Section 1.6: Relative Motion

Tutorial 1 Practice, page 47–48

1. (a) Given: $\vec{v}_{SW} = 2.8$ m/s [forward]; $\vec{v}_{TS} = 1.1$ m/s [forward]

Required: $\vec{v}_{TS}$

Analysis: Use $\vec{v}_{TW} = \vec{v}_{TS} + \vec{v}_{SW}$, with forward as the positive direction.

Solution:
$$\begin{aligned}\vec{v}_{TW} &= \vec{v}_{TS} + \vec{v}_{SW}\\ &= 1.1 \text{ m/s [forward]} + 2.8 \text{ m/s [forward]}\\ \vec{v}_{TW} &= 3.9 \text{ m/s [forward]}\end{aligned}$$

Statement: The velocity of the teenagers with respect to the water is 3.9 m/s [forward].

(b) Given: $\vec{v}_{SW} = 2.8$ m/s [forward]; $\vec{v}_{TS} = 1.1$ m/s [backward]

Required: $\vec{v}_{TS}$

Analysis: Use $\vec{v}_{TW} = \vec{v}_{TS} + \vec{v}_{SW}$, with forward as the positive direction.

Solution:
$$\begin{aligned}\vec{v}_{TW} &= \vec{v}_{TS} + \vec{v}_{SW}\\ &= 1.1 \text{ m/s [backward]} + 2.8 \text{ m/s [forward]}\\ &= -1.1 \text{ m/s [forward]} + 2.8 \text{ m/s [forward]}\\ \vec{v}_{TW} &= 1.7 \text{ m/s [forward]}\end{aligned}$$

Statement: The velocity of the teenagers with respect to the water is 1.7 m/s [forward].

2. Given: $\vec{v}_{PA} = 235$ km/h [N]; $\vec{v}_{AG} = 65$ km/h [E 45° N]

Required: $\vec{v}_{PG}$

Component Method:

Analysis: Use $(v_{PG})_x = (v_{PA})_x + (v_{AG})_x$ and $(v_{PG})_y = (v_{PA})_y + (v_{AG})_y$, with east and north as positive.

Solution: x-components:
$$\begin{aligned}(v_{PG})_x &= (v_{PA})_x + (v_{AG})_x\\ &= 0 \text{ km/h} + (65 \text{ km/h})(\cos 45°)\\ &= 0 \text{ km/h} + 45.96 \text{ km/h}\\ (v_{PG})_y &= 45.96 \text{ km/h} \quad \text{(two extra digits carried)}\end{aligned}$$

y-components:
$$\begin{aligned}(v_{PG})_y &= (v_{PA})_y + (v_{AG})_y\\ &= 235 \text{ km/h} + (65 \text{ km/h})(\sin 45°)\\ &= 235 \text{ km/h} + 45.96 \text{ km/h}\\ (v_{PG})_y &= 281.0 \text{ km/h} \quad \text{(two extra digits carried)}\end{aligned}$$

Now use these components to determine $\vec{v}_{PG}$.

$$\left|\vec{v}_{PG}\right| = \sqrt{\left|(v_{PG})_x\right|^2 + \left|(v_{PG})_y\right|^2}$$
$$= \sqrt{(45.96 \text{ km/h})^2 + (281.0 \text{ km/h})^2}$$
$$= 284.7 \text{ km/h (two extra digits carried)}$$
$$\left|\vec{v}_{PG}\right| = 280 \text{ km/h}$$

$$\theta_2 = \tan^{-1}\left(\frac{281.0 \cancel{\text{km/h}}}{45.96 \cancel{\text{km/h}}}\right)$$
$$\theta_2 = 81°$$

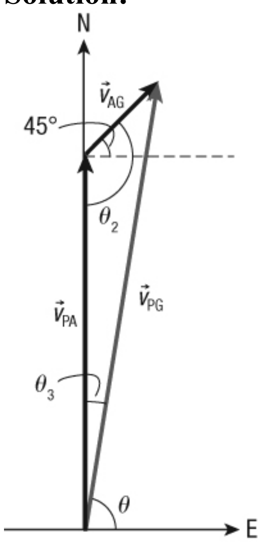
Statement: The speed and direction of the plane with respect to the ground are 280 km/h [E 81° N].

Geometry Method:

Analysis: I know two sides of the vector addition triangle, and the angle that lies in between. Draw a diagram of the situation. Use the cosine and sine laws to determine the speed and direction of the plane.

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Solution:



From the vector addition diagram, $\theta_2 = 90° + 45° = 135°$.

The cosine law gives

$$v_{PG}^2 = v_{PA}^2 + v_{AG}^2 - 2v_{PA}v_{AG}\cos\theta_2$$
$$= (235 \text{ km/h})^2 + (65 \text{ km/h})^2 - 2(235 \text{ km/h})(65 \text{ km/h})(\cos 135°)$$
$$v_{PG}^2 = 81\ 054 \text{ km/h}$$
$$v_{PG} = 284.7 \text{ km/h (two extra digits carried)}$$
$$v_{PG} = 280 \text{ km/h}$$

The sine law gives

$$\frac{\sin\theta_3}{v_{AG}} = \frac{\sin\theta_2}{v_{PG}}$$

$$\sin\theta_3 = \frac{(65\text{ km/h})(\sin 135°)}{284.7\text{ km/h}}$$

$$\theta_3 = 9.29°$$

$$\theta = 90° - \theta_3$$
$$= 90° - 9.29°$$
$$\theta = 81°$$

Statement: The velocity of the plane with respect to the ground is 280 km/h [E 81°N].

3. Given: $\vec{v}_{HA} = 175$ km/h [S]; $\vec{v}_{AG} = 85$ km/h [E]

Required: $\vec{v}_{HG}$

Analysis: The directions of the helicopter and the wind form a right angle with $\vec{v}_{HG}$ as the hypotenuse of a right-angled triangle. Use the Pythagorean theorem and the tangent ratio to determine $\vec{v}_{HG}$.

Solution: Determine the magnitude of $\vec{v}_{HG}$.

$$v_{HG} = \sqrt{v_{HA}^2 + v_{AG}^2}$$
$$= \sqrt{(175\text{ km/h})^2 + (85\text{ km/h})^2}$$
$$v_{HG} = 190\text{ km/h}$$

Determine the direction of $\vec{v}_{HG}$

$$\theta = \tan^{-1}\left(\frac{|\vec{v}_{HA}|}{|\vec{v}_{AG}|}\right)$$
$$= \tan^{-1}\left(\frac{175\ \cancel{\text{km/h}}}{85\ \cancel{\text{km/h}}}\right)$$
$$\theta = 64°$$

Statement: The velocity of the helicopter with respect to the ground is 190 km/h [E 64°S].

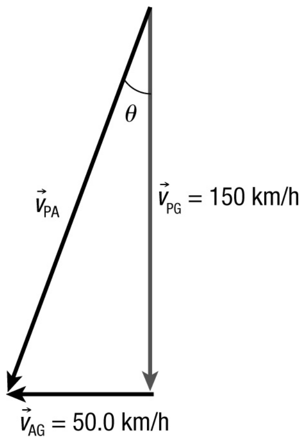
4. Given: $\Delta\vec{d} = 450$ km [S]; $\Delta t = 3.0$ h; $\vec{v}_{AG} = 50.0$ km/h [E]

Required: $\vec{v}_{PA}$

Analysis: The displacement of the plane is due south, so the direction of $\vec{v}_{PG}$ is also south. Determine the magnitude of $\vec{v}_{PG}$ using the given displacement and time. Then draw the vector addition diagram for $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. Since this is a right-angled triangle, use the Pythagorean theorem and the tangent ratio to determine $\vec{v}_{PA}$.

