Schrödinger Cats, Maxwell's Demon and Quantum Error Correction

<u>Experiment</u>

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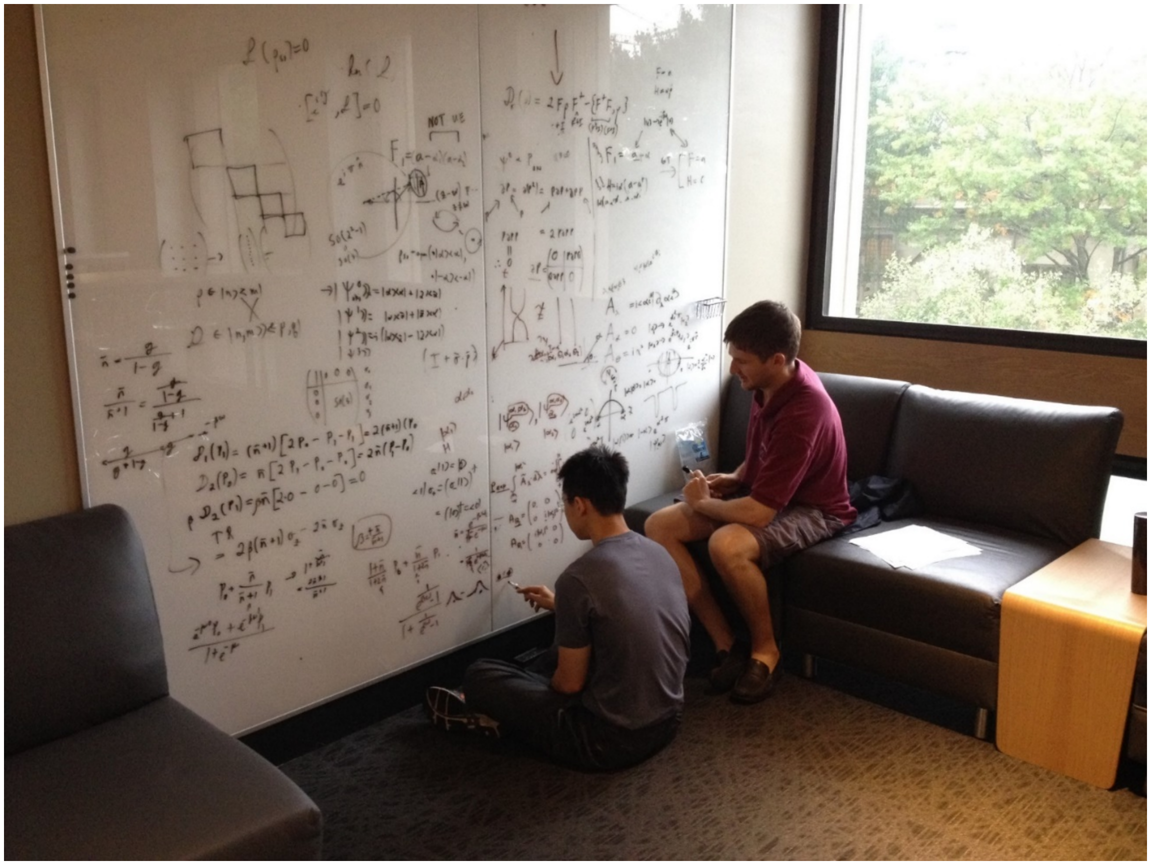
+…..



<u>Theory</u>

SMG
Liang Jiang
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Shruti Puri
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Victor Albert**
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Huaixui Zheng
Marios Michael
+…..

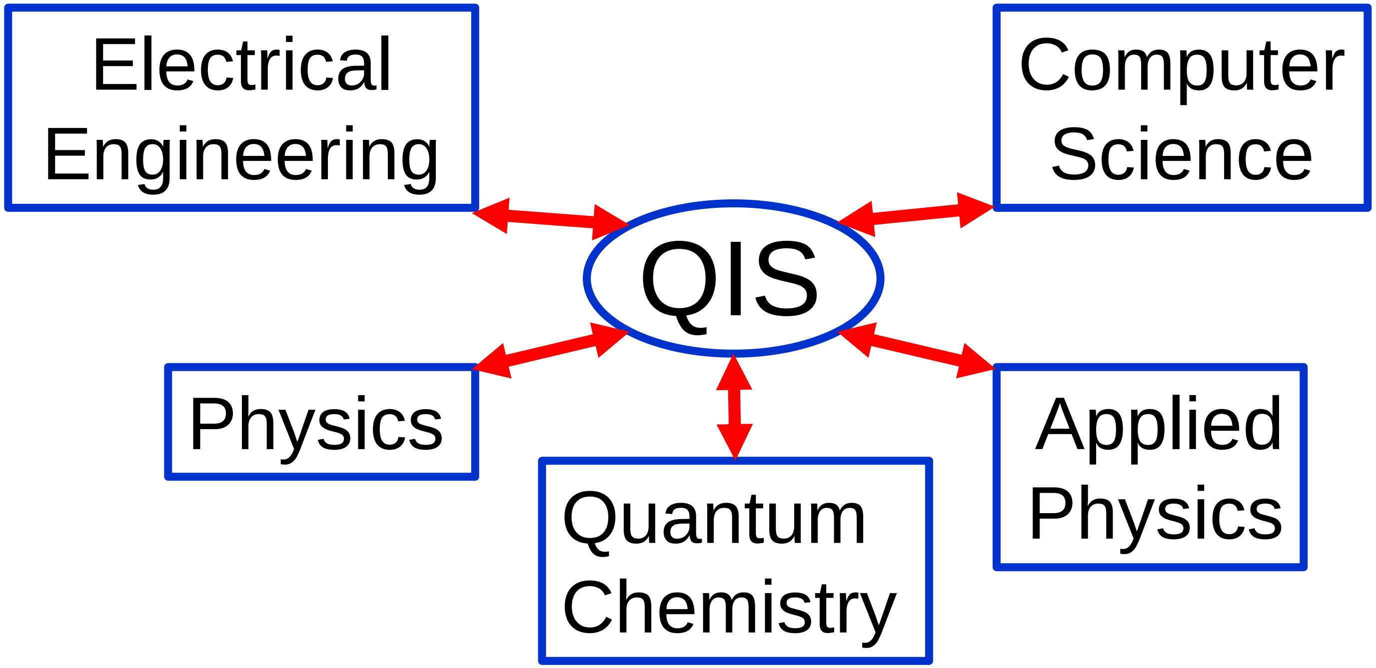
QuantumInstitute.yale.edu

Quantum Information Science

Control theory, Coding theory,
Computational Complexity theory,
Networks, Systems, Information theory

Ultra-high-speed digital, analog,
microwave electronics, FPGA

Programming languages, Compilers



Quantum mechanics, Quantum optics,
Circuit QED, Materials Science

http://quantuminstitute.yale.edu/

Is quantum information carried by waves or by particles?

YES!

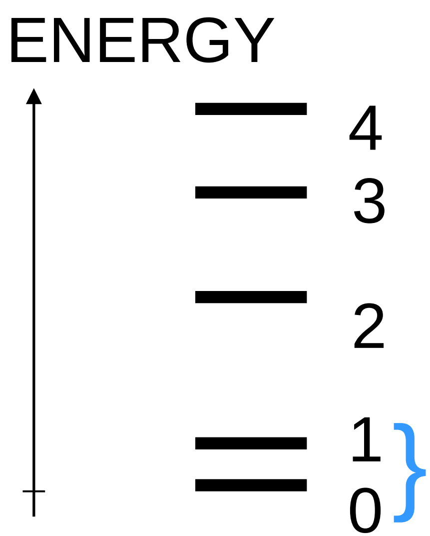
Is quantum information analog or digital?

YES!

Quantum information is digital:

Energy levels of a quantum system are discrete.

We use only the lowest two.



Measurement of the state of a qubit yields (only) 1 classical bit of information.

$$\text{excited state } 1 = |e\rangle = |\uparrow\rangle$$

$$\text{ground state } 0 = |g\rangle = |\downarrow\rangle$$

Quantum information is analog:

A quantum system with two distinct states can exist in an Infinite number of physical states ('superpositions') *intermediate* between $|\downarrow\rangle$ and $|\uparrow\rangle$.

Bug:
We are uncertain which state the bit is in. Measurement results are truly random.

Feature:
Qubit is in both states at once, so we can do parallel processing. Potential exponential speed up.

Quantum Computing is a New Paradigm

- Quantum computing: a completely new way to store and process information.
- **Superposition**: each quantum bit can be **BOTH** a zero and a one.
- **Entanglement**: Bits can have non-classical correlations.

  (Einstein: ‘Spooky action at a distance.’)
- **Massive parallelism**: carry out computations that are impossible

Daily routine engineering and calibration test:
Carry out spooky operations that Einstein said were impossible!

Register of N conventional bits can be in 1 of 2^N states.

000, 001, 010, 011, 100, 101, 110, 111

Register of N quantum bits can be in an

arbitrary superposition of 2^N states

Just this set of different superpositions represents $\sim 2^{2^N}$ states!

$$|\Psi\rangle \sim |000\rangle \pm |001\rangle \pm |010\rangle \pm |011\rangle \pm |100\rangle \pm |101\rangle \pm |110\rangle \pm |111\rangle$$

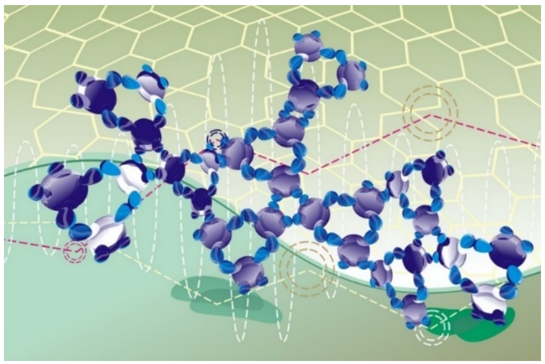
Even a small quantum computer of 50 qubits will be so powerful its operation would be difficult to simulate on a conventional supercomputer.

Quantum computers are good for problems that have simple input and simple output but must explore a large space of states at intermediate stages of the calculation.

Applications of Quantum Computing

Solving the Schrödinger Eqn. (even with fermions!)

Quantum materials

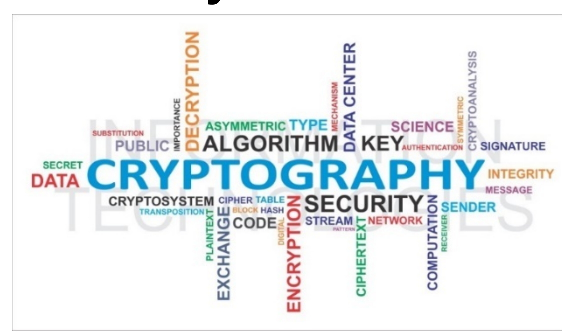


Quantum chemistry

Machine learning*

***read the fine print**

Cryptography and Privacy Enhancement



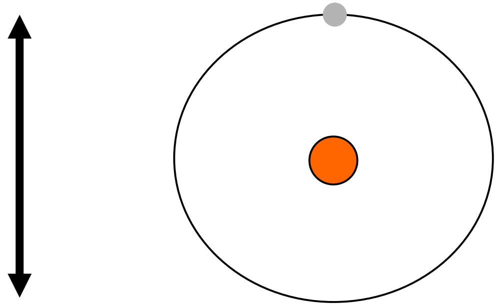
Storing information in quantum states sounds great…,

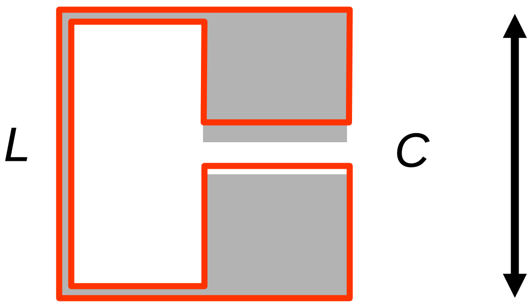
but how on earth do you build a quantum computer?

ATOM vs CIRCUIT

Hydrogen atom

Superconducting
circuit oscillator





10^{-7} mm

0.1 - 1 mm

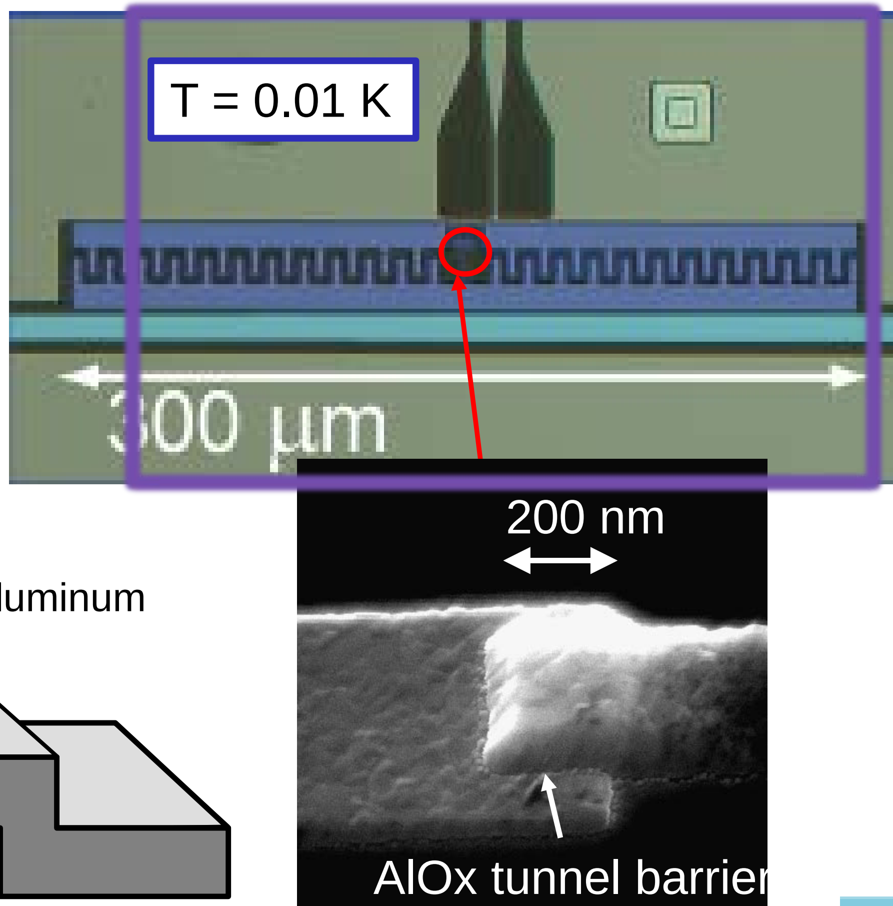
(Not to scale!)

1 electron

$\sim 10^{12}$ electrons

'Artificial atom'

How to Build a Qubit with an Artificial Atom…



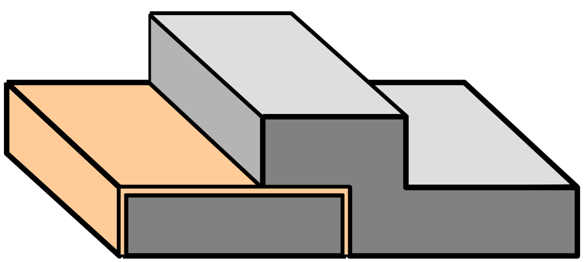
Superconducting integrated circuits are a promising technology for <u>scalable</u> quantum computing

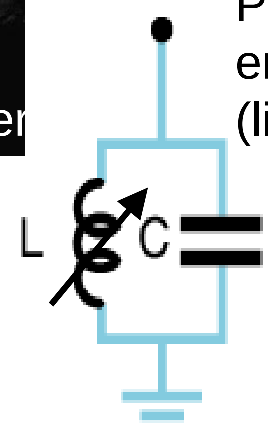
<u>Josephson junction:</u>

The "transistor of quantum computing"

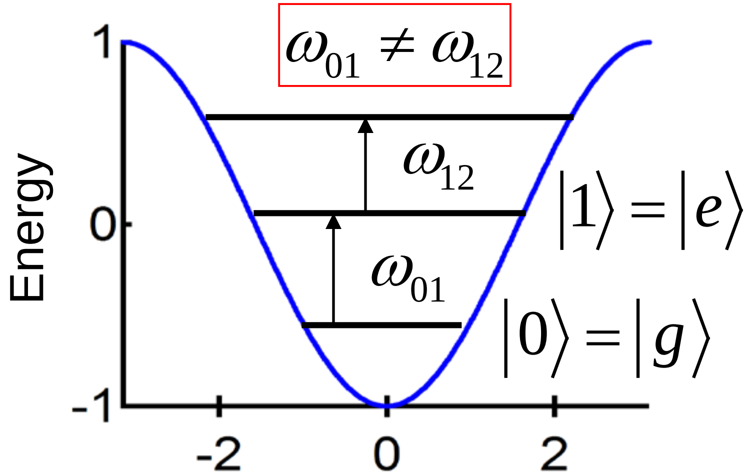
Provides <u>anharmonic</u> energy level structure (like an atom)

Aluminum/AlOx/Aluminum



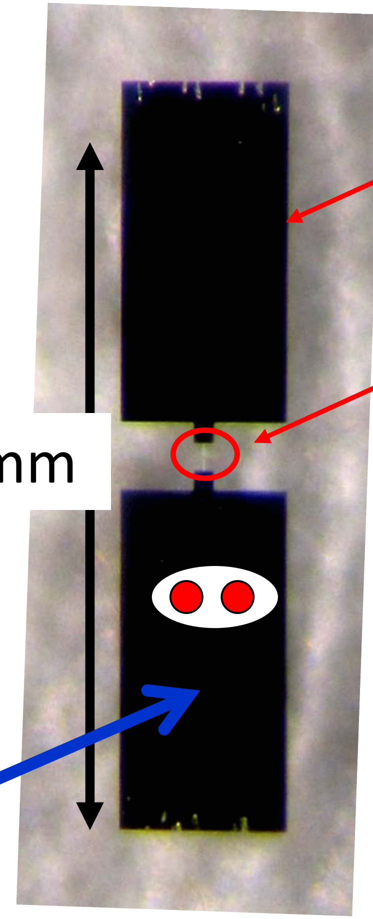


Transmon Qubit



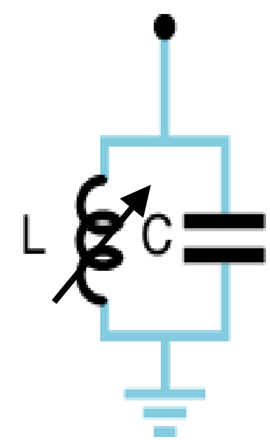
$\omega_{01} \sim 5-10$ **GHz**

10^{12} mobile electrons



Antenna pads are capacitor plates

Josephson tunnel junction



Superconductivity gaps out single-particle excitations

Quantized energy level spectrum is <u>*simpler*</u> than hydrogen

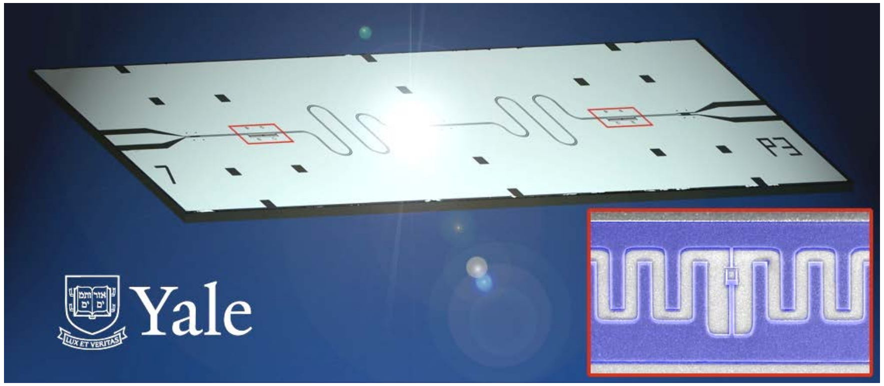
Quality factor $Q = \omega T_1$ <u>*comparable*</u> *to* that of hydrogen 1s-2p

<u>Enormous</u> transition dipole moment:
ultra-strong coupling to microwave photons

"Circuit QED"

The first electronic quantum processor (2009) was based on 'Circuit QED'

Executed Deutsch-Josza and Grover search algorithms



Lithographically produced integrated circuit with semiconductors replaced by superconductors.

Michel Devoret

Rob Schoelkopf

DiCarlo et al., Nature **460**, 240 (2009)

The huge information content of quantum superpositions

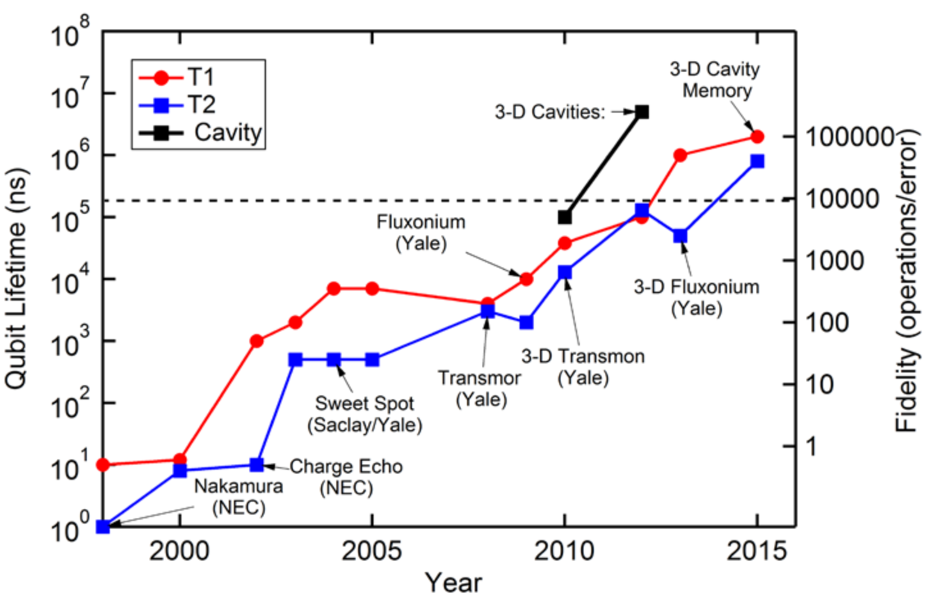
comes with a price:

Great sensitivity to noise perturbations and dissipation.

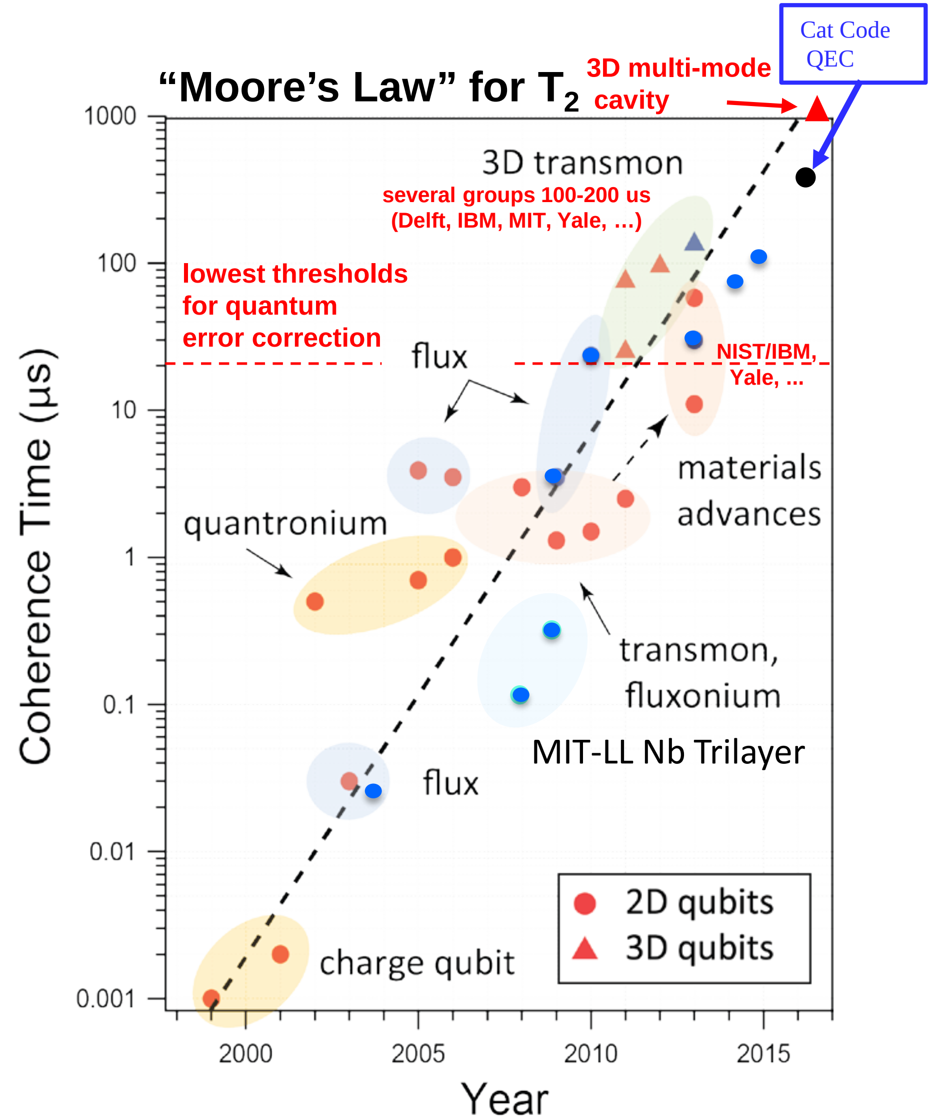
The quantum phase of superposition states is well-defined only for a finite ‘coherence time’ T_2

Despite this sensitivity, we have made exponential progress in qubit coherence times.

Exponential Growth in SC Qubit Coherence



R. Schoelkopf and M. Devoret



Oliver & Welander, MRS Bulletin (2013)

Girvin's Law:

There is no such thing as too much coherence.

We need quantum error correction!

The Quantum Error Correction Problem

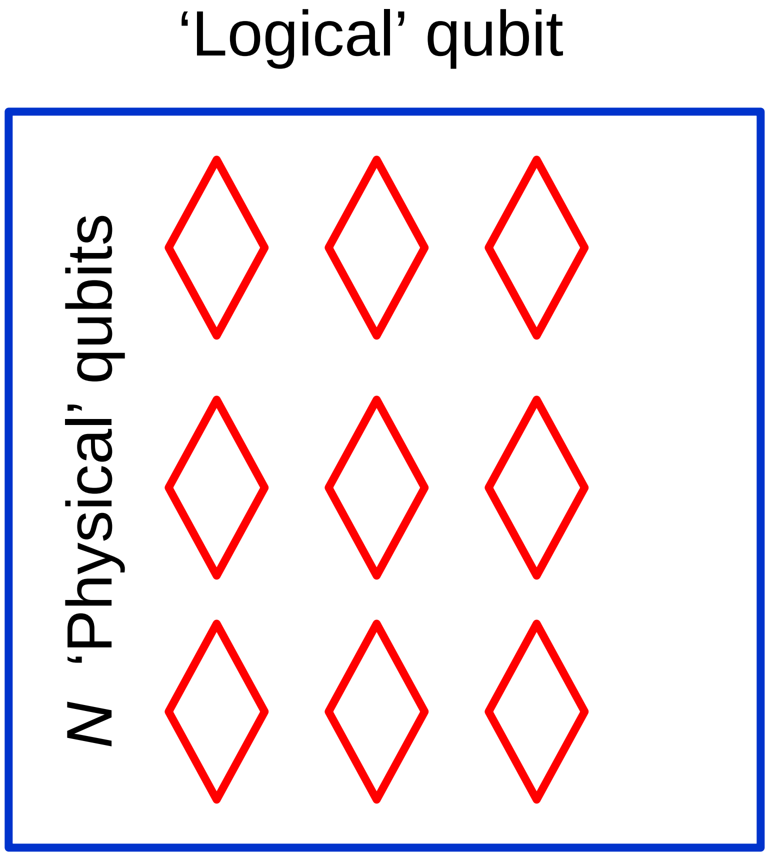
I am going to give you an unknown quantum state.

If you measure it, it will change randomly due to state collapse (‘back action’).

If it develops an error, please fix it.

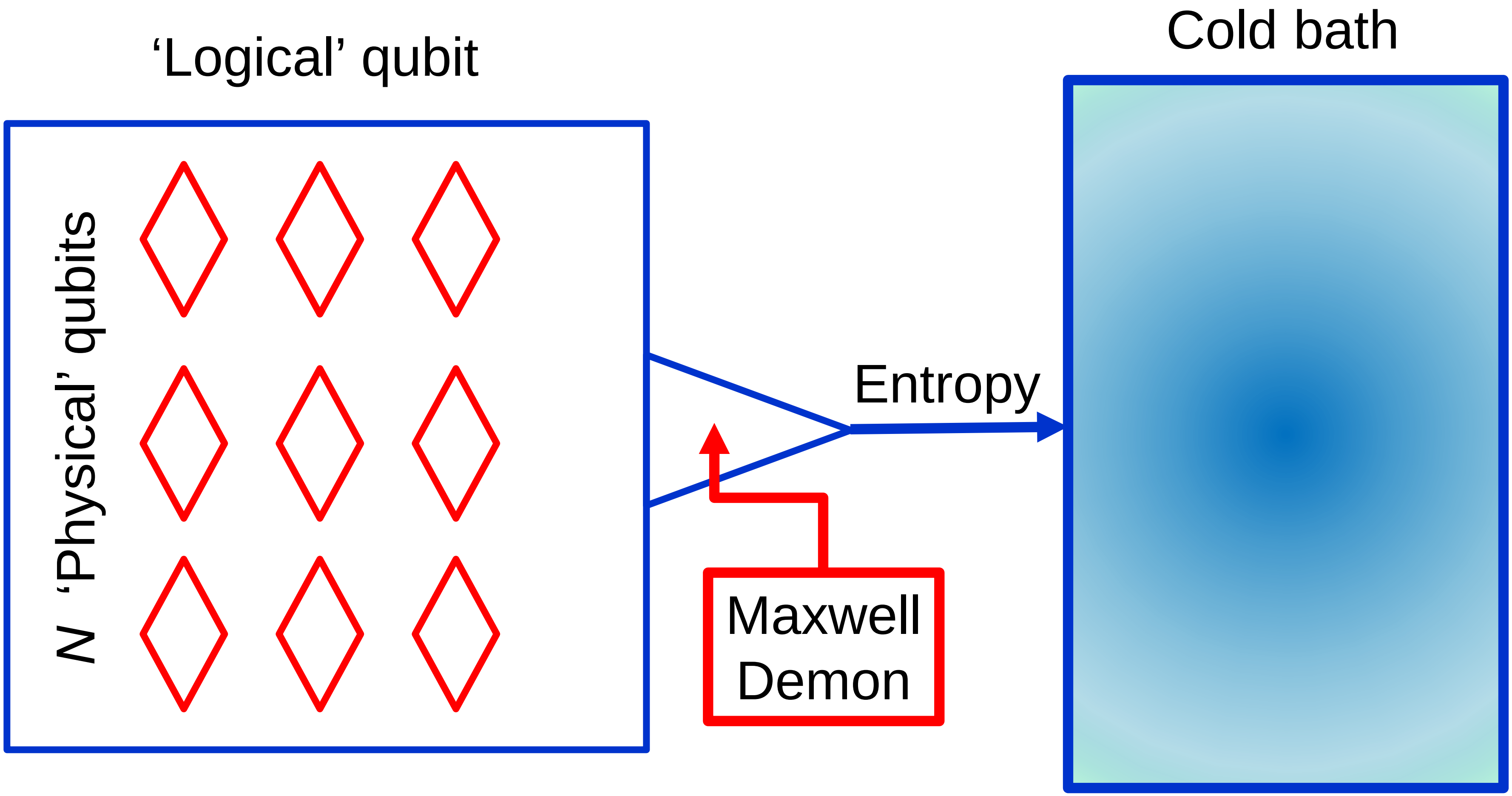
***Mirable dictu*: It can be done!**

Quantum Error Correction for an unknown state requires storing the quantum information non-locally in (non-classical) *correlations* over multiple physical qubits.



Non-locality: No single physical qubit can "know" the state of the logical qubit.

Quantum Error Correction



N qubits have errors *N* times faster. Maxwell demon must overcome this factor of *N* – *and not introduce errors of its own! (or at least not uncorrectable errors)*

1. Quantum Information
2. Quantum Measurements
3. Quantum Error Heralding
4. Quantum Error Correction

1. **Quantum Information**

2. Quantum Measurements

3. Quantum Error Heralding

4. Quantum Error Correction

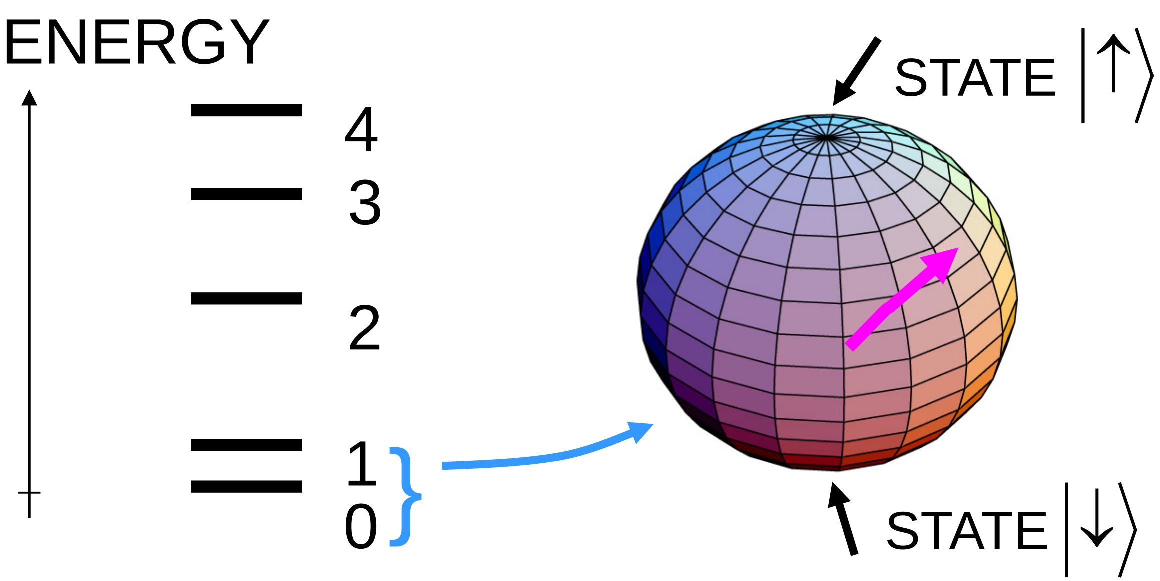
Quantum information is analog:

A quantum system with two distinct states can exist in an Infinite number of physical states (‘superpositions’) *intermediate* between $|\downarrow\rangle$ and $|\uparrow\rangle$.

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|\downarrow\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|\uparrow\rangle$$

$\theta =$ latitude

$\varphi =$ longitude

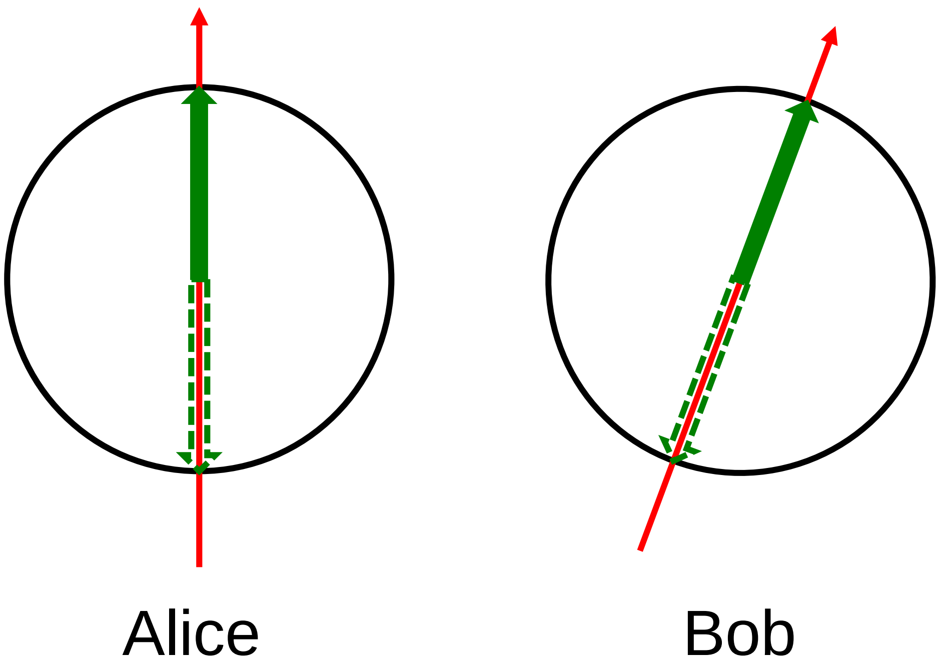


State defined by ‘spin polarization vector’ on Bloch sphere

Quantum information is <u>analog/digital</u>:

Equivalently: a quantum bit is like a classical bit except there are an infinite number of encodings (aka 'quantization axes').

$Z = \pm 1$ $\quad\quad$ $Z' = \pm 1$



If Alice gives Bob a $Z = +1$,

Bob measures:

$$Z' = +1 \text{ with probability } P_+ = \cos^2 \frac{\theta}{2}$$

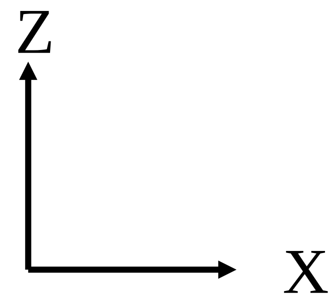
$$Z' = -1 \text{ with probability } P_- = \sin^2 \frac{\theta}{2}$$

'Back action' of Bob's measurement changes the state, but it is invisible to Bob.

1. Quantum Information
2. **Quantum Measurements**
3. Quantum Error Heralding
4. Quantum Error Correction

What is knowable?

Consider just 4 states:



We are allowed to ask only one of two possible questions:

Does the spin lie along the Z axis? Answer is always yes! (± 1)
Does the spin lie along the X axis? Answer is always yes! (± 1)

BUT WE CANNOT ASK BOTH!
Z and X are ***INCOMPATIBLE*** OBSERVABLES

What is knowable?

We are allowed to ask only one of two possible questions:

Does the spin lie along the Z axis? Answer is always yes! (± 1)
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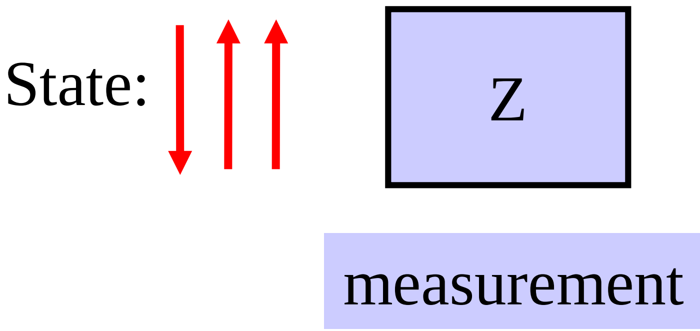
BUT WE CANNOT ASK BOTH!
Z and X are ***INCOMPATIBLE*** OBSERVABLES

Heisenberg Uncertainty Principle

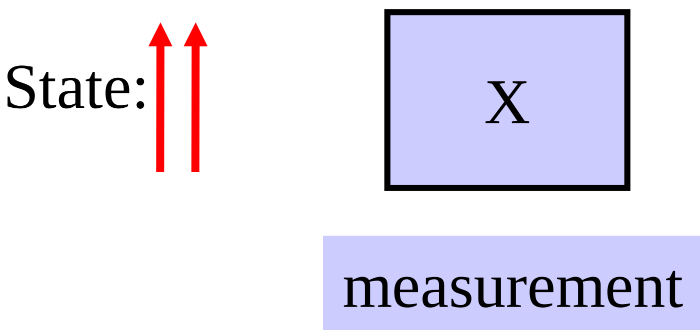
If you know the answer to the Z question
you cannot know the answer to the X question
and vice versa.

(If you know position you cannot know momentum.)

Measurements 1.



Result: quantum state
is unaffected.

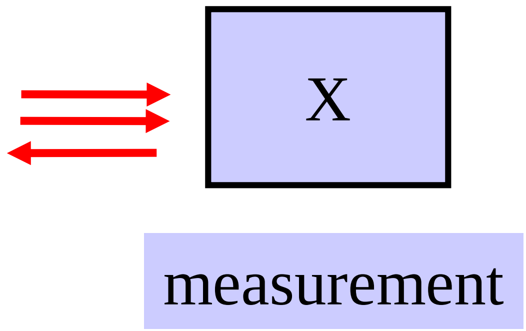


Result: ± 1 randomly!
State is *changed by measurement* to lie along X axis.

Unpredictable result

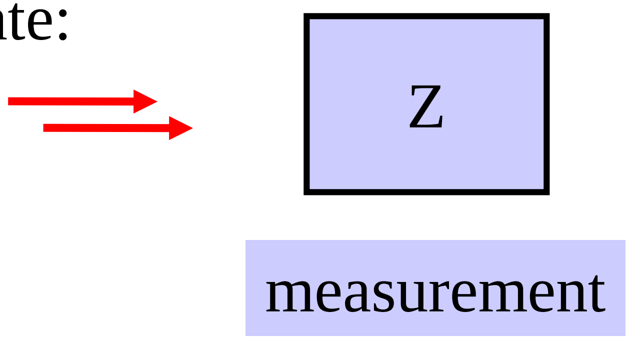
Measurements 2.

State:



Result: quantum state
is unaffected.

State:



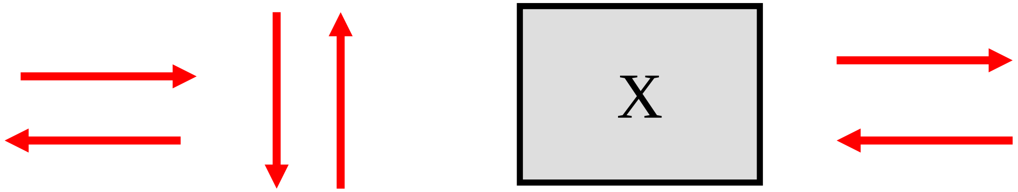
Result: $\underline{+}1$ randomly!
State is <u>*changed by measurement*</u> to lie along Z axis.

Unpredictable result

No Cloning Theorem

Given an unknown quantum state, it is ***impossible*** to make multiple copies

Unknown state:



Guess which measurement to make
---if you guess wrong you change the state
and you have no way of knowing if you did….

No Cloning Theorem

Given an unknown quantum state, it is ***impossible*** to make multiple copies

Big Problem:

Classical error correction is based on cloning!
(or at least on measuring)

Replication code:

$$0 \to 000$$

$$1 \to 111$$

Majority Rule voting corrects single bit flip errors.

1. Quantum Information

2. Quantum Measurements

3. **Quantum Error Heralding**

4. Quantum Error Correction

Let's start with classical error heralding

Classical duplication code: $0 \to 00 \quad 1 \to 11$

Herald error if bits do not match.

In	Out	# of Errors	Probability	Herald?
00	00	0	$(1-p)^2$	Yes
00	01	1	$(1-p)p$	Yes
00	10	1	$(1-p)p$	Yes
00	11	2	p^2	Fail

And similarly for 11 input.

Using duplicate bits:

-lowers channel bandwidth by factor of 2 (bad)

-lowers the fidelity from $(1-p)$ to $(1-p)^2$ (bad)

-improves unheralded error rate from p to p^2 (good)

In	Out	# of Errors	Probability	Herald?
00	00	0	$(1-p)^2$	Yes
00	01	1	$(1-p)p$	Yes
00	10	1	$(1-p)p$	Yes
00	11	2	p^2	Fail

And similarly for 11 input.

Quantum Duplication Code

No cloning prevents duplication

$$U\underbrace{\left(\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\right)}_{\text{Unknown quantum state}}\otimes\underbrace{\left|\downarrow\right\rangle}_{\text{Ancilla initialized to ground state}}=\left(\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\right)\otimes\left(\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\right)$$

Unknown quantum state

Ancilla initialized to ground state

Proof of no-cloning theorem:

α and β are unknown; Hence U cannot depend on them.

No such unitary can exist if QM is linear.

Q.E.D.

Don’t clone – entangle!

$$U\left(\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\right)\otimes\left|\downarrow\right\rangle=\alpha\left|\downarrow\right\rangle\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\left|\uparrow\right\rangle$$

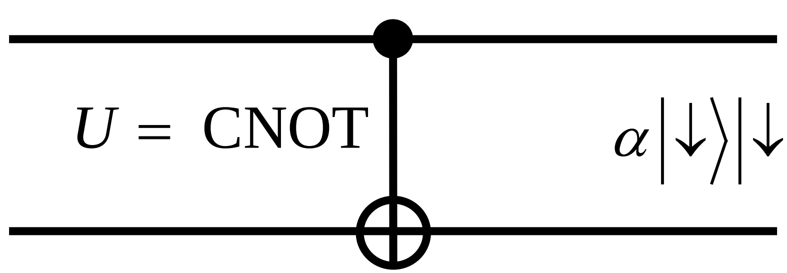
Unknown quantum state: $\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle$

Ancilla initialized to ground state: $\left|\downarrow\right\rangle$

Quantum circuit notation:

$\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle$

$\left|\downarrow\right\rangle$



$\alpha\left|\downarrow\right\rangle\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\left|\uparrow\right\rangle$

Heralding Quantum Errors

$$Z_1, Z_2 = \pm 1$$

Measure the
Joint Parity operator:

$$\Pi_{12} = Z_1 Z_2$$

$$\Pi_{12} \left|\uparrow\right\rangle\left|\uparrow\right\rangle = +\left|\uparrow\right\rangle\left|\uparrow\right\rangle$$

$$\Pi_{12} \left|\downarrow\right\rangle\left|\downarrow\right\rangle = +\left|\downarrow\right\rangle\left|\downarrow\right\rangle$$

$$\Pi_{12} \left|\uparrow\right\rangle\left|\downarrow\right\rangle = -\left|\uparrow\right\rangle\left|\downarrow\right\rangle$$

$$\Pi_{12} \left|\downarrow\right\rangle\left|\uparrow\right\rangle = -\left|\downarrow\right\rangle\left|\uparrow\right\rangle$$

$$\Pi_{12}\left(\alpha\left|\downarrow\right\rangle\left|\downarrow\right\rangle + \beta\left|\uparrow\right\rangle\left|\uparrow\right\rangle\right) = +\left(\alpha\left|\downarrow\right\rangle\left|\downarrow\right\rangle + \beta\left|\uparrow\right\rangle\left|\uparrow\right\rangle\right)$$

$\Pi_{12} = -1$ heralds single bit flip errors

Heralding Quantum Errors

$$\Pi_{12} = Z_1 Z_2$$

Not easy to measure a joint operator while not accidentally measuring individual operators!

(Typical 'natural' coupling is $M_Z = Z_1 + Z_2$)

$\left|\uparrow\right\rangle\left|\uparrow\right\rangle$ and $\left|\downarrow\right\rangle\left|\downarrow\right\rangle$ are very different,

yet we must make that difference invisible

But it can be done if you know the right experimentalists...

Heralding Quantum Errors

Example of error heralding:

$$|\Psi\rangle = \alpha|\downarrow\rangle|\downarrow\rangle + \beta|\uparrow\rangle|\uparrow\rangle$$

Introduce single qubit error on 1 (over rotation, say)

$$e^{i\frac{\theta}{2}X_1}|\Psi\rangle = \cos\frac{\theta}{2}|\Psi\rangle + i\sin\frac{\theta}{2}X_1|\Psi\rangle$$

Relative weight of α, β is untouched.

Probability of error: $\sin^2\frac{\theta}{2}$

If no error is heralded, state collapses to $|\Psi\rangle$

and <u>there is no error!</u>

Heralding Quantum Errors

Example of error heralding:

$$|\Psi\rangle = \alpha|\downarrow\rangle|\downarrow\rangle + \beta|\uparrow\rangle|\uparrow\rangle$$

Introduce single qubit rotation error on 1 (say)

$$e^{i\frac{\theta}{2}X_1}|\Psi\rangle = \cos\frac{\theta}{2}|\Psi\rangle + i\sin\frac{\theta}{2}X_1|\Psi\rangle$$

Relative weight of α, β is untouched.

Probability of error: $\sin^2\frac{\theta}{2}$

If error is heralded, state collapses to $X_1|\Psi\rangle$

and there is a full bitflip error. We cannot correct it

because we don't know which qubit flipped.

Heralding Quantum Errors

Quantum errors are continuous (analog!).

But the detector result is discrete.

The measurement back action renders the error discrete (digital!)

– either no error or full bit flip.

1. Quantum Information

2. Quantum Measurements

3. Quantum Error Heralding

4. **Quantum Error Correction**

Correcting Quantum Errors

Extension to 3-qubit code allows full correction of bit-flip errors

$$|\Psi\rangle = \alpha|\downarrow\rangle|\downarrow\rangle|\downarrow\rangle + \beta|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle$$

$$\Pi_{12} = Z_1 Z_2 \text{ and } \Pi_{32} = Z_3 Z_2$$

Provide two classical bits of information to diagnose and correct all 4 possible bit-flip errors:

$$I, X_1, X_2, X_3$$

Correcting Quantum Errors

Joint parity measurements provide two classical bits of information to diagnose and correct all 4 possible bit-flip errors:

$$|\Psi\rangle = \alpha|\downarrow\rangle|\downarrow\rangle|\downarrow\rangle + \beta|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle$$

Error	Z_1Z_2	Z_2Z_3
I	+1	+1
X_1	-1	-1
X_2	-1	-1
X_3	+1	-1

Correcting Quantum Errors

Extension to 5,7,or 9-qubit code allows full correction of ALL single qubit errors

I (no error)

$X_1,\ldots,X_N$ (single bit flip)

$Z_1,\ldots,Z_N$ (single phase flip; no classical analog)

$Y_1,\ldots,Y_N$ (single bit AND phase flip; no classical analog)

For *N=5,* there are 16 errors and 32 states

32= 16 x 2

Just enough room to encode which error occurred and still have one qubit left to hold the quantum information.

Full Steane Code – Arbitrary Errors

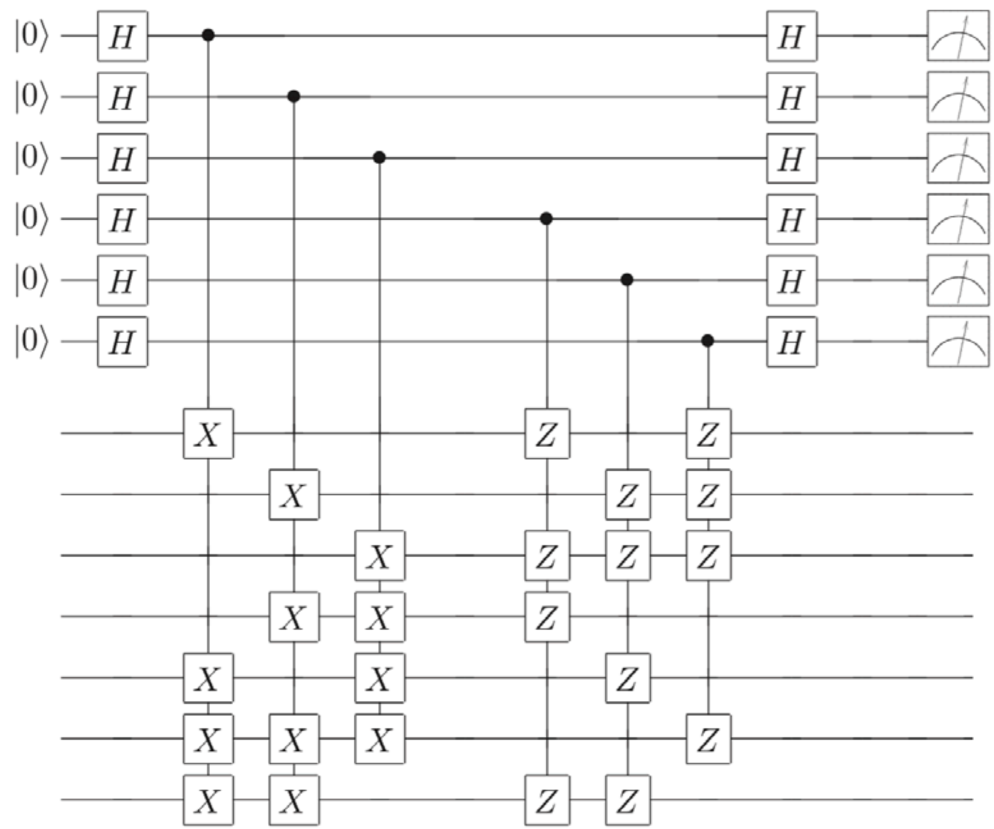
$$|0_L\rangle = \frac{1}{\sqrt{8}}\Big[|0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle\Big]$$

$$|1_L\rangle = \frac{1}{\sqrt{8}}\Big[|1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle + |1110000\rangle + |0100101\rangle + |1000011\rangle + |0010110\rangle\Big]$$

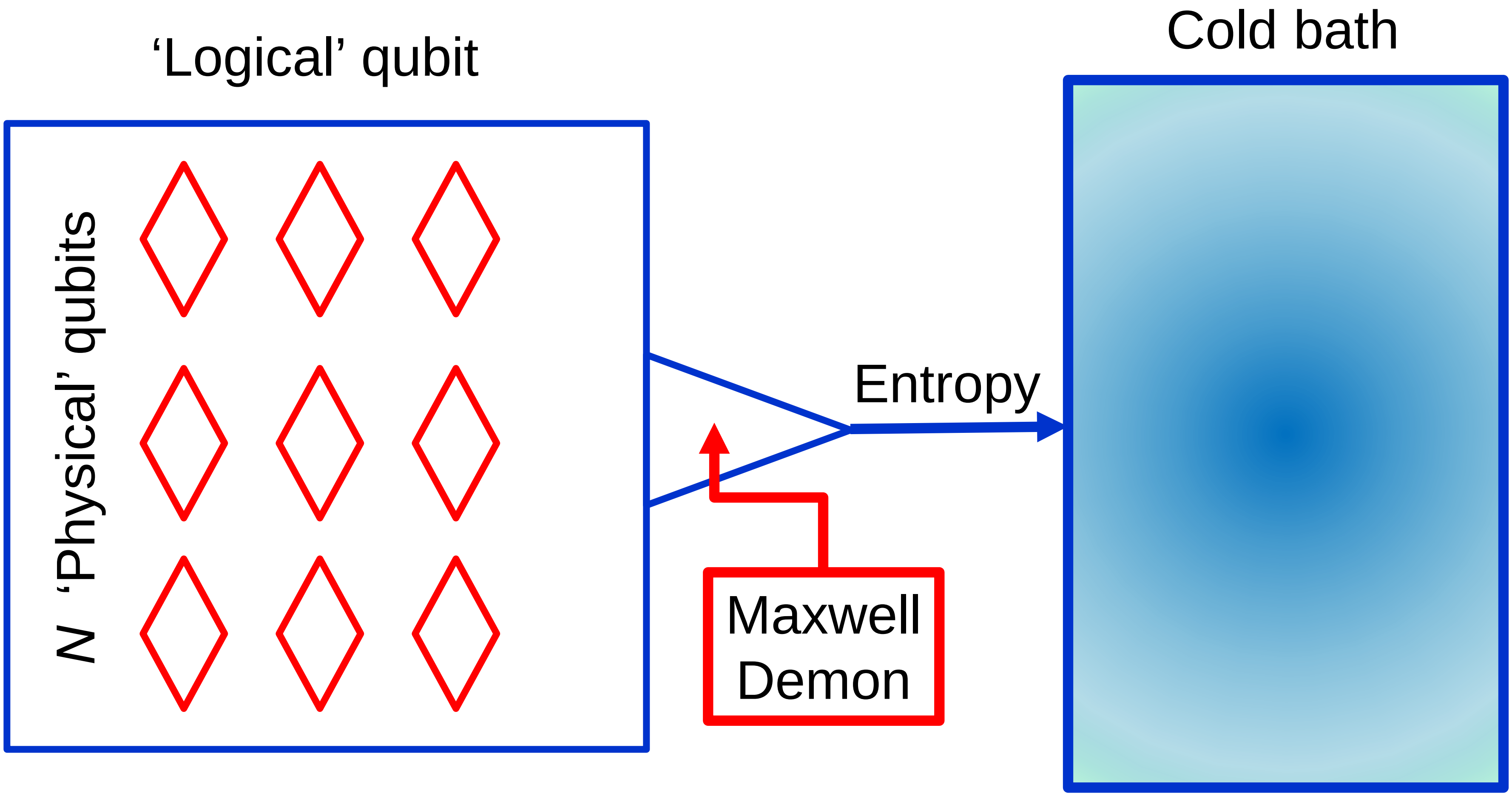
Single round of error correction

6 ancillae

7 qubits



Quantum Error Correction



N qubits have errors *N* times faster. Maxwell demon must overcome this factor of *N* – *and not introduce errors of its own! (or at least not uncorrectable errors)*

All previous attempts to overcome the factor of *N* *and reach the 'break even' point of QEC have failed.*

Lecture 2 will describe first QEC to reach break even.

Are We There Yet?

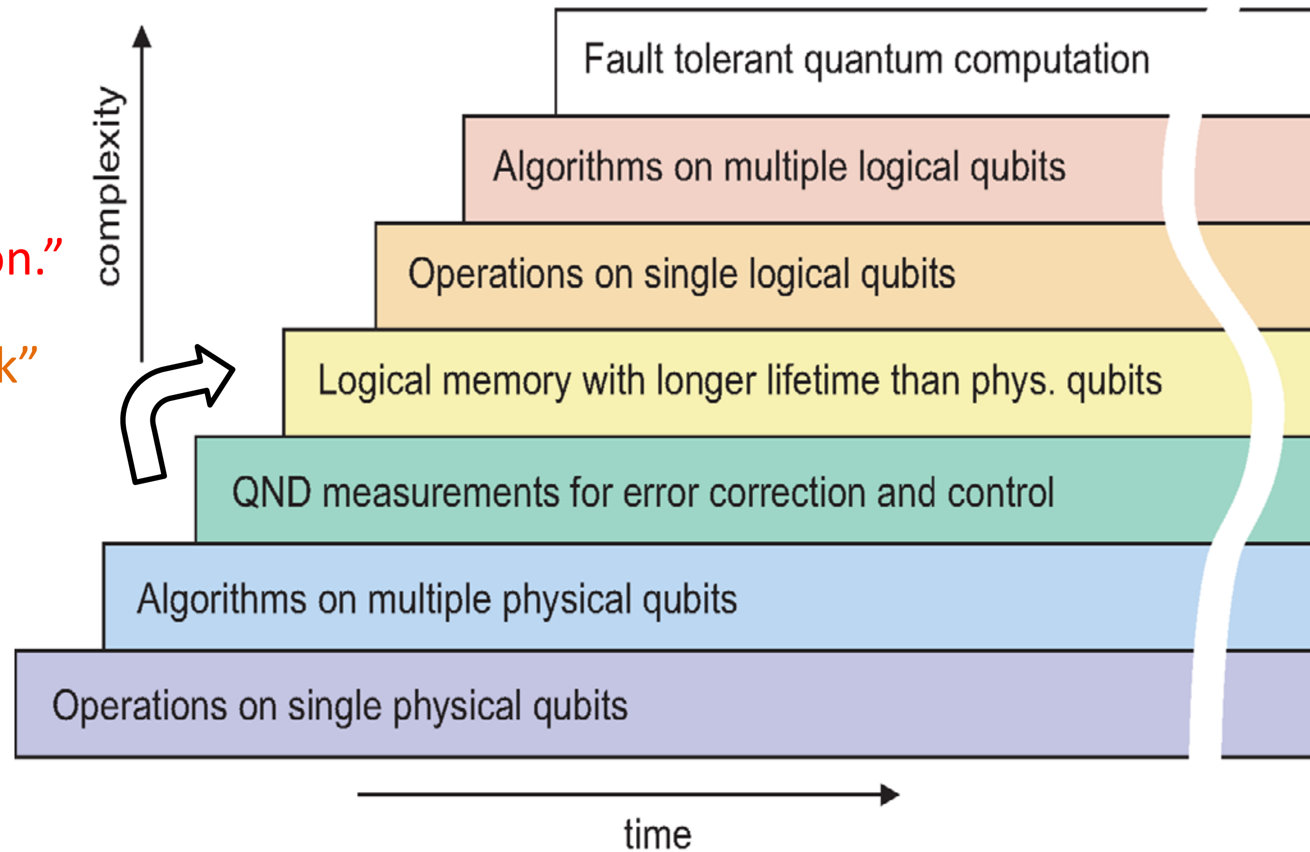
“Age of Qu. Error Correction.”

“Age of Quantum Feedback”

“Age of Measurement”

“Age of Entanglement”

“Age of Coherence”



Goal of next stage: reaching “break-even” point for error correction

“We” are ~ here (also ions, Rydbergs, q-dots, …)

M. Devoret and RS, Science (2013)