

level and (b) the $n = 4$ level? In each case, compare the de Broglie wavelength to the circumference $2\pi r_n$ of the orbit.

39.6 • (a) A nonrelativistic free particle with mass m has kinetic energy K . Derive an expression for the de Broglie wavelength of the particle in terms of m and K . (b) What is the de Broglie wavelength of an 800-eV electron?

39.7 • **Why Don't We Diffract?** (a) Calculate the de Broglie wavelength of a typical person walking through a doorway. Make reasonable approximations for the necessary quantities. (b) Will the person in part (a) exhibit wavelike behavior when walking through the "single slit" of a doorway? Why?

39.8 •• What is the de Broglie wavelength for an electron with speed (a) $v = 0.480c$ and (b) $v = 0.960c$? (*Hint:* Use the correct relativistic expression for linear momentum if necessary.)

39.9 • (a) If a photon and an electron each have the same energy of 20.0 eV, find the wavelength of each. (b) If a photon and an electron each have the same wavelength of 250 nm, find the energy of each. (c) You want to study an organic molecule that is about 250 nm long using either a photon or an electron microscope. Approximately what wavelength should you use, and which probe, the electron or the photon, is likely to damage the molecule the least?

39.10 • How fast would an electron have to move so that its de Broglie wavelength is 1.00 mm?

39.11 • **Wavelength of a Bullet.** Calculate the de Broglie wavelength of a 5.00-g bullet that is moving at 340 m/s. Will the bullet exhibit wavelike properties?

39.12 •• Find the wavelengths of a photon and an electron that have the same energy of 25 eV. (*Note:* The energy of the electron is its kinetic energy.)

39.13 •• (a) What accelerating potential is needed to produce electrons of wavelength 5.00 nm? (b) What would be the energy of photons having the same wavelength as these electrons? (c) What would be the wavelength of photons having the same energy as the electrons in part (a)?

39.14 •• Through what potential difference must electrons be accelerated so they will have (a) the same wavelength as an x ray of wavelength 0.150 nm and (b) the same energy as the x ray in part (a)?

39.15 • (a) Approximately how fast should an electron move so it has a wavelength that makes it useful to measure the distance between adjacent atoms in typical crystals (about 0.10 nm)? (b) What is the kinetic energy of the electron in part (a)? (c) What would be the energy of a photon of the same wavelength as the electron in part (b)? (d) Which would make a more effective probe of small-scale structures: electrons or photons? Why?

39.16 •• **CP** A beam of electrons is accelerated from rest through a potential difference of 0.100 kV and then passes through a thin slit. The diffracted beam shows its first diffraction minima at $\pm 11.5^\circ$ from the original direction of the beam when viewed far from the slit. (a) Do we need to use relativity formulas? How do you know? (b) How wide is the slit?

39.17 •• A beam of neutrons that all have the same energy scatters from atoms that have a spacing of 0.0910 nm in the surface plane of a crystal. The $m = 1$ intensity maximum occurs when the angle θ in Fig. 39.2 is 28.6° . What is the kinetic energy (in electron volts) of each neutron in the beam?

39.18 • A beam of 188-eV electrons is directed at normal incidence onto a crystal surface as shown in Fig. 39.3b. The $m = 2$ intensity maximum occurs at an angle $\theta = 60.6^\circ$. (a) What is the spacing between adjacent atoms on the surface? (b) At what other angle or angles is there an intensity maximum? (c) For what electron

energy (in electron volts) would the $m = 1$ intensity maximum occur at $\theta = 60.6^\circ$? For this energy, is there an $m = 2$ intensity maximum? Explain.

39.19 • A CD-ROM is used instead of a crystal in an electron-diffraction experiment. The surface of the CD-ROM has tracks of tiny pits with a uniform spacing of $1.60 \mu\text{m}$. (a) If the speed of the electrons is $1.26 \times 10^4 \text{ m/s}$, at which values of θ will the $m = 1$ and $m = 2$ intensity maxima appear? (b) The scattered electrons in these maxima strike at normal incidence a piece of photographic film that is 50.0 cm from the CD-ROM. What is the spacing on the film between these maxima?

39.20 • (a) In an electron microscope, what accelerating voltage is needed to produce electrons with wavelength 0.0600 nm? (b) If protons are used instead of electrons, what accelerating voltage is needed to produce protons with wavelength 0.0600 nm? (*Hint:* In each case the initial kinetic energy is negligible.)

39.21 •• You want to study a biological specimen by means of a wavelength of 10.0 nm, and you have a choice of using electromagnetic waves or an electron microscope. (a) Calculate the ratio of the energy of a 10.0-nm-wavelength photon to the kinetic energy of a 10.0-nm-wavelength electron. (b) In view of your answer to part (a), which would be less damaging to the specimen you are studying: photons or electrons?

Section 39.2 The Nuclear Atom and Atomic Spectra

39.22 •• **CP** A 4.78-MeV alpha particle from a ^{226}Ra decay makes a head-on collision with a uranium nucleus. A uranium nucleus has 92 protons. (a) What is the distance of closest approach of the alpha particle to the center of the nucleus? Assume that the uranium nucleus remains at rest and that the distance of closest approach is much greater than the radius of the uranium nucleus. (b) What is the force on the alpha particle at the instant when it is at the distance of closest approach?

39.23 • A beam of alpha particles is incident on a target of lead. A particular alpha particle comes in "head-on" to a particular lead nucleus and stops $6.50 \times 10^{-14} \text{ m}$ away from the center of the nucleus. (This point is well outside the nucleus.) Assume that the lead nucleus, which has 82 protons, remains at rest. The mass of the alpha particle is $6.64 \times 10^{-27} \text{ kg}$. (a) Calculate the electrostatic potential energy at the instant that the alpha particle stops. Express your result in joules and in MeV. (b) What initial kinetic energy (in joules and in MeV) did the alpha particle have? (c) What was the initial speed of the alpha particle?

Section 39.3 Energy Levels and the Bohr Model of the Atom

39.24 • The silicon-silicon single bond that forms the basis of the mythical silicon-based creature the Horta has a bond strength of 3.80 eV. What wavelength of photon would you need in a (mythical) phasor disintegration gun to destroy the Horta?

39.25 •• A hydrogen atom is in a state with energy -1.51 eV . In the Bohr model, what is the angular momentum of the electron in the atom, with respect to an axis at the nucleus?

39.26 • A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of the photon.

39.27 • A triply ionized beryllium ion, Be^{3+} (a beryllium atom with three electrons removed), behaves very much like a hydrogen atom except that the nuclear charge is four times as great. (a) What is the ground-level energy of Be^{3+} ? How does this compare to the ground-level energy of the hydrogen atom? (b) What is the ionization energy of Be^{3+} ? How does this compare to the ionization energy of the

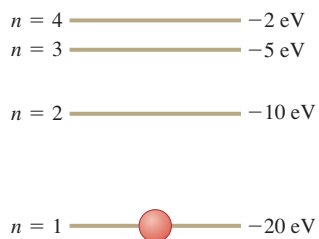
hydrogen atom? (c) For the hydrogen atom, the wavelength of the photon emitted in the $n = 2$ to $n = 1$ transition is 122 nm (see Example 39.6). What is the wavelength of the photon emitted when a Be^{3+} ion undergoes this transition? (d) For a given value of n , how does the radius of an orbit in Be^{3+} compare to that for hydrogen?

39.28 •• (a) Show that, as n gets very large, the energy levels of the hydrogen atom get closer and closer together in energy. (b) Do the radii of these energy levels also get closer together?

39.29 • (a) Using the Bohr model, calculate the speed of the electron in a hydrogen atom in the $n = 1, 2,$ and 3 levels. (b) Calculate the orbital period in each of these levels. (c) The average lifetime of the first excited level of a hydrogen atom is 1.0×10^{-8} s. In the Bohr model, how many orbits does an electron in the $n = 2$ level complete before returning to the ground level?

39.30 • CP The energy-level scheme for the hypothetical one-electron element Searsium is shown in Fig. E39.30. The potential energy is taken to be zero for an electron at an infinite distance from the nucleus. (a) How much energy (in electron volts) does it take to ionize an electron from the ground level? (b) An 18-eV photon is absorbed

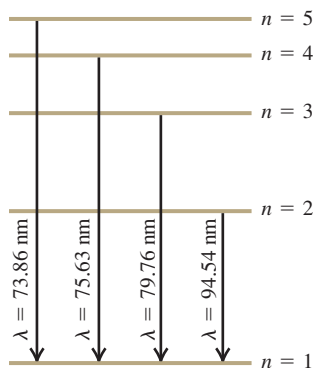
Figure E39.30



by a Searsium atom in its ground level. As the atom returns to its ground level, what possible energies can the emitted photons have? Assume that there can be transitions between all pairs of levels. (c) What will happen if a photon with an energy of 8 eV strikes a Searsium atom in its ground level? Why? (d) Photons emitted in the Searsium transitions $n = 3 \rightarrow n = 2$ and $n = 3 \rightarrow n = 1$ will eject photoelectrons from an unknown metal, but the photon emitted from the transition $n = 4 \rightarrow n = 3$ will not. What are the limits (maximum and minimum possible values) of the work function of the metal?

39.31 • In a set of experiments on a hypothetical one-electron atom, you measure the wavelengths of the photons emitted from transitions ending in the ground state ($n = 1$), as shown in the energy-level diagram in Fig. E39.31. You also observe that it takes 17.50 eV to ionize this atom.

Figure E39.31



(a) What is the energy of the atom in each of the levels ($n = 1, n = 2,$ etc.) shown in the figure? (b) If an electron made a transition from the $n = 4$ to the $n = 2$ level, what wavelength of light would it emit?

39.32 • Find the longest and shortest wavelengths in the Lyman and Paschen series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

39.33 • (a) An atom initially in an energy level with $E = -6.52$ eV absorbs a photon that has wavelength 860 nm. What is the internal energy of the atom after it absorbs the photon? (b) An atom initially in an energy level with $E = -2.68$ eV emits a photon that has wavelength 420 nm. What is the internal energy of the atom after it emits the photon?

39.34 •• Use Balmer's formula to calculate (a) the wavelength, (b) the frequency, and (c) the photon energy for the H_γ line of the Balmer series for hydrogen.

Section 39.4 The Laser

39.35 • BIO Laser Surgery. Using a mixture of $\text{CO}_2, \text{N}_2,$ and sometimes He, CO_2 lasers emit a wavelength of $10.6 \mu\text{m}$. At power outputs of 0.100 kW, such lasers are used for surgery. How many photons per second does a CO_2 laser deliver to the tissue during its use in an operation?

39.36 • BIO Removing Birthmarks. Pulsed dye lasers emit light of wavelength 585 nm in 0.45-ms pulses to remove skin blemishes such as birthmarks. The beam is usually focused onto a circular spot 5.0 mm in diameter. Suppose that the output of one such laser is 20.0 W. (a) What is the energy of each photon, in eV? (b) How many photons per square millimeter are delivered to the blemish during each pulse?

39.37 • How many photons per second are emitted by a 7.50-mW CO_2 laser that has a wavelength of $10.6 \mu\text{m}$?

39.38 • BIO PRK Surgery. Photorefractive keratectomy (PRK) is a laser-based surgical procedure that corrects near- and farsightedness by removing part of the lens of the eye to change its curvature and hence focal length. This procedure can remove layers 0.25 μm thick using pulses lasting 12.0 ns from a laser beam of wavelength 193 nm. Low-intensity beams can be used because each individual photon has enough energy to break the covalent bonds of the tissue. (a) In what part of the electromagnetic spectrum does this light lie? (b) What is the energy of a single photon? (c) If a 1.50-mW beam is used, how many photons are delivered to the lens in each pulse?

39.39 • A large number of neon atoms are in thermal equilibrium. What is the ratio of the number of atoms in a $5s$ state to the number in a $3p$ state at (a) 300 K; (b) 600 K; (c) 1200 K? The energies of these states, relative to the ground state, are $E_{5s} = 20.66$ eV and $E_{3p} = 18.70$ eV. (d) At any of these temperatures, the rate at which a neon gas will spontaneously emit 632.8-nm radiation is quite low. Explain why.

39.40 • Figure 39.19a shows the energy levels of the sodium atom. The two lowest excited levels are shown in columns labeled $^2P_{3/2}$ and $^2P_{1/2}$. Find the ratio of the number of atoms in a $^2P_{3/2}$ state to the number in a $^2P_{1/2}$ state for a sodium gas in thermal equilibrium at 500 K. In which state are more atoms found?

Section 39.5 Continuous Spectra

39.41 •• A 100-W incandescent light bulb has a cylindrical tungsten filament 30.0 cm long, 0.40 mm in diameter, and with an emissivity of 0.26. (a) What is the temperature of the filament? (b) For what wavelength does the spectral emittance of the bulb peak? (c) Incandescent light bulbs are not very efficient sources of visible light. Explain why this is so.

39.42 • Determine λ_m , the wavelength at the peak of the Planck distribution, and the corresponding frequency f , at these temperatures: (a) 3.00 K; (b) 300 K; (c) 3000 K.

39.43 • Radiation has been detected from space that is characteristic of an ideal radiator at $T = 2.728$ K. (This radiation is a relic of the Big Bang at the beginning of the universe.) For this temperature, at what wavelength does the Planck distribution peak? In what part of the electromagnetic spectrum is this wavelength?

39.44 • The shortest visible wavelength is about 400 nm. What is the temperature of an ideal radiator whose spectral emittance peaks at this wavelength?

39.45 •• Two stars, both of which behave like ideal blackbodies, radiate the same total energy per second. The cooler one has a surface temperature T and a diameter 3.0 times that of the hotter star. (a) What is the temperature of the hotter star in terms of T ? (b) What is the ratio of the peak-intensity wavelength of the hot star to the peak-intensity wavelength of the cool star?

39.46 • Sirius B. The brightest star in the sky is Sirius, the Dog Star. It is actually a binary system of two stars, the smaller one (Sirius B) being a white dwarf. Spectral analysis of Sirius B indicates that its surface temperature is 24,000 K and that it radiates energy at a total rate of 1.0×10^{25} W. Assume that it behaves like an ideal blackbody. (a) What is the total radiated intensity of Sirius B? (b) What is the peak-intensity wavelength? Is this wavelength visible to humans? (c) What is the radius of Sirius B? Express your answer in kilometers and as a fraction of our sun's radius. (d) Which star radiates more *total* energy per second, the hot Sirius B or the (relatively) cool sun with a surface temperature of 5800 K? To find out, calculate the ratio of the total power radiated by our sun to the power radiated by Sirius B.

39.47 •• Blue Supergiants. A typical blue supergiant star (the type that explodes and leaves behind a black hole) has a surface temperature of 30,000 K and a visual luminosity 100,000 times that of our sun. Our sun radiates at the rate of 3.86×10^{26} W. (Visual luminosity is the total power radiated at visible wavelengths.) (a) Assuming that this star behaves like an ideal blackbody, what is the principal wavelength it radiates? Is this light visible? Use your answer to explain why these stars are blue. (b) If we assume that the power radiated by the star is also 100,000 times that of our sun, what is the radius of this star? Compare its size to that of our sun, which has a radius of 6.96×10^5 km. (c) Is it really correct to say that the visual luminosity is proportional to the total power radiated? Explain.

Section 39.6 The Uncertainty Principle Revisited

39.48 • A pesky 1.5-mg mosquito is annoying you as you attempt to study physics in your room, which is 5.0 m wide and 2.5 m high. You decide to swat the bothersome insect as it flies toward you, but you need to estimate its speed to make a successful hit. (a) What is the maximum uncertainty in the horizontal position of the mosquito? (b) What limit does the Heisenberg uncertainty principle place on your ability to know the horizontal velocity of this mosquito? Is this limitation a serious impediment to your attempt to swat it?

39.49 • By extremely careful measurement, you determine the x -coordinate of a car's center of mass with an uncertainty of only $1.00 \mu\text{m}$. The car has a mass of 1200 kg. (a) What is the minimum uncertainty in the x -component of the velocity of the car's center of mass as prescribed by the Heisenberg uncertainty principle? (b) Does the uncertainty principle impose a practical limit on our ability to make simultaneous measurements of the positions and velocities of ordinary objects like cars, books, and people? Explain.

39.50 • A 10.0-g marble is gently placed on a horizontal tabletop that is 1.75 m wide. (a) What is the maximum uncertainty in the horizontal position of the marble? (b) According to the Heisenberg uncertainty principle, what is the minimum uncertainty in the horizontal velocity of the marble? (c) In light of your answer to part (b), what is the longest time the marble could remain on the table? Compare this time to the age of the universe, which is approximately 14 billion years. (*Hint:* Can you know that the horizontal velocity of the marble is *exactly* zero?)

39.51 • A scientist has devised a new method of isolating individual particles. He claims that this method enables him to detect

simultaneously the position of a particle along an axis with a standard deviation of 0.12 nm and its momentum component along this axis with a standard deviation of 3.0×10^{-25} kg \cdot m/s. Use the Heisenberg uncertainty principle to evaluate the validity of this claim.

39.52 • (a) The x -coordinate of an electron is measured with an uncertainty of 0.20 mm. What is the x -component of the electron's velocity, v_x , if the minimum percentage uncertainty in a simultaneous measurement of v_x is 1.0%? (b) Repeat part (a) for a proton.

39.53 • An atom in a metastable state has a lifetime of 5.2 ms. What is the uncertainty in energy of the metastable state?

39.54 • (a) The uncertainty in the y -component of a proton's position is 2.0×10^{-12} m. What is the minimum uncertainty in a simultaneous measurement of the y -component of the proton's velocity? (b) The uncertainty in the z -component of an electron's velocity is 0.250 m/s. What is the minimum uncertainty in a simultaneous measurement of the z -coordinate of the electron?

PROBLEMS

39.55 •• The negative muon has a charge equal to that of an electron but a mass that is 207 times as great. Consider a hydrogenlike atom consisting of a proton and a muon. (a) What is the reduced mass of the atom? (b) What is the ground-level energy (in electron volts)? (c) What is the wavelength of the radiation emitted in the transition from the $n = 2$ level to the $n = 1$ level?

39.56 • An atom with mass m emits a photon of wavelength λ . (a) What is the recoil speed of the atom? (b) What is the kinetic energy K of the recoiling atom? (c) Find the ratio K/E , where E is the energy of the emitted photon. If this ratio is much less than unity, the recoil of the atom can be neglected in the emission process. Is the recoil of the atom more important for small or large atomic masses? For long or short wavelengths? (d) Calculate K (in electron volts) and K/E for a hydrogen atom (mass 1.67×10^{-27} kg) that emits an ultraviolet photon of energy 10.2 eV. Is recoil an important consideration in this emission process?

39.57 • (a) What is the smallest amount of energy in electron volts that must be given to a hydrogen atom initially in its ground level so that it can emit the H_α line in the Balmer series? (b) How many different possibilities of spectral-line emissions are there for this atom when the electron starts in the $n = 3$ level and eventually ends up in the ground level? Calculate the wavelength of the emitted photon in each case.

39.58 • A large number of hydrogen atoms are in thermal equilibrium. Let n_2/n_1 be the ratio of the number of atoms in an $n = 2$ excited state to the number of atoms in an $n = 1$ ground state. At what temperature is n_2/n_1 equal to (a) 10^{-12} ; (b) 10^{-8} ; (c) 10^{-4} ? (d) Like the sun, other stars have continuous spectra with dark absorption lines (see Fig. 39.9). The absorption takes place in the star's atmosphere, which in all stars is composed primarily of hydrogen. Explain why the Balmer absorption lines are relatively weak in stars with low atmospheric temperatures such as the sun (atmosphere temperature 5800 K) but strong in stars with higher atmospheric temperatures.

39.59 ••• A sample of hydrogen atoms is irradiated with light with wavelength 85.5 nm, and electrons are observed leaving the gas. (a) If each hydrogen atom were initially in its ground level, what would be the maximum kinetic energy in electron volts of these photoelectrons? (b) A few electrons are detected with energies as much as 10.2 eV greater than the maximum kinetic energy calculated in part (a). How can this be?

39.60 • CP Bohr Orbits of a Satellite. A 20.0-kg satellite circles the earth once every 2.00 h in an orbit having a radius of 8060 km. (a) Assuming that Bohr's angular-momentum result ($L = nh/2\pi$) applies to satellites just as it does to an electron in the hydrogen atom, find the quantum number n of the orbit of the satellite. (b) Show from Bohr's angular momentum result and Newton's law of gravitation that the radius of an earth-satellite orbit is directly proportional to the square of the quantum number, $r = kn^2$, where k is the constant of proportionality. (c) Using the result from part (b), find the distance between the orbit of the satellite in this problem and its next "allowed" orbit. (Calculate a numerical value.) (d) Comment on the possibility of observing the separation of the two adjacent orbits. (e) Do quantized and classical orbits correspond for this satellite? Which is the "correct" method for calculating the orbits?

39.61 •• The Red Supergiant Betelgeuse. The star Betelgeuse has a surface temperature of 3000 K and is 600 times the diameter of our sun. (If our sun were that large, we would be inside it!) Assume that it radiates like an ideal blackbody. (a) If Betelgeuse were to radiate all of its energy at the peak-intensity wavelength, how many photons per second would it radiate? (b) Find the ratio of the power radiated by Betelgeuse to the power radiated by our sun (at 5800 K).

39.62 •• CP Light from an ideal spherical blackbody 15.0 cm in diameter is analyzed using a diffraction grating having 3850 lines/cm. When you shine this light through the grating, you observe that the peak-intensity wavelength forms a first-order bright fringe at $\pm 11.6^\circ$ from the central bright fringe. (a) What is the temperature of the blackbody? (b) How long will it take this sphere to radiate 12.0 MJ of energy?

39.63 • What must be the temperature of an ideal blackbody so that photons of its radiated light having the peak-intensity wavelength can excite the electron in the Bohr-model hydrogen atom from the ground state to the third excited state?

39.64 • CP An ideal spherical blackbody 24.0 cm in diameter is maintained at 225°C by an internal electrical heater and is immersed in a very large open-faced tank of water that is kept boiling by the energy radiated by the sphere. You can neglect any heat transferred by conduction and convection. Consult Table 17.4 as needed. (a) At what rate, in g/s, is water evaporating from the tank? (b) If a physics-wise thermophile organism living in the hot water is observing this process, what will it measure for the peak-intensity (i) wavelength and (ii) frequency of the electromagnetic waves emitted by the sphere?

39.65 ••• When a photon is emitted by an atom, the atom must recoil to conserve momentum. This means that the photon and the recoiling atom share the transition energy. (a) For an atom with mass m , calculate the correction $\Delta\lambda$ due to recoil to the wavelength of an emitted photon. Let λ be the wavelength of the photon if recoil is not taken into consideration. (*Hint:* The correction is very small, as Problem 39.56 suggests, so $|\Delta\lambda|/\lambda \ll 1$. Use this fact to obtain an approximate but very accurate expression for $\Delta\lambda$.) (b) Evaluate the correction for a hydrogen atom in which an electron in the n th level returns to the ground level. How does the answer depend on n ?

39.66 •• An Ideal Blackbody. A large cavity with a very small hole and maintained at a temperature T is a good approximation to an ideal radiator or blackbody. Radiation can pass into or out of the cavity only through the hole. The cavity is a perfect absorber, since any radiation incident on the hole becomes trapped inside the cavity. Such a cavity at 200°C has a hole with area 4.00 mm^2 . How

long does it take for the cavity to radiate 100 J of energy through the hole?

39.67 •• CALC (a) Write the Planck distribution law in terms of the frequency f , rather than the wavelength λ , to obtain $I(f)$. (b) Show that

$$\int_0^\infty I(\lambda) d\lambda = \frac{2\pi^5 k^4}{15c^2 h^3} T^4$$

where $I(\lambda)$ is the Planck distribution formula of Eq. (39.24). (*Hint:* Change the integration variable from λ to f . You will need to use the following tabulated integral:

$$\int_0^\infty \frac{x^3}{e^{ax} - 1} dx = \frac{1}{240} \left(\frac{2\pi}{a} \right)^4$$

(c) The result of part (b) is I and has the form of the Stefan-Boltzmann law, $I = \sigma T^4$ (Eq. 39.19). Evaluate the constants in part (b) to show that σ has the value given in Section 39.5.

39.68 •• CP A beam of 40-eV electrons traveling in the $+x$ -direction passes through a slit that is parallel to the y -axis and $5.0\ \mu\text{m}$ wide. The diffraction pattern is recorded on a screen 2.5 m from the slit. (a) What is the de Broglie wavelength of the electrons? (b) How much time does it take the electrons to travel from the slit to the screen? (c) Use the width of the central diffraction pattern to calculate the uncertainty in the y -component of momentum of an electron just after it has passed through the slit. (d) Use the result of part (c) and the Heisenberg uncertainty principle (Eq. 39.29 for y) to estimate the minimum uncertainty in the y -coordinate of an electron just after it has passed through the slit. Compare your result to the width of the slit.

39.69 • (a) What is the energy of a photon that has wavelength $0.10\ \mu\text{m}$? (b) Through approximately what potential difference must electrons be accelerated so that they will exhibit wave nature in passing through a pinhole $0.10\ \mu\text{m}$ in diameter? What is the speed of these electrons? (c) If protons rather than electrons were used, through what potential difference would protons have to be accelerated so they would exhibit wave nature in passing through this pinhole? What would be the speed of these protons?

39.70 • CP Electrons go through a single slit 150 nm wide and strike a screen 24.0 cm away. You find that at angles of $\pm 20.0^\circ$ from the center of the diffraction pattern, no electrons hit the screen but electrons hit at all points closer to the center. (a) How fast were these electrons moving when they went through the slit? (b) What will be the next larger angles at which no electrons hit the screen?

39.71 •• CP A beam of electrons is accelerated from rest and then passes through a pair of identical thin slits that are 1.25 nm apart. You observe that the first double-slit interference dark fringe occurs at $\pm 18.0^\circ$ from the original direction of the beam when viewed on a distant screen. (a) Are these electrons relativistic? How do you know? (b) Through what potential difference were the electrons accelerated?

39.72 •• CP A beam of protons and a beam of alpha particles (of mass $6.64 \times 10^{-27}\text{ kg}$ and charge $+2e$) are accelerated from rest through the same potential difference and pass through identical circular holes in a very thin, opaque film. When viewed far from the hole, the diffracted proton beam forms its first dark ring at 15° with respect to its original direction. When viewed similarly, at what angle will the alpha particle form its first dark ring?

39.73 •• CP An electron beam and a photon beam pass through identical slits. On a distant screen, the first dark fringe occurs at the same angle for both of the beams. The electron speeds are much

slower than that of light. (a) Express the energy of a photon in terms of the kinetic energy K of one of the electrons. (b) Which is greater, the energy of a photon or the kinetic energy of an electron?

39.74 • CP Coherent light is passed through two narrow slits whose separation is $40.0 \mu\text{m}$. The second-order bright fringe in the interference pattern is located at an angle of 0.0300 rad . If electrons are used instead of light, what must the kinetic energy (in electron volts) of the electrons be if they are to produce an interference pattern for which the second-order maximum is also at 0.0300 rad ?

39.75 • BIO What is the de Broglie wavelength of a red blood cell, with mass $1.00 \times 10^{-11} \text{ g}$, that is moving with a speed of 0.400 cm/s ? Do we need to be concerned with the wave nature of the blood cells when we describe the flow of blood in the body?

39.76 • Calculate the energy in electron volts of (a) an electron that has de Broglie wavelength 400 nm and (b) a photon that has wavelength 400 nm .

39.77 • High-speed electrons are used to probe the interior structure of the atomic nucleus. For such electrons the expression $\lambda = h/p$ still holds, but we must use the relativistic expression for momentum, $p = mv/\sqrt{1 - v^2/c^2}$. (a) Show that the speed of an electron that has de Broglie wavelength λ is

$$v = \frac{c}{\sqrt{1 + (mc\lambda/h)^2}}$$

(b) The quantity h/mc equals $2.426 \times 10^{-12} \text{ m}$. (As we saw in Section 38.3, this same quantity appears in Eq. (38.7), the expression for Compton scattering of photons by electrons.) If λ is small compared to h/mc , the denominator in the expression found in part (a) is close to unity and the speed v is very close to c . In this case it is convenient to write $v = (1 - \Delta)c$ and express the speed of the electron in terms of Δ rather than v . Find an expression for Δ valid when $\lambda \ll h/mc$. [Hint: Use the binomial expansion $(1 + z)^n = 1 + nz + n(n-1)z^2/2 + \dots$, valid for the case $|z| < 1$.] (c) How fast must an electron move for its de Broglie wavelength to be $1.00 \times 10^{-15} \text{ m}$, comparable to the size of a proton? Express your answer in the form $v = (1 - \Delta)c$, and state the value of Δ .

39.78 • Suppose that the uncertainty of position of an electron is equal to the radius of the $n = 1$ Bohr orbit for hydrogen. Calculate the simultaneous minimum uncertainty of the corresponding momentum component, and compare this with the magnitude of the momentum of the electron in the $n = 1$ Bohr orbit. Discuss your results.

39.79 • CP (a) A particle with mass m has kinetic energy equal to three times its rest energy. What is the de Broglie wavelength of this particle? (Hint: You must use the relativistic expressions for momentum and kinetic energy: $E^2 = (pc)^2 + (mc^2)^2$ and $K = E - mc^2$.) (b) Determine the numerical value of the kinetic energy (in MeV) and the wavelength (in meters) if the particle in part (a) is (i) an electron and (ii) a proton.

39.80 • Proton Energy in a Nucleus. The radii of atomic nuclei are of the order of $5.0 \times 10^{-15} \text{ m}$. (a) Estimate the minimum uncertainty in the momentum of a proton if it is confined within a nucleus. (b) Take this uncertainty in momentum to be an estimate of the magnitude of the momentum. Use the relativistic relationship between energy and momentum, Eq. (37.39), to obtain an estimate of the kinetic energy of a proton confined within a nucleus. (c) For a proton to remain bound within a nucleus, what must the magnitude of the (negative) potential energy for a proton be within the nucleus? Give your answer in eV and in MeV. Compare to the potential energy for an electron in a hydrogen atom, which has a magnitude of a few tens of eV. (This shows why the

interaction that binds the nucleus together is called the “strong nuclear force.”)

39.81 • Electron Energy in a Nucleus. The radii of atomic nuclei are of the order of $5.0 \times 10^{-15} \text{ m}$. (a) Estimate the minimum uncertainty in the momentum of an electron if it is confined within a nucleus. (b) Take this uncertainty in momentum to be an estimate of the magnitude of the momentum. Use the relativistic relationship between energy and momentum, Eq. (37.39), to obtain an estimate of the kinetic energy of an electron confined within a nucleus. (c) Compare the energy calculated in part (b) to the magnitude of the Coulomb potential energy of a proton and an electron separated by $5.0 \times 10^{-15} \text{ m}$. On the basis of your result, could there be electrons within the nucleus? (Note: It is interesting to compare this result to that of Problem 39.80.)

39.82 • In a TV picture tube the accelerating voltage is 15.0 kV , and the electron beam passes through an aperture 0.50 mm in diameter to a screen 0.300 m away. (a) Calculate the uncertainty in the component of the electron’s velocity perpendicular to the line between aperture and screen. (b) What is the uncertainty in position of the point where the electrons strike the screen? (c) Does this uncertainty affect the clarity of the picture significantly? (Use nonrelativistic expressions for the motion of the electrons. This is fairly accurate and is certainly adequate for obtaining an estimate of uncertainty effects.)

39.83 • The neutral pion (π^0) is an unstable particle produced in high-energy particle collisions. Its mass is about 264 times that of the electron, and it exists for an average lifetime of $8.4 \times 10^{-17} \text{ s}$ before decaying into two gamma-ray photons. Using the relationship $E = mc^2$ between rest mass and energy, find the uncertainty in the mass of the particle and express it as a fraction of the mass.

39.84 • Quantum Effects in Daily Life? A 1.25-mg insect flies through a 4.00-mm -diameter hole in an ordinary window screen. The thickness of the screen is 0.500 mm . (a) What should be the approximate wavelength and speed of the insect for her to show wave behavior as she goes through the hole? (b) At the speed found in part (a), how long would it take the insect to pass through the 0.500-mm thickness of the hole in the screen? Compare this time to the age of the universe (about 14 billion years). Would you expect to see “insect diffraction” in daily life?

39.85 • Doorway Diffraction. If your wavelength were 1.0 m , you would undergo considerable diffraction in moving through a doorway. (a) What must your speed be for you to have this wavelength? (Assume that your mass is 60.0 kg .) (b) At the speed calculated in part (a), how many years would it take you to move 0.80 m (one step)? Will you notice diffraction effects as you walk through doorways?

39.86 • Atomic Spectra Uncertainties. A certain atom has an energy level 2.58 eV above the ground level. Once excited to this level, the atom remains in this level for $1.64 \times 10^{-7} \text{ s}$ (on average) before emitting a photon and returning to the ground level. (a) What is the energy of the photon (in electron volts)? What is its wavelength (in nanometers)? (b) What is the smallest possible uncertainty in energy of the photon? Give your answer in electron volts. (c) Show that $|\Delta E/E| = |\Delta\lambda/\lambda|$ if $|\Delta\lambda/\lambda| \ll 1$. Use this to calculate the magnitude of the smallest possible uncertainty in the wavelength of the photon. Give your answer in nanometers.

39.87 •• You intend to use an electron microscope to study the structure of some crystals. For accurate resolution, you want the electron wavelength to be 1.00 nm . (a) Are these electrons relativistic? How do you know? (b) What accelerating potential is needed? (c) What is the kinetic energy of the electrons you are using? To see if it is great enough to damage the crystals you are

studying, compare it to the potential energy of a typical NaCl molecule, which is about 6.0 eV. (d) If you decided to use electromagnetic waves as your probe, what energy should their photons have to provide the same resolution as the electrons? Would this energy damage the crystal?

39.88 •• For x rays with wavelength 0.0300 nm, the $m = 1$ intensity maximum for a crystal occurs when the angle θ in Fig. 39.2 is 35.8° . At what angle θ does the $m = 1$ maximum occur when a beam of 4.50-keV electrons is used instead? Assume that the electrons also scatter from the atoms in the surface plane of this same crystal.

39.89 •• CP Electron diffraction can also take place when there is interference between electron waves that scatter from atoms on the surface of a crystal and waves that scatter from atoms in the next plane below the surface, a distance d from the surface (see Fig. 36.23c). (a) Find an equation for the angles θ at which there is an intensity maximum for electron waves of wavelength λ . (b) The spacing between crystal planes in a certain metal is 0.091 nm. If 71.0-eV electrons are used, find the angle at which there is an intensity maximum due to interference between scattered waves from adjacent crystal planes. The angle is measured as shown in Fig. 36.23c. (c) The actual angle of the intensity maximum is slightly different from your result in part (b). The reason is the work function ϕ of the metal (see Section 38.1), which changes the electron potential energy by $-e\phi$ when it moves from vacuum into the metal. If the effect of the work function is taken into account, is the angle of the intensity maximum larger or smaller than the value found in part (b)? Explain.

39.90 •• A certain atom has an energy level 3.50 eV above the ground state. When excited to this state, it remains 4.0 μs , on the average, before emitting a photon and returning to the ground state. (a) What is the energy of the photon? What is its wavelength? (b) What is the smallest possible uncertainty in energy of the photon?

39.91 •• BIO Structure of a Virus. To investigate the structure of extremely small objects, such as viruses, the wavelength of the probing wave should be about one-tenth the size of the object for sharp images. But as the wavelength gets shorter, the energy of a photon of light gets greater and could damage or destroy the object being studied. One alternative is to use electron matter waves instead of light. Viruses vary considerably in size, but 50 nm is not unusual. Suppose you want to study such a virus, using a wave of wavelength 5.00 nm. (a) If you use light of this wavelength, what would be the energy (in eV) of a single photon? (b) If you use an electron of this wavelength, what would be its kinetic energy (in eV)? Is it now clear why matter waves (such as in the electron microscope) are often preferable to electromagnetic waves for studying microscopic objects?

39.92 •• CALC Zero-Point Energy. Consider a particle with mass m moving in a potential $U = \frac{1}{2}kx^2$, as in a mass-spring system. The total energy of the particle is $E = p^2/2m + \frac{1}{2}kx^2$. Assume that p and x are approximately related by the Heisenberg uncertainty principle, so $px \approx h$. (a) Calculate the minimum possible value of the energy E , and the value of x that gives this minimum E . This lowest possible energy, which is not zero, is called the

zero-point energy. (b) For the x calculated in part (a), what is the ratio of the kinetic to the potential energy of the particle?

39.93 •• CALC A particle with mass m moves in a potential $U(x) = A|x|$, where A is a positive constant. In a simplified picture, quarks (the constituents of protons, neutrons, and other particles, as will be described in Chapter 44) have a potential energy of interaction of approximately this form, where x represents the separation between a pair of quarks. Because $U(x) \rightarrow \infty$ as $x \rightarrow \infty$, it's not possible to separate quarks from each other (a phenomenon called *quark confinement*). (a) Classically, what is the force acting on this particle as a function of x ? (b) Using the uncertainty principle as in Problem 39.92, determine approximately the zero-point energy of the particle.

39.94 •• Imagine another universe in which the value of Planck's constant is 0.0663 J \cdot s, but in which the physical laws and all other physical constants are the same as in our universe. In this universe, two physics students are playing catch. They are 12 m apart, and one throws a 0.25-kg ball directly toward the other with a speed of 6.0 m/s. (a) What is the uncertainty in the ball's horizontal momentum, in a direction perpendicular to that in which it is being thrown, if the student throwing the ball knows that it is located within a cube with volume 125 cm³ at the time she throws it? (b) By what horizontal distance could the ball miss the second student?

CHALLENGE PROBLEMS

39.95 ••• (a) Show that in the Bohr model, the frequency of revolution of an electron in its circular orbit around a stationary hydrogen nucleus is $f = me^4/4\epsilon_0^2 n^3 h^3$. (b) In classical physics, the frequency of revolution of the electron is equal to the frequency of the radiation that it emits. Show that when n is very large, the frequency of revolution does indeed equal the radiated frequency calculated from Eq. (39.5) for a transition from $n_1 = n + 1$ to $n_2 = n$. (This illustrates Bohr's *correspondence principle*, which is often used as a check on quantum calculations. When n is small, quantum physics gives results that are very different from those of classical physics. When n is large, the differences are not significant, and the two methods then "correspond." In fact, when Bohr first tackled the hydrogen atom problem, he sought to determine f as a function of n such that it would correspond to classical results for large n .)

39.96 ••• CP CALC You have entered a contest in which the contestants drop a marble with mass 20.0 g from the roof of a building onto a small target 25.0 m below. From uncertainty considerations, what is the typical distance by which you will miss the target, given that you aim with the highest possible precision? (*Hint:* The uncertainty Δx_f in the x -coordinate of the marble when it reaches the ground comes in part from the uncertainty Δx_i in the x -coordinate initially and in part from the initial uncertainty in v_x . The latter gives rise to an uncertainty Δv_x in the horizontal motion of the marble as it falls. The values of Δx_i and Δv_x are related by the uncertainty principle. A small Δx_i gives rise to a large Δv_x , and vice versa. Find the value of Δx_i that gives the smallest total uncertainty in x at the ground. Ignore any effects of air resistance.)

Answers

Chapter Opening Question ?

The smallest detail visible in an image is comparable to the wavelength used to make the image. Electrons can easily be given a large momentum p and hence a short wavelength $\lambda = h/p$, and so can be used to resolve extremely fine details. (See Section 39.1.)

Test Your Understanding Questions

39.1 Answers: (a) (i), (b) no From Example 39.2, the speed of a particle is $v = h/\lambda m$ and the kinetic energy is $K = \frac{1}{2}mv^2 = (m/2)(h/\lambda m)^2 = h^2/2\lambda^2 m$. This shows that for a given wavelength, the kinetic energy is inversely proportional to the mass. Hence the proton, with a smaller mass, has more kinetic energy than the neutron. For part (b), the energy of a photon is $E = hf$, and the frequency of a photon is $f = c/\lambda$. Hence $E = hc/\lambda$ and $\lambda = hc/E = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})/(54 \text{ eV}) = 2.3 \times 10^{-8} \text{ m}$. This is more than 100 times greater than the wavelength of an electron of the same energy. While both photons and electrons have wavelike properties, they have different relationships between their energy and momentum and hence between their frequency and wavelength.

39.2 Answer: (iii) Because the alpha particle is more massive, it won't bounce back in even a head-on collision with a proton that's initially at rest, any more than a bowling ball would when colliding with a Ping-Pong ball at rest (see Fig. 8.22b). Thus there would be *no* large-angle scattering in this case. Rutherford saw large-angle scattering in his experiment because gold nuclei are more massive than alpha particles (see Fig. 8.22a).

39.3 Answer: (iv) Figure 39.27 shows that many (though *not* all) of the energy levels of He^+ are the same as those of H. Hence photons emitted during transitions between corresponding pairs of levels in He^+ and H have the same energy E and the same wavelength $\lambda = hc/E$. An H atom that drops from the $n = 2$ level to the $n = 1$ level emits a photon of energy 10.20 eV and wavelength 122 nm (see Example 39.6); a He^+ ion emits a photon of the same energy and wavelength when it drops from the $n = 4$ level to the

$n = 2$ level. Inspecting Fig. 39.27 will show you that every even-numbered level in He^+ matches a level in H, while none of the odd-numbered He^+ levels do. The first three He^+ transitions given in the question ($n = 2$ to $n = 1$, $n = 3$ to $n = 2$, and $n = 4$ to $n = 3$) all involve an odd-numbered level, so none of their wavelengths match a wavelength emitted by H atoms.

39.4 Answer: (i) In a neon light fixture, a large potential difference is applied between the ends of a neon-filled glass tube. This ionizes some of the neon atoms, allowing a current of electrons to flow through the gas. Some of the neon atoms are struck by fast-moving electrons, making them transition to an excited level. From this level the atoms undergo *spontaneous* emission, as depicted in Fig. 39.28b, and emit 632.8-nm photons in the process. No population inversion occurs and the photons are not trapped by mirrors as shown in Fig. 39.29d, so there is no stimulated emission. Hence there is no laser action.

39.5 Answers: (a) yes, (b) yes The Planck radiation law, Eq. (39.24), shows that an ideal blackbody emits radiation at *all* wavelengths: The spectral emittance $I(\lambda)$ is equal to zero only for $\lambda = 0$ and in the limit $\lambda \rightarrow \infty$. So a blackbody at 2000 K does indeed emit both x rays and radio waves. However, Fig. 39.32 shows that the spectral emittance for this temperature is very low for wavelengths much shorter than $1 \mu\text{m}$ (including x rays) and for wavelengths much longer than a few μm (including radio waves). Hence such a blackbody emits very little in the way of x rays or radio waves.

39.6 Answer: (i) and (iii) (tie), (ii) and (iv) (tie) According to the Heisenberg uncertainty principle, the smaller the uncertainty Δx in the x -coordinate, the greater the uncertainty Δp_x in the x -momentum. The relationship between Δx and Δp_x does not depend on the mass of the particle, and so is the same for a proton as for an electron.

Bridging Problem

Answers: (a) 192 nm; ultraviolet **(b)** $n = 4$

(c) $\lambda_2 = 0.665 \text{ nm}$, $\lambda_3 = 0.997 \text{ nm}$

40

QUANTUM MECHANICS

LEARNING GOALS

By studying this chapter, you will learn:

- About the wave function that describes the behavior of a particle and the Schrödinger equation that this function must satisfy.
- How to calculate the wave functions and energy levels for a particle confined to a box.
- How to analyze the quantum-mechanical behavior of a particle in a potential well.
- How quantum mechanics makes it possible for particles to go where Newtonian mechanics says they cannot.
- How to use quantum mechanics to analyze a harmonic oscillator.



? These containers hold solutions of microscopic semiconductor particles of different sizes. The particles glow when exposed to ultraviolet light; the smallest particles glow blue and the largest particles glow red. Why?

In Chapter 39 we found that particles can behave like waves. In fact, it turns out that we can use the wave picture to completely describe the behavior of a particle. This approach, called *quantum mechanics*, is the key to understanding the behavior of matter on the molecular, atomic, and nuclear scales. In this chapter we'll see how to find the *wave function* of a particle by solving the *Schrödinger equation*, which is as fundamental to quantum mechanics as Newton's laws are to mechanics or as Maxwell's equations are to electromagnetism.

We'll begin with a quantum-mechanical analysis of a *free particle* that moves along a straight line without being acted on by forces of any kind. We'll then consider particles that are acted on by forces and are trapped in *bound states*, just as electrons are bound within an atom. We'll see that solving the Schrödinger equation automatically gives the possible energy levels for the system.

Besides energies, solving the Schrödinger equation gives us the probabilities of finding a particle in various regions. One surprising result is that there is a nonzero probability that microscopic particles will pass through thin barriers, even though such a process is forbidden by Newtonian mechanics.

In this chapter we'll consider the Schrödinger equation for one-dimensional motion only. In Chapter 41 we'll see how to extend this equation to three-dimensional problems such as the hydrogen atom. The hydrogen-atom wave functions will in turn form the foundation for our analysis of more complex atoms, of the periodic table of the elements, of x-ray energy levels, and of other properties of atoms.

40.1 Wave Functions and the One-Dimensional Schrödinger Equation

We have now seen compelling evidence that on an atomic or subatomic scale, an object such as an electron cannot be described simply as a classical, Newtonian point particle. Instead, we must take into account its *wave* characteristics. In the

Bohr model of the hydrogen atom (Section 39.3) we tried to have it both ways: We pictured the electron as a classical particle in a circular orbit around the nucleus, and used the de Broglie relation between particle momentum and wavelength to explain why only orbits of certain radii are allowed. As we saw in Section 39.6, however, the Heisenberg uncertainty principle tells us that a hybrid description of this kind can't be wholly correct. In this section we'll explore how to describe the state of a particle by using *only* the language of waves. This new description, called **quantum mechanics**, will replace the classical scheme of describing the state of a particle by its coordinates and velocity components.

Our new quantum-mechanical scheme for describing a particle has a lot in common with the language of classical wave motion. In Section 15.3 of Chapter 15, we described transverse waves on a string by specifying the position of each point in the string at each instant of time by means of a *wave function* $y(x, t)$ that represents the displacement from equilibrium, at time t , of a point on the string at a distance x from the origin (Fig. 40.1). Once we know the wave function for a particular wave motion, we know everything there is to know about the motion. For example, we can find the velocity and acceleration of any point on the string at any time. We worked out specific forms for these functions for *sinusoidal* waves, in which each particle undergoes simple harmonic motion.

We followed a similar pattern for sound waves in Chapter 16. The wave function $p(x, t)$ for a wave traveling along the x -direction represented the pressure variation at any point x and any time t . We used this language once more in Section 32.3, where we used *two* wave functions to describe the electric and magnetic fields in an electromagnetic wave.

Thus it's natural to use a wave function as the central element of our new language of quantum mechanics. The customary symbol for this wave function is the Greek letter psi, Ψ or ψ . In general, we'll use an uppercase Ψ to denote a function of all the space coordinates and time, and a lowercase ψ for a function of the space coordinates only—*not* of time. Just as the wave function $y(x, t)$ for mechanical waves on a string provides a complete description of the motion, so the wave function $\Psi(x, y, z, t)$ for a particle contains all the information that can be known about the particle.

CAUTION **Particle waves vs. mechanical waves** Unlike for mechanical waves on a string or sound waves in air, the wave function for a particle is *not* a mechanical wave that needs some material medium in order to propagate. The wave function describes the particle, but we cannot define the function itself in terms of anything material. We can only describe how it is related to physically observable effects. ■

Waves in One Dimension

The wave function of a particle depends in general on all three dimensions of space. For simplicity, however, we'll begin our study of these functions by considering *one-dimensional* motion, in which a particle of mass m moves parallel to the x -axis and the wave function Ψ depends on the coordinate x and the time t only. (In the same way, we studied one-dimensional kinematics in Chapter 2 before going on to study two- and three-dimensional motion in Chapter 3.)

What does a one-dimensional quantum-mechanical wave look like, and what determines its properties? We can answer this question by first recalling the properties of a wave on a string. We saw in Section 15.3 that any wave function $y(x, t)$ that describes a wave on a string must satisfy the *wave equation*:

$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2} \quad (\text{wave equation for waves on a string}) \quad (40.1)$$

In Eq. (40.1) v is the speed of the wave, which is the same no matter what the wavelength. As an example, consider the following wave function for a

40.1 These children are talking over a cup-and-string “telephone.” The displacement of the string is completely described by a wave function $y(x, t)$. In an analogous way, a particle is completely described by a quantum-mechanical wave function $\Psi(x, y, z, t)$.



wave of wavelength λ and frequency f moving in the positive x -direction along a string:

$$y(x, t) = A \cos(kx - \omega t) + B \sin(kx - \omega t) \quad \begin{array}{l} \text{(sinusoidal wave} \\ \text{on a string)} \end{array} \quad (40.2)$$

Here $k = 2\pi/\lambda$ is the *wave number* and $\omega = 2\pi f$ is the *angular frequency*. (We used these same quantities for mechanical waves in Chapter 15 and electromagnetic waves in Chapter 32.) The quantities A and B are constants that determine the amplitude and phase of the wave. The expression in Eq. (40.2) is a valid wave function if and only if it satisfies the wave equation, Eq. (40.1). To check this, take the first and second derivatives of $y(x, t)$ with respect to x and take the first and second derivatives with respect to t :

$$\frac{\partial y(x, t)}{\partial x} = -kA \sin(kx - \omega t) + kB \cos(kx - \omega t) \quad (40.3a)$$

$$\frac{\partial^2 y(x, t)}{\partial x^2} = -k^2 A \cos(kx - \omega t) - k^2 B \sin(kx - \omega t) \quad (40.3b)$$

$$\frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t) - \omega B \cos(kx - \omega t) \quad (40.3c)$$

$$\frac{\partial^2 y(x, t)}{\partial t^2} = -\omega^2 A \cos(kx - \omega t) - \omega^2 B \sin(kx - \omega t) \quad (40.3d)$$

If we substitute Eqs. (40.3b) and (40.3d) into the wave equation, Eq. (40.1), we get

$$\begin{aligned} & -k^2 A \cos(kx - \omega t) - k^2 B \sin(kx - \omega t) \\ &= \frac{1}{v^2} [-\omega^2 A \cos(kx - \omega t) - \omega^2 B \sin(kx - \omega t)] \end{aligned} \quad (40.4)$$

For Eq. 40.4 to be satisfied at all coordinates x and all times t , the coefficients of the cosine function must be the same on both sides of the equation, and likewise for the coefficients of the sine function. You can see that both of these conditions will be satisfied if

$$k^2 = \frac{\omega^2}{v^2} \quad \text{or} \quad \omega = vk \quad \text{(waves on a string)} \quad (40.5)$$

From the definitions of angular frequency ω and wave number k , Eq. (40.5) is equivalent to

$$2\pi f = v \frac{2\pi}{\lambda} \quad \text{or} \quad v = \lambda f \quad \text{(waves on a string)}$$

This equation is just the familiar relationship among wave speed, wavelength, and frequency for waves on a string. So our calculation shows that Eq. (40.2) is a valid wave function for waves on a string for any values of A and B , provided that ω and k are related by Eq. (40.5).

What we need is a quantum-mechanical version of the wave equation, Eq. (40.1), valid for particle waves. We expect this equation to involve partial derivatives of the wave function $\Psi(x, t)$ with respect to x and with respect to t . However, this new equation *cannot* be the same as Eq. (40.1) for waves on a string because the relationship between ω and k is different. We can show this by considering a **free particle**, one that experiences no force at all as it moves along the x -axis. For such a particle the potential energy $U(x)$ has the same value for all x (recall from Chapter 7 that $F_x = -dU(x)/dx$, so zero force means the potential energy has zero derivative). For simplicity let $U = 0$ for all x . Then the energy of the free particle is equal to its kinetic energy, which we can express in terms of its momentum p :

$$E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \quad \text{(energy of a free particle)} \quad (40.6)$$

The de Broglie relations that we introduced in Section 39.1 tell us that the energy E is proportional to the angular frequency ω and the momentum p is proportional to the wave number:

$$E = hf = \frac{h}{2\pi} 2\pi f = \hbar\omega \quad (40.7a)$$

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \frac{2\pi}{\lambda} = \hbar k \quad (40.7b)$$

Remember that $\hbar = h/2\pi$. If we substitute Eqs. (40.7) into Eq. (40.6), we find that the relationship between ω and k for a free particle is

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} \quad (\text{free particle}) \quad (40.8)$$

Equation (40.8) is *very* different from the corresponding relationship for waves on a string, Eq. (40.5): The angular frequency ω for particle waves is proportional to the *square* of the wave number, while for waves on a string ω is directly proportional to k . Our task is therefore to construct a quantum-mechanical version of the wave equation whose free-particle solutions satisfy Eq. (40.8).

We'll attack this problem by assuming a sinusoidal wave function $\Psi(x, t)$ of the same form as Eq. (40.2) for a sinusoidal wave on a string. For a wave on a string, Eq. (40.2) represents a wave of wavelength $\lambda = 2\pi/k$ and frequency $f = \omega/2\pi$ propagating in the positive x -direction. By analogy, our sinusoidal wave function $\Psi(x, t)$ represents a free particle of mass m , momentum $p = \hbar k$, and energy $E = \hbar\omega$ moving in the positive x -direction:

$$\Psi(x, t) = A \cos(kx - \omega t) + B \sin(kx - \omega t) \quad \begin{array}{l} \text{(sinusoidal wave} \\ \text{function representing} \\ \text{a free particle)} \end{array} \quad (40.9)$$

The wave number k and angular frequency ω in Eq. (40.9) must satisfy Eq. (40.8). If you look at Eq. (40.3b), you'll see that taking the second derivative of $\Psi(x, t)$ in Eq. (40.9) with respect to x gives us $\Psi(x, t)$ multiplied by $-k^2$. Hence if we multiply $\partial^2\Psi(x, t)/\partial x^2$ by $-\hbar^2/2m$, we get

$$\begin{aligned} -\frac{\hbar^2}{2m} \frac{\partial^2\Psi(x, t)}{\partial x^2} &= -\frac{\hbar^2}{2m} [-k^2 A \cos(kx - \omega t) - k^2 B \sin(kx - \omega t)] \\ &= \frac{\hbar^2 k^2}{2m} [A \cos(kx - \omega t) + B \sin(kx - \omega t)] \\ &= \frac{\hbar^2 k^2}{2m} \Psi(x, t) \end{aligned} \quad (40.10)$$

Equation (40.10) suggests that $(-\hbar^2/2m)\partial^2\Psi(x, t)/\partial x^2$ should be one side of our quantum-mechanical wave equation, with the other side equal to $\hbar\omega\Psi(x, t)$ in order to satisfy Eq. (40.8). If you look at Eq. (40.3c), you'll see that taking the *first* time derivative of $\Psi(x, t)$ in Eq. (40.9) brings out a factor of ω . So we'll make the educated guess that the right-hand side of our quantum-mechanical wave equation involves $\hbar = h/2\pi$ times $\partial\Psi(x, t)/\partial t$. So our tentative equation is

$$-\frac{\hbar^2}{2m} \frac{\partial^2\Psi(x, t)}{\partial x^2} = C\hbar \frac{\partial\Psi(x, t)}{\partial t} \quad (40.11)$$

At this point we include a constant C as a “fudge factor” to make sure that everything turns out right. Now let's substitute the wave function from Eq. (40.9) into Eq. (40.11). From Eq. (40.10) and Eq. (40.3c), we get

$$\begin{aligned} \frac{\hbar^2 k^2}{2m} [A \cos(kx - \omega t) + B \sin(kx - \omega t)] \\ = C\hbar\omega [A \sin(kx - \omega t) - B \cos(kx - \omega t)] \end{aligned} \quad (40.12)$$

From Eq. (40.8), $\hbar\omega = \hbar^2 k^2 / 2m$, so we can cancel these factors on the two sides of Eq. (40.12). What remains is

$$\begin{aligned} A \cos(kx - \omega t) + B \sin(kx - \omega t) = CA \sin(kx - \omega t) \\ - CB \cos(kx - \omega t) \end{aligned} \quad (40.13)$$

As in our discussion above of the wave equation for waves on a string, in order for Eq. (40.13) to be satisfied for all values of x and all values of t , the coefficients of the cosine function must be the same on both sides of the equation, and likewise for the coefficients of the sine function. Hence we have the following relationships among the coefficients A and B in Eq. (40.9) and the coefficient C in Eq. (40.11):

$$A = -CB \quad (40.14a)$$

$$B = CA \quad (40.14b)$$

If we use Eq. (40.14b) to eliminate B from Eq. (40.14a), we get $A = -C^2 A$, which means that $C^2 = -1$. Thus C is equal to the *imaginary* number $i = \sqrt{-1}$, and Eq. (40.11) becomes

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x, t)}{\partial t} \quad \text{(one-dimensional Schrödinger equation for a free particle)} \quad (40.15)$$

Equation (40.15) is the one-dimensional **Schrödinger equation** for a free particle, developed in 1926 by the Austrian physicist Erwin Schrödinger (Fig. 40.2). The presence of the imaginary number i in Eq. (40.15) means that the solutions to the Schrödinger equation are complex quantities, with a real part and an imaginary part. (The imaginary part of $\Psi(x, t)$ is a real function multiplied by the imaginary number $i = \sqrt{-1}$.) An example is our free-particle wave function from Eq. (40.9). Since we found $C = i$ in Eqs. (40.14), it follows from Eq. (40.14b) that $B = iA$. Then Eq. (40.9) becomes

$$\Psi(x, t) = A[\cos(kx - \omega t) + i \sin(kx - \omega t)] \quad \begin{array}{l} \text{(sinusoidal wave} \\ \text{function representing} \\ \text{a free particle)} \end{array} \quad (40.16)$$

The real part of $\Psi(x, t)$ is $\text{Re}\Psi(x, t) = A \cos(kx - \omega t)$ and the imaginary part is $\text{Im}\Psi(x, t) = A \sin(kx - \omega t)$. Figure 40.3 graphs the real and imaginary parts of $\Psi(x, t)$ at $t = 0$, so $\Psi(x, 0) = A \cos kx + iA \sin kx$.

We can rewrite Eq. (40.16) using *Euler's formula*, which states that for any angle θ ,

$$\begin{aligned} e^{i\theta} &= \cos \theta + i \sin \theta \\ e^{-i\theta} &= \cos(-\theta) + i \sin(-\theta) = \cos \theta - i \sin \theta \end{aligned} \quad (40.17)$$

Thus our sinusoidal free-particle wave function becomes

$$\Psi(x, t) = A e^{i(kx - \omega t)} = A e^{ikx} e^{-i\omega t} \quad \begin{array}{l} \text{(sinusoidal wave function} \\ \text{representing a free particle)} \end{array} \quad (40.18)$$

40.2 Erwin Schrödinger (1887–1961) developed the equation that bears his name in 1926, an accomplishment for which he shared (with the British physicist P. A. M. Dirac) the 1933 Nobel Prize in physics. His grave marker is adorned with a large letter ψ .



If k is positive in Eq. (40.16), the wave function represents a free particle moving in the positive x -direction with momentum $p = \hbar k$ and energy $E = \hbar\omega = \hbar^2 k^2 / 2m$. If k is negative, the momentum and hence the motion are in the negative x -direction. (With a negative value of k , the wavelength is $\lambda = 2\pi/|k|$).

Interpreting the Wave Function

The complex nature of the wave function for a free particle makes this function challenging to interpret. (We certainly haven't needed imaginary numbers before this point to describe real physical phenomena.) Here's how to think about this function: $\Psi(x, t)$ describes the *distribution* of a particle in space, just as the wave functions for an electromagnetic wave describe the distribution of the electric and magnetic fields. When we worked out interference and diffraction patterns in Chapters 35 and 36, we found that the intensity I of the radiation at any point in a pattern is proportional to the square of the electric-field magnitude—that is, to E^2 . In the photon interpretation of interference and diffraction (see Section 38.4), the intensity at each point is proportional to the number of photons striking around that point or, alternatively, to the *probability* that any individual photon will strike around the point. Thus the square of the electric-field magnitude at each point is proportional to the probability of finding a photon around that point.

In exactly the same way, the square of the wave function of a particle at each point tells us about the probability of finding the particle around that point. More precisely, we should say the square of the *absolute value* of the wave function, $|\Psi|^2$. This is necessary because, as we have seen, the wave function is a complex quantity with real and imaginary parts.

For a particle that can move only along the x -direction, the quantity $|\Psi(x, t)|^2 dx$ is the probability that the particle will be found at time t at a coordinate in the range from x to $x + dx$. The particle is most likely to be found in regions where $|\Psi|^2$ is large, and so on. This interpretation, first made by the German physicist Max Born (Fig. 40.4), requires that the wave function Ψ be *normalized*. That is, the integral of $|\Psi(x, t)|^2 dx$ over all possible values of x must equal exactly 1. In other words, the probability is exactly 1, or 100%, that the particle is *somewhere*.

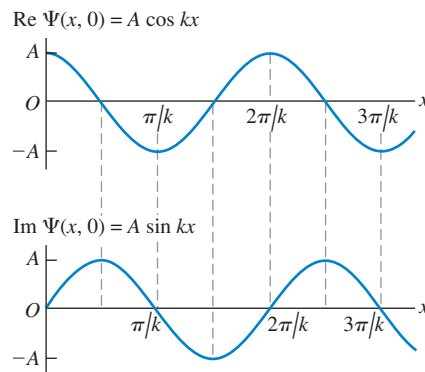
CAUTION Interpreting $|\Psi|^2$ Note that $|\Psi(x, t)|^2$ itself is *not* a probability. Rather, $|\Psi(x, t)|^2 dx$ is the probability of finding the particle between position x and position $x + dx$ at time t . If the length dx is made smaller, it becomes less likely that the particle will be found within that length, so the probability decreases. A better name for $|\Psi(x, t)|^2$ is the **probability distribution function**, since it describes how the probability of finding the particle at different locations is distributed over space. Another common name for $|\Psi(x, t)|^2$ is the *probability density*. **I**

We can use the probability interpretation of $|\Psi|^2$ to get a better understanding of Eq. (40.18), the wave function for a free particle. This function describes a particle that has a definite momentum $p = \hbar k$ in the x -direction and *no* uncertainty in momentum: $\Delta p_x = 0$. The Heisenberg uncertainty principle for position and momentum, Eq. (39.29), says that $\Delta x \Delta p_x \geq \hbar/2$. If Δp_x is zero, then Δx must be infinite, and we have no idea whatsoever where along the x -axis the particle can be found. (We saw a similar result for photons in Section 38.4.) We can show this by calculating the probability distribution function $|\Psi(x, t)|^2$. This is the product of Ψ and its *complex conjugate* Ψ^* . To find the complex conjugate of a complex number, we simply replace all i with $-i$. For example, the complex conjugate of $c = a + ib$, where a and b are real, is $c^* = a - ib$, so $|c|^2 = c^* c = (a + ib)(a - ib) = a^2 + b^2$ (recall that $i^2 = -1$). The complex conjugate of Eq. (40.18) is

$$\Psi^*(x, t) = A^* e^{-i(kx - \omega t)} = A^* e^{-ikx} e^{i\omega t}$$

(We have to allow for the possibility that the coefficient A is itself a complex number.) Hence the probability distribution function is

40.3 The spatial wave function $\Psi(x, t) = Ae^{i(kx - \omega t)}$ for a free particle of definite momentum $p = \hbar k$ is a complex function: It has both a real part and an imaginary part. These are graphed here as functions of x for $t = 0$.



40.4 In 1926, the German physicist Max Born (1882–1970) devised the interpretation that $|\Psi|^2$ is the probability distribution function for a particle that is described by the wave function Ψ . He also coined the term “quantum mechanics” (in the original German, *Quantenmechanik*). For his contributions, Born shared (with Walther Bothe) the 1954 Nobel Prize in physics.



$$|\Psi(x, t)|^2 = \Psi^*(x, t)\Psi(x, t) = (A^*e^{-ikx}e^{i\omega t})(Ae^{ikx}e^{-i\omega t}) \\ = A^*Ae^0 = |A|^2$$

The probability distribution function doesn't depend on position, which says that we are equally likely to find the particle *anywhere* along the x -axis! Mathematically, this is because the wave function $\Psi(x, t) = Ae^{i(kx - \omega t)} = A[\cos(kx - \omega t) + i\sin(kx - \omega t)]$ is a sinusoidal function that extends all the way from $x = -\infty$ to $x = +\infty$ with the same amplitude A . This also means that the wave function can't be normalized: The integral of $|\Psi(x, t)|^2$ over all space would be infinite for any value of A .

Note also that the wave function in Eq. (40.18) describes a particle with a definite energy $E = \hbar\omega$, so there is zero uncertainty in energy: $\Delta E = 0$. The Heisenberg uncertainty principle for energy and time interval, $\Delta t \Delta E \geq \hbar$ [Eq. (39.30)], tells us that the time uncertainty Δt for this particle is infinite. In other words, we can have no idea *when* the particle will pass a given point on the x -axis. That also agrees with our result $|\Psi(x, t)|^2 = |A|^2$; the probability distribution function has the same value at all times.

Since we always have some idea of where a particle is, the wave function given in Eq. (40.18) isn't a realistic description. In our study of light in Section 38.4, we saw that we can make a wave function that's more *localized* in space by superimposing two or more sinusoidal functions. (This would be a good time to review that section.) As an illustration, let's calculate $|\Psi(x, t)|^2$ for a wave function of this kind.

Example 40.1 A localized free-particle wave function

The wave function $\Psi(x, t) = Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)}$ is a superposition of *two* free-particle wave functions of the form given by Eq. (40.18). Both k_1 and k_2 are positive. (a) Show that this wave function satisfies the Schrödinger equation for a free particle of mass m . (b) Find the probability distribution function for $\Psi(x, t)$.

SOLUTION

IDENTIFY and SET UP: The wave functions $Ae^{i(k_1x - \omega_1t)}$ and $Ae^{i(k_2x - \omega_2t)}$ both represent particles moving in the positive x -direction, but with different momenta and kinetic energies: $p_1 = \hbar k_1$ and $E_1 = \hbar\omega_1 = \hbar^2 k_1^2 / 2m$ for the first function, $p_2 = \hbar k_2$ and $E_2 = \hbar\omega_2 = \hbar^2 k_2^2 / 2m$ for the second function. To test whether a superposition of these is also a valid wave function for a free particle, we'll see whether our function $\Psi(x, t)$ satisfies the free-particle Schrödinger equation, Eq. (40.15). It's useful to remember the derivatives of the exponential function: $(d/du)e^{au} = ae^{au}$ and $(d^2/du^2)e^{au} = a^2e^{au}$. The probability distribution function $|\Psi(x, t)|^2$ is the product of $\Psi(x, t)$ and its complex conjugate.

EXECUTE: (a) If we substitute $\Psi(x, t)$ into Eq. (40.15), the left-hand side of the equation is

$$\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} = \frac{\hbar^2}{2m} \frac{\partial^2 (Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)})}{\partial x^2} \\ = -\frac{\hbar^2}{2m} [(ik_1)^2 Ae^{i(k_1x - \omega_1t)} + (ik_2)^2 Ae^{i(k_2x - \omega_2t)}] \\ = \frac{\hbar^2 k_1^2}{2m} Ae^{i(k_1x - \omega_1t)} + \frac{\hbar^2 k_2^2}{2m} Ae^{i(k_2x - \omega_2t)}$$

The right-hand side is

$$\frac{\partial \Psi(x, t)}{\partial t} = \frac{\partial (Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)})}{\partial t} \\ = i\hbar [(-i\omega_1)Ae^{i(k_1x - \omega_1t)} + (-i\omega_2)Ae^{i(k_2x - \omega_2t)}] \\ = \hbar\omega_1 Ae^{i(k_1x - \omega_1t)} + \hbar\omega_2 Ae^{i(k_2x - \omega_2t)}$$

The two sides are equal, provided that $\hbar\omega_1 = \hbar^2 k_1^2 / 2m$ and $\hbar\omega_2 = \hbar^2 k_2^2 / 2m$. These are just the relationships that we noted above. So we conclude that $\Psi(x, t) = Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)}$ is a valid free-particle wave function. In general, if we take any two wave functions that are solutions of the Schrödinger equation and then make a superposition of these to create a third wave function $\Psi(x, t)$, then $\Psi(x, t)$ is also a solution of the Schrödinger equation.

(b) The complex conjugate of $\Psi(x, t)$ is

$$\Psi^*(x, t) = A^* e^{-i(k_1x - \omega_1t)} + A^* e^{-i(k_2x - \omega_2t)}$$

Hence

$$|\Psi(x, t)|^2 \\ = \Psi^*(x, t)\Psi(x, t) \\ = (A^* e^{-i(k_1x - \omega_1t)} + A^* e^{-i(k_2x - \omega_2t)})(Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)}) \\ = A^* A \left[e^{-i(k_1x - \omega_1t)} e^{i(k_1x - \omega_1t)} + e^{-i(k_2x - \omega_2t)} e^{i(k_2x - \omega_2t)} \right. \\ \left. + e^{-i(k_1x - \omega_1t)} e^{i(k_2x - \omega_2t)} + e^{-i(k_2x - \omega_2t)} e^{i(k_1x - \omega_1t)} \right] \\ = |A|^2 [e^0 + e^0 + e^{i[(k_2 - k_1)x - (\omega_2 - \omega_1)t]} + e^{-i[(k_2 - k_1)x - (\omega_2 - \omega_1)t]}]$$

To simplify this expression, recall that $e^0 = 1$. From Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$ and $e^{-i\theta} = \cos\theta - i\sin\theta$, so $e^{i\theta} + e^{-i\theta} = 2\cos\theta$. Hence

$$|\Psi(x, t)|^2 = |A|^2 \{2 + 2\cos[(k_2 - k_1)x - (\omega_2 - \omega_1)t]\} \\ = 2|A|^2 \{1 + \cos[(k_2 - k_1)x - (\omega_2 - \omega_1)t]\}$$

EVALUATE: Figure 40.5 is a graph of the probability distribution function $|\Psi(x, t)|^2$ at $t = 0$. The value of $|\Psi(x, t)|^2$ varies between 0 and $4|A|^2$; probabilities can never be negative! The particle has become *somewhat* localized: The particle is most likely to be found near a point where $|\Psi(x, t)|^2$ is maximum (where the functions $Ae^{i(k_1x - \omega_1t)}$ and $Ae^{i(k_2x - \omega_2t)}$ interfere constructively) and is very unlikely to be found near a point where $|\Psi(x, t)|^2 = 0$ (where $Ae^{i(k_1x - \omega_1t)}$ and $Ae^{i(k_2x - \omega_2t)}$ interfere destructively). This is very similar to the phenomenon of beats for sound waves (see Section 16.7).

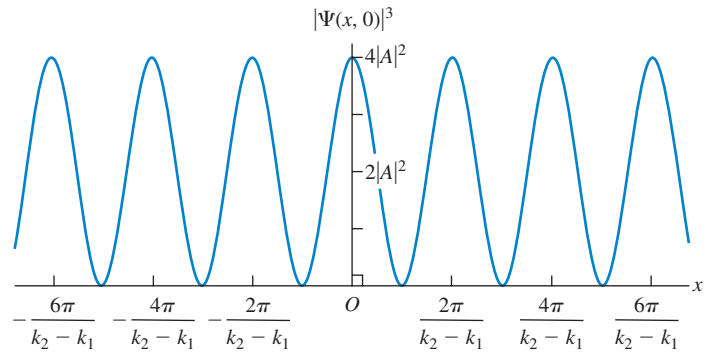
Note also that the probability distribution function is not stationary, but moves in the positive x -direction like the particle that it represents. To see this, recall from Section 15.3 that a sinusoidal wave given by $y(x, t) = A \cos(kx - \omega t)$ moves in the positive x -direction with velocity $v = \omega/k$; since $|\Psi(x, t)|^2$ includes a term $\cos[(k_2 - k_1)x - (\omega_2 - \omega_1)t]$, the probability distribution

moves at a velocity $v_{\text{av}} = (\omega_2 - \omega_1)/(k_2 - k_1)$. The subscript “av” reminds us that v_{av} represents the *average* value of the particle’s velocity.

The price we pay for localizing the particle somewhat is that, unlike a particle represented by Eq. (40.18), it no longer has either a definite momentum or a definite energy. That’s consistent with the Heisenberg uncertainty principles: If we decrease the uncertainties about where a particle is and when it passes a certain point, the uncertainties in its momentum and energy must increase.

The average momentum of the particle is $p_{\text{av}} = (\hbar k_2 + \hbar k_1)/2$, the average of the momenta associated with the free-particle wave functions we added to create $\Psi(x, t)$. This corresponds to the particle having an average velocity $v_{\text{av}} = p_{\text{av}}/m = (\hbar k_2 + \hbar k_1)/2m$. Can you show that this is equal to the expression $v_{\text{av}} = (\omega_2 - \omega_1)/(k_2 - k_1)$ that we found above?

40.5 The probability distribution function at $t = 0$ for $\Psi(x, t) = Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)}$.



Wave Packets

The wave function that we examined in Example 40.1 is not very well localized: The probability distribution function still extends from $x = -\infty$ to $x = +\infty$. Hence this wave function can’t be normalized, either. To make a wave function that’s more highly localized, imagine superposing two additional sinusoidal waves with different wave numbers and amplitudes so as to reinforce alternate maxima of $|\Psi(x, t)|^2$ in Fig. 40.5 and cancel out the in-between ones. Finally, if we superpose waves with a very large number of different wave numbers, we can construct a wave with only *one* maximum of $|\Psi(x, t)|^2$ (Fig. 40.6). Then, finally, we have something that begins to look like both a particle and a wave. It is a particle in the sense that it is localized in space; if we look from a distance, it may look like a point. But it also has a periodic structure that is characteristic of a wave.

