# Experiment 1: Bernoulli's Equation and Air Duct Design

## Introduction

Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho V_1{}^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2{}^2 + \rho g h_2$$

The Bernoulli's equation in fluid dynamics states that an increase in the speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy. It can be used to analyse air duct design and many other fluid flow issues in HVAC systems. This experiment provides an opportunity to practically study the principles of Bernoulli's equation and how it is applied to air duct design.

As shown in Figure 1, the Optional Bernoulli's Equation F100B investigation duct has been designed for operation with the Hilton Airflow System F100. The duct allows students to quantitatively investigate Bernoulli's equation relating total pressure and dynamic pressure in an air stream. The unit also introduces students to the pitot-static tube, an essential tool for aerodynamic investigation and velocity measurement.

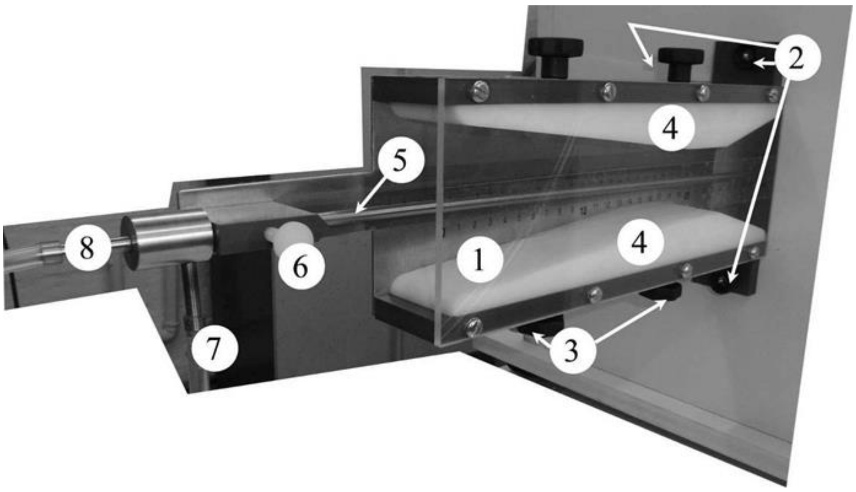


Diagram Key: 1-Position Measuring Scale; 2-Mounting Nuts; 3-Profile Retaining Nuts; 4-Duct Profiles; 5-Pitot-Static Tube; 6-Locking Nut; 7-Static Pressure Tapping; 8-Total Pressure Tapping

*Figure 1. Bernoulli's Equation F100B investigation duct mounted on the Hilton Airflow System F100*

## Objectives

- To study the principles of Bernoulli's equation.
- To use the pitot-static tube to measure air velocity for air duct design.

## Theory and Principles

This optional duct demonstrates the use of a pitot-static tube and the application of Bernoulli's equation along a convergent-divergent passage. The pitot-static tube head detail is shown in Figure 2. The static pressure (7) tapping is 25mm behind the total pressure (8) tapping. The total pressure tapping brings the flow immediately in front of it to a halt. By locating the pitot-static tube head in different positions, the static pressure and total pressure can be measured, as shown in Figure 3.

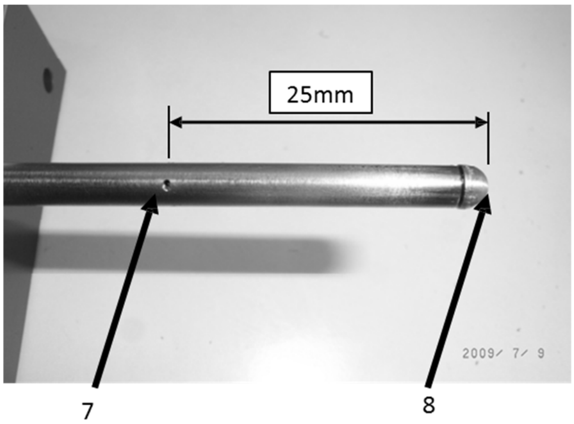


*Figure 2. The **pitot-static tube(5)** head detail*

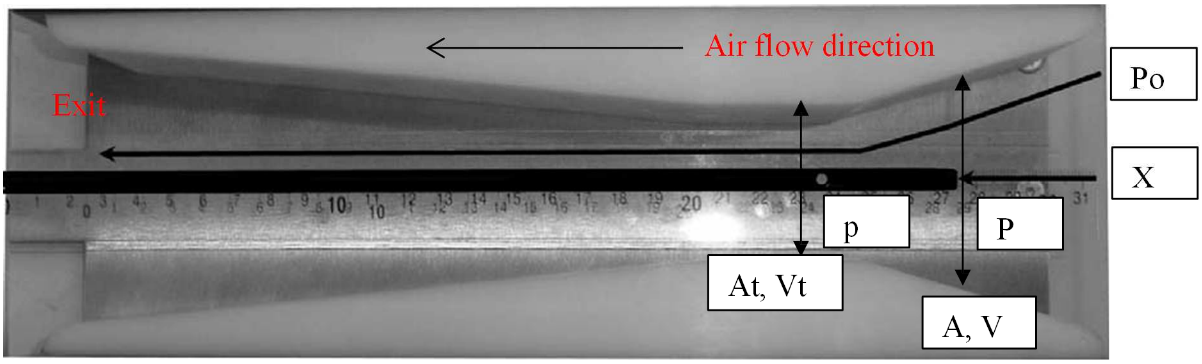


*Figure 3. Measurement of total and static pressure*

According to Bernoulli's equation the total pressure P is defined as,

$$P = p + \frac{1}{2}\rho V^2 \tag{1}$$

where $p$ is the static pressure (N m$^{-2}$) measured in a flow field moving at velocity $V$ (m s$^{-1}$).

