Ministry of Higher Education
& Scientific Research
University of Anbar
College of Science
Department of Applied Mathematics

lectures
Subject: **Vector analysis.**
2020-2021.
Stage: 2st.
The lecturer: Assist. Prof. Dr. Ali Rashid Ibrahim

Introduction to vectors:

scalar quantities and vector quantities:

Many physical quantities, such as length, area, volume …, can be completely specified by a single real number, these quantities are often called **scalars**.

Other quantities, such as directed distances, velocities and forces …, require for their complete specification both a magnitude and direction, these quantities are called **vectors**.

Vectors are widely used in many areas of science and engineering.

A vector is a quantity that has both magnitude and direction.

Definition (1): (**Vector in mathematics**): In mathematics, a vector is any object that has a definable length known as **magnitude** and **direction**.

Geometric vectors and vector addition:

Geometric vector:

Vectors can be represented geometrically as a directed line segments or arrows in 2-space or 3-space. The direction of the arrow specifies the direction of the vector, and the length of arrow describes its magnitude. The tail of arrow is called the **initial point** of the vector, and the tip of arrow is called the **terminal point**. We shall denote vectors in **lowercase boldface type** (as **a, k, v, w,** and **x**). When discussing vectors, we shall refer to numbers as **scalars** and these scalars will be real numbers and will be denoted in **lowercase italic type** (as a, k, v, w, and x). A vector **v** with an initial point A and terminal point B also is denoted by $\overrightarrow{AB}$ or $\vec{\mathbf{v}}$ and we write $\mathbf{v} = \overrightarrow{AB}$ as shown in **(figure 1)**. Because it is difficult to write boldface on paper, we suggest that you use an arrow over a single letter, such as ($\vec{\mathbf{v}}$) when you want the letter to denote a vector.

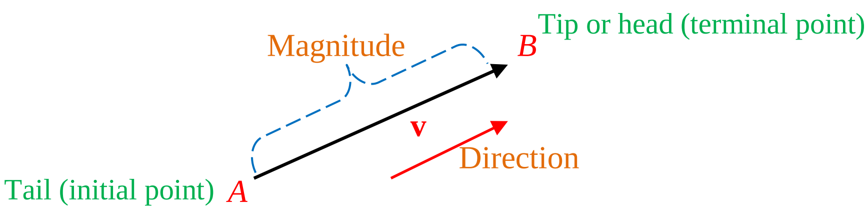

Figure 1
(Vector $\overrightarrow{AB}$ or **v** or $\vec{\mathbf{v}}$)

The magnitude of the vector $\overrightarrow{AB}$, denoted by $|\overrightarrow{AB}|$, $|\vec{\mathbf{v}}|$ or $|\mathbf{v}|$ is the length of the directed line segment.

Definition (2): **(Vector addition):**
The sum of two vectors **u** and **v** can be defined using the **tail-to-tip rule.**

Translate **v** so that its tail end (initial point) is at the tip end (terminal point) of **u**, then the vector from the tail end of **u** to the tip end **v** is the **sum** and denoted by **u+v** of the vectors **u** and **v**, as shown in **(figure 2)**.

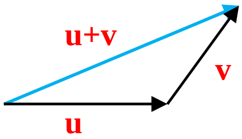

Figure 2

The sum of two nonparallel vectors also can be defined using the **parallelogram rule**.

The sum of two nonparallel vectors **u** and **v** is the diagonal of the parallelogram formed using **u** and **v** as adjacent sides as shown in **(figure 3)**.

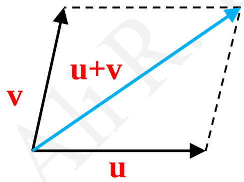

Figure 3

If **u** and **v** are parallel, use the tail-to-tip rule.

Both rules give the same sum. The choice of which rule to use depends on the situation and what seems most natural.

The vector **u+v** is also called the resultant of the two vectors **u** and **v**, which are called vector components of **u+v**.

It is useful to observe that vector addition is commutative and associative. That is **u+v = v+u** and **u+ (v+w) = (u+v) +w**.

