black Ballpoint Pen to darken the circle. Do not use Pencil to darken the circle for Final Answer.
3. Please darken the whole circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

   ***Example* :**

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	~~ⓐ~~ ⓑ ⓒ ⓓ	~~ⓐ~~ ⓑ ⓒ ~~ⓓ~~	◉ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is possible.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth of the marks assigned to that question.**
10. Write your six-digit Roll Number in small boxes provided for the purpose; and also darken the appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    ***Example* :**

ROLL NUMBER

1	3	5	7	2	0	2
●	①	①	①	①	①	①
②	②	②	②	●	②	●
③	●	③	③	③	③	③
④	④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●	⓪

PART—A

1. Every Cauchy sequence is convergent on

a. real line

b. complex plane

c. both real line and complex plane

d. None of the above

2. The value of $\lim\limits_{h\to 0}\frac{1}{h}\int_2^{2+h}\sqrt{t^2+2}\,dt$ is

a. 0

b. $\sqrt{6}$

c. $\sqrt{2}-2$

d. $\sqrt{3}$

3. The value of

$$\lim_{h\to 0}\left\{\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n\right\}^{1/n}$$

is

a. e

b. π

c. $1/e$

d. $1/\pi$

4. Let $f(x)=\sum\limits_{n=1}^{\infty}\frac{x^n}{n^2}$ be a function defined on $[-1, 1]$. Then which of the following statements is ***not*** true?

a. f is a continuous function.

b. f converges uniformly.

c. f represents a bounded function.

d. None of the above

5. Let $f(x)=\frac{\alpha x}{x+1}$, $x\neq -1$. Then the value of α for which $f(f(x))=x$ is

a. $\sqrt{2}$

b. $-\sqrt{2}$

c. 1

d. -1

6. Let $f(x)=[x^2-3]$, where [.] denotes the greatest integer function. Then the number of points in the interval (1, 2), where the function is discontinuous, is

a. 4

b. 6

c. 2

d. ∞

7. Which one of the following d is ***not*** a metric on the set $\mathbb{R}$ of real numbers?

a. $d(x, y)=\frac{|x-y|}{1+|x-y|}$

b. $d(x, y)=\min(1, |x-y|)$

c. $d(x, y)=\max(1, |x-y|)$

d. $d(x, y)=\left|\frac{x}{1+|x|}-\frac{y}{1+|y|}\right|$

8. If $L:\mathbb{R}^2\to\mathbb{R}^2$ be a linear transformation given by

$$L\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

then

a. $\ker(L)=\{0\}$

b. $\ker(L)=\left\{\begin{bmatrix}1\\0\end{bmatrix}\right\}$

c. $\ker(L)=\left\{\begin{bmatrix}0\\1\end{bmatrix}\right\}$

d. $\ker(L)=\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\}$

9. If in ring $\mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} : a, b \in \mathbb{Z}\}$, then which of the following is correct?

a. 2 and $1+\sqrt{5}$ are irreducible and prime

b. 2 and $1+\sqrt{5}$ are irreducible but not prime

c. 2 and $1+\sqrt{5}$ are not irreducible but prime

d. 2 and $1+\sqrt{5}$ are neither irreducible nor prime

10. The rank of the matrix

$$\begin{pmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{pmatrix}$$

is

a. 1

b. 2

c. 3

d. 4

11. Let k be the sum of all the eigenvalues of a square matrix

$$\begin{pmatrix} -1 & 2 & -2 & 3 \\ 1 & 2 & 1 & 10 \\ -1 & -1 & 0 & 5 \\ 6 & -7 & -8 & 1 \end{pmatrix}$$

Then the value of k is

a. 2

b. 4

c. 6

d. 8

12. If in a group G, $a^3 = e$ and e is the identity of the group G and $aba^{-1} = b^2$ for a, b in G, then the order of b equals

a. 31

b. 23

c. 11

d. 7

13. In the ring $\mathbb{Z}$ of integers, let $A = \{0\}$. Then which of the following is incorrect?

a. A is a maximal ideal of $\mathbb{Z}$ but not a prime ideal of $\mathbb{Z}$

b. A is a prime ideal of $\mathbb{Z}$ but not a maximal ideal of $\mathbb{Z}$

c. A is neither maximal nor prime ideal of $\mathbb{Z}$

d. A is both maximal and prime ideal of $\mathbb{Z}$

14. Which of the following sets is ***not*** a vector sub-space of the vector space of 3×3 matrices over the field $\mathbb{R}$ of real numbers?

a. All upper triangular matrices of order 3

b. All symmetric matrices of order 3

c. All non-singular matrices of order 3

d. All matrices of order 3, the sum of whose diagonal elements is zero

15. The ordinary differential equation

$$\frac{dy}{dx} = \frac{2y}{x}$$

with the initial condition $y(0) = 0$ has

a. no solution

b. a unique solution

c. exactly two solutions

d. infinitely many solutions

16. For the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$$

the characteristic coordinates are given by

a. $\xi = x + 2t, \eta = x - 2t$

b. $\xi = x + 4t, \eta = x - 4t$

c. $\xi = x + 16t, \eta = x - 16t$

d. $\xi = x + 256t, \eta = x - 256t$

17. A homogenous linear differential equation with real constant coefficients, which has $y = xe^{-3x}\cos 2x + e^{-3x}\sin 2x$ as one of its solutions, is given by

a. $(D^2 - 6D + 13)\,y = 0$

b. $(D^2 + 6D + 13)\,y = 0$

c. $(D^2 - 6D + 13)^2\,y = 0$

d. $(D^2 + 6D + 13)^2\,y = 0$

18. The orthogonal trajectory to the family of circles $x^2 + y^2 = 2cx$ (c arbitrary) is described by the differential equation

a. $(y^2 - x^2)\,y' = xy$

b. $(y^2 - x^2)\,y' = 2xy$

c. $(x^2 - y^2)\,y' = 2xy$

d. $(x^2 + y^2)\,y' = 2xy$

19. The partial differential equation $y^3 u_{xx} - (x^2 + 1) u_{yy} = 0$ is

a. hyperbolic in $\{(x, y) : y > 0\}$

b. parabolic in $\{(x, y) : y < 0\}$

c. parabolic in $\{(x, y) : y > 0\}$

d. elliptic in $\mathbb{R}^2$

20. In the solution of the partial differential equation $(D^2 - 6DD' + 9D^2)z = 6x + 2y$, the complementary function is given by

a. $\phi_1(y+3x) + \phi_2(y-3x)$

b. $\phi_1(3y+x) + \phi_2(3y-x)$

c. $\phi_1(y+3x) + x\phi_2(y+3x)$

d. $\phi_1(3y+x) + x\phi_2(3y+x)$

21. If $J_n(x)$ denotes the Bessel's function of first kind of order n, then which one of the following is true?

a. $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$

b. $xJ_n'(x) = nJ_n(x) + xJ_{n+1}(x)$

c. $xJ_n'(x) = nJ_{n-1}(x) - xJ_{n+1}(x)$

d. $xJ_n'(x) = nJ_{n-1}(x) + xJ_{n+1}(x)$

22. The iterative formula to compute the reciprocal of a given positive real number α, using Newton-Raphson method is

a. $x_{n+1} = x_n(2 + \alpha x_n)$

b. $x_{n+1} = x_n^2(2 + \alpha x_n)$

c. $x_{n+1} = x_n^2(2 - \alpha x_n)$

d. $x_{n+1} = x_n(2 - \alpha x_n)$

23. The quadrature formula $\int_{-1}^{1} f(x)\,dx = f(\alpha) + \beta f(1)$ is exact for all polynomials of degree ≤ 1 for

a. $\alpha = 1, \beta = 1$

b. $\alpha = 1, \beta = -1$

c. $\alpha = -1, \beta = 1$

d. $\alpha = -1, \beta = -1$

24. The backward Euler method for solving the differential equation $y' = f(t, y)$ is

a. $y_{n+1} = y_n + hf(t_n, y_n)$

b. $y_{n+1} = y_n + hf(t_{n+1}, y_{n+1})$

c. $y_{n+1} = y_{n-1} + 2hf(t_n, y_n)$

d. $y_{n+1} = y_{n-1} + 2hf(t_n, y_{n+1})$

25. In geometric distribution, the relationship between the mean and variance is

a. variance < mean

b. 3 variance < 2 mean

c. variance = mean

d. variance > mean

26. If the density function of a random variable X is given by $f(x) = kx(2-x)$, $0 \leq x \leq 2$, then variance of the random variable is

a. 1/4

b. 1/5

c. 1/6

d. 1/7

27. A coin is tossed until a head appears. Expectation of the number of tosses is

a. 2

b. 6

c. 4

d. 8

28. A basic solution of a linear programming problem is called degenerate if

a. the value of at least one of the basic variables is non-zero

b. the value of all the basic variables is zero

c. the value of at least one of the basic variables is zero

d. the value of all the basic variables is non-zero

29. Which one of the following statements is ***not*** true about primal and dual?

a. The solution of the primal problem can be interpreted from the last simple table.

b. The decision variables in both primal and dual are positive.

c. If the primal problem is of maximization-type, then the dual will be of minimization-type.

d. The dual of dual is primal.

