Section 2: Equations and Inequalities

Section 2 – Topic 1

Equations: True or False?

Consider the statement $4 + 5 = 2 + 7$. This is a grammatically correct sentence.

Is the sentence true or false?

True

Consider the statement $1 + 3 = 8 + 6$. This statement is also a grammatically correct sentence.

Is the sentence true or false?

False

The previous statements are examples of ***number sentences***.

- A number sentence is a statement of equality between two __numerical__ expressions.
- A number sentence is said to be true if both numerical expressions are __equal__.
- If both numerical expressions don't equal the same number, we say the number sentence is __false__.
- True and false statements are called ***truth values***.

Let's Practice!

1. Determine whether the following number sentences are true or false. Justify your answer.

   a. $13 + 4 = 7 + 11$

   False. $\mathbf{13 + 4 = 17}$ **and** $\mathbf{7 + 11 = 18}$.

   b. $\frac{1}{2} + \frac{5}{8} = 1.4 - 0.275$

   True. $\frac{1}{2} + \frac{5}{8} = \frac{9}{8} = 1\frac{1}{8} = 1.125$ **and** $1.4 - 0.275 = 1.125$

Try It!

2. Determine whether the following number sentences are true or false. Justify your answer.

   a. $(83 \cdot 401) \cdot 638 = 401 \cdot (638 \cdot 83)$

   True. Associative property and commutative property show that these are equivalent.

   b. $(6 + 4)^2 = 6^2 + 4^2$

   False. $(6 + 4)^2 = (10)^2 = 100$ **and**
   $6^2 + 4^2 = 36 + 16 = 52$

A number sentence is an example of an ***algebraic equation***.

- An algebraic equation is a statement of equality between two ____expressions____.

- Algebraic equations can be number sentences (when both expressions contain only numbers), but often they contain ____variables____ whose values have not been determined.

Consider the algebraic equation $4(x+2) = 4x + 8$.

Are the expressions on each side of the equal sign equivalent? Justify your answer.

Yes. We can use the distributive property to show that $\mathbf{4(x+2) = 4x+8}$.

What does this tell you about the numbers we can substitute for x?

You can substitute any number for x and it would make the sentence true.

Let's Practice!

3. Consider the algebraic equation $x + 3 = 9$.

   a. What value can we substitute for x to make it a true number sentence?
   6

   b. How many values could we substitute for x and have a true number sentence?
   Only one

4. Consider the algebraic equation $x + 6 = x + 9$. What values could we substitute for x to make it a true number sentence?
   There are no values that would make the number sentence true.

Try It!

5. Complete the following sentences.

   a. $d^2 = 4$ is true for **$d = 2$ and $d = -2$**.

   b. $2m = m + m$ is true for **any value for m**.

   c. $d + 67 = d + 68$ is true for **no value for d**.

BEAT THE TEST!

1. Which of the following equations have the correct solution? Select all that apply.

   ☒ $2x + 5 = 19;\ x = 7$

   ☐ $3 + x + 2 - x = 16;\ x = 3$

   ☒ $\dfrac{x + 2}{5} = 2;\ x = 8$

   ☒ $6 = 2x - 8;\ x = 7$

   ☐ $14 = \frac{1}{3}x + 5;\ x = 18$

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Section 2 – Topic 2
Identifying Properties When Solving Equations

The following equations are equivalent. Describe the operation that occurred in the second equation.

$3 + 5 = 8$ and $3 + 5 - 5 = 8 - 5$

Subtracted 5 from both sides of the equation

$x - 3 = 7$ and $x - 3 + 3 = 7 + 3$

Added 3 to both sides of the equation

$2(4) = 8$ and $\frac{2(4)}{2} = \frac{8}{2}$

Divided both sides of the equation by 2

$\frac{x}{2} = 3$ and $2 \cdot \frac{x}{2} = 2 \cdot 3$

Multiplied both sides of the equation by 2

This brings us to some more properties that we can use to write equivalent equations.

Properties of Equality

If x is a solution to an equation, then x will also be a solution to the new equation formed when the same number is added to each side of the original equation.

These are the ***addition and subtraction properties of equality***.

- If $a = b$, then $a + c = b + c$ and $a - c = b - c$.
- Give examples of this property.

If $x - 2 = 5$, **then** $x - 2 + 2 = 5 + 2$.
If $x + 2 = 5$, **then** $x + 2 - 2 = 5 + 2 - 2$.

If x is a solution to an equation, x will also be a solution to the new equation formed when each side of the original equation is multiplied by the same number.

These are the ***multiplication and division properties of equality***.

- If $a = b$, then $a \cdot c = b \cdot c$ and $\frac{a}{c} = \frac{b}{c}$.
- Give examples of this property.

If $\frac{x}{2} = 30$, **then** $2 \cdot \frac{x}{2} = 2 \cdot 30$

If $2x = 30$, **then** $\frac{2x}{2} = \frac{30}{2}$.

Let's Practice!

1. The following equations are equivalent. Determine the property that was used to create the second equation.

   a. $x - 5 = 3x + 7$ and $x - 5 + 5 = 3x + 7 + 5$

   Addition property of equality

   b. $x = 3x + 12$ and $x - 3x = 3x - 3x + 12$

   Subtraction property of equality or Addition property of equality if you consider that you are adding $-3x$ both sides.

   c. $-2x = 12$ and $\frac{-2x}{-2} = \frac{12}{-2}$

   Division property of equality

   or Multiplication property of equality because you multiply by the reciprocal of -2, which is $-\frac{1}{2}$

Try It!

2. The following pairs of equations are equivalent. Determine the property that was used to create the second equation in each pair.

   a. $2(x+4) = 14 - 6x$ and $2x + 8 = 14 - 6x$

      Distributive property of equality

   b. $2x + 8 = 14 - 6x$ and $2x + 8 + 6x = 14 - 6x + 6x$

      Addition property of equality

   c. $2x + 8 + 6x = 14$ and $2x + 6x + 8 = 14$

      Commutative property of addition

   d. $8x + 8 = 14$ and $8x + 8 - 8 = 14 - 8$

      Subtraction property of equality

   e. $8x = 6$ and $\frac{1}{8} \cdot 8x = \frac{1}{8} \cdot 6$

      Division property of equality

BEAT THE TEST!

1. For each algebraic equation, select the property or properties that could be used to solve it.

Algebraic Equation	Addition or Subtraction Property of Equality	Multiplication or Division Property of Equality	Distributive Property	Commutative Property
$\frac{x}{2} = 5$	☐	☒	☐	☐
$2x + 7 = 13$	☒	☒	☐	☐
$4x = 23$	☐	☒	☐	☐
$x - 3 = -4$	☒	☐	☐	☐
$4(x + 5) = 40$	☒	☒	☒	☐
$10 + x = 79$	☒	☐	☐	☐
$-8 - x = 19$	☒	☒	☐	☐
$2(x - 8) + 7x = 9$	☒	☒	☒	☒

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Section 2 – Topic 3
Solving Equations

Sometimes you will be required to justify the steps to solve an equation. The following equation is solved for x. Use the properties to justify the reason for each step in the chart below.

Statements	Reasons
a. $5(x+3)-2=2-x+9$	**a.** Given
b. $5x+15-2=2-x+9$	**b.** Distributive Property
c. $5x+15-2=2+9-x$	**c.** Commutative Property of Addition
d. $5x+13=11-x$	**d.** Equivalent Equation
e. $5x+13-13=11-13-x$	**e.** Subtraction Property of Equality
f. $5x=-2-x$	**f.** Equivalent Equation
g. $5x+x=-2-x+x$	**g.** Addition Property of Equality
h. $6x=-2$	**h.** Equivalent Equation
i. $\frac{6x}{6}=\frac{-2}{6}$	**i.** Division Property of Equality
j. $x=-\frac{1}{3}$	**j.** Equivalent Equation

Other times, a word problem or situation may require you to write and solve an equation.

