UNIT 5: Analogies for Transport processes

Contents

5.1 Analogy of Mass Transfer

Mass transfer by convection involves the transport of material between a boundary surface (such as solid or liquid surface) and a moving fluid or between two relatively immiscible, moving fluids.

There are two different cases of convective mass transfer:

1. Mass transfer takes place only in a single phase either to or from a phase boundary, as in sublimation of naphthalene (solid form) into the moving air.

2. Mass transfer takes place in the two contacting phases as in extraction and absorption.

Convective Mass Transfer Coefficient

In the study of convective heat transfer, the heat flux is connected to heat transfer coefficient as

$$Q/A = q = h(t_s - t_m) \quad \text{--------------------- (1.1)}$$

The analogous situation in mass transfer is handled by an equation of the form

$$N_A = k_c(C_{As} - C_A) \quad \text{--------------------- (1.2)}$$

The molar flux N_A is measured relative to a set of axes fixed in space. The driving force is the difference between the concentration at the phase boundary, C_{AS} (a solid surface or a fluid interface) and the concentration at some arbitrarily defined point in the fluid medium, C_A . The convective mass transfer coefficient k_C is a function of geometry of the system and the velocity and properties of the fluid similar to the heat transfer coefficient, h.

Significant Parameters in Convective Mass Transfer

Dimensionless parameters are often used to correlate convective transfer data. In momentum transfer Reynolds number and friction factor play a major role. In the correlation of convective heat transfer data, Prandtl and Nusselt numbers are important. Some of the same parameters, along with some newly defined dimensionless numbers, will be useful in the correlation of convective mass-transfer data.

The molecular diffusivities of the three transport process (momentum, heat and mass) have been defined as:

$$\text{Momentum diffusivity} \quad \nu = \frac{\mu}{\rho} \quad \text{------------------------------} \ (1.3)$$

$$\text{Thermal diffusivity} \quad \alpha = \frac{k}{\rho C_p} \quad \text{----------------------------} \ (1.4)$$

and

$$\text{Mass diffusivity} \quad D_{AB} \quad \text{----------------------------} \ (1.5)$$

It can be shown that each of the diffusivities has the dimensions of L^2 / t, hence, a ratio of any of the two of these must be dimensionless.

The ratio of the molecular diffusivity of momentum to the molecular diffusivity of heat (thermal diffusivity) is designated as the Prandtl Number

$$\frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \Pr = \frac{\nu}{\alpha} = \frac{C_p\,\mu}{K} \quad \text{-------------------------} \ (1.6)$$

The analogous number in mass transfer is Schmidt number given as

$$\frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}} = Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}} \text{ -------------- (1.7)}$$

The ratio of the molecular diffusivity of heat to the molecular diffusivity of mass is designated the Lewis Number, and is given by

$$\frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}} = Le = \frac{\alpha}{D_{AB}} = \frac{k}{\rho C_p D_{AB}} \text{ ------------- (1.8)}$$

Lewis number is encountered in processes involving simultaneous convective transfer of mass and energy.

Let us consider the mass transfer of solute A from a solid to a fluid flowing past the surface of the solid. The concentration and velocity profile is depicted .For such a case, the mass transfer between the solid surface and the fluid may be written as

$$N_A = k_c \left(C_{As} - C_{A\infty}\right) \text{ ---------------------- (1 a)}$$

Since the mass transfer at the surface is by molecular diffusion, the mass transfer may also described by

$$N_A = -D_{AB} \left.\frac{dC_A}{dy}\right|_{y=0} \text{ ------------------------- (1.9)}$$

When the boundary concentration, C_{As} is constant, equation (9) may be written as

$$N_A = -D_{AB} \left.\frac{d\left(C_A - C_{As}\right)}{dy}\right|_{y=0} \text{ ---------------------- (1.10)}$$

Equation (4.1a) and (4.10) may be equated, since they define the same flux of component A leaving the surface and entering the fluid

$$k_c\left(C_{As} - C_{A\infty}\right) = -D_{AB}\frac{d}{dy}\left(C_A - C_{As}\right)\Bigg|_{y=0} \text{ --------------- (1.11)}$$

This relation may be rearranged into the following form:

$$\frac{k_c}{D_{AB}} = -\left.\frac{d\left(C_A - C_{As}\right)/dy}{\left(C_A - C_{A\infty}\right)}\right|_{y=0} \text{ -------------------- (1..12)}$$

Multiplying both sides of equation(4.12) by a characteristic length, L we obtain the following dimensionless expression:

$$\frac{k_c L}{D_{AB}} = -\frac{d\left(C_A - C_{As}\right)/dy\,\big|_{y=0}}{\left(C_{AS} - C_{A\infty}\right)/L} \text{ ----------------- (1.13)}$$

The right hand side of equation (4.13) is the ratio of the concentration gradient at the surface to an overall or reference concentration gradient; accordingly, it may be considered as the ratio of molecular mass-transport resistance to the convective mass-transport resistance of the fluid. This ratio is generally known as the Sherwood number, Sh and analogous to the Nusselt number Nu, in heat transfer.

5.2 Application of Dimensional Analysis to Mass Transfer

One of the method of obtaining equations for predicting mass-transfer coefficients is the use of dimensionless analysis. Dimensional analysis predicts the various dimensionless parameters which are helpful in correlating experimental data.

There are two important mass transfer processes, which we shall consider, the transfer of mass into a steam flowing under forced convection and the transfer of mass into a phase which is moving as the result of natural convection associated with density gradients.