30. If the primal constraint is originally in equation form, then the corresponding dual variable is necessarily

a. non-negative

b. positive

c. negative

d. unrestricted

PART—B

31. If

$$f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3(3z+2)^2}$$

then which one of the following is true?

a. $z = 1$, $z = -2/3$ and $z = \infty$ are the poles of order 3, 2 and 3 respectively

b. $z = 1$, $z = -2/3$ are the poles of order 3 and 2 respectively

c. $z = \infty$, $z = 1$ and $z = -2/3$ are the poles of order 2, 3 and 2 respectively

d. $z = 1$, $z = -2/3$ and $z = \infty$ are the poles of order 3, 2 and 1 respectively

32. If $\mathbb{R}$ is a metric space with usual metric and $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$, then the boundary of A is the set

a. $\{0\}$

b. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$

c. $\{1, \infty\}$

d. $\{0, \infty\}$

33. Which one of the following is a compact subset of $\mathbb{R}$ but is ***not*** connected?

a. $(0, 1)$

b. $[1, 2)$

c. $[1, 2]$

d. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$

34. Let $d(x, y) = |x - y|$ be the metric defined on the set $\mathbb{N}$ of natural numbers. The open sphere $S_2(3)$ in $\mathbb{N}$ is the set

a. $\{1, 5\}$

b. $\{1, 10\}$

c. $\{2, 3, 4\}$

d. $\{1, 2, 3, 4, 5\}$

35. If $f_n(x) = \frac{x}{1+nx^2}$, and if $f(x) = 0$ for all real x, then which of the following is ***not*** true?

a. $< f_n'(x) >$ is point-wise convergent to some non-differentiable function

b. $f_n'(x) \to f'(x)$ for all x

c. $< f_n >$ converges point-wise to f

d. $< f_n >$ converges uniformly to f

36. The number of roots of $z^7 - 4z^3 + z + 1 = 0$ which lie interior to the unit circle $|z| = 1$ is

a. 3

b. 4

c. 5

d. 7

37. Which one of the following statements is incorrect?

a. Every subspace of T_0 topological space is T_0.

b. Every subspace of T_1 topological space is T_1.

c. Every subspace of normal topological space is normal.

d. Every subspace of Hausdorff topological space is Hausdorff.

38. Which one of the following statements is ***not*** true with reference to Lebesgue measurability?

a. Every continuous function is measurable.

b. Every characteristic function is measurable.

c. Every constant function is measurable.

d. Every monotonic function is measurable.

39. The value of the Lebesgue integral

$$\int_0^\infty \frac{dx}{\left(1+\frac{x}{n}\right)^n x^{\frac{1}{n}}}$$

as $n \to \infty$ equals

a. 1

b. ∞

c. e

d. 0

40. If $L : V \to W$ be a linear transformation with V and W finite dimensional, then which one of the following is correct?

a. If L is one-one, then $\dim(V) \geq \dim(W)$

b. If L is onto, then $\dim(V) \leq \dim(W)$

c. If L is onto, then $\dim(V) = \dim(W)$

d. If L is onto, then $\dim(V) \geq \dim(W)$

41. If $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by $L(1, 1) = (0, 1)$, $L(-1, 1) = (2, 3)$, then the matrix of L with respect to standard basis of $\mathbb{R}^2$ is

a. $\begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$

d. $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

42. In a complex inner product space, which one of the following relations is always satisfied?

a. $(u, z_1v_1 + z_2v_2) = \bar{z}_1(u, v_1) + \bar{z}_2(u, v_2)$

b. $(u, z_1v_1 + z_2v_2) = z_1(u, v_1) + \bar{z}_2(u, v_2)$

c. $(u, z_1v_1 + z_2v_2) = \bar{z}_1(u, v_1) + z_2(u, v_2)$

d. $(u, z_1v_1 + z_2v_2) = z_1(u, v_1) + z_2(u, v_2)$

43. Which one of the following is ***not*** true?

a. The homomorphic image of an Abelian group is Abelian

b. The homomorphic image of a cyclic group is cyclic

c. The homomorphic image of a finite group is finite

d. The homomorphic image of an infinite group is infinite

44. If G be a finite group and simple, and if $O(G) = n$, then n can be which one of the following?

a. 33

b. 32

c. 31

d. 30

45. Let R be a ring with unity. Then R may ***not*** be commutative in which of the following situations?

a. When $(x+y)^2 = x^2 + y^2 + 2xy, \quad \forall x, y \in R$

b. When $x^2 = x, \quad \forall x \in R$

c. When $(xy)^2 = x^2y^2, \quad \forall x, y \in R$

d. When $2x = 0, \quad \forall x \in R$

46. Let V be the set of all 2×2 matrices over $\mathbb{R}$ and let $W_1 = \left\{\begin{pmatrix} x & z \\ 0 & y \end{pmatrix} : x, y, z \in \mathbb{R}\right\}$ and $W_2 = \left\{\begin{pmatrix} x & 0 \\ z & y \end{pmatrix} : x, y, z \in \mathbb{R}\right\}$. Then which of the following is ***not*** true?

a. $W_1 \cap W_2 = \{0\}$

b. V is a vector space over $\mathbb{R}$ and $\dim(V) = 2$

c. W_1 and W_2 are subspaces of V each of dimension 3

d. $V = W_1 + W_2$

47. Which one of the following sets of vectors is linearly dependent over $\mathbb{R}$?

a. $\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$

b. $\{(2, -2, 5), (0, 8, -15), (3, 1, 0)\}$

c. $\{(0, 1, -2), (1, -1, 1), (1, 2, 1)\}$

d. $\{(1, -1, 1), (0, 1, 1), (1, 1, 1)\}$

48. The solution of $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$ is

a. $(x + y + z)e^x = c$

b. $(x + y + z^2)e^x = c$

c. $(x + y + z^2)e^{x^2} = c$

d. $(x + y + z)e^{-x} = c$

49. Let n be a non-negative integer. The eigenvalues of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0$$

with boundary conditions $y(0) = y(\pi) = 0$ is

a. n

b. n^2

c. $n\pi$

d. $n^2\pi^2$

50. For the equation

$$x^3(x-2)\frac{d^2y}{dx^2} + x^3\frac{dy}{dx} + 6y = 0$$

which one of the following is true?