Transition from geometric vectors to algebraic vectors:

<u>**Definition (3):**</u> **(Standard vector):**

The transition from geometric vectors to algebraic (standard) vectors is begun by placing geometric vector in a rectangular coordinate system. A geometric vector $\overrightarrow{AB}$ in a rectangular coordinate system translated so that its initial point is at the origin point O is said to be in **standard position**. The vector $\overrightarrow{OP}$ such that $\overrightarrow{OP} = \overrightarrow{AB}$ is said to be the **standard vector** for $\overrightarrow{AB}$, as shown in **(figure 4).**

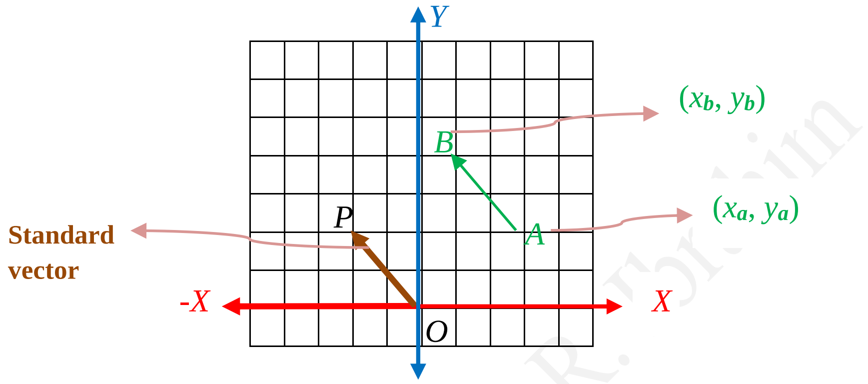

Figure 4

($\overrightarrow{OP}$is the standard vector of $\overrightarrow{AB}$)

The point P is the terminal point of the standard vector $\overrightarrow{OP}$.

Note that the vector $\overrightarrow{OP}$ in this figure is **the standard vector for infinitely many vectors and all vectors with the same magnitude and direction as** $\overrightarrow{OP}$.

Finding the corresponding standard vector of a geometric vector:

Let $\overrightarrow{AB}$ any geometric vector, such that $A= (x_a, y_a)$ and $B= (x_b, y_b)$, then the standard vector $\overrightarrow{OP}$ of this geometric vector $\overrightarrow{AB}$, is the vector that has initial point $O= (0,0)$ (origin point) and terminal point $P= (x_p, y_p)$, such that, $P= (x_p, y_p) = (x_b - x_a, y_b - y_a)$, $x_a - x_b$ and $y_a - y_b$ are the **components** of the vector $\overrightarrow{OP}$ such that, $\overrightarrow{OP}= <x_a - x_b, y_a - y_b>$.

Example (1): Given the geometric vector $\overrightarrow{AB}$ with initial point A=(3, 4) and terminal point B=(7, -1), find the standard vector $\overrightarrow{OP}$ of $\overrightarrow{AB}$. That is, find the coordinates of the point p such that $\overrightarrow{OP} = \overrightarrow{AB}$.

Solution:

The coordinates of P given by:

$(x_p, y_p) = (x_b - x_a, y_b - y_a)$

= (7-3, -1-4)

= (4, -5) as shown in the following **(figure 5).**

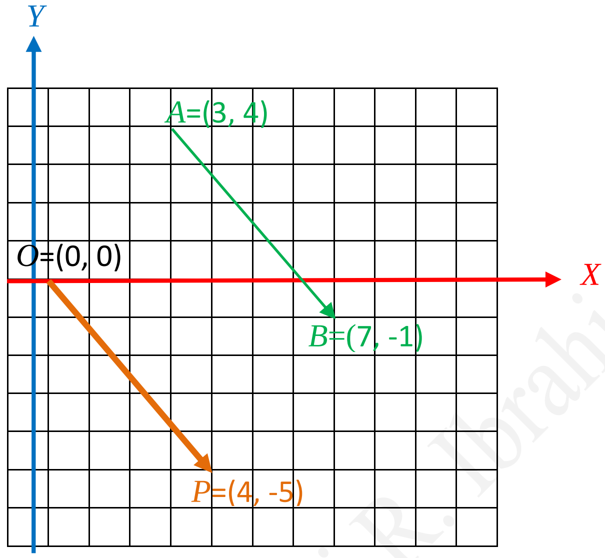

Figure 5

We not in this figure, that if we start at A, then move to the right four units and down five units, we will be at B and if we start at the origin, then move to the right four units and down five units, we will be at P.

