# Schema Refinement \& Normalization Theory 

## INFS 614

## Overview of Normal Forms

* If a relation is placed in a normal form (BCNF, 3NF etc.), certain problems are avoided or minimized.
- redundancy and its associated update/insert/delete anomolies
: Normal forms include: 1NF, 2NF, 3NF, BCNF, 4NF, 5NF, ...
- we focus on BCNF in this lecture
* Normal forms are achieved by decomposing a schema to isolate certain dependencies
- we focus only on functional dependencies (FDs)
- there are other types: e.g., multivalued dependencies (4NF), join dependencies (5NF)


## Boyce-Codd* Normal Form (BCNF)

* Reln $R$ with FDs $F$ is in BCNF if, for each $X \rightarrow A$ in $F^{+}$ either:
- 1) $A \subseteq X$ (i.e, $A$ 's attributes are all in $X$, a trivial FD), or
- 2) $X$ is a (super) key for $R$ (i.e., $X \rightarrow R$ is in $F^{+}$).
$\therefore$ In other words, $R$ is in BCNF if the only non-trivial FDs involve a key for $R$.
* If R is in BCNF , there is no redundancy solely due to FD's
* A relation in BCNF corresponds well to an "entity set" or a "relationship set" in a (good) ER design.
* Remember him? (he invented the relational model in 1970)


## Testing For BCNF

* The BCNF conditions must hold for ALL FD's!
- not just those in $F$, but those in $F+$
* The hard way:
* 1) generate F+ from F by Armstrong's Axioms
- a lot of work, potentially!!
* 2) check every FD in F+:
- is $X \rightarrow Y$ trivial?
- is $X$ is superkey for $R$ ?
* If, for every FD in $\mathrm{F}+$, the answer is "yes" to one of these questions, then $R$ is in BCNF


## Testing for BCNF (Easier Way)

* Strategy: For each non-trivial FD $X \rightarrow A$ in $F$
- is $X$ a superkey for $R$ ?
- use the attribute closure algorithm: i.e., does $X^{+}$ contain all attributes in R?
* Armstrong's Axioms preserve "superkeyness"!
- (see next slide)

Thus, for every FD in F+ generated from $X \rightarrow A$, its left hand side is also superkey

- and thus $R$ is in BCNF
- and we didn' t have to compute $\mathrm{F}^{+}$!
* Armstrong's axioms also preserve "trivialness" - adding trivial FD's has no impact on BCNF.


## Remember These?

* Armstrong' s Axioms (X, Y, Z are sets of attributes):
- Reflexivity: If $Y \subseteq X$ (i.e., $X \supseteq Y$ ), then $X \rightarrow Y$
- Augmentation: If $\mathrm{X} \rightarrow \mathrm{Y}$, then $\mathrm{XZ} \rightarrow \mathrm{YZ}$ for any Z
- Transitivity: If $\mathrm{X} \rightarrow \mathrm{Y}$ and $\mathrm{Y} \rightarrow \mathrm{Z}$, then $\mathrm{X} \rightarrow \mathrm{Z}$


## Testing for BCNF: Example

* Given $A B C D$ and $F=\{B \rightarrow C, C \rightarrow D, C \rightarrow A\}$
- (Exercise 19.7 \#1)
* Is relation $A B C D$ in $B C N F$ ?
- none of these FDs are trivial (must check all)
- $B \rightarrow C: B^{+}=\{A B C D\} O K$
- $C \rightarrow D: C^{+}=\{A C D\}$ Whoops! (Not in BCNF)
* We will have to decompose $A B C D$ to achieve BCNF!
* Strong Hint: Know how to test for BCNF!


## Decomposition of a Relation Schema

* When a relation schema is not in BCNF: decompose.
* Decompose $R$ into $R_{1} \ldots R_{n}$, where
- each $R_{i}$ is a projection of $R$ (contains a subset of $R_{N} s$ attributes)
- $\bigcup_{i=1}$ Atts $\left(R_{i}\right)=$ Atts $(R)$ (the union of the attributes ${ }_{i=1}$ of each $R_{i}=$ the attributes of $R$ )
- each $R_{i}$ is in BCNF
- intuitively, "rogue dependencies" are broken out into their own tables