A localized wave pulse like that shown in Fig. 40.6 is called a **wave packet**. We can represent a wave packet by an expression such as

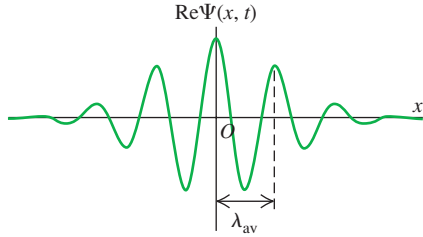
$$\Psi(x, t) = \int_{-\infty}^{\infty} A(k)e^{i(kx - \omega t)} dk \quad (40.19)$$

This integral represents a superposition of a very large number of waves, each with a different wave number k and angular frequency $\omega = \hbar k^2/2m$, and each with an amplitude $A(k)$ that depends on k .

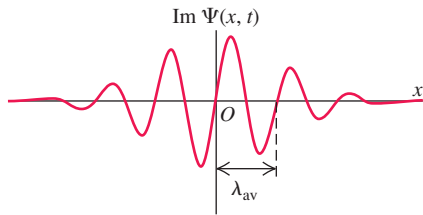
There is an important relationship between the two functions $\Psi(x, t)$ and $A(k)$, which we show qualitatively in Fig. 40.7. If the function $A(k)$ is sharply peaked, as in Fig. 40.7a, we are superposing only a narrow range of wave numbers. The resulting wave pulse is then relatively broad (Fig. 40.7b). But if we use

40.6 Superposing a large number of sinusoidal waves with different wave numbers and appropriate amplitudes can produce a wave pulse that has a wavelength $\lambda_{av} = 2\pi/k_{av}$ and is localized within a region of space of length Δx . This localized pulse has aspects of both particle and wave.

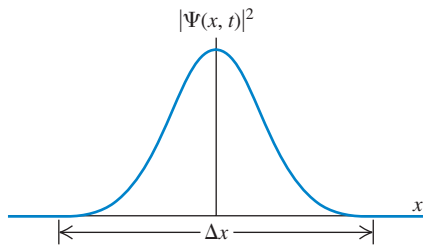
(a) Real part of the wave function at time t



(b) Imaginary part of the wave function at time t



(c) Probability distribution function at time t



a wider range of wave numbers, so that the function $A(k)$ is broader (Fig. 40.7c), then the wave pulse is more narrowly localized (Fig. 40.7d). This is simply the uncertainty principle in action. A narrow range of k means a narrow range of $p_x = \hbar k$ and thus a small Δp_x ; the result is a relatively large Δx . A broad range of k corresponds to a large Δp_x , and the resulting Δx is smaller. You can see that the uncertainty principle for position and momentum, $\Delta x \Delta p_x \geq \hbar/2$, is really just a consequence of the properties of integrals like Eq. (40.19).

CAUTION **Matter waves versus light waves in vacuum** We can regard both a wave packet that represents a particle and a short pulse of light from a laser as superpositions of waves of different wave numbers and angular frequencies. An important difference is that the speed of light in vacuum is the same for all wavelengths λ and hence all wave numbers $k = 2\pi/\lambda$, but the speed of a matter wave is *different* for different wavelengths. You can see this from the formula for the speed of the wave crests in a periodic wave, $v = \lambda f = \omega/k$. For a matter wave, $\omega = \hbar k^2/2m$, so $v = \hbar k/2m = h/2m\lambda$. Hence matter waves with longer wavelengths and smaller wave numbers travel more slowly than those with short wavelengths and large wave numbers. (This shouldn't be too surprising. The de Broglie relations that we learned in Section 39.1 tell us that shorter wavelength corresponds to greater momentum and hence a greater speed.) Since the individual sinusoidal waves that make up a wave packet travel at different speeds, the shape of the packet changes as it moves. That's why we've specified the time for which the wave packets in Figs. 40.6 and 40.7 are drawn; at later times, the packets become more spread out. By contrast, a pulse of light waves in vacuum retains the same shape at all times because all of its constituent sinusoidal waves travel together at the same speed. ■

The One-Dimensional Schrödinger Equation with Potential Energy

The one-dimensional Schrödinger equation that we presented in Eq. (40.15) is valid only for free particles, for which the potential energy function is zero: $U(x) = 0$. But for an electron within an atom, a proton within an atomic nucleus, and many other real situations, the potential energy plays an important role. To study the behavior of matter waves in these situations, we need a version of the Schrödinger equation that describes a particle moving in the presence of a nonzero potential energy function $U(x)$. This equation is

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + U(x)\Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t} \quad \text{(general one-dimensional Schrödinger equation)} \quad (40.20)$$

Note that if $U(x) = 0$, Eq. (40.20) reduces to the free-particle Schrödinger equation given in Eq. (40.15).

Here's the motivation behind Eq. (40.20). If $\Psi(x, t)$ is a sinusoidal wave function for a free particle, $\Psi(x, t) = Ae^{i(kx - \omega t)} = Ae^{ikx}e^{-i\omega t}$, the derivative terms in Eq. (40.20) become

$$\begin{aligned} -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} &= -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} (Ae^{ikx}e^{-i\omega t}) = -\frac{\hbar^2}{2m} (ik)^2 (Ae^{ikx}e^{-i\omega t}) \\ &= \frac{\hbar^2 k^2}{2m} \Psi(x, t) \\ i\hbar \frac{\partial \Psi(x, t)}{\partial t} &= i\hbar \frac{\partial}{\partial t} (Ae^{ikx}e^{-i\omega t}) = i\hbar (-i\omega) (Ae^{ikx}e^{-i\omega t}) = \hbar\omega \Psi(x, t) \end{aligned}$$

In these expressions $(\hbar^2 k^2/2m)\Psi(x, t)$ is just the kinetic energy $K = p^2/2m = \hbar^2 k^2/2m$ multiplied by the wave function, and $\hbar\omega \Psi(x, t)$ is the total energy $E = \hbar\omega$ multiplied by the wave function. So for a wave function of this kind, Eq. (40.20) says that kinetic energy times $\Psi(x, t)$ plus potential energy times $\Psi(x, t)$ equals total energy times $\Psi(x, t)$. That's equivalent to the statement in

classical physics that the sum of kinetic energy and potential energy equals total mechanical energy: $K + U = E$.

The observations we've just made certainly aren't a *proof* that Eq. (40.20) is correct. The real reason we know this equation *is* correct is that it works: Predictions made with this equation agree with experimental results. In the remaining sections of this chapter we'll apply Eq. (40.20) to several physical situations, each with a different form of the function $U(x)$.

Stationary States

We saw in our discussion of wave packets that any free-particle wave function can be built up as a superposition of sinusoidal wave functions of the form $\Psi(x, t) = Ae^{ikx}e^{-i\omega t}$. Each such sinusoidal wave function corresponds to a state of definite energy $E = \hbar\omega = \hbar^2k^2/2m$ and definite angular frequency $\omega = E/\hbar$, so we can rewrite these functions as $\Psi(x, t) = Ae^{ikx}e^{-iEt/\hbar}$. If the potential energy function $U(x)$ is nonzero, these sinusoidal wave functions do not satisfy the Schrödinger equation, Eq. (40.20), and so these functions cannot be the basic “building blocks” of more complicated wave functions. However, we can still write the wave function for a state of definite energy E in the form

$$\Psi(x, t) = \psi(x)e^{-iEt/\hbar} \quad \text{(time-dependent wave function for a state of definite energy)} \quad (40.21)$$

That is, the wave function $\Psi(x, t)$ for a state of definite energy is the product of a *time-independent* wave function $\psi(x)$ and a factor $e^{-iEt/\hbar}$. (For the free-particle sinusoidal wave function, $\psi(x) = Ae^{ikx}$.) States of definite energy are of tremendous importance in quantum mechanics. For example, for each energy level in a hydrogen atom (Section 39.3) there is a specific wave function. It is possible for an atom to be in a state that does not have a definite energy. The wave function for any such state can be written as a combination of definite-energy wave functions, in precisely the same way that a free-particle wave packet can be written as a superposition of sinusoidal wave functions of definite energy as in Eq. (40.19).

A state of definite energy is commonly called a **stationary state**. To see where this name comes from, let's multiply Eq. (40.21) by its complex conjugate to find the probability distribution function $|\Psi|^2$:

$$\begin{aligned} |\Psi(x, t)|^2 &= \Psi^*(x, t)\Psi(x, t) = [\psi^*(x)e^{+iEt/\hbar}][\psi(x)e^{-iEt/\hbar}] \\ &= \psi^*(x)\psi(x)e^{(+iEt/\hbar)+(-iEt/\hbar)} = |\psi(x)|^2e^0 \\ &= |\psi(x)|^2 \end{aligned} \quad (40.22)$$

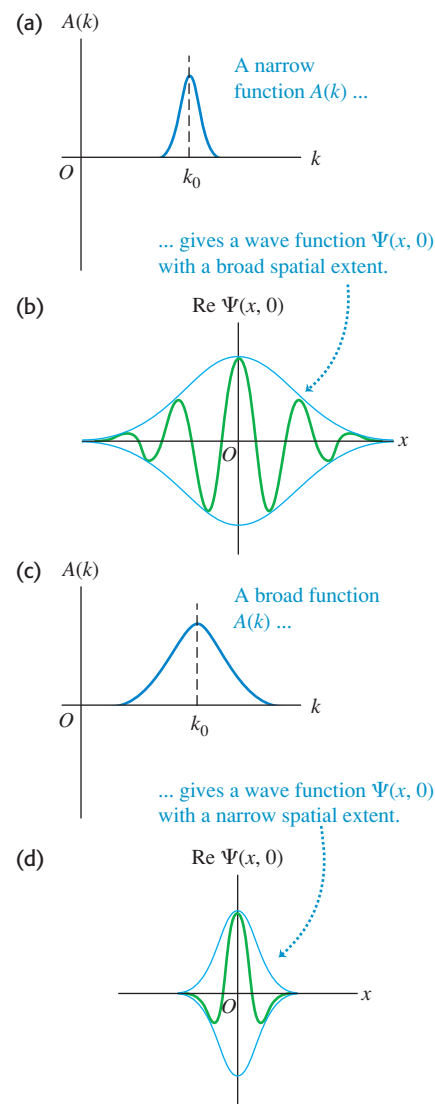
Since $|\psi(x)|^2$ does not depend on time, Eq. (40.22) shows that the same must be true for the probability distribution function $|\Psi(x, t)|^2$. This justifies the term “stationary state” for a state of definite energy.

CAUTION A stationary state does not mean a stationary particle The name *stationary state* may lead you to think that the particle is not in motion if it is described by such a wave function. That's not the case. It's the *probability distribution* (that is, the relative likelihood of finding the particle at various positions), not the particle itself, that's stationary. **I**

The Schrödinger equation, Eq. (40.20), becomes quite a bit simpler for stationary states. To see this, we substitute Eq. (40.21) into Eq. (40.20):

$$-\frac{\hbar^2}{2m} \frac{\partial^2 [\psi(x)e^{-iEt/\hbar}]}{\partial x^2} + U(x)\psi(x)e^{-iEt/\hbar} = i\hbar \frac{\partial [\psi(x)e^{-iEt/\hbar}]}{\partial t}$$

40.7 How varying the function $A(k)$ in the wave-packet expression, Eq. (40.19), changes the character of the wave function $\Psi(x, t)$ (shown here at a specific time $t = 0$).



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The derivative on the first term on the left-hand side is with respect to x , so the factor of $e^{-iEt/\hbar}$ comes outside of the derivative. Now we take the derivative with respect to t on the right-hand side of the equation:

$$\begin{aligned} -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} e^{-iEt/\hbar} + U(x)\psi(x)e^{-iEt/\hbar} &= i\hbar \left(\frac{-iE}{\hbar} \right) [\psi(x)e^{-iEt/\hbar}] \\ &= E\psi(x)e^{-iEt/\hbar} \end{aligned}$$

If we divide both sides of this equation by $e^{-iEt/\hbar}$, we get

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x) \quad \begin{array}{l} \text{(time-independent} \\ \text{Schrödinger equation)} \end{array} \quad (40.23)$$

This is called the **time-independent Schrödinger equation**. The time-dependent factor $e^{-iEt/\hbar}$ does not appear, and Eq. (40.23) is an equation that involves only the time-independent wave function $\psi(x)$. We'll devote much of this chapter to solving this equation to find the definite-energy, stationary-state wave functions $\psi(x)$ and the corresponding values of E —that is, the energies of the allowed levels—for different physical situations.

Example 40.2 A stationary state

Consider the wave function $\psi(x) = A_1e^{ikx} + A_2e^{-ikx}$, where k is positive. Is this a valid time-independent wave function for a free particle in a stationary state? What is the energy corresponding to this wave function?

SOLUTION

IDENTIFY and SET UP: A valid stationary-state wave function for a free particle must satisfy the time-independent Schrödinger equation, Eq. (40.23), with $U(x) = 0$. To test the given function $\psi(x)$, we simply substitute it into the left-hand side of the equation. If the result is a constant times $\psi(x)$, then the wave function is indeed a solution and the constant is equal to the particle energy E .

EXECUTE: Substituting $\psi(x) = A_1e^{ikx} + A_2e^{-ikx}$ and $U(x) = 0$ into Eq. (40.23), we obtain

$$\begin{aligned} \frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} &= \frac{\hbar^2}{2m} \frac{d^2(A_1e^{ikx} + A_2e^{-ikx})}{dx^2} \\ &= -\frac{\hbar^2}{2m} [(ik)^2 A_1e^{ikx} + (-ik)^2 A_2e^{-ikx}] \\ &= \frac{\hbar^2 k^2}{2m} (A_1e^{ikx} + A_2e^{-ikx}) = \frac{\hbar^2 k^2}{2m} \psi(x) \end{aligned}$$

The result is a constant times $\psi(x)$, so this $\psi(x)$ is indeed a valid stationary-state wave function for a free particle. Comparing with Eq. (40.23) shows that the constant on the right-hand side is the particle energy: $E = \hbar^2 k^2 / 2m$.

EVALUATE: Note that $\psi(x)$ is a *superposition* of two different wave functions: one function (A_1e^{ikx}) that represents a particle with magnitude of momentum $p = \hbar k$ moving in the positive x -direction, and one function (A_2e^{-ikx}) that represents a particle with the same magnitude of momentum moving in the negative x -direction. So while the combined wave function $\psi(x)$ represents a stationary state with a definite energy, this state does *not* have a definite momentum. We'll see in Section 40.2 that such a wave function can represent a *standing wave*, and we'll explore situations in which such standing matter waves can arise.

Test Your Understanding of Section 40.1 Does a wave packet given by Eq. (40.19) represent a stationary state? I

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ActivPhysics 20.2: Particle in a Box

40.2 Particle in a Box

An important problem in quantum mechanics is how to use the time-independent Schrödinger equation, Eq. (40.23), to determine the possible energy levels and the corresponding wave functions for various systems. The fundamental problem is then the following: For a given potential energy function $U(x)$, what are the possible stationary-state wave functions $\psi(x)$, and what are the corresponding energies E ?

In Section 40.1 we solved this problem for the case $U(x) = 0$, corresponding to a *free* particle. The allowed wave functions and corresponding energies are

$$\psi(x) = Ae^{ikx} \quad E = \frac{\hbar^2 k^2}{2m} \quad (\text{free particle}) \quad (40.24)$$

The wave number k is equal to $2\pi/\lambda$, where λ is the wavelength. We found that k can have any real value, so the energy E of a free particle can have any value from zero to infinity. Furthermore, the particle can be found with equal probability at any value of x from $-\infty$ to $+\infty$.

Now let's look at a simple model in which a particle is *bound* so that it cannot escape to infinity, but rather is confined to a restricted region of space. Our system consists of a particle confined between two rigid walls separated by a distance L (Fig. 40.8). The motion is purely one dimensional, with the particle moving along the x -axis only and the walls at $x = 0$ and $x = L$. The potential energy corresponding to the rigid walls is infinite, so the particle cannot escape; between the walls, the potential energy is zero (Fig. 40.9). This situation is often described as a “**particle in a box.**” This model might represent an electron that is free to move within a long, straight molecule or along a very thin wire.

Wave Functions for a Particle in a Box

To solve the Schrödinger equation for this system, we begin with some restrictions on the particle's stationary-state wave function $\psi(x)$. Because the particle is confined to the region $0 \leq x \leq L$, we expect the probability distribution function $|\Psi(x, t)|^2 = |\psi(x)|^2$ and the wave function $\psi(x)$ to be zero outside that region. This agrees with the Schrödinger equation: If the term $U(x)\psi(x)$ in Eq. (40.23) is to be finite, then $\psi(x)$ must be zero where $U(x)$ is infinite.

Furthermore, $\psi(x)$ must be a *continuous* function to be a mathematically well-behaved solution to the Schrödinger equation. This implies that $\psi(x)$ must be zero at the region's boundary, $x = 0$ and $x = L$. These two conditions serve as *boundary conditions* for the problem. They should look familiar, because they are the same conditions that we used to find the normal modes of a vibrating string in Section 15.8 (Fig. 40.10); you should review that discussion.

An additional condition is that to calculate the second derivative $d^2\psi(x)/dx^2$ in Eq. (40.23), the *first* derivative $d\psi(x)/dx$ must also be continuous except at points where the potential energy becomes infinite (as it does at the walls of the box). This is analogous to the requirement that a vibrating string, like those shown in Fig. 40.10, can't have any kinks in it (which would correspond to a discontinuity in the first derivative of the wave function) except at the ends of the string.

We now solve for the wave functions in the region $0 \leq x \leq L$ subject to the above conditions. In this region $U(x) = 0$, so the wave function in this region must satisfy

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x) \quad (\text{particle in a box}) \quad (40.25)$$

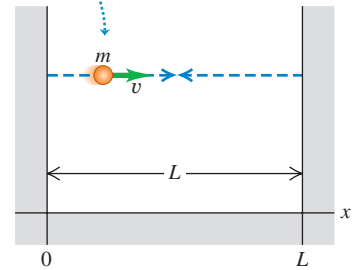
Equation (40.25) is the *same* Schrödinger equation as for a free particle, so it is tempting to conclude that the wave functions and energies are given by Eq. (40.24). It is true that $\psi(x) = Ae^{ikx}$ satisfies the Schrödinger equation with $U(x) = 0$, is continuous, and has a continuous first derivative $d\psi(x)/dx = ikAe^{ikx}$. However, this wave function does *not* satisfy the boundary conditions that $\psi(x)$ must be zero at $x = 0$ and $x = L$: At $x = 0$ the wave function in Eq. (40.24) is equal to $Ae^0 = A$, and at $x = L$ it is equal to Ae^{ikL} . (These would be equal to zero if $A = 0$, but then the wave function would be zero and there would be no particle at all!)

The way out of this dilemma is to recall Example 40.2 (Section 40.1), in which we found that a more general stationary-state solution to the time-independent Schrödinger equation with $U(x) = 0$ is

$$\psi(x) = A_1e^{ikx} + A_2e^{-ikx} \quad (40.26)$$

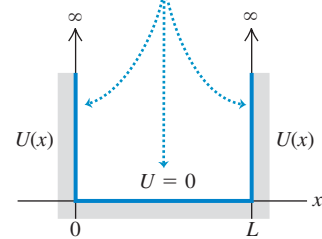
40.8 The Newtonian view of a particle in a box.

A particle with mass m moves along a straight line at constant speed, bouncing between two rigid walls a distance L apart.



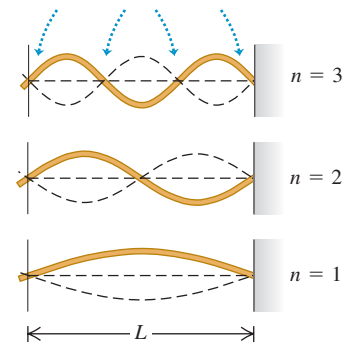
40.9 The potential-energy function for a particle in a box.

The potential energy U is zero in the interval $0 < x < L$ and is infinite everywhere outside this interval.



40.10 Normal modes of vibration for a string with length L , held at both ends.

Each end is a node, and there are $n - 1$ additional nodes between the ends.



The length is an integral number of half-wavelengths: $L = n\lambda_n/2$.

This wave function is a superposition of two waves: one traveling in the $+x$ -direction of amplitude A_1 , and one traveling in the $-x$ -direction with the same wave number but amplitude A_2 . This is analogous to a standing wave on a string (Fig. 40.10), which we can regard as the superposition of two sinusoidal waves propagating in opposite directions (see Section 15.7). The energy that corresponds to Eq. (40.26) is $E = \hbar^2 k^2 / 2m$, just as for a single wave.

To see whether the wave function given by Eq. (40.26) can satisfy the boundary conditions, let's first rewrite it in terms of sines and cosines using Euler's formula, Eq. (40.17):

$$\begin{aligned}\psi(x) &= A_1(\cos kx + i \sin kx) + A_2[\cos(-kx) + i \sin(-kx)] \\ &= A_1(\cos kx + i \sin kx) + A_2(\cos kx - i \sin kx) \\ &= (A_1 + A_2)\cos kx + i(A_1 - A_2)\sin kx\end{aligned}\quad (40.27)$$

At $x = 0$ this is equal to $\psi(0) = A_1 + A_2$, which must equal zero if we are to satisfy the boundary condition at that point. Hence $A_2 = -A_1$, and Eq. (40.27) becomes

$$\psi(x) = 2iA_1 \sin kx = C \sin kx \quad (40.28)$$

We have simplified the expression by introducing the constant $C = 2iA_1$. (We'll come back to this constant later.) We can also satisfy the second boundary condition that $\psi = 0$ at $x = L$ by choosing values of k such that $kL = n\pi$ ($n = 1, 2, 3, \dots$). Hence Eq. (40.28) does indeed give the stationary-state wave functions for a particle in a box in the region $0 \leq x \leq L$. (Outside this region, $\psi(x) = 0$.) The possible values of k and the wavelength $\lambda = 2\pi/k$ are

$$k = \frac{n\pi}{L} \quad \text{and} \quad \lambda = \frac{2\pi}{k} = \frac{2L}{n} \quad (n = 1, 2, 3, \dots) \quad (40.29)$$

Just as for the string in Fig. 40.10, the length L of the region is an integral number of half-wavelengths.

Energy Levels for a Particle in a Box

The possible energy levels for a particle in a box are given by $E = \hbar^2 k^2 / 2m = p^2 / 2m$, where $p = \hbar k = (h/2\pi)(2\pi/\lambda) = h/\lambda$ is the magnitude of momentum of a free particle with wave number k and wavelength λ . This makes sense, since inside the region $0 \leq x \leq L$ the potential energy is zero and the energy is all kinetic. For each value of n , there are corresponding values of p , λ , and E ; let's call them p_n , λ_n , and E_n . Putting the pieces together, we get

$$p_n = \frac{h}{\lambda_n} = \frac{nh}{2L} \quad (40.30)$$

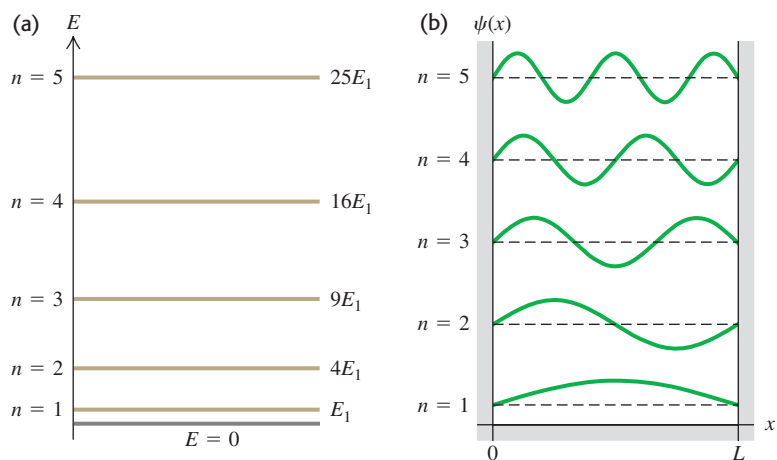
and so the energy levels for a particle in a box are

$$E_n = \frac{p_n^2}{2m} = \frac{n^2 h^2}{8mL^2} = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad (n = 1, 2, 3, \dots) \quad \text{(energy levels, particle in a box)} \quad (40.31)$$

Each energy level has its own value of the quantum number n and a corresponding wave function, which we denote by ψ_n . When we replace k in Eq. (40.28) by $n\pi/L$ from Eq. (40.29), we find

$$\psi_n(x) = C \sin \frac{n\pi x}{L} \quad (n = 1, 2, 3, \dots) \quad (40.32)$$

The energy-level diagram in Fig. 40.11a shows the five lowest levels for a particle in a box. The energy levels are proportional to n^2 , so successively higher levels are spaced farther and farther apart. There are an infinite number of levels because the walls are perfectly rigid; even a particle of infinitely great kinetic



40.11 (a) Energy-level diagram for a particle in a box. Each energy is n^2E_1 , where E_1 is the ground-level energy. (b) Wave functions for a particle in a box, with $n = 1, 2, 3, 4,$ and 5 . **CAUTION:** The five graphs have been displaced vertically for clarity, as in Fig. 40.10. Each of the horizontal dashed lines represents $\psi = 0$ for the respective wave function.

energy is confined within the box. Figure 40.11b shows graphs of the wave functions $\psi_n(x)$ for $n = 1, 2, 3, 4,$ and 5 . Note that these functions look identical to those for a standing wave on a string (see Fig. 40.10).

CAUTION **A particle in a box cannot have zero energy** Note that the energy of a particle in a box *cannot* be zero. Equation (40.31) shows that $E = 0$ would require $n = 0$, but substituting $n = 0$ into Eq. (40.32) gives a zero wave function. Since a particle is described by a *nonzero* wave function, this means that there cannot be a particle with $E = 0$. This is a consequence of the Heisenberg uncertainty principle: A particle in a zero-energy state would have a definite value of momentum (precisely zero), so its position uncertainty would be infinite and the particle could be found anywhere along the x -axis. But this is impossible, since a particle in a box can be found only between $x = 0$ and $x = L$. Hence $E = 0$ is not allowed. By contrast, the allowed stationary-state wave functions with $n = 1, 2, 3, \dots$ do not represent states of definite momentum (each is an equal mixture of a state of x -momentum $+p_n = nh/2L$ and a state of x -momentum $-p_n = -nh/2L$). Hence each stationary state has a nonzero momentum uncertainty, consistent with having a finite position uncertainty. **|**

Example 40.3 Electron in an atom-size box

Find the first two energy levels for an electron confined to a one-dimensional box 5.0×10^{-10} m across (about the diameter of an atom).

SOLUTION

IDENTIFY and SET UP: This problem uses what we have learned in this section about a particle in a box. The first two energy levels correspond to $n = 1$ and $n = 2$ in Eq. (40.31).

EXECUTE: From Eq. (40.31),

$$E_1 = \frac{h^2}{8mL^2} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2}{8(9.109 \times 10^{-31} \text{ kg})(5.0 \times 10^{-10} \text{ m})^2}$$

$$= 2.4 \times 10^{-19} \text{ J} = 1.5 \text{ eV}$$

$$E_2 = \frac{2^2 h^2}{8mL^2} = 4E_1 = 9.6 \times 10^{-19} \text{ J} = 6.0 \text{ eV}$$

EVALUATE: The difference between the first two energy levels is $E_2 - E_1 = 4.5$ eV. An electron confined to a box is different from an electron bound in an atom, but it is reassuring that this result is of the same order of magnitude as the difference between actual atomic energy levels.

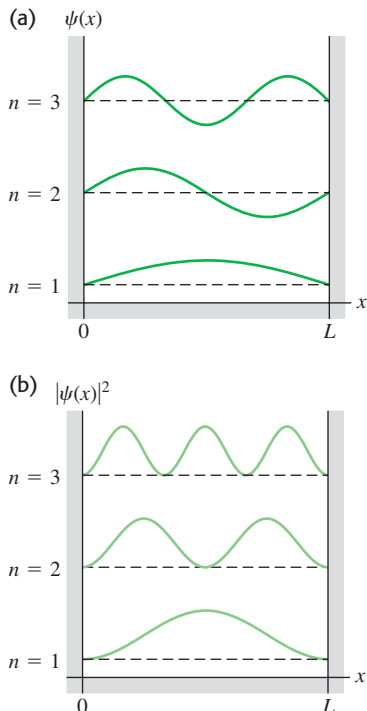
You can also show that for a proton or neutron ($m = 1.67 \times 10^{-27}$ kg) confined to a box 1.1×10^{-14} m across (the width of a medium-sized atomic nucleus), the energies of the first two levels are about a million times larger: $E_1 = 1.7 \times 10^6$ eV = 1.7 MeV, $E_2 = 4E_1 = 6.8$ MeV, $E_2 - E_1 = 5.1$ MeV. This suggests why nuclear reactions (which involve transitions between energy levels in nuclei) release so much more energy than chemical reactions (which involve transitions between energy levels of electrons in atoms).

Finally, you can show (see Exercise 40.11) that the energy levels of a billiard ball ($m = 0.2$ kg) confined to a box 1.3 m across—the width of a billiard table—are separated by about 5×10^{-67} J. Quantum effects won't disturb a game of billiards.

Probability and Normalization

Let's look a bit more closely at the wave functions for a particle in a box, keeping in mind the *probability* interpretation of the wave function ψ that we discussed in Section 40.1. In our one-dimensional situation the quantity $|\psi(x)|^2 dx$ is proportional

40.12 Graphs of (a) $\psi(x)$ and (b) $|\psi(x)|^2$ for the first three wave functions ($n = 1, 2, 3$) for a particle in a box. The horizontal dashed lines represent $\psi(x) = 0$ and $|\psi(x)|^2 = 0$ for each of the three levels. The value of $|\psi(x)|^2 dx$ at each point is the probability of finding the particle in a small interval dx about the point. As in Fig. 40.11b, the three graphs in each part have been displaced vertically for clarity.



to the probability that the particle will be found within a small interval dx about x . For a particle in a box,

$$|\psi(x)|^2 dx = C^2 \sin^2 \frac{n\pi x}{L} dx$$

Figure 40.12 shows graphs of both $\psi(x)$ and $|\psi(x)|^2$ for $n = 1, 2$, and 3 . Note that not all positions are equally likely. By contrast, in classical mechanics the particle is equally likely to be found at any position between $x = 0$ and $x = L$. We see from Fig. 40.12b that $|\psi(x)|^2 = 0$ at some points, so there is zero probability of finding the particle at exactly these points. Don't let that bother you; the uncertainty principle has already shown us that we can't measure position exactly. The particle is localized only to be somewhere between $x = 0$ and $x = L$.

The particle must be *somewhere* on the x -axis—that is, somewhere between $x = -\infty$ and $x = +\infty$. So the *sum* of the probabilities for all the dx 's everywhere (the *total* probability of finding the particle) must equal 1. That's the normalization condition that we discussed in Section 40.1:

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \quad (\text{normalization condition}) \quad (40.33)$$

A wave function is said to be *normalized* if it has a constant such as C in Eq. (40.32) that is calculated to make the total probability equal 1 in Eq. (40.33). For a normalized wave function, $|\psi(x)|^2 dx$ is not merely proportional to, but *equals*, the probability of finding the particle between the coordinates x and $x + dx$. That's why we call $|\psi(x)|^2$ the probability distribution function. (In Section 40.1 we called $|\Psi(x, t)|^2$ the probability distribution function. For the case of a stationary-state wave function, however, $|\Psi(x, t)|^2$ is equal to $|\psi(x)|^2$.)

Let's normalize the particle-in-a-box wave functions $\psi_n(x)$ given by Eq. (40.32). Since $\psi_n(x)$ is zero except between $x = 0$ and $x = L$, Eq. (40.33) becomes

$$\int_0^L C^2 \sin^2 \frac{n\pi x}{L} dx = 1 \quad (40.34)$$

You can evaluate this integral using the trigonometric identity $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$; the result is $C^2 L/2$. Thus our probability interpretation of the wave function demands that $C^2 L/2 = 1$, or $C = (2/L)^{1/2}$; the constant C is *not* arbitrary. (This is in contrast to the classical vibrating string problem, in which C represents an amplitude that depends on initial conditions.) Thus the normalized stationary-state wave functions for a particle in a box are

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad (n = 1, 2, 3, \dots) \quad (\text{particle in a box}) \quad (40.35)$$

Example 40.4 A nonsinusoidal wave function?

(a) Show that $\psi(x) = Ax + B$, where A and B are constants, is a solution of the Schrödinger equation for an $E = 0$ energy level of a particle in a box. (b) What constraints do the boundary conditions at $x = 0$ and $x = L$ place on the constants A and B ?

SOLUTION

IDENTIFY and SET UP: To be physically reasonable, a wave function must satisfy both the Schrödinger equation and the appropriate boundary conditions. In part (a) we'll substitute $\psi(x)$ into the

Schrödinger equation for a particle in a box, Eq. (40.25), to determine whether it is a solution. In part (b) we'll see what restrictions on $\psi(x)$ arise from applying the boundary conditions that $\psi(x) = 0$ at $x = 0$ and $x = L$.

EXECUTE: (a) From Eq. (40.25), the Schrödinger equation for an $E = 0$ energy level of a particle in a box is

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x) = 0$$

in the region $0 \leq x \leq L$. Differentiating $\psi(x) = Ax + B$ twice with respect to x gives $d^2\psi(x)/dx^2 = 0$, so the left side of the equation is zero, and so $\psi(x) = Ax + B$ is a solution of this Schrödinger equation for $E = 0$. (Note that both $\psi(x)$ and its derivative $d\psi(x)/dx = A$ are continuous functions, as they must be.)

(b) Applying the boundary condition at $x = 0$ gives $\psi(0) = B = 0$, and so $\psi(x) = Ax$. Applying the boundary condition at $x = L$ gives $\psi(L) = AL = 0$, so $A = 0$. Hence $\psi(x) = 0$ both

inside the box ($0 \leq x \leq L$) and outside: There is zero probability of finding the particle anywhere with this wave function, and so $\psi(x) = Ax + B$ is not a physically valid wave function.

EVALUATE: The moral is that there are many functions that satisfy the Schrödinger equation for a given physical situation, but most of these—including the function considered here—have to be rejected because they don't satisfy the appropriate boundary conditions.

Time Dependence

Finally, we note that the wave functions $\psi_n(x)$ in Eq. (40.35) depend only on the spatial coordinate x . Equation (40.21) shows that if $\psi(x)$ is the wave function for a state of definite energy E , the full time-dependent wave function is $\Psi(x, t) = \psi(x)e^{-iEt/\hbar}$. Hence the time-dependent stationary-state wave functions for a particle in a box are

$$\Psi_n(x, t) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) e^{-iE_n t/\hbar} \quad (n = 1, 2, 3, \dots) \quad (40.36)$$

In this expression the energies E_n are given by Eq. (40.31). The higher the quantum number n , the greater the angular frequency $\omega_n = E_n/\hbar$ at which the wave function oscillates. Note that since $|e^{-iE_n t/\hbar}|^2 = e^{+iE_n t/\hbar} e^{-iE_n t/\hbar} = e^0 = 1$, the probability distribution function $|\Psi_n(x, t)|^2 = (2/L) \sin^2(n\pi x/L)$ is independent of time and does not oscillate. (Remember, this is why we say that these states of definite energy are stationary.)

Test Your Understanding of Section 40.2 If a particle in a box is in the n th energy level, what is the average value of its x -component of momentum p_x ? (i) $nh/2L$; (ii) $(\sqrt{2}/2)nh/L$; (iii) $(1/\sqrt{2})nh/L$; (iv) $[1/(2\sqrt{2})]nh/L$; (v) zero.



40.3 Potential Wells

A **potential well** is a potential-energy function $U(x)$ that has a minimum. We introduced this term in Section 7.5, and we also used it in our discussion of periodic motion in Chapter 14. In Newtonian mechanics a particle trapped in a potential well can vibrate back and forth with periodic motion. Our first application of the Schrödinger equation, the particle in a box, involved a rudimentary potential well with a function $U(x)$ that is zero within a certain interval and infinite everywhere else. As we mentioned in Section 40.2, this function corresponds to a few situations found in nature, but the correspondence is only approximate.

A better approximation to several actual physical situations is a **finite well**, which is a potential well with straight sides but finite height. Figure 40.13 shows a potential-energy function that is zero in the interval $0 \leq x \leq L$ and has the value U_0 outside this interval. This function is often called a **square-well potential**. It could serve as a simple model of an electron within a metallic sheet with thickness L , moving perpendicular to the surfaces of the sheet. The electron can move freely inside the metal but has to climb a potential-energy barrier with height U_0 to escape from either surface of the metal. The energy U_0 is related to the *work function* that we discussed in Section 38.1 in connection with the photoelectric effect. In three dimensions, a spherical version of a finite well gives an approximate description of the motions of protons and neutrons within a nucleus.

Bound States of a Square-Well Potential

In Newtonian mechanics, the particle is trapped (localized) in a well if the total mechanical energy E is less than U_0 . In quantum mechanics, such a trapped state is often called a **bound state**. All states are bound for an infinitely deep well like

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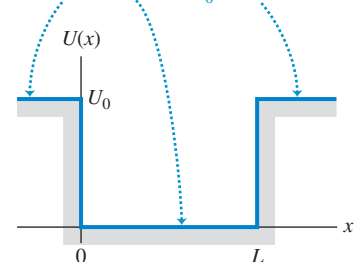
PhET: Quantum Bound States

ActivPhysics 20.1: Potential Energy Diagrams

ActivPhysics 20.3: Potential Wells

40.13 A square-well potential.

The potential energy U is zero within the potential well (in the interval $0 \leq x \leq L$) and has the constant value U_0 outside this interval.



the one we described in Section 40.2. For a finite well like that shown in Fig. 40.13, if E is greater than U_0 , the particle is *not* bound.

Let's see how to solve the Schrödinger equation for the bound states of a square-well potential. Our goal is to find the energies and wave functions for which $E < U_0$. The easiest approach is to consider separately the regions where $U = 0$ and where $U = U_0$. Where $U = 0$, the time-independent Schrödinger equation is

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x) \quad \text{or} \quad \frac{d^2\psi(x)}{dx^2} = -\frac{2mE}{\hbar^2} \psi(x) \quad (40.37)$$

This is the same as Eq. (40.25) from Section 40.2, which describes a particle in a box. As in Section 40.2, we can express the solutions of this equation as combinations of $\cos kx$ and $\sin kx$, where $E = \hbar^2 k^2 / 2m$. We can rewrite the relationship between E and k as $k = \sqrt{2mE}/\hbar$. Hence inside the square well ($0 \leq x \leq L$) we have

$$\psi(x) = A \cos\left(\frac{\sqrt{2mE}}{\hbar} x\right) + B \sin\left(\frac{\sqrt{2mE}}{\hbar} x\right) \quad (\text{inside the well}) \quad (40.38)$$

where A and B are constants. So far, this looks a lot like the particle-in-a-box analysis in Section 40.2. The difference is that for the square-well potential, the potential energy outside the well is not infinite, so the wave function $\psi(x)$ outside the well is *not* zero.

For the regions outside the well ($x < 0$ and $x > L$) the potential-energy function in the time-independent Schrödinger equation is $U = U_0$:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U_0\psi(x) = E\psi(x) \quad \text{or} \quad \frac{d^2\psi(x)}{dx^2} = \frac{2m(U_0 - E)}{\hbar^2} \psi(x) \quad (40.39)$$

The quantity $U_0 - E$ is positive, so the solutions of this equation are exponential. Using κ (the Greek letter kappa) to represent the quantity $[2m(U_0 - E)]^{1/2}/\hbar$ and taking κ as positive, we can write the solutions as

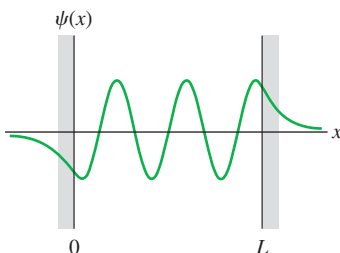
$$\psi(x) = Ce^{\kappa x} + De^{-\kappa x} \quad (\text{outside the well}) \quad (40.40)$$

where C and D are constants with different values in the two regions $x < 0$ and $x > L$. Note that ψ can't be allowed to approach infinity as $x \rightarrow +\infty$ or $x \rightarrow -\infty$. [If it did, we wouldn't be able to satisfy the normalization condition, Eq. (40.33).] This means that in Eq. (40.40), we must have $D = 0$ for $x < 0$ and $C = 0$ for $x > L$.

Our calculations so far show that the bound-state wave functions for a finite well are sinusoidal inside the well [Eq. (40.38)] and exponential outside it [Eq. (40.40)]. We have to *match* the wave functions inside and outside the well so that they satisfy the boundary conditions that we mentioned in Section 40.2: $\psi(x)$ and $d\psi(x)/dx$ must be continuous at the boundary points $x = 0$ and $x = L$. If the wave function $\psi(x)$ or the slope $d\psi(x)/dx$ were to change discontinuously at a point, the second derivative $d^2\psi(x)/dx^2$ would be *infinite* at that point. That would violate the time-independent Schrödinger equation, Eq. (40.23), which says that at every point $d^2\psi(x)/dx^2$ is proportional to $U - E$. For a finite well $U - E$ is finite everywhere, so $d^2\psi(x)/dx^2$ must also be finite everywhere.

Matching the sinusoidal and exponential functions at the boundary points so that they join smoothly is possible only for certain specific values of the total energy E , so this requirement determines the possible energy levels of the finite square well. There is no simple formula for the energy levels as there was for the infinitely deep well. Finding the levels is a fairly complex mathematical problem that requires solving a transcendental equation by numerical approximation; we won't go into the details. Figure 40.14 shows the general shape of a possible wave function. The most striking features of this wave function are the

40.14 A possible wave function for a particle in a finite potential well. The function is sinusoidal inside the well ($0 \leq x \leq L$) and exponential outside it. It approaches zero asymptotically at large $|x|$. The functions must join smoothly at $x = 0$ and $x = L$; the wave function and its derivative must be continuous.



“exponential tails” that extend outside the well into regions that are forbidden by Newtonian mechanics (because in those regions the particle would have negative kinetic energy). We see that there is some probability for finding the particle *outside* the potential well, which would be impossible in classical mechanics. In Section 40.4 we’ll discuss an amazing result of this effect.

Example 40.5 Outside a finite well

(a) Show that Eq. (40.40), $\psi(x) = Ce^{\kappa x} + De^{-\kappa x}$, is indeed a solution of the time-independent Schrödinger equation outside a finite well of height U_0 . (b) What happens to $\psi(x)$ in the limit $U_0 \rightarrow \infty$?

SOLUTION

IDENTIFY and SET UP: In part (a), we try the given function $\psi(x)$ in the time-independent Schrödinger equation for $x < 0$ and for $x > L$, Eq. (40.39). In part (b), we note that in the limit $U_0 \rightarrow \infty$ the finite well becomes an *infinite* well, like those we considered in Section 40.2 for a particle in a box. So in this limit the wave functions outside a finite well must reduce to the wave functions outside the box.

EXECUTE: (a) We must show that $\psi(x) = Ce^{\kappa x} + De^{-\kappa x}$ satisfies $d^2\psi(x)/dx^2 = [2m(U_0 - E)/\hbar^2]\psi(x)$. We recall that $(d/du)e^{au} = ae^{au}$ and $(d^2/du^2)e^{au} = a^2e^{au}$; the left-hand side of the Schrödinger equation is then

$$\begin{aligned} \frac{d^2\psi(x)}{dx^2} &= \frac{d^2}{dx^2}(Ce^{\kappa x}) + \frac{d^2}{dx^2}(De^{-\kappa x}) \\ &= C\kappa^2 e^{\kappa x} + D(-\kappa)^2 e^{-\kappa x} \\ &= \kappa^2(Ce^{\kappa x} + De^{-\kappa x}) \\ &= \kappa^2\psi(x) \end{aligned}$$

Since from Eq. (40.40) $\kappa^2 = 2m(U_0 - E)/\hbar^2$, this is equal to the right-hand side of the equation. The equation is satisfied, and $\psi(x)$ is a solution.

(b) As U_0 approaches infinity, κ also approaches infinity. In the region $x < 0$, $\psi(x) = Ce^{\kappa x}$; as $\kappa \rightarrow \infty$, $\kappa x \rightarrow -\infty$ (since x is negative) and $e^{\kappa x} \rightarrow 0$, so the wave function approaches zero for all $x < 0$. Likewise, we can show that the wave function also approaches zero for all $x > L$. This is just what we found in Section 40.2; the wave function for a particle in a box must be zero outside the box.

EVALUATE: Our result in part (b) shows that the infinite square well is a *limiting case* of the finite well. We’ve seen many cases in Newtonian mechanics where it’s important to consider limiting cases (such as Examples 5.11 and 5.13 in Section 5.2). Limiting cases are no less important in quantum mechanics.

Comparing Finite and Infinite Square Wells

Let’s continue the comparison of the finite-depth potential well with the infinitely deep well, which we began in Example 40.5. First, because the wave functions for the finite well don’t go to zero at $x = 0$ and $x = L$, the wavelength of the sinusoidal part of each wave function is *longer* than it would be with an infinite well. This increase in λ corresponds to a reduced magnitude of momentum $p = h/\lambda$ and therefore a reduced energy. Thus each energy level, including the ground level, is *lower* for a finite well than for an infinitely deep well with the same width.

Second, a well with finite depth U_0 has only a *finite* number of bound states and corresponding energy levels, compared to the *infinite* number for an infinitely deep well. How many levels there are depends on the magnitude of U_0 in comparison with the ground-level energy for the infinitely deep well (IDW), which we call E_{1-IDW} . From Eq. (40.31),

$$E_{1-IDW} = \frac{\pi^2 \hbar^2}{2mL^2} \quad (\text{ground-level energy, infinitely deep well}) \quad (40.41)$$

When the well is very deep so U_0 is much larger than E_{1-IDW} , there are many bound states and the energies of the lowest few are nearly the same as the energies for the infinitely deep well. When U_0 is only a few times as large as E_{1-IDW} there are only a few bound states. (There is always at least *one* bound state, no matter how shallow the well.) As with the infinitely deep well, there is no state with $E = 0$; such a state would violate the uncertainty principle.

40.15 (a) Wave functions for the three bound states for a particle in a finite potential well with depth U_0 , for the case $U_0 = 6E_{1-IDW}$. (Here E_{1-IDW} is the ground-level energy for an infinite well of the same width.) The horizontal brown line for each wave function corresponds to $\psi = 0$; the vertical placement of these lines indicates the energy of each bound state (compare Fig. 40.11). (b) Energy-level diagram for this system. The energies are expressed both as multiples of E_{1-IDW} and as fractions of U_0 . All energies greater than U_0 are possible; states with $E > U_0$ form a continuum.

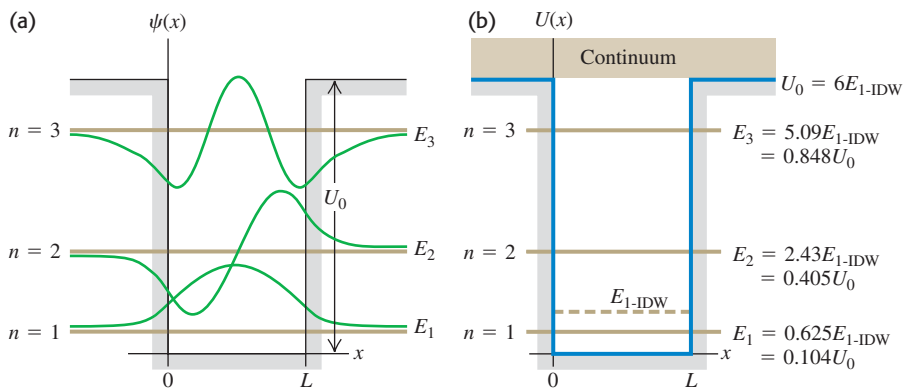
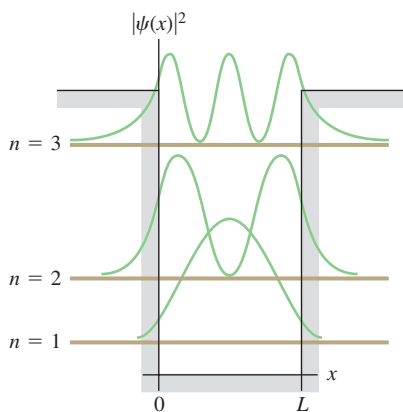


Figure 40.15 shows the case $U_0 = 6E_{1-IDW}$; for this particular case there are three bound states. In the figure, we express the energy levels both as fractions of the well depth U_0 and as multiples of E_{1-IDW} . Note that if the well were infinitely deep, the lowest three levels, as given by Eq. (40.31), would be E_{1-IDW} , $4E_{1-IDW}$, and $9E_{1-IDW}$. Figure 40.15 also shows the wave functions for the three bound states.

40.16 Probability distribution functions $|\psi(x)|^2$ for the square-well wave functions shown in Fig. 40.15. The horizontal brown line for each wave function corresponds to $|\psi|^2 = 0$.



It turns out that when U_0 is less than E_{1-IDW} , there is only one bound state. In the limit when U_0 is *much smaller* than E_{1-IDW} (a very shallow well), the energy of this single state is approximately $E = 0.68U_0$.

Figure 40.16 shows graphs of the probability distributions—that is, the values of $|\psi|^2$ —for the wave functions shown in Fig. 40.15a. As with the infinite well, not all positions are equally likely. Unlike the infinite well, there is some probability of finding the particle outside the well in the classically forbidden regions.

There are also states for which E is *greater* than U_0 . In these *free-particle states* the particle is not bound but is free to move through all values of x . Any energy E greater than U_0 is possible, so the free-particle states form a *continuum* rather than a discrete set of states with definite energy levels. The free-particle wave functions are sinusoidal both inside and outside the well. The wavelength is shorter inside the well than outside, corresponding to greater kinetic energy inside the well than outside it.

Figure 40.17 shows a graphic demonstration of particles in a *two-dimensional* finite potential well. Example 40.6 describes another application of the square-well potential.

Example 40.6 An electron in a finite well

An electron is trapped in a square well 0.50 nm across (roughly five times a typical atomic diameter). (a) Find the ground-level energy E_{1-IDW} if the well is infinitely deep. (b) Find the energy levels if the actual well depth U_0 is six times the ground-level energy found in part (a). (c) Find the wavelength of the photon emitted when the electron makes a transition from the $n = 2$ level to the $n = 1$ level. In what region of the electromagnetic spectrum does the photon wavelength lie? (d) If the electron is in the $n = 1$ (ground) level and absorbs a photon, what is the minimum photon

energy that will free the electron from the well? In what region of the spectrum does the wavelength of this photon lie?

SOLUTION

IDENTIFY and SET UP: Equation (40.41) gives the ground-level energy E_{1-IDW} for an infinitely deep well, and Fig. 40.15b shows the energies for a square well with $U_0 = 6E_{1-IDW}$. The energy of the photon emitted or absorbed in a transition is equal to the difference

in energy between two levels involved in the transition; the photon wavelength is given by $E = hc/\lambda$ (see Chapter 38).

EXECUTE: (a) From Eq. (40.41),

$$E_{1-\text{IDW}} = \frac{\pi^2 \hbar^2}{2mL^2} = \frac{\pi^2 (1.055 \times 10^{-34} \text{ J}\cdot\text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(0.50 \times 10^{-9} \text{ m})^2} \\ = 2.4 \times 10^{-19} \text{ J} = 1.5 \text{ eV}$$

(b) We have $U_0 = 6E_{1-\text{IDW}} = 6(1.5 \text{ eV}) = 9.0 \text{ eV}$. We can read off the energy levels from Fig. 40.15b:

$$E_1 = 0.625E_{1-\text{IDW}} = 0.625(1.5 \text{ eV}) = 0.94 \text{ eV}$$

$$E_2 = 2.43E_{1-\text{IDW}} = 2.43(1.5 \text{ eV}) = 3.6 \text{ eV}$$

$$E_3 = 5.09E_{1-\text{IDW}} = 5.09(1.5 \text{ eV}) = 7.6 \text{ eV}$$

(c) The photon energy and wavelength for the $n = 2$ to $n = 1$ transition are

$$E_2 - E_1 = 3.6 \text{ eV} - 0.94 \text{ eV} = 2.7 \text{ eV} \\ \lambda = \frac{hc}{E} = \frac{(4.136 \times 10^{-15} \text{ eV}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{2.7 \text{ eV}} \\ = 460 \text{ nm}$$

in the blue region of the visible spectrum.

(d) We see from Fig. 40.15b that the minimum energy needed to free the electron from the well from the $n = 1$ level is $U_0 - E_1 = 9.0 \text{ eV} - 0.94 \text{ eV} = 8.1 \text{ eV}$, which is three times the 2.7-eV photon energy found in part (c). Hence the corresponding photon wavelength is one-third of 460 nm, or (to two significant figures) 150 nm, which is in the ultraviolet region of the spectrum.

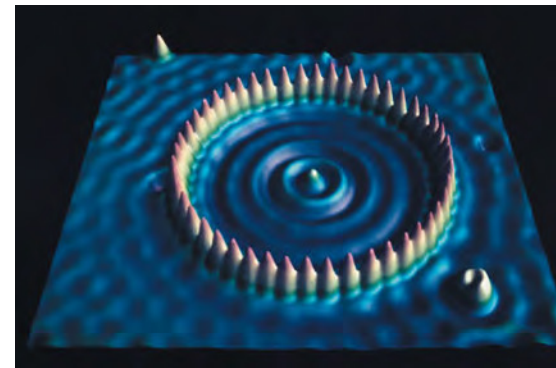
EVALUATE: As a check, you can also calculate the bound-state energies by using the formulas $E_1 = 0.104U_0$, $E_2 = 0.405U_0$, and $E_3 = 0.848U_0$ given in Fig. 40.15b. As an additional check, note that the first three energy levels of an infinitely deep well of the same width are $E_{1-\text{IDW}} = 1.5 \text{ eV}$, $E_{2-\text{IDW}} = 4E_{1-\text{IDW}} = 6.0 \text{ eV}$, and $E_{3-\text{IDW}} = 9E_{1-\text{IDW}} = 13.5 \text{ eV}$. The energies we found in part (b) are less than these values: As we mentioned earlier, the finite depth of the well lowers the energy levels compared to the levels for an infinitely deep well.

One application of these ideas is to *quantum dots*, which are nanometer-sized particles of a semiconductor such as cadmium selenide (CdSe). An electron within a quantum dot behaves much like a particle in a finite potential well of width L equal to the size of the dot. When quantum dots are illuminated with ultraviolet light, the electrons absorb the ultraviolet photons and are excited into high energy levels, such as the $n = 3$ level described in this example. If the electron returns to the ground level ($n = 1$) in two or more steps (for example, from $n = 3$ to $n = 2$ and from $n = 2$ to $n = 1$), one of the steps will involve emitting a visible-light photon, as we have calculated here. (We described this process of *fluorescence* in Section 39.3.) Increasing the value of L decreases the energies of the levels and hence the spacing between them, and thus decreases the energy and increases the wavelength of the emitted photons. The photograph that opens this chapter shows quantum dots of different sizes in solution: Each emits a characteristic wavelength that depends on the dot size. Quantum dots can be injected into living tissue and their fluorescent glow used as a tracer for biological research and for medicine. They may also be the key to a new generation of lasers and ultrafast computers.

Test Your Understanding of Section 40.3 Suppose that the width of the finite potential well shown in Fig. 40.15 is reduced by one-half. How must the value of U_0 change so that there are still just three bound energy levels whose energies are the fractions of U_0 shown in Fig. 40.15b? U_0 must: (i) increase by a factor of four; (ii) increase by a factor of two; (iii) remain the same; (iv) decrease by a factor of one-half; (v) decrease by a factor of one-fourth.



40.17 To make this image, 48 iron atoms (shown as yellow peaks) were placed in a circle on a copper surface. The “elevation” at each point inside the circle indicates the electron density within the circle. The standing-wave pattern is very similar to the probability distribution function for a particle in a one-dimensional finite potential well. (This image was made with a scanning tunneling microscope, discussed in Section 40.4.)



40.4 Potential Barriers and Tunneling

A **potential barrier** is the opposite of a potential well; it is a potential-energy function with a *maximum*. Figure 40.18 shows an example. In classical Newtonian mechanics, if a particle (such as a roller coaster) is located to the left of the barrier (which might be a hill), and if the total mechanical energy of the system is E_1 , the particle cannot move farther to the right than $x = a$. If it did, the potential energy U would be greater than the total energy E and the kinetic energy $K = E - U$ would be negative. This is impossible in classical mechanics since $K = \frac{1}{2}mv^2$ can never be negative.

A quantum-mechanical particle behaves differently: If it encounters a barrier like the one in Fig. 40.18 and has energy less than E_2 , it *may* appear on the other side. This phenomenon is called *tunneling*. In quantum-mechanical tunneling, unlike macroscopic, mechanical tunneling, the particle does not actually push through the barrier and loses no energy in the process.

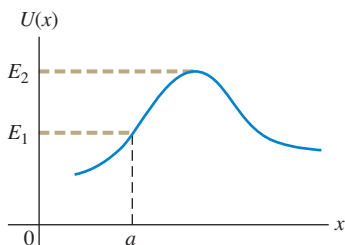
Tunneling Through a Rectangular Barrier

To understand how tunneling can occur, let's look at the potential-energy function $U(x)$ shown in Fig. 40.19. It's like Fig. 40.13 turned upside-down; the potential energy is zero everywhere except in the range $0 \leq x \leq L$, where it has the value U_0 . This might represent a simple model for the potential energy of an

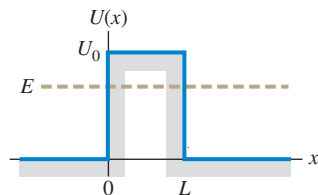
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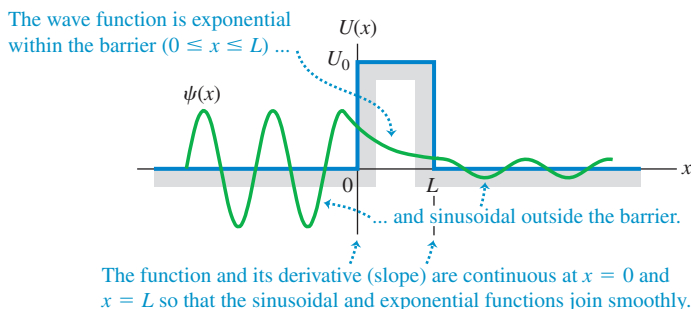
40.18 A potential-energy barrier. According to Newtonian mechanics, if the total energy of the system is E_1 , a particle to the left of the barrier can go no farther than $x = a$. If the total energy is greater than E_2 , the particle can pass over the barrier.



40.19 A rectangular potential-energy barrier with width L and height U_0 . According to Newtonian mechanics, if the total energy E is less than U_0 , a particle cannot pass over this barrier but is confined to the side where it starts.



40.20 A possible wave function for a particle tunneling through the potential-energy barrier shown in Fig. 40.19.



electron in the presence of two slabs of metal separated by an air gap of thickness L . The potential energy is lower within either slab than in the gap between them.

Let's consider solutions of the Schrödinger equation for this potential-energy function for the case in which E is less than U_0 . We can use our results from Section 40.3. In the regions $x < 0$ and $x > L$, where $U = 0$, the solution is sinusoidal and is given by Eq. (40.38). Within the barrier ($0 \leq x \leq L$), $U = U_0$ and the solution is exponential as in Eq. (40.40). Just as with the finite potential well, the functions have to join smoothly at the boundary points $x = 0$ and $x = L$, which means that both $\psi(x)$ and $d\psi(x)/dx$ have to be continuous at these points.

These requirements lead to a wave function like the one shown in Fig. 40.20. The function is *not* zero inside the barrier (the region forbidden by Newtonian mechanics). Even more remarkable, a particle that is initially to the *left* of the barrier has some probability of being found to the *right* of the barrier. How great this probability is depends on the width L of the barrier and the particle's energy E in comparison with the barrier height U_0 . The **tunneling probability** T that the particle gets through the barrier is proportional to the square of the ratio of the amplitudes of the sinusoidal wave functions on the two sides of the barrier. These amplitudes are determined by matching wave functions and their derivatives at the boundary points, a fairly involved mathematical problem. When T is much smaller than unity, it is given approximately by

$$T = Ge^{-2\kappa L} \text{ where } G = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right) \text{ and } \kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar} \quad (40.42)$$

(probability of tunneling)

The probability decreases rapidly with increasing barrier width L . It also depends critically on the energy difference $U_0 - E$, which in Newtonian physics is the additional kinetic energy the particle would need to be able to climb over the barrier.

Example 40.7 Tunneling through a barrier

A 2.0-eV electron encounters a barrier 5.0 eV high. What is the probability that it will tunnel through the barrier if the barrier width is (a) 1.00 nm and (b) 0.50 nm?

SOLUTION

IDENTIFY and SET UP: This problem uses the ideas of tunneling through a rectangular barrier, as in Figs. 40.19 and 40.20. Our target variable is the tunneling probability T in Eq. (40.42), which we evaluate for the given values $E = 2.0$ eV (electron energy), $U = 5.0$ eV (barrier height), $m = 9.11 \times 10^{-31}$ kg (mass of the electron), and $L = 1.00$ nm or 0.50 nm (barrier width).

EXECUTE: First we evaluate G and κ in Eq. (40.42), using $E = 2.0$ eV:

$$G = 16 \left(\frac{2.0 \text{ eV}}{5.0 \text{ eV}} \right) \left(1 - \frac{2.0 \text{ eV}}{5.0 \text{ eV}} \right) = 3.8$$

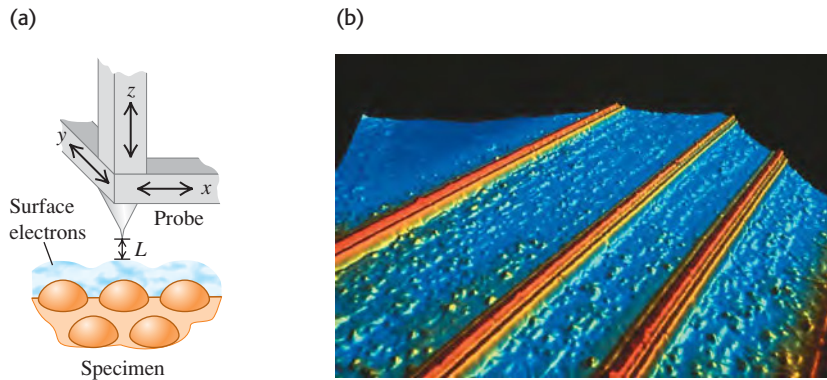
$$U_0 - E = 5.0 \text{ eV} - 2.0 \text{ eV} = 3.0 \text{ eV} = 4.8 \times 10^{-19} \text{ J}$$

$$\kappa = \frac{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(4.8 \times 10^{-19} \text{ J})}}{1.055 \times 10^{-34} \text{ J}\cdot\text{s}} = 8.9 \times 10^9 \text{ m}^{-1}$$

(a) When $L = 1.00$ nm $= 1.00 \times 10^{-9}$ m, $2\kappa L = 2(8.9 \times 10^9 \text{ m}^{-1})(1.00 \times 10^{-9} \text{ m}) = 17.8$ and $T = Ge^{-2\kappa L} = 3.8e^{-17.8} = 7.1 \times 10^{-8}$.

(b) When $L = 0.50$ nm, one-half of 1.00 nm, $2\kappa L$ is one-half of 17.8, or 8.9. Hence $T = 3.8e^{-8.9} = 5.2 \times 10^{-4}$.

EVALUATE: Halving the width of this barrier increases the tunneling probability T by a factor of $(5.2 \times 10^{-4})/(7.1 \times 10^{-8}) = 7.3 \times 10^3$, or nearly ten thousand. The tunneling probability is an *extremely* sensitive function of the barrier width.



40.21 (a) Schematic diagram of the probe of a scanning tunneling microscope (STM). As the sharp conducting probe is scanned across the surface in the x - and y -directions, it is also moved in the z -direction to maintain a constant tunneling current. The changing position of the probe is recorded and used to construct an image of the surface. (b) This colored STM image shows “quantum wires”: thin strips, just 10 atoms wide, of a conductive rare-earth silicide atop a silicon surface. Such quantum wires may one day be the basis of ultraminiaturized circuits.

Applications of Tunneling

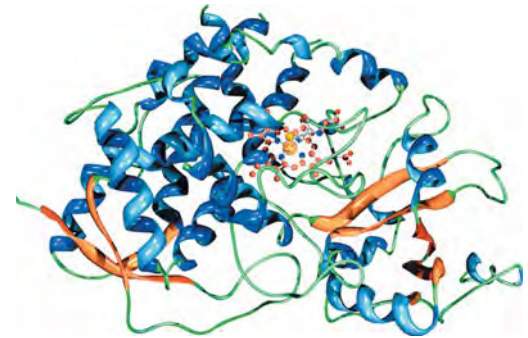
Tunneling has a number of practical applications, some of considerable importance. When you twist two copper wires together or close the contacts of a switch, current passes from one conductor to the other despite a thin layer of nonconducting copper oxide between them. The electrons tunnel through this thin insulating layer. A *tunnel diode* is a semiconductor device in which electrons tunnel through a potential barrier. The current can be switched on and off very quickly (within a few picoseconds) by varying the height of the barrier. A *Josephson junction* consists of two superconductors separated by an oxide layer a few atoms (1 to 2 nm) thick. Electron pairs in the superconductors can tunnel through the barrier layer, giving such a device unusual circuit properties. Josephson junctions are useful for establishing precise voltage standards and measuring tiny magnetic fields, and they play a crucial role in the developing field of quantum computing.

The *scanning tunneling microscope* (STM) uses electron tunneling to create images of surfaces down to the scale of individual atoms. An extremely sharp conducting needle is brought very close to the surface, within 1 nm or so (Fig. 40.21a). When the needle is at a positive potential with respect to the surface, electrons can tunnel through the surface potential-energy barrier and reach the needle. As Example 40.7 shows, the tunneling probability and hence the tunneling current are very sensitive to changes in the width L of the barrier (the distance between the surface and the needle tip). In one mode of operation the needle is scanned across the surface and at the same time is moved perpendicular to the surface to maintain a constant tunneling current. The needle motion is recorded, and after many parallel scans, an image of the surface can be reconstructed. Extremely precise control of needle motion, including isolation from vibration, is essential. Figure 40.21b shows an STM image. (Figure 40.17 is also an STM image.)

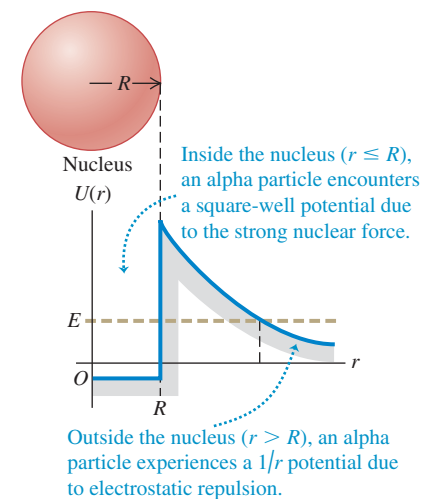
Tunneling is also of great importance in nuclear physics. A fusion reaction can occur when two nuclei tunnel through the barrier caused by their electrical repulsion and approach each other closely enough for the attractive nuclear force to cause them to fuse. Fusion reactions occur in the cores of stars, including the sun; without tunneling, the sun wouldn’t shine. The emission of alpha particles from unstable nuclei such as radium also involves tunneling. An alpha particle is a cluster of two protons and two neutrons (the same as a nucleus of the most common form of helium). Such clusters form naturally within larger atomic nuclei. An alpha particle trying to escape from a nucleus encounters a potential barrier that results from the combined effect of the attractive nuclear force and the electrical repulsion of the remaining part of the nucleus (Fig. 40.22). The alpha particle can escape only by tunneling through this barrier. Depending on the barrier height and width for a given kind of alpha-emitting nucleus, the tunneling probability can be low or high, and the alpha-emitting material will have low or high radioactivity. Recall from Section 39.2 that Ernest Rutherford used alpha particles

Application Electron Tunneling in Enzymes

Protein molecules play essential roles as enzymes in living organisms. Enzymes like the one shown here are large molecules, and in many cases their function depends on the ability of electrons to tunnel across the space that separates one part of the molecule from another. Without tunneling, life as we know it would be impossible!



40.22 Approximate potential-energy function for an alpha particle interacting with a nucleus of radius R . If an alpha particle inside the nucleus has energy E greater than zero, it can tunnel through the barrier and escape from the nucleus.



from a radioactive source to discover the atomic nucleus. Although Rutherford did not know it, tunneling by these alpha particles made his experiments possible! We'll learn more about alpha decay in Chapter 43.

Test Your Understanding of Section 40.4 Is it possible for a particle undergoing tunneling to be found *within* the barrier rather than on either side of it? I

40.5 The Harmonic Oscillator

Systems that *oscillate* are of tremendous importance in the physical world, from the oscillations of your eardrums in response to a sound wave to the vibrations of the ground caused by an earthquake. Oscillations are equally important on the microscopic scale where quantum effects dominate. The molecules of the air around you can be set into vibration when they collide with each other, the protons and neutrons in an excited atomic nucleus can oscillate in opposite directions, and a microwave oven transfers energy to food by making water molecules in the food flip back and forth. In this section we'll look at the solutions of the Schrödinger equation for the simplest kind of vibrating system, the quantum-mechanical harmonic oscillator.

As we learned in Chapter 14, a **harmonic oscillator** is a particle with mass m that moves along the x -axis under the influence of a conservative force $F_x = -k'x$. The constant k' is called the *force constant*. (In Chapter 14 we used the symbol k for the force constant. In this section we'll use the symbol k' instead to minimize confusion with the wave number $k = 2\pi/\lambda$.) The force is proportional to the particle's displacement x from its equilibrium position, $x = 0$. The corresponding potential-energy function is $U = \frac{1}{2}k'x^2$ (Fig. 40.23). In Newtonian mechanics, when the particle is displaced from equilibrium, it undergoes sinusoidal motion with frequency $f = (1/2\pi)(k'/m)^{1/2}$ and angular frequency $\omega = 2\pi f = (k'/m)^{1/2}$. The amplitude (that is, the maximum displacement from equilibrium) of these Newtonian oscillations is A , which is related to the energy E of the oscillator by $E = \frac{1}{2}k'A^2$.

Let's make an enlightened guess about the energy levels of a quantum-mechanical harmonic oscillator. In classical physics an electron oscillating with angular frequency ω emits electromagnetic radiation with that same angular frequency. It's reasonable to guess that when an excited quantum-mechanical harmonic oscillator with angular frequency $\omega = (k'/m)^{1/2}$ (according to Newtonian mechanics, at least) makes a transition from one energy level to a lower level, it would emit a photon with this same angular frequency ω . The energy of such a photon is $hf = (h/2\pi)(\omega/2\pi) = \hbar\omega$. So we would expect that the spacing between adjacent energy levels of the harmonic oscillator would be

$$hf = \hbar\omega = \hbar \sqrt{\frac{k'}{m}} \quad (40.43)$$

That's the same spacing between energy levels that Planck assumed in deriving his radiation law (see Section 39.5). It was a good assumption; as we'll see, the energy levels are in fact half-integer $(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots)$ multiples of $\hbar\omega$.

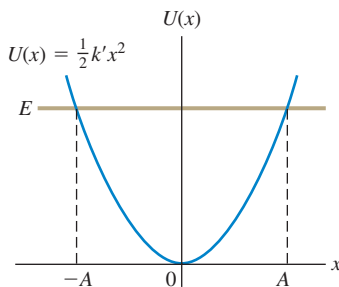
Wave Functions, Boundary Conditions, and Energy Levels

We'll begin our quantum-mechanical analysis of the harmonic oscillator by writing down the one-dimensional time-independent Schrödinger equation, Eq. (40.23), with $\frac{1}{2}k'x^2$ in place of U :

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + \frac{1}{2}k'x^2\psi(x) = E\psi(x) \quad \text{(Schrödinger equation for the harmonic oscillator)} \quad (40.44)$$

The solutions of this equation are wave functions for the physically possible states of the system.

40.23 Potential-energy function for the harmonic oscillator. In Newtonian mechanics the amplitude A is related to the total energy E by $E = \frac{1}{2}k'A^2$, and the particle is restricted to the range from $x = -A$ to $x = A$. In quantum mechanics the particle can be found at $x > A$ or $x < -A$.



In the discussion of square-well potentials in Section 40.2 we found that the energy levels are determined by boundary conditions at the walls of the well. However, the harmonic-oscillator potential has no walls as such; what, then, are the appropriate boundary conditions? Classically, $|x|$ cannot be greater than the amplitude A given by $E = \frac{1}{2}k'A^2$. Quantum mechanics does allow some penetration into classically forbidden regions, but the probability decreases as that penetration increases. Thus the wave functions must approach zero as $|x|$ grows large.

Satisfying the requirement that $\psi(x) \rightarrow 0$ as $|x| \rightarrow \infty$ is not as trivial as it may seem. To see why this is, let's rewrite Eq. (40.44) in the form

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2} \left(\frac{1}{2}k'x^2 - E \right) \psi(x) \quad (40.45)$$

Equation (40.45) shows that when x is large enough (either positive or negative) to make the quantity $(\frac{1}{2}k'x^2 - E)$ positive, the function $\psi(x)$ and its second derivative $d^2\psi(x)/dx^2$ have the same sign. Figure 40.24 shows four possible kinds of behavior of $\psi(x)$ beginning at a point where x is greater than the classical amplitude A , so that $\frac{1}{2}k'x^2 - \frac{1}{2}k'A^2 = \frac{1}{2}k'x^2 - E > 0$. Let's look at these four cases more closely. Note that if $\psi(x)$ is positive as shown in Fig. 40.24, Eq. (40.45) tells us that $d^2\psi(x)/dx^2$ is also positive and the function is *concave upward*. Note also that $d^2\psi(x)/dx^2$ is the rate of change of the *slope* of $\psi(x)$; this will help us understand how our four possible wave functions behave.

