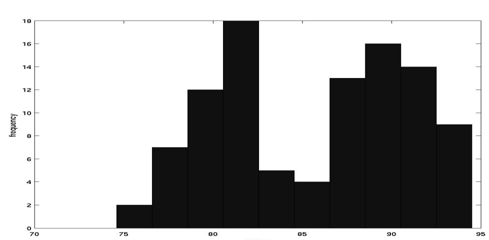
Section 1.1/1.2
Graphical and Numerical Summaries of Data

- **Shape of a Distribution**
  - **Modes**
  - **Symmetric vs. Skewed**
  - **Outliers**
- **Measures of the Center**
  - **mean**
  - **median**
- **Measures of Spread**
  - **IQR**
  - **standard deviation**
- **Choosing Summaries of Distributions**
- **Changing the Units of Measurement**

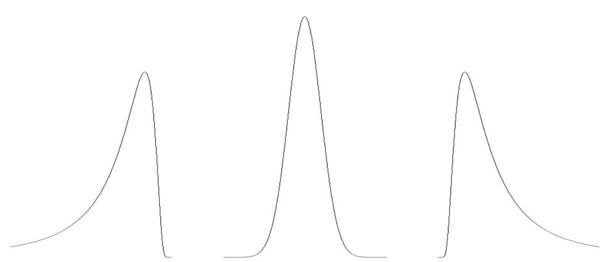
Modes

- Question: Does the distribution have one or several major peaks?
  → Look at histograms and stemplots.
- A distribution with one major peak is called **unimodal**. A distribution with two major peaks is called **bimodal**.
- Example of a bimodal distribution: scores on an exam

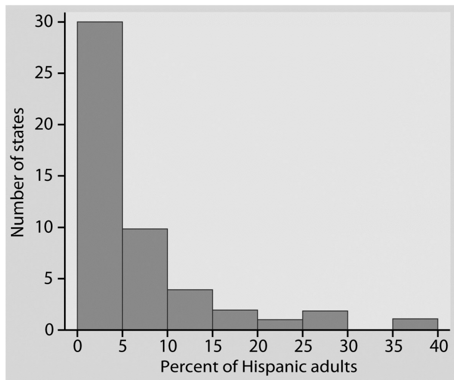


Symmetric vs. Skewed

- A distribution is **symmetric** if the values larger or smaller than the midpoint are mirror images of each other.
- A distribution is **skewed to the right** if the right tail (larger values) is much longer than the left tail (smaller values).
- A distribution is **skewed to the left** if the left tail (smaller values) is much longer than the right tail (larger values).



Left Skewed Symmetric Right Skewed



Outliers

Outliers – values that fall outside the overall pattern and are far from the bulk of the data

- Can be a result of natural variation.
- Or, can be evidence of a mistake (equipment failure, incorrect recording of an observation, etc.).

Removing an outlier? $\longrightarrow$ **Big Decision**

Measures of the Center

Two different ideas for the "center" of a distribution - can be very different.

- **Mean - "**average value**"**

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

or,
$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- **Median** - "middle value"

a) sort observations from smallest to largest

b) <u>if n is odd</u> (n = number of observations)

median = middle value of the sorted list

= $(n+1)/2^{th}$ observation up from the bottom of the list

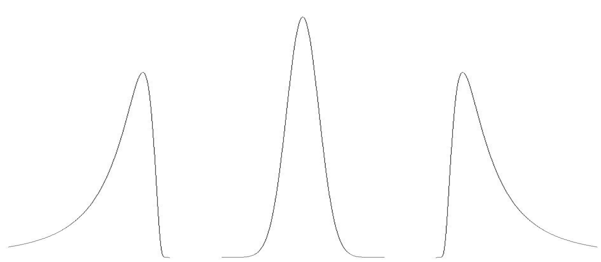
c) <u>if *n* is even</u>

median = mean of the middle two observations

Mean vs. Median

- The median is a more resistant measure of the center of a distribution, i.e., the median is not as affected by extreme observations (long tails, outliers)