Transfer into a stream flowing under forced convection

Consider the transfer of mass from the walls of a circular conduit to a fluid flowing through the conduit. The mass transfer is due to the concentration driving force $C_{As} - C_A$.

These variables include terms descriptive of the system geometry, the flow and fluid properties and the quantity of importance, k_c.

By the Buckingham method of grouping the variables, the number of dimensionless π groups is equal to the number of variables minus the number of fundamental dimensions. Hence the number of dimensionless group for this problem will be three.

With D_{AB}, ρ and D as the core variables, the three π groups to be formed are

$$\pi_1 = D_{AB}^a \rho^b D^c k_c \text{ ---------------------------- (1.14)}$$

$$\pi_2 = D_{AB}^d \rho^e D^f \vartheta \text{ ---------------------------- (1.15)}$$

and $$\pi_3 = D_{AB}^g \rho^h D^i \mu \text{ ---------------------------- (1.16)}$$

Substituting the dimensions for π ,

$$\pi_1 = D_{AB}^a \rho^b D^c k_c \text{ ---------------------------- (1.17)}$$

$$1 = \left[\frac{L^2}{t}\right]^a \left[\frac{M}{L^3}\right]^b (L)^c \left[\frac{L}{t}\right] \text{ ---------------------------- (1.18)}$$

Equating the exponents of the fundamental dimensions on both sides of the equation, we have

L : $0 = 2a - 3b + c + 1$

t : $0 = -a - 1$

M : $0 = b$

Solving these equations,

$a = -1, \quad b = 0 \quad \text{and} \quad c = 1$

Thus $\pi_1 = \dfrac{k_c D}{D_{AB}}$ which is the Sherwood number.

The other two π groups could be determined in the same manner, yielding

$$\pi_2 = \frac{D v}{D_{AB}} \text{ --------------------------- (1.19)}$$

and $\pi_3 = \dfrac{\mu}{\rho D_{AB}} = S_c$ -------------------------------- (1.20)

which is termed as Schmidt Number

Dividing π_2 by π_3, we get

$$\frac{\pi_2}{\pi_3} = \left(\frac{D v}{D_{AB}}\right) \Big/ \left(\frac{\mu}{\rho D_{AB}}\right) = \frac{D v \rho}{\mu} = \mathrm{Re} \text{ ----------------- (1.21)}$$

which is the Reynolds Number

The result of the dimensional analysis of mass transfer by forced convection in a circular conduit indicates that a correlating relation could be of the form,

$$Sh = \psi\left(\text{Re}, Sc\right) \text{ ---------------------------- (1.22)}$$

Which is analogous to the heat transfer correlation

$$Nu = \psi\left(\text{Re}, \text{Pr}\right) \text{ ---------------------------- (1.23)}$$

Transfer into a phase whose motion is due to Natural Convection

Natural convection currents develop if there exists any variation in density within the fluid phase. The density variation may be due to temperature differences or to relatively large concentration differences.

According to Buckingham theorem, there will be three dimensionless groups. Choosing D_{AB}, L and μ as the core variables, the π groups to be formed are

$$\pi_1 = D_{AB}^a L^b \mu^c k_c \text{ ---------------------------- (4.24)}$$

$$\pi_2 = D_{AB}^d L^e \mu^f \rho \text{ ---------------------------- (4.25)}$$

and $$\pi_3 = D_{AB}^g L^h \mu^i g \Delta \rho_A \text{ ---------------------------- (4.26)}$$

Solving for the dimensionless groups, we obtain

$$\pi_1 = \frac{k_c L}{D_{AB}} = Nu, \text{ the Nusselt number ---------------------- (4.27)}$$

$$\pi_2 = \frac{\rho D_{AB}}{\mu} = \frac{1}{Sc}, \text{ the reciprocal of Schmidt number} \quad \text{-------- (4.28)}$$

and $$\pi_3 = \frac{L^3 g \Delta \rho_A}{\mu D_{AB}} \quad \text{------------------------------- (4.29)}$$

With the multiplication of π_2 and π_3, we obtain a dimensionless parameter analogous to the Grashof number in heat transfer by natural convection

$$\pi_2 \pi_3 = \left(\frac{\rho D_{AB}}{\mu} \right) \left(\frac{L^3 g \Delta \rho_A}{\mu D_{AB}} \right)$$

$$= \frac{L^3 \rho g \Delta \rho_A}{\mu^2} = Gr_{AB} \quad \text{-------------------------- (4.30)}$$

The result of the dimensional analysis of mass transfer by natural convection indicates that a correlating relation could be of the form,

$$Sh = \psi \left(Gr_{AB}, Sc \right) \quad \text{---------------------------- (4.31)}$$

5.3 Analogy among Mass, Heat and Momentum Transfer

Analogies among mass, heat and momentum transfer have their origin either in the mathematical description of the effects or in the physical parameters used for quantitative description.

To explore those analogies, it could be understood that the diffusion of mass and conduction of heat obey very similar equations. In particular, diffusion in one dimension is described by the Fick's Law as

$$J_A = - D_{AB} \frac{d C_A}{d z} \quad \text{------------------------------ (4.32)}$$

Similarly, heat conduction is described by Fourier's law as

$$q = -k \frac{dT}{dz} \text{ ---------------------------------- (4.33)}$$

Where k is the thermal conductivity.

The similar equation describing momentum transfer as given by Newton's law is

$$\tau = -\mu \frac{dv}{dz} \text{ ----------------------------- (4.34)}$$

Where τ is the momentum flux (or shear stress) and μ is the viscosity of fluid.