- **Curve a:** The slope of $\psi(x)$ is positive at point x . Since $d^2\psi(x)/dx^2 > 0$, the function curves upward increasingly steeply and goes to infinity. This violates the boundary condition that $\psi(x) \rightarrow 0$ as $|x| \rightarrow \infty$, so this isn't a viable wave function.
- **Curve b:** The slope of $\psi(x)$ is negative at point x , and $d^2\psi(x)/dx^2$ has a large positive value. Hence the slope changes rapidly from negative to positive and keeps on increasing—so, again, the wave function goes to infinity. This wave function isn't viable either.
- **Curve c:** As for curve b , the slope is negative at point x . However, $d^2\psi(x)/dx^2$ now has a *small* positive value, so the slope increases only gradually as $\psi(x)$ decreases to zero and crosses over to negative values. Equation (40.45) tells us that once $\psi(x)$ becomes negative, $d^2\psi(x)/dx^2$ also becomes negative. Hence the curve becomes *concave downward* and heads for *negative* infinity. This wave function, too, fails to satisfy the requirement that $\psi(x) \rightarrow 0$ as $|x| \rightarrow \infty$ and thus isn't viable.
- **Curve d:** If the slope of $\psi(x)$ at point x is negative, and the positive value of $d^2\psi(x)/dx^2$ at this point is neither too large nor too small, the curve bends just enough to glide in asymptotically to the x -axis. In this case $\psi(x)$, $d\psi(x)/dx$, and $d^2\psi(x)/dx^2$ all approach zero at large x . This case offers the only hope of satisfying the boundary condition that $\psi(x) \rightarrow 0$ as $|x| \rightarrow \infty$, and it occurs only for certain very special values of the energy E .

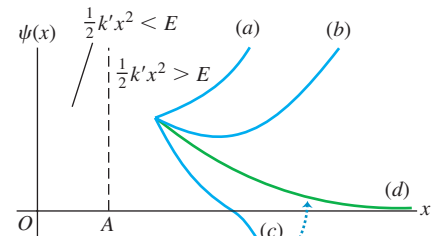
This qualitative discussion suggests how the boundary conditions as $|x| \rightarrow \infty$ determine the possible energy levels for the quantum-mechanical harmonic oscillator. It turns out that these boundary conditions are satisfied only if the energy E is equal to one of the values E_n , given by the simple formula

$$E_n = \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} = \left(n + \frac{1}{2}\right)\hbar\omega \quad (n = 0, 1, 2, \dots) \quad (40.46)$$

(energy levels, harmonic oscillator)

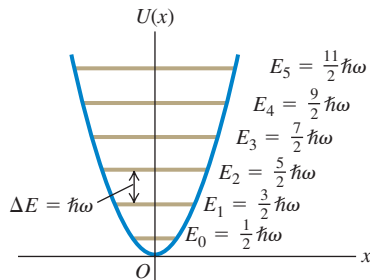
where n is the quantum number identifying each state and energy level. Note that the ground level of energy $E_0 = \frac{1}{2}\hbar\omega$ is denoted by $n = 0$, *not* $n = 1$.

40.24 Possible behaviors of harmonic-oscillator wave functions in the region $\frac{1}{2}k'x^2 > E$. In this region, $\psi(x)$ and $d^2\psi(x)/dx^2$ have the same sign. The curve is concave upward when $d^2\psi(x)/dx^2$ is positive and concave downward when $d^2\psi(x)/dx^2$ is negative.



Only curve d , which approaches the x -axis asymptotically for large x , is an acceptable wave function for this system.

40.25 Energy levels for the harmonic oscillator. The spacing between any two adjacent levels is $\Delta E = \hbar\omega$. The energy of the ground level is $E_0 = \frac{1}{2}\hbar\omega$.



Equation (40.46) confirms our guess [Eq. 40.43] that adjacent energy levels are separated by a constant interval of $\hbar\omega = hf$, as Planck assumed in 1900. There are infinitely many levels; this shouldn't be surprising because we are dealing with an infinitely deep potential well. As $|x|$ increases, $U = \frac{1}{2}k'x^2$ increases without bound.

Figure 40.25 shows the lowest six energy levels and the potential-energy function $U(x)$. For each level n , the value of $|x|$ at which the horizontal line representing the total energy E_n intersects $U(x)$ gives the amplitude A_n of the corresponding Newtonian oscillator.

Example 40.8 Vibration in a crystal

A sodium atom of mass 3.82×10^{-26} kg vibrates within a crystal. The potential energy increases by 0.0075 eV when the atom is displaced 0.014 nm from its equilibrium position. Treat the atom as a harmonic oscillator. (a) Find the angular frequency of the oscillations according to Newtonian mechanics. (b) Find the spacing (in electron volts) of adjacent vibrational energy levels according to quantum mechanics. (c) What is the wavelength of a photon emitted as the result of a transition from one level to the next lower level? In what region of the electromagnetic spectrum does this lie?

SOLUTION

IDENTIFY and SET UP: We'll find the force constant k' from the expression $U = \frac{1}{2}k'x^2$ for potential energy. We'll then find the angular frequency $\omega = (k'/m)^{1/2}$ and use this in Eq. (40.46) to find the spacing between adjacent energy levels. We'll calculate the wavelength of the emitted photon as in Example 40.6.

EXECUTE: We are given that $U = 0.0075$ eV $= 1.2 \times 10^{-21}$ J when $x = 0.014 \times 10^{-9}$ m, so we can solve $U = \frac{1}{2}k'x^2$ for k' :

$$k' = \frac{2U}{x^2} = \frac{2(1.2 \times 10^{-21} \text{ J})}{(0.014 \times 10^{-9} \text{ m})^2} = 12.2 \text{ N/m}$$

(a) The Newtonian angular frequency is

$$\omega = \sqrt{\frac{k'}{m}} = \sqrt{\frac{12.2 \text{ N/m}}{3.82 \times 10^{-26} \text{ kg}}} = 1.79 \times 10^{13} \text{ rad/s}$$

(b) From Eq. (40.46) and Fig. 40.25, the spacing between adjacent energy levels is

$$\begin{aligned} \hbar\omega &= (1.054 \times 10^{-34} \text{ J}\cdot\text{s})(1.79 \times 10^{13} \text{ s}^{-1}) \\ &= 1.88 \times 10^{-21} \text{ J} \left(\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) = 0.0118 \text{ eV} \end{aligned}$$

(c) The energy E of the emitted photon is equal to the energy lost by the oscillator in the transition, 0.0118 eV. Then

$$\begin{aligned} \lambda &= \frac{hc}{E} = \frac{(4.136 \times 10^{-15} \text{ eV}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{0.0118 \text{ eV}} \\ &= 1.05 \times 10^{-4} \text{ m} = 105 \mu\text{m} \end{aligned}$$

This photon wavelength is in the infrared region of the spectrum.

EVALUATE: This example shows us that interatomic force constants are a few newtons per meter, about the same as those of household springs or spring-based toys such as the Slinky. It also suggests that we can learn about the vibrations of molecules by measuring the radiation that they emit in transitioning to a lower vibrational state. We will explore this idea further in Chapter 42.

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ActivPhysics 20.1.6: Potential Energy Diagrams, Question 6

Comparing Quantum and Newtonian Oscillators

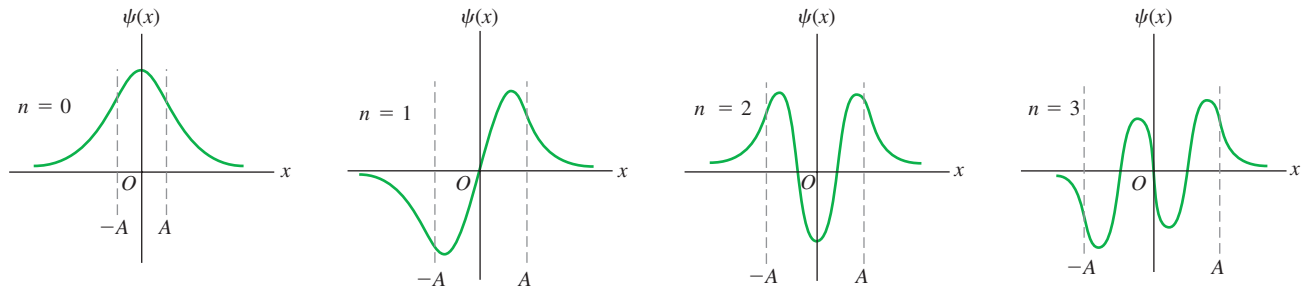
The wave functions for the levels $n = 0, 1, 2, \dots$ of the harmonic oscillator are called *Hermite functions*; they aren't encountered in elementary calculus courses but are well known to mathematicians. Each Hermite function is an exponential function multiplied by a polynomial in x . The harmonic-oscillator wave function corresponding to $n = 0$ and $E = E_0$ (the ground level) is

$$\psi(x) = Ce^{-\sqrt{mk'}x^2/2\hbar} \quad (40.47)$$

The constant C is chosen to normalize the function—that is, to make $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$. (We're using C rather than A as a normalization constant in this section, since we've already appropriated the symbol A to denote the Newtonian amplitude of a harmonic oscillator.) You can find C using the following result from integral tables:

$$\int_{-\infty}^{\infty} e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{a}$$

40.26 The first four wave functions for the harmonic oscillator. The amplitude A of a Newtonian oscillator with the same total energy is shown for each. Each wave function penetrates somewhat into the classically forbidden regions $|x| > A$. The total number of finite maxima and minima for each function is $n + 1$, one more than the quantum number.



To confirm that $\psi(x)$ as given by Eq. (40.47) really *is* a solution of the Schrödinger equation for the harmonic oscillator, we invite you to calculate the second derivative of this wave function, substitute it into Eq. (40.44), and verify that the equation is satisfied if the energy E is equal to $E_0 = \frac{1}{2}\hbar\omega$ (see Exercise 40.38). It's a little messy, but the result is satisfying and worth the effort.

Figure 40.26 shows the the first four harmonic-oscillator wave functions. Each graph also shows the amplitude A of a Newtonian harmonic oscillator with the same energy—that is, the value of A determined from

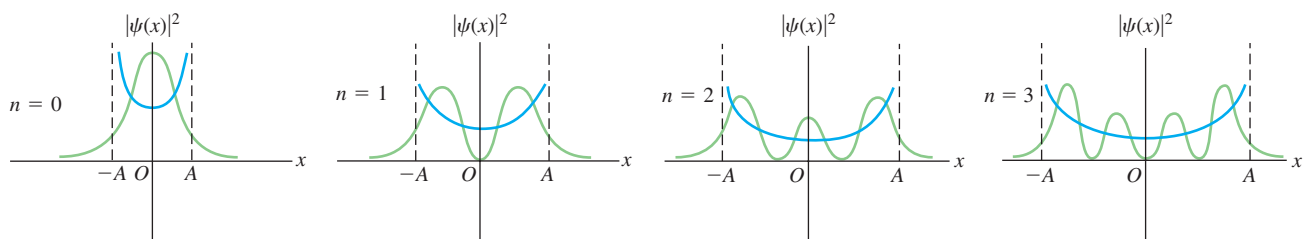
$$\frac{1}{2}k'A^2 = \left(n + \frac{1}{2}\right)\hbar\omega \quad (40.48)$$

In each case there is some penetration of the wave function into the regions $|x| > A$ that are forbidden by Newtonian mechanics. This is similar to the effect that we noted in Section 40.3 for a particle in a finite square well.

Figure 40.27 shows the probability distributions $|\psi(x)|^2$ for these same states. Each graph also shows the probability distribution determined from Newtonian physics, in which the probability of finding the particle near a randomly chosen point is inversely proportional to the particle's speed at that point. If we average out the wiggles in the quantum-mechanical probability curves, the results for $n > 0$ resemble the Newtonian predictions. This agreement improves with increasing n ; Fig. 40.28 shows the classical and quantum-mechanical probability functions for $n = 10$. Notice that the spacing between zeros of $|\psi(x)|^2$ in Fig. 40.28 increases with increasing distance from $x = 0$. This makes sense from the Newtonian perspective: As a particle moves away from $x = 0$, its kinetic energy K and the magnitude p of its momentum both decrease. Thinking quantum-mechanically, this means that the wavelength $\lambda = h/p$ increases, so the spacing between zeros of $\psi(x)$ (and hence of $|\psi(x)|^2$) also increases.

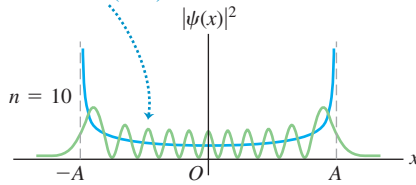
In the Newtonian analysis of the harmonic oscillator the minimum energy is zero, with the particle at rest at its equilibrium position $x = 0$. This is not possible in quantum mechanics; no solution of the Schrödinger equation has $E = 0$ and satisfies the boundary conditions. Furthermore, if there were such a state, it

40.27 Probability distribution functions $|\psi(x)|^2$ for the harmonic-oscillator wave functions shown in Fig. 40.26. The amplitude A of the Newtonian motion with the same energy is shown for each. The blue lines show the corresponding probability distributions for the Newtonian motion. As n increases, the averaged-out quantum-mechanical functions resemble the Newtonian curves more and more.

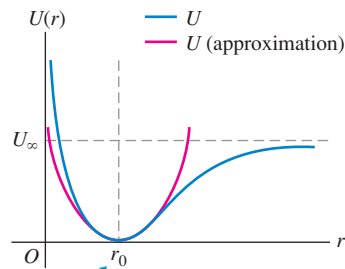


40.28 Newtonian and quantum-mechanical probability distribution functions for a harmonic oscillator for the state $n = 10$. The Newtonian amplitude A is also shown.

The larger the value of n , the more closely the quantum-mechanical probability distribution (green) matches the Newtonian probability distribution (blue).



40.29 A potential-energy function describing the interaction of two atoms in a diatomic molecule. The distance r is the separation between the centers of the atoms, and the equilibrium separation is $r = r_0$. The energy needed to dissociate the molecule is U_∞ .



When r is near r_0 , the potential-energy curve is approximately parabolic (as shown by the red curve) and the system behaves approximately like a harmonic oscillator.

would violate the Heisenberg uncertainty principle because there would be no uncertainty in either position or momentum. The energy must be at least $\frac{1}{2}\hbar\omega$ for the system to conform to the uncertainty principle. To see qualitatively why this is so, consider a Newtonian oscillator with total energy $\frac{1}{2}\hbar\omega$. We can find the amplitude A and the maximum velocity just as we did in Section 14.3. When the particle is at its maximum displacement ($x = \pm A$) and instantaneously at rest, $K = 0$ and $E = U = \frac{1}{2}k'A^2$. When the particle is at equilibrium ($x = 0$) and moving at its maximum speed, $U = 0$ and $E = K = \frac{1}{2}mv_{\max}^2$. Setting $E = \frac{1}{2}\hbar\omega$, we find

$$E = \frac{1}{2}k'A^2 = \frac{1}{2}\hbar\omega = \frac{1}{2}\hbar\left(\frac{k'}{m}\right)^{1/2} \quad \text{so} \quad A = \frac{\hbar^{1/2}}{k'^{1/4}m^{1/4}}$$

$$E = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}k'A^2 \quad \text{so} \quad v_{\max} = A\left(\frac{k'}{m}\right)^{1/2} = \frac{\hbar^{1/2}k'^{1/4}}{m^{3/4}}$$

The maximum *momentum* of the particle is

$$p_{\max} = mv_{\max} = \hbar^{1/2}k'^{1/4}m^{1/4}$$

Here's where the Heisenberg uncertainty principle comes in. It turns out that the uncertainties in the particle's position and momentum (calculated as standard deviations) are, respectively, $\Delta x = A/\sqrt{2} = A/2^{1/2}$ and $\Delta p_x = p_{\max}/\sqrt{2} = p_{\max}/2^{1/2}$. Then the product of the two uncertainties is

$$\Delta x \Delta p_x = \left(\frac{\hbar^{1/2}}{2^{1/2}k'^{1/4}m^{1/4}}\right)\left(\frac{\hbar^{1/2}k'^{1/4}m^{1/4}}{2^{1/2}}\right) = \frac{\hbar}{2}$$

This product equals the minimum value allowed by Eq. (39.29), $\Delta x \Delta p_x \geq \hbar/2$, and thus satisfies the uncertainty principle. If the energy had been less than $\frac{1}{2}\hbar\omega$, the product $\Delta x \Delta p_x$ would have been less than $\hbar/2$, and the uncertainty principle would have been violated.

Even when a potential-energy function isn't precisely parabolic in shape, we may be able to approximate it by the harmonic-oscillator potential for sufficiently small displacements from equilibrium. Figure 40.29 shows a typical potential-energy function for an interatomic force in a molecule. At large separations the curve of $U(r)$ versus r levels off, corresponding to the absence of force at great distances. But the curve is approximately parabolic near the minimum of $U(r)$ (the equilibrium separation of the atoms). Near equilibrium the molecular vibration is approximately simple harmonic with energy levels given by Eq. (40.46), as we assumed in Example 40.8.

Test Your Understanding of Section 40.5 A quantum-mechanical system initially in its ground level absorbs a photon and ends up in the first excited state. The system then absorbs a second photon and ends up in the second excited state. For which of the following systems does the second photon have a longer wavelength than the first one? (i) a harmonic oscillator; (ii) a hydrogen atom; (iii) a particle in a box.



Wave functions: The wave function for a particle contains all of the information about that particle. If the particle moves in one dimension in the presence of a potential energy function $U(x)$, the wave function $\Psi(x, t)$ obeys the one-dimensional Schrödinger equation. (For a *free* particle on which no forces act, $U(x) = 0$.) The quantity $|\Psi(x, t)|^2$, called the probability distribution function, determines the relative probability of finding a particle near a given position at a given time. If the particle is in a state of definite energy, called a stationary state, $\Psi(x, t)$ is a product of a function $\psi(x)$ that depends only on spatial coordinates and a function $e^{-iEt/\hbar}$ that depends only on time. For a stationary state, the probability distribution function is independent of time.

A spatial stationary-state wave function $\psi(x)$ for a particle that moves in one dimension in the presence of a potential-energy function $U(x)$ satisfies the time-independent Schrödinger equation. More complex wave functions can be constructed by superposing stationary-state wave functions. These can represent particles that are localized in a certain region, thus representing both particle and wave aspects. (See Examples 40.1 and 40.2.)

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + U(x)\Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t} \quad (40.20)$$

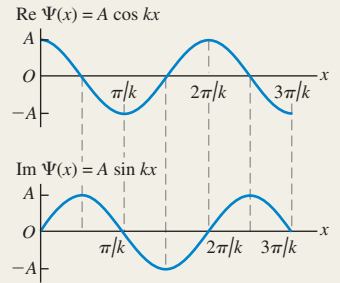
(general 1-D Schrödinger equation)

$$\Psi(x, t) = \psi(x)e^{-iEt/\hbar} \quad (40.21)$$

(time-dependent wave function for a state of definite energy)

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x) \quad (40.23)$$

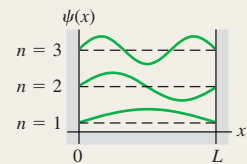
(time-independent Schrödinger equation)



Particle in a box: The energy levels for a particle of mass m in a box (an infinitely deep square potential well) with width L are given by Eq. (40.31). The corresponding normalized stationary-state wave functions of the particle are given by Eq. (40.35). (See Examples 40.3 and 40.4.)

$$E_n = \frac{p_n^2}{2m} = \frac{n^2 h^2}{8mL^2} = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad (n = 1, 2, 3, \dots) \quad (40.31)$$

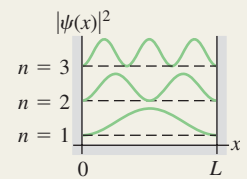
$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad (n = 1, 2, 3, \dots) \quad (40.35)$$



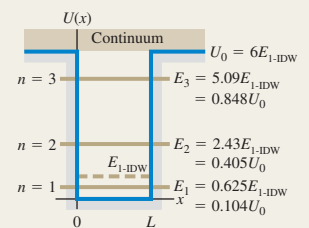
Wave functions and normalization: To be a solution of the Schrödinger equation, the wave function $\psi(x)$ and its derivative $d\psi(x)/dx$ must be continuous everywhere except where the potential-energy function $U(x)$ has an infinite discontinuity. Wave functions are usually normalized so that the total probability of finding the particle somewhere is unity.

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \quad (40.33)$$

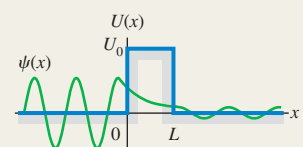
(normalization condition)



Finite potential well: In a potential well with finite depth U_0 , the energy levels are lower than those for an infinitely deep well with the same width, and the number of energy levels corresponding to bound states is finite. The levels are obtained by matching wave functions at the well walls to satisfy the continuity of $\psi(x)$ and $d\psi(x)/dx$. (See Examples 40.5 and 40.6.)

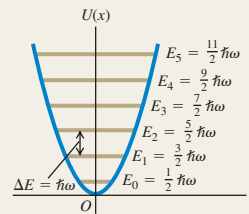


Potential barriers and tunneling: There is a certain probability that a particle will penetrate a potential-energy barrier even though its initial energy is less than the barrier height. This process is called tunneling. (See Example 40.7.)



Quantum harmonic oscillator: The energy levels for the harmonic oscillator (for which $U(x) = \frac{1}{2}k'x^2$) are given by Eq. (40.46). The spacing between any two adjacent levels is $\hbar\omega$, where $\omega = \sqrt{k'/m}$ is the oscillation angular frequency of the corresponding Newtonian harmonic oscillator. (See Example 40.8.)

$$E_n = \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} = \left(n + \frac{1}{2}\right)\hbar\omega \quad (n = 0, 1, 2, 3, \dots) \quad (40.46)$$



BRIDGING PROBLEM

A Packet in a Box

A particle of mass m in an infinitely deep well has the following wave function in the region from $x = 0$ to $x = L$:

$$\Psi(x, t) = \frac{1}{\sqrt{2}}\psi_1(x)e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\psi_2(x)e^{-iE_2t/\hbar}$$

Here $\psi_1(x)$ and $\psi_2(x)$ are the normalized stationary-state wave functions for the first two levels ($n = 1$ and $n = 2$), given by Eq. (40.35). E_1 and E_2 , given by Eq. (40.31), are the energies of these levels. The wave function is zero for $x < 0$ and for $x > L$.

(a) Find the probability distribution function for this wave function. (b) Does $\Psi(x, t)$ represent a stationary state of definite energy? How can you tell? (c) Show that the wave function $\Psi(x, t)$ is normalized. (d) Find the angular frequency of oscillation of the probability distribution function. What is the interpretation of this oscillation? (e) Suppose instead that $\Psi(x, t)$ is a combination of the wave functions of the two lowest levels of a finite well of length L and height U_0 equal to six times the energy of the lowest-energy bound state of an infinite well of length L . What would be the angular frequency of the probability distribution function in this case?

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.



IDENTIFY and SET UP

1. In Section 40.1 we saw how to interpret a combination of two free-particle wave functions of different energies. In this problem you need to apply these same ideas to a combination of wave functions for the infinite well (Section 40.2) and the finite well (Section 40.3).

EXECUTE

- Write down the full time-dependent wave function $\Psi(x, t)$ and its complex conjugate $\Psi^*(x, t)$ using the functions $\psi_1(x)$ and $\psi_2(x)$ from Eq. (40.35). Use these to calculate the probability distribution function, and decide whether or not this function depends on time.
- To check for normalization, you'll need to verify that when you integrate the probability distribution function from step 2 over all values of x , the integral is equal to 1. [Hint: The trigonometric identities $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ and $\sin\theta \sin\phi = \frac{1}{2}(\cos(\theta - \phi) - \cos(\theta + \phi))$ may be helpful.]
- To find the answer to part (d) you'll need to identify the oscillation angular frequency ω_{osc} in your expression from step 2 for the probability distribution function. To interpret the oscillations, draw graphs of the probability distribution functions at times $t = 0$, $t = T/4$, $t = T/2$, and $t = 3T/4$, where $T = 2\pi/\omega_{\text{osc}}$ is the oscillation period of the probability distribution function.
- For the finite well you do not have simple expressions for the first two stationary-state wave functions $\psi_1(x)$ and $\psi_2(x)$. However, you can still find the oscillation angular frequency ω_{osc} , which is related to the energies E_1 and E_2 in the same way as for the infinite-well case. (Can you see why?)

EVALUATE

- Why are the factors of $1/\sqrt{2}$ in the wave function $\Psi(x, t)$ important?
- Why do you suppose the oscillation angular frequency for a finite well is lower than for an infinite well of the same width?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q40.1 If quantum mechanics replaces the language of Newtonian mechanics, why don't we have to use wave functions to describe the motion of macroscopic bodies such as baseballs and cars?

Q40.2 A student remarks that the relationship of ray optics to the more general wave picture is analogous to the relationship of New-

tonian mechanics, with well-defined particle trajectories, to quantum mechanics. Comment on this remark.

Q40.3 As Eq. (40.21) indicates, the time-dependent wave function for a stationary state is a complex number having a real part and an imaginary part. How can this function have any physical meaning, since part of it is *imaginary*?

Q40.4 Why must the wave function of a particle be normalized?

Q40.5 If a particle is in a stationary state, does that mean that the particle is not moving? If a particle moves in empty space with constant momentum \vec{p} and hence constant energy $E = p^2/2m$, is it in a stationary state? Explain your answers.

Q40.6 For the particle in a box, we chose $k = n\pi/L$ with $n = 1, 2, 3, \dots$ to fit the boundary condition that $\psi = 0$ at $x = L$. However, $n = 0, -1, -2, -3, \dots$ also satisfy that boundary condition. Why didn't we also choose those values of n ?

Q40.7 If ψ is normalized, what is the physical significance of the area under a graph of $|\psi|^2$ versus x between x_1 and x_2 ? What is the total area under the graph of $|\psi|^2$ when all x are included? Explain.

Q40.8 For a particle in a box, what would the probability distribution function $|\psi|^2$ look like if the particle behaved like a classical (Newtonian) particle? Do the actual probability distributions approach this classical form when n is very large? Explain.

Q40.9 In Chapter 15 we represented a standing wave as a superposition of two waves traveling in opposite directions. Can the wave functions for a particle in a box also be thought of as a combination of two traveling waves? Why or why not? What physical interpretation does this representation have? Explain.

Q40.10 A particle in a box is in the ground level. What is the probability of finding the particle in the right half of the box? (Refer to Fig. 40.12, but don't evaluate an integral.) Is the answer the same if the particle is in an excited level? Explain.

Q40.11 The wave functions for a particle in a box (see Fig. 40.12a) are zero at certain points. Does this mean that the particle can't move past one of these points? Explain.

Q40.12 For a particle confined to an infinite square well, is it correct to say that each state of definite energy is also a state of definite wavelength? Is it also a state of definite momentum? Explain. (*Hint:* Remember that momentum is a vector.)

Q40.13 For a particle in a finite potential well, is it correct to say that each bound state of definite energy is also a state of definite wavelength? Is it a state of definite momentum? Explain.

Q40.14 In Fig. 40.12b, the probability function is zero at the points $x = 0$ and $x = L$, the "walls" of the box. Does this mean that the particle never strikes the walls? Explain.

Q40.15 A particle is confined to a finite potential well in the region $0 < x < L$. How does the area under the graph of $|\psi|^2$ in the region $0 < x < L$ compare to the total area under the graph of $|\psi|^2$ when including all possible x ?

Q40.16 Compare the wave functions for the first three energy levels for a particle in a box of width L (see Fig. 40.12a) to the corresponding wave functions for a finite potential well of the same width (see Fig. 40.15a). How does the wavelength in the interval $0 \leq x \leq L$ for the $n = 1$ level of the particle in a box compare to the corresponding wavelength for the $n = 1$ level of the finite potential well? Use this to explain why E_1 is less than $E_{1-\text{IDW}}$ in the situation depicted in Fig. 40.15b.

Q40.17 It is stated in Section 40.3 that a finite potential well always has at least one bound level, no matter how shallow the well. Does this mean that as $U_0 \rightarrow 0$, $E_1 \rightarrow 0$? Does this violate the Heisenberg uncertainty principle? Explain.

Q40.18 Figure 40.15a shows that the higher the energy of a bound state for a finite potential well, the more the wave function extends outside the well (into the intervals $x < 0$ and $x > L$). Explain why this happens.

Q40.19 In classical (Newtonian) mechanics, the total energy E of a particle can never be less than the potential energy U because the kinetic energy K cannot be negative. Yet in barrier tunneling (see Section 40.4) a particle passes through regions where E is less than U . Is this a contradiction? Explain.

Q40.20 Figure 40.17 shows the scanning tunneling microscope image of 48 iron atoms placed on a copper surface, the pattern indicating the density of electrons on the copper surface. What can you infer about the potential-energy function inside the circle of iron atoms?

Q40.21 Qualitatively, how would you expect the probability for a particle to tunnel through a potential barrier to depend on the height of the barrier? Explain.

Q40.22 The wave function shown in Fig. 40.20 is nonzero for both $x < 0$ and $x > L$. Does this mean that the particle splits into two parts when it strikes the barrier, with one part tunneling through the barrier and the other part bouncing off the barrier? Explain.

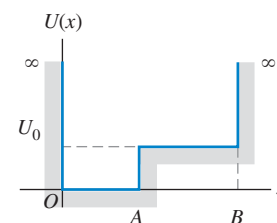
Q40.23 The probability distributions for the harmonic oscillator wave functions (see Figs. 40.27 and 40.28) begin to resemble the classical (Newtonian) probability distribution when the quantum number n becomes large. Would the distributions become the same as in the classical case in the limit of very large n ? Explain.

Q40.24 In Fig. 40.28, how does the probability of finding a particle in the center half of the region $-A < x < A$ compare to the probability of finding the particle in the outer half of the region? Is this consistent with the physical interpretation of the situation?

Q40.25 Compare the allowed energy levels for the hydrogen atom, the particle in a box, and the harmonic oscillator. What are the values of the quantum number n for the ground level and the second excited level of each system?

Q40.26 Sketch the wave function for the potential-energy well shown in Fig. Q40.26 when E_1 is less than U_0 and when E_3 is greater than U_0 .

Figure Q40.26



EXERCISES

Section 40.1 Wave Functions and the One-Dimensional Schrödinger Equation

40.1 • An electron is moving as a free particle in the $-x$ -direction with momentum that has magnitude 4.50×10^{-24} kg \cdot m/s. What is the one-dimensional time-dependent wave function of the electron?

40.2 • A free particle moving in one dimension has wave function

$$\Psi(x, t) = A[e^{i(kx - \omega t)} - e^{i(2kx - 4\omega t)}]$$

where k and ω are positive real constants. (a) At $t = 0$ what are the two smallest positive values of x for which the probability function $|\Psi(x, t)|^2$ is a maximum? (b) Repeat part (a) for time $t = 2\pi/\omega$. (c) Calculate v_{av} as the distance the maxima have moved divided by the elapsed time. Compare your result to the expression $v_{\text{av}} = (\omega_2 - \omega_1)/(k_2 - k_1)$ from Example 40.1.

40.3 • Consider the free-particle wave function of Example 40.1. Let $k_2 = 3k_1 = 3k$. At $t = 0$ the probability distribution function $|\Psi(x, t)|^2$ has a maximum at $x = 0$. (a) What is the smallest positive value of x for which the probability distribution function has a maximum at time $t = 2\pi/\omega$, where $\omega = \hbar k^2/2m$. (b) From your result in part (a), what is the average speed with which the probability distribution is moving in the $+x$ -direction? Compare your result to the expression $v_{\text{av}} = (\omega_2 - \omega_1)/(k_2 - k_1)$ from Example 40.1.

40.4 • Consider the free particle of Example 40.1. Show that $v_{\text{av}} = (\omega_2 - \omega_1)/(k_2 - k_1)$ can be written as $v_{\text{av}} = p_{\text{av}}/m$, where $p_{\text{av}} = (\hbar k_2 + \hbar k_1)/2$.

40.5 • Consider a wave function given by $\psi(x) = A \sin kx$, where $k = 2\pi/\lambda$ and A is a real constant. (a) For what values of x is there the highest probability of finding the particle described by this wave function? Explain. (b) For which values of x is the probability zero? Explain.

40.6 •• Compute $|\Psi|^2$ for $\Psi = \psi \sin \omega t$, where ψ is time independent and ω is a real constant. Is this a wave function for a stationary state? Why or why not?

40.7 • CALC Let ψ_1 and ψ_2 be two solutions of Eq. (40.23) with energies E_1 and E_2 , respectively, where $E_1 \neq E_2$. Is $\psi = A\psi_1 + B\psi_2$, where A and B are nonzero constants, a solution to Eq. (40.23)? Explain your answer.

40.8 • A particle is described by a wave function $\psi(x) = Ae^{-\alpha x^2}$, where A and α are real, positive constants. If the value of α is increased, what effect does this have on (a) the particle's uncertainty in position and (b) the particle's uncertainty in momentum? Explain your answers.

40.9 • CALC Linear Combinations of Wave Functions. Let ψ_1 and ψ_2 be two solutions of Eq. (40.23) with the same energy E . Show that $\psi = B\psi_1 + C\psi_2$ is also a solution with energy E , for any values of the constants B and C .

Section 40.2 Particle in a Box

40.10 •• CALC A particle moving in one dimension (the x -axis) is described by the wave function

$$\psi(x) = \begin{cases} Ae^{-bx}, & \text{for } x \geq 0 \\ Ae^{bx}, & \text{for } x < 0 \end{cases}$$

where $b = 2.00 \text{ m}^{-1}$, $A > 0$, and the $+x$ -axis points toward the right. (a) Determine A so that the wave function is normalized. (b) Sketch the graph of the wave function. (c) Find the probability of finding this particle in each of the following regions: (i) within 50.0 cm of the origin, (ii) on the left side of the origin (can you first guess the answer by looking at the graph of the wave function?), (iii) between $x = 0.500 \text{ m}$ and $x = 1.00 \text{ m}$.

40.11 • Ground-Level Billiards. (a) Find the lowest energy level for a particle in a box if the particle is a billiard ball ($m = 0.20 \text{ kg}$) and the box has a width of 1.3 m, the size of a billiard table. (Assume that the billiard ball slides without friction rather than rolls; that is, ignore the *rotational* kinetic energy.) (b) Since the energy in part (a) is all kinetic, to what speed does this correspond? How much time would it take at this speed for the ball to move from one side of the table to the other? (c) What is the difference in energy between the $n = 2$ and $n = 1$ levels? (d) Are quantum-mechanical effects important for the game of billiards?

40.12 • A proton is in a box of width L . What must the width of the box be for the ground-level energy to be 5.0 MeV, a typical value for the energy with which the particles in a nucleus are bound? Compare your result to the size of a nucleus—that is, on the order of 10^{-14} m .

40.13 •• Find the width L of a one-dimensional box for which the ground-state energy of an electron in the box equals the absolute value of the ground state of a hydrogen atom.

40.14 •• When a hydrogen atom undergoes a transition from the $n = 2$ to the $n = 1$ level, a photon with $\lambda = 122 \text{ nm}$ is emitted. (a) If the atom is modeled as an electron in a one-dimensional box, what is the width of the box in order for the $n = 2$ to $n = 1$ transition to correspond to emission of a photon of this energy? (b) For a box with the width calculated in part (a), what is the ground-state energy? How does this correspond to the ground-state energy of a hydrogen atom? (c) Do you think a one-dimensional box is a good

model for a hydrogen atom? Explain. (*Hint:* Compare the spacing between adjacent energy levels as a function of n .)

40.15 •• A certain atom requires 3.0 eV of energy to excite an electron from the ground level to the first excited level. Model the atom as an electron in a box and find the width L of the box.

40.16 • An electron in a one-dimensional box has ground-state energy 1.00 eV. What is the wavelength of the photon absorbed when the electron makes a transition to the second excited state?

40.17 • CALC Show that the time-dependent wave function given by Eq. (40.35) is a solution to the one-dimensional Schrödinger equation, Eq. (40.23).

40.18 • Recall that $|\psi|^2 dx$ is the probability of finding the particle that has normalized wave function $\psi(x)$ in the interval x to $x + dx$. Consider a particle in a box with rigid walls at $x = 0$ and $x = L$. Let the particle be in the ground level and use ψ_n as given in Eq. (40.35). (a) For which values of x , if any, in the range from 0 to L is the probability of finding the particle zero? (b) For which values of x is the probability highest? (c) In parts (a) and (b) are your answers consistent with Fig. 40.12? Explain.

40.19 • Repeat Exercise 40.18 for the particle in the first excited level.

40.20 • CALC (a) Show that $\psi = A \sin kx$ is a solution to Eq. (40.25) if $k = \sqrt{2mE}/\hbar$. (b) Explain why this is an acceptable wave function for a particle in a box with rigid walls at $x = 0$ and $x = L$ only if k is an integer multiple of π/L .

40.21 • CALC (a) Repeat Exercise 40.20 for $\psi = A \cos kx$. (b) Explain why this cannot be an acceptable wave function for a particle in a box with rigid walls at $x = 0$ and $x = L$ no matter what the value of k .

40.22 • (a) Find the excitation energy from the ground level to the third excited level for an electron confined to a box that has a width of 0.125 nm. (b) The electron makes a transition from the $n = 1$ to $n = 4$ level by absorbing a photon. Calculate the wavelength of this photon.

40.23 • An electron is in a box of width $3.0 \times 10^{-10} \text{ m}$. What are the de Broglie wavelength and the magnitude of the momentum of the electron if it is in (a) the $n = 1$ level; (b) the $n = 2$ level; (c) the $n = 3$ level? In each case how does the wavelength compare to the width of the box?

40.24 •• CALC Normalization of the Wave Function. Consider a particle moving in one dimension, which we shall call the x -axis. (a) What does it mean for the wave function of this particle to be *normalized*? (b) Is the wave function $\psi(x) = e^{ax}$, where a is a positive real number, normalized? Could this be a valid wave function? (c) If the particle described by the wave function $\psi(x) = Ae^{bx}$, where A and b are positive real numbers, is confined to the range $x \geq 0$, determine A (including its units) so that the wave function is normalized.

Section 40.3 Potential Wells

40.25 • CALC (a) Show that $\psi = A \sin kx$, where k is a real (not complex) constant, is *not* a solution of Eq. (40.23) for $U = U_0$ and $E < U_0$. (b) Is this ψ a solution for $E > U_0$?

40.26 •• An electron is moving past the square well shown in Fig. 40.13. The electron has energy $E = 3U_0$. What is the ratio of the de Broglie wavelength of the electron in the region $x > L$ to the wavelength for $0 < x < L$?

40.27 • An electron is bound in a square well of depth $U_0 = 6E_{1-\text{IDW}}$. What is the width of the well if its ground-state energy is 2.00 eV?

40.28 •• An electron is bound in a square well of width 1.50 nm and depth $U_0 = 6E_{1-\text{IDW}}$. If the electron is initially in the ground

level and absorbs a photon, what maximum wavelength can the photon have and still liberate the electron from the well?

40.29 • CALC Calculate $d^2\psi/dx^2$ for the wave function of Eq. (40.38), and show that the function is a solution of Eq. (40.37).

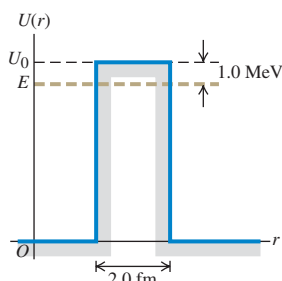
40.30 •• An electron is bound in a square well with a depth equal to six times the ground-level energy E_{1-IDW} of an infinite well of the same width. The longest-wavelength photon that is absorbed by the electron has a wavelength of 400.0 nm. Determine the width of the well.

40.31 •• A proton is bound in a square well of width 4.0 fm = 4.0×10^{-15} m. The depth of the well is six times the ground-level energy E_{1-IDW} of the corresponding infinite well. If the proton makes a transition from the level with energy E_1 to the level with energy E_3 by absorbing a photon, find the wavelength of the photon.

Section 40.4 Potential Barriers and Tunneling

40.32 •• Alpha Decay. In a simple model for a radioactive nucleus, an alpha particle ($m = 6.64 \times 10^{-27}$ kg) is trapped by a square barrier that has width 2.0 fm and height 30.0 MeV. (a) What is the tunneling probability when the alpha particle encounters the barrier if its kinetic energy is 1.0 MeV below the top of the barrier (Fig. E40.32)? (b) What is the tunneling probability if the energy of the alpha particle is 10.0 MeV below the top of the barrier?

Figure E40.32



40.33 • An electron with initial kinetic energy 6.0 eV encounters a barrier with height 11.0 eV. What is the probability of tunneling if the width of the barrier is (a) 0.80 nm and (b) 0.40 nm?

40.34 • An electron with initial kinetic energy 5.0 eV encounters a barrier with height U_0 and width 0.60 nm. What is the transmission coefficient if (a) $U_0 = 7.0$ eV; (b) $U_0 = 9.0$ eV; (c) $U_0 = 13.0$ eV?

40.35 •• An electron is moving past the square barrier shown in Fig. 40.19, but the energy of the electron is *greater* than the barrier height. If $E = 2U_0$, what is the ratio of the de Broglie wavelength of the electron in the region $x > L$ to the wavelength for $0 < x < L$?

40.36 • A proton with initial kinetic energy 50.0 eV encounters a barrier of height 70.0 eV. What is the width of the barrier if the probability of tunneling is 3.0×10^{-3} ? How does this compare with the barrier width for an electron with the same energy tunneling through a barrier of the same height with the same probability?

40.37 •• (a) An electron with initial kinetic energy 32 eV encounters a square barrier with height 41 eV and width 0.25 nm. What is the probability that the electron will tunnel through the barrier? (b) A proton with the same kinetic energy encounters the same barrier. What is the probability that the proton will tunnel through the barrier?

Section 40.5 The Harmonic Oscillator

40.38 • CALC Show that $\psi(x)$ given by Eq. (40.47) is a solution to Eq. (40.44) with energy $E_0 = \hbar\omega/2$.

40.39 • A wooden block with mass 0.250 kg is oscillating on the end of a spring that has force constant 110 N/m. Calculate the ground-level energy and the energy separation between adjacent levels. Express your results in joules and in electron volts. Are quantum effects important?

40.40 • A harmonic oscillator absorbs a photon of wavelength 8.65×10^{-6} m when it undergoes a transition from the ground state to the first excited state. What is the ground-state energy, in electron volts, of the oscillator?

40.41 • Chemists use infrared absorption spectra to identify chemicals in a sample. In one sample, a chemist finds that light of wavelength 5.8 μm is absorbed when a molecule makes a transition from its ground harmonic oscillator level to its first excited level. (a) Find the energy of this transition. (b) If the molecule can be treated as a harmonic oscillator with mass 5.6×10^{-26} kg, find the force constant.

40.42 •• The ground-state energy of a harmonic oscillator is 5.60 eV. If the oscillator undergoes a transition from its $n = 3$ to $n = 2$ level by emitting a photon, what is the wavelength of the photon?

40.43 • In Section 40.5 it is shown that for the ground level of a harmonic oscillator, $\Delta x \Delta p_x = \hbar/2$. Do a similar analysis for an excited level that has quantum number n . How does the uncertainty product $\Delta x \Delta p_x$ depend on n ?

40.44 •• For the ground-level harmonic oscillator wave function $\psi(x)$ given in Eq. (40.47), $|\psi|^2$ has a maximum at $x = 0$. (a) Compute the ratio of $|\psi|^2$ at $x = +A$ to $|\psi|^2$ at $x = 0$, where A is given by Eq. (40.48) with $n = 0$ for the ground level. (b) Compute the ratio of $|\psi|^2$ at $x = +2A$ to $|\psi|^2$ at $x = 0$. In each case is your result consistent with what is shown in Fig. 40.27?

40.45 •• For the sodium atom of Example 40.8, find (a) the ground-state energy, (b) the wavelength of a photon emitted when the $n = 4$ to $n = 3$ transition occurs; (c) the energy difference for any $\Delta n = 1$ transition.

PROBLEMS

40.46 • The discussion in Section 40.1 shows that the wave function $\Psi = \psi e^{-i\omega t}$ is a stationary state, where ψ is time independent and ω is a real (not complex) constant. Consider the wave function $\Psi = \psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t}$, where ψ_1 and ψ_2 are different time-independent functions and ω_1 and ω_2 are different real constants. Assume that ψ_1 and ψ_2 are real-valued functions, so that $\psi_1^* = \psi_1$ and $\psi_2^* = \psi_2$. Is this Ψ a wave function for a stationary state? Why or why not?

40.47 •• A particle of mass m in a one-dimensional box has the following wave function in the region $x = 0$ to $x = L$:

$$\Psi(x, t) = \frac{1}{\sqrt{2}}\psi_1(x)e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\psi_3(x)e^{-iE_3t/\hbar}$$

Here $\psi_1(x)$ and $\psi_3(x)$ are the normalized stationary-state wave functions for the $n = 1$ and $n = 3$ levels, and E_1 and E_3 are the energies of these levels. The wave function is zero for $x < 0$ and for $x > L$. (a) Find the value of the probability distribution function at $x = L/2$ as a function of time. (b) Find the angular frequency at which the probability distribution function oscillates.

40.48 •• CALC Consider the wave packet defined by

$$\psi(x) = \int_0^{\infty} B(k) \cos kx \, dk$$

Let $B(k) = e^{-\alpha^2 k^2}$. (a) The function $B(k)$ has its maximum value at $k = 0$. Let k_h be the value of k at which $B(k)$ has fallen to half its maximum value, and define the width of $B(k)$ as $w_k = k_h$. In terms of α , what is w_k ? (b) Use integral tables to evaluate the integral that gives $\psi(x)$. For what value of x is $\psi(x)$ maximum? (c) Define the width of $\psi(x)$ as $w_x = x_h$, where x_h is the positive

value of x at which $\psi(x)$ has fallen to half its maximum value. Calculate w_x in terms of α . (d) The momentum p is equal to $\hbar k/2\pi$, so the width of B in momentum is $w_p = \hbar w_k/2\pi$. Calculate the product $w_p w_x$ and compare to the Heisenberg uncertainty principle.

40.49 •• CALC (a) Using the integral in Problem 40.48, determine the wave function $\psi(x)$ for a function $B(k)$ given by

$$B(k) = \begin{cases} 0 & k < 0 \\ 1/k_0, & 0 \leq k \leq k_0 \\ 0, & k > k_0 \end{cases}$$

This represents an equal combination of all wave numbers between 0 and k_0 . Thus $\psi(x)$ represents a particle with average wave number $k_0/2$, with a total spread or uncertainty in wave number of k_0 . We will call this spread the *width* w_k of $B(k)$, so $w_k = k_0$. (b) Graph $B(k)$ versus k and $\psi(x)$ versus x for the case $k_0 = 2\pi/L$, where L is a length. Locate the point where $\psi(x)$ has its maximum value and label this point on your graph. Locate the two points closest to this maximum (one on each side of it) where $\psi(x) = 0$, and define the distance along the x -axis between these two points as w_x , the width of $\psi(x)$. Indicate the distance w_x on your graph. What is the value of w_x if $k_0 = 2\pi/L$? (c) Repeat part (b) for the case $k_0 = \pi/L$. (d) The momentum p is equal to $\hbar k/2\pi$, so the width of B in momentum is $w_p = \hbar w_k/2\pi$. Calculate the product $w_p w_x$ for each of the cases $k_0 = 2\pi/L$ and $k_0 = \pi/L$. Discuss your results in light of the Heisenberg uncertainty principle.

40.50 • CALC Show that the wave function $\psi(x) = Ae^{ikx}$ is a solution of Eq. (40.23) for a particle of mass m , in a region where the potential energy is a constant $U_0 < E$. Find an expression for k , and relate it to the particle's momentum and to its de Broglie wavelength.

40.51 •• CALC Wave functions like the one in Problem 40.50 can represent free particles moving with velocity $v = p/m$ in the x -direction. Consider a beam of such particles incident on a potential-energy step $U(x) = 0$, for $x < 0$, and $U(x) = U_0 < E$, for $x > 0$. The wave function for $x < 0$ is $\psi(x) = Ae^{ik_1x} + Be^{-ik_1x}$, representing incident and reflected particles, and for $x > 0$ is $\psi(x) = Ce^{ik_2x}$, representing transmitted particles. Use the conditions that both ψ and its first derivative must be continuous at $x = 0$ to find the constants B and C in terms of k_1 , k_2 , and A .

40.52 • Let ΔE_n be the energy difference between the adjacent energy levels E_n and E_{n+1} for a particle in a box. The ratio $R_n = \Delta E_n/E_n$ compares the energy of a level to the energy separation of the next higher energy level. (a) For what value of n is R_n largest, and what is this largest R_n ? (b) What does R_n approach as n becomes very large? How does this result compare to the classical value for this quantity?

40.53 • Photon in a Dye Laser. An electron in a long, organic molecule used in a dye laser behaves approximately like a particle in a box with width 4.18 nm. What is the wavelength of the photon emitted when the electron undergoes a transition (a) from the first excited level to the ground level and (b) from the second excited level to the first excited level?

40.54 • CALC A particle is in the ground level of a box that extends from $x = 0$ to $x = L$. (a) What is the probability of finding the particle in the region between 0 and $L/4$? Calculate this by integrating $|\psi(x)|^2 dx$, where ψ is normalized, from $x = 0$ to $x = L/4$. (b) What is the probability of finding the particle in the region $x = L/4$ to $x = L/2$? (c) How do the results of parts (a) and (b) compare? Explain. (d) Add the probabilities calculated in parts (a) and (b). (e) Are your results in parts (a), (b), and (d) consistent with Fig. 40.12b? Explain.

40.55 •• CALC What is the probability of finding a particle in a box of length L in the region between $x = L/4$ and $x = 3L/4$ when the particle is in (a) the ground level and (b) the first excited level? (*Hint:* Integrate $|\psi(x)|^2 dx$, where ψ is normalized, between $L/4$ and $3L/4$.) (c) Are your results in parts (a) and (b) consistent with Fig. 40.12b? Explain.

40.56 •• Consider a particle in a box with rigid walls at $x = 0$ and $x = L$. Let the particle be in the ground level. Calculate the probability $|\psi|^2 dx$ that the particle will be found in the interval x to $x + dx$ for (a) $x = L/4$; (b) $x = L/2$; (c) $x = 3L/4$.

40.57 •• Repeat Problem 40.56 for a particle in the first excited level.

40.58 •• CP A particle is confined within a box with perfectly rigid walls at $x = 0$ and $x = L$. Although the magnitude of the instantaneous force exerted on the particle by the walls is infinite and the time over which it acts is zero, the impulse (that involves a product of force and time) is both finite and quantized. Show that the impulse exerted by the wall at $x = 0$ is $(nh/L)\hat{i}$ and that the impulse exerted by the wall at $x = L$ is $-(nh/L)\hat{i}$. (*Hint:* You may wish to review Section 8.1.)

40.59 •• CALC A fellow student proposes that a possible wave function for a free particle with mass m (one for which the potential-energy function $U(x)$ is zero) is

$$\psi(x) = \begin{cases} e^{+\kappa x}, & x < 0 \\ e^{-\kappa x}, & x \geq 0 \end{cases}$$

where κ is a positive constant. (a) Graph this proposed wave function. (b) Show that the proposed wave function satisfies the Schrödinger equation for $x < 0$ if the energy is $E = -\hbar^2\kappa^2/2m$ —that is, if the energy of the particle is *negative*. (c) Show that the proposed wave function also satisfies the Schrödinger equation for $x \geq 0$ with the same energy as in part (b). (d) Explain why the proposed wave function is nonetheless *not* an acceptable solution of the Schrödinger equation for a free particle. (*Hint:* What is the behavior of the function at $x = 0$?) It is in fact impossible for a free particle (one for which $U(x) = 0$) to have an energy less than zero.

40.60 •• The *penetration distance* η in a finite potential well is the distance at which the wave function has decreased to $1/e$ of the wave function at the classical turning point:

$$\psi(x = L + \eta) = \frac{1}{e}\psi(L)$$

The penetration distance can be shown to be

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

The probability of finding the particle beyond the penetration distance is nearly zero. (a) Find η for an electron having a kinetic energy of 13 eV in a potential well with $U_0 = 20$ eV. (b) Find η for a 20.0-MeV proton trapped in a 30.0-MeV-deep potential well.

40.61 • CALC (a) For the finite potential well of Fig. 40.13, what relationships among the constants A and B of Eq. (40.38) and C and D of Eq. (40.40) are obtained by applying the boundary condition that ψ be continuous at $x = 0$ and at $x = L$? (b) What relationships among A , B , C , and D are obtained by applying the boundary condition that $d\psi/dx$ be continuous at $x = 0$ and at $x = L$?

40.62 • An electron with initial kinetic energy 5.5 eV encounters a square potential barrier with height 10.0 eV. What is the width of

the barrier if the electron has a 0.10% probability of tunneling through the barrier?

40.63 •• A particle with mass m and total energy E tunnels through a square barrier of height U_0 and width L . When the transmission coefficient is *not* much less than unity, it is given by

$$T = \left[1 + \frac{(U_0 \sinh \kappa L)^2}{4E(U_0 - E)} \right]^{-1}$$

where $\sinh \kappa L = (e^{\kappa L} - e^{-\kappa L})/2$ is the hyperbolic sine of κL . (a) Show that if $\kappa L \gg 1$, this expression for T approaches Eq. (40.42). (b) Explain why the restriction $\kappa L \gg 1$ in part (a) implies either that the barrier is relatively wide or that the energy E is relatively low compared to U_0 . (c) Show that as the particle's incident kinetic energy E approaches the barrier height U_0 , T approaches $[1 + (kL/2)^2]^{-1}$, where $k = \sqrt{2mE}/\hbar$ is the wave number of the incident particle. (*Hint:* If $|z| \ll 1$, then $\sinh z \approx z$.)

40.64 • CP A harmonic oscillator consists of a 0.020-kg mass on a spring. Its frequency is 1.50 Hz, and the mass has a speed of 0.360 m/s as it passes the equilibrium position. (a) What is the value of the quantum number n for its energy level? (b) What is the difference in energy between the levels E_n and E_{n+1} ? Is this difference detectable?

40.65 • For small amplitudes of oscillation the motion of a pendulum is simple harmonic. For a pendulum with a period of 0.500 s, find the ground-level energy and the energy difference between adjacent energy levels. Express your results in joules and in electron volts. Are these values detectable?

40.66 •• Some 164.9-nm photons are emitted in a $\Delta n = 1$ transition within a solid-state lattice. The lattice is modeled as electrons in a box having length 0.500 nm. What transition corresponds to the emitted light?

40.67 •• CALC Show that for $\psi(x)$ given by Eq. (40.47), the probability distribution function has a maximum at $x = 0$.

40.68 •• CALC (a) Show by direct substitution in the Schrödinger equation for the one-dimensional harmonic oscillator that the wave function $\psi_1(x) = A_1 x e^{-\alpha^2 x^2/2}$, where $\alpha^2 = m\omega/\hbar$, is a solution with energy corresponding to $n = 1$ in Eq. (40.46). (b) Find the normalization constant A_1 . (c) Show that the probability density has a minimum at $x = 0$ and maxima at $x = \pm 1/\alpha$, corresponding to the classical turning points for the ground state $n = 0$.

40.69 •• CP (a) The wave nature of particles results in the quantum-mechanical situation that a particle confined in a box can assume only wavelengths that result in standing waves in the box, with nodes at the box walls. Use this to show that an electron confined in a one-dimensional box of length L will have energy levels given by

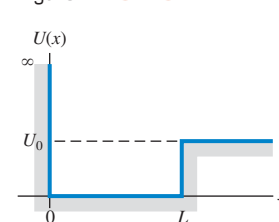
$$E_n = \frac{n^2 \hbar^2}{8mL^2}$$

(*Hint:* Recall that the relationship between the de Broglie wavelength and the speed of a nonrelativistic particle is $mv = h/\lambda$. The energy of the particle is $\frac{1}{2}mv^2$.) (b) If a hydrogen atom is modeled as a one-dimensional box with length equal to the Bohr radius, what is the energy (in electron volts) of the lowest energy level of the electron?

40.70 ••• Consider a potential well defined as $U(x) = \infty$ for $x < 0$, $U(x) = 0$ for $0 < x < L$, and $U(x) = U_0 > 0$ for $x > L$ (Fig. P40.70). Consider a particle with mass m and kinetic energy $E < U_0$ that is trapped in the well. (a) The boundary condition at the infinite wall ($x = 0$) is $\psi(0) = 0$. What must the form of

the function $\psi(x)$ for $0 < x < L$ be in order to satisfy both the Schrödinger equation and this boundary condition? (b) The wave function must remain finite as $x \rightarrow \infty$. What must the form of the function $\psi(x)$ for $x > L$ be in order to satisfy both the Schrödinger equation and this

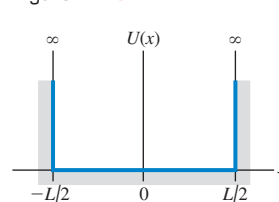
Figure P40.70



boundary condition at infinity? (c) Impose the boundary conditions that ψ and $d\psi/dx$ are continuous at $x = L$. Show that the energies of the allowed levels are obtained from solutions of the equation $k \cot kL = -\kappa$, where $k = \sqrt{2mE}/\hbar$ and $\kappa = \sqrt{2m(U_0 - E)}/\hbar$.

40.71 ••• Section 40.2 considered a box with walls at $x = 0$ and $x = L$. Now consider a box with width L but centered at $x = 0$, so that it extends from $x = -L/2$ to $x = +L/2$ (Fig. P40.71). Note that this box is symmetric about $x = 0$. (a) Consider possible wave functions of the form $\psi(x) =$

Figure P40.71



$A \sin kx$. Apply the boundary conditions at the wall to obtain the allowed energy levels. (b) Another set of possible wave functions are functions of the form $\psi(x) = A \cos kx$. Apply the boundary conditions at the wall to obtain the allowed energy levels. (c) Compare the energies obtained in parts (a) and (b) to the set of energies given in Eq. (40.31). (d) An odd function f satisfies the condition $f(x) = -f(-x)$, and an even function g satisfies $g(x) = g(-x)$. Of the wave functions from parts (a) and (b), which are even and which are odd?

CHALLENGE PROBLEMS

40.72 ••• CALC The WKB Approximation. It can be a challenge to solve the Schrödinger equation for the bound-state energy levels of an arbitrary potential well. An alternative approach that can yield good approximate results for the energy levels is the *WKB approximation* (named for the physicists Gregor Wentzel, Hendrik Kramers, and Léon Brillouin, who pioneered its application to quantum mechanics). The WKB approximation begins from three physical statements: (i) According to de Broglie, the magnitude of momentum p of a quantum-mechanical particle is $p = h/\lambda$. (ii) The magnitude of momentum is related to the kinetic energy K by the relationship $K = p^2/2m$. (iii) If there are no non-conservative forces, then in Newtonian mechanics the energy E for a particle is constant and equal at each point to the sum of the kinetic and potential energies at that point: $E = K + U(x)$, where x is the coordinate. (a) Combine these three relationships to show that the wavelength of the particle at a coordinate x can be written as

$$\lambda(x) = \frac{h}{\sqrt{2m[E - U(x)]}}$$

Thus we envision a quantum-mechanical particle in a potential well $U(x)$ as being like a free particle, but with a wavelength $\lambda(x)$ that is a function of position. (b) When the particle moves into a region of increasing potential energy, what happens to its wavelength? (c) At a point where $E = U(x)$, Newtonian mechanics says that the particle has zero kinetic energy and must be instantaneously at rest. Such a point is called a *classical turning point*, since this is where a Newtonian particle must stop its motion and

reverse direction. As an example, an object oscillating in simple harmonic motion with amplitude A moves back and forth between the points $x = -A$ and $x = +A$; each of these is a classical turning point, since there the potential energy $\frac{1}{2}k'x^2$ equals the total energy $\frac{1}{2}k'A^2$. In the WKB expression for $\lambda(x)$, what is the wavelength at a classical turning point? (d) For a particle in a box with length L , the walls of the box are classical turning points (see Fig. 40.8). Furthermore, the number of wavelengths that fit within the box must be a half-integer (see Fig. 40.10), so that $L = (n/2)\lambda$ and hence $L/\lambda = n/2$, where $n = 1, 2, 3, \dots$ [Note that this is a restatement of Eq. (40.29).] The WKB scheme for finding the allowed bound-state energy levels of an *arbitrary* potential well is an extension of these observations. It demands that for an allowed energy E , there must be a half-integer number of wavelengths between the classical turning points for that energy. Since the wavelength in the WKB approximation is not a constant but depends on x , the number of wavelengths between the classical turning points a and b for a given value of the energy is the integral of $1/\lambda(x)$ between those points:

$$\int_a^b \frac{dx}{\lambda(x)} = \frac{n}{2} \quad (n = 1, 2, 3, \dots)$$

Using the expression for $\lambda(x)$ you found in part (a), show that the *WKB condition for an allowed bound-state energy* can be written as

$$\int_a^b \sqrt{2m[E - U(x)]} dx = \frac{nh}{2} \quad (n = 1, 2, 3, \dots)$$

(e) As a check on the expression in part (d), apply it to a particle in a box with walls at $x = 0$ and $x = L$. Evaluate the integral and show that the allowed energy levels according to the WKB approximation are the same as those given by Eq. (40.31). (*Hint:* Since the walls of the box are infinitely high, the points $x = 0$ and $x = L$ are classical turning points for *any* energy E . Inside the box, the potential energy is zero.) (f) For the finite square well shown in Fig. 40.13, show that the WKB expression given in part (d) predicts the *same* bound-state energies as for an infinite square well of the same width. (*Hint:* Assume $E < U_0$. Then the classical turning points are at $x = 0$ and $x = L$.) This shows that the WKB approximation does a poor job when the potential-energy function changes discontinuously, as for a finite potential well. In the next two problems we consider situations in which the potential-energy function changes gradually and the WKB approximation is much more useful.

40.73 ••• CALC The WKB approximation (see Challenge Problem 40.72) can be used to calculate the energy levels for a harmonic oscillator. In this approximation, the energy levels are the solutions to the equation

$$\int_a^b \sqrt{2m[E - U(x)]} dx = \frac{nh}{2} \quad n = 1, 2, 3, \dots$$

Here E is the energy, $U(x)$ is the potential-energy function, and $x = a$ and $x = b$ are the classical turning points (the points at

which E is equal to the potential energy, so the Newtonian kinetic energy would be zero). (a) Determine the classical turning points for a harmonic oscillator with energy E and force constant k' . (b) Carry out the integral in the WKB approximation and show that the energy levels in this approximation are $E_n = \hbar\omega$, where $\omega = \sqrt{k'/m}$ and $n = 1, 2, 3, \dots$ (*Hint:* Recall that $\hbar = h/2\pi$. A useful standard integral is

$$\int \sqrt{A^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{A^2 - x^2} + A^2 \arcsin\left(\frac{x}{A}\right) \right]$$

where \arcsin denotes the inverse sine function. Note that the integrand is even, so the integral from $-x$ to x is equal to twice the integral from 0 to x .) (c) How do the approximate energy levels found in part (b) compare with the true energy levels given by Eq. (40.46)? Does the WKB approximation give an underestimate or an overestimate of the energy levels?

40.74 ••• CALC Protons, neutrons, and many other particles are made of more fundamental particles called *quarks* and *antiquarks* (the antimatter equivalent of quarks). A quark and an antiquark can form a bound state with a variety of different energy levels, each of which corresponds to a different particle observed in the laboratory. As an example, the ψ particle is a low-energy bound state of a so-called charm quark and its antiquark, with a rest energy of 3097 MeV; the $\psi(2S)$ particle is an excited state of this same quark–antiquark combination, with a rest energy of 3686 MeV. A simplified representation of the potential energy of interaction between a quark and an antiquark is $U(x) = A|x|$, where A is a positive constant and x represents the distance between the quark and the antiquark. You can use the WKB approximation (see Challenge Problem 40.72) to determine the bound-state energy levels for this potential-energy function. In the WKB approximation, the energy levels are the solutions to the equation

$$\int_a^b \sqrt{2m[E - U(x)]} dx = \frac{nh}{2} \quad (n = 1, 2, 3, \dots)$$

Here E is the energy, $U(x)$ is the potential-energy function, and $x = a$ and $x = b$ are the classical turning points (the points at which E is equal to the potential energy, so the Newtonian kinetic energy would be zero). (a) Determine the classical turning points for the potential $U(x) = A|x|$ and for an energy E . (b) Carry out the above integral and show that the allowed energy levels in the WKB approximation are given by

$$E_n = \frac{1}{2m} \left(\frac{3mA\hbar}{4} \right)^{2/3} n^{2/3} \quad (n = 1, 2, 3, \dots)$$

(*Hint:* The integrand is even, so the integral from $-x$ to x is equal to twice the integral from 0 to x .) (c) Does the difference in energy between successive levels increase, decrease, or remain the same as n increases? How does this compare to the behavior of the energy levels for the harmonic oscillator? For the particle in a box? Can you suggest a simple rule that relates the difference in energy between successive levels to the shape of the potential-energy function?

Answers

Chapter Opening Question ?

When an electron in one of these particles—called *quantum dots*—makes a transition from an excited level to a lower level, it emits a photon whose energy is equal to the difference in energy between the levels. The smaller the quantum dot, the larger the energy spacing between levels and hence the shorter (bluer) the wavelength of the emitted photons. See Example 40.6 (Section 40.3) for more details.

Test Your Understanding Questions

40.1 Answer: no Equation (40.19) represents a superposition of wave functions with different values of wave number k and hence different values of energy $E = \hbar^2 k^2 / 2m$. The state that this combined wave function represents is not a state of definite energy, and therefore not a stationary state. Another way to see this is to note that there is a factor $e^{-iEt/\hbar}$ inside the integral in Eq. (40.19), with a different value of E for each value of k . This wave function therefore has a very complicated time dependence, and the probability distribution function $|\Psi(x, t)|^2$ does depend on time.

40.2 Answer: (v) Our derivation of the stationary-state wave functions for a particle in a box shows that they are superpositions of waves propagating in opposite directions, just like a standing wave on a string. One wave has momentum in the positive x -direction, while the other wave has an equal magnitude of momentum in the negative x -direction. The *total* x -component of momentum is zero.

40.3 Answer: (i) The energy levels are arranged as shown in Fig. 40.15b if $U_0 = 6E_{1-IDW}$, where $E_{1-IDW} = \pi^2 \hbar^2 / 2mL^2$ is the

ground-level energy of an infinite well. If the well width L is reduced to one-half of its initial value, E_{1-IDW} increases by a factor of four and so U_0 must also increase by a factor of four. The energies E_1 , E_2 , and E_3 shown in Fig. 40.15b are all specific fractions of U_0 , so they will also increase by a factor of four.

40.4 Answer: yes Figure 40.20 shows a possible wave function $\psi(x)$ for tunneling. Since $\psi(x)$ is not zero within the barrier ($0 \leq x \leq L$), there is some probability that the particle can be found there.

40.5 Answer: (ii) If the second photon has a longer wavelength and hence lower energy than the first photon, the difference in energy between the first and second excited levels must be less than the difference between the ground level and the first excited level. This is the case for the hydrogen atom, for which the energy difference between levels decreases as the energy increases (see Fig. 39.24). By contrast, the energy difference between successive levels increases for a particle in a box (see Fig. 40.11b) and is constant for a harmonic oscillator (see Fig. 40.25).

Bridging Problem

Answers: (a)

$$|\Psi(x, t)|^2 = \frac{1}{L} \left[\sin^2 \frac{\pi x}{L} + \sin^2 \frac{2\pi x}{L} + 2 \sin \frac{\pi x}{L} \sin \frac{2\pi x}{L} \cos \left(\frac{(E_2 - E_1)t}{\hbar} \right) \right]$$

(b) no **(c)** yes **(d)** $\frac{3\pi^2 \hbar}{2mL^2}$ **(e)** $\frac{0.905\pi^2 \hbar}{mL^2}$

41

ATOMIC STRUCTURE

LEARNING GOALS

By studying this chapter, you will learn:

- How to extend quantum-mechanical calculations to three-dimensional problems.
- How to solve the Schrödinger equation for a particle trapped in a cubical box.
- How to describe the states of a hydrogen atom in terms of quantum numbers.
- How magnetic fields affect the orbital motion of atomic electrons.
- How we know that electrons have their own intrinsic angular momentum.
- How to analyze the structure of many-electron atoms.
- How x rays emitted by atoms reveal their inner structure.



? Lithium (with three electrons per atom) is a metal that burns spontaneously in water, while helium (with two electrons per atom) is a gas that undergoes almost no chemical reactions. How can one extra electron make these two elements so dramatically different?

Some physicists claim that all of chemistry is contained in the Schrödinger equation. This is somewhat of an exaggeration, but this equation can teach us a great deal about the chemical behavior of elements and the nature of chemical bonds. It provides insight into the periodic table of the elements and the microscopic basis of magnetism.

In order to learn about the quantum-mechanical structure of atoms, we'll first construct a three-dimensional version of the Schrödinger equation. We'll try this equation out by looking at a three-dimensional version of a particle in a box: a particle confined to a cubical volume.

We'll then see that we can learn a great deal about the structure and properties of *all* atoms from the solutions to the Schrödinger equation for the hydrogen atom. These solutions have quantized values of angular momentum; we don't need to make a separate statement about quantization as we did with the Bohr model. We label the states with a set of quantum numbers, which we'll use later with many-electron atoms as well. We'll find that the electron also has an intrinsic *spin* angular momentum in addition to the orbital angular momentum associated with its motion.

We'll also encounter the exclusion principle, a kind of microscopic zoning ordinance that is the key to understanding many-electron atoms. This principle says that no two electrons in an atom can have the same quantum-mechanical state. Finally, we'll use the principles of this chapter to explain the characteristic x-ray spectra of atoms.

41.1 The Schrödinger Equation in Three Dimensions

We have discussed the Schrödinger equation and its applications only for *one-dimensional* problems, the analog of a Newtonian particle moving along a straight line. The straight-line model is adequate for some applications, but to understand atomic structure, we need a three-dimensional generalization.

It's not difficult to guess what the three-dimensional Schrödinger equation should look like. First, the wave function Ψ is a function of time and all three space coordinates (x, y, z) . In general, the potential-energy function also depends on all three coordinates and can be written as $U(x, y, z)$. Next, recall from Section 40.1 that the term $-(\hbar^2/2m)\partial^2\Psi/\partial x^2$ in the one-dimensional Schrödinger equation, Eq. (40.20), is related to the kinetic energy of the particle in the state described by the wave function Ψ . For example, if we insert into this term the wave function $\Psi(x, t) = Ae^{ikx}e^{-i\omega t}$ for a free particle with magnitude of momentum $p = \hbar k$ and kinetic energy $K = p^2/2m$, we obtain $-(\hbar^2/2m)(ik)^2 Ae^{ikx}e^{-i\omega t} = (\hbar^2 k^2/2m) Ae^{ikx}e^{-i\omega t} = (p^2/2m)\Psi(x, t) = K\Psi(x, t)$. If the particle can move in three dimensions, its momentum has three components (p_x, p_y, p_z) and its kinetic energy is

$$K = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} \quad (41.1)$$

These observations, taken together, suggest that the correct generalization of the Schrödinger equation to three dimensions is

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2\Psi(x, y, z, t)}{\partial x^2} + \frac{\partial^2\Psi(x, y, z, t)}{\partial y^2} + \frac{\partial^2\Psi(x, y, z, t)}{\partial z^2} \right) + U(x, y, z)\Psi(x, y, z, t) = i\hbar \frac{\partial\Psi(x, y, z, t)}{\partial t} \quad (41.2)$$

(general three-dimensional Schrödinger equation)

The three-dimensional wave function $\Psi(x, y, z, t)$ has a similar interpretation as in one dimension. The wave function itself is a complex quantity with both a real part and an imaginary part, but $|\Psi(x, y, z, t)|^2$ —the square of its absolute value, equal to the product of $\Psi(x, y, z, t)$ and its complex conjugate $\Psi^*(x, y, z, t)$ —is real and either positive or zero at every point in space. We interpret $|\Psi(x, y, z, t)|^2 dV$ as the *probability* of finding the particle within a small volume dV centered on the point (x, y, z) at time t , so $|\Psi(x, y, z, t)|^2$ is the *probability distribution function* in three dimensions. The *normalization condition* on the wave function is that the probability that the particle is *somewhere* in space is exactly 1. Hence the integral of $|\Psi(x, y, z, t)|^2$ over all space must equal 1:

$$\int |\Psi(x, y, z, t)|^2 dV = 1 \quad \begin{array}{l} \text{(normalization condition} \\ \text{in three dimensions)} \end{array} \quad (41.3)$$

If the wave function $\Psi(x, y, z, t)$ represents a state of a definite energy E —that is, a stationary state—we can write it as the product of a spatial wave function $\psi(x, y, z)$ and a function of time $e^{-iEt/\hbar}$:

$$\Psi(x, y, z, t) = \psi(x, y, z)e^{-iEt/\hbar} \quad \begin{array}{l} \text{(time-dependent wave function} \\ \text{for a state of definite energy)} \end{array} \quad (41.4)$$

(Compare this to Eq. (40.21) for a one-dimensional state of definite energy.) If we substitute Eq. (41.4) into Eq. (41.2), the right-hand side of the equation becomes $i\hbar\psi(x, y, z)(-iE/\hbar)e^{-iEt/\hbar} = E\psi(x, y, z)e^{-iEt/\hbar}$. We can then divide

both sides by the factor $e^{-iEt/\hbar}$, leaving the *time-independent* Schrödinger equation in three dimensions for a stationary state:

$$\begin{aligned}
 & -\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} + \frac{\partial^2 \psi(x, y, z)}{\partial z^2} \right) + U(x, y, z) \psi(x, y, z) \\
 & = E \psi(x, y, z) \quad \text{(three-dimensional time-independent Schrödinger equation)} \quad (41.5)
 \end{aligned}$$

The probability distribution function for a stationary state is just the square of the absolute value of the spatial wave function: $|\psi(x, y, z)e^{-iEt/\hbar}|^2 = \psi^*(x, y, z)e^{+iEt/\hbar} \psi(x, y, z)e^{-iEt/\hbar} = |\psi(x, y, z)|^2$. Note that this doesn't depend on time. (As we discussed in Section 40.1, that's why we call these states *stationary*.) Hence for a stationary state the wave function normalization condition, Eq. (41.3), becomes

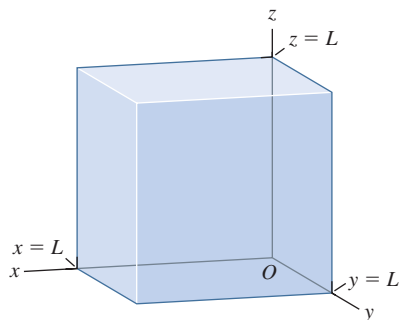
$$\int |\psi(x, y, z)|^2 dV = 1 \quad \text{(normalization condition for a stationary state in three dimensions)} \quad (41.6)$$

We won't pretend that we have *derived* Eqs. (41.2) and (41.5). Like their one-dimensional versions, these equations have to be tested by comparison of their predictions with experimental results. Happily, Eqs. (41.2) and (41.5) both pass this test with flying colors, so we are confident that they *are* the correct equations.

