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Science and Engineering Students' Difficulties With Fractions At Entry-Level To University

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ABSTRACT

This study was carried out at a South African university. The aim of the study was to test entry-level students' fractions skills in order to facilitate teaching at appropriate levels. The sample consisted of 94 first-year entry level students (54 mainstream and 40 extended stream) who were enrolled for national diplomas in science and engineering, out of a population of 120 students. The instrument had 20 items, including three multiple choice questions (MCQs). The data were analyzed using Microsoft Excel 2013. The main finding was that entry-level students enrolled for engineering and science diplomas performed poorly in a test of numeracy skills. The average score (47.8%) was regarded as a cause for concern, especially considering that the test was pitched at Grade 8 level. The study also found that students struggled to apply proportional reasoning when dealing with word problems. Mathematics teachers and lecturers need to be aware of students' difficulties and ought to attempt to assist them to overcome such challenges. It is hoped that this paper will be useful to mathematics curriculum implementers at school level, subject advisors at the district level, pre-service teacher educators at Teachers' colleges and universities, and university lecturers teaching mathematics at first year level.

KEYWORDS
mathematics education; numeracy; fractions; ratios; proportions

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Introduction

A current theme in the mathematics literature revolves around the question whether students' difficulties with fractions have been resolved by the time they enrol at university (Booth et al., 2014; Duffin, 2003; Gabaldon, 2015;

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Jukes & Gilchrist, 2006; Livy & Herbert, 2013; Schneider & Siegler, 2010). Duffin (2003, p. 1) asserts that the terms “numeracy” and “higher education” seem anomalous. However, evidence points to the contrary (Schneider & Siegler, 2010). Researchers have reported students to be underprepared for the numeracy demands in courses such as nursing and health sciences (Jukes & Gilchrist, 2006), business (Boreland, 2016) and law (Gabaldon, 2015). Although research reports on numeracy demands on science and engineering students have been sparse, this research area has recently begun gaining momentum. The inadequacy might probably have been due to the assumption that competency in mathematics necessarily encompasses high levels of quantitative literacy. Interestingly, some people regard this as a reasonable assumption, especially for students who register for science and engineering courses. This is however not necessarily the case (Barwell, 2004; Houston et al., 2015; Wilson & MacGillivray, 2007). While discussing challenges in teaching, the authors and their colleagues have on several occasions reflected on what they perceived as their science and engineering students’ poor proficiency in fractions. According to Torbeyns et al. (2015), teachers should be informed of their students’ difficulties with fractions in order to adjust their teaching to their students’ current skills levels. The aim of the study was therefore to identify science and engineering students’ prior knowledge in fractions at university entry level. The terms “numeracy” and “quantitative literacy” are considered as synonyms and used interchangeably in this paper.

Despite a strong association between numeracy and mathematics, these two are not equivalent. Roohr et al. (2014) assert that quantitative literacy is embedded in real-world contexts, and therefore lacks the more abstract and general nature of mathematics. Because of the context-based nature, quantitative literacy articulates the “power of practicality, whereas mathematics in turn articulates the ‘power of abstraction” (Bowie & Frith, 2006, p. 29). Numeracy skills are however required by students enrolled for science and engineering diplomas. The United States National Mathematics Advisory Panel (NMAP) described fractions as “the most important foundational skill not presently developed” (NMAP, 2008, p. 18). Poor numeracy skills will detrimentally affect students’ performance in mathematics, engineering, statistics and other related subjects (Cetin & Ertekin, 2011; Torbeyns et al., 2015; Wilson & MacGillivray, 2007).

The importance of mathematics cannot be overemphasised. Mathematics is a compulsory subject for further studies in science, technology and engineering (STEM). Furthermore, skills level in high school mathematics can be used to predict future qualifications and levels of job satisfaction and income (Rivera-Batiz, 1992). More specifically, research shows that there is a strong link between understanding of fractions and success in mathematics. Fractions play a central role in algebra (Booth et al., 2014; Siegler et al., 2012a; Siegler & Lortie-Forgues, 2015) and influence performance in more advanced courses in mathematics (Booth & Newton, 2012; Siegler et al., 2012b; Watts et al., 2014). “Fractions (along with the closely related concepts of ratios and proportion) are ubiquitous in algebra” (Bailey et al., 2015), in specific topics in mathematics such as geometry, probability and trigonometry (Pienaar, 2014, p. 2). Lesh et al. (1988, p. 93) argue that proportional reasoning is especially important, calling it the "capstone of elementary math" and the "cornerstone of high school math". Bone et al. (1984) assert that ratio and proportion are the two skills most frequently characterized by instructors of technical courses as essential. Knowledge of fractions indirectly determines career choices and eventual income levels (Titus, 1995). Quantitative literacy has been shown to influence the prospect of full-time employment (Naureen & Vicki, 2012; Rivera-Batiz, 1992). Also, quantitative literacy has been found to influence the quality of judgement and decision making in consumerism and medical and financial contexts in people's personal lives, even amongst highly qualified communities (Reyna et al., 2009).

Difficulties with mathematics may commence early in the educational process (Bailey et al., 2015; Cai, 1995; Torbeyns et al., 2015). It is well-known that "mathematics is ruthlessly cumulative all the way back to counting to ten" (Pinker, 1998, p. 342), and even if lower level procedures are mastered, learning without understanding them will impair future performance and subsequent learning. Spaull and Kotze (2015, p. 14) describe this structure as "a hierarchy of knowledge and intellectual skill", and proceed to quote the description of the hierarchical nature of the subject given by Schollar (2008, p. 1):

> Mathematics, however, is an hierarchical subject in which the development of increasingly complex cognitive abilities at each succeeding level is dependent on the progressive and cumulative mastery of its conceptual frameworks, starting with the absolutely fundamental basics of place value (the base-10 number system) and the four operations (calculation).

It therefore stands to reason that some of the problems with mathematics can be traced back to primary school mathematics, and more specifically, to the learning and teaching of fractions. The national report of the Department of Basic Education of South Africa pointed out in 2012 that the lack of understanding in basics of fraction was one of the factors that contributed to the low achievement in matriculation mathematics examinations in that year (DoBE, 2012). Problems with conceptual understanding of fractions are common amongst students and may last into adulthood (Siegler et al., 2012b; Siegler & Thompson, 2014). In a survey conducted on adult skills, the Organization for Economic Co-operation and Development (OECD, 2013) found that only one in three adults was able to understand and interpret simple data and statistics in tables and graphs. Even in first world countries such as Italy and Spain, almost a third of the adults performed at or below the lowest level of proficiency in both literacy and numeracy (OECD, 2013). Proficiency in literacy and numeracy is closely related and also related to proficiency in problem solving in digital environments (OECD, 2013).

