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Introduction

The South Dakota State Standards for Mathematics specify what students should know and be able to do as learners of mathematics at the end of each grade level or course. The order of the standards at any grade level is not meant to imply a sequence of topics and should be considered flexible for the organization of coherent learning progressions. The standards are written in a vertical progression that respects what is known about how students learn and how students' mathematical knowledge, skill, and understanding develop over time. The South Dakota State Mathematics Standards set a path for all students to become mathematically proficient and literate by emphasizing and engaging students in problem solving, communicating, reasoning and proof, making connections, using representations, and using mathematics to make sense of the world around them.

The South Dakota State Standards for Mathematics set grade-specific standards and targets for learning, but do not dictate curriculum or teaching methods. The standards also do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The South Dakota State Standards for Mathematics should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs.

The South Dakota State Standards for Mathematics have two components: the Standards for Mathematical Practice and the Standards for Mathematical Content. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important processes and proficiencies with longstanding importance in mathematics education. The first of these are the NCTM (National Council of Teachers of Mathematics) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. It is important to take advantage of opportunities in the standards to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the

Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**
   Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**
   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not

generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**
   Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**
   Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**
   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$ in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics | Kindergarten

Grade K Overview

Counting and Cardinality

- Know number names and the count sequence
- Count to tell the number of objects
- Compare numbers

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11 – 19 to gain foundations for place value

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in each category.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

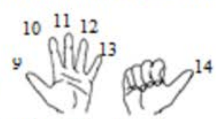
Important Definitions and Resources

- Fluency – skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- Know from Memory – quick, effortless recall of facts. (**Notice there are no Kindergarten standards that require students to "know from memory.")

K-2 Common Addition and Subtraction Situations – Addition and Subtraction Problem Types Chart **(see page 9)**

	Result Unknown	Change Unknown	Start Unknown
Add to (Join) (Combining)	**Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?** **2 + 3 = ?** ***(K)***	**Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?** **2 + ? = 5** ***(1st)***	**Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?** **? + 3 = 5** ***One-Step Problem (2nd)***
Take from (Separate) (Separating)	**Five apples were on the table. I ate two apples. How many apples are on the table now?** **5 – 2 = ?** ***(K)***	**Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?** **5 – ? = 3** ***(1st)***	**Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3** ***One-Step Problem (2nd)***
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[2]
Put Together/ Take Apart[3] (Part-Part Whole)	**Three red apples and two green apples are on the table. How many apples are on the table?** **3 + 2 = ?** ***(K)***	**Five apples are on the table. Three are red and the rest are green. How many apples are green?** **3 + ? = 5, 5 – 3 = ?** ***(K)***	**Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?** **5 = 0 + 5, 5 = 5 + 0** **5 = 1 + 4, 5 = 4 + 1** **5 = 2 + 3, 5 = 3 + 2** ***(1st)***
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	**("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?** ***(1st)***	**(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?** ***One-Step Problem (1st)***	**(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have?** **5 – 3 = ? ? + 3 = 5** **One-Step Problem** ***(2nd)***
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? **2 + ? = 5, 5 – 2 = ?** ***(1st)***	**(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?** **2 + 3 = ?, 3 + 2 = ?** ***One-Step Problem (2nd)***	**(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have?** ***One-Step Problem (1st)***

K: Problem types to be mastered by the end of the Kindergarten year.1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, First Grade students should have experiences with all 12 problem types.2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous year(s).

Levels	8 + 6 = 14	14 – 8 = 6
Level 1: **Count all** **(Direct Modeling)**	Count All a 1 2 3 4 5 6 7 8 b 1 2 3 4 5 6 ○○○○○ ○○○ ○○○○○ ○ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 c	Take Away a 1 2 3 4 5 6 7 8 9 10 11 12 13 14 ~~○○○○○ ○○○~~ ○○ ○○○○ 1 2 3 4 5 6 7 8 1 2 3 4 5 6 b c
Level 2: Counting Strategies • **Counting On** • **Counting Up to** • Counting Back • Counting Back to	Count On 8 ○○○○○ ○○○ ○○○○○ ○ 8 9 10 11 12 13 14	To solve 14 – 8 I count on 8 + ? = 14 I took away 8  8 to 14 is 6 so 14 – 8 = 6
Level 3: **Use Known Facts** **Use Derived Facts** **ADDITION** • Make a Ten • Doubles • Commutative Property **Use Derived Facts** **SUBTRACTION** • Think Addition • Build up thru 10 • Build down thru 10	Make a Ten (Recompose) ○○○○○ ○○○ ○○○○○ ○ 10 + 4 ○○○○○ ○○○ ○○○○○ ○ 10 + 4 Think Addition – I know that 8 + 6 = 14, so 14 – 8 = 6	14 – 8: I make a ten for 8 + ? = 14 ~~○○○○○ ○○○~~ ○○ ○○○○ 8 + 2 + 4 6 8 + 6 = 14 Build up thru 10 (14 – 6, I know that 8 + 2 = 10, 10 + 4 = 14. 2 + 4 = 6) Build down thru 10 (14 – 6, I know that 14 – 4 = 10. 10 – 2 = 8. 4 + 2 = 6

Note:
Many children attempt to count down for subtraction, but counting down is difficult and error-prone.
Children are much more successful with counting on; it makes subtraction as easy as addition.

Grade K Introduction

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Grade K Standards

Counting and Cardinality — K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from any given number within 100 (instead of having to begin at 1). Count backwards beginning from any given number within 20.
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (one-to-one correspondence)
   b. Understand that the last number name said tells the number of objects counted. (cardinality) The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?"
   a. When counting, answer questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or and as many as 10 things in a scattered configuration.
   b. Given a number(s) from 1–20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. Include groups with up to ten objects.
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking — K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problem.)
2. Solve addition and subtraction word problems.
   a. Solve addition and subtraction word problems (within 10), involving result unknown problems, put together/take apart total unknown, and put together/take apart addend unknown, e.g., using objects or drawings to represent the problem. (see appendix for K-2 Common Addition and Subtraction Situations)
   b. Add and subtract within 10, eg., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5. (See strategies chart on page 9.)

