02 Scalars and Vectors

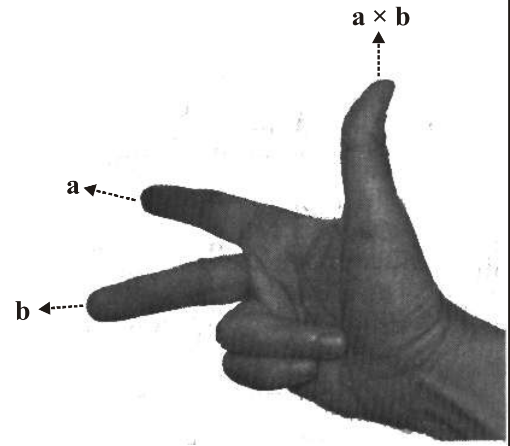
Syllabus

2.0 Introduction

2.1 Addition and substraction of vectors

2.2 Product of vectors

Right hand screw rule in vectors



If we have any two skew vectors $\vec{a}$ and $\vec{b}$, the right-hand rule is used to determine the direction of third vector $\vec{c}$. Vector $\vec{c}$ is normal to the plane containing vectors a and b such that $\vec{c} = \vec{a} \times \vec{b}$.

Formulae

1. **Magnitude of resultant of two vectors $\vec{P}$ and $\vec{Q}$:**

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

2. **Direction of resultant vector :**

$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

Where $\vec{P}$ and $\vec{Q}$ are two adjacent vectors

3. **Commutative law of vector addition :**

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

4. **Associative law of vector addition :**

$$\vec{P}(\vec{Q} + \vec{R}) = (\vec{P} + \vec{Q}) + \vec{R}$$

5. **Distributve law of multiplication over addition :**

i. $\vec{P}\cdot(\vec{Q} + \vec{R}) = \vec{P}\cdot\vec{Q} + \vec{P}\cdot\vec{R}$

ii. $\vec{P}\times(\vec{Q} + \vec{R}) = \vec{P}\times\vec{Q} + \vec{P}\times\vec{R}$

6. **Distributive law of multiplication over subtraction :**

i. $\vec{P}\cdot(\vec{Q} - \vec{R}) = \vec{P}\cdot\vec{Q} - \vec{P}\cdot\vec{R}$

ii. $\vec{P}\times(\vec{Q} - \vec{R}) = \vec{P}\times\vec{Q} - \vec{P}\times\vec{R}$

7. **Magnitude of resolution of a vector :**

i. along two dimensional rectangular components

$$R = \sqrt{R_x^2 + R_y^2}$$

ii. along three dimensional rectangular components

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

8. **Angle of inclination of resultant with positive direction of X–axis :**

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

9. **Scalar (dot) product of two vectors :**

i. $\vec{P}.\vec{Q} = PQ\cos\theta$

ii. $\cos\theta = \dfrac{\vec{P}.\vec{Q}}{|\vec{P}||\vec{Q}|}$

iii. $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

iv. $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

v. If $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

and $\vec{Q} = Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k}$,

$\vec{P}\cdot\vec{Q} = P_xQ_x + P_yQ_y + P_zQ_z$

10. Vector (cross) product of two vectors :

i. $\vec{P}\times\vec{Q} = PQ\sin\theta$

ii. $\sin\theta = \dfrac{\vec{P}\times\vec{Q}}{|\vec{P}||\vec{Q}|}$

iii. Unit vector perpendicular to the cross product

$\hat{n} = \dfrac{\vec{P}\times\vec{Q}}{PQ\sin\theta}$

iv. Cross product of unit vectors

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$

v. If $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

and $\vec{Q} = Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k}$,

$$\vec{P}\times\vec{Q} = (P_yQ_z - P_zQ_y)\hat{i} - (P_xQ_z - P_zQ_x)\hat{j} + (P_xQ_y - P_yQ_x)\hat{k}$$

$$\therefore \vec{P}\times\vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

11. Direction cosine of a vector :

i. $\cos\alpha = \dfrac{R_x}{R}$ ii. $\cos\beta = \dfrac{R_y}{R}$

iii. $\cos\gamma = \dfrac{R_z}{R}$

12. Area of parallelogram :

| cross product of two vectors representing adjacent sides |

13. Area of triangle :

$\dfrac{1}{2}\times$ | cross product of two adjacent sides |

Notes

1. Scalars are added, subtracted and divided algebraically.
2. Vectors are added and subtracted geometrically.
3. The vectors acting in the same plane are called coplanar vectors. The vectors which are perpendicular to each other are called orthogonal vectors.
4. Commutative and associative laws are true for vector addition but not true for subtraction of vectors.
5. Two vectors can be added by using either triangle law or parallelogram law of vector addition.
6. If two vectors $\vec{P}$ and $\vec{Q}$ lie at an angle θ, then the magnitude of their resultant is given by

   $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

   i. If $\theta = 0^0$ i.e., two vectors are parallel then $R = P + Q$

   ii. If $\theta = 90^0$ i.e., two vectors are perpendicular then

   $R = \sqrt{P^2 + Q^2}$

   iii. If $\theta = 180^0$ i.e., they have common point but acting in opposite direction then $R = P - Q$

   iv. Range of resultant of two vectors is,

   $|\vec{P} - \vec{Q}| \le |\vec{R}| \le |\vec{P} + \vec{Q}|$

7. If a vector $\vec{P}$ splits up into two rectangular components then it is given by

   $\vec{P} = \vec{P}_x + \vec{P}_y$

   Horizontal component of $\vec{P} = \vec{P}_x = P\cos\theta$

   Vertical component of $\vec{P} = \vec{P}_y = P\sin\theta$

8. For three rectangular components i.e., along x, y and z axis

   $\vec{P} = \vec{P}_x + \vec{P}_y + \vec{P}_z$

9. Division of vectors is not allowed as directions cannot be divided.
10. Angle between two vectors can be determined either by using dot product or cross product of two vectors.

θ	$\vec{P}.\vec{Q}(PQ\cos\theta)$	$\vec{P}\times\vec{Q}(PQ\sin\theta)$
0^0	PQ	0
90^0	0	PQ
180^0	– PQ	0

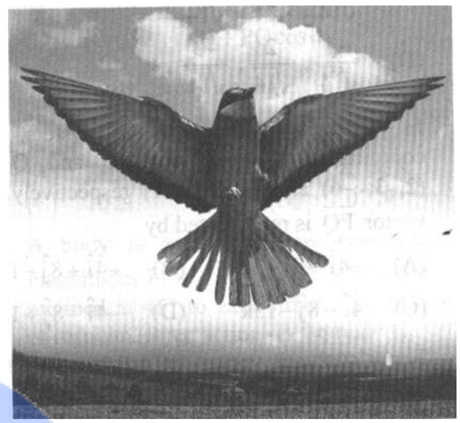
Mindbenders

1. The magnitude of a vector is a scalar while component of a vector is a vector.
2. A quantity having magnitude and direction is not necessarily a vector. eg.: time, finite, angular displacement and electric current. These quantities have magnitude and direction but they are scalars. This is because they do not obey the laws of vector addition.
3. The resultant of two vectors of unequal magnitude can never be a null vector.
4. The magnitude of rectangular components of a vector is always less than the magnitude of the vector.
5. Unit vector does not have any dimensions and unit. It is only used to specify direction.
6. If the frame of reference is rotated or displaced, the vector will not change. If the frame of reference is translated only the components of the vector change. If the frame of reference is rotated the components as well as direction cosines of the vector change.
7. A physical quantity which has different values in different directions is called a tensor.
   e.g.: Moment of inertia

Shortcuts

1. If $\vec{A}_1+\vec{A}_2+\vec{A}_3+....+\vec{A}_n=\vec{0}$ and $A_1 = A_2 = A_3 = ... = A_n$ then the adjacent vectors are inclined to each other at an angle $\frac{2\pi}{n}$.
2. If $\vec{A}+\vec{B}=\vec{C}$ and $A^2 + B^2 = C^2$, then the angle between $\vec{A}$ and $\vec{B}$ is 90^0. Also some values that A, B and C can have are:
   i. A = 3, B = 4, C = 5
   ii. A = 5, B = 12, C = 13
   iii. A = 8, B = 15, C = 17
3. If $|\vec{A}\times\vec{B}|=|\vec{A}-\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is 90^0.
4. Angle between $(\vec{A}+\vec{B})$ and $(\vec{A}\times\vec{B})$ is 90^0.
5. Angle between $(\hat{i}+\hat{j})$ and $\hat{i}$ is 45^0 and that between $(\hat{i}+\hat{j})$ and $\hat{k}$ is 90^0.
6. Unit vector :
   i. along $(\hat{i}+\hat{j})$ is $(\hat{i}+\hat{j})/\sqrt{2}$
   ii. along $(\hat{i}+\hat{j}+\hat{k})$ is $(\hat{i}+\hat{j}+\hat{k})/\sqrt{3}$
   iii. along $(\hat{i}-\hat{j})$ is $(\hat{i}-\hat{j})/\sqrt{2}$
7. Two vectors are collinear if their dot product equals the product of their magnitudes and their cross product is zero.

Flight of bird



When a bird has to fly, it presses air with its two wings along on oblique direction. The resultant of the reactions on the two wings of the bird enables it to fly up in the sky. Thus, it is an example of composition of vectors.

Multiple Choice Questions

Classical Thinking

2.0 Introduction

1. Vectors are physical quantities which are completely specified by _______ .
   a) magnitude only
   b) number only
   c) direction only
   d) both magnitude and direction
2. The magnitude of a vector cannot be _______
   a) zero b) negative
   c) positive d) unity
3. Which of the following is a scalar ?
   a) Displacement b) Kinetic energy
   c) Couple d) Momentum
4. Which of the following is a scalar ?
   a) Torque b) Linear momentum
   c) Electric field d) Electric potential
5. Out of the following physical quantities which is NOT a scalar ?
   a) Angular velocity b) Angular frequency
   c) Number of moles d) Total path length
6. Which of the following quantity is a vector ?
   a) pressure b) time
   c) impulse d) charge

7. The vectors of the same quantity having same magnitude and same direction are called _______
   a) parallel vectors b) equal vectors
   c) zero vectors d) negative vectors
8. A single vector which produces the same effect of two or more vectors is called _______
   a) position vector b) resultant vector
   c) positive vector d) equal vector

2.1 Addition and subtraction of vectors

9. Choose the INCORRECT statement.
   a) Vectors having same direction can be added.
   b) Vectors having same magnitude can be added.
   c) Vectors having different physical quantities can be added.
   d) Vectors representing same physical quantity can be added.
10. Vector subtraction is _______
   a) non-commutative only
   b) non-associative only
   c) neither non-commutative nor non-associative
   d) neither commutative nor associative
11. The process of finding the resultant of two or more vectors is called _______
   a) resolution of vectors
   b) addition of vectors only
   c) subtraction of vectors only
   d) composition of vectors
12. The resultant of two vectors will be maximum, if they are _______
   a) equal vectors b) parallel vectors
   c) coplanar vectors d) orthogonal vectors
13. The resultant of two vectors will be minimum, if they are
   a) equal vectors
   b) parallel to each other
   c) coplanar vectors
   d) perpendicular to each other
14. The process of finding the components of a given vector is called as _______
   a) composition of vector
   b) multiplication of vector
   c) addition of vector
   d) resolution of vector
15. If the component of one vector in the direction of another vector is zero, then those two vectors are _______.
   a) parallel to each other
   b) opposite to each other
   c) coplanar vectors
   d) perpendicular to each other
16. Under what condition $|\vec{A}+\vec{B}|=|\vec{A}|+|\vec{B}|$ holds good ?
   a) $\vec{A}$ and $\vec{B}$ act in the same direction
   b) $\vec{A}$ and $\vec{B}$ act in the opposite direction
   c) $\vec{A}$ and $\vec{B}$ are different physical quantities
   d) $\vec{A}$ and $\vec{B}$ have same magnitude
17. Law of polygon of vectors is a repeated use of
   a) triangle law of vectors
   b) parallelogram law of vectors
   c) addition of vectors in one dimension
   d) multiplication law of vectors
18. In parallelogram law of vectors, the direction of resultant vector is given by
   a) $\tan\alpha = \dfrac{Q\cos\theta}{P+Q\sin\theta}$
   b) $\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}$
   c) $\tan\alpha = \dfrac{P\sin\theta}{P+Q\cos\theta}$
   d) $\tan\alpha = \dfrac{P\cos\theta}{P+Q\cos\theta}$
19. If vector $\vec{A} = 3\hat{i}+2\hat{j}-4\hat{k}$, its magnitude is
   a) 1 b) $\sqrt{3}$
   c) $\sqrt{9}$ d) $\sqrt{29}$
20. A vector is represented by $\vec{P} = 3\hat{i}+\hat{j}+2\hat{k}$, its length in XY plane is
   a) 2 unit b) $\sqrt{5}$ unit
   c) $\sqrt{10}$ unit d) $\sqrt{15}$ unit
21. The direction cosines of $\vec{A} = -\hat{i}+2\hat{j}+3\hat{k}$ is
   a) $\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}$ b) $-\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}$
   c) $\dfrac{1}{\sqrt{14}}, \dfrac{-2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}$ d) $\dfrac{-1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{-3}{\sqrt{14}}$
22. In a cartesian coordinate system, the coordinate of two points P and Q are (2, 3, –6) and (–2, –5, 7) respectively, the vector $\overline{PQ}$ is represented by
   a) $-4\hat{i}-8\hat{j}-13\hat{k}$ b) $-4\hat{i}+8\hat{j}-13\hat{k}$
   c) $4\hat{i}-8\hat{j}-13\hat{k}$ d) $-4\hat{i}-8\hat{j}+13\hat{k}$

