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SEARCH FOR VIOLATIONS OF QUANTUM MECHANICS*

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ABSTRACT

The treatment of quantum effects in gravitational fields indicates that pure states may evolve into mixed states, and Hawking has proposed modifications of the axioms of field theory which incorporate the corresponding violation of quantum mechanics. In this paper we propose a modified Hamiltonian equation of motion for density matrices and use it to interpret upper bounds on the violation of quantum mechanics in different phenomenological situations. We apply our formalism to the $K^0 - \bar{K}^0$ system and to long baseline neutron interferometry experiments. In both cases we find upper bounds of about 2×10^{-21} GeV on contributions to the single particle "Hamiltonian" which violate quantum mechanical coherence. We discuss how these limits might be improved in the future, and consider the relative significance of other successful tests of quantum mechanics. An Appendix contains model estimates of the magnitude of effects violating quantum mechanics.

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1. INTRODUCTION

Physicists have long been reconciled to the loss of classical predictability inherent to quantum mechanics. Satisfactory quantized versions exist of field theories describing all the fundamental interactions except gravity. Although a full theory of quantized gravity still escapes us, many calculations have been made of quantum effects in curved gravitational background fields. These calculations have indicated that quantum effects cause black holes to radiate particles with a thermal spectrum [1]. This phenomenon is associated with the existence of an event horizon [2]: one can envisage pair creation taking place near the horizon, with one particle of the pair falling in while the other escapes. Phase information is lost with the infalling particle, and this is reflected in the mixed nature of the thermal final state. The evolution of a pure initial state into a mixed final state is forbidden in conventional quantum mechanics. If the final density matrix $\rho_{+}{}^{A}{}_{B}$ is related linearly to the initial density matrix $\rho_{-}{}^{C}{}_{D}$ by a superscattering operator $\not{S}^{A}{}_{BC}{}^{D}$:

$$\rho_{+}{}^{A}{}_{B} = \not{S}^{A}{}_{BC}{}^{D} \; \rho_{-}{}^{C}{}_{D} \tag{1.1}$$

then the factorization of the superscattering operator as the product of two S-matrices:

$$\not{S}^{A}{}_{BC}{}^{D} = S^{A}_{C} \; S^{*D}_{B} \tag{1.2}$$

and the unitarity $SS^{\dagger} = 1$ of the S-matrix together guarantee that Tr $\rho_{+}^{2} = 1$ if Tr $\rho_{-}^{2} = 1$, so that purity stays eternal. The calculations [1] of quantum black hole radiance suggest that macrophysics may violate this romantic proposition. If so, we would have to reconcile ourselves to a second loss of predictability.

What are the microphysical consequences of this macrophysical deduction? It is difficult or impossible to know in the absence of a theory of quantum gravity, and we must rely for the moment on intuition. Our intuition starts from the observation that information can be lost across any event horizon. It seems inescapable [3] that space-time should have a foamy structure on the Planck scale, with Planck radius black holes appearing and disappearing on a Planck time scale. Hence space-time should be topologically complicated with many evanescent microhorizons [4]. These imply mathematically that one cannot maintain globally hyperbolic boundary conditions. Therefore we believe [5] that our microphysical laws should also accommodate violations of quantum mechanics such as the evolution of pure states into mixed states.

This point of view has been argued by Hawking [5] and collaborators. They have presented a number of model calculations [4] to support their point of view, and while none is wholly convincing and some do not respect essential physical boundary conditions, we are sympathetic to the intuitive reasoning behind them. Hawking [5] has proceeded to postulate a set of modified quantum mechanical axioms. They treat the density matrix as basic and allow the superscattering operator $\not{S}$ to be unfactorizable, in which case Tr $\rho_-^2 = 1$ does not imply that Tr $\rho_+^2 = 1$.

So far there has been no contact between these theoretical speculations and experiment. In view of the many experimental tests of quantum mechanics, it appears to us desirable to develop a phenomenological framework for discussing violations of quantum mechanics. A unified formalism would enable us to compare the significance of different classes of experiment, as well as to confront possible theoretical estimates of the magnitude of effects violating quantum mechanics. Among the tests of quantum mechanics whose significance might be sharpened in this way we would include studies of simple systems such as the $K^o - \bar{K}^o$ system [6], long baseline neutron interference experiments [7], and complex systems such as long-lived superconductors, systems at ultra-low temperatures, or lasers.

Our first step in this paper is to build upon Hawking's discussion [5] of the asymptotic superscattering operator formalism (1.1) and (1.2). In sect. 2 we propose an extension to describe the time evolution of the density matrix

$$\dot{\rho}^A_{\ B} = \not{H}^{A\ \ D}_{\ BC}\ \rho^C_{\ D} \tag{1.3}$$

where the linear operator $\not{H}$ contains an addition to the usual quantum mechanical piece:

$$\not{H}^{A\ \ D}_{\ BC}\ \rho^C_{\ D} = i[\rho, H]^A_{\ B} + \delta\not{H}^{A\ \ D}_{\ BC}\ \rho^C_{\ D} \quad . \tag{1.4}$$

We show how the eq. (1.3) can be used for a simple finite N-dimensional system, and argue that the real parts of the eigenvalues κ of its solutions must either be zero (the usual quantum mechanical case) or else negative. In this latter case the system evolves towards a mixed state: $\mathrm{Tr}\,\rho^2 \to 1/N$ on a characteristic time-scale

$$t = (-\ \mathrm{Re}\ \kappa)^{-1} \quad . \tag{1.5}$$

Note that our formalism (1.3), (1.4) bears no relation to proposals [8] to modify quantum mechanics by adopting a nonlinear Scrödinger equation. We abandon [5] the usual quantum mechanical assumption of asymptotic completeness, namely that the asymptotic "in" and "out" states span the Hilbert space of the theory. For us the density matrix is fundamental. Our formalism (1.3), (1.4) resembles the description used [9] for quantum-mechanical systems in contact with a thermal bath. However, our philosophical orientation is different, since we regard quantum mechanics as fundamentally incomplete, whereas it has normally been assumed to be fundamentally correct. Conventional conservation laws such as those of energy and angular momentum are imposed [5] in the asymptotic limit (1.1, 1.2), and we retain them in our time evolution formalism (1.3, 1.4). Thus our treatment differs from that of a heat bath [9] which is allowed to transfer these quantities to and from the microscopic system under study. For this reason, among others, it is not possible to regard our quantum mechanics violating (QMV) effects as somehow equivalent to a finite nonzero temperature.

While complete realistic calculations may demonstrate that conventional conservation laws are automatically respected, at our level of sophistication the conservation of energy or angular momentum must be put in by hand as a statistical constraint. As we point out explicitly in sect. 2.1, symmetry rules do not necessarily lead to conservation laws. The imposition of angular momentum conservation means that many beautiful quantum mechanics probes, such as the classic EPR-type experiments [10] or the precessing muon spin in (g-2) experiments [11], are not sensitive to the form of QMV that we propose.

In sect. 3 we apply our formalism to two 2-dimensional systems. One is the type of neutron beam used in interferometry experiments [7]. It is split into two parts which follow separated paths and are then observed to interfere after a fraction of a millisecond has elapsed. From the successful observation [7] of such interference fringes we deduce that for the neutron system

$$|\mathrm{Re}\ \kappa|_n < 2 \times 10^{-21}\ \mathrm{GeV} \quad . \tag{1.6}$$

We then apply a similar formalism to the 2-dimensional $K^o - \bar{K}^o$ system [6]. The application is slightly more complicated because the kaons decay, which is conveniently treated by a nonhermitian generalization of the original Hamiltonian formalism [12]. By comparing carefully measurements of CP violation in the $\pi\pi$ final state far downstream, where the beam should be predominantly K_L^o, with information from semileptonic K^o decays, we deduce by remarkable coincidence that also

$$|\mathrm{Re}\ \kappa|_{K^o} < 2 \times 10^{-21}\ \mathrm{GeV} \quad . \tag{1.7}$$

Section 4 contains discussions of possible future improvements in the current experimental limits (1.6) and (1.7), and mentions possible tests of quantum mechanics in various other systems. These include baryon decay [13] – difficult to produce a pure initial state or to find a testing final state observable; long-lived supercurrents – QMV effects are unlikely to be observable in low temperature systems which do not have degenerate ground states; very low temperature physics – can one prepare a degenerate system for which thermal effects are negligible, in which case any mixing could be ascribed to QMV?; and lasers – the calculations of Hawking et al. [4] suggest that QMV effects vanish for massless particles. An Appendix contains some order-of-magnitude estimates of QMV effects in currently fashionable models: they do not encourage hopes of imminent discovery but we do not consider them to be very reliable.

