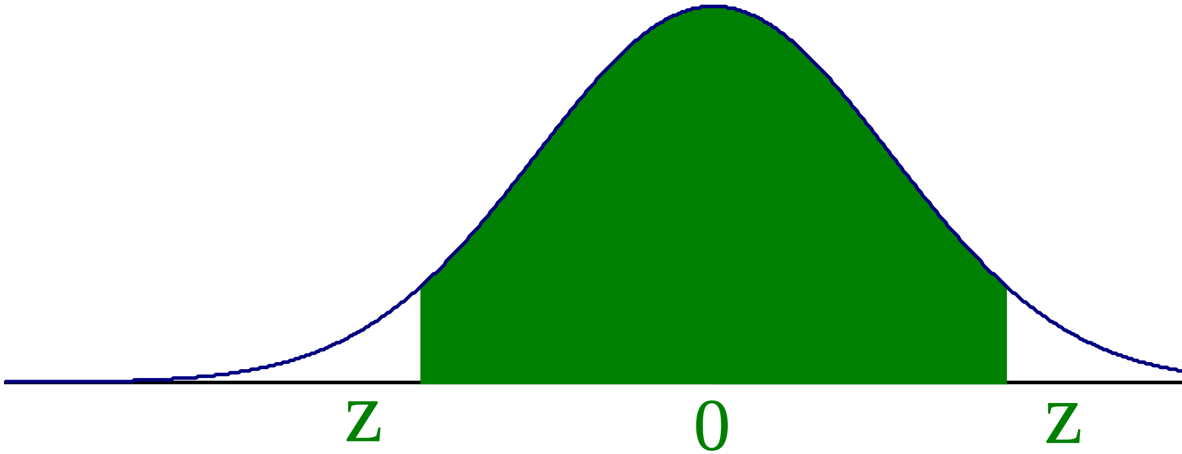
Chapter 7

Confidence Intervals and Sample Size

Review: Find the z values; the shaded area is given. The graph is symmetrical.



CI	z
90%	1.65
95%	1.96
98%	2.33
99%	2.58

A) 90%
B) 95%
C) 98%
D) 99%

Chapter 7 Overview

Introduction

- 7-1 Confidence Intervals for the Mean When σ Is Known and Sample Size
- 7-2 Confidence Intervals for the Mean When σ Is Unknown
- 7-3 Confidence Intervals and Sample Size for Proportions
- 7-4 Confidence Intervals and Sample Size for Variances and Standard Deviations

Chapter 7 Objectives

1. Find the confidence interval for the mean when σ is known.
2. Determine the minimum sample size for finding a confidence interval for the mean.
3. Find the confidence interval for the mean when σ is unknown.
4. Find the confidence interval for a proportion.

Chapter 7 Objectives

5. Determine the minimum sample size for finding a confidence interval for a proportion.
6. Find a confidence interval for a variance and a standard deviation.

Funny stuff!

- Logic is a systematic method for getting the wrong conclusion with confidence.
- Statistics is a systematic method for getting the wrong conclusion with 95% confidence!

Common symbols

	Sample	Population
Mean	$\bar{x}$	μ
Standard Deviation	s	σ
Proportion	$\hat{p}$	p

Jamie Foxx

Age

Take a guess of how old Jamie Foxx is.

Guess Jamie Foxx's age within 1 year.

Guess Jamie Foxx's age within 2 years.

Guess Jamie Foxx's age within 5 years.

ERIC MARLON BISHOP (BORN DECEMBER 13, 1967)

7.1 Confidence Intervals for the Mean When σ Is Known and Sample Size

- A point estimate is a specific numerical value estimate of a parameter.
- The best point estimate of the population mean μ is the sample mean $\bar{X}$.

Two examples of point and interval values

- People living with HIV/AIDS in 2008
  - Point estimate: 33.4 million
  - Interval: 31.1-35.8 million
- AIDS deaths in 2008
  - Point estimate: 0.28 million
  - Interval: 0.15-0.41 million

Three Properties of a Good Estimator

1. The estimator should be an **unbiased estimator**. That is, the expected value or the mean of the estimates obtained from samples of a given size is equal to the parameter being estimated.