The total pressure $P$ should be constant along the duct provided that the flow is steady and that the air is incompressible and inviscid. If the pressure in the plenum chamber of the F100 unit is $P_o$ then the pressure along the streamline shown above should be everywhere the same as $P_o$. This pressure can be measured using a tapping in the top wall of the box before the contraction as the velocity $V$ inside the box is a fraction of that in the F100B duct.

As the flow along the streamline X is brought to a halt at the total pressure tapping (8) this tube will measure the total Pressure P at that point. The static pressure p can be measured by the static pressure tapings in the wall of the pitot-static tube as the air is moving at velocity $V$ (m s$^{-1}$) at this point. In order to not be affected by the presence of the tip of the tube (disturbing the streamlines) the static pressure holes are located at a position approximately 5 diameters downstream of the tip (25 mm).

If the flow is assumed to be one dimensional (assuming that the velocity over any chosen cross section to be uniform across that section) then the continuity equation may be written as,

$$\dot{Q} = A_t V_t = AV \tag{2}$$

where, $\dot{Q}$ is the volume flow (m$^3$ s$^{-1}$), $A_t$ is the area at the throat (m$^2$), $V_t$ is the velocity at the throat (m s$^{-1}$), A is the area at any point in the duct (m$^2$), V is the velocity at any point in the duct (m s$^{-1}$).

Re-arranging (2), the velocity distribution along the duct may be written as the ratio,

$$\frac{V}{V_t} = \frac{A_t}{A} \qquad (3)$$

The depth of the duct is constant (along the duct) and hence the area will be proportional to the duct height H. Hence,

$$\frac{V}{V_t} = \frac{H_t}{H} \qquad (4)$$

Therefore from the continuity equation, the theoretical velocity ratio (relative to the velocity at the contraction) at any point can be calculated purely from the height ratio.

From Bernoulli's equation the velocity at any point can be determined from the following,

$$V = \sqrt{\frac{2(P-p)}{\rho}} \qquad (5)$$

The velocity at the throat $V_t$ is,

$$V_t = \sqrt{\frac{2(P_t-p_t)}{\rho}} \qquad (6)$$

The actual velocity ratio in the duct may be determined from the following,

$$\frac{V}{V_t} = \sqrt{\frac{(P-p)}{(P_t-p_t)}} \qquad (7)$$

Note that as the total pressure P will be the same ($P_t = P$) at all points along the duct the equation may be written as,

$$\frac{V}{V_t} = \sqrt{\frac{(P-p)}{(P-p_t)}} \qquad (8)$$

Hence it is possible to measure the total and static pressure along the duct and compare the resulting velocity ratio with the velocity ratio calculated from the duct dimensions.

## Equipment, Instruments and Useful Data

- Hilton Airflow System F100
- Optional Bernoulli's Equation F100B investigation duct
- Useful Data:
  The atmospheric pressure is $1.01325 \times 10^5$ N $m^{-2}$;
  F100A Manometer Fluid Density: 800 kg $m^{-3}$;
  Duct Depth: 50 mm;
  Duct Throat Height Ht: 44 mm;
  Duct Height to Throat Ratio: Table 1.

**Table 1. Duct height to throat ratio**

| | Liners Normal Configuration | Liners Reversed Configuration |
|---|---|---|
| X-Distance from Exit Plane mm | $H_t/H$ | $H_t/H$ |
| 315 | 0.440 | 0.440 |
| 300 | 0.573 | 0.551 |
| 290 | 0.620 | 0.571 |
| 280 | 0.676 | 0.586 |
| 270 | 0.743 | 0.602 |
| 260 | 0.824 | 0.619 |
| 250 | 0.926 | 0.636 |
| 240 | 1.000 | 0.655 |
| 230 | 1.000 | 0.675 |
| 220 | 1.000 | 0.697 |
| 210 | 1.000 | 0.720 |
| 200 | 1.000 | 0.744 |
| 190 | 0.965 | 0.770 |
| 180 | 0.919 | 0.798 |
| 170 | 0.880 | 0.828 |
| 160 | 0.846 | 0.860 |
| 150 | 0.815 | 0.894 |
| 140 | 0.786 | 0.932 |
| 130 | 0.759 | 0.973 |
| 120 | 0.734 | 1.000 |
| 110 | 0.710 | 1.000 |
| 100 | 0.688 | 1.000 |
| 90 | 0.667 | 1.000 |
| 80 | 0.648 | 1.000 |
| 70 | 0.633 | 0.882 |
| 60 | 0.612 | 0.790 |
| 50 | 0.595 | 0.715 |
| 40 | 0.580 | 0.652 |
| 30 | 0.565 | 0.600 |
| 20 | 0.550 | 0.556 |
| 10 | 0.440 | 0.440 |
| 0 | 0.440 | 0.440 |