A class is raising funds to go ice skating at the Rink at Campus Martius in Detroit. The class plans to rent one bus. It costs $150.00 to rent a school bus for the day, plus $11.00 per student for admission to the rink, including skates.

What is the variable in the situation?

The number of students

Write an expression to represent the amount of money the school needs to raise.

Let x represent the number of students.

$150 + 11x$ represents the total cost.

The class raised $500.00 for the trip. Write an equation to represent the number of students who can attend the trip.

$$150 + 11x = 500$$

Solve the equation to determine the number of students who can attend the trip.

$$150 + 11x = 500$$

$$150 - 150 + 11x = 500 - 150$$

$$\frac{11x}{11} = \frac{350}{11}$$

$x = 31.8$ **31 students can attend the trip.**

Let's Practice!

1. Consider the equation $2x - 3(2x - 1) = 3 - 4x$. Solve the equation for x. For each step, identify the property used to write an equivalent equation.

$2x - 3(2x - 1) = 3 - 4x$	
$2x - 6x + 3 = 3 - 4x$	Distributive Property
$-4x + 3 - 3 = 3 - 3 - 4x$	Subtraction Prop. of Equality
$-4x = -4x$	Equivalent Equation

All real numbers are solutions.

STUDY EDGE TIP

Some equations, such as $2x = 2x$, have ***all real numbers*** as the solution. No matter what number we substitute for x, the equation will still be true.

Try It!

2. Consider the equation $3(4x + 1) = 3 + 12x - 5$. Solve the equation for x. For each step, identify the property used to convert the equation.

$3(4x + 1) = 3 + 12x - 5$	
$12x + 3 = 3 + 12x - 5$	Distributive Property
$12x + 3 = 3 - 5 + 12x$	Commutative Prop. of Add.
$12x + 3 - 3 = -2 - 3 + 12x$	Subtraction Prop. of Equality
$12x = -5 + 12x$	Equivalent Equation

No Solution

STUDY EDGE TIP

Some equations, such as $2x + 5 = 2x - 1$, have ***no solution***. There is no number that we could substitute for x that will make the equation true.

3. Brooklyn Technical High School surveyed its students about their favorite sports. The 487 students who listed soccer as their favorite sport represented 17 fewer students than three times the number of students who listed basketball as their favorite sport. Write and solve an equation to determine how many students listed basketball as their favorite sport.

Let b represent the number of students who listed basketball as their favorite sport.

$$487 = 3b - 17$$
$$487 + 17 = 3b - 17 + 17$$
$$\frac{504}{3} = \frac{3b}{3}$$
$$b = 168 \text{ students}$$

BEAT THE TEST!

1. The following equation is solved for x. Use the properties to justify the reason for each step in the chart below.

Statements	Reasons
a. $2(x+5)-3=15$	**a.** Given
b. $2x+10-3=15$	**b.** Distributive Property
c. $2x+7=15$	**c.** Equivalent Equation
d. $2x+7-7=15-7$	**d.** Addition Property of Equality
e. $2x=8$	**e.** Equivalent Equation
f. $\frac{2x}{2}=\frac{8}{2}$	**f.** Multiplication Property of Equality
g. $x=4$	**g.** Equivalent Equation

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<u>Section 2 – Topic 4</u>
<u>Solving Equations Using the Zero Product Property</u>

If someone told you that the product of two numbers is 10, what could you say about the two numbers?

The two numbers must be factors of 10.

If someone told you that the product of two numbers is zero, what could you say about the two numbers?

The two numbers are factors of zero.
One of the numbers <u>must</u> be zero.
Both of the numbers <u>could</u> be zero.

This is the ***zero product property***.

- If $ab = 0$, then either $a = 0$ or $b = 0$.

Describe how to use the zero product property to solve the equation $(x-3)(x+9) = 0$. Then, identify the solutions.

$(x-3)$ and $(x+9)$ are factors of zero. So, one or both of the factors could equal zero. So we will set both factors equal to zero and solve for x.

$x - 3 = 0$	$x + 9 = 0$
$x - 3 + 3 = 0 + 3$	$x + 9 - 9 = 0 - 9$
$x = 3$	$x = -9$

Solution set: $\{-9, 3\}$

Let's Practice!

1. Identify the solution(s) to $2x(x+4)(x+5)=0$.

$\frac{2x}{2}=\frac{0}{2}$ **or** $x+4=0$ **or** $x+5=0$

$x+4-4=0-4$ $x+5-5=0-5$

$x=0$ $x=-4$ $x=-5$

Solution Set: $\{-5,-4,0\}$

2. Identify the solution(s) to $(2x-5)(x+11)=0$.

$2x-5=0$ **or** $x+11=0$

$2x-5+5=0+5$ $x+11-11=0-11$

$\frac{2x}{2}=\frac{5}{2}$ $x=-11$

Solution Set: $\left\{-11,\frac{5}{2}\right\}$

Try It!

3. Michael was given the equation $(x+7)(x-11)=0$ and asked to find the zeros. His solution set was $\{-11, 7\}$. Explain whether you agree or disagree with Michael.

Disagree. Michael did not set both factors equal to zero and solve. He should have gotten $\{-7, 11\}$.

4. Identify the solution(s) to $2(y-3)\cdot 6(-y-3)=0$.

$$\frac{\mathbf{12(y-3)(-y-3)}}{\mathbf{12}}=\frac{\mathbf{0}}{\mathbf{12}}$$

$$\mathbf{(y-3)(-y-3)=0}$$

$\mathbf{y-3=0}$	**or**	$\mathbf{-y-3=0}$
$\mathbf{y-3+3=0+3}$		$\mathbf{-y-3+3=0+3}$
$\mathbf{y=3}$		$\dfrac{\mathbf{-y}}{\mathbf{-1}}=\dfrac{\mathbf{3}}{\mathbf{-1}}$
		$\mathbf{y=-3}$

Solution Set: $\{-3, 3\}$

BEAT THE TEST!

1. Use the values below to determine the solutions for each equation.

0	2	3	$\frac{4}{5}$
$\frac{2}{7}$	$-\frac{1}{2}$	$-\frac{3}{4}$	-14
6	0	$-\frac{1}{4}$	-2

$(2y+1)(y+14)=0$	$-\frac{1}{2}$	-14	
$(7n-2)(5n-4)=0$	$\frac{2}{7}$	$\frac{4}{5}$	
$(4x+3)(x-6)=0$	$-\frac{3}{4}$	6	
$x(x+2)(x-3)=0$	0	-2	3
$t(4t+1)(t-2)=0$	0	$-\frac{1}{4}$	2

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Section 2 – Topic 5
Solving Power Equations

We can use the rules of exponents and what we know about rational exponents to solve power equations.

Let's Practice!

1. $x^{\frac{1}{2}} = 4$

$\left(x^{\frac{1}{2}}\right)^2 = 4^2$

$x = 16$

2. $\sqrt{x} = 4$

$\left(\sqrt{x}\right)^2 = (4)^2$

$x = 16$

3. $x^{\frac{1}{2}} + 2 = 11$

$x^{\frac{1}{2}} + 2 - 2 = 11 - 2$

$\left(x^{\frac{1}{2}}\right)^2 = 9^2$

$x = 81$

4. $\sqrt{x} + 2 = 11$

$\sqrt{x} + 2 - 2 = 11 - 2$

$\left(\sqrt{x}\right)^2 = (9)^2$

$x = 81$

Try It!