Three Properties of a Good Estimator

2. The estimator should be consistent. For a **consistent estimator**, as sample size increases, the value of the estimator approaches the value of the parameter estimated.

Three Properties of a Good Estimator

3. The estimator should be a **relatively efficient estimator**; that is, of all the statistics that can be used to estimate a parameter, the relatively efficient estimator has the smallest variance.

Confidence Intervals for the Mean When σ Is Known

- An **interval estimate** of a parameter is an interval or a range of values used to estimate the parameter.
- This estimate may or may not contain the value of the parameter being estimated.

Confidence Level of an Interval Estimate

- The **confidence level** of an interval estimate of a parameter is the probability that the interval estimate will contain the parameter, assuming that a large number of samples are selected and that the estimation process on the same parameter is repeated.

Confidence Interval

- A **confidence interval** is a specific interval estimate of a parameter determined by using data obtained from a sample and by using the specific confidence level of the estimate.

Confidence Interval of the Mean for a Specific α

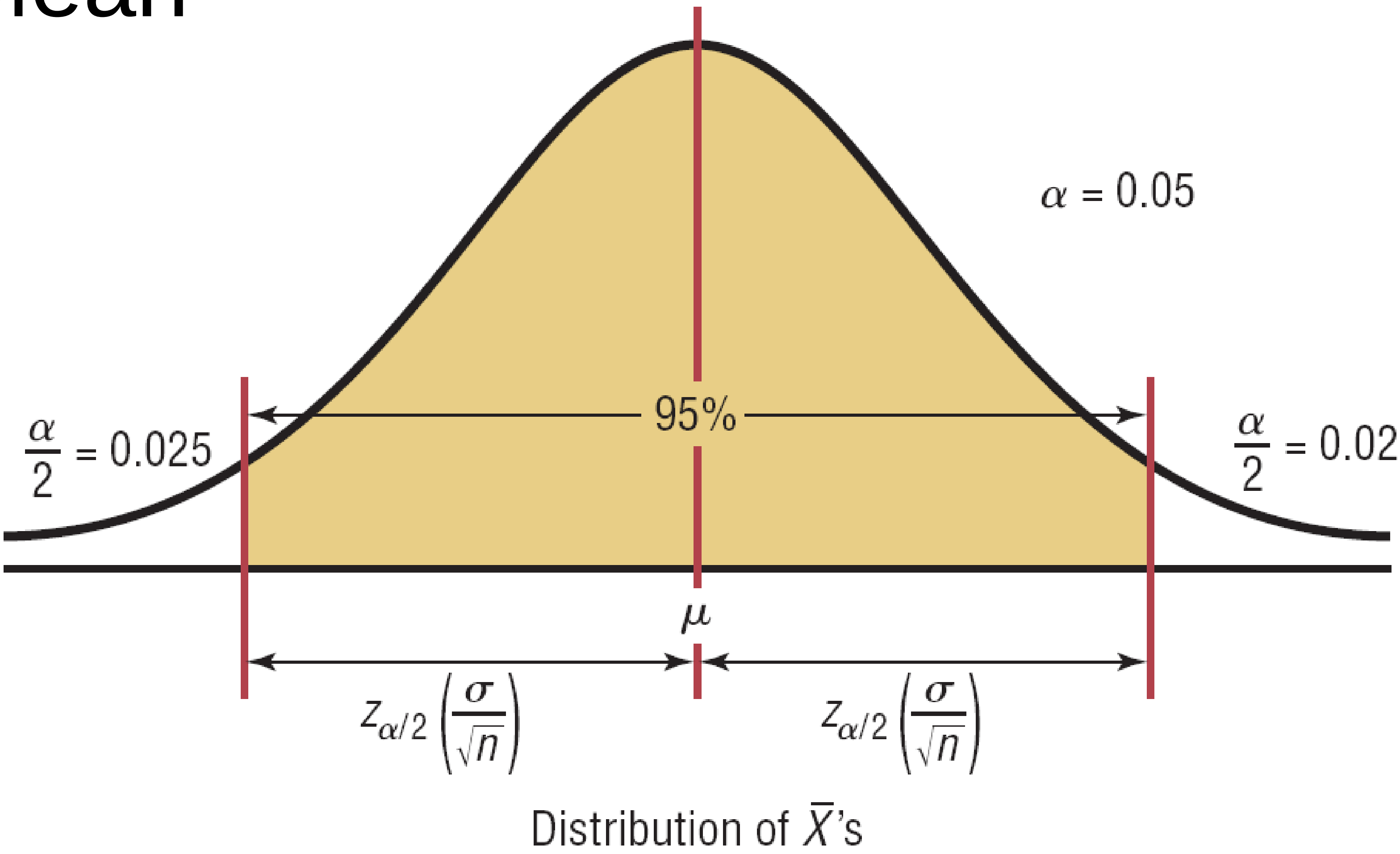
$$\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Common confidence intervals, CI, and z scores associated with them.