## Procedure

1) Set the manometer to the vertical or inclined condition as required and adjust the reservoir to about mid-height. Record the atmospheric datum or zero level.
   Start the fan and slowly increase the speed, at the same time monitoring the manometer levels. As the pressures in the various tubes move up and/or down, adjust the reservoir level also up or down, so that the liquid levels are kept within the range of the manometer.
   Once the fan is running at the desired speed, make any final adjustments to the reservoir level to set the atmospheric datum to a convenient value using the two outer tubes as a reference. Record this atmospheric datum as the reference value. It is this value that will be either taken from, or added to the other levels recorded on the manometer tubes.
2) Once the fan is at the desired operating speed (record the fan frequency), loosen the locking nut (6) and carefully slide the pitot-static tube along the length of the duct while monitoring the manometer tubes

that are connected. Ensure that the static pressure stays within the limits of the manometer. Then set the manometer so that the static pressure tapping is located at the intake position (approximately x = 315mm from the duct exit) and record the following: $P_o$-Plenum Chamber Pressure, P-Total Pressure, and p-Static Pressure.

3) Refer to the useful data and retract the pitot-static tube a convenient distance, for which towards the discharge (say 10 or 15mm), record the location X and repeat the three pressure measurements $P_o$, P, and p.
4) Continue retracting the pitot-static tube at regular intervals (with reference to the Table 1), record the location X and the three pressures until the tube is at the exit plane of the duct.

## Results

Tables 2 and 3 provided in the Appendix are for recording the measurement data and calculation. After the experiment, the following information should be established to report the findings.

- Clear presentation of the measurement data
- Calculations to determine plenum pressure $P_o$
- Calculations to determine the total pressure **P**, static pressure p, Height ratios $H/H_t$, and velocity ratio $V/V_t$ (from test). Show the variation of the above parameter with the distance from discharge plane on a single diagram.
- Calculations to determine the velocity V at the selected points. Draw a diagram to show the variation of velocity with the distance from discharge plane.

## Discussions

The following issues shall be evaluated and discussed.

- Confirm the validity of Bernoulli's equation.
- Comment on the velocity profile along the duct.

## Laboratory Report

Each student should prepare their own report based on the data and information obtained during the experiment. While the results from the observations and measurements can be shared among the members in the same student group, each student shall generate information to show his/her own understanding and ideas. Students making direct copy of the information in other's report (plagiarism), if found, will be disqualified.

The laboratory report in PDF format shall be submitted to the Moodle before the deadline. Late submission will receive reduction in marks.

## References

ASHRAE, 2017. *ASHRAE Handbook Fundamentals 2017*, SI edition, Chp. 21: Duct Design, American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Atlanta, GA.

Elger, D. F., LeBret, B. A., Crowe, C. T. and Roberson, J. A. 2016. *Engineering Fluid Mechanics*, Eleventh Edition, John Wiley & Sons, Hoboken, NJ.

## Web Links

Bernoulli Equation http://hyperphysics.phy-astr.gsu.edu/hbase/pber.html
What is Bernoulli's equation? http://www.khanacademy.org/science/physics/fluids/fluid-dynamics/a/what-is-bernoullis-equation

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## Appendix

**Table 2. Observed data**

**Fan frequency= Hz**

| Distance from Exit Plane | Liners Normal Configuration | Total Pressure P | Static Pressure p | Plenum Pressure $P_o$ | Atmospheric Datum |
|---|---|---|---|---|---|
| X mm | $H_t/H$ | mm | mm | mm | mm |
| 315 | 0.440 | | | | |
| 280 | 0.676 | | | | |
| 250 | 0.926 | | | | |
| 220 | 1.000 | | | | |
| 190 | 0.965 | | | | |
| 160 | 0.846 | | | | |
| 130 | 0.759 | | | | |
| 100 | 0.688 | | | | |
| 70 | 0.633 | | | | |
| 40 | 0.580 | | | | |
| 10 | 0.440 | | | | |
| 0 | 0.440 | | | | |

**Table 3. Table for calculation**

| Distance from Exit Plane | Total Pressure P | Static Pressure p | Plenum Pressure $P_o$ | Liners Normal Configuration | $\sqrt{\frac{(P-p)}{(P-p_t)}}$ | Air Velocity |
|---|---|---|---|---|---|---|
| X mm | $N/m^2$ | $N/m^2$ | $N/m^2$ | $H_t/H$ | $V/V_t$ | m/s |
| 315 | | | | | | |
| 280 | | | | | | |
| 250 | | | | | | |
| 220 | | | | | | |
| 190 | | | | | | |
| 160 | | | | | | |
| 130 | | | | | | |
| 100 | | | | | | |
| 70 | | | | | | |
| 40 | | | | | | |
| 10 | | | | | | |
| 0 | | | | | | |