An important topic that we will address in this chapter is the solutions for Eq. (41.5) for the stationary states of the hydrogen atom. The potential-energy function for an electron in a hydrogen atom is *spherically symmetric*; it depends only on the distance $r = (x^2 + y^2 + z^2)^{1/2}$ from the origin of coordinates. To take advantage of this symmetry, it's best to use *spherical coordinates* rather than the Cartesian coordinates (x, y, z) to solve the Schrödinger equation for the hydrogen atom. Before introducing these new coordinates and investigating the hydrogen atom, it's useful to look at the three-dimensional version of the particle in a box that we considered in Section 40.2. Solving this simpler problem will give us insight into the more complicated stationary states found in atomic physics.

Test Your Understanding of Section 41.1 In a certain region of space the potential-energy function for a quantum-mechanical particle is zero. In this region the wave function $\psi(x, y, z)$ for a certain stationary state satisfies $\partial^2 \psi / \partial x^2 > 0$, $\partial^2 \psi / \partial y^2 > 0$, and $\partial^2 \psi / \partial z^2 > 0$. The particle has a definite energy E that is positive. What can you conclude about $\psi(x, y, z)$ in this region? (i) It must be positive; (ii) it must be negative; (iii) it must be zero; (iv) not enough information given to decide. **I**

41.1 A particle is confined in a cubical box with walls at $x = 0$, $x = L$, $y = 0$, $y = L$, $z = 0$, and $z = L$.



41.2 Particle in a Three-Dimensional Box

Consider a particle enclosed within a cubical box of side L . This could represent an electron that's free to move anywhere within the interior of a solid metal cube but cannot escape the cube. We'll choose the origin to be at one corner of the box, with the x -, y -, and z -axes along edges of the box. Then the particle is confined to the region $0 \leq x \leq L$, $0 \leq y \leq L$, $0 \leq z \leq L$ (Fig. 41.1). What are the stationary states of this system?

As for the particle in a one-dimensional box that we considered in Section 40.2, we'll say that the potential energy is zero inside the box but infinite outside. Hence the spatial wave function $\psi(x, y, z)$ must be zero outside the box in order that the term $U(x, y, z)\psi(x, y, z)$ in the time-independent Schrödinger equation, Eq. (41.5), not be infinite. Hence the probability distribution function $|\psi(x, y, z)|^2$ is zero outside the box, and there is zero probability that the particle will be found

there. Inside the box, the spatial wave function for a stationary state obeys the time-independent Schrödinger equation, Eq. (41.5), with $U(x, y, z) = 0$:

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} + \frac{\partial^2 \psi(x, y, z)}{\partial z^2} \right) = E \psi(x, y, z)$$

(particle in a three-dimensional box) (41.7)

In order for the wave function to be continuous from the inside to the outside of the box, $\psi(x, y, z)$ must equal zero on the walls. Hence our boundary conditions are that $\psi(x, y, z) = 0$ at $x = 0$, $x = L$, $y = 0$, $y = L$, $z = 0$, and $z = L$.

Guessing a solution to a complicated partial differential equation like Eq. (41.7) seems like quite a challenge. To make progress, recall that we wrote the time-dependent wave function for a stationary state as the product of one function that depends only on the spatial coordinates x , y , and z and a second function that depends only on the time t : $\Psi(x, y, z, t) = \psi(x, y, z)e^{-iEt/\hbar}$. In the same way, let's try a technique called *separation of variables*: We'll write the spatial wave function $\psi(x, y, z)$ as a product of one function X that depends only on x , a second function Y that depends only on y , and a third function Z that depends only on z :

$$\psi(x, y, z) = X(x)Y(y)Z(z) \quad (41.8)$$

If we substitute Eq. (41.8) into Eq. (41.7), we get

$$-\frac{\hbar^2}{2m} \left(Y(y)Z(z) \frac{d^2 X(x)}{dx^2} + X(x)Z(z) \frac{d^2 Y(y)}{dy^2} + X(x)Y(y) \frac{d^2 Z(z)}{dz^2} \right) = EX(x)Y(y)Z(z) \quad (41.9)$$

The partial derivatives in Eq. (41.7) have become ordinary derivatives since they act on functions of a single variable. Now we divide both sides of Eq. (41.9) by the product $X(x)Y(y)Z(z)$:

$$\left(-\frac{\hbar^2}{2m} \frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} \right) + \left(-\frac{\hbar^2}{2m} \frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} \right) + \left(-\frac{\hbar^2}{2m} \frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} \right) = E \quad (41.10)$$

The right-hand side of Eq. (41.10) is the energy of the stationary state, which does not and cannot depend on the values of x , y , or z . For this to be true, the left-hand side of the equation must also be independent of the values of x , y , and z . Hence the first term in parentheses on the left-hand side of Eq. (41.10) must equal a constant that doesn't depend on x , the second term in parentheses must equal another constant that doesn't depend on y , and the third term in parentheses must equal a third constant that doesn't depend on z . Let's call these constants E_X , E_Y , and E_Z , respectively. We then have a separate equation for each of the three functions $X(x)$, $Y(y)$, and $Z(z)$:

$$-\frac{\hbar^2}{2m} \frac{d^2 X(x)}{dx^2} = E_X X(x) \quad (41.11a)$$

$$-\frac{\hbar^2}{2m} \frac{d^2 Y(y)}{dy^2} = E_Y Y(y) \quad (41.11b)$$

$$-\frac{\hbar^2}{2m} \frac{d^2 Z(z)}{dz^2} = E_Z Z(z) \quad (41.11c)$$

To satisfy the boundary conditions that $\psi(x, y, z) = X(x)Y(y)Z(z)$ be equal to zero on the walls of the box, we demand that $X(x) = 0$ at $x = 0$ and $x = L$, $Y(y) = 0$ at $y = 0$ and $y = L$, and $Z(z) = 0$ at $z = 0$ and $z = L$.

How can we interpret the three constants E_X , E_Y , and E_Z in Eqs. (41.11)? From Eq. (41.10), they are related to the energy E by

$$E_X + E_Y + E_Z = E \quad (41.12)$$

Equation (41.12) should remind you of Eq. (41.1) in Section 41.1, which states that the kinetic energy of a particle is the sum of contributions coming from its x -, y -, and z -components of momentum. Hence the constants E_X , E_Y , and E_Z tell us how much of the particle's energy is due to motion along each of the three coordinate axes. (Inside the box the potential energy is zero, so the particle's energy is purely kinetic.)

Equations (41.11) represent an enormous simplification; we've reduced the problem of solving a fairly complex *partial* differential equation with three independent variables to the much simpler problem of solving three separate *ordinary* differential equations with one independent variable each. What's more, each of these ordinary differential equations is just the same as the time-independent Schrödinger equation for a particle in a *one-dimensional* box, Eq. (40.25), and with exactly the same boundary conditions at 0 and L . (The only differences are that some of the quantities are labeled by different symbols.) By comparing with our work in Section 40.2, you can see that the solutions to Eqs. (41.11) are

$$X_{n_X}(x) = C_X \sin \frac{n_X \pi x}{L} \quad (n_X = 1, 2, 3, \dots) \quad (41.13a)$$

$$Y_{n_Y}(y) = C_Y \sin \frac{n_Y \pi y}{L} \quad (n_Y = 1, 2, 3, \dots) \quad (41.13b)$$

$$Z_{n_Z}(z) = C_Z \sin \frac{n_Z \pi z}{L} \quad (n_Z = 1, 2, 3, \dots) \quad (41.13c)$$

where C_X , C_Y , and C_Z are constants. The corresponding values of E_X , E_Y , and E_Z are

$$E_X = \frac{n_X^2 \pi^2 \hbar^2}{2mL^2} \quad (n_X = 1, 2, 3, \dots) \quad (41.14a)$$

$$E_Y = \frac{n_Y^2 \pi^2 \hbar^2}{2mL^2} \quad (n_Y = 1, 2, 3, \dots) \quad (41.14b)$$

$$E_Z = \frac{n_Z^2 \pi^2 \hbar^2}{2mL^2} \quad (n_Z = 1, 2, 3, \dots) \quad (41.14c)$$

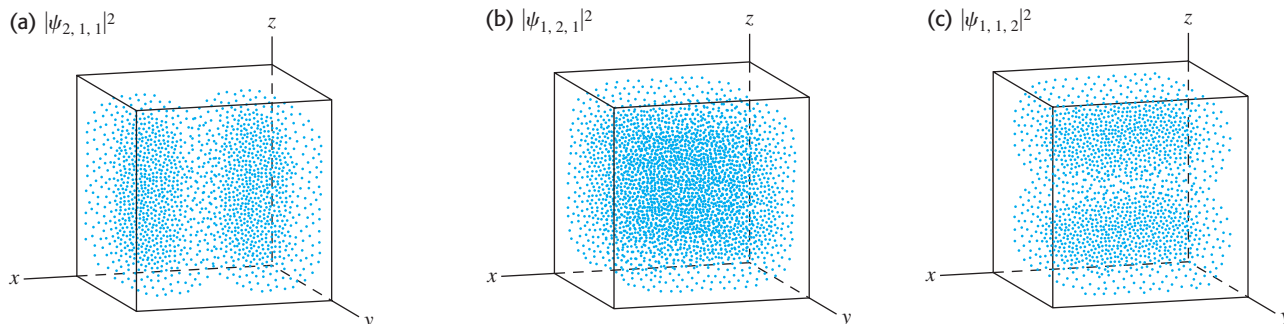
There is only one quantum number n for the one-dimensional particle in a box, but *three* quantum numbers n_X , n_Y , and n_Z for the three-dimensional box. If we substitute Eqs. (41.13) back into Eq. (41.8) for the total spatial wave function, $\psi(x, y, z) = X(x)Y(y)Z(z)$, we get the following stationary-state wave functions for a particle in a three-dimensional cubical box:

$$\psi_{n_X, n_Y, n_Z}(x, y, z) = C \sin \frac{n_X \pi x}{L} \sin \frac{n_Y \pi y}{L} \sin \frac{n_Z \pi z}{L} \\ (n_X = 1, 2, 3, \dots; n_Y = 1, 2, 3, \dots; n_Z = 1, 2, 3, \dots) \quad (41.15)$$

where $C = C_X C_Y C_Z$. The value of the constant C is determined by the normalization condition, Eq. (41.6).

In Section 40.2 we saw that the stationary-state wave functions for a particle in a one-dimensional box were analogous to standing waves on a string. In a similar way, the *three-dimensional* wave functions given by Eq. (41.15) are analogous to standing electromagnetic waves in a cubical cavity like the interior of a microwave oven (see Section 32.5). In a microwave oven there are “dead spots” where the wave intensity is zero, corresponding to the nodes of the standing wave. (The rotating platform in a microwave oven ensures even cooking by making sure that no part of the food sits at any “dead spot.”) In a similar fashion, the probability distribution function corresponding to Eq. (41.15) can have “dead

41.2 Probability distribution function $|\psi_{n_x, n_y, n_z}(x, y, z)|^2$ for (n_x, n_y, n_z) equal to (a) (2, 1, 1), (b) (1, 2, 1), and (c) (1, 1, 2). The value of $|\psi|^2$ is proportional to the density of dots. The wave function is zero on the walls of the box and on the midplane of the box, so $|\psi|^2 = 0$ at these locations.



spots” where there is zero probability of finding the particle. As an example, consider the case $(n_x, n_y, n_z) = (2, 1, 1)$. From Eq. (41.15), the probability distribution function for this case is

$$|\psi_{2,1,1}(x, y, z)|^2 = |C|^2 \sin^2 \frac{2\pi x}{L} \sin^2 \frac{\pi y}{L} \sin^2 \frac{\pi z}{L}$$

As Fig. 41.2a shows, the probability distribution function is zero on the plane $x = L/2$, where $\sin^2(2\pi x/L) = \sin^2 \pi = 0$. The particle is most likely to be found near where all three of the sine-squared functions are greatest, at $(x, y, z) = (L/4, L/2, L/2)$ or $(x, y, z) = (3L/4, L/2, L/2)$. Figures 41.2b and 41.2c show the similar cases $(n_x, n_y, n_z) = (1, 2, 1)$ and $(n_x, n_y, n_z) = (1, 1, 2)$. For higher values of the quantum numbers n_x , n_y , and n_z there are additional planes on which the probability distribution function equals zero, just as the probability distribution function $|\psi(x)|^2$ for a one-dimensional box has more zeros for higher values of n (see Fig. 40.12).

Example 41.1 Probability in a three-dimensional box

(a) Find the value of the constant C that normalizes the wave function of Eq. (41.15). (b) Find the probability that the particle will be found somewhere in the region $0 \leq x \leq L/4$ (Fig. 41.3) for the cases (i) $(n_x, n_y, n_z) = (1, 2, 1)$, (ii) $(n_x, n_y, n_z) = (2, 1, 1)$, and (iii) $(n_x, n_y, n_z) = (3, 1, 1)$.

SOLUTION

IDENTIFY and SET UP: Equation (41.6) tells us that to normalize the wave function, we have to choose the value of C so that the

integral of the probability distribution function $|\psi_{n_x, n_y, n_z}(x, y, z)|^2$ over the volume within the box equals 1. (The integral is actually over *all* space, but the particle-in-a-box wave functions are zero outside the box.)

The probability of finding the particle within a certain volume within the box equals the integral of the probability distribution function over that volume. Hence in part (b) we’ll integrate $|\psi_{n_x, n_y, n_z}(x, y, z)|^2$ for the given values of (n_x, n_y, n_z) over the volume $0 \leq x \leq L/4$, $0 \leq y \leq L$, $0 \leq z \leq L$.

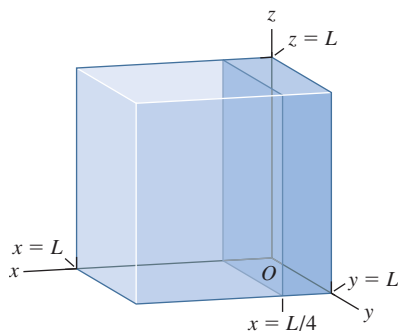
EXECUTE: (a) From Eq. (41.15),

$$|\psi_{n_x, n_y, n_z}(x, y, z)|^2 = |C|^2 \sin^2 \frac{n_x \pi x}{L} \sin^2 \frac{n_y \pi y}{L} \sin^2 \frac{n_z \pi z}{L}$$

Hence the normalization condition is

$$\begin{aligned} & \int |\psi_{n_x, n_y, n_z}(x, y, z)|^2 dV \\ &= |C|^2 \int_{x=0}^{x=L} \int_{y=0}^{y=L} \int_{z=0}^{z=L} \sin^2 \frac{n_x \pi x}{L} \sin^2 \frac{n_y \pi y}{L} \sin^2 \frac{n_z \pi z}{L} dx dy dz \\ &= |C|^2 \left(\int_{x=0}^{x=L} \sin^2 \frac{n_x \pi x}{L} dx \right) \left(\int_{y=0}^{y=L} \sin^2 \frac{n_y \pi y}{L} dy \right) \\ & \quad \times \left(\int_{z=0}^{z=L} \sin^2 \frac{n_z \pi z}{L} dz \right) = 1 \end{aligned}$$

41.3 What is the probability that the particle is in the dark-colored quarter of the box?



Continued

We can use the identity $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ and the variable substitution $\theta = n_X\pi x/L$ to show that

$$\begin{aligned}\int \sin^2 \frac{n_X\pi x}{L} dx &= \frac{L}{2n_X\pi} \left[\frac{n_X\pi x}{L} - \frac{1}{2} \sin\left(\frac{2n_X\pi x}{L}\right) \right] \\ &= \frac{x}{2} - \frac{L}{4n_X\pi} \sin\left(\frac{2n_X\pi x}{L}\right)\end{aligned}$$

If we evaluate this integral between $x = 0$ and $x = L$, the result is $L/2$ (recall that $\sin 0 = 0$ and $\sin 2n_X\pi = 0$ for any integer n_X). The y - and z -integrals each yield the same result, so the normalization condition is

$$|C|^2 \left(\frac{L}{2}\right) \left(\frac{L}{2}\right) \left(\frac{L}{2}\right) = |C|^2 \left(\frac{L}{2}\right)^3 = 1$$

or $|C|^2 = (2/L)^3$. If we choose C to be real and positive, then $C = (2/L)^{3/2}$.

(b) We have the same y - and z -integrals as in part (a), but now the limits of integration on the x -integral are $x = 0$ and $x = L/4$:

$$\begin{aligned}P &= \int_{0 \leq x \leq L/4} |\psi_{n_X, n_Y, n_Z}|^2 dV = |C|^2 \left(\int_{x=0}^{x=L/4} \sin^2 \frac{n_X\pi x}{L} dx \right) \\ &\quad \times \left(\int_{y=0}^{y=L} \sin^2 \frac{n_Y\pi y}{L} dy \right) \left(\int_{z=0}^{z=L} \sin^2 \frac{n_Z\pi z}{L} dz \right)\end{aligned}$$

The x -integral is

$$\begin{aligned}\int_{x=0}^{x=L/4} \sin^2 \frac{n_X\pi x}{L} dx &= \left(\frac{x}{2} - \frac{L}{4n_X\pi} \sin\left(\frac{2n_X\pi x}{L}\right) \right) \Big|_{x=0}^{x=L/4} \\ &= \frac{L}{8} - \frac{L}{4n_X\pi} \sin\left(\frac{n_X\pi}{2}\right)\end{aligned}$$

Hence the probability of finding the particle somewhere in the region $0 \leq x \leq L/4$ is

$$\begin{aligned}P &= \left(\frac{2}{L}\right)^3 \left(\frac{L}{8} - \frac{L}{4n_X\pi} \sin\left(\frac{n_X\pi}{2}\right)\right) \left(\frac{L}{2}\right) \left(\frac{L}{2}\right) \\ &= \frac{1}{4} - \frac{1}{2n_X\pi} \sin\left(\frac{n_X\pi}{2}\right)\end{aligned}$$

This depends only on the value of n_X , not on n_Y or n_Z . Hence for the three cases we have

$$\begin{aligned}\text{(i) } n_X = 1: P &= \frac{1}{4} - \frac{1}{2(1)\pi} \sin\left(\frac{\pi}{2}\right) = \frac{1}{4} - \frac{1}{2\pi}(1) \\ &= \frac{1}{4} - \frac{1}{2\pi} = 0.091\end{aligned}$$

$$\begin{aligned}\text{(ii) } n_X = 2: P &= \frac{1}{4} - \frac{1}{2(2)\pi} \sin\left(\frac{2\pi}{2}\right) = \frac{1}{4} - \frac{1}{4\pi} \sin \pi \\ &= \frac{1}{4} - 0 = 0.250\end{aligned}$$

$$\begin{aligned}\text{(iii) } n_X = 3: P &= \frac{1}{4} - \frac{1}{2(3)\pi} \sin\left(\frac{3\pi}{2}\right) = \frac{1}{4} - \frac{1}{6\pi}(-1) \\ &= \frac{1}{4} + \frac{1}{6\pi} = 0.303\end{aligned}$$

EVALUATE: You can see why the probabilities in part (b) are different by looking at part (b) of Fig. 40.12, which shows $\sin^2 n_X\pi x/L$ for $n_X = 1, 2$, and 3 . For $n_X = 2$ the area under the curve between $x = 0$ and $x = L/4$ (equal to the integral between these two points) is exactly $\frac{1}{4}$ of the total area between $x = 0$ and $x = L$. For $n_X = 1$ the area between $x = 0$ and $x = L/4$ is less than $\frac{1}{4}$ of the total area, and for $n_X = 3$ it is greater than $\frac{1}{4}$ of the total area.

Energy Levels, Degeneracy, and Symmetry

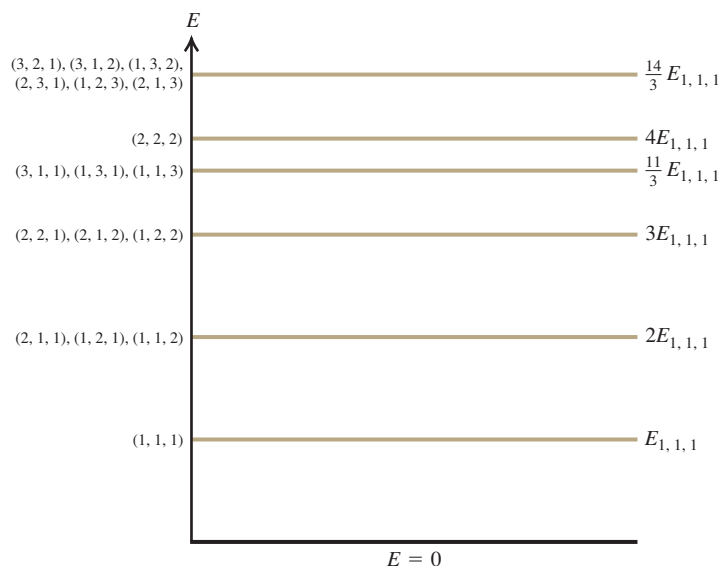
From Eqs. (41.12) and (41.14), the allowed energies for a particle of mass m in a cubical box of side L are

$$E_{n_X, n_Y, n_Z} = \frac{(n_X^2 + n_Y^2 + n_Z^2)\pi^2\hbar^2}{2mL^2} \quad (n_X = 1, 2, 3, \dots; n_Y = 1, 2, 3, \dots; n_Z = 1, 2, 3, \dots) \quad (41.16)$$

(energy levels, particle in a three-dimensional cubical box)

Figure 41.4 shows the six lowest energy levels given by Eq. (41.16). Note that most energy levels correspond to more than one set of quantum numbers (n_X, n_Y, n_Z) and hence to more than one quantum state. Having two or more distinct quantum states with the same energy is called **degeneracy**, and states with the same energy are said to be **degenerate**. For example, Fig. 41.4 shows that the states $(n_X, n_Y, n_Z) = (2, 1, 1), (1, 2, 1),$ and $(1, 1, 2)$ are degenerate. By comparison, for a particle in a one-dimensional box there is just one state for each energy level (see Fig. 40.11a) and no degeneracy.

The reason the cubical box exhibits degeneracy is that it is *symmetric*: All sides of the box have the same dimensions. As an illustration, Fig. 41.2 shows the probability distribution functions for the three states $(n_X, n_Y, n_Z) = (2, 1, 1), (1, 2, 1),$ and $(1, 1, 2)$. You can transform any one of these three states into a different one by simply rotating the cubical box by 90° . This rotation doesn't change the energy, so the three states are degenerate.



41.4 Energy-level diagram for a particle in a three-dimensional cubical box. We label each level with the quantum numbers of the states (n_x, n_y, n_z) with that energy. Several of the levels are degenerate (more than one state has the same energy). The lowest (ground) level, $(n_x, n_y, n_z) = (1, 1, 1)$, has energy $E_{1,1,1} = (1^2 + 1^2 + 1^2)\pi^2\hbar^2/2mL^2 = 3\pi^2\hbar^2/2mL^2$; we show the energies of the other levels as multiples of $E_{1,1,1}$.

Since degeneracy is a consequence of symmetry, we can remove the degeneracy by making the box asymmetric. We do this by giving the three sides of the box different lengths L_x , L_y , and L_z . If we repeat the steps that we followed to solve the time-independent Schrödinger equation, we find that the energy levels are given by

$$E_{n_x, n_y, n_z} = \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right) \frac{\pi^2 \hbar^2}{2m} \quad (n_x = 1, 2, 3, \dots; n_y = 1, 2, 3, \dots; n_z = 1, 2, 3, \dots) \quad (41.17)$$

(energy levels, particle in a three-dimensional box with sides of length L_x , L_y , and L_z)

If L_x , L_y , and L_z are all different, the states $(n_x, n_y, n_z) = (2, 1, 1)$, $(1, 2, 1)$, and $(1, 1, 2)$ have different energies and hence are no longer degenerate. Note that Eq. (41.17) reduces to Eq. (41.16) if the lengths are all the same ($L_x = L_y = L_z = L$).

Let's summarize the key differences between the three-dimensional particle in a box and the one-dimensional case that we examined in Section 40.2:

- We can write the wave function for a three-dimensional stationary state as a product of three functions, one for each spatial coordinate. Only a single function of the coordinate x is needed in one dimension.
- In the three-dimensional case, three quantum numbers are needed to describe each stationary state. Only one quantum number is needed in the one-dimensional case.
- Most of the energy levels for the three-dimensional case are degenerate: More than one stationary state has this energy. There is no degeneracy in the one-dimensional case.
- For a stationary state of the three-dimensional case, there are surfaces on which the probability distribution function $|\psi|^2$ is zero. In the one-dimensional case there are positions on the x -axis where $|\psi|^2$ is zero.

We'll see these same features in the following section as we examine a three-dimensional situation that's more realistic than a particle in a box: a hydrogen atom in which a negatively charged electron orbits a positively charged nucleus.

Test Your Understanding of Section 41.2 Rank the following states of a particle in a cubical box of side L in order from highest to lowest energy: (i) $(n_x, n_y, n_z) = (2, 3, 2)$; (ii) $(n_x, n_y, n_z) = (4, 1, 1)$; (iii) $(n_x, n_y, n_z) = (2, 2, 3)$; (iv) $(n_x, n_y, n_z) = (1, 3, 3)$.

41.3 The Hydrogen Atom

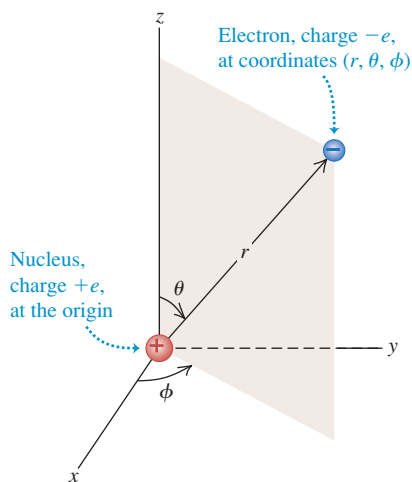
Let's continue the discussion of the hydrogen atom that we began in Chapter 39. In the Bohr model, electrons move in circular orbits like Newtonian particles, but with quantized values of angular momentum. While this model gave the correct energy levels of the hydrogen atom, as deduced from spectra, it had many conceptual difficulties. It mixed classical physics with new and seemingly contradictory concepts. It provided no insight into the process by which photons are emitted and absorbed. It could not be generalized to atoms with more than one electron. It predicted the wrong magnetic properties for the hydrogen atom. And perhaps most important, its picture of the electron as a localized point particle was inconsistent with the more general view we developed in Chapters 39 and 40. To go beyond the Bohr model, let's apply the Schrödinger equation to find the wave functions for stationary states (states of definite energy) of the hydrogen atom. As in Section 39.3, we include the motion of the nucleus by simply replacing the electron mass m with the reduced mass m_r .

The Schrödinger Equation for the Hydrogen Atom

We discussed the three-dimensional version of the Schrödinger equation in Section 41.1. The potential-energy function is *spherically symmetric*: It depends only on the distance $r = (x^2 + y^2 + z^2)^{1/2}$ from the origin of coordinates:

$$U(r) = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad (41.18)$$

41.5 The Schrödinger equation for the hydrogen atom can be solved most readily using spherical coordinates.



The hydrogen-atom problem is best formulated in spherical coordinates (r, θ, ϕ) , shown in Fig. 41.5; the spherically symmetric potential-energy function depends only on r , not on θ or ϕ . The Schrödinger equation with this potential-energy function can be solved exactly; the solutions are combinations of familiar functions. Without going into a lot of detail, we can describe the most important features of the procedure and the results.

First, we find the solutions using the same method of separation of variables that we employed for a particle in a cubical box in Section 41.2. We express the wave function $\psi(r, \theta, \phi)$ as a product of three functions, each one a function of only one of the three coordinates:

$$\psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi) \quad (41.19)$$

That is, the function $R(r)$ depends only on r , $\Theta(\theta)$ depends only on θ , and $\Phi(\phi)$ depends only on ϕ . Just as for a particle in a three-dimensional box, when we substitute Eq. (41.19) into the Schrödinger equation, we get three separate ordinary differential equations. One equation involves only r and $R(r)$, a second involves only θ and $\Theta(\theta)$, and a third involves only ϕ and $\Phi(\phi)$:

$$-\frac{\hbar^2}{2m_r r^2} \frac{d}{dr} \left(r^2 \frac{dR(r)}{dr} \right) + \left(\frac{\hbar^2 l(l+1)}{2m_r r^2} + U(r) \right) R(r) = ER(r) \quad (41.20a)$$

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d\Theta(\theta)}{d\theta} \right) + \left(l(l+1) - \frac{m_l^2}{\sin^2 \theta} \right) \Theta(\theta) = 0 \quad (41.20b)$$

$$\frac{d^2\Phi(\phi)}{d\phi^2} + m_l^2 \Phi(\phi) = 0 \quad (41.20c)$$

In Eqs. (41.20) E is the energy of the stationary state and l and m_l are constants that we'll discuss later. (Be careful! Don't confuse the constant m_l with the reduced mass m_r .)

We won't attempt to solve this set of three equations, but we can describe how it's done. As for the particle in a cubical box, the physically acceptable solutions of these three equations are determined by boundary conditions. The radial function $R(r)$ in Eq. (41.20a) must approach zero at large r , because we are describing

bound states of the electron that are localized near the nucleus. This is analogous to the requirement that the harmonic-oscillator wave functions (see Section 40.5) must approach zero at large x . The angular functions $\Theta(\theta)$ and $\Phi(\phi)$ in Eqs. (41.20b) and (41.20c) must be *finite* for all relevant values of the angles. For example, there are solutions of the Θ equation that become infinite at $\theta = 0$ and $\theta = \pi$; these are unacceptable, since $\psi(r, \theta, \phi)$ must be normalizable. Furthermore, the angular function $\Phi(\phi)$ in Eq. (41.20c) must be *periodic*. For example, (r, θ, ϕ) and $(r, \theta, \phi + 2\pi)$ describe the same point, so $\Phi(\phi + 2\pi)$ must equal $\Phi(\phi)$.

The allowed radial functions $R(r)$ turn out to be an exponential function $e^{-\alpha r}$ (where α is positive) multiplied by a polynomial in r . The functions $\Theta(\theta)$ are polynomials containing various powers of $\sin\theta$ and $\cos\theta$, and the functions $\Phi(\phi)$ are simply proportional to $e^{im_l\phi}$, where $i = \sqrt{-1}$ and m_l is an integer that may be positive, zero, or negative.

In the process of finding solutions that satisfy the boundary conditions, we also find the corresponding energy levels. We denote the energies of these levels [E in Eq. (41.20a)] by E_n ($n = 1, 2, 3, \dots$). These turn out to be *identical* to those from the Bohr model, as given by Eq. (39.15), with the electron rest mass m replaced by the reduced mass m_r . Rewriting that equation using $\hbar = h/2\pi$, we have

$$E_n = -\frac{1}{(4\pi\epsilon_0)^2} \frac{m_r e^4}{2n^2\hbar^2} = -\frac{13.60 \text{ eV}}{n^2} \quad (\text{energy levels of hydrogen}) \quad (41.21)$$

As in Section 39.3, we call n the **principal quantum number** for the level of energy E_n .

Equation (41.21) is an important validation of our Schrödinger-equation analysis of the hydrogen atom. The Schrödinger analysis is quite different from the Bohr model, both mathematically and conceptually, yet both yield the same energy-level scheme—a scheme that agrees with the energies determined from spectra. As we will see, the Schrödinger analysis can explain many more aspects of the hydrogen atom than can the Bohr model.

Quantization of Orbital Angular Momentum

The solutions to Eqs. (41.20) that satisfy the boundary conditions mentioned above also have quantized values of *orbital angular momentum*. That is, only certain discrete values of the magnitude and components of orbital angular momentum are permitted. In discussing the Bohr model in Section 39.3, we mentioned that quantization of angular momentum was a result with little fundamental justification. With the Schrödinger equation it appears automatically.

The possible values of the magnitude L of orbital angular momentum \vec{L} are determined by the requirement that the $\Theta(\theta)$ function in Eq. (41.20b) must be finite at $\theta = 0$ and $\theta = \pi$. In a level with energy E_n and principal quantum number n , the possible values of L are

$$L = \sqrt{l(l+1)}\hbar \quad (l = 0, 1, 2, \dots, n-1) \quad (\text{magnitude of orbital angular momentum}) \quad (41.22)$$

The *orbital angular-momentum quantum number* l , which is the same l that appears in Eqs. (41.20a) and (41.20b), is called the **orbital quantum number** for short. In the Bohr model, each energy level corresponded to a single value of angular momentum. Equation (41.22) shows that in fact there are n different possible values of L for the n th energy level.

An interesting feature of Eq. (41.22) is that the orbital angular momentum is *zero* for $l = 0$ states. This result disagrees with the Bohr model, in which the electron always moved in a circle of definite radius and L was never zero. The $l = 0$ wave functions ψ depend only on r ; for these states, the functions $\Theta(\theta)$ and $\Phi(\phi)$ are constants. Thus the wave functions for $l = 0$ states are spherically

symmetric. There is nothing in their probability distribution $|\psi|^2$ to favor one direction over any other, and there is no orbital angular momentum.

The permitted values of the *component* of \vec{L} in a given direction, say the z -component L_z , are determined by the requirement that the $\Phi(\phi)$ function must equal $\Phi(\phi + 2\pi)$. The possible values of L_z are

$$L_z = m_l \hbar \quad (m_l = 0, \pm 1, \pm 2, \dots, \pm l) \quad \begin{array}{l} \text{(components of} \\ \text{orbital angular} \\ \text{momentum)} \end{array} \quad (41.23)$$

The quantum number m_l is the same as that in Eqs. (41.20b) and (41.20c). We see that m_l can be zero or a positive or negative integer up to, but no larger in magnitude than, l . That is, $|m_l| \leq l$. For example, if $l = 1$, m_l can equal 1, 0, or -1 . For reasons that will emerge later, we call m_l the *orbital magnetic quantum number*, or **magnetic quantum number** for short.

The component L_z can never be quite as large as L (unless both are zero). For example, when $l = 2$, the largest possible value of m_l is also 2; then Eqs. (41.22) and (41.23) give

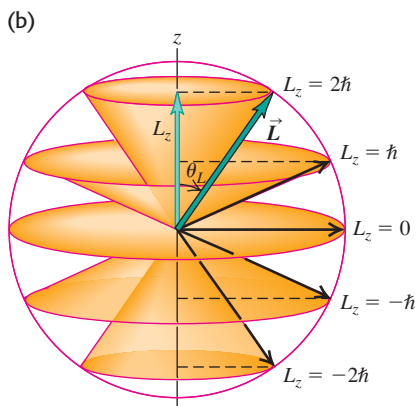
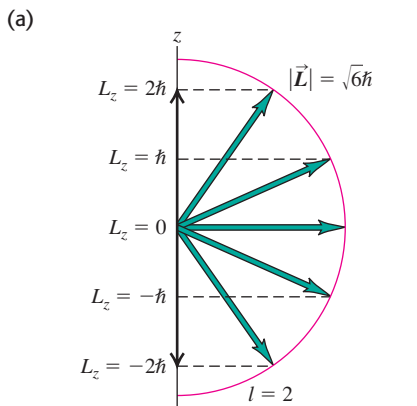
$$L = \sqrt{2(2 + 1)}\hbar = \sqrt{6}\hbar = 2.45\hbar$$

$$L_z = 2\hbar$$

Figure 41.6 shows the situation. The minimum value of the angle θ_L between the vector \vec{L} and the z -axis is given by

$$\theta_L = \arccos \frac{L_z}{L} = \arccos \frac{2}{2.45} = 35.3^\circ$$

41.6 (a) When $l = 2$, the magnitude of the angular momentum vector \vec{L} is $\sqrt{6}\hbar = 2.45\hbar$, but \vec{L} does not have a definite direction. In this semiclassical vector picture, \vec{L} makes an angle of 35.3° with the z -axis when the z -component has its maximum value of $2\hbar$. (b) These cones show the possible directions of \vec{L} for different values of L_z .



That $|L_z|$ is always less than L is also required by the uncertainty principle. Suppose we could know the precise *direction* of the orbital angular momentum vector. Then we could let that be the direction of the z -axis, and L_z would equal L . This corresponds to a particle moving in the xy -plane only, in which case the z -component of the linear momentum \vec{p} would be zero with no uncertainty Δp_z . Then the uncertainty principle $\Delta z \Delta p_z \geq \hbar$ requires infinite uncertainty Δz in the coordinate z . This is impossible for a localized state; we conclude that we can't know the direction of \vec{L} precisely. Thus, as we've already stated, the component of \vec{L} in a given direction can never be quite as large as its magnitude L . Also, if we can't know the direction of \vec{L} precisely, we can't determine the components L_x and L_y precisely. Thus we show *cones* of possible directions for \vec{L} in Fig. 41.6b.

You may wonder why we have singled out the z -axis for special attention. There's no fundamental reason for this; the atom certainly doesn't care what coordinate system we use. The point is that we can't determine all three components of orbital angular momentum with certainty, so we arbitrarily pick one as the component we want to measure. When we discuss interactions of the atom with a magnetic field, we will consistently choose the positive z -axis to be in the direction of \vec{B} .

Quantum Number Notation

The wave functions for the hydrogen atom are determined by the values of three quantum numbers n , l , and m_l . (Compare this to the particle in a three-dimensional box that we considered in Section 41.2. There, too, three quantum numbers were needed to describe each stationary state.) The energy E_n is determined by the principal quantum number n according to Eq. (41.21). The magnitude of orbital angular momentum is determined by the orbital quantum number l , as in Eq. (41.22). The component of orbital angular momentum in a specified axis direction (customarily the z -axis) is determined by the magnetic quantum number m_l , as in Eq. (41.23). The energy does not depend on the values of l or m_l (Fig. 41.7), so for each energy level E_n given by Eq. (41.21), there is more than one distinct state having the same energy but different quantum numbers. That is, these states are *degenerate*, just like most of the states of a particle in a

three-dimensional box. As for the three-dimensional box, degeneracy arises because the hydrogen atom is symmetric: If you rotate the atom through any angle, the potential-energy function at a distance r from the nucleus has the same value.

States with various values of the orbital quantum number l are often labeled with letters, according to the following scheme:

$l = 0$: s states

$l = 1$: p states

$l = 2$: d states

$l = 3$: f states

$l = 4$: g states

$l = 5$: h states

and so on alphabetically. This seemingly irrational choice of the letters s , p , d , and f originated in the early days of spectroscopy and has no fundamental significance. In an important form of *spectroscopic notation* that we'll use often, a state with $n = 2$ and $l = 1$ is called a $2p$ state; a state with $n = 4$ and $l = 0$ is a $4s$ state; and so on. Only s states ($l = 0$) are spherically symmetric.

Here's another bit of notation. The radial extent of the wave functions increases with the principal quantum number n , and we can speak of a region of space associated with a particular value of n as a **shell**. Especially in discussions of many-electron atoms, these shells are denoted by capital letters:

$n = 1$: K shell

$n = 2$: L shell

$n = 3$: M shell

$n = 4$: N shell

and so on alphabetically. For each n , different values of l correspond to different *subshells*. For example, the L shell ($n = 2$) contains the $2s$ and $2p$ subshells.

Table 41.1 shows some of the possible combinations of the quantum numbers n , l , and m_l for hydrogen-atom wave functions. The spectroscopic notation and the shell notation for each are also shown.

Table 41.1 Quantum States of the Hydrogen Atom

n	l	m_l	Spectroscopic Notation	Shell
1	0	0	$1s$	K
2	0	0	$2s$	L
2	1	$-1, 0, 1$	$2p$	
3	0	0	$3s$	M
3	1	$-1, 0, 1$	$3p$	
3	2	$-2, -1, 0, 1, 2$	$3d$	
4	0	0	$4s$	N

and so on

41.7 The energy for an orbiting satellite such as the Hubble Space Telescope depends on the average distance between the satellite and the center of the earth. It does *not* depend on whether the orbit is circular (with a large orbital angular momentum L) or elliptical (in which case L is smaller). In the same way, the energy of a hydrogen atom does not depend on the orbital angular momentum.



Problem-Solving Strategy 41.1 Atomic Structure



IDENTIFY *the relevant concepts:* Many problems in atomic structure can be solved simply by reference to the quantum numbers n , l , and m_l that describe the total energy E , the magnitude of the orbital angular momentum \vec{L} , the z -component of \vec{L} , and other properties of an atom.

SET UP *the problem:* Identify the target variables and choose the appropriate equations, which may include Eqs. (41.21), (41.22), and (41.23).

EXECUTE *the solution* as follows:

1. Be sure you understand the possible values of the quantum numbers n , l , and m_l for the hydrogen atom. They are all integers; n is always greater than zero, l can be zero or positive up

to $n - 1$, and m_l can range from $-l$ to l . You should know how to count the number of (n, l, m_l) states in each shell (K , L , M , and so on) and subshell ($3s$, $3p$, $3d$, and so on). Be able to *construct* Table 41.1, not just to write it from memory.

2. Solve for the target variables.

EVALUATE *your answer:* It helps to be familiar with typical magnitudes in atomic physics. For example, the electric potential energy of a proton and electron 0.10 nm apart (typical of atomic dimensions) is about -15 eV. Visible light has wavelengths around 500 nm and frequencies around 5×10^{14} Hz. Problem-Solving Strategy 39.1 (Section 39.1) gives other typical magnitudes.

Example 41.2 Counting hydrogen states

How many distinct (n, l, m_l) states of the hydrogen atom with $n = 3$ are there? What are their energies?

SOLUTION

IDENTIFY and SET UP: This problem uses the relationships among the principal quantum number n , orbital quantum number l , magnetic quantum number m_l , and energy of a state for the hydrogen atom. We use the rule that l can have n integer values, from 0 to $n - 1$, and that m_l can have $2l + 1$ values, from $-l$ to l . Equation (41.21) gives the energy of any particular state.

EXECUTE: When $n = 3$, l can be 0, 1, or 2. When $l = 0$, m_l can be only 0 (1 state). When $l = 1$, m_l can be $-1, 0$, or 1 (3 states). When $l = 2$, m_l can be $-2, -1, 0, 1$, or 2 (5 states). The total number of

(n, l, m_l) states with $n = 3$ is therefore $1 + 3 + 5 = 9$. (In Section 41.5 we'll find that the total number of $n = 3$ states is in fact twice this, or 18, because of electron spin.)

The energy of a hydrogen-atom state depends only on n , so all 9 of these states have the same energy. From Eq. (41.21),

$$E_3 = \frac{-13.60 \text{ eV}}{3^2} = -1.51 \text{ eV}$$

EVALUATE: For a given value of n , the total number of (n, l, m_l) states turns out to be n^2 . In this case $n = 3$ and there are $3^2 = 9$ states. Remember that the ground level of hydrogen has $n = 1$ and $E_1 = -13.6 \text{ eV}$; the $n = 3$ excited states have a higher (less negative) energy.

Example 41.3 Angular momentum in an excited level of hydrogen

Consider the $n = 4$ states of hydrogen. (a) What is the maximum magnitude L of the orbital angular momentum? (b) What is the maximum value of L_z ? (c) What is the minimum angle between \vec{L} and the z -axis? Give your answers to parts (a) and (b) in terms of \hbar .

SOLUTION

IDENTIFY and SET UP: We again need to relate the principal quantum number n and the orbital quantum number l for a hydrogen atom. We also need to relate the value of l and the magnitude and possible directions of the orbital angular momentum vector. We'll use Eq. (41.22) in part (a) to determine the maximum value of L ; then we'll use Eq. (41.23) in part (b) to determine the maximum value of L_z . The angle between \vec{L} and the z -axis is minimum when L_z is maximum (so that \vec{L} is most nearly aligned with the positive z -axis).

EXECUTE: (a) When $n = 4$, the maximum value of the orbital angular-momentum quantum number l is $(n - 1) = (4 - 1) = 3$; from Eq. (41.22),

$$L_{\max} = \sqrt{3(3 + 1)}\hbar = \sqrt{12}\hbar = 3.464\hbar$$

(b) For $l = 3$ the maximum value of m_l is 3. From Eq. (41.23),

$$(L_z)_{\max} = 3\hbar$$

(c) The *minimum* allowed angle between \vec{L} and the z -axis corresponds to the *maximum* allowed values of L_z and m_l (Fig. 41.6b shows an $l = 2$ example). For the state with $l = 3$ and $m_l = 3$,

$$\theta_{\min} = \arccos \frac{(L_z)_{\max}}{L} = \arccos \frac{3\hbar}{3.464\hbar} = 30.0^\circ$$

EVALUATE: As a check, you can verify that θ is greater than 30.0° for all states with smaller values of l .

Electron Probability Distributions

Rather than picturing the electron as a point particle moving in a precise circle, the Schrödinger equation gives an electron *probability distribution* surrounding the nucleus. The hydrogen-atom probability distributions are three-dimensional, so they are harder to visualize than the two-dimensional circular orbits of the Bohr model. It's helpful to look at the *radial probability distribution function* $P(r)$ —that is, the probability per radial length for the electron to be found at various distances from the proton. From Section 41.1 the probability for finding the electron in a small volume element dV is $|\psi|^2 dV$. (We assume that ψ is normalized in accordance with Eq. (41.6)—that is, that the integral of $|\psi|^2 dV$ over all space equals unity so that there is 100% probability of finding the electron somewhere in the universe.) Let's take as our volume element a thin spherical shell with inner radius r and outer radius $r + dr$. The volume dV of this shell is approximately its area $4\pi r^2$ multiplied by its thickness dr :

$$dV = 4\pi r^2 dr \quad (41.24)$$

We denote by $P(r) dr$ the probability of finding the particle within the radial range dr ; then, using Eq. (41.24),

$$P(r) dr = |\psi|^2 dV = |\psi|^2 4\pi r^2 dr \quad (\text{probability that the electron is between } r \text{ and } r + dr) \quad (41.25)$$

For wave functions that depend on θ and ϕ as well as r , we use the value of $|\psi|^2$ averaged over all angles in Eq. (41.25).

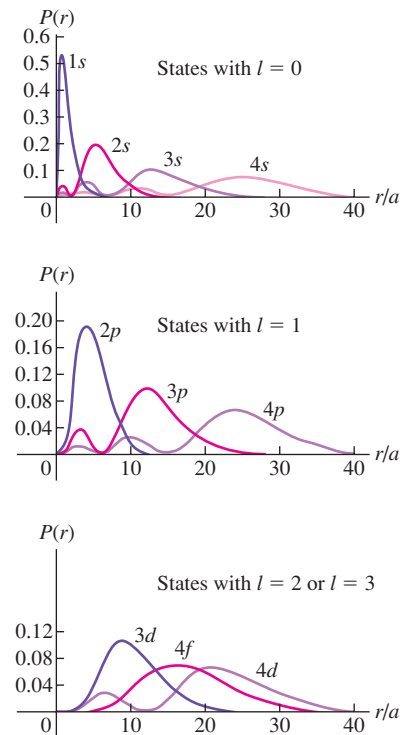
Figure 41.8 shows graphs of $P(r)$ for several hydrogen-atom wave functions. The r scales are labeled in multiples of a , the smallest distance between the electron and the nucleus in the Bohr model:

$$a = \frac{\epsilon_0 \hbar^2}{\pi m_e e^2} = \frac{4\pi \epsilon_0 \hbar^2}{m_e e^2} = 5.29 \times 10^{-11} \text{ m} \quad (\text{smallest } r, \text{ Bohr model}) \quad (41.26)$$

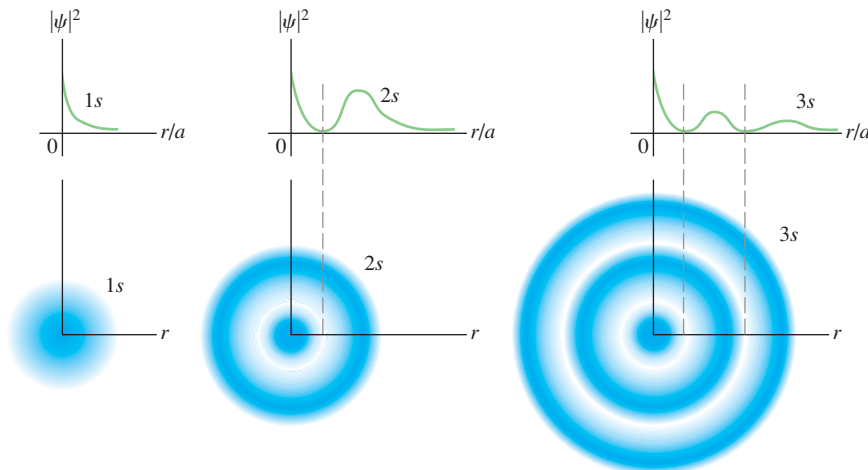
Just as for a particle in a cubical box (see Section 41.2), there are some locations where the probability is zero. These surfaces are planes for a particle in a box; for a hydrogen atom these are spherical surfaces (that is, surfaces of constant r). But again, the uncertainty principle tells us not to worry; we can't localize the electron exactly anyway. Note that for the states having the largest possible l for each n (such as $1s$, $2p$, $3d$, and $4f$ states), $P(r)$ has a single maximum at $n^2 a$. For these states, the electron is most likely to be found at the distance from the nucleus that is predicted by the Bohr model, $r = n^2 a$.

Figure 41.8 shows *radial* probability distribution functions $P(r) = 4\pi r^2 |\psi|^2$, which indicate the relative probability of finding the electron within a thin spherical shell of radius r . By contrast, Figs. 41.9 and 41.10 show the *three-dimensional* probability distribution functions $|\psi|^2$, which indicate the relative probability of finding the electron within a small box at a given position. The darker the blue “cloud,” the greater the value of $|\psi|^2$. (These are similar to the “clouds” shown in Fig. 41.2.) Figure 41.9 shows cross sections of the spherically symmetric probability clouds for the lowest three s subshells, for which $|\psi|^2$ depends only on the radial coordinate r . Figure 41.10 shows cross sections of the clouds for other electron states for which $|\psi|^2$ depends on both r and θ . For these states the probability distribution function is zero for certain values of θ as well as for certain values of r . In *any* stationary state of the hydrogen atom, $|\psi|^2$ is independent of ϕ .

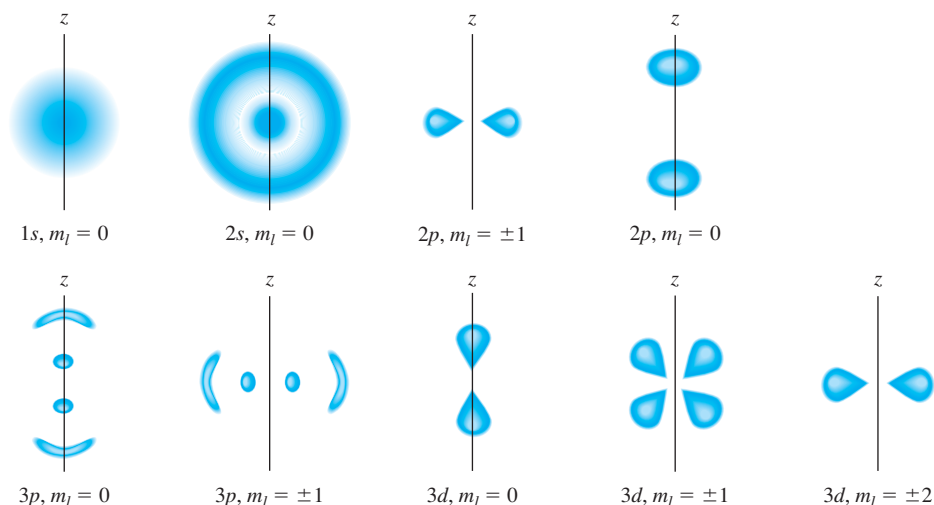
41.8 Radial probability distribution functions $P(r)$ for several hydrogen-atom wave functions, plotted as functions of the ratio r/a [see Eq. (41.26)]. For each function, the number of maxima is $(n - l)$. The curves for which $l = n - 1$ ($1s$, $2p$, $3d$, ...) have only one maximum, located at $r = n^2 a$.



41.9 Three-dimensional probability distribution functions $|\psi|^2$ for the spherically symmetric $1s$, $2s$, and $3s$ hydrogen-atom wave functions.



41.10 Cross sections of three-dimensional probability distributions for a few quantum states of the hydrogen atom. They are not to the same scale. Mentally rotate each drawing about the z -axis to obtain the three-dimensional representation of $|\psi|^2$. For example, the $2p, m_l = \pm 1$ probability distribution looks like a fuzzy donut.



Example 41.4 A hydrogen wave function

The ground-state wave function for hydrogen (a $1s$ state) is

$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

(a) Verify that this function is normalized. (b) What is the probability that the electron will be found at a distance less than a from the nucleus?

SOLUTION

IDENTIFY and SET UP: This example is similar to Example 41.1 in Section 41.2. We need to show that this wave function satisfies the condition that the probability of finding the electron *somewhere* is 1. We then need to find the probability that it will be found in the region $r < a$. In part (a) we'll carry out the integral $\int |\psi|^2 dV$ over all space; if it is equal to 1, the wave function is normalized. In part (b) we'll carry out the same integral over a spherical volume that extends from the origin (the nucleus) out to a distance a from the nucleus.

EXECUTE: (a) Since the wave function depends only on the radial coordinate r , we can choose our volume elements to be spherical shells of radius r , thickness dr , and volume dV given by Eq. (41.24). We then have

$$\begin{aligned} \int_{\text{all space}} |\psi_{1s}|^2 dV &= \int_0^\infty \frac{1}{\pi a^3} e^{-2r/a} (4\pi r^2 dr) \\ &= \frac{4}{a^3} \int_0^\infty r^2 e^{-2r/a} dr \end{aligned}$$

You can find the following indefinite integral in a table of integrals or by integrating by parts:

$$\int r^2 e^{-2r/a} dr = \left(-\frac{ar^2}{2} - \frac{a^2 r}{2} - \frac{a^3}{4} \right) e^{-2r/a}$$

Evaluating this between the limits $r = 0$ and $r = \infty$ is simple; it is zero at $r = \infty$ because of the exponential factor, and at $r = 0$ only the last term in the parentheses survives. Thus the value of the definite integral is $a^3/4$. Putting it all together, we find

$$\int_0^\infty |\psi_{1s}|^2 dV = \frac{4}{a^3} \int_0^\infty r^2 e^{-2r/a} dr = \frac{4}{a^3} \frac{a^3}{4} = 1$$

The wave function *is* normalized.

(b) To find the probability P that the electron is found within $r < a$, we carry out the same integration but with the limits 0 and a . We'll leave the details to you (Exercise 41.15). From the upper limit we get $-5e^{-2}a^3/4$; the final result is

$$\begin{aligned} P &= \int_0^a |\psi_{1s}|^2 4\pi r^2 dr = \frac{4}{a^3} \left(-\frac{5a^3 e^{-2}}{4} + \frac{a^3}{4} \right) \\ &= (-5e^{-2} + 1) = 1 - 5e^{-2} = 0.323 \end{aligned}$$

EVALUATE: Our results tell us that in a ground state we expect to find the electron at a distance from the nucleus less than a about $\frac{1}{3}$ of the time and at a greater distance about $\frac{2}{3}$ of the time. It's hard to tell, but in Fig. 41.8, about $\frac{2}{3}$ of the area under the $1s$ curve is at distances greater than a (that is, $r/a > 1$).

Hydrogenlike Atoms

Two generalizations that we discussed with the Bohr model in Section 39.3 are equally valid in the Schrödinger analysis. First, if the "atom" is not composed of a single proton and a single electron, using the reduced mass m_r of the system in Eqs. (41.21) and (41.26) will lead to changes that are substantial for some exotic

systems. One example is *positronium*, in which a positron and an electron orbit each other; another is a *muonic atom*, in which the electron is replaced by an unstable particle called a muon that has the same charge as an electron but is 207 times more massive. Second, our analysis is applicable to single-electron ions, such as He^+ , Li^{2+} , and so on. For such ions we replace e^2 by Ze^2 in Eqs. (41.21) and (41.26), where Z is the number of protons (the **atomic number**).

Test Your Understanding of Section 41.3 Rank the following states of the hydrogen atom in order from highest to lowest probability of finding the electron in the vicinity of $r = 5a$: (i) $n = 1, l = 0, m_l = 0$; (ii) $n = 2, l = 1, m_l = +1$; (iii) $n = 2, l = 1, m_l = 0$.



41.4 The Zeeman Effect

The **Zeeman effect** is the splitting of atomic energy levels and the associated spectral lines when the atoms are placed in a magnetic field (Fig. 41.11). This effect confirms experimentally the quantization of angular momentum. The discussion in this section, which assumes that the only angular momentum is the *orbital* angular momentum of a single electron, also shows why we call m_l the magnetic quantum number.

Atoms contain charges in motion, so it should not be surprising that magnetic forces cause changes in that motion and in the energy levels. As early as the middle of the 19th century, physicists speculated that the sources of visible light might be vibrating electric charge on an atomic scale. In 1896 the Dutch physicist Pieter Zeeman was the first to show that in the presence of a magnetic field, some spectral lines were split into groups of closely spaced lines (Fig. 41.12). This effect now bears his name.

Magnetic Moment of an Orbiting Electron

Let's begin our analysis of the Zeeman effect by reviewing the concept of *magnetic dipole moment* or *magnetic moment*, introduced in Section 27.7. A plane current loop with vector area \vec{A} carrying current I has a magnetic moment $\vec{\mu}$ given by

$$\vec{\mu} = I\vec{A} \quad (41.27)$$

When a magnetic dipole of moment $\vec{\mu}$ is placed in a magnetic field \vec{B} , the field exerts a torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ on the dipole. The potential energy U associated with this interaction is given by Eq. (27.27):

$$U = -\vec{\mu} \cdot \vec{B} \quad (41.28)$$

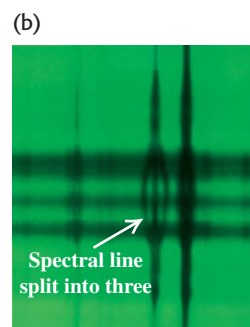
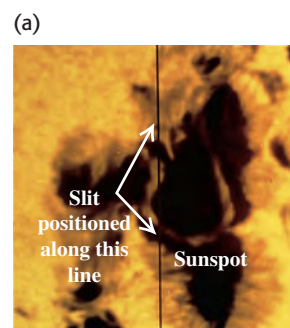
Now let's use Eqs. (41.27) and (41.28) and the Bohr model to look at the interaction of a hydrogen atom with a magnetic field. The orbiting electron with speed v is equivalent to a current loop with radius r and area πr^2 . The average current I is the average charge per unit time that passes a given point of the orbit. This is equal to the charge magnitude e divided by the time T for one revolution, given by $T = 2\pi r/v$. Thus $I = ev/2\pi r$, and from Eq. (41.27) the magnitude μ of the magnetic moment is

$$\mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2} \quad (41.29)$$

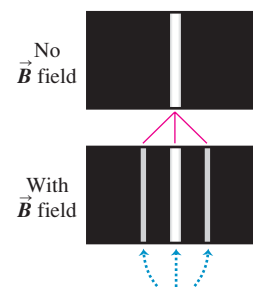
We can also express this in terms of the magnitude L of the orbital angular momentum. From Eq. (10.28) the angular momentum of a particle in a circular orbit is $L = mvr$, so Eq. (41.29) becomes

$$\mu = \frac{e}{2m}L \quad (41.30)$$

41.11 Magnetic effects on the spectrum of sunlight. (a) The slit of a spectrograph is positioned along the black line crossing a portion of a sunspot. (b) The 0.4-T magnetic field in the sunspot (a thousand times greater than the earth's field) splits the middle spectral line into three lines.



41.12 The normal Zeeman effect. Compare this to the magnetic splitting in the solar spectrum shown in Fig. 41.11b.



When an excited gas is placed in a magnetic field, the interaction of orbital magnetic moments with the field splits individual spectral lines of the gas into sets of three lines.

The ratio of the magnitude of $\vec{\mu}$ to the magnitude of \vec{L} is $\mu/L = e/2m$ and is called the *gyromagnetic ratio*.

In the Bohr model, $L = nh/2\pi = n\hbar$, where $n = 1, 2, \dots$. For an $n = 1$ state (a ground state), Eq. (41.30) becomes $\mu = (e/2m)\hbar$. This quantity is a natural unit for magnetic moment; it is called one **Bohr magneton**, denoted by μ_B :

$$\mu_B = \frac{e\hbar}{2m} \quad (\text{definition of the Bohr magneton}) \quad (41.31)$$

Evaluating Eq. (41.31) gives

$$\mu_B = 5.788 \times 10^{-5} \text{ eV/T} = 9.274 \times 10^{-24} \text{ J/T or A} \cdot \text{m}^2$$

Note that the units J/T and $\text{A} \cdot \text{m}^2$ are equivalent. We defined this quantity previously in Section 28.8.

While the Bohr model suggests that the orbital motion of an atomic electron gives rise to a magnetic moment, this model does *not* give correct predictions about magnetic interactions. As an example, the Bohr model predicts that an electron in a hydrogen-atom ground state has an orbital magnetic moment of magnitude μ_B . But the Schrödinger picture tells us that such a ground-state electron is in an s state with zero angular momentum, so the orbital magnetic moment must be *zero!* To get the correct results, we must describe the states by using Schrödinger wave functions.

It turns out that in the Schrödinger formulation, electrons have the same ratio of μ to L (gyromagnetic ratio) as in the Bohr model—namely, $e/2m$. Suppose the magnetic field \vec{B} is directed along the $+z$ -axis. From Eq. (41.28) the interaction energy U of the atom's magnetic moment with the field is

$$U = -\mu_z B \quad (41.32)$$

where μ_z is the z -component of the vector $\vec{\mu}$.

Now we use Eq. (41.30) to find μ_z , recalling that e is the *magnitude* of the electron charge and that the actual charge is $-e$. Because the electron charge is negative, the orbital angular momentum and magnetic moment vectors are opposite. We find

$$\mu_z = -\frac{e}{2m}L_z \quad (41.33)$$

For the Schrödinger wave functions, $L_z = m_l\hbar$, with $m_l = 0, \pm 1, \pm 2, \dots, \pm l$, so Eq. (41.33) becomes

$$\mu_z = -\frac{e}{2m}L_z = -m_l \frac{e\hbar}{2m} \quad (41.34)$$

CAUTION Two uses of the symbol m Be careful not to confuse the electron mass m with the magnetic quantum number m_l .

Finally, we can express the interaction energy, Eq. (41.32), as

$$U = -\mu_z B = m_l \frac{e\hbar}{2m} B \quad (m_l = 0, \pm 1, \pm 2, \dots, \pm l)$$

(orbital magnetic interaction energy) (41.35)

In terms of the Bohr magneton $\mu_B = e\hbar/2m$, we can write Eq. (41.35) as

$$U = m_l \mu_B B \quad (\text{orbital magnetic interaction energy}) \quad (41.36)$$

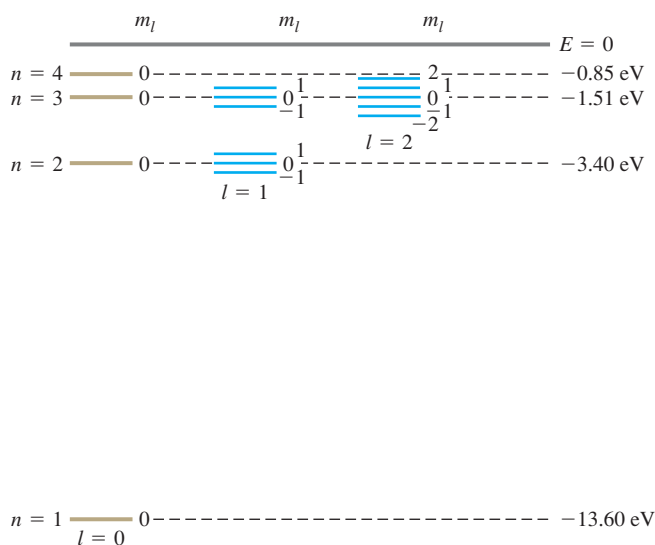
The magnetic field shifts the energy of each orbital state by an amount U . The interaction energy U depends on the value of m_l because m_l determines the

orientation of the orbital magnetic moment relative to the magnetic field. This dependence is the reason m_l is called the magnetic quantum number.

The values of m_l range from $-l$ to $+l$ in steps of one, so an energy level with a particular value of the orbital quantum number l contains $(2l + 1)$ different orbital states. Without a magnetic field these states all have the same energy; that is, they are degenerate. The magnetic field removes this degeneracy. In the presence of a magnetic field they are split into $2l + 1$ distinct energy levels; adjacent levels differ in energy by $(e\hbar/2m)B = \mu_B B$. We can understand this in terms of the connection between degeneracy and symmetry. With a magnetic field applied along the z -axis, the atom is no longer completely symmetric under rotation: There is a preferred direction in space. By removing the symmetry, we remove the degeneracy of states.

Figure 41.13 shows the effect on the energy levels of hydrogen. Spectral lines corresponding to transitions from one set of levels to another set are correspondingly split and appear as a series of three closely spaced spectral lines replacing a single line. As the following example shows, the splitting of spectral lines is quite small because the value of $\mu_B B$ is small even for substantial magnetic fields.

41.13 This energy-level diagram for hydrogen shows how the levels are split when the electron's orbital magnetic moment interacts with an external magnetic field. The values of m_l are shown adjacent to the various levels. The relative magnitudes of the level splittings are exaggerated for clarity. The $n = 4$ splittings are not shown; can you draw them in?



Example 41.5 An atom in a magnetic field

An atom in a state with $l = 1$ emits a photon with wavelength 600.000 nm as it decays to a state with $l = 0$. If the atom is placed in a magnetic field with magnitude $B = 2.00$ T, what are the shifts in the energy levels and in the wavelength that result from the interaction between the atom's orbital magnetic moment and the magnetic field?

SOLUTION

IDENTIFY and SET UP: This problem concerns the splitting of atomic energy levels by a magnetic field (the Zeeman effect). We use Eq. (41.35) or (41.36) to determine the energy-level shifts. The relationship $E = hc/\lambda$ between the energy and wavelength of a photon then lets us calculate the wavelengths emitted during transitions from the $l = 1$ states to the $l = 0$ state.

EXECUTE: The energy of a 600-nm photon is

$$E = \frac{hc}{\lambda} = \frac{(4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{600 \times 10^{-9} \text{ m}} = 2.07 \text{ eV}$$

If there is no external magnetic field, that is the difference in energy between the $l = 0$ and $l = 1$ levels.

With a 2.00-T field present, Eq. (41.36) shows that there is no shift of the $l = 0$ state (which has $m_l = 0$). For the $l = 1$ states, the splitting of levels is given by

$$U = m_l \mu_B B = m_l (5.788 \times 10^{-5} \text{ eV/T})(2.00 \text{ T}) \\ = m_l (1.16 \times 10^{-4} \text{ eV}) = m_l (1.85 \times 10^{-23} \text{ J})$$

The possible values of m_l for $l = 1$ are -1 , 0 , and $+1$, and the three corresponding levels are separated by equal intervals of

Continued

1.16×10^{-4} eV. This is a small fraction of the 2.07-eV photon energy:

$$\frac{\Delta E}{E} = \frac{1.16 \times 10^{-4} \text{ eV}}{2.07 \text{ eV}} = 5.60 \times 10^{-5}$$

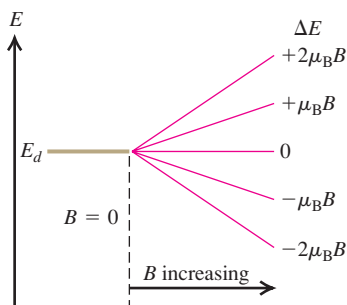
The possible values of m_l for $l = 1$ are $-1, 0,$ and $+1$, and the three corresponding levels are separated by equal intervals of 1.16×10^{-4} eV. This is a small fraction of the 2.07-eV photon energy:

$$\frac{1.16 \times 10^{-4} \text{ eV}}{2.07 \text{ eV}} = 5.60 \times 10^{-5}$$

The corresponding *wavelength* shifts are approximately $(5.60 \times 10^{-5})(600 \text{ nm}) = 0.034 \text{ nm}$. The original 600.000-nm line is split into a triplet with wavelengths 599.966, 600.000, and 600.034 nm.

EVALUATE: Even though 2.00 T would be a strong field in most laboratories, the wavelength splittings are extremely small. Nonetheless, modern spectrographs have more than enough chromatic resolving power to measure these splittings (see Section 36.5).

41.14 This figure shows how the splitting of the energy levels of a d state ($l = 2$) depends on the magnitude B of an external magnetic field, assuming only an orbital magnetic moment.



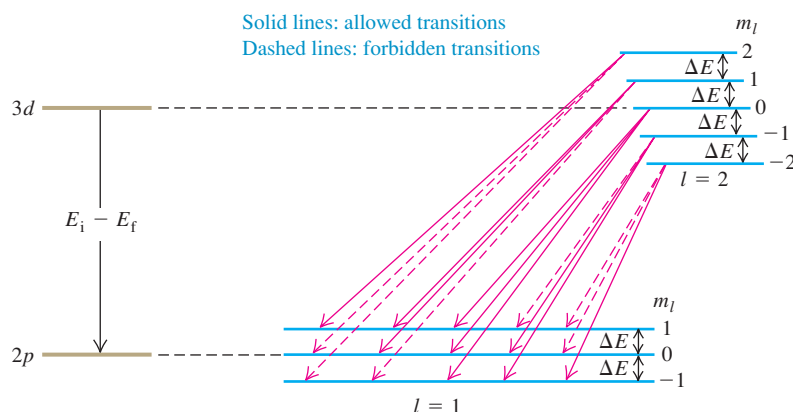
Selection Rules

Figure 41.14 shows what happens to a set of d states ($l = 2$) as the magnetic field increases. With zero field the five states $m_l = -2, -1, 0, 1,$ and 2 are degenerate (have the same energy), but the applied field spreads the states out. Figure 41.15 shows the splittings of both the $3d$ and $2p$ states. Equal energy differences $(e\hbar/2m)B = \mu_B B$ separate adjacent levels. In the absence of a magnetic field, a transition from a $3d$ to a $2p$ state would yield a single spectral line with photon energy $E_i - E_f$. With the levels split as shown, it might seem that there are five possible photon energies.

In fact, there are only three possibilities. Not all combinations of initial and final levels are possible because of a restriction associated with conservation of angular momentum. The photon ordinarily carries off one unit (\hbar) of angular momentum, which leads to the requirements that in a transition l must change by 1 and m_l must change by 0 or ± 1 . These requirements are called **selection rules**. Transitions that obey these rules are called *allowed transitions*; those that don't are *forbidden transitions*. In Fig. 41.15 we show the allowed transitions by solid arrows. You should count the possible transition energies to convince yourself that the nine solid arrows give only three possible energies; the zero-field value $E_i - E_f$, and that value plus or minus $\Delta E = (e\hbar/2m)B = \mu_B B$. Figure 41.12 shows the corresponding spectral lines.

What we have described is called the *normal Zeeman effect*. It is based entirely on the orbital angular momentum of the electron. However, it leaves out a very important consideration: the electron *spin* angular momentum, the subject of the next section.

41.15 The cause of the normal Zeeman effect. The magnetic field splits the levels, but selection rules allow transitions with only three different energy changes, giving three different photon frequencies and wavelengths.



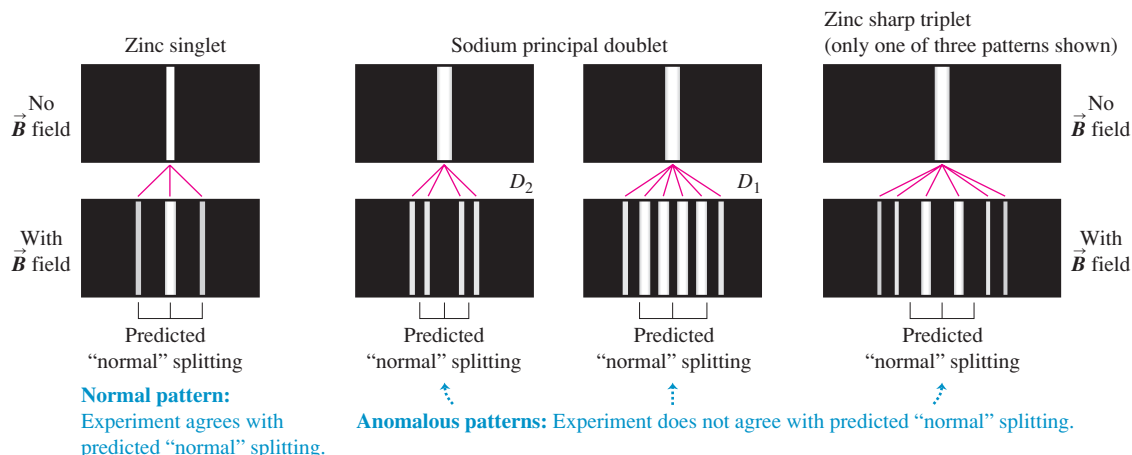
Test Your Understanding of Section 41.4 In this section we assumed that the magnetic field points in the positive z -direction. Would the results be different if the magnetic field pointed in the positive x -direction?