a. $x = 0$ is an ordinary point

b. $x = 2$ is an ordinary point

c. $x = 0$ is a regular singular point

d. $x = 2$ is a regular singular point

51. The region in which the partial differential equation

$$u_{xx} - \sqrt{y}\,u_{xy} + xu_{yy} = \cos(x^2 - 2y),\ y \geq 0$$

is hyperbolic is

a. $\{(x, y) : y > 4x\}$

b. $\mathbb{R}^2$

c. $\{(x, y) : y = 4x\}$

d. $\{(x, y) : y < 4x\}$

52. If $P_n(x)$ is the Legendre polynomial of degree n, then the value of $P_2(x)$ is

a. 1

b. x

c. $\frac{1}{2}(3x^2 - 1)$

d. $\frac{1}{2}(5x^3 - 3x)$

53. The solution of

$$\frac{(y-z)}{yz}\frac{\partial z}{\partial x} + \frac{(z-x)}{zx}\frac{\partial z}{\partial y} = \frac{x-y}{xy}$$

is

a. $x + y + z = f(xyz)$

b. $x^2 + y^2 + z^2 = f(xyz)$

c. $x - y + z = f(x^2 yz)$

d. $x^2 - y^2 + z^2 = f(xyz)$

54. The d'Alembert solution of the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},\ u(x, 0) = \sin x,\ u_t(x, 0) = 0$$

is

a. $u(x, t) = -\sin x \cos ct$

b. $u(x, t) = -\cos x \sin ct$

c. $u(x, t) = \sin x \cos ct$

d. $u(x, t) = \cos x \sin ct$

55. The general solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is

a. $u(x, y) = f(x + y) + g(x - y)$

b. $u(x, y) = f(x + iy) + g(x - iy)$

c. $u(x, y) = e^x (f(x + iy) + g(x - iy))$

d. $u(x, y) = e^{-x} (f(x + iy) + g(x - iy))$

56. Let $g : [0, 1] \to \mathbb{R}$ be a three-times continuously differentiable function and the iterates defined by $x_{n+1} = g(x_n)$, $n \geq 0$ converge to the fixed point ξ of g. If the order of convergence is three, then

a. $g'(\xi) \neq 0$, $g''(\xi) \neq 0$

b. $g'(\xi) \neq 0$, $g''(\xi) = 0$

c. $g'(\xi) = 0$, $g''(\xi) \neq 0$

d. $g'(\xi) = 0$, $g''(\xi) = 0$

57. If $f : [0, 4] \to \mathbb{R}$ be a three-times continuously differentiable function, then the value of the divided difference $f[1, 2, 3, 4]$ is

a. $\frac{f''(\eta)}{3}$ for $\eta \in (0, 4)$

b. $\frac{f''(\eta)}{6}$ for $\eta \in (0, 4)$

c. $\frac{f'''(\eta)}{3}$ for $\eta \in (0, 4)$

d. $\frac{f'''(\eta)}{6}$ for $\eta \in (0, 4)$

58. A quadratic polynomial $f(x)$ is constructed by interpolating the data points $(0, 1)$, $(1, e)$ and $(2, e^2)$. If $\sqrt{e}$ is approximated by using $f(x)$, then its approximate value is

a. $(3-6e-e^2)/8$

b. $(3+6e-e^2)/8$

c. $(3+6e-2e^2)/8$

d. $(3-6e+2e^2)/8$

59. For a sufficiently smooth function $f(x)$, a formula for estimating its derivative is given by

$$f'(x) = \frac{f(x+2h)-f(x-2h)}{4h} + \text{error term}$$

Which of the following expressions is correct for the error term?

a. $-f'(\xi)h$

b. $\frac{-f''(\xi)h^2}{2}$

c. $\frac{-2f'''(\xi)h^2}{3}$

d. $\frac{-f^{(4)}(\xi)h^4}{12}$

60. Using Euler's method and taking the step size as 0·5, the approximate solution corresponding to $x = 2$ for the initial value problem

$$\frac{dy}{dx} = 1 + \frac{y}{x},\ y(1) = 1$$

is

a. 3·167

b. 2·048

c. 4·456

d. 2·218

61. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is six. The probability that it is actually a six is

a. 3/8

b. 1/5

c. 3/4

d. 5/6

62. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $(7^m + 7^n)$ is divisible by 5 is

a. 1/7

b. 1/8

c. 1/4

d. 1/49

63. If (X, Y) is uniformly distributed over the semi-circle bounded by $y = \sqrt{1-x^2}$ and $y = 0$, then the value of $E(X/Y)$ is

a. 1

b. 0

c. $\frac{4}{3\pi}$

d. $\frac{2}{3\pi}$

64. Suppose a random variable X has a binomial distribution $B(6, \frac{1}{2})$. The most likely outcome for X is

a. 6

b. 4

c. 5

d. 3

65. If

$$f(x) = \begin{cases} 0 & , \quad x \le -1 \\ m(x+1) & , \quad -1 < x \le 3 \\ 4m & , \quad 3 < x \le 4 \\ 0 & , \quad x > 4 \end{cases}$$

represents the density function, then the value of x, when the mean deviation of this distribution is the least, is

a. $\sqrt{3}+1$

b. $\sqrt{3}-1$

c. $2\sqrt{3}-1$

d. $2\sqrt{3}+1$

66. Consider the primal problem

Maximize $Z = 4x + 3y$

subject to

$$\begin{aligned} x + y &\le 8 \\ 2x + y &\le 10 \\ x &\ge 0 \\ y &\ge 10 \end{aligned}$$

together with its dual. Then which one of the following statements is correct?

a. Primal and dual both are infeasible

b. Primal and dual both are feasible

c. Primal is feasible but dual is infeasible

d. Primal is infeasible but dual is feasible

67. If $S_1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\}$ and $S_2 = \{(x, y) \in \mathbb{R}^2 : y \le x^2\}$, then which one of the following statements is correct?

a. S_1 is a convex set but S_2 is not a convex set

b. S_2 is a convex set but S_1 is not a convex set

c. Neither S_1 nor S_2 is a convex set

d. S_1 and S_2 are both convex sets

68. If a primal linear programming problem admits an optimal solution, then the corresponding dual problem

a. does not have a feasible solution

b. has a feasible solution but does not have any optimal solution

c. does not have a convex feasible region

d. has an optimal solution

69. For a linear programming problem

Minimize $Z = x - y$

subject to

$$2x + 3y \le 6$$
$$0 \le x \le 3$$
$$0 \le y \le 3$$

which of the following represents the correct number of extreme points of its feasible region and of basic feasible solutions respectively?

a. 3 and 3

b. 4 and 4

c. 3 and 5

d. 4 and 5

70. Consider the linear programming problem

Minimize $Z = -2x - 5y$

subject to

$$3x + 4y \le 5$$
$$x \ge 0$$
$$y \ge 0$$

Then which one of the following statements is correct?

a. Set of feasible solutions is empty.

b. Set of feasible solutions is non-empty but there is no optimal solution.

c. Optimal value is attained at (0, 5/4).

d. Optimal value is attained at (5/3, 0).

Test Centre : ______________________

Roll No. : ______________________

Name of the Candidate : ______________________

S A U

Entrance Test for Ph.D. (Applied Mathematics) 2018

[PROGRAMME CODE : 50005]

Question Paper Series Code : A

QUESTION PAPER

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (objective-type) has 30 questions of **1** mark each. All questions are compulsory. Part—B (objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(iv) **A wrong answer will lead to the deduction of one-fourth (¼) of the marks assigned to that question.**

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(vii) All questions should be answered on the OMR sheet.

(viii) Answers written inside the Question Paper will **NOT** be evaluated.

(ix) **Non-programmable calculators and Log Tables may be used. Mobile Phones are NOT allowed.**

(x) Pages at the end of the Question Paper have been provided for rough work.

(xi) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR SHEET.**

/14-A

INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Question Paper Series Code

   Write Question Paper Series Code **A** or **B** in the box and darken the appropriate circle.

   ☐ **A** or **B**

   ●

   Ⓑ

2. Use only Blue/Black Ballpoint Pen to darken the circle. Do not use Pencil to darken the circle for Final Answer.
3. Please darken the whole circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

   ***Example* :**

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	~~ⓐ~~ ⓑ ⓒ ⓓ	~~ⓐ~~ ⓑ ⓒ ~~ⓓ~~	⊙ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is possible.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth of the marks assigned to that question.**
10. Write your seven-digit Roll Number in small boxes provided for the purpose; and also darken the appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    ***Example* :**

    ROLL NUMBER

1	3	5	7	2	0	2
●	①	①	①	①	①	①
②	②	②	②	●	②	●
③	●	③	③	③	③	③
④	④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●	⓪

PART—A

1. The sequence $\{f_n\}$ of functions is defined by $f_n(x) = 2^n x^n$. In order that the sequence $\{f_n\}$ is uniformly convergent in the interval $[0, k]$

a. $k < 1$

b. $k < 1/2$

c. $k < 3/2$

d. $k < 2$

2. If $\langle x_n \rangle_{n=1}^{\infty}$ is a sequence defined as $x_n = \sum_{m=1}^{n} \frac{1}{m^2}$, then the sequence $\langle x_n \rangle$ ***does not*** satisfy which one of the following statements?