5. $x^{\frac{1}{2}} = 64$

$\left(x^{\frac{1}{2}}\right)^2 = (64)^2$

$x = 4096$

6. $x^{\frac{1}{2}} + 2 = 27$

$x^{\frac{1}{2}} + 2 - 2 = 27 - 2$

$\left(x^{\frac{1}{2}}\right)^2 = (25)^2$

$x = 625$

7. $(x+2)^{\frac{1}{2}} = 6$

$\left((x+2)^{\frac{1}{2}}\right)^2 = 6^2$

$x + 2 = 36$

$x = 34$

8. $(x-3)^{\frac{2}{3}} = 9$

$\left((x-3)^{\frac{1}{2}}\right)^2 = 9^2$

$x - 3 = 81$

$x = 84$

9. Zaira is taking a sail theory class with The Great Lakes Sailing Company. She determines that the hull speed, s, in nautical miles per hour, of a sailboat can be modeled by the formula $s = 1.34 \cdot l^{\frac{1}{2}}$, where l is the length in feet of the sailboat's waterline. Find the length of the sailboat's

waterline if the speed of the boat is 4.2 nautical miles per hour.

$$\mathbf{4.2 = 1.34 \cdot l^{\frac{1}{2}}}$$

$$\mathbf{\frac{4.2}{1.34} = \frac{1.34 \cdot l^{\frac{1}{2}}}{1.34}}$$

$$\mathbf{\left(\frac{4.2}{1.34}\right)^2 = \left(l^{\frac{1}{2}}\right)^2}$$

About $\mathbf{9.8}$ nautical miles per hour

BEAT THE TEST!

1. The average amount of bananas consumed by Americans (in pounds per person), y, can be modeled by the equation $y = (22x + 275)^{\frac{1}{2}}$, where x is the number of years since 1993. In which year were about 20 pounds of bananas consumed per person?

$$20 = (22x + 275)^{\frac{1}{2}}$$
$$(20)^2 = \left((22x + 275)^{\frac{1}{2}}\right)^2$$
$$400 = 22x + 275$$
$$125 = 22x$$
$$x \approx 5.7$$

During the year 1999, the average amount of bananas consumed per person was 20 pounds.

2. The average amount of sewage discharged by a factory, y, can be modeled by the equation $y = 235(2x + 0.75)^{\frac{1}{2}}$, where x is the number of years since 2000 and y is the average sewage discharged per month, in tons. In which year were approximately 610 tons of sewage discharged per month?

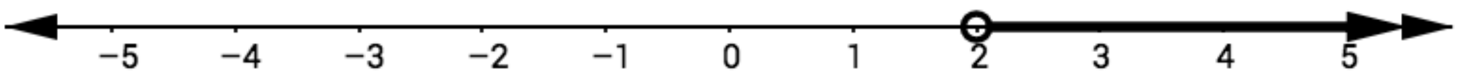
$$610 = 235(2x + 0.75)^{\frac{1}{2}}$$
$$\left(\frac{610}{235}\right)^2 = \left((2x + 0.75)^{\frac{1}{2}}\right)^2$$

$$x \approx 2.99$$

During the year 2003, the average sewage discharged by the factory was 610 tons.

Section 2 – Topic 6
Solving Inequalities – Part 1

Let's start by reviewing how to graph inequalities.



- When the endpoint is a(n) **open** dot or circle, the number represented by the endpoint **is** **not** a part of the solution set.

Describe the numbers that are graphed in the example above.

The numbers graphed are greater than 2.

Can you list all the numbers graphed in the example above? Explain your answer.

No, there are infinitely many numbers greater than 2.

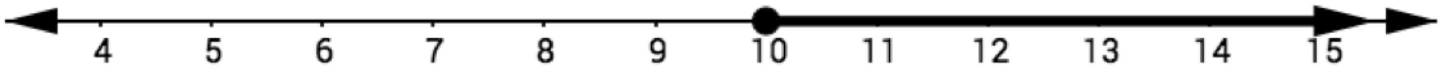
Write an inequality that represents the graph above.

$x > 2$

Write the solution set that represents the graph above.

$\{x|x > 2\}$

Consider the following graph.



- When the endpoint is a(n) ___closed___ dot or circle, the number represented by the endpoint ___is___ a part of the solution set.

Write an inequality that represents the graph above.

$\boldsymbol{x \geq 10}$

Write the solution set that represents the graph above.

$\boldsymbol{\{x | x \geq 10\}}$

Why is "or equal to" included in the solution set?

Because $\mathbf{10}$ is included in the solution.

Just like there are Properties of Equality, there are also ***Properties of Inequality***.

If $x > 5$, is $x + 1 > 5 + 1$? Substitute values for x to justify your answer.

Let $\boldsymbol{x = 10}$, $\mathbf{10 > 5}$ and $\mathbf{10 + 1 > 5 + 1}$.

Addition and Subtraction Property of Inequality

- If $a > b$, then $a + c > b + c$ and $a - c > b - c$ for any real number c.

Consider $(2x - 1) + 2 > x + 1$. Use the addition or subtraction property of inequality to solve for x.

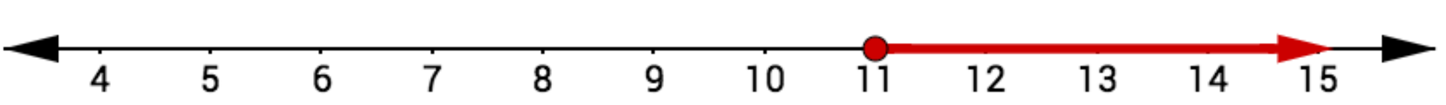
$\mathbf{(2x - 1) + 2 > x + 1}$
$\mathbf{2x + 1 > x + 1}$
$\mathbf{2x + 1 - 1 > x + 1 - 1}$
$\mathbf{2x > x}$
$\mathbf{2x - x > x - x}$
$\mathbf{x > 0}$

Let's Practice!

1. Consider the inequality $(4 + x) - 5 \geq 10$. Use the addition or subtraction property of inequality to solve for x. Express the solution in set notation and graphically on a number line.

   $\mathbf{(4 + x) - 5 \geq 10}$
   $\mathbf{x - 1 \geq 10}$
   $\mathbf{x - 1 + 1 \geq 10 + 1}$
   $\mathbf{x \geq 11}$

   $\mathbf{\{x | x \geq 11\}}$



Try It!

2. Consider the inequality $4x + 8 < 1 + (2x - 5)$. Use the addition or subtraction property of inequality to solve for x. Express the solution in set notation and graphically on a number line.

$$4x + 8 < 1 + (2x - 5)$$
$$4x + 8 < -4 + 2x$$
$$4x + 8 - 8 < -4 - 8 + 2x$$
$$4x < -12 + 2x$$
$$4x - 2x < -12 + 2x - 2x$$
$$\frac{2x}{2} < -\frac{12}{2}$$
$$x < -6$$

$$\{x|x < -6\}$$

3. Peter deposited \$27 into his savings account, bringing the total to over \$234. Write and solve an inequality to represent the amount of money in Peter's account before the \$27 deposit.

Let x represent amount of money in account before deposit.

$$x + 27 > 234$$
$$x + 27 - 27 > 234 - 27$$
$$x > 207$$

Peter had more than \$207 in his account before the deposit.

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Section 2 – Topic 7
Solving Inequalities – Part 2

Consider $x > 5$ and $2 \cdot x > 2 \cdot 5$. Identify a solution to the first inequality. Show that this solution also makes the second inequality true.

Let $x = 6$.
$6 > 5$ and $2(6) > 10$.

Consider $x > 5$ and $-2 \cdot x > -2 \cdot 5$. Identify a solution to the first inequality. Show that this solution makes the second inequality false.

Let $x = 6$
$6 > 5$ and $-2(6) \ngtr -10$.

How can we change the second inequality so that the solution makes it true?

$$-2x < -10$$
$$-2(6) < -10$$

Consider $-q > 5$. Use the addition and/or subtraction property of inequality to solve.

$$-q + q > 5 + q$$
$$0 > 5 + q$$
$$0 - 5 > 5 - 5 + q$$
$$-5 > q$$
$$q < -5$$

Multiplication Property of Inequality

- If $a > b$, then for any positive real number k, ak ___>___ bk.
- If $a < b$, then for any positive real number k, ak ___<___ bk.
- If $a > b$, then for any negative real number k, ak ___<___ bk.
- If $a < b$, then for any negative real number k, ak ___>___ bk.