At this point it has become conventional to draw an analogy among mass, heat and momentum transfer. Each process uses a simple law combined with a mass or energy or momentum balance.

In this section, we shall consider several analogies among transfer phenomenon which has been proposed because of the similarity in their mechanisms. The analogies are useful in understanding the transfer phenomena and as a satisfactory means for predicting behaviour of systems for which limited quantitative data are available.

The similarity among the transfer phenomena and accordingly the existence of the analogies require that the following five conditions exist within the system

1. The physical properties are constant

2. There is no mass or energy produced within the system. This implies that there is no chemical reaction within the system

3. There is no emission or absorption of radiant energy.

4. There is no viscous dissipation of energy.

5. The velocity profile is not affected by the mass transfer. This implies there should be a low rate of mass transfer.

5.4 Reynolds Analogy

The first recognition of the analogous behaviour of mass, heat and momentum transfer was reported by Osborne Reynolds in 1874. Although his analogy is limited in application, it served as the base for seeking better analogies.

Reynolds postulated that the mechanisms for transfer of momentum, energy and mass are identical. Accordingly,

$$\frac{k_c}{v_\infty} = \frac{h}{\rho v_\infty C_p} = \frac{f}{2} \quad \text{--------------------------------} \quad (4.35)$$

Here h is heat transfer coefficient

f is friction factor

v_∞ is velocity of free stream

The Reynolds analogy is interesting because it suggests a very simple relation between different transport phenomena. This relation is found to be accurate when Prandtl and Schmidt numbers are equal to one. This is applicable for mass transfer by means of turbulent eddies in gases. In this situation, we can estimate mass transfer coefficients from heat transfer coefficients or from friction factors.

5.5 Chilton – Colburn Analogy

Because the Reynold's analogy was practically useful, many authors tried to extend it to liquids. Chilton and Colburn, using experimental data, sought modifications to the Reynold's analogy that would not have the restrictions that Prandtl and Schmidt numbers must be equal to one. They defined for the j factor for mass transfer as

$$j_D = \frac{k_c}{v_\infty}(Sc)^{2/3} \text{ ---------------------------- } (4.36)$$

The analogous j factor for heat transfer is

$$j_H = St\ \Pr^{2/3} \text{ ------------------------------ } (4.37)$$

where St is Stanton number = $\frac{Nu}{\text{Re Pr}} = \frac{h}{\rho \vartheta_\infty C_p}$

Based on data collected in both laminar and turbulent flow regimes, they found

$$j_D = j_H = \frac{f}{2} \text{ ----------------------------- } (4.38)$$

This analogy is valid for gases and liquids within the range of 0.6 < Sc < 2500 and 0.6 < Pr < 100.

The Chilton-Colburn analogy has been observed to hold for many different geometries for example, flow over flat plates, flow in pipes, and flow around cylinders.

5.6 The Prandtl analogy

In the turbulent core the transport is mainly by eddies and near the wall, that is laminar sub-layer, the transport is by molecular diffusion. Therefore, Prandtl modified the above two analogies using universal velocity profile while driving the analogy

$$St = \frac{\frac{f}{2}}{1+5\sqrt{\frac{f}{2}}(Pr-1)} \qquad (4.36)$$

5.7 The Van Karman analogy

Though Prandtl considered the laminar and turbulent laminar sublayers but did not consider the buffer zone. Thus, Van Karman included the buffer zone into the Prandtl analogy to further improve the analogy.

$$St = \frac{\frac{f}{2}}{1+5\sqrt{\frac{f}{2}}\left[(Pr-1)+\ln\left\{1+\frac{5}{6}(Pr-1)\right\}\right]} \qquad (4.37)$$

Problem. A stream of air at 100 kPa pressure and 300 K is flowing on the top surface of a thin flat sheet of solid naphthalene of length 0.2 m with a velocity of 20 m/sec. The other data are:

Mass diffusivity of naphthalene vapor in air = $6 * 10^{-6}$ m^2/sec

Kinematic viscosity of air = $1.5 * 10^{-5}$ m^2.sc

Concentration of naphthalene at the air-solid naphthalene interface = $1 * 10^{-5}$ kmol/m^3

Calculate:

(a) the overage mass transfer coefficient over the flat plate
(b) the rate of loss of naphthalene from the surface per unit width

Note: For heat transfer over a flat plate, convective heat transfer coefficient for laminar flow can be calculated by the equation.

$$Nu = 0.664\ Re_L^{1/2}\ Pr^{1/3}$$

you may use analogy between mass and heat transfer.

Solution:

Given: Correlation for heat transfer

$$Nu = 0.664\ Re_L^{1/2}\ Pr^{1/3}$$

The analogous relation for mass transfer is

$$Sh = 0.664\ Re_L^{1/2}\ Sc^{1/3} \quad \text{--(1)}$$

where

Sh = Sherwood number = kL/D_{AB}

Re_L = Reynolds number = $L\upsilon\rho/\mu$

Sc = Schmidt number = $\mu / (\rho\ D_{AB})$

k = overall mass transfer coefficient

L = length of sheet

D_{AB} = diffusivity of A in B

υ = velocity of air

μ = viscosity of air

ρ = density of air, and

μ/ρ = kinematic viscosity of air.