Solution: The ground speed is

$$v_{PG} = \frac{\Delta d}{\Delta t}$$
$$= \frac{450 \text{ km}}{3.0 \text{ h}}$$
$$v_{PG} = 150 \text{ km/h}$$



Determine the magnitude of $\vec{v}_{PA}$.

$$v_{PA} = \sqrt{(v_{PG})^2 + (v_{AG})^2}$$
$$= \sqrt{(150 \text{ km/h})^2 + (50.0 \text{ km/h})^2}$$
$$v_{PA} = 160 \text{ km/h}$$

Determine the direction of $\vec{v}_{PA}$.

$$\theta = \tan^{-1}\left(\frac{|\vec{v}_{AG}|}{|\vec{v}_{PG}|}\right)$$
$$= \tan^{-1}\left(\frac{50.0 \cancel{\text{km/h}}}{150 \cancel{\text{km/h}}}\right)$$
$$\theta = 18^\circ$$

Statement: The plane should head [S 18° W] with an airspeed of 160 km/h.

5. (a) Given: $\vec{v}_{FE} = 4.0$ m/s [N]; $\vec{v}_{CF} = 3.0$ m/s [N]

Required: $\vec{v}_{CE}$

Analysis: Use $\vec{v}_{CE} = \vec{v}_{CF} + \vec{v}_{FE}$, with north as the positive direction.

Solution:
$$\vec{v}_{CE} = \vec{v}_{CF} + \vec{v}_{FE}$$
$$= (+4.0 \text{ m/s}) + (+3.0 \text{ m/s})$$
$$= +7.0 \text{ m/s}$$
$$\vec{v}_{CE} = 7.0 \text{ m/s [N]}$$

Statement: When the child is running north, the velocity of the child with respect to Earth is 7.0 m/s [N].

(b) Given: $\vec{v}_{\text{FE}} = 4.0\text{ m/s [N]}$; $\vec{v}_{\text{CF}} = 3.0\text{ m/s [S]}$

Required: $\vec{v}_{\text{CE}}$

Analysis: Use $\vec{v}_{\text{CE}} = \vec{v}_{\text{CF}} + \vec{v}_{\text{FE}}$, with north as the positive direction.

Solution:
$$\begin{aligned}\vec{v}_{\text{CE}} &= \vec{v}_{\text{CF}} + \vec{v}_{\text{FE}} \\ &= (+4.0\text{ m/s}) + (-3.0\text{ m/s}) \\ &= +1.0\text{ m/s} \\ \vec{v}_{\text{CE}} &= 1.0\text{ m/s [N]}\end{aligned}$$

Statement: When the child is running south, the velocity of the child with respect to Earth is 1.0 m/s [N].

(c) Given: $\vec{v}_{\text{FE}} = 4.0\text{ m/s [N]}$; $\vec{v}_{\text{CF}} = 3.0\text{ m/s [E]}$

Required: $\vec{v}_{\text{CE}}$

Analysis: Use $\vec{v}_{\text{CE}} = \vec{v}_{\text{CF}} + \vec{v}_{\text{FE}}$. This is a right-angled triangle, so use the Pythagorean theorem and the tangent ratio.

Solution: Determine the magnitude of $\vec{v}_{\text{CE}}$.
$$\begin{aligned}v_{\text{CE}} &= \sqrt{(v_{\text{CF}})^2 + (v_{\text{FE}})^2} \\ &= \sqrt{(3.0\text{ m/s})^2 + (4.0\text{ m/s})^2} \\ v_{\text{CE}} &= 5.0\text{ m/s}\end{aligned}$$

Determine the direction of $\vec{v}_{\text{CE}}$.
$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{|\vec{v}_{\text{FE}}|}{|\vec{v}_{\text{CF}}|}\right) \\ &= \tan^{-1}\left(\frac{4.0\ \cancel{\text{m}}/\text{s}}{3.0\ \cancel{\text{m}}/\text{s}}\right) \\ \theta &= 53^\circ\end{aligned}$$

Statement: When the child is running east, the child's velocity with respect to Earth is 5.0 m/s [E 53° N], or equivalently 5.0 m/s [N 37° E].

6. (a) Given: $\vec{v}_{\text{PA}} = 3.5 \times 10^2\text{ km/h [N 35° W]}$; $\vec{v}_{\text{AG}} = 62\text{ km/h [S]}$

Required: $\vec{v}_{\text{PG}}$

Analysis: Since one of the given velocities points due south, use the component method of solution. Use $(v_{\text{PG}})_x = (v_{\text{PA}})_x + (v_{\text{AG}})_x$ and $(v_{\text{PG}})_y = (v_{\text{PA}})_y + (v_{\text{AG}})_y$, with east and north as positive.

Solution: *x*-components:
$$\begin{aligned}(v_{\text{PG}})_x &= (v_{\text{PA}})_x + (v_{\text{AG}})_x \\ &= (-350\text{ km/h})(\sin 35^\circ) + 0\text{ km/h} \\ (v_{\text{PG}})_x &= -200.8\text{ km/h}\ \ \text{(two extra digits carried)}\end{aligned}$$

y-components:

$$(v_{\text{PG}})_y = (v_{\text{PA}})_y + (v_{\text{AG}})_y$$
$$= (350 \text{ km/h})(\cos 35°) + (-62 \text{ km/h})$$
$$(v_{\text{PG}})_y = 224.7 \text{ km/h} \text{ (two extra digits carried)}$$

Now use these components to determine $\vec{v}_{\text{PG}}$.

$$|\vec{v}_{\text{PG}}| = \sqrt{|(v_{\text{PG}})_x|^2 + |(v_{\text{PG}})_y|^2}$$
$$= \sqrt{(200.75 \text{ km/h})^2 + (224.70 \text{ km/h})^2}$$
$$= 301.3 \text{ km/h} \text{ (two extra digits carried)}$$
$$|\vec{v}_{\text{PG}}| = 3.0 \times 10^2 \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{200.8 \text{ km/h}}{224.7 \text{ km/h}}\right)$$
$$\theta = 42°$$

Statement: The velocity of plane with respect to the ground is 3.0×10^2 km/h [N 42° W].

(b) Given: $\vec{v}_{\text{PG}} = 301.3$ km/h [N 42º W]; $\Delta t = 1.2$ h

Required: $\Delta\vec{d}$

Analysis: Since the plane moves at constant ground velocity, use $\vec{v}_{\text{av}} = \frac{\Delta\vec{d}}{\Delta t}$; $\Delta\vec{d} = \vec{v}_{\text{av}}\Delta t$.

Solution: $\Delta\vec{d} = \vec{v}_{\text{PG}}\Delta t$

$$= (301.3 \text{ km/h [N 42° W]})(1.2 \text{ h})$$
$$\Delta\vec{d} = 3.6 \times 10^2 \text{ km/h [N 42° W]}$$

Statement: The plane's displacement after 1.2 h is 3.6×10^2 km/h [N 42° W].

7. (a) Given: $\vec{v}_{\text{PW}} = 0.70$ m/s [N]; $\vec{v}_{\text{WE}} = 0.40$ m/s [E]

Required: $\vec{v}_{\text{PE}}$

Analysis: The directions of the current and the swimmer form a right angle with $\vec{v}_{\text{PE}}$ as the hypotenuse of a right-angled triangle. Use the Pythagorean theorem and the tangent ratio to determine $\vec{v}_{\text{PE}}$.

Solution: Determine the magnitude of $\vec{v}_{\text{PE}}$.

$$v_{\text{PE}} = \sqrt{(v_{\text{PW}})^2 + (v_{\text{WE}})^2}$$
$$= \sqrt{(0.70 \text{ m/s})^2 + (0.40 \text{ m/s})^2}$$
$$v_{\text{PE}} = 0.81 \text{ m/s}$$

Determine the direction of $\vec{v}_{\text{PE}}$.

$$\theta = \tan^{-1}\left(\frac{|\vec{v}_{\text{WE}}|}{|\vec{v}_{\text{PW}}|}\right)$$
$$= \tan^{-1}\left(\frac{0.40 \text{ m/s}}{0.70 \text{ m/s}}\right)$$
$$\theta = 30°$$

Statement: The velocity of the swimmer with respect to Earth is 0.81 m/s [N 30° E].

(b) Given: $\Delta\vec{d} = 84 \text{ m [N]}$

Required: time to cross river, Δt

Analysis: The component of $\vec{v}_{\text{PE}}$ pointing north is $\vec{v}_{\text{PW}}$. Use $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$; $\Delta t = \dfrac{\Delta d}{v_{\text{av}}}$.

Solution:
$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{\text{PW}}}\\ &= \frac{84 \text{ m}}{0.70 \text{ m/s}}\\ \Delta t &= 1.2\times10^{2} \text{ s}\end{aligned}$$

Statement: It takes the swimmer 1.2×10^2 s to cross the river.

(c) Given: $\vec{v}_{\text{WE}} = 0.40 \text{ m/s [E]}$; $\Delta t = 120 \text{ s}$

Required: distance downstream, Δd

Analysis: The component of $\vec{v}_{\text{PE}}$ pointing east is $\vec{v}_{\text{WE}}$. Use $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$; $\Delta\vec{d} = \vec{v}_{\text{av}}\Delta t$.

Solution:
$$\begin{aligned}\Delta d &= v_{\text{WE}}\Delta t\\ &= (0.40 \text{ m/s})(120 \text{ s})\\ \Delta d &= 48 \text{ m}\end{aligned}$$

Statement: The swimmer lands 48 m downstream.

(d) Given: $v_{\text{PW}} = 0.70 \text{ m/s}$; $\vec{v}_{\text{WE}} = 0.40 \text{ m/s [E]}$; $\vec{v}_{\text{PE}} = ?$ [N]

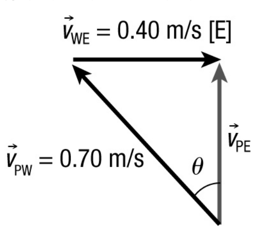
Required: direction of $\vec{v}_{\text{PW}}$

Analysis: The vectors form a right-angled triangle with $\vec{v}_{\text{PW}}$ as the hypotenuse. is a right-angled triangle. Use the sine ratio to determine the direction of $\vec{v}_{\text{PW}}$ with respect to $\vec{v}_{\text{PE}}$ [N]:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypoteuse}}$$

$$\theta = \sin^{-1}\left(\frac{\text{opposite side}}{\text{hypoteuse}}\right)$$

Solution: Determine the direction of $\vec{v}_{\text{PW}}$.



$$\theta = \sin^{-1}\left(\frac{|\vec{v}_{WE}|}{|\vec{v}_{PW}|}\right)$$
$$= \sin^{-1}\left(\frac{0.40\ \cancel{m/s}}{0.70\ \cancel{m/s}}\right)$$
$$\theta = 35°$$

Statement: The swimmer should head [N 35° W] to land directly north of her starting point.

8. Given: $|\vec{v}_{C_1W}| = |\vec{v}_{C_2W}|$; $\vec{v}_{C_1E} = 1.2$ m/s [upstream]; $\vec{v}_{C_2E} = 2.9$ m/s [downstream]

Required: v_{WE}

Analysis: I know the relative velocity equations, $\vec{v}_{C_1E} = \vec{v}_{C_1W} + \vec{v}_{WE}$ and $\vec{v}_{C_2E} = \vec{v}_{C_2W} + \vec{v}_{WE}$.

Switching to a simpler notation, $v = |\vec{v}_{C_1W}| = |\vec{v}_{C_2W}|$ and $w = \vec{v}_{WE}$. I will rewrite the relative velocity equations, using downstream as the positive direction. Then, I can solve for the required speed.

Solution: The relative velocity equations are

$$\vec{v}_{C_1E} = \vec{v}_{C_1W} + \vec{v}_{WE}$$
$$-1.2\text{ m/s} = (-v) + w \qquad \text{(Equation 1)}$$
$$\vec{v}_{C_2E} = \vec{v}_{C_2W} + \vec{v}_{WE}$$
$$+2.9\text{ m/s} = v + w \qquad \text{(Equation 2)}$$

Adding Equations 1 and 2,

$$(-1.2\text{ m/s}) + (+2.9\text{ m/s}) = (-v) + w + v + w$$
$$1.7\text{ m/s} = 2w$$
$$w = 0.85\text{ m/s}$$

Statement: The speed of the water relative to Earth is 0.85 m/s.

(b) Given: $\vec{v}_{C_2E} = 2.9$ m/s [downstream]; $v_{WE} = w = 0.85$ m/s

Required: $v = |\vec{v}_{C_1W}| = |\vec{v}_{C_2W}|$

Analysis: Substitute the speed of the water from part (a) into either Equation 1 or Equation 2.

Solution: Using Equation 2,

$$(+2.9\text{ m/s}) = v + w$$
$$v = (+2.9\text{ m/s}) - w$$
$$= (+2.9\text{ m/s}) - (0.85\text{ m/s})$$
$$v = 2.0\text{ m/s}$$

Statement: The canoeists paddle at 2.0 m/s with respect to the water.

9. (a) Given: $\vec{v}_{PA} = 630$ km/h [N]; $\vec{v}_{AG} = 35$ km/h [S]; $\Delta\vec{d} = 750$ km [N]

Required: Δt

Analysis: Use $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, with north as the positive direction, to calculate the ground velocity of the plane. Then use $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$ to determine the time $\Delta t = \frac{\Delta d}{v_{av}}$.

Solution: The ground velocity is

$$\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$$
$$= (+630 \text{ km/h}) + (-35 \text{ km/h})$$
$$= +595 \text{ km/h}$$
$$\vec{v}_{\text{PG}} = 595 \text{ km/h [N]} \ \text{(one extra digit carried)}$$

The required time, Δt, is

$$\Delta t = \frac{\Delta d}{v_{\text{PG}}}$$
$$= \frac{750 \ \cancel{\text{km}}}{595 \ \cancel{\text{km}}\text{/h}}$$
$$\Delta t = 1.3 \text{ h}$$

Statement: The flight time is 1.3 h when the wind is blowing south.