CI	z
90%	1.65
95%	1.96
98%	2.33
99%	2.58

- When $n \geq 30$, s can be substituted for σ.

95% Confidence Interval of the Mean



Maximum Error of the Estimate

The maximum error of the estimate is the maximum likely difference between the point estimate of a parameter and the actual value of the parameter.
The common term you are likely to know is *margin of error.*

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Confidence Interval for a Mean

Rounding Rule

When you are computing a confidence interval for a population mean by using *raw data,* round off to one more decimal place than the number of decimal places in the original data.

When you are computing a confidence interval for a population mean by using a sample mean and a standard deviation, round off to the same number of decimal places as given for the mean.

Chapter 7
Confidence Intervals and Sample Size

Section 7-1

Example 7-1

Page #360

Example 7-1: Days to Sell an Aveo

A researcher wishes to estimate the number of days it takes an automobile dealer to sell a Chevrolet Aveo. A sample of 50 cars had a mean time on the dealer's lot of 54 days. Assume the population standard deviation to be 6.0 days. Find the best point estimate of the population mean and the 95% confidence interval of the population mean.

The best point estimate of the mean is 54 days.

$$\bar{X} = 54, s = 6.0, n = 50, 95\% \rightarrow z = 1.96$$

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Example 7-1: Days to Sell an Aveo

$$\bar{X} = 54, s = 6.0, n = 50, 95\% \rightarrow z = 1.96$$

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$54 - 1.96\left(\frac{6.0}{\sqrt{50}}\right) < \mu < 54 + 1.96\left(\frac{6.0}{\sqrt{50}}\right)$$

$$54 - 1.7 < \mu < 54 + 1.7$$

$$52.3 < \mu < 55.7$$

$$52 < \mu < 56$$

One can say with 95% confidence that the interval between 52 and 56 days contains the population mean, based on a sample of 50 automobiles.

Chapter 7
Confidence Intervals and Sample Size

Section 7-1

Example 7-2

Page #360

Example 7-2: Ages of Automobiles

A survey of 30 adults found that the mean age of a person's primary vehicle is 5.6 years. Assuming the standard deviation of the population is 0.8 year, find the best point estimate of the population mean and the 99% confidence interval of the population mean.