2. FORMALISM

2.1 PRELIMINARIES

The main purpose of this section is to set up a modified Hamiltonian formalism for the time evolution of density matrices which includes QMV. Before doing this, we warm up with a few remarks based on Hawking's [5] asymptotic formalism (1.1). Let us recall what occurs with a quantum mechanical system of two widely separated particles (of spin 1/2, for convenience' sake) which are in a pure state of total spin zero [10]. The spin part of the density matrix of this system is

$$\rho_{-} = \frac{1}{4}\left(I - \underline{\sigma}_1 \cdot \underline{\sigma}_2\right) \tag{2.1}$$

where $\underline{n} \cdot \underline{\sigma}_1 \, (\underline{n}' \cdot \underline{\sigma}_2)$ measures the spin of particle 1 (2) along the unit vector $\underline{n} \, (\underline{n}')$: $\underline{n} \cdot \underline{\sigma}_1$ and $\underline{n}' \cdot \underline{\sigma}_2$ have eigenvalues ± 1. The density matrix ρ_{-} (2.1) is pure with $\mathrm{Tr}\ \rho = 1 = \mathrm{Tr}\ \rho^2$. This is the classic system for which Bell [14] derived his famous inequalities.

Suppose experimentalist A in the vicinity of particle 1 measures $\underline{a} \cdot \underline{\sigma}_1$ where $\underline{a}$ is a unit vector of her choice. Meanwhile experimentalist B measures $\underline{b} \cdot \underline{\sigma}_2$ and each gets a result ± 1. Later they compare notes and compute the experimental value of $\langle(\underline{a}\cdot\underline{\sigma}_1)(\underline{b}\cdot\underline{\sigma}_2)\rangle$. According to conventional quantum mechanics they should find

$$\langle(\underline{a}\cdot\underline{\sigma}_1)(\underline{b}\cdot\underline{\sigma}_2)\rangle = \mathrm{Tr}\left[(\underline{a}\cdot\underline{\sigma}_1)(\underline{b}\cdot\underline{\sigma}_2)\rho_-\right] = -\underline{a}\cdot\underline{b} \quad . \tag{2.2}$$

What is allowed by QMV? The density matrix ρ_+ is now permitted to be that of a mixed state. We assume that the dynamics violating quantum mechanics is rotationally invariant so that ρ_+ only contains the unit matrix and $\underline{\sigma}_1\cdot\underline{\sigma}_2$:

$$\rho_+ = \frac{1}{4}\left(\alpha I - \beta\,\underline{\sigma}_1\cdot\underline{\sigma}_2\right) \quad . \tag{2.3}$$

Requiring Tr $\rho_+ = 1$ fixes $\alpha = 1$, while requiring Tr $\rho_+^2 \leq 1$ means that $\beta \leq 1$:

$$\rho_+ = \frac{1}{4}\left(I - \beta\,\underline{\sigma}_1\cdot\underline{\sigma}_2\right) \quad . \tag{2.4}$$

If we wish, we can visualize ρ_+ as resulting from QMV during the decay or as a final-state interaction:

$$\rho_+ = \not{S}\ \rho_- \tag{2.5}$$

where $\not{S}$ is rotationally invariant. If our experimentalist friends compute $\langle(\underline{a}\cdot\underline{\sigma}_1)(\underline{b}\cdot\underline{\sigma}_2)\rangle$ they will obtain

$$\langle(\underline{a}\cdot\underline{\sigma}_1)(\underline{a}\cdot\underline{\sigma}_2)\rangle = \mathrm{Tr}\left[(\underline{a}\cdot\underline{\sigma}_1)(\underline{b}\cdot\underline{\sigma}_2)\rho_+\right] = -\beta(\underline{a}\cdot\underline{b}) \tag{2.6}$$

which is also rotationally invariant. Nevertheless, ***angular momentum is not conserved***. This is directly seen when our experimentalist friends choose $\underline{a} = \underline{b}$, in which case quantum mechanics would tell them to expect perfect anticorrelation. Here they get it only if $\beta = 1$. The result (2.6) could not come from any nonconservation of angular momentum within the context of conventional quantum mechanics. In quantum mechanics, rotational invariance would necessarily be violated if angular momentum were not conserved. This little example illustrates a general result that if we abandon quantum mechanics, invariance principles are no longer equivalent to conservation laws, and they need to be imposed by hand. Hawking [5] imposes the conservation of energy, momentum and angular momentum on his $\not{S}$ operator axiomatically. However, these conservation laws have not been derived from any path integral or other quantum field-theoretical formalism. Indeed, it is unlikely * that energy conservation can be imposed without sacrificing locality: energy has in any case no gauge-invariant local meaning.

We will return to the $\not{S}$ matrix formalism in sect. 4 when we discuss the possibility [13] of observing QMV in baryon decay. However, for the applications of sect. 3 we need a different formalism.

2.2 Time Evolution

We have already observed that the mixed final state may either arise from some interaction during the decay process itself, or else as a final state interaction. One picturesque possibility is that one of the particles 1 or 2 may experience a QMV effect due to interaction with a virtual black hole. Clearly, the longer A and B wait to perform their measurements, the more likely it is that one or the other of the two particles will have been affected. Thus β should in general be time-dependent, and it is necessary to develop a time-dependent formalism.

In conventional quantum mechanics we have

$$\dot{\rho} = i\, [\rho, H] \tag{2.7}$$

where the dot denotes time differentiation and H is the Hamiltonian. Both the density matrix ρ and the Hamiltonian H are hermitian. To see why pure states stay pure in

* We thank M. E. Peskin and L. Susskind for emphasizing this point to us.

quantum mechanics, we compute

$$\frac{d}{dt}\,\mathrm{Tr}\,(\rho^2) = \mathrm{Tr}\,(\rho\,\dot\rho + \dot\rho\,\rho)$$
$$= i\ \mathrm{Tr}\,(\rho^2 H - H\rho^2) \quad . \tag{2.8}$$

Expression (2.8) vanishes because of the cyclic property of the trace. This need not hold for infinite-dimensional systems, but we will restrict ourselves to finite-dimensional systems for which Tr ρ^2 is therefore constant in conventional quantum mechanics. In particular, pure states remain pure with Tr $\rho^2 = 1$. We propose replacing (2.7) by a more general linear equation

$$\dot\rho^A{}_B = \not{H}^A{}_{BC}{}^D\ \rho^C{}_D \tag{2.9a}$$

where

$$(\not{H}\,\rho)^A{}_B = i[\rho, H]^A{}_B + \delta\not{H}^A{}_{BC}{}^D\ \rho^C{}_D \quad . \tag{2.9b}$$

The extra term $\delta\not{H}$ represents the fact that in general the superscattering operator $\not{S}$ cannot be factorized as it is (1.2) in conventional quantum mechanics. We consider the most general linear operator $\delta\not{H}$ which has four indices and maps hermitian matrices into hermitian matrices.

Our QMV equation (2.9) has the same structure as the Markovian master equation reviewed in ref. [9]. This equation has been shown to control the time evolution of a reduced quantum mechanical system in contact with a reservoir whose unseen states are summed over, and whose characteristic correlation time is much shorter than the time constant of the reduced system. The physical QMV that we wish to describe by eq. (2.9) is very reminiscent of this quantum mechanical situation. In our case the role of the reservoir is played by space-time foam which is correlated on a Planck time scale, so that eq. (2.9) would be valid on larger time scales such as those we consider in sect. 3. In our case this foamy "reservoir" is supposed to be intrinsically unobservable, so that $\delta\not{H}$ in eq. (2.9) is an essential modification, and not the artefact of a voluntary choice to observe only a reduced part of the world.

We must demand that $\delta\not{H}$ obey certain restrictions. In order to conserve probability by keeping Tr $\rho = 1$, we need Tr $\dot{\rho} = 0$ and hence must impose

$$\delta\not{H}^{A\ \ D}_{\ AC} = 0 \quad . \tag{2.10}$$

Secondly, we must ensure that Tr ρ^2 can never exceed unity, and thereby avoid states with complex entropy. This means that

$$\frac{1}{2}\,\frac{d}{dt}\ \mathrm{Tr}\,(\rho^2) = \mathrm{Tr}\,(\rho\,\dot{\rho}) = \rho^{B}_{\ A}\ \delta\not{H}^{A\ \ D}_{\ BC}\ \rho^{C}_{\ D} \leq 0 \tag{2.11}$$

for all ρ with Tr $\rho^2 = 1$. Thus if we think of possible matrices ρ as defining a vector space, $\delta\not{H}$ is a negative semidefinite quadratic form on that space. This condition is in fact sufficient to ensure that

$$\frac{d}{dt}\ \mathrm{Tr}\,(\rho^2) \leq 0 \tag{2.12}$$

for all ρ. For the simple two-dimensional systems we study, the condition (2.11) also guarantees that the entropy of a system never decreases and that the eigenvalues P_i of the density matrix stay positive semidefinite. In more complicated systems these requirements of entropy increase and of positivity lead to additional constraints [9],[15].