23. Three coplanar vectors in arbitrary units are given by $\vec{A}=4\hat{i}+2\hat{j}-3\hat{k}$, $\vec{B}=\hat{i}+\hat{j}+3\hat{k}$ and $\vec{C}=4\hat{i}+5\hat{j}+3\hat{k}$, the resultant is

a) $8\hat{i}+3\hat{j}+3\hat{k}$ b) $5\hat{i}+3\hat{j}-3\hat{k}$

c) $9\hat{i}+8\hat{j}+12\hat{k}$ d) $9\hat{i}+8\hat{j}+3\hat{k}$

24. The unit vector parallel to the resultant of the vectors $\vec{A}=4\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{B}=-\hat{i}+3\hat{j}-8\hat{k}$ is

a) $\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$ b) $\frac{1}{7}(3\hat{i}+6\hat{j}+2\hat{k})$

c) $\frac{1}{49}(3\hat{i}+6\hat{j}-2\hat{k})$ d) $\frac{1}{49}(3\hat{i}-6\hat{j}+2\hat{k})$

25. If $\vec{A}=3\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{B}=5\hat{i}-7\hat{j}+2\hat{k}$, which vector when added to $\vec{A}+\vec{B}$ will give unit vector along X–axis ?

a) $7\hat{i}+5\hat{j}+2\hat{k}$ b) $-7\hat{i}-5\hat{j}+2\hat{k}$

c) $-7\hat{i}+5\hat{j}+2\hat{k}$ d) $7\hat{i}+5\hat{j}-2\hat{k}$

26. The magnitude of the resultant of two vectors $\vec{P}$ and $\vec{Q}$ is R. It is given by

a) $R=\sqrt{P^2+Q^2+2PQ\sin\theta}$

b) $R=\sqrt{P^2+Q^2+2PQ\cos\theta}$

c) $R=\sqrt{P^2+Q^2+PQ\sin\theta}$

d) $R=\sqrt{P^2+Q^2+PQ\cos\theta}$

27. Two equal forces acting at a point, at right angle to each other have a resultant of 14.14 N. The magnitude of each force is

a) 28.28 N b) 24.14 N

c) 10 N d) 7.07 N

28. A body is acted upon by two forces of magnitudes $F_1=\sqrt{2}$ N and $F_2=3$ N which are inclined at 45^0 to each other. The magnitude of resultant force acting on the body is

a) 17 N b) 11 N

c) $\sqrt{17}$ N d) $\sqrt{11}$ N

29. The velocity of a body is 20 m/s making an angle of 30^0 with the horizontal, the vertical component of velocity is

a) 20 m/s b) 17.32 m/s

c) 10 m/s d) 7 m/s

30. A body of mass 10 kg is placed on a smooth inclined plane making an angle of 30^0 with the horizontal, the component of the force of gravity trying to move the body down the inclined plane is ($g = 9.8$ m/s^2)

a) 98 N b) 49 N

c) 10 N d) 5 N

2.2 Product of vectors

31. The vectors $\vec{A}=6\hat{i}+9\hat{j}-3\hat{k}$ and $\vec{B}=2\hat{i}+3\hat{j}-\hat{k}$ are

a) parallel b) antiparallel

c) perpendicular d) identical

32. Two vectors of different physical quantities can be _____ to obtain a scalar

a) added b) subtracted

c) multiplied d) divided

33. Choose the WRONG statement.

a) Scalar product of two vectors is a scalar quantity

b) Dot product of two vectors obeys the distributive law of multiplication

c) Dot product of a vector with itself is zero

d) Scalar product of vector with itself is equal to square of its magnitude

34. The scalar product of electric field intensity and area vector through which the line of force emerges is _______ .

a) electric potential

b) electric current

c) electric charge density

d) electric flux

35. The example of dot product is ______ .

a) angular momentum

b) moment of force

c) linear velocity of terms of angular velocity

d) magnetic flux linked with the surface of mangetic induction

36. Two vectors $\vec{A}$ and $\vec{B}$ are at right angles to each other then

a) $\vec{A}.\vec{B}=0$ b) $\vec{A}\times\vec{B}=0$

c) $\frac{\vec{A}}{\vec{B}}=0$ d) $\frac{\vec{B}}{\vec{A}}=0$

37. Two vectors $\vec{P}$ and $\vec{Q}$ are given by $\vec{P}=5\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{Q}=2\hat{i}+2\hat{j}-a\hat{k}$, If they are mutually perpendicular then value of 'a' is
a) 8 b) 5
c) 3 d) – 8

38. A force of $(5\hat{i}+6\hat{j})N$ makes a body to move on a rough surface with a velocity of $(4\hat{i}-2\hat{k})m/s$. What is the power ?
a) 8 unit b) 13 unit
c) 14 unit d) 24 unit

39. A constant force of $(2\hat{i}+3\hat{j}+5\hat{k})N$ produces a displacement of $(3\hat{i}+2\hat{j}+2\hat{k})m$. Then work done is
a) 5 J b) 15 J
c) 22 J d) 50 J

40. The angle between the vectors $\vec{P}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{Q}=\hat{i}-2\hat{j}+3\hat{k}$ is
a) 120^0 b) 90^0
c) 60^0 d) 45^0

41. The angle between the following pair of vectors $\vec{A}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}=-\hat{i}-\hat{j}+2\hat{k}$ is
a) 150^0 b) 120^0
c) 90^0 d) 30^0

42. What is the dot product of two vectors of magnitude 3 and 5, if the angle between them is 60^0 ?
a) 15 b) 8
c) 7.5 d) 5.3

43. The vector product of two vectors is a vector whose direction is given by
a) Left hand thumb rule
b) Right hand thumb rule
c) Fleming's left hand rule
d) Biot–Savart's rule

44. The magnitude of self cross product is
a) zero
b) magnitude of vector
c) square of the magnitude of vector
d) half the magnitude of vector

45. The vector product of two non-zero vectors is zero
a) only when they are in the same direction
b) only when they are making angle 60^0
c) only when they are perpendicular
d) whey they are parallel or antiparallel

46. The example of cross product is ________ .
a) power b) torque
c) work d) electric flux

47. If $\vec{A}=-2\hat{i}+3\hat{j}-4\hat{k}$ and $\vec{B}=-3\hat{i}-4\hat{j}+5\hat{k}$ then $\vec{A}\times\vec{B}$ is
a) $\hat{i}-2\hat{j}-\hat{k}$ b) $-\hat{i}+2\hat{j}-\hat{k}$
c) $-\hat{i}-2\hat{j}+\hat{k}$ d) $-\hat{i}-2\hat{j}-\hat{k}$

48. Determine a vector product of $\vec{A}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}=-3\hat{i}+\hat{j}-2\hat{k}$
a) $3\hat{i}-\hat{j}+4\hat{k}$ b) $-3\hat{i}+\hat{j}+4\hat{k}$
c) $3\hat{i}+\hat{j}-4\hat{k}$ d) $-3\hat{i}-\hat{j}+4\hat{k}$

49. If $\vec{P}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{Q}=3\hat{i}+\hat{j}-\hat{k}$ then $\vec{P}\times\vec{Q}$ is
a) $-3\hat{i}+4\hat{j}-5\hat{k}$ b) $3\hat{i}-4\hat{j}+5\hat{k}$
c) $3\hat{i}+4\hat{j}-5\hat{k}$ d) $3\hat{i}-4\hat{j}-5\hat{k}$

50. Linear momentum $\vec{p}=2\hat{i}+4\hat{j}+5\hat{k}$ and position vector is $\vec{r}=3\hat{i}-\hat{j}+2\hat{k}$, the angular momentum is given by
a) $3\hat{i}-19\hat{j}+14\hat{k}$ b) $13\hat{i}+19\hat{j}+14\hat{k}$
c) $-3\hat{i}-19\hat{j}+14\hat{k}$ d) $-13\hat{i}-11\hat{j}+14\hat{k}$

51. The area of triangle formed by the sides of vector $\vec{A}$ and $\vec{B}$ is
a) $|\vec{A}\times\vec{B}|$ b) $|\vec{A}.\vec{B}|$
c) $\frac{1}{2}|\vec{A}.\vec{B}|$ d) $\frac{1}{2}|\vec{A}\times\vec{B}|$

52. The area of the triangle having two sides $\vec{A}=\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{B}=2\hat{i}+2\hat{j}+3\hat{k}$ is
a) $\sqrt{45}$ sq. unit b) 22.5 sq. unit
c) 4.717 sq. unit d) 9.43 sq. unit

53. Area of parallelogram whose adjacent sides are $(\hat{i}+2\hat{j}+3\hat{k})m$ and $(\hat{i}-3\hat{j}+\hat{k})m$ is

a) $\sqrt{50}\ m^2$ b) $\sqrt{150}\ m^2$
c) $25\ m^2$ d) $\sqrt{75}\ m^2$

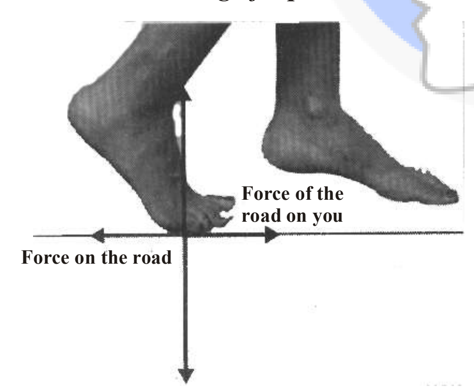
Miscellaneous

54. If $\vec{P} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{Q} = \hat{i} + 2\hat{j} - \hat{k}$ then $(\vec{P}+\vec{Q})\cdot(\vec{P}-\vec{Q})$ is
a) 5 b) 15
c) 25 d) 115

55. If $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$. The value of $(2\vec{A} - \vec{B})\cdot(\vec{A} + 2\vec{B})$ is
a) 30 b) 40
c) 55 d) 90

56. A particle moves from position $\vec{s}_1 = (3\hat{i} + 3\hat{j} - 6\hat{k})m$ to position of $\vec{s}_2 = (14\hat{i} + 13\hat{j} + 9\hat{k})m$, under the action of a force $\vec{F} = (4\hat{i} + \hat{j} + 3\hat{k})N$. The work done by the force is
a) 200 J b) 100 J
c) 75 J d) 50 J

Walking of a person



Walking on a person is an example of resolution of a vector. When a person walks, he presses the ground obliquely, in the backward direction. The ground offers an equal and opposite reaction in the opposite direction. The vertical component of this reaction balances the weight of the person. The horizontal component helps the person to walk.