The same property is true when dealing with ≤ or ≥.

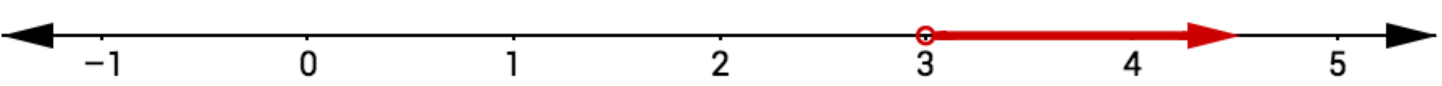
Let's Practice!

1. Find the solution set to each inequality. Express the solution in set notation and graphically on a number line.

   a. $-9y + 4 < -7y - 2$

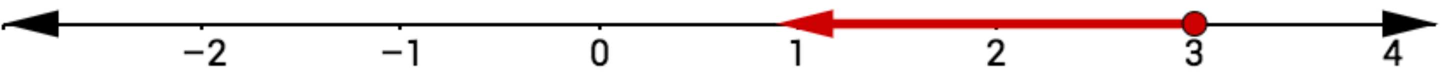
$$-9y + 4 - 4 < -7y - 2 - 4$$
$$-9y < -7y - 6$$
$$-9y + 7y < -7y + 7y - 6$$
$$-2y < -6$$
$$\frac{-2y}{-2} > \frac{-6}{-2}$$
$$y > 3$$

$$\{y|y > 3\}$$



b. $\frac{m}{3}+8\leq 9$

$$\frac{m}{3}+8-8\leq 9-8$$
$$\frac{m}{3}\leq 1$$
$$\frac{m}{3}\cdot 3\leq 1\cdot 3$$
$$m\leq 3$$
$$\{m|m\leq 3\}$$



2. At 5:00 PM in Atlanta, Georgia, Ethan noticed the temperature outside was 72°F. The temperature decreased at a steady rate of 2°F per hour. At what time was the temperature below 64°F?

Let h represent the number of hours since 5:00 PM.

$$72-2h<64$$
$$72-72-2h<64-72$$
$$-2h<-8$$
$$\frac{-2h}{-2}>\frac{-8}{-2}$$
$$h>4$$

After 9:00 PM, the temperature was less than 64°F.

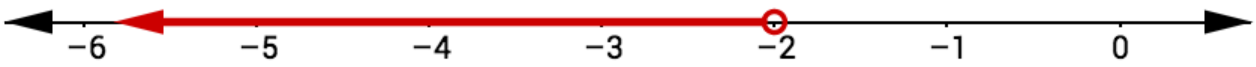
Try It!

3. Find the solution set to the inequality. Express the solution in set notation and graphically on a number line.

a. $-6\,(x-5) > 42$

$$\mathbf{-6x + 30 > 42}$$
$$\mathbf{-6x + 30 - 30 > 42 - 30}$$
$$\mathbf{-6x > 12}$$
$$\mathbf{\frac{-6x}{-6} < \frac{12}{-6}}$$
$$\mathbf{x < -2}$$

$$\{x|x < -2\}$$



b. $4(x+3) \geq 2(2x-2)$

$$\mathbf{4x + 12 \geq 4x - 4}$$
$$\mathbf{4x + 12 - 12 \geq 4x - 4 - 12}$$
$$\mathbf{4x \geq 4x - 16}$$

All real numbers for x will make the inequality true.

$$\{x|x = all\,real\,numbers\}$$

BEAT THE TEST!

1. Ulysses is spending his vacation in South Carolina. He rents a car and is offered two different payment options. He can either pay \$25.00 each day plus \$0.15 per mile (option A) or pay \$10.00 each day plus \$0.40 per mile (option B). Ulysses rents the car for one day.

Part A: Write an inequality representing the number of miles where option A will be the cheaper plan.

Let x represent the number of miles driven.

Option A: $25 + 0.15x$

Option B: $10 + 0.40x$

$$25 + 0.15x < 10 + 0.40x$$

Part B: How many miles will Ulysses have to drive for option A to be the cheaper option?

$$25 - 25 + 0.15x < 10 - 25 + 0.40x$$

$$0.15x < -15 + 0.40x$$

$$0.15x - 0.40x < -15 + 0.40x - 0.40x$$

$$-0.25x < -15$$

$$\frac{-0.25x}{-0.25} > \frac{-15}{-0.25}$$

$$x > 60$$

If Ulysses drove more than 60 miles, Option A would be cheaper.

2. Stephanie has just been given a new job in the sales department of Frontier Electric Authority. She has two salary options. She can either receive a fixed salary of $500.00 per week or a salary of $200.00 per week plus a 5% commission on her weekly sales. The variable s represents Stephanie's weekly sales. Which solution set represents the dollar amount of sales that she must generate in a week in order for the option with commission to be the better choice?

A $\{s|s > \$300.00\}$
B $\{s|s > \$700.00\}$
C $\{s|s > \$3{,}000.00\}$
D $\{s|s > \$6{,}000.00\}$

Answer: D

Let s represent the weekly sales.

$$200 + 0.05s > 500$$

$$200 - 200 + 0.05s > 500 - 200$$

$$\frac{0.05s}{0.05} > \frac{300}{0.05}$$

$$s > 6000$$

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Section 2 – Topic 8
Solving Compound Inequalities

Consider the following options.

Option A: You get to play *NBA 2K* after you clean your room and do the dishes.

Option B: You get to play *NBA 2K* after you clean your room or do the dishes.

What is the difference between Option A and B?

In Option A, you have to do both chores. In Option B, you only have to do one chore.

Circle the statements that are true.

$2 + 9 = 11$ and $10 < 5 + 6$

$4 + 5 \neq 9$ and $2 + 3 > 0$

$0 > 4 - 6$ or $3 + 2 = 6$

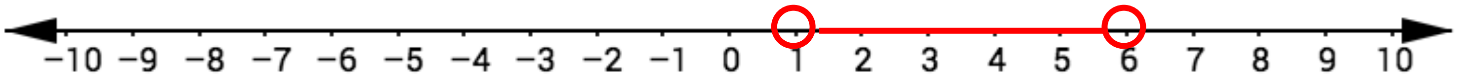
$15 - 20 > 0$ or $2.5 + 3.5 = 7$

These are called ***compound equations*** or ***inequalities***.

- When the two statements in the previous sentences were joined by the word *AND*, the compound equation or inequality is true only if __both__ statements are true.

- When the two statements in the previous sentences were joined by the word *OR*, the compound equation or inequality is true if at least __one__ of the statements is true. Therefore, it is also considered true if __both__ statements are true.

Let's graph $x < 6$ and $x > 1$.



This is the __graphical__ __solution__ to the compound inequality.

How many solutions does this inequality have?

Infinitely many

Many times this is written as $1 < x < 6$. This notation denotes the conjunction "and."

We read this as "x is greater than one __and__ less than six."

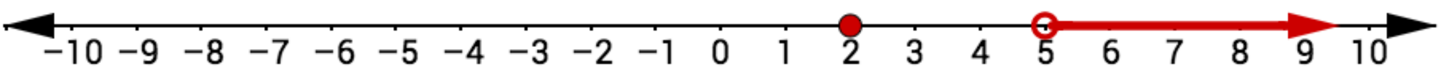
Let's Practice!

1. Consider $x < 1$ or $x > 6$. Could we write the inequalities above as $1 > x > 6$? Explain your answer.

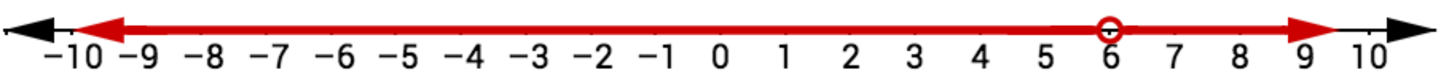
   No, that would say that $x < 1$ and $x > 6$. x cannot be both of those things.

2. Graph the solution set to each compound inequality on a number line.

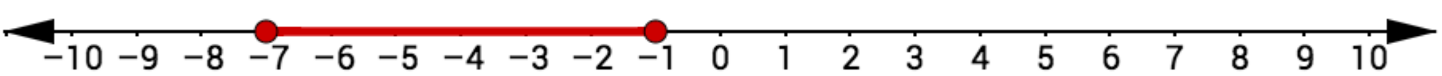
   a. $x = 2$ or $x > 5$



   b. $x > 6$ or $x < 6$



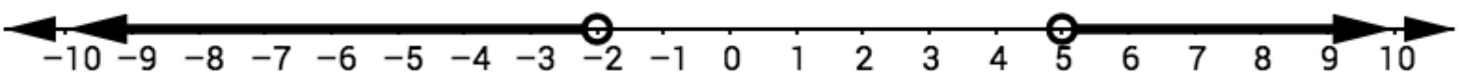
   c. $1 \leq -x \leq 7$



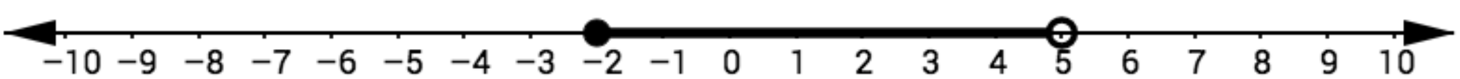
STUDY EDGE TIP

Be on the lookout for negative coefficients. When solving inequalities, you will need to reverse the inequality symbol when you multiply or divide by a negative value.

3. Write a compound inequality for the following graphs.



a. Compound inequality: $x < -2$ **or** $x > 5$



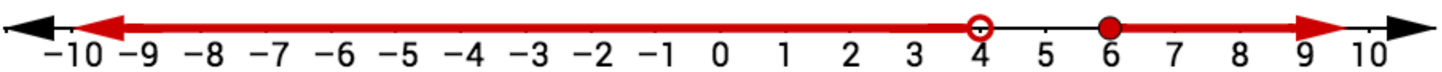
b. Compound inequality: $-2 \leq x < 5$

Try It!

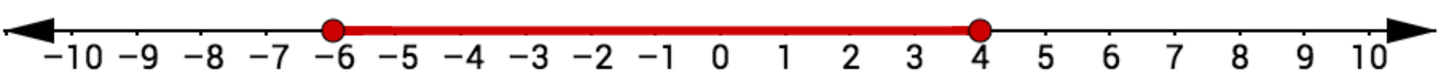
4. Graph the solution set to each compound inequality on a number line.

   a. $x < 1$ or $x > 8$

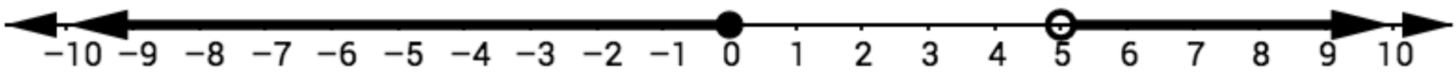
   b. $x \geq 6$ or $x < 4$



   c. $-6 \leq x \leq 4$



5. Write a compound inequality for the following graphs.



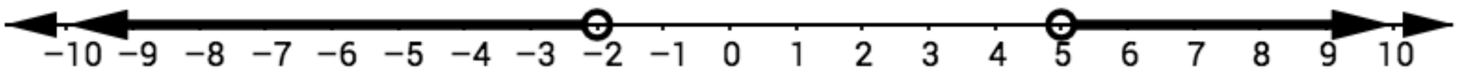
a. Compound inequality: $x \leq 0$ or $x > 5$

b. Compound inequality: $0 \leq x \leq 5$

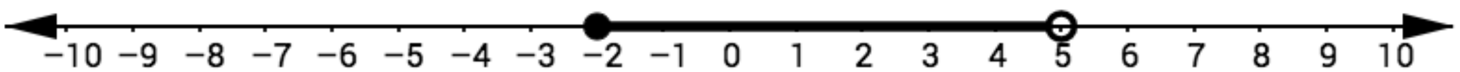
BEAT THE TEST!

1. Use the terms and symbols in the table to write a compound inequality for each of the following graphs. You may only use each term once, but you do not have to use all of them.

$3x$	-14	-6	$\geq$	$-$	17	15	$<$
$7x$	$<$	2	or	$\leq$	$3x$	$+$	$>$



Compound Inequality: $7x < -14$ or $3x > 15$



Compound Inequality: $-6 \leq 3x < 17 - 2$

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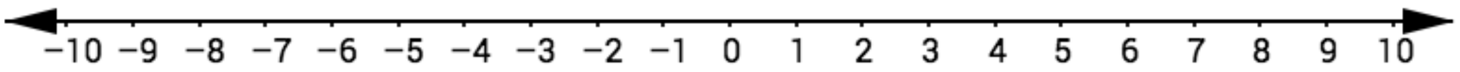
Section 2 – Topic 9
Solving Absolute Value Equations and Inequalities

Absolute value represents the distance of a number from zero on a number line.



How far away is "9" from zero on the number line? **9 units**

This is written as $\mathbf{|9| = 9}$.

How far away is "-9" from zero on the number line? **9 units**

This is written as $\mathbf{|-9| = 9}$.

This is the ***absolute value*** of a number.

- For any real numbers c and d, if $|c| = d$, then $c = d$ or $c = -d$.

For example, $|f| = 5$, so $f = \mathbf{5}$ or $f = \mathbf{-5}$.

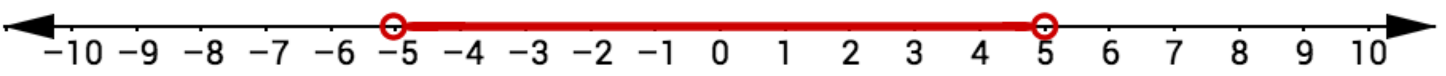
Consider $|c| < 5$.

Using our definition of absolute value, this says that c represents all the numbers **less than** five units from zero on the number line.

What are some numbers that could be represented by c?

$\mathbf{-4.99, -3, -2, 0, \frac{1}{2}, 3.2, 4.99}$

Graph all the numbers represented by c on a number line.



What is the solution set for c?

$\{\boldsymbol{c} | \mathbf{-5} < \boldsymbol{c} < \mathbf{5}\}$

- For any real numbers c and d, if $|c| < d$, then $-d < c < d$.

- For any real numbers c and d, if $|c| \leq d$, then $-d \leq c \leq d$.

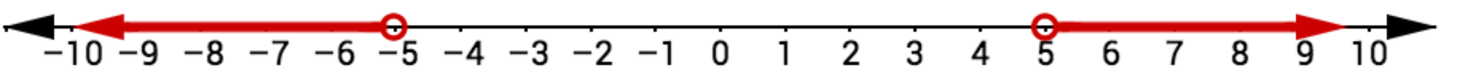
Consider $|c| > 5$.

Using our definition of absolute value, c represents all the numbers **greater than** five units from zero on the number line.

What are some numbers that could be represented by c?

$\mathbf{-10, -8, -5.1, -5.001, 5.002, 5.4, 7, 12}$

Graph all the numbers represented by c on a number line.



What is the solution set for c?

$\{c | c < -5 \text{ or } c > 5\}$

- For any real numbers c and d, if $|c| > d$, then $c > d$ or $c < -d$.
- For any real numbers c and d, if $|c| \geq d$, then $c \geq d$ or $c \leq -d$.