(b) Given: $\vec{v}_{\text{PA}} = 630$ km/h [N]; $\vec{v}_{\text{AG}} = 35$ km/h [N]; $\Delta\vec{d} = 750$ km [N]

Required: Δt

Analysis: Use $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$, with north as the positive direction, to calculate the ground velocity of the plane. Then use $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$ to determine the time Δt.

Solution: The ground velocity is

$$\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$$
$$= (+630 \text{ km/h}) + (+35 \text{ km/h})$$
$$= +665 \text{ km/h}$$
$$\vec{v}_{\text{PG}} = 665 \text{ km/h [N]} \ \text{(one extra digit carried)}$$

The required time Δt is

$$v_{\text{av}} = \frac{\Delta d}{\Delta t}$$
$$\Delta t = \frac{\Delta d}{v_{\text{PG}}}$$
$$= \frac{750 \ \cancel{\text{km}}}{665 \ \cancel{\text{km}}\text{/h}}$$
$$\Delta t = 1.1 \text{ h}$$

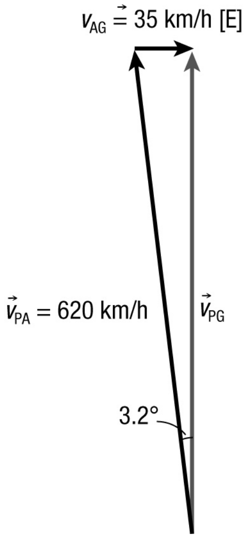
Statement: The flight time is 1.1 h when there is a tail wind. This time is shorter than when there is an opposing wind because the plane moves more quickly with respect to the ground.

(c) Given: $v_{\text{PA}} = 630$ km/h; $\vec{v}_{\text{AG}} = 35$ km/h [E]; $\Delta\vec{d} = 750$ km [N]

Required: direction of $\vec{v}_{\text{PA}}$, Δt

Analysis: The pilot needs to head somewhat west of north to compensate for the wind that heads east. Sketch the relative velocities, $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. Since the vector addition triangle is a right-angled triangle, use the Pythagorean theorem and the sine ratio to determine the direction of the air velocity and the magnitude of the ground velocity. Then determine the flight time using $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$; $\Delta t = \dfrac{\Delta d}{v_{\text{av}}}$.

Solution:



Determine the direction of $\vec{v}_{\text{PA}}$.

$$\theta = \sin^{-1}\left(\frac{|\vec{v}_{\text{AG}}|}{|\vec{v}_{\text{PA}}|}\right)$$

$$= \sin^{-1}\left(\frac{35\ \cancel{\text{km/h}}}{630\ \cancel{\text{km/h}}}\right)$$

$$\theta = 3.2°$$

Determine the magnitude of $\vec{v}_{\text{PG}}$.

$$(v_{\text{PA}})^2 = (v_{\text{PG}})^2 + (v_{\text{AG}})^2$$

$$v_{\text{PG}} = \sqrt{(v_{\text{PA}})^2 - (v_{\text{AG}})^2}$$

$$= \sqrt{(630\ \text{km/h})^2 - (35\ \text{km/h})^2}$$

$$= 629.0\ \text{km/h} \ \ \text{(two extra digits carried)}$$

$$\Delta t = \frac{\Delta d}{v_{PG}}$$
$$= \frac{750\ \cancel{km}}{629.0\ \cancel{km}/h}$$
$$\Delta t = 1.2\ h$$
Statement: The pilot's heading needs to be [N 3.2° W]. The new flight time is 1.2 h.

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1. (a) Given: $v_{PW} = 1.2$ m/s; $\vec{v}_{WE} = 0.50$ m/s [E]; $\Delta\vec{d}_1 = 1.0$ km [W]; $\Delta\vec{d}_2 = 1.0$ km [E]

Required: $\Delta t = \Delta t_1 + \Delta t_2$

Analysis: Look first at the upstream motion. Determine the speed of the person with respect to Earth using $\vec{v}_{PE} = \vec{v}_{PW} + \vec{v}_{WE}$. Then, rearrange the equation $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$ to determine the time required; $\Delta t = \frac{\Delta d}{v_{av}}$. Repeat this procedure for the downstream motion. Finally, use $\Delta t = \Delta t_1 + \Delta t_2$. Throughout, use east as the positive direction.

Solution:

upstream motion:
$$\vec{v}_{PE} = \vec{v}_{PW} + \vec{v}_{WE}$$
$$= (-1.2\ \text{m/s}) + (+0.50\ \text{m/s})$$
$$= -0.70\ \text{m/s}$$
$$\Delta t_1 = \frac{\Delta d}{v_{PW}}$$
$$= \frac{1000\ \cancel{m}}{0.70\ \cancel{m}/s}$$
$$\Delta t_1 = 1429\ \text{s} \text{ (two extra digits carried)}$$

downstream motion:
$$\vec{v}_{PE} = \vec{v}_{PW} + \vec{v}_{WE}$$
$$= (+1.2\ \text{m/s}) + (+0.50\ \text{m/s})$$
$$= +1.70\ \text{m/s}$$
$$\Delta t_2 = \frac{\Delta d}{v_{PW}}$$
$$= \frac{1000\ \cancel{m}}{1.70\ \cancel{m}/s}$$
$$\Delta t_2 = 588.2\ \text{s} \text{ (two extra digits carried)}$$

The total time for the swim is
$$\Delta t = \Delta t_1 + \Delta t_2$$
$$= 1429\ \text{s} + 588.2\ \text{s}$$
$$= 2017\ \cancel{s} \times \frac{1\ \text{min}}{60\ \cancel{s}}$$
$$\Delta t = 34\ \text{min}$$
Statement: The swim upstream and back takes 34 min.

(b) Answers may vary. Sample answer: No, the time will not change. The total time downstream and back will also be 34 min. The first leg of the swim will be fast (588 s) and the second leg slow (1429 s), but the whole swim takes the same time.

(c) Given: $v = 1.2$ m/s; $\Delta d = 2.0$ km
Required: Δt
Analysis: $v_{av} = \dfrac{\Delta d}{\Delta t}$; $\Delta t = \dfrac{\Delta d}{v_{av}}$

Solution:
$$\begin{aligned}\Delta t &= \frac{\Delta d}{v} \\ &= \frac{2000\ \cancel{m}}{1.2\ \cancel{m}/s} \\ &= 1667\ \cancel{s} \times \frac{1\ \text{min}}{60\ \cancel{s}}\ \text{(two extra digits carried)} \\ \Delta t &= 28\ \text{min}\end{aligned}$$

Statement: The total swim would take 28 min in still water. This time is less than for the swims in parts (a) and (b). Swimming against the current is a great disadvantage that is not compensated for fully by swimming with the current for the same distance. The trip takes longer when there is a current because the current slows the swimmer when swimming against it.

