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Scheduling Project Crashing Time using Linear Programming Technique

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Abstract

In this paper, some concepts of linear programming and critical path method were reviewed to describe recent modeling structures that have been of great value in analyzing extended planning horizon project time-cost crashes problems. A simplified representation of a small project and a linear programming model was formulated to represent this system. Procedures to solve these various problems formulations were cited and the final solution was obtained by using LINDO program. The model developed represents many restrictions and management considerations of the project. It could be used by construction managers in a planning stage to explore numerous possible opportunities to the contractor and predict the effect of a decision on the construction to facilitate a preferred operating policy given different management objectives. An implementation using this method was shown to outperform several other techniques and a large class of test problems. Linear programming shows that the algorithm is very promising in practice on a wide variety of time-cost crashes problems. This method is simple, applicable to a large network, and generates a shorter computational time at low cost, along with an increase in robustness.

Keywords
Linear programming, project crashing, time-cost trade-off, CPM.

1. Introduction
Project management is the process of the application of knowledge, skills, tools, and techniques to project activities to meet project requirements. That is to say, project management is an interrelated group of processes that enables the project team to achieve a successful project. The functions of project management include:

a) Planning – Planning the project and establishing its life cycle.
b) Organizing – Organizing resources: personnel, equipment, materials, facilities, and finances.
c) Leading – Assigning the right people to the right job, motivating people, setting the course and goals for the project.
d) Controlling – Evaluating progress of project and, when necessary, applying hangs to get it back on track. Performing these functions in an organized framework of processes is the job of the Project Manager. The project engineer is assigned by an organization the responsibility and authority to manage a technical project or program in a manner that will result in meeting the project objectives, the three basic objectives for which the contractor‘s project engineer is responsible are as follows:

a) Deliver a project that meets the requirements of the specification.
b) Deliver a project that meets the requirements of the contract delivery schedule.
c) Meet the company’s profit objectives for the contract.
From a project management perspective, a project is considered a success if:
a) The resulting information system is acceptable to the customer.
b) The system is delivered “on time”
c) The system is delivered “within budget”

Project management has evolved as a new field with the development of two analytical techniques for planning, scheduling, and controlling of projects. These are the critical path method (CPM) and the project evaluation and review technique (PERT). These techniques are neither suitable for the scheduling of linear projects nor adequate for addressing typical challenge related to time-cost trade-off [1].

Optimal schedule cost can be determined by trial and error for small project, but in a realistic project consisting of many activities, such trial-and error becomes extremely tedious and impossible. A very limited number of computer programs are available but far from perfect. Such programs have a limited capacity to accept time-cost data and at a very high price. Thus in this paper, LP (Linear Programming) as an optimization technique has been developed to aid in the quick determination of the minimum cost for every possible value of project duration. Clearly, the use of this optimization techniques incorporated with time-cost trade-off becomes an economic necessity [1].

2. Literature Review

The cost of the network activities had been optimized for various overall durations. The optimum crash of time against cost had been made. This approach was an acceptable tool of management and proving to be not only superior method for planning, scheduling and controlling project progress, but also was very real and valuable assets to contractors in convincing the owner of their potentials and abilities [2].

Project scheduling under uncertainty by using survey and research potentials was carried out. In that survey they reviewed the fundamental approaches for scheduling under uncertainty, reactive scheduling, stochastic project scheduling, fuzzy project scheduling, robust (proactive) scheduling and sensitivity analysis. The potentials of those approaches for scheduling under uncertainty projects with deterministic network evolution structure were discussed. They offered a review of the major approaches to deal with scheduling risk and uncertainty. The methodologies for stochastic project scheduling basically view the project scheduling problem as a multi-stage decision process [3] .

Project crashing and costs laws in the knowledge age was studied. That study seeks to add contributions of the innovation and industrial economics to more used techniques of crashing in the projects management domain. First, it presented the brute force method, improved for the use of the MS Project. Second, it developed the models for determining the earliest crash completion time and for determining a least costly crash schedule. Third, it established costs laws, which allow inferring that the cost of a project does not depend only on the production rate but depending also on the time were the first unit of production will be available, on the global volume of production and on the project completion time. Projects management might considerably help organizations that search for a better market position, providing a higher significance to the competitive advantages to be developed by the company along the time [4] .