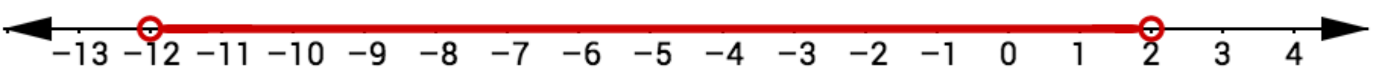
Let's Practice!

1. Solve each absolute value inequality and graph the solution set.

   a. $|n + 5| < 7$

   $\boldsymbol{-7 < n + 5 < 7}$
   $\boldsymbol{-7 - 5 < n + 5 - 5 < 7 - 5}$
   $\boldsymbol{-12 < n < 2}$

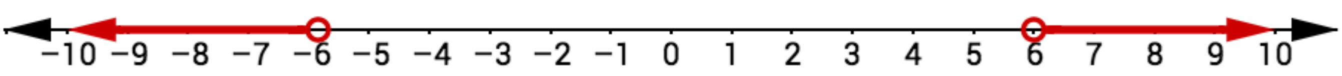
   $\boldsymbol{\{n | -12 < n < 2\}}$



   b. $|a| + 3 > 9$

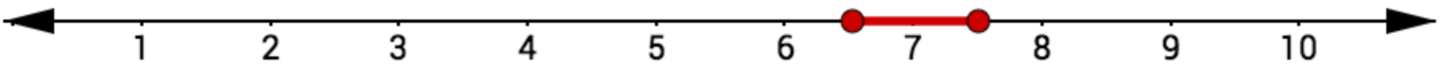
   $\boldsymbol{|a| + 3 - 3 > 9 - 3}$
   $\boldsymbol{|a| > 6}$
   $\boldsymbol{a < -6 \text{ or } a > 6}$

   $\boldsymbol{\{a | a < -6 \text{ or } a > 6\}}$



2. Tammy purchased a pH meter to measure the acidity of her freshwater aquarium. The ideal pH level for a freshwater aquarium is between 6.5 and 7.5 inclusive.

a. Graph an inequality that represents the possible pH levels needed for Tammy's aquarium.



b. Define the variable and write an absolute value inequality that represents the possible pH levels needed for Tammy's aquarium.

Let x represent pH level.
The absolute difference between the pH level and 7 must be less than or equal to 0.5.

$$|x - 7| \leq 0.5$$

Try It!

3. Solve each equation or inequality and graph the solution set.

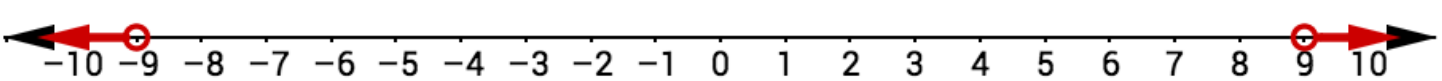
   a. $|p + 7| = -13$

   No Solution

   b. $2|x| - 4 < 14$

$$\mathbf{2|x| - 4 + 4 < 14 + 4}$$
$$\mathbf{\frac{2|x|}{2} < \frac{18}{2}}$$
$$\mathbf{|x| < 9}$$
$$\mathbf{-9 < x < 9}$$

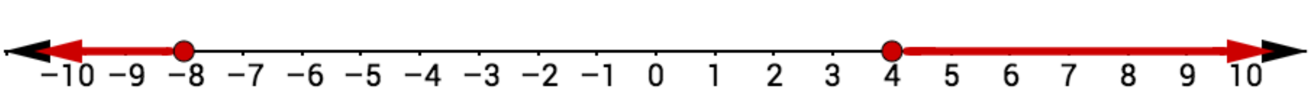
$$\mathbf{\{x | -9 < x < 9\}}$$



   c. $|2m + 4| \geq 12$

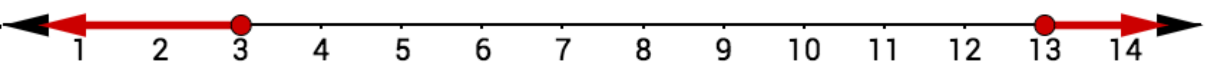
$$\mathbf{|2m + 4| \geq 12}$$
$$\mathbf{2m + 4 \geq 12 \ or \ 2m + 4 \leq -12}$$
$$\mathbf{2m + 4 - 4 \geq 12 - 4 \ or \ 2m + 4 - 4 \leq -12 - 4}$$
$$\mathbf{\frac{2m}{2} \geq \frac{8}{2} \ or \ \frac{2m}{2} \leq -\frac{16}{2}}$$
$$\mathbf{m \geq 4 \ or \ m \leq -8}$$

$$\mathbf{\{m | m \geq 4 \ or \ m \leq -8\}}$$



4. Baseball fans often leave a baseball game if their team is ahead or behind by five runs or more. Toronto Blue Jays fans follow this pattern, and the Blue Jays have scored eight runs in a particular game.

a. Graph an inequality that represents the possible runs, r, scored by the opposing team if Toronto fans are leaving the game.



b. Write an absolute value inequality that represents the possible runs, r, scored by the opposing team if Toronto fans are leaving the game.

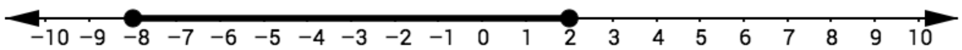
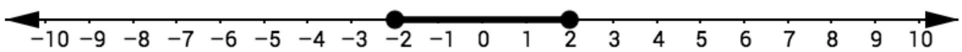
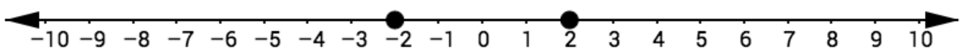
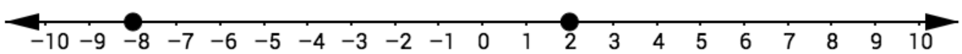
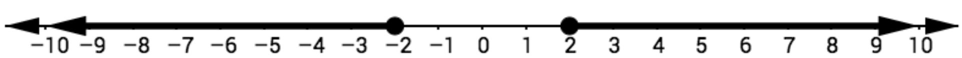
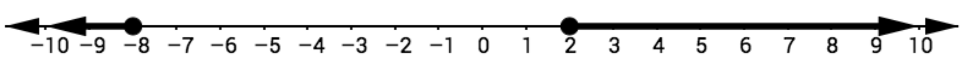
Let r represent the number of runs scored by the opposing team.

The absolute difference between the number of runs scored by the opposing team and the number of runs Toronto scores is greater than or equal to 5.

$$|r - 8| \geq 5$$

BEAT THE TEST!

1. Match the following absolute value equations and inequalities to the graph that represents their solution set.

D		A. $\lvert x\rvert = 2$
C		B. $\lvert x\rvert \geq 2$
A		C. $\lvert x\rvert \leq 2$
F		D. $\lvert x+3\rvert \leq 5$
B		E. $\lvert x+3\rvert \geq 5$
E		F. $\lvert x+3\rvert = 5$

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Section 2 – Topic 10
Rearranging Formulas

Solve each equation for x.

$2x + 4 = 12$

$\mathbf{2x + 4 - 4 = 12 - 4}$

$\frac{2x}{2} = \frac{8}{2}$

$\mathbf{x = 4}$

$2x + y = z$

$\mathbf{2x + y - y = z - y}$

$\frac{2x}{2} = \frac{z-y}{2}$

$x = \frac{z-y}{2}$

Did we use different properties when we solved the two equations? **No**

Consider the formula for the perimeter of a rectangle:
$P = 2l + 2w$.

Sometimes, we might need the formula solved for length.

$\mathbf{P - 2w = 2l + 2w - 2w}$

$\mathbf{P - 2w = 2l}$

$$\frac{P - 2w}{2} = \frac{2l}{2}$$

$$l = \frac{P - 2w}{2}$$

STUDY EDGE TIP

When solving for a variable, it's helpful to circle that variable.