Example(2): Given the geometric vector $\overrightarrow{AB}$ with initial point A=(8, -3) and terminal point B=(4, 5), find the standard vector $\overrightarrow{OP}$ of $\overrightarrow{AB}$. **(Homework).**

The preceding discussion suggests another way of looking at vectors. Because given any geometric vector $\overrightarrow{AB}$ in a rectangular coordinate system, there always exists a point $P= (x_p, y_p)$ such that $\overrightarrow{OP} = \overrightarrow{AB}$, the point $P= (x_p, y_p)$ completely specifies the vector $\overrightarrow{AB}$, except for its position and we are not concerned about its position, because we are free to translate $\overrightarrow{AP}$ anywhere we please. Conversely, given any point $P= (x_p, y_p)$ in a rectangular coordinate system, the directed line segment joining O to P forms the geometric $\overrightarrow{OP}$.

This leads us to define an algebraic vector (in 2-space) as an ordered pair of real numbers. To avoid confusing a point (a, b) with a vector (a, b), we use $<a, b>$ (or $\begin{bmatrix}a\\b\end{bmatrix}$, $\begin{pmatrix}a\\b\end{pmatrix}$) to represent an algebraic vector.

Geometrically, the algebraic vector $\mathbf{v} = <a, b>$ corresponds to the standard (geometric) vector $\overrightarrow{OP}$ with terminal point $P= (a, b)$ and initial point $O= (0, 0)$, as illustrated in the following **(figure 6).**

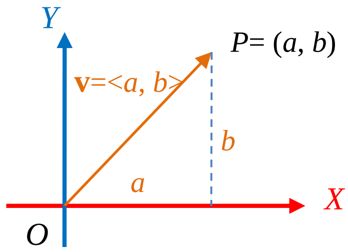

Figure 6

The real numbers *a* and *b* are **scalar components** of the vector <*a*, *b*>.

The word scalar means real number and is often used in the context of vectors in reference to "**scalar quantities**" as opposed to "**vector quantities**". Thus, we talk about "**scalar components**" and "**vector components**" of a given vector.

The word scalar and vector are often dropped if the meaning of component is clear from the context.

Geometric vectors are limited to spaces that we can visualize, that is, to two- and three-dimensional spaces. Algebraic vector does not have these restrictions. The following are algebraic vectors from two-, three-, four- and five dimensional spaces:

<-2, 5>, <3, 0, -8>, <5, 1, 1, -2>, <-1, 0, 1, 3, 4>.

Definition (4): (Parallel vectors and Anti-parallel vectors):

Vectors are **parallel** if they have the same direction **(figure 7-a)**.

Two vectors have the same direction if they are parallel and point in the **same direction** and the angle between these parallel vectors is zero degree. Both components of one vector must be in the same ratio to the corresponding components of the **parallel vector**.

Two vectors have opposite direction if they are parallel and point in the **opposite direction** and the angle between these vectors is 180°, these vectors are called **anti-Parallel Vectors (figure 7-b)**.

Definition (5): (Collinear vectors):

Two or more **vectors** that lie on the same line or on a parallel line to this are called **collinear vectors**. Two **collinear vectors** may point in either same or opposite direction. But, they cannot be inclined at some angle from each other for sure. Angle between collinear vectors is either zero degree or 180°, **(figure 7-c)**.

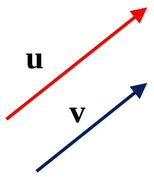

Figure 7-a

Parallel vectors

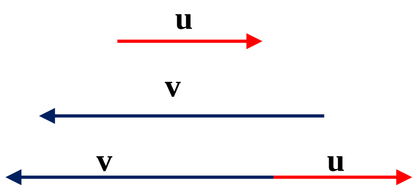

Figure 7-b

Anti-parallel vectors

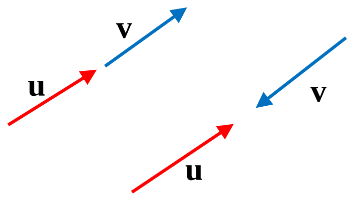

Figure 7-c

Collinear vectors

Definition (6): (Equality of Vectors):

If the vectors have the same value and direction, then we say that they are equal. Positioning of the vectors does not matter **(figure 5).**

Definition (7): Zero vector or (Null vector):

The zero vector is a vector having magnitude equal to zero and denoted by $\vec{0}$ or $\mathbf{0}$ = <0, 0>, this vector has no direction or it may have any direction (an arbitrary direction).