41.5 Electron Spin

Despite the success of the Schrödinger equation in predicting the energy levels of the hydrogen atom, experimental observations indicate that it doesn't tell the whole story of the behavior of electrons in atoms. First, spectroscopists have found magnetic-field splitting into other than the three lines we've explained, sometimes unequally spaced. Before this effect was understood, it was called the *anomalous* Zeeman effect to distinguish it from the "normal" effect discussed in the preceding section. Figure 41.16 shows both kinds of splittings.

Second, some energy levels show splittings that resemble the Zeeman effect even when there is *no* external magnetic field. For example, when the lines in the hydrogen spectrum are examined with a high-resolution spectrograph, some lines are found to consist of sets of closely spaced lines called *multiplets*. Similarly, the orange-yellow line of sodium, corresponding to the transition $4p \rightarrow 3s$ of the outer electron, is found to be a doublet ($\lambda = 589.0, 589.6 \text{ nm}$), suggesting that the $4p$ level might in fact be two closely spaced levels. The Schrödinger equation in its original form didn't predict any of this.

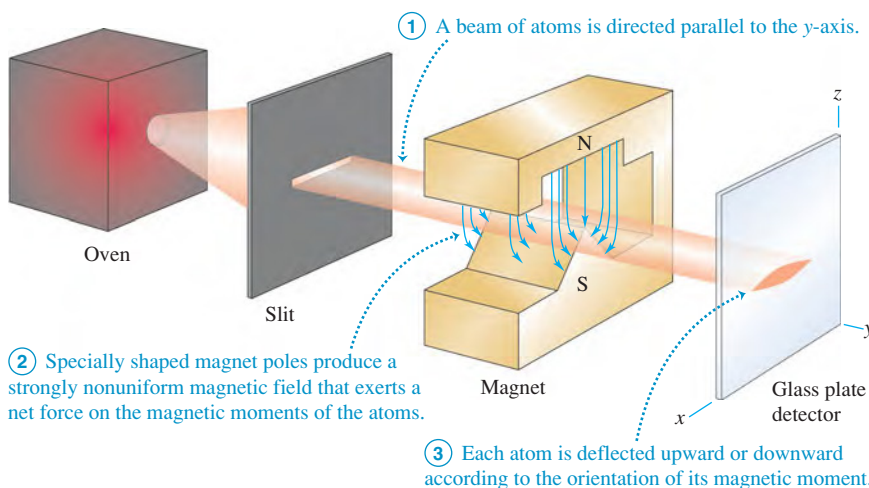
41.16 Illustrations of the normal and anomalous Zeeman effects for two elements, zinc and sodium. The brackets under each illustration show the "normal" splitting predicted by neglecting the effect of electron spin.



The Stern–Gerlach Experiment

Similar anomalies appeared in 1922 in atomic-beam experiments performed in Germany by Otto Stern and Walter Gerlach. When they passed a beam of neutral atoms through a nonuniform magnetic field (Fig. 41.17), atoms were deflected

MasteringPHYSICS®
PhET: Stern–Gerlach Experiment



41.17 The Stern–Gerlach experiment.

according to the orientation of their magnetic moments with respect to the field. These experiments demonstrated the quantization of angular momentum in a very direct way. If there were only orbital angular momentum, the deflections would split the beam into an odd number $(2l + 1)$ of different components. However, some atomic beams were split into an *even* number of components. If we use a different symbol j for an angular momentum quantum number, setting $2j + 1$ equal to an even number gives $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$, suggesting a half-integer angular momentum. This can't be understood on the basis of the Bohr model and similar pictures of atomic structure.

In 1925 two graduate students in the Netherlands, Samuel Goudsmit and George Uhlenbeck, proposed that the electron might have some additional motion. Using a semiclassical model, they suggested that the electron might behave like a spinning sphere of charge instead of a particle. If so, it would have an additional *spin* angular momentum and magnetic moment. If these were quantized in much the same way as *orbital* angular momentum and magnetic moment, they might help to explain the observed energy-level anomalies.

An Analogy for Electron Spin

To introduce the concept of **electron spin**, let's start with an analogy. The earth travels in a nearly circular orbit around the sun, and at the same time it *rotates* on its axis. Each motion has its associated angular momentum, which we call the *orbital* and *spin* angular momentum, respectively. The total angular momentum of the earth is the vector sum of the two. If we were to model the earth as a single point, it would have no moment of inertia about its spin axis and thus no spin angular momentum. But when our model includes the finite size of the earth, spin angular momentum becomes possible.

In the Bohr model, suppose the electron is not just a point charge but a small spinning sphere moving in orbit. Then the electron has not only orbital angular momentum but also spin angular momentum associated with the rotation of its mass about its axis. The sphere carries an electric charge, so the spinning motion leads to current loops and to a magnetic moment, as we discussed in Section 27.7. In a magnetic field, the *spin* magnetic moment has an interaction energy in addition to that of the *orbital* magnetic moment (the normal Zeeman-effect interaction that we discussed in Section 41.4). We should see additional Zeeman shifts due to the spin magnetic moment.

As we mentioned, such shifts *are* indeed observed in precise spectroscopic analysis. This and a variety of other experimental evidence have shown conclusively that the electron *does* have a spin angular momentum and a spin magnetic moment that do not depend on its orbital motion but are intrinsic to the electron itself. The origin of this spin angular momentum is fundamentally quantum-mechanical, so it's not correct to model the electron as a spinning charged sphere. But just as the Bohr model can be a useful conceptual picture for the motion of an electron in an atom, the spinning-sphere analogy can help you visualize the intrinsic spin angular momentum of an electron.

Spin Quantum Numbers

Like orbital angular momentum, the spin angular momentum of an electron (denoted by \vec{S}) is found to be quantized. Suppose we have an apparatus that measures a particular component of \vec{S} , say the z -component S_z . We find that the only possible values are

$$S_z = \pm \frac{1}{2} \hbar \text{ (components of spin angular momentum)} \quad (41.37)$$

This relationship is reminiscent of the expression $L_z = m_l \hbar$ for the z -component of orbital angular momentum, except that $|S_z|$ is *one-half* of \hbar instead of an

integer multiple. Equation (41.37) also suggests that the magnitude S of the spin angular momentum is given by an expression analogous to Eq. (41.22) with the orbital quantum number l replaced by the **spin quantum number** $s = \frac{1}{2}$:

$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2} + 1\right)}\hbar = \sqrt{\frac{3}{4}}\hbar \quad \text{(magnitude of spin angular momentum)} \quad (41.38)$$

The electron is often called a “spin $-\frac{1}{2}$ particle.”

To visualize the quantized spin of an electron in a hydrogen atom, think of the electron probability distribution function $|\psi|^2$ as a cloud surrounding the nucleus like those shown in Figs. 41.9 and 41.10. Then imagine many tiny spin arrows distributed throughout the cloud, either all with components in the $+z$ -direction or all with components in the $-z$ -direction. But don't take this picture too seriously.

To label completely the state of the electron in a hydrogen atom, we now need a fourth quantum number m_s to specify the electron spin orientation. In analogy to the orbital magnetic quantum number m_l , we call m_s the **spin magnetic quantum number**. For an electron we give m_s the value $+\frac{1}{2}$ or $-\frac{1}{2}$ to agree with Eq. (41.37):

$$S_z = m_s \hbar \quad \left(m_s = \pm \frac{1}{2}\right) \quad \text{(allowed values of } m_s \text{ and } S_z \text{ for an electron)} \quad (41.39)$$

The spin angular momentum vector \vec{S} can have only two orientations in space relative to the z -axis: “spin up” with a z -component of $+\frac{1}{2}\hbar$ and “spin down” with a z -component of $-\frac{1}{2}\hbar$.

The z -component of the associated spin magnetic moment (μ_z) turns out to be related to S_z by

$$\mu_z = -(2.00232)\frac{e}{2m}S_z \quad (41.40)$$

where $-e$ and m are (as usual) the charge and mass of the electron. When the atom is placed in a magnetic field, the interaction energy $-\vec{\mu} \cdot \vec{B}$ of the spin magnetic dipole moment with the field causes further splittings in energy levels and in the corresponding spectral lines.

Equation (41.40) shows that the gyromagnetic ratio for electron spin is approximately *twice* as great as the value $e/2m$ for *orbital* angular momentum and magnetic dipole moment. This result has no classical analog. But in 1928 Paul Dirac developed a relativistic generalization of the Schrödinger equation for electrons. His equation gave a spin gyromagnetic ratio of exactly $2(e/2m)$. It took another two decades to develop the area of physics called *quantum electrodynamics*, abbreviated QED, which predicts the value we've given to “only” six significant figures as 2.00232. QED now predicts a value that agrees with a recent (2006) measurement of 2.00231930436170(152), making QED the most precise theory in all science.

Example 41.6 Energy of electron spin in a magnetic field

Calculate the interaction energy for an electron in an $l = 0$ state in a magnetic field with magnitude 2.00 T.

EXECUTE: Combining Eqs. (41.37) and (41.40), we have

SOLUTION

IDENTIFY and SET UP: For $l = 0$ the electron has zero orbital angular momentum and zero orbital magnetic moment. Hence the only magnetic interaction is that between the \vec{B} field and the spin magnetic moment $\vec{\mu}$. From Eq. (41.28), the interaction energy is $U = -\vec{\mu} \cdot \vec{B}$. As in Section 41.4, we take \vec{B} to be in the positive z -direction so that $U = -\mu_z B$ [Eq. (41.32)]. Equation (41.40) gives μ_z in terms of S_z , and Eq. (41.37) gives S_z .

$$\begin{aligned} \mu_z &= -(2.00232)\left(\frac{e}{2m}\right)\left(\pm\frac{1}{2}\hbar\right) \\ &= \mp\frac{1}{2}(2.00232)\left(\frac{e\hbar}{2m}\right) = \mp(1.00116)\mu_B \\ &= \mp(1.00116)(9.274 \times 10^{-24} \text{ J/T}) \\ &= \mp 9.285 \times 10^{-24} \text{ J/T} \\ &= \mp 5.795 \times 10^{-5} \text{ eV/T} \end{aligned}$$

Continued

Then from Eq. (41.32),

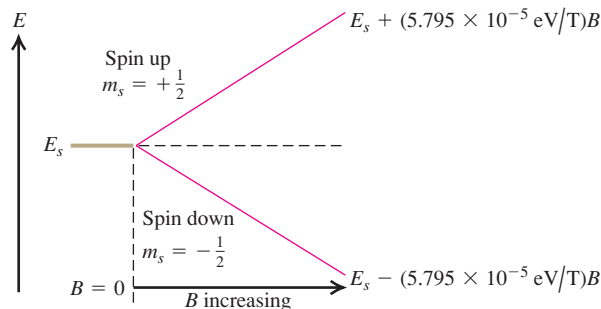
$$U = -\mu_z B = \pm(9.285 \times 10^{-24} \text{ J/T})(2.00 \text{ T}) \\ = \pm 1.86 \times 10^{-23} \text{ J} = \pm 1.16 \times 10^{-4} \text{ eV}$$

The positive value of U and the negative value of μ_z correspond to $S_z = +\frac{1}{2}\hbar$ (spin up); the negative value of U and the positive value of μ_z correspond to $S_z = -\frac{1}{2}\hbar$ (spin down).

EVALUATE: Let's check the *signs* of our results. If the electron is spin down, \vec{S} points generally opposite to \vec{B} . Then the magnetic moment $\vec{\mu}$ (which is opposite to \vec{S} because the electron charge is negative) points generally parallel to \vec{B} , and μ_z is positive. From Eq. (41.28), $U = -\vec{\mu} \cdot \vec{B}$, the interaction energy is negative if $\vec{\mu}$ and \vec{B} are parallel. Our results show that U is indeed negative in this case. We can similarly confirm that U must be positive and μ_z negative for a spin-up electron.

The red lines in Fig. 41.18 show how the interaction energies for the two spin states vary with the magnetic field magnitude B . The graphs are straight lines because, from Eq. (41.32), U is proportional to B .

41.18 An $l = 0$ level of a single electron is split by interaction of the spin magnetic moment with an external magnetic field. The greater the magnitude B of the magnetic field, the greater the splitting. The quantity $5.795 \times 10^{-5} \text{ eV/T}$ is just $(1.00116)\mu_B$.



Spin-Orbit Coupling

We mentioned earlier that the spin magnetic dipole moment also gives splitting of energy levels even when there is *no* external field. One cause involves the orbital motion of the electron. In the Bohr model, observers moving with the electron would see the positively charged nucleus revolving around them (just as to earthbound observers the sun seems to be orbiting the earth). This apparent motion of charge causes a magnetic field at the location of the electron, as measured in the electron's moving frame of reference. The resulting interaction with the spin magnetic moment causes a twofold splitting of this level, corresponding to the two possible orientations of electron spin.

Discussions based on the Bohr model can't be taken too seriously, but a similar result can be derived from the Schrödinger equation. The interaction energy U can be expressed in terms of the scalar product of the angular momentum vectors \vec{L} and \vec{S} . This effect is called **spin-orbit coupling**; it is responsible for the small energy difference between the two closely spaced, lowest excited levels of sodium shown in Fig. 39.19a and for the corresponding doublet (589.0, 589.6 nm) in the spectrum of sodium.

Example 41.7 An effective magnetic field

To six significant figures, the wavelengths of the two spectral lines that make up the sodium doublet are $\lambda_1 = 588.995 \text{ nm}$ and $\lambda_2 = 589.592 \text{ nm}$. Calculate the effective magnetic field experienced by the electron in the $3p$ levels of the sodium atom.

SOLUTION

IDENTIFY and SET UP: The two lines in the sodium doublet result from transitions from the two $3p$ levels, which are split by spin-orbit coupling, to the $3s$ level, which is *not* split because it has $L = 0$. We picture the spin-orbit coupling as an interaction between the electron spin magnetic moment and an effective magnetic field due to the nucleus. This example is like Example 41.6 in reverse: There we were given B and found the difference between the energies of the two spin states, while here we use the energy difference to find the target variable B . The difference in energy between the two $3p$ levels is equal to the difference in energy

between the two photons of the sodium doublet. We use this relationship and the results of Example 41.6 to determine B .

EXECUTE: The energies of the two photons are $E_1 = hc/\lambda_1$ and $E_2 = hc/\lambda_2$. Here $E_1 > E_2$ because $\lambda_1 < \lambda_2$, so the difference in their energies is

$$\Delta E = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = hc \left(\frac{\lambda_2 - \lambda_1}{\lambda_2 \lambda_1} \right) \\ = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s}) \\ \times \frac{(589.592 \times 10^{-9} \text{ m}) - (588.995 \times 10^{-9} \text{ m})}{(589.592 \times 10^{-9} \text{ m})(588.995 \times 10^{-9} \text{ m})} \\ = 0.00213 \text{ eV} = 3.41 \times 10^{-22} \text{ J}$$

This equals the energy difference between the two $3p$ levels. The spin-orbit interaction raises one level by $1.70 \times 10^{-22} \text{ J}$ (one-half

of 3.41×10^{-22} J) and lowers the other by 1.70×10^{-22} J. From Example 41.6, the amount each state is raised or lowered is $|U| = (1.00116)\mu_B B$, so

$$B = \left| \frac{U}{(1.00116)\mu_B} \right| = \frac{1.70 \times 10^{-22} \text{ J}}{9.28 \times 10^{-24} \text{ J/T}} = 18.0 \text{ T}$$

EVALUATE: The electron experiences a *very* strong effective magnetic field. To produce a steady, macroscopic field of this magnitude in the laboratory requires state-of-the-art electromagnets.

Combining Orbital and Spin Angular Momenta

The orbital and spin angular momenta (\vec{L} and \vec{S} , respectively) can combine in various ways. The vector sum of \vec{L} and \vec{S} is the *total* angular momentum \vec{J} :

$$\vec{J} = \vec{L} + \vec{S} \quad (41.41)$$

The possible values of the magnitude J are given in terms of a quantum number j by

$$J = \sqrt{j(j+1)}\hbar \quad (41.42)$$

We can then have states in which $j = |l \pm \frac{1}{2}|$. The $l + \frac{1}{2}$ states correspond to the case in which the vectors \vec{L} and \vec{S} have parallel z -components; for the $l - \frac{1}{2}$ states, \vec{L} and \vec{S} have antiparallel z -components. For example, when $l = 1$, j can be $\frac{1}{2}$ or $\frac{3}{2}$. In another spectroscopic notation these p states are labeled ${}^2P_{1/2}$ and ${}^2P_{3/2}$, respectively. The superscript is the number of possible spin orientations, the letter P (now capitalized) indicates states with $l = 1$, and the subscript is the value of j . We used this scheme to label the energy levels of the sodium atom in Fig. 39.19a.

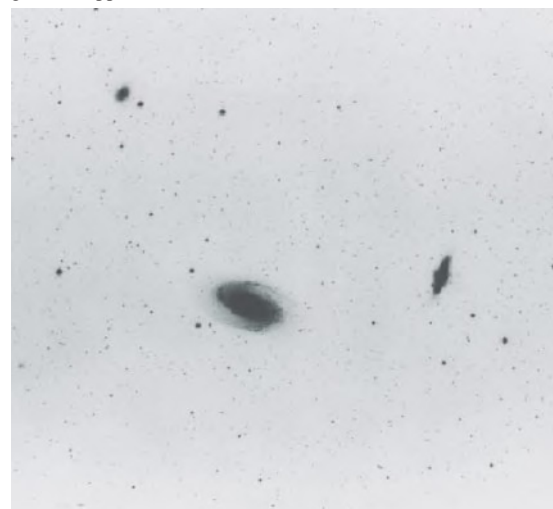
The various line splittings resulting from magnetic interactions are collectively called *fine structure*. There are also additional, much smaller splittings associated with the fact that the *nucleus* of the atom has a magnetic dipole moment that interacts with the orbital and/or spin magnetic dipole moments of the electrons. These effects are called *hyperfine structure*. For example, the ground level of hydrogen is split into two states, separated by only 5.9×10^{-6} eV. The photon that is emitted in the transitions between these states has a wavelength of 21 cm. Radio astronomers use this wavelength to map clouds of interstellar hydrogen gas that are too cold to emit visible light (Fig. 41.19).

Test Your Understanding of Section 41.5 In which of the following situations is the magnetic moment of an electron perfectly aligned with a magnetic field that points in the positive z -direction? (i) $m_s = +\frac{1}{2}$; (ii) $m_s = -\frac{1}{2}$; (iii) both (i) and (ii); (iv) neither (i) nor (ii).

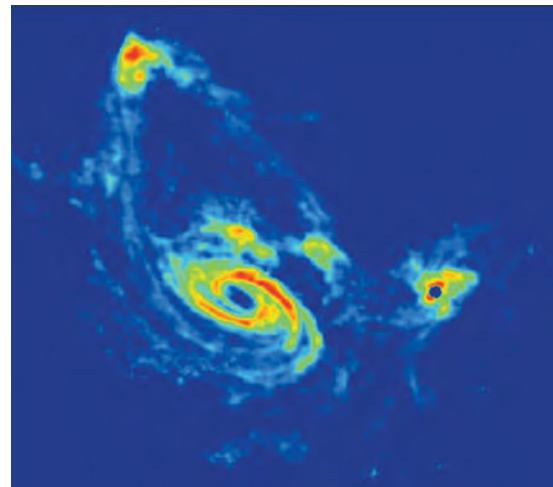


41.19 In a visible-light image (top), these three distant galaxies appear to be unrelated. But in fact these galaxies are connected by immense streamers of hydrogen gas. This is revealed by the false-color image (bottom) made with a radio telescope tuned to the 21-cm wavelength emitted by hydrogen atoms.

Galaxies in visible light (negative image); galaxies appear dark)



Radio image at wavelength 21 cm



41.6 Many-Electron Atoms and the Exclusion Principle

So far our analysis of atomic structure has concentrated on the hydrogen atom. That's natural; neutral hydrogen, with only one electron, is the simplest atom. If we can't understand hydrogen, we certainly can't understand anything more complex. But now let's move on to many-electron atoms.

In general, an atom in its normal (electrically neutral) state has Z electrons and Z protons. Recall from Section 41.3 that we call Z the *atomic number*. The total electric charge of such an atom is exactly zero because the neutron has no charge while the proton and electron charges have the same magnitude but opposite sign.

We can apply the Schrödinger equation to this general atom. However, the complexity of the analysis increases very rapidly with increasing Z . Each of the Z electrons interacts not only with the nucleus but also with every other electron.

The wave functions and the potential energy are functions of $3Z$ coordinates, and the equation contains second derivatives with respect to all of them. The mathematical problem of finding solutions of such equations is so complex that it has not been solved exactly even for the neutral helium atom, which has only two electrons.

Fortunately, various approximation schemes are available. The simplest approximation is to ignore all interactions between electrons and consider each electron as moving under the action only of the nucleus (considered to be a point charge). In this approximation the wave function for each individual electron is a function like that for the hydrogen atom, specified by four quantum numbers (n, l, m_l, m_s) . The nuclear charge is Ze instead of e , so we replace every factor of e^2 in the wave functions and the energy levels by Ze^2 . In particular, the energy levels are given by Eq. (41.21) with e^4 replaced by Z^2e^4 :

$$E_n = -\frac{1}{(4\pi\epsilon_0)^2} \frac{m_r Z^2 e^4}{2n^2 \hbar^2} = -\frac{Z^2}{n^2} (13.6 \text{ eV}) \quad (41.43)$$

This approximation is fairly drastic; when there are many electrons, their interactions with each other are as important as the interaction of each with the nucleus. So this model isn't very useful for quantitative predictions.

The Central-Field Approximation

A less drastic and more useful approximation is to think of all the electrons together as making up a charge cloud that is, on average, *spherically symmetric*. We can then think of each individual electron as moving in the total electric field due to the nucleus and this averaged-out cloud of all the other electrons. There is a corresponding spherically symmetric potential-energy function $U(r)$. This picture is called the **central-field approximation**; it provides a useful starting point for understanding atomic structure.

In the central-field approximation we can again deal with one-electron wave functions. The Schrödinger equation differs from the equation for hydrogen only in that the $1/r$ potential-energy function is replaced by a different function $U(r)$. But it turns out that $U(r)$ does not enter the differential equations for $\Theta(\theta)$ and $\Phi(\phi)$, so those angular functions are exactly the same as for hydrogen, and the orbital angular-momentum *states* are also the same as before. The quantum numbers l , m_l , and m_s have the same meanings as before, and Eqs. (41.22) and (41.23) again give the magnitude and z -component of the orbital angular momentum.

The radial wave functions and probabilities are different than for hydrogen because of the change in $U(r)$, so the energy levels are no longer given by Eq. (41.21). We can still label a state using the four quantum numbers (n, l, m_l, m_s) . In general, the energy of a state now depends on both n and l , rather than just on n as with hydrogen. The restrictions on values of the quantum numbers are the same as before:

$$n \geq 1 \quad 0 \leq l \leq n - 1 \quad |m_l| \leq l \quad m_s = \pm \frac{1}{2} \quad (\text{allowed values of quantum numbers}) \quad (41.44)$$

The Exclusion Principle

To understand the structure of many-electron atoms, we need an additional principle, the *exclusion principle*. To see why this principle is needed, let's consider the lowest-energy state, or *ground state*, of a many-electron atom. In the one-electron states of the central-field model, there is a lowest-energy state (corresponding to an $n = 1$ state of hydrogen). We might expect that in the ground state of a complex atom, *all* the electrons should be in this lowest state. If so, then we should see only gradual changes in physical and chemical properties when we look at the behavior of atoms with increasing numbers of electrons (Z).

Such gradual changes are *not* what is observed. Instead, properties of elements vary widely from one to the next, with each element having its own distinct personality. For example, the elements fluorine, neon, and sodium have 9, 10, and 11 electrons, respectively, per atom. Fluorine ($Z = 9$) is a *halogen*; it tends strongly to form compounds in which each fluorine atom acquires an extra electron. Sodium ($Z = 11$) is an *alkali metal*; it forms compounds in which each sodium atom *loses* an electron. Neon ($Z = 10$) is a *noble gas*, forming no compounds at all. Such observations show that in the ground state of a complex atom the electrons *cannot* all be in the lowest-energy states. But why not?

The key to this puzzle, discovered by the Austrian physicist Wolfgang Pauli (Fig. 41.20) in 1925, is called the **exclusion principle**. This principle states that **no two electrons can occupy the same quantum-mechanical state** in a given system. That is, **no two electrons in an atom can have the same values of all four quantum numbers (n, l, m_l, m_s)**. Each quantum state corresponds to a certain distribution of the electron “cloud” in space. Therefore the principle also says, in effect, that no more than two electrons with opposite values of the quantum number m_s can occupy the same region of space. We shouldn’t take this last statement too seriously because the electron probability functions don’t have sharp, definite boundaries. But the exclusion principle limits the amount by which electron wave functions can overlap. Think of it as the quantum-mechanical analog of a university rule that allows only one student per desk.

CAUTION **The meaning of the exclusion principle** Don’t confuse the exclusion principle with the electric repulsion between electrons. While both effects tend to keep electrons within an atom separated from each other, they are very different in character. Two electrons can always be pushed closer together by adding energy to combat electric repulsion; in contrast, *nothing* can overcome the exclusion principle and force two electrons into the same quantum-mechanical state. **I**

Table 41.2 lists some of the sets of quantum numbers for electron states in an atom. It’s similar to Table 41.1 (Section 41.3), but we’ve added the number of states in each subshell and shell. Because of the exclusion principle, the “number of states” is the same as the *maximum* number of electrons that can be found in those states. For each state, m_s can be either $+\frac{1}{2}$ or $-\frac{1}{2}$.

As with the hydrogen wave functions, different states correspond to different spatial distributions; electrons with larger values of n are concentrated at larger distances from the nucleus. Figure 41.8 (Section 41.3) shows this effect. When an atom has more than two electrons, they can’t all huddle down in the low-energy $n = 1$ states nearest to the nucleus because there are only two of these states; the exclusion principle forbids multiple occupancy of a state. Some electrons are forced into states farther away, with higher energies. Each value of n corresponds roughly to a region of space around the nucleus in the form of a spherical *shell*. Hence we speak of the *K* shell as the region that is occupied by the electrons in the $n = 1$ states, the *L* shell as the region of the $n = 2$ states, and so on. States with the same n but different l form *subshells*, such as the $3p$ subshell.

Table 41.2 Quantum States of Electrons in the First Four Shells

n	l	m_l	Spectroscopic Notation	Number of States	Shell	
1	0	0	1s	2	K	
2	0	0	2s	2		
2	1	-1, 0, 1	2p	6	8	L
3	0	0	3s	2		
3	1	-1, 0, 1	3p	6	18	M
3	2	-2, -1, 0, 1, 2	3d	10		
4	0	0	4s	2	32	N
4	1	-1, 0, 1	4p	6		
4	2	-2, -1, 0, 1, 2	4d	10		
4	3	-3, -2, -1, 0, 1, 2, 3	4f	14		

41.20 The key to understanding the periodic table of the elements was the discovery by Wolfgang Pauli (1900–1958) of the exclusion principle. Pauli received the 1945 Nobel Prize in physics for his accomplishment. This photo shows Pauli (on the left) and Niels Bohr watching the physics of a toy top spinning on the floor—a macroscopic analog of a microscopic electron with spin.



The Periodic Table

We can use the exclusion principle to derive the most important features of the structure and chemical behavior of multielectron atoms, including the periodic table of the elements. Let's imagine constructing a neutral atom by starting with a bare nucleus with Z protons and then adding Z electrons, one by one. To obtain the ground state of the atom as a whole, we fill the lowest-energy electron states (those closest to the nucleus, with the smallest values of n and l) first, and we use successively higher states until all the electrons are in place. The chemical properties of an atom are determined principally by interactions involving the outermost, or *valence*, electrons, so we particularly want to learn how these electrons are arranged.

Let's look at the ground-state electron configurations for the first few atoms (in order of increasing Z). For hydrogen the ground state is $1s$; the single electron is in a state $n = 1$, $l = 0$, $m_l = 0$, and $m_s = \pm\frac{1}{2}$. In the helium atom ($Z = 2$), both electrons are in $1s$ states, with opposite spins; one has $m_s = -\frac{1}{2}$ and the other has $m_s = +\frac{1}{2}$. We denote the helium ground state as $1s^2$. (The superscript 2 is not an exponent; the notation $1s^2$ tells us that there are two electrons in the $1s$ subshell. Also, the superscript 1 is understood, as in $2s$.) For helium the K shell is completely filled, and all others are empty. Helium is a noble gas; it has no tendency to gain or lose an electron, and it forms no compounds.

Lithium ($Z = 3$) has three electrons. In its ground state, two are in $1s$ states and one is in a $2s$ state, so we denote the lithium ground state as $1s^2 2s$. On average, the $2s$ electron is considerably farther from the nucleus than are the $1s$ electrons (Fig. 41.21). According to Gauss's law, the *net* charge Q_{encl} attracting the $2s$ electron is nearer to $+e$ than to the value $+3e$ it would have without the two $1s$ electrons present. As a result, the $2s$ electron is loosely bound; only 5.4 eV is required to remove it, compared with the 30.6 eV given by Eq. (41.43) with $Z = 3$ and $n = 2$. In chemical behavior, lithium is an *alkali metal*. It forms ionic compounds in which each lithium atom loses an electron and has a valence of $+1$.

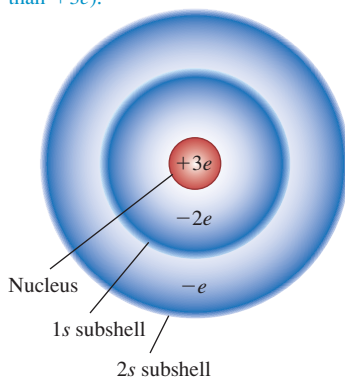
Next is beryllium ($Z = 4$); its ground-state configuration is $1s^2 2s^2$, with its two valence electrons filling the s subshell of the L shell. Beryllium is the first of the *alkaline earth* elements, forming ionic compounds in which the valence of the atoms is $+2$.

Table 41.3 shows the ground-state electron configurations of the first 30 elements. The L shell can hold eight electrons. At $Z = 10$, both the K and L shells are filled, and there are no electrons in the M shell. We expect this to be a particularly stable configuration, with little tendency to gain or lose electrons. This element is neon, a noble gas with no known compounds. The next element after neon is sodium ($Z = 11$), with filled K and L shells and one electron in the M shell. Its "noble-gas-plus-one-electron" structure resembles that of lithium; both are alkali metals. The element *before* neon is fluorine, with $Z = 9$. It has a vacancy in the L shell and has an affinity for an extra electron to fill the shell. Fluorine forms ionic compounds in which it has a valence of -1 . This behavior is characteristic of the *halogens* (fluorine, chlorine, bromine, iodine, and astatine), all of which have "noble-gas-minus-one" configurations (Fig. 41.22).

Proceeding down the list, we can understand the regularities in chemical behavior displayed by the **periodic table of the elements** (Appendix D) on the basis of electron configurations. The similarity of elements in each *group* (vertical column) of the periodic table is the result of similarity in outer-electron configuration. All the noble gases (helium, neon, argon, krypton, xenon, and radon) have filled-shell or filled-shell plus filled p subshell configurations. All the alkali metals (lithium, sodium, potassium, rubidium, cesium, and francium) have "noble-gas-plus-one" configurations. All the alkaline earth metals (beryllium, magnesium, calcium, strontium, barium, and radium) have "noble-gas-plus-two" configurations, and, as we just mentioned, all the halogens (fluorine, chlorine, bromine, iodine, and astatine) have "noble-gas-minus-one" structures.

41.21 Schematic representation of the charge distribution in a lithium atom. The nucleus has a charge of $+3e$.

On average, the $2s$ electron is considerably farther from the nucleus than the $1s$ electrons. Therefore, it experiences a net nuclear charge of approximately $+3e - 2e = +e$ (rather than $+3e$).



41.22 Salt (sodium chloride, NaCl) dissolves readily in water, making seawater salty. This is due to the electron configurations of sodium and chlorine: Sodium can easily lose an electron to form an Na^+ ion, and chlorine can easily gain an electron to form a Cl^- ion. These ions are held in solution because they are attracted to the polar ends of water molecules (see Fig. 21.30a).



Table 41.3 Ground-State Electron Configurations

Element	Symbol	Atomic Number (Z)	Electron Configuration
Hydrogen	H	1	$1s$
Helium	He	2	$1s^2$
Lithium	Li	3	$1s^2 2s$
Beryllium	Be	4	$1s^2 2s^2$
Boron	B	5	$1s^2 2s^2 2p$
Carbon	C	6	$1s^2 2s^2 2p^2$
Nitrogen	N	7	$1s^2 2s^2 2p^3$
Oxygen	O	8	$1s^2 2s^2 2p^4$
Fluorine	F	9	$1s^2 2s^2 2p^5$
Neon	Ne	10	$1s^2 2s^2 2p^6$
Sodium	Na	11	$1s^2 2s^2 2p^6 3s$
Magnesium	Mg	12	$1s^2 2s^2 2p^6 3s^2$
Aluminum	Al	13	$1s^2 2s^2 2p^6 3s^2 3p$
Silicon	Si	14	$1s^2 2s^2 2p^6 3s^2 3p^2$
Phosphorus	P	15	$1s^2 2s^2 2p^6 3s^2 3p^3$
Sulfur	S	16	$1s^2 2s^2 2p^6 3s^2 3p^4$
Chlorine	Cl	17	$1s^2 2s^2 2p^6 3s^2 3p^5$
Argon	Ar	18	$1s^2 2s^2 2p^6 3s^2 3p^6$
Potassium	K	19	$1s^2 2s^2 2p^6 3s^2 3p^6 4s$
Calcium	Ca	20	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$
Scandium	Sc	21	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d$
Titanium	Ti	22	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^2$
Vanadium	V	23	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^3$
Chromium	Cr	24	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^5$
Manganese	Mn	25	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^5$
Iron	Fe	26	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^6$
Cobalt	Co	27	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^7$
Nickel	Ni	28	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^8$
Copper	Cu	29	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10}$
Zinc	Zn	30	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10}$

A slight complication occurs with the M and N shells because the $3d$ and $4s$ subshell levels ($n = 3, l = 2$, and $n = 4, l = 0$, respectively) have similar energies. (We'll discuss in the next subsection why this happens.) Argon ($Z = 18$) has all the $1s, 2s, 2p, 3s$, and $3p$ subshells filled, but in potassium ($Z = 19$) the additional electron goes into a $4s$ energy state rather than a $3d$ state (because the $4s$ state has slightly lower energy).

The next several elements have one or two electrons in the $4s$ subshell and increasing numbers in the $3d$ subshell. These elements are all metals with rather similar chemical and physical properties; they form the first *transition series*, starting with scandium ($Z = 21$) and ending with zinc ($Z = 30$), for which all the $3d$ and $4s$ subshells are filled.

Something similar happens with $Z = 57$ through $Z = 71$, which have one or two electrons in the $6s$ subshell but only partially filled $4f$ and $5d$ subshells. These are the *rare earth* elements; they all have very similar physical and chemical properties. Yet another such series, called the *actinide* series, starts with $Z = 91$.

Screening

We have mentioned that in the central-field picture, the energy levels depend on l as well as n . Let's take sodium ($Z = 11$) as an example. If 10 of its electrons fill its K and L shells, the energies of some of the states for the remaining electron are found experimentally to be

$$\begin{aligned} 3s \text{ states:} & -5.138 \text{ eV} \\ 3p \text{ states:} & -3.035 \text{ eV} \\ 3d \text{ states:} & -1.521 \text{ eV} \\ 4s \text{ states:} & -1.947 \text{ eV} \end{aligned}$$

Application Electron Configurations and Bone Cancer Radiotherapy

The orange spots in this colored x-ray image are bone cancer tumors. One method of treating bone cancer is to inject a radioactive isotope of strontium (^{89}Sr) into a patient's vein. Strontium is chemically similar to calcium because in both atoms the two outer electrons are in an s state (the structures are $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2$ for strontium and $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$ for calcium). Hence the strontium is readily taken up by the tumors, where calcium turnover is more rapid than in healthy bone. Radiation from the strontium helps to destroy the tumors.



The $3s$ states are the lowest (most negative); one is the ground state for the 11th electron in sodium. The energy of the $3d$ states is quite close to the energy of the $n = 3$ state in hydrogen. The surprise is that the $4s$ state energy is 0.426 eV *below* the $3d$ state, even though the $4s$ state has larger n .

We can understand these results using Gauss's law and the radial probability distribution. For any spherically symmetric charge distribution, the electric-field magnitude at a distance r from the center is $Q_{\text{encl}}/4\pi\epsilon_0 r^2$, where Q_{encl} is the total charge enclosed within a sphere with radius r . Mentally remove the outer (valence) electron atom from a sodium atom. What you have left is a spherically symmetric collection of 10 electrons (filling the K and L shells) and 11 protons, so $Q_{\text{encl}} = -10e + 11e = +e$. If the 11th electron is completely outside this collection of charges, it is attracted by an effective charge of $+e$, not $+11e$. This is a more extreme example of the effect depicted in Fig. 41.21.

This effect is called **screening**; the 10 electrons *screen* 10 of the 11 protons, leaving an effective net charge of $+e$. In general, an electron that spends all its time completely outside a positive charge $Z_{\text{eff}}e$ has energy levels given by the hydrogen expression with e^2 replaced by $Z_{\text{eff}}e^2$. From Eq. (41.43) this is

$$E_n = -\frac{Z_{\text{eff}}^2}{n^2} (13.6 \text{ eV}) \quad (\text{energy levels with screening}) \quad (41.45)$$

If the 11th electron in the sodium atom is completely outside the remaining charge distribution, then $Z_{\text{eff}} = 1$.

CAUTION **Different equations for different atoms** Equations (41.21), (41.43), and (41.45) all give values of E_n in terms of $(13.6 \text{ eV})/n^2$, but they don't apply in general to the same atoms. Equation (41.21) is *only* for hydrogen. Equation (41.43) is only for the case in which there is no interaction with any other electron (and is thus accurate only when the atom has just one electron). Equation (41.45) is useful when one electron is screened from the nucleus by other electrons. ■

Now let's use the radial probability functions shown in Fig. 41.8 to explain why the energy of a sodium $3d$ state is approximately the same as the $n = 3$ value of hydrogen, -1.51 eV . The distribution for the $3d$ state (for which l has the maximum value $n - 1$) has one peak, and its most probable radius is *outside* the positions of the electrons with $n = 1$ or 2. (Those electrons also are pulled closer to the nucleus than in hydrogen because they are less effectively screened from the positive charge $11e$ of the nucleus.) Thus in sodium a $3d$ electron spends most of its time well outside the $n = 1$ and $n = 2$ states (the K and L shells). The 10 electrons in these shells screen about ten-elevenths of the charge of the 11 protons, leaving a net charge of about $Z_{\text{eff}}e = (1)e$. Then, from Eq. (41.45), the corresponding energy is approximately $-(1)^2(13.6 \text{ eV})/3^2 = -1.51 \text{ eV}$. This approximation is very close to the experimental value of -1.521 eV .

Looking again at Fig. 41.8, we see that the radial probability density for the $3p$ state (for which $l = n - 2$) has two peaks and that for the $3s$ state ($l = n - 3$) has three peaks. For sodium the first small peak in the $3p$ distribution gives a $3p$ electron a higher probability (compared to the $3d$ state) of being *inside* the charge distributions for the electrons in the $n = 2$ states. That is, a $3p$ electron is less completely screened from the nucleus than is a $3d$ electron because it spends some of its time within the filled K and L shells. Thus for the $3p$ electrons, Z_{eff} is greater than unity. From Eq. (41.45) the $3p$ energy is lower (more negative) than the $3d$ energy of -1.521 eV . The actual value is -3.035 eV . A $3s$ electron spends even more time within the inner electron shells than a $3p$ electron does, giving an even larger Z_{eff} and an even more negative energy.

Example 41.8 Determining Z_{eff} experimentally

The measured energy of a $3s$ state of sodium is -5.138 eV. Calculate the value of Z_{eff} .

SOLUTION

IDENTIFY and SET UP: Sodium has a single electron in the M shell outside filled K and L shells. The ten K and L electrons partially screen the single M electron from the $+11e$ charge of the nucleus; our goal is to determine the extent of this screening. We are given $n = 3$ and $E_n = -5.138$ eV, so we can use Eq. (41.45) to determine Z_{eff} .

EXECUTE: Solving Eq. (41.45) for Z_{eff} , we have

$$Z_{\text{eff}}^2 = -\frac{n^2 E_n}{13.6 \text{ eV}} = -\frac{3^2(-5.138 \text{ eV})}{13.6 \text{ eV}} = 3.40$$

$$Z_{\text{eff}} = 1.84$$

EVALUATE: The effective charge attracting a $3s$ electron is $1.84e$. Sodium's 11 protons are screened by an average of $11 - 1.84 = 9.16$ electrons instead of 10 electrons because the $3s$ electron spends some time within the inner (K and L) shells.

Each alkali metal (lithium, sodium, potassium, rubidium, and cesium) has one more electron than the corresponding noble gas (helium, neon, argon, krypton, and xenon). This extra electron is mostly outside the other electrons in the filled shells and subshells. Therefore all the alkali metals behave similarly to sodium.

Example 41.9 Energies for a valence electron

The valence electron in potassium has a $4s$ ground state. Calculate the approximate energy of the $n = 4$ state having the smallest Z_{eff} , and discuss the relative energies of the $4s$, $4p$, $4d$, and $4f$ states.

SOLUTION

IDENTIFY and SET UP: The state with the smallest Z_{eff} is the one in which the valence electron spends the most time outside the inner filled shells and subshells, so that it is most effectively screened from the charge of the nucleus. Once we have determined which state has the smallest Z_{eff} , we can use Eq. (41.45) to determine the energy of this state.

EXECUTE: A $4f$ state has $n = 4$ and $l = 3 = 4 - 1$. Thus it is the state of greatest orbital angular momentum for $n = 4$, and thus the state in which the electron spends the most time outside the electron charge clouds of the inner filled shells and subshells. This makes Z_{eff} for a $4f$ state close to unity. Equation (41.45) then gives

$$E_4 = -\frac{Z_{\text{eff}}^2}{n^2}(13.6 \text{ eV}) = -\frac{1}{4^2}(13.6 \text{ eV}) = -0.85 \text{ eV}$$

This approximation agrees with the measured energy of the sodium $4f$ state to the precision given.

An electron in a $4d$ state spends a bit more time within the inner shells, and its energy is therefore a bit more negative (measured to be -0.94 eV). For the same reason, a $4p$ state has an even lower energy (measured to be -2.73 eV) and a $4s$ state has the lowest energy (measured to be -4.339 eV).

EVALUATE: We can extend this analysis to the *singly ionized alkali earth elements*: Be^+ , Mg^+ , Ca^+ , Sr^+ , and Ba^+ . For any allowed value of n , the highest- l state ($l = n - 1$) of the one remaining outer electron sees an effective charge of almost $+2e$, so for these states, $Z_{\text{eff}} = 2$. A $3d$ state for Mg^+ , for example, has an energy of about $-2^2(13.6 \text{ eV})/3^2 = -6.0$ eV.

Test Your Understanding of Section 41.6 If electrons did *not* obey the exclusion principle, would it be easier or more difficult to remove the first electron from sodium?

41.7 X-Ray Spectra

X-ray spectra provide yet another example of the richness and power of the Schrödinger equation and of the model of atomic structure that we derived from it in the preceding section. In Section 38.2 we discussed x-ray production on the basis of the photon concept. With the development of x-ray diffraction techniques (see Section 36.6) by von Laue, Bragg, and others, beginning in 1912, it became possible to measure x-ray wavelengths quite precisely (to within 0.1% or less).

Detailed studies of x-ray spectra showed a continuous spectrum of wavelengths (see Fig. 38.8 in Section 38.2), with minimum wavelength (corresponding to maximum frequency and photon energy) determined by the accelerating

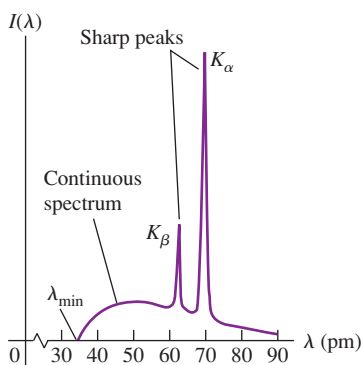
voltage V_{AC} in the x-ray tube, according to the relationship derived in Section 38.2 for *bremsstrahlung* processes:

$$\lambda_{\min} = \frac{hc}{eV_{AC}} \quad (41.46)$$

This continuous-spectrum radiation is nearly independent of the target material in the x-ray tube.

Moseley's Law and Atomic Energy Levels

41.23 Graph of intensity per unit wavelength as a function of wavelength for x rays produced with an accelerating voltage of 35 kV and a molybdenum target. The curve is a smooth function similar to the *bremsstrahlung* spectra in Fig. 38.8 (Section 38.2), but with two sharp spikes corresponding to part of the characteristic x-ray spectrum for molybdenum.



Depending on the accelerating voltage and the target element, we may find sharp peaks superimposed on this continuous spectrum, as in Fig. 41.23. These peaks are at different wavelengths for different elements; they form what is called a *characteristic x-ray spectrum* for each target element. In 1913 the British scientist Henry G. J. Moseley studied these spectra in detail using x-ray diffraction techniques. He found that the most intense short-wavelength line in the characteristic x-ray spectrum from a particular target element, called the K_α line, varied smoothly with that element's atomic number Z (Fig. 41.24). This is in sharp contrast to optical spectra, in which elements with adjacent Z -values have spectra that often bear no resemblance to each other.

Moseley found that the relationship could be expressed in terms of x-ray frequencies f by a simple formula called *Moseley's law*:

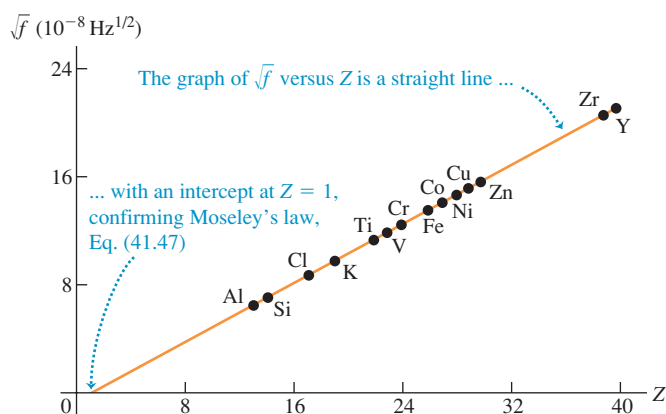
$$f = (2.48 \times 10^{15} \text{ Hz})(Z - 1)^2 \quad (\text{Moseley's law}) \quad (41.47)$$

Moseley went far beyond this empirical relationship; he showed how characteristic x-ray spectra could be understood on the basis of energy levels of atoms in the target. His analysis was based on the Bohr model, published in the same year. We will recast it somewhat, using the ideas of atomic structure that we discussed in Section 41.6. First recall that the *outer* electrons of an atom are responsible for optical spectra. Their excited states are usually only a few electron volts above their ground state. In transitions from excited states to the ground state, they usually emit photons in or near the visible region.

Characteristic x rays, by contrast, are emitted in transitions involving the *inner* shells of a complex atom. In an x-ray tube the electrons may strike the target with enough energy to knock electrons out of the inner shells of the target atoms. These inner electrons are much closer to the nucleus than are the electrons in the outer shells; they are much more tightly bound, and hundreds or thousands of electron volts may be required to remove them.

Suppose one electron is knocked out of the K shell. This process leaves a vacancy, which we'll call a *hole*. (One electron remains in the K shell.) The hole can then be filled by an electron falling in from one of the outer shells, such as the L , M , N , ... shell. This transition is accompanied by a decrease in the energy of the

41.24 The square root of Moseley's measured frequencies of the K_α line for 14 elements.



atom (because *less* energy would be needed to remove an electron from an L , M , N , ... shell), and an x-ray photon is emitted with energy equal to this decrease. Each state has definite energy, so the emitted x rays have definite wavelengths; the emitted spectrum is a *line* spectrum.

We can estimate the energy and frequency of K_α x-ray photons using the concept of screening from Section 41.6. A K_α x-ray photon is emitted when an electron in the L shell ($n = 2$) drops down to fill a hole in the K shell ($n = 1$). As the electron drops down, it is attracted by the Z protons in the nucleus screened by the one remaining electron in the K shell. We therefore approximate the energy by Eq. (41.45), with $Z_{\text{eff}} = Z - 1$, $n_i = 2$, and n_f . The energy before the transition is

$$E_i \approx -\frac{(Z-1)^2}{2^2}(13.6 \text{ eV}) = -(Z-1)^2(3.4 \text{ eV})$$

and the energy after the transition is

$$E_f \approx -\frac{(Z-1)^2}{1^2}(13.6 \text{ eV}) = -(Z-1)^2(13.6 \text{ eV})$$

The energy of the K_α x-ray photon is $E_{K_\alpha} = E_i - E_f \approx (Z-1)^2(-3.4 \text{ eV} + 13.6 \text{ eV})$. That is,

$$E_{K_\alpha} \approx (Z-1)^2(10.2 \text{ eV}) \quad (41.48)$$

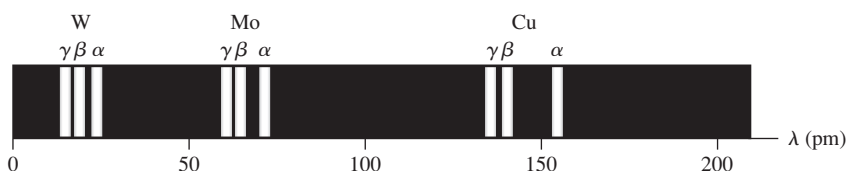
The frequency of the photon is its energy divided by Planck's constant:

$$f = \frac{E}{h} \approx \frac{(Z-1)^2(10.2 \text{ eV})}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = (2.47 \times 10^{15} \text{ Hz})(Z-1)^2$$

This relationship agrees almost exactly with Moseley's experimental law, Eq. (41.47). Indeed, considering the approximations we have made, the agreement is better than we have a right to expect. But our calculation does show how Moseley's law can be understood on the bases of screening and transitions between energy levels.

The hole in the K shell may also be filled by an electron falling from the M or N shell, assuming that these are occupied. If so, the x-ray spectrum of a large group of atoms of a single element shows a series, named the K series, of three lines, called the K_α , K_β , and K_γ lines. These three lines result from transitions in which the K -shell hole is filled by an L , M , or N electron, respectively. Figure 41.25 shows the K series for tungsten ($Z = 74$), molybdenum ($Z = 42$), and copper ($Z = 29$).

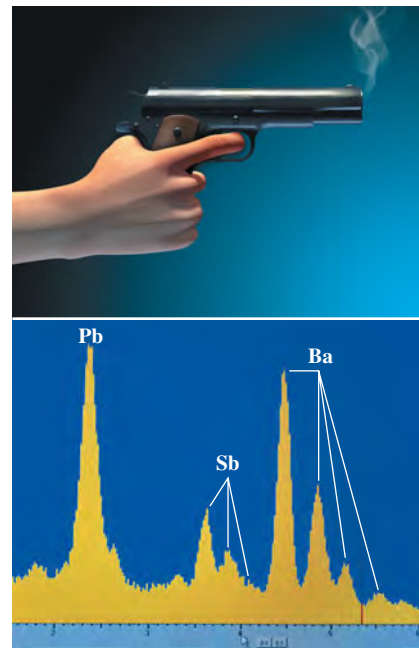
There are other series of x-ray lines, called the L , M , and N series, that are produced after the ejection of electrons from the L , M , and N shells rather than the K shell. Electrons in these outer shells are farther away from the nucleus and are not held as tightly as are those in the K shell, so removing these outer electrons requires less energy. Hence the x-ray photons that are emitted when these vacancies are filled have lower energy than those in the K series.



The three lines in each series are called the K_α , K_β , and K_γ lines. The K_α line is produced by the transition of an L electron to the vacancy in the K shell, the K_β line by an M electron, and the K_γ line by an N electron.

Application X Rays in Forensic Science

When a handgun is fired, a cloud of gunshot residue (GSR) is ejected from the barrel. The x-ray emission spectrum of GSR includes characteristic peaks from lead (Pb), antimony (Sb), and barium (Ba). If a sample taken from a suspect's skin or clothing has an x-ray emission spectrum with these characteristics, it indicates that the suspect recently fired a gun.



41.25 Wavelengths of the K_α , K_β , and K_γ lines of tungsten (W), molybdenum (Mo), and copper (Cu).

Example 41.10 Chemical analysis by x-ray emission

You measure the K_α wavelength for an unknown element, obtaining the value 0.0709 nm. What is the element?

SOLUTION

IDENTIFY and SET UP: To determine which element this is, we need to know its atomic number Z . We can find this using Moseley's law, which relates the frequency of an element's K_α x-ray emission line to that element's atomic number Z . We'll use the relationship $f = c/\lambda$ to calculate the frequency for the K_α line, and then use Eq. (41.47) to find the corresponding value of the atomic number Z . We'll then consult the periodic table (Appendix D) to determine which element has this atomic number.

EXECUTE: The frequency is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{0.0709 \times 10^{-9} \text{ m}} = 4.23 \times 10^{18} \text{ Hz}$$

Solving Moseley's law for Z , we get

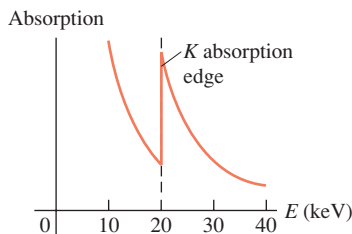
$$Z = 1 + \sqrt{\frac{f}{2.48 \times 10^{15} \text{ Hz}}} = 1 + \sqrt{\frac{4.23 \times 10^{18} \text{ Hz}}{2.48 \times 10^{15} \text{ Hz}}} = 42.3$$

We know that Z has to be an integer; we conclude that $Z = 42$, corresponding to the element molybdenum.

EVALUATE: If you're worried that our calculation did not give an integer for Z , remember that Moseley's law is an empirical relationship. There are slight variations from one atom to another due to differences in the structure of the electron shells. Nonetheless, this example suggests the power of Moseley's law.

Niels Bohr commented that it was Moseley's observations, not the alpha-particle scattering experiments of Rutherford, Geiger, and Marsden (see Section 39.2), that truly convinced physicists that the atom consists of a positive nucleus surrounded by electrons in motion. Unlike Bohr or Rutherford, Moseley did not receive a Nobel Prize for his important work; these awards are given only to living scientists, and Moseley was killed in combat during the First World War.

41.26 When a beam of x rays is passed through a slab of molybdenum, the extent to which the beam is absorbed depends on the energy E of the x-ray photons. A sharp increase in absorption occurs at the K absorption edge at 20 keV. The increase occurs because photons with energies above this value can excite an electron from the K shell of a molybdenum atom into an empty state.

**X-Ray Absorption Spectra**

We can also observe x-ray *absorption* spectra. Unlike optical spectra, the absorption wavelengths are usually not the same as those for emission, especially in many-electron atoms, and do not give simple line spectra. For example, the K_α emission line results from a transition from the L shell to a hole in the K shell. The reverse transition doesn't occur in atoms with $Z \geq 10$ because in the atom's ground state, there is no vacancy in the L shell. To be absorbed, a photon must have enough energy to move an electron to an empty state. Since empty states are only a few electron volts in energy below the free-electron continuum, the minimum absorption energies in many-electron atoms are about the same as the minimum energies that are needed to remove an electron from its shell. Experimentally, if we gradually increase the accelerating voltage and hence the maximum photon energy, we observe sudden increases in absorption when we reach these minimum energies. These sudden jumps of absorption are called *absorption edges* (Fig. 41.26).

Characteristic x-ray spectra provide a very useful analytical tool. Satellite-borne x-ray spectrometers are used to study x-ray emission lines from highly excited atoms in distant astronomical sources. X-ray spectra are also used in air-pollution monitoring and in studies of the abundance of various elements in rocks.

Test Your Understanding of Section 41.7 A beam of photons is passed through a sample of high-temperature atomic hydrogen. At what photon energy would you expect there to be an absorption edge like that shown in Fig. 41.26? (i) 13.60 eV; (ii) 3.40 eV; (iii) 1.51 eV; (iv) all of these; (v) none of these.



Three-dimensional problems: The time-independent Schrödinger equation for three-dimensional problems is given by Eq. (41.5).

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} + \frac{\partial^2 \psi(x, y, z)}{\partial z^2} \right) + U(x, y, z) \psi(x, y, z) = E \psi(x, y, z)$$

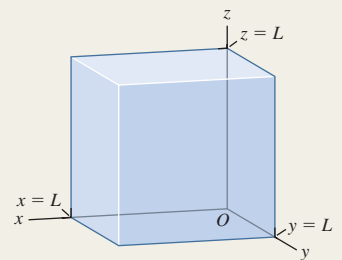
(three-dimensional time-independent Schrödinger equation) (41.5)

Particle in a three-dimensional box: The wave function for a particle in a cubical box is the product of a function of x only, a function of y only, and a function of z only. Each stationary state is described by three quantum numbers (n_x, n_y, n_z). Most of the energy levels given by Eq. (41.16) exhibit degeneracy: More than one quantum state has the same energy. (See Example 41.1.)

$$E_{n_x, n_y, n_z} = \frac{(n_x^2 + n_y^2 + n_z^2) \pi^2 \hbar^2}{2mL^2}$$

$$(n_x = 1, 2, 3, \dots; n_y = 1, 2, 3, \dots; n_z = 1, 2, 3, \dots)$$

(energy levels, particle in a three-dimensional cubical box) (41.16)



The hydrogen atom: The Schrödinger equation for the hydrogen atom gives the same energy levels as the Bohr model. If the nucleus has charge Ze , there is an additional factor of Z^2 in the numerator of Eq. (41.21). The possible magnitudes L of orbital angular momentum are given by Eq. (41.22), and the possible values of the z -component of orbital angular momentum are given by Eq. (41.23). (See Examples 41.2 and 41.3.)

$$E_n = -\frac{1}{(4\pi\epsilon_0)^2} \frac{m_r e^4}{2n^2 \hbar^2} = -\frac{13.60 \text{ eV}}{n^2}$$

(energy levels of hydrogen) (41.21)

$$L = \sqrt{l(l+1)} \hbar$$

($l = 0, 1, 2, \dots, n-1$) (41.22)

$$L_z = m_l \hbar$$

($m_l = 0, \pm 1, \pm 2, \dots, \pm l$) (41.23)

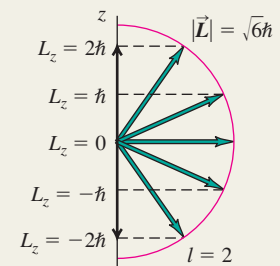
$$P(r) dr = |\psi|^2 dV = |\psi|^2 4\pi r^2 dr$$

(41.25)

$$a = \frac{\epsilon_0 \hbar^2}{\pi m_r e^2} = \frac{4\pi\epsilon_0 \hbar^2}{m_r e^2}$$

$$= 5.29 \times 10^{-11} \text{ m}$$

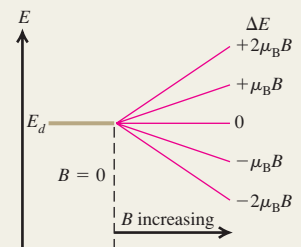
(41.26)



The Zeeman effect: The interaction energy of an electron (mass m) with magnetic quantum number m_l in a magnetic field \vec{B} along the $+z$ -direction is given by Eq. (41.35) or (41.36), where $\mu_B = e\hbar/2m$ is called the Bohr magneton. (See Example 41.5.)

$$U = -\mu_z B = m_l \frac{e\hbar}{2m} B = m_l \mu_B B$$

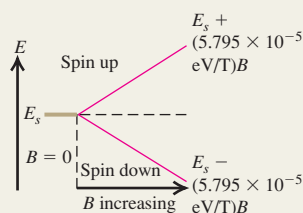
($m_l = 0, \pm 1, \pm 2, \dots, \pm l$) (41.35), (41.36)



Electron spin: An electron has an intrinsic spin angular momentum of magnitude S , given by Eq. (41.38). The possible values of the z -component of the spin angular momentum are $S_z = m_s \hbar$, where $m_s = \pm \frac{1}{2}$. (See Examples 41.6 and 41.7.)

$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2} + 1\right)}\hbar = \sqrt{\frac{3}{4}}\hbar \quad (41.38)$$

$$S_z = \pm \frac{1}{2}\hbar \quad (41.37)$$

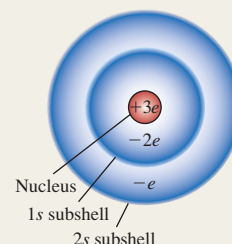


Many-electron atoms: In a hydrogen atom, the quantum numbers n , l , m_l , and m_s of the electron have certain allowed values given by Eq. (41.44). In a many-electron atom, the allowed quantum numbers for each electron are the same as in hydrogen, but the energy levels depend on both n and l because of screening, the partial cancellation of the field of the nucleus by the inner electrons. If the effective (screened) charge attracting an electron is $Z_{\text{eff}}e$, the energies of the levels are given approximately by Eq. (41.45). (See Examples 41.8 and 41.9.)

$$n \geq 1 \quad 0 \leq l \leq n - 1 \quad (41.44)$$

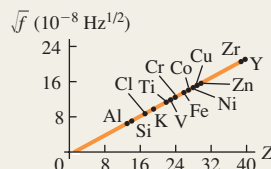
$$|m_l| \leq l \quad m_s = \pm \frac{1}{2}$$

$$E_n = -\frac{Z_{\text{eff}}^2}{n^2}(13.6 \text{ eV}) \quad (41.45)$$



X-ray spectra: Moseley's law states that the frequency of a K_α x ray from a target with atomic number Z is given by Eq. (41.47). Characteristic x-ray spectra result from transitions to a hole in an inner energy level of an atom. (See Example 41.10.)

$$f = (2.48 \times 10^{15} \text{ Hz})(Z - 1)^2 \quad (41.47)$$



BRIDGING PROBLEM

A Many-Electron Atom in a Box

An atom of titanium (Ti) has 22 electrons and has a radius of 1.47×10^{-10} m. As a simple model of this atom, imagine putting 22 electrons into a cubical box that has the same volume as a titanium atom. (a) What is the length of each side of the box? (b) What will be the configuration of the 22 electrons? (c) Find the energies of each of the levels occupied by the electrons. (Ignore the electric forces that the electrons exert on each other.) (d) You remove one of the electrons from the lowest level. As a result, one of the electrons from the highest occupied level drops into the lowest level to fill the hole, emitting a photon in the process. What is the energy of this photon? How does this compare to the energy of the K_α photon for titanium as predicted by Moseley's law?

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.



IDENTIFY and SET UP

- In this problem you'll use ideas from Section 41.2 about a particle in a cubical box. You'll also apply the exclusion principle from Section 41.6 to find the electron configuration of this cubical "atom." The ideas about x-ray spectra from Section 41.7 are also important.
- The target variables are (a) the dimensions of the box, (b) the electron configurations (like those given in Table 41.3 for real atoms), (c) the occupied energy levels of the cubical box, and (d) the energy of the emitted photon.

EXECUTE

- Use your knowledge of geometry to find the length of each side of the box.
- Each electron state is described by four quantum numbers: n_x , n_y , and n_z as described in Section 41.2 and the spin magnetic quantum number m_s described in Section 41.5. Use the exclusion principle to determine the quantum numbers of each of the 22 electrons in the "atom." (*Hint:* Figure 41.4 in Section 41.2 shows the first several energy levels of a cubical box relative to the ground level $E_{1,1,1}$.)
- Use your results from steps 3 and 4 to find the energies of each of the occupied levels.
- Use your result from step 5 to find the energy of the photon emitted when an electron makes a transition from the highest occupied level to the ground level. Compare this to the energy calculated for titanium using Moseley's law.

EVALUATE

- Is this cubical "atom" a useful model for titanium? Why or why not?
- In this problem you ignored the electric interactions between electrons. To estimate how large these are, find the electrostatic potential energy of two electrons separated by half the length of the box. How does this compare to the energy levels you calculated in step 5? Is it a good approximation to ignore these interactions?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q41.1 Particle A is described by the wave function $\psi(x, y, z)$. Particle B is described by the wave function $\psi(x, y, z)e^{i\phi}$, where ϕ is a real constant. How does the probability of finding particle A within a volume dV around a certain point in space compare with the probability of finding particle B within this same volume?

Q41.2 What are the most significant differences between the Bohr model of the hydrogen atom and the Schrödinger analysis? What are the similarities?

Q41.3 For a body orbiting the sun, such as a planet, comet, or asteroid, is there any restriction on the z -component of its orbital angular momentum such as there is with the z -component of the electron's orbital angular momentum in hydrogen? Explain.

Q41.4 Why is the analysis of the helium atom much more complex than that of the hydrogen atom, either in a Bohr type of model or using the Schrödinger equation?

Q41.5 The Stern–Gerlach experiment is always performed with beams of *neutral* atoms. Wouldn't it be easier to form beams using *ionized* atoms? Why won't this work?

Q41.6 (a) If two electrons in hydrogen atoms have the same principal quantum number, can they have different orbital angular momentum? How? (b) If two electrons in hydrogen atoms have the same orbital angular-momentum quantum number, can they have different principal quantum numbers? How?

Q41.7 In the Stern–Gerlach experiment, why is it essential for the magnetic field to be *inhomogeneous* (that is, nonuniform)?

Q41.8 In the ground state of the helium atom one electron must have “spin down” and the other “spin up.” Why?

Q41.9 An electron in a hydrogen atom is in an s level, and the atom is in a magnetic field $\vec{B} = B\hat{k}$. Explain why the “spin up” state ($m_s = +\frac{1}{2}$) has a higher energy than the “spin down” state ($m_s = -\frac{1}{2}$).

Q41.10 The central-field approximation is more accurate for alkali metals than for transition metals such as iron, nickel, or copper. Why?

Q41.11 Table 41.3 shows that for the ground state of the potassium atom, the outermost electron is in a $4s$ state. What does this tell you about the relative energies of the $3d$ and $4s$ levels for this atom? Explain.

Q41.12 Do gravitational forces play a significant role in atomic structure? Explain.

Q41.13 Why do the transition elements ($Z = 21$ to 30) all have similar chemical properties?

Q41.14 Use Table 41.3 to help determine the ground-state electron configuration of the neutral gallium atom (Ga) as well as the ions Ga^+ and Ga^- . Gallium has an atomic number of 31.

Q41.15 On the basis of the Pauli exclusion principle, the structure of the periodic table of the elements shows that there must be a fourth quantum number in addition to n , l , and m_l . Explain.

Q41.16 A small amount of magnetic-field splitting of spectral lines occurs even when the atoms are not in a magnetic field. What causes this?

Q41.17 The ionization energies of the alkali metals (that is, the lowest energy required to remove one outer electron when the

atom is in its ground state) are about 4 or 5 eV, while those of the noble gases are in the range from 11 to 25 eV. Why is there a difference?

Q41.18 The energy required to remove the $3s$ electron from a sodium atom in its ground state is about 5 eV. Would you expect the energy required to remove an additional electron to be about the same, or more, or less? Why?

Q41.19 What is the “central-field approximation” and why is it only an approximation?

Q41.20 The nucleus of a gold atom contains 79 protons. How does the energy required to remove a $1s$ electron completely from a gold atom compare with the energy required to remove the electron from the ground level in a hydrogen atom? In what region of the electromagnetic spectrum would a photon with this energy for each of these two atoms lie?

Q41.21 (a) Can you show that the orbital angular momentum of an electron in any given direction (e.g., along the z -axis) is *always* less than or equal to its total orbital angular momentum? In which cases would the two be equal to each other? (b) Is the result in part (a) true for a classical object, such as a spinning top or planet?

Q41.22 An atom in its ground level absorbs a photon with energy equal to the K absorption edge. Does absorbing this photon ionize this atom? Explain.

Q41.23 Can a hydrogen atom emit x rays? If so, how? If not, why not?

EXERCISES

Section 41.2 Particle in a Three-Dimensional Box

41.1 • For a particle in a three-dimensional box, what is the degeneracy (number of different quantum states with the same energy) of the following energy levels: (a) $3\pi^2\hbar^2/2mL^2$ and (b) $9\pi^2\hbar^2/2mL^2$?

41.2 • **CP** Model a hydrogen atom as an electron in a cubical box with side length L . Set the value of L so that the volume of the box equals the volume of a sphere of radius $a = 5.29 \times 10^{-11}$ m, the Bohr radius. Calculate the energy separation between the ground and first excited levels, and compare the result to this energy separation calculated from the Bohr model.

41.3 • **CP** A photon is emitted when an electron in a three-dimensional box of side length 8.00×10^{-11} m makes a transition from the $n_X = 2, n_Y = 2, n_Z = 1$ state to the $n_X = 1, n_Y = 1, n_Z = 1$ state. What is the wavelength of this photon?

41.4 • For each of the following states of a particle in a three-dimensional box, at what points is the probability distribution function a maximum: (a) $n_X = 1, n_Y = 1, n_Z = 1$ and (b) $n_X = 2, n_Y = 2, n_Z = 1$?

41.5 •• A particle is in the three-dimensional box of Section 41.1. For the state $n_X = 2, n_Y = 2, n_Z = 1$, for what planes (in addition to the walls of the box) is the probability distribution function zero? Compare this number of planes to the corresponding number of planes where $|\psi|^2$ is zero for the lower-energy state $n_X = 2, n_Y = 1, n_Z = 1$ and for the ground state $n_X = 1, n_Y = 1, n_Z = 1$.

41.6 • What is the energy difference between the two lowest energy levels for a proton in a cubical box with side length 1.00×10^{-14} m, the approximate diameter of a nucleus?

Section 41.3 The Hydrogen Atom

41.7 •• Consider an electron in the N shell. (a) What is the smallest orbital angular momentum it could have? (b) What is the largest orbital angular momentum it could have? Express your answers in terms of \hbar and in SI units. (c) What is the largest orbital angular momentum this electron could have in any chosen direction? Express your answers in terms of \hbar and in SI units. (d) What is the largest spin angular momentum this electron could have in any chosen direction? Express your answers in terms of \hbar and in SI units. (e) For the electron in part (c), what is the ratio of its spin angular momentum in the z -direction to its orbital angular momentum in the z -direction?

41.8 • An electron is in the hydrogen atom with $n = 5$. (a) Find the possible values of L and L_z for this electron, in units of \hbar . (b) For each value of L , find all the possible angles between \vec{L} and the z -axis. (c) What are the maximum and minimum values of the magnitude of the angle between \vec{L} and the z -axis?

41.9 • The orbital angular momentum of an electron has a magnitude of 4.716×10^{-34} kg \cdot m²/s. What is the angular-momentum quantum number l for this electron?

41.10 • Consider states with angular-momentum quantum number $l = 2$. (a) In units of \hbar , what is the largest possible value of L_z ? (b) In units of \hbar , what is the value of L ? Which is larger: L or the maximum possible L_z ? (c) For each allowed value of L_z , what angle does the vector \vec{L} make with the $+z$ -axis? How does the minimum angle for $l = 2$ compare to the minimum angle for $l = 3$ calculated in Example 41.3?

41.11 • Calculate, in units of \hbar , the magnitude of the maximum orbital angular momentum for an electron in a hydrogen atom for states with a principal quantum number of 2, 20, and 200. Compare each with the value of $n\hbar$ postulated in the Bohr model. What trend do you see?

41.12 • (a) Make a chart showing all the possible sets of quantum numbers l and m_l for the states of the electron in the hydrogen atom when $n = 5$. How many combinations are there? (b) What are the energies of these states?

41.13 •• (a) How many different $5g$ states does hydrogen have? (b) Which of the states in part (a) has the largest angle between \vec{L} and the z -axis, and what is that angle? (c) Which of the states in part (a) has the smallest angle between \vec{L} and the z -axis, and what is that angle?

41.14 •• **CALC** (a) What is the probability that an electron in the $1s$ state of a hydrogen atom will be found at a distance less than $a/2$ from the nucleus? (b) Use the results of part (a) and of Example 41.4 to calculate the probability that the electron will be found at distances between $a/2$ and a from the nucleus.

41.15 • **CALC** In Example 41.4 fill in the missing details that show that $P = 1 - 5e^{-2}$.

41.16 • Show that $\Phi(\phi) = e^{im_l\phi} = \Phi(\phi + 2\pi)$ (that is, show that $\Phi(\phi)$ is periodic with period 2π) if and only if m_l is restricted to the values $0, \pm 1, \pm 2, \dots$ (*Hint*: Euler's formula states that $e^{i\phi} = \cos\phi + i\sin\phi$.)

Section 41.4 The Zeeman Effect

41.17 • A hydrogen atom in a $3p$ state is placed in a uniform external magnetic field \vec{B} . Consider the interaction of the magnetic field with the atom's orbital magnetic dipole moment. (a) What field magnitude B is required to split the $3p$ state into multiple lev-

els with an energy difference of 2.71×10^{-5} eV between adjacent levels? (b) How many levels will there be?

41.18 • A hydrogen atom is in a d state. In the absence of an external magnetic field the states with different m_l values have (approximately) the same energy. Consider the interaction of the magnetic field with the atom's orbital magnetic dipole moment. (a) Calculate the splitting (in electron volts) of the m_l levels when the atom is put in a 0.400-T magnetic field that is in the $+z$ -direction. (b) Which m_l level will have the lowest energy? (c) Draw an energy-level diagram that shows the d levels with and without the external magnetic field.

41.19 • A hydrogen atom in the $5g$ state is placed in a magnetic field of 0.600 T that is in the z -direction. (a) Into how many levels is this state split by the interaction of the atom's orbital magnetic dipole moment with the magnetic field? (b) What is the energy separation between adjacent levels? (c) What is the energy separation between the level of lowest energy and the level of highest energy?

41.20 •• **CP** A hydrogen atom undergoes a transition from a $2p$ state to the $1s$ ground state. In the absence of a magnetic field, the energy of the photon emitted is 122 nm. The atom is then placed in a strong magnetic field in the z -direction. Ignore spin effects; consider only the interaction of the magnetic field with the atom's orbital magnetic moment. (a) How many different photon wavelengths are observed for the $2p \rightarrow 1s$ transition? What are the m_l values for the initial and final states for the transition that leads to each photon wavelength? (b) One observed wavelength is exactly the same with the magnetic field as without. What are the initial and final m_l values for the transition that produces a photon of this wavelength? (c) One observed wavelength with the field is longer than the wavelength without the field. What are the initial and final m_l values for the transition that produces a photon of this wavelength? (d) Repeat part (c) for the wavelength that is shorter than the wavelength in the absence of the field.

