1 — Searching for Patterns
Pacing: 29 Days

Topic 1: Quantities and Relationships

Students analyze scenarios and graphs representing the functions they will study throughout the course. They learn to write equations for functions in function notation. Students recognize that different function families have different key characteristics, and they use graphical behavior to classify functions according to their function families.

Standards: N.Q.1, N.Q.2, A.REI.10, F.IF.1, F.IF.4, F.IF.5   **Pacing:** 8 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**A Picture Is Worth a Thousand Words** Understanding Quantities and Their Relationships	N.Q.1 N.Q.2 A.REI.10 F.IF.1 F.IF.4	1	Students identify the independent and dependent quantities for various real-world scenarios, match a graph to the scenario, and interpret the scale of the axes. They observe similarities and differences in the graphs, and then focus on key characteristics, such as intercepts, increasing and decreasing intervals, and relative maximum and minimum points.	• There are two quantities that change in problem situations. • When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity. • The independent quantity is used to label the x-axis. The dependent quantity is used to label the y-axis. • The domain includes the values that make sense for the independent quantity. The range includes the values that make sense for the dependent quantity. • Graphs can be used to model problem situations.
2	**A Sort of Sorts** Analyzing and Sorting Graphs	F.IF.4	1	Students sort a variety of graphs based on their own rationale, compare their groupings with their classmates', and discuss the reasoning behind their choices. Next, four different groups of graphs are given and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations.	• A relationship between two quantities can be graphed on the coordinate plane. • Graphical behaviors can reveal important information about a relationship. • A graph of a relationship can have a minimum or maximum or no minimum or maximum. A graph can pass through one or more quadrants. A graph can exhibit vertical or horizontal symmetry. And a graph can be increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**F of X** Recognizing Functions and Function Families	F.IF.1 F.IF.4 F.IF.5	2	Function notation is introduced. The terms *increasing function*, *decreasing function*, and *constant function* are defined. Students sort the graphs from the previous lesson into groups using these terms and match each graph with its appropriate equation written in function notation. The terms *function family*, *linear function*, and *exponential function* are then defined. Next, the terms *absolute minimum* and *absolute maximum* are defined. Students sort the remaining graphs into groups using these terms and match each graph with its appropriate equation written in function notation. The terms *quadratic function* and *linear absolute value function* are then defined. Linear piecewise functions are defined, and students match the remaining graphs to their appropriate functions. In the final activity, students demonstrate how the families differ with respect to their intercepts.	• A function is a relation that assigns to each element of the domain exactly one element of the range. • The family of linear functions includes functions of the form $f(x) = mx + b$, where a and b are real numbers. • The family of exponential functions includes functions of the form $f(x) = ab^x + c$, where a, b, and c are real numbers, and b is greater than 0 but is not equal to 1. • The family of quadratic functions includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0. • The family of linear absolute value functions includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0. • Linear piecewise functions include functions that have equation changes for different parts, or pieces, of the domain.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Function Families for 200, Alex** Recognizing Functions by Characteristics	F.IF.4	2	Given characteristics describing the graphical behavior of specific functions, students name the possible function family/families that fit each description. Students revisit the scenarios and graphs from the first lesson, name the function family associated with each scenario, identify the domain, and describe the graph. Students then write equations and sketch graphs to satisfy a list of characteristics. They conclude by determining that a function or equation, not just a list of characteristics, is required to generate a unique graph.	• The graph of an exponential or quadratic function is a curve. • The graph of a linear or linear absolute value function is a line or pair of lines, respectively. • The graph of a linear or exponential function is either increasing or decreasing. • The graph of a quadratic function or a linear absolute value function has intervals where it is increasing and intervals where it is decreasing. Each function also has an absolute maximum or absolute minimum. • Key characteristics of graphs help to determine the function family to which it belongs.
	Learning Individually with MATHia or Skills Practice	N.Q.2 F.IF.1	2	Students answer questions related to two animations — one discussing dependent and independent quantities and slope in a real-world context, and the other investigating the shapes of graphs of functions, which show the linear and non-linear relationships between different quantities in real-world contexts. They study numberless graphs of functions and match the graphs to various situations. Students then answer questions related to an animation describing different function families, their graphs, equations, and general characteristics. **MATHia Unit:** Understanding Quantities and Their Relationships **MATHia Workspaces:** Identifying Quantities **MATHia Unit:** Recognizing Functions and Function Families **MATHia Workspaces:** Evaluating Linear Functions / Identifying Domain and Range / Identifying Key Characteristics of Graphs of Functions / Introduction to Function Families	

*Pacing listed in 45-minute days

Topic 2: Sequences

Students explore sequences represented as lists of numbers, as tables of values, as equations, and as graphs modeled on the coordinate plane. Students recognize that all sequences are functions. They also recognize the characteristics of arithmetic and geometric sequences and learn to write recursive and explicit formulas for both. They learn to use a modeling process as a structure for approaching real-world mathematical problems.

Standards: F.IF.3, F.IF.5, F.BF.1a, F.BF.2   **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Is There a Pattern Here?** Recognizing Patterns and Sequences	F.IF.3 F.IF.5 F.BF.1a	2	Given ten contexts or geometric patterns, students write a numeric sequence to represent each problem. They represent each sequence as a table of values, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. They determine that all sequences are functions and have a domain that includes only positive integers. Infinite sequence and finite sequence are defined.	• A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. • A term of a sequence is an individual number, figure, or letter in the sequence. • A sequence can be written as a function. The domain includes only positive integers. • An infinite sequence is a sequence that continues forever, or never ends. • A finite sequence is a sequence that terminates, or has an end term.
2	**The Password Is … Operations!** Arithmetic and Geometric Sequences	F.BF.2	2	Given 16 numeric sequences, students generate additional terms and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics and explain their rationale. The terms arithmetic sequence, common difference, geometric sequence, and common ratio are defined with examples. They then categorize the given sequences based on the definitions and identify the common difference or common ratio where appropriate. Students then practice writing sequences with given characteristics.	• An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a positive or negative constant. This constant is called the common difference and is represented by the variable d. • A geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant. This constant is called the common ratio and is represented by the variable r . • The graph of a sequence is a set of discrete points. • The points of an arithmetic sequence lie on a line. When the common difference is a positive, the graph is increasing, and when the common difference is a negative, the graph is decreasing. • The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.
3	**Did You Mean: *Recursion*?** Determining Recursive and Explicit Expressions from Contexts	F.BF.1a	1	Scenarios are presented that can be represented by arithmetic and geometric sequences. Students determine the value of different terms in each sequence. As the term number increases it becomes more timeconsuming to generate the term value, which sets the stage for explicit formulas to be defined and used. Students practice using these formulas to determine the values of terms in both arithmetic and geometric sequences.	• A recursive formula expresses each new term of a sequence based on a preceding term of the sequence. • An explicit formula for a sequence is a formula for calculating each term of the sequence using the term's position in the sequence. • The explicit formula for determining the nth term of an arithmetic sequence is $a_n = a_1 + d(n-1)$, where n is the term number, a_1 is the first term in the sequence, a_n is the nth term in the sequence, and d is the common difference. • The explicit formula for determining the nth term of a geometric sequence is $g_n = g_1 \cdot r^{n-1}$, where n is the term number, g_1 is the first term in the sequence, g_n is the nth term in the sequence, and r is the common ratio.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**3 Pegs, N Discs** Modeling Using Sequences	F.BF.2	2	Students are introduced to the process of mathematical modeling, with each of the four activities representing a specific step in the process. Students are invited to play a puzzle game, observe patterns, and think about a mathematical question. Students then organize their information and pursue a given question by representing the patterns they noticed using mathematical notation. As a third step, students analyze their recursive and explicit formulas and use them to make predictions. Finally, students test their predictions and interpret their results.	• Mathematical modeling involves noticing patterns and formulating mathematical questions, organizing information and representing this information using appropriate mathematical notation, analyzing mathematical representations and using them to make predictions, and then testing these predictions and interpreting the results. • Both recursive and explicit formulas can be used for sequences that model situations. • Sequence formulas can be used to make predictions for real-world situations.
	Learning Individually with MATHia or Skills Practice	F.IF.3 F.BF.1a	3	In the MATHia software, students determine and classify the patterns in sequences and identify the next terms. They then determine the recursive and explicit formulas for given sequences. **MATHia Unit:** Recognizing Patterns and Sequences **MATHia Workspaces:** Describing Patterns in Sequences / Graphs of Sequences **MATHia Unit:** Determining Recursive and Explicit Expressions **MATHia Workspaces:** Writing Recursive Formulas / Writing Explicit Formulas	

*Pacing listed in 45-minute days

Topic 3: Linear Regressions

Students formalize their understanding of how to use lines of best fit to model bivariate data. Students use technology to generate linear regressions. To determine the appropriateness of a line to model a given data set, they analyze the shape of the scatterplot, calculate and assess the correlation coefficient, and build and evaluate residual plots. Students differentiate between correlation and causation, recognizing that a correlation between two quantities does not necessarily mean that there is also a causal relationship.