Substituting for the known quantities in equation (1)

$$\frac{k\,(0.2)}{6 * 10^{-6}} = 0.664 \left[\frac{(0.2)(20)}{1.5 * 10^{-5}}\right]^{1/2} \left[\frac{1.5 * 10^{-5}}{6 * 10^{-6}}\right]^{1/3}$$

k = 0.014 m/sec

Rate of loss of naphthalene = $k\,(C_{Ai} - C_{A\infty})$

$= 0.014\,(1 * 10^{-5} - 0) = 1.4024 * 10^{-7}$ kmol/m^2 sec

Rate of loss per meter width = $(1.4024 * 10^{-7})\,(0.2) = 2.8048 * 10^{-8}$ kmol/m.sec

= 0.101 gmol/m.hr.

5.8 Convective Mass Transfer Correlations

Extensive data have been obtained for the transfer of mass between a moving fluid and certain shapes, such as flat plates, spheres and cylinders. The techniques include sublimation of a solid, vapourization of a liquid into a moving stream of air and the dissolution of a solid into water.

These data have been correlated in terms of dimensionless parameters and the equations obtained are used to estimate the mass transfer coefficients in other moving fluids and geometrically similar surfaces.

Flat Plate

From the experimental measurements of rate of evaporation from a liquid surface or from the sublimation rate of a volatile solid surface into a controlled air-stream, several correlations are available. These correlation have been found to satisfy the equations obtained by theoretical analysis on boundary layers,

$$Sh = 0.664\ \mathrm{Re}_L^{1/2} Sc^{1/3}\ (\text{laminar})\ \mathrm{Re}_L < 3*10^5 \quad \text{-------------} \ (4.39)$$

$$Sh = 0.036\ \mathrm{Re}_L^{0.8}\ Sc^{1/3}\ (\text{turbulent})\ \mathrm{Re}_L > 3*10^5 \quad \text{-----------} \ (4.40)$$

Using the definition of j factor for mass transfer on equation (4.39) and (4.40) we obtain

$$j_D = 0.664\ \mathrm{Re}_L^{-1/2}\ (\text{laminar})\ \mathrm{Re}_L < 3*10^5 \quad \text{-------------} \ (4.41)$$

$$J_D = 0.037\ \mathrm{Re}_L^{-0.2}\ (\text{turbulent})\ \mathrm{Re}_L > 3*10^5 \quad \text{-----------} \ (4.42)$$

These equations may be used if the Schmidt number in the range 0.6 < Sc < 2500.

7. If the local Nusselt number for the laminar boundary layer that is formed over a flat plate is

$$Nu_x = 0.332\ \mathrm{Re}_x^{1/2}\ Sc^{1/3}$$

Obtain an expression for the average film-transfer coefficient $\bar{k}_c$, when the Reynolds number for the plate is

a) $Re_L = 100\,000$
b) $Re_L = 1500\,000$

The transition from laminar to turbulent flow occurs at $Re_x = 3 * 10^5$.

Derivation:

By definition : $$\bar{k}_c = \frac{\int_o^L k_c\, dx}{\int_o^L dx}$$

and $Nu_x = \dfrac{k_c\, x}{D_{AB}}$; $\quad Re_x = \dfrac{x\, v\, \rho}{\mu}$; $\quad Sc = \dfrac{\mu}{\rho\, D_{AB}}$;

For $Re_L = 100\,000$; (which is less than the Reynolds number corresponding to Transition value of $3 * 10^5$)

$$\bar{k}_c = \frac{\int_o^L 0.332 \left(\frac{x\, v\, \rho}{\mu}\right)^{\frac{1}{2}} (Sc)^{\frac{1}{3}} \frac{D_{AB}}{x}\, d\,x}{L}$$

$$= \frac{0.332\, (Sc)^{1/3} \left(\frac{v\, \rho}{\mu}\right)^{1/2}}{L} D_{AB} \int_o^L \frac{d\,x}{x^{1/2}}$$

$$= \frac{0.332}{\frac{1}{2} L} Sc^{1/3} \left(\frac{v\, \rho}{\mu}\right)^{1/2} D_{AB} \left[x^{1/2}\right]_o^L$$

(i.e.) $\dfrac{\bar{k}_c\, L}{D_{AB}} = 0.664\ Re_L^{1/2}\ Sc^{1/3}$ [answer (a)]

For $Re_L = 1500\,000$ ($> 3 * 10^5$)

$$\bar{k}_c = D_{AB}\,\frac{\left[\int_o^{L_t} 0.332\,\mathrm{Re}_x^{1/2}\,Sc^{1/3}\,\frac{dx}{x} + \int_{L_t}^{L} 0.0292\,\mathrm{Re}_x^{4/5}\,Sc^{1/3}\,\frac{dx}{x}\right]}{L}$$

where L_t is the distance from the leading edge of the plane to the transition point where $\mathrm{Re}_x = 3 * 10^5$.

$$\bar{k}_c = D_{AB}\,\frac{\left[0.332\,Sc^{1/3}\left(\frac{v\rho}{\mu}\right)^{1/2}\int_o^{L_t}\frac{dx}{x^{1/2}} + 0.0292\,Sc^{1/3}\left(\frac{v\rho}{\mu}\right)^{4/5}\int_{L_t}^{L}\frac{dx}{x^{1/5}}\right]}{L}$$

$$\frac{\bar{k}_c L}{D_{AB}} = 0.664\,\mathrm{Re}_t^{1/2}\,Sx^{1/3} + \frac{0.0292}{4/5}\,Sc^{1/3}\left[x^{4/5}\right]_{L_t}^{L}\left(\frac{V\rho}{\mu}\right)^{4/5}$$

$$= 0.664\,\mathrm{Re}_t^{1/2}\,Sc^{1/3} + 0.0365\,Sc^{1/3}\left(\mathrm{Re}_L^{4/5} - \mathrm{Re}_t^{4/5}\right)$$

$$\frac{\bar{k}_c L}{D_{AB}} = 0.664\,\mathrm{Re}_t^{1/2}\,Sc^{1/3} + 0.0365\,\mathrm{Re}_L^{4/5}\,Sc^{1/3} - 0.0365\,\mathrm{Re}_t^{4/5}\,Sc^{1/3}$$ where $\mathrm{Re}_t = 3 * 10^5$