a. $\langle x_n \rangle$ is a Cauchy sequence

b. $\langle x_n \rangle$ is a unbounded sequence

c. $\langle x_n \rangle$ is a monotonically increasing sequence

d. $\langle x_n \rangle$ is a convergent sequence

3. At the point (0, 0), the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

a. is differentiable

b. does not have partial derivatives

c. is continuous but not differentiable

d. is not continuous

4. The series $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$, $x > 0$, ***does not*** satisfy which one of the following?

a. Divergent if $x > 1$

b. Convergent if $x < 1$

c. Divergent if $x = 1$

d. Convergent if $x = 1$

5. Which one of the following sequences $\langle f_n(x) \rangle$, $n = 1, 2, 3, \cdots$, of functions is uniformly convergent over $[0, 1]$?

a. $f_n(x) = \dfrac{nx}{1+n^2x^2}$

b. $f_n(x) = \dfrac{nx}{1+n^3x^3}$

c. $f_n(x) = \dfrac{nx}{1+n^3x^2}$

d. $f_n(x) = \dfrac{nx}{1+n^2x^3}$

6. If $f(z) = \sqrt{|xy|}$, for all $z = x + iy$, a complex number, then which one of the following is ***not*** satisfied?

a. f is continuous at $z = 0$

b. Cauchy-Riemann equations are satisfied at $z = 0$

c. $|f(z)| \leq 1$ for all z satisfying $|z| \leq 1$

d. f is analytic at $z = 0$

7. Which one of the following is a metric on R?

a. $d(x, y) = \min\{1, |x - y|\}$

b. $d(x, y) = |x^2 - y^2|$

c. $d(x, y) = \sin|x - y|$

d. $d(x, y) = |x - y|^2$

8. The system of equations

$$5x + 7y = b_1$$
$$2x + 3y = b_2$$

is consistent for

a. at least one b_1 and b_2

b. all b_1 and b_2

c. no b_1 and b_2

d. exactly one b_1 and b_2

9. Which one of the following is ***not*** true for a square matrix A?

a. The eigenvalues of A and its transpose are the same

b. The sum of the eigenvalues of A is the sum of the elements on its principal diagonal

c. The product of the eigenvalues of A is the product of the elements on its principal diagonal

d. If A is idempotent then its eigenvalues are either 0 or 1

10. If G is a cyclic group of order 20, then the number of generators of G is

a. 7

b. 8

c. 9

d. 10

11. There exists a field having

a. 8 elements

b. 7 elements

c. 6 elements

d. 5 elements

12. In the ring $\mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} \mid a, b \in \mathbb{Z}\}$, which one of the following is correct?

a. 2 and $1+\sqrt{5}$ are irreducible and prime

b. 2 and $1+\sqrt{5}$ are irreducible but not prime

c. 2 and $1+\sqrt{5}$ are not irreducible but prime

d. 2 and $1+\sqrt{5}$ are neither irreducible nor prime

13. If V be the vector space of 5×5 matrices over the field of real numbers, then which one of the following sets is ***not*** a subspace of V?

a. All lower triangular matrices of order 5

b. All nilpotent matrices of order 5

c. All diagonal matrices of order 5

d. All scalar matrices of order 5

14. If the subspace W is defined as $W = \{(a, b, 0) \mid a, b \in R\}$, then

a. $W^{\perp} = \{(x, 0, 0) \mid x \in R\}$

b. $W^{\perp} = \{(0, y, 0) \mid y \in R\}$

c. $W^{\perp} = \{(0, 0, z) \mid z \in R\}$

d. None of the above

15. The solution of the initial value problem

$$\frac{dy}{dx} = 1 + y^2, \; y(0) = 1$$

will be given by

a. $y = \tan\left(x - \frac{\pi}{2}\right)$

b. $y = \tan\left(x + \frac{\pi}{2}\right)$

c. $y = \tan\left(x - \frac{\pi}{4}\right)$

d. $y = \tan\left(x + \frac{\pi}{4}\right)$

16. One particular solution of

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = -e^x$$

is a constant multiple of

a. xe^{-x}

b. xe^{x}

c. x^2e^{-x}

d. x^2e^{x}

17. The general solution of the differential equation

$$(2x+y+1)\,dx+(4x+2y-1)\,dy=0$$

is

a. $\ln(2x+y-1)+x+2y=k$

b. $\ln(2x+y-1)-x-2y=k$

c. $\ln(2x+y-1)-x+y=k$

d. $\ln(2x+y+1)+x-2y=k$

18. The solution of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^3$$

is

a. $c_1x+\frac{c_2}{x}+\frac{x}{2}$

b. $c_1x+\frac{c_2}{x}+\frac{x^2}{4}$

c. $c_1x+\frac{c_2}{x}+\frac{x^3}{8}$

d. $c_1x+\frac{c_2}{x}+\frac{x^3}{16}$

19. If $u(x, t)$ is the solution of the wave equation

$$\frac{\partial^2u}{\partial x^2}=\frac{\partial^2u}{\partial t^2},\ u(x,\ 0)=\cos(5\pi x),\ \frac{\partial u}{\partial t}(x,\ 0)=0$$

then the value of $u(1, 1)$ is

a. -1

b. 0

c. 1

d. 2

20. If P_n denotes Legendre polynomial defined on the interval $[-1, 1]$, then the value of

$$\int_{-1}^{1} (P_n(x))^2 \, dx$$

is

a. 0

b. 1

c. $2n^2$

d. $\frac{2}{2n+1}$

21. The partial differential equation arising from the surface $z = f(x^2 + y^2)$ is

a. $yq - xp = 0$

b. $yq + xp = 0$

c. $yp - xq = 0$

d. $yp + xq = 0$

22. For $f(x) = \frac{1}{x^2}$, the first divided difference $f[a, b]$ with respect to the points a and b is equal to

a. $\frac{a-b}{a^2b^2}$

b. $\frac{a+b}{a^2b^2}$

c. $\frac{-(a+b)}{ab}$

d. $\frac{-(a+b)}{a^2b^2}$

23. The method $x_{n+1} = x_n - \left(\frac{x_n - x_0}{f_n - f_0}\right) f_n$, $n = 1, 2, \cdots$ is used to find a simple root of $f(x) = 0$. The order of convergence of the method is

a. 1
b. 2
c. 3
d. 1·68

24. The value of integral $\int_0^1 \frac{x}{x^2+10} dx$ using Simpson's $\frac{1}{3}$rd rule with $h = 0{\cdot}5$ is

a. $\frac{47}{902}$
b. $\frac{45}{902}$
c. $\frac{43}{902}$
d. $\frac{41}{902}$

25. A bag X contains 2 white and 3 red balls, and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. The probability that it was drawn from Y is

a. 22/52
b. 25/52
c. 27/52
d. 15/52

26. Two numbers a and b are chosen at random from the set of the first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is

a. 9/87
b. 12/87
c. 15/87
d. 47/87

27. If X is a continuous random variable with probability density function given by

$$f(x) = \begin{vmatrix} kx & , & 0 \le x < 2 \\ 2k & , & 2 \le x < 4 \\ 6k - kx, & & 4 \le x < 6 \end{vmatrix}$$

The mean value of X is

a. 3
b. 2
c. 1
d. 4

28. In a maximization linear programming problem the variable corresponding to minimum ratio with solution column leaves the basis. This ensures

a. largest rise in the objective function

b. that the next solution will be a basic feasible solution

c. that the next solution will not be unbounded

d. None of the above

29. Consider the following linear programming problem :

$$\text{Maximize } Z = x_1 + 3x_2$$

subject to

$$x_1 + x_2 \leq 2$$
$$-x_1 + x_2 \leq 4$$

x_1 is unrestricted; $x_2 \geq 0$

Then which of the following is the best basic feasible objective function value?

a. 8

b. 11

c. 6

d. 15

30. For the problem

$$\text{Maximize } Z = (2x_1 + x_2 + 5x_3 + 6x_4)$$

subject to

$$2x_1 + x_2 + x_4 \leq 8$$
$$2x_1 + 2x_2 + x_3 + 2x_4 \leq 12$$
$$\text{for all} \quad x_i > 0$$

It is known that $x_1 = 0, x_2 = 0, x_3 = 4, x_4 = 4$ is the optimal solution. The optimal of the dual is (if y_1 and y_2 are dual variables)

a. $y_1 = 1, \; y_2 = 4$

b. $y_1 = 4, \; y_2 = 4$

c. $y_1 = 4, \; y_2 = 1$

d. $y_1 = 0, \; y_2 = 0$

PART—B

31. The dual of the space $L^{2/3}$ is the space

a. $L^{3/2}$

b. L^2

c. L^{-2}

d. does not exist

32. The operator $T: R^2 \to R^2$ defined by $T(x_1, x_2) = (0, x_2)$ is

a. bounded but not continuous

b. continuous but not bounded

c. bounded as well as continuous

d. neither bounded nor continuous

33. Defined $f(z) = z^3 \sin\frac{1}{z}$, $z \neq 0$ and $f(0) = 0$, where z is a complex number. Then for $f(z)$, which one of the following is true?

a. $z = 0$ is a removable singularity

b. $z = 0$ is a pole of order 2

c. $z = 0$ is a pole of order 3

d. $z = 0$ is an essential singularity

34. Let S be a unit sphere in R^3, $x^2 + y^2 + z^2 = 1$. Let P be a plane in R^3, $lx + my + nz = p$ intersecting the sphere in a circle Γ. If Γ passes through the point $N(0, 0, 1)$, then its stereographic projection on the complex plane is

a. a straight line

b. a circle

c. an arc of a circle

d. an ellipse with latus rectum twice the radius of the circle Γ

35. If C be the unit circle centered at the origin, then the value of the integral $\int_C \frac{e^z}{z^3}dz$ is

a. $e^{\pi i}$

b. πi

c. $-\pi i$

d. $e^{-\pi i}$

36. If (X, T) be a topological space, then which one of the following is ***not*** equivalent to others?

a. X can be written as the disjoint union of two nonempty closed subsets

b. X can be written as the disjoint union of two nonempty open sets

c. There exists a subset A of X, $A \neq \phi$, $A \neq X$; A is both open and closed

d. X is a connected space

37. Which one of the following statements is ***not*** true?

a. Composite of two measurable functions is measurable.

b. If f is measurable, then $|f|$ is also measurable.

c. The sum of two measurable functions is measurable.

d. If f and g are two measurable functions, then both $\max(f(x), 1)$ and $\min(g(x), 1)$ are measurable functions.

38. Let A denote a set of algebraic numbers. Consider the function $f : [0, 2] \to R$ defined by

$$f(x) = \begin{cases} 0, & x \in A \cap [0, 2] \\ 1, & x \in [0, 2] - A \end{cases}$$

Then f is

a. only Riemann integrable

b. only Lebesgue integrable

c. Riemann as well as Lebesgue integrable

d. neither Riemann nor Lebesgue integrable

39. Which one of the following is ***not*** true for projections on Hilbert spaces?

a. The sum of two projections P and Q is a projection if and only if $PQ = QP$

b. The product of two projections P and Q is a projection if and only if $PQ = QP$

c. P-Q is a projection for projections P and Q if and only if $Q \leq P$

d. P is a projection if and only if I-P is so

40. S_3, the symmetric group of degree 3 has

a. no Sylow 2-subgroup

b. two Sylow 2-subgroup

c. three Sylow 2-subgroup

d. four Sylow 2-subgroup

41. Which of the following is ***not*** true for a group G?

a. If $O(G) = 4$, then G is cyclic

b. If $O(G) = 5$, then G is cyclic

c. If $O(G) = 6$, then either G is Abelian or isomorphic to S_3

d. If $O(G) = 8$, then either G is Abelian or isomorphic to group of quaternions

42. If $R = \{0, 2, 4, 6\}$ be the ring of integers modulo 8, and $M = \{0, 4\}$, then which one of the following is true?

a. M is a maximal ideal of R but not prime

b. M is a prime ideal of R but not maximal

c. M is both maximal and prime ideal of R

d. M is neither maximal nor prime ideal of R

43. If Q is a field of rationals, then which of the following is ***not*** true?

a. x^2+1 is irreducible over Q

b. x^3-9x+5 is irreducible over Q

c. x^2+2x+3 is irreducible over Z_5

d. x^3-9 is irreducible over Z_{11}

44. If U and W are distinct four-dimensional subspaces of a vector space V with $\dim(V)=6$, then

a. $\dim(U\cap W)$ is 3 or 2

b. $\dim(U\cap W)$ is 3 or 4

c. $\dim(U\cap W)$ is 3 or 5

d. $\dim(U\cap W)$ is 3 or 6

45. If $L: M_{22}\to R$ is a linear transformation given by $L(A)=\text{trace}(A)$, then

a. $\dim(\ker(L))=1$

b. $\dim(\ker(L))=2$

c. $\dim(\ker(L))=3$

d. $\dim(\ker(L))=4$

46. Which of the following is ***not*** true?

a. The ring of integers has characteristic 0

b. The ring of even integers has characteristic 2

c. The field of rationals has characteristic 0

d. The field $Z_2=\{0, 1\}$ of integers modulo 2 has characteristic 2

47. If $T : R^4 \to R^4$ is a linear mapping defined by

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{bmatrix}$$

then T is diagonalizable, when

a. $a = 0,\ b = c = 1$

b. $b = 0,\ a = c = 1$

c. $a = b = c = 1$

d. $a = b = c = 0$

48. The complete integral of the partial differential equation $p^2y(1+x^2) = qx^2$ is

a. $z = a\sqrt{1+x^2} + \frac{1}{2}a^2y^2 + b$

b. $z = a(1+x^2) + \frac{1}{2}ay + b$

c. $z = a(1+x^2) + \frac{1}{2}a^2y^2 + b$

d. $z = \frac{1}{2}a^2\sqrt{1+x^2} + ay + b$

49. If the initial value problem

$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(0,\ y) = 4e^{-2y}$$

then the value of $u(1,\ 1)$ is

a. $4e^{-4}$

b. $4e^{-2}$

c. $4e^{2}$

d. $4e^{4}$

50. The general solution of the system

$$y_1' = 3y_2$$
$$y_2' = 12y_1$$

is

a. $y_1 = c_1e^{-3t} + c_2e^{3t},\ y_2 = -2c_1e^{-3t} + 2c_2e^{3t}$

b. $y_1 = c_1e^{-6t} + c_2e^{6t},\ y_2 = -2c_1e^{-6t} + 2c_2e^{6t}$

c. $y_1 = c_1e^{-3t} + c_2e^{6t},\ y_2 = -2c_1e^{-3t} + 2c_2e^{6t}$

d. $y_1 = c_1e^{3t} + c_2e^{6t},\ y_2 = -2c_1e^{3t} + 2c_2e^{6t}$

51. Consider the diffusion equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ 0 < x < \pi,\ t > 0$$

subject to the initial and boundary conditions $u(x, 0) = 4\sin 2x,\ \ 0 < x < \pi$ and $u(0, t) = u(\pi, t) = 0,\ t > 0$. Then the value of $u(\pi/8, 1)$ is

a. $\frac{4}{\sqrt{e}}$

b. $\frac{4}{e^2}$

c. $\frac{4e^{-4}}{\sqrt{2}}$

d. $\frac{4e^{-9}}{\sqrt{2}}$

52. If the partial differential equation is given by

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

then which one of the following is ***not*** correct?