***Mean vs. Median* Applet** - example of a **dot plot**
(http://bcs.whfreeman.com/ips4e/default.asp)



Left Skewed	Symmetric	Right Skewed
Mean < Median	**Mean = Median**	**Mean > Median**

Example: Phyllis received 6 HW grades in her statistics class:

86 88 92 44 89 90

Her mean grade is:

$$\frac{86+88+92+44+89+90}{6} = 81.5$$

Her median grade is:

44 86 88 89 90 92

$$\frac{88+89}{2} = 88.5$$

Question: Does the mean, 81.5, give a good idea of her "typical" grade?

No, it is lower than all but one of her grades.

Question: What about the median, 88.5?

88.5 is more typical.

Measures of Spread

The **p^{th} percentile** of a distribution is the value such that p percent of the observations fall at or below it.

Most common percentiles: QUARTILES (25%, 50% (median), 75%)

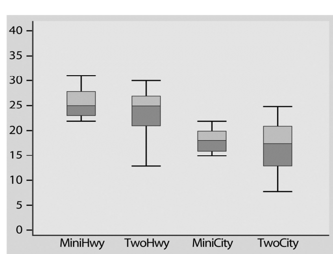
Q_1 (1st Quartile) - the median of the observations whose position in the ordered list is to the left of the location of the overall median.

Q_3 (3rd Quartile) - the median of the observations whose position in the ordered list is to the right of the location of the overall median.

Five-Number Summary: Minimum Q_1 Median Q_3 Maximum

Boxplots

- Boxplots are graphs of five-number summaries.
  - A central box spans the quartiles Q_1 and Q_3
  - A line in the box marks the median.
  - Lines extend from the box out to the largest and smallest observations.
- Boxplots are good for side-by-side comparison of a few variables.



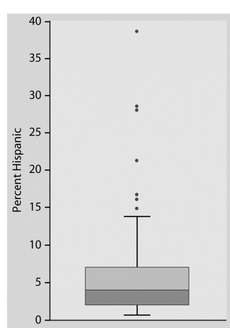
Measures of Spread

IQR vs. Standard Deviation

- **Inter Quartile Range (IQR)** = Q_3 - Q_1
  - Resistant to outliers.
  - Not very useful for describing skewed distribution (as are all measures of spread).
- **1.5 X IQR criterion for outliers -** call an observation an outlier if it falls more than 1.5 X IQR above Q_3 or below Q_1.

Statistics 528 - Lecture 3 15
Prof. Kate Calder

Modified Boxplot: lines extend out from the central box only to the smallest and largest observations that are not suspected outliers.



Statistics 528 - Lecture 3 16
Prof. Kate Calder

Variance (s^2) - average of the squares of the deviations of the observations from their mean

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2\right]$$

Standard deviation (s) - square root of the variance (has the same units as the data)

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Properties of the Standard Deviation

- s measures the spread about the mean and should only be used when the mean is chosen as the measure of the center of a distribution.
- s = 0 only when all the observations take on the same values. Otherwise, s > 0.
- s, like the mean $\bar{x}$, is not resistant to outliers. A few outliers can make s very large.

Choosing a Summary

- The median, IQR, or five-number summary are better than the mean and the standard deviation for describing a skewed distribution or a distribution with outliers.
- The mean and standard deviation should only be used for describing symmetric distributions with no outliers.
- Why should we ever used the mean and standard deviation?
  Answer: They completely specify a normal distribution which allows us to easily perform statistical inference.

Changing the Unit of Measurement

Linear Transformations: $x_{new} = a + bx$

- a (constant) shifts all of the values of x up or down by the same amount
- b (positive constant) changes the size of the unit of measurement

- A linear transformation will not change the shape of a distribution.
- Multiplying each observation by a positive constant b multiplies both measures of the center (mean and median) and measures of spread (IQR and standard deviation) by b.
- Adding the same number a (either positive or negative) to each observation adds a to the measures of the center (mean and median) and to the quartiles (and other percentiles) but does not change measures of spread.