Single Sphere

Correlations for mass transfer from single spheres are represented as addition of terms representing transfer by purely molecular diffusion and transfer by forced convection, in the form

$$Sh = Sh_o + C\,\mathrm{Re}^m\,Sc^n \quad \text{----------------------- (4.43)}$$

Where C, m and n are constants, the value of n is normally taken as 1/3

For very low Reynold's number, the Sherwood number should approach a value of 2. This value has been derived in earlier sections by theoretical consideration of molecular diffusion from a sphere into a large volume of stagnant fluid. Therefore the generalized equation becomes

$$Sh = 2 + C\,\mathrm{Re}^m\,Sc^{1/3} \quad \text{-------------------------- (4.44)}$$

For mass transfer into liquid streams, the equation given by Brain and Hales

$$Sh = \left(4 + 1.21\ Pe_{AB}^{2/3}\right)^{1/2} \text{ -------------------------- (4.45)}$$

correlates the data that are obtained when the mass transfer Peclet number, Pe_{AB} is less than 10,000. This Peclet number is equal to the product of Reynolds and Schmidt numbers (i.e.)

$$Pe_{AB} = \mathrm{Re}\ Sc \text{ ---------------------------------- (4.46)}$$

For Peclet numbers greater than 10,000, the relation given by Levich is useful

$$Sh = 1.01\ Pe_{AB}^{1/3} \text{ --------------------------- (4.47)}$$

The relation given by Froessling

$$Sh = 2 + 0.552\ \mathrm{Re}^{1/2}\ Sc^{1/3} \text{ ----------------------- (4.48)}$$

correlates the data for mass transfer into gases for at Reynold's numbers ranging from 2 to 800 and Schmidt number ranging 0.6 to 2.7.

For natural convection mass transfer the relation given by Schutz

$$Sh = 2 + 0.59\ \left(Gr_{AB}\ Sc\right)^{1/4} \text{ ----------------------- (4.49)}$$

is useful over the range

$2 * 10^{8} < Gr_{AB}\ Sc < 1.5 * 10^{10}$

Problem. The mass flux from a 5 cm diameter naphthalene ball placed in stagnant air at 40°C and atmospheric pressure, is $1.47 * 10^{-3}$ mol/m^2. sec. Assume the vapor pressure of naphthalene to be 0.15 atm at 40°C and negligible bulk concentration of naphthalene in air. If air starts blowing across the surface of naphthalene ball at 3 m/s by what factor will the mass transfer rate increase, all other conditions remaining the same?

For spheres : $Sh = 2.0 + 0.6\,(Re)^{0.5}\,(Sc)^{0.33}$

Where Sh is the Sherwood number and Sc is the Schmids number. The viscosity and density of air are $1.8 * 10^{-5}$ kg/m.s and 1.123 kg/m^3, respectively and the gas constant is 82.06 cm^3 . atm/mol.K.

Calculations:

$Sh = \dfrac{k_c L}{D_{AB}}$ where L is the characteristic dimension for sphere L = Diameter.

$$Sc = \frac{\mu}{\rho D_{AB}}$$

$$R_c = \frac{D v \rho}{\mu}$$

$Mass\ flux,\ N_A = K_c \Delta c$ -------------------------------(1)

$Sh = 2.0 + 0.6\,(Re)^{0.5}\,(Sc)^{0.33}$

$\dfrac{k_c D}{D_{AB}} = 2.0 + 0.6 \left(\dfrac{D V \rho}{\mu}\right)^{0.5} \left(\dfrac{\mu}{\rho D_{AB}}\right)^{0.33}$ ----------------------- (2)

also $N = K_G \Delta \overline{p}_A$

Therefore $\dfrac{k_c}{RT} = K_G$

Given:

$$N = 1.47 * 10^{-3} \frac{mol}{m^2.\sec} = \frac{K_c}{RT} \Delta \bar{p}_A$$

$$\frac{k_c}{RT}\left[\frac{0.15}{1} - 0\right] = 1.47 * 10^{-3} * 10^{-4} \frac{mol}{cm^2.\sec}$$

$$k_c = \frac{1.47 * 10^{-7}}{0.15} * 82.06 * (273 + 40)$$

= 0.0252 cm/sec

$k_c = 2.517 * 10^{-4}$ m/sec ------------------------------(3)

Estimation of D_{AB}:

From (2),

$$\frac{2.517 * 10^{-4} * 5 * 10^{-2}}{D_{AB}} = 2 \quad \text{(since v = 0)}$$

Therefore $D_{AB} = 6.2925 * 10^{-6}$ m²/sec.