A new method for determining which project activities to crash was presented. The calculation of resource elasticity required no new information, and was computed using the project manager's conventional estimated of normal and crash duration times as well as normal and crash costs. Instead of assuming a linear relationship between crash spending and time saved, a linear relationship between the additional resources used per day and the time saved were assumed. That assumption allowed computing a resource elasticity measurement that tolled which activities would respond most significantly to increases in the resources allocated to them. That elasticity measure was a practical, easily implemented metric that allows the manager to identify which activities' duration times were more or less sensitive to daily spending rates. That metric could be used for determining which activities should be crashed, which should be slowed (if the money for crashing had to come from another activity), and for determining which activities need to be most closely monitored [5].

The simulation approach for optimization project cost and schedule was one of a variety of tools that could use to bring projects back under control and reinforce the use of project management in organizations. The use of simulation to crash project management networks in order to reduce time and cost overruns was a worth endeavor. The project manager, in collaboration with the IT division, could routinely submit each developed network to crashing (using the simulation program), before major work commences [6]. Hence, the optimization of time and cost process technique could be incorporated as a standard procedure for every project was concluded, the time spent on the actual crashing was minimal and the project management schedule could be reduced to a minimum optimum level to save time and money [6].

3. Linear Programming Technique

Linear programming is a tool for decision making under certain situation. So, the basic assumption of this approach is that we have to know some relevant data with certainty. The basic data requirements are as follows:

a) We have to know the project network with activity time, which can be achieved from PERT and CPM.
b) To what extent an activity can be crashed.

c) The crash cost associated with per unit of time for all activities. Before formulating the model, let us define some relevant terms. We know, a project is the combination of some activities, which are interrelated in a logical sequence in the sense that the starting of some activities is dependent upon the completion of some other activities. These activities are jobs which require time and resources to be completed. The relationship between the activities is specified by using event. As an event represents a point in time that implies the completion of some activities and the beginning of new ones, the beginning and end point of an activity are thus expressed by two events. To reduce the time to complete the activity, more resources are applied in the form of additional personnel and overtime. As more resources are applied, the duration is shortened, but the cost rises. The maximum effort is applied so that the activity can be completed in the shortest possible time. The equation for the cost slope is

$$U_i = \frac{C_C - C_n}{T_n - T_c} \tag{1}$$

Where:
Ui,Cc and Cn are the cost slop, the crash cost and normal costs, respectively. Tc and Tn are the crash and normal times, respectively. The cost slope shows by how much the cost of the job would change if activities were speed up or slowed down. Before formulating the model, some relevant terms will be defined. It is very well known, a project is the combination of some activities, which are interrelated in a logical sequence in the sense that the starting of some activities is dependent upon the completion of some other activities. These activities are jobs, which require time and resources to be completed. The relationship between the activities is specified by using event. As an event represents a point in time that implies the completion of some activities and the beginning of new ones, the beginning and end point of an activity are thus, expressed by two events.

Now let's define the variable of the problem.

Yi = The time when an event i will occur, measured since the beginning of the project,
where i = (1, 2, 3,…,n).
Xi = Amount of times (measured in terms of days, weeks, months or some other units) that each activity i will be crashed, where i = (1, 2, 3…n).

The objective is to minimize the cost of crashing total project via minimize the durations of crashing activities that multiplied by their associated costs slope, then adding the resultant cost to the normal cost of project completion. The LP objective function will be:

$$\min z = \sum_{i=1}^{n} U_i X_i \tag{2}$$

This objective function is subject to some constraints. These constraints can be classified in to three categories.

Crash time constraints: We can reduce the time to complete an activity by simply increasing the resources or by improving the productivity, which also requires the commitment of additional resources. But, it is not possible to reduce the required time to complete an activity after a certain threshold limit. Strive for such intention will result in superfluous resources employment which will be an inefficient approach. That is why, the allowable time to crash an activity has a limit. Constraints unfolding the network: These set of constraints describe the structure of the network. As we mention earlier that the activities of a project are interrelated, the starting of some activities is dependent upon the completion of some other activities; we must have to establish research sequence of the activities through constraints.

Nonnegative constraints: All decision variables must ≥ 0. So, the constraints are:

Crash time constraints: Xi≤ Allowable crashing time for activity i measured in terms of days, weeks, months or some other units.
Constraints unfolding the network: There will be one or more constraints for each event depending on the predecessor activities of that event. As the event 1 will start at the beginning of the project, we begin by setting the occurrence time for event 1 equals to zero. Thus, Y1 = 0. The other events will be expressed as follows:

Start time of this activity (Yi) = (start time + normal duration -crash duration) for this immediate predecessor.

Project completion constraints: Ym≤ project deadline after being stretched, where m indicates the last event of that project. This constraint will recognize that the last event (completion of last activities) must take place before the project deadline date.

3. A Prototype Example

In order to illustrate how to implement linear programming technique in scheduling project crashing time Al jabal construction company, road infrastructure project has been selected. The project deals with the construction of a rod of 463.875 meter long and it is consists of two phases, the first phase is known as foul and storm drainage works phase and the second phase is known as rods and side walks phase. Table 4.1a and Table 4.1b show the list of the various activities needed for each phase. The first column represents the activity identification number, the description of each activity is shown in second column, and the third column provides the activity duration. Table 4.2 shows a project with hypothetical normal time - cost data and crash time –cost data.

Table 4.1a Project activities with duration for the first phase

ID	Activity description	Duration (Day)
Road (463.875 m)		
1	Excavation foul	10
2	Foul pipe installation, backfilling & testing	10
3	Foul manhole construction	10
4	Foul laterals & hours connections	10
5	Excavation storm	10
6	Storm pipe installation, backfilling & testing	10
7	Storm manhole construction	10
8	Storm pipe junctions backfilling	10

Table 4.1b Project activities with duration for the second phase

ID	Activity description	Duration (Day)
Road (463.875 m)		
9	Excavation to formation	30
10	Water house connections	30
11	Cable crossing ducts	30
12	Gully & gully connection to storm	30
13	Road endings	30
14	Compaction to formation	30
15	Sub base laying & compaction	30
16	Granular base coarse laying & compaction	30
17	Asphalt binder to roads	10
18	House concrete steps and ramps	12
19	Sidewalk backfill & base coarse	10
20	Sidewalk wearing	10
21	Asphalt wearing to roads	10
22	Finish roads & sidewalks road	0

Table 4.2 Activities list with hypothetical normal and crash data

Activity code	Depends on	Normal time (days)	Crash time(days)	Normal cost $	Crash cost $
1	-	10	9	4,400.00	4,840.00
2	1	10	9	3,035.00	33,385,00
3	1	10	9	4,200.00	4,620.00
4	2,3	10	8	87,600.00	105,120.00
5	4	10	9	4,400.00	4,840.00
6	5	10	9	30,350.00	33,385.00
7	5	10	9	4,200.00	4,620.00
8	5	10	9	2,400.00	2,640.00
9	6,7,8	30	28	7,650,00	8,160.00
10	9	30	28	10,950.00	11,680.00
11	9	30	28	12,570.00	13,408.00
12	9	30	28	10,400.00	11.093.00
13	10	30	28	29,643.00	31,620.00
14	13	30	28	20,161.00	21,505.00
15	14	30	28	28,292.00	30,178.00
16	15	30	28	33,133.00	35,342.00
17	16	10	10	33,416.00	33,416.00
18	16	12	10	5,460.00	6,370.00
19	17	10	10	7,400.00	7,400.00
20	11,12,19	10	10	12,600.00	12,600.00
21	18,20	10	10	38,567.00	38,567.00
22	21	0	0	0	0

Now, the question arises what will be or what will be the minimum completion date of the project after being crashed. To determine this extent, we have to develop a CPM based on crash time of the activities that will provide us the minimum time beyond which the project cannot be crashed. So, the adjusted deadline of the project must be greater than or equal to the project.

The total duration for the completion of the project is 97 days based on the critical path. Assume, the project manager decides to complete the project within 91 days. Now, the project manager should know how to crash the activities of the project (with minimum additional cost) so that the total cost will be minimized.

Figure 4.9 shows AON network with maximum reduction time for each activity. The cost-time slopes of the activities are shown in Table 4.3

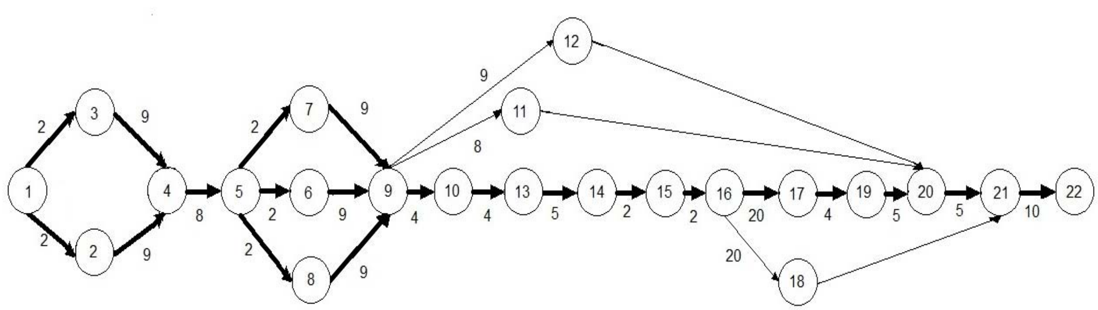

Figure 4.9 Maximum reduction time of the network activities.

Table 4.3 Cost-time slope of the activities

Activity Code[1]	Activity time that can be crashed day [2]	Additional Cost for crashing $ [3]	Crash cost/time [4] = 3/2 $/day
1	1	440	440
2	1	3,035.00	3,035.00
3	1	420,00	420,00
4	2	17,400.00	8,700.00
5	1	440,00	440,00
6	1	440,00	440,00
7	1	420,00	420,00
8	1	240,00	240,00
9	2	510,00	255,00
10	2	730	365,00
11	2	438,00	419,00
12	2	692,00	346,00
13	2	1,976.00	988,00
14	2	1,344.00	672,00
15	2	1,884.00	942,00
16	2	2,208.00	1,104.00
17	0	0.00	0.00
18	2	910,00	455,00
19	0	0.00	0.00
20	0	0.00	0.00
21	0	0.00	0.00
22	0	0.00	0.00

4. Linear Programming Model for Crashing the Hypothetical Project

Using the normal times from Table 4.5, the duration of each activity should be given by the following formula:
Duration of activity i = (ESj – Esi) – Xi (4.1)
To illustrate these relationships, consider activity 13 in the project network (Fig. 4.8)
Activity 13 represents immediate predecessor of activity 14:
Activity 13, which has Esi = 20/2/2012.
Activity 14, which has ESj= 25/2/2012.So, duration of activity 13= (25 – 20) – Xi = 5 – Xi. That is to say, activity 14 start after 5 days from starting activity 13.

By including this relationship for all the activities as constraints, the complete linear programming model is given below.

Minimize Z=440 X1 + 3035 X2 + 420 X3 + 8760 X4 + 440 X5 + 3035 X6 +420 X7 + 240 X8 + 255 X9 + 365 X10 + 419 X11 + 346.67 X12 + 988.13 X13 + 672.06 X14 + 943.09 X15 + 1104.46 X16 + 0 X17 + 455 X18 + 0 X19 + 0 X20 + 0 X21

Subject to :
a. Maximum reduction constraints :

X1 <= 1 , X2 <= 1 , X3 <= 1 , X4 <= 2 , X5 <= 1 , X6 <= 1 , X7 <= 1 , X8 <= 1 , X9 <= 2, X10 <= 2 , X11 <= 2 , X12 <= 2 , X13 <= 2 , X14 <= 2 , X15 <= 2 , X16 <= 2 , X17 <= 0, X18 <= 2 , X19 <= 0 , X20 <= 0 , X21 <= 0

b. Nonnegativity constraints:

X1 >= 0, X2 >= 0, X3 >= 0, ……………….. , X21 >= 0.

Y1=0 , Y2 >= 0 , Y3 >= 0 , ……………………, Y21 >= 0.

c. Start time constraints:

Y2 + X1 >= 2
Y3 + X1 >= 2
Y4 – Y2 + X2 >= 10
Y4 – Y3 + X3 >= 10
Y5 – Y4 + X4 >= 10
Y6 – Y5 + X5 >= 2
Y7 – Y5 + X5 >= 2
Y8 – Y5 + X5 >= 2
Y9 – Y6 + X6 >= 10
Y9 – Y7 + X7 >= 10
Y9 – Y8 + X8 >= 10
Y10 – Y9 + X9 >= 4
Y11 – Y9 + X9 >= 8
Y12 – Y9 + X9 >= 9
Y13 – Y10 + X10 >= 4
Y14 – Y13 + X13 >= 5
Y15 – Y14 + X14 >= 2
Y16 – Y15 + X15 >= 2
Y17 – Y16 + X16 >= 22
Y18 – Y16 + X16 >= 22
Y19 – Y17 + X17 >= 4
Y20 – Y11 + X11 >= 30
Y20 – Y12 + X12 >= 30
Y20 – Y19 + X10 >= 5
Y21 – Y20 + X20 >= 5
Y21 – Y18 + X18 >= 12
YFinish – Y21 + X21 >= 10

d. Project duration constraint:
YFinish <= 91, which is means that the manager need to crash the project time 6 days.