a. It is a second-order parabolic equation

b. The characteristic curves are given by $y = cx$, a family of straight lines passing through origin

c. The canonical form is $\frac{\partial^2 u}{\partial \eta^2} = 0$

d. The canonical form is $\frac{\partial^2 u}{\partial \eta^2} = 1$

53. The two linearly independent solutions f_1 and f_2 of the second-order homogeneous linear differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ which are such that $f_1(0) = 1$, $f_1'(0) = 0$, $f_2(0) = 0$ and $f_2'(0) = 1$ are

a. $f_1(x) = 2e^{x} - e^{2x}$, $f_2(x) = -e^{x} + e^{2x}$

b. $f_1(x) = 2e^{-x} - e^{2x}$, $f_2(x) = -e^{-x} + e^{2x}$

c. $f_1(x) = 2e^{x} - e^{-2x}$, $f_2(x) = -e^{x} + e^{-2x}$

d. $f_1(x) = e^{x} + 2e^{2x}$, $f_2(x) = e^{x} + e^{2x}$

54. The initial boundary value problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ 0 < x < 1,\ t > 0,\ u(0, t) = u(1, t) = 0;\ u(x, 0) = x(1 - x),\ 0 \le x \le 1$$

has solution $u(x, t) = \sum_{n=1}^{\infty} b_n e^{-n^2 \pi t} \sin(n\pi x)$, where b_n is equal to

a. $\frac{4}{n^3 \pi^3}$, if n is odd and 0 if n is even

b. $\frac{4}{n^3 \pi^3}$, if n is even and 0 if n is odd

c. $\frac{8}{n^3 \pi^3}$, if n is odd and 0 if n is even

d. $\frac{8}{n^3 \pi^3}$, if n is even and 0 if n is odd

55. The initial value problem $\frac{dy}{dx} = y^{1/3}$, $y(0) = 0$ has

a. no solution

b. infinitely many solutions

c. more than one but only finitely many solutions

d. a unique solution

56. If

$$\begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & p \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & -53 \end{pmatrix} \begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

then the value of p is

a. -2

b. -1

c. 1

d. 2

57. If Δ and ∇ are the forward and the backward difference operators, respectively, then $\Delta + \nabla$ is equal to

a. $-\Delta\nabla$

b. Δ / ∇

c. $\Delta / \nabla - \nabla / \Delta$

d. $\Delta / \nabla + \nabla / \Delta$

58. A Runge-Kutta method for solving the initial value problem $\frac{dy}{dx} = f(x, y), \; y(x_0) = f(x_0)$ is given by $y(x+h) = y(x) + a_1 hf(x, y) + a_2 hf(x + p_1 h, y + p_2 hf(x, y))$. The values of a_1, a_2, p_1 and p_2 for this formula to be second-order accurate are

a. $a_1 = \frac{1}{3}, a_2 = \frac{2}{3}, p_1 = \frac{3}{2}, p_2 = \frac{3}{4}$

b. $a_1 = \frac{3}{4}, a_2 = \frac{1}{4}, p_1 = \frac{1}{2}, p_2 = \frac{1}{2}$

c. $a_1 = 2, a_2 = 1, p_1 = \frac{1}{2}, p_2 = \frac{1}{2}$

d. $a_1 = \frac{1}{2}, a_2 = \frac{1}{2}, p_1 = 1, p_2 = 1$

59. The iterative formula to obtain the approximate value of $\alpha^{1/3}$ of a given positive real number α using Newton-Raphson method is

a. $x_{n+1} = \frac{2}{3}x_n + \frac{\alpha}{3x_n^2}$

b. $x_{n+1} = \frac{2}{3}x_n - \frac{\alpha}{3x_n^2}$

c. $x_{n+1} = \frac{3}{2}x_n + \frac{\alpha}{3x_n^2}$

d. $x_{n+1} = \frac{3}{2}x_n - \frac{\alpha}{3x_n^2}$

60. If $f(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$ is a polynomial, where $\alpha_0, \alpha_1, \alpha_2$ are real numbers with $\alpha_2 \neq 0$ and

$$E_1 = \int_0^1 f(x)\,dx - f\left(\frac{1}{2}\right), \quad E_2 = \int_0^1 f(x)\,dx - \frac{1}{2}(f(0) + f(1))$$

then

a. $|E_1| = |E_2|$ b. $|E_1| = 2|E_2|$

c. $|E_1| > |E_2|$ d. $|E_2| > |E_1|$

61. The probability of obtaining 3 defectives in a sample of size 10 taken without replacement from a box of 20 components containing 4 defectives is

a. 25/100 b. 20/100

c. 30/100 d. 15/100

62. If X and Y are standardized random variables where $r(2X + 3Y, 3X + 2Y) = 1$, then the coefficient of correlation $r(X, Y)$ is

a. 1/3 b. 1

c. 2/3 d. 1/2

63. If X is a continuous random variable with probability density function defined by

$$f(x) = \begin{cases} ax & , \ 0 \le x \le 1 \\ a & , \ 1 \le x \le 2 \\ -ax + 3a & , \ 2 \le x \le 3 \\ 0 & , \ \text{elsewhere} \end{cases}$$

then $P(x \le 1{\cdot}5)$ is given by

a. 1/4

b. 1/2

c. 2/3

d. 3/4

64. A coin is tossed until a head appears. The expectation of the number of tosses required will be

a. 2

b. 8

c. 16

d. 64

65. The regression equation of Y on X for the following distribution

$$f(x, y) = \frac{y e^{\frac{-y}{1+x}}}{(1+x)^4}, \ x, y > 0$$

is

a. $y = 3(1+x)$

b. $y = 2(1+x)$

c. $y = (1+3x)$

d. $y = 3(1+2x)$

66. If in a simplex table the relative cost $(z_j - c_j)$ is zero for a non-basic variable, then there exists an alternative optimal solution, provided

a. it is a starting simplex table

b. it is an optimal simplex table

c. it can be any simplex table

d. zero relative cost $(z_j - c_j)$ for a non-basic variable, does not imply an alternative optimal solution

67. If in Phase-1 of the simplex method, an artificial variable remains at positive level in the optimal table of Phase-1, then

a. the solution is unbounded

b. there exists an optimal solution

c. there exists no solution

d. the solution is bounded

68. In canonical form of a linear programming problem, the availability of vector ***b***

a. is restricted to > 0

b. is restricted to < 0

c. is equal to 0

d. has no restriction on > 0, < 0 or = 0

69. For a linear programming problem, which of the following is a correct relation between number of basic feasible solutions (BFS) and number of vertices?

a. Number of BFS ≤ Number of vertices

b. Number of BFS ≥ Number of vertices

c. Number of BFS = Number of vertices

d. None of the above

70. The intersection of all convex sets of which a set S is a subset, is known as

a. convex hull

b. hyperplane

c. supporting hyperplane

d. None of the above

SAU PhD Mathematics Entrance Exam 2019 || Sample

- This is only a sample paper and only meant to be indicative of the type of questions that will be asked.

1. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y=f(x)$ and $y=g(x)$

   a. intersect exactly once
   b. intersect no more than once
   c. do not intersect
   d. have a common tangent at each point of intersection

2. If a function f is continuous for all real x and if f has a relative maximum 4 at x=-1 and a relative minimum -2 at x=3 then which of the following statements must be true?

   a. $f'(-1) = 0$
   b. The graph of f has a horizontal asymptote
   c. The graph of f has a horizontal tangent line at x = 3
   d. The graph of f intersects both axes

3. If $\frac{1}{u} = \sqrt{x^2 + y^2 + z^2}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to

   a. u
   b. $-u$
   c. 0
   d. 1/2

4. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x))$ is

   a. $5x^3 + 15x^2 + 20x + 25$
   b. 1125
   c. 225
   d. 5

5. Which one of the following equations has a graph that is symmetric with respect to the origin?

   a. $y = \frac{x+1}{x}$
   b. $y = -x^5 + 3x$
   c. $y = x^4 - 2x^2 + 6$
   d. $y = (x-1)^3 + 1$

6. If f is a continuous function on $[a,b]$, which one of the following is necessarily true?

   a. f' exists on (a,b).
   b. The graph of f' is a straight line.
   c. $\lim_{x\to x_0} f(x) = f\left(\lim_{x\to x_0} x\right)$ for $x_0 \in (a,b)$
   d. $f'(x)=0$ for some $x\in[a,b]$

7. Consider $L = \left(\frac{z}{\underline{z}}\right)^2$. The value of L is

   a. 1 if $z \to 0$ along real axis
   b. -1 if $z \to 0$ along imaginary axis
   c. 1 if $z \to 0$ along line $y = x$
   d. none of these.

8. The function $f(z) = 2x^2 + y + i(y^2 - x)$ is

   a. analytic at one point.
   b. analytic at two points.
   c. nowhere analytic.
   d. none of the above.

9. The derivative of the principal value z^i at the point $z = 1 + i$ is

   a. $\frac{1+i}{2}e^{-\pi/4+i(ln2)/2}$
   b. $\frac{1-i}{2}e^{-\pi/4+i(ln2)/2}$
   c. $\frac{1+i}{2}e^{-\pi/4-i(ln2)/2}$
   d. $\frac{1-i}{2}e^{-\pi/4-i(ln2)/2}$

10. The integral $\int_C \quad (x^2 + iy^2)dz$, where C is the contour starting from (0,0) along $y = x$ to (1,1) and then along line $x = 1$ to (1,2) is

   a. $7/3 - i5/3$.
   b. $-7/3 + i5/3$.

c. $7/3 + i5/3$.

d. $-7/3 - i5/3$.