Definition (8): (magnitude of a vector in 2-space):

The magnitude or **norm** of a vector **v**= <*a*, *b*>, is denoted by |**v**| and is given by |**v**|=$\sqrt{a^2+b^2}$ or ||**v**|| = $\sqrt{a^2+b^2}$.

Geometrically, using the theorem of **Pythagoras** we note that $\sqrt{a^2+b^2}$ is the length of the standard geometric vector $\overrightarrow{OP}$ associated with the algebraic vector <*a*, *b*> as shown in **(figure 8).** ***(The same method will be for the vector in 3-space)***.

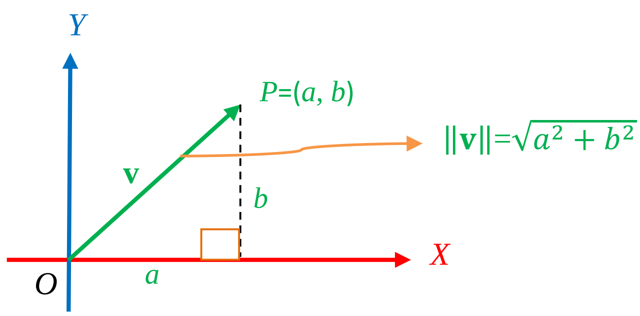

Figure 8

(Geometrically: magnitude of a vector)

The definition of magnitude is readily generalized to higher- dimensional vector space.

<u>For example</u>: If **v**=<a, b, c, d>, then the magnitude, or norm, is given by ||**v**||=$\sqrt{a^2+b^2+c^2+d^2}$.

<u>Example (3):</u> Find the magnitude of the vector **v**=<3, -5>.

Solution: ||**v**||= $\sqrt{3^2+(-5)^2}$ = $\sqrt{34}$

Geometric explanation of vector addition:

If **u**= <u_1, u_2> and **v**=<v_1, v_2>, then **u**+**v**=<u_1+v_1, u_2+v_2> as shown below in **(figure 9).**

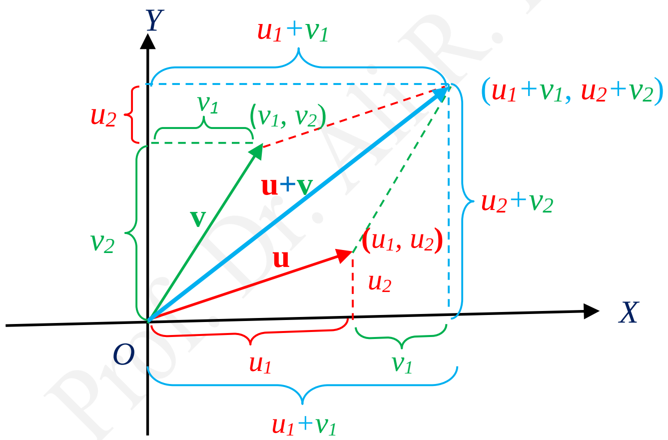

Figure 9
(Geometric explanation of vector addition)

<u>Example (4):</u> If **u**=<-3, 2> and **v**=<7, 3>, then find **u**+**v**.

Solution: **u**+**v**= <-3+7, 2+3>= <4, 5>.

<u>References</u>

1- Introductory linear algebra with applications, Bernard Kolman, first edition, 1976.

2- Elementary Linear Algebra Subsequent Edition, Arthur Wayne Roberts,1985.

3- Elementary Linear Algebra, Ninth Edition, Howard Anton, Chris Rorres, 2005.

4- Student Solutions Manuals for use with College Algebra with Trigonometry: graphs and models, by Raymond A. Barnett, Michael R. Ziegler and Karl E. Byleen, 2005.