Section 41.5 Electron Spin

41.21 •• **CP Classical Electron Spin.** (a) If you treat an electron as a classical spherical object with a radius of 1.0×10^{-17} m, what angular speed is necessary to produce a spin angular momentum of magnitude $\sqrt{\frac{3}{4}}\hbar$? (b) Use $v = r\omega$ and the result of part (a) to calculate the speed v of a point at the electron's equator. What does your result suggest about the validity of this model?

41.22 •• A hydrogen atom in the $n = 1, m_s = -\frac{1}{2}$ state is placed in a magnetic field with a magnitude of 0.480 T in the $+z$ -direction. (a) Find the magnetic interaction energy (in electron volts) of the electron with the field. (b) Is there any orbital magnetic dipole moment interaction for this state? Explain. Can there be an orbital magnetic dipole moment interaction for $n \neq 1$?

41.23 • Calculate the energy difference between the $m_s = \frac{1}{2}$ ("spin up") and $m_s = -\frac{1}{2}$ ("spin down") levels of a hydrogen atom in the $1s$ state when it is placed in a 1.45-T magnetic field in the negative z -direction. Which level, $m_s = \frac{1}{2}$ or $m_s = -\frac{1}{2}$, has the lower energy?

41.24 • **CP** The hyperfine interaction in a hydrogen atom between the magnetic dipole moment of the proton and the spin magnetic dipole moment of the electron splits the ground level into two levels separated by 5.9×10^{-6} eV. (a) Calculate the wavelength and frequency of the photon emitted when the atom makes a transition between these states, and compare your answer to the value given at the end of Section 41.5. In what part of the electromagnetic spectrum does this lie? Such photons are emitted by cold hydrogen clouds in interstellar space; by detecting these photons,

astronomers can learn about the number and density of such clouds. (b) Calculate the effective magnetic field experienced by the electron in these states (see Fig. 41.18). Compare your result to the effective magnetic field due to the spin-orbit coupling calculated in Example 41.7.

41.25 • A hydrogen atom in a particular orbital angular momentum state is found to have j quantum numbers $\frac{7}{2}$ and $\frac{9}{2}$. What is the letter that labels the value of l for the state?

Section 41.6 Many-Electron Atoms and the Exclusion Principle

41.26 • For germanium (Ge, $Z = 32$), make a list of the number of electrons in each subshell ($1s, 2s, 2p, \dots$). Use the allowed values of the quantum numbers along with the exclusion principle; do *not* refer to Table 41.3.

41.27 • Make a list of the four quantum numbers $n, l, m_l,$ and m_s for each of the 10 electrons in the ground state of the neon atom. Do *not* refer to Table 41.2 or 41.3.

41.28 •• (a) Write out the ground-state electron configuration ($1s^2, 2s^2, \dots$) for the carbon atom. (b) What element of next-larger Z has chemical properties similar to those of carbon? Give the ground-state electron configuration for this element.

41.29 •• (a) Write out the ground-state electron configuration ($1s^2, 2s^2, \dots$) for the beryllium atom. (b) What element of next-larger Z has chemical properties similar to those of beryllium? Give the ground-state electron configuration of this element. (c) Use the procedure of part (b) to predict what element of next-larger Z than in (b) will have chemical properties similar to those of the element you found in part (b), and give its ground-state electron configuration.

41.30 • For magnesium, the first ionization potential is 7.6 eV. The second ionization potential (additional energy required to remove a second electron) is almost twice this, 15 eV, and the third ionization potential is much larger, about 80 eV. How can these numbers be understood?

41.31 • The $5s$ electron in rubidium (Rb) sees an effective charge of $2.771e$. Calculate the ionization energy of this electron.

41.32 • The energies of the $4s, 4p,$ and $4d$ states of potassium are given in Example 41.9. Calculate Z_{eff} for each state. What trend do your results show? How can you explain this trend?

41.33 • (a) The doubly charged ion N^{2+} is formed by removing two electrons from a nitrogen atom. What is the ground-state electron configuration for the N^{2+} ion? (b) Estimate the energy of the least strongly bound level in the L shell of N^{2+} . (c) The doubly charged ion P^{2+} is formed by removing two electrons from a phosphorus atom. What is the ground-state electron configuration for the P^{2+} ion? (d) Estimate the energy of the least strongly bound level in the M shell of P^{2+} .

41.34 • (a) The energy of the $2s$ state of lithium is -5.391 eV. Calculate the value of Z_{eff} for this state. (b) The energy of the $4s$ state of potassium is -4.339 eV. Calculate the value of Z_{eff} for this state. (c) Compare Z_{eff} for the $2s$ state of lithium, the $3s$ state of sodium (see Example 41.8), and the $4s$ state of potassium. What trend do you see? How can you explain this trend?

41.35 • Estimate the energy of the highest- l state for (a) the L shell of Be^+ and (b) the N shell of Ca^+ .

Section 41.7 X-Ray Spectra

41.36 • A K_α x ray emitted from a sample has an energy of 7.46 keV. Of which element is the sample made?

41.37 • Calculate the frequency, energy (in keV), and wavelength of the K_α x ray for the elements (a) calcium (Ca, $Z = 20$); (b) cobalt (Co, $Z = 27$); (c) cadmium (Cd, $Z = 48$).

41.38 •• The energies for an electron in the $K, L,$ and M shells of the tungsten atom are $-69,500$ eV, $-12,000$ eV, and -2200 eV, respectively. Calculate the wavelengths of the K_α and K_β x rays of tungsten.

PROBLEMS

41.39 • In terms of the ground-state energy $E_{1,1,1}$, what is the energy of the highest level occupied by an electron when 10 electrons are placed into a cubical box?

41.40 •• CALC A particle in the three-dimensional box of Section 41.2 is in the ground state, where $n_x = n_y = n_z = 1$. (a) Calculate the probability that the particle will be found somewhere between $x = 0$ and $x = L/2$. (b) Calculate the probability that the particle will be found somewhere between $x = L/4$ and $x = L/2$. Compare your results to the result of Example 41.1 for the probability of finding the particle in the region $x = 0$ to $x = L/4$.

41.41 •• CALC A particle is in the three-dimensional box of Section 41.2. (a) Consider the cubical volume defined by $0 \leq x \leq L/4, 0 \leq y \leq L/4,$ and $0 \leq z \leq L/4$. What fraction of the total volume of the box is this cubical volume? (b) If the particle is in the ground state ($n_x = 1, n_y = 1, n_z = 1$) calculate the probability that the particle will be found in the cubical volume defined in part (a). (c) Repeat the calculation of part (b) when the particle is in the state $n_x = 2, n_y = 1, n_z = 1$.

41.42 •• CALC A particle is described by the normalized wave function $\psi(x, y, z) = A x e^{-\alpha x^2} e^{-\beta y^2} e^{-\gamma z^2}$, where $A, \alpha, \beta,$ and γ are all real, positive constants. The probability that the particle will be found in the infinitesimal volume $dx dy dz$ centered at the point (x_0, y_0, z_0) is $|\psi(x_0, y_0, z_0)|^2 dx dy dz$. (a) At what value of x_0 is the particle most likely to be found? (b) Are there values of x_0 for which the probability of the particle being found is zero? If so, at what x_0 ?

41.43 •• CALC A particle is described by the normalized wave function $\psi(x, y, z) = A e^{-\alpha(x^2+y^2+z^2)}$, where A and α are real, positive constants. (a) Determine the probability of finding the particle at a distance between r and $r + dr$ from the origin. (*Hint:* See Problem 41.42. Consider a spherical shell centered on the origin with inner radius r and thickness dr .) (b) For what value of r does the probability in part (a) have its maximum value? Is this the same value of r for which $|\psi(x, y, z)|^2$ is a maximum? Explain any differences.

41.44 •• CP CALC A Three-Dimensional Isotropic Harmonic Oscillator. An isotropic harmonic oscillator has the potential-energy function $U(x, y, z) = \frac{1}{2}k'(x^2 + y^2 + z^2)$. (*Isotropic* means that the force constant k' is the same in all three coordinate directions.) (a) Show that for this potential, a solution to Eq. (41.5) is given by $\psi = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z)$. In this expression, $\psi_{n_x}(x)$ is a solution to the one-dimensional harmonic oscillator Schrödinger equation, Eq. (40.44), with energy $E_{n_x} = (n_x + \frac{1}{2})\hbar\omega$. The functions $\psi_{n_y}(y)$ and $\psi_{n_z}(z)$ are analogous one-dimensional wave functions for oscillations in the y - and z -directions. Find the energy associated with this ψ . (b) From your results in part (a) what are the ground-level and first-excited-level energies of the three-dimensional isotropic oscillator? (c) Show that there is only one state (one set of quantum numbers $n_x, n_y,$ and n_z) for the ground level but three states for the first excited level.

41.45 •• CP CALC Three-Dimensional Anisotropic Harmonic Oscillator. An oscillator has the potential-energy function $U(x, y, z) = \frac{1}{2}k_1'(x^2 + y^2) + \frac{1}{2}k_2'z^2$, where $k_1' > k_2'$. This oscillator is called *anisotropic* because the force constant is not the same in all three coordinate directions. (a) Find a general expression

for the energy levels of the oscillator (see Problem 41.44). (b) From your results in part (a), what are the ground-level and first-excited-level energies of this oscillator? (c) How many states (different sets of quantum numbers n_x , n_y , and n_z) are there for the ground level and for the first excited level? Compare to part (c) of Problem 41.44.

41.46 •• An electron in hydrogen is in the $5f$ state. (a) Find the largest possible value of the z -component of its angular momentum. (b) Show that for the electron in part (a), the corresponding x - and y -components of its angular momentum satisfy the equation $\sqrt{L_x^2 + L_y^2} = \hbar\sqrt{3}$.

41.47 •• (a) Show that the total number of atomic states (including different spin states) in a shell of principal quantum number n is $2n^2$. [Hint: The sum of the first N integers $1 + 2 + 3 + \cdots + N$ is equal to $N(N + 1)/2$.] (b) Which shell has 50 states?

41.48 •• (a) What is the lowest possible energy (in electron volts) of an electron in hydrogen if its orbital angular momentum is $\sqrt{12}\hbar$? (b) What are the largest and smallest values of the z -component of the orbital angular momentum (in terms of \hbar) for the electron in part (a)? (c) What are the largest and smallest values of the spin angular momentum (in terms of \hbar) for the electron in part (a)? (d) What are the largest and smallest values of the orbital angular momentum (in terms of \hbar) for an electron in the M shell of hydrogen?

41.49 • Consider an electron in hydrogen having total energy -0.5440 eV. (a) What are the possible values of its orbital angular momentum (in terms of \hbar)? (b) What wavelength of light would it take to excite this electron to the next higher shell? Is this photon visible to humans?

41.50 • (a) Show all the distinct states for an electron in the N shell of hydrogen. Include all four quantum numbers. (b) For an f electron in the N shell, what is the largest possible orbital angular momentum and the greatest positive value for the component of this angular momentum along any chosen direction (the z -axis)? What is the magnitude of its spin angular momentum? Express these quantities in units of \hbar . (c) For an electron in the d state of the N shell, what are the maximum and minimum angles between its angular momentum vector and any chosen direction (the z -axis)? (d) What is the largest value of the orbital angular momentum for an f electron in the M shell?

41.51 • (a) The energy of an electron in the $4s$ state of sodium is -1.947 eV. What is the effective net charge of the nucleus “seen” by this electron? On the average, how many electrons screen the nucleus? (b) For an outer electron in the $4p$ state of potassium, on the average 17.2 inner electrons screen the nucleus. (i) What is the effective net charge of the nucleus “seen” by this outer electron? (ii) What is the energy of this outer electron?

41.52 • CALC For a hydrogen atom, the probability $P(r)$ of finding the electron within a spherical shell with inner radius r and outer radius $r + dr$ is given by Eq. (41.25). For a hydrogen atom in the $1s$ ground state, at what value of r does $P(r)$ have its maximum value? How does your result compare to the distance between the electron and the nucleus for the $n = 1$ state in the Bohr model, Eq. (41.26)?

41.53 •• CALC Consider a hydrogen atom in the $1s$ state. (a) For what value of r is the potential energy $U(r)$ equal to the total energy E ? Express your answer in terms of a . This value of r is called the *classical turning point*, since this is where a Newtonian particle would stop its motion and reverse direction. (b) For r greater than the classical turning point, $U(r) > E$. Classically, the particle cannot be in this region, since the kinetic energy cannot be negative. Calculate the probability of the electron being found in this classically forbidden region.

41.54 • CP Rydberg Atoms. Rydberg atoms are atoms whose outermost electron is in an excited state with a very large principal quantum number. Rydberg atoms have been produced in the laboratory and detected in interstellar space. (a) Why do all neutral Rydberg atoms with the same n value have essentially the same ionization energy, independent of the total number of electrons in the atom? (b) What is the ionization energy for a Rydberg atom with a principal quantum number of 350? What is the radius in the Bohr model of the Rydberg electron’s orbit? (c) Repeat part (b) for $n = 650$.

41.55 •• CALC The wave function for a hydrogen atom in the $2s$ state is

$$\psi_{2s}(r) = \frac{1}{\sqrt{32\pi a^3}} \left(2 - \frac{r}{a} \right) e^{-r/2a}$$

(a) Verify that this function is normalized. (b) In the Bohr model, the distance between the electron and the nucleus in the $n = 2$ state is exactly $4a$. Calculate the probability that an electron in the $2s$ state will be found at a distance less than $4a$ from the nucleus.

41.56 •• CALC The normalized wave function for a hydrogen atom in the $2s$ state is given in Problem 41.55. (a) For a hydrogen atom in the $2s$ state, at what value of r is $P(r)$ maximum? How does your result compare to $4a$, the distance between the electron and the nucleus in the $n = 2$ state of the Bohr model? (b) At what value of r (other than $r = 0$ or $r = \infty$) is $P(r)$ equal to zero, so that the probability of finding the electron at that separation from the nucleus is zero? Compare your result to Fig. 41.9.

41.57 •• (a) For an excited state of hydrogen, show that the smallest angle that the orbital angular momentum vector \vec{L} can have with the z -axis is

$$(\theta_L)_{\min} = \arccos\left(\frac{n-1}{\sqrt{n(n-1)}}\right)$$

(b) What is the corresponding expression for $(\theta_L)_{\max}$, the largest possible angle between \vec{L} and the z -axis?

41.58 •• (a) If the value of L_z is known, we cannot know either L_x or L_y precisely. But we can know the value of the quantity $\sqrt{L_x^2 + L_y^2}$. Write an expression for this quantity in terms of l , m_l , and \hbar . (b) What is the meaning of $\sqrt{L_x^2 + L_y^2}$? (c) For a state of nonzero orbital angular momentum, find the maximum and minimum values of $\sqrt{L_x^2 + L_y^2}$. Explain your results.

41.59 •• CALC The normalized radial wave function for the $2p$ state of the hydrogen atom is $R_{2p} = (1/\sqrt{24a^5})re^{-r/2a}$. After we average over the angular variables, the radial probability function becomes $P(r) dr = (R_{2p})^2 r^2 dr$. At what value of r is $P(r)$ for the $2p$ state a maximum? Compare your results to the radius of the $n = 2$ state in the Bohr model.

41.60 •• CP Stern–Gerlach Experiment. In a Stern–Gerlach experiment, the deflecting force on the atom is $F_z = -\mu_z(dB_z/dz)$, where μ_z is given by Eq. (41.40) and dB_z/dz is the magnetic-field gradient. In a particular experiment the magnetic-field region is 50.0 cm long; assume the magnetic-field gradient is constant in this region. A beam of silver atoms enters the magnetic field with a speed of 525 m/s. What value of dB_z/dz is required to give a separation of 1.0 mm between the two spin components as they exit the field? (Note: The magnetic dipole moment of silver is the same as that for hydrogen, since its valence electron is in an $l = 0$ state.)

41.61 • Consider the transition from a $3d$ to a $2p$ state of hydrogen in an external magnetic field. Assume that the effects of electron

spin can be ignored (which is not actually the case) so that the magnetic field interacts only with the orbital angular momentum. Identify each allowed transition by the m_l values of the initial and final states. For each of these allowed transitions, determine the shift of the transition energy from the zero-field value and show that there are three different transition energies.

41.62 •• An atom in a $3d$ state emits a photon of wavelength 475.082 nm when it decays to a $2p$ state. (a) What is the energy (in electron volts) of the photon emitted in this transition? (b) Use the selection rules described in Section 41.4 to find the allowed transitions if the atom is now in an external magnetic field of 3.500 T. Ignore the effects of the electron's spin. (c) For the case in part (b), if the energy of the $3d$ state was originally -8.50000 eV with no magnetic field present, what will be the energies of the states into which it splits in the magnetic field? (d) What are the allowed wavelengths of the light emitted during transition in part (b)?

41.63 •• CALC Spectral Analysis. While studying the spectrum of a gas cloud in space, an astronomer magnifies a spectral line that results from a transition from a p state to an s state. She finds that the line at 575.050 nm has actually split into three lines, with adjacent lines 0.0462 nm apart, indicating that the gas is in an external magnetic field. (Ignore effects due to electron spin.) What is the strength of the external magnetic field?

41.64 •• A hydrogen atom makes a transition from an $n = 3$ state to an $n = 2$ state (the Balmer H_α line) while in a magnetic field in the $+z$ -direction and with magnitude 1.40 T. (a) If the magnetic quantum number is $m_l = 2$ in the initial ($n = 3$) state and $m_l = 1$ in the final ($n = 2$) state, by how much is each energy level shifted from the zero-field value? (b) By how much is the wavelength of the H_α line shifted from the zero-field value? Is the wavelength increased or decreased? Disregard the effect of electron spin. [*Hint:* Use the result of Problem 39.86(c).]

41.65 • CP A large number of hydrogen atoms in $1s$ states are placed in an external magnetic field that is in the $+z$ -direction. Assume that the atoms are in thermal equilibrium at room temperature, $T = 300$ K. According to the Maxwell-Boltzmann distribution (see Section 39.4), what is the ratio of the number of atoms in the $m_s = \frac{1}{2}$ state to the number in the $m_s = -\frac{1}{2}$ state when the magnetic-field magnitude is (a) 5.00×10^{-5} T (approximately the earth's field); (b) 0.500 T; (c) 5.00 T?

41.66 •• Effective Magnetic Field. An electron in a hydrogen atom is in the $2p$ state. In a simple model of the atom, assume that the electron circles the proton in an orbit with radius r equal to the Bohr-model radius for $n = 2$. Assume that the speed v of the orbiting electron can be calculated by setting $L = mvr$ and taking L to have the quantum-mechanical value for a $2p$ state. In the frame of the electron, the proton orbits with radius r and speed v . Model the orbiting proton as a circular current loop, and calculate the magnetic field it produces at the location of the electron.

41.67 •• Weird Universe. In another universe, the electron is a spin- $\frac{3}{2}$ rather than a spin- $\frac{1}{2}$ particle, but all other physics are the same as in our universe. In this universe, (a) what are the atomic numbers of the lightest two inert gases? (b) What is the ground-state electron configuration of sodium?

41.68 • For an ion with nuclear charge Z and a single electron, the electric potential energy is $-Ze^2/4\pi\epsilon_0 r$ and the expression for the energies of the states and for the normalized wave functions are obtained from those for hydrogen by replacing e^2 by Ze^2 . Consider the N^{6+} ion, with seven protons and one electron. (a) What is the ground-state energy in electron volts? (b) What is the ionization energy, the energy required to remove the electron from the N^{6+}

ion if it is initially in the ground state? (c) What is the distance a [given for hydrogen by Eq. (41.26)] for this ion? (d) What is the wavelength of the photon emitted when the N^{6+} ion makes a transition from the $n = 2$ state to the $n = 1$ ground state?

41.69 •• A hydrogen atom in an $n = 2$, $l = 1$, $m_l = -1$ state emits a photon when it decays to an $n = 1$, $l = 0$, $m_l = 0$ ground state. (a) In the absence of an external magnetic field, what is the wavelength of this photon? (b) If the atom is in a magnetic field in the $+z$ -direction and with a magnitude of 2.20 T, what is the shift in the wavelength of the photon from the zero-field value? Does the magnetic field increase or decrease the wavelength? Disregard the effect of electron spin. [*Hint:* Use the result of Problem 39.86(c).]

41.70 •• A lithium atom has three electrons, and the $^2S_{1/2}$ ground-state electron configuration is $1s^2 2s$. The $1s^2 2p$ excited state is split into two closely spaced levels, $^2P_{3/2}$ and $^2P_{1/2}$, by the spin-orbit interaction (see Example 41.7 in Section 41.5). A photon with wavelength 67.09608 μm is emitted in the $^2P_{3/2} \rightarrow ^2S_{1/2}$ transition, and a photon with wavelength 67.09761 μm is emitted in the $^2P_{1/2} \rightarrow ^2S_{1/2}$ transition. Calculate the effective magnetic field seen by the electron in the $1s^2 2p$ state of the lithium atom. How does your result compare to that for the $3p$ level of sodium found in Example 41.7?

41.71 • Estimate the minimum and maximum wavelengths of the characteristic x rays emitted by (a) vanadium ($Z = 23$) and (b) rhenium ($Z = 45$). Discuss any approximations that you make.

41.72 •• CP Electron Spin Resonance. Electrons in the lower of two spin states in a magnetic field can absorb a photon of the right frequency and move to the higher state. (a) Find the magnetic-field magnitude B required for this transition in a hydrogen atom with $n = 1$ and $l = 0$ to be induced by microwaves with wavelength λ . (b) Calculate the value of B for a wavelength of 3.50 cm.

CHALLENGE PROBLEMS

41.73 ••• Each of $2N$ electrons (mass m) is free to move along the x -axis. The potential-energy function for each electron is $U(x) = \frac{1}{2}k'x^2$, where k' is a positive constant. The electric and magnetic interactions between electrons can be ignored. Use the exclusion principle to show that the minimum energy of the system of $2N$ electrons is $\hbar N^2 \sqrt{k'/m}$. (*Hint:* See Section 40.5 and the hint given in Problem 41.47.)

41.74 ••• CP Consider a simple model of the helium atom in which two electrons, each with mass m , move around the nucleus (charge $+2e$) in the same circular orbit. Each electron has orbital angular momentum \hbar (that is, the orbit is the smallest-radius Bohr orbit), and the two electrons are always on opposite sides of the nucleus. Ignore the effects of spin. (a) Determine the radius of the orbit and the orbital speed of each electron. [*Hint:* Follow the procedure used in Section 39.3 to derive Eqs. (39.8) and (39.9). Each electron experiences an attractive force from the nucleus and a repulsive force from the other electron.] (b) What is the total kinetic energy of the electrons? (c) What is the potential energy of the system (the nucleus and the two electrons)? (d) In this model, how much energy is required to remove both electrons to infinity? How does this compare to the experimental value of 79.0 eV?

41.75 ••• CALC Repeat the calculation of Problem 41.53 for a one-electron ion with nuclear charge Z . (See Problem 41.68.) How does the probability of the electron being found in the classically forbidden region depend on Z ?

Answers

Chapter Opening Question ?

Helium is inert because it has a filled K shell, while sodium is very reactive because its third electron is loosely bound in an L shell. See Section 41.6 for more details.

Test Your Understanding Questions

41.1 Answer: (iv) If $U(x, y, z) = 0$ in a certain region of space, we can rewrite the time-independent Schrödinger equation [Eq. (41.5)] for that region as $\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \partial^2\psi/\partial z^2 = (-2mE/\hbar^2)\psi$. We are told that all of the second derivatives of $\psi(x, y, z)$ are positive in this region, so the left-hand side of this equation is positive. Hence the right-hand side $(-2mE/\hbar^2)\psi$ must also be positive. Since $E > 0$, the quantity $-2mE/\hbar^2$ is negative, and so $\psi(x, y, z)$ must be negative.

41.2 Answer: (iv), (ii), (i) and (iii) (tie) Equation (41.16) shows that the energy levels for a cubical box are proportional to the quantity $n_x^2 + n_y^2 + n_z^2$. Hence ranking in order of this quantity is the same as ranking in order of energy. For the four cases we are given, we have (i) $n_x^2 + n_y^2 + n_z^2 = 2^2 + 3^2 + 2^2 = 17$; (ii) $n_x^2 + n_y^2 + n_z^2 = 4^2 + 1^2 + 1^2 = 18$; (iii) $n_x^2 + n_y^2 + n_z^2 = 2^2 + 2^2 + 3^2 = 17$; and (iv) $n_x^2 + n_y^2 + n_z^2 = 1^2 + 3^2 + 3^2 = 19$. The states $(n_x, n_y, n_z) = (2, 3, 2)$ and $(n_x, n_y, n_z) = (2, 2, 3)$ have the same energy (they are degenerate).

41.3 Answer: (ii) and (iii) (tie), (i) An electron in a state with principal quantum number n is most likely to be found at $r = n^2a$. This result is independent of the values of the quantum numbers l and m_l . Hence an electron with $n = 2$ (most likely to be found at $r = 4a$) is more likely to be found near $r = 5a$ than an electron with $n = 1$ (most likely to be found at $r = a$).

41.4 Answer: no All that matters is the component of the electron's orbital magnetic moment along the direction of \vec{B} . We called this quantity μ_z in Eq. (41.32) because we *defined* the positive z -axis to be in the direction of \vec{B} . In reality, the names of the axes are entirely arbitrary.

41.5 Answer: (iv) For the magnetic moment to be perfectly aligned with the z -direction, the z -component of the spin vector \vec{S} would have to have the same absolute value as \vec{S} . However, the possible values of S_z are $\pm \frac{1}{2}\hbar$ [Eq. (41.37)], while the magnitude of the spin vector is $S = \sqrt{\frac{3}{4}}\hbar$ [Eq. (41.38)]. Hence \vec{S} can never be perfectly aligned with any one direction in space.

41.6 Answer: more difficult If there were no exclusion principle, all 11 electrons in the sodium atom would be in the level of lowest energy (the $1s$ level) and the configuration would be $1s^{11}$. Consequently, it would be more difficult to remove the first electron. (In a real sodium atom the valence electron is in a screened $3s$ state, which has a comparatively high energy.)

41.7 Answer: (iv) An absorption edge appears if the photon energy is just high enough to remove an electron in a given energy level from the atom. In a sample of high-temperature hydrogen we expect to find atoms whose electron is in the ground level ($n = 1$), the first excited level ($n = 2$), and the second excited level ($n = 3$). From Eq. (41.21) these levels have energies $E_n = (-13.60 \text{ eV})/n^2 = -13.60 \text{ eV}$, -3.40 eV , and -1.51 eV (see Fig. 38.9b).

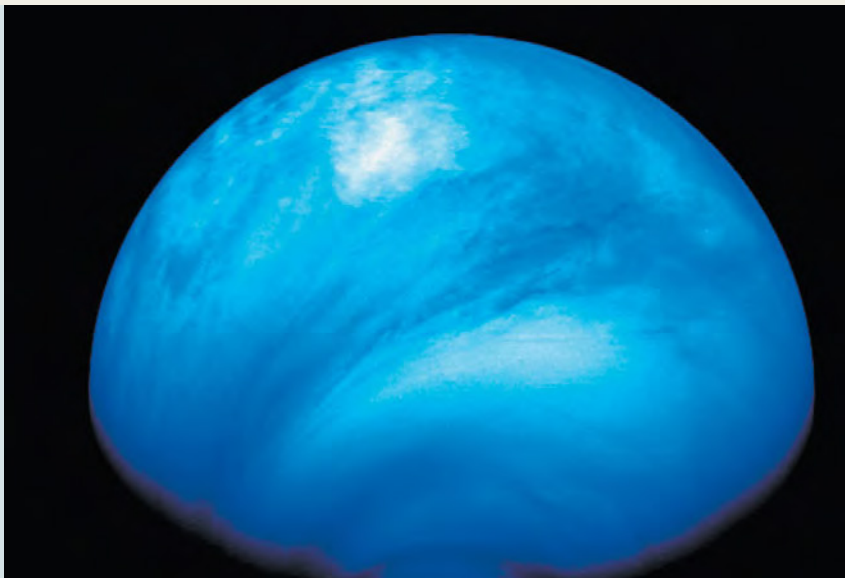
Bridging Problem

Answers: (a) $2.37 \times 10^{-10} \text{ m}$

(b) Values of (n_x, n_y, n_z, m_s) for the 22 electrons: $(1, 1, 1, +\frac{1}{2})$, $(1, 1, 1, -\frac{1}{2})$, $(2, 1, 1, +\frac{1}{2})$, $(2, 1, 1, -\frac{1}{2})$, $(1, 2, 1, +\frac{1}{2})$, $(1, 2, 1, -\frac{1}{2})$, $(1, 1, 2, +\frac{1}{2})$, $(1, 1, 2, -\frac{1}{2})$, $(2, 2, 1, +\frac{1}{2})$, $(2, 2, 1, -\frac{1}{2})$, $(2, 1, 2, +\frac{1}{2})$, $(2, 1, 2, -\frac{1}{2})$, $(1, 2, 2, +\frac{1}{2})$, $(1, 2, 2, -\frac{1}{2})$, $(3, 1, 1, +\frac{1}{2})$, $(3, 1, 1, -\frac{1}{2})$, $(1, 3, 1, +\frac{1}{2})$, $(1, 3, 1, -\frac{1}{2})$, $(1, 1, 3, +\frac{1}{2})$, $(1, 1, 3, -\frac{1}{2})$, $(2, 2, 2, +\frac{1}{2})$, $(2, 2, 2, -\frac{1}{2})$

(c) 20.1 eV, 40.2 eV, 60.3 eV, 73.7 eV, and 80.4 eV

(d) 60.3 eV versus $4.52 \times 10^3 \text{ eV}$



? This false-color image of Venus shows its thick, cloud-shrouded atmosphere, which is 96.5% carbon dioxide (CO_2). The atmosphere raises the surface temperature of Venus to 735 K ($462^\circ\text{C} = 863^\circ\text{F}$)—hotter even than Mercury, the closest planet to the sun. What property of CO_2 molecules makes them a potent agent for raising Venus's temperature?

In Chapter 41 we discussed the structure and properties of isolated atoms. But such atoms are the exception; usually we find atoms combined to form molecules or more extended structures we call condensed matter (liquid or solid). It's the attractive forces between atoms, called molecular bonds, that cause them to combine. In this chapter we'll study several kinds of bonds as well as the energy levels and spectra associated with diatomic molecules. We will see that just as atoms have quantized energies determined by the quantum-mechanical state of their electrons, so molecules have quantized energies determined by their rotational and vibrational states.

The same physical principles behind molecular bonds also apply to the study of condensed matter, in which various types of bonding occur. We'll explore the concept of energy bands and see how it helps us understand the properties of solids. Then we'll look more closely at the properties of a special class of solids called semiconductors. Devices using semiconductors are found in every radio, TV, pocket calculator, and computer used today; they have revolutionized the entire field of electronics during the past half-century.

42.1 Types of Molecular Bonds

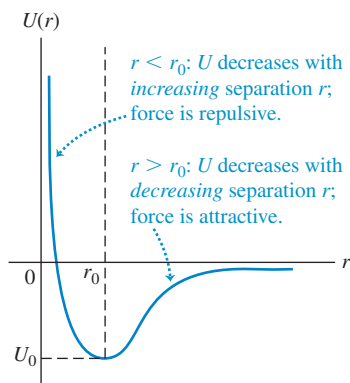
We can use our discussion of atomic structure in Chapter 41 as a basis for exploring the nature of *molecular bonds*, the interactions that hold atoms together to form stable structures such as molecules and solids.

LEARNING GOALS

By studying this chapter, you will learn:

- The various types of bonds that hold atoms together.
- How the rotational and vibrational dynamics of molecules are revealed by molecular spectra.
- How and why atoms form into crystalline structures.
- How to use the energy-band concept to explain the electrical properties of solids.
- A simple model for metals that explains many of their physical properties.
- How the character of a semiconductor can be radically transformed by adding small amounts of an impurity.
- Some of the technological applications of semiconductor devices.
- Why certain materials become superconductors at low temperature.

42.1 When the separation r between two oppositely charged ions is large, the potential energy $U(r)$ is proportional to $1/r$ as for point charges and the force is attractive. As r decreases, the charge clouds of the two atoms overlap and the force becomes less attractive. If r is less than the equilibrium separation r_0 , the force is repulsive.



Ionic Bonds

The **ionic bond** is an interaction between oppositely charged *ionized* atoms. The most familiar example is sodium chloride (NaCl), in which the sodium atom gives its one $3s$ electron to the chlorine atom, filling the vacancy in the $3p$ subshell of chlorine.

Let's look at the energy balance in this transaction. Removing the $3s$ electron from a neutral sodium atom requires 5.138 eV of energy; this is called the *ionization energy* of sodium. The neutral chlorine atom can attract an extra electron into the vacancy in the $3p$ subshell, where it is incompletely screened by the other electrons and therefore is attracted to the nucleus. This state has 3.613 eV lower energy than a neutral chlorine atom and a distant free electron; 3.613 eV is the magnitude of the *electron affinity* of chlorine. Thus creating the well-separated Na^+ and Cl^- ions requires a net investment of only $5.138 \text{ eV} - 3.613 \text{ eV} = 1.525 \text{ eV}$. When the two oppositely charged ions are brought together by their mutual attraction, the magnitude of their negative potential energy is determined by how closely they can approach each other. This in turn is limited by the exclusion principle, which forbids extensive overlap of the electron clouds of the two ions. As the distance decreases, the exclusion principle distorts the charge clouds, so the ions no longer interact like point charges and the interaction eventually becomes repulsive (Fig. 42.1).

The minimum electric potential energy for NaCl turns out to be -5.7 eV at a separation of 0.24 nm. The net energy released in creating the ions and letting them come together to the equilibrium separation of 0.24 nm is $5.7 \text{ eV} - 1.525 \text{ eV} = 4.2 \text{ eV}$. Thus, if the kinetic energy of the ions is neglected, 4.2 eV is the *binding energy* of the NaCl molecule, the energy that is needed to dissociate the molecule into separate neutral atoms.

Ionic bonds can involve more than one electron per atom. For instance, alkaline earth elements form ionic compounds in which an atom loses *two* electrons; an example is magnesium chloride, or $\text{Mg}^{2+}(\text{Cl}^-)_2$. Ionic bonds that involve a loss of more than two electrons are relatively rare. Instead, a different kind of bond, the *covalent* bond, comes into operation. We'll discuss this type of bond below.

Example 42.1 Electric potential energy of the NaCl molecule

Find the electric potential energy of an Na^+ ion and a Cl^- ion separated by 0.24 nm. Consider the ions as point charges.

SOLUTION

IDENTIFY and SET UP: Equation (23.9) in Section 23.1 tells us that the electric potential energy of two point charges q and q_0 separated by a distance r is $U = qq_0/4\pi\epsilon_0 r$.

EXECUTE: We have $q = +e$ (for Na^+), $q_0 = -e$ (for Cl^-), and $r = 0.24 \text{ nm} = 0.24 \times 10^{-9} \text{ m}$. From Eq. (23.9),

$$U = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_0} = -(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.6 \times 10^{-19} \text{ C})^2}{0.24 \times 10^{-9} \text{ m}}$$

$$= -9.6 \times 10^{-19} \text{ J} = -6.0 \text{ eV}$$

EVALUATE: This result agrees fairly well with the observed value of -5.7 eV . The reason for the difference is that when the two ions are at their equilibrium separation of 0.24 nm, the outer regions of their electron clouds overlap. Hence the two ions don't behave exactly like point charges.

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PHET: Double Wells and Covalent Bonds

Covalent Bonds

The **covalent bond** is characterized by a more egalitarian participation of the two atoms than occurs with the ionic bond. The simplest covalent bond is found in the hydrogen molecule, a structure containing two protons and two electrons. As the separate atoms (Fig. 42.2a) come together, the electron wave functions are distorted and become more concentrated in the region between the two protons (Fig. 42.2b). The net attraction of the electrons for each proton more than balances the repulsion of the two protons and of the two electrons.

The attractive interaction is then supplied by a *pair* of electrons, one contributed by each atom, with charge clouds that are concentrated primarily in the region between the two atoms. The energy of the covalent bond in the hydrogen molecule H_2 is -4.48 eV .

As we saw in Chapter 41, the exclusion principle permits two electrons to occupy the same region of space (that is, to be in the same spatial quantum state) only when they have opposite spins. When the spins are parallel, the exclusion principle forbids the molecular state that would be most favorable from energy considerations (with both electrons in the region between atoms). Opposite spins are an essential requirement for a covalent bond, and no more than two electrons can participate in such a bond.

However, an atom with several electrons in its outermost shell can form several covalent bonds. The bonding of carbon and hydrogen atoms, of central importance in organic chemistry, is an example. In the *methane* molecule (CH_4) the carbon atom is at the center of a regular tetrahedron, with a hydrogen atom at each corner. The carbon atom has four electrons in its L shell, and each of these four electrons forms a covalent bond with one of the four hydrogen atoms (Fig. 42.3). Similar patterns occur in more complex organic molecules.

Because of the role played by the exclusion principle, covalent bonds are highly directional. In the methane molecule the wave function for each of carbon's four valence electrons is a combination of the $2s$ and $2p$ wave functions called a *hybrid wave function*. The probability distribution for each one has a lobe protruding toward a corner of a tetrahedron. This symmetric arrangement minimizes the overlap of wave functions for the electron pairs, minimizing their repulsive potential energy.

Ionic and covalent bonds represent two extremes in molecular bonding, but there is no sharp division between the two types. Often there is a *partial* transfer of one or more electrons from one atom to another. As a result, many molecules that have dissimilar atoms have electric dipole moments—that is, a preponderance of positive charge at one end and of negative charge at the other. Such molecules are called *polar* molecules. Water molecules have large electric dipole moments; these are responsible for the exceptionally large dielectric constant of liquid water (see Sections 24.4 and 24.5).

van der Waals Bonds

Ionic and covalent bonds, with typical bond energies of 1 to 5 eV, are called *strong bonds*. There are also two types of weaker bonds. One of these, the **van der Waals bond**, is an interaction between the electric dipole moments of atoms or molecules; typical energies are 0.1 eV or less. The bonding of water molecules in the liquid and solid states results partly from dipole–dipole interactions.

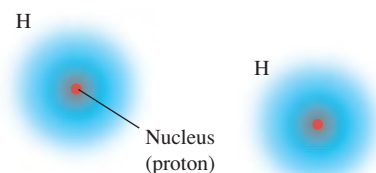
No atom has a permanent electric dipole moment, nor do many molecules. However, fluctuating charge distributions can lead to fluctuating dipole moments; these in turn can induce dipole moments in neighboring structures. Overall, the resulting dipole–dipole interaction is attractive, giving a weak bonding of atoms or molecules. The interaction potential energy drops off very quickly with distance r between molecules, usually as $1/r^6$. The liquefaction and solidification of the inert gases and of molecules such as H_2 , O_2 , and N_2 are due to induced-dipole van der Waals interactions. Not much thermal-agitation energy is needed to break these weak bonds, so such substances usually exist in the liquid and solid states only at very low temperatures.

Hydrogen Bonds

In the other type of weak bond, the **hydrogen bond**, a proton (H^+ ion) gets between two atoms, polarizing them and attracting them by means of the induced dipoles. This bond is unique to hydrogen-containing compounds because only hydrogen has a singly ionized state with no remaining electron cloud; the hydrogen ion is a bare proton, much smaller than any other singly ionized atom. The bond energy is usually less than 0.5 eV. The hydrogen bond is responsible for the cross-linking of long-chain organic molecules such as polyethylene (used in plastic bags). Hydrogen bonding also plays a role in the structure of ice.

42.2 Covalent bond in a hydrogen molecule.

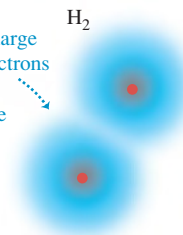
(a) Separate hydrogen atoms



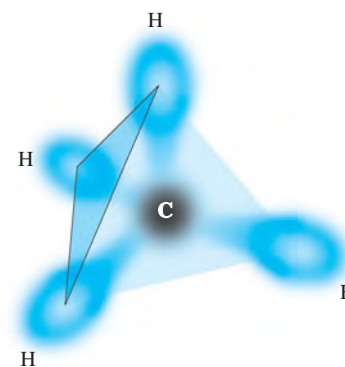
Individual H atoms are usually widely separated and do not interact.

(b) H_2 molecule

Covalent bond: the charge clouds for the two electrons with opposite spins are concentrated in the region between the nuclei.

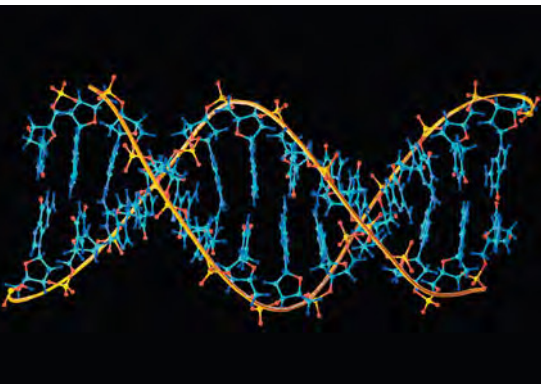


42.3 Schematic diagram of the methane (CH_4) molecule. The carbon atom is at the center of a regular tetrahedron and forms four covalent bonds with the hydrogen atoms at the corners. Each covalent bond includes two electrons with opposite spins, forming a charge cloud that is concentrated between the carbon atom and a hydrogen atom.

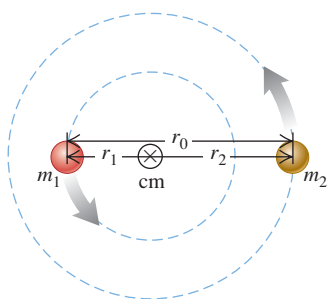


Application Molecular Zipper

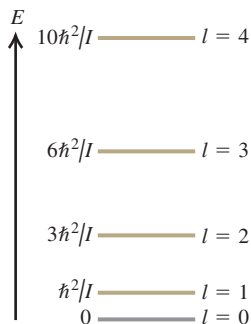
A DNA molecule functions like a twisted zipper. Each of the two strands of the “zipper” consists of an outer backbone and inward-facing nucleotide “teeth”; hydrogen bonds between facing teeth “zip” the strands together. The covalent bonds that hold together the atoms of each strand are strong, whereas the hydrogen bonds are relatively weak, so that the cell’s biochemical machinery can easily separate the strands for reading or copying.



42.4 A diatomic molecule modeled as two point masses m_1 and m_2 separated by a distance r_0 . The distances of the masses from the center of mass are r_1 and r_2 , where $r_1 + r_2 = r_0$.



42.5 The ground level and first four excited rotational energy levels for a diatomic molecule. The levels are not equally spaced.



All these bond types hold the atoms together in *solids* as well as in molecules. Indeed, a solid is in many respects a giant molecule. Still another type of bonding, the *metallic bond*, comes into play in the structure of metallic solids. We’ll return to this subject in Section 42.3.

Test Your Understanding of Section 42.1 If electrons obeyed the exclusion principle but did *not* have spin, how many electrons could participate in a covalent bond? (i) one; (ii) two; (iii) three; (iv) more than three.

**42.2 Molecular Spectra**

Molecules have energy levels that are associated with rotational motion of a molecule as a whole and with vibrational motion of the atoms relative to each other. Just as transitions between energy levels in atoms lead to atomic spectra, transitions between rotational and vibrational levels in molecules lead to *molecular spectra*.

Rotational Energy Levels

In this discussion we’ll concentrate mostly on *diatomic* molecules, to keep things as simple as possible. In Fig. 42.4 we picture a diatomic molecule as a rigid dumbbell (two point masses m_1 and m_2 separated by a constant distance r_0) that can *rotate* about axes through its center of mass, perpendicular to the line joining them. What are the energy levels associated with this motion?

We showed in Section 10.5 that when a rigid body rotates with angular speed ω about a perpendicular axis through its center of mass, the magnitude L of its angular momentum is given by Eq. (10.28), $L = I\omega$, where I is its moment of inertia about that symmetry axis. Its kinetic energy is given by Eq. (9.17), $K = \frac{1}{2}I\omega^2$. Combining these two equations, we find $K = L^2/2I$. There is no potential energy U , so the kinetic energy K is equal to the total mechanical energy E :

$$E = \frac{L^2}{2I} \quad (42.1)$$

Zero potential energy means that U does not depend on the angular coordinate of the molecule. But the potential-energy function U for the hydrogen atom (see Section 41.3) also has no dependence on angular coordinates. Thus the angular solutions to the Schrödinger equation for rigid-body rotation are the same as for the hydrogen atom, and the angular momentum is quantized in the same way. As in Eq. (41.21),

$$L = \sqrt{l(l+1)}\hbar \quad (l = 0, 1, 2, \dots) \quad (42.2)$$

Combining Eqs. (42.1) and (42.2), we obtain the *rotational energy levels*:

$$E_l = l(l+1)\frac{\hbar^2}{2I} \quad (l = 0, 1, 2, \dots) \quad \text{(rotational energy levels, diatomic molecule)} \quad (42.3)$$

Figure 42.5 is an energy-level diagram showing these rotational levels. The ground level has zero quantum number l , corresponding to zero angular momentum (no rotation and zero rotational energy E). The spacing of adjacent levels increases with increasing l .

We can express the moment of inertia I in Eqs. (42.1) and (42.3) in terms of the *reduced mass* m_r of the molecule:

$$m_r = \frac{m_1 m_2}{m_1 + m_2} \quad (42.4)$$

We introduced the reduced mass in Section 39.3 to accommodate the finite nuclear mass of the hydrogen atom. In Fig. 42.4 the distances r_1 and r_2 are the

distances from the center of mass to the nuclei of the atoms. By definition of the center of mass, $m_1 r_1 = m_2 r_2$, and the figure also shows that $r_0 = r_1 + r_2$. Solving these equations for r_1 and r_2 , we find

$$r_1 = \frac{m_2}{m_1 + m_2} r_0 \quad r_2 = \frac{m_1}{m_1 + m_2} r_0 \quad (42.5)$$

The moment of inertia is $I = m_1 r_1^2 + m_2 r_2^2$; substituting Eq. (42.5), we find

$$I = m_1 \frac{m_2^2}{(m_1 + m_2)^2} r_0^2 + m_2 \frac{m_1^2}{(m_1 + m_2)^2} r_0^2 = \frac{m_1 m_2}{m_1 + m_2} r_0^2 \quad \text{or}$$

$$I = m_r r_0^2 \quad (\text{moment of inertia of a diatomic molecule}) \quad (42.6)$$

The reduced mass enables us to reduce this two-body problem to an equivalent one-body problem (a particle of mass m_r moving around a circle with radius r_0), just as we did with the hydrogen atom. Indeed, the only difference between this problem and the hydrogen atom is the difference in the radial forces. To conserve angular momentum and account for the angular momentum of the emitted or absorbed photon, the allowed transitions between rotational states must satisfy the same selection rule that we discussed in Section 41.4 for allowed transitions between the states of an atom: l must change by exactly one unit, that is, $\Delta l = \pm 1$.

Example 42.2 Rotational spectrum of carbon monoxide

The two nuclei in the carbon monoxide (CO) molecule are 0.1128 nm apart. The mass of the most common carbon atom is 1.993×10^{-26} kg; that of the most common oxygen atom is 2.656×10^{-26} kg. (a) Find the energies of the lowest three rotational energy levels of CO. Express your results in meV ($1 \text{ meV} = 10^{-3} \text{ eV}$). (b) Find the wavelength of the photon emitted in the transition from the $l = 2$ to the $l = 1$ level.

SOLUTION

IDENTIFY and SET UP: This problem uses the ideas developed in this section about the rotational energy levels of molecules. We are given the distance r_0 between the atoms and their masses m_1 and m_2 . We find the reduced mass m_r using Eq. (42.4), the moment of inertia I using Eq. (42.6), and the energies E_l using Eq. (42.3). The energy E of the emitted photon is equal to the difference in energy between the $l = 2$ and $l = 1$ levels. (This transition obeys the $\Delta l = \pm 1$ selection rule, since $\Delta l = 1 - 2 = -1$.) We determine the photon wavelength using $E = hc/\lambda$.

EXECUTE: (a) From Eqs. (42.4) and (42.6), the reduced mass and moment of inertia of the CO molecule are:

$$\begin{aligned} m_r &= \frac{m_1 m_2}{m_1 + m_2} \\ &= \frac{(1.993 \times 10^{-26} \text{ kg})(2.656 \times 10^{-26} \text{ kg})}{(1.993 \times 10^{-26} \text{ kg}) + (2.656 \times 10^{-26} \text{ kg})} \\ &= 1.139 \times 10^{-26} \text{ kg} \\ I &= m_r r_0^2 \\ &= (1.139 \times 10^{-26} \text{ kg})(0.1128 \times 10^{-9} \text{ m})^2 \\ &= 1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

The rotational levels are given by Eq. (42.3):

$$\begin{aligned} E_l &= l(l+1) \frac{\hbar^2}{2I} = l(l+1) \frac{(1.0546 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2)} \\ &= l(l+1)(3.838 \times 10^{-23} \text{ J}) = l(l+1)0.2395 \text{ meV} \end{aligned}$$

($1 \text{ meV} = 10^{-3} \text{ eV}$.) Substituting $l = 0, 1, 2$, we find

$$E_0 = 0 \quad E_1 = 0.479 \text{ meV} \quad E_2 = 1.437 \text{ meV}$$

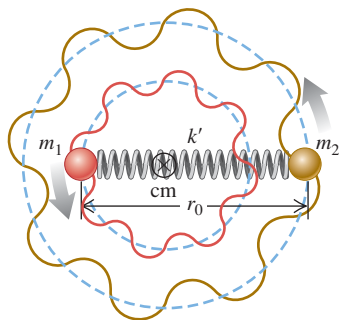
(b) The photon energy and wavelength are:

$$\begin{aligned} E &= E_2 - E_1 = 0.958 \text{ meV} \\ \lambda &= \frac{hc}{E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{0.958 \times 10^{-3} \text{ eV}} \\ &= 1.29 \times 10^{-3} \text{ m} = 1.29 \text{ mm} \end{aligned}$$

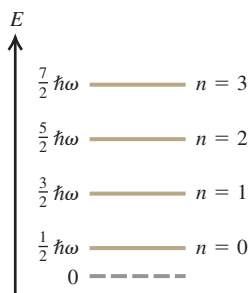
EVALUATE: The differences between the first few rotational energy levels of CO are very small (about $1 \text{ meV} = 10^{-3} \text{ eV}$) compared to the differences between atomic energy levels (typically a few eV). Hence a photon emitted by a CO molecule in a transition from the $l = 2$ to the $l = 1$ level has very low energy and a very long wavelength compared to the visible light emitted by excited atoms. Photon wavelengths for rotational transitions in molecules are typically in the microwave and far infrared regions of the spectrum.

In this example we were given the equilibrium separation between the atoms, also called the *bond length*, and we used it to calculate one of the wavelengths emitted by excited CO molecules. In actual experiments, scientists work this problem backward: By measuring the long-wavelength emissions of a sample of diatomic molecules, they determine the moment of inertia of the molecule and hence the bond length.

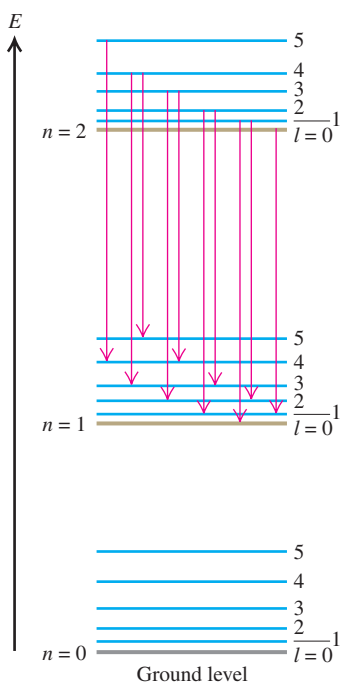
42.6 A diatomic molecule modeled as two point masses m_1 and m_2 connected by a spring with force constant k' .



42.7 The ground level and first three excited vibrational levels for a diatomic molecule, assuming small displacements from equilibrium so we can treat the oscillations as simple harmonic. (Compare Fig. 40.25.)



42.8 Energy-level diagram for vibrational and rotational energy levels of a diatomic molecule. For each vibrational level (n) there is a series of more closely spaced rotational levels (l). Several transitions corresponding to a single band in a band spectrum are shown. These transitions obey the selection rule $\Delta l = \pm 1$.



Vibrational Energy Levels

Molecules are never completely rigid. In a more realistic model of a diatomic molecule we represent the connection between atoms not as a rigid rod but as a spring (Fig. 42.6). Then, in addition to rotating, the atoms of the molecule can *vibrate* about their equilibrium positions along the line joining them. For small oscillations the restoring force can be taken as proportional to the displacement from the equilibrium separation r_0 (like a spring that obeys Hooke's law with a force constant k'), and the system is a harmonic oscillator. We discussed the quantum-mechanical harmonic oscillator in Section 40.5. The energy levels are given by Eq. (40.46) with the mass m replaced by the reduced mass m_r :

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega = \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m_r}} \quad (n = 0, 1, 2, \dots) \quad (42.7)$$

(vibrational energy levels of a diatomic molecule)

This represents a series of levels equally spaced in energy, with an energy separation of

$$\Delta E = \hbar\omega = \hbar\sqrt{\frac{k'}{m_r}} \quad (42.8)$$

Figure 42.7 is an energy-level diagram showing these vibrational levels. As an example, for the carbon monoxide molecule of Example 42.2 the spacing $\hbar\omega$ between vibrational energy levels is 0.2690 eV. From Eq. (42.8) this corresponds to a force constant of 1.90×10^3 N/m, which is a fairly loose spring. (To stretch a macroscopic spring with this value of k' by 1.0 cm would require a force of only 19 N, or about 4 lb.) Force constants for diatomic molecules are typically about 100 to 2000 N/m.

CAUTION Watch out for k , k' , and K As in Section 40.5 we're again using k' for the force constant, this time to minimize confusion with Boltzmann's constant k , the gas constant per molecule (introduced in Section 18.3). Besides the quantities k and k' , we also use the absolute temperature unit $1 \text{ K} = 1$ kelvin. ■

Rotation and Vibration Combined

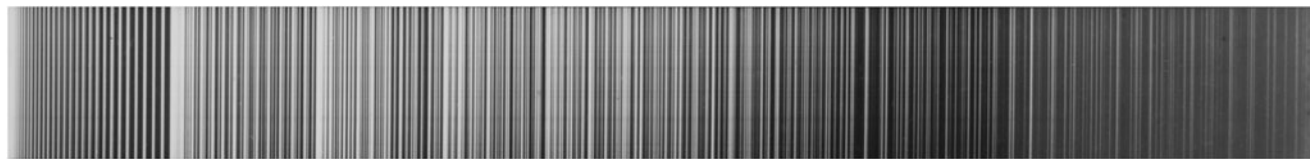
Visible-light photons have energies between 1.65 eV and 3.26 eV. The 0.2690-eV energy difference between vibrational levels for carbon monoxide corresponds to a photon of wavelength 4.613 μm , in the infrared region of the spectrum. This is much closer to the visible region than is the photon in the rotational transition in Example 42.2. In general the energy differences for molecular *vibration* are much smaller than those that produce atomic spectra, but much larger than the energy differences for molecular *rotation*.

When we include *both* rotational and vibrational energies, the energy levels for our diatomic molecule are

$$E_{nl} = l(l+1)\frac{\hbar^2}{2I} + \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m_r}} \quad (42.9)$$

Figure 42.8 shows the energy-level diagram. For each value of n there are many values of l , forming a series of closely spaced levels.

The red arrows in Fig. 42.8 show several possible transitions in which a molecule goes from a level with $n = 2$ to a level with $n = 1$ by emitting a photon. As we mentioned above, these transitions must obey the selection rule $\Delta l = \pm 1$ in order to conserve angular momentum. An additional selection rule states that if the vibrational level changes, the vibrational quantum number n in Eq. (42.9) must increase by 1 ($\Delta n = 1$) if a photon is absorbed or decrease by 1 ($\Delta n = -1$) if a photon is emitted.

42.9 A typical molecular band spectrum.

As an illustration of these selection rules, Fig. 42.8 shows that a molecule in the $n = 2, l = 4$ level can emit a photon and drop into the $n = 1, l = 5$ level ($\Delta n = -1, \Delta l = +1$) or the $n = 1, l = 3$ level ($\Delta n = -1, \Delta l = -1$), but is forbidden from making a $\Delta n = -1, \Delta l = 0$ transition into the $n = 1, l = 4$ level.

Transitions between states with various pairs of n -values give different series of spectrum lines, and the resulting spectrum has a series of *bands*. Each band corresponds to a particular vibrational transition, and each individual line in a band represents a particular rotational transition, with the selection rule $\Delta l = \pm 1$. Figure 42.9 shows a typical *band spectrum*.

All molecules can have excited states of the *electrons* in addition to the rotational and vibrational states that we have described. In general, these lie at higher energies than the rotational and vibrational states, and there is no simple rule relating them. When there is a transition between electronic states, the $\Delta n = \pm 1$ selection rule for the vibrational levels no longer holds.

Example 42.3 Vibration-rotation spectrum of carbon monoxide

Consider again the CO molecule of Example 42.2. Find the wavelength of the photon emitted by a CO molecule when its vibrational energy changes and its rotational energy is (a) initially zero and (b) finally zero.

SOLUTION

IDENTIFY and SET UP: We need to use the selection rules for the vibrational and rotational transitions of a diatomic molecule. Since a photon is emitted as the vibrational energy changes, the selection rule $\Delta n = -1$ tells us that the vibrational quantum number n decreases by 1 in both parts (a) and (b). In part (a) the initial value of l is zero; the selection rule $\Delta l = \pm 1$ tells us that the *final* value of l is 1, so the rotational energy increases in this case. In part (b) the *final* value of l is zero; $\Delta l = \pm 1$ then tells us that the *initial* value of l is 1, and the rotational energy decreases.

In each case the energy E of the emitted photon is the difference between the initial and final energies of the molecule, accounting for the change in both vibrational and rotational energy. In part (a) E equals the difference $\hbar\omega$ between adjacent vibrational energy levels *minus* the rotational energy that the molecule *gains*; in part (b) E equals $\hbar\omega$ *plus* the rotational energy that the molecule *loses*. Example 42.2 tells us that the difference between the $l = 0$ and $l = 1$ rotational energy levels is $0.479 \text{ meV} = 0.000479 \text{ eV}$, and we learned above that the

vibrational energy-level separation for CO is $\hbar\omega = 0.2690 \text{ eV}$. We use $E = hc/\lambda$ to determine the corresponding wavelengths (our target variables).

EXECUTE: (a) In this transition the CO molecule loses $\hbar\omega = 0.2690 \text{ eV}$ of vibrational energy and gains 0.000479 eV of rotational energy. Hence the energy E that goes into the emitted photon equals 0.2690 eV *less* 0.000479 eV , or 0.2685 eV . The photon wavelength is

$$\begin{aligned}\lambda &= \frac{hc}{E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{0.2685 \text{ eV}} \\ &= 4.618 \times 10^{-6} \text{ m} = 4.618 \mu\text{m}\end{aligned}$$

(b) Now the CO molecule loses $\hbar\omega = 0.2690 \text{ eV}$ of vibrational energy and also loses 0.000479 eV of rotational energy, so the energy that goes into the photon is $E = 0.2690 \text{ eV} + 0.000479 \text{ eV} = 0.2695 \text{ eV}$. The wavelength is

$$\begin{aligned}\lambda &= \frac{hc}{E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{0.2695 \text{ eV}} \\ &= 4.601 \times 10^{-6} \text{ m} = 4.601 \mu\text{m}\end{aligned}$$

EVALUATE: In part (b) the molecule loses more energy than it does in part (a), so the emitted photon must have greater energy and a shorter wavelength. That's just what our results show.

Complex Molecules

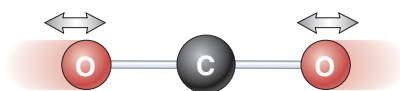
We can apply these same principles to more complex molecules. A molecule with three or more atoms has several different kinds or *modes* of vibratory motion. Each mode has its own set of energy levels, related to its frequency by Eq. (42.7).

42.10 The carbon dioxide molecule can vibrate in three different modes. For clarity, the atoms are not shown to scale: The separation between atoms is actually comparable to their diameters.

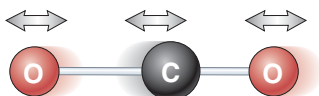
(a) Bending mode



(b) Symmetric stretching mode



(c) Asymmetric stretching mode



In nearly all cases the associated radiation lies in the infrared region of the electromagnetic spectrum.

Infrared spectroscopy has proved to be an extremely valuable analytical tool. It provides information about the strength, rigidity, and length of molecular bonds and the structure of complex molecules. Also, because every molecule (like every atom) has its characteristic spectrum, infrared spectroscopy can be used to identify unknown compounds.

One molecule that can readily absorb and emit infrared radiation is carbon dioxide (CO_2). Figure 42.10 shows the three possible modes of vibration of a CO_2 molecule. A number of transitions are possible between excited levels of the same vibrational mode as well as between levels of different vibrational modes. The energy differences are less than 1 eV in all of these transitions, and so involve infrared photons of wavelength longer than $1 \mu\text{m}$. Hence a gas of CO_2 can readily absorb light at a number of different infrared wavelengths. This makes CO_2 a very effective greenhouse gas (see Section 17.7) even on earth, where carbon dioxide is just 0.04% of the atmosphere by volume. On Venus, however, the atmosphere has more than 90 times the total mass of our atmosphere and is almost entirely CO_2 . The resulting greenhouse effect is tremendous: The surface temperature on Venus is more than 400 kelvins greater than what it would be if the planet had no atmosphere at all.

Test Your Understanding of Section 42.2 A rotating diatomic molecule emits a photon when it makes a transition from level l to level $l - 1$. If the value of l increases, does the wavelength of the emitted photon (i) increase, (ii) decrease, or (iii) remain unchanged?



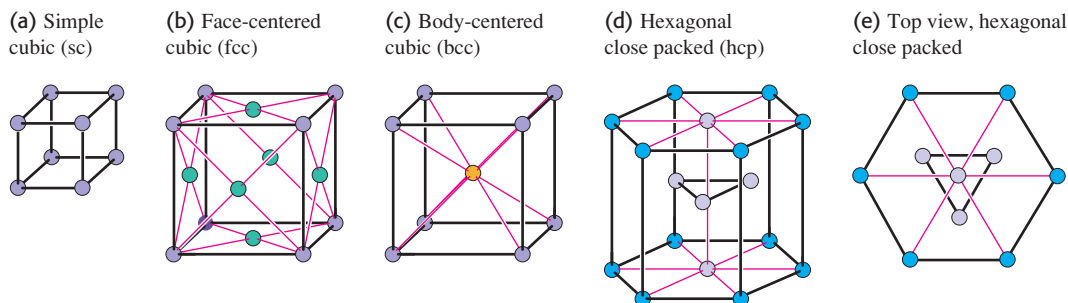
42.3 Structure of Solids

The term *condensed matter* includes both solids and liquids. In both states, the interactions between atoms or molecules are strong enough to give the material a definite volume that changes relatively little with applied stress. In condensed matter, adjacent atoms attract one another until their outer electron charge clouds begin to overlap significantly. Thus the distances between adjacent atoms in condensed matter are about the same as the diameters of the atoms themselves, typically 0.1 to 0.5 nm. Also, when we speak of the distances between atoms, we mean the center-to-center (nucleus-to-nucleus) distances.

Ordinarily, we think of a liquid as a material that can flow and of a solid as a material with a definite shape. However, if you heat a horizontal glass rod in the flame of a burner, you'll find that the rod begins to sag (flow) more and more easily as its temperature rises. Glass has no definite transition from solid to liquid, and no definite melting point. On this basis, we can consider glass at room temperature as being an extremely viscous liquid. Tar and butter show similar behavior.

What is the microscopic difference between materials like glass or butter and solids like ice or copper, which do have definite melting points? Ice and copper are examples of *crystalline solids* in which the atoms have *long-range order*, a recurring pattern of atomic positions that extends over many atoms. This pattern is called the *crystal structure*. In contrast, glass at room temperature is an example of an *amorphous solid*, one that has no long-range order, but only *short-range order* (correlations between neighboring atoms or molecules). Liquids also have only short-range order. The boundaries between crystalline solid, amorphous solid, and liquid may be sometimes blurred. Some solids, crystalline when perfect, can form with so many imperfections in their structure that they have almost no long-range order. Conversely, some liquid crystals (organic compounds composed of cylindrical molecules that tend to line up parallel to each other) have a fairly high degree of long-range order.

Nearly everything we know about crystal structure has been learned from diffraction experiments, initially with x rays and later with electrons and neutrons.

42.11 Portions of some common types of crystal lattices.

A typical distance between atoms is of the order of 0.1 nm. You can show that 12.4-keV x rays, 150-eV electrons, and 0.0818-eV neutrons all have wavelengths $\lambda = 0.1$ nm.

Crystal Lattices and Structures

A *crystal lattice* is a repeating pattern of mathematical points that extends throughout space. There are 14 general types of such patterns; Fig. 42.11 shows small portions of some common examples. The *simple cubic lattice* (sc) has a lattice point at each corner of a cubic array (Fig. 42.11a). The *face-centered cubic lattice* (fcc) is like the simple cubic but with an additional lattice point at the center of each cube face (Fig. 42.11b). The *body-centered cubic lattice* (bcc) is like the simple cubic but with an additional point at the center of each cube (Fig. 42.11c). The *hexagonal close-packed lattice* has layers of lattice points in hexagonal patterns, each hexagon made up of six equilateral triangles (Figs. 42.11d and 42.11e).

CAUTION A perfect crystal lattice is infinitely large. Figure 42.11 shows just enough lattice points so you can easily visualize the pattern; the lattice, a mathematical abstraction, extends throughout space. Thus the lattice points shown repeat endlessly in all directions. **|**

In a crystal structure, a single atom or a group of atoms is associated with each lattice point. The group may contain the same or different kinds of atoms. This atom or group of atoms is called a *basis*. Thus a complete description of a crystal structure includes both the lattice and the basis. We initially consider *perfect crystals*, or *ideal single crystals*, in which the crystal structure extends uninterrupted throughout space.

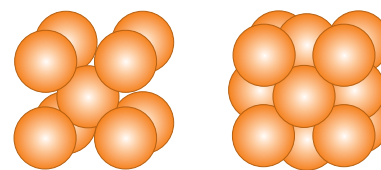
The bcc and fcc structures are two common simple crystal structures. The alkali metals have a bcc structure—that is, a bcc lattice with a basis of one atom at each lattice point. Each atom in a bcc structure has eight nearest neighbors (Fig. 42.12a). The elements Al, Ca, Cu, Ag, and Au have an fcc structure—that is, an fcc lattice with a basis of one atom at each lattice point. Each atom in an fcc structure has 12 nearest neighbors (Fig. 42.12b).

Figure 42.13 shows a representation of the structure of sodium chloride (NaCl, ordinary salt). It may look like a simple cubic structure, but it isn't. The sodium and chloride ions each form an fcc structure, so we can think roughly of the sodium chloride structure as being composed of two interpenetrating fcc structures. More correctly, the sodium chloride crystal structure of Fig. 42.13 has an fcc lattice with one chloride ion at each lattice point and one sodium ion half a cube length above it. That is, its basis consists of one chloride and one sodium ion.

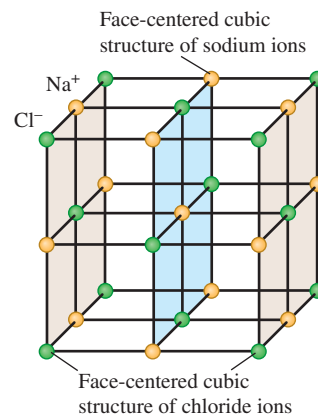
Another example is the *diamond structure*; it's called that because it is the crystal structure of carbon in the diamond form. It's also the crystal structure of silicon, germanium, and gray tin. The diamond lattice is fcc; the basis consists of one atom at each lattice point and a second *identical* atom displaced a quarter of a cube length in each of the three cube-edge directions. Figure 42.14 will help you

42.12 (a) The bcc structure is composed of a bcc lattice with a basis of one atom for each lattice point. (b) The fcc structure is composed of an fcc lattice with a basis of one atom for each lattice point. These structures repeat precisely to make up perfect crystals.

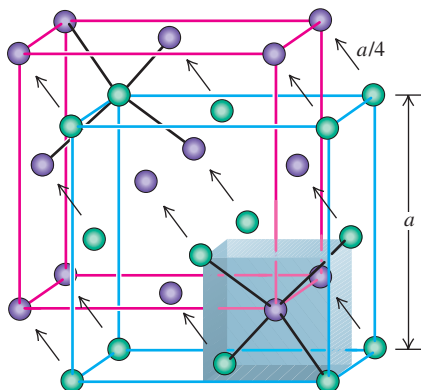
(a) The bcc structure (b) The fcc structure



42.13 Representation of part of the sodium chloride crystal structure. The distances between ions are exaggerated.



42.14 The diamond structure, shown as two interpenetrating face-centered cubic structures with distances between atoms exaggerated. Relative to the corresponding green atom, each purple atom is shifted up, back, and to the left by a distance $a/4$.



visualize this. The shaded volume in Fig. 42.14 shows the bottom right front eighth of the basic cube; the four atoms at alternate corners of this cube are at the corners of a regular tetrahedron, and there is an additional atom at the center. Thus each atom in the diamond structure is at the center of a regular tetrahedron with four nearest-neighbor atoms at the corners.

In the diamond structure, both the purple and green spheres in Fig. 42.14 represent *identical* atoms—for example, both carbon or both silicon. In the cubic zinc sulfide structure, the purple spheres represent one type of atom and the green spheres represent a *different* type. For example, in zinc sulfide (ZnS) each zinc atom (purple in Fig. 42.14) is at the center of a regular tetrahedron with four sulfur atoms (green in Fig. 42.14) at its corners, and vice versa. Gallium arsenide (GaAs) and similar compounds have this same structure.

Bonding in Solids

The forces that are responsible for the regular arrangement of atoms in a crystal are the same as those involved in molecular bonds, plus one additional type. Not surprisingly, *ionic* and *covalent* molecular bonds are found in ionic and covalent crystals, respectively. The most familiar *ionic crystals* are the alkali halides, such as ordinary salt (NaCl). The positive sodium ions and the negative chloride ions occupy alternate positions in a cubic arrangement (see Fig. 42.13). The attractive forces are the familiar Coulomb's-law forces between charged particles. These forces have no preferred direction, and the arrangement in which the material crystallizes is partly determined by the relative sizes of the two ions. Such a structure is *stable* in the sense that it has lower total energy than the separated ions (see the following example). The negative potential energies of pairs of opposite charges are greater in absolute value than the positive energies of pairs of like charges because the pairs of unlike charges are closer together, on average.

Example 42.4 Potential energy of an ionic crystal

Imagine a one-dimensional ionic crystal consisting of a very large number of alternating positive and negative ions with charges e and $-e$, with equal spacing a along a line. Show that the total interaction potential energy is negative, which means that such a “crystal” is stable.

SOLUTION

IDENTIFY and SET UP: We treat each ion as a point charge and use our results from Section 23.1 for the electric potential energy of a collection of point charges. Equations (23.10) and (23.11) tell us to consider the electric potential energy U of each pair of charges. The total potential energy of the system is the sum of the values of U for every possible pair; we take the number of pairs to be infinite.

EXECUTE: Let's pick an ion somewhere in the middle of the line and add the potential energies of its interactions with all the ions to one side of it. From Eq. (23.11), that sum is

$$\begin{aligned}\sum U &= -\frac{e^2}{4\pi\epsilon_0 a} + \frac{e^2}{4\pi\epsilon_0 2a} - \frac{e^2}{4\pi\epsilon_0 3a} + \dots \\ &= -\frac{e^2}{4\pi\epsilon_0 a} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots\right)\end{aligned}$$

You may notice that the series in parentheses resembles the Taylor series for the function $\ln(1+x)$:

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

When $x = 1$ this becomes the series in parentheses above, so

$$\sum U = -\frac{e^2}{4\pi\epsilon_0 a} \ln 2$$

This is certainly a negative quantity. The atoms on the other side of the ion we're considering make an equal contribution to the potential energy. And if we include the potential energies of all pairs of atoms, the sum is certainly negative.

EVALUATE: We conclude that this one-dimensional ionic “crystal” is stable: It has lower energy than the zero electric potential energy that is obtained when all the ions are infinitely far apart from each other.

Types of Crystals

Carbon, silicon, germanium, and tin in the diamond structure are simple examples of *covalent crystals*. These elements are in Group IV of the periodic table,

meaning that each atom has four electrons in its outermost shell. Each atom forms a covalent bond with each of four adjacent atoms at the corners of a tetrahedron (Fig. 42.14). These bonds are strongly directional because of the asymmetric electron distributions dictated by the exclusion principle, and the result is the tetrahedral diamond structure.

A third crystal type, less directly related to the chemical bond than are ionic or covalent crystals, is the **metallic crystal**. In this structure, one or more of the outermost electrons in each atom become detached from the parent atom (leaving a positive ion) and are free to move through the crystal. These electrons are not localized near the individual ions. The corresponding electron wave functions extend over many atoms.

Thus we can picture a metallic crystal as an array of positive ions immersed in a sea of freed electrons whose attraction for the positive ions holds the crystal together (Fig. 42.15). These electrons also give metals their high electrical and thermal conductivities. This sea of electrons has many of the properties of a gas, and indeed we speak of the *electron-gas model* of metallic solids. The simplest version of this model is the *free-electron model*, which ignores interactions with the ions completely (except at the surface). We'll return to this model in Section 42.5.