5. Analysis and Results

The linear programming model will not only take into account the activities on the critical path, but will also consider the noncritical activities, which in their turn become critical as the project time decreases. Using LINDO software, the solution of the model is presented in table 5.1, which indicates that the adding cost (crash cost) by crashing critical activities to reduce the project duration within 91 Days is $. 2120 and the cost to complete the project in normal duration (97 Days) is $. 418,145. So, the final project cost is computed by adding crash cost of the crashing critical activities $2,120 to the normal cost $418,145. So, the final project cost is $420,265.

Subtracting the crash-time amounts from the normal completion, the result indicates that activity 1 should be crashed one day. That is to say, activity 2 is starting after activity 1 by one day. Activity 5 crashed one day, 9 crashed two days and 10 crashed two days. The linear programming solution also provides a valuable sensitivity analysis. As shown in the computer output in Table 5.1, the "reduced cost" column represents the save cost if other activities "other than the activities that have small slope" were chosen at the same project deadline.

The total cost for crashing will be $ 2120 from this approach. First of all, the manual approach of crashing time is a time consuming erroneous process. It requires trial and error method to get the optimal result. Linear Programming model solution also gives us some flexibility by providing sensitivity report of the mathematical model. The linear programming analysis carried out to determine the optimal policy of investing in extra resources in order to meet the deadline is obtained. It is important for project manager to recognize the flexibility of the system that can be used to explore numerous possible opportunities to the contractor. Moreover, this approach allows the user to easily manipulate different project networks of various difficulties representing real world applications, and to study the effectiveness of the model in the case of large projects. The implementation of the developed model showed more efficient and reliable results and generated a considerable computational savings along with an increase in robustness.

Table 5.1. Solution of the model

Objective value	Final value $	Reduced cost $
Z	2120.00	-
X1	1.00	0.00
X2	0.00	2,362
X3	0.00	420
X4	0.00	8,087.93
X5	1.00	0.00
X6	0.00	2,362
X7	0.00	420
X8	0.00	240
X9	2.00	0.00
X10	0.00	0.00
X11	0.00	419
X12	0.00	346.67
X13	0.00	316.07
X14	0.00	0.00
X15	0.00	217.03
X16	0.00	432.39
X17	0.00	0.00
X18	0.00	455
X19	0.00	0.00
X20	0.00	0.00
X21	0.00	0.00
Y1	0.00	0.00
Y2	1.00	2.00
Y3	1.00	0.00
Y4	11.00	0.00
Y5	21.00	0.00
Y6	22.00	0.00
Y7	22.00	0.00
Y8	22.00	0.00
Y9	32.00	0.00
Y10	34.00	0.00
Y11	40.00	0.00
Y12	41.00	0.00
Y13	36.00	0.00
Y14	41.00	0.00
Y15	43.00	0.00
Y16	45.00	0.00
Y17	67.00	0.00
Y18	69.00	0.00
Y19	71.00	2.00
Y20	76.00	0.00
Y21	81.00	0.00
YFinsh	91.00	0.00

6. Conclusion

The data needed for crashing project activities by means of linear programming technique are the time and cost for each activity when it is done in the normal way and then when it is fully crashed (expedited). The project manager can investigate the effect on total cost of changing the estimated duration of the project to various alternative values. Using linear programming model, the project manager will be able to determine how much (if any) to crash each activity in order to minimize the total cost of meeting any specified deadline for the project. An algorithmic model based on linear programming incorporated with a minimal time-cost crash in a construction project was introduced. The format of the model lends itself to a wide range of variables and considerations. The introduced modeling strategy which showed the resources of this interactive approach including a bulk of data to completely analyze the project is easily possible. It allowed a great number of parameters to simulate project conditions and contractor's preference and provided potentially useful tool for decision making on project scheduling.

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