Standards: N.Q.3, S.ID.6, S.ID.6a, S.ID.6b, S.ID.6c, S.ID.7, S.ID.8, S.ID.9   **Pacing:** 11 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Like a Glove** Least Squares Regressions	N.Q.3 S.ID.6a S.ID.6c S.ID.7	2	Students informally approximate a line of best fit for a given data set, write an equation for their line, and then use their function to make predictions, learning about interpolation and extrapolation. They are then introduced to a formal method to determine the linear regression line of a data set via graphing technology.	• Interpolation is the process of using a regression equation to make predictions within the data set. • Extrapolation is the process of using a regression equation to make predictions beyond the data set. • A least squares regression line is the line of best fit that minimizes the squares of the distanced of the points from the line.
2	**Gotta Keep It Correlatin'** Correlation	N.Q.3 S.ID.6a S.ID.6c S.ID.8 S.ID.9	2	Students analyze graphs and estimate a reasonable correlation coefficient based on visual evidence. They then use technology to determine a linear regression and interpret the correlation coefficient. Next, students analyze several problem situations to determine whether correlation is always connected to causation.	• A correlation is a measure of how well a regression model fits a data set. • The correlation coefficient, r, is a value between 21 and 1 that indicates the type (positive or negative) of association and the strength of the relationship. Values close to 1 or 21 demonstrate a strong association, while a value of 0 signifies no association. • Causation is when one event causes a second event. A correlation is a necessary condition for causation, but not a sufficient condition for causation. • Two relationships are that are often mistaken for causation are a common response, when some other reason may cause the same result, and a confounding variable, when there are other variables that are unknown or unobserved.
3	**The Residual Effect** Creating Residual Plots	S.ID.6 S.ID.6a S.ID.6b S.ID.6c	2	Students calculate a linear regression for a real-world problem and analyze the correlation coefficient to conclude whether the linear model is a good fit. The terms *residual* and *residual plot* are defined. Students calculate the residuals, construct a residual scatter plot, and conclude by its shape that there may be a more appropriate model. They are given a second data plot. Students create a scatter plot and determine the equation for the least squares regression line and the correlation coefficient with respect to the problem situation. They then calculate residuals and create a residual plot to conclude how the data are related.	• A residual is the distance between an observed data value and its predicted value using the regression equation. • Analyzing residuals is a method to determine whether a linear model is appropriate for a data set. • A residual plot is a scatter plot of the independent variable on the *x*-axis and the residuals on the *y*-axis. • The shape of a residual plot can be useful to determine whether there may be a more appropriate model than a line of best fit for a data set.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**To Fit or Not to Fit? That Is the Question!** Using Residual Plots	N.Q.3 S.ID.6 S.ID.6a S.ID.6b S.ID.6c	2	Students construct a scatter plot, determine a linear regression equation, compute the correlation coefficient, determine the residuals, and create a residual plot for a data set with one variable. They use all of the given information to decide whether a linear model is appropriate. A quadratic function is given and students conclude that this type of function appears to be a better fit. Finally, students summarize how the shape of a scatter plot, the correlation coefficient, and the residual plot help determine whether a linear model is an appropriate fit for the data set. The lesson emphasizes the importance of using more than one measure to determine if a linear model is a good fit.	• Scatter plots, regression functions, correlation coefficients, residuals, and residual plots are used to determine the appropriate model of best fit. • It is important to use several measures to determine the appropriate model of best fit.
	Learning Individually with MATHia or Skills Practice	S.ID.6 S.ID.6a S.ID.6b S.ID.6c S.ID.7 S.ID.8 S.ID.9	3	Students investigate linear regression functions. Students enter data related to various real-world contexts and use an Explore Tool to analyze the linear trend present in the data set, as given by the regression function. Students investigate how moving the points of the data set affects the slope of the regression line, and they analyze the effect of outliers on the regression function. Students are given a table of data and a linear regression equation that represents the line of best fit, which they use to interpolate and extrapolate values. Students solve problems in context, giving rough estimates of the value of r, stating how the estimate is reflected in the table of values, and determining whether the linear regression equation is appropriate for the data set. Students analyze a scatter plot and line of best fit, a table comparing the data with the residuals, and a residual plot. **MATHia Unit:** Least Squares Regression **MATHia Workspaces:** Exploring Linear Regression / Using Linear Regression **MATHia Unit:** Correlation **MATHia Workspaces:** Interpreting Lines of Best Fit / Correlation and Causation **MATHia Unit:** Creating Residual Plots **MATHia Workspaces:** Analyzing Residuals of Lines of Best Fit	

*Pacing listed in 45-minute days

2 Exploring Constant Change
Pacing: 44 Days

Topic 1: Linear Functions

Students connect arithmetic sequences to linear functions, proving that the constant difference, d, is always equal to the slope, m, of the corresponding linear function. Students examine the structure of equations representing functions and compare the graphs to determine what their differences indicate about the functions and the scenarios they model. Students are introduced to transformation notation, $y = A \cdot f(B(x - C)) + D$, although only the A-value (vertical dilations and/or reflections across the x-axis) and the D-value (vertical translations) are explicitly addressed.

The topic concludes with students comparing key characteristics of linear functions presented in different forms.

Standards: N.Q.1, A.SSE.1, A.CED.1, A.REI.10, F.IF.1, F.IF.2, F.IF.3, F.IF.4, F.IF.6, F.IF.7a, F.IF.9, F.BF.3, F.LE.1a, F.LE.1b, F.LE.2   **Pacing:** 10 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Connecting the Dots** Making Connections Between Arithmetic Sequences and Linear Functions	F.IF.1 F.IF.3 F.IF.6 F.LE.1a F.LE.1b F.LE.2	2	The lesson builds from what students know about arithmetic sequences to a general understanding of linear functions. Students connect an arithmetic sequence written in explicit form to a linear function in slope-intercept form. They compare the terms of each equation and prove that the common difference and the slope are always constant and equal. First differences is defined as a strategy to determine whether a table represents a linear relationship. Average rate of change is defined and presented graphically. Finally, students use what they know about arithmetic sequences to complete a graphic organizer to summarize the characteristics and representations of linear functions.	• The explicit formula of an arithmetic sequence can be rewritten as the slope-intercept form of a linear function using algebraic properties. • The explicit formula of an arithmetic sequence, $a_n = a_1 + d(n - 1)$, includes the first term of the sequence, $f(1)$, and the common difference. The slope-intercept form of a linear function, $f(x) = mx + b$, includes $f(0)$ and the slope. • Both the average rate of change formula and slope formula calculate the unit rate over a given interval. The average rate of change formula refers to the dependent variable as $f(x)$, while the slope formula uses y. • First differences is a strategy to determine whether a table of values can be modeled by a linear function. First differences are the values determined by subtracting consecutive output values when the input values have an interval of 1. • The domain of an arithmetic sequence is consecutive integers beginning with 1, while the domain of a linear function may include all real numbers.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
2	**Fun with Functions, Linear Ones** Making Sense of Different Representations of a Linear Function	N.Q.1 A.SSE.1a A.CED.1 A.REI.10 F.IF.2 F.IF.4 F.BF.3	2	Students determine whether functions represented as scenarios, equations, or graphs are linear functions. They extend what they learned about first differences to analyze tables with input values that are not consecutive integers. Students then analyze a scenario and graph that can be represented by a function in the form $f(x) = ax$. A new scenario requires an equation in the form $f(x) = a(x - c)$. They analyze the meaning of this shift in the graph in terms of the context and compare the structure to that of $f(x) = ax + b$. The scenario changes a second time, and the students explore an equation in the form $f(x) = a(x - c) + d$.	• If a table represents a linear function, the slope, or average rate of change, is constant between all given points. • Using an equation to solve for the independent value given the dependent value always results in an exact answer. Using a graph or a table to determine the independent value sometimes results in an exact answer. • The graph of an equation plotted on the coordinate plane represents the set of all its solutions. • The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient, which describes the steepness and direction of the line. The b-value describes the y-intercept. • The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. When a polynomial is in factored form, the value of x that makes each factor equal to zero is the x-intercept. This value is called the zero of the function. • A linear function is a polynomial with a degree of one.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Get Your Move On** Transforming Linear Functions	F.IF.4 F.IF.7a F.BF.3	2	Students identify key characteristics of several linear functions. A graph and a table of values for the basic linear function $f(x) = x$ are given, and students investigate $f(x) + D$ and $A \cdot f(x)$. Given a function $g(x)$ in terms of $f(x)$, students graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$. Finally, students use their knowledge of linear function transformations to test a video game that uses linear functions to hit targets. Students write the function transformations several ways and identify the domains, ranges, slopes, and y-intercepts of the new functions.	• For the basic function $f(x) = x$, the transformed function $y = f(x) + D$ affects the output values of the function. For $D > 0$, the graph vertically shifts up. For $D < 0$, the graph vertically shifts down. The amount of shift is given by $|D|$. • For the basic function $f(x) = x$, the transformed function $y = Af(x)$ affects the output values of the function. For $|A| > 1$, the graph stretches vertically by a factor of A units. For $0 < |A| < 1$, the graph compresses vertically by a factor of A units. For $A < 0$, the graph reflects across the x-axis.
4	**Connect Four** Comparing Linear Functions in Different Forms	N.Q.1 F.IF.9	1	Students compare linear functions represented in different forms to answer questions about real-world scenarios. They also identify the scale and origin on the graph of a function given a situation description. Finally, students generate and compare their own linear functions using tables, graphs, and equations.	• Functions can be represented using tables, equations, graphs, and with verbal descriptions. • Features of linear functions such as y-intercepts, slope, independent quantities, and dependent quantities can be determined from different representations of functions.

*Pacing listed in 45-minute days

Learning Individually with MATHia or Skills Practice	A.REI.D.10 F.IF.1 F.IF.2 F.IF.6 F.IF.9 F.BF.3	3	Students use an Explore Tool to investigate linear functions. They analyze and compare the *x*- and *y*-intercepts, domains, ranges, and slopes of linear functions. They evaluate functions using function notation. Students use an interactive function machine and graph to identify transformations of functions. **MATHia Unit:** Connecting Arithmetic Sequences and Linear Functions **MATHia Workspaces:** Writing Sequences as Linear Functions / Understanding Linear Functions / Equal Differences Over Equal Intervals **MATHia Unit:** Multiple Representations of Linear Functions **MATHia Workspaces:** Multiple Representations of Linear Functions / Modeling Linear Relationships Using Multiple Representations **MATHia Unit:** Transforming Linear Functions **MATHia Workspaces:** Exploring Graphs of Linear Functions / Vertically Translating Linear Functions / Vertically Dilating Linear Functions / Multiple Transformations of Linear Functions **MATHia Unit:** Comparing Linear Functions in Different Forms **MATHia Workspaces:** Comparing Linear Functions in Different Forms

Topic 2: Solving Linear Equations and Inequalities

Students use the Properties of Equality to justify the steps to solve one-variable equations. They examine and compare the structure of equations that have one, zero, and infinitely many solutions. Students use what they know about solving equations to solve literal equations for variables of interest and to convert between common formulas. When constraints are put on a scenario, students connect what they know about equations to solve linear inequalities. They learn how a negative coefficient on the variable affects the inequality sign and solve more complex inequalities. Finally, students solve compound inequalities and represent their solutions on number lines.

Standard: N.Q.1, N.Q.3, A.CED.1, A.CED.3, A.CED.4, A.REI.1, A.REI.3   **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Strike a Balance** Solving Linear Equations	A.CED.1 A.REI.1 A.REI.3	1	Students are given a mathematical sentence that is always true and one that is always false. They choose any variable or constant and use the Properties of Equality to investigate ways to change the outcome of the given number sentences. Students reason that the mathematical sentence that is always true is still always true and that one that is false is still false. The terms *no solution* and *infinite solutions* are defined. Finally, students play Tic-Tac-Bingo as they work together to create equations with given solution types from assigned expressions.	• A solution to an equation is any variable value that makes that equation true. • Solving equations requires the use of number properties and the Properties of Equality. • The Properties of Equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality. • When the Properties of Equality are applied to an equation, the transformed equation will have the same solution as the original equation. • Equations with infinite solutions are created by equating two equivalent expressions. • Equations with no solution are created by equating expressions of the form $ax + b$ with the same value for a and different values for b. • Equations with a solution $x = 0$ are created by equating expressions of the form $ax + b$ with different values for a and the same value for b.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
2	**It's Literally About Literal Equations** Literal Equations	N.Q.1 A.CED.4	1	Students identify the slope and intercepts of functions in general, factored, and standard form. They determine the same characteristics for the equation $Ax + By = C$. They then explain which form is more efficient in determining the slope and the x- and y-intercepts. Next, the term *literal equation* is defined. Students rewrite different literal equations to solve for given variables.	• The general form of a linear equation is $y = ax + b$, where a and b are real numbers; a represents the slope, and b represents the y-intercept. • The factored form of a linear equation is $y = a(x - c)$, where a and b are real numbers; a represents the slope, and c represents the x-intercept. • The standard form of a linear equation is $Ax + By = C$ where A is a positive integer, B and C are integers and both A and $B \neq 0$. It can be rewritten in general form as $y = (-A/B)x + C/B$; $-A/B$ represents the slope, C/B represents the y-intercept, and C/A represents the x-intercept. • General form of a linear equation is most useful form to identify the slope and y-intercept. Factored form of a linear equation is the most useful form to identify the slope and x-intercept. • Literal equations can be rewritten to highlight a specific variable.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Not All Statements Are Made Equal** Modeling Linear Inequalities	N.Q.3 A.CED.1 A.CED.3 A.REI.3	2	Students use the graph of a function modeling a scenario with a positive rate of change to determine solutions to linear inequalities. The term solve an inequality is defined. Students write and solve two-step inequalities algebraically, choosing the most accurate solution in the context of the problem situation. Students solve linear inequalities for a scenario with a negative rate of change that affects the sign of the inequality. Finally, they solve linear inequalities that require more than two steps to solve.	• A linear inequality context can be modeled with a table of values, a graph on a coordinate plane, a graph on a number line, and with an inequality statement. • Solutions to linear inequalities can be determined both graphically and algebraically; they can be expressed using a number line or inequality statement. • The steps to solving a linear inequality algebraically are the same steps to solve a linear equation, except that when solving a linear inequality with a negative rate of change, the inequality sign of the solution must be reversed to accurately reflect the relationship.
4	**Don't Confound Your Compounds** Solving and Graphing Compound Inequalities	N.Q.3 A.CED.1 A.REI.3	2	The term compound inequality is defined. Students determine the inequality symbols that complete statements about a scenario represented by compound inequalities and express them in compact form. Given a scenario, they express the inequalities using symbols, then solve and graph the inequalities. The terms *solution of a compound inequality*, *conjunction*, and *disjunction* are defined. Students solve and graph compound inequalities, including those written in compact form.	• A compound inequality is an inequality that is formed by the union, "or," or the intersection, "and" of two simple inequalities. • Certain compound inequalities can be written in compact form. • The solution of a compound inequality (conjunction) written in the form $a < x < b$, where a and b are any real numbers, is the part or parts of the solutions that satisfy both of the inequalities. • The solution of a compound inequality (disjunction) written in the form $x < a$ or $x > b$, where a and b are any real numbers, is the part or parts of the solutions that satisfy either of the inequalities.

*Pacing listed in 45-minute days

Learning Individually with MATHia or Skills Practice	A.CED.1 A.REI.3	3	Students use linear equations to model situations given two points or an initial point. They solve for unknown values and represent the solutions as a graph. Students solve two-step linear inequalities and graph simple inequalities involving rational numbers on a number line. They differentiate between conjunctions and disjunctions and write and solve compound inequalities, using graphs to interpret solutions. **MATHia Unit:** Solving Linear and Literal Equations **MATHia Workspaces:** Reasoning About Solving Equations / Solving Linear Equations in a Variety of Forms / Extending Equations to Literal Equations / Solving Literal Equations **MATHia Unit:** Modeling Linear Inequalities **MATHia Workspaces:** Graphing Inequalities with Rational Numbers / Solving Two-Step Linear Inequalities / Representing Compound Inequalities

Topic 3: Systems of Equations and Inequalities

Students build on their current tools for solving systems of equations. They first use equations in standard form to solve systems of equations using linear combinations. Students analyze the structure of equations to select an efficient method to solve: inspection, graphing, substitution, or linear combinations. Students are introduced to two-variable inequalities. They recognize that just as the solutions to a one-variable inequality are a set of numbers, the solutions to a two-variable inequality are a set of ordered pairs. Students solve systems of linear inequalities graphically. They write systems of equations and inequalities for real-world situations and use function notation to solve linear programming problems.

Standards: A.CED.2, A.CED.3, A.REI.5, A.REI.6, A.REI.10, A.REI.11, A.REI.12, F.IF.2, F.IF.7a   **Pacing:** 13 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Double the Fun** Introduction to Systems of Equations	A.CED.2 A.REI.6 A.REI.10 A.REI.11 F.IF.7a	1	Students explore a scenario that can be modeled with a system of linear equations in standard form. They graph the equations using the intercepts. They determine the intersection of the lines graphically and algebraically using substitution. Finally, students write a system of equations for given scenarios and analyze the slopes and y-intercepts and their relevance to the problem situation. They solve each system of equations graphically and algebraically, concluding that for any system there is no solution, one solution, or an infinite number of solutions.	• The standard form of a linear equation is $Ax + By = C$ where A, B, and C are constants and A and B are not both zero. Linear functions written in standard form can be graphed using the x- and y-intercepts. • Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane. • A linear system of equations is two or more linear equations that define a relationship between quantities. The solution of a linear system is an ordered pair that makes both equations in the system true. • Lines that do not intersect describe a system of equations in which each linear equation has the same slope and there is no solution. • Lines intersecting at a single point describe a system of equations in which each linear equation has a different slope and there is one solution. • Lines intersecting at an infinite number of points describe a system of equations in which each linear equation is the same equation and there are an infinite number of solutions. • Consistent systems of equations are systems that have one or many solutions. Inconsistent systems of equations are systems that have no solutions.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
2	**The Elimination Round** Using Linear Combinations to Solve a System of Linear Equations	A.CED.2 A.REI.5 A.REI.6	2	Students explore a system of equations with opposite y-coefficients that is solved for x by adding the equations together. The term *linear combinations method* is defined, and students analyze systems that are solved by multiplying either one or both equations by a constant to rewrite the system with a single variable. Students analyze different systems of equations to determine how they would rewrite the equations to solve for one variable. Next, they apply the linear combinations method to two realworld problems, one with fractional coefficients.	• The linear combination method is a process to solve a system of linear equations by adding two equations together, resulting in an equation with one variable. • When using the linear combination method, it is often necessary to multiply one or both equations by a constant to create two equations in which the coefficients of one of the variables are additive inverses.
3	**Throwing Shade** Graphing Inequalities in Two Variables	A.CED.2 A.CED.3 A.REI.12	2	Scenarios are used that are represented by two-variable inequalities. Students write the inequality, complete a table of values, and use the table of values to graph the situation. The terms *half-plane* and *boundary line* are defined. Students use shading and solid or dashed lines to indicate which regions on the coordinate plane represent solution sets to the problem situation. Multiple representations such as equations, tables, and graphs are used to represent inequalities and their solutions.	• The graph of a linear inequality is a half-plane, or half of a coordinate plane. • Shading is used to indicate which half-plane describes the solution to the inequality. • Dashed and solid lines are used to indicate if the line itself is included in the solution set of an inequality. • Linear inequalities and their graphs can be used to represent and solve problems in context.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Working with Constraints** System of Linear Inequalities	A.CED.3 A.REI.12	2	The term *constraints* is defined. Students write a system of linear inequalities to model a scenario, and graph the system, determining that overlapping shaded regions identify the possible solutions to the system. They practice graphing several systems of inequalities and determining the solution set. Finally, students match systems, graphs, and possible solutions of systems.	• In a system of linear inequalities, the inequalities are known as constraints because the values of the expressions are constrained to lie within a certain region. • The solution of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersecting region satisfies all inequalities in the system.
5	**Working the System** Solving Systems of Equations and Inequalities	A.CED.3 A.REI.6 A.REI.12	1	Students write a system of linear equations for each of three different scenarios: one in the form $y = ax + b$, one in the form $y = a(x - c) + b$, and one in the form $y = a(cx) + b$. They use any method to solve the system before reasoning about the solution in terms of the problem context. Students write a system composed of four linear inequalities to model a scenario and graph the system. Students determine the correct region that contains the solution set that satisfies all of the inequalities in the system.	• Contexts about choosing between two options can sometimes be modeled by a system of linear equations or inequalities. • The point of intersection of two lines separates the input values, with x-values less than and x-values greater than the x-value of the point of intersection. The solution to a problem in context may be dependent upon where the input values lie relative to the point of intersection. • Based upon a context, the solution of a system may be represented by inequalities rather than a single coordinate pair.

*Pacing listed in 45-minute days

CARNEGIE LEARNING

6	**Take It to the Max … or Min** Linear Programming	A.CED.3 A.REI.11 A.REI.12 F.IF.2	1	Students are introduced to function notation for two variables and the term *linear programming* is defined. They define variables and identify the constraints as a system of linear inequalities for different scenarios. Students then graph the solution region of the system and label all points of intersection of the boundary lines, identifying the vertices of the solution region. They write a function to represent the profit or cost and substitute each of the four vertices into the equation of the function to determine a maximum profit or a minimum cost.	• Linear programming is a branch of mathematics that determines the maximum and minimum value of linear expressions on a region produced by a system of linear inequalities. • Real-world problems that involve determining maximum profit or minimum costs may be solved using linear programming. • Linear programming involves determining the solution to a system of linear inequalities, identifying the vertices of its solution region, and substituting the coordinates of each vertex into an algebraic expression to determine a maximum or minimum value.
	Learning Individually with MATHia or Skills Practice	A.REI.5 A.REI.6 A.REI.11 A.REI.12	4	Students reason with linear functions and their graphs to solve systems of two linear functions in real-world contexts. They then solve systems of linear equations using substitution or linear combinations. Students graph and solve linear inequalities in two variables graphically by determining the correct half-planes for the solution sets. Students solve systems of linear inequalities by determining the intersections of two inequalities in two variables. **MATHia Unit:** Introduction to Systems of Linear Equations **MATHia Workspaces:** Representing Systems of Linear Functions / Modeling Linear Systems Involving Integers / Solving Linear Systems Using Substitution **MATHia Unit:** Using Linear Combinations to Solve a System of Linear Equations **MATHia Workspaces:** Solving Linear Systems Using Linear Combinations / Solving Linear Systems Using Any Method **MATHia Unit:** Graphing Linear Inequalities in Two Variables **MATHia Workspaces:** Exploring Linear Inequalities / Graphing Linear Inequalities in Two Variables **MATHia Unit:** Graphing a System of Linear Inequalities **MATHia Workspaces:** Systems of Linear Inequalities / Interpreting Solutions to Systems of Inequalities	

*Pacing listed in 45-minute days

Topic 4: Functions Derived from Linear Relationships

Students connect the absolute value of a number to the absolute value of a function. They examine the graph of absolute value functions, noting the reflection of the negative *y*-values across the *x*-axis. The visual representation allows students to see that each output corresponds to two input values. From this intuition, students solve absolute value equations and inequalities. Students extend their understanding of transforming linear functions to transform absolute value functions, now considering the effect of adding a constant to the argument. Linear piecewise functions are introduced, and students analyze the restricted domains of each piece. They then write absolute value functions as piecewise functions to connect the two concepts. Step functions are introduced as a special case of piecewise functions. Finally, students reflect functions across $y = x$ to generate the inverse of a linear function.

Standards: A.CED.2, A.CED.3, A.REI.11, F.IF.4, F.IF.7b, F.BF.3, F.BF.4a   **Pacing:** 12 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Putting the V in Absolute Value** Defining Absolute Value Functions and Transformations	F.IF.7b F.BF.3	2	Students model absolute value functions and their transformations on a human coordinate plane. They explore and analyze different transformations of absolute value functions, their graphs, and equations, and summarize the effects of these transformations. By transforming absolute value functions, students distinguish between the effects of changing values inside the argument of the function vs. changing values outside the function.	• An absolute value function is a function of the form $f(x) = |x|$. • A function $g(x)$ of the form $g(x) = f(x) + D$ is a vertical translation of the function $f(x)$. The value $|D|$ describes the number of units the graph of $f(x)$ is translated up or down. If $D > 0$, the graph is translated up; if $D < 0$, the graph is translated down. • A function $g(x)$ of the form $g(x) = Af(x)$ is a vertical dilation of the function $f(x)$. For $|A| > 1$, the graph is vertically stretched by a factor of A units; for $0 < |A| < 1$, the graph vertically compresses by a factor of A units. For $A < 0$, the graph also reflects across the *x*-axis. • A function $g(x)$ of the form $g(x) = f(x - C)$ is a horizontal translation of the function $f(x)$. The value $|C|$ describes the number of units the graph of $f(x)$ is translated right or left. If $C > 0$, the graph is translated to the right; if $C < 0$, the graph is translated to the left. • Transforming a function by changing the A- or D-values affects the output of the function, y. Transforming a function by changing the C-value affects the input of the function, x.
2	**Play Ball** Absolute Value Equations and Inequalities	A.CED.2 A.CED.3 A.REI.11 F.IF.7b	2	Students create a linear absolute value function to model a scenario and use the graph of the function to estimate solutions in the context of the problem situation. They then solve a linear absolute value equation algebraically by first rewriting it as two separate linear equations. One equation represents the case where the value of the expression inside the absolute value is positive, and the second represents the case where it is negative. Students model a scenario with a linear absolute value inequality and its corresponding graph. Finally, they solve linear absolute value inequalities algebraically by first rewriting them as equivalent compound inequalities.	• Linear absolute value equations have 0, 1, or 2 solutions. The solution set for linear absolute value inequalities may contain all real numbers, a subset of the real numbers represented by a compound inequality, or no solutions. • Linear absolute value inequalities can be rewritten as equivalent compound inequalities. • Linear absolute value equations and inequalities can be used to represent real-world situations.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**I Graph in Pieces** Linear Piecewise Functions	F.IF.4 F.IF.7b	2	Students develop a piecewise function from a scenario. The terms *piecewise function* and *linear piecewise function* are defined. They analyze a piecewise graph and write a scenario and piecewise function to represent the graph. Students analyze statements that correspond to different pieces of the graphed function. They then write a scenario that can be modeled with a piecewise function, graph a partner's scenario, and work with their partner to write a piecewise function for each.	• A piecewise function is a function that can be represented by more than one function, where each function corresponds to a specific part of the domain. • To write a piecewise function, you must write the equation and domain for each piece of the function. • Piecewise functions can be represented by scenarios, equations, tables, and graphs. • A linear absolute value function can be constructed using a piecewise function. • Graphing technology can be used to model piecewise functions.
4	**Step by Step** Step Functions	F.IF.7b	2	Students analyze the graph of a special piecewise function. The terms *discontinuous graph* and *step function* are defined, and students interpret those definitions through examination of graphs. Students are then given a context and must provide both the piecewise function and the graph that models it, explaining why the function is a step function. Next, they are introduced to the greatest integer function (floor function) and least integer function (ceiling function) through their definitions, notation, meaning of individual values, graphs and real-world examples.	• A discontinuous graph is a graph that is continuous for some values of the domain with at least one disjoint area between consecutive x-values. • A step function is a piecewise function on a given interval whose pieces are discontinuous constant functions. • The greatest integer function, also known as the floor function, is a special type of step function. The greatest integer function $G(x) = \lfloor x \rfloor$ is defined as the greatest integer less than or equal to x. • The least integer function, also known as the ceiling function, is a special type of step function. The least integer function $L(x) = \lceil x \rceil$ is defined as the least integer greater than or equal to x.
5	**A Riddle Wrapped in a Mystery** Inverses of Linear Functions	F.BF.4a	1	Students use a table of values to determine the inverse of a given problem situation. The term *inverse of a function* is defined and students are shown how to algebraically determine the inverse of a function. They then create the graph of the inverse of a linear function by reflecting the original function across the line $y = x$ using patty paper. The term *one-to-one function* is defined, and students determine whether given functions are one-to-one functions.	• Inverses of functions can be determined algebraically and graphically. • The inverse of a function is determined by replacing $f(x)$ with y, switching the x and y variables, and solving for y. • The graph of the inverse of a function is a reflection of that function across the line $y = x$. • A one-to-one function is a function in which its inverse is also a function. • For a one-to-one function $f(x)$, the notation for its inverse is $f^{-1}(x)$.

*Pacing listed in 45-minute days

Learning Individually with MATHia or Skills Practice	A.CED.3 F.IF.7b	3	Students solve and graph simple absolute value equations in one variable. They use graphical representations to solve absolute value inequalities and learn to write equivalent compound inequalities for absolute value inequalities. Students analyze a linear piecewise function from a real-world scenario. They then match sketches of graphs of linear piecewise functions to given scenarios and identify the graph of a linear piecewise function given an equation. Students identify the domain in both noncontinuous and continuous piecewise functions and determine the equation given a problem situation and a graph. Students identify the step function that represents a given problem situation and graph. **MATHia Unit:** Defining Absolute Value Functions and Transformations **MATHia Workspaces:** Building Absolute Value Functions / Vertically Dilating Absolute Value Functions / Vertically Translating Absolute Value Functions / Horizontally Translating Absolute Value Functions / Multiple Transformations of Absolute Value Functions **MATHia Unit:** Absolute Value Equations and Inequalities **MATHia Workspaces:** Reasoning About Absolute Value Functions / Graphing Simple Absolute Value Equations Using Number Lines / Introduction to Absolute Value Equations / Solving Absolute Value Equations / Reasoning About Absolute Value Inequalities **MATHia Unit:** Linear Piecewise Functions **MATHia Workspaces:** Introduction to Piecewise Functions / Graphing Linear Piecewise Functions / Interpreting Piecewise Functions / Using Linear Piecewise Functions **MATHia Unit:** Step Functions **MATHia Workspaces:** Analyzing Step Functions

*Pacing listed in 45-minute days

3 Investigating Growth and Decay
Pacing: 20 Days

Topic 1: Introduction to Exponential Functions

Students recall geometric sequences and explore their graphs. They learn that some geometric sequences belong to the exponential function family. They learn to rewrite geometric sequences as exponential functions in the form $f(x) = a \cdot b^x$. They examine the structure of exponential expressions, concluding that $2^{(1/2)} = \sqrt{2}$ and $2^{(1/3)} = \sqrt[3]{2}$. Finally, students expand their experiences with function transformations, now including horizontal transformations and dilations in their repertoire. They generalize the effect of these translations on points, sketch graphs of transformed functions, and write equations based on a given sequence of transformations.

Standards: N.RN.1, N.RN.2, N.RN.3, A.CED.1, A.REI.3, A.REI.10, F.IF.4, F.IF.7e, F.IF.8b, F.BF.1a, F.BF.3, F.LE.1a, F.LE.2, F.LE.5   **Pacing: 12 Days**

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**A Constant Ratio** Geometric Sequences and Exponential Functions	A.REI.10 F.BF.1a F.LE.1a F.LE.2 F.LE.5	2	Students learn through investigation that while all geometric sequences are functions, only some geometric sequences can be represented as exponential functions. They identify the constant ratio in different representations of exponential functions and then show algebraically that the constant ratio between output values of an exponential function is represented by the variable b in the function form $f(x) = ab^x$. Students also identify the a-value of that form as the y-intercept of the graph of the function. They learn to write an exponential equation from two given points. The lesson concludes with a comparison of the base of the power in the equation $f(x) = ab^x$, the expression $(f(x + 1))/(f(x))$, and the common ratio of the corresponding geometric sequence.	• All geometric sequences are functions; however, only geometric sequences with a positive constant ratio are exponential functions. • If a geometric sequence represents an exponential function, then the Product of Powers Rule and the definition of negative exponents can be used to rewrite the explicit formula for the sequence as an exponential function. • The form of an exponential function is $f(x) = a \cdot b^x$, where b represents the constant ratio and $(0, a)$ represents the y-intercept. • For an exponential function in the form $f(x) = a \cdot b^x$, the ratio $(f(x + 1))/(f(x))$ is constant and equal to b.
2	**The Power Within** Rational Exponents and Graphs of Exponential Functions	N.RN.1 N.RN.2 N.RN.3 A.CED.1 A.REI.3 F.IF.8b F.LE.2	2	Students explore a scenario that can be represented by the function $f(x) = 2^x$. They use the rules of exponents and their understanding of a constant ratio to determine output values for the exponential function when the input values are non-integers. Students use this exploration to connect expressions with rational exponents to those in radical notation. Students learn the term *horizontal asymptote* and explore this concept on different graphs, analyzing end behavior, particularly as the x-values approach negative infinity. Finally, students practice converting between expressions with rational exponents and those in radical notation and make generalizations about the constant ratio for exponential functions.	• If the difference in the input values is the same, an exponential function shows a constant ratio between output values, no matter how large or how small the gap between input values. • Multiple representations such as tables, equations, and graphs are used to represent exponential problem situations. • Properties of powers can be used to rewrite numeric and algebraic expressions involving integer and rational exponents. • Because rational exponents can be rewritten as radicals, the properties of powers apply to radical expressions, as well. • A rational expression of the form $a^{m/n}$ can be written as a radical expression $\sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$. • Common bases and properties of exponents are used to solve simple exponential equations.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Now I Know My A, B, C, Ds** Transformations of Exponential Functions	F.IF.4 F.IF.7e F.IF.8b F.BF.3	2	Students explore a variety of different transformations of exponential functions, including vertical translations, horizontal translations, vertical reflections and dilations, and horizontal reflections and dilations. For each transformation, students sketch graphs of the transformation, compare characteristics of the transformed graphs with the graph of the parent function, including the horizontal asymptote when appropriate, and write the transformations using coordinate notation. They also consider different ways to rewrite and interpret equations of function transformations. Finally, students summarize the effects of the different transformations at the end of the lesson.	• A reflection of a graph is a mirror image of the graph about a line. The line that the graph is reflected across is called the line of reflection. • Reflections across the x-axis can be expressed using the notation $(x, y) \rightarrow (x, -y)$. It affects the y-coordinate of each point on the graph. • Reflections across the y-axis can be expressed using the notation $(x, y) \rightarrow (-x, y)$. It affects the *x*-coordinate of each point on the graph.
Learning Individually with MATHia or Skills Practice		N.RN.1 N.RN.2 F.IF.3 F.IF.4 F.IF.5 F.IF.6 F.IF.8b F.IF.9 F.BF.3 F.LE.1b F.LE.1c	6	Students write sequences as functions. They compare linear and exponential sequences, functions, tables and graphs, and then sort examples of these based upon whether they show linear or exponential growth. Students identify key characteristics of exponential functions (asymptotes, *x*-intercepts, *y*-intercepts, domain, range, and intervals of increase or decrease) from a function, table or graph. Students analyze exponential functions and calculate the average rate of change for given intervals. They solve problems related to the independent and dependent variables of both linear and exponential functions using graphs and equations. They recognize the difference between scenarios of exponential decay and exponential growth. Finally, students explore transformations of linear and exponential functions, including vertical and horizontal shifts, reflections, and dilations. They transform graphs and functions based on tables, verbal descriptions, graphs, and algebraic representations. Students review the rules of exponents and practice rewriting expressions with radical notation using rational exponents, and then reverse the process and rewrite expressions with rational exponents using radical notation. **MATHia Unit:** Geometric Sequences and Exponential Functions **MATHia Workspaces:** Writing Sequences as Exponential Functions **MATHia Unit:** Rational Exponents **MATHia Workspaces:** Using the Properties of Exponents / Properties of Rational Exponents / Rewriting Expressions with Radical and Rational Exponents / Solving Contextual Exponential Equations Using Common Bases **MATHia Unit:** Transformations of Exponential Transformations **MATHia Workspaces:** Introduction to Transforming Exponential Functions / Vertically Translating Exponential Functions / Horizontally Translating Exponential Functions / Reflecting and Dilating Exponential Functions Using Graphs/ Transforming Exponential Functions Using Tables / Multiple Transformations of Exponential Functions	

*Pacing listed in 45-minute days

Topic 2: Using Exponential Equations

In this topic, students use exponential equations to solve problems. They solve equations of the form $a \cdot b^x = d$ by graphing $y = a \cdot b^x$ and $y = d$ and locating the point of intersection. They examine the structure of two equivalent exponential equations, revealing how rewriting the B-value of an exponential function affects the b-value of $f(x) = a \cdot b^x$. Students then solve real-world problems that can be modeled by exponential functions. They use technology to calculate regression equations and use them to make predictions. Students are reminded of the modeling process for problem solving.

Standards: N.Q.2, N.Q.3, A.SSE.1b, A.SSE.3c, A.CED.1, A.CED.2, A.REI.10, A.REI.11, F.IF.8b, F.BF.1b, F.LE.1c, F.LE.3, F.LE.5, S.ID.6a   **Pacing:** 8 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Uptown and Downtown** Exponential Equations for Growth and Decay	N.Q.3 A.SSE.1b A.CED.1 A.CED.2 F.IF.6 F.LE.1c F.LE.3 F.LE.5	2	Students compare linear and exponential functions in the context of simple interest and compound interest situations. They identify the values in the exponential function equation that indicate whether an exponential function is a growth or decay function, and they apply this reasoning in context.	• Simple interest can be represented by a linear function. Compound interest can be represented by an exponential function. • An exponential growth function is of the form $y = a \cdot (1 + r)^x$, where r is the rate of growth. • An exponential decay function is of the form $y = a \cdot (1 - r)^x$, where r is the rate of decay.
2	**Powers and the Horizontal Line** Interpreting Parameters in Context	A.SSE.3c A.CED.1 A.CED.2 A.REI.10 A.REI.11 F.IF.8b F.LE.5	1	Students match exponential equations to their graphs to discern that the horizontal asymptote is always represented by $y = D$. For exponential growth and decay scenarios, students complete tables of values, graph the functions, and write exponential equations using function notation. Students use graphs to estimate the solutions to equations by graphing both sides of the equation and locating the point of intersection. They use the properties of exponents to rewrite the band B-values of exponential functions in equivalent forms to reveal properties of the quantity represented in the function. This allows them to re-interpret an equation showing annual rates of increase for a mutual fund to show monthly and quarterly rates of increase for the same fund.	• Given an exponential function, $f(x) = A \cdot b^{B(x-C)} + D$, the horizontal asymptote is always represented by the equation $y = D$, and the y-intercept is the A-value plus any vertical translation. • Multiple representations such as tables, equations, and graphs can be used to represent and compare exponential problem situations. • Graphs can be used to solve exponential equations by graphing both sides of the equation and estimating the point of intersection. • A quantity increasing exponentially eventually exceeds a quantity increasing linearly. • Properties of exponential functions can be compared using different representations. • Transforming exponential functions into equivalent forms can reveal different properties of the quantities represented.

*Pacing listed in 45-minute days

3	**Savings, Tea, and Carbon Dioxide** Modeling Using Exponential Functions	N.Q.2 F.BF.1b S.ID.6a	2	Students model two savings scenarios, one by means of an exponential function $f(x)$ and one by a constant function $g(x)$. They then create a third function, $h(x) = f(x) + g(x)$, graph all three functions on the same graph, and explain how they are related. Given a data set, students create a scatter plot, write a regression equation, use the function to make predictions, and interpret the reasonableness of a prediction. The lesson concludes with students generalizing about the common features of scenarios that are modeled by exponential functions. They also describe the shape of a scatter plot representing an exponential function and sketch possible graphs of exponential functions.	• An exponential function and a constant function can be added to create a third function that is the sum of the two functions, resulting in a graph that is a vertical translation of the original exponential function. • Technology can be used to determine exponential regression equations to model real-world situations. The regression equation can then be used to make predictions. • Sometimes referring to the scenario or obtaining further information may be required to determine whether a scatter plot is best modeled by a linear or exponential function.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**BAC Is BAD News** Choosing a Function to Model Data	N.Q.2 S.ID.6a	1	Students are given a context involving the blood alcohol content (BAC) of a driver and the driver's likelihood of causing an accident. Students are then given data from a study connecting BAC and the relative probability of causing an accident. They apply the relationship from the data to create a model predicting the likelihood of a person causing an accident based on their BAC. They summarize their learning by writing an article for a newsletter about the seriousness of drinking and driving. The lesson concludes with students connecting their process for solving the problem to the steps in the mathematical modeling process.	• Determining and using a regression equation is sometimes a step in the process of solving a more complex mathematical problem, rather than the final solution. • The mathematical modeling process is an effective structure to solve complex mathematical problems.
	Learning Individually with MATHia or Skills Practice	N.Q.2 A.CED.1 A.REI.3 A.REI.11	2	Students solve exponential equations in context using common bases and horizontal lines. **MATHia Unit:** Exponential Equations for Growth and Decay **MATHia Workspaces:** Recognizing Linear and Exponential Models / Calculating and Interpreting Average Rate of Change / Recognizing Growth and Decay / Comparing Exponential Functions in Different Forms **MATHia Unit:** Solving Exponential Equations **MATHia Workspaces:** Modeling Equations with a Starting Point of 1 / Modeling Equations with a Starting Point Other Than 1 / Solving Exponential Equations Using a Graph **MATHia Unit:** Modeling Using Exponential Functions **MATHia Workspaces:** Relating the Domain to Exponential Functions / Exploring Exponential Regressions	

*Pacing listed in 45-minute days

4 Describing Distributions
Pacing: 16 Days

Topic 1: One-Variable Statistics

Students create and analyze dot plots, histograms, and box-and-whisker plots to analyze different sets of data. They learn that different data displays are useful with different types of data sets. They learn formal notation for the mean and the formula for standard deviation. Students recognize and extract outliers in skewed data sets, noting that the outliers do not have much impact on the mean and interquartile range of a data set. Students have multiple opportunities to use the statistical process to compare two data sets using shape and measures of center and spread.

Standards: S.ID.1, S.ID.2, S.ID.3   **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Way to Represent** Graphically Representing Data	S.ID.1 S.ID.2	2	The statistical process is reviewed. Students analyze a small data set given by creating a dot plot. Next, a much larger data set for the same scenario is presented in a frequency table. Students construct and analyze a histogram for the data. A worked example shows how to use a five-number summary to create a box-and-whisker plot. Students are given two five-number summaries of data comparing the same variable and use them to construct and analyze two box-and-whisker plots. They then write an analysis comparing the two data sets.	• Discrete data are data that have a finite number of values, or data that can be counted, while continuous data are data that can take any numerical value within a range. • A dot plot is best used to organize and display a small amount of discrete data points. • A histogram is effective in displaying large amounts of continuous data using vertical bars, or bins, representing intervals of data. • A box-and-whisker plot displays the spread of data based on a fivenumber summary consisting of the minimum value, the first quartile (Q1), the median, the third quartile (Q3), and the maximum value. • A box-and-whisker plot is helpful when comparing two large data sets.
2	**A Skewed Reality** Determining the Better Measure of Center and Spread for a Data Set	S.ID.1 S.ID.2 S.ID.3	2	Students are presented with various data displays and predict the location of the mean and median in each display. A worked example presents the formula for calculating the arithmetic mean and introduces students to the formal notation. Students construct a box-andwhisker plot that overlays a given dot plot to analyze the spread of the data points. The term *interquartile range (IQR)* is introduced, and students calculate the IQR for the same data set. They remove any outliers and reanalyze the IQR of the data set. Next, students compare two new data sets displayed in a table and in box-and-whisker plots, removing possible outliers. They then calculate and interpret the standard deviation to compare three symmetric data sets. At the end of the lesson, students know when and how to use mean and standard deviation vs. mean and IQR to describe the center and spread of a data set.	• Extremely high or low data values have a greater effect on the mean than the median of a data set. • The data distribution is the overall shape of a graph. • The median and IQR are the most appropriate measures of center and spread when data have a skewed distribution. • Removing outliers from a data set has a minimal effect, in any, on the IQR. • The mean and standard deviation are the most appropriate measures of center and spread when data have a symmetric distribution.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas

*Pacing listed in 45-minute days

| 3 | **Dare to Compare**

Comparing Data Sets | S.ID.1 S.ID.2 S.ID.3 | 2 | Students conclude that when comparing two data sets, if one data set is skewed, then the median and IQR should be used to compare the sets. Next, students are provided with three scenarios that each compare two different data sets. In the each, students are provided with a table comparing two data sets and must decide which measure of center and spread to use in their comparison. | When analyzing data, it must first be determined whether the data distribution is symmetric or skewed in order to know what measure of center and measure of spread is appropriate. The median and IQR are the appropriate measures of center and spread when comparing data sets where at least one distribution is skewed. Depending upon the context, a smaller or larger measure of center may be desired; however, a smaller measure of spread, either the IQR or standard deviation, is always preferred. If the distribution of a data set appears symmetric once the outliers are removed, it is appropriate to use the mean and standard deviation to analyze the center and spread of the data. |
| **Learning Individually with MATHia or Skills Practice** | | S.ID.2 | 3 | Students practice calculating variance and standard deviation. They then use shape, center, and spread to compare two data sets.

MATHia Unit: Graphically Representing Data
MATHia Workspaces: Creating Frequency Plots / Describing Data Sets

MATHia Unit: Comparing Measures of Center and Spread
MATHia Workspaces Determining Appropriate Measures of Center / Measuring the Effects of Changing Data Sets / Creating Box Plots and Identifying Outliers / Calculating Standard Deviation

MATHia Unit: Comparing Data Sets
MATHia Workspaces Comparing and Interpreting Measures of Center / Comparing Data Sets Using Center and Spread | |

*Pacing listed in 45-minute days

Topic 2: Two-Variable Categorical Data

Students display and analyze categorical data using a number of different distribution types. They begin with frequency and marginal frequency tables. To better compare the joint frequencies, students create relative frequency tables and relative marginal frequency tables. They answer questions about the values in the tables and use the tables to make decisions about the relationship between the variables. To help discern associations between categorical data sets, students create conditional relative frequency tables. Finally, students are presented with a statistical question and data set that they need to organize, analyze, and summarize for a final report.

Standard: S.ID.5   **Pacing:** 7 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**It Takes Two** Creating and Interpreting Frequency Distributions	S.ID.5	2	Students differentiate between questions that are answered with numeric data from those answered with categorical data. They are presented with data expressed as categories rather than numerical values. The terms *two-way frequency table*, *frequency distribution*, *joint frequency*, and *marginal frequency distribution* are defined. Students organize data into a two-way frequency table and create a marginal frequency distribution and bar graphs to answer questions related to the given scenario. Finally, students interpret the data analyzed in the context of the scenario.	• Data sets can be categorical or numerical. • A frequency distribution displays the joint frequencies for categorical data in a two-way table. • A marginal frequency distribution displays the total of the joint frequencies of the rows and columns of a frequency distribution. • A bar graph displays the frequency of categorical data.
2	**Relatively Speaking** Relative Frequency Distribution	S.ID.5	1	Students construct a relative frequency distribution and marginal relative frequency distribution using data for a scenario. They analyze the distributions and answer questions about the problem situation. Next, students are shown stacked bar graphs that represent the relative frequency distribution in two different ways. They compare the graphs to the tables in the previous activity and explain the advantages of graphing the data each way. Finally, students analyze and interpret the data represented by the stacked bar graphs in terms of the problem situation.	• A relative frequency distribution provides the ratio of occurrences for each category to the total number of occurrences. • A marginal relative frequency distribution includes the ratio of total occurrences for each category to the total number of occurrences. • A stacked bar graph is a bar graph in which the bars are stacked on top of each other as opposed to sitting next to each other.
3	**On One Condition … or More** Conditional Relative Frequency Distribution	S.ID.5	1	Students consider what different joint frequencies in a marginal relative frequency distribution represent. They construct a stacked bar graph and analyze the percentages shown in the graph before the term conditional relative frequency distribution is introduced. Students construct a conditional relative frequency distribution and use it to answer questions related to the given scenario. They construct a second conditional relative frequency distribution in terms of the other variable. Finally, students construct a conditional relative frequency distribution and interpret the data in terms of the problem situation.	• A conditional relative frequency distribution is the percent of proportion of occurrences of a category given the specific value of another category.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Data Jam** Drawing Conclusions from Data	S.ID.5	1	Students synthesize what they know about analyzing and interpreting two-variable categorical data to make a recommendation in a real-world scenario. They organize a given data set by creating a frequency distribution and a stacked bar graph, and then use conditional relative frequency distributions to determine whether there is an association between the two categories. Students formulate conclusions for specified subsets of the data and use statistics to support their conclusions.	• A marginal frequency distribution is used to formulate and support conclusions to a real-world problem. • A stacked bar graph is a visual display used to formulate and support conclusions to a real-world problem. • A conditional relative frequency distribution compares occurrences within a category is used to formulate and support conclusions to a real-world problem.
Learning Individually with MATHia or Skills Practice			2	Students construct frequency distributions and relative frequency distributions. They use these distributions to answer questions about joint frequencies and marginal frequencies. Students then create conditional relative frequency tables and discern any possible associations. **MATHia Unit:** Two-Variable Categorical Data **MATHia Workspaces:** Creating Marginal Frequency Distributions / Using Marginal Frequency Distributions / Creating Marginal Relative Frequency Distributions / Using Marginal Relative Frequency Distributions / Creating Conditional Relative Frequency Distributions / Using Conditional Relative Frequency Distributions	

*Pacing listed in 45-minute days

5 Maximizing and Minimizing
Pacing: 46 Days

Topic 1: Introduction to Quadratic Functions

Students explore the structure of quadratic functions through four real-world situations. They learn key characteristics of quadratic functions through explorations with their graphs and equations: intercepts, absolute maximum/minimum, vertex, axis of symmetry, domain, range, and intervals of increasing and decreasing. Students transform functions using function transformation form $y = A \cdot f(B(x - C)) + D$ and learn how coordinates are affected; any point (x, y) on the graph $y = f(x)$ maps to the point $((1/B)x - C, Ay + D)$ on the graph $y = A \cdot f(B(x - C)) + D$.
Given quadratic functions represented in different forms (table, graph, equation, or scenario), students compare key characteristics.