It is often convenient to work with a hermitian basis $\{\lambda_\alpha\} \equiv \{\lambda_0 \equiv 1/\sqrt{2N}\,, \lambda_a\}$ for the $N \times N$ density matrices of an N-dimensional system, where the $\{\lambda_a\}$ are the $N^2 - 1$ adjoint matrices of SU(N) normalized: Tr $(\lambda_a \lambda_b) = (1/2)\delta_{ab}$. Thus

$$\rho = \frac{1}{2}\ \rho_\alpha \lambda_\alpha = \frac{1}{2}\,(\rho_0 \lambda_0 + \rho_a \lambda_a) \tag{2.13}$$

where probability conservation Tr $\rho = 1$ implies

$$\rho_0 = \sqrt{2/N} \tag{2.14}$$

as in the simple examples (2.1) and (2.4), and hermiticity requires that the ρ_a be real. We can write the conventional quantum-mechanical Hamiltonian H (2.9) as

$$H = \frac{1}{2}\, h_\alpha \lambda_\alpha = \frac{1}{2}\, (h_0 \lambda_0 + h_a \lambda_a) \tag{2.15}$$

where hermiticity also requires that the h_α be real, in which case

$$i[\rho, H] = \frac{1}{2}\, h_a\, \rho_b\, f_{abc}\, \lambda_c \tag{2.16}$$

where the f_{abc} are the structure constants of SU(N). We can also write

$$\delta\!\!/H\, \rho = \frac{1}{2}\, \lambda_\alpha\, h\!\!/_{\alpha\beta}\, \rho_\beta \tag{2.17}$$

where hermiticity requires the coefficients $h\!\!/_{\alpha\beta}$ to be real, and the practical constraints (2.10) and (2.11) become

$$h\!\!/_{0\beta} = 0 \tag{2.10$'$}$$

$$\rho_\alpha\, h\!\!/_{\alpha\beta}\, \rho_\beta \leq 0 \tag{2.11$'$}$$

where (2.11$'$) says that the symmetric part of the matrix $h\!\!/_{\alpha\beta}$ is negative semidefinite. Additional restrictions may be imposed by conservation laws. If a hermitian operator $O \equiv O_\alpha \lambda_\alpha$ commutes with the Hamiltonian H and hence is quantum-mechanically conserved, it will only be statistically conserved in the presence of QMV if

$$\frac{d}{dt}\, \mathrm{Tr}\,(O\rho) = 0 \tag{2.18}$$

which requires

$$\mathrm{Tr}(O\,\dot{\rho}) = 0 \tag{2.19}$$

and hence

$$\mathrm{Tr}(O\,\delta\not{H}\rho) = 0 \tag{2.20}$$

which implies in our hermitian basis that

$$O_\alpha\,\not{K}_{\alpha\beta} = 0 \quad . \tag{2.21}$$

The statistical conservation of any observable O must be imposed using eq. (2.21): this condition is a consequence of (2.10′) only if all states of the system have the same value of O so that $O_a = 0$. Thus, as we saw in sect. 2.1, we find that invariance principles applied to $\not{H}$ alone do not lead to conservation laws.

In our hermitian basis the time evolution eq. (2.9) becomes

$$\dot{\rho}_0 = 0 \quad ; \quad \dot{\rho}_a = h_{ab}\,\rho_b = f_{abc}\,h_b\,\rho_c + \not{K}_{ab}\,\rho_b \quad . \tag{2.22}$$

where we have used the fact that eqs. (2.10′) and (2.11′) together imply that $h_{\alpha 0} = 0$. The general solution of this set of linear homogeneous equations is

$$\rho_0 = \sqrt{2/N} \quad ; \quad \rho_a = \sum_{i=1}^{N^2-1} r_i\,v_{ia}\,exp(\kappa_i t) \tag{2.23}$$

in terms of eigenvectors $V_i = v_{ia}\lambda_a$, where the κ_i are eigenvalue solutions of the characteristic equation

$$|h_{ab} - \kappa\delta_{ab}| = 0 \tag{2.24}$$

and the r_i are arbitrary coefficients. Hermiticity requires that the entries h_{ab} be real, in which case the eigenvalues κ_i are either real or occur in complex conjugate pairs.

In conventional quantum mechanics (2.16) the matrix h_{ab} is totally antisymmetric and the nonzero eigenvalues κ_i occur in complex conjugate pairs

$$\kappa_{\pm} = i(E_A - E_B),\ i(E_B - E_A) \tag{2.25}$$

given by differences between the energies of possible states $|A\rangle, |B\rangle$ of the system. In our form of QMV the imaginary parts of the κ_i need not obey this Rayleigh-Ritz combination principle when the system has more than two states. In general, the condition (2.11) that Tr ρ^2 stay ≤ 1 implies that

$$\text{Re}\ \kappa < 0 \quad . \tag{2.26}$$

In general the density matrix ρ will relax to a block-diagonal form which commutes with the conserved observables O, and will relax to the unit matrix if there are no such conservation laws. The positivity requirement that all the eigenvalues P_i of the density matrix satisfy $0 \leq P_i \leq 1$ is simple to enforce in a 2-dimensional system, where it is an automatic consequence of eq. (2.11), but is nontrivial [9] in systems with more than two states.

To gain some intuition, consider a simple 2-dimensional system in which the hermitian matrices λ_a are just the Pauli matrices $(1/2)\sigma_{x,y,z}$. Suppose that the system has two energy levels $E \pm \Delta E/2$ so that the quantum mechanical Hamiltonian is just

$$H = E + \Delta E \sigma_z : h_0 = 2E,\ h_3 = \Delta E \quad . \tag{2.27}$$

The evolution eq. (2.22) now becomes

$$\begin{aligned} \dot{\rho}_x &= -\Delta E\ \rho_y \qquad & \dot{\rho}_z &= 0 \\ \dot{\rho}_y &= \ \ \Delta E\ \rho_x \qquad & \dot{\rho}_0 &= 0 \quad . \end{aligned} \tag{2.28}$$

The vanishing of $\dot{\rho}_0$ ensures that the total probability is conserved: Tr $\rho = 1$. The vanishing of $\dot{\rho}_z$ ensures that the individual probabilities of the higher and lower energy eigenstates are also constant, reflecting energy conservation. However, in the two-dimensional complex plane with ρ_x and ρ_y defined as axes, the time evolution of $\rho_x + i\rho_y$ is counter-clockwise circular motion with frequency $\omega = \Delta E$ as indicated in fig. 1(a). It is therefore apparent that

$$\text{Tr } \rho^2 = \frac{1}{2} \rho_\alpha \rho_\alpha = \frac{1}{2}\left(1 + \rho_x^2 + \rho_y^2 + \rho_z^2\right)$$

is constant so that entropy is conserved, and in particular pure states remain pure states in quantum mechanics, orbiting around the unit circle in fig. 1(a). In terms of ρ this corresponds to the familiar solution

$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\Delta Et} \\ e^{i\Delta Et} & 1 \end{pmatrix} \quad . \tag{2.29}$$

Let us now see what happens when quantum mechanics is violated. Probability conservation and the impossibility of entropy decrease (2.10′) and (2.11′) tell us that $\not{h}_{0\beta} = 0 = \not{h}_{\alpha 0}$, while energy conservation (2.21) combined with (2.11′) tells us that

$$\not{h}_{3\beta} = 0 = \not{h}_{\alpha 3} \quad . \tag{2.30}$$

We can treat $\not{h}_{ab}$ as symmetric, since any antisymmetric piece has a form indistinguishable from that of conventional quantum mechanics for this simple 2-dimensional system. We therefore have

$$\not{h}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\alpha & -\beta & 0 \\ 0 & -\beta & -\gamma & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.31}$$

in the $\alpha = 0, 1, 2, 3$ basis. Negativity (2.11′) also requires

$$\alpha > 0 \quad , \quad \gamma > 0 \quad , \quad \alpha\gamma > \beta^2 \quad . \tag{2.32}$$

The modified QMV forms of the evolution equations (2.28) are

$$\begin{aligned} \dot{\rho}_x &= -\alpha\rho_x + (-\Delta E - \beta)\rho_y & \dot{\rho}_z &= 0 \\ \dot{\rho}_y &= (\Delta E - \beta)\rho_x - \gamma\rho_y & \dot{\rho}_0 &= 0 \end{aligned} \tag{2.33}$$

which describe a quasi-circular spiral orbit in the ρ_x, ρ_y plane as illustrated in fig. 1(b):

$$\rho(t) \simeq \frac{1}{2} \begin{pmatrix} 1 & e^{-(\alpha+\gamma)t/2}\, e^{-i\Delta Et} \\ e^{-(\alpha+\gamma)t/2}\, e^{i\Delta Et} & 1 \end{pmatrix} \tag{2.34}$$

As $|\alpha + \gamma|t \to \infty$, ρ becomes completely mixed and contains no further information.