## Testing for BCNF: Hard Case

* What if the atts of F are not contained entirely within the relation $R$ we are testing?
* This can happen in a decomposition of $R$ :
- E.g. Consider $R_{1}(A, B, C, D)$, with $F=\{A \rightarrow B, B \rightarrow C\}$
- Now decompose $R_{1}$ into $R_{2}(A, B)$ and $R_{3}(A, C, D)$
- Although neither dependency in $F$ contains only attributes from (A,C,D) $R_{3}$ does not satisfy BCNF!
- Dependency $A \rightarrow C$ in $F^{+}$shows $R_{3}$ is not in BCNF.
* To test if a decomposed relation $\mathrm{R}_{\mathrm{d}}$ is in BCNF:
- 1) compute the key for $R_{d}$,
- 2) for all (non-trival) FDs $X \rightarrow Y$ in $\mathrm{F}+$ whose attrs iNFS614 are entirely within $R_{d}$, is $X$ a superkey for $R_{d}$ ?


## Decomposition example

| S | N | L | R | W | H |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $123-22-3666$ | Attishoo | 48 | 8 | 10 | 40 |
| $231-31-5368$ | Smiley | 22 | 8 | 10 | 30 |
| $131-24-3650$ | Smethurst | 35 | 5 | 7 | 30 |
| $434-26-3751$ | Guldu | 35 | 5 | 7 | 32 |
| 612-67-4134 | Madayan | 35 | 8 | 10 | 40 |

## Original relation

## Decomposition

$=$| S | N | L | R | H |
| :--- | :--- | :--- | :--- | :--- |
| $123-22-3666$ | Attishoo | 48 | 8 | 40 |
| $231-31-5368$ | Smiley | 22 | 8 | 30 |
| $131-24-3650$ | Smethurst | 35 | 5 | 30 |
| $434-26-3751$ | Guldu | 35 | 5 | 32 |
| $612-67-4134$ | Madayan | 35 | 8 | 40 |


$\triangleright \triangleleft$|  |  |
| :--- | :--- |
| 8 | $W$ |
|  | 10 |
| 5 | 7 |

## Problems with Decompositions

$\therefore$ There are at least four potential problems to consider:

1) The decomposed relation instances are not "join lossless":

- The original $R$ cannot be recaptured via joins (very bad!)

2) The decomposed instances are not "dependency preserving"

- Checking some FDs may require joins

3) Some queries become more expensive.

- e.g., How much did Attishoo earn? (earn $=W *$ H, requires a join)

4) Some decompositions fragment important conceptual entities - "overnormalization"; classic example: an address

* Design considerations: Compare these costs vs. tolerating some redundancy (and associated anomolies)
- BCNF avoids 1), and is conceptually easy.
- 3NF avoids both 1) and 2), but permits some redundancy
- sometimes we can use views cleverly (especially materialized) to compensate for 3) and 4)


## Example: Join

## 

| Student_ID | Name | Dcode | Cno | Grade |
| :--- | :--- | :--- | :--- | :--- |
| $123-22-3666$ | Attishoo | INFS | 501 | A |
| 231-31-5368 | Guldu | CS | 102 | B |
| $131-24-3650$ | Smethurst | INFS | 614 | B |
| $434-26-3751$ | Guldu | INFS | 614 | A |
| $434-26-3751$ | Guldu | INFS | 612 | C |


| Name | Dcode | Cno | Grade |
| :--- | :--- | :--- | :--- |
| Attishoo | INFS | 501 | A |
| Guldu | CS | 102 | B |
| Smethurst | INFS | 614 | B |
| Guldu | INFS | 614 | A |
| Guldu | INFS | 612 | C |$\quad \triangleright \triangleleft$| Student_ID | Name |
| :--- | :--- |
| $123-22-3666$ | Attishoo |
| $231-31-5368$ | Guldu |
| $131-24-3650$ | Smethurst |
| $434-26-3751$ | Guldu |

## Lossless Join Decompositions

* Decomposition of $R$ into $R_{1}$ and $R_{2}$ is joinlossless w.r.t. a set of FDs F if, for every instance rof $R$ that satisfies $F$, we have:

$$
\pi_{R_{1}}(r) \triangleright \Delta \pi_{R_{2}}(r)=r
$$

$\because$ It is always true that

$$
r \subseteq \pi_{R_{1}}(r) \triangleright \triangleleft \pi_{R_{2}}(r)
$$

* The other direction does not always hold!
- unless the common (join) attributes of the decomposed tables form a key for at least one of those tables


## Further Example: Lossy



Note: $B$ is not a key for $A B$ or $B C$ !

## Further Example: Lossless



Now, $B$ is a key for $B C$.

## Lossless Join Decomposition

* The decomposition of $R$ into $R_{1}$ and $R_{2}$ wrt $F$ is join-lossless if:
- Atts of $\left(R_{1} \cap R_{2}\right) \rightarrow R_{1}$, or
- Atts of $\left(R_{1} \cap R_{2}\right) \rightarrow R_{2}$
- I.e.: if the join attributes form a key for $R_{1}$ or $R_{2}$
: Important special case: if $X \rightarrow Y$ holds on $R$, the decomposition of $R$ into ( $R-Y$ ) and ( $X Y$ ) is join-lossless
- X and Y do not share attributes.
- X is common to both tables, and is a key for XY


## A Simple "Tree" Algorithm for Decomposition into BCNF

* Consider relation R with $\mathrm{FD} s \mathrm{~F}$. If $\mathrm{X} \rightarrow \mathrm{A}$ in $\mathrm{F}^{+}$ over $R$, violates $B C N F$, i.e.,
- the attributes of $X A$ are all in $R$, and yet:
- $X \rightarrow R$ is not in $\mathrm{F}^{+}(X$ is not a key for $R$ )
- $A$ is not in $X(X \rightarrow A$ is not trivial)
$\%$ Then: decompose $R$ into two new relations
- XA
- R-A
* Repeated application of this decomposition:
- results in a collection of relations that are in BCNF
- produces a lossless join decomposition
- is guaranteed to terminate.


## BCNF Decomposition Example

* Assume relation schema CSJDPQV
- key C, JP $\rightarrow$ C, SD $\rightarrow P, J \rightarrow S$ (call the last $2 \mathrm{~F}_{\mathrm{BAD}}$ )
$\therefore$ Remove SD $\rightarrow P$ from $F_{B A D}$, and decompose into SDP ("XA"), CSJDQV ("R-A").
- Test if these are individually in BCNF.
- If not, recurse! (on either side; see below)
: Remove J $\rightarrow$ S from $\mathrm{F}_{\mathrm{BAD}}$, decompose into JS, CJDQV.
- We are done now. WHY? (The LHS of all fds in F , and $\mathrm{F}^{+}$, is a key)
* Note: it helps to draw this tree as you go!



## BCNF Decomposition

* In general: several dependencies may cause violation of BCNF.
* The order in which we "deal with" them could lead to different (valid) sets of BCNF relations.


## Dependency Preservation

* Consider CSJDPQV, $C$ is key, JP $\rightarrow C$ and SD $\rightarrow P$.
- BCNF decomposition: CSJDQV and SDP
- Problem: Checking JP $\rightarrow C$ requires a join!
- e.g., does an insertion to SDP violate JP $\rightarrow C$ ? How can we tell?
* Dependency preservation problem:
- If $R$ is decomposed into $X$, and $Y$, then all FDs that were given to hold on $R$ should still hold
- but we can' $\dagger$ (easily) enforce them if their attributes are "scattered" across multiple relations!


## BCNF and Dependency Preservation

* In general, there may not be a dependency preserving decomposition into BCNF.


## The Projection of a set of FDs

* Given $F$, let $R$ be decomposed into $X$ and $Y$
* The projection of $F$ onto $X$ (denoted $F_{X}$ ) is the set of $\mathrm{FDs} \mathrm{U} \rightarrow \mathrm{V}$ in $\mathrm{F}^{+}$such that the attributes of $U, V$ are entirely in $X$.


## Dependency Preserving Decompositions

$\because$ Decomposition of R into X and Y is dependency preserving if $\mathrm{F}^{+}=\left(\mathrm{F}_{\mathrm{X}} \text { union } \mathrm{F}_{\mathrm{Y}}\right)^{+}$

- Example: $R=A B C, F=\{A \rightarrow B, B \rightarrow C, C \rightarrow A\}$, and $R$ is decomposed into $A B$ and $B C$.
- Is this dependency preserving? Yes.
- (Note: Consider $F^{+}$, not just $F$, in definition of $F_{x}$ and $F_{y}$ !)
- By transitivity, $F+$ also includes $A \rightarrow C, B \rightarrow A, C \rightarrow B$
- Thus $F_{X}=\{A \rightarrow B$ and $B \rightarrow A\}$
- Thus $F_{Y}=\{B \rightarrow C$ and $C \rightarrow B\}$
- Thus ( $F_{X}$ union $F_{Y}$ ) ${ }^{+}$includes $C \rightarrow A$ (by transitivity)
- So the $A B, B C$ decomposition will enforce/preserve $C \rightarrow A!!$
* Dependency preserving does not imply lossless join:
- $A B C, A \rightarrow B$, decomposed into $A B$ and $B C$.
* And vice-versa! (Example?)


## Example

* Assume CSJDPQV is decomposed into SDP, JS, CJDQV
this is not dependency preserving w.r.t. the FDs: $\mathrm{JP} \rightarrow \mathrm{C}, \mathrm{SD} \rightarrow \mathrm{P}$ and $\mathrm{J} \rightarrow \mathrm{S}$.
* However, it is a lossless join decomposition.
* Adding table JPC gives us a dependency preserving decomposition.
- Note: JPC tuples stored only for checking FD!
- But it is redundant ...
* Is there a general way to ensure dep. pres?


## Third Normal Form (3NF)

$\therefore R$ with FDs $F$ is in $3 N F$ if, for all $X \rightarrow A$ in $F^{+}$

- $A$ in $X$ (i.e., FD is trivial), or
- X contains a key for $R$, or
- A is part of some (candidate, not super) key for $R$.
- a bit weaker than BCNF!
* It is always possible to decompose into 3NF and preserve dependencies
- and be join-lossless
- but you may have some FD-induced redundancy
$\therefore 3 N F$ is a compromise
- if you really want dependency preservation
- or (more likely) if BCNF is just too hard to achieve


## Third Normal Form (3NF)

* A relation in 3NF is free of:
- partial dependencies: e.g. given $R(A B C), A B \rightarrow C(A B$ is a key), $A \rightarrow C$
- transitive dependencies: e.g. given $R(A B C), A \rightarrow B, B$ $\rightarrow C$
: 2NF is free of partial dependencies only


## 1NF

: First normal form requires every column in a relation to be atomic (i.e. not a repeating group)
: This is the default in the relational model.
: XML DTD's can define non-1NF models. !!DOCTYPE BOOKLIST [ «!ELEMENT LISTNAME (\#PCDATA)> «!ELEMENT LIST (BOOK)*> «!ELEMENT BOOK (AUTHOR, TITLE)> «!ELEMENT AUTHOR (\#PCDATA)> «ELEMENT TITLE (\#PCDATA)> «!ELEMENT TITLE (\#PCDATA) ]

## Overnormalization

* Address entity:
- Bob Jones, 12 Main Street, Springfield IL, 12345
* Address (Firstname, Lastname, Number, Street, City, State, Zip)
- FLNSCTZ
- NSCT $\rightarrow$ Z

: Do we really want to do this?
- so what if zip codes are a bit redundant
- insert, update, delete anomolies are managable!


## Summary of Schema Refinement

: If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.

* If a relation is not in BCNF, we can try to decompose it into a collection of $B C N F$ relations.
- Must consider whether all FDs are preserved. If a lossless-join, dependency preserving decomposition into BCNF is not possible (or unsuitable, given typical queries), should consider decomposition into 3NF.
- Decompositions should be carried out and/or re-examined while keeping performance requirements in mind.


## Three Normalization Lessons

1) Intuitively, the left hand side of every FD wants to be the key of some relation.
2) "Perfect" decompositions don' $\dagger$ always happen.

- But you should still know BCNF decomposition for the final :-)

3) You can overnormalize.

## Big Picture Takeaways

- What can FDs \& normalization do for you?
- Reduce redundancy, reducing storage costs
- Improve data consistency (by having only one copy)
- Improve concurrency (less needs to be locked)
- Provide insight into any database design (e.g., causal links)
- How could normalization hurt you?
- Requiring (potentially lossy) joins to put your data back together
- Require you to understand all FDs before you can design, use your database (big issue in analytic databases).
- This takes time (doesn't lend itself to rapid prototyping).
- If you don't know BCNF on the final ©
- Normalization is a tool (like ER design). Designing great infs614 ${ }^{\text {databases is up to you! }}$


## Backup

## What Does 3NF Achieve?

* If 3NF is violated by $X \rightarrow A$, one of the following holds:
- $X$ is a proper subset of some key $K$
- We store $(X, A)$ pairs redundantly.
E.g.: SBDC, SBD is the only key, $S \rightarrow C$. Pairs (S,C) redundant.
- $X$ is not a proper subset of any key.
- There is a chain of FDs $K \rightarrow X \rightarrow A$, which means that we canno $\dagger$ associate an $X$ value with a $K$ value unless we also associate an $A$ value with an $X$ value.
E.g.: Hourly_Emps SNLRWH, S is the only key, $\mathrm{R} \rightarrow \mathrm{W}$.
* But: even if reln is in 3NF, these problems could arise.
- e.g., Reserves SBDC, $S \rightarrow C, C \rightarrow S$ is in 3NF, but for each reservation of sailor $S$, same ( $S, C$ ) pair is stored.
: Thus, $3 N F$ is indeed a compromise with respect to $B C N F$.


## Testing for 3NF

* Optimization: Need to check only FDs in F, need not check all FDs in $\mathrm{F}^{+}$.
* Use attribute closure to check, for each dependency $X \rightarrow A$, if $X$ is a superkey.
* If $X$ is not a superkey, we have to verify if $A$ is part of some candidate key of $R$
- this test is rather expensive, since it involves finding candidate keys


## Decomposition into 3NF

* Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier).
* To ensure dependency preservation, one idea:
- If $X \rightarrow Y$ is not preserved, add relation $X Y$.
- Problem is that XY may violate 3NF!
* Refinement: Instead of the given set of FDs F, use a minimal cover for $F$.


## Minimal Cover for a Set of FDs

: Minimal cover $G$ for a set of FDs F:

- Closure of $F=$ closure of $G$.
- Right hand side of each FD in $G$ is a single attribute.
- If we modify $G$ by deleting an FD or by deleting attributes from an FD in $G$, the closure changes.
* Intuitively, every FD in G is needed and" as small as possible' in order to get the same closure as F.
$\%$ e.g., $A \rightarrow B, A B C D \rightarrow E, E F \rightarrow G H, A C D F \rightarrow E G$ has the following minimal cover:
$-A \rightarrow B, A C D \rightarrow E, E F \rightarrow G$ and $E F \rightarrow H$


## Dependency-Preserving Decomposition into 3NF

$*$ Given $R$ with a set $F$ of $F D$ s that is a minimal cover. Let R1,...,Rn be a lossless-join decomposition of $R$ in 3NF. Let Fi be the projection of F onto Ri. Do:

- Identify the set $N$ of dependencies in $F$ that is not preserved, i.e. not included in the closure of the union of F1,...Fn;
- For each FD $X \rightarrow A$ in $N$, create a relation schema XA and add it to the decomposition of $R$.
* The resulting decomposition is a lossless-join and dependency-preserving decomposition of R into 3NF relations.


## Problem

* Given a relation schema $\mathrm{R}(\mathrm{ABCDE})$ with FD's

$$
F=\{A \rightarrow B, C \rightarrow D, B \rightarrow A E\}
$$

Find all the candidate keys for $R$.

## Problem (contd.)

Proceed as follows:
(a) Identify the attributes that are in the relation schema and not in the set of functional dependencies: None.
(b) Identify the attributes that appear only on the left hand-side in F (these attributes will belong to any single key of the relation schema): C.
(c) Identify the attributes that appear only on the right hand-side in F (these attributes are not part of any key): DE.
(d) Combine the set of attributes obtained in (a) and (b) with the attributes that are not obtained in (c).
Compute their closure to check whether they are a key.

## Problem (contd.)

- From (a) and (b), we get C;
- The attributes not obtained in (c) are: A,B,C.
- We compute the closure of $\mathrm{C}: \mathrm{C}+=\mathrm{CD}$. Thus: $\underline{\mathrm{C} \text { is not a key. }}$
- We compute the closure of $\mathrm{CA}: \mathrm{CA}+=\mathrm{CABDE}$. Thus: $\underline{C A}$ is a key.
- We compute the closure of $\mathrm{CB}: \mathrm{CB}+=\mathrm{CBDAE}$. Thus: $\underline{C B}$ is a key.
- Note: We know from (c) that CD and CE are not keys;
- Then, we consider combinations of three attributes that don' $t$ contain CA or CB (since they are keys), and include C. The only possible combination is CDE, but D and E are not part of any key.
- Therefore, CA and CB are the only keys in R.