11. Let $z_1 = \frac{(4n+1)\pi}{2} - iln(5 + 2\sqrt{6})$ and $z_2 = \frac{(4n+1)\pi}{2} - iln(5 - 2\sqrt{6})$. The solution of the equation $sinz = 5$ is

a. z_1 only

b. z_2 only

c. both z_1 and z_2

d. either $z_1$or z_2.

12. The value of the integral $\oint_C \frac{e^z}{z^4+5z^3} dz$ where C is the circle $|z| = 2$ is

a. $-\frac{17\pi}{125} i$.

b. $\frac{17\pi}{125} i$.

c. $-\frac{17\pi}{123} i$.

d. $\frac{17\pi}{123} i$

13. The value of the integral $\oint_C \frac{1}{(z-1)^2(z-3)} dz$ where contour C is the rectangle defined by $x = 0, x = 4, y = -1$ and $y = 1$ in anticlockwise direction is

a. -2.

b. 2.

c. 1.

d. 0.

14. If A and B are two disjoint sets with cardinality α and β, respectively, then the cardinality of the set $\{f : f : A \to B\}$ is

a. $\alpha\beta$

b. β^α

c. α^β

d. $\alpha + \beta$

15. Eigenvalues of a real symmetric matrix are always

a. positive only

b. negative only

c. real

d. real or imaginary

16. The dimension of **R**over **Q**is

a. 1

b. 2
c. n
d. infinite

17. Let $V = C^2$, with the standard inner product and define $T: C^2 \to C^2$ by
$$T([z_1 z_2]) = [2z_1 + iz_2(1-i)z_1],$$

then the adjoint of operator T is

$$a. T([z_1 z_2]) = [2z_1 + (1+i)z_2 - iz_1]$$
$$b.\ T([z_1 z_2]) = [-2z_1 + (1+i)z_2 iz_1]$$

c. $T([z_1 z_2]) = [2z_1 + (1-i)z_2 - iz_1]$
d. $T([z_1 z_2]) = [2z_1 + (1+i)z_2 iz_1]$

18. The orthonormal basis of column space of the matrix $[1 - 110011 - 1001 - 1]$ is

a. $\{[\sqrt{2}/2,0,\sqrt{2}/2,0]^T, [0,0,0,1]^T, [-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6,0]^T}\}$.
b. $\{[\sqrt{2}/2,0,-\sqrt{2}/2,0]^T, [0,0,0,1]^T, [\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6,0]^T}\}$.
c. $\{[-\sqrt{2}/2,0,\sqrt{2}/2,0]^T, [0,0,0,1]^T, [\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6,0]^T}\}$.
d. $\{[\sqrt{2}/2,0,\sqrt{2}/2,0]^T, [0,0,0,1]^T, [\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6,0]^T}\}$.

19. As $t \to \infty$, the solution of the differential equation $\frac{dy}{dt} = y - y^3$ with initial condition $y(0) = 0.1$ will approach to

a. 1
b. -1
c. 0
d. -2

20. The general solution of the differential equation $y'' + 4y = 3\ cos\ cos 2x + 2x^2$ is

a. $y = c_1\ sin\ sin 2x + c_2\ cos\ cos 2x + \frac{3}{4}x\ sin\ sin 2x + \frac{1}{2}x^2 - \frac{1}{4}$

b. $y = c_1\ sin\ sin 2x + c_2\ cos\ cos 2x - \frac{3}{4}x\ sin\ sin 2x + \frac{1}{2}x^2 - \frac{1}{4}$

c. $y = c_1\ sin\ sin 2x + c_2\ cos\ cos 2x + \frac{3}{4}x\ sin\ sin 2x - \frac{1}{2}x^2 - \frac{1}{4}$

d. $y = c_1\ sin\ sin 2x + c_2\ cos\ cos 2x + \frac{3}{4}x\ sin\ sin 2x + \frac{1}{2}x^2 + \frac{1}{4}$

21. The system of equations $\frac{dx_1}{dt} = x_2, \frac{dx_2}{dt} = x_3, \frac{dx_3}{dt} = x_3 - 2tx_2 + 3x_1 - 6$ is equivalent to

a. x’’’+ x’’ +2tx’-3x-6 =0

b. x’’’- x’’ +2tx’-3x-6 =0

c. x’’’+ x’’ +2tx’-3x+6=0

d. x’’’+ x’’ +2tx’-3x-6 =0.

22. The solution of the heat equation $u_t = 3\,u_{xx}, u(x,0) = x(\pi - x),\ u(0,t) = u(\pi,t) = 0$ is

a. $u(x,t) = \frac{8}{\pi}\sum_{k=1}^{\infty} \frac{1}{(2k-1)^3} e^{-3(2k+1)^2 t}\, sin\, sin(2k-1)x.$

b. $u(x,t) = \frac{8}{\pi}\sum_{k=1}^{\infty} \frac{1}{(2k+1)^3} e^{-3(2k-1)^2 t}\, sin\, sin(2k+1)x.$

c. $u(x,t) = \frac{8}{\pi}\sum_{k=1}^{\infty} \frac{1}{(2k-1)^3} e^{-3(2k-1)^2 t}\, sin\, sin(2k-1)x.$

d. $u(x,t) = \frac{8}{\pi}\sum_{k=1}^{\infty} \frac{1}{(2k+1)^3} e^{-3(2k+1)^2 t}\, sin\, sin(2k+1)x.$

23. The Lagrange polynomial that passes through three data points is given by

$f(x) = 24L_0(x) + 37L_1(x) + 22L_2(x)$, where $f(15) = 24, f(18) = 37, f(22) = 25$. The value of $L_1(x)$ at $x = 16$ is

a. 4
b. 2
c. ½
d. 1/4

24. The solution of the partial differential equation $u_x - u_y - 2y = 0$, $u(x, 2x+1) = e^x$ is

a. $u(x,y) = x^2 - 2xy - e^{\frac{x+y-1}{3}} - 6\left(\frac{x+y-1}{3}\right)^2 - 2\left(\frac{x+y-1}{3}\right).$
b. $u(x,y) = x^2 + 2xy + e^{\frac{x+y-1}{3}} + 6\left(\frac{x+y-1}{3}\right)^2 + 2\left(\frac{x+y-1}{3}\right).$
c. $u(x,y) = x^2 + 2xy - e^{\frac{x+y-1}{3}} + 6\left(\frac{x+y-1}{3}\right)^2 - 2\left(\frac{x+y-1}{3}\right).$
d. $u(x,y) = x^2 + 2xy + e^{\frac{x+y-1}{3}} - 6\left(\frac{x+y-1}{3}\right)^2 - 2\left(\frac{x+y-1}{3}\right).$

25. The solution of heat equation $u_{tt} = 5\,u_{xx}, u(x,0) =$ $sin\, sinx$, $u_t(x,0) =$ $sin\, sin3x$ is

a. $u(x,t) = \frac{1}{2}\left[sin\, sin\left(x + \sqrt{5}t\right) - sin\, sin(x - \sqrt{5}t)\right] - \frac{1}{30}\left[cos\, cos\left(x + \sqrt{5}t\right) - cos\, cos(x - \sqrt{5}t)\right]$

b. $u(x,t) = \frac{1}{2}\left[\left(x - \sqrt{5}t\right) - sin\, sin(x + \sqrt{5}t)\right] - \frac{1}{30}\left[cos\, cos\left(x - \sqrt{5}t\right) - cos\, cos(x - \sqrt{5}t)\right]$.

c. $u(x,t) = \frac{1}{2}\left[sin\, sin\left(x - \sqrt{5}t\right) - sin\, sin(x - \sqrt{5}t)\right] - \frac{1}{30}\left[cos\, cos\left(x + \sqrt{5}t\right) - cos\, cos(x - \sqrt{5}t)\right]$.

d. $u(x,t) = \frac{1}{2}\left[sin\, sin\left(x - \sqrt{5}t\right) - sin\, sin(x - \sqrt{5}t)\right] + \frac{1}{30}\left[cos\, cos\left(x - \sqrt{5}t\right) - cos\, cos(x - \sqrt{5}t)\right]$.