And

$$\frac{k_c * 5 * 10^{-2}}{6.2925 * 10^{-6}} = 2 + 0.6\left[\frac{5 * 10^{-2} * 3 * 1.123}{1.8 * 10^{-5}}\right]^{0.5}\left[\frac{1.8 * 10^{-5}}{1.123 * 6.2925 * 10^{-6}}\right]^{0.33}$$

$7946\ k_c = 2 + 0.6 * (96.74) * (1.361)$

$k_c = 0.0102$ m/sec. -- (4)

$$\frac{(4)}{(3)} \Rightarrow \frac{N_{A2}}{N_{A1}} = \frac{0.0102}{2.517 * 10^{-4}} = 40.5$$

Therefore, rate of mass transfer increases by 40.5 times the initial conditions.

Single Cylinder

Several investigators have studied the rate of sublimation from a solid cylinder into air flowing normal to its axis. Bedingfield and Drew correlated the available data in the form

$$\frac{k_G P Sc^{0.56}}{G_m} = 0.281 \left(Re^{/}\right)^{-0.4} \text{ ------------------------ (4.50)}$$

which is valid for $400 < Re^{/} < 25000$

and $0.6 < Sc < 2.6$

Where $Re^{/}$ is the Reynold's number in terms of the diameter of the cylinder, G_m is the molar mass velocity of gas and P is the pressure.

5.9 Flow Through Pipes

Mass transfer from the inner wall of a tube to a moving fluid has been studied extensively. Gilliland and Sherwood, based on the study of rate of vapourization of nine different liquids into air given the correlation

$$Sh \frac{p_{B,lm}}{P} = 0.023 \ Re^{0.83} \ Sc^{0.44} \text{ -------------- (4.51)}$$

Where $p_{B,\,lm}$ is the log mean composition of the carrier gas, evaluated between the surface and bulk stream composition. P is the total pressure. This expression has been found to be valid over the range

$2000 < Re < 35000$

$0.6 < Sc < 2.5$

Linton and Sherwood modified the above relation making it suitable for large ranges of Schmidt number. Their relation is given as

$$Sh = 0.023\ \mathrm{Re}^{0.83}\ Sc^{1/3} \text{ --------------------- } (4.52)$$

and found to be valid for

$2000 < Re < 70000$

and $1000 < Sc < 2260$

Problem. A solid disc of benzoic acid 3 cm in diameter is spin at 20 rpm and 25°C. Calculate the rate of dissolution in a large volume of water. Diffusivity of benzoic acid in water is $1.0 * 10^{-5}$ cm^2/sec, and solubility is 0.003 g/cc. The following mass transfer correlation is applicable:

$$Sh = 0.62\ Re^{1/2}\ Sc^{1/3}$$

Where $\mathrm{Re} = \dfrac{D^2 \omega \rho}{\mu}$ and ω is the angular speed in radians/time.

Calculations:

$$\text{Dissolution rate} = N_A\ S \text{ ----------------------------} (1)$$

Where N_A = mass flux, and

S = surface area for mass transfer

$$N_A = k_c\ (C_{As} - C_{A\infty}) \text{ ------------------------------- } (2)$$

Where C_{As} is the concentration of benzoic and at in water at the surface of the dose.

$C_{A\infty}$ is the concentration benzoic acid in wate for an from the surface of the disc.

Given:

Sh = 0.62 $Re^{1/2}$ $Sc^{1/3}$

(i.e.) $\frac{k_c D}{D_{AB}} = 0.62 \left(\frac{D^2 \omega \rho}{\mu}\right)^{\frac{1}{2}} \left(\frac{\mu}{\rho D_{AB}}\right)^{\frac{1}{3}}$ ------------------ (3)

1 rotation = 2 π radian

Therefore 20 rotation per minute = 20 * 2 π radian/min

$$= \frac{20}{60} * 2\pi \ radian/\sec$$

For water ρ = 1 g/cm^3 μ = 1 centipoise = 0.01 g/cm.sec.

From (3),

$$k_c = 0.62\ D_{AB} \left(\frac{\omega \rho}{\mu}\right)^{\frac{1}{2}} \left(\frac{\mu}{\rho D_{AB}}\right)^{\frac{1}{3}}$$

$$= 0.62 * 1.0 * 10^5 * \left(\frac{(40\pi/60) * 1}{0.01}\right)^{\frac{1}{2}} \left(\frac{0.01}{1 * 1.0 * 10^{-5}}\right)^{\frac{1}{3}}$$

= $8.973 * 10^{-4}$ cm/sec.

From (2),

N_A = $8.973 * 10^{-4}$ (0.003 – 0)

= $2.692 * 10^{-6}$ g/cm^2.sec

From (1),

$N_A S = N_A * (2\pi r^2)$

= $2.692 * 10^{-6} * (2\pi * 1.5^2)$

= $3.805 * 10^{-5}$ g/sec

= 0.137 g/hr.