The above discussion could easily be extended to multi-dimensional systems, but the 2-dimensional case is sufficient for our phenomenological purposes, as we will see in the next section.

3. PHENOMENOLOGICAL APPLICATIONS

3.1 Neutron Inteferometry

Some of the most sensitive experimental tests of quantum mechanics have been carried out with a slow neutron beam [7]. The requirements of energy and momentum conservation imply that a neutron in a momentum eigenstate must be pure and cannot be a statistical ensemble. We consider experiments in which the beam is split into two components which travel along different paths and are subsequently brought together to interfere. The paths may have different heights, and the expected quantum mechanical phase shift induced by gravity has been verified, as has the effect of the Earth's rotation [7]. We can apply the formalism of the previous section directly to this system, taking the two basis states to be the two components of the split beam. The interference measurement corresponds to computing the expectation value Tr $(O\rho)$ of an observable

$$O(\theta) = \frac{1}{2} \begin{pmatrix} 1 & e^{i\theta} \\ e^{-i\theta} & 1 \end{pmatrix} \tag{3.1}$$

where θ is a relative phase which depends on the experimental configuration. This observable takes the value

$$\mathrm{Tr}(O\rho) = \frac{1}{2} + \frac{1}{2}\cos(\Delta E t + \theta) \tag{3.2}$$

when the density matrix is purely quantum mechanical as in eq. (2.29). Since this density matrix describes a pure state, the magnitude of the interference is complete – the variation in intensity with θ (3.1) is from 0 to 1. This quantum-mechanical prediction is well supported by experimental studies [7] which observe almost complete interference between the split neutron beams.

If we now compute the same interference observable (3.1) using the unquantum mechanical density matrix (2.24) we get

$$\mathrm{Tr}(O\rho) = \frac{1}{2} + \frac{1}{2}\, exp - \left(\frac{\alpha+\gamma}{2}\right)t\, \cos(\Delta E t + \theta) \quad . \tag{3.3}$$

We see that the interference pattern decays exponentially as t increases. As $t \to \infty$ we recover the incoherent superposition of two neutron beams expected classically: the historic quantum-mechanical two-slit experiment is undone. The fact that at most a 20% attenuation of the interference pattern has been observed [7] in experiments where the neutron beam propagates for $t \sim 1/3000$ seconds allows us to infer that $(\alpha+\gamma)t \lesssim 1$ so that

$$\alpha + \gamma \lesssim 2 \times 10^{-21}\ \mathrm{GeV} \quad . \tag{3.4}$$

One possible interpretation of any nonzero effect of this type would be some contribution to the effective apparent mass of a neutron due to an interaction with virtual black holes which was incoherent, and so destroyed the initial coherence of the pure neutron beam after a time $t \gtrsim O(\delta m^{-1})$. To help us assess the significance of the limit (3.4), we recall that while the natural hadronic scale of the neutron is O(1) GeV, it has been suggested [16] in the context of conventional quantum field theory that gravitodynamic effects might contribute O(10) MeV to u and d quark masses and hence to the neutron mass. Our limit (3.4) is 19 orders of magnitude smaller, but as discussed in the Appendix we expect [5] QMV effects to have a very different structure from such conventional coherent effects. We make model estimates in the Appendix which fall both above and below the experimental limit (3.4) that we have just deduced, but we do not regard these estimates as very reliable.

3.2 Neutral Kaons

A second very interesting and sensitive probe for violations of quantum mechanics is the neutral kaon system [6,12]. In the usual quantum-mechanical framework this system is described by a phenomenological Hamiltonian with hermitian (mass) and antihermitian (decay) components:

$$H = \begin{pmatrix} M - \frac{i}{2}\,\Gamma & M_{12}^* - \frac{i}{2}\,\Gamma_{12}^* \\ M_{12} - \frac{i}{2}\,\Gamma_{12} & M - \frac{i}{2}\,\Gamma \end{pmatrix} \tag{3.5}$$

in the $(K^o, \bar{K}^o)$ basis. The antihermitian decay matrix is a convenience which enables one to avoid treating the 2π, 3π and other final states explicitly. It presents no special difficulty here, since the violations of quantum mechanics we consider have an entirely different form.

When H is not hermitian, the time-evolution of ρ is ordinarily given by

$$\dot{\rho} = -i(H\rho - \rho H^\dagger) \tag{3.6}$$

and the state is pure if Tr $\rho^2 = (\mathrm{Tr}\ \rho)^2$. As before, we will define

$$\begin{aligned} \rho &\equiv \frac{1}{2}\ \rho_\alpha\ \sigma_\alpha \\ H &\equiv \frac{1}{2}\ h_\beta\ \sigma_\beta \end{aligned} \tag{3.7}$$

where the ρ_α are real but where the h_β are now complex. Equation (3.6) leads to a quantum mechanical time-evolution matrix $h_{\alpha\beta}$ [as defined in eq. (2.22) where $K_{\alpha\beta}$ is set equal to zero] of the form

$$h_{\alpha\beta} = \begin{pmatrix} \mathrm{Im}\ h_0 & \mathrm{Im}\ h_1 & \mathrm{Im}\ h_2 & \mathrm{Im}\ h_3 \\ \mathrm{Im}\ h_1 & \mathrm{Im}\ h_0 & -\mathrm{Re}\ h_3 & \mathrm{Re}\ h_2 \\ \mathrm{Im}\ h_2 & \mathrm{Re}\ h_3 & \mathrm{Im}\ h_0 & -\mathrm{Re}\ h_1 \\ \mathrm{Im}\ h_3 & -\mathrm{Re}\ h_2 & \mathrm{Re}\ h_1 & \mathrm{Im}\ h_0 \end{pmatrix} . \tag{3.8}$$

The 'hermitian' contribution ($\propto \mathrm{Re}\ h_\alpha$) consists of the antisymmetric components h_{ab}, as was found previously. The 'antihermitian' contribution ($\propto \mathrm{Im}\ h_\alpha$) includes the symmetric components $h_{0a} = h_{a0}$ and a piece proportional to the identity matrix. For the system under consideration in eq. (3.5), the observed CPT invariance of quantum mechanics requires $h_3 = 0$, and $h_{\alpha\beta}$ takes the special form [here we switch to the more convenient CP eigenstate basis $K_{1,2} = (K^o \pm \bar{K}^o)/\sqrt{2}$, which are CP odd and CP even respectively, by letting $\sigma_3 \leftrightarrow \sigma_1$, $\sigma_2 \leftrightarrow -\sigma_2$]:

$$h_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\mathrm{Re}\ \Gamma_{12} & \mathrm{Im}\ \Gamma_{12} & 0 \\ -\mathrm{Re}\ \Gamma_{12} & -\Gamma & 0 & -2\,\mathrm{Im}\ M_{12} \\ \mathrm{Im}\ \Gamma_{12} & 0 & -\Gamma & -2\,\mathrm{Re}\ M_{12} \\ 0 & 2\,\mathrm{Im}\ M_{12} & 2\,\mathrm{Re}\ M_{12} & -\Gamma \end{pmatrix} . \tag{3.9}$$

If we define

$$\rho \equiv \begin{pmatrix} \rho_{11} & \rho_{12}^* \\ \rho_{12} & \rho_{22} \end{pmatrix} \tag{3.10}$$

in the (K_1, K_2) basis then the equations of motions (2.9) and (3.9) for ρ can be written

$$\dot\rho_{11} = -(\Gamma + \mathrm{Re}\ \Gamma_{12})\rho_{11} - 2\,\mathrm{Im}\ M_{12}\ \mathrm{Re}\ \rho_{12} - \mathrm{Im}\ \Gamma_{12}\ \mathrm{Im}\ \rho_{12} \tag{3.11a}$$

$$\dot\rho_{12} = -(\Gamma - 2i\,\mathrm{Re}\ M_{12})\rho_{12} + \left(\mathrm{Im}\ M_{12} - \frac{i}{2}\ \mathrm{Im}\ \Gamma_{12}\right)\rho_{11}$$
$$- \left(\mathrm{Im}\ M_{12} + \frac{i}{2}\ \mathrm{Im}\ \Gamma_{12}\right)\rho_{22} \tag{3.11b}$$

$$\dot\rho_{22} = -(\Gamma - \mathrm{Re}\ \Gamma_{12})\rho_{22} + 2\,\mathrm{Im}\ M_{12}\ \mathrm{Re}\ \rho_{12} - \mathrm{Im}\ \Gamma_{12}\ \mathrm{Im}\ \rho_{12} . \tag{3.11c}$$

The coefficient ρ_{11} corresponds to the long-lived CP eigenstate (K_1) since $(\Gamma+\text{Re }\Gamma_{12}) \ll \Gamma < (\Gamma - \text{Re }\Gamma_{12})$. In the phenomenologically interesting limit $\rho_{11} \gg \rho_{12} \gg \rho_{22}$ the equations further simplify:

$$\dot{\rho}_{11} = -(\Gamma + \text{Re}\,\Gamma_{12})\rho_{11} \tag{3.12a}$$

$$\dot{\rho}_{12} = -(\Gamma - 2i\,\text{Re } M_{12})\rho_{12} + \left(\text{Im } M_{12} - \frac{i}{2} \text{ Im }\Gamma_{12}\right)\rho_{11} \tag{3.12b}$$

$$\dot{\rho}_{22} = -(\Gamma - \text{Re}\,\Gamma_{12})\rho_{22} + 2\,\text{Im } M_{12} \text{ Re } \rho_{12} - \text{Im }\Gamma_{12} \text{ Im } \rho_{12} \quad . \tag{3.12c}$$

It is now straightforward to check that for large t, ρ decays exponentially to:

$$\rho \propto \begin{pmatrix} 1 & \epsilon^* \\ \epsilon & |\epsilon|^2 \end{pmatrix} \tag{3.13}$$

which corresponds to the long-lived mass eigenstate K_L, where

$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}} \,[(1+\epsilon)K^o + (1-\epsilon)\bar{K}^o] \tag{3.14a}$$

and where the CP impurity parameter ϵ is given by

$$\epsilon = \frac{(i/2)\text{ Im }\Gamma_{12} - \text{Im } M_{12}}{(1/2)\,\Delta\Gamma - i\Delta M} \tag{3.14b}$$

and where $\Delta M \equiv M_L - M_S$ is positive, $\Delta\Gamma \equiv \Gamma_L - \Gamma_S$ is negative.

We now consider how the time-evolution matrix $h_{\alpha\beta}$ could be generalized. Let us consider the case where the violations of quantum mechanics are approximately flavor conserving, which could correspond to a violation of quantum mechanics in the strong dynamics of quark binding or to flavor-conserving quark mass terms. Even in these cases, violations of quantum mechanics could still affect strangeness-changing processes such as Γ and M_{12}, but these are already so small compared to the dominant $\Delta S = 0$ dynamics that small corrections to them would be unobservable. With this simplifying assumption $\not{h}_{1\alpha} = 0$, since ρ_1 measures strangeness in the (K_1, K_2) basis.

Moreover $\not{h}_{0\alpha} = 0$ to conserve probability, since interactions that conserve strangeness will not contribute to decay. One can then show that the constraint (2.11′) on the negativity of $\not{h}_{\alpha\beta}$ requires $\not{h}_{\alpha 1} = \not{h}_{\alpha 0} = 0$. Thus we can write

$$\not{h}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} \tag{3.15}$$

where the negativity of $\not{h}_{\alpha\beta}$ again imposes $\alpha > 0,\ \gamma > 0,\ \alpha\gamma > \beta^2$. The equations of motion (3.11) become

$$\begin{aligned}\dot{\rho}_{11} = &-(\Gamma + \text{Re}\,\Gamma_{12})\rho_{11} - \gamma(\rho_{11} - \rho_{22}) - 2\,\text{Im}\ M_{12}\ \text{Re}\ \rho_{12} \\ &- (\text{Im}\ \Gamma_{12} + 2\beta)\ \text{Im}\ \rho_{12}\end{aligned} \tag{3.16a}$$

$$\begin{aligned}\dot{\rho}_{12} = &-(\Gamma - 2i\,\text{Re}\ M_{12})\rho_{12} - 2i\alpha\ \text{Im}\ \rho_{12} + \left(\text{Im}\ M_{12} - \frac{i}{2}\ \text{Im}\,\Gamma_{12} - i\beta\right)\rho_{11} \\ &- \left(\text{Im}\ M_{12} + \frac{i}{2}\ \text{Im}\ \Gamma_{12} - i\beta\right)\rho_{22}\end{aligned} \tag{3.16b}$$

$$\begin{aligned}\dot{\rho}_{22} = &-(\Gamma - \text{Re}\,\Gamma_{12})\rho_{22} + \gamma(\rho_{11} - \rho_{22}) + 2\,\text{Im}\ M_{12}\ \text{Re}\ \rho_{12} \\ &- (\text{Im}\ \Gamma_{12} - 2\beta)\ \text{Im}\ \rho_{12}\end{aligned} \tag{3.16c}$$

which simplify in the phenomenologically interesting limit where $\rho_{11} \gg \rho_{12} \gg \rho_{22}$ to:

$$\dot{\rho}_{11} = -(\Gamma + \text{Re}\,\Gamma_{12})\rho_{11} - \gamma\rho_{11} \tag{3.17a}$$

$$\dot{\rho}_{12} = -(\Gamma - 2i\,\text{Re}\ M_{12})\rho_{12} + \left(\text{Im}\ M_{12} - \frac{i}{2}\ \text{Im}\,\Gamma_{12} - i\beta\right)\rho_{11} \tag{3.17b}$$

$$\dot{\rho}_{22} = -(\Gamma - \text{Re}\,\Gamma_{12})\rho_{22} + \gamma\rho_{11} + 2\,\text{Im}\ M_{12}\ \text{Re}\ \rho_{12} - (\text{Im}\ \Gamma_{12} - 2\beta)\ \text{Im}\ \rho_{12}\ . \tag{3.17c}$$

One can now easily check that for large t, ρ decays exponentially to:

$$\rho \propto \begin{pmatrix} 1 & \frac{-(i/2)(\text{Im}\,\Gamma_{12}+2\beta)-\text{Im}\ M_{12}}{(1/2)\Delta\Gamma+i\Delta M} \\ \frac{(i/2)(\text{Im}\,\Gamma_{12}+2\beta)-\text{Im}\ M_{12}}{(1/2)\Delta\Gamma+i\Delta M} & |\epsilon|^2 + \frac{\gamma}{|\Delta\Gamma|} - \frac{4\beta\,\text{Im}\ M_{12}(\Delta M/\Delta\Gamma)+\beta^2}{(1/4)\Delta\Gamma^2+\Delta M^2} \end{pmatrix} \tag{3.18}$$

which is different from the quantum mechanical solution (3.13) in several important respects. First, eq. (3.18) no longer represents a pure state, since Tr $\rho^2 < 1$ when ρ is normalized so that Tr $\rho = 1$. Instead, ρ corresponds to a mixture of a K_L beam plus a low intensity K_S beam. The origin of this new, long-lived K_S component is evident from eqs. (2.17a) and (2.17c). The terms proportional to γ show a direct 'decay' of the K_1 into the K_2, which constantly 'regenerates' the K_S beam. This is exactly what one might expect if the definite phase relationship between the K^o and $\bar{K}^o$ components of a K_L beam were to become mixed by gravity. The $K_L \rightarrow K_S$ 'decay' is consistent with our intention to conserve energy, since the $K_L - K_S$ mass splitting is comparable to the natural width of the K_S. The K_S simply propagates off-shell until it decays into 2π, or makes a transition to an on-shell K_S by emitting a 35 cm photon.

The interpretation of eq. (3.18) becomes considerably simpler if we drop the terms proportional to β in ρ_{22}. These terms are negligible if β and γ are of the same order of magnitude, and we will show later that β cannot be much larger than γ. The term in ρ_{22} proportional to γ is a potentially large new source of 'downstream' $K \rightarrow 2\pi$ decays. The observable corresponding to 2π decay is

$$O_{2\pi} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.19}$$

since if we neglect 'direct' CP violation it is only the K_2 state that decays into 2π, and thus

$$\mathrm{Tr}\,(O_{2\pi}\rho) = |\epsilon|^2 + \frac{\gamma}{|\Delta\Gamma|} \quad . \tag{3.20}$$

The two contributions to downstream 2π decays from the CP impurity of the K_L and the 'spontaneous regeneration' of K_S are easily distinguished. There is a well measured interference effect between $K_S \rightarrow 2\pi$ and $K_L \rightarrow 2\pi$ which leads to an oscillation in the 2π yield shortly after regeneration when the two contributions are comparable, as can be deduced from eqs. (3.12b) and (3.12c). If spontaneous regeneration were the true origin of downstream $K \to 2\pi$ decays, no oscillation would be observed, as can be deduced from eq. (3.17c).

Furthermore, there is an independent measurement of CP violation from $K_L \to \pi\ell\nu$ decays which severely restricts the magnitude of γ. Since the K^o decays only into $\pi^-\ell^+\nu$ and the $\bar{K}^o$ decays only into $\pi^+\ell^-\bar{\nu}$, the observable corresponding to $\pi^-\ell^+\nu$ in the (K_1, K_2) basis is

$$O_{\pi^-\ell^+\nu} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{3.21}$$

and the observable corresponding to $\pi^+\ell^-\bar{\nu}$ is

$$O_{\pi^+\ell^-\bar{\nu}} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad . \tag{3.2}$$

Thus the CP violating charge asymmetry δ is given by

$$\delta \equiv \frac{\Gamma(\pi^-\ell^+\nu) - \Gamma(\pi^+\ell^-\bar{\nu})}{\Gamma(\pi^-\ell^+\nu) + \Gamma(\pi^+\ell^-\bar{\nu})} \simeq 2\,\mathrm{Re}\,\rho_{12} \quad . \tag{3.23}$$

Apart from a term proportional to β, ρ_{12} is just the usual CP impurity parameter ϵ. However, if β is comparable to γ which is $\lesssim |\epsilon|^2 \Delta\Gamma$, then its contribution to ρ_{12} is negligible as can be seen from eq. (3.18). * Therefore we can simply compare the experimental value [17] for δ

$$\delta \simeq 2\,\mathrm{Re}\,\epsilon = (3.3 \pm 0.12) \times 10^{-2} \tag{3.24}$$

and the theoretically known phase of ϵ (43.7^o with a negligible error) with the experimental value for

$$|\eta_{+-}|^2 = \rho_{22} = |\epsilon|^2 + \frac{\gamma}{|\Delta\Gamma|} = [(2.274 \pm 0.022) \times 10^{-3}]^2 \tag{3.25}$$

---

* A separate bound on β comes from noting that its presence in ρ_{22} [eq. (3.17c)] affects the oscillation in $K \to 2\pi$ decay and the measured phase of ϵ. The relatively good agreement between the experimental value [12] ($\phi = 44.6 \pm 1.2^o$) and the theoretical value obtained by setting $\beta = \mathrm{Im}\,\Gamma_{12} = 0$ ($\phi = 43.7^o$) puts an upper bound on the magnitude of β which is of order 10^{-20} GeV.

and conclude * that

$$\gamma < 2 \times 10^{-21} \ \text{GeV} \quad . \tag{3.26}$$

This bound on the presence of quantum mechanics violating effects in the $K^o - \bar{K}^o$ system is the same as the bound derived from slow neutrons. A remarkable coincidence!

4. DISCUSSION

In the previous section we applied the formalism proposed in sect. 2 to two interesting experimental tests of quantum mechanics, and found two very similar limits (3.4) and (3.26) on QMV in these two hadronic systems. In this section we discuss possible future improvements in these limits as well as other experimental probes for QMV.

<u>Neutron Interferometry</u>

The interference patterns observed in this experiment [7] exhibited some attenuation, but the amount was dependent on the experimental conditions and presumably has a perfectly mundane explanation. It is unlikely that analogous experiments in the future could be much more than an order of magnitude larger than the previous experiment, but there are two possible improvements which could improve sensitivity to QMV. One would be to use slower neutrons which take a longer time to traverse the same apparatus – the effects we seek increase linearly with time. Ultra-cold neutrons are now being made available for $n - \bar{n}$ oscillation experiments [18]. However, the fact that these neutrons may bounce off the walls of the containment vessel would disturb their purity. Another interesting possibility for neutron interferometry might be to

* Strictly speaking, δ should be compared with $|\frac{1}{3}(2\eta_{+-} + \eta_{00})|$ rather than $|\eta_{+-}|$ if the difference between η_{+-} and η_{00} due to 'direct' CP violation becomes experimentally observable.

control the experimental conditions sufficiently precisely (do experiments in vacuo?) that one could take seriously even a small attenuation of the maximal quantum mechanical interference. In these ways an improvement in sensitivity to QMV by a few orders of magnitude might be feasible.

Neutral Kaons

A new generation of experiments probing CP violation in the neutral kaon system is now underway [19], and might be expected to improve our limit (3.26). Unfortunately, there is not an ideal match between the central focus of these experiments and our special interests. They seek to measure and compare $K_L \to \pi^+\pi^-$ and $\pi^o\pi^o$ decays but do not plan to look closely at the interference between $K_S \to \pi\pi$ and $K_L \to \pi\pi$, or to look at semileptonic $K \to \pi\ell\nu$ decays. Our best limit (3.26) came from combining $K_L \to \pi\pi$ and $K \to \pi\ell\nu$ data. However, perhaps our experimental colleagues could be inspired by our discussion to plan their experimental running so as to allow for direct comparisons of $K_S/K_L \to \pi\pi$ interference and downstream $K \to \pi\ell\nu$ decays as well as $K_L \to \pi\pi$ decays all in the same experiment so as to minimize systematic errors. It is well-known that the neutral kaon system is a great laboratory for verifying quantum mechanics [7,12], and we should exploit it to the utmost.

Baryon Decay

As was mentioned in ref. [13], the trigger for our interest in QMV was the realization [20] that B-violating Planck mass dynamics could be observable in baryon decay. Unfortunately, for the reasons discussed in the Appendix, it is expected that the dominant gravitodynamic effects [13,19] would respect the laws of conventional quantum mechanics. This suspicion is borne out by phenomenological arguments based on the bounds (3.4) and (3.26). It has been argued elsewhere [16] that B-conserving gravitodynamic effects might contribute $O(10^{-2})$ GeV to the masses of quarks, which is 19 orders of magnitude larger than our bounds (3.4) and (3.26) on the magnitude of QMV effects:

$$\frac{QMVB\checkmark\,(\mathrm{exp})}{QMB\checkmark\,(\mathrm{th})} \le O(10^{-19}) \quad . \tag{4.1}$$

Thus, if B-conserving gravitodynamic effects are as large as we suggest, then they must be predominantly quantum mechanical. In point of fact, the negative results [21] of baryon decay experiments already suggest [13] that B-violating gravitodynamic amplitudes are at least 6 orders of magnitude smaller than was suggested by our dimensional arguments:

$$\frac{QMB\times\,(\mathrm{exp})}{QMB\times\,(\mathrm{th})} < O(10^{-6}) \quad . \tag{4.2}$$

It is inconceivable that baryon decay experiments could reach down the additional 26 orders of magnitude in rate (13 orders of magnitude in amplitude) which would be necessary to reach comparable sensitivity:

$$\frac{QMVB\times\,(\mathrm{exp})}{QMB\times\,(\mathrm{th})} = \frac{QMVB\times\,(\mathrm{exp})}{QMB\times\,(\mathrm{exp})} \cdot \frac{QMB\times\,(\mathrm{exp})}{QMB\times\,(\mathrm{th})} = \frac{QMVB\checkmark\,(\mathrm{exp})}{QMB\checkmark\,(\mathrm{th})} \tag{4.3}$$

to the B-conserving tests of quantum mechanics.

Nevertheless one can contemplate performing experimental tests of QMV in baryon decay. We disfavor the idea of looking for unquantum-mechanical effects in correlations between the spins of baryon decay products, since as mentioned in sect. 2.1 any such QMV would necessarily violate angular momentum conservation. There would in any case be severe practical difficulties [13] since one would require a polarized initial state and an analyzable final state. Polarized targets or sources with masses in the kiloton range are not available, and the only analyzable final states we know of [13] are $B \rightarrow \mu$ + vector meson, which do not allow spin determinations as unambiguous as the classic Einstein-Podolsky-Rosen [10] set-up. An alternative possibility one might entertain is to look for mixtures in the final state density matrix in flavor space. Unfortunately, we only know how to measure diagonal elements of the flavor density matrix, and to discriminate between pure and mixed states one needs to be able to measure off-diagonal elements of the density matrix. We therefore abandon the search for QMV in baryon decay.

Long – Lived Supercurrents

There are macroscopic phenomena such as superconductivity and superfluidity whose existence is traced to quantum mechanics. Some of the these phenomena can be very persistent; for example, supercurrents may be maintained for years. One might wonder whether one can deduce from this stability a stringent upper limit on violations of quantum mechanics of the type we have been discussing. It is not obvious how to relate effects in macroscopic systems to those in microscopic systems, but it does not seem likely to us that one could get a limit on QMV which would be competitive with our previous microscopic limits (3.4) and (3.26). We presume that QMV effects respect familiar conservation laws such as those of energy and angular momentum. This means that the mixing and related entropy increase that we propose can only occur when there is more than one degenerate ground state. Such is not the case for superfluid or superconducting systems.

Very Low Temperature Physics

The conservation of energy that we impose means that our QMV effects are not strictly analogous to embedding a quantum mechanical system in a thermal bath [9]. Therefore our QMV effects cannot be re-expressed globally as a minimum temperature. As emphasized above, we expect mixing to occur only when there is more than one degenerate ground state. It might be possible to use ultra-low temperature technology to prepare such a degenerate system for which thermal effects could be neglected.

Accordingly, any mixing observed could be ascribed to QMV.

Lasers

Another macroscopic arena for quantum mechanics is provided by coherent electromagnetic wave phenomena such as lasers. Unfortunately, preliminary calculations [9] suggest that Planck mass gravitodynamic effects vanish for on-shell massless vector particles, so it seems unlikely that any substantial QMV effects should be expected.

We infer from the above discussion that the best present limits on QMV may indeed be those (3.4) and (3.26) deduced in sect. 3 from neutron interferometry and from the neutral kaon system. We have also seen that these limits can perhaps be improved by a few orders of magnitude without undue effort. Although the model estimates of the Appendix do not encourage one to expect QMV to be detectable imminently, we believe that the ideas discussed in this paper should give fresh heart to those who are already in the business of testing quantum mechanics. While it is not overwhelmingly probable that violations of quantum mechanics will be detectable in the near future, the interest of such a discovery would not be negligible.

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APPENDIX

Model Estimates of QMV Effects

We are interested in the possibility [5] that the superscattering operator $\not{S}$ cannot be factorized as a product of two S-matrices. In conventional quantum field theory each S-matrix element may be expressed in leading order as a matrix element of the action $A = \int d^4x\ \mathcal{L}(x)$. Therefore we can represent an $\not{S}$-matrix element in the schematic form

$$\langle \not{S} \rangle = \left\langle \int d^4x\ \mathcal{L}(x) \int d^4x'\ \mathcal{L}(x') \right\rangle \quad . \tag{A.1}$$

The suggestion of Hawking and collaborators [4,5] is that formula (A.1) must be modified when $|x - x'| = O(m_P^{-1})$ where m_P is the Planck mass $= 0\ (10^{19})$ GeV. We can therefore expect QMV effects to be suppressed by $O(m_P^{-n})$. If $n = 0$ the effect is probably too large, if $n = 1$ we might be interestingly close to the bounds (3.4) and (3.26), and if $n \geq 2$ the QMV effects would be unobservably small. One expects a nonfactorizable contribution to $\not{S}$ from $x' \sim x$ of the schematic form

$$\langle \delta\not{S} \rangle = \left\langle \int d^4x\ \not{\mathcal{L}}(x) \right\rangle \quad . \tag{A.2}$$

If we are interested in a process involving M external legs, then (A.1) is valid if $\mathcal{L}(x)$ contains a piece with the appropriate M fields to match the external legs. In the case of the proposed modification (A.2) we need a term in $\not{\mathcal{L}}$ with $2M$ fields – the previous M together with their complex conjugates. The large number of fields required in $\not{\mathcal{L}}$ suggests anew that such QMV effects will tend to be suppressed by inverse powers of m_P. This can be traced to the belief that the QMV occurs when $|x - x'| = O(m_P^{-1})$ in eq. (A.1). We presume that $\not{\mathcal{L}}$ in eq. (A.2) can take the most general form consistent with sacred symmetries such as gauge invariance and possibly supersymmetry, even though existing calculations [4] may have additional restrictions and hence suggest smaller QMV effects in some specific circumstances.

Estimates of the magnitude of QMV effects are clearly very model-dependent, since they involve all physics between here and the Planck scale. It it even possible that quantum mechanics may break down before the Planck scale, which would presumably imply larger QMV effects. Furthermore, our estimation efforts are hindered by the lack of a suitable unquantum field-theoretical formalism. We develop our estimates in three stages, starting with just the known low-mass quark and lepton fields, continuing by adding in conjectural low-mass scalar fields, and finishing by imposing supersymmetry. To minimize the required dimensionality of $\not{\mathcal{L}}$ we consider effects with $M = 2$ which correspond to quark masses in conventional field theory. Our first model for $\not{\mathcal{L}}$ is therefore

$$\not{\mathcal{L}}_1 \ni O\left(\frac{1}{m_P^2}\right) |\bar{q}_L q_R \, , \, \bar{q}_R q_L|^2 \quad . \tag{A.3}$$

When we compute $\delta\not{S}$ or $\delta\not{H}$ for a neutron or K^o from (A.3) we expect typical hadronic factors $O(1)$ GeV to appear in the numerator. Therefore we expect that

$$|\delta\not{H}_1| \equiv \alpha, \beta, \gamma = O\left(\frac{1\,(\mathrm{GeV})^3}{m_P^2}\right) = O(10^{-38})\ \mathrm{GeV} \tag{A.4}$$

in this model, which should probably be regarded as an upper bound. Gauge and chiral symmetry keep quark masses small in field theory. No analogous chiral suppressions are necessary in (A.3) because of its real structure, but we could be unlucky. The extimate (A.4) is in any case many orders of magnitude below the present experimental upper limits (3.4),(3.26) and hence discouragingly inaccessible.

It is probably also unrealistic in that one expects low-mass scalar fields ϕ to exist: certainly Higgses H and possibly scalar partners $\phi_{q,\ell}$ of quarks and leptons. In this case one might expect a contribution

$$\not{\mathcal{L}}_2 \ni O(1)\,(|\phi|^2)^2 \quad . \tag{A.5}$$

To make contact with external hadrons we need to dress $\not{\mathcal{L}}_2$ (A.5) by two-loop diagrams. If the ϕ in (A.5) are Higgs fields these are of the form shown in fig. 2(a) and introduce external factors yielding a four-fermion operator of order

$$\left(\frac{\alpha}{4\pi}\right)^2 \frac{m_q^6}{m_W^8} \, |\bar{q}_L q_R \, , \, \bar{q}_R q_L|^2 \quad . \tag{A.6}$$

Putting in the hadronic factors $O(1)$ GeV we get

$$|\delta \not{H}_2| = O\left[\left(\frac{\alpha}{4\pi}\right)^2 \frac{m_q^6}{m_W^8} \times 1 \text{ GeV}^3\right] = O(10^{-28}) \text{ GeV} \tag{A.7}$$

if we take $m_q \approx 100$ MeV. This estimate is also smaller than our upper limits (3.4) and (3.26) despite the absence (A.5) of any m_P^{-n} factors. We would get an estimate much larger than (A.7) if there were light squark fields: $m_{\tilde{q}} = O(m_W)$. In this case the analogous two-loop diagram of fig. 2(b) with gluinos can be estimated to give

$$|\delta \not{H}_2| = O\left[\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{1}{m_W^2} \times 1 \text{ GeV}^3\right] = O(10^{-8}) \text{ GeV} \tag{A.8}$$

which is catastrophically larger than our phenomenological bounds (3.4) and (3.26). It might happen that the gluino loops in fig. 2(b) vanish for some reason [13] along with other gaugino loops. In that case the dominant contribution might come from Higgsino loops of order

$$|\delta \not{H}_2| = O\left[\left(\frac{\alpha}{4\pi}\right)^2 \frac{m_q^4}{m_W^6} \times 1 \text{ GeV}^3\right] = O(10^{-22}) \text{ GeV} \tag{A.9}$$

which is encouragingly close to our bounds (3.4) and (3.26) despite the absence of any explicit inverse m_P factor.

We have no reason to expect light squark fields to exist unless we adopt a supersymmetric framework which would disfavor a contribution to $\not{\mathcal{L}}$ of the form (A.5). Surely we would anticipate that any Planck scale QMV effects in a supersymmetric theory should also obey supersymmetric constraints. Supersymmetric contributions to

$\not{L}$ can come from integrals over superspace of the form $\int d^4\theta$ or $\int d^2\theta$. The former would have the structure of generalized kinetic terms with the generic structure

$$O\left(\frac{1}{m_P^{2n}}\right) |\partial\phi|^2 \ \phi^{2n} \tag{A.10}$$

which are too small to be interesting. Alternatively, we could pick up a term analogous to a quartic term in a superpotential:

$$\int d^2\theta \ P(\phi) : P \ni O\left(\frac{1}{m_P}\right) qqq^c q^c \tag{A.11}$$

which would yield the interesting interaction

$$\not{L}_3 \ni O\left(\frac{1}{m_P}\right) qq^c \ \phi_q \ \phi_{q^c} \quad . \tag{A.12}$$

This may be dressed into the one-loop diagram of fig. 2(c) which gives us the estimate

$$|\delta\not{H}_3| = O\left(\frac{\alpha}{4\pi} \frac{m_q^2}{m_W^3} \frac{1 \ \mathrm{GeV}^3}{m_P}\right) > O(10^{-30}) \ \mathrm{GeV} \quad . \tag{A.13}$$

This may be the most reasonable estimate we have found, even if it is not the most encouraging. We should perhaps warn that even this may be an over-estimate. It has been argued elsewhere [13] that any quartic superpotential term of the form $qqq\ell$ can have a coefficient no larger than $O(10^{-6}/m_P)$ if it is to be compatible with the experimental lower limit on the nucleon lifetime. An analogous suppression in the expression (A.11) would suppress the estimate (A.13) of $|\delta\not{H}_3|$ by another six orders of magnitude. This is unfortunate, but we do not think that this or any other of our estimates should be taken too seriously, and certainly the intelligent reader will not have read this far.