2. (a) Given: $\vec{v}_{PA} = 200$ m/s [W]; $\vec{v}_{AG} = 60$ m/s [N]
Required: $\vec{v}_{CE}$
Analysis: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. This is a right-angled triangle, so use the Pythagorean theorem and the tangent ratio.
Solution: Determine the magnitude of $\vec{v}_{PG}$.
$$\begin{aligned}(v_{PG})^2 &= (v_{PA})^2 + (v_{AG})^2 \\ &= (200\ \text{m/s})^2 + (60\ \text{m/s})^2 \\ (v_{PG})^2 &= 43\ 600\ \text{m}^2/\text{s}^2 \\ v_{PG} &= 209\ \text{m/s (two extra digits carried)} \\ v_{PG} &= 200\ \text{m/s}\end{aligned}$$
Determine the direction of $\vec{v}_{PG}$.
$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{|\vec{v}_{AG}|}{|\vec{v}_{PG}|}\right) \\ &= \tan^{-1}\left(\frac{60\ \cancel{m/s}}{209\ \cancel{m/s}}\right) \\ \theta &= 20^\circ\end{aligned}$$
Statement: The ground velocity of the plane is 200 m/s [W 20° N].

(b) Given: $\vec{v}_{PA} = 200$ m/s [W]; $\vec{v}_{AG} = 60$ m/s [E]; $\Delta\vec{d} = 300$ km [W]
Required: Δt
Analysis: First, use $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ to determine the ground velocity. Then rearrange the equation $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$ to calculate the flight time; $\Delta t = \dfrac{\Delta d}{v_{av}}$.
Solution: The ground velocity is

$$\begin{aligned}\vec{v}_{PG} &= \vec{v}_{PA} + \vec{v}_{AG} \\ &= 200 \text{ m/s [W]} + 60 \text{ m/s [E]} \\ &= 200 \text{ m/s [W]} + (-60 \text{ m/s [W]}) \\ \vec{v}_{PG} &= 140 \text{ m/s [W]} \ \text{ (one extra digit carried)}\end{aligned}$$

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{PG}} \\ &= \frac{300 \ \cancel{\text{km}}}{140 \ \cancel{\text{m}}\text{/s}} \times \frac{1000 \ \cancel{\text{m}}}{1 \ \cancel{\text{km}}} \\ &= 2143 \ \cancel{\text{s}} \times \frac{1 \text{ min}}{60 \ \cancel{\text{s}}} \\ \Delta t &= 36 \text{ min}\end{aligned}$$

Statement: The flight time is 36 min.
3. (a) Given: $\vec{v}_{HA} = 62$ m/s [N]; $\vec{v}_{AE} = 18$ m/s [N]
Required: $\vec{v}_{HE}$
Analysis: $\vec{v}_{HE} = \vec{v}_{HA} + \vec{v}_{AE}$
Solution: $\vec{v}_{HE} = \vec{v}_{HA} + \vec{v}_{AE}$

$$\begin{aligned} &= 62 \text{ m/s [N]} + 18 \text{ m/s [N]} \\ \vec{v}_{HE} &= 8.0 \times 10^1 \text{ m/s [N]}\end{aligned}$$

Statement: The ground velocity of the helicopter is 8.0×10^1 m/s [N].
(b) Given: $\vec{v}_{HA} = 62$ m/s [N]; $\vec{v}_{AE} = 18$ m/s [S]
Required: $\vec{v}_{HE}$
Analysis: $\vec{v}_{HE} = \vec{v}_{HA} + \vec{v}_{AE}$
Solution: $\vec{v}_{HE} = \vec{v}_{HA} + \vec{v}_{AE}$

$$\begin{aligned} &= 62 \text{ m/s [N]} + 18 \text{ m/s [S]} \\ &= 62 \text{ m/s [N]} + (-18 \text{ m/s [N]}) \\ \vec{v}_{HE} &= 44 \text{ m/s [N]}\end{aligned}$$

Statement: The ground velocity of the helicopter is 44 m/s [N].

(c) Given: $\vec{v}_{\text{HA}} = 62\text{ m/s [N]}; \vec{v}_{\text{AE}} = 18\text{ m/s [W]}$

Required: $\vec{v}_{\text{HE}}$

Analysis: The vectors $\vec{v}_{\text{HE}} = \vec{v}_{\text{HA}} + \vec{v}_{\text{AE}}$ form a right-angled triangle with $\vec{v}_{\text{HE}}$ as the hypotenuse. Use the Pythagorean theorem and the tangent ratios to determine $\vec{v}_{\text{HE}}$.

Solution:
$$\begin{aligned} v_{\text{HE}} &= \sqrt{(v_{\text{HA}})^2 + (v_{\text{AE}})^2} \\ &= \sqrt{(62\text{ m/s})^2 + (18\text{ m/s})^2} \\ v_{\text{HE}} &= 65\text{ m/s} \end{aligned}$$

Determine the direction of $\vec{v}_{\text{HE}}$.

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{|\vec{v}_{\text{AE}}|}{|\vec{v}_{\text{HA}}|}\right) \\ &= \tan^{-1}\left(\frac{18\ \cancel{\text{m/s}}}{62\ \cancel{\text{m/s}}}\right) \\ \theta &= 16^\circ \end{aligned}$$

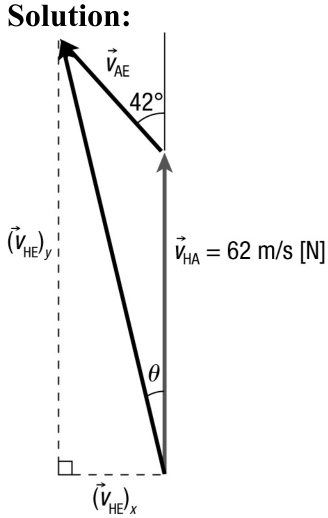
Statement: The velocity of the helicopter with respect to Earth is 65 m/s [N 16° W].

(d) Given: $\vec{v}_{\text{HA}} = 62\text{ m/s [N]}; \vec{v}_{\text{AE}} = 18\text{ m/s [N 42° W]}$

Required: $\vec{v}_{\text{HE}}$

Analysis: Draw the vector addition diagram for the situation. Use components to determine $\vec{v}_{\text{HE}}$. Use east and north as positive.

Solution:



x-components:

$$\begin{aligned} (v_{\text{HE}})_x &= (v_{\text{HA}})_x + (v_{\text{AE}})_x \\ &= 0\text{ m/s} + (-(18\text{ m/s})\sin 42^\circ) \\ &= 0\text{ m/s} + (-12.04\text{ m/s}) \\ (v_{\text{HE}})_x &= -12.04\text{ m/s}\ \text{(two extra digits carried)} \end{aligned}$$

y-components:

$$(v_{HE})_y = (v_{HA})_y + (v_{AE})_y$$
$$= 62 \text{ m/s} + (18 \text{ m/s})(\cos 42°)$$
$$= 62 \text{ m/s} + 13.38 \text{ m/s}$$
$$(v_{HE})_y = 75.38 \text{ m/s} \text{ (two extra digits carried)}$$

Determine the magnitude of $\vec{v}_{HE}$.

$$v_{HE} = \sqrt{\left|(v_{HE})_x\right|^2 + \left|(v_{HE})_y\right|^2}$$
$$= \sqrt{(12.04 \text{ m/s})^2 + (75.38 \text{ m/s})^2}$$
$$v_{HE} = 76 \text{ m/s}$$

Determine the direction of $\vec{v}_{HE}$.

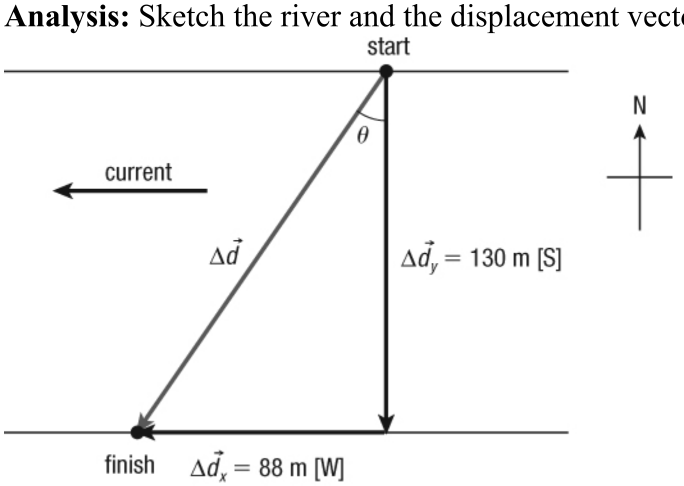
$$\theta = \tan^{-1}\left(\frac{\left|(\vec{v}_{HE})_x\right|}{\left|(\vec{v}_{HE})_y\right|}\right)$$
$$= \tan^{-1}\left(\frac{12.04 \cancel{\text{m/s}}}{75.38 \cancel{\text{m/s}}}\right)$$
$$\theta = 9.1°$$

Statement: The velocity of the helicopter with respect to the ground is 76 m/s [N 9.1° W].

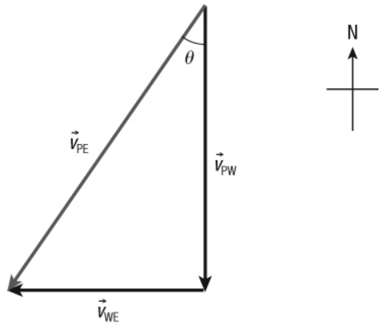
4. (a) Given: $\vec{v}_{PW} = 0.65$ m/s [S]; $\Delta d_x = 88$ m [W]; $\Delta d_y = 130$ m [S]; river flows [W]

Required: $\vec{v}_{WE}$

Analysis: Sketch the river and the displacement vectors.



Also draw $\vec{v}_{\text{PE}} = \vec{v}_{\text{PW}} + \vec{v}_{\text{WE}}$.



Vectors $\vec{v}_{\text{PW}}$ and $\vec{v}_{\text{WE}}$ are perpendicular components of $\vec{v}_{\text{PE}}$.
I know the river flows west and that the swimmer drifts 88 m west during her crossing. I can determine the time Δt of the crossing because she swims heading straight across the 130 m at 0.65 m/s. Use $v_{\text{av}} = \dfrac{\Delta d_y}{\Delta t}$ to determine the crossing time and the $v_{\text{av}} = \dfrac{\Delta d_x}{\Delta t}$ to determine the speed of the current. Use south and west as the positive directions.

Solution:

Crossing the river:

$$v_{\text{av}} = \frac{\Delta d_y}{\Delta t}$$

$$\Delta t = \frac{\Delta d_y}{v_{\text{PW}}} = \frac{130 \text{ m}}{0.65 \text{ m/s}}$$

$$\Delta t = 200 \text{ s}$$

Drifting downstream:

$$v_{\text{av}} = \frac{\Delta d_{\text{x}}}{\Delta t}$$

$$v_{\text{WE}} = \frac{88 \text{ m}}{200 \text{ s}}$$

$$v_{\text{WE}} = 0.44 \text{ m/s}$$

Statement: The water moves at 0.44 m/s [W].

(b) Given: $\Delta d_x = 88$ m [W]; $\Delta d_y = 130$ m [S]

Required: $\vec{v}_{\text{PE}}$

Analysis: The displacement triangle and the velocity triangle are similar, right-angled triangles. Use the displacement triangle to determine the angle θ, and the velocity triangle to calculate $\vec{v}_{\text{PE}}$.

Solution: In the displacement triangle:

$$\theta = \tan^{-1}\left(\frac{|\Delta\vec{d}_x|}{|\Delta\vec{d}_y|}\right) = \tan^{-1}\left(\frac{88 \text{ m}}{130 \text{ m}}\right)$$

$$\theta = 34°$$

In the velocity triangle:

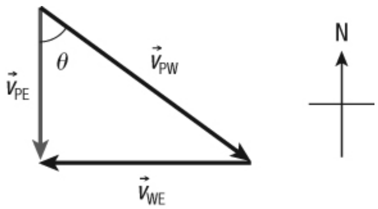
$$(v_{PE})^2 = (v_{PW})^2 + (v_{WE})^2$$
$$= (0.65\text{ m/s})^2 + (0.44\text{ m/s})^2$$
$$v_{PE} = 0.78\text{ m/s}$$

Statement: The velocity of the swimmer with respect to Earth is 0.78 m/s [S 34° W].

(c) Given: $v_{PW} = 0.65$ m/s ; $\vec{v}_{WE} = 0.44$ m/s [W]; $\vec{v}_{PE}$ = ? [S]

Required: direction of $\vec{v}_{PW}$

Analysis: Use $\vec{v}_{PE} = \vec{v}_{PW} + \vec{v}_{WE}$ to draw the relative velocities. This is a right-angled triangle. Use the sine ratio to determine the direction of $\vec{v}_{PW}$.



$$\sin\theta = \frac{\left|\vec{v}_{WE}\right|}{\left|\vec{v}_{PW}\right|};\ \theta = \sin^{-1}\left(\frac{\left|\vec{v}_{WE}\right|}{\left|\vec{v}_{PW}\right|}\right)$$

Solution: $\theta = \sin^{-1}\left(\frac{\left|\vec{v}_{WE}\right|}{\left|\vec{v}_{PW}\right|}\right)$

$$= \sin^{-1}\left(\frac{0.44\ \cancel{\text{m/s}}}{0.65\ \cancel{\text{m/s}}}\right)$$
$$\theta = 43°$$

Statement: The swimmer should head [S 43° E].

5. Given: $\Delta\vec{d} = 1.4 \times 10^3$ km [S 43° E]; $\Delta t = 3.4$ h; $\vec{v}_{AG} = 55$ km/h [S]

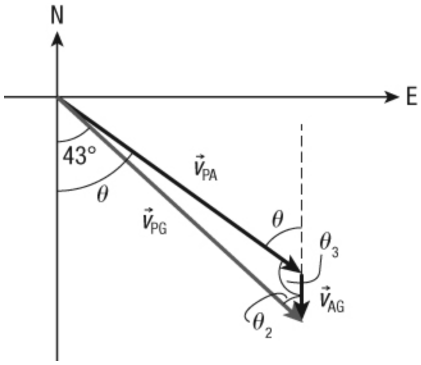
Required: $\vec{v}_{PA}$

Analysis: Draw the relative velocities using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. Solve the triangle using the cosine and sine laws:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Solution:



$$\vec{v}_{PG} = \frac{\Delta\vec{d}}{\Delta t}$$

$$= \frac{1.4\times10^3\ \text{km [S 43° E]}}{3.4\ \text{h}}$$

$$= 411.8\ \text{km/h [S 43° E]} \quad \text{(two extra digits carried)}$$

$$(v_{PA})^2 = (v_{PG})^2 + (v_{AG})^2 - 2(v_{PG})(v_{AG})\cos\theta_2$$

$$= (v_{PG})^2 + (v_{AG})^2 - 2(v_{PG})(v_{AG})\cos\theta_2$$

$$(v_{PA})^2 = (411.8\ \text{km/h})^2 + (55\ \text{km/h})^2 - 2(411.8\ \text{km/h})(55\ \text{km/h})(\cos 43°)$$

$$v_{PA} = 373.4\ \text{km/h} \ \ \text{(two extra digits carried)}$$

$$v_{PA} = 370\ \text{km/h}$$

$$\frac{\sin\theta_3}{v_{PG}} = \frac{\sin\theta_2}{v_{PA}}$$

$$\sin\theta_3 = \frac{v_{PG}\sin\theta_2}{v_{PA}}$$

$$= \frac{(411.8\ \cancel{\text{km/h}})(\sin 43°)}{373.4\ \cancel{\text{km/h}}}$$

$$\theta_3 = 131.2°$$

$$\theta = 180° - \theta_3$$

$$= 180° - 131.2°$$

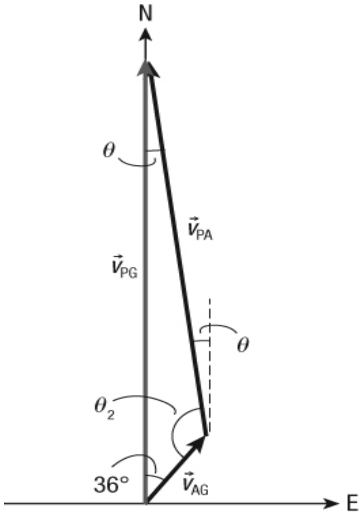
$$\theta = 49°$$

Statement: The plane must maintain an air velocity of 3.7×10^2 km/h [S 49° E].

6. (a) Given: $\Delta\vec{d} = 220$ km [N]; $\vec{v}_{AG} = 42$ km/h [N 36° E]; $v_{PA} = 230$ km/h [?]

Required: direction of $\vec{v}_{PA}$

Analysis: The direction of the ground velocity (the track) of the plane is [N] and I know the wind direction. So I can draw $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$.



The direction of the air speed (the heading), angle θ in the velocity triangle, can be determined using the sine law.

Solution: $\dfrac{\sin\theta}{v_{AG}} = \dfrac{\sin 36°}{v_{PA}}$

$$\sin\theta = \frac{v_{AG}\sin 36°}{v_{PA}}$$

$$= \frac{(42 \text{ km/h})(\sin 36°)}{(230 \text{ km/h})}$$

$$\theta = 6.167° \text{ (two extra digits carried)}$$

$$\theta = 6.2°$$

Statement: The heading of the plane should be [N 6.2° W].

(b) Given: $\Delta\vec{d} = 220$ km [N]; $\vec{v}_{AG} = 42$ km/h [N 36° E]; $v_{PA} = 230$ km/h; $\theta = 6.167°$

Required: Δt

Analysis: Use the vector triangle and the sine law to calculate ground speed, v_{PG}.

The calculate Δt using $v_{PG} = \dfrac{\Delta d}{\Delta t}$; $\Delta t = \dfrac{\Delta d}{v_{PG}}$.

Solution: Determine the third angle in the vector triangle, ϕ.

$$\phi + \theta + 36° = 180°$$

$$\phi = 180° - \theta - 36°$$

$$\phi = 180° - 6.167° - 36°$$

$$\phi = 137.8° \text{ (two extra digits carried)}$$

$$\frac{\sin\phi}{v_{PG}} = \frac{\sin 36°}{v_{PA}}$$

$$v_{PG} = \frac{v_{PA}\sin\phi}{\sin 36°}$$

$$= \frac{(230\text{ km/h})(\sin 137.8°)}{\sin 36°}$$

$$v_{PG} = 262.6\text{ km/h (two extra digits carried)}$$

$$\Delta t = \frac{\Delta d}{v_{PG}}$$

$$= \frac{220\ \cancel{\text{km}}}{262.6\ \cancel{\text{km}}\text{/h}}$$

$$\Delta t = 0.84\text{ h}$$

Statement: The trip will take 0.84 h.

7. (a) Given: $\vec{v}_{PA} = 250$ m/s [W]; $\vec{v}_{AG} = 50.0$ m/s [E]

Required: v_{PG}

Analysis: Use $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ to determine v_{PG}.

Solution: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$

$$= 250\text{ m/s [W]} + 50.0\text{ m/s [E]}$$

$$= 250\text{ m/s [W]} + (–50.0\text{ m/s [W]})$$

$$\vec{v}_{PG} = 2.0 \times 10^2\text{ m/s [W]}$$

Statement: The ground speed of the plane on its way west is 2.0×10^2 m/s.

(b) Given: $\vec{v}_{PA} = 250$ m/s [E]; $\vec{v}_{AG} = 50.0$ m/s [E]

Required: v_{PG}

Analysis: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$

Solution: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$

$$= 250\text{ m/s [E]} + 50.0\text{ m/s [E]}$$

$$\vec{v}_{PG} = 3.0 \times 10^2\text{ m/s [E]}$$

Statement: The ground speed of the plane on its way east is 3.0×10^2 m/s.

8. (a) Given: $\vec{v}_{PS} = 2.0$ m/s [up]; $\vec{v}_{SW} = 3.2$ m/s [E]

Required: $\vec{v}_{PW}$

Analysis: The relative velocity triangle, $\vec{v}_{PW} = \vec{v}_{PS} + \vec{v}_{SW}$, is right-angled with $\vec{v}_{PW}$ as the hypotenuse. Use the Pythagorean theorem and the tangent ratio to determine the magnitude and direction of $\vec{v}_{PW}$.

Solution: $(v_{PW})^2 = (v_{PS})^2 + (v_{SW})^2$

$$v_{PW} = \sqrt{(v_{PS})^2 + (v_{SW})^2}$$
$$= \sqrt{(2.0\text{ m/s})^2 + (3.2\text{ m/s})^2}$$
$$v_{PW} = 3.8\text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_{PS}}{v_{SW}}\right)$$
$$= \tan^{-1}\left(\frac{2.0\ \cancel{\text{m/s}}}{3.2\ \cancel{\text{m/s}}}\right)$$
$$\theta = 32°$$

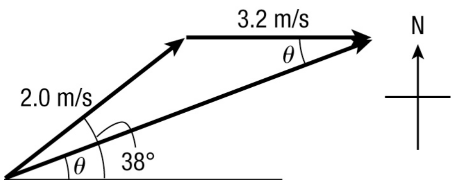
Statement: The people who take the elevator move at 3.8 m/s [E 32° up] relative to the water.

(b) Given: $\vec{v}_{PS} = 2.0$ m/s [E 38° up]; $\vec{v}_{SW} = 3.2$ m/s [E]

Required: $\vec{v}_{PW}$

Analysis: Sketch the relative velocity triangle, $\vec{v}_{PW} = \vec{v}_{PS} + \vec{v}_{SW}$. Use components to determine $\vec{v}_{PW}$, with east and up as positive.