5.10 Hot Wire Anemometer

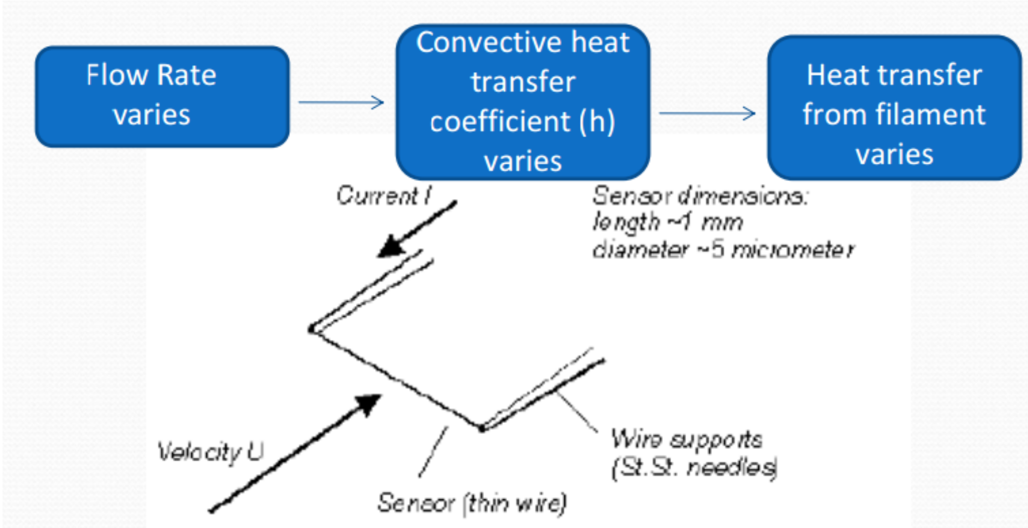
A hot-wire anemometer measures local instantaneous velocity based on principles of heat transfer. However, it requires that the fluid itself be at a uniform temperature. It can be used to measure three components of velocity and velocity fluctuations arising in turbulent flow. This is possible because of the high speed of response of the hot-wire probe and the associated feedback circuit. A hot-wire probe is used in gas flows, while a hot-film is used for liquid flow. The hot-wire has a limitation that it is insensitive to the flow direction. Further, it has a non-linear input-output relationship which makes its sensitivity non-uniform over any velocity range. In particular, the sensitivity decreases with increasing velocity. The hot-wire probe is a platinum-coated tungsten wire, typically of 5 μm diameter and about mm length, supported between highly conducting prongs. Tungsten has high temperature coefficient of resistance (i.e., resistance increases rapidly with temperature) and the platinum coating affords strength as well as protection against corrosion of the thin wire.

Characteristics of Hot Wire Anemometry

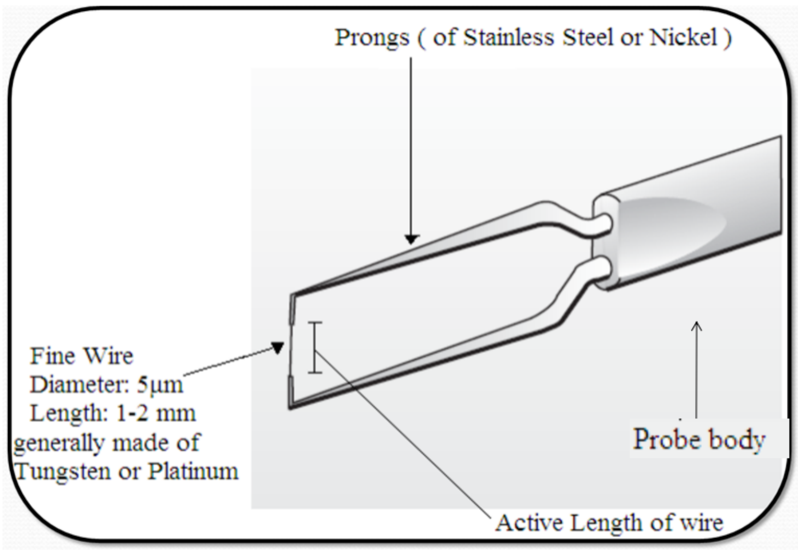
- Intrusive Technique
- Measurement of instantaneous velocities and temperature at a point in a flow.
- Hot wire anemometry is an ideal tool for measurement of velocity fluctuations in time domain in turbulent flows.
- Principal tool for basic studies of physics of turbulent flows.

Advantages of HWA

- Good Frequency response
- Measurements to several hundred kHz possible, 1 MHz also feasible
- Velocity Measurement: measures magnitude and direction of velocity and velocity fluctuations, Wide velocity range Temperature Measurements
- Two Phase Flow: Measurements in flows containing continuous turbulent phase and distributed bubbles