26. The solution of the partial differential equation $9\, u_{xx} + 12\, u_{xy} + 4u_{yy} = 0$ is given by

$a.\ u = xf(2x + 3y) + g(2x - 3y).$

$b.\ u = xf(2x - 3y) + g(2x - 3y).$

$c.\ u = xf(2x - 3y) + g(2x + 3y).$

$d.\ u = xf(2x - 3y) - g(2x + 3y).$

27. The Green's function for the BVP $y'' = -f, y(0) = y(\pi) = 0\ ,\ 0 < x < \pi$ is

$a.\ G(x;\xi) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{sinsinnxsinsinn\xi}{n^2}.$

$b.\ G(x;\xi) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{sinsinnxcoscosn\xi}{n^2}.$

$c.\ G(x;\xi) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{coscosnxsinsinn\xi}{n^2}.$

$$d.\ G(x;\xi) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{cos\ cosnx\ cos\ cosn\xi}{n^2}$$

28. The hospital period, in days, for patients following treatment for a certain type of kidney disorder is a random variable $Y = X + 4$, where X has the density function

$$f(x) = \frac{32}{(x+4)^3}, \qquad x > 0{,}0, \qquad otherwise.$$

Find the probability that the hospital period for a patient following this treatment will exceed 8 days.

a. $\frac{1}{4}$

b. $\frac{1}{5}$

c. $\frac{2}{9}$

d.$\frac{3}{4}$

29. Consider two independent random variables $X$and Z with means 3 and 4, and variances 9 and 16, respectively. For a random variable $Y = \frac{X}{Z,}$ the variance of Y is approximately equal to

a. $\frac{140}{9}$

b. 10

c. $\frac{153}{16}$

d. 9

30. Let$f$be a function defined by $f(x) = \int_1^x \quad t(t^2 - 3t + 2)dt, \quad x \in [1,3]$. If a number is randomly selected from the domain of f, then the chance that the selected number also belongs to range of f is

a. $\frac{1}{6}$

b.$\frac{1}{2}$

c. $\frac{1}{4}$

d. $\frac{1}{8}$

31. If $xy = a^2$ and $S = b^2x + c^2y$, where a, b and c are randomly selected from intervals[0,1], [2,4] and [6,9], respectively, then the probability of minimum value of S lying in the interval [0,72] is

a. 0

b.$\frac{1}{2}$

c. 1

d. $\frac{2}{3}$

32. Suppose that the two-dimensional random variable (X, Y) is uniformly distributed over the region

$$R = \{0 < x < y < 1\}.$$

The correlation coefficient between X and Y is given by

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{2}{3}$

d.$\frac{1}{4.}$

33. Which of the following is unique factorization domain (UFD) but not principal ideal domain (PID)

a. Z[X]
b. Q[x]
c. Z
d. R[x].

34. If G is a group of order 175, then which of the following statements is false?

a. G is abelian

b. G is cyclic

c. G is simple

d. G has a Sylow 5 subgroup of order 25

35. Which of the following groups is simple?

a. A group of order 108

b. A group of order 148

c. A group of order 56

d. A group of order 17

36. The Order of 5 + <6>in **Z**18\ <6>is

a. 1

b. 2

c. 3

d. 6

37. Given $\frac{d^2y}{dx^2} = 6x - \frac{1}{2}x^2, y(0) = 0, y(12) = 0$, the value of $\frac{d^2y}{dx^2}$ at $x = 4$ using the finite difference method and a step size $h = 4$ can be approximated by

a. $\frac{y(0)+y(8)-2y(4)}{16}$

b. $\frac{y(0)-y(8)-2y(4)}{16}$

c. $\frac{y(0)+y(8)+2y(4)}{16}$

d. $\frac{y(0)+y(8)-2y(4).}{4}$

38. The sufficient condition for Jacobi iterative method $X^{(k+1)} = BX^{(k)} + C, k = 0,1,2 \ldots \ldots$ for solving the system of linear equations is given by

a. $\|B\| < 1/2$

b. $\|B\| < 3$

c. $\|B\| < 1$

d. $\|B\| > 1.$

39. The approximated value of $y(1.1)$ for the initial value problem $\frac{dy}{dx} + 2xy^2 = 0,\ y(1) = 1$ by Euler method using step size $h = 0.1$ is given by

a. 0.1

b. 0.7

c. 0.9

d. 0.8

40. If Δ and ∇ denote forward and backward difference operators, then which of the following is true?

a. $\Delta + \nabla = \frac{\Delta}{\nabla} + \frac{\nabla}{\Delta}$

b. $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

c. $\Delta + \nabla = -\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

d. $\Delta + \nabla = \frac{\Delta}{\nabla} / \frac{\nabla}{\Delta}$

41. The boundary value problem $\frac{d^2y}{dx^2} = y^2, y(0) = 1, y(1) = 2$, is approximated by the difference equation $y_0 = 1, y_{n+1} = 2, y_{k-1} - 2y_k + y_{k+1} = h^2 y_k^2, k = 1(1)n$. The order of the difference equation is

a. 1

b. 2

c. 3

d. 4

42. Picard's method of successive integration fails if

a. the function is not measurable.

b. the function is not monotone.

c. the function is not bounded.

d. the function is not integrable.

43. Using Simpson's 1/3 rd rule of integration with step size , the value of integral correct upto three decimal places is

a. 0.695

b. 0.691

c. 0.694

d. 0.693

44. A variable which does not appear in the basis variable column of simplex table is

a. Never zero
b. Always equals to zero
c. Called basic variable
d. None of these.

45. Suppose that the cost of performing an experiment is 1000 rupees. If the experiment fails, an additional cost of 300 rupees occurs because of certain changes that have to be made before the next trial is attempted. If the probability of success on any given trial is 0.2, if the individual trials are independent, and if the experiments are continued until the first successful result is achieved, what is the expected cost of the entire procedure?

a. 6500 Rs

b. 6200 Rs

c. 5600 Rs

d. 6000 Rs

46. The following four inequalities define a feasible region. Which one of these could be removed from the list without changing the region?

a. $x-2y \geq 8$
b. $y \geq 0$
c. $-x-y \leq 10$
d. $x+y \leq 20$.

47. If an artificial variable is present in the basic variable column of an optimal simplex table, then the solution is

a. unbounded
b. infeasible
c. optimal

d. None of these

48. A solved LP problem indicated that the optimal solution was $x_1 = 10$ and $x_2 = 20$. One of the constraints was $4\ x_1 + 2\ x_2 \leq\ 80$. This constraint has

a. surplus greater than zero.
b. slack greater than zero.
c. surplus equal to zero.
d. slack equal to zero.

49. If in a standard form LPP, we have a linear system with 20 nonnegative variables and 10 equations, then the maximum number of basic solutions is given by

a. 200
b. 2
c. 184756
d. 670442572800

50. Which of the following is not a feasible solution of the dual of the given problem:

$Maxx_1\ +\ 2x_2$ subject to

$$x_1\ +\ x_2\ \leq\ 4$$

$$-x_1\ +\ x_2\ \leq\ 1$$

$$2x_1 - x_2 \leq\ 3$$

$$x_1, x_2\ \geq\ 0$$

a. (0, 1/2, 1/2)

b. (2, 1, 1)

c. (3/2, 1/2, 0)

d. (2, 1, 2)

Some Useful Links:

1. Free Maths Study Materials (https://pkalika.in/2020/04/06/free-maths-study-materials/**)**

2. BSc/MSc Free Study Materials (https://pkalika.in/2019/10/14/study-material/**)**

3. MSc Entrance Exam Que. Paper: (https://pkalika.in/2020/04/03/msc-entrance-exam-paper/**)**
[JAM(MA), JAM(MS), BHU, CUCET, ...etc]

4. PhD Entrance Exam Que. Paper: (https://pkalika.in/que-papers-collection/**)**
[CSIR-NET, GATE(MA), BHU, CUCET,IIT, NBHM, ...etc]

5. CSIR-NET Maths Que. Paper: (https://pkalika.in/2020/03/30/csir-net-previous-yr-papers/**)**
[Upto 2019 Dec]

6. Practice Que. Paper: (https://pkalika.in/2019/02/10/practice-set-for-net-gate-set-jam/**)**
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