In a metallic crystal the freed electrons are not localized but are shared among *many* atoms. This gives a bonding that is neither localized nor strongly directional. The crystal structure is determined primarily by considerations of *close packing*—that is, the maximum number of atoms that can fit into a given volume. The two most common metallic crystal lattices are the face-centered cubic and hexagonal close-packed (see Figs. 42.11b, 42.11d, and 42.11e). In structures composed of these lattices with a basis of one atom, each atom has 12 nearest neighbors.

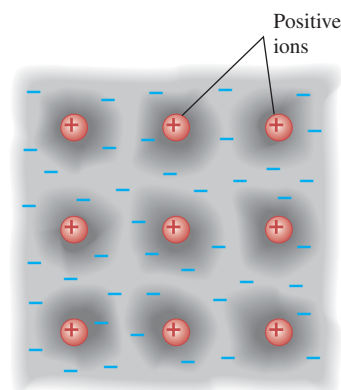
As we mentioned in Section 42.1, van der Waals interactions and hydrogen bonding also play a role in the structure of some solids. In polyethylene and similar polymers, covalent bonding of atoms forms long-chain molecules, and hydrogen bonding forms cross-links between adjacent chains. In solid water, both van der Waals forces and hydrogen bonds are significant in determining the crystal structures of ice.

Our discussion has centered on perfect crystals, or ideal single crystals. Real crystals show a variety of departures from this idealized structure. Materials are often *polycrystalline*, composed of many small single crystals bonded together at *grain boundaries*. There may be *point defects* within a single crystal: *Interstitial* atoms may occur in places where they do not belong, and there may be *vacancies*, positions that should be occupied by an atom but are not. A point defect of particular interest in semiconductors, which we will discuss in Section 42.6, is the *substitutional impurity*, a foreign atom replacing a regular atom (for example, arsenic in a silicon crystal).

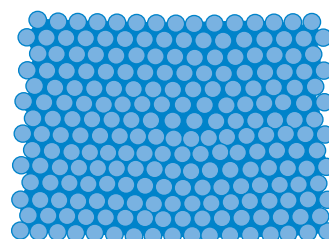
There are several basic types of extended defects called *dislocations*. One type is the *edge dislocation*, shown schematically in Fig. 42.16, in which one plane of atoms slips relative to another. The mechanical properties of metallic crystals are influenced strongly by the presence of dislocations. The ductility and malleability of some metals depend on the presence of dislocations that can move through the crystal during plastic deformations. Solid-state physicists often point out that the biggest extended defect of all, present in *all* real crystals, is the surface of the material with its dangling bonds and abrupt change in potential energy.

Test Your Understanding of Section 42.3 If a is the distance in an NaCl crystal from an Na^+ ion to one of its nearest-neighbor Cl^- ions, what is the distance from an Na^+ ion to one of its *next-to-nearest-neighbor* Cl^- ions? (i) $a\sqrt{2}$; (ii) $a\sqrt{3}$; (iii) $2a$; (iv) none of these.

42.15 In a metallic solid, one or more electrons are detached from each atom and are free to wander around the crystal, forming an “electron gas.” The wave functions for these electrons extend over many atoms. The positive ions vibrate around fixed locations in the crystal.



42.16 An edge dislocation in two dimensions. In three dimensions an edge dislocation would look like an extra plane of atoms slipped partway into the crystal.



The irregularity is seen most easily by viewing the figure from various directions at a grazing angle with the page.

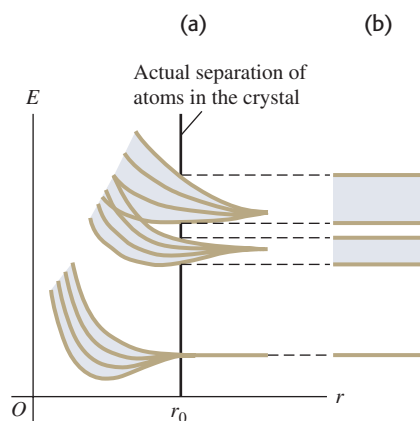


42.17 The concept of energy bands was first developed by the Swiss-American physicist Felix Bloch (1905–1983) in his doctoral thesis. Our modern understanding of electrical conductivity stems from that landmark work. Bloch's work in nuclear physics brought him (along with Edward Purcell) the 1952 Nobel Prize in physics.



42.18 Origin of energy bands in a solid.

(a) As the distance r between atoms decreases, the energy levels spread into bands. The vertical line at r_0 shows the actual atomic spacing in the crystal. (b) Symbolic representation of energy bands.



42.4 Energy Bands

The **energy-band** concept, introduced in 1928 (Fig. 42.17), is a great help in understanding several properties of solids. To introduce the idea, suppose we have a large number N of identical atoms, far enough apart that their interactions are negligible. Every atom has the same energy-level diagram. We can draw an energy-level diagram for the *entire system*. It looks just like the diagram for a single atom, but the exclusion principle, applied to the entire system, permits each state to be occupied by N electrons instead of just one.

Now we begin to push the atoms uniformly closer together. Because of the electrical interactions and the exclusion principle, the wave functions begin to distort, especially those of the outer, or *valence*, electrons. The corresponding energies also shift, some upward and some downward, by varying amounts, as the valence electron wave functions become less localized and extend over more and more atoms. Thus the valence states that formerly gave the *system* a state with a sharp energy level that could accommodate N electrons now give a *band* containing N closely spaced levels (Fig. 42.18). Ordinarily, N is very large, somewhere near the order of Avogadro's number (10^{24}), so we can accurately treat the levels as forming a *continuous* distribution of energies within a band. Between adjacent energy bands are gaps or forbidden regions where there are *no* allowed energy levels. The inner electrons in an atom are affected much less by nearby atoms than are the valence electrons, and their energy levels remain relatively sharp.

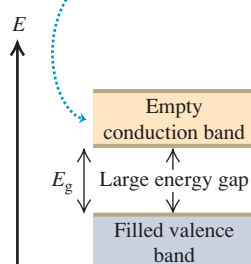
Insulators, Semiconductors, and Conductors

The nature of the energy bands determines whether the material is an electrical insulator, a semiconductor, or a conductor. In particular, what matters are the extent to which the states in each band are occupied and the spacing, or *energy gap*, between adjacent bands. A crucial factor is the exclusion principle (see Section 41.6), which states that only one electron can occupy a given quantum-mechanical state.

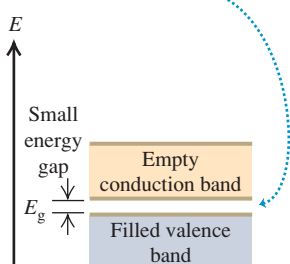
In an *insulator* at absolute zero temperature, the highest band that is completely filled, called the **valence band**, is also the highest band that has *any* electrons in it. The next higher band, called the **conduction band**, is completely empty; there are no electrons in its states (Fig. 42.19a). Imagine what happens if an electric field is applied to a material of this kind. To move in response to the field, an electron would have to go into a different quantum state with a slightly different energy. It can't do that, however, because all the neighboring states are already occupied. The only way such an electron can move is to jump across the energy gap into the conduction band, where there are plenty of nearby unoccupied states. At any temperature above absolute zero

42.19 Three types of energy-band structure.

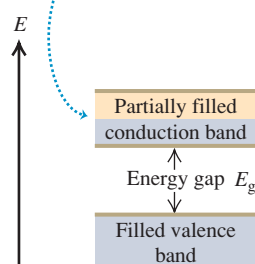
(a) In an insulator at absolute zero, there are no electrons in the conduction band.



(b) A semiconductor has the same band structure as an insulator but a smaller gap between the valence and conduction bands.



(c) A conductor has a partially filled conduction band.



there is some probability this jump can happen, because an electron can gain energy from thermal motion. In an insulator, however, the energy gap between the valence and conduction bands can be 5 eV or more, and that much thermal energy is not ordinarily available. Hence little or no current flows in response to an applied electric field, and the electric conductivity (Section 25.2) is low. The thermal conductivity (Section 17.7), which also depends on mobile electrons, is likewise low.

We saw in Section 24.4 that an insulator becomes a conductor if it is subjected to a large enough electric field; this is called *dielectric breakdown*. If the electric field is of order 10^{10} V/m, there is a potential difference of a few volts over a distance comparable to atomic sizes. In this case the field can do enough work on a valence electron to boost it across the energy gap and into the conduction band. (In practice dielectric breakdown occurs for fields much less than 10^{10} V/m, because imperfections in the structure of an insulator provide some more accessible energy states *within* the energy gap.)

As in an insulator, a *semiconductor* at absolute zero has an empty conduction band above the full valence band. The difference is that in a semiconductor the energy gap between these bands is relatively small and electrons can more readily jump into the conduction band (Fig. 42.19b). As the temperature of a semiconductor increases, the population in the conduction band increases very rapidly, as does the electric conductivity. For example, in a semiconductor near room temperature with an energy gap of 1 eV, the number of conduction electrons doubles when the temperature rises by just 10°C . We will use the concept of energy bands to explore semiconductors in more depth in Section 42.6.

In a *conductor* such as a metal, there are electrons in the conduction band even at absolute zero (Fig. 42.19c). The metal sodium is an example. An analysis of the atomic energy-level diagram for sodium (see Fig. 39.19a) shows that for an isolated sodium atom, the six lowest excited states (all $3p$ states) are about 2.1 eV above the two $3s$ ground states. In solid sodium, however, the atoms are so close together that the $3s$ and $3p$ bands spread out and overlap into a single band. Each sodium atom contributes one electron to the band, leaving an Na^+ ion behind. Each atom also contributes eight states to that band (two $3s$, six $3p$), so the band is only one-eighth occupied. We call this structure a *conduction* band because it is only partially occupied. Electrons near the top of the filled portion of the band have many adjacent unoccupied states available, and they can easily gain or lose small amounts of energy in response to an applied electric field. Therefore these electrons are mobile, giving solid sodium its high electrical and thermal conductivity. A similar description applies to other conducting materials.

Example 42.5 Photoconductivity in germanium

At room temperature, pure germanium has an almost completely filled valence band separated by a 0.67-eV gap from an almost completely empty conduction band. It is a poor electrical conductor, but its conductivity increases greatly when it is irradiated with electromagnetic waves of a certain maximum wavelength. What is that wavelength?

SOLUTION

IDENTIFY and SET UP: The conductivity of a semiconductor increases greatly when electrons are excited from the valence band into the conduction band. In germanium, the excitation occurs when an electron absorbs a photon with an energy of at least $E_{\min} = 0.67$ eV. From the relationship $E = hc/\lambda$, the *maximum* wavelength λ_{\max} (our target variable) corresponds to this *minimum* photon energy.

EXECUTE: The wavelength of a photon with energy $E_{\min} = 0.67$ eV is

$$\begin{aligned}\lambda_{\max} &= \frac{hc}{E_{\min}} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{0.67 \text{ eV}} \\ &= 1.9 \times 10^{-6} \text{ m} = 1.9 \mu\text{m} = 1900 \text{ nm}\end{aligned}$$

EVALUATE: This wavelength is in the infrared part of the spectrum, so visible-light photons (which have shorter wavelength and higher energy) will also induce conductivity in germanium. As we'll see in Section 42.7, semiconductor crystals are widely used as photocells and for many other applications.

MasteringPHYSICS®

PhET: Band Structure
PhET: Conductivity

Test Your Understanding of Section 42.4 One type of thermometer works by measuring the temperature-dependent electrical resistivity of a sample. Which of the following types of material displays the greatest change in resistivity for a given temperature change? (i) insulator; (ii) semiconductor; (iii) resistor.



42.5 Free-Electron Model of Metals

Studying the energy states of electrons in metals can give us a lot of insight into their electrical and magnetic properties, the electron contributions to heat capacities, and other behavior. As we discussed in Section 42.3, one of the distinguishing features of a metal is that one or more valence electrons are detached from their home atom and can move freely within the metal, with wave functions that extend over many atoms.

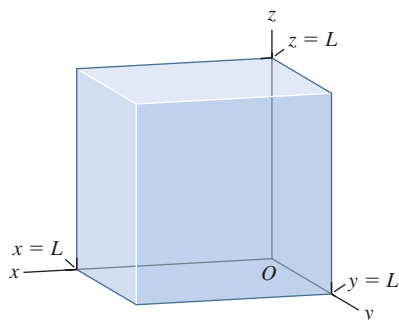
The **free-electron model** assumes that these electrons are completely free inside the material, that they don't interact at all with the ions or with each other, but that there are infinite potential-energy barriers at the surfaces. The idea is that a typical electron moves so rapidly within the metal that it "sees" the effect of the ions and other electrons as a uniform potential-energy function, whose value we can choose to be zero.

We can represent the surfaces of the metal by the same cubical box that we analyzed in Section 41.2 (the three-dimensional version of the particle in a box studied in Section 40.2). If the box has sides of length L (Fig. 42.20), the energies of the stationary states (quantum states of definite energy) are

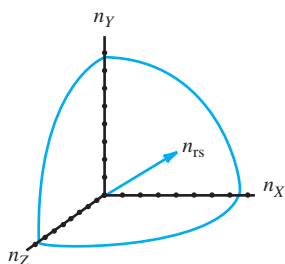
$$E_{n_x, n_y, n_z} = \frac{(n_x^2 + n_y^2 + n_z^2)\pi^2\hbar^2}{2mL^2} \quad (n_x = 1, 2, 3, \dots; n_y = 1, 2, 3, \dots; n_z = 1, 2, 3, \dots) \quad (42.10)$$

Each state is labeled by the three positive-integer quantum numbers (n_x, n_y, n_z) .

42.20 A cubical box with side length L . We studied this three-dimensional version of the infinite square well in Section 41.2. The energy levels for a particle in this box are given by Eq. (42.10).



42.21 The allowed values of n_x , n_y , and n_z are positive integers for the electron states in the free-electron gas model. Including spin, there are two states for each unit volume in n space.



Density of States

Later we'll need to know the *number* dn of quantum states that have energies in a given range dE . The number of states per unit energy range dn/dE is called the **density of states**, denoted by $g(E)$. We'll begin by working out an expression for $g(E)$. Think of a three-dimensional space with coordinates (n_x, n_y, n_z) (Fig. 42.21). The radius n_{rs} of a sphere centered at the origin in that space is $n_{rs} = (n_x^2 + n_y^2 + n_z^2)^{1/2}$. Each point with integer coordinates in that space represents one spatial quantum state. Thus each point corresponds to one unit of volume in the space, and the total number of points with integer coordinates inside a sphere equals the volume of the sphere, $\frac{4}{3}\pi n_{rs}^3$. Because all our n 's are positive, we must take only one *octant* of the sphere, with $\frac{1}{8}$ the total volume, or $(\frac{1}{8})(\frac{4}{3}\pi n_{rs}^3) = \frac{1}{6}\pi n_{rs}^3$. The particles are electrons, so each point corresponds to *two* states with opposite spin components ($m_s = \pm\frac{1}{2}$), and the total number n of electron states corresponding to points inside the octant is twice $\frac{1}{6}\pi n_{rs}^3$, or

$$n = \frac{\pi n_{rs}^3}{3} \quad (42.11)$$

The energy E of states at the surface of the sphere can be expressed in terms of n_{rs} . Equation (42.10) becomes

$$E = \frac{n_{rs}^2 \pi^2 \hbar^2}{2mL^2} \quad (42.12)$$

We can combine Eqs. (42.11) and (42.12) to get a relationship between E and n that doesn't contain n_{rs} . We'll leave the details as an exercise (Exercise 42.24); the result is

$$n = \frac{(2m)^{3/2} V E^{3/2}}{3\pi^2 \hbar^3} \quad (42.13)$$

where $V = L^3$ is the volume of the box. Equation (42.13) gives the total number of states with energies of E or less.

To get the number of states dn in an energy interval dE , we treat n and E as continuous variables and take differentials of both sides of Eq. (42.13). We get

$$dn = \frac{(2m)^{3/2}VE^{1/2}}{2\pi^2\hbar^3} dE \tag{42.14}$$

The density of states $g(E)$ is equal to dn/dE , so from Eq. (42.14) we get

$$g(E) = \frac{(2m)^{3/2}V}{2\pi^2\hbar^3} E^{1/2} \quad (\text{density of states, free-electron model}) \tag{42.15}$$

Fermi–Dirac Distribution

Now we need to know how the electrons are distributed among the various quantum states at any given temperature. The Maxwell–Boltzmann distribution states that the average number of particles in a state of energy E is proportional to $e^{-E/kT}$ (see Sections 18.5 and 39.4). However, there are two very important reasons why it wouldn't be right to use the Maxwell–Boltzmann distribution. The first reason is the exclusion principle. At absolute zero the Maxwell–Boltzmann function predicts that *all* the electrons would go into the two ground states of the system, with $n_x = n_y = n_z = 1$ and $m_s = \pm \frac{1}{2}$. But the exclusion principle allows only one electron in each state. At absolute zero the electrons can fill up the lowest *available* states, but there's not enough room for *all* of them to go into the lowest states. Thus a reasonable guess as to the shape of the distribution would be Fig. 42.22. At absolute zero temperature the states are filled up to some value E_{F0} , and all states above this value are empty.

The second reason we can't use the Maxwell–Boltzmann distribution is more subtle. That distribution assumes that we are dealing with *distinguishable* particles. It might seem that we could put a tag on each electron and know which is which. But overlapping electrons in a system such as a metal are *indistinguishable*. Suppose we have two electrons; a state in which the first is in energy level E_1 and the second is in level E_2 is not distinguishable from a state in which the two electrons are reversed, because we can't tell which electron is which.

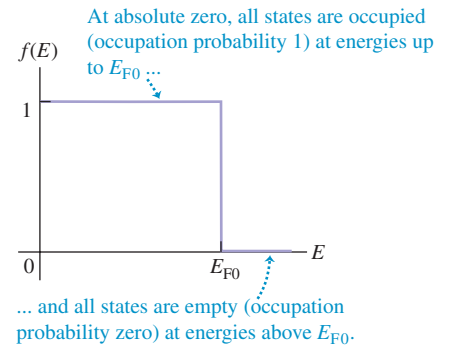
The statistical distribution function that emerges from the exclusion principle and the indistinguishability requirement is called (after its inventors) the **Fermi–Dirac distribution**. Because of the exclusion principle, the probability that a particular state with energy E is occupied by an electron is the same as $f(E)$, the fraction of states with that energy that are occupied:

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad (\text{Fermi–Dirac distribution}) \tag{42.16}$$

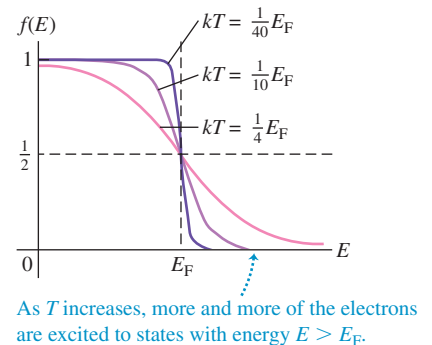
The energy E_F is called the **Fermi energy** or the *Fermi level*; we'll discuss its significance below. We use E_{F0} for its value at absolute zero ($T = 0$) and E_F for other temperatures. We can accurately let $E_F = E_{F0}$ for metals because the Fermi energy does not change much with temperature for solid conductors. However, it is not safe to assume that $E_F = E_{F0}$ for semiconductors, in which the Fermi energy usually does change with temperature.

Figure 42.23 shows graphs of Eq. (42.16) for three temperatures. The trend of this function as kT approaches zero confirms our guess. When $E = E_F$, the exponent is zero and $f(E_F) = \frac{1}{2}$. That is, the probability is $\frac{1}{2}$ that a state at the Fermi energy contains an electron. Alternatively, at $E = E_F$, half the states are filled (and half are empty).

42.22 The probability distribution for occupation of free-electron energy states at absolute zero.



42.23 Graphs of the Fermi–Dirac distribution function for various values of kT , assuming that the Fermi energy E_F is independent of the temperature T .



For $E < E_F$ the exponent is negative, and $f(E) > \frac{1}{2}$. For $E > E_F$ the exponent is positive, and $f(E) < \frac{1}{2}$. The shape depends on the ratio E_F/kT . At $T \ll E_F/k$ this ratio is very large. Then for $E < E_F$ the curve very quickly approaches 1, and for $E > E_F$ it quickly approaches zero. When T is larger, the changes are more gradual. When T is zero, all the states up to the Fermi level E_{F0} are filled, and all states above that level are empty (Fig. 42.22).

Example 42.6 Probabilities in the free-electron model

For free electrons in a solid, at what energy is the probability that a particular state is occupied equal to (a) 0.01 and (b) 0.99?

SOLUTION

IDENTIFY and SET UP: This problem asks us to explore the Fermi–Dirac distribution. Equation (42.16) gives the occupation probability $f(E)$ for a given energy E . If we solve this equation for E , we get an expression for the energy that corresponds to a given occupation probability—which is just what we need to solve this problem.

EXECUTE: Using Eq. (42.16), you can show that

$$E = E_F + kT \ln \left(\frac{1}{f(E)} - 1 \right)$$

(a) When $f(E) = 0.01$,

$$E = E_F + kT \ln \left(\frac{1}{0.01} - 1 \right) = E_F + 4.6kT$$

The probability that a state $4.6kT$ above the Fermi level is occupied is only 0.01, or 1%.

(b) When $f(E) = 0.99$,

$$E = E_F + kT \ln \left(\frac{1}{0.99} - 1 \right) = E_F - 4.6kT$$

The probability that a state $4.6kT$ below the Fermi level is occupied is 0.99, or 99%.

EVALUATE: At very low temperatures, $4.6kT$ is much less than E_F . Then the occupation probability of levels even slightly below E_F is nearly 1 (100%), and that for levels even slightly above E_F is nearly zero (see Fig. 42.23). In general, if the probability is P that a state with an energy ΔE above E_F is occupied, then the probability is $1 - P$ that a state ΔE below E_F is occupied. We leave the proof to you (Problem 42.50).

Electron Concentration and Fermi Energy

Equation (42.16) gives the probability that any specific state with energy E is occupied at a temperature T . To get the actual number of electrons in any energy range dE , we have to multiply this probability by the number dn of states in that range $g(E) dE$. Thus the number dN of electrons with energies in the range dE is

$$dN = g(E)f(E) dE = \frac{(2m)^{3/2}VE^{1/2}}{2\pi^2\hbar^3} \frac{1}{e^{(E-E_F)/kT} + 1} dE \quad (42.17)$$

The Fermi energy E_F is determined by the total number N of electrons; at any temperature the electron states are filled up to a point at which all electrons are accommodated. At absolute zero there is a simple relationship between E_{F0} and N . All states below E_{F0} are filled; in Eq. (42.13) we set n equal to the total number of electrons N and E to the Fermi energy at absolute zero E_{F0} :

$$N = \frac{(2m)^{3/2}VE_{F0}^{3/2}}{3\pi^2\hbar^3} \quad (42.18)$$

Solving Eq. (42.18) for E_{F0} , we get

$$E_{F0} = \frac{3^{2/3}\pi^4/3\hbar^2}{2m} \left(\frac{N}{V} \right)^{2/3} \quad (42.19)$$

The quantity N/V is the number of free electrons per unit volume. It is called the *electron concentration* and is usually denoted by n .

If we replace N/V with n , Eq. (42.19) becomes

$$E_{F0} = \frac{3^{2/3}\pi^4/3\hbar^2 n^{2/3}}{2m} \quad (42.20)$$

CAUTION **Electron concentration and number of electrons** Don't confuse the electron concentration n with any quantum number n . Furthermore, the number of states is *not* in general the same as the total number of electrons N . **|**

Example 42.7 The Fermi energy in copper

At low temperatures, copper has a free-electron concentration $n = 8.45 \times 10^{28} \text{ m}^{-3}$. Using the free-electron model, find the Fermi energy for solid copper, and find the speed of an electron with a kinetic energy equal to the Fermi energy.

SOLUTION

IDENTIFY and SET UP: This problem uses the relationship between Fermi energy and free-electron concentration. Because copper is a solid conductor, its Fermi energy changes very little with temperature and we can use the expression for the Fermi energy at absolute zero, Eq. (42.20). We'll find the *Fermi speed* v_F that corresponds to kinetic energy E_F using the nonrelativistic formula $E_F = \frac{1}{2}mv_F^2$.

EXECUTE: Using the given value of n , we solve for E_F and v_F :

$$\begin{aligned} E_F &= \frac{3^{2/3}\pi^{4/3}(1.055 \times 10^{-34} \text{ J}\cdot\text{s})^2(8.45 \times 10^{28} \text{ m}^{-3})^{2/3}}{2(9.11 \times 10^{-31} \text{ kg})} \\ &= 1.126 \times 10^{-18} \text{ J} = 7.03 \text{ eV} \end{aligned}$$

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2(1.126 \times 10^{-18} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 1.57 \times 10^6 \text{ m/s}$$

EVALUATE: Our values of E_F and v_F are within the ranges of typical values for metals, 1.6–14 eV and $0.8\text{--}2.2 \times 10^6 \text{ m/s}$, respectively. Note that the calculated Fermi speed is far less than the speed of light $c = 3.00 \times 10^8 \text{ m/s}$, which justifies our use of the nonrelativistic formula $\frac{1}{2}mv_F^2 = E_F$.

Our calculated Fermi energy is much larger than kT at ordinary temperatures. (At room temperature $T = 20^\circ\text{C} = 293 \text{ K}$, the quantity kT equals $(1.381 \times 10^{-23} \text{ J/K})(293 \text{ K}) = 4.04 \times 10^{-21} \text{ J} = 0.0254 \text{ eV}$.) So it is a good approximation to take almost all the states below E_F as completely full and almost all those above E_F as completely empty (see Fig. 42.22).

We can also use Eq. (42.15) to find $g(E)$ if E and V are known. You can show that if $E = 7.03 \text{ eV}$ and $V = 1 \text{ cm}^3$, $g(E)$ is about $2 \times 10^{22} \text{ states/eV}$. This huge number shows why we were justified in treating n and E as continuous variables in our density-of-states derivation.

Average Free-Electron Energy

We can calculate the *average* free-electron energy in a metal at absolute zero by using the same ideas that we used to find E_{F0} . From Eq. (42.17) the number dN of electrons with energies in the range dE is $g(E)f(E) dE$. The energy of these electrons is $E dN = Eg(E)f(E) dE$. At absolute zero we substitute $f(E) = 1$ from $E = 0$ to $E = E_{F0}$ and $f(E) = 0$ for all other energies. Therefore the total energy E_{tot} of all the N electrons is

$$E_{\text{tot}} = \int_0^{E_{F0}} Eg(E)(1) dE + \int_{E_{F0}}^{\infty} Eg(E)(0) dE = \int_0^{E_{F0}} Eg(E) dE$$

The simplest way to evaluate this expression is to compare Eqs. (42.15) and (42.19), noting that

$$g(E) = \frac{3NE^{1/2}}{2E_{F0}^{3/2}}$$

Substituting this expression into the integral and using $E_{\text{av}} = E_{\text{tot}}/N$, we get

$$E_{\text{av}} = \frac{3}{2E_{F0}^{3/2}} \int_0^{E_{F0}} E^{3/2} dE = \frac{3}{5}E_{F0} \quad (42.21)$$

At absolute zero the average free-electron energy equals $\frac{3}{5}$ of the Fermi energy.

Example 42.8 Free-electron gas versus ideal gas

(a) Find the average energy of the free electrons in copper at absolute zero (see Example 42.7). (b) What would be the average kinetic energy of electrons if they behaved like an ideal gas at

room temperature, 20°C (see Section 18.3)? What would be the speed of an electron with this kinetic energy? Compare these ideal-gas values with the (correct) free-electron values.

Continued

SOLUTION

IDENTIFY and SET UP: Free electrons in a metal behave like a kind of gas. In part (a) we use Eq. (42.21) to determine the average kinetic energy of free electrons in terms of the Fermi energy at absolute zero, which we know for copper from Example 42.7. In part (b) we treat electrons as an ideal gas at room temperature: Eq. (18.16) then gives the average kinetic energy per electron as $E_{\text{av}} = \frac{3}{2}kT$, and $E_{\text{av}} = \frac{1}{2}mv^2$ gives the corresponding electron speed v .

EXECUTE: (a) From Example 42.7, the Fermi energy in copper at absolute zero is $1.126 \times 10^{-18} \text{ J} = 7.03 \text{ eV}$. According to Eq. (42.21), the average energy is $\frac{3}{5}$ of this, or $6.76 \times 10^{-19} \text{ J} = 4.22 \text{ eV}$.

(b) In Example 42.7 we found that $kT = 4.04 \times 10^{-21} \text{ J} = 0.0254 \text{ eV}$ at room temperature $T = 20^\circ\text{C} = 293 \text{ K}$. If electrons behaved like an ideal gas at this temperature, the average kinetic energy per electron would be $\frac{3}{2}$ of this, or $6.07 \times 10^{-21} \text{ J} = 0.0379 \text{ eV}$. The speed of an electron with this kinetic energy would be

$$v = \sqrt{\frac{2E_{\text{av}}}{m}} = \sqrt{\frac{2(6.07 \times 10^{-21} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 1.15 \times 10^5 \text{ m/s}$$

EVALUATE: The ideal-gas model predicts an average energy that is about 1% of the value given by the free-electron model, and

a speed that is about 7% of the free-electron Fermi speed $v_F = 1.57 \times 10^6 \text{ m/s}$ that we found in Example 42.7. Thus temperature plays a *very* small role in determining the properties of electrons in metals; their average energies are determined almost entirely by the exclusion principle.

A similar analysis allows us to determine the contributions of electrons to the heat capacities of a solid metal. If there is one conduction electron per atom, the principle of equipartition of energy (see Section 18.4) would predict that the kinetic energies of these electrons contribute $3R/2$ to the molar heat capacity at constant volume C_V . But when kT is much smaller than E_F , which is usually the situation in metals, only those few electrons near the Fermi level can find empty states and change energy appreciably when the temperature changes. The number of such electrons is proportional to kT/E_F , so we expect that the electron molar heat capacity at constant volume is proportional to $(kT/E_F)(3R/2) = (3kT/2E_F)R$. A more detailed analysis shows that the actual electron contribution to C_V for a solid metal is $(\pi^2 kT/2E_F)R$, not far from our prediction. You can verify that if $T = 293 \text{ K}$ and $E_F = 7.03 \text{ eV}$, the electron contribution to C_V is $0.018R$, which is only 1.2% of the (incorrect) $3R/2$ prediction of the equipartition principle. Because the electron contribution is so small, the overall heat capacity of most solid metals is due primarily to vibration of the atoms in the crystal structure (see Fig. 18.18 in Section 18.4).

Test Your Understanding of Section 42.5 An ideal gas obeys the relationship $pV = nRT$ (see Section 18.1). That is, for a given volume V and a number of moles n , as the temperature T decreases, the pressure p decreases proportionately and tends to zero as T approaches absolute zero. Is this also true of the free-electron gas in a solid metal?

MasteringPHYSICS

PhET: Semiconductors
PhET: Conductivity

42.6 Semiconductors

A **semiconductor** has an electrical resistivity that is intermediate between those of good conductors and of good insulators. The tremendous importance of semiconductors in present-day electronics stems in part from the fact that their electrical properties are very sensitive to very small concentrations of impurities. We'll discuss the basic concepts using the semiconductor elements silicon (Si) and germanium (Ge) as examples.

Silicon and germanium are in Group IV of the periodic table. Both have four electrons in the outermost atomic subshells ($3s^2 3p^2$ for silicon, $4s^2 4p^2$ for germanium), and both crystallize in the covalently bonded diamond structure discussed in Section 42.3 (see Fig. 42.14). Because all four of the outer electrons are involved in the bonding, at absolute zero the band structure (see Section 42.4) has a completely empty conduction band (see Fig. 42.19b). As we discussed in Section 42.4, at very low temperatures electrons cannot jump from the filled valence band into the conduction band. This property makes these materials insulators at very low temperatures; their electrons have no nearby states available into which they can move in response to an applied electric field.

However, in semiconductors the energy gap E_g between the valence and conduction bands is small in comparison to the gap of 5 eV or more for many insulators; room-temperature values are 1.12 eV for silicon and only 0.67 eV for germanium. Thus even at room temperature a substantial number of electrons can gain enough energy to jump the gap to the conduction band, where they are dissociated from their parent atoms and are free to move about the crystal. The number of these electrons increases rapidly with temperature.

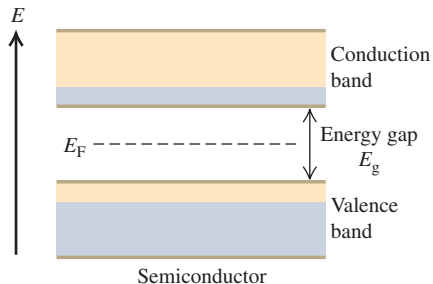
Example 42.9 Jumping a band gap

Consider a material with the band structure described above, with its Fermi energy in the middle of the gap (Fig. 42.24). Find the probability that a state at the bottom of the conduction band is occupied at $T = 300$ K, and compare that with the probability at $T = 310$ K, for band gaps of (a) 0.200 eV; (b) 1.00 eV; (c) 5.00 eV.

SOLUTION

IDENTIFY and SET UP: The Fermi–Dirac distribution function gives the probability that a state of energy E is occupied at temperature T . Figure 42.24 shows that the state of interest at the bottom of the conduction band has an energy $E = E_F + E_g/2$ that is greater than the Fermi energy E_F , with $E - E_F = E_g/2$.

42.24 Band structure of a semiconductor. At absolute zero a completely filled valence band is separated by a narrow energy gap E_g of 1 eV or so from a completely empty conduction band. At ordinary temperatures, a number of electrons are excited to the conduction band.



In principle, we could continue the calculation in Example 42.9 to find the actual density $n = N/V$ of electrons in the conduction band at any temperature. To do this, we would have to evaluate the integral $\int g(E)f(E) dE$ from the bottom of the conduction band to its top. First we would need to know the density of states function $g(E)$. It wouldn't be correct to use Eq. (42.15) because the energy-level structure and the density of states for real solids are more complex than those for the simple free-electron model. However, there are theoretical methods for predicting what $g(E)$ should be near the bottom of the conduction band, and such calculations have been carried out. Once we know n , we can *begin* to determine the resistivity of the material (and its temperature dependence) using the analysis of Section 25.2, which you may want to review. But next we'll see that the electrons in the conduction band don't tell the whole story about conduction in semiconductors.

Holes

When an electron is removed from a covalent bond, it leaves a vacancy behind. An electron from a neighboring atom can move into this vacancy, leaving the neighbor with the vacancy. In this way the vacancy, called a **hole**, can travel through the material and serve as an additional current carrier. It's like describing the motion of a bubble in a liquid. In a pure, or *intrinsic*, semiconductor, valence-band holes and conduction-band electrons are always present in equal numbers. When an electric field is applied, they move in opposite directions (Fig. 42.25). Thus a hole in the valence band behaves like a positively charged particle, even though the moving charges in that band are electrons. The conductivity that we

Figure 42.23 shows that the higher the temperature, the larger the fraction of electrons with energies greater than the Fermi energy.

EXECUTE: (a) When $E_g = 0.200$ eV,

$$\frac{E - E_F}{kT} = \frac{E_g}{2} \frac{0.100 \text{ eV}}{(8.617 \times 10^{-5} \text{ eV/K})(300 \text{ K})} = 3.87$$

$$f(E) = \frac{1}{e^{3.87} + 1} = 0.0205$$

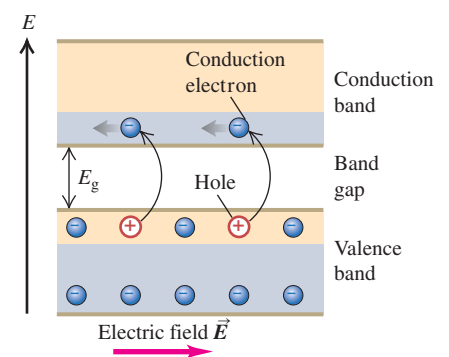
For $T = 310$ K, the exponent is 3.74 and $f(E) = 0.0231$, a 13% increase in probability for a temperature rise of 10 K.

(b) For $E_g = 1.00$ eV, both exponents are five times as large as in part (a), namely 19.3 and 18.7; the values of $f(E)$ are 4.0×10^{-9} and 7.4×10^{-9} . In this case the (low) probability nearly doubles with a temperature rise of 10 K.

(c) For $E_g = 5.0$ eV, the exponents are 96.7 and 93.6; the values of $f(E)$ are 1.0×10^{-42} and 2.3×10^{-41} . The (extremely low) probability increases by a factor of 23 for a 10 K temperature rise.

EVALUATE: This example illustrates two important points. First, the probability of finding an electron in a state at the bottom of the conduction band is extremely sensitive to the width of the band gap. At room temperature, the probability is about 2% for a 0.200-eV gap, a few in a thousand million for a 1.00-eV gap, and essentially zero for a 5.00-eV gap. (Pure diamond, with a 5.47-eV band gap, has essentially no electrons in the conduction band and is an excellent insulator.) Second, for any given band gap the probability depends strongly on temperature, and even more strongly for large gaps than for small ones.

42.25 Motion of electrons in the conduction band and of holes in the valence band of a semiconductor under the action of an applied electric field \vec{E} .



just described for a pure semiconductor is called *intrinsic conductivity*. Another kind of conductivity, to be discussed in the next subsection, is due to impurities.

An analogy helps to picture conduction in an intrinsic semiconductor. The valence band at absolute zero is like a floor of a parking garage that's filled bumper to bumper with cars (which represent electrons). No cars can move because there is nowhere for them to go. But if one car is moved to the vacant floor above, it can move freely, just as electrons can move freely in the conduction band. Also, the empty space that it leaves permits cars to move on the nearly filled floor, thereby moving the empty space just as holes move in the normally filled valence band.

Impurities

Suppose we mix into melted germanium ($Z = 32$) a small amount of arsenic ($Z = 33$), the next element after germanium in the periodic table. This deliberate addition of impurity elements is called *doping*. Arsenic is in Group V; it has *five* valence electrons. When one of these electrons is removed, the remaining electron structure is essentially identical to that of germanium. The only difference is that it is smaller; the arsenic nucleus has a charge of $+33e$ rather than $+32e$, and it pulls the electrons in a little more. An arsenic atom can comfortably take the place of a germanium atom as a substitutional impurity. Four of its five valence electrons form the necessary nearest-neighbor covalent bonds.

The fifth valence electron is very loosely bound (Fig. 42.26a); it doesn't participate in the covalent bonds, and it is screened from the nuclear charge of $+33e$ by the 32 electrons, leaving a net effective charge of about $+e$. We might guess that the binding energy would be of the same order of magnitude as the energy of the $n = 4$ level in hydrogen—that is, $(\frac{1}{4})^2(13.6 \text{ eV}) = 0.85 \text{ eV}$. In fact, it is much smaller than this, only about 0.01 eV, because the electron probability distribution actually extends over many atomic diameters and the polarization of intervening atoms provides additional screening.

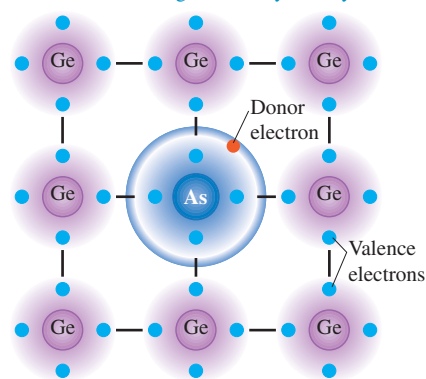
The energy level of this fifth electron corresponds in the band picture to an isolated energy level lying in the gap, about 0.01 eV below the bottom of the conduction band (Fig. 42.26b). This level is called a *donor level*, and the impurity atom that is responsible for it is simply called a *donor*. All Group V elements, including N, P, As, Sb, and Bi, can serve as donors. At room temperature, kT is about 0.025 eV. This is substantially greater than 0.01 eV, so at ordinary temperatures, most electrons can gain enough energy to jump from donor levels into the conduction band, where they are free to wander through the material. The remaining ionized donor stays at its site in the structure and does not participate in conduction.

Example 42.9 shows that at ordinary temperatures and with a band gap of 1.0 eV, only a very small fraction (of the order of 10^{-9}) of the states at the bottom of the conduction band in a pure semiconductor contain electrons to participate in intrinsic conductivity. Thus we expect the conductivity of such a semiconductor to be about 10^{-9} as great as that of good metallic conductors, and measurements bear out this prediction. However, a concentration of donors as small as one part in 10^8 can increase the conductivity so drastically that conduction due to impurities becomes by far the dominant mechanism. In this case the conductivity is due almost entirely to *negative* charge (electron) motion. We call the material an ***n-type semiconductor***, with *n-type* impurities.

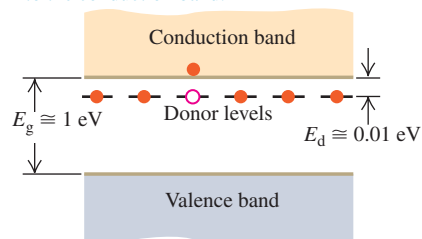
Adding atoms of an element in Group III (B, Al, Ga, In, Tl), with only *three* valence electrons, has an analogous effect. An example is gallium ($Z = 31$); as a substitutional impurity in germanium, the gallium atom would like to form four covalent bonds, but it has only three outer electrons. It can, however, steal an electron from a neighboring germanium atom to complete the required four covalent bonds (Fig. 42.27a). The resulting atom has the same electron configuration as Ge but is somewhat larger because gallium's nuclear charge is smaller, $+31e$ instead of $+32e$.

42.26 An *n*-type semiconductor.

(a) A donor (*n*-type) impurity atom has a fifth valence electron that does not participate in the covalent bonding and is very loosely bound.



(b) Energy-band diagram for an *n*-type semiconductor at a low temperature. One donor electron has been excited from the donor levels into the conduction band.



This theft leaves the neighboring atom with a *hole*, or missing electron. The hole acts as a positive charge that can move through the crystal just as with intrinsic conductivity. The stolen electron is bound to the gallium atom in a level called an *acceptor level* about 0.01 eV above the top of the valence band (Fig. 42.27b). The gallium atom, called an *acceptor*, thus accepts an electron to complete its desire for four covalent bonds. This extra electron gives the previously neutral gallium atom a net charge of $-e$. The resulting gallium ion is *not* free to move. In a semiconductor that is doped with acceptors, we consider the conductivity to be almost entirely due to *positive* charge (hole) motion. We call the material a ***p*-type semiconductor**, with *p*-type impurities. Some semiconductors are doped with *both* *n*- and *p*-type impurities. Such materials are called *compensated* semiconductors.

CAUTION The meaning of “*p*-type” and “*n*-type” Saying that a material is a *p*-type semiconductor does *not* mean that the material has a positive charge; ordinarily, it would be neutral. Rather, it means that its *majority carriers* of current are positive holes (and therefore its *minority carriers* are negative electrons). The same idea holds for an *n*-type semiconductor; ordinarily, it will *not* have a negative charge, but its majority carriers are negative electrons. |

We can verify the assertion that the current in *n*- and *p*-type semiconductors really *is* carried by electrons and holes, respectively, by using the Hall effect (see Section 27.9). The sign of the Hall emf is opposite in the two cases. Hall-effect devices constructed from semiconductor materials are used in probes to measure magnetic fields and the currents that cause those fields.

Test Your Understanding of Section 42.6 Would there be any advantage to adding *n*-type or *p*-type impurities to copper?

42.7 Semiconductor Devices

Semiconductor devices play an indispensable role in contemporary electronics. In the early days of radio and television, transmitting and receiving equipment relied on vacuum tubes, but these have been almost completely replaced in the last six decades by solid-state devices, including transistors, diodes, integrated circuits, and other semiconductor devices. The only surviving vacuum tubes in consumer electronics are the picture tubes in older TV receivers and computer monitors; these are rapidly being replaced by flat-screen displays.

One simple semiconductor device is the *photocell* (Fig. 42.28). When a thin slab of semiconductor is irradiated with an electromagnetic wave whose photons have at least as much energy as the band gap between the valence and conduction bands, an electron in the valence band can absorb a photon and jump to the conduction band, where it and the hole it left behind contribute to the conductivity (see Example 42.5 in Section 42.4). The conductivity therefore increases with wave intensity, thus increasing the current I in the photocell circuit of Fig. 42.28. Hence the ammeter reading indicates the intensity of the light.

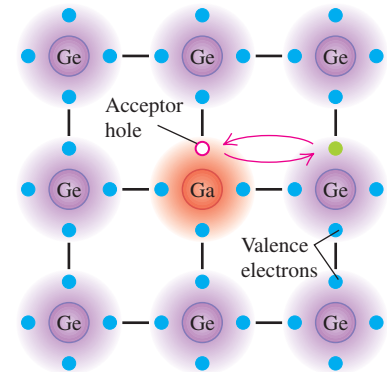
Detectors for charged particles operate on the same principle. An external circuit applies a voltage across a semiconductor. An energetic charged particle passing through the semiconductor collides inelastically with valence electrons, exciting them from the valence to the conduction band and creating pairs of holes and conduction electrons. The conductivity increases momentarily, causing a pulse of current in the external circuit. Solid-state detectors are widely used in nuclear and high-energy physics research.

The *p*-*n* Junction

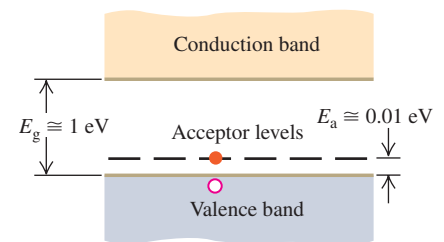
In many semiconductor devices the essential principle is the fact that the conductivity of the material is controlled by impurity concentrations, which can be varied

42.27 A *p*-type semiconductor.

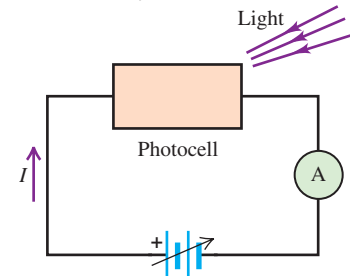
(a) An acceptor (*p*-type) impurity atom has only three valence electrons, so it can borrow an electron from a neighboring atom. The resulting hole is free to move about the crystal.



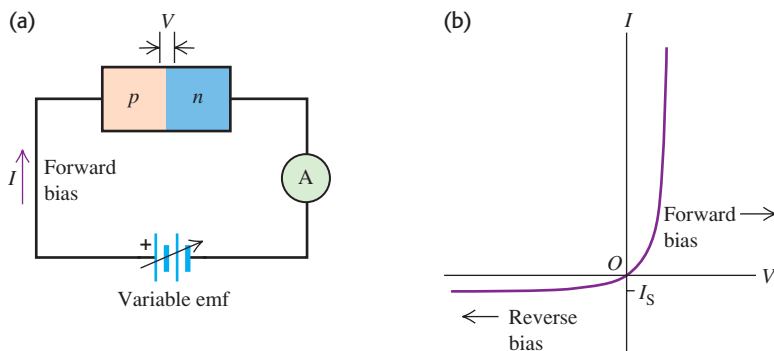
(b) Energy-band diagram for a *p*-type semiconductor at a low temperature. One acceptor level has accepted an electron from the valence band, leaving a hole behind.



42.28 A semiconductor photocell in a circuit. The more intense the light falling on the photocell, the greater the conductivity of the photocell and the greater the current measured by the ammeter (A).



42.29 (a) A semiconductor p - n junction in a circuit. (b) Graph showing the asymmetric current–voltage relationship. The curve is described by Eq. (42.22).



within wide limits from one region of a device to another. An example is the **p - n junction** at the boundary between one region of a semiconductor with p -type impurities and another region containing n -type impurities. One way of fabricating a p - n junction is to deposit some n -type material on the *very* clean surface of some p -type material. (We can't just stick p - and n -type pieces together and expect the junction to work properly because of the impossibility of matching their surfaces at the atomic level.)

When a p - n junction is connected to an external circuit, as in Fig. 42.29a, and the potential difference $V_p - V_n = V$ across the junction is varied, the current I varies as shown in Fig. 42.29b. In striking contrast to the symmetric behavior of resistors that obey Ohm's law and give a straight line on an I - V graph, a p - n junction conducts much more readily in the direction from p to n than the reverse. Such a (mostly) one-way device is called a **diode rectifier**. Later we'll discuss a simple model of p - n junction behavior that predicts a current–voltage relationship in the form

$$I = I_S(e^{eV/kT} - 1) \quad (\text{current through a } p\text{-}n \text{ junction}) \quad (42.22)$$

In the exponent, $e = 1.602 \times 10^{-19}$ C is the quantum of charge, k is Boltzmann's constant, and T is absolute temperature.

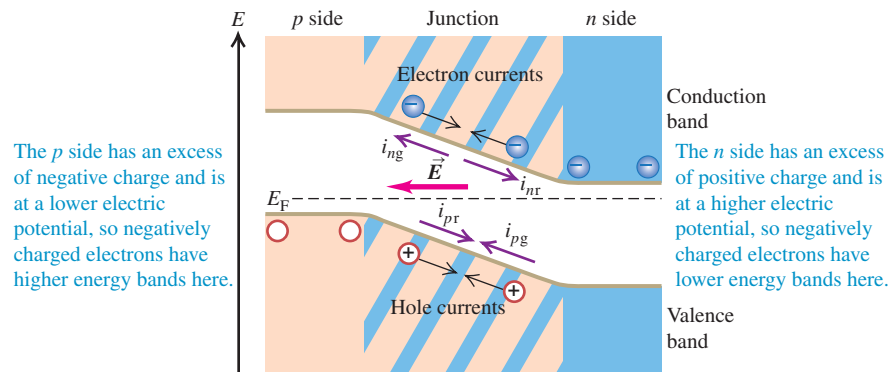
CAUTION Two different uses of e In $e^{eV/kT}$ the base of the exponent also uses the symbol e , standing for the base of the natural logarithms, 2.71828... This e is quite different from $e = 1.602 \times 10^{-19}$ C in the exponent. **I**

Equation (42.22) is valid for both positive and negative values of V ; note that V and I always have the same sign. As V becomes very negative, I approaches the value $-I_S$. The magnitude I_S (always positive) is called the *saturation current*.

Currents Through a p - n Junction

We can understand the behavior of a p - n junction diode qualitatively on the basis of the mechanisms for conductivity in the two regions. Suppose, as in Fig. 42.29a, you connect the positive terminal of the battery to the p region and the negative terminal to the n region. Then the p region is at higher potential than the n region, corresponding to positive V in Eq. (42.22), and the resulting electric field is in the direction p to n . This is called the *forward* direction, and the positive potential difference is called *forward bias*. Holes, plentiful in the p region, flow easily across the junction into the n region, and free electrons, plentiful in the n region, easily flow into the p region; these movements of charge constitute a *forward* current. Connecting the battery with the opposite polarity gives *reverse bias*, and the field tends to push electrons from p to n and holes from n to p . But there are very few free electrons in the p region and very few holes in the n region. As a result, the current in the *reverse* direction is much smaller than that with the same potential difference in the forward direction.

42.30 A p - n junction in equilibrium, with no externally applied field or potential difference. The generation and recombination currents exactly balance. The Fermi energy E_F is the same on both sides of the junction. The excess positive and negative charges on the n and p sides produce an electric field \vec{E} in the direction shown.



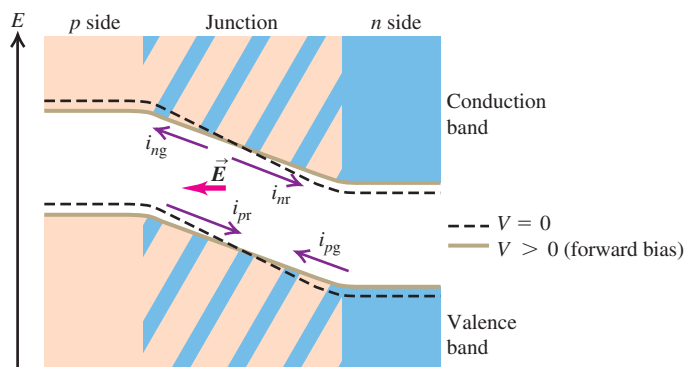
Suppose you have a box with a barrier separating the left and right sides: You fill the left side with oxygen gas and the right side with nitrogen gas. What happens if the barrier leaks? Oxygen diffuses to the right, and nitrogen diffuses to the left. A similar diffusion occurs across a p - n junction. First consider the equilibrium situation with no applied voltage (Fig. 42.30). The many holes in the p region act like a hole gas that diffuses across the junction into the n region. Once there, the holes recombine with some of the many free electrons. Similarly, electrons diffuse from the n region to the p region and fall into some of the many holes there. The hole and electron diffusion currents lead to a net positive charge in the n region and a net negative charge in the p region, causing an electric field in the direction from n to p at the junction. The potential energy associated with this field raises the electron energy levels in the p region relative to the same levels in the n region.

There are four currents across the junction, as shown. The diffusion processes lead to *recombination currents* of holes and electrons, labeled i_{pr} and i_{nr} in Fig. 42.30. At the same time, electron-hole pairs are generated in the junction region by thermal excitation. The electric field described above sweeps these electrons and holes out of the junction; electrons are swept opposite the field to the n side, and holes are swept in the same direction as the field to the p side. The corresponding currents, called *generation currents*, are labeled i_{pg} and i_{ng} . At equilibrium the magnitudes of the generation and recombination currents are equal:

$$|i_{pg}| = |i_{pr}| \quad \text{and} \quad |i_{ng}| = |i_{nr}| \quad (42.23)$$

In thermal equilibrium the Fermi energy is the same at each point across the junction.

Now we apply a forward bias—that is, a positive potential difference V across the junction. A forward bias *decreases* the electric field in the junction region. It also decreases the difference between the energy levels on the p and n sides (Fig. 42.31)



42.31 A p - n junction under forward-bias conditions. The potential difference between p and n regions is reduced, as is the electric field within the junction. The recombination currents increase but the generation currents are nearly constant, causing a net current from left to right. (Compare Fig. 42.30.)

by an amount $\Delta E = -eV$. It becomes easier for the electrons in the n region to climb the potential-energy hill and diffuse into the p region and for the holes in the p region to diffuse into the n region. This effect increases both recombination currents by the Maxwell–Boltzmann factor $e^{-\Delta E/kT} = e^{eV/kT}$. (We don't have to use the Fermi–Dirac distribution because most of the available states for the diffusing electrons and holes are empty, so the exclusion principle has little effect.) The generation currents don't change appreciably, so the net hole current is

$$\begin{aligned} i_{p\text{tot}} &= i_{pr} - |i_{pg}| \\ &= |i_{pg}|e^{eV/kT} - |i_{pg}| \\ &= |i_{pg}|(e^{eV/kT} - 1) \end{aligned} \quad (42.24)$$

The net electron current $i_{n\text{tot}}$ is given by a similar expression, so the total current $I = i_{p\text{tot}} + i_{n\text{tot}}$ is

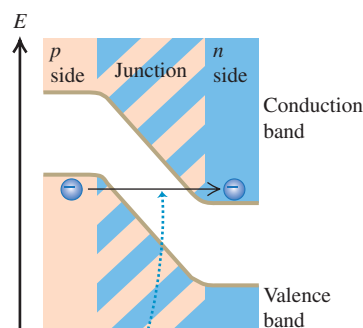
$$I = I_S(e^{eV/kT} - 1) \quad (42.25)$$

in agreement with Eq. (42.22). We can repeat this entire discussion for reverse bias (negative V and I) with the same result. Therefore Eq. (42.22) is valid for both positive and negative values.

Several effects make the behavior of practical p - n junction diodes more complex than this simple analysis predicts. One effect, *avalanche breakdown*, occurs under large reverse bias. The electric field in the junction is so great that the carriers can gain enough energy between collisions to create electron–hole pairs during inelastic collisions. The electrons and holes then gain energy and collide to form more pairs, and so on. (A similar effect occurs in dielectric breakdown in insulators, discussed in Section 42.4.)

A second type of breakdown begins when the reverse bias becomes large enough that the top of the valence band in the p region is just higher in energy than the bottom of the conduction band in the n region (Fig. 42.32). If the junction region is thin enough, the probability becomes large that electrons can *tunnel* from the valence band of the p region to the conduction band of the n region. This process is called *Zener breakdown*. It occurs in Zener diodes, which are used for voltage regulation and protection against voltage surges.

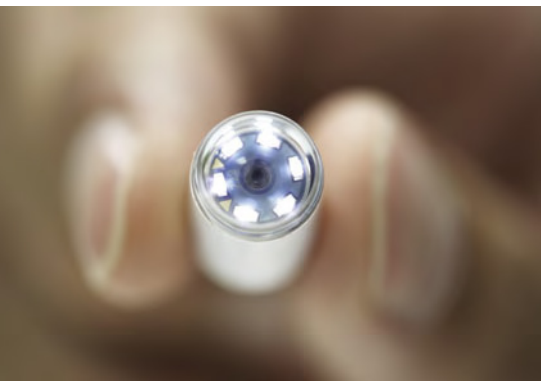
42.32 Under reverse-bias conditions the potential-energy difference between the p and n sides of a junction is greater than at equilibrium. If this difference is great enough, the bottom of the conduction band on the n side may actually be below the top of the valence band on the p side.



If a p - n junction under reverse bias is thin enough, electrons can tunnel from the valence band to the conduction band (a process called *Zener breakdown*).

Application Swallow This Semiconductor Device

This tiny capsule—designed to be swallowed by a patient—contains a miniature camera with a CCD light detector, plus six LEDs to illuminate the subject. The capsule radios high-resolution images to an external recording unit as it passes painlessly through the patient's stomach and intestines. This technique makes it possible to examine the small intestine, which is not readily accessible with conventional endoscopy.



Semiconductor Devices and Light

A *light-emitting diode (LED)* is a p - n junction diode that emits light. When the junction is forward biased, many holes are pushed from their p region to the junction region, and many electrons are pushed from their n region to the junction region. In the junction region the electrons fall into holes (recombine). In recombining, the electron can emit a photon with energy approximately equal to the band gap. This energy (and therefore the photon wavelength and the color of the light) can be varied by using materials with different band gaps. Light-emitting diodes are very energy-efficient light sources and have many applications, including automobile lamps, traffic signals, and large stadium displays.

The reverse process is called the *photovoltaic effect*. Here the material absorbs photons, and electron–hole pairs are created. Pairs that are created in the p - n junction, or close enough to migrate to it without recombining, are separated by the electric field we described above that sweeps the electrons to the n side and the holes to the p side. We can connect this device to an external circuit, where it becomes a source of emf and power. Such a device is often called a *solar cell*, although sunlight isn't required. Any light with photon energies greater than the band gap will do. You might have a calculator powered by such cells. Production of low-cost photovoltaic cells for large-scale solar energy conversion is a very active field of research. The same basic physics is used in charge-coupled device (CCD) image detectors, digital cameras, and video cameras.

Transistors

A *bipolar junction transistor* includes two p - n junctions in a “sandwich” configuration, which may be either p - n - p or n - p - n . Figure 42.33 shows such a p - n - p transistor. The three regions are called the emitter, base, and collector, as shown. When there is no current in the left loop of the circuit, there is only a very small current through the resistor R because the voltage across the base–collector junction is in the reverse direction. But when a forward bias is applied between emitter and base, as shown, most of the holes traveling from emitter to base travel *through* the base (which is typically both narrow and lightly doped) to the second junction, where they come under the influence of the collector-to-base potential difference and flow on through the collector to give an increased current to the resistor.

In this way the current in the collector circuit is *controlled* by the current in the emitter circuit. Furthermore, V_c may be considerably larger than V_e , so the *power* dissipated in R may be much larger than the power supplied to the emitter circuit by the battery V_e . Thus the device functions as a *power amplifier*. If the potential drop across R is greater than V_e , it may also be a voltage amplifier.

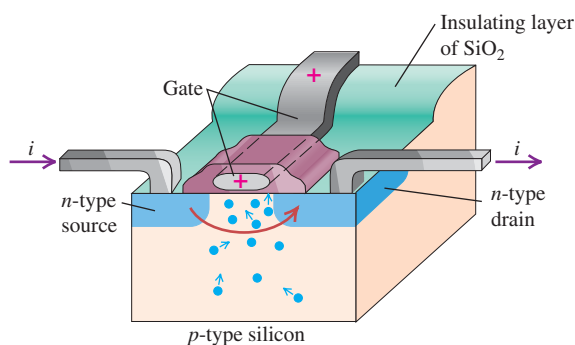
In this configuration the *base* is the common element between the “input” and “output” sides of the circuit. Another widely used arrangement is the *common-emitter* circuit, shown in Fig. 42.34. In this circuit the current in the collector side of the circuit is much larger than that in the base side, and the result is current amplification.

The *field-effect transistor* (Fig. 42.35) is an important type. In one variation a slab of p -type silicon is made with two n -type regions on the top, called the *source* and the *drain*; a metallic conductor is fastened to each. A third electrode called the *gate* is separated from the slab, source, and drain by an insulating layer of SiO_2 . When there is no charge on the gate and a potential difference of either polarity is applied between the source and the drain, there is very little current because one of the p - n junctions is reverse biased.

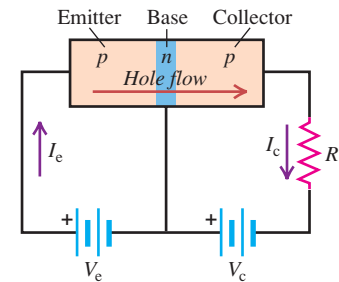
Now we place a positive charge on the gate. With dimensions of the order of 10^{-6} m, it takes little charge to provide a substantial electric field. Thus there is very little current into or out of the gate. There aren’t many free electrons in the p -type material, but there are some, and the effect of the field is to attract them toward the positive gate. The resulting greatly enhanced concentration of electrons near the gate (and between the two junctions) permits current to flow between the source and the drain. The current is very sensitive to the gate charge and potential, and the device functions as an amplifier. The device just described is called an *enhancement-type MOSFET* (metal-oxide-semiconductor field-effect transistor).

Integrated Circuits

A further refinement in semiconductor technology is the *integrated circuit*. By successively depositing layers of material and etching patterns to define current paths, we can combine the functions of several MOSFETs, capacitors, and resistors on a single square of semiconductor material that may be only a few millimeters on a side. An elaboration of this idea leads to *large-scale integrated circuits*. The resulting integrated circuit chips are the heart of all pocket calculators and present-day computers, large and small (Fig. 42.36).

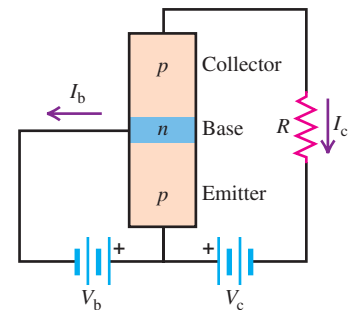


42.33 Schematic diagram of a p - n - p transistor and circuit.



- When $V_e = 0$, the current is very small.
- When a potential V_e is applied between emitter and base, holes travel from the emitter to the base.
- When V_c is sufficiently large, most of the holes continue into the collector.

42.34 A common-emitter circuit.



- When $V_b = 0$, I_c is very small, and most of the voltage V_c appears across the base–collector junction.
- As V_b increases, the base–collector potential decreases, and more holes can diffuse into the collector; thus, I_c increases. Ordinarily, I_c is much larger than I_b .

42.35 A field-effect transistor. The current from source to drain is controlled by the potential difference between the source and the drain and by the charge on the gate; no current flows through the gate.

42.36 An integrated circuit chip the size of your thumb can contain millions of transistors.



The first semiconductor devices were invented in 1947. Since then, they have completely revolutionized the electronics industry through miniaturization, reliability, speed, energy usage, and cost. They have found applications in communications, computer systems, control systems, and many other areas. In transforming these areas, they have changed, and continue to change, human civilization itself.

Test Your Understanding of Section 42.7 Suppose a negative charge is placed on the gate of the MOSFET shown in Fig. 42.35. Will a substantial current flow between the source and the drain?

42.8 Superconductivity

Superconductivity is the complete disappearance of all electrical resistance at low temperatures. We described this property at the end of Section 25.2 and the magnetic properties of type-I and type-II superconductors in Section 29.8. In this section we'll relate superconductivity to the structure and energy-band model of a solid.

Although superconductivity was discovered in 1911, it was not well understood on a theoretical basis until 1957. In that year, the American physicists John Bardeen, Leon Cooper, and Robert Schrieffer published the theory of superconductivity, now called the BCS theory, that was to earn them the Nobel Prize in physics in 1972. (It was Bardeen's second Nobel Prize; he shared his first for his work on the development of the transistor.) The key to the BCS theory is an interaction between *pairs* of conduction electrons, called *Cooper pairs*, caused by an interaction with the positive ions of the crystal. Here's a rough qualitative picture of what happens. A free electron exerts attractive forces on nearby positive ions, pulling them slightly closer together. The resulting slight concentration of positive charge then exerts an attractive force on another free electron with momentum opposite to the first. At ordinary temperatures this electron-pair interaction is very small in comparison to energies of thermal motion, but at very low temperatures it becomes significant.

Bound together this way, the pairs of electrons cannot *individually* gain or lose very small amounts of energy, as they would ordinarily be able to do in a partly filled conduction band. Their pairing gives an energy gap in the allowed electron quantum levels, and at low temperatures there is not enough collision energy to jump this gap. Therefore the electrons can move freely through the crystal without any energy exchange through collisions—that is, with zero resistance.

Researchers have not yet reached a consensus on whether some modification of the BCS theory can explain the properties of the high- T_C superconductors that have been discovered since 1986. There *is* evidence for pairing, but of a different sort than for conventional superconductors. Furthermore, the original pairing mechanism of the BCS theory seems too weak to explain the high transition temperatures and critical fields of these new superconductors.

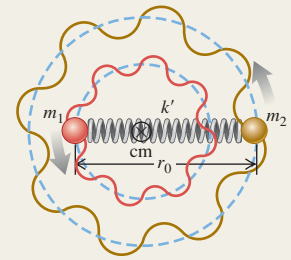
Molecular bonds and molecular spectra: The principal types of molecular bonds are ionic, covalent, van der Waals, and hydrogen bonds. In a diatomic molecule the rotational energy levels are given by Eq. (42.3), where I is the moment of inertia of the molecule, m_r is its reduced mass, and r_0 is the distance between the two atoms. The vibrational energy levels are given by Eq. (42.7), where k' is the effective force constant of the interatomic force. (See Examples 42.1–42.3.)

$$E_l = l(l + 1) \frac{\hbar^2}{2I} \quad (l = 0, 1, 2, \dots) \quad (42.3)$$

$$I = m_r r_0^2 \quad (42.6)$$

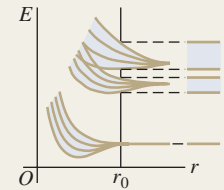
$$m_r = \frac{m_1 m_2}{m_1 + m_2} \quad (42.4)$$

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega = \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{k'}{m_r}} \quad (n = 0, 1, 2, \dots) \quad (42.7)$$



Solids and energy bands: Interatomic bonds in solids are of the same types as in molecules plus one additional type, the metallic bond. Associating the basis with each lattice point gives the crystal structure. (See Example 42.4.)

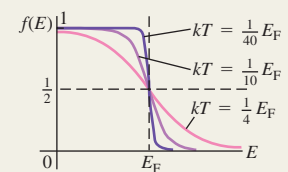
When atoms are bound together in condensed matter, their outer energy levels spread out into bands. At absolute zero, insulators and conductors have a completely filled valence band separated by an energy gap from an empty conduction band. Conductors, including metals, have partially filled conduction bands. (See Example 42.5.)



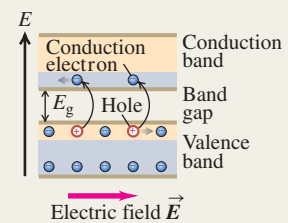
Free-electron model of metals: In the free-electron model of the behavior of conductors, the electrons are treated as completely free particles within the conductor. In this model the density of states is given by Eq. (42.15). The probability that an energy state of energy E is occupied is given by the Fermi–Dirac distribution, Eq. (42.16), which is a consequence of the exclusion principle. In Eq. (42.16), E_F is the Fermi energy. (See Examples 42.6–42.8.)

$$g(E) = \frac{(2m)^{3/2} V}{2\pi^2 \hbar^3} E^{1/2} \quad (42.15)$$

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad (42.16)$$

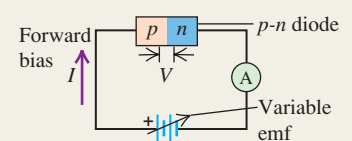


Semiconductors: A semiconductor has an energy gap of about 1 eV between its valence and conduction bands. Its electrical properties may be drastically changed by the addition of small concentrations of donor impurities, giving an n -type semiconductor, or acceptor impurities, giving a p -type semiconductor. (See Example 42.9.)



Semiconductor devices: Many semiconductor devices, including diodes, transistors, and integrated circuits, use one or more p - n junctions. The current–voltage relationship for an ideal p - n junction diode is given by Eq. (42.22).

$$I = I_S (e^{eV/kT} - 1) \quad (42.22)$$



BRIDGING PROBLEM

Detecting Infrared Photons

At 80 K, the band gap in the semiconductor indium antimonide (InSb) is 0.230 eV. A photon emitted by a hydrogen fluoride (HF) molecule undergoing a vibration-rotation transition from $(n = 1, l = 0)$ to $(n = 0, l = 1)$ is absorbed by an electron at the top of the valence band of InSb. (a) How far above the top of the band gap (in eV) is the final state of the electron? (b) What is the probability that the final state was already occupied? The vibration frequency for HF is 1.24×10^{14} Hz, the mass of a hydrogen atom is 1.67×10^{-27} kg, the mass of a fluorine atom is 3.15×10^{-26} kg, and the equilibrium distance between the two nuclei is 0.092 nm. Assume that the Fermi energy for InSb is in the middle of the gap.

SOLUTION GUIDE

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IDENTIFY and SET UP

1. This problem involves what you learned about molecular transitions in Section 42.2, about the Fermi–Dirac distribution in Section 42.5, and about semiconductors in Section 42.6.
2. Equation (42.9) gives the combined vibrational-rotational energy in the initial and final molecular states. The difference between the initial and final molecular energies equals the energy E of the emitted photon, which is in turn equal to the energy gained by the InSb valence electron when it absorbs that

photon. The probability that the final state is occupied is given by the Fermi–Dirac distribution, Eq. (42.16).

EXECUTE

3. Before you can use Eq. (42.9), you'll first need to use the data given to calculate the moment of inertia I and the quantity $\hbar\omega$ for the HF molecule. (*Hint:* Be careful not to confuse frequency f and angular frequency ω .)
4. Use your results from step 3 to calculate the initial and final energies of the HF molecule. (*Hint:* Does the vibrational energy increase or decrease? What about the rotational energy?)
5. Use your result from step 4 to find the energy imparted to the InSb electron. Determine the final energy of this electron relative to the bottom of the conduction band.
6. Use your result from step 5 to determine the probability that the InSb final state is already occupied.

EVALUATE

7. Is the molecular transition of the HF molecule allowed? Which is larger: the vibrational energy change or the rotational energy change?
8. Is it likely that the excited InSb electron will be blocked from entering a state in the conduction band?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

- Q42.1** Ionic bonds result from the electrical attraction of oppositely charged particles. Are other types of molecular bonds also electrical in nature, or is some other interaction involved? Explain.
- Q42.2** In ionic bonds, an electron is transferred from one atom to another and thus no longer “belongs” to the atom from which it came. Are there similar transfers of ownership of electrons with other types of molecular bonds? Explain.
- Q42.3** Van der Waals bonds occur in many molecules, but hydrogen bonds occur only with materials that contain hydrogen. Why is this type of bond unique to hydrogen?
- Q42.4** The bonding of gallium arsenide (GaAs) is said to be 31% ionic and 69% covalent. Explain.
- Q42.5** The H_2^+ molecule consists of two hydrogen nuclei and a single electron. What kind of molecular bond do you think holds this molecule together? Explain.
- Q42.6** The moment of inertia for an axis through the center of mass of a diatomic molecule calculated from the wavelength emitted in an $l = 19 \rightarrow l = 18$ transition is different from the moment of inertia calculated from the wavelength of the photon emitted in an $l = 1 \rightarrow l = 0$ transition. Explain this difference. Which transition corresponds to the larger moment of inertia?
- Q42.7** Analysis of the photon absorption spectrum of a diatomic molecule shows that the vibrational energy levels for small values

of n are very nearly equally spaced but the levels for large n are not equally spaced. Discuss the reason for this observation. Do you expect the adjacent levels to move closer together or farther apart as n increases? Explain.

Q42.8 Discuss the differences between the rotational and vibrational energy levels of the deuterium (“heavy hydrogen”) molecule D_2 and those of the ordinary hydrogen molecule H_2 . A deuterium atom has twice the mass of an ordinary hydrogen atom.

Q42.9 Various organic molecules have been discovered in interstellar space. Why were these discoveries made with radio telescopes rather than optical telescopes?

Q42.10 The air you are breathing contains primarily nitrogen (N_2) and oxygen (O_2). Many of these molecules are in excited rotational energy levels ($l = 1, 2, 3, \dots$), but almost all of them are in the vibrational ground level ($n = 0$). Explain this difference between the rotational and vibrational behaviors of the molecules.

Q42.11 In what ways do atoms in a diatomic molecule behave as though they were held together by a spring? In what ways is this a poor description of the interaction between the atoms?

Q42.12 Individual atoms have discrete energy levels, but certain solids (which are made up of only individual atoms) show energy bands and gaps. What causes the solids to behave so differently from the atoms of which they are composed?

Q42.13 What factors determine whether a material is a conductor of electricity or an insulator? Explain.

Q42.14 Ionic crystals are often transparent, whereas metallic crystals are always opaque. Why?

Q42.15 Speeds of molecules in a gas vary with temperature, whereas speeds of electrons in the conduction band of a metal are nearly independent of temperature. Why are these behaviors so different?

Q42.16 Use the band model to explain how it is possible for some materials to undergo a semiconductor-to-metal transition as the temperature or pressure varies.