Torbeyns et al. (2015) found that difficulties with fractions are common, even amongst prospective teachers from various countries. It is therefore understandable that research on students' difficulties with fractions has been an ongoing activity. There is consensus amongst educational researchers that fractions are a complex and multifaceted construct (Brousseau et al., 2004; Lamon, 2001) Learners find fractions difficult to learn (Booth & Newton, 2012; Lortie-Forgues et al., 2015) and teachers likewise find it a difficult topic to teach (Clarke, 2006; Ma, 1999). Ratio and proportion especially cause difficulties for students and teachers alike (Livy et al., 2013). In a study done in England on undergraduate nursing students' abilities to calculate drug dosages, only 7% correctly answered the questions on ratio and proportion, although the average score on the test was 55% (Jukes & Gilchrist, 2006).

Research Method

The present study involved entry-level Diploma students from a comprehensive university in the South Africa. In this university, the students are streamed as mainstream or extended stream, depending on their prior academic performance. If applicants do not qualify academically for the entrance requirements of their chosen diploma, they are placed in the extended stream, and allowed extra time to complete the qualification. Extended stream courses qualify for additional government subsidies. The sample consisted of university entry level students who were enrolled for mathematics, a service course for national diploma studies in engineering and science. Based on a request from the researchers, out of a population of 120 students in three cohorts (Electrical Engineering, Civil Engineering and Analytical Chemistry), 94 students (54 mainstream and 40 extended stream) voluntarily took part in the study.

Adopting a positivist paradigm and a quantitative research approach, the study applied a survey design. The instrument collected general data such as age, Grade 12 mathematics scores, gender, course and stream. The fractions part in the instrument consisted of 20 items, of which three were multiple choice questions (MCQs) and the rest open-ended calculations. The lead researcher compiled the instrument after a study of the pertinent literature, including various Trends in International Mathematics and Science studies (TIMSS). Questions were selected to cover the following topics: notation, magnitude and magnitude on a number line (B1); operations on fractions (B2); operations combined with SI unit conversions (B3); ratio and proportion (B4) and percentage and percentage increase and decrease (B5). The skills tested are required in all engineering and science courses. An attempt was made to cover four levels of skills in the test, namely knowing; performing routine procedures and/or measurements; using complex procedures and lastly, solving problems as envisaged by (DoBE, 2011a). The allocation of a question to one of the four categories mentioned above was not always made apparent. Long et al. (2014, p.8) assert that such allocation "depends on the level of knowledge acquired by the learner". The first category was 'knowing' and the allocation of questions to this category seemed simple, but in retrospect it was found to be more difficult than expected as Long et al. (2014) posited:

> Even a seemingly simple category such as "knowing", can be problematic…. Whilst there is an element of memory involved, in that recalling facts, terms, basic concepts and answers forms part of this component, this component also embraces knowledge of specifics (terminology and specific facts), knowledge of ways and means of dealing with specifics and knowledge of the universal abstractions in a field (principles and generalisations, theories and structures) (Long et al., 2014, p. 4).

Hence, experienced mathematics lecturers from the same faculty were requested to analyse the test. They rated the degree of difficulty of each question in the questionnaire on a five point scale as either very easy, easy, moderate, difficult or very difficult. The vast majority of questions were rated as very easy, easy or moderate (85.5%). Only 14.5% of the ratings indicated a difficult or very difficult option. The test was then piloted and slight modifications were made.

Data were collected during the third week of the semester. Data collection was managed by field workers, who emphasized that participation in the research project was voluntary and the upholding of the anonymity of respondents. Since data was required on every question, no time limit was set to complete the test. Furthermore, a decision was taken to allow calculators, since students were permitted to use calculators in all their assessments.

Results and Dıscussıon

Microsoft Excel 2013 was used in the analysis of the data. The majority of the students participating in the study were in the mainstream (54 or 57%) and the rest (40 or 43%) in the extended stream. The gender division of the sample was almost even at forty-eight (51%) males and forty-six (49%) females. The sample was composed of more engineering (47.9%) than science (52.1%) students. Majority (55 or 58.5%) were in the 20-24 years age category. About a third (32 or 34.0%) was considered to be "at risk" of failing mathematics based on Grade 12 mathematics scores below 50% (Table 1). Another twenty-seven (28.7%) students with Grade 12 mathematics scores of between 50-59% were considered to be in need of support. Only 37.2% reported Grade 12 mathematics scores of above 59%.

The Electrical engineering cohort's Grade 12 results were superior to those of the other two (Table 1). Also, seven students among the Electrical engineering cohort had Grade 12 Mathematics scores of 70% or above, whereas none in the science cohort had a similar score. Despite this, those in the Civil engineering cohort had a higher number of scores in the top category than the other two cohorts (Table 3).

Table 1. Contingency Table – Cohort and Self-Reported Mathematics Grade12 Score

Percentage range	Cohort						Total	
Mathematics Grade 12	Analytical Chemistry		Civil Engineering		Electrical Engineering			
0-29%	0	0%	0	0%	0	0%	0	0%
30-39%	0	0%	1	4%	0	0%	1	1%
40-49%	31	63%	0	0%	1	5%	32	34%
50-59%	9	18%	15	60%	3	15%	27	29%
60-69%	9	18%	6	24%	9	45%	24	26%
70-79%	0	0%	3	12%	5	25%	8	9%
80-100%	0	0%	0	0%	2	10%	2	2%
Total	49	100%	25	100%	20	100%	94	100%

Not many students were confident when working with fractions. Only 40 (42.6%) reported that they were confident, whilst 44 (46.8%) were unsure. These figures show nexus to the test scores-the average test score was similar (47.8%). Considering their choice of career, it stands to reason that most of the students from this sample (86 or 91.5%) regarded mathematics as very important. This is significant, since Thomson and Hillman (2010) assert that students who value mathematics are more likely to be successful in their tertiary study endeavours.

The students' average scores per question have been summarised in Table 2, sorted in the descending order. B1-B5 in Table 2 refer to the topics covered, B1 for notation, magnitude and magnitude on a number line; B2 for operations on fractions; B3 for operations combined with SI unit conversions; B4 for ratio and proportion and B5 for percentage and percentage increase and decrease. The skills level in Table 2 refers to one of four levels of skills, namely knowing (K), performing routine procedures and/or measurements (R), using complex procedures (C) and lastly solving problems (S). There were five questions in each of these categories.

Table 2[1]. Frequency Distributions: Correct answers to Test Questions (n = 94)

Question Number	Skills level	Topics covered	Analytical Chemistry		Civil Engineering		Electrical Engineering		Total	
3e	K	B2	40	87%	17	68%	15	75%	72	77%
5	R	B3	32	70%	19	76%	16	80%	67	72%
3b	R	B2	31	67%	19	76%	17	85%	67	71%
3d	R	B2	29	63%	24	96%	14	70%	67	71%
1	K	B1	29	63%	18	72%	19	95%	66	70%
7	C	B4	28	61%	17	68%	8	40%	53	60%
4	R	B3	27	59%	15	60%	13	65%	55	59%
11	K	B5	27	59%	15	60%	11	55%	53	58%
8	C	B3	28	61%	12	48%	11	55%	51	54%
2a	C	B1	22	48%	15	60%	13	65%	50	54%
2b	C	B1	23	50%	14	56%	13	65%	50	54%
10	C	B4	21	46%	16	64%	7	35%	44	47%
3a	R	B2	20	43%	10	40%	8	40%	38	41%
13	S	B5	13	28%	12	48%	9	45%	34	37%

[1]*Percentages in this table were calculated as a proportion of the number of students who answered each question. In the rest of the paper, percentages were calculated as a fraction of 94, the sample size. It was assumed that students who did not offer an answer,*

did not know how to do the question, since they had almost unlimited time (Wilson & MacGillivray 2007).

The average score (47.8%) was disappointing indicating that most entry-level engineering and science diploma students at this particular university in South Africa still struggle with fractions. The scores had a wide range (7%-86%). The standard deviation and the median were 19.6% and 50.5%, respectively.

Table 3. Mainstream Averages compared to Extended Stream Averages

Description	Mainstream (54)			Extended Stream (40)	
average overall scores	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
average percentage	50.8%	57%	51%	44%	39%
standard deviation	15.9%	17%	15%	20%	19%
per stream	53.5% (S.D. = 16.6%)			40.1% (S.D. = 20.9%)	
sample	47.8%				

The spread of the B scores (the average test scores) is tabulated (Table 4). Only 30 out of the 94 (31.9%) students scored above 60% for the test.

Table 4. Contingency Table-Cohort and B Score

B Score	Cohort						Total	
	Analytical Chemistry		Civil Engineering		Electrical Engineering			
0 to 39	24	49.0%	6	24.0%	4	20.0 %	34	36.2%
40 to 60	12	24.5%	8	32.0%	10	50.0 %	30	31.9%
61 to 100	13	26.5%	11	44.0%	6	30.0 %	30	31.9%
Total	49	100%	25	100%	20	100 %	94	100%
Chi^2 (d.f. = 4, n = 94) = 9.20; p = .056								

Of the five sections in the test, namely notation, magnitude and magnitude on a number line (B1), operations on fractions (B2), operations combined with SI unit conversions (B3), ratio and proportion (B4) and percentage and percentage increase and decrease (B5), students performed best in operations combined with SI unit conversions (B3) and worst in the section on percentages (B5). The average score for only two of the sections, namely B1 and B3, were higher than 50%, while the mean scores for B4 and B5 were below 40% (Table 5).

Table 5. Central tendency & dispersion: test scores in percentages (n = 94)

Scores	Mean	S.D.	Min	Quartile 1	Median	Quartile 3	Max
B1	58.8	38.2	0.0	33.0	67.0	100.0	100.0
B2	46.9	20.2	0.0	29.0	43.0	57.0	86.0
B3	61.4	33.4	0.0	33.0	67.0	100.0	100.0
B4	39.6	29.4	0.0	25.0	50.0	50.0	100.0
B5	32.2	24.7	0.0	0.0	33.0	33.0	100.0
B	47.8	19.6	7.0	31.0	50.5	63.0	86.0

The data revealed a significant relationship between the self-reported Grade 12 Mathematics score and the B1 Score and a statistically significant relationship between the self-reported Grade 12 Mathematics score and both the B3 Score and the average score for the test (the B score) (Figure 1 and Tables 6 and 7). According to Gravetter and Wallnau (2009), correlations are statistically significant at the 0.05 level for n = 94 if $|r| \geq 0.203$ and practically significant if $|r| \geq 0.300$.

Table 6. Pearson Product Moment Correlations–B1 (Notation, magnitude and magnitude on a number line) score to B score and Mathematics Grade 12 (n = 94) score

	Mathematics Grade 12
B1 score	0.347
B2 score	0.110
B3 score	0.228
B4 score	0.169
B5 score	0.030
B score	0.291

Table 7. Contingency Table - Mathematics Grade 12 and B Scores

B Score	Mathematics Grade 12							
	30 - 49%		50 - 59%		60 - 100%		Total	
0 - 39	19	58%	9	33%	6	18%	34	36%
40 - 100	14	42%	18	67%	28	82%	60	64%
Total	33	100%	27	100%	34	100%	94	100%
Chi^2 (d.f. = 2, n = 94) = 11.70; p = .003; V = 0.35 Medium								

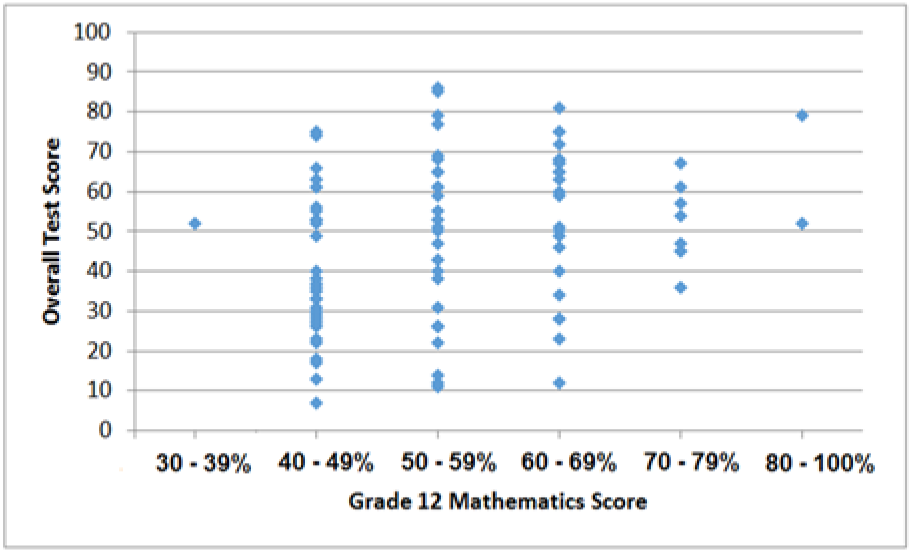

Figure 1. Relationship between Grade12 Mathematics Score and Overall Test Score

A medium size difference (Cohen's d = 0.73) was detected between the average test scores (B score) of the mainstream and the extended stream (Table 8). Medium size differences were also indicated (Cohen's d = 0.66; Cohen's d = 0.67 respectively) between the B1 and B3 scores of the two cohorts.

Table 8. t-Tests: B1 to B Score by Mainstream (n = 54) and Extended Stream (n = 40)

Variable	Stream	Mean	S.D	Difference	t	d.f.	p (d.f.=92)	Cohen's d
B1 Score	Main	69.07	35.48	24.07	3.18	92	0.002	0.66 Medium
	Extended	45.00	37.48					
B2 Score	Main	48.76	19.28	4.46	1.06	92	0.292	n/a
	Extended	44.30	21.32					
B3 Score	Main	70.46	27.27	21.34	3.22	92	0.002	0.67 Medium
	Extended	49.13	37.05					
B4 Score	Main	43.52	29.61	9.14	1.50	92	0.137	n/a
	Extended	34.38	28.69					
B5 Score	Main	35.69	25.06	8.29	1.62	92	0.109	n/a
	Extended	27.40	23.80					
B Score	Main	53.52	16.62	13.47	3.48	92	.001	0.73 Medium
	Extended	40.05	20.91					

The meta-cognition of the students was probed in the last question (Q15) by asking them to rate their scores in the test:

Please indicate what, in your opinion, you most likely scored on this test by writing down the letter of the score category:

A: 0 – 19%	*B: 20 – 39%*	*C: 40 - 59%*	*D: 60 - 79%*	*E: 80 - 100%*

A statistically significant correlation was measured between the answers (B15 scores, Figure 2) and the B4 and B scores (Table 9).

Table 9. Contingency Table – Cohort and B15

B15	Cohort						Total	
	Analytical Chemistry		Civil Engineering		Electrical Engineering.			
0 - 19%	1	2.1%	2	8.0%	0	0.0%	3	3.2%
20 - 39%	9	18.8%	2	8.0%	1	5.0%	12	12.9%
40 - 59%	18	37.5%	10	40.0%	6	30.0%	34	36.6%
60 - 79%	11	22.9%	7	28.0%	7	35.0%	25	26.9%
80 - 100%	9	18.8%	4	16.0%	6	30.0%	19	20.4%
Total	48	100%	25	100%	20	100%	93	100%

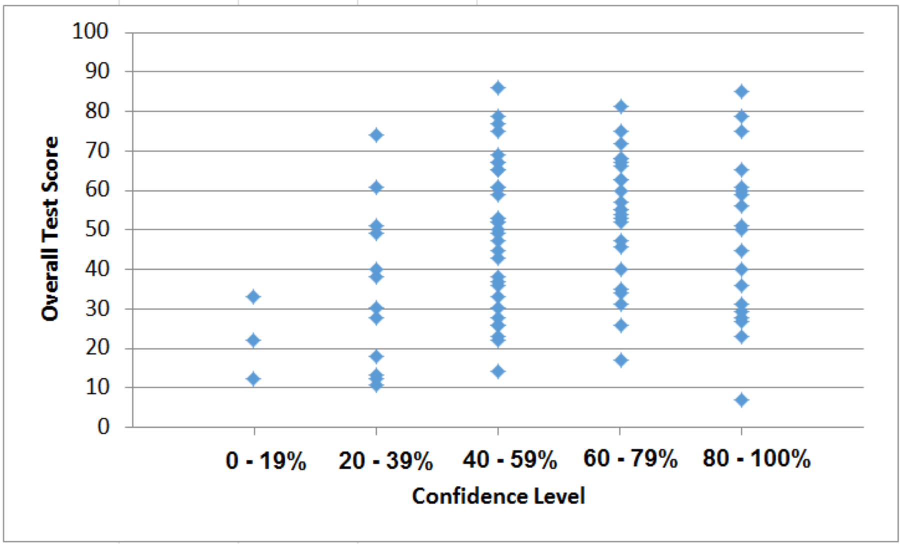

Figure 2. Relationship between Meta-cognition or Confidence level (B15) and Overall Test Score (B Score)

NOTATION, MAGNITUDE & MAGNITUDE ON A NUMBER LINE (B1, 58.8% AVERAGE) (Q1 & 2A, 2B)

The first question, along with four others, namely Questions 3a ,3e and Questions 3c and 3d, were discussed in another article (Coetzee & Mammen, 2016), since the data gathered from these questions pointed to language challenges with fractions terminology.

The second question tested knowledge of decimal fraction magnitude on a number line. This section is mentioned in the Grade 6 National Curriculum and Assessment Policy Statement (CAPS) document under "Recognizing, ordering and place value of decimal fractions" (DoBE, 2011c, p. 17). A portion of the number line was provided, with the endpoints indicated as zero and 0.4 (Figure 3). The line segment was subdivided into 10 equal parts. Two values, *x* and *y*, had to be read from the line segment.

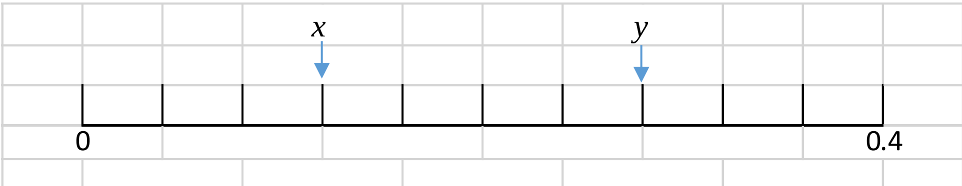

Figure 3. Line segment for answering Question 2

Only 50% of the students in the sample reported the value of *x* correctly (Table 10). Although the Civil engineering mainstream scored higher than the other cohorts (64.7%), it remains disconcerting that 34.5% of the mainstream Civil engineering students did not manage to answer this question correctly, since these students are expected to be able to work with scale measures in subjects such as Drawing and Survey. The correct answers for the first number read from the line segment correlated almost perfectly with the correct answers given for the second number. The errors committed are thus interpreted as different versions of incorrect subdivisions of the given interval. In such a case, it stands to reason that a mistake in reading the value of *x*, will necessarily lead to a mistake in reading the value of *y*. Despite an odd number of subdivisions (5) between zero and 0.2, which meant that the midpoint between zero and 0.2 did not coincide with a subdivision, some students indicated it as such and hence concluded that this point was 0.1. A number of students indicated the first of the subdivisions following zero as 0.22 and the subsequent ones as 0.24; 0.26; 0.28; and so forth, up to 0.38 followed by 0.4 (Figure 4).

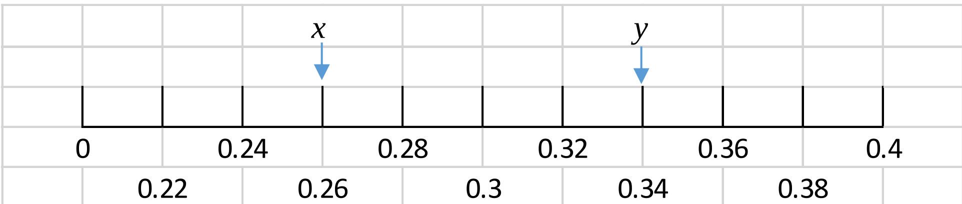

Figure 4. Incorrect subdivision for Question 2

This division seemed to work out perfectly, but for the fact that the distance between zero and the first subdivision was $\frac{22}{100}$ units and the distance between each successive subdivision, was substantially smaller at $\frac{2}{100}$ units.

Table 10. Scores for Question 2

Question 2		Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers for Q2a	13 (65.0%)	11 (64.7%)	10 (58.8%)	4 (50%)	12 (37.5%)
correct answers per stream	34 (63%)			16 (40%)	
sample Q2a	50 (53.2%)				
correct answers for Q2b	13 (65%)	9 (52.9%)	10 (58.8%)	5 (62.5%)	13 (40.6%)
correct answers per stream	32 (59.3%)			18 (45%)	
sample Q2b	50 (53.2%)				

The summary of the scores for Cohort for B1 (Notation, magnitude and magnitude on a number line) is given in Table 11.

Table 11. Contingency Table – Cohort and B1 score

B1 Score	Cohort						Total	
	Analytical Chemistry (49)		Civil Engineering (25)		Electrical Engineering (20)			
0 to 39	26	53%	9	36%	7	35%	42	45%
40 to 100	23	47%	16	64%	13	65%	52	55%
Total	49	100%	25	100%	20	100%	94	100%
	Chi² (d.f. = 2, n = 94) = 2.91; p = .233							

OPERATIONS ON FRACTIONS (B2, 46.8% AVERAGE) (Q 3A-F & Q9)

In Question 3b, students had to calculate $\frac{1}{5}$ of a decimal fraction - a Grade 6 skill, which is also revised in Grades 7 and 8 (DoBE, 2011c, p. 17).

Only 71% of the students were able to calculate the answer correctly, which is disconcertingly low for such a basic skill, especially considering that all the students had access to calculators.

In the last question of this section (Q9), students had to calculate how much sugar remained in a dish if a fraction ($\frac{1}{4}$) of the sugar was spilt. This skill is prescribed in the Grade 6 syllabus (DoBE, 2011c, p. 15), yet only 24.5% of the students in the sample managed to answer correctly (Table 12). Students who gave the incorrect answer, mostly failed to calculate the amount of sugar spilt as a ratio of the original amount before subtracting from the original amount, i.e. they calculated 0.7 kg – $\frac{1}{4}$, instead of calculating 0.7 kg – ($\frac{1}{4}$ of 0.7 kg). They therefore also committed the error of subtracting a unit-less amount ($\frac{1}{4}$) from an amount representing units (0.7 kg). The scores for these questions are similar to those achieved by university calculus students in a study involving units conducted in the United States. Of the 169 students tested in the former study, only forty five (26,6%) gave correct units for all of the tasks (Dorko & Speer, 2014).

Table 12. Scores for Question 9

Question 9	Mainstream			Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	7 (35%)	5 (29.4%)	4 (23.5%)	2 (25%)	5 (15.6%)
correct answers per stream	16 (29.6%)			7 (17.5%)	
sample	23 (24.5%)				

In Question 3f, students' conceptual understanding of $3 \div \frac{1}{2}$ was tested by providing a picture of three apples. Appropriate drawings were made by 36% of the students, depicting the correct answer to the question (Table 13). Students were required to draw 6 half apples, as depicted by Vuyo's suitable drawing in Figure 5.

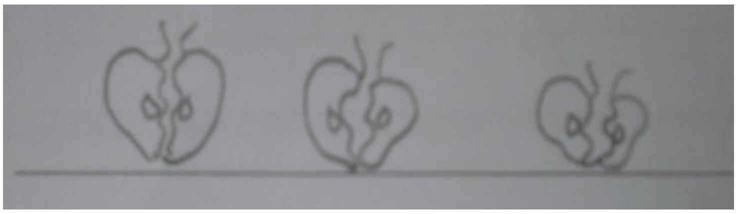

Figure 5. Vuyo's drawing made in response to Question 3f

Although many students realised that the answer was supposed to be six, they did not necessarily relate the six to halves. Numerous answers depicted one apple divided into six portions, which apparently satisfied the need to have six elements in the answer.

Peter realised that an answer consisting of 6 whole units was incorrect, and that fractional portions were called for, and thus changed his answer to reflect these (Figure 6).

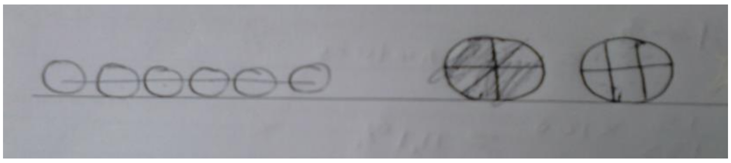

Figure 6. Peter's drawing in response to Question 3f

Peter hence realised that the fractions in his first circular drawing were unequal, and attempted to correct it. He however ended up with four parts equal in size, but unequal in size to the other two parts.

Sam in turn reported the answer as six halves, but Sam's drawing (Figure 7) did not correspond to his written answer, and contained six parts, but not six halves. The drawing he made was similar to Peter's.

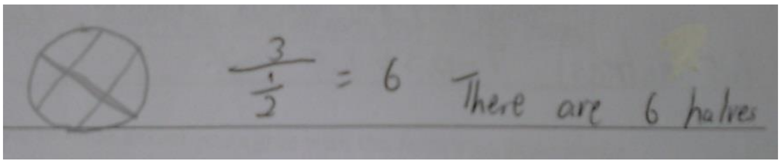

Figure 7. Sam's drawing in response to Question 3f

Sam was able to find the correct answer, but did not display accurate conceptual understanding of the answer.

Table 13. Scores for Question 3f

Description		Mainstream		Extended Stream	
Scores for Question 3f	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)

correct answers	1 (5%)	6 (35%)	5 (29.4%)	1 (13%)	14 (35%)
correct answers per stream	12 (22.2%)			15 (37.5%)	
Sample	27 (28.7%)				

The summary of the scores for cohort for B2 (Operations on fractions) is given in Table 14.

Table 14. Contingency Table – Cohort and B2 Score

B2 Score	Cohort						Total	
	Analytical Chemistry (49)		Civil Engineering (25)		Electrical Engineering (20)			
0 to 39	17	(35%)	6	(24%)	6	(30%)	29	(31%)
40 to 100	32	65%	19	76%	14	70%	65	69%
Total	49	100%	25	100%	20	100%	94	100%
Chi² (d.f. = 2, n = 94) = 0.90; p = .639								

OPERATIONS COMBINED WITH SI UNIT CONVERSIONS (B3, 61.4% AVERAGE) (Q 4, 5 & 8)

Both Questions four and five involved SI unit conversions, combined with an operation on a decimal fraction; division for Question four and addition for Question five (DoBE 2011c, p. 26). In Question four, students had to add two measurements of which one was an integer and the other a fraction. One measurement was however given in grams whereas the other was given in kilograms. In Question five, students had to calculate how many smaller containers of oil were required to fill one bigger container. The smaller container's volume was given in millilitres, whereas the bigger one's was given in litres. In both questions students needed to do unit conversions, but students clearly found the addition in Question five easier than the division in Question four. The proportion of correct answers for these two questions differed substantially at 57% and 72%, respectively.

In Question eight two prices were provided for sugar - one for a 500 g pack of sugar and another for a 2.5 kg pack of sugar. Students had to calculate which option was cheaper, and were required to convert to a common unit. One method was to convert 2.5 kg to 2500 g and hence multiply the first price by 5, since $\frac{2500}{500}=5$. Just over 50% of the students managed to do this correctly (Table 15). A common mistake was to divide by 1,000, instead of multiplying by 1,000, when converting from kg to g. This mistake points to rote manipulation that lacks conceptual understanding. Another incorrect procedure was to multiply the mass (500 g) by the price (R 5.27) to arrive at R 2, 635. Students following this incorrect procedure hence understood the given price to be per one gram of sugar, instead of per packet of sugar. The correct answer involved multiple steps. Some students executed the first step correctly, but did not carry through, and therefore could not make the correct decision. An example of such a method is to first convert the price for the 2.5 kg pack of sugar to price per kg, and then to stop. The second step, which was missing, would have been to also convert the price for the 500 g pack to a price per kg, and then to compare the prices.

A study conducted in Italy on consumer choice (Graffeo et al., 2015) used a field experiment that showed marked similarities to Question eight in this study. During the experiment a product was made available, with different initial prices and discounts, at two shops. One of the deals was better than the other, and consumers had to pick the better one and describe the arithmetic operations used in their decision. The researchers classified the approaches used by the consumers as either “complete” or “partial”. “Complete” refers to decisions taken after all the arithmetic operations required to solve the problem were calculated. “Partial” referred to decisions taken after only some of the operations were calculated. The researchers came to the conclusion that higher levels of numeracy were associated with the “complete” decision approach, which enabled the consumers to make a better quality purchase decision. The students involved in the current study, who had incomplete answers to Question eight, therefore used the “partial” decision approach, possibly demonstrating lower levels of numeracy. Wilson and MacGillivray (2007) assert that success rates fall rapidly when answers require multiple steps to be performed.

Table 15. Scores for Question 8

Question 8		Mainstream		Extended Stream	
Scores for Question 8	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	11	8 (65%)	13 (76%)	4 (75%)	15 (38%)
correct answers per stream	32 (59.3%)			19 (47.5%)	
sample	51 (54.3%)				

A summary of the scores for Cohort on B3 (Operations combined with SI units) is given in Table 16.

Table 16. Contingency Table - Cohort and B3 Score

B3 Score	Cohort						Total	
	Analytical Chemistry		Civil Engineering		Electrical Engineering			
0 to 39	18	37%	9	36%	6	30%	33	35%
40 to 80	17	35%	8	32%	7	35%	32	34%
81 to 100	14	29%	8	32%	7	35%	29	31%
Total	49	100%	25	100%	20	100%	94	100

								%
	Chi² (d.f. = 4, n = 94) = 0.44; p = .979							

RATIO AND PROPORTION (B4, 39.6% AVERAGE) (Q 6, 7, 10 & 12)

An amount for rent (R1,000) had to be proportionally shared amongst three people according to the number of cows each kept in a shared field (Q6). Only 30% of the students could do this correctly. Most incorrect answers were reached by dividing the amount by the number of people (three), instead of calculating the proportion according to the total number of cows each kept in the field. Although the total amount to be paid jointly by the three men was specified as R1,000, many answers exceeded this amount, and incorrect answers like R15,000.00, R3,300.33, R30,000.00, R130,000.00 and R36,000.00 were common. R30,000.00 was a popular answer, and it was reached by interpreting the joint amount payable, as an individual amount payable, depending on the number of cows each person kept in the field. Michael, who had 30 cows, would therefore have to pay R1,000.00 multiplied by 30, which is R30,000.00. It seems that students taking part in this study, often executed operations perfunctorily without performing reality checks, which is common internationally (Blais & Bath, 1992). Graffeo et al., (2015, p. 6) ascribe this phenomenon to low cognitive reflection levels, or "cognitive impulsivity", as measured by a cognitive reflection test (CRT). The researchers assert that this effect is partially counteracted by high numeracy levels.

In a subsequent question (Q7), students had to convert petrol consumption per 165 km, to consumption per 100 km, which involves ratio and proportion (DoBE, 2011a, p. 14). Correct answers were supplied by more than half (56.4%) of the students (Table 17). Six students did not attempt the question at all. Since questions seven and twelve were similar, and could be solved with similar procedures, it is unclear why there is such a significant difference between the proportions of correct answers for the two questions. Only 24.5% of the students could answer Question 12 correctly (Table 19), whereas 56.4% of the students answered Question seven correctly.

Table 17. Scores for Question 7

Question 7		Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	8 (40%)	11 (65%)	13 (76%)	6 (75%)	15 (38%)
correct answers per stream	32 (59.3%)			21(52.5%)	
sample	53 (56.4%)				

In Question 10, students were required to convert a rate into an equivalent rate (DoBE, 2011a, p. 77).

If I can walk $1\frac{1}{5}$ kilometres in twelve minutes, how long will it take me at that rate to walk five kilometres? Answer in minutes.

The results are summarised in Table 18. Fewer than half of the students (46.8%) answered correctly. Some students attempted to convert the rate to a rate per hour, by multiplying by 5. Few of these students however proceeded to answer the question correctly. Another common mistake, especially amongst the extended stream students, was to translate a mixed fraction incorrectly to a decimal fraction, that is, $1\frac{1}{5}$ was incorrectly converted to 1.5. Most of the students, who presented incorrect answers for this question, showed no steps, and it was therefore difficult to analyse thought processes without interviewing the students.

Table 18. Scores for Question 10

Question 10		Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	7 (35%)	13 (76.5%)	8 (47.1%)	3 (38.5%)	13 (40.6%)
correct answers per stream	28 (51.9%)			16 (40%)	
sample	44 (46.8%)				

Question 12 tested ratio and direct proportion (DoBE 2011a, p. 14). A girl's height was provided together with the length of the girl's shadow. Also, the length of a nearby pole's shadow was provided, and students had to calculate the height of the pole.

Very few of the students could present a correct answer (Table 19). Most students who presented incorrect answers, attempted to solve this problem by means of subtraction, i.e. they calculated the difference between the length of the girl's shadow and the girl's height, and then subtracted that amount from the length of the pole's shadow to obtain the pole's height, a process that yielded the incorrect answer of 6.5 m, offered by 17 (18.1%) of the students.

Table 19. Scores for Question 12

Question 12		Mainstream		Extended Stream	
	Electrical Engineerin g (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	2 (10%)	11 (64.7%)	3 (17.6%)	3 (37.5%)	4 (12.5%)
correct	16 (29.6%)			7 (17.5%)	

answers per stream		
sample	23 (24.5%)	

A summary of the scores for Cohort for B4 (Ratio and proportion) is given in Table 20.

Table 20. Contingency Table - Cohort and B4 Score

Mathematics Grade 12	Cohort						Total	
	Analytical Chemistry		Civil Engineering		Electrical Engineering			
0-19%	12	24.5%	2	8.0%	7	35.0%	21	22.3%
20-39%	14	28.6%	5	20.0%	5	25.0%	24	25.5%
40-60%	14	28.6%	8	32.0%	5	25.0%	27	28.7%
61-100	9	18.4%	10	40.0%	3	15.0%	22	23.4%
Total	49	100%	25	100%	20	100%	94	100%
Chi^2 (d.f. = 6, n = 94) = 8.58; p = 0.198								

PERCENTAGE AND PERCENTAGE INCREASE AND DECREASE (B5, 32.2% AVERAGE) (Q 11, 13 &14)

In the first question of this section (Q11), the total number of workers in a factory was given as the variable *P*, and the number of absentees was given as *N*. Students were expected to choose a formula, representing the percentage of absentees, from a list. This skill is prescribed in the Grade 7 syllabus as solving “problems in context involving percentages”, but may very well be of a higher degree of difficulty because of the variables involved (DoBE, 2011a, p. 18). The majority of the incorrect answers (18 of 31 incorrect answers) were choice *a*, which reflected the percentage of workers absent, not those present. There was a noticeable difference between the mainstream (59.3% of the answers correct) and the extended stream (52.5% correct) (Table 21).

Table 21. Scores for Question 11

Question 11		Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineerin g (8)	Analyti cal Chemis try (32)
correct answers	11 (55%)	12 (70.6%)	9 (52.9%)	3 (37.5%)	18 (45%)
correct answers per stream	32 (59.3%)			21 (52.5%)	
sample	53 (56.4%)				

In the second question of this section (Q13), students had to calculate the percentage increase in a price. Solving problems in contexts involving percentages, is a skill prescribed by the Grade 7 syllabus (DoBE, 2011a, p. 18). It is perturbing that so few students could answer this question correctly (Table 22), since lack of basic financial skills will most certainly hamper their "meaningful participation in society as citizens of a free country" (DoBE, 2011c, p. 18). These students will have to enter the free market and will encounter price increases on a daily basis as part of their work and personal environment. One of the common mistakes made was to calculate the difference between the original price and the raised price, (R15) and to assume this amount to be a percentage (incorrect answer 15%). In some instances, students divided the difference (R15) by the new price (R70) when calculating a percentage, instead of dividing by the original price. Another common incorrect answer was 80%. These students presumably divided the original price by the raised price (R60/R75) and then multiplied by 100 to convert to a percentage.

Table 22. Scores for Question 13

Question 13		Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	9 (45%)	7 (41%)	7 (41%)	5 (63%)	6 (15%)
correct answers per stream	23 (43%)			11(27.5%)	
sample	34 (36.2%)				

The final question (Q14) was:

The price of fuel has dropped by to R10.89. What was the price of fuel before the price decrease?

This question yielded by far the worst results (4.3% correct answers, Table 23), which is disturbing, since the topic of financial mathematics features in the Grade 8 syllabus as Finance and Growth, emphasising how 'to solve problems, including annual interest, hire purchase, inflation, population growth and other real-life problems' (DoBE, 2011a, p. 18). Furthermore, the topic is repeated in the Grade 10 Mathematics syllabus, which prescribes that learners should understand the implication of 'fluctuating foreign exchange rates, for example on petrol price, imports, exports, overseas travel' (DoBE, 2011b, p. 26). The topic is examined in the first of two compulsory mathematics examination papers in Grade 8 and the question should be worth 10 ± 3 of the 100 marks for Grade 10, and 15 ± 3 marks for Grade 11 and 12.

Table 23. Scores for Question 14

Question 14		*Mainstream*		*Extended Stream*	
	Electrical Engineering	*Civil Engineering*	*Analytical Chemistry*	*Civil Engineering*	*Analytical Chemistry*
correct answers	1 (5%)	1 (6%)	1 (6%)	0	1 (3%)
correct	3 (5.6%)			1 (2.5%)	

answers per stream		
sample	4 (4.3%)	

A summary of the scores for Cohort for B5 (Percentage and percentage increase and decrease) is given in Table 24.

Table 24. Contingency Table - Cohort and B5 Score

B5 Score	Cohort						Total	
	Analytical Chemistry		Civil Engineering		Electrical Engineering			
0 to 19	15	30.6%	6	24.0%	4	20.0%	25	26.6%
20 to 39	27	55.1%	11	44.0%	11	55.0%	49	52.1%
60 to 100	7	14.3%	8	32.0%	5	25.0%	20	21.3%
Total	49	100%	25	100%	20	100%	94	100%
Chi² (d.f. = 4, n = 94) = 3.73; p = 0.444								

The data in this study indicated that most first-year entry level science and engineering diploma students at this particular university in South Africa still struggle with fractions. The average score of 47.8% was disappointing. So was the highest score-only 86%. Several colleagues opined that they expected students to score at least 90% for this particular test. In a study done at the University of Johannesburg, 90% was expected as a pass rate for a test based on Grade 6 work (Fonseca & Petersen 2015). For a diagnostic test to assess the mathematical knowledge of students enrolling in its B.Ed. program, the University of New England expected a mastery level of 80%. The Khan online academy encourages 100% mastery for basic numeracy (Fonseca & Petersen 2015). In nursing education, 90% is usually regarded as a competent score for numerical drug calculations in research, although no justification for this figure is given and Jukes and Gilchrist (2006) are of the opinion that 100% mastery should be achieved in tests on the administration of medicines. The authors of the present research anticipated a mastery score of above 90% for numeracy tests for engineering and science students at entry level.