The most level-headed attitude towards these estimates may be to observe that they vary over a typical range of order $(1 \ \mathrm{GeV}/m_P)$ or 19 orders of magnitude. Looking at this logarithmically, an experiment which aims to improve on the existing (3.4) and (3.26) limits by two orders of magnitude may have a likelihood of order 10% to uncover a violation of quantum mechanics.

REFERENCES

[1] S. Hawking, Commun. Math. Phys. 43 (1975) 199; R. M. Wald, Commun. Math. Phys. 45 (1975) 9.

[2] J. H. Bardeen, B. Carter and S. Hawking, Commun. Math. Phys. 31 (1973) 181.

[3] J. A. Wheeler, *Relativity, Groups and Topology*, ed. B. S. and C. M. deWitt (Gordon and Breach, N.Y., 1963).

[4] S. Hawking, D. N. Page and C. N. Pope, Phys. Lett. 86B (1979) 175; and Nucl. Phys. B170 (FS1) (1980) 283.

[5] S. Hawking, Commun. Math. Phys. 87 (1982) 395.

[6] K. Kleinknecht, Proceedings of the XVII International Conference on High Energy Physics, London 1974, ed. J. R. Smith (S.R.C., 1974), p. III-23.

[7] R. Colella, A. W. Overhauser and S. A. Werner, Phys. Rev. Lett. 34 (1975) 1472; J.-L. Staudenmann, S. A. Werner, R. Colella and A. W. Overhauser, Phys. Rev. 21A (1979) 1419.

[8] See for example, P. Pearle, Phys. Rev. D13 (1976) 857; I. Bialynicki-Birula and J. Mycielski, Ann. Phys. (N.Y.) 100 (1976) 62; and Phys. Scr. 20 (1979) 539.

[9] V. Gorini, A. Frigerio, M. Verri, A. Kossakowski and E. C. G. Sudarshan, Rep. on Math. Phys. 13 (1978) 149.

[10] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47 (1935) 777.

[11] F. Combley and E. Picasso, Phys. Rep. 14C (1974) 1.

[12] J. S. Bell and J. Steinberger, Proceedings of the 1965 Oxford Conference on Elementary Particles (Rutherford Laboratory, 1966), p. 195.

[13] J. Ellis, J. S. Hagelin, D. V. Nanopoulos and K. A. Tamvakis, Phys. Lett. 124B (1983) 484.

[14] J. S. Bell, Physics 1 (1965) 195.

[15] T. Banks, M. E. Peskin and L. Susskind, work in preparation (1983).

[16] J. Ellis and M. K. Gaillard, Phys. Lett. 88B (1979) 315; D. V. Nanopoulos and M. Srednicki, Phys. Lett. 124B (1983) 37.

[17] Particle Data Group, Phys. Lett. 111B (1982) 1.

[18] V. A. Kuzmin, Zh. Eksp. Teor. Fiz. 13 (1970) 335; for an experimental review, see M. Baldo-Ceolin, Proceedings of the International Conference on Unified Theories and Their Experimental Tests, Venice (1982).

[19] R. Bernstein et al., FNAL Experimental Proposal 617 (1979); D. Cundy et al., CERN Experimental Proposal SPSC/P174 (1981).

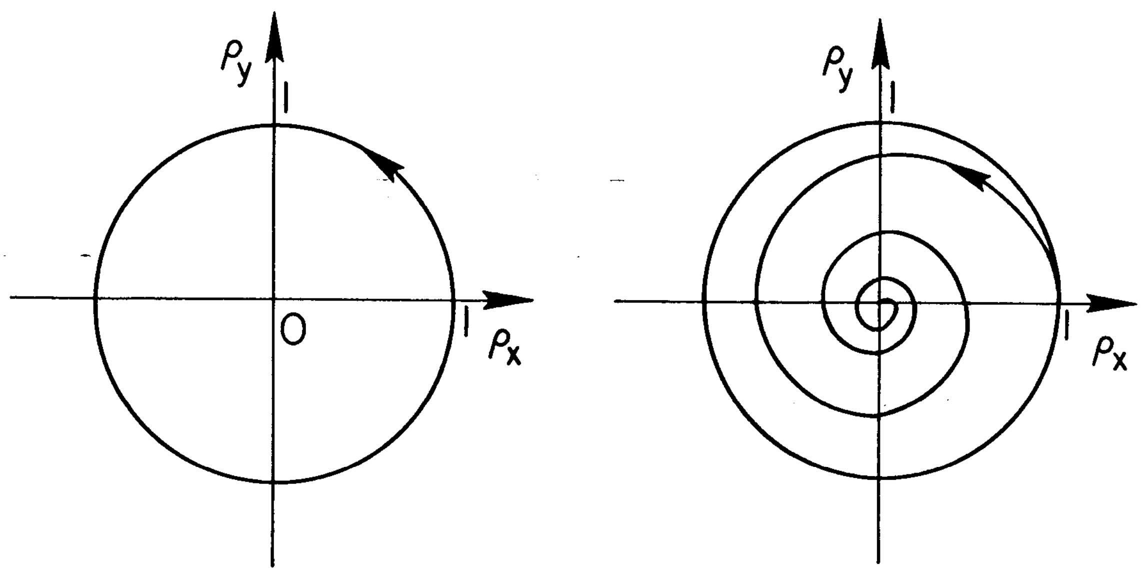
[20] J. Ellis, D. V. Nanopoulos and K. A. Tamvakis, Phys. Lett. 121B (1983) 123.

[21] R. Bionta et al., IMB collaboration Phys. Rev. Lett. 51, 27 (1983).

FIGURE CAPTURES

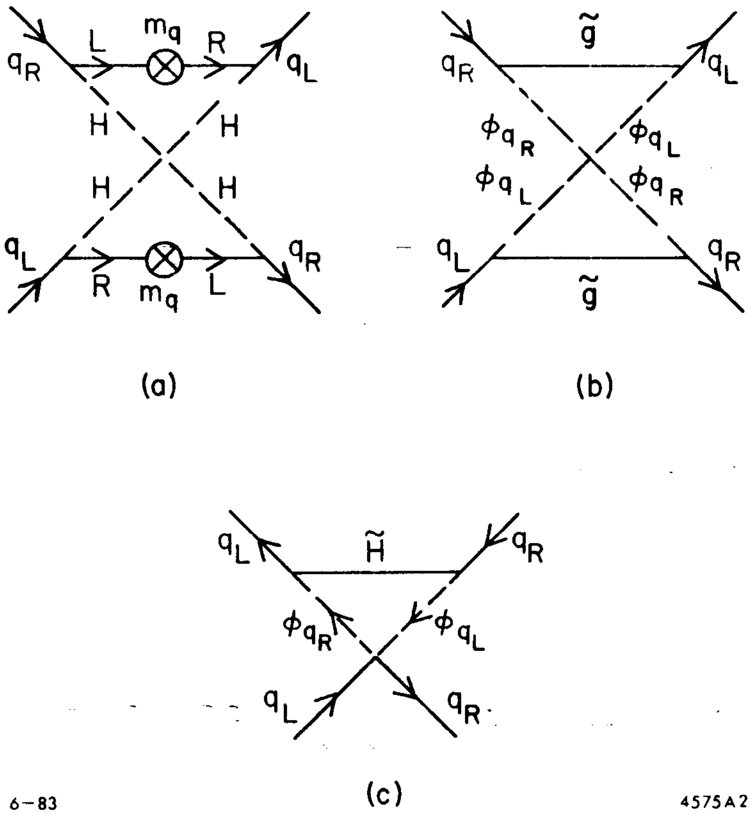
Fig. 1. Time evolution of density matrix elements (a) in quantum mechanics, (b) in unquantum mechanics.

Fig. 2. Diagrams which might give rise to unquantum-mechanical analogues of quark mass terms including (a) a 4-Higgs H vertex (b) a 4-squark $\phi_{\tilde{q}}$ vertex, and (c) a supersymmetric 2-quark, 2-squark vertex; all are dressed by appropriate external loops.



6-83 (a) (b) 4575A1

Fig. 1



q_R
L
m_q
R
q_L
H
H
H
H
q_L
R
m_q
L
q_R
(a)
g̃
q_R
q_L
φq_R
φq_L
φq_L
φq_R
q_L
q_R
g̃
(b)
H̃
q_L
q_R
φq_R
φq_L
q_L
q_R
(c)
6–83
4575A2

Fig. 2