Standards: A.SSE.1a, A.SSE.3a, A.CED.4, A.APR.3, A.REI.10, A.REI.11, F.IF.4, F.IF.5, F.IF.6, F.IF.7a, F.IF.8a, F.IF.9, F.BF.3, F.LE.3   **Pacing:** 16 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Up and Down or Down and Up** Exploring Quadratic Functions	A.REI.10 A.REI.11 F.IF.4 F.IF.5 F.IF.7a	2	Students are introduced to quadratic functions and their growth pattern through a sequence of pennies. They are then provided four different contexts that can be modeled by quadratic functions. For each function, students address the key characteristics of the graphs and interpret them in terms of the context. They also compare the domain and range of the functions and the context they represent. The first context involves area and is used to compare and contrast linear and quadratic relationships. The second context involves handshakes and has the student write the function. The third context involves catapulting a pumpkin. Students analyze this function written in general form. The final context involves revenue and demonstrates how a quadratic function can be written as the product of two linear functions.	• Quadratic functions can be used to model certain real-world situations. • The graph of a quadratic function is called a parabola. • A parabola is a smooth curve with reflectional symmetry. • A parabola has an absolute maximum or absolute minimum point and an interval where it is increasing and an interval where it is decreasing. • A parabola has one y-intercept and at most two x-intercepts. • The domain of a quadratic function is the set of all real numbers. The range is a subset of the real numbers that is limited based upon the y-coordinate of the absolute maximum or absolute minimum point.
2	**Endless Forms Most Beautiful** Key Characteristics of Quadratic Functions	A.SSE.1a A.SSE.3a A.APR.3 F.IF.4 F.IF.6 F.IF.7a F.IF.8a	3	Students revisit the four scenarios from the previous lesson as a way to introduce equivalent quadratic equations with different structures to reveal different characteristics of their graphs. They learn that a table of values represents a quadratic function if its second differences are constant. Students analyze the effect of the leading coefficient on whether the parabola opens up or down. They identify the axis of symmetry and vertex for graphs using the equations in each form. Finally, students determine the x- and y-intercepts along with intervals of increase and decrease, using a combination of technology, symmetry, and equations.	• A table of values representing a quadratic function has constant second differences. • A quadratic function may be written in general form $f(x) = ax^2 + bx + c$, where $a \neq 0$, and in factored form $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$. • When the a-value of a quadratic function in general form or factored form is positive, the graph opens upward and has an absolute minimum; when the leading coefficient is negative, the graph opens downward and has an absolute maximum. • The vertex (or maximum/minimum) of a parabola lies on its axis of symmetry. The axis of symmetry can be determined by the formula $x = (r_1 + r_2)/2$ from the factored form or by $x = -b/(2a)$ from the general form of a quadratic equation. • In factored form, $f(x) = a(x - r_1)(x - r_2)$, the values of $(r_1, 0)$ and $(r_2, 0)$ are the x-intercepts of the quadratic function. In general form, $f(x) = ax^2 + bx + c$, $(0, c)$ is the y-intercept of the quadratic function.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**More Than Meets the Eye** Transformations of Quadratic Functions	A.SSE.3a F.BF.3	2	Students explore a variety of different transformations of quadratic functions, including vertical translations, horizontal translations, vertical dilations and reflections, and horizontal dilations and reflections. For each transformation, students sketch graphs of the transformation, compare characteristics of the transformed graphs with those of the graph of the basic function, and write the transformations using coordinate notation. Students write quadratic equations in vertex form using the coordinates of the vertex and another point on the graph and in factored form using the zeros and another point on the graph.	• The general transformation equation is $y = Af(B(x - C)) + D$, where the A-value describes a vertical dilation or reflection, the B-value describes a horizontal dilation or reflection, the C-value describes a horizontal translation, and the D-value describes a vertical translation. • Given $f(x) = x^2$ as a basic quadratic function, reference points can be used to graph $y = Af(B(x - C)) + D$ such that any point (x, y) on $f(x)$ maps to the point $((1/B)x + C, Ay + D)$. • The vertex form for a quadratic function is $f(x) = a(x - h)^2 + k$, where (h, k) is the location of the vertex, and the sign of a indicates whether the parabola opens upward or downward. • A function written in equivalent forms can reveal different characteristics of the function it defines. • You can write a quadratic function in vertex form if you know the coordinates of the vertex and another point on the graph. • You can write a quadratic function in factored form if you know the zeros and another point on the graph.
4	**You Lose Some, You Lose Some** Comparing Functions Using Key Characteristics and Average Rate of Change	A.SSE.1a A.CED.4 F.IF.9 F.LE.3	1	Students compare quadratic functions in standard form, factored form, and vertex form, then analyze the properties of each form. Students then answer questions to compare linear, quadratic, and exponential functions. They compute average rates of change for the functions across different intervals and then compare the change in the average rates of change across the different intervals. Quadratic equations in different forms are compared by identifying key characteristics of their representations.	• The average rate of change of any function over an interval is the slope of a linear function passing through the beginning and end points of the interval. • A quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. • The average rate of change of an increasing exponential function will eventually exceed the average rate of change for an increasing linear and quadratic function. • A function written in equivalent forms can reveal different characteristics of the function it defines. • Quadratic equations in different forms are compared by identifying key characteristics of their representations or by changing one representation to match the other for comparison purposes.

*Pacing listed in 45-minute days

Learning Individually with MATHia or Skills Practice	A.CED.1 F.IF.3 F.IF.4	8	Students complete a table of values and graph from a scenario represented by a quadratic model. Students construct the quadratic function for the scenario as a product of a monomial and a binomial or a product of two binomials. Students use an interactive Explore Tool to investigate how a vertical motion graph changes when the different values in the vertex, factored, and general form of the quadratic function change. They then use vertical motion graphs to identify the maximum, x-intercepts, y-intercept, domain, range, axis of symmetry, and vertex of a quadratic function. Students transform linear and quadratic functions. Given a representation of a transformed function, students determine how the basic linear and quadratic functions were transformed to create the new function. **MATHia Unit:** Exploring Quadratic Functions **MATHia Workspaces:** Introduction to Quadratic Functions / Modeling Area as Product of Monomial and Binomial / Modeling Area as Product of Two Binomials / Modeling Projectile Motion / Interpreting Maximums of Quadratic Models **MATHia Unit:** Key Characteristics of Quadratic Functions **MATHia Workspaces:** Recognizing Quadratic Functions from Tables / Identifying Properties of Quadratic Functions **MATHia Unit:** Transformations of Quadratic Functions **MATHia Workspaces:** Vertically Translating Quadratic Functions / Horizontally Translating Quadratic Functions / Reflecting and Dilating Quadratic Functions using Graphs / Transforming Quadratic Functions Using Tables / Multiple Transformations of Quadratic Functions **MATHia Unit:** Sketching and Comparing Quadratic Functions **MATHia Workspaces:** Comparing Increasing Linear, Exponential, and Quadratic Functions / Sketching Quadratic Functions / Comparing Quadratic Functions in Different Forms

*Pacing listed in 45-minute days

Topic 2: Solving Quadratic Equations

Students begin by solving simple quadratic equations in the form $x^2 = n$ using horizontal lines and the Properties of Equality, skills they learned in middle school. They recognize that the solutions to a quadratic equation are equidistant to the axis of symmetry on a graph of the function. They use these tools to solve increasingly complex quadratic equations, each time making a connection to the location of the solutions on a graphical representation. When they encounter an equation in general form, students learn to use the Zero Product Property to solve via factoring. For trinomials that cannot be factored, students learn to complete the square and to use the Quadratic Formula. In deriving the Quadratic Formula, students recognize that its structure supports the fact that all solutions are equidistant from the axis of symmetry for any quadratic equation. Students are provided multiple opportunities to solve quadratic equations using efficient methods.

Standards: N.RN.3, N.CN.1, A.SSE.1a, A.SSE.1b, A.SSE.2, A.SSE.3a, A.SSE.3b, A.APR.1, A.REI.4, A.REI.4a, A.REI.4b, F.IF.8a   **Pacing:** 21 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**This Time, with Polynomials** Adding, Subtracting, and Multiplying Polynomials	A.SSE.1a A.APR.1	2	Students are introduced to polynomials and identify the terms and coefficients of polynomials. Students sort polynomials by the number of terms, rewrite in general form if possible, and identify the degree. Students add and subtract polynomial functions algebraically and graphically and then determine that polynomials are closed under addition and subtraction. Students use area models and the Distributive Property to determine the product of binomials. They explore special products and are introduced to the terms difference of two squares and perfect square trinomial.	• A polynomial is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. A polynomial in one variable is the sum of terms of the form ax^k, where a is any real number and k is a non-negative integer. In general, a polynomial is of the form $a_1x^k + a_2x^{k-1} + \ldots a_nx^0$. • Polynomials with only one term are monomials. Polynomials with exactly two terms are binomials. Polynomials with exactly three terms are trinomials. • Polynomials can be added, subtracted and multiplied using algebraic operations. • Polynomials are closed under addition, subtraction and multiplication. • The difference of two squares and perfect square trinomials are special products that can be recognized when multiplying binomials.
2	**Solutions, More or Less** Representing Solutions to Quadratic Equations	N.RN.2 A.SSE.2 A.SSE.3a A.REI.4b A.REI.10	2	Students use the Properties of Equality and square roots to solve simple quadratic equations. They express solutions in terms of the distance from the axis of symmetry to the parabola. Students identify double roots, estimate square roots, and extract perfect roots from the square roots of products. They show graphically that a quadratic function is the product of two linear functions with the same zeros. Students then use the Zero Product Property to explain that the zeros of a quadratic function are the same as the zeros of its linear factors. Finally, they rewrite any quadratic in the form $f(x) = ax^2 - c$ as the product of two linear factors.	• Every whole number has two square roots, a positive principal square root and a negative square root. • A quadratic function is a polynomial of degree 2. Thus, a quadratic function has two zeros or two solutions at $f(x) = 0$. If both solutions are the same, the quadratic function is said to have a double zero. • The x-coordinates of the x-intercepts of a graph of a quadratic function are called the zeros of the quadratic function. The zeros are called the roots of the quadratic equation. • The real solutions to a quadratic equation can be represented as the x-value of the axis of symmetry plus or minus a constant. • The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. Therefore, the zeros of a quadratic function are the same as the zeros of its linear factors. • A quadratic function in the form $f(x) = ax^2 - c$ can be rewritten as the product of two linear factors, $(\sqrt{a}x + \sqrt{c})(\sqrt{a}x - \sqrt{c})$.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Transforming Solutions** Solutions to Quadratic Equations in Vertex Form	A.SSE.2 A.SSE.3a	2	Students solve quadratic equations that are squares of binomials, recognizing them as horizontal translations of the function. They use the Properties of Equality and square roots to solve quadratic equations of the form $y = a(x - c)^2$ and determine how the dilation affects the solutions. Finally, students solve quadratic functions of the form $y = a(x - c)^2 + d$ and determine how the translation affects the solutions. Students learn that a quadratic function can have one unique real zero, two real zeros, or no real zeros, and how the number of real zeros relates to the graph of the function.	• The solutions to a quadratic equation can be represented as the x-value of the axis of symmetry plus or minus a constant. • A quadratic function written in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$, is in vertex form. • The solutions for a quadratic equation of the form $y = (x - c)^2$ are $c \pm \sqrt{y}$. • The solutions for a quadratic equation of the form $y = a(x - c)^2$ are $c \pm \sqrt{y/a}$. • The solutions for a quadratic equation of the form $y = a(x - c)^2 + d$ are $c \pm \sqrt{(y - d)/a}$.
4	**The Missing Link** Factoring and Completing the Square	A.SSE.3b A.REI.4a F.IF.8a	2	Students recall how to factor out the GCF from different polynomials. They follow examples to factor quadratic trinomials, first using area models and then recognizing patterns in the coefficients. Students use the Zero Product Property to solve quadratic equations by factoring. They are then introduced to completing the square, a method they can use to convert a quadratic equation given in general form to vertex form. Students complete the square to solve quadratic equations that cannot be solved using other methods.	• One method of solving some quadratic equations in the form $y = ax^2 + bx + c$ is to set the equation equal to zero, factor the trinomial expression, and use the Zero Product Property to determine the roots. • Completing the square is a method for rewriting a quadratic equation in the form $y = ax^2 + bx + c$ as a quadratic equation in vertex form. • When a quadratic equation in the form $y = ax^2 + bx + c$ is not factorable, completing the square is an alternative method of determining the roots of the equation. • Completing the square is a useful method for converting a quadratic function written as $f(x) = ax^2 + bx + c$ to vertex form for graphing purposes and determining the maximum or minimum in problem situations. • Given a quadratic equation in the form $y = ax^2 + bx + c$, the vertex of the function is located at (x, y) such that $x = (-b)/(2a)$ and $y = c - (b^2)/(4a)$.

*Pacing listed in 45-minute days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
5	**Ladies and Gentlemen, Please Welcome the Quadratic Formula!** The Quadratic Formula	N.RN.2 N.RN.3 N.CN.1 A.SSE.1b A.REI.4 A.REI.4a A.REI.4b	3	The first activity focuses on the graphical interpretation of $x = -(b/2a) +/- (sqrt(b^2 - 4ac)/2a)$ as the distance from $(-b/2a, 0)$ to each root. Students are introduced to the Quadratic Formula as a method to calculate the solutions to any quadratic equation written in general form. Students use the discriminant to determine the number and type of roots for a given function. Students learn why rational numbers are closed under addition and that the sum or product of a rational number and an irrational number is an irrational number. Students reason about the solution to a function with no x-intercepts. Students practice simplifying expressions with negative roots.	• The Quadratic Formula, $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$, can be used to calculate the solutions to any quadratic equation written in general form, $f(x) = ax^2 + bx + c$, where a, b, and c represent real numbers and $a \neq 0$. • On the graph of a quadratic function, $\pm (\sqrt{b^2 - 4ac})/(2a)$ is the distance from $((-b)/(2a), 0)$ to each root. • When a quadratic equation is in the form, $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$, the discriminant is $b^2 - 4ac$. When $b^2 - 4ac < 0$, the quadratic equation has no real roots. When $b^2 - 4ac = 0$, the quadratic equation has one real root. When $b^2 - 4ac > 0$, the quadratic equation has two real roots. • When $b^2 - 4ac$ is a positive perfect square number, the roots are rational numbers. When $b^2 - 4ac$ is positive, but not a perfect square number, the roots are irrational numbers. • A rational number plus a rational number equals a rational number; rational numbers are closed under addition. A rational number plus an irrational number equals an irrational number. • The number i is a number such that $i^2 = -1$. The set of imaginary numbers is the set of all numbers written in the form $a + bi$, where a and b are real numbers and b is not equal to 0.
	Learning Individually with MATHia or Skills Practice	A.SSE.3a A.SSE.3b A.APR.1 A.APR.6 A.REI.4a A.REI.4b A.REI.11 F.IF.7a F.IF.8a F.IF.9	10	Students are introduced to polynomials and use an Explore Tool to investigate combining like terms when adding polynomial expressions. They practice adding, subtracting, multiplying, and factoring quadratic expressions. Students learn to complete the square using area models and algebraically. Students differentiate among general form, factored form, and vertex form of a quadratic equation; they learn the characteristics of the graph that are apparent in each form. Students convert between the forms of a quadratic equation and sketch the corresponding graphs. Students solve quadratic equations by factoring and applying the Zero Product Property or by using the Quadratic Formula. **MATHia Unit:** Adding, Subtracting, and Multiplying Polynomials **MATHia Workspaces:** Introduction to Polynomial Arithmetic / Ide4ntifying Parts of Complex Algebraic Expressions / Operating with Functions on the Coordinate Plane / Adding Polynomials / Subtracting Polynomials / Using a Factor Table to Multiply Binomials / Multiplying Binomials **MATHia Unit:** Representing Solutions to Quadratic Equations **MATHia Workspaces:** Making Sense of Roots and Zeros / Factoring Using Difference of Squares **MATHia Unit:** Solutions to Quadratic Equations in Vertex Form **MATHia Workspaces:** Using Properties of Equality to Solve Quadratic Equations **MATHia Unit:** Factoring and Completing the Square **MATHia Workspaces:** Introduction to Factoring / Factoring Trinomials with Coefficients of One / Factoring Trinomials with Coefficients Other than One / Factoring Quadratic Expressions / Solving Quadratic Equations by Factoring / Completing the Square / Problem Solving Using Completing the Square **MATHia Unit:** The Quadratic Formula **MATHia Workspaces:** Deriving the Quadratic Formula / Solving Quadratic Equations	

*Pacing listed in 45-minute days

Topic 3: Applications of Quadratics

The final topic incorporates skills and key understandings from throughout the course. Students use the structure of a parabola and a given context to solve quadratic inequalities. They use what they know about solutions to functions on a graphical representation to solve systems of equations comprised of a quadratic and a linear function or two quadratic functions. Given a data set, students use technology to determine a regression curve that best fits the data and to make predictions for given input values. Finally, students reflect quadratics across $y = x$ to identify the graphical inverse of a function and then learn how to determine the equation of the inverse algebraically. Because quadratic functions are not one-to-one, students restrict the domain of quadratic functions to write their inverse functions.

Standards: A.CED.1, A.CED.2, A.CED.3, A.REI.4, A.REI.4a, A.REI.4b, A.REI.7, F.IF.7b, F.BF.4a, F.BF.4d, S.ID.6a   **Pacing:** 9 Days

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Ahead of the Curve** Solving Quadratic Inequalities	A.CED.1 A.CED.2 A.CED.3 A.REI.4 A.REI.4a A.REI.4b	1	Students use the graph of a vertical motion model to approximate the times when an object is at given heights. They identify regions on the graph that are less than or greater than a given height and write a quadratic inequality to represent the situation. Next, students are shown how to solve a quadratic inequality algebraically. They determine the solution set of the inequality by dividing the graph into intervals defined by the roots of the quadratic equation, and then test values in each interval to determine which intervals satisfy the inequality. Finally, with a second scenario, students write the function that represents the situation, sketch a graph of the function, and write and solve a quadratic inequality related to the solution set of the quadratic function.	• A horizontal line drawn across the graph of a quadratic function intersects the parabola at exactly two points, except at the vertex, where it intersects the parabola at exactly one point. • The solution set of a quadratic inequality is determined by first solving for the roots of the quadratic equation, and then determining which interval(s) created by the roots will satisfy the inequality. A combination of algebraic and graphical methods may be the most efficient solution method. • Quadratic inequalities can be used to model some real-world contexts. The effects of translations of quadratic functions can be used to make comparisons within a context.
2	**All Systems Are Go!** Systems of Quadratic Equations	A.CED.2 A.CED.3 A.REI.7 A.REI.11	2	Students are presented with a scenario that can be modeled with a quadratic and a linear equation and reason about the intersections of the two equations in the context of the problem. Next, they solve systems of equations composed of a linear equation and a quadratic equation algebraically using substitution, factoring, and the Quadratic Formula. They then verify their algebraic solutions graphically by determining the coordinates of the points of intersection. Finally, students solve a system composed of two quadratic equations using the same methods. They conclude that a system of equations consisting of a linear and a quadratic equation can have one solution, two solutions, or no solutions, while a system of two quadratic equations can have one solution, two solutions, no solutions, or infinite solutions.	• Systems of equations involving a linear equation and a quadratic equation or two quadratic equations can be solved both algebraically and graphically. • A system of equations containing a linear equation and a quadratic equation may have no solution, one solution, or two solutions. • A system of equations containing two quadratic equations may have no solution, one solution, two solution, or an infinite number of solutions. • The number of solutions for a system of equations depends on the number of points where the graphs of the two equations intersect. • A system of equations involving a linear equation and a quadratic equation may be used to model real-world problems.
Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas

*Pacing listed in 45-minute days

3	**Model Behavior** Using Quadratic Functions to Model Data	F.IF.7b F.BF.4a F.BF.4d S.ID.6a	2	Students determine a quadratic regression that best models a table of data. They answer questions and make predictions using the regression equation. Students then analyze another data set and determine the quadratic regression. Next, the same algebraic process is used to determine the inverse of a quadratic function. Students graph equations containing square roots, identify the domain and range of each graph, and determine which graphs describe functions. Using only the equation of the inverse of a function, students then determine the original function and identify its domain and range. The term restrict the domain is introduced. Students determine the restrictions on the domain of a quadratic function based on the problem situation and graph the function with the restricted domain. They write the equation for the inverse function and interpret it with respect to the problem situation. Finally, students determine whether certain types of functions are oneto-one functions.	• Some data in context can be modeled by a quadratic regression equation. The regression equation can be used to make predictions; however there may be limitations on the domain depending on the context. • To determine the inverse of a function, replace $f(x)$ with y, switch the x and y variables, then solve for y. • When a problem requires using a given function to determine the independent quantity when a dependent quantity is given, determining the inverse of the original function may be a more efficient way to handle the situation. • The inverse of a function may or may not be a function. A function is a one-to-one function if its inverse is also a function. • To restrict the domain of a function means to define a new domain for the function that is a subset of the original domain.
	Learning Individually with MATHia or Skills Practice	F.BF.1b F.BF.3 F.BF.4 S.ID.6a	4	Students use equations of quadratic regression models, the solver, and graphs to answer questions. They then watch an animation about operating with functions on the coordinate plane before adding and subtracting constant functions, linear functions, and a linear and a quadratic function. Given two functions in function notation, students determine the sum or difference of the functions and verify the sum or difference by evaluating the new function at a given value. Given the graphs of two relations, they decide if the relations are inverses. Students determine the equation of the inverse function for a given function and use composition of functions to verify that the functions are inverses. **MATHia Unit:** Using Quadratic Functions to Model Data **MATHia Workspace:** Using Quadratic Models / Introduction to Inverses / Recognizing Graphs of Inverses	

Total Days: 155

Learning Together: 99
Learning Individually: 56

*Pacing listed in 45-minute days