Critical Thinking

2.0 Introduction

1. Scalars are physical quantities which are completely specified by _______ .
a) number and unit b) number only
c) unit only d) neither number nor unit

2. A vector is not changed if _______ .
a) it is divided by a scalar
b) it is multiplied by a scalar
c) it slides parallel to itself
d) all of these

3. The velocity vector of a stationary particle is
a) zero vector
b) vector with magnitude of velocity vector
c) scalar
d) scalar with magnitude of velocity vector

4. If the angle between two collinear vectors is π radians, vectors are said to be _______ .
a) antiparallel vectors
b) parallel vectors
c) similar vectors
d) identical vectors

5. If the angular displacement is large, it is a scalar quantity because
a) its magnitude for large values cannot be calculated
b) it is not coplanar for large values
c) it will not obey the commutative law of vector addition
d) it will not obey principle of homogeneity

6. Angular momentum is
a) a scalar b) a polar vector
c) an axial vector d) none of these

2.1 Addition and subtraction of vectors

7. The component of a vector may be
a) equal to its magnitude
b) double its magnitude
c) greater than its magnitude
d) either greater or equal to its magnitude

8. Which of the following is NOT essential for three forces to produce zero resultant ?
a) They should be in same plane
b) It should be possible to represent then by the three sides of a triangle taken in the same order
c) They should act along the sides of parallelogram
d) The resultant of any two forces should be equal and opposite to the third force

9. Following sets of three forces act on a body. Whose resultant cannot be zero ?
a) 10, 10, 10 b) 10, 10, 20
c) 10, 20, 30 d) 10, 20, 40

10. If more than three forces are acting on a heavy rigid body such that the body is in balanced state, then all the forces are
a) collinear
b) coplanar

c) acting in random direction
d) represented by the sides of a polygon of vectors

11. The vector projection of a vector $3\hat{i}+4\hat{k}$ on Y–axis is
a) five b) four
c) three d) zero

12. A vector is represented by $3\hat{i}+\hat{j}+2\hat{k}$. Its length in XY plane is
a) 2 b) $\sqrt{14}$
c) $\sqrt{10}$ d) $\sqrt{5}$

13. A particle is simultaneously acted by two forces equal to 4 N and 3 N. The net force on the particle is
a) 7 N b) 5 N
c) 1 N d) between 1 N and 7 N

14. The vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$. The angle between the two vectors is
a) 90^0 b) 180^0
c) 0^0 d) 45^0

15. The resultant of two vectors at right angles is 5 N. If the angle between them is 120^0 and the resultant is $\sqrt{13}$ then the vectors are
a) $\sqrt{3}$ N, $\sqrt{4}$ N b) $\sqrt{2}$ N, $\sqrt{5}$ N
c) 3 N, 4 N d) 7 N, 3 N

16. $\vec{A}$ is a vector with magnitude A, then the unit vector $\hat{A}$ in the direction of $\vec{A}$ is
a) $A\vec{A}$ b) $\vec{A}.\vec{A}$
c) $\vec{A}\times\vec{A}$ d) $\dfrac{\vec{A}}{A}$

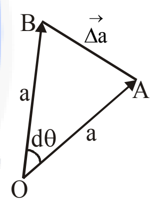
17. If a unit vector is represented by $0.5\hat{i}-0.8\hat{j}+c\hat{k}$ then the value of c is
a) $\sqrt{0.01}$ b) $\sqrt{0.11}$
c) 1 d) $\sqrt{0.39}$

18. A unit vector in the direction of the negative of the vector $(-\hat{i}+\hat{j}-\hat{k})$ is
a) $\dfrac{-1}{\sqrt{3}}(-\hat{i}+\hat{j}-\hat{k})$ b) $\sqrt{3}(\hat{i}+\hat{j}-\hat{k})$
c) $\dfrac{1}{\sqrt{2}}(\hat{i}+\hat{j}-\hat{k})$ d) $\dfrac{-1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$

19. If $\vec{A}=2\hat{i}+6\hat{j}$ and $\vec{B}=4\hat{i}+3\hat{j}$, the vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$ is
a) $\dfrac{5}{2}(2\hat{i}-6\hat{j})$ b) $\dfrac{\sqrt{10}}{4}(\hat{i}-3\hat{j})$
c) $\dfrac{\sqrt{10}}{4}(4\hat{i}+3\hat{j})$ d) $\dfrac{\sqrt{10}}{2}(\hat{i}+3\hat{j})$

20. If the sum of two unit vectors is a unit vector, then magnitude of difference is
a) $\sqrt{2}$ b) $\sqrt{3}$
c) $\dfrac{1}{\sqrt{2}}$ d) $\sqrt{5}$

21. A vector $\vec{a}$ is turned without a change in its length through a small angle dθ. The value of $|\Delta\vec{a}|$ and Δa are respectively



a) 0, adθ b) adθ, 0
c) 0, 0 d) adθ, adθ

22. A vector of magnitude a is rotated through angle θ. What is the magnitude of the change in the vector ?
a) 2 a sin θ/2 b) 2 a cos θ/2
c) 2 a sin θ d) 2 a cos θ

23. The resultant of two vector $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant becomes perpendicular to $\vec{P}$. Then the magnitude of $\vec{R}$ is
a) P + Q b) Q
c) P d) $\dfrac{P+Q}{2}$

2.2 Product of vectors

24. A force vector applied on a mass is represented as $\vec{F}=6\hat{i}-8\hat{j}+10\hat{k}$ and accelerates with 1 m/s². What will be the mass of the body in kg ?

a) $10\sqrt{2}$ b) 20
c) $2\sqrt{10}$ d) 10

25. Vectors $\vec{A} = 2\hat{i} - 3\hat{j} + a\hat{k}$ and $\vec{B} = 12\hat{i} - b\hat{j} + 6\hat{k}$ are parallel to each other, then value of 'a' and 'b' are
a) 1, 18 b) 1, – 18
c) – 1, 18 d) – 1, – 18

26. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{i} - 4\hat{j} + m\hat{k}$, then the value of m is
a) $\frac{1}{2}$ b) $-\frac{1}{2}$
c) 1 d) – 1

27. A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle displaces it. The displacement is given by $\vec{s} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in its own direction. If the work done is 6 J, then value of 'c' is
a) 0 b) 1
c) – 6 d) 12

28. Work done when a force of $(7\hat{i} - 4\hat{j} - 4\hat{k})$N moves a body through a distance of 10 metre in its own direction is
a) 160 J b) 120 J
c) 90 J d) 10 J

29. If $\vec{P} = \hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + 6\hat{k}$, the angle made by $\vec{P} + \vec{Q}$ with X–axis is
a) 30^0 b) 45^0
c) 60^0 d) 90^0

30. Choose the CORRECT statement.
a) The vector product does not obey commutative law but obeys distributive law of multiplication
b) The vector product obeys commutative law of multiplication but does not obey distributive law of multiplication
c) The vector product does not obey both commutative and distributive law of multiplication
d) The vector product obeys both commutative and distributive law of multiplication

31. The sine of the angle between $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$ is
a) 1 b) 0.91
c) 0.76 d) 0.67

32. If $\vec{A}.\vec{B} = 0$ and $\vec{A} \times \vec{B} = 0$, then which of the following conditions is necessary ?
a) A = 1, B = 0 b) A = 0 and B = 0
c) A = 0 or B = 0 d) A = 0, B = 1

33. If the ratio of the dot product of two vectors and cross product of same two vectors is $\sqrt{3}$, the two vectors make angle
a) 30^0 b) 45^0
c) 90^0 d) 120^0

Miscellaneous

34. Select the WRONG one.
a) $\vec{P} \times \vec{Q} \neq \vec{Q} \times \vec{P}$
b) $\vec{P} \times \left(\vec{Q} \times \vec{R}\right) = \left(\vec{P} \times \vec{Q}\right) \times \vec{R}$
c) $\vec{P}.\vec{Q} = \vec{Q}.\vec{P}$
d) $\vec{P} \times \left(\vec{Q} + \vec{R}\right) = \vec{P} \times \vec{Q} + \vec{P} \times \vec{R}$

35. If $\vec{A}$ and $\vec{B}$ are two vectors then $\left(\vec{A} + \vec{B}\right) \times \left(\vec{A} - \vec{B}\right)$ is
a) $2\left(\vec{B} \times \vec{A}\right)$ b) $\left(\vec{B} \times \vec{A}\right)$
c) $2\left(\vec{A} + \vec{B}\right)$ d) $2\left(\vec{A} - \vec{B}\right)$

36. Given $\vec{P}.\left(\vec{P} + \vec{Q}\right) = P^2$ then the angle between $\vec{P}$ and $\vec{Q}$ is
a) 0^0 b) 30^0
c) 45^0 d) 90^0

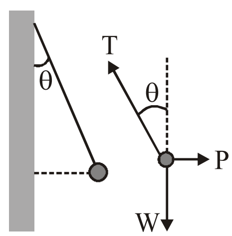
37. **Assertion :** If dot product and cross product of $\vec{A}$ and $\vec{B}$ are zero, it implies that one of the vector $\vec{A}$ and $\vec{B}$ must be a null vector.
Reason : Null vector is a vector with zero magnitude.
a) Assertion is True, Reason is True; Reason is a correct explanation for Assertion.
b) Assertion is True, Reason is True; Reason is not a correct explanation for Assertion
c) Assertion is True, Reason is False
d) Assertion is False, Reason is True

38. **Assertion :** $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}+\vec{B}$ as well as $\vec{A}-\vec{B}$

Reason : $\vec{A}+\vec{B}$ as well as $\vec{A}-\vec{B}$ lie in the plane.

a) Assertion is True, Reason is True; Reason is a correct explanation for Assertion.
b) Assertion is True, Reason is True; Reason is not a correct explanation for Assertion
c) Assertion is True, Reason is False
d) Assertion is False, Reason is True

39. A small ball is hung by a string fixed to a wall. The ball is pushed away from the wall. The forces acting on the ball are shown in the diagram. Which of the following statements is wrong ?



a) $P = W \tan\theta$ b) $\vec{T}+\vec{P}+\vec{W}=0$
c) $T^2 = P^2 + W^2$ d) $T = P + W$

Competitive Thinking

2.0 Introduction

1. Surface area is
a) Scalar
b) Vector
c) Neither scalar nor vector
d) Both scalar and vector

2.1 Addition and subtraction of vectors

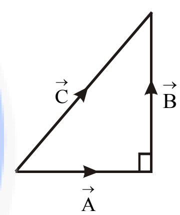
2. Can the resultant of 2 vectors be zero ?
a) Yes, when the 2 vectors are same in magnitude and direction
b) No
c) Yes, when the 2 vectors are same in magnitude but opposite in sense
d) Yes, when the 2 vectors are same is magnitude making an angle of $\frac{2\pi}{3}$ with each other.

3. Two vectors $\vec{A}$ and $\vec{B}$ are acting in the same plane and the vector $\vec{C}$ is perpendicular to the plane. The resultant of these vectors
a) may be zero
b) can not be zero
c) lies between $\vec{A}$ and $\vec{B}$
d) lies between $\vec{A}$ and $-\vec{B}$

4. If $|\vec{A}+\vec{B}|=|\vec{A}|+|\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be
a) 90^0 b) 120^0
c) 0^0 d) 60^0

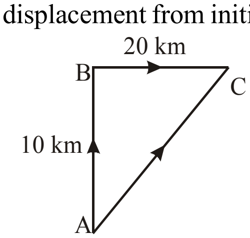
5. A bird flies from (–3 m, 4 m, –3 m) to (7 m, –2 m, –3 m) in XYZ co–ordinates. The bird's displacement in unit vectors is given by
a) $(4\hat{i}+\hat{j}-6\hat{k})$ b) $(10\hat{i}+6\hat{j})$
c) $(10\hat{i}-6\hat{j})$ d) $(10\hat{i}+6\hat{j}-6\hat{k})$

6. The magnitudes of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 3, 4 and 5 unit respectively. If $\vec{A}+\vec{B}=\vec{C}$, the angle between $\vec{A}$ and $\vec{B}$ is



a) $\frac{\pi}{2}$ b) $\cos^{-1}(0.6)$
c) $\tan^{-1}\left(\frac{7}{5}\right)$ d) $\frac{\pi}{4}$

7. A person goes 10 km north and 20 km west. What will be the displacement from initial point ?



a) 22.36 km b) 2 km
c) 5 km d) 20 km

8. A particle has displacement of 12 m towards east and 5 m towards north then 6 m vertically upward. The sun of these displacement is
a) 12 m b) 10.04 m
c) 14.31 m d) None of these

9. Two equal forces are acting at a point with an angle of 60^0 between them. If the resultant force is equal to $40\sqrt{3}$ N, the magnitude of each force is
a) 40 N b) 20 N
c) 80 N d) 30 N

10. The resultant force of 5 N and 10 N cannot be
a) 12 N b) 8 N
c) 4 N d) 5 N

11. The maximum and minimum magnitude of the resultant of two given vectors are 17 units and 7 units respectively. If these two vectors are at right angles to each other, the magnitude of their resultant is
a) 18 b) 16
c) 14 d) 13

12. If $\vec{a}=4\hat{i}-\hat{j}$, $\vec{b}=-3\hat{i}+2\hat{j}$ and $\vec{c}=-\hat{k}$. Then the unit vector $\hat{r}$ along the direction of sum of these vectors will be
a) $\hat{r}=\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ b) $\hat{r}=\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}-\hat{k})$
c) $\hat{r}=\frac{1}{3}(\hat{i}-\hat{j}+\hat{k})$ d) $\hat{r}=\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$

13. The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is
a) 60^0 b) 120^0
c) 150^0 d) 90^0

14. Two forces are such that the sum of their magnitudes is 18 N and their resultant is perpendicular to the smaller force and magnitude of resultant is 12 N. Then the magnitudes of the forces are
a) 12 N, 6 N b) 13 N, 5 N
c) 10 N, 8 N d) 16 N, 2 N

15. Two forces with equal magnitudes F act on a body and the magnitude of the resultant force is $\frac{F}{3}$. The angle between the two forces is
a) $\cos^{-1}\left(-\frac{17}{18}\right)$ b) $\cos^{-1}\left(-\frac{1}{3}\right)$
c) $\cos^{-1}\left(-\frac{2}{3}\right)$ d) $\cos^{-1}\left(-\frac{8}{9}\right)$

16. Two forces of equal magnitude F are at a point. If θ is the angle between two forces then magnitude of the resultant force will be
a) $2F\cos\frac{\theta}{2}$ b) $F\cos\frac{\theta}{2}$
c) 2F cos θ d) $\frac{F}{2}\cos\frac{\theta}{2}$

17. Two equal vectors have a resultant equal to either of them. The angle between them is
a) 60^0 b) 90^0
c) 100^0 d) 120^0

18. Two forces 3 N and 2 N are at an angle θ such that the resultant is R. The first force is now increased to 6 N and the resultant because 2R. The value of θ is
a) 30^0 b) 60^0
c) 90^0 d) 120^0

19. The resultant of two forces 3P and 2P is R. If the first force is doubled then the resultant is also doubled. The angle between the two forces is
a) 60^0 b) 120^0
c) 70^0 d) 180^0

20. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A}+\vec{B}$ is equal to n times the magnitude of $\vec{A}-\vec{B}$, then angle between $\vec{A}$ and $\vec{B}$, then angle between $\vec{A}$ and $\vec{B}$ is
a) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$ b) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$
c) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$ d) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

21. The angle between two vectors A and B is θ. Vector R is the resultant of the two vectors. If R makes an angle $\frac{\theta}{2}$ with A, then
a) A = 2B b) $A=\frac{B}{2}$
c) A = B d) AB = 1

2.2 Product of vectors

22. The magnitude of the component of the vector $2\hat{i}+3\hat{j}+\hat{k}$ along $3\hat{i}+4\hat{k}$ is
a) $\frac{1}{2}$ b) $\frac{14}{5}$
c) 2 d) $\frac{6}{5}$

23. $\vec{A}$ and $\vec{B}$ are two vectors given by $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}$. The magnitude of the component of $\vec{A}$ along $\vec{B}$ is
a) $\frac{5}{\sqrt{2}}$ b) $\frac{3}{\sqrt{2}}$

c) $\frac{7}{\sqrt{2}}$ d) $\frac{1}{\sqrt{2}}$

24. If the two vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - x\hat{k}$ are perpendicular then the value of x is
a) 1 b) 2
c) 3 d) 4

25. The vector $\vec{P} = a\hat{i} - a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other. The positive value of a is
a) 3 b) 4
c) 9 d) 13

26. Consider two vectors, $\vec{F}_1 = 2\hat{j} + 5\hat{k}$ and $\vec{F}_2 = 3\hat{j} + 4\hat{k}$. The magnitude of the scalar product of these vectors is
a) 20 b) 23
c) $5\sqrt{33}$ d) 26

27. A force $(4\hat{i} + \hat{j} - 2\hat{k})N$ acting on a body maintains its velocity at $(2\hat{i} + 2\hat{j} + 3\hat{k})ms^{-1}$. The power exerted is
a) 4 W b) 5 W
c) 2 W d) 8 W

28. When $\vec{A}.\vec{B} = -|\vec{A}||\vec{B}|$, then
a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other
b) $\vec{A}$ and $\vec{B}$ act in the same direction
c) $\vec{A}$ and $\vec{B}$ act in the opposite direction
d) $\vec{A}$ and $\vec{B}$ can act in any direction

29. The angle between the two vectors, $\left(\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}\right)$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ m will be
a) zero b) 45^0
c) 90^0 d) 180^0

30. The angle θ between the vector $\vec{p} = \hat{i} + \hat{j} + \hat{k}$ and unit vector along X-axis is
a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ b) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$
c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ d) $\cos^{-1}\left(\frac{1}{2}\right)$

31. The angle between the vectors $(\hat{i} + \hat{j})$ and $(\hat{j} + \hat{k})$ is
a) 30^0 b) 45^0
c) 60^0 d) 90^0

32. A particle moves in the x–y plane under the action of a force $\vec{F}$ such that the value of its linear momentum $(\vec{P})$ at any time t is $P_x = 2\cos t$, $P_y = 2\sin t$. The angle θ between $\vec{F}$ and $\vec{P}$ at a given time t will be
a) $\theta = 0^0$ b) $\theta = 30^0$
c) $\theta = 90^0$ d) $\theta = 180^0$

33. In an clockwise system
a) $\hat{j} \times \hat{k} = \hat{i}$ b) $\hat{i}.\hat{i} = 0$
c) $\hat{j} \times \hat{j} = 1$ d) $\hat{k}.\hat{j} = 1$

34. For vectors $\vec{A}$ and $\vec{B}$ making an angle θ which one of the following relations is correct ?
a) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ b) $\vec{A} \times \vec{B} = AB\sin\theta$
c) $\vec{A} \times \vec{B} = AB\cos\theta$ d) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

35. A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is
a) zero b) along west
c) along east d) vertically downward

36. Which of the following relation is not correct ?
a) $\vec{V} = \vec{\omega} \times \vec{r}$ b) $\vec{V} = \vec{r} \times \vec{\omega}$
c) $\vec{\delta s} = \vec{\delta\theta} \times \vec{r}$ d) $V = r\omega$

37. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$
a) $6\hat{i} - 2\hat{j} + 3\hat{k}$ b) $6\hat{i} - 2\hat{j} + 8\hat{k}$
c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ d) $18\hat{i} + 13\hat{j} - 2\hat{k}$

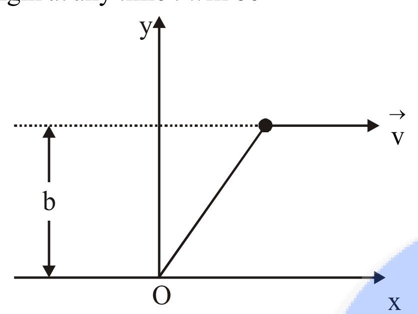
38. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then angle between $\vec{A}$ and $\vec{B}$ is
a) π b) π/3
c) π/2 d) π/4

39. What is the torque of force $F=(-3\hat{i}+\hat{j}+5\hat{k})$ acting at point whose position vector is $r=7\hat{i}+3\hat{j}+\hat{k}$?

a) $14\hat{i}-38\hat{j}+16\hat{k}$ b) $4\hat{i}+4\hat{j}+\hat{k}$

c) $-14\hat{i}+38\hat{j}+16\hat{k}$ d) $-21\hat{i}+3\hat{j}+5\hat{k}$

40. If a particle of mass m is moving with constant velocity v parallel to x-axis in x-y plane as shown in figure, its angular momentum with respect to origin at any time t will be



a) $mvb\hat{k}$ b) $-mvb\hat{k}$

c) $mvb\hat{i}$ d) $mv\hat{i}$

41. Two adjacent sides of a parallelogram are represented by the two vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$. What is the area of parallelogram ?

a) 8 b) $8\sqrt{3}$

c) $3\sqrt{8}$ d) 192

42. Three vectors satisfy the relation $\vec{A}.\vec{B}$ and $\vec{A}.\vec{C}=0$, then $\vec{A}$ is parallel to

a) $\vec{B}\times\vec{C}$ b) $\vec{B}.\vec{C}$

c) $\vec{C}$ d) $\vec{B}$

Miscellaneous

43. If $|\vec{A}\times\vec{B}|=\sqrt{3}\,\vec{A}.\vec{B}$ then the value of $|\vec{A}+\vec{B}|$ is

a) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$

b) $A+B$

c) $(A^2+B^2+\sqrt{3}AB)^{1/2}$

d) $(A^2+B^2+AB)^{1/2}$

44. If $|\vec{V}_1+\vec{V}_2|=|\vec{V}_1-\vec{V}_2|$ and V_2 is finite, then

a) V_1 is parallel to V_2

b) $\vec{V}_1=\vec{V}_2$

c) V_1 and V_2 are mutually perpendicular

d) $|\vec{V}_1|=|\vec{V}_2|$

45. A particle moves from position $3\hat{i}+2\hat{j}-6\hat{k}$ to $14\hat{i}+13\hat{j}+9\hat{k}$ due to a uniform force of $(4\hat{i}+\hat{j}+3\hat{k})N$. If the displacement is in metre then work done will be

a) 100 J b) 200 J

c) 300 J d) 250 J

46. A force $\vec{F}=-K(y\hat{i}+x\hat{j})$ (where K is a positive constant) acts on a particle moving in the X–Y plane. Starting from the origin, the particle is taken along the positive X–axis to the point (a, 0) and then parallel to the Y–axis to the point (a, a). The total work done by the force $\vec{F}$ on the particle is

a) $-2Ka^2$ b) $2Ka^2$

c) $-Ka^2$ d) Ka^2

47. The vector sum of two forces is perpendicular to their vector difference. In that case, the forces

a) are not equal to each other in magnitude

b) cannot be predicted

c) are equal to each other

d) are equal to each other in magnitude

48. Which of the following statement is true ?

a) When the coordinate axes are translated the component of a vector in a plane changes

b) When the coordinate axes are rotated through some angle components of the vector change but the vector's magnitude remains constant

c) Sum of $\vec{a}$ and $\vec{b}$ is $\vec{R}$. If the magnitude of $\vec{a}$ alone is increased, angle between $\vec{b}$ and $\vec{R}$ decreases

d) The cross product of $3\hat{i}$ and $4\hat{j}$ is 12

49. If $\vec{a}$ and $\vec{b}$ are two vectors then the value of $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$ is

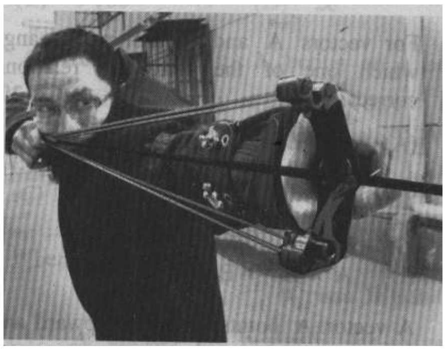
a) $2(\vec{b}\times\vec{a})$ b) $-2(\vec{b}\times\vec{a})$

c) $(\vec{b}\times\vec{a})$ d) $\vec{a}\times\vec{b}$

50. The angle between the vectors $\vec{A}$ and $\vec{B}$ is θ. The value of the triple product $\vec{A}.\left(\vec{B}\times\vec{A}\right)$ is

a) A^2B
b) Zero
c) $A^2B \sin\theta$
d) $A^2B \cos\theta$.

Slingshot



A slingshot works on the principle of parallelogram law of vectors addition. Tension in both sides of rubber forms two arms of the parallelogram. When an arrow is released, it moves under the action of resultant tension in forward direction with gram speed.

Answer Key

Classical Thinking

1. (D)	2. (B)	3. (B)	4. (D)	5. (A)	6. (C)	7. (B)	8. (B)	9. (D)	10. (D)
11. (D)	12. (B)	13. (D)	14. (D)	15. (D)	16. (A)	17. (A)	18. (B)	19. (D)	20. (C)
21. (B)	22. (D)	23. (D)	24. (A)	25. (C)	26. (B)	27. (C)	28. (C)	29. (C)	30. (B)
31. (A)	32. (C)	33. (C)	34. (D)	35. (D)	36. (A)	37. (D)	38. (D)	39. (C)	40. (C)
41. (C)	42. (C)	43. (B)	44. (A)	45. (D)	46. (B)	47. (D)	48. (D)	49. (A)	50. (D)
51. (D)	52. (C)	53. (B)	54. (B)	55. (D)	56. (B)				

Critical Thinking

1. (A)	2. (C)	3. (A)	4. (A)	5. (C)	6. (C)	7. (A)	8. (C)	9. (D)	10. (D)
11. (D)	12. (C)	13. (D)	14. (A)	15. (C)	16. (D)	17. (B)	18. (A)	19. (D)	20. (B)
21. (B)	22. (A)	23. (B)	24. (A)	25. (A)	26. (B)	27. (C)	28. (C)	29. (B)	30. (A)
31. (C)	32. (C)	33. (A)	34. (B)	35. (A)	36. (D)	37. (B)	38. (A)	39. (D)	

Competitive Thinking

1. (B)	2. (C)	3. (B)	4. (C)	5. (C)	6. (A)	7. (A)	8. (C)	9. (A)	10. (C)
11. (D)	12. (A)	13. (B)	14. (B)	15. (A)	16. (A)	17. (D)	18. (D)	19. (B)	20. (B)
21. (C)	22. (C)	23. (A)	24. (B)	25. (A)	26. (D)	27. (A)	28. (C)	29. (C)	30. (A)
31. (C)	32. (C)	33. (A)	34. (D)	35. (B)	36. (A)	37. (D)	38. (A)	39. (A)	40. (B)
41. (B)	42. (A)	43. (D)	44. (C)	45. (A)	46. (C)	47. (D)	48. (B)	49. (A)	50. (B)

Hints

Classical Thinking

19. $\vec{A} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$|\vec{A}| = \sqrt{(3)^2 + (2)^2 + (-4)^2} = \sqrt{29}$

20. $\vec{P} = 3\hat{i} + \hat{j} + 2\hat{k}$

Length in XY plane $= \sqrt{(3)^2 + (1)^2} = \sqrt{10}$ unit

21. Magnitude of $\vec{A} = |\vec{A}| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{14}$

$\therefore$ Direction cosine $= \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}$ and $\frac{3}{\sqrt{14}}$

22. $\overrightarrow{PQ} = \vec{Q} - \vec{P}$

$= \left(-2\hat{i} - 5\hat{j} + 7\hat{k}\right) - \left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$

$= -4\hat{i} - 8\hat{j} + 13\hat{k}$

23. Resultant vector $= \vec{A} + \vec{B} + \vec{C}$

$= \left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) + \left(\hat{i} + \hat{j} + 3\hat{k}\right) + \left(4\hat{i} + 5\hat{j} + 3\hat{k}\right)$

$= 9\hat{i} + 8\hat{j} + 3\hat{k}$

24. Resultant of vectors $\vec{A}$ and $\vec{B}$

$\vec{R} = \vec{A} + \vec{B}$

$= 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$

$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}}$

$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

25. $\vec{A} + \vec{B} = \left(3\hat{i} + 2\hat{j} - 4\hat{k}\right) + \left(5\hat{i} - 7\hat{j} + 2\hat{k}\right)$

$\vec{A} + \vec{B} = 8\hat{i} - 5\hat{j} - 2\hat{k}$

Let $\vec{P}$ be the vector when added to $\vec{A} + \vec{B}$ gives a unit vector along X-axis.

$\therefore \quad \vec{P} + 8\hat{i} - 5\hat{j} - 2\hat{k} = \hat{i}$

$\Rightarrow \vec{P} = -7\hat{i} + 5\hat{j} + 2\hat{k}$

27. $14.14 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^\circ}$

But $F_1 = F_2 = F$

$\therefore \quad 14.14 = \sqrt{2F^2}$

$\therefore \quad 199.94 = 2F^2$

$\therefore \quad F = 9.99 \approx 10$ N

$\therefore \quad F_1 = F_2 = 10$ N

28. $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{\left(\sqrt{2}\right)^2 + (3)^2 + 2\left(\sqrt{2}\right)(3)\cos 45^\circ}$

$F = \sqrt{2+9+6}$

$F = \sqrt{17}$ N

29. Vertical component of velocity,

$v_y = v\sin\theta = 20 \times \sin 30^\circ$

$v_y = 10$ m/s

30. Component of force of gravity $= F_y = F\sin\theta$

$F_y = mg\sin 30^\circ = 10 \times 9.8 \times \frac{1}{2} = 49$ N

31. $\vec{A} = 3(2\hat{i} + 3\hat{j} - \hat{k}) = 2\vec{B}$

As $\vec{A}$ is scalar multiple of $\vec{B}$, $\vec{A}$ and $\vec{B}$ are parallel.

34. Electric flux $d\phi = \vec{E} \cdot \overline{ds}$

35. $\phi = \vec{B} \cdot \vec{A}$

where, $\vec{B}$ is magnetic induction and $\vec{A}$ is area vector.

37. $\vec{P} \cdot \vec{Q} = 0 \qquad \left(\because \vec{P} \perp \vec{Q}\right)$

$\left(5\hat{i} + 7\hat{j} - 3\hat{k}\right) \cdot \left(2\hat{i} + 2\hat{j} - a\hat{k}\right) = 0$

$(5)(2) + (7)(2) + (-3)(-a) = 0$

$10 + 14 + 3a = 0$

$\therefore \quad a = -8$

38. Power $= \vec{F} \cdot \vec{v}$

$= \left(5\hat{i} + 6\hat{j}\right) \cdot \left(4\hat{j} - 2\hat{k}\right) = 24$ unit

39. $W = \vec{F} \cdot \vec{s}$

$= \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) \cdot \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$= 6 + 6 + 10$

$\therefore \quad W = 22$ J

40. $\cos\theta = \dfrac{\vec{P} \cdot \vec{Q}}{|\vec{P}||\vec{Q}|}$

$= \dfrac{\left(3\hat{i} + \hat{j} + 2\hat{k}\right)\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)}{\left(\sqrt{(3)^2 + (1)^2 + (2)^2}\right)\left(\sqrt{(1)^2 + (-2)^2 + (3)^2}\right)}$

$= \dfrac{3 - 2 + 6}{\sqrt{14}\sqrt{14}} = \dfrac{7}{14} = \dfrac{1}{2}$

$\therefore \quad \cos\theta = \dfrac{1}{2}$

$\therefore \quad \theta = 60^\circ$

41. $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$

$= \dfrac{\left(\hat{i} + \hat{j} + \hat{k}\right) \cdot \left(-\hat{i} - \hat{j} + 2\hat{k}\right)}{\sqrt{(1)^2 + (1)^2 + (1)^2} \cdot \sqrt{(-1)^2 + (-1)^2 + (2)^2}}$

$= \dfrac{-1 - 1 + 2}{\sqrt{18}} = 0$

$\therefore \quad \theta = 90^\circ$

42. $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\vec{A} \cdot \vec{B} = 3 \times 5 \times \cos 60^\circ = 15 \times \dfrac{1}{2}$

$\therefore \quad \vec{A} \cdot \vec{B} = 7.5$

46. $\vec{\tau} = \vec{r} \times \vec{F}$

47. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & -4 \\ 3 & -4 & 5 \end{vmatrix}$

$= \hat{i}(15 - 16) - \hat{j}(-10 + 12) + \hat{k}(8 - 9)$

$= -\hat{i} - 2\hat{j} - \hat{k}$

48. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -3 & 1 & -2 \end{vmatrix}$

$= \hat{i}(-2 - 1) - \hat{j}(-2 + 3) + \hat{k}(1 + 3)$

$= -3\hat{i} - \hat{j} + 4\hat{k}$

49. $\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 3 & 1 & -1 \end{vmatrix}$

$= \hat{i}(-2-1) - \hat{j}(-1-3) + \hat{k}(1-6)$

$= -3\hat{i} + 4\hat{j} - 5\hat{k}$

50. Angular momentum

$= \vec{r} \times \vec{p}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 2 & 4 & 5 \end{vmatrix}$

$= \hat{i}(-5-8) - \hat{j}(15-4) + \hat{k}(12+2)$

$= -13\hat{i} - 11\hat{j} + 14\hat{k}$

52. Area of triangle $= \dfrac{1}{2}|\vec{A} \times \vec{B}|$

$\therefore \quad \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ 2 & 2 & 3 \end{vmatrix}$

$= \hat{i}(-6+4) - \hat{j}(3+4) + \hat{k}(2+4)$

$= -2\hat{i} - 7\hat{j} + 6\hat{k}$

$\therefore \quad |\vec{A} \times \vec{B}| = \sqrt{(-2)^2 + (-7)^2 + (6)^2} = \sqrt{89}$

Area of triangle $= \dfrac{\sqrt{89}}{2} = 4.717$ sq. unit

53. Let $\vec{P} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{Q} = (\hat{i} - 3\hat{j} + \hat{k})$

Area of parallelogram $= |\vec{P} \times \vec{Q}|$

$\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -3 & 1 \end{vmatrix}$

$= \hat{i}(2+9) - \hat{j}(1-3) + \hat{k}(-3-2)$

$= 11\hat{i} + 2\hat{j} - 5\hat{k}$

$\therefore \quad |\vec{P} \times \vec{Q}| = \sqrt{(11)^2 + (2)^2 + (-5)^2}$

$= \sqrt{121 + 4 + 25} = \sqrt{150}\ m^2$

54. $\vec{P} + \vec{Q} = (\hat{i} + 2\hat{j} - 4\hat{k}) + (\hat{i} + 2\hat{j} - \hat{k})$

$= 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{P} - \vec{Q} = (\hat{i} + 2\hat{j} - 4\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) = -3\hat{k}$

$(\vec{P} + \vec{Q}).(\vec{P} - \vec{Q}) = (2\hat{i} + 4\hat{j} - 5\hat{k})(-3\hat{k})$

$= 15$

55. $(2\vec{A} - \vec{B}) = 2(2\hat{i} + 3\hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= 3\hat{i} + 4\hat{j} + 5\hat{k}$

$(\vec{A} + 2\vec{B}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) + 2(\hat{i} + 2\hat{j} + 3\hat{k})$

$= 4\hat{i} + 7\hat{j} + 10\hat{k}$

$(2\vec{A} - \vec{B}).(\vec{A} + 2\vec{B})$

$= (3\hat{i} + 4\hat{j} + 5\hat{k})(4\hat{i} + 7\hat{j} + 10\hat{k})$

$= 12 + 28 + 50 = 90$

56. $\vec{s} = \vec{s}_2 - \vec{s}_1$

$= (14\hat{i} + 13\hat{j} + 9\hat{k}) - (3\hat{i} + 2\hat{j} - 6\hat{k})$

$= 11\hat{i} + 11\hat{j} + 15\hat{k}$

$W = \vec{F}.\vec{s}$

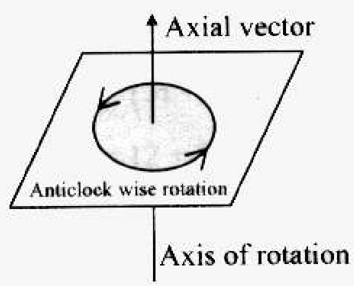
$= (4\hat{i} + \hat{j} + 3\hat{k})(11\hat{i} + 11\hat{j} + 15\hat{k})$

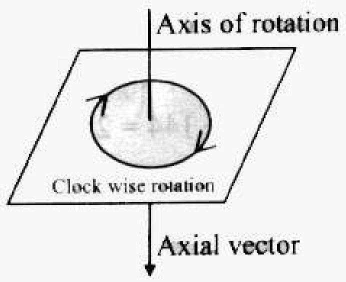
$= 44 + 11 + 45 = 100$ J

Critical Thinking

4. The vectors acting along parallel straight lines are called collinear vectors. When they are in same direction, angle between them is 0^c and they are said to be parallel vectors. When they are in opposite direction, angle between them is π^c and they are said to be antiparallel vectors.

6. A vector representing rotational effects and is always along the axis of rotation in accordance with right hand screw rule is called an axial vector.
eg.: Angular velocity, torque





9. Resultant of forces will be zero when they can be represented by the sides of a triangle taken in same order. In such a case, the sum of the two smaller sides of the triangle is more than the third side.
Only in option (D), sum of the first two forces is smaller than third force, thus not forming a possible triangle.

11. As the multiple of $\hat{j}$ in the given vector is zero therefore this vector lies in XZ plane and projection of this vector on Y-axis is zero.

12. $\vec{R} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\therefore$ Length in XY plane $= \sqrt{R_x^2 + R_y^2} = \sqrt{3^2 + 1^2}$
$= \sqrt{10}$

13. If two vectors $\vec{A}$ and $\vec{B}$ are given then the resultant $R_{max} = A + B = 7N$ and $R_{min} = 4 - 3 = 1N$ i.e., net force on the particle is in between 1 N and 7 N.

14. As $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

$\therefore$ $A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$\therefore$ $4AB\cos\theta = 0 \Rightarrow \cos\theta = 0,$

$\therefore$ $\theta = 90°$

15. $5 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2.\cos 90°}$

$25 = F_1^2 + F_2^2$ .....(i)

When $\theta = 120°$

$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$

$13 = 25 + 2F_1F_2\left(-\frac{1}{2}\right)$

$13 = 25 - F_1F_2$

$F_1F_2 = 12$

$F_2 = \frac{12}{F_1}$ .....(ii)

Substituting equation (ii) in (i)

$F_1^2 + \frac{144}{F_1^2} = 25$

$F_1^4 + 144 = 25F_1^2$

$F_1^4 - 25F_1^2 + 144 = 0$

$(F_1^2 - 9)(F_1^2 - 16) = 0$

$F_1, F_2 = 3, 4$

16. $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$

17. Magnitude of vector = 1

$\sqrt{a_x^2 + a_y^2 + a_z^2} = 1$

$\therefore$ $\sqrt{0.5^2 + 0.8^2 + c^2} = 1$

$\sqrt{c^2 + 0.89} = 1$

$\therefore$ $c^2 = 0.11$

$\therefore$ $c = \sqrt{0.11}$

18. Negative of the given vector be $\vec{A}$.

$\therefore$ $\vec{A} = -\left(-\hat{i} + \hat{j} - \hat{k}\right)$

Unit vector in direction of $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

$= \frac{-\left(-\hat{i} + \hat{j} - \hat{k}\right)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}}$

$= \frac{-1}{\sqrt{3}}\left(-\hat{i} + \hat{j} - \hat{k}\right)$

19. Magnitude of vector $\vec{A} = |\vec{A}|$

$= \sqrt{(2)^2 + (6)^2}$

$= \sqrt{4 + 36}$

$= \sqrt{40}$

Unit vector parallel to $\vec{A}$ is $\frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{i} + 6\hat{j}}{\sqrt{40}}$

Magnitude of vector $\vec{B} = |\vec{B}|$

$= \sqrt{(4)^2 + (3)^2}$

$= 5$

Let $\vec{p}$ be the required vector then $\frac{\vec{p}}{p} = \hat{p}$

$\vec{p} = \hat{p}\,p = \left(\frac{2\hat{i} + 6\hat{j}}{\sqrt{40}}\right)5$

$= \frac{\sqrt{10}}{4}\left[2\left(\hat{i} + 3\hat{j}\right)\right] = \frac{\sqrt{10}}{2}\left(\hat{i} + 3\hat{j}\right)$

20. Let $\hat{n}_1$ and $\hat{n}_2$ be the two unit vectors, then the sum is

$\hat{n}_S = \hat{n}_1 + \hat{n}_2$

$n_S^2 = n_1^2 + n_2^2 + 2n_1n_2 \cos\theta = 1 + 1 + 2\cos\theta$

As n_S is also a unit vector,

$\Rightarrow 1 = 1 + 1 + 2\cos\theta$

$\therefore \quad \cos\theta = -\dfrac{1}{2} \;\Rightarrow\; \theta = 120°$

Let the difference vector be $\hat{n}_d = \hat{n}_1 - \hat{n}_2$

$n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta$

$= 1 + 1 - 2\cos(120°)$

$\therefore \quad n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3$

$\therefore \quad n_d = \sqrt{3}$

21. From the figure, $|\overline{OA}| = a$ and $|\overline{OB}| = a$

Also from triangle rule, $\overline{OB} - \overline{OA} = \overline{AB} = \Delta\vec{a}$

$\therefore \quad |\Delta\vec{a}| = AB$

since $d\theta = \dfrac{\text{arc}}{\text{radius}} \Rightarrow AB = a\,d\theta$

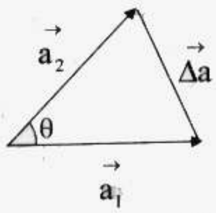
$\therefore \quad |\Delta\vec{a}| = a d\theta$

Δa means change in magnitude of vector i.e., $|\overline{OB}| - |\overline{OA}|$

$\therefore \quad a - a = 0$

Hence, $\Delta a = 0$

22. From the figure,



$\vec{a}_2 = \vec{a}_1 + \Delta\vec{a}$

$\Rightarrow \Delta\vec{a} = \vec{a}_2 - \vec{a}_1$

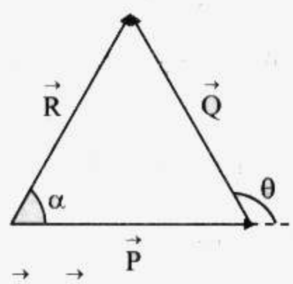
Also $|\vec{a}_2| = |\vec{a}_1| = a$

$\therefore \quad \Delta a = \left|\vec{a}_2 - \vec{a}_1\right| = \left[a_2^2 + a_1^2 - 2a_2a_1\cos\theta\right]^{1/2}$

$= \left[2a^2(1-\cos\theta)\right]^{1/2}$

$= \left[2a^2(2\sin^2\theta/2)\right]^{\frac{1}{2}} = 2\,a\sin\theta/2.$

23.



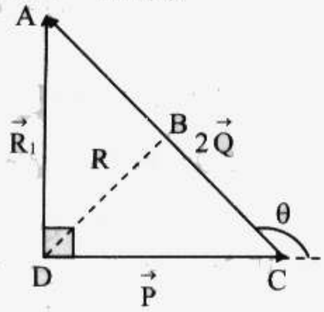
$\vec{P} + \vec{Q} = \vec{R}$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta \quad \ldots(i)$

and $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

When Q is doubled, resultant is perpendicular to $\vec{P}$

$\therefore \quad R_1^2 = P^2 + 4Q^2 + 4PQ\cos\theta \quad \ldots(ii)$



From right angled triangle ADC

$4Q^2 = R_1^2 + P^2$

$R_1^2 = 4Q^2 - P^2$

Substituting in (ii) and solving,

$P^2 + 2PQ\cos\theta = 0 \quad \ldots(iii)$

Substituting (iii) in (i),

$R = Q$

24. Mass $= \dfrac{\text{Force}}{\text{Acceleration}} = \dfrac{|\vec{F}|}{a}$

$= \dfrac{\sqrt{36+64+100}}{1} = 10\sqrt{2}$ kg

25. $\vec{A}$ and $\vec{B}$ are parallel to each other. This implies $\vec{A} = m\vec{B}$. comparing X-component, $m = \dfrac{1}{6}$. Comparing Y-component, b = 18 and comparing Z-component a = 1.

26. Let $\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k}$ and $\vec{B} = -4\hat{i} + 4\hat{j} + m\hat{k}$.

For $\vec{A}$ perpendicular to $\vec{B}$,

$\vec{A}\cdot\vec{B} = 0$

$\therefore \quad \left(2\hat{i} + 3\hat{j} + 8\hat{k}\right)\left(-4\hat{i} + 4\hat{j} + m\hat{k}\right) = 0$

$\therefore \quad -8 + 12 + 8m = 0$

$\therefore \quad m = -\dfrac{1}{2}$

27. $W = \vec{F}\cdot\vec{s}$

$\therefore \quad 6 = \left(3\hat{i} + c\hat{j} + 2\hat{k}\right)\left(4\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$\therefore \quad 6 = 12 + 2c + 6$

$6 = 18 + 2c$

$2c = -12$

$c = -6$

28. $W = \vec{F}\cdot\vec{s} = Fs\cos\theta$

For force causing displacement in its own direction $\theta = 0°$

$\therefore \quad W = Fs = \left(\sqrt{(7)^2+(-4)^2+(-4)^2}\right)\times 10$

$= \left(\sqrt{49+16+16}\right)\times 10 = 9\times 10 = 90\ J$

29. $\vec{P}+\vec{Q} = 5\hat{i}-4\hat{j}+3\hat{k}$

Let α be the angle made by $\vec{P}+\vec{Q}$ with X-axis

$\therefore \quad \cos\alpha = \dfrac{\left(\vec{P}+\vec{Q}\right)\cdot\hat{i}}{|\vec{P}+\vec{Q}||\hat{i}|}$

$= \dfrac{5}{\sqrt{5^2+(-4^2)+3^2}} = \dfrac{5}{\sqrt{50}} = \dfrac{1}{\sqrt{2}}$

$\therefore \quad \alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45°$

31. $\vec{A} = 3\hat{i}+\hat{j}+2\hat{k}$, $\vec{B} = 2\hat{i}-2\hat{j}+4\hat{k}$

$\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$

$$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ 2 & -2 & 4\end{vmatrix}$$

$= \hat{i}(4+4) - \hat{j}(12-4) + \hat{k}(-6-2)$

$= 8\hat{i} - 8\hat{j} - 8\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{8^2+(-8)^2+(-8)^2} = \sqrt{192}$

$|\vec{A}| = \sqrt{(3)^2+(1)^2+(2)^2} = \sqrt{14}$

$|\vec{B}| = \sqrt{(2)^2+(-2)^2+(4)^2} = \sqrt{24}$

$\sin\theta = \dfrac{\sqrt{192}}{\sqrt{14}\sqrt{24}} \approx 0.76$

33. Let the two vectors be $\vec{A}$ and $\vec{B}$.

$\sin\theta = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}||\vec{B}|}$

$\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

$\cot\theta = \dfrac{\vec{A}.\vec{B}}{\vec{A}\times\vec{B}} = \sqrt{3} \quad \therefore \quad \theta = 30°$

35. $\left(\vec{A}+\vec{B}\right)\times\left(\vec{A}-\vec{B}\right) = \left(\vec{A}\times\vec{A}\right)-\left(\vec{A}\times\vec{B}\right)$

$+\left(\vec{B}\times\vec{A}\right)-\left(\vec{B}\times\vec{B}\right)$

$= -\left(\vec{A}\times\vec{B}\right)+\left(\vec{B}\times\vec{A}\right)$

$+\left(\vec{B}\times\vec{A}\right)+\left(\vec{B}\times\vec{A}\right)$

$= 2\left(\vec{B}\times\vec{A}\right)$

36. $\vec{P}.\left(\vec{P}+\vec{Q}\right) = P^2$

$\therefore \quad \vec{P}.\vec{P} + \vec{P}.\vec{Q} = P^2 \quad \Rightarrow \quad P^2 + \vec{P}.\vec{Q} = P^2$

$\therefore \quad \vec{P}.\vec{Q} = 0 \quad \Rightarrow \quad PQ\cos\theta = 0$

$\therefore \quad \cos\theta = 0 \quad \Rightarrow \quad \theta = 90°$

37. $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta = 0$ and

$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta = 0$

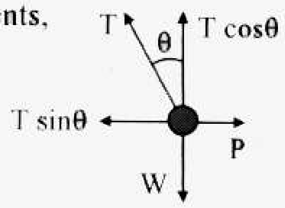
If $\vec{A}$ and $\vec{B}$ are not null vectors then $\sin\theta$ and $\cos\theta$ both should be zero simultaneously. This is not possible so it is essential that one of the vector must be null vector.

38. Cross product of two vectors is perpendicular to the plane containing both the vectors.

39. As the ball is in equilibrium under the effect of three forces,

$\vec{T}+\vec{P}+\vec{W} = 0$. Hence option (B) is true.

Resolving tension into two rectangular components,



From the figure,

$T\cos\theta = W$ and $T\sin\theta = P$

$\dfrac{T\cos\theta}{T\sin\theta} = \dfrac{W}{P} \Rightarrow P = W\tan\theta$. Hence option (A) is true.

Also, $(T\sin\theta)^2 + (T\cos\theta)^2 = T^2$

$\Rightarrow \quad P^2 + W^2 = T^2$

Hence option (C) is true.

But $T = \sqrt{(T\sin\theta)^2+(T\cos\theta)^2} = \sqrt{P^2+W^2}$

Hence option (D) is wrong.

Competitive Thinking

4. Resultant of two vectors $\vec{A}$ and $\vec{B}$ can be given by, $\vec{R} = \vec{A} + \vec{B}$

$|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

If $\theta = 0$ then $|\vec{R}| = A + B = |\vec{A}| + |\vec{B}|$

5. Initial position vector

$\vec{r}_1 = (-3\hat{i} + 4\hat{j} - 3\hat{k})$ m

Final position vector

$\vec{r}_2 = (7\hat{i} - 2\hat{j} - 3\hat{k})$ m

Displacement $\vec{r} = \vec{r}_1 - \vec{r}_2$

$= (7\hat{i} - 2\hat{j} - 3\hat{k}) - (-3\hat{i} + 4\hat{j} - 3\hat{k}) = 10\hat{i} - 6\hat{j}$

6. $C = \sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2} = 5$

$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is $\dfrac{\pi}{2}$

7. $\overline{AC} = \overline{AB} + \overline{BC}$

$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(10)^2 + (20)^2}$

$= \sqrt{100 + 400} = \sqrt{500} = 22.36$ km

8. $R = \sqrt{12^2 + 5^2 + 6^2} = \sqrt{144 + 25 + 36}$

$= \sqrt{205} \approx 14.31$ m

9. $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$\therefore$ $40\sqrt{3} = \sqrt{F^2 + F^2 + 2F^2\cos 60°} = \sqrt{3F^2}$

$\Rightarrow F = 40$ N

10. $F_{max} = 5 + 10 = 15$ N and $F_{min} = 10 - 5 = 5$ N

Range of resultant force is $5 \leq F \leq 15$

11. $R_{max} = A + B = 17$ when $\theta = 0°$

$R_{max} = A - B = 7$ when $\theta = 180°$

by solving, $A = 12$ and $B = 5$

When $\theta = 90°$

then $R = \sqrt{A^2 + B^2}$

$\Rightarrow R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$

12. $\vec{r} = \vec{a} + \vec{b} + \vec{c} = 4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - \hat{k}$

$= \hat{i} + \hat{j} - \hat{k}$

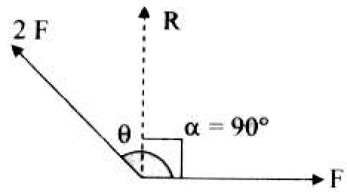
$\hat{r} = \dfrac{\vec{r}}{|r|} = \dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

13. $\tan\alpha = \dfrac{2F\sin\theta}{F + 2F\cos\theta} = \infty$ (as $\alpha = 90°$)

$F + 2F\cos\theta = 0$

$\cos\theta = -\dfrac{1}{2}$

$\theta = 120°$



14. $|\vec{A}| + |\vec{B}| = 18$ ....(i)

$12 = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ....(ii)

$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta} = \tan 90°$

$\Rightarrow \cos\theta = -\dfrac{A}{B}$ ....(iii)

By solving (i), (ii) and (iii),

$A = 13$ N and $B = 5$ N

15. $F_{net}^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$\left(\dfrac{F}{3}\right)^2 = F^2 + F^2 + 2F^2\cos\theta$

$\dfrac{F^2}{9} = 2F^2(1 + \cos\theta)$

$\therefore$ $1 + \cos\theta = \dfrac{1}{18}$

$\cos\theta = \left(-\dfrac{17}{18}\right)$

16. $|\vec{F}_R| = |\vec{F} + \vec{F}| = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$

$= \left[2F^2(1 + \cos\theta)\right]^{\frac{1}{2}}$

$= \left[2F^2(2\cos^2\theta/2\right]^{\frac{1}{2}}$

$= 2F\cos\theta/2$

17. Since, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$A = B = R$

$\therefore$ $A^2 = 2A^2 + 2A^2\cos\theta$

$\cos\theta = -\dfrac{1}{2} = \cos 120°$

$\therefore$ $\theta = 120°$

18. Let $A = 3$ N and $B = 2$ N then

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$R = \sqrt{9 + 4 + 12\cos\theta}$ ....(i)

Now $A = 6$ N, $B = 2$ N then

$2R = \sqrt{36 + 4 + 24\cos\theta}$ ....(ii)

From (i) and (ii), $\cos\theta = -\dfrac{1}{2}$

$\therefore$ $\theta = 120°$

19. $R^2 = (3P)^2 + (2P)^2 + 2 \times 3P \times 2P \times \cos\theta$
$R^2 = 9P^2 + 4P^2 + 12P^2 \cos\theta$
$R^2 = 13P^2 + 12P^2 \cos\theta \quad \ldots(i)$
$(2R)^2 = (6P)^2 + (2P)^2 \times 2 \times 6P \times 2P \times \cos\theta$
$4R^2 = 40P^2 + 24P^2 \cos\theta$
$R^2 = 10P^2 + 6P^2 \cos\theta \quad \ldots(ii)$
From (i) and (ii)
$13P^2 + 12P^2 \cos\theta = 10P^2 + 6P^2 \cos\theta$
$3P^2 = -6P^2 \cos\theta$
$\therefore \quad \cos\theta = -\frac{1}{2}$
$\theta = 120°$

20. Let θ be the angle between $\vec{A}$ and $\vec{B}$. Given,
$|\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$
$\therefore \quad |\vec{A}+\vec{B}|^2 = n^2|\vec{A}-\vec{B}|^2$
$A^2 + B^2 + 2AB\cos\theta = n^2[A^2 + B^2 - 2AB\cos\theta]$
$A^2 + A^2 + 2A^2\cos\theta = n^2[A^2 + A^2 - 2A^2\cos\theta]$
$(\because A = B)$
$2A^2(1+\cos\theta) = n^2 2A^2(1-\cos\theta)$
$1 + \cos\theta = n^2(1-\cos\theta)$
$(n^2+1)\cos\theta = (n^2-1)$
$\cos\theta = \left(\frac{n^2-1}{n^2+1}\right)$
$\therefore \quad \theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

21. The angle α which the resultant R makes with A is given by
$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$
$\tan\left(\frac{\theta}{2}\right) = \frac{B\sin\theta}{A+B\cos\theta} \quad \left(\because \alpha = \frac{\theta}{2}\right)$
$\Rightarrow \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{2B\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{A+B\cos\theta}$
Which gives $A + B\cos\theta = 2B\cos^2\left(\frac{\theta}{2}\right)$
$\Rightarrow A + B\left[2\cos^2\left(\frac{\theta}{2}\right)-1\right] = 2B\cos^2\left(\frac{\theta}{2}\right)$
Which gives A = B.

22. Let $\vec{A} = 2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{B} = 3\hat{i}+4\hat{k}$
$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$
$\therefore \quad |\vec{A}|\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$
$= \frac{(2\hat{i}+3\hat{j}+\hat{k}).(3\hat{i}+4\hat{k})}{\sqrt{3^2+4^2}} = \frac{10}{5} = 2$

23. $\vec{A}.\vec{B} = AB\cos\theta$
$\therefore \quad |\vec{A}|\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

24. $\vec{A}.\vec{B} = 0$
$\therefore \quad (2\times1) + (3\times2) + [4\times(-x)] = 0$
$\therefore \quad 2 + 6 - 4x = 0$
$\therefore \quad 8 - 4x = 0$
$\therefore \quad x = \frac{8}{4} = 2$

25. $\vec{P}\cdot\vec{Q} = 0$
$\therefore \quad a^2 - 2a - 3 = 0 \Rightarrow a = 3$

26. $\vec{F_1}.\vec{F_2} = (2\hat{j}+5\hat{k})(3\hat{j}+4\hat{k}) = 6 + 20 = 26$

27. $P = \vec{F}.\vec{v}$
$= (4\hat{i}+\hat{j}-2\hat{k}).(2\hat{i}+2\hat{j}+3\hat{k})$
$= (8 + 2 - 6)\ W = 4\ W$

28. $\vec{A}\cdot\vec{B} = AB\cos\theta$
Given, $\vec{A}\cdot\vec{B} = -|\vec{A}||\vec{B}|$
i.e., $\cos\theta = -1$
$\therefore \quad \theta = 180°$
i.e., $\vec{A}$ and $\vec{B}$ act in the opposite direction.

29. $\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$
$= \frac{(3\hat{i}+4\hat{j}+5\hat{k})\cdot(3\hat{i}+4\hat{j}-5\hat{k})}{\sqrt{3^2+4^2+5^2}\sqrt{3^2+4^2+(-5^2)}}$
$= \frac{9+16-25}{\sqrt{25}\sqrt{25}} = 0$
$\Rightarrow \quad \theta = 90°$

30. Unit vector along X-axis is $\hat{i}$.

$$\cos\theta = \frac{\vec{P}.\hat{i}}{|\vec{P}||\hat{i}|} = \frac{(\hat{i}+\hat{j}+\hat{k}).(\hat{i})}{\sqrt{1^2+1^2+1^2}\sqrt{1^2}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

31. $(\hat{i}+\hat{j}).(\hat{j}+\hat{k}) = 0+0+1+0 = 1$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{1}{\sqrt{2}\times\sqrt{2}} = \frac{1}{2}$$

$\therefore$ $\theta = 60°$

32. $P_x = 2\cos t$, $P_y = 2\sin t$

$\therefore$ $\vec{P} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

$$\vec{F} = \frac{d\vec{P}}{dt} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$$

$\vec{F}.\vec{P} = (-2\sin t\,\hat{i} + 2\cos t\,\hat{j})(2\cos t\,\hat{i} + 2\sin t\,\hat{j})$

$\vec{F}\cdot\vec{P} = 0$

$\therefore$ $\theta = 90°$

34. $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

Where, $\hat{n}$ is a unit vector indicating the direction of $\vec{A}\times\vec{B}$.

Vector product is non commutative,

$\therefore$ $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

35. Direction of vector A is along Z-axis

$\therefore$ $\vec{A} = a\hat{k}$

Direction of vector B is towards north

$\therefore$ $\vec{B} = b\hat{j}$

Now $\vec{A}\times\vec{B} = a\hat{k}\times b\hat{j} = ab(-\hat{i})$

$\therefore$ The direction of $\vec{A}\times\vec{B}$ is along west.

36. Vector product is non commutative,

$\therefore$ $\vec{V} = \vec{r}\times\vec{\omega}$ and $\vec{V} = -(\vec{\omega}\times\vec{r})$

37. $$\vec{v} = \hat{r}\times\hat{\omega} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & -6 & 6\\ 3 & -4 & 1\end{vmatrix}$$

$$= \hat{i}(-6+24) - \hat{j}(5-18) + \hat{k}(-20+18)$$

$$\vec{v} = 18\hat{i} + 13\hat{j} - 2\hat{k}$$

38. $AB\sin\theta = -AB\sin\theta$

$2AB\sin\theta = 0$

$\sin\theta = 0$ or $\theta = 180°$

39. $\vec{\tau} = \vec{r}\times\vec{F} = (7\hat{i}+3\hat{j}+k)\times(-3\hat{i}+\hat{j}+5\hat{k})$

$$\vec{\tau} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & 3 & 1\\ -3 & 1 & 5\end{vmatrix}$$

$$= \hat{i}(15-1) - \hat{j}(35+3) + \hat{k}(7+9)$$

$$\vec{\tau} = 14\hat{i} - 38\hat{j} + 16\hat{k}$$

40. Angular momentum

$\vec{L} = \vec{r}\times\vec{p}$ in terms of component becomes

$$\vec{L} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ p_x & p_y & p_z\end{vmatrix}$$

As motion is in x-y plane ($z = 0$ and $p_z = 0$), hence

$\vec{L} = \vec{k}(xp_y - yp_x)$

Here $x = vt$, $y = b$, $p_x = mv$ and $p_y = 0$

$\therefore$ $\vec{L} = \vec{k}[vt\times 0 - bmv] = -mvb\hat{k}$

41. Area of parallelogram $= |\vec{A}\times\vec{B}|$

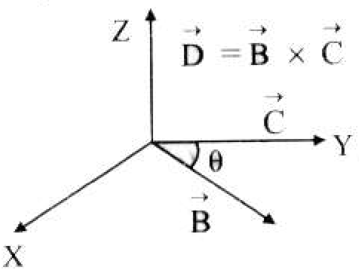
$$\vec{A}\times\vec{B} = (\hat{i}+2\hat{j}+3\hat{k})\times(3\hat{i}-2\hat{j}+\hat{k})$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 3 & -2 & 1\end{vmatrix}$$

$$= (8)\hat{i} + (8)\hat{j} - (8)\hat{k}$$

$\therefore$ $|\vec{A}\times\vec{B}| = \sqrt{64+64+64} = 8\sqrt{3}$

42. $\vec{A}.\vec{B} = 0$; $\vec{A}.\vec{C} = 0$



$\vec{A}$ is perpendicular to $\vec{B}$ as well as $\vec{C}$.