Let's Practice!

1. Consider the equation $rx - sx + y = z$; solve for x.

$$\boldsymbol{rx - sx + y - y = z - y}$$

$$\boldsymbol{x(r - s) = z - y}$$

$$\boldsymbol{\frac{x(r-s)}{r-s} = \frac{z-y}{r-s}}$$

$$\boldsymbol{x = \frac{z-y}{r-s}}$$

Try It!

2. Consider the equation $8c + 6j = 5p$; solve for c.

$$\boldsymbol{8c + 6j - 6j = 5p - 6j}$$

$$\boldsymbol{\frac{8c}{8} = \frac{5p-6j}{8}}$$

$$\boldsymbol{c = \frac{5p - 6j}{8}}$$

$$\boldsymbol{c = \frac{5p}{8} - \frac{6j}{8}}$$

$$\boldsymbol{c = \frac{5p}{8} - \frac{3j}{4}}$$

3. Consider the equation $\frac{x-c}{2} = d$; solve for c.

$$2 \cdot \frac{x-c}{2} = d \cdot 2$$

$$x - c = 2d$$

$$x - x - c = 2d - x$$

$$\frac{-c}{-1} = \frac{2d - x}{-1}$$

$$c = -2d + x$$

$$c = x - 2d$$

BEAT THE TEST!

1. Isaiah planted a seedling in his garden and recorded its height every week. The equation shown can be used to estimate the height, h, of the seedling after w weeks since he planted the seedling.

$$h = \frac{3}{4}w + \frac{9}{4}$$

Solve the formula for w, the number of weeks since he planted the seedling.

$$4 \cdot h = 4\left(\frac{3}{4}w + \frac{9}{4}\right)$$

$$4h = 4 \cdot \frac{3}{4}w + 4 \cdot \frac{9}{4}$$

$$4h = 3w + 9$$

$$4h - 9 = 3w + 9 - 9$$

$$\frac{(4h-9)}{3} = \frac{3w}{3}$$

$$w = \frac{4h-9}{3}$$

$$w = \frac{1}{3} \cdot (4h - 9)$$

$$w = \frac{4}{3}h - 3$$

2. Under the Brannock device method, shoe size and foot length for women are related by the formula $S = 3F - 21$, where S represents the shoe size and F represents the length of the foot in inches. Solve the formula for F.

$$\boldsymbol{S + 21 = 3F - 21 + 21}$$

$$\boldsymbol{\frac{S + 21}{3} = \frac{3F}{3}}$$

$$\boldsymbol{\frac{S}{3} + \frac{21}{3} = F}$$

$$\boldsymbol{F = \frac{S}{3} + 7}$$

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Section 2 – Topic 11
Solution Sets to Equations with Two Variables

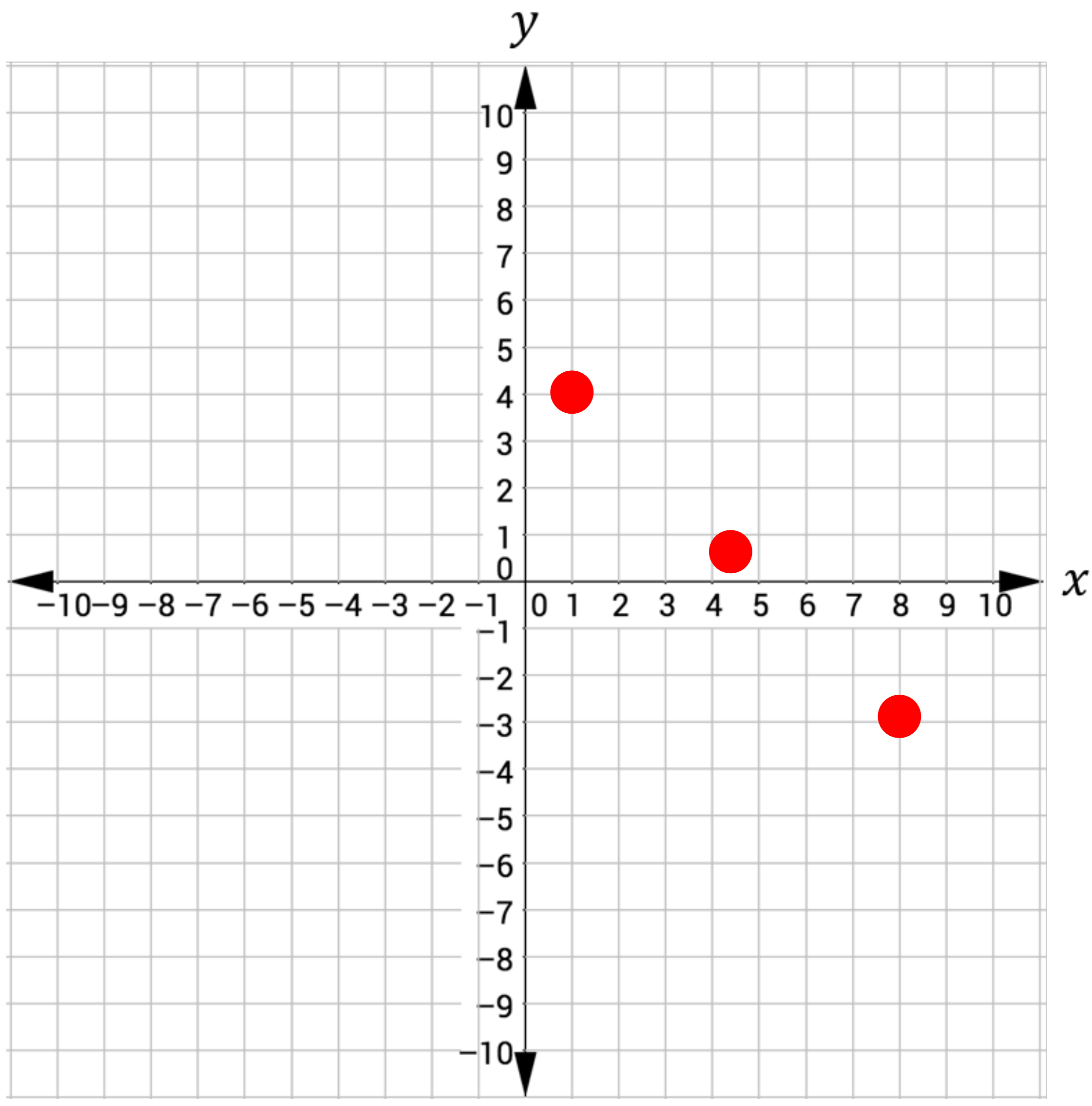
Consider $x + 2 = 5$. What is the only possible value of x that makes the equation a true statement?

Now consider $x + y = 5$. What are some solutions for x and y that would make the equation true?

$\boldsymbol{x = 1, y = 4}$
$\boldsymbol{x = 4.5, y = 0.5}$
$\boldsymbol{x = 8, y = -3}$

Possible solutions can be listed as ***ordered pairs***.

Graph each of the ordered pairs from the previous problem on the graph below.



What do you notice about the points you graphed?

They form a line

How many solutions are there to the equation $x + y = 5$?

Infinitely many

Let's Practice!

1. Taylor has 10 songs on her phone's playlist. The playlist features songs from her two favorite artists, Beyoncé and Pharrell.

   a. Create an equation using two variables to represent this situation.

      Let x represent number of Beyoncé songs and y represent number of Pharrell songs.
      $\mathbf{x + y = 10}$

   b. List at least three solutions to the equation that you created.