Number and Operation in Base Ten — K.NBT

Work with numbers 11 – 19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data — K.MD

Describe and compare measureable attributes.

1. Describe measurable attributes of a single object or objects, such as length, weight, or size.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the number of objects in each category and sort the categories by count. Limit category counts to be less than or equal to 10.

Work with time and money.

4. Identify a penny and understand that the value is one. Count pennies within 20.

Geometry — K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes.

Mathematics | Grade 1

Grade 1 Overview

Operations and Algebraic Thinking	Mathematical Practices
• Represent and solve problems involving addition and subtraction. • Understand and apply properties of operations and the relationship between addition and subtraction. • Add and subtract within 20. • Work with addition and subtraction equations.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Number and Operations Base Ten	
• Extend the counting sequence. • Understand place value. • Use place value understanding and properties of operations to add and subtract.	
Measurement and Data	
• Measure lengths indirectly and by iterating length units. • Tell and write time. • Represent and interpret data. • Identify and count pennies and dimes.	
Geometry	
• Reason with shapes and their attributes.	

Important Definitions and Resources

- Fluency – skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- Know from Memory – quick, effortless recall of facts.

K-2 Common Addition and Subtraction Situations – Addition and Subtraction Problem Types Chart (see page 9)

Grade 1 Introduction

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction. Fluency is defined as the skill of carrying out procedures flexibly, accurately, efficiently, and appropriately. There is no time element associated with fluency.

2. Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understanding of properties such as congruence and symmetry.

Grade 1 Standards

Operation and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between additions and subtraction.

3. Apply commutative, associative, and additive identity properties of operations as strategies to add. (Students need not use formal terms for these properties.) Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) 8 + 0 = 8 (Additive Identity property)
4. Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

Add and Subtract with in 20.

5. Understand counting on as addition and counting back as subtraction e.g. 5, (6,7,8) means 5 + 3 and 5, (4,3,2) means 5-3.
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
8. Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 +? = 11, 5 = ? – 3, 6 + 6 = ? .

Number and Operation in Base Ten 1.NBT

Extend the counting sequence.

1. In the range of 0 - 120
   a. Count on from any given number.
   b. Read and write numerals.
   c. Represent a number of objects with a written numeral.

Understand Place Value

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

   a. 10 can be thought of as a bundle of ten ones — called a “ten.”
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols **<, =, and >.**

Use place value understanding and properties of operation to add and subtract

4. Add and subtract within 100.
   a. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
   b. Understand that in adding two-digit numbers (sums within 100) add tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

Work with time and money.

3. Tell and write time in hours and half-hours using analog and digital clocks.
5. Identify nickels and understand that five pennies can be thought of as a nickel. Identify dimes and understand ten pennies can be thought of as a dime. Count the value of a set of coins comprised of pennies, nickels, and dimes.

Represent and interpret data

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry 1.G

Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose and Identify regular and irregular two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) and compose three-dimensional shapes (cubes, spheres, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to master formal names such as "right rectangular prism.")
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Mathematics | Grade 2

Grade 2 Overview

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Work with equal groups of objects to gain foundations for multiplication.
Number and Operations in Base Ten
Understand place value.
Use place value understanding and properties of operations to add and subtract.
Measurement and Data
Measure and estimate lengths in standard units.
Relate addition and subtraction to length.
Work with time and money.
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Reason with shapes and their attributes.</td>
<td>Make sense of problems and persevere in solving them.
Reason abstractly and quantitatively.
Construct viable arguments and critique the reasoning of others.
Model with mathematics.
Use appropriate tools strategically.
Attend to precision.
Look for and make use of structure.
Look for and express regularity in repeated reasoning.</td></tr>
<tr><th colspan="2">Important Definitions and Resources</th></tr>
<tr><td colspan="2"><u>Fluency</u>-skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
<u>Know from memory</u>-quick, effortless, recall of facts.
K-2 Common Addition and Subtraction Situations – Addition and Subtraction Problem Types Chart (see page 9)</td></tr>
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Grade 2 Introduction

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iteration they need to cover a given length.
4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Grade 2 Standards

Operation and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

2. Add and subtract within 20.
   a. Fluently add and subtract within 20 using mental strategies. (See standard 1.OA.6 for a list of mental strategies.)
   b. By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operation in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s, starting from any number in its skip counting sequence.
3. Read and write numbers to 1000 using base-ten numerals (standard form), number names (word form), and expanded form.
4. Compare, two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
5.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by words, drawings or objects.)

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
8. Identify and count coins and bills and apply that understanding to solve word problems.
    a. Recognize and know the value of coins up to one dollar.
    b. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph.

Geometry 2.G

Reason with shapes and their attributes.

1. Recognize, identify, and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces; to include triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Mathematics |Grade 3

Grade 3 Overview

Operations and Algebraic Thinking	Mathematical Practices
• Represent and solve problems involving multiplication and division. • Understand properties of multiplication and the relationship between multiplication and division. • Multiply and divide within 100. • Solve problems involving the four operations, and identify and explain patterns in arithmetic.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Number and Operations Base Ten	
• Use place value understanding and properties of operations to perform multi-digit arithmetic.	
Number and Operations-- Fractions	
• Develop understanding of fractions as numbers.	
Measurement and Data	
• Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. • Represent and interpret data. • Geometric measurement: understand concepts of area and relate area to multiplication and to addition. • Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	
Geometry	
• Reason with shapes and their attributes.	

Important Definitions and Resources

- Fluency – skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- Know from Memory – quick, effortless recall of facts. (Note: The know from memory standard has been moved to 4th grade.)
- Strategy – Purposeful manipulations that may be chosen for specific problems, my not have a fixed order, and may be aimed at converting one problem into another.
- Algorithm – A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Grade 3 Introduction

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1. Students develop and understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, ½ of the paint in a small bucket could be less paint than ⅓ of the paint in a larger bucket, but ⅓ of a ribbon is longer than ⅕ of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

4. Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3 Standards

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5x7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5x7.
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)
6. Understand division as an unknown-factor problem. For example, find 32÷8 by finding the number that makes 32 when multiplied by 8.

Multiply and divide within 100.

7. Multiply and divide within 100.
   a. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.
   b. Demonstrate fluency (skill in carrying out procedures flexibly, appropriately, efficiently, and accurately) for all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order [Order of Operations]).
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Number and Operation in Base Ten 3.NBT

Use place value understanding and properties of operation to perform multi-digit arithmetic (A range of algorithms may be used).

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[5] 3.NF

Develop understanding of Fractions as numbers.

1. Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts (example: 1 part out of 4 equal parts is the same as 1/4); understand a fraction a/b as the quantity formed by a parts of size 1/b. (example:3/4 is the same as 3 one-fourths (1/4, 1/4, 1/4)
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
   b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. *Note - Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.*
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols < , =, or >.

Measurement and Data 3.MD

Solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes, using an analog and digital clock. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems [problems involving notions of “times as much”; see Table, page 34])

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
    a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
    b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of *n* square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
    a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
    b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
    c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
    d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
9. Determine the value of a collection of money using dollar sign and decimal point appropriately. Understand that the digits to the right of the decimal represent parts of a whole dollar.

Geometry 3.G

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of a shape.

Mathematics | Grade 4

Grade 4 Overview

Operations and Algebraic Thinking	Mathematical Practices
• Use the four operations with whole numbers to solve problems. • Gain familiarity with factors and multiples. • Generate and analyze patterns.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Number and Operations in Base Ten	
• Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic.	
Number and Operations - Fractions	
• Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions, and compare decimal fractions.	
Measurement and Data	
• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. • Represent and interpret data. • Geometric measurement: understand concepts of angle and measure angles.	
Geometry	
• Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	

Important Definitions and Resources

- Fluency – skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- Know from Memory – quick, effortless recall of facts.
- Strategy: Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.
- Algorithm: A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Grade 4 Introduction

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. Students describe, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Grade 4 Standards

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. Use and interpret multiplicative equations.
    a. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal or written statements of multiplicative comparisons as multiplication equations. Example: Tom has 7 toy cars; Joe has 5 times as many. How many toy cars does Joe have? Answer: 35, because 7 x 5 = 35 or 5 x 7 = 35.
    b. Know from memory (quick effortless recall of facts) all products of two one-digit numbers.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem), and distinguish multiplicative comparison from additive comparison.
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Using whole number in the range 1–100.
    a. Find all factor pairs for a given whole number.
    b. Recognize that a whole number is a multiple of each of its factors.
    c. Determine whether a given whole number is a multiple of each of a given one-digit number.
    d. Determine whether a given whole number is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number is 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Number and Operation in Base Ten 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that the 7 in 700 is 10 times greater than the 7 in 70 because 700 ÷ 70 =10 and 70 x 10=700.
2. Read and write multi-digit whole numbers.
    a. Read and write multi-digit whole numbers using base-ten numerals (standard form), number names (word form), and expanded form.
    b. Compare two multi-digit numbers based on values of the digits in each place, using <, >, and = symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using an algorithm including, but not limited to, the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions 4.NF

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction *a/b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols <, >, =, and justify the conclusions.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with a > 1 as a sum of fractions 1/b. For example, 4/5 = 1/5 + 1/5 + 1/5 + 1/5
   a. Add and subtract of fractions e.g., joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with like denominators in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4).
   b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5. (In general, n x (a/b) = (n x a)/b = (n x a) x 1/b.)
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.
6. Read and write decimal notation for fractions with denominators 10 or 100. Locate these decimals on a number

line.

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, <, or =, and justify the conclusions.

Measurement and Data 4.MD

Solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. For example, know that 1ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),...
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.
   b. An angle that turns through in one-degree angles is said to have an angle measure of n degrees.
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize, and identify categories of right, acute, and obtuse triangles.
3. Recognize and draw lines of symmetry for two-dimensional figures.

Mathematics | Grade 5

Grade 5 Overview

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place-value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations - Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Important Definitions and Resources

- Fluency-skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- Know from memory-quick, effortless, recall of facts.
- Strategy: Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.
- Algorithm: A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.
- Trapezoid: A quadrilateral with exactly ONE pair of parallel sides.
- Attribute: A quality or feature regarded as a characteristic or inherent part of something.

See table on page 33 for common addition and subtraction problem types and table on page 34 for common multiplication and division problem types.

Common addition and subtraction. [1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? **2 + 3 = ?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? **2 + ? = 5**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **? + 3 =5**
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?**5- 2 = ?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?**5 – ? = 3**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?**? -2 = 3**
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN [2]**
PUT TOGETHER / TAKE APART [3]	Three red apples and two green apples are on the table. How many apples are on the table? **3 + 2 = ?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? **3 + ? = 5, 5-3 = ?**	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? **5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2**
COMPARE	**DIFFERENCE UKNOWN**	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? **2 + ? = 5, 5 – 2 = ?**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? **2 + 3 = ?, 3 + 2 = ?**	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **5 – 3 = ?, ? + 3 = 5**

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Common multiplication and division situations. [1]

	UNKNOWN PRODUCT	**GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)**	**NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)**
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Grade 5 Introduction

In Grade 5, instructional time should focus on three critical areas: (1) fractions, (2) decimals, and (3) volume. The following table indicates the instructional focus for each of the following areas:

Developing Fluency	Developing Understanding
Add and subtract fractions	Division of whole numbers - 2 digit divisors
Add, subtract, multiply whole numbers	Multiply and divide fractions
	The value of the digit in relation to the decimal point (5.NBT.1 and 5.NBT.2)
	Adding, subtracting, multiplying, and dividing decimals
	Volume

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, and multiplication. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Grade 5 Standards

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

1. Use and explain parentheses, in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers to represent real world problems, and interpret numerical expressions without evaluating them.(For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.)

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. Identify the relationship between the two patterns. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Number and Operation in Base Ten 5.NBT

Understand the place value system

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain and apply patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain and apply patterns in the placement of the decimal point with respect to the values of the digits in the product or the quotient, when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and, < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole number and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using an algorithm, including but not limited to the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Explain the calculation by using equations, rectangular arrays, illustrations, area models, or other representations based on place value.
7. Use the four operations with decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; justify the reasoning used with a written explanation.
   a. Add and subtract decimals
   b. Multiply and divide decimals.

Number and Operations—Fractions[5] 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference with a like denominator. It is not necessary at this grade level to simplify the sum or difference. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, $a/b + c/d = (ad + bc)/bd$.)
2. Solve word problems involving addition and subtraction of fractions.
   a. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.
   b. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, $(a/b) \times (c/d) = ac/bd$.)
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.

b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. Convert customary and metric measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m). Use these conversions in solving multi-step, real world problems involving distances, intervals of time, liquid volumes, masses of objects, and money (including problems involving simple fractions or decimals). For example, 3.6 liters and 4.1 liters can be combined as 7.7 liters or 7700 milliliters.

Represent and interpret data.

2. Make a line plot to display a data set.
   a. Use operations on fractions of a unit (1/2, 1/4, 1/8) for this grade to solve problems involving information presented in line plots.
   b. Use information from a line plot representing an unequal situation and redistribute whole or fractional parts to create an equal distribution. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
   a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
   b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
   a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base.
   b. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
   c. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ (where B is the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.
   d. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and y-coordinate).
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.
4. Classify two-dimensional figures in a hierarchy based on properties. *For example, all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides.*

Mathematics | Grade 6

Grade 6 Overview

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Multiply and divide multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Important Definitions and Resources

- Fluency-skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- Know from memory-quick, effortless, recall of facts.
- Strategy: Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.
- Algorithm: A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Grade 6 Introduction

In grade 6, instructional time should focus on five critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; (4) developing understanding of statistical thinking; and (5) reasoning about relationships among shapes.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios to fractions. Students solve a wide variety of problems involving ratios and rates.

2. Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

5. Students develop reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Grade 6 Standards

Rations and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For examples, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with b not equal to 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?
2. Fluently divide multi-digit numbers using an algorithm including but not limited to the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using an algorithm including but not limited to the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Apply and extend previous understands of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world

contexts, explaining the meaning of 0 in each situation.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
7. Understand ordering and absolute value of rational numbers.
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.
   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
   d. Distinguish comparisons of absolute value from statements about order.
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations 6.EE

Apply and extend previous understandings of numbers to the system of rational numbers.

1. Write and evaluate numerical expressions involving whole-number exponents (e.g. parentheses, brackets, or braces).
2. Write, read, and evaluate expressions in which letters stand for numbers.
   a. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract *y* from 5" as 5-*y*.
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 +7) as both a single entity and a sum of two terms.
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.
   d. Perform arithmetic operations following the order of operations with and without parentheses, including those involving whole-number exponents.
3. Apply the properties of operations to generate equivalent expressions with an emphasis on the distributive property.
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality is a process in which you determine values from a set that make an equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form $x > c$, $x \geq c$, $x < c$ or $x \leq c$ which represents a condition or constraint in a real-world or mathematical problem. Recognize that inequalities have infinitely many solutions; represent solutions of inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Geometry 6.G

Solve real-world and mathematical problems involved area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ where B is the area of the base to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability 6.SP

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center spread, and overall shape.
3. Recognize that a measure of center (mean and/or median) for a numerical data set summarizes all of its values with a single number, while a measure of variation (such as mean absolute deviation and/or range) summarizes data points' distances from the mean or each other.
4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
   a. Reporting the number of observations.
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Mathematics | Grade 7

Grade 7 Overview

<table>
<tr><th>Ratios and Proportional Relationships</th><th>Mathematical Practices</th></tr>
<tr><td>Analyze proportional relationships and use them to solve real-world and mathematical problems.</td><td rowspan="9">Make sense of problems and persevere in solving them.
Reason abstractly and quantitatively.
Construct viable arguments and critique the reasoning of others.
Model with mathematics.
Use appropriate tools strategically.
Attend to precision.
Look for and make use of structure.
Look for and express regularity in repeated reasoning.</td></tr>
<tr><th>The Number System</th></tr>
<tr><td>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</td></tr>
<tr><th>Expressions and Equations</th></tr>
<tr><td>Use properties of operations to generate equivalent expressions.
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</td></tr>
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<tr><td>Draw, construct, and describe geometrical figures and describe the relationships between them.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</td></tr>
<tr><th>Statistics and Probability</th></tr>
<tr><td>Use random sampling to draw inferences about a population.
Draw informal comparative inferences about two populations.
Investigate the chance processes.
Develop, use, and evaluate probability models.</td></tr>
<tr><th colspan="2">Important Definitions and Resources</th></tr>
<tr><td colspan="2"><u>Fluency</u>-skill in carrying out procedures flexibly, accurately, efficiently and appropriately. The four tenets of fluency are: (1) flexibility, (2) appropriate strategy use, (3) efficiency, and (4) accuracy.</td></tr>
</table>

Grade 7 Introduction

In grade 7, instructional time should focus on ***three critical content areas***: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers; (3) working with expressions and linear equations; ***AND two supporting content areas***: (4) solving problems involving scale drawings and informal geometric constructions, and working with two and three-dimensional shapes to solve problems involving area, surface area, and volume; and (5) drawing inferences about populations based on samples and investigate the chance processes and develop, use, and evaluate probability models.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single-and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and relationships between addition and subtraction, and multiplication and division.

3. By applying these properties of operations as strategies, students explore working with expressions, equations, and inequalities. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve multi-step real-world problems. They use variables to represent quantities and construct simple equations and inequalities to solve problems by reasoning about the quantities.

4. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

5. Students continue their work from 6th grade in order to build a strong foundation for statistics and probability needed for high school. Students understand that statistics can be used to gain information about a population through sampling. They work with drawing inferences about a population based on a sample and use measures of center and of variability to draw informal comparative inferences about two populations. Students investigate the chance processes and develop, use, and evaluate probability models. Student summarize numerical data sets with respect to their context using quantitative measures and describe an overall patterns or deviation from the overall pattern.

Grade 7 Standards

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.
2. Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship. For example, by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items. Purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.
   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. For example, simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. For example, if you get paid $5 for babysitting but you owe your friend $5, you have $0.
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real world contexts.

  c. Apply properties of operations as strategies to multiply and divide rational numbers.
  d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
3. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Expressions and Equations 7.EE

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple grouping symbols (parentheses, brackets, and/or braces).
2. Understand the reason for rewriting an expression in different forms in contextual problems is to provide multiple ways of interpreting the problem, and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that increase by 5% is the same as "multiply by 1.05".
3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.
   a. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate. For example, if a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50.
   b. Assess the reasonableness of answers using mental computation and estimation strategies. For example, if you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
   b. Solve word problems leading to inequalities of the form $px + q > r$, $px + q \geq r$, $px + q < r$, and $px + q \leq r$ where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

Geometry 7.G

Draw, construct and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Draw (freehand, with ruler and protractor/angle ruler, and/or with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, using quantitative measures of center (focusing on mean and median) and variability (interquartile range, mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Investigate chance processes and develop, use and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
    a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
    b. Develop a probability model (which may not be uniform) by observing frequencies in data generated

from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes from the spinning penny appear to be equally likely based on the observed frequencies?

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
    a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
    b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
    c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Mathematics | Grade 8

Grade 8 Overview

The Number Systesm

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Important Definitions and Resources

- Function notation is not required in grade 8.

Grade 8 Introduction

In grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

The Number System — 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats or terminates. Convert a decimal expansion which repeats or terminates into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers. Locate irrational numbers approximately on a number line diagram, and estimate the value of expressions such as (π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions and Equations — 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
    b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.
8. Analyze and solve pairs of simultaneous linear equations.
    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For an inspection example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
c. Solve real-world and mathematical problems involving leading to two linear equations in one and/or two variables.

Functions 8.F

Define, evaluate and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8).
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations.
   a. Lines are mapped to lines, and line segments to line segments of the same length.
   b. Angles are mapped to angles of the same measure.
   c. Parallel lines are mapped to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and

mathematical problems in two and three dimensions.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line (i.e. line of fit), and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and the y-intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
4. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Mathematics Standards for High School

The high school standards are listed by course in conceptual categories:

- Number and Quality
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Some standards are shared between Algebra I and Algebra II, where one part of the standard is taught in Algebra I and then the concepts are extended and/or applied in Algebra II. When this occurs, the standard has been split to clarify which parts of the standards are to be learned in each course. Shared standards are indicated by (i) for Algebra I and (ii) for Algebra II.

Mathematics | High School—Number and Quantity

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Mathematics | High School—Algebra

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1 + b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

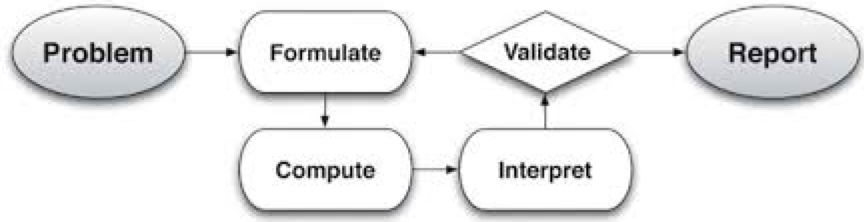
Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).*

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions which must include dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Mathematics | High School—Statistics and Probability*

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as interquartile range, mean absolute deviation, or standard deviation). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to completely describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Mathematics | Algebra I

Algebra I Overview

The Real Number System	Mathematical Practices
• Extend the properties of exponents to rational exponents • Use properties of rational and irrational numbers.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. Look for and express regularity in repeated reasoning.
Quantities	
• Reason quantitatively and use units to solve problems	
Seeing Structure in Expressions	
• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems	
Arithmetic with Polynomials and Rational Expressions	
• Perform arithmetic operations on polynomials	
Creating Equations	
• Create equations that describe numbers or relationships	
Reasoning with Equations and Inequalities	
• Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically	
Interpreting Functions	
• Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations	
Building Functions	
• Build a function that models a relationship between two quantities • Build new functions from existing functions	
Linear, Quadratic and Exponential Models	
• Construct and compare linear, quadratic, and exponential models and solve problems • Interpret expressions for functions in terms of the situation they model	
Interpreting Categorical and Quantitative Data	
• Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Interpret linear models	

Algebra I Introduction

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. These standards are the baseline expectations of students completing this course. Individual school districts or teachers are welcome to expand on these standards as they see fit to meet the needs of their students. Students will deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

In previous courses, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Statistical understanding builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Algebra I Standards

The Real Number System — N.RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of rational exponents follows from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; the sum of a rational and an irrational number is irrational; and the product of a nonzero rational and an irrational number is irrational.

Quantities * — N.Q

Reason quantitatively and use units to solve problems.

1. Use unit analysis to understand and guide the process of solving multi-step problems; choose and interpret units consistently in formulas; and choose and interpret the scale and origin in graphs and data displays. *
2. Define appropriate quantities for the purpose of descriptive modeling. *
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *

Seeing Structure in Expression — A.SSE

Interpret the structure of expressions.

1. (i) Interpret expressions that represent a quantity in terms of its context. *
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity in context.
2. (i) Recognize and use the structure of an expression to identify ways to rewrite it.

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to write equivalent expressions for exponential functions.

Arithmetic with Polynomials and Rational Expressions — A.APR

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Creating Equations * — A.CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable arising from situations in which linear, quadratic, and exponential functions are appropriate and use them to solve problems. *
2. (i)Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *
3. (i) Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *
4. (i) Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *

Reasoning With Equations and Inequalities A.REI

Understand solving equations as a process of reasoning.

1. Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. (i) Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions.
    b. Derive the quadratic formula from this form completing the square.
    c. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Solve systems of equations.

5. Understand the principles of the elimination method.
6. Solve systems of linear equations exactly and approximately by graphing, focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. (i) Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, including but not limited to using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic and exponential.
12. Graph a linear inequality (strict or inclusive) in two variables; graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions F.IF

Understand the concept of a function and use functions notation.

1. Understand that a function maps each element of the domain to exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph off is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Interpret functions that arise in applications in terms of the context.

4. (i) For functions, including linear, quadratic, and exponential, that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing or decreasing, including using interval notation; maximums and minimums; symmetries.* *
5. (i) Relate the domain of a function to its graph and find an appropriate domain in the context of the problem. *
6. Calculate and interpret the average rate of change of a function, both symbolically and from a table over a specified interval. Estimate the rate of change from a graph. *

Analyze functions using different representations.

7. (i) Graph parent functions and their transformations expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *
   a. Graph linear, exponential, and quadratic functions and show intercepts, maxima, and minima.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of graphing, factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Interpret expressions for exponential growth and decay.
9. (i) Compare properties of two functions (linear, quadratic and exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions F.BF

Build a function that models a relationship between two quantities.

1. Write a function (linear, quadratic, and exponential) that describes a relationship between two quantities. *
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context. *
   b. Determine an explicit expression from a graph. *
   c. Combine standard function types using arithmetic operations. *
2. Write arithmetic and geometric sequences both recursively and with an explicit formula and use them to model situations. *

Build new functions from existing functions.

3. (i) Identify the effect on the graph of $f(x)$ (linear, exponential, quadratic) replaced with $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology.

Linear, Quadratic and Exponential Models F.LE

Construct and compare linear and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to

another.
    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *
3. Recognize, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. *

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context. *

Interpreting Categorical and Quantitative Data S.ID

Summarize, represent and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape and context of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    a. Determine the function (linear, quadratic, or exponential model) that best fits a set of data and use that function fitted to data to solve problems within context.
    b. Informally and using technology assess the fit of a function by plotting and analyzing residuals.
    c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Mathematics | Geometry

Geometry Overview

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a circle
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension

- Explain volume and surface area formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry

- Apply geometric concepts in modeling situations

Condition Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Important Definitions

- Trapezoid - Quadrilateral with exactly ONE pair of parallel lines

Geometry Introduction

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, high school mathematics is devoted to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate that through a point not on a given line there is exactly one parallel line.

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that maps one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. Special attention is also paid to traditional geometric constructions using a compass and straightedge in order to strengthen conceptual understandings.

Geometry Standards

Congruence G-CO

Experiment with transformations in the plane.

1. State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane.
2. Represent transformations in the plane. (e.g., using transparencies and/or geometry software);
    a. Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
    b. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that map the figure onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, (e.g., using graph paper, tracing paper, or geometry software). Specify a sequence of transformations that will map a given figure onto another.

Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures.
    a. Predict the effect of a given rigid motion on a given figure.
    b. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

9. Prove theorems about lines and angles. Theorems must include but not limited to: vertical angles are congruent; when a transversal intersects parallel lines, alternate interior angles are congruent and same side interior angles are supplementary (using corresponding angles postulate); points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints.
10. Prove congruence theorems about triangles. Theorems must include but not limited to: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the mid segment of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
11. Prove theorems about parallelograms. Theorems must include but not limited to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions.

12. Perform geometric constructions with a compass and straightedge. including copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines/segments, constructing a line parallel to a given line through a point not on the line.
13. Construct an equilateral triangle, a square, and a regular hexagon.

Similarity, Right Triangles and Trigonometry — G-SRT

Understand similarity in terms of similarity transformations.

1. Verify experimentally and apply the properties of dilations as determined by a center and a scale factor.
2. Determine whether figures are similar, using the definition of similarity and using similarity transformations.
3. Use the properties of similarity transformations to establish similarity theorems. Theorems must include AA, SAS, and SSS.

Prove theorems involving similarity.

4. Prove theorems about triangles involving similarity. Theorems must include but not limited to: a line parallel to one side of a triangle divides the other two proportionally, and its converse; the Pythagorean Theorem proved using triangle similarity.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles.

6. Define, using similarity, that side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios (sine, cosine, and tangent) for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. *

Circles — G-C

Understand and apply theorems about circles.

1. Prove that all circles are similar.
2. Identify and describe relationships among central angles, inscribed angles, circumscribed angles, radii, and chords.
3. Construct, using a compass and straight edge, the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

5. Derive using similarity the length of the arc intercepted by an angle is proportional to the radius.
    a. Define the radian measure of the angle as the constant of proportionality;
    b. Derive and apply the formula for the area of a sector.

Expressing Geometric Properties with Equations — G-GPE

Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Use coordinates to prove geometric relationships algebraically. For example, determine whether a figure defined by four given points in the coordinate plane is a rectangle; determine whether the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).
3. Define and use the slope criteria for parallel and perpendicular lines. (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Use coordinates to prove simple geometric theorems algebraically.

4. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. e.g. Determine the point(s) that divide the segment with endpoints of (-4, 7) and (6, 3) into the ratio 2:3
5. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. *

Geometric Measurement and Dimension G-GMD

Explain volume and surface area formulas and use them to solve problems.

1. Give an informal argument for the formulas for the volume of a cylinder, pyramid, sphere, and cone. Use dissection arguments, and informal limit arguments.
2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures
3. Know and apply volume and surface area formulas for cylinders, pyramids, cones, and spheres for composite figures to solve problems. *

Visualize relationships between two-dimensional and three- dimensional objects.

4. Identify two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry G-MG

Applying geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). *
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). *
3. Apply geometric concepts to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). *

Statistics and Probability- Conditions Probability and Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space or as unions, intersections, or complements of other events
2. Determine whether two events *A* and *B* are independent.
3. Determine conditional probabilities and interpret independence by analyzing conditional probability
4. Construct and interpret two-way frequency tables of data. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
6. Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*, and interpret the result.

7. Apply the Addition Rule, P(A or B), and interpret the result.
8. Apply the general Multiplication Rule, P(A and B), and interpret the result.

Mathematics | Algebra II

Algebra II Overview

Domains and Clusters	Mathematical Practices
Seeing Structure in Expressions • Interpret the structure of expressions • Write expressions in equivalent forms to solve problems **Arithmetic with Polynomials and Rational Expressions** • Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions **Creating Equations** • Create equations that describe numbers or relationships **Reasoning with Equations and Inequalities** • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically **Interpreting Functions** • Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations **Building functions** • Build a function that models a relationship between two quantities • Build new functions from existing functions **Linear, Quadratic and Exponential Models** • Construct and compare linear, quadratic, and exponential models and solve problems • Interpret expressions for functions in terms of the situation they model **Trigonometric Functions** • Extend the domain of trigonometric functions using the unit circle • Model periodic phenomena with trigonometric functions • Prove and apply trigonometric identities **The Complex Number System** • Perform arithmetic operations with complex numbers • Represent complex numbers and their operations on the complex plane	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. Look for and express regularity in repeated reasoning

• Use complex numbers in polynomial identities and equations	
Making Inferences and Justifying Conclusions	
• Understand and evaluate random processes underlying statistical experiments • Make inferences and justify conclusions from sample surveys, experiments and observational studies	
Interpreting Categorical and Quantitative Data	
• Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Interpret linear models	

Algebra II Introduction

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. These standards are the baseline expectations of students completing this course. Since repeated exposure and practice is needed in order to master concepts, some standards introduced in Algebra I will be expanded upon in Algebra II. Individual school districts or teachers are welcome to expand on these standards as they see fit to meet the needs of their students. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Develop the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Algebra II Standards

Seeing Structure in Expressions — A.SSE

Interpret the structure of expressions.

1. (ii) Interpret expressions that represent a quantity in terms of its context. *
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity in context.
2. (ii) Recognize and use the structure of an expression to identify ways to rewrite it.

Arithmetic with Polynomials and Rational Expressions — A.APR

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem.
3. Identify zeros of polynomials by factoring.
   a. When suitable factorizations are available, use the zeros to construct a rough graph of the related function.
   b. When given a graph, use the zeros to construct a possible factorization of a polynomial.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; using inspection, synthetic division, long division, box method or, for the more complicated examples, a computer algebra system.

Creating Equations * — A.CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *
2. (ii) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *
3. (ii) Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *
4. (ii) Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *

Reasoning with Equations and Inequalities — A.REI

Understand solving equations as a process of reason and explain the reasoning.

2. Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. *Rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2. Radical functions are limited to square roots or cube roots of at most quadratic polynomials.*

Solve equations and inequalities in one variable.

4. (ii) Select, justify and apply appropriate methods to solve quadratic equations in one variable. Recognize complex solutions and write them as a +/- bi for real numbers a and b.

Represent and solve equations and inequalities graphically.

11. (ii) Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, including but not limited to using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. *

Interpreting Functions F.IF

Interpret functions that arise in applications in terms of the context.

4. (ii) For functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries (including even, odd, or neither); end behavior; and periodicity.**
5. (ii) Relate the domain of a function to its graph and find an appropriate domain in the context of the problem. *

Analyze functions using different representations.

7. (ii) Graph parent functions and their transformations expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.
   e. Graph logarithmic functions, showing intercepts and end behavior.
   f. Graph trigonometric functions (sine and cosine), showing period, midline, and amplitude.
9. (ii) Compare properties of <u>two functions</u> each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions F.BF

Build new functions from existing functions.

1. Write a function that describes a relationship between two quantities. *
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context. *
   b. Determine an explicit expression from a graph. *
   c. Combine standard function types using arithmetic operations. *
   d. Compose functions. *
3. (ii) Identify the effect on the graph of $f(x)$ replaced with $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology.
4. Find inverse functions.
   a. Solve an equation for the independent variable of a function f that has an inverse function and write an expression for the inverse.
   b. Verify by composition that one function is the inverse of another.
   c. Read values of an inverse function from a graph or a table, given that the function has an inverse.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic and Exponential Models — F.LE

Construct and compare linear and exponential models and solve

4. For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. *

Trigonometric Functions — F.TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions (sine and cosine) to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline. *

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(A) + \cos^2(A) = 1$ and use it to calculate trigonometric ratios.

The Complex Number System — N.CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ where *a* and *b* are real numbers.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomials identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

Interpreting Categorical and Quantitative Data — S.ID

Summarize, represent and interpret data on a single count or measurement variable.

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Making Inferences and Justifying Conclusions — S.IC

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

2. Determine whether a specified model is consistent with results from a given data-generating process.

Make interferences and justify conclusions from sample surveys, experiments and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatment groups; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Mathematics |4th Year

4th Year Overview

The Complex Number System	Mathematical Practices
• Perform arithmetic operations with complex numbers • Represent complex numbers and their operations on the complex plane • Use complex numbers in polynomial identities and equations	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Vector and Matric Quantities	
• Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications.	
Seeing Structure in Expressions	
• Write expressions in equivalent forms to solve problems	
Arithmetic with Polynomials	
• Use polynomial identities to solve problems • Rewrite rational functions	
Reasoning with Equations and Inequalities	
• Solve systems of equations • Solve equations and inequalities	
Interpreting Functions	
• Analyze functions using different representations	
Building Functions	
• Build a function that models a relationship between two quantities • Build new functions from existing functions	
Trigonometric Functions	
• Extend the domain of trigonometric functions using the unit circle • Model periodic phenomena with trigonometric functions • Prove and apply trigonometric identities	
Similarity, Right Triangle and Trigonometry	
• Apply trigonometry to general triangles	
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4th Year Introduction

A fourth year course can look different at each school. The standards listed are the (+) standards that would be appropriate for a variety of courses. These standards could be used as a basis for a precalculus course or could be selectively used to create a different 4th year course.

A precalculus course combines concepts of trigonometry, geometry, and algebra that are needed to prepare students for the study of calculus. This course is intended to strengthen students' conceptual understanding of problems and mathematical reasoning in solving problems. Facility with these topics is especially important for students who intend to study calculus, physics, other sciences, and engineering in college. The main topics in the precalculus course are complex numbers, rational functions, trigonometric functions and their inverses, inverse functions, vectors, matrices, parametric and polar equations, and conic sections. Students will continue their work with functions, using composition, inverses, exponents, trigonometry, and logarithms to build, model, and interpret functions along with careful examination of the domain and restrictions that apply.

Because the standards that comprise this course are (+) standards, students who enroll in precalculus should have met the college- and career-ready standards of the previous courses in an Integrated Pathway or Traditional Pathway. It is recommended that students complete precalculus before taking an Advanced Placement (AP) calculus course.

Since not all students will need a precalculus course, school districts could develop a fourth year course utilizing the (+) standards that best meet the scholastic needs of their students.

The Mathematical Practice standards apply throughout the Fourth Year Course and together with the content standards prescribe that students experience math as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

4th Year Standards

The Complex Number System — N.CN

Perform arithmetic operations with complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities — N.VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes
2. (+) Write a vector in component form.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. (+) Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
   a. (+) Represent scalar multiplication graphically by scaling vectors and/or reversing their direction; perform scalar multiplication component-wise.
   b. (+) Compute the magnitude of a scalar multiple cv. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data.
7. (+) Multiply matrices by scalars to produce new matrices.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. Discover that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Arithmetic with Polynomials and Rational Expressions — A.APR

Use polynomial identities to solve problems.

5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions.

7. (+) Discover that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Reasoning with Equations and Inequalities — A.REI

Solve systems of equations.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Use matrices to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
10. (+) Solve linear, quadratic, polynomial, and rational inequalities in two variables algebraically and graphically.

Seeing Structure in Expressions — A.SSE

Write expressions in equivalent forms to solve problems.

4. (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *
5. (+) Use summation notation to describe the sums in a series.

Interpreting Functions — F.IF

Analyze functions using different representations.

7. (+) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *
   d. (+) Graph rational functions, identify zeros and vertical, horizontal, and slant asymptotes, and determine end behavior.
   e. (+) Graph exponential and logarithmic functions, showing relationships, intercepts and end behavior.
   f. (+) Graph all trigonometric functions, showing key features and applying transformations.

Building Functions F.BF

Build a function that models a relationship between two quantities.

1. (+) Write a function that describes a relationship between two quantities. *
    c. (+) Compose functions in context.
4. (+) Find inverse functions.
    b. (+)Verify by composition that one function is the inverse of another.
    c. (+)Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+)Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
6. (+) Use reciprocal properties to develop definitions for cotangent, cosecant, and secant.

Trigonometric Functions F.TF

Extend the domain of trigonometric functions using the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. *

Prove and apply trigonometric identities.

9. (+) Prove the addition and subtraction, half-angle, and double-angle formulas for sine, cosine, and tangent and use them to solve problems.
10. (+) Use fundamental trigonometric identities.
    a. (+) Verify trigonometric identities
    b. (+) Evaluate trigonometric functions
    c. (+) Write equivalent trigonometric expressions
    d. (+) Solve trigonometric equations.

Similarity, Right Triangles and Trigonometry G.SRT

Apply trigonometry to general triangles.

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side and use the formula to solve problems.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems involving right and non-right triangles.

Expressing Geometric Properties G.GPE

Translate between the geometric description and the equation for a conic section.

3. (+) Analyze conic sections using equations and graphs.
    a. (+) Given a quadratic equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ (where $B = 0$), determine whether the graph is a circle, parabola, ellipse, or hyperbola
    b. (+) Use the process of completing the square to put the equation in standard form
    c. (+) When given a graph, be able to write the equation of the conic section, and vice versa.

Conditional Probability and the Rules of Probability S.CP

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S.MD

Calculate expected values and use them to solve problems.

1. (+) Assign a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; understand that it is the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; calculate the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; calculate the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding calculating the expected values.
    a. (+) Calculate the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
    b. (+) Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Polar Coordinates PC.PC

Define polar coordinates and the relationship between polar coordinates and Cartesian coordinates.
Limits

1. (+) Define polar coordinates and the relationship between polar coordinates and Cartesian coordinates with and without the use of technology.
2. (+) Use polar equations to model and solve problems using graphs and algebraic properties.

Parametric Equations PC.PE

Define parametric equations.

1. (+) Given equations for a parametric function, plot the graph and make conclusions about the geometric figure that result.
2. (+) Convert between a pair of parametric equations and an equation in x and y. Model and solve problems using parametric equations.

Limits PC.L

Define a continuous function.

1. (+) Determine if a function is continuous at a point. Find the types of discontinuities of a function and relate them to finding limits of a function. Use the concept of limits to describe discontinuity and end-behavior of the function.

Define limits.

2. (+) Demonstrate knowledge of both the definition and graphical interpretation of limits of values of functions and sequences. Verify and estimate limits using graphs, tables, and technology.
3. (+) Evaluate limits of functions and apply properties of limits, including one-sided limits and limits at infinity using algebra.

Sequences PC.S

Define sequences.

1. (+) Define arithmetic and geometric sequences and series. Model and solve word problems involving applications of sequences and series, interpret the solutions and determine whether the solutions are reasonable.