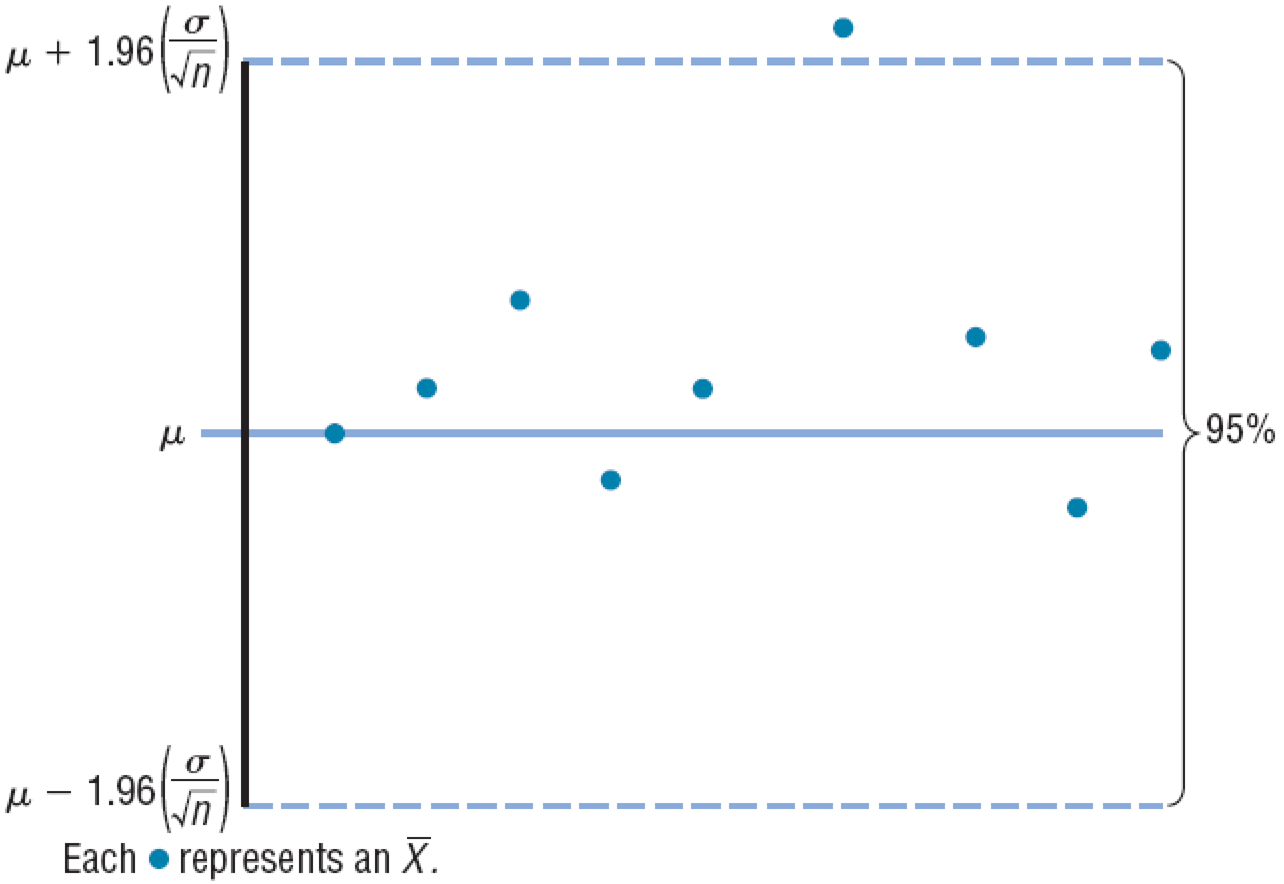
The best point estimate of the mean is 5.6 years.

$$5.6 - 2.58\left(\frac{0.8}{\sqrt{30}}\right) < \mu < \mathbf{5.6} + \mathbf{2.58}\left(\frac{0.8}{\sqrt{30}}\right)$$

$$5.2 < \mu < 6.0$$

One can be 99% confident that the mean age of all primary vehicles is between 5.2 and 6.0 years, based on a sample of 30 vehicles.

95% Confidence Interval of the Mean



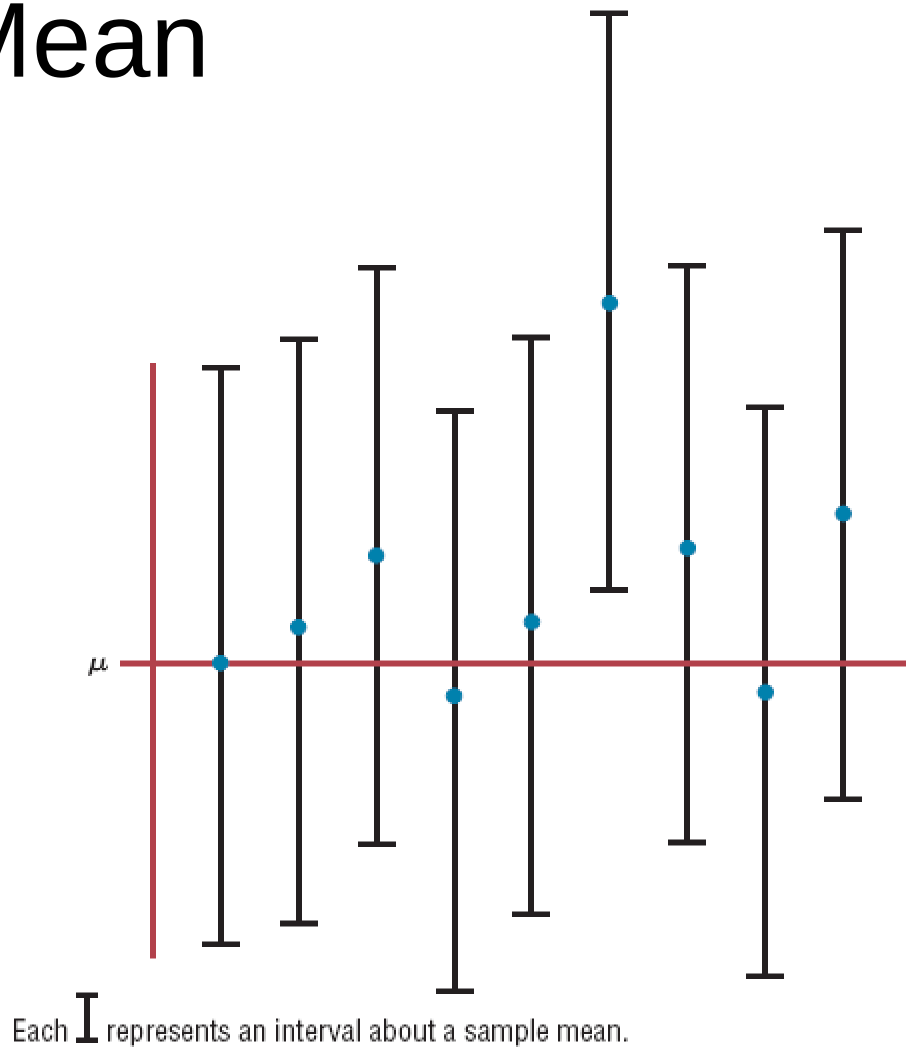
Confidence Interval of the Mean for a Specific α

$$\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Common confidence intervals, CI, and z scores associated with them.

CI	z
90%	1.65
95%	1.96
98%	2.33
99%	2.58

95% Confidence Interval of the Mean



One can be 95% confident that an interval built around a specific sample mean would contain the population mean.

Finding $z_{\alpha/2}$ for 98% CL.

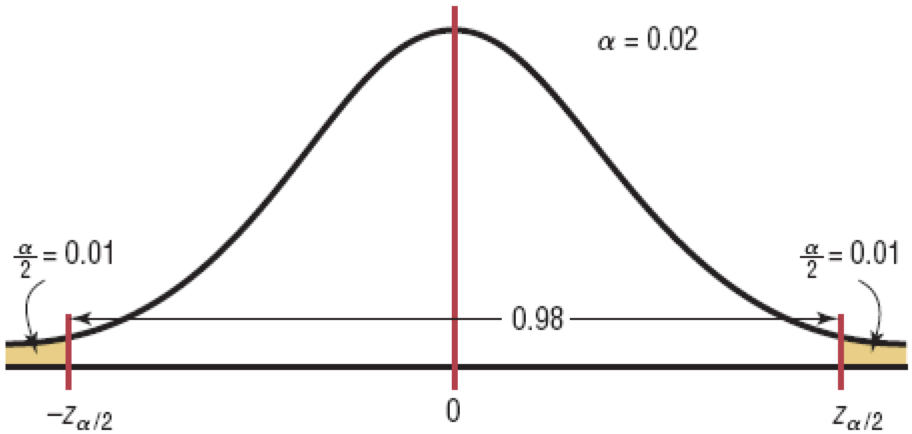


Table E
The Standard Normal Distribution

z	.00	.01	.02	.03	...	.09
0.0						
0.1						
⋮						
2.3				0.9901		

$$z_{\alpha/2} = 2.33$$

Chapter 7
Confidence Intervals and Sample Size

Section 7-1

Example 7-3

Page #362

Example 7-3: Credit Union Assets

The following data represent a sample of the assets (in millions of dollars) of 30 credit unions in southwestern Pennsylvania. Find the 90% confidence interval of the mean.

12.23	16.56	4.39
2.89	1.24	2.17
13.19	9.16	1.42
73.25	1.91	14.64
11.59	6.69	1.06
8.74	3.17	18.13
7.92	4.78	16.85
40.22	2.42	21.58
5.01	1.47	12.24
2.27	12.77	2.76

Example 7-3: Credit Union Assets

Step 1: Find the mean and standard deviation. Using technology, we find $\bar{X}$ = 11.091 and *s* = 14.405.

Step 2: Find $\alpha/2$. 90% CL → $\alpha/2 = 0.05$.

Step 3: Find $z_{\alpha/2}$. 90% CL → $\alpha/2 = 0.05$ → $z_{.05} = 1.65$

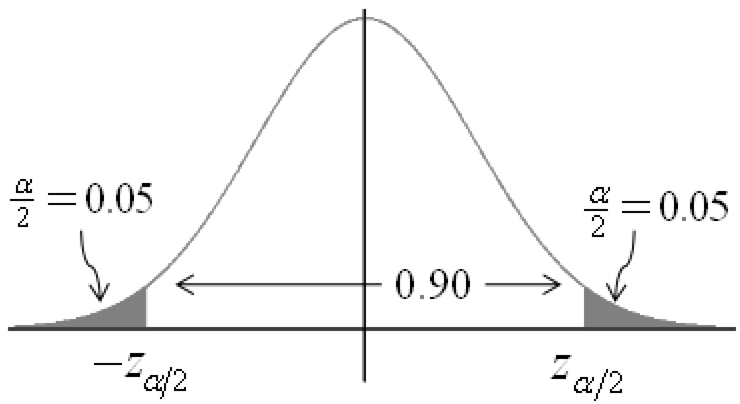


Table E

The Standard Normal Distribution

z	.00	…	.04	.05	…	.09
0.0						
0.1						
⋮						
1.6			0.9495	0.9505		

Example 7-3: Credit Union Assets

Step 4: Substitute in the formula.

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$11.091 - 1.65\left(\frac{14.405}{\sqrt{30}}\right) < \mu < 11.091 + 1.65\left(\frac{14.405}{\sqrt{30}}\right)$$

$$11.091 - 4.339 < \mu < 11.091 + 4.339$$

$$6.751 < \mu < 15.430$$

One can be 90% confident that the population mean of the assets of all credit unions is between \$6.752 million and \$15.430 million, based on a sample of 30 credit unions.

Technology Note

This chapter and subsequent chapters include examples using raw data. If you are using computer or calculator programs to find the solutions, the answers you get may vary somewhat from the ones given in the textbook.

This is so because computers and calculators do not round the answers in the intermediate steps and can use 12 or more decimal places for computation. Also, they use more exact values than those given in the tables in the back of this book.

These discrepancies are part and parcel of statistics.

Formula for Minimum Sample Size Needed for an Interval Estimate of the Population Mean

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

where *E* is the maximum error of estimate. If necessary, round the answer up to obtain a whole number. That is, if there is any fraction or decimal portion in the answer, use the next whole number for sample size *n.*

Chapter 7
Confidence Intervals and Sample Size

Section 7-1

Example 7-4

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Example 7-4: Depth of a River

A scientist wishes to estimate the average depth of a river. He wants to be 99% confident that the estimate is accurate within 2 feet. From a previous study, the standard deviation of the depths measured was 4.33 feet. What is the minimum sample size?

$$n = \left(\frac{2.58 \cdot 4.33}{2}\right)^2$$

n= 32

Therefore, to be 99% confident that the estimate is within 2 feet of the true mean depth, the scientist needs at least a sample of 32 measurements.

$$5.6 - 2.58\left(\frac{0.8}{\sqrt{30}}\right) < \mu < 5.6 + 2.58(\frac{0.8}{\sqrt{30}})$$

homework

- Sec 7-1 page 366
- #1-8 all, #11, 12, 17, 21, 23, 25

- **Optional: if you have a TI 83 or 84 see page 368**