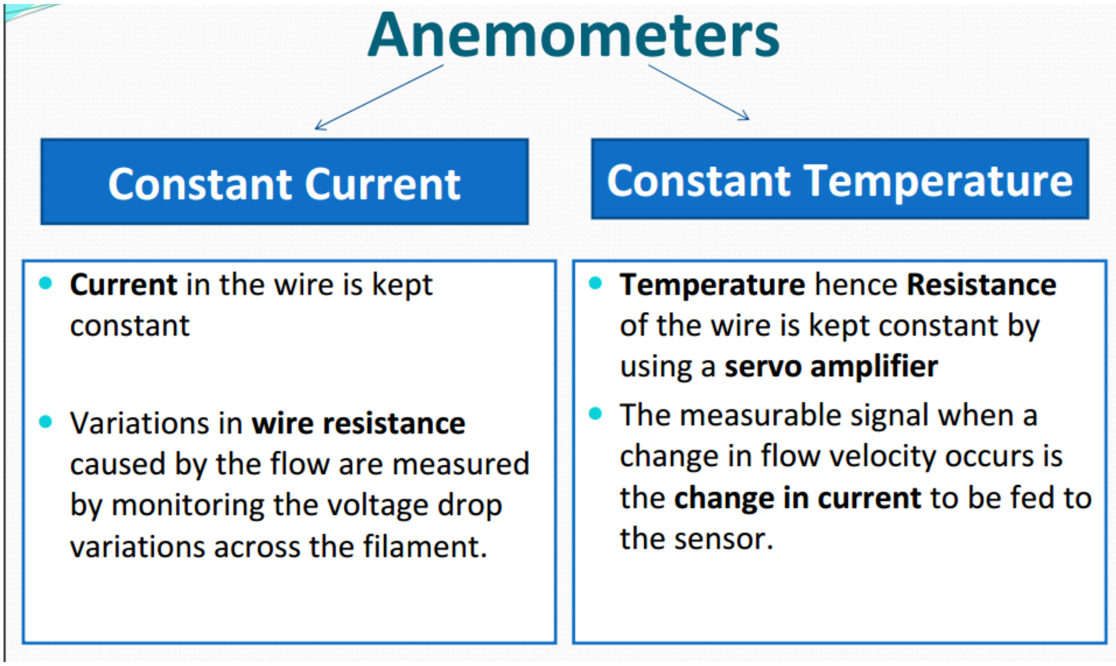
Operation of Hot Wire Sensor

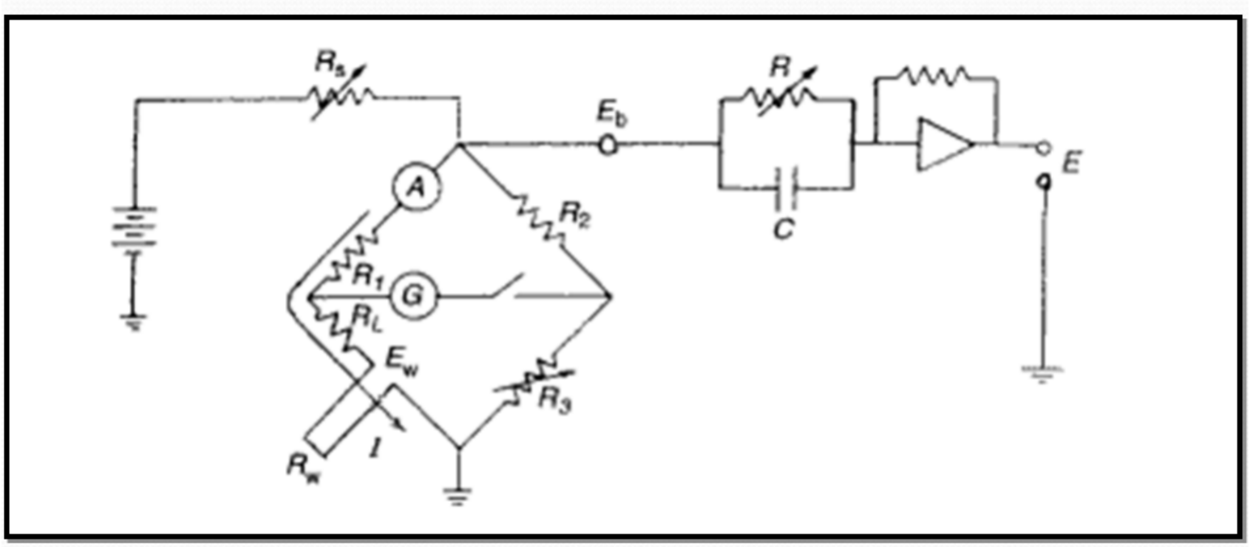


Structure of hot wire probe

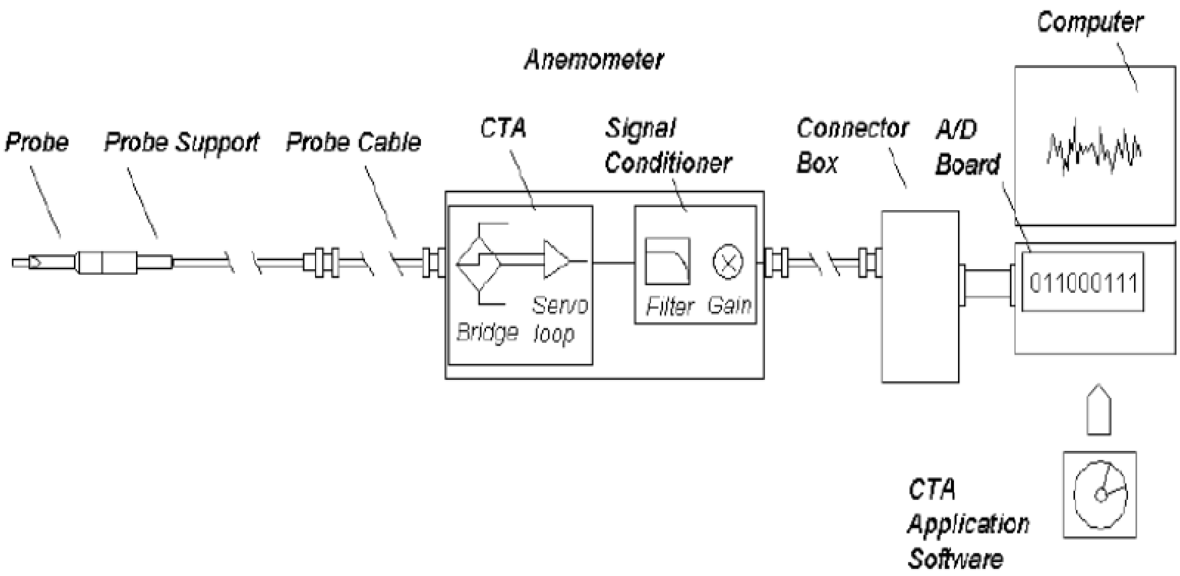
Mode of Operation

1. **Constant Current Anemometer**
2. **Constant Temperature Anemometer**





Circuit Diagram of Constant Current Anemometer



Basic CTA Measuring Chain