Solution:



x-components:

$$(v_{PW})_x = (v_{PS})_x + (v_{SW})_x$$
$$= +(2.0\text{ m/s})\cos 38° + (+3.2\text{ m/s})$$
$$= 1.576\text{ m/s} + 3.2\text{ m/s}$$
$$(v_{PW})_x = 4.776\text{ m/s (two extra digits carried)}$$

y-components:

$$(v_{PW})_y = (v_{PS})_y + (v_{SW})_y$$
$$= (2.0\text{ m/s})\sin 38° + 0\text{ m/s}$$
$$= 1.231\text{ m/s} + 0\text{ m/s}$$
$$(v_{PW})_y = 1.231\text{ m/s (two extra digits carried)}$$

Now use these components to determine $\vec{v}_{PW}$.

$$|\vec{v}_{PW}| = \sqrt{|(v_{PW})_x|^2 + |(v_{PW})_y|^2}$$
$$= \sqrt{(4.776\text{ m/s})^2 + (1.231\text{ m/s})^2}$$
$$|\vec{v}_{PG}| = 4.9\text{ m/s}$$

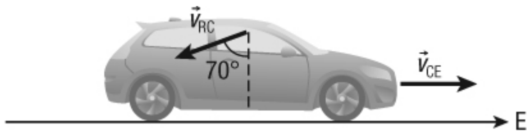
$$\theta = \tan^{-1}\left(\frac{1.231\ \cancel{\text{m/s}}}{4.776\ \cancel{\text{m/s}}}\right)$$
$$\theta = 14°$$

Statement: The people who take the stairs move at 4.9 m/s [E 14° up] relative to the water.

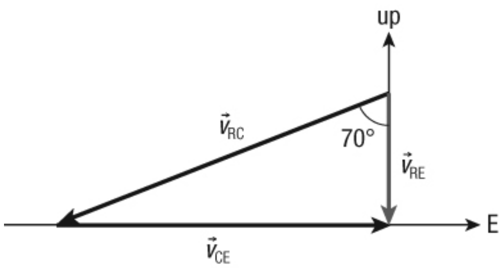
9. (a) Given: $\vec{v}_{\text{CE}} = 60.0$ km/h [E]; $\vec{v}_{\text{RC}}$ = ? [down 70.0° W]

Required: $\vec{v}_{\text{RC}}$

Analysis: Make a sketch of the car with the rain.



Also draw $\vec{v}_{\text{RE}} = \vec{v}_{\text{RC}} + \vec{v}_{\text{CE}}$.



Convert kilometres per hour to metres per second.

$$60\ \frac{\text{km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 16.667\text{ m/s (two extra digits carried)}$$

Solve for v_{RC} using the sine ratio.

Solution: $\sin 70° = \dfrac{v_{\text{CE}}}{v_{\text{RC}}}$

$$\begin{aligned} v_{\text{RC}} &= \frac{v_{\text{CE}}}{\sin 70.0°} \\ &= \frac{16.667\text{ m/s}}{\sin 70.0°} \\ &= 17.737\text{ m/s (two extra digits carried)} \\ v_{\text{RC}} &= 17.7\text{ m/s} \end{aligned}$$

Statement: The rain moves at 17.7 m/s [down 70.0° W] with respect to the car.

(b) Given: $\vec{v}_{\text{RC}}$ = 17.737 m/s [down 70.0° W]

Required: $\vec{v}_{\text{RE}}$

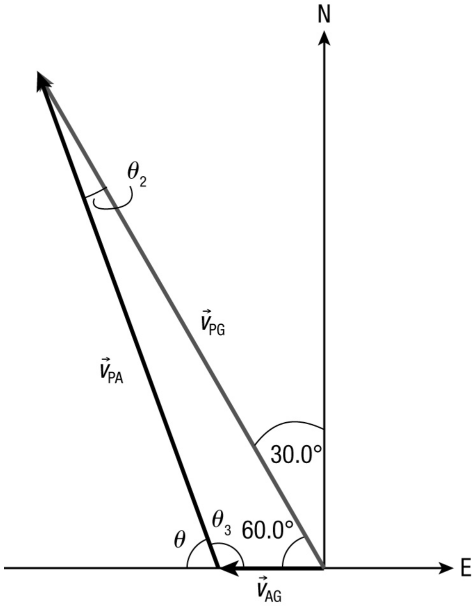
Analysis: Look at the triangles in part (a). Solve for v_{RE} using the cosine ratio.

Solution: $\cos 70° = \dfrac{v_{\text{RE}}}{v_{\text{RC}}}$

$$\begin{aligned} v_{\text{RE}} &= v_{\text{RC}} \cos 70.0° \\ &= (17.737\text{ m/s})(\cos 70.0°) \\ v_{\text{RE}} &= 6.07\text{ m/s} \end{aligned}$$

Statement: The velocity of the rain with respect to Earth is 6.07 m/s [down].

10. (a) Given: $\vec{v}_{PG}$ = ? [N 30.0° W]; $\vec{v}_{AG} = 48$ km/h [W]; $v_{PA} = 260$ km/h
Required: direction of $\vec{v}_{PA}$
Analysis: The direction of the ground velocity (the track) of the plane is [N 30.0° W], and I know the wind direction, so I can draw $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. The angle opposite $\vec{v}_{PA}$ in the relative velocity triangle is 60.0º (since [N 30.0° W] is equivalent to [W 60.0° N]).



Calculate the direction of the air speed (the heading) from the angles θ_2 and θ_3 in the scalene relative-velocity triangle using the sine law.
Solution: Using the sine law,

$$\frac{\sin\theta_2}{v_{AG}} = \frac{\sin 60.0°}{v_{PA}}$$

$$\sin\theta_2 = \frac{v_{AG}\sin 60.0°}{v_{PA}}$$

$$= \frac{(48 \text{ km/h})(\sin 60.0°)}{260 \text{ km/h}}$$

$$\theta_2 = 9.2°$$

$$\theta_3 = 180° - 60.0° - \theta_2$$

$$= 180° - 60.0° - 9.2°$$

$$\theta_3 = 110.8°$$

$$\theta = 180° - \theta_3$$

$$= 180° - 110.8°$$

$$\theta = 69°$$

The heading of the plane should be [W 69° N], which is equivalent to [N 21° W].
Statement: The heading of the plane should be [N 21° W].

(b) Given: $\vec{v}_{\text{PG}}$ = ? [W 60.0° N]; $v_{\text{PA}} = 260$ km/h; $\theta_3 = 110.8°$

Required: v_{PG}

Analysis: Use the relative-velocity triangle from part (a) and the sine law to determine v_{PG}.

Solution: $\dfrac{\sin\theta_3}{v_{\text{PG}}} = \dfrac{\sin 60.0°}{v_{\text{PA}}}$

$$v_{\text{PG}} = \frac{v_{\text{PA}} \sin\theta_3}{\sin 60.0°}$$

$$= \frac{(260 \text{ km/h})(\sin 110.8°)}{\sin 60.0°}$$

$$v_{\text{PG}} = 280 \text{ km/h}$$

Statement: The ground speed of the plane is 280 km/h.