These results compare poorly to the results of an international study by Siegler et al. (2011) in which mathematics and science students answered consistently correct on questions on multiplications and division of fractions. However, results from the present study are better than those Akyuz (2015) reported from a study conducted by a university in Turkey on students who were in a two-year program at a technical vocational school. They were tested on numeracy and basic algebraic skills and their average score was only 6.32 out of a total of twenty points (31.6%).

There was a marked difference in the performance of the mainstream and the extended stream students (Table 3). Numeracy seems to be a stumbling block for those in the extended stream. Students should be assisted to overcome these problems. "Giving access to students from disadvantaged backgrounds without providing the environment in which they can succeed academically is commonly termed the *revolving door syndrome*" (Case, 2006, p. 25). Academic support programs should include quantitative skills. A study by Campbell (2009) in mathematics, and another by Kremmer et al. (2010) in the field of business, showed that remedial programmes at tertiary education institutions had a positive impact on students' success at university.

Data collected pointed to lack of conceptual understanding of operations on fractions. This is a common problem internationally (Siegler & Lortie-Forgues, 2015). Especially in the case where no calculators are allowed, procedural mastery does not necessarily imply conceptual understanding. Students may remember the procedure without ever having understood the theoretical underpinning for the procedure. This may also apply to teachers (Ma, 1999). Consistent with this interpretation, Ma (1999) found that most United States teachers in her study could not generate any explanation of what $1\frac{3}{4} \div \frac{1}{2}$ means, or resorted to explaining a different problem, i.e. $1\frac{3}{4} \div 2$. A study by Ball (1990) revealed similar results. Teachers in other local and international studies have demonstrated weak conceptual understanding of fraction arithmetic (Lin et al., 2013; Ma, 1999; Rizvi & Lawson, 2007). A study conducted at five South African universities reported that prospective teachers enter university programmes with reasonable procedural knowledge of mathematics but poor conceptual knowledge (Bowie, 2014).

Similar problems occur when students have to multiply by fractions with magnitude smaller than one. Teachers should emphasize that multiplication and division produce different outcomes depending on whether the numbers involved are greater or lesser than 1, and should discuss why this is true. 'Chinese textbooks include such instruction' (Siegler & Lortie-Forgues, 2015). To strengthen their case, these researchers referred to an example given by Sun and Wang (2005) in which Chinese students were asked to solve and discuss answers to the following three problems: 4.9 * 1.01; 4.9 * 1; 4.9 * 0.99.

Jukes and Gilchrist (2006) observe that a lack of retention may be to blame. Some of the students involved in this research may have understood the concepts at the time when these were explained to them, but may have forgotten at the time of this study. Johnson and Johnson (2002) claim that the educational institution and the student should share the responsibility for both attainment and retention of skills.

Reports of studies done by various researchers (Siegler et al., 2011; Booth & Newton, 2012; Torbeyns et al., 2014; Wu, 2001) emphasise the importance of magnitude representations of fractions on number lines when teaching fractions, as opposed to the part-of-a-whole approach. Some countries focus almost exclusively on the part-whole approach and neglect the number-line approach. South African teaching seems to fall into the latter category. Both methods should be incorporated into the pedagogy of teaching fractions in order to supplement and complement each other.

Conclusions and Suggestions

The current study revealed that entry-level students enrolled for engineering and science diplomas performed poorly in a test of numeracy skills.

The average score (47.8%) was regarded as a cause for concern, especially considering that the test was pitched at Grade 8 level. The average score was far below the researchers' expectations. Furthermore, students displayed a lack of conceptual understanding of the procedures. This study also revealed a marked difference between the performance of the mainstream and the extended stream students.

Mathematics lecturers at universities and mathematics teachers at secondary schools should take note of the results of this study. First year university lecturers at universities need to offer remedial action, especially for students in the extended stream. Furthermore, problems with conceptual understanding of fractions could possibly be ascribed to conceptual problems that teachers might have had with fractions. Hence, one approach to addressing students' difficulties with fractions would be to conduct in-service workshops to refresh practicing teachers' subject content knowledge and pedagogical content knowledge in order to enhance teachers' conceptual knowledge of fractions so as to promote effective teaching and learning activities.

Even people who are proficient in fraction procedures often possess weak conceptual understanding of multiplication and division of fractions less than 1. Literature highlights that specific teaching approaches may yield improved results, especially in this instance. Teachers should point out to students that multiplication and division produce different outcomes depending on whether the numbers involved are greater than or less than 1, and should discuss why this is so.

Lastly, the importance of magnitude representations of fractions on number lines, as opposed to the part-of-a-whole approach to teaching fractions, does need emphasis. Both methods should be incorporated into the pedagogy when teaching fractions.

Further research should examine whether a cause-effect relationship exists between reflective reasoning and mathematics scores in general, and if so, whether it is possible to improve students' cognitive reflection in mathematics. Also, various studies have found fraction knowledge to be an early and accurate predictor of later mathematics achievement. These predictions may extend to predictions of tertiary mathematics success. Possible links between fraction test scores and tertiary students' pass rates in mathematics should be explored. It may well transpire that scores on fraction tests could be used as a predictive measure for the eventual academic success in mathematics service courses at university. If a positive correlation does exist, further research should be conducted on whether remedial measures on fraction skills will positively influence students' ensuing academic achievement in higher education. Mathematics educators have asserted that this may indeed be the case, but more research is required to confirm that the conceptual mastery of lower-level fraction skills will positively impact students' achievement in mathematics service courses at tertiary level. Pre-service mathematics educators also need to note the results of this study so as to equip student teachers with both subject content knowledge and pedagogical content knowledge of fractions.

The current study did not make use of interviews, and interpretation of student errors were therefore limited. Interviews would elicit the processes students used to reach answers, and the reasoning underpinning these. The researcher would then be able to reach more detailed conclusions regarding the levels of conceptual understanding that cause university entry level students' incorrect answers.

The sample in this study was relatively small, but there is no reason to assume that the findings cannot be generalised to other diploma students in science and engineering, provided that the admission requirements for the diplomas are compatible.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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