Let $\vec{D} = \vec{B}\times\vec{C}$

The direction of $\vec{D}$ is perpendicular to the plane containing $\vec{B}$ and $\vec{C}$.

Hence, $\vec{A}$ is parallel to $\vec{D}$ i.e., $\vec{A}$ is parallel to $\vec{B} \times \vec{C}$.

43. $|\vec{A} \times \vec{B}| = \sqrt{3}\ \vec{A}.\vec{B}$

$AB \sin\theta = \sqrt{3}\ AB\cos\theta$

$\therefore \quad \tan\theta = \sqrt{3}$

$\therefore \quad \theta = 60°$

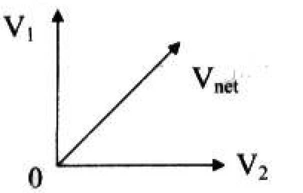
Now $|\vec{R}| = |\vec{A} + \vec{B}|$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\left(\frac{1}{2}\right)}$$

$$= (A^2 + B^2 + AB)^{1/2}$$

44.



If $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$

then $|\vec{V}_1 + \vec{V}_2|^2 = |\vec{V}_1 - \vec{V}_2|^2$

i.e., $\left(\vec{V}_1 + \vec{V}_2\right)\cdot\left(\vec{V}_1 + \vec{V}_2\right) = \left(\vec{V}_1 - \vec{V}_2\right)\cdot\left(\vec{V}_1 - \vec{V}_2\right)$

On solving, $4V_1V_2\cos\theta = 0$

$\theta = 90°$

So V_1 and V_2 will be mutually perpendicular.

45. $s = \vec{r}_2 - \vec{r}_1 = (11\hat{i} + 11\hat{j} + 15\hat{k})$

$W = \vec{F}.\vec{s}$

$= (4\hat{i} + \hat{j} + 3\hat{k})\ (11\hat{i} + 11\hat{j} + 15\hat{k})$

$= (4 \times 11 + 1 \times 11 + 3 \times 15)$

$= 100$ J

46. For motion of the particle from (0, 0) to (a, 0)

$\vec{F} = -K(y\hat{i} + x\hat{j}) \Rightarrow \vec{F} = -Ka\hat{j}$

Displacement $\vec{r} = (a\hat{i} + 0\hat{j}) - (0\hat{i} + 0\hat{j}) = a\hat{i}$

So work done from (0, 0) to (a, 0) is given by

$W = \vec{F}\cdot\vec{r} = -Ka\hat{j}.a\hat{i} = 0$

For motion (a, 0) to (a, a)

$\vec{F} = K(a\hat{i} + a\hat{j})$ and displacement

$\vec{r} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j}) = a\hat{j}$

So work done from (a, 0) to (a, a),

$W = \vec{F}.\vec{r}$

$= -K(a\hat{i} + a\hat{j}).a\hat{j}$

$= -Ka^2$

So total work done $= -Ka^2$

47. The sum of the two forces be,

$\vec{F}_1 = \vec{A} + \vec{B}$ ....(i)

The difference of the two forces be,

$\vec{F}_2 = \vec{A} - \vec{B}$ ....(ii)

Since sum of the two forces is perpendicula to their difference,

$\vec{F}_1 \cdot \vec{F}_2 = 0$

$\Rightarrow (A + B)\cdot(A - B) = 0$

$\Rightarrow A^2 - A\cdot B + B\cdot A - B^2 = 0$

$\therefore \quad A^2 = B^2$

$\Rightarrow |A| = |B|$

Thus, the forces are equal to each other is magnitude.

49. $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$= \vec{a} \times \vec{a} - \vec{a} \times \vec{b} + \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$ ....(i)

Because, cross product of parallel vectors is zero

Therefore,

$\vec{a} \times \vec{a} = \vec{b} \times \vec{b}$

$= \vec{0}$ Because

$\vec{a} \times \vec{b} = (ab\sin\theta)\,\hat{n} = -[(ba\sin\theta)\,\hat{n}]$

$= -\vec{b} \times \vec{a}$

Substituting the values in relation (i),

$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = 2(\vec{b} \times \vec{a})$

50. Let $\vec{A}\cdot\left(\vec{B} \times \vec{A}\right) = \vec{A}\cdot\vec{C}$

$\vec{C} = \vec{B} \times \vec{A}$ which is perpendicular to both vectors $\vec{A}$ and $\vec{B}$

$\therefore \quad \vec{A}\cdot\vec{C} = 0$

Evaluation Test

1. A force $\vec{F} = 4\hat{i} + 3\hat{j} - 2\hat{k}$ is passing through the origin. Its moment about point (1, 1, 0) is
   (A) $-\hat{i} + \hat{j} + \hat{k}$
   (B) zero
   (C) $2\hat{i} + 3\hat{j}$
   (D) $-2\hat{i} + 2\hat{j} - \hat{k}$

2. **Assertion:** If $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, then $|\vec{a}| \neq |\vec{b}|$.
   Reason: Two unequal vectors can never have same magnitude.
   (A) Assertion is True, Reason is True; Reason is a correct explanation for Assertion.
   (B) Assertion is True, Reason is True; Reason is not a correct explanation for Assertion.
   (C) Assertion is True, Reason is False.
   (D) Assertion is False, Reason is True.

3. Two forces of magnitudes 3 N and 5 N act at the same point on an object. Which one of the following equations will satisfy the magnitude of the resultant force R in newtons?
   (A) $2 \le R \le 5$
   (B) $2 \le R \le 8$
   (C) $3 \le R \le 5$
   (D) $2 \le R \le 3$

4. If $\vec{A}$ is a vector of magnitude 3 units due east. What is the magnitude and direction of a vector $-4\vec{A}$?
   (A) 3 units due east
   (B) 4 units due east
   (C) 12 units due east
   (D) 12 units due west

5. A body constrained to move in Y direction, is subjected to a force given by $\vec{F} = (-2\hat{i} + 15\hat{j} + 6\hat{k})$ N. What is the work done by this force in moving the body through a distance of 10 m along Z axis?
   (A) 190 J (B) 60 J
   (C) 150 J (D) 20 J

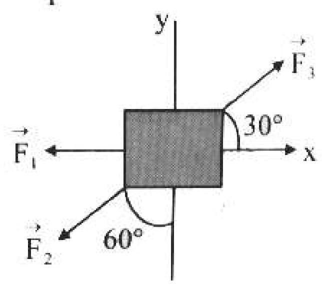
6. Choose the incorrect option.
   The two vectors $\vec{P}$ and $\vec{Q}$ are drawn from a common point and $\vec{R} = \vec{P} + \vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is
   (A) 90° if $R^2 = P^2 + Q^2$
   (B) less than 90° if $R^2 > P^2 + Q^2$
   (C) greater than 90° if $R^2 < P^2 + Q^2$
   (D) greater than 90° if $R^2 > P^2 + Q^2$

7. When vector $\hat{n} = a\hat{i} + b\hat{j}$ is perpendicular to $\left(2\hat{i} + \hat{j}\right)$, then a and b are
   (A) $\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$ (B) −2, 0
   (C) 0, −2 (D) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

8. A force of $-4F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, −1) is
   (A) $-4F\left(\hat{i} - \hat{j}\right)$ (B) $4F\left(\hat{i} - \hat{j}\right)$
   (C) $-4F\left(\hat{i} + \hat{j}\right)$ (D) $4F\left(\hat{i} + \hat{j}\right)$

9. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively, the angle θ between the vector $\hat{i} + \hat{j} + \hat{k}$ and vector $\hat{j}$ is given by
   (A) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
   (C) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (D) $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

10. Figure shows an overhead view of three horizontal forces acting on a cargo canister that was initially stationary but now moving across a frictionless floor. The force magnitudes are F_1 = 3 N, F_2 = 4 N and F_3 = 10 N. What is the net work done on the canister by the three forces during the first 5 m of displacement?
   (A) 3.813 J
   (B) 1.53 J
   (C) 18.6 J
   (D) 38.13 J



11. $\vec{A}$ and $\vec{B}$ are the two vectors such that ratio of their dot product to magnitude of their cross product is $\frac{1}{\sqrt{3}}$. Then the angle between $\vec{A}$ and $\vec{B}$ is

(A) $\frac{\pi^c}{2}$ (B) $\frac{\pi^c}{3}$

(C) 0^c (D) $\frac{\pi^c}{6}$

12. Two vectors $\vec{P}$ and $\vec{Q}$ lie in one plane. Vector $\vec{R}$ lies in a different plane. In such a case $\vec{P}+\vec{Q}+\vec{R}$

(A) can be zero
(B) must be zero
(C) lies in the same plane as $\vec{P}$ or $\vec{Q}$
(D) lies in the plane different from any of the three vectors.

13. A particle acted upon by constant forces $5\hat{i}+\hat{j}-2\hat{k}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is displaced from the point $2\hat{i}+2\hat{j}-4\hat{k}$ to point $6\hat{i}+4\hat{j}-2\hat{k}$. The total work done by the forces in SI unit is

(A) $20\sqrt{2}$ (B) 47
(C) 24 (D) 33

14. The x and y components of vectors $\vec{A}$ are 4 m and 6 m respectively. The x and y components of vector $(\vec{A}+\vec{B})$ are 12 m and 10 m respectively. Then what are the x and y component of vector $\vec{B}$?

(A) 8 m, 4 m (B) 3 m, 6 m
(C) 4 m, 8 m (D) 4 m, 6 m

15. The angle subtended by the vector $A = 6\hat{i}+3\hat{j}+4\hat{k}$ with the y-axis is

(A) $\sin^{-1}\left(\frac{3}{61}\right)$

(B) $\sin^{-1}\left(\frac{3}{\sqrt{61}}\right)$

(C) $\cos^{-1}\left(\frac{3}{\sqrt{61}}\right)$

(D) $\cos^{-1}\left(\frac{4}{\sqrt{61}}\right)$

16. A particle moves in the x-y plane under the action of a force $\vec{F}$ such that the components of its linear momentum $\vec{p}$ at any time t are $p_x = 3\cos t$ and $p_y = 3\sin t$. What is the magnitude of the vector $\vec{F}$?

(A) $2\sqrt{2}$ (B) 5
(C) 3 (D) 4

17. Given $\vec{A} = 3\hat{i}+2\hat{j}$ and $\vec{B} = \hat{i}+\hat{j}$. The component of vector $\vec{A}$ along vector $\vec{B}$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{3}{\sqrt{2}}$

(C) $\frac{5}{\sqrt{2}}$ (D) $\frac{7}{\sqrt{2}}$

18. A vector $\vec{A}$ is along the positive x-axis and its vector product with another vector $\vec{B}$ is zero, then vector $\vec{B}$ could be

(A) $\hat{i}+\hat{j}$ (B) $4\hat{i}$
(C) $\hat{j}+\hat{k}$ (D) $-7\hat{k}$

19. What is the area of the triangle formed by sides $\vec{A} = 2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{B} = \hat{i}-2\hat{k}$?

(A) $\sqrt{13.5}$ unit (B) 13.5 unit
(C) $\sqrt{109}$ unit (D) 5.22 unit

20. The component of vector $\vec{A} = a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$ along the direction of $\hat{j}-\hat{k}$ is

(A) $a_x - a_y + a_z$ (B) $a_z - a_y$
(C) $(a_x - a_y)/\sqrt{2}$ (D) $\frac{a_y - a_z}{\sqrt{2}}$

Answers to Evaluation Test

1.	(D)	2.	(C)	3.	(B)	4.	(D)
5.	(B)	6.	(D)	7.	(A)	8.	(D)
9.	(A)	10.	(C)	11.	(B)	12.	(D)
13.	(C)	14.	(A)	15.	(B)	16.	(C)
17.	(C)	18.	(B)	19.	(D)	20.	(D)