      $\mathbf{x = 3, y = 7}$
      $\mathbf{x = 9, y = 1}$
      $\mathbf{x = 5, y = 5}$

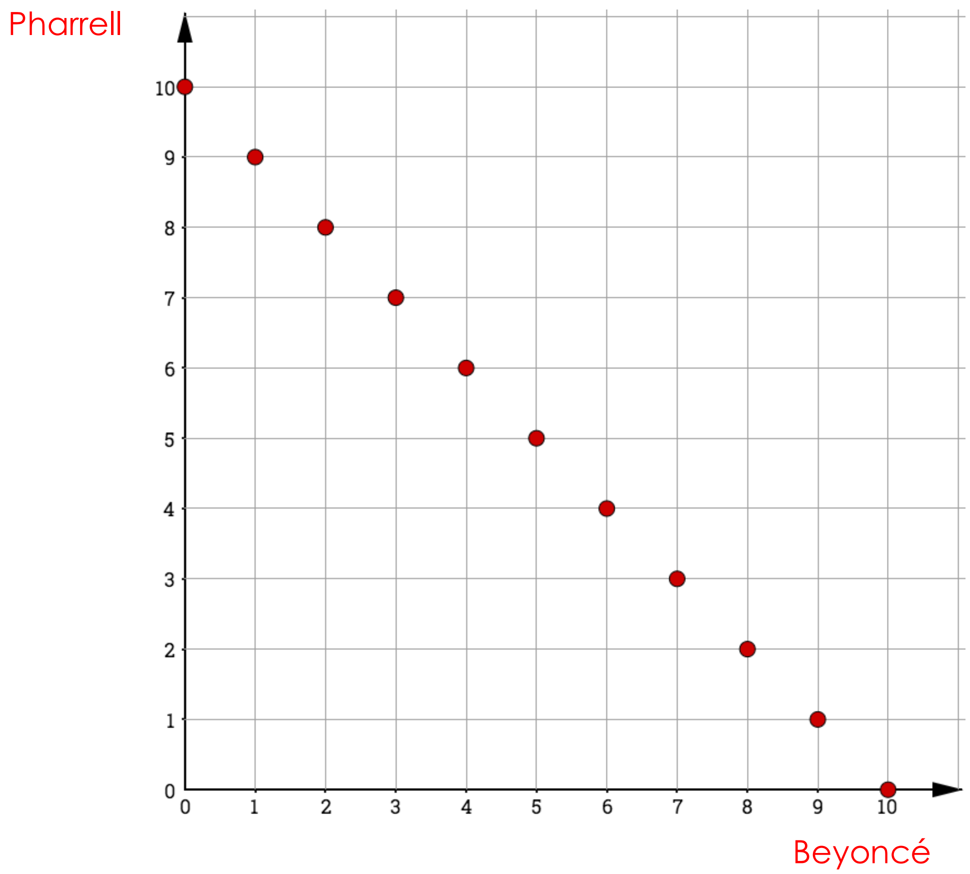
   c. Do we have infinitely many solutions to this equation? Why or why not?

      No. You cannot have half of a song.

STUDY EDGE TIP

In this case, our solutions must be natural numbers. This is called a ***discrete function***. Notice that the solutions follow a linear pattern. However, they do not form a line.

d. Create a graph that represents the solution set to your equation.



e. Why are there only positive values on this graph?

Because you cannot have a negative number of songs.

Try It!

2. The sum of two numbers is 15.

   a. Create an equation using two variables to represent this situation.

   $x + y = 15$

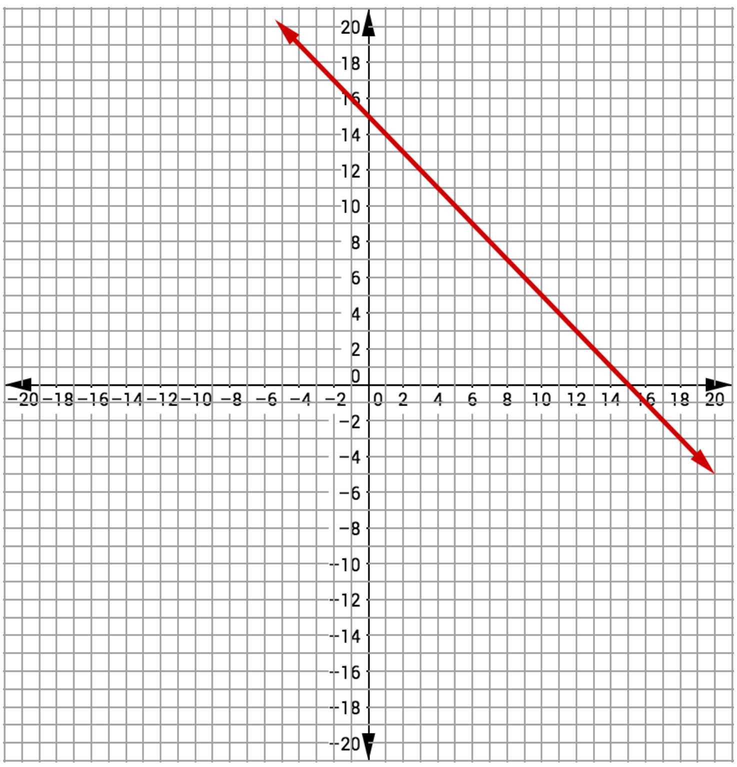
   b. List at least three possible solutions.

   $5 \text{ and } 10,\ 20 \text{ and } -5, 12.5 \text{ and } 2.5$

   c. How many solutions are there to this equation?

   Infinitely many

   d. Create a visual representation of all the possible solutions on the graph.



STUDY EDGE TIP

In this case, we have a ***continuous function***. Notice the solutions are rational numbers and they form a line.

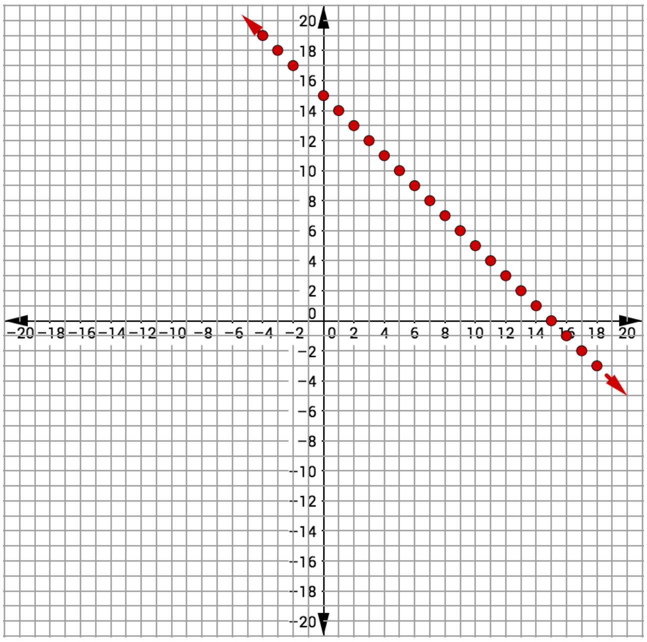
3. What if we changed the problem to say the sum of two integers is 15?

   a. Create an equation using two variables to represent this situation.
   $x + y = 15$

   b. Is this function discrete or continuous? Explain your answer.

   Discrete because the solution set only consist of integers.

   c. Represent the solution on the graph below.



BEAT THE TEST!

1. Elizabeth's tablet has a combined total of 20 apps and movies. Let x represent the number of apps and y represent the number of movies. Which of the following could represent the number of apps and movies on Elizabeth's tablet? Select all that apply.

   ☒ $x + y = 20$
   ☐ 7 apps and 14 movies
   ☐ $x - y = 20$
   ☒ $y = -x + 20$
   ☒ 8 apps and 12 movies
   ☐ $xy = 20$

Test Yourself! Practice Tool

Great job! You have reached the end of this section. Now it's time to try the "Test Yourself! Practice Tool," where you can practice all the skills and concepts you learned in this section. Log in to Algebra Nation and try out the "Test Yourself! Practice Tool" so you can see how well you know these topics!