Q42.17 An isolated zinc atom has a ground-state electron configuration of filled $1s$, $2s$, $2p$, $3s$, $3p$, and $4s$ subshells. How can zinc be a conductor if its valence subshell is full?

Q42.18 The assumptions of the *free-electron model* of metals may seem contrary to reason, since electrons exert powerful electrical forces on each other. Give some reasons why these assumptions actually make physical sense.

Q42.19 Why are materials that are good thermal conductors also good electrical conductors? What kinds of problems does this pose for the design of appliances such as clothes irons and electric heaters? Are there materials that do not follow this general rule?

Q42.20 What is the essential characteristic for an element to serve as a donor impurity in a semiconductor such as Si or Ge? For it to serve as an acceptor impurity? Explain.

Q42.21 There are several methods for removing electrons from the surface of a semiconductor. Can holes be removed from the surface? Explain.

Q42.22 A student asserts that silicon and germanium become good insulators at very low temperatures and good conductors at very high temperatures. Do you agree? Explain your reasoning.

Q42.23 The electrical conductivities of most metals decrease gradually with increasing temperature, but the intrinsic conductivity of semiconductors always *increases* rapidly with increasing temperature. What causes the difference?

Q42.24 How could you make compensated silicon that has twice as many acceptors as donors?

Q42.25 For electronic devices such as amplifiers, what are some advantages of transistors compared to vacuum tubes? What are some disadvantages? Are there any situations in which vacuum tubes *cannot* be replaced by solid-state devices? Explain your reasoning.

Q42.26 Why does tunneling limit the miniaturization of MOSFETs?

Q42.27 The saturation current I_S for a p - n junction, Eq. (42.22), depends strongly on temperature. Explain why.

EXERCISES

Section 42.1 Types of Molecular Bonds

42.1 • If the energy of the H_2 covalent bond is -4.48 eV, what wavelength of light is needed to break that molecule apart? In what part of the electromagnetic spectrum does this light lie?

42.2 • **An Ionic Bond.** (a) Calculate the electric potential energy for a K^+ ion and a Br^- ion separated by a distance of 0.29 nm, the equilibrium separation in the KBr molecule. Treat the ions as point charges. (b) The ionization energy of the potassium atom is 4.3 eV. Atomic bromine has an electron affinity of 3.5 eV. Use these data and the results of part (a) to estimate the binding energy of the KBr molecule. Do you expect the actual binding energy to be higher or lower than your estimate? Explain your reasoning.

42.3 • We know from Chapter 18 that the average kinetic energy of an ideal-gas atom or molecule at Kelvin temperature T is $\frac{3}{2}kT$. For what value of T does this energy correspond to (a) the bond energy of the van der Waals bond in He_2 (7.9×10^{-4} eV) and (b) the bond energy of the covalent bond in H_2 (4.48 eV)? (c) The kinetic energy in a collision between molecules can go into dissociating one or both molecules, provided the kinetic energy is higher than the bond energy. At room temperature (300 K), is it likely that He_2 molecules will remain intact after a collision? What about H_2 molecules? Explain.

42.4 •• Light of wavelength 3.10 nm strikes and is absorbed by a molecule. Is this process most likely to alter the rotational, vibrational, or atomic energy levels of the molecule? Explain your reasoning. (b) If the light in part (a) had a wavelength of 207 nm, which energy levels would it most likely affect? Explain.

42.5 • For the H_2 molecule the equilibrium spacing of the two protons is 0.074 nm. The mass of a hydrogen atom is 1.67×10^{-27} kg. Calculate the wavelength of the photon emitted in the rotational transition $l = 2$ to $l = 1$.

42.6 • (a) A molecule decreases its vibrational energy by 0.250 eV by giving up a photon of light. What wavelength of light does it give up during this process, and in what part of the electromagnetic spectrum does that wavelength of light lie? (b) An atom decreases its energy by 8.50 eV by giving up a photon of light. What wavelength of light does it give up during this process, and in what part of the electromagnetic spectrum does that wavelength of light lie? (c) A molecule decreases its rotational energy by 3.20×10^{-3} eV by giving up a photon of light. What wavelength of light does it give up during this process, and in what part of the electromagnetic spectrum does that wavelength of light lie?

Section 42.2 Molecular Spectra

42.7 • A hypothetical NH molecule makes a rotational-level transition from $l = 3$ to $l = 1$ and gives off a photon of wavelength 1.780 nm in doing so. What is the separation between the two atoms in this molecule if we model them as point masses? The mass of hydrogen is 1.67×10^{-27} kg, and the mass of nitrogen is 2.33×10^{-26} kg.

42.8 • The water molecule has an $l = 1$ rotational level 1.01×10^{-5} eV above the $l = 0$ ground level. Calculate the wavelength and frequency of the photon absorbed by water when it undergoes a rotational-level transition from $l = 0$ to $l = 1$. The magnetron oscillator in a microwave oven generates microwaves with a frequency of 2450 MHz. Does this make sense, in view of the frequency you calculated in this problem? Explain.

42.9 • In Example 42.2 the moment of inertia for CO was calculated using Eq. (42.6). (a) In CO, how far is each atom from the center of mass of the molecule? (b) Use $I = m_1 r_1^2 + m_2 r_2^2$ to calculate the moment of inertia of CO about an axis through the center of mass and perpendicular to the line joining the centers of the two atoms. Does your result agree with the value obtained in Example 42.2?

42.10 • Two atoms of cesium (Cs) can form a Cs_2 molecule. The equilibrium distance between the nuclei in a Cs_2 molecule is 0.447 nm. Calculate the moment of inertia about an axis through the center of mass of the two nuclei and perpendicular to the line joining them. The mass of a cesium atom is 2.21×10^{-25} kg.

42.11 •• **CP** The rotational energy levels of CO are calculated in Example 42.2. If the energy of the rotating molecule is described by the classical expression $K = \frac{1}{2}I\omega^2$, for the $l = 1$ level what are (a) the angular speed of the rotating molecule; (b) the linear speed

of each atom (use the result of Exercise 42.9); (c) the rotational period (the time for one rotation)?

42.12 • If a sodium chloride (NaCl) molecule could undergo an $n \rightarrow n - 1$ vibrational transition with no change in rotational quantum number, a photon with wavelength $20.0 \mu\text{m}$ would be emitted. The mass of a sodium atom is $3.82 \times 10^{-26} \text{ kg}$, and the mass of a chlorine atom is $5.81 \times 10^{-26} \text{ kg}$. Calculate the force constant k' for the interatomic force in NaCl.

42.13 • A lithium atom has mass $1.17 \times 10^{-26} \text{ kg}$, and a hydrogen atom has mass $1.67 \times 10^{-27} \text{ kg}$. The equilibrium separation between the two nuclei in the LiH molecule is 0.159 nm . (a) What is the difference in energy between the $l = 3$ and $l = 4$ rotational levels? (b) What is the wavelength of the photon emitted in a transition from the $l = 4$ to the $l = 3$ level?

42.14 •• When a hypothetical diatomic molecule having atoms 0.8860 nm apart undergoes a rotational transition from the $l = 2$ state to the next lower state, it gives up a photon having energy $8.841 \times 10^{-4} \text{ eV}$. When the molecule undergoes a vibrational transition from one energy state to the next lower energy state, it gives up 0.2560 eV . Find the force constant of this molecule.

42.15 • (a) Show that the energy difference between rotational levels with angular-momentum quantum numbers l and $l - 1$ is $l\hbar^2/I$. (b) In terms of l , \hbar , and I , what is the frequency of the photon emitted in the pure rotation transition $l \rightarrow l - 1$?

42.16 • The vibrational and rotational energies of the CO molecule are given by Eq. (42.9). Calculate the wavelength of the photon absorbed by CO in each of the following vibration-rotation transitions: (a) $n = 0, l = 1 \rightarrow n = 1, l = 2$; (b) $n = 0, l = 2 \rightarrow n = 1, l = 1$; (c) $n = 0, l = 3 \rightarrow n = 1, l = 2$.

Section 42.3 Structure of Solids

42.17 • **Density of NaCl.** The spacing of adjacent atoms in a crystal of sodium chloride is 0.282 nm . The mass of a sodium atom is $3.82 \times 10^{-26} \text{ kg}$, and the mass of a chlorine atom is $5.89 \times 10^{-26} \text{ kg}$. Calculate the density of sodium chloride.

42.18 • Potassium bromide (KBr) has a density of $2.75 \times 10^3 \text{ kg/m}^3$ and the same crystal structure as NaCl. The mass of a potassium atom is $6.49 \times 10^{-26} \text{ kg}$, and the mass of a bromine atom is $1.33 \times 10^{-25} \text{ kg}$. (a) Calculate the average spacing between adjacent atoms in a KBr crystal. (b) How does the value calculated in part (a) compare with the spacing in NaCl (see Exercise 42.17)? Is the relationship between the two values qualitatively what you would expect? Explain.

Section 42.4 Energy Bands

42.19 • The maximum wavelength of light that a certain silicon photocell can detect is $1.11 \mu\text{m}$. (a) What is the energy gap (in electron volts) between the valence and conduction bands for this photocell? (b) Explain why pure silicon is opaque.

42.20 • The gap between valence and conduction bands in diamond is 5.47 eV . (a) What is the maximum wavelength of a photon that can excite an electron from the top of the valence band into the conduction band? In what region of the electromagnetic spectrum does this photon lie? (b) Explain why pure diamond is transparent and colorless. (c) Most gem diamonds have a yellow color. Explain how impurities in the diamond can cause this color.

42.21 • The gap between valence and conduction bands in silicon is 1.12 eV . A nickel nucleus in an excited state emits a gamma-ray photon with wavelength $9.31 \times 10^{-4} \text{ nm}$. How many electrons can be excited from the top of the valence band to the bottom of the conduction band by the absorption of this gamma ray?

Section 42.5 Free-Electron Model of Metals

42.22 • Calculate v_{rms} for free electrons with average kinetic energy $\frac{3}{2}kT$ at a temperature of 300 K . How does your result compare to the speed of an electron with a kinetic energy equal to the Fermi energy of copper, calculated in Example 42.7? Why is there such a difference between these speeds?

42.23 • Calculate the density of states $g(E)$ for the free-electron model of a metal if $E = 7.0 \text{ eV}$ and $V = 1.0 \text{ cm}^3$. Express your answer in units of states per electron volt.

42.24 • Supply the details in the derivation of Eq. (42.13) from Eqs. (42.11) and (42.12).

42.25 • **CP** Silver has a Fermi energy of 5.48 eV . Calculate the electron contribution to the molar heat capacity at constant volume of silver, C_V , at 300 K . Express your result (a) as a multiple of R and (b) as a fraction of the actual value for silver, $C_V = 25.3 \text{ J/mol} \cdot \text{K}$. (c) Is the value of C_V due principally to the electrons? If not, to what is it due? (*Hint:* See Section 18.4.)

42.26 • The Fermi energy of sodium is 3.23 eV . (a) Find the average energy E_{av} of the electrons at absolute zero. (b) What is the speed of an electron that has energy E_{av} ? (c) At what Kelvin temperature T is kT equal to E_F ? (This is called the *Fermi temperature* for the metal. It is approximately the temperature at which molecules in a classical ideal gas would have the same kinetic energy as the fastest-moving electron in the metal.)

42.27 •• For a solid metal having a Fermi energy of 8.500 eV , what is the probability, at room temperature, that a state having an energy of 8.520 eV is occupied by an electron?

Section 42.6 Semiconductors

42.28 • Pure germanium has a band gap of 0.67 eV . The Fermi energy is in the middle of the gap. (a) For temperatures of 250 K , 300 K , and 350 K , calculate the probability $f(E)$ that a state at the bottom of the conduction band is occupied. (b) For each temperature in part (a), calculate the probability that a state at the top of the valence band is empty.

42.29 • Germanium has a band gap of 0.67 eV . Doping with arsenic adds donor levels in the gap 0.01 eV below the bottom of the conduction band. At a temperature of 300 K , the probability is 4.4×10^{-4} that an electron state is occupied at the bottom of the conduction band. Where is the Fermi level relative to the conduction band in this case?

Section 42.7 Semiconductor Devices

42.30 •• (a) Suppose a piece of very pure germanium is to be used as a light detector by observing, through the absorption of photons, the increase in conductivity resulting from generation of electron-hole pairs. If each pair requires 0.67 eV of energy, what is the maximum wavelength that can be detected? In what portion of the spectrum does it lie? (b) What are the answers to part (a) if the material is silicon, with an energy requirement of 1.14 eV per pair, corresponding to the gap between valence and conduction bands in that element?

42.31 • **CP** At a temperature of 290 K , a certain p - n junction has a saturation current $I_S = 0.500 \text{ mA}$. (a) Find the current at this temperature when the voltage is (i) 1.00 mV , (ii) -1.00 mV , (iii) 100 mV , and (iv) -100 mV . (b) Is there a region of applied voltage where the diode obeys Ohm's law?

42.32 • For a certain p - n junction diode, the saturation current at room temperature (20°C) is 0.750 mA . What is the resistance of this diode when the voltage across it is (a) 85.0 mV and (b) -50.0 mV ?

42.33 •• (a) A forward-bias voltage of 15.0 mV produces a positive current of 9.25 mA through a p - n junction at 300 K. What does the positive current become if the forward-bias voltage is reduced to 10.0 mV? (b) For reverse-bias voltages of -15.0 mV and -10.0 mV, what is the reverse-bias negative current?

42.34 •• A p - n junction has a saturation current of 3.60 mA. (a) At a temperature of 300 K, what voltage is needed to produce a positive current of 40.0 mA? (b) For a voltage equal to the negative of the value calculated in part (a), what is the negative current?

PROBLEMS

42.35 •• A hypothetical diatomic molecule of oxygen (mass = 2.656×10^{-26} kg) and hydrogen (mass = 1.67×10^{-27} kg) emits a photon of wavelength $2.39 \mu\text{m}$ when it makes a transition from one vibrational state to the next lower state. If we model this molecule as two point masses at opposite ends of a massless spring, (a) what is the force constant of this spring, and (b) how many vibrations per second is the molecule making?

42.36 • When a diatomic molecule undergoes a transition from the $l = 2$ to the $l = 1$ rotational state, a photon with wavelength $63.8 \mu\text{m}$ is emitted. What is the moment of inertia of the molecule for an axis through its center of mass and perpendicular to the line connecting the nuclei?

42.37 •• **CP** (a) The equilibrium separation of the two nuclei in an NaCl molecule is 0.24 nm. If the molecule is modeled as charges $+e$ and $-e$ separated by 0.24 nm, what is the electric dipole moment of the molecule (see Section 21.7)? (b) The measured electric dipole moment of an NaCl molecule is 3.0×10^{-29} C·m. If this dipole moment arises from point charges $+q$ and $-q$ separated by 0.24 nm, what is q ? (c) A definition of the *fractional ionic character* of the bond is q/e . If the sodium atom has charge $+e$ and the chlorine atom has charge $-e$, the fractional ionic character would be equal to 1. What is the actual fractional ionic character for the bond in NaCl? (d) The equilibrium distance between nuclei in the hydrogen iodide (HI) molecule is 0.16 nm, and the measured electric dipole moment of the molecule is 1.5×10^{-30} C·m. What is the fractional ionic character for the bond in HI? How does your answer compare to that for NaCl calculated in part (c)? Discuss reasons for the difference in these results.

42.38 • The binding energy of a potassium chloride molecule (KCl) is 4.43 eV. The ionization energy of a potassium atom is 4.3 eV, and the electron affinity of chlorine is 3.6 eV. Use these data to estimate the equilibrium separation between the two atoms in the KCl molecule. Explain why your result is only an estimate and not a precise value.

42.39 • (a) For the sodium chloride molecule (NaCl) discussed at the beginning of Section 42.1, what is the maximum separation of the ions for stability if they may be regarded as point charges? That is, what is the largest separation for which the energy of an Na^+ ion and a Cl^- ion, calculated in this model, is lower than the energy of the two separate atoms Na and Cl? (b) Calculate this distance for the potassium bromide molecule, described in Exercise 42.2.

42.40 •• The rotational spectrum of HCl contains the following wavelengths (among others): $60.4 \mu\text{m}$, $69.0 \mu\text{m}$, $80.4 \mu\text{m}$, $96.4 \mu\text{m}$, and $120.4 \mu\text{m}$. Use this spectrum to find the moment of inertia of the HCl molecule about an axis through the center of mass and perpendicular to the line joining the two nuclei.

42.41 • (a) Use the result of Problem 42.40 to calculate the equilibrium separation of the atoms in an HCl molecule. The mass of a chlorine atom is 5.81×10^{-26} kg, and the mass of a hydrogen

atom is 1.67×10^{-27} kg. (b) The value of l changes by ± 1 in rotational transitions. What is the value of l for the upper level of the transition that gives rise to each of the wavelengths listed in Problem 42.40? (c) What is the longest-wavelength line in the rotational spectrum of HCl? (d) Calculate the wavelengths of the emitted light for the corresponding transitions in the deuterium chloride (DCl) molecule. In this molecule the hydrogen atom in HCl is replaced by an atom of deuterium, an isotope of hydrogen with a mass of 3.34×10^{-27} kg. Assume that the equilibrium separation between the atoms is the same as for HCl.

42.42 • When a NaF molecule makes a transition from the $l = 3$ to the $l = 2$ rotational level with no change in vibrational quantum number or electronic state, a photon with wavelength 3.83 mm is emitted. A sodium atom has mass 3.82×10^{-26} kg, and a fluorine atom has mass 3.15×10^{-26} kg. Calculate the equilibrium separation between the nuclei in a NaF molecule. How does your answer compare with the value for NaCl given in Section 42.1? Is this result reasonable? Explain.

42.43 •• **CP** Consider a gas of diatomic molecules (moment of inertia I) at an absolute temperature T . If E_g is a ground-state energy and E_{ex} is the energy of an excited state, then the Maxwell-Boltzmann distribution (see Section 39.4) predicts that the ratio of the numbers of molecules in the two states is

$$\frac{n_{\text{ex}}}{n_g} = e^{-(E_{\text{ex}} - E_g)/kT}$$

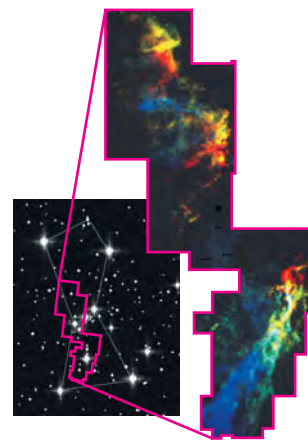
(a) Explain why the ratio of the number of molecules in the l th rotational energy level to the number of molecules in the ground ($l = 0$) rotational level is

$$\frac{n_l}{n_0} = (2l + 1)e^{-[l(l+1)\hbar^2]/2lIkT}$$

(Hint: For each value of l , how many states are there with different values of m_l ?) (b) Determine the ratio n_l/n_0 for a gas of CO molecules at 300 K for the cases (i) $l = 1$; (ii) $l = 2$; (iii) $l = 10$; (iv) $l = 20$; (v) $l = 50$. The moment of inertia of the CO molecule is given in Example 42.2 (Section 42.2). (c) Your results in part (b) show that as l is increased, the ratio n_l/n_0 first increases and then decreases. Explain why.

42.44 •• Our galaxy contains numerous *molecular clouds*, regions

Figure P42.44



many light-years in extent in which the density is high enough and the temperature low enough for atoms to form into molecules. Most of the molecules are H_2 , but a small fraction of the molecules are carbon monoxide (CO). Such a molecular cloud in the constellation Orion is shown in Fig. P42.44. The left-hand image was made with an ordinary visible-light telescope; the right-hand image shows the molecular cloud in Orion as imaged with a radio telescope tuned to a wavelength emitted by CO in a rotational transition. The different colors in the radio image indicate regions of the cloud that are moving either toward us (blue) or away from us (red) relative to the motion of the cloud as a whole, as determined by the Doppler shift of the radiation. (Since a

molecular cloud has about 10,000 hydrogen molecules for each CO molecule, it might seem more reasonable to tune a radio telescope to emissions from H_2 than to emissions from CO. Unfortunately, it turns out that the H_2 molecules in molecular clouds do not radiate in either the radio or visible portions of the electromagnetic spectrum.) (a) Using the data in Example 42.2 (Section 42.2), calculate the energy and wavelength of the photon emitted by a CO molecule in an $l = 1 \rightarrow l = 0$ rotational transition. (b) As a rule, molecules in a gas at temperature T will be found in a certain excited rotational energy level provided the energy of that level is no higher than kT (see Problem 42.43). Use this rule to explain why astronomers can detect radiation from CO in molecular clouds even though the typical temperature of a molecular cloud is a very low 20 K.

42.45 • Spectral Lines from Isotopes. The equilibrium separation for NaCl is 0.2361 nm. The mass of a sodium atom is 3.8176×10^{-26} kg. Chlorine has two stable isotopes, ^{35}Cl and ^{37}Cl , that have different masses but identical chemical properties. The atomic mass of ^{35}Cl is 5.8068×10^{-26} kg, and the atomic mass of ^{37}Cl is 6.1384×10^{-26} kg. (a) Calculate the wavelength of the photon emitted in the $l = 2 \rightarrow l = 1$ and $l = 1 \rightarrow l = 0$ transitions for Na^{35}Cl . (b) Repeat part (a) for Na^{37}Cl . What are the differences in the wavelengths for the two isotopes?

42.46 • When an OH molecule undergoes a transition from the $n = 0$ to the $n = 1$ vibrational level, its internal vibrational energy increases by 0.463 eV. Calculate the frequency of vibration and the force constant for the interatomic force. (The mass of an oxygen atom is 2.66×10^{-26} kg, and the mass of a hydrogen atom is 1.67×10^{-27} kg.)

42.47 • The force constant for the internuclear force in a hydrogen molecule (H_2) is $k' = 576$ N/m. A hydrogen atom has mass 1.67×10^{-27} kg. Calculate the zero-point vibrational energy for H_2 (that is, the vibrational energy the molecule has in the $n = 0$ ground vibrational level). How does this energy compare in magnitude with the H_2 bond energy of -4.48 eV?

42.48 • Suppose the hydrogen atom in HF (see the Bridging Problem for this chapter) is replaced by an atom of deuterium, an isotope of hydrogen with a mass of 3.34×10^{-27} kg. The force constant is determined by the electron configuration, so it is the same as for the normal HF molecule. (a) What is the vibrational frequency of this molecule? (b) What wavelength of light corresponds to the energy difference between the $n = 1$ and $n = 0$ levels? In what region of the spectrum does this wavelength lie?

42.49 • The hydrogen iodide (HI) molecule has equilibrium separation 0.160 nm and vibrational frequency 6.93×10^{13} Hz. The mass of a hydrogen atom is 1.67×10^{-27} kg, and the mass of an iodine atom is 2.11×10^{-25} kg. (a) Calculate the moment of inertia of HI about a perpendicular axis through its center of mass. (b) Calculate the wavelength of the photon emitted in each of the following vibration-rotation transitions: (i) $n = 1, l = 1 \rightarrow n = 0, l = 0$; (ii) $n = 1, l = 2 \rightarrow n = 0, l = 1$; (iii) $n = 2, l = 2 \rightarrow n = 1, l = 3$.

42.50 •• Prove this statement: For free electrons in a solid, if a state that is at an energy ΔE above E_F has probability P of being occupied, then the probability is $1 - P$ that a state at an energy ΔE below E_F is occupied.

42.51 •• Compute the Fermi energy of potassium by making the simple approximation that each atom contributes one free electron. The density of potassium is 851 kg/m³, and the mass of a single potassium atom is 6.49×10^{-26} kg.

42.52 •• Hydrogen is found in two naturally occurring isotopes; normal hydrogen (containing a single proton in its nucleus) and deu-

terium (having a proton and a neutron). Assuming that both molecules are the same size and that the proton and neutron have the same mass (which is almost the case), find the ratio of (a) the energy of any given rotational state in a diatomic hydrogen molecule to the energy of the same state in a diatomic deuterium molecule and (b) the energy of any given vibrational state in hydrogen to the same state in deuterium (assuming that the force constant is the same for both molecules). Why is it physically reasonable that the force constant would be the same for hydrogen and deuterium molecules?

42.53 ••• Metallic lithium has a bcc crystal structure. Each unit cell is a cube of side length $a = 0.35$ nm. (a) For a bcc lattice, what is the number of atoms per unit volume? Give your answer in terms of a . (*Hint:* How many atoms are there per unit cell?) (b) Use the result of part (a) to calculate the zero-temperature Fermi energy E_{F0} for metallic lithium. Assume there is one free electron per atom.

42.54 •• CALC The one-dimensional calculation of Example 42.4 (Section 42.3) can be extended to three dimensions. For the three-dimensional fcc NaCl lattice, the result for the potential energy of a pair of Na^+ and Cl^- ions due to the electrostatic interaction with all of the ions in the crystal is $U = -\alpha e^2 / 4\pi\epsilon_0 r$, where $\alpha = 1.75$ is the *Madelung constant*. Another contribution to the potential energy is a repulsive interaction at small ionic separation r due to overlap of the electron clouds. This contribution can be represented by A/r^8 , where A is a positive constant, so the expression for the total potential energy is

$$U_{\text{tot}} = -\frac{\alpha e^2}{4\pi\epsilon_0 r} + \frac{A}{r^8}$$

(a) Let r_0 be the value of the ionic separation r for which U_{tot} is a minimum. Use this definition to find an equation that relates r_0 and A , and use this to write U_{tot} in terms of r_0 . For NaCl, $r_0 = 0.281$ nm. Obtain a numerical value (in electron volts) of U_{tot} for NaCl. (b) The quantity $-U_{\text{tot}}$ is the energy required to remove a Na^+ ion and a Cl^- ion from the crystal. Forming a pair of neutral atoms from this pair of ions involves the release of 5.14 eV (the ionization energy of Na) and the expenditure of 3.61 eV (the electron affinity of Cl). Use the result of part (a) to calculate the energy required to remove a pair of neutral Na and Cl atoms from the crystal. The experimental value for this quantity is 6.39 eV; how well does your calculation agree?

42.55 •• CALC Consider a system of N free electrons within a volume V . Even at absolute zero, such a system exerts a pressure p on its surroundings due to the motion of the electrons. To calculate this pressure, imagine that the volume increases by a small amount dV . The electrons will do an amount of work $p dV$ on their surroundings, which means that the total energy E_{tot} of the electrons will change by an amount $dE_{\text{tot}} = -p dV$. Hence $p = -dE_{\text{tot}}/dV$. (a) Show that the pressure of the electrons at absolute zero is

$$p = \frac{3^{2/3} \pi^{4/3} \hbar^2}{5m} \left(\frac{N}{V} \right)^{5/3}$$

(b) Evaluate this pressure for copper, which has a free-electron concentration of 8.45×10^{28} m⁻³. Express your result in pascals and in atmospheres. (c) The pressure you found in part (b) is extremely high. Why, then, don't the electrons in a piece of copper simply explode out of the metal?

42.56 •• CALC When the pressure p on a material increases by an amount Δp , the volume of the material will change from V to $V + \Delta V$, where ΔV is negative. The *bulk modulus* B of the mate-

rial is defined to be the ratio of the pressure change Δp to the absolute value $|\Delta V/V|$ of the fractional volume change. The greater the bulk modulus, the greater the pressure increase required for a given fractional volume change, and the more incompressible the material (see Section 11.4). Since $\Delta V < 0$, the bulk modulus can be written as $B = -\Delta p/(\Delta V/V_0)$. In the limit that the pressure and volume changes are very small, this becomes

$$B = -V \frac{dp}{dV}$$

(a) Use the result of Problem 42.55 to show that the bulk modulus for a system of N free electrons in a volume V at low temperatures is $B = \frac{5}{3}p$. (Hint: The quantity p in the expression $B = -V(dp/dV)$ is the external pressure on the system. Can you explain why this is equal to the internal pressure of the system itself, as found in Problem 42.55?) (b) Evaluate the bulk modulus for the electrons in copper, which has a free-electron concentration of $8.45 \times 10^{28} \text{ m}^{-3}$. Express your result in pascals. (c) The actual bulk modulus of copper is $1.4 \times 10^{11} \text{ Pa}$. Based on your result in part (b), what fraction of this is due to the free electrons in copper? (This result shows that the free electrons in a metal play a major role in making the metal resistant to compression.) What do you think is responsible for the remaining fraction of the bulk modulus?

42.57 •• In the discussion of free electrons in Section 42.5, we assumed that we could ignore the effects of relativity. This is not a safe assumption if the Fermi energy is greater than about $\frac{1}{100}mc^2$ (that is, more than about 1% of the rest energy of an electron). (a) Assume that the Fermi energy at absolute zero, as given by Eq. (42.19), is equal to $\frac{1}{100}mc^2$. Show that the electron concentration is

$$\frac{N}{V} = \frac{2^{3/2}m^3c^3}{3000\pi^2\hbar^3}$$

and determine the numerical value of N/V . (b) Is it a good approximation to ignore relativistic effects for electrons in a metal such as copper, for which the electron concentration is $8.45 \times 10^{28} \text{ m}^{-3}$? Explain. (c) A *white dwarf star* is what is left behind by a star like the sun after it has ceased to produce energy by nuclear reactions. (Our own sun will become a white dwarf star in another 6×10^9 years or so.) A typical white dwarf has mass $2 \times 10^{30} \text{ kg}$ (comparable to the sun) and radius 6000 km (comparable to that of the earth). The gravitational attraction of different parts of the white dwarf for each other tends to compress the star; what prevents it from compressing is the pressure of free electrons within the star (see Problem 42.55). Estimate the electron concentration within a typical white dwarf star using the following assumptions: (i) the white dwarf star is made of carbon, which has a mass per atom of

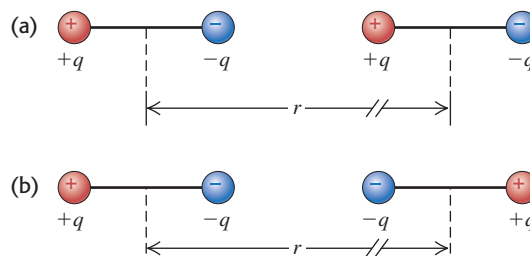
$1.99 \times 10^{-26} \text{ kg}$; and (ii) all six of the electrons from each carbon atom are able to move freely throughout the star. (d) Is it a good approximation to ignore relativistic effects in the structure of a white dwarf star? Explain.

42.58 •• CP A variable DC battery is connected in series with a $125\text{-}\Omega$ resistor and a p - n junction diode that has a saturation current of 0.625 mA at room temperature (20°C). When a voltmeter across the $125\text{-}\Omega$ resistor reads 35.0 V , what are (a) the voltage across the diode and (b) the resistance of the diode?

CHALLENGE PROBLEMS

42.59 ••• CP Van der Waals bonds arise from the interaction between two permanent or induced electric dipole moments in a pair of atoms or molecules. (a) Consider two identical dipoles, each consisting of charges $+q$ and $-q$ separated by a distance d and oriented as shown in Fig. P42.59a. Calculate the electric potential energy, expressed in terms of the electric dipole moment $p = qd$, for the situation where $r \gg d$. Is the interaction attractive or repulsive, and how does this potential energy vary with r , the separation between the centers of the two dipoles? (b) Repeat part (a) for the orientation of the dipoles shown in Fig. P42.59b. The dipole interaction is more complicated when we have to average over the relative orientations of the two dipoles due to thermal motion or when the dipoles are induced rather than permanent.

Figure P42.59



42.60 ••• CP CALC (a) Consider the hydrogen molecule (H_2) to be a simple harmonic oscillator with an equilibrium spacing of 0.074 nm , and estimate the vibrational energy-level spacing for H_2 . The mass of a hydrogen atom is $1.67 \times 10^{-27} \text{ kg}$. (Hint: Estimate the force constant by equating the change in Coulomb repulsion of the protons, when the atoms move slightly closer together than r_0 , to the “spring” force. That is, assume that the chemical binding force remains approximately constant as r is decreased slightly from r_0 .) (b) Use the results of part (a) to calculate the vibrational energy-level spacing for the deuterium molecule, D_2 . Assume that the spring constant is the same for D_2 as for H_2 . The mass of a deuterium atom is $3.34 \times 10^{-27} \text{ kg}$.

Answers

Chapter Opening Question ?

Venus must radiate energy into space at the same rate that it receives energy in the form of sunlight. However, carbon dioxide (CO_2) molecules in the atmosphere absorb infrared radiation

emitted by the surface of Venus and re-emit it toward the ground. To compensate for this and to maintain the balance between emitted and received energy, the surface temperature of Venus and hence the rate of blackbody radiation from the surface both increase.

Test Your Understanding Questions

42.1 Answer: (i) The exclusion principle states that only one electron can be in a given state. Real electrons have spin, so two electrons (one spin up, one spin down) can be in a given *spatial* state and hence two can participate in a given covalent bond between two atoms. If electrons obeyed the exclusion principle but did not have spin, that state of an electron would be completely described by its spatial distribution and only *one* electron could participate in a covalent bond. (We will learn in Chapter 44 that this situation is wholly imaginary: There are subatomic particles without spin, but they do *not* obey the exclusion principle.)

42.2 Answer: (ii) Figure 42.5 shows that the difference in energy between adjacent rotational levels increases with increasing l . Hence, as l increases, the energy E of the emitted photon increases and the wavelength $\lambda = hc/E$ decreases.

42.3 Answer: (ii) In Fig. 42.13 let a be the distance between adjacent Na^+ and Cl^- ions. This figure shows that the Cl^- ion that is the next nearest neighbor to a Na^+ ion is on the opposite corner of a cube of side a . The distance between these two ions is $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = a\sqrt{3}$.

42.4 Answer: (ii) A small temperature change causes a substantial increase in the population of electrons in a semiconductor's conduction band and a comparably substantial increase in conductivity. The conductivity of conductors and insulators varies more gradually with temperature.

42.5 Answer: no The kinetic-molecular model of an ideal gas (see Section 18.3) shows that the gas pressure is proportional to the

average translational kinetic energy E_{av} of the particles that make up the gas. In a classical ideal gas, E_{av} is directly proportional to the average temperature T , so the pressure decreases as T decreases. In a free-electron gas, the average kinetic energy per electron is *not* related simply to T ; as Example 42.8 shows, for the free-electron gas in a metal, E_{av} is almost completely a consequence of the exclusion principle at room temperature and colder. Hence the pressure of a free-electron gas in a solid metal does *not* change appreciably between room temperature and absolute zero.

42.6 Answer: no Pure copper is already an excellent conductor since it has a partially filled conduction band (Fig. 42.19c). Furthermore, copper forms a metallic crystal (Fig. 42.15) as opposed to the covalent crystals of silicon or germanium, so the scheme of using an impurity to donate or accept an electron does not work for copper. In fact, adding impurities to copper *decreases* the conductivity because an impurity tends to scatter electrons, impeding the flow of current.

42.7 Answer: no A negative charge on the gate will repel, not attract, electrons in the p -type silicon. Hence the electron concentration in the region between the two p - n junctions will be made even smaller. With so few charge carriers present in this region, very little current will flow between the source and the drain.

Bridging Problem

Answers: (a) 0.278 eV
(b) 1.74×10^{-25}

NUCLEAR PHYSICS



? This sculpture of a woolly mammoth, just 3.7 cm (1.5 in.) in length, was carved from a mammoth's ivory tusk by an artist who lived in southwestern Germany 35,000 years ago. What physical principles make it possible to date biological specimens such as these?

During the past century, applications of nuclear physics have had enormous effects on humankind, some beneficial, some catastrophic. Many people have strong opinions about applications such as bombs and reactors. Ideally, those opinions should be based on understanding, not on prejudice or emotion, and we hope this chapter will help you to reach that ideal.

Every atom contains at its center an extremely dense, positively charged *nucleus*, which is much smaller than the overall size of the atom but contains most of its total mass. We will look at several important general properties of nuclei and of the nuclear force that holds them together. The stability or instability of a particular nucleus is determined by the competition between the attractive nuclear force among the protons and neutrons and the repulsive electrical interactions among the protons. Unstable nuclei *decay*, transforming themselves spontaneously into other nuclei by a variety of processes. Nuclear reactions can also be induced by impact on a nucleus of a particle or another nucleus. Two classes of reactions of special interest are *fission* and *fusion*. We could not survive without the energy released by one nearby fusion reactor, our sun.

43.1 Properties of Nuclei

As we described in Section 39.2, Rutherford found that the nucleus is tens of thousands of times smaller in radius than the atom itself. Since Rutherford's initial experiments, many additional scattering experiments have been performed, using high-energy protons, electrons, and neutrons as well as alpha particles (helium-4 nuclei). These experiments show that we can model a nucleus as a sphere with a radius R that depends on the total number of *nucleons* (neutrons

LEARNING GOALS

By studying this chapter, you will learn:

- Some key properties of atomic nuclei, including radii, densities, spins, and magnetic moments.
- How the binding energy of a nucleus depends on the numbers of protons and neutrons that it contains.
- The most important ways in which unstable nuclei undergo radioactive decay.
- How the decay rate of a radioactive substance depends on time.
- Some of the biological hazards and medical uses of radiation.
- How to analyze some important types of nuclear reactions.
- What happens in a nuclear fission chain reaction, and how it can be controlled.
- The sequence of nuclear reactions that allow the sun and stars to shine.

and protons) in the nucleus. This number is called the **nucleon number** A . The radii of most nuclei are represented quite well by the equation

$$R = R_0 A^{1/3} \quad (\text{radius of a nucleus}) \quad (43.1)$$

where R_0 is an experimentally determined constant:

$$R_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm}$$

The nucleon number A in Eq. (43.1) is also called the **mass number** because it is the nearest whole number to the mass of the nucleus measured in unified atomic mass units (u). (The proton mass and the neutron mass are both approximately 1 u.) The best current conversion factor is

$$1 \text{ u} = 1.660538782(83) \times 10^{-27} \text{ kg}$$

In Section 43.2 we'll discuss the masses of nuclei in more detail. Note that when we speak of the masses of nuclei and particles, we mean their *rest* masses.

Nuclear Density

The volume V of a sphere is equal to $4\pi R^3/3$, so Eq. (43.1) shows that the *volume* of a nucleus is proportional to A . Dividing A (the approximate mass in u) by the volume gives us the approximate density and cancels out A . Thus *all nuclei have approximately the same density*. This fact is of crucial importance in understanding nuclear structure.

Example 43.1 Calculating nuclear properties

The most common kind of iron nucleus has mass number $A = 56$. Find the radius, approximate mass, and approximate density of the nucleus.

The volume V of the nucleus (which we treat as a sphere of radius R) and its density ρ are

$$\begin{aligned} V &= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A = \frac{4}{3}\pi(4.6 \times 10^{-15} \text{ m})^3 \\ &= 4.1 \times 10^{-43} \text{ m}^3 \\ \rho &= \frac{m}{V} \approx \frac{9.3 \times 10^{-26} \text{ kg}}{4.1 \times 10^{-43} \text{ m}^3} = 2.3 \times 10^{17} \text{ kg/m}^3 \end{aligned}$$

SOLUTION

IDENTIFY and SET UP: Equation (43.1) tells us how the nuclear radius R depends on the mass number A . The mass of the nucleus in atomic mass units is approximately equal to the value of A , and the density ρ is mass divided by volume.

EXECUTE: The radius and approximate mass are

$$\begin{aligned} R &= R_0 A^{1/3} = (1.2 \times 10^{-15} \text{ m})(56)^{1/3} \\ &= 4.6 \times 10^{-15} \text{ m} = 4.6 \text{ fm} \\ m &\approx (56 \text{ u})(1.66 \times 10^{-27} \text{ kg/u}) = 9.3 \times 10^{-26} \text{ kg} \end{aligned}$$

EVALUATE: As we mentioned above, *all* nuclei have approximately this same density. The density of solid iron is about 7000 kg/m^3 ; the iron nucleus is more than 10^{13} times as dense as iron in bulk. Such densities are also found in *neutron stars*, which are similar to gigantic nuclei made almost entirely of neutrons. A 1-cm cube of material with this density would have a mass of $2.3 \times 10^{11} \text{ kg}$, or 230 million metric tons!

Nuclides and Isotopes

The building blocks of the nucleus are the proton and the neutron. In a neutral atom, the nucleus is surrounded by one electron for every proton in the nucleus. We introduced these particles in Section 21.1; we'll recount the discovery of the neutron and proton in Chapter 44. The masses of these particles are

$$\begin{aligned} \text{Proton:} \quad m_p &= 1.007276 \text{ u} = 1.672622 \times 10^{-27} \text{ kg} \\ \text{Neutron:} \quad m_n &= 1.008665 \text{ u} = 1.674927 \times 10^{-27} \text{ kg} \\ \text{Electron:} \quad m_e &= 0.000548580 \text{ u} = 9.10938 \times 10^{-31} \text{ kg} \end{aligned}$$

The number of protons in a nucleus is the **atomic number** Z . The number of neutrons is the **neutron number** N . The nucleon number or mass number A is the sum of the number of protons Z and the number of neutrons N :

$$A = Z + N \quad (43.2)$$

A single nuclear species having specific values of both Z and N is called a **nuclide**. Table 43.1 lists values of A , Z , and N for some nuclides. The electron structure of an atom, which is responsible for its chemical properties, is determined by the charge Ze of the nucleus. The table shows some nuclides that have the same Z but different N . These nuclides are called **isotopes** of that element; they have different masses because they have different numbers of neutrons in their nuclei. A familiar example is chlorine (Cl, $Z = 17$). About 76% of chlorine nuclei have $N = 18$; the other 24% have $N = 20$. Different isotopes of an element usually have slightly different physical properties such as melting and boiling temperatures and diffusion rates. The two common isotopes of uranium with $A = 235$ and 238 are usually separated industrially by taking advantage of the different diffusion rates of gaseous uranium hexafluoride (UF_6) containing the two isotopes.

Table 43.1 also shows the usual notation for individual nuclides: the symbol of the element, with a pre-subscript equal to Z and a pre-superscript equal to the mass number A . The general format for an element El is ${}^A_Z\text{El}$. The isotopes of chlorine mentioned above, with $A = 35$ and 37, are written ${}^{35}_{17}\text{Cl}$ and ${}^{37}_{17}\text{Cl}$ and pronounced “chlorine-35” and “chlorine-37,” respectively. This name of the element determines the atomic number Z , so the pre-subscript Z is sometimes omitted, as in ${}^{35}\text{Cl}$.

Table 43.2 gives the masses of some common atoms, including their electrons. Note that this table gives masses of *neutral* atoms (with Z electrons) rather than masses of *bare* nuclei, because it is much more difficult to measure masses of bare nuclei with high precision. The mass of a neutral carbon-12 atom is exactly 12 u; that’s how the unified atomic mass unit is defined. The masses of other atoms are *approximately* equal to A atomic mass units, as we stated earlier. In fact, the atomic masses are *less* than the sum of the masses of their parts (the Z protons, the Z electrons, and the N neutrons). We’ll explain this very important mass difference in the next section.

Table 43.1 Compositions of Some Common Nuclides

Z = atomic number (number of protons)

N = neutron number

$A = Z + N$ = mass number (total number of nucleons)

Nucleus	Z	N	$A = Z + N$
${}^1_1\text{H}$	1	0	1
${}^2_1\text{H}$	1	1	2
${}^4_2\text{He}$	2	2	4
${}^6_3\text{Li}$	3	3	6
${}^7_3\text{Li}$	3	4	7
${}^9_4\text{Be}$	4	5	9
${}^{10}_5\text{B}$	5	5	10
${}^{11}_5\text{B}$	5	6	11
${}^{12}_6\text{C}$	6	6	12
${}^{13}_6\text{C}$	6	7	13
${}^{14}_7\text{N}$	7	7	14
${}^{16}_8\text{O}$	8	8	16
${}^{23}_{11}\text{Na}$	11	12	23
${}^{63}_{29}\text{Cu}$	29	36	65
${}^{200}_{80}\text{Hg}$	80	120	200
${}^{235}_{92}\text{U}$	92	143	235
${}^{238}_{92}\text{U}$	92	146	238

Table 43.2 Neutral Atomic Masses for Some Light Nuclides

Element and Isotope	Atomic Number, Z	Neutron Number, N	Atomic Mass (u)	Mass Number, A
Hydrogen (${}^1_1\text{H}$)	1	0	1.007825	1
Deuterium (${}^2_1\text{H}$)	1	1	2.014102	2
Tritium (${}^3_1\text{H}$)	1	2	3.016049	3
Helium (${}^3_2\text{He}$)	2	1	3.016029	3
Helium (${}^4_2\text{He}$)	2	2	4.002603	4
Lithium (${}^6_3\text{Li}$)	3	3	6.015122	6
Lithium (${}^7_3\text{Li}$)	3	4	7.016004	7
Beryllium (${}^9_4\text{Be}$)	4	5	9.012182	9
Boron (${}^{10}_5\text{B}$)	5	5	10.012937	10
Boron (${}^{11}_5\text{B}$)	5	6	11.009305	11
Carbon (${}^{12}_6\text{C}$)	6	6	12.000000	12
Carbon (${}^{13}_6\text{C}$)	6	7	13.003355	13
Nitrogen (${}^{14}_7\text{N}$)	7	7	14.003074	14
Nitrogen (${}^{15}_7\text{N}$)	7	8	15.000109	15
Oxygen (${}^{16}_8\text{O}$)	8	8	15.994915	16
Oxygen (${}^{17}_8\text{O}$)	8	9	16.999132	17
Oxygen (${}^{18}_8\text{O}$)	8	10	17.999160	18

Source: A. H. Wapstra and G. Audi, *Nuclear Physics* **A595**, 4 (1995).

Nuclear Spins and Magnetic Moments

Like electrons, protons and neutrons are also spin- $\frac{1}{2}$ particles with spin angular momenta given by the same equations as in Section 41.5. The magnitude of the spin angular momentum \vec{S} of a nucleon is

$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2} + 1\right)}\hbar = \sqrt{\frac{3}{4}}\hbar \quad (43.3)$$

and the z -component is

$$S_z = \pm \frac{1}{2}\hbar \quad (43.4)$$

In addition to the spin angular momentum of the nucleons, there may be *orbital* angular momentum associated with their motions within the nucleus. The orbital angular momentum of the nucleons is quantized in the same way as that of electrons in atoms.

The *total* angular momentum \vec{J} of the nucleus is the vector sum of the individual spin and orbital angular momenta of all the nucleons. It has magnitude

$$J = \sqrt{j(j + 1)}\hbar \quad (43.5)$$

and z -component

$$J_z = m_j\hbar \quad (m_j = -j, -j + 1, \dots, j - 1, j) \quad (43.6)$$

When the total number of nucleons A is *even*, j is an integer; when it is *odd*, j is a half-integer. All nuclides for which both Z and N are even have $J = 0$, which suggests that pairing of particles with opposite spin components may be an important consideration in nuclear structure. The total nuclear angular momentum quantum number j is usually called the *nuclear spin*, even though in general it refers to a combination of the orbital and spin angular momenta of the nucleons that make up the nucleus.

Associated with nuclear angular momentum is a *magnetic moment*. When we discussed *electron* magnetic moments in Section 41.4, we introduced the Bohr magneton $\mu_B = e\hbar/2m_e$ as a natural unit of magnetic moment. We found that the

magnitude of the z -component of the electron-spin magnetic moment is almost exactly equal to μ_B ; that is, $|\mu_{sz}|_{\text{electron}} \approx \mu_B$. In discussing *nuclear* magnetic moments, we can define an analogous quantity, the **nuclear magneton** μ_n :

$$\mu_n = \frac{e\hbar}{2m_p} = 5.05078 \times 10^{-27} \text{ J/T} = 3.15245 \times 10^{-8} \text{ eV/T} \quad (43.7)$$

(nuclear magneton)

where m_p is the proton mass. Because the proton mass m_p is 1836 times larger than the electron mass m_e , the nuclear magneton μ_n is 1836 times smaller than the Bohr magneton μ_B .

We might expect the magnitude of the z -component of the spin magnetic moment of the proton to be approximately μ_n . Instead, it turns out to be

$$|\mu_{sz}|_{\text{proton}} = 2.7928\mu_n \quad (43.8)$$

Even more surprising, the neutron, which has zero charge, has a spin magnetic moment; its z -component has magnitude

$$|\mu_{sz}|_{\text{neutron}} = 1.9130\mu_n \quad (43.9)$$

The proton has a positive charge; as expected, its spin magnetic moment $\vec{\mu}$ is parallel to its spin angular momentum \vec{S} . However, $\vec{\mu}$ and \vec{S} are opposite for a neutron, as would be expected for a *negative* charge distribution. These *anomalous* magnetic moments arise because the proton and neutron aren't really fundamental particles but are made of simpler particles called *quarks*. We'll discuss quarks in some detail in Chapter 44.

The magnetic moment of an entire nucleus is typically a few nuclear magnetons. When a nucleus is placed in an external magnetic field \vec{B} , there is an interaction energy $U = -\vec{\mu} \cdot \vec{B} = -\mu_z B$ just as with atomic magnetic moments. The components of the magnetic moment in the direction of the field μ_z are quantized, so a series of energy levels results from this interaction.

Example 43.2 Proton spin flips

Protons are placed in a 2.30-T magnetic field that points in the positive z -direction. (a) What is the energy difference between states with the z -component of proton spin angular momentum parallel and antiparallel to the field? (b) A proton can make a transition from one of these states to the other by emitting or absorbing a photon with the appropriate energy. Find the frequency and wavelength of such a photon.

SOLUTION

IDENTIFY and SET UP: The proton is a spin- $\frac{1}{2}$ particle with a magnetic moment $\vec{\mu}$ in the same direction as its spin \vec{S} , so its energy depends on the orientation of its spin relative to an applied magnetic field \vec{B} . If the z -component of \vec{S} is aligned with \vec{B} , then μ_z is equal to the positive value given in Eq. (43.8). If the z -component of \vec{S} is opposite \vec{B} , then μ_z is the negative of this value. The interaction energy in either case is $U = -\mu_z B$; the difference between these energies is our target variable in part (a). We find the photon frequency and wavelength using $E = hf = hc/\lambda$.

EXECUTE: (a) When the z -components of \vec{S} and $\vec{\mu}$ are parallel to \vec{B} , the interaction energy is

$$\begin{aligned} U &= -|\mu_z|B = -(2.7928)(3.152 \times 10^{-8} \text{ eV/T})(2.30 \text{ T}) \\ &= -2.025 \times 10^{-7} \text{ eV} \end{aligned}$$

When the z -components of \vec{S} and $\vec{\mu}$ are antiparallel to the field, the energy is $+2.025 \times 10^{-7} \text{ eV}$. Hence the energy *difference* between the states is

$$\Delta E = 2(2.025 \times 10^{-7} \text{ eV}) = 4.05 \times 10^{-7} \text{ eV}$$

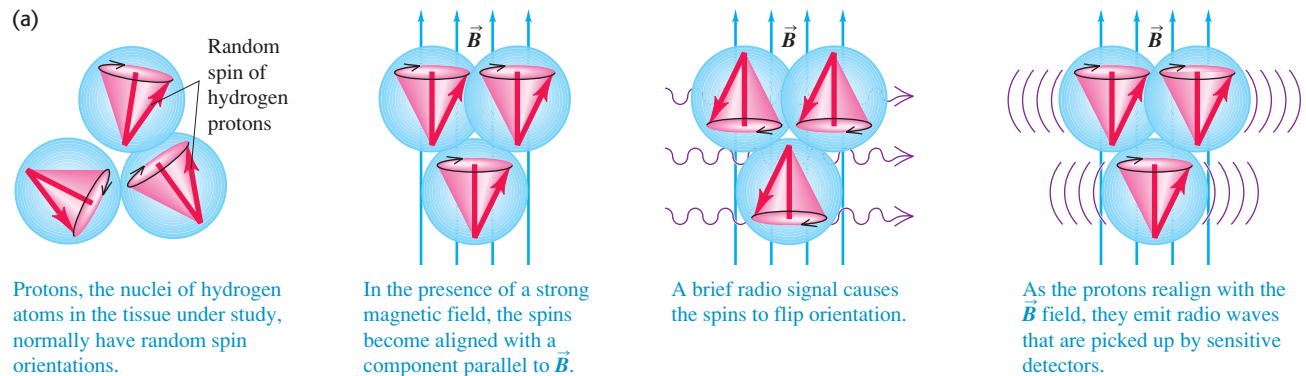
(b) The corresponding photon frequency and wavelength are

$$f = \frac{\Delta E}{h} = \frac{4.05 \times 10^{-7} \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 9.79 \times 10^7 \text{ Hz} = 97.9 \text{ MHz}$$

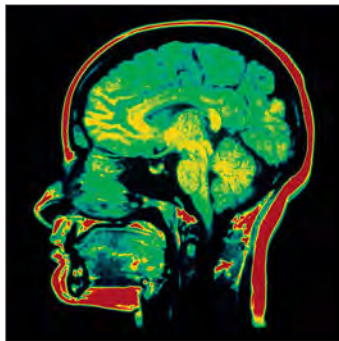
$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{9.79 \times 10^7 \text{ s}^{-1}} = 3.06 \text{ m}$$

EVALUATE: This frequency is in the middle of the FM radio band. When a hydrogen specimen is placed in a 2.30-T magnetic field and irradiated with radio waves of this frequency, proton *spin flips* can be detected by the absorption of energy from the radiation.

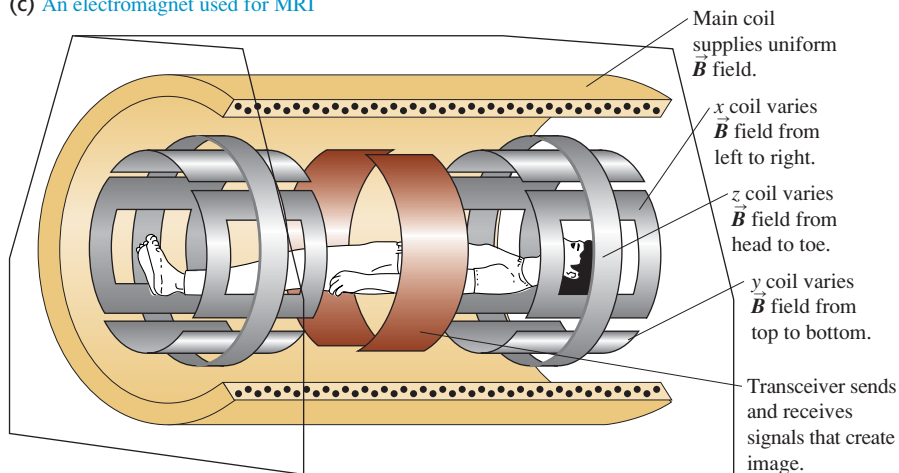
43.1 Magnetic resonance imaging (MRI).



(b) Since \vec{B} has a different value at different locations in the tissue, the radio waves from different locations have different frequencies. This makes it possible to construct an image.



(c) An electromagnet used for MRI



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Nuclear Magnetic Resonance and MRI

Spin-flip experiments of the sort referred to in Example 43.2 are called *nuclear magnetic resonance* (NMR). They have been carried out with many different nuclides. Frequencies and magnetic fields can be measured very precisely, so this technique permits precise measurements of nuclear magnetic moments. An elaboration of this basic idea leads to *magnetic resonance imaging* (MRI), a noninvasive imaging technique that discriminates among various body tissues on the basis of the differing environments of protons in the tissues (Fig. 43.1).

The magnetic moment of a nucleus is also the *source* of a magnetic field. In an atom the interaction of an electron's magnetic moment with the field of the nucleus's magnetic moment causes additional splittings in atomic energy levels and spectra. We called this effect *hyperfine structure* in Section 41.5. Measurements of the hyperfine structure may be used to directly determine the nuclear spin.

Test Your Understanding of Section 43.1 (a) By what factor must the mass number of a nucleus increase to double its volume? (i) $\sqrt[3]{2}$; (ii) $\sqrt{2}$; (iii) 2; (iv) 4; (v) 8. (b) By what factor must the mass number increase to double the radius of the nucleus? (i) $\sqrt[3]{2}$; (ii) $\sqrt{2}$; (iii) 2; (iv) 4; (v) 8.



I

43.2 Nuclear Binding and Nuclear Structure

Because energy must be added to a nucleus to separate it into its individual protons and neutrons, the total rest energy E_0 of the separated nucleons is greater than the rest energy of the nucleus. The energy that must be added to separate the

nucleons is called the **binding energy** E_B ; it is the magnitude of the energy by which the nucleons are bound together. Thus the rest energy of the nucleus is $E_0 - E_B$. Using the equivalence of rest mass and energy (see Section 37.8), we see that the total mass of the nucleons is always greater than the mass of the nucleus by an amount E_B/c^2 called the *mass defect*. The binding energy for a nucleus containing Z protons and N neutrons is defined as

$$E_B = (ZM_H + Nm_n - \frac{A}{Z}M)c^2 \quad (\text{nuclear binding energy}) \quad (43.10)$$

where $\frac{A}{Z}M$ is the mass of the *neutral atom* containing the nucleus, the quantity in the parentheses is the mass defect, and $c^2 = 931.5 \text{ MeV/u}$. Note that Eq. (43.10) does not include Zm_p , the mass of Z protons. Rather, it contains ZM_H , the mass of Z protons and Z electrons combined as Z neutral ${}^1_1\text{H}$ atoms, to balance the Z electrons included in $\frac{A}{Z}M$, the mass of the neutral atom.

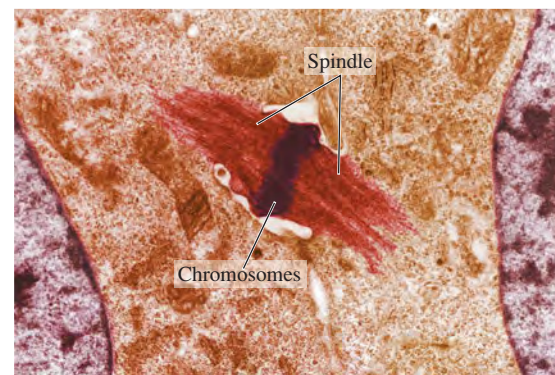
The simplest nucleus is that of hydrogen, a single proton. Next comes the nucleus of ${}^2_1\text{H}$, the isotope of hydrogen with mass number 2, usually called *deuterium*. Its nucleus consists of a proton and a neutron bound together to form a particle called the *deuteron*. By using values from Table 43.2 in Eq. (43.10), we find that the binding energy of the deuteron is

$$\begin{aligned} E_B &= (1.007825 \text{ u} + 1.008665 \text{ u} - 2.014102 \text{ u})(931.5 \text{ MeV/u}) \\ &= 2.224 \text{ MeV} \end{aligned}$$

This much energy would be required to pull the deuteron apart into a proton and a neutron. An important measure of how tightly a nucleus is bound is the *binding energy per nucleon*, E_B/A . At $(2.224 \text{ MeV})/(2 \text{ nucleons}) = 1.112 \text{ MeV}$ per nucleon, ${}^2_1\text{H}$ has the lowest binding energy per nucleon of all nuclides.

Application Deuterium and Heavy Water Toxicity

A crucial step in plant and animal cell division is the formation of a spindle, which separates the two sets of daughter chromosomes. If a plant is given only heavy water—in which one or both of the hydrogen atoms in an H_2O molecule are replaced with a deuterium atom—cell division stops and the plant stops growing. The reason is that deuterium is more massive than ordinary hydrogen, so the O–H bond in heavy water has a slightly different binding energy and heavy water has slightly different properties as a solvent. The biochemical reactions that occur during cell division are very sensitive to these solvent properties, so a spindle never forms and the cell cannot reproduce.



Problem-Solving Strategy 43.1 Nuclear Properties

IDENTIFY *the relevant concepts:* The key properties of a nucleus are its mass, radius, binding energy, mass defect, binding energy per nucleon, and angular momentum.

SET UP *the problem:* Once you have identified the target variables, assemble the equations needed to solve the problem. A relatively small number of equations from this section and Section 43.1 are all you need.

EXECUTE *the solution:* Solve for the target variables. Binding-energy calculations using Eq. (43.10) often involve subtracting two nearly equal quantities. To get enough precision in the difference, you may need to carry as many as nine significant figures, if that many are available.

EVALUATE *your answer:* It's useful to be familiar with the following benchmark magnitudes. Protons and neutrons are about 1840 times as massive as electrons. Nuclear radii are of the order of 10^{-15} m . The electric potential energy of two protons in a nucleus is roughly 10^{-13} J or 1 MeV, so nuclear interaction energies are typically a few MeV rather than a few eV as with atoms. The binding energy per nucleon is about 1% of the nucleon rest energy. (The ionization energy of the hydrogen atom is only 0.003% of the electron's rest energy.) Angular momenta are determined only by the value of \hbar , so they are of the same order of magnitude in both nuclei and atoms. Nuclear magnetic moments, however, are about a factor of 1000 *smaller* than those of electrons in atoms because nuclei are so much more massive than electrons.

Example 43.3 The most strongly bound nuclide

Find the mass defect, the total binding energy, and the binding energy per nucleon of ${}^{62}_{28}\text{Ni}$, which has the highest binding energy per nucleon of all nuclides (Fig. 43.2). The neutral atomic mass of ${}^{62}_{28}\text{Ni}$ is 61.928349 u.

SOLUTION

IDENTIFY and SET UP: The mass defect ΔM is the difference between the mass of the nucleus and the combined mass of its constituent nucleons. The binding energy E_B is this quantity

multiplied by c^2 , and the binding energy per nucleon is E_B divided by the mass number A . We use Eq. (43.10), $\Delta M = ZM_H + Nm_n - \frac{A}{Z}M$, to determine both the mass defect and the binding energy.

EXECUTE: With $Z = 28$, $M_H = 1.007825 \text{ u}$, $N = A - Z = 62 - 28 = 34$, $m_n = 1.008665 \text{ u}$, and $\frac{A}{Z}M = 61.928349 \text{ u}$, Eq. (43.10) gives $\Delta M = 0.585361 \text{ u}$. The binding energy is then

$$E_B = (0.585361 \text{ u})(931.5 \text{ MeV/u}) = 545.3 \text{ MeV}$$

Continued

The binding energy *per nucleon* is $E_B/A = (545.3 \text{ MeV})/62$, or 8.795 MeV per nucleon.

EVALUATE: Our result means that it would take a minimum of 545.3 MeV to pull a ${}^{62}_{28}\text{Ni}$ completely apart into 28 protons and 34 neutrons. The mass defect of ${}^{62}_{28}\text{Ni}$ is about 1% of the atomic

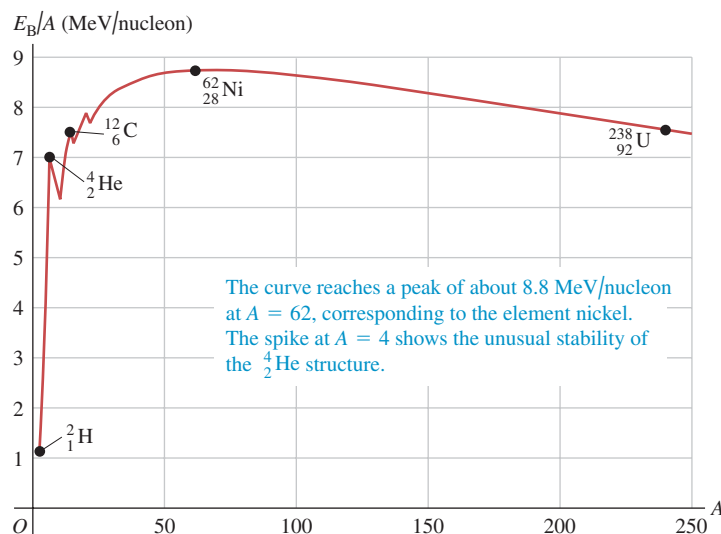
(or the nuclear) mass. The binding energy is therefore about 1% of the rest energy of the nucleus, and the binding energy per nucleon is about 1% of the rest energy of a nucleon. Note that the mass defect is more than half the mass of a nucleon, which suggests how tightly bound nuclei are.

Nearly all stable nuclides, from the lightest to the most massive, have binding energies in the range of 7–9 MeV per nucleon. Figure 43.2 is a graph of binding energy per nucleon as a function of the mass number A . Note the spike at $A = 4$, showing the unusually large binding energy per nucleon of the ${}^4_2\text{He}$ nucleus (alpha particle) relative to its neighbors. To explain this curve, we must consider the interactions among the nucleons.

The Nuclear Force

The force that binds protons and neutrons together in the nucleus, despite the electrical repulsion of the protons, is an example of the *strong interaction* that we mentioned in Section 5.5. In the context of nuclear structure, this interaction is called the *nuclear force*. Here are some of its characteristics. First, it does not depend on charge; neutrons as well as protons are bound, and the binding is the same for both. Second, it has short range, of the order of nuclear dimensions—that is, 10^{-15} m. (Otherwise, the nucleus would grow by pulling in additional protons and neutrons.) But within its range, the nuclear force is much stronger than electrical forces; otherwise, the nucleus could never be stable. It would be nice if we could write a simple equation like Newton's law of gravitation or Coulomb's law for this force, but physicists have yet to fully determine its dependence on the separation r . Third, the nearly constant density of nuclear matter and the nearly constant binding energy per nucleon of larger nuclides show that a particular nucleon cannot interact simultaneously with *all* the other nucleons in a nucleus, but only with those few in its immediate vicinity. This is different from electrical forces; *every* proton in the nucleus repels every other one. This limited number of interactions is called *saturation*; it is analogous to covalent bonding in molecules and solids. Finally, the nuclear force favors binding of *pairs* of protons or neutrons with opposite spins and of *pairs of pairs*—that is, a pair of protons and a pair of neutrons, each pair having opposite spins. Hence the alpha particle (two protons and two neutrons) is an exceptionally stable nucleus for its mass number. We'll see other evidence for pairing effects in nuclei in the

43.2 Approximate binding energy per nucleon as a function of mass number A (the total number of nucleons) for stable nuclides.



next subsection. (In Section 42.8 we described an analogous pairing that binds opposite-spin electrons in Cooper pairs in the BCS theory of superconductivity.)

The analysis of nuclear structure is more complex than the analysis of many-electron atoms. Two different kinds of interactions are involved (electrical and nuclear), and the nuclear force is not yet completely understood. Even so, we can gain some insight into nuclear structure by the use of simple models. We'll discuss briefly two rather different but successful models, the *liquid-drop model* and the *shell model*.

The Liquid-Drop Model

The **liquid-drop model**, first proposed in 1928 by the Russian physicist George Gamow and later expanded on by Niels Bohr, is suggested by the observation that all nuclei have nearly the same density. The individual nucleons are analogous to molecules of a liquid, held together by short-range interactions and surface-tension effects. We can use this simple picture to derive a formula for the estimated total binding energy of a nucleus. We'll include five contributions:

1. We've remarked that nuclear forces show *saturation*; an individual nucleon interacts only with a few of its nearest neighbors. This effect gives a binding-energy term that is proportional to the number of nucleons. We write this term as C_1A , where C_1 is an experimentally determined constant.
2. The nucleons on the surface of the nucleus are less tightly bound than those in the interior because they have no neighbors outside the surface. This decrease in the binding energy gives a *negative* energy term proportional to the surface area $4\pi R^2$. Because R is proportional to $A^{1/3}$, this term is proportional to $A^{2/3}$; we write it as $-C_2A^{2/3}$, where C_2 is another constant.
3. Every one of the Z protons repels every one of the $(Z - 1)$ other protons. The total repulsive electric potential energy is proportional to $Z(Z - 1)$ and inversely proportional to the radius R and thus to $A^{1/3}$. This energy term is negative because the nucleons are less tightly bound than they would be without the electrical repulsion. We write this correction as $-C_3Z(Z - 1)/A^{1/3}$.
4. To be in a stable, low-energy state, the nucleus must have a balance between the energies associated with the neutrons and with the protons. This means that N is close to Z for small A and N is greater than Z (but not too much greater) for larger A . We need a negative energy term corresponding to the difference $|N - Z|$. The best agreement with observed binding energies is obtained if this term is proportional to $(N - Z)^2/A$. If we use $N = A - Z$ to express this energy in terms of A and Z , this correction is $-C_4(A - 2Z)^2/A$.
5. Finally, the nuclear force favors *pairing* of protons and of neutrons. This energy term is positive (more binding) if both Z and N are even, negative (less binding) if both Z and N are odd, and zero otherwise. The best fit to the data occurs with the form $\pm C_5A^{-4/3}$ for this term.

The total estimated binding energy E_B is the sum of these five terms:

$$E_B = C_1A - C_2A^{2/3} - C_3 \frac{Z(Z - 1)}{A^{1/3}} - C_4 \frac{(A - 2Z)^2}{A} \pm C_5A^{-4/3} \quad (43.11)$$

(nuclear binding energy)

The constants C_1 , C_2 , C_3 , C_4 , and C_5 , chosen to make this formula best fit the observed binding energies of nuclides, are

$$\begin{aligned} C_1 &= 15.75 \text{ MeV} \\ C_2 &= 17.80 \text{ MeV} \\ C_3 &= 0.7100 \text{ MeV} \\ C_4 &= 23.69 \text{ MeV} \\ C_5 &= 39 \text{ MeV} \end{aligned}$$

The constant C_1 is the binding energy per nucleon due to the saturated nuclear force. This energy is almost 16 MeV per nucleon, about double the *total* binding energy per nucleon in most nuclides.

If we estimate the binding energy E_B using Eq. (43.11), we can solve Eq. (43.10) to use it to estimate the mass of any neutral atom:

$${}^A_ZM = ZM_H + Nm_n - \frac{E_B}{c^2} \quad (\text{semiempirical mass formula}) \quad (43.12)$$

Equation (43.12) is called the *semiempirical mass formula*. The name is apt; it is *empirical* in the sense that the C 's have to be determined empirically (experimentally), yet it does have a sound theoretical basis.

Example 43.4 Estimating binding energy and mass

For the nuclide ${}^{62}_{28}\text{Ni}$ of Example 43.3, (a) calculate the five terms in the binding energy and the total estimated binding energy, and (b) find the neutral atomic mass using the semiempirical mass formula.

SOLUTION

IDENTIFY and SET UP: We use the liquid-drop model of the nucleus and its five contributions to the binding energy, as given by Eq. (43.11), to calculate the total binding energy E_B . We then use Eq. (43.12) to find the neutral atomic mass ${}^{62}_{28}M$.

EXECUTE: (a) With $Z = 28$, $A = 62$, and $N = 34$, the five terms in Eq. (43.11) are

- $C_1A = (15.75 \text{ MeV})(62) = 976.5 \text{ MeV}$
- $-C_2A^{2/3} = -(17.80 \text{ MeV})(62)^{2/3} = -278.8 \text{ MeV}$
- $-C_3\frac{Z(Z-1)}{A^{1/3}} = -(0.7100 \text{ MeV})\frac{(28)(27)}{(62)^{1/3}}$
 $= -135.6 \text{ MeV}$

$$4. -C_4\frac{(A-2Z)^2}{A} = -(23.69 \text{ MeV})\frac{(62-56)^2}{62} = -13.8 \text{ MeV}$$

$$5. +C_5A^{-4/3} = (39 \text{ MeV})(62)^{-4/3} = 0.2 \text{ MeV}$$

The pairing correction (term 5) is by far the smallest of all the terms; it is positive because both Z and N are even. The sum of all five terms is the total estimated binding energy, $E_B = 548.5 \text{ MeV}$.

(b) We use $E_B = 548.5 \text{ MeV}$ in Eq. (43.12):

$$\begin{aligned} {}^{62}_{28}M &= 28(1.007825 \text{ u}) + 34(1.008665 \text{ u}) \\ &\quad - \frac{548.5 \text{ MeV}}{931.5 \text{ MeV/u}} = 61.925 \text{ u} \end{aligned}$$

EVALUATE: The binding energy of ${}^{62}_{28}\text{Ni}$ calculated in part (a) is only about 0.6% larger than the true value of 545.3 MeV found in Example 43.3, and the mass calculated in part (b) is only about 0.005% smaller than the measured value of 61.928349 u. The semiempirical mass formula can be quite accurate!

The liquid-drop model and the mass formula derived from it are quite successful in correlating nuclear masses, and we will see later that they are a great help in understanding decay processes of unstable nuclides. Some other aspects of nuclei, such as angular momentum and excited states, are better approached with different models.

The Shell Model

The **shell model** of nuclear structure is analogous to the central-field approximation in atomic physics (see Section 41.6). We picture each nucleon as moving in a potential that represents the averaged-out effect of all the other nucleons. This may not seem to be a very promising approach; the nuclear force is very strong, very short range, and therefore strongly distance dependent. However, in some respects, this model turns out to work fairly well.

The potential-energy function for the nuclear force is the same for protons as for neutrons. Figure 43.3a shows a reasonable assumption for the shape of this function: a spherical version of the square-well potential we discussed in Section 40.3. The corners are somewhat rounded because the nucleus doesn't have a sharply defined surface. For protons there is an additional potential energy associated with electrical repulsion. We consider each proton to interact with a sphere of uniform charge density, with radius R and total charge $(Z-1)e$. Figure 43.3b shows the nuclear, electric, and total potential energies for a proton as functions of the distance r from the center of the nucleus.

In principle, we could solve the Schrödinger equation for a proton or neutron moving in such a potential. For any spherically symmetric potential energy, the angular-momentum states are the same as for the electrons in the central-field approximation in atomic physics. In particular, we can use the concept of *filled shells and subshells* and their relationship to stability. In atomic structure we found that the values $Z = 2, 10, 18, 36, 54,$ and 86 (the atomic numbers of the noble gases) correspond to particularly stable electron arrangements.

A comparable effect occurs in nuclear structure. The numbers are different because the potential-energy function is different and the nuclear spin-orbit interaction is much stronger and of opposite sign than in atoms, so the subshells fill up in a different order from those for electrons in an atom. It is found that when the number of neutrons *or* the number of protons is $2, 8, 20, 28, 50, 82,$ or 126 , the resulting structure is unusually stable—that is, has an unusually high binding energy. (Nuclides with $Z = 126$ have not been observed in nature.) These numbers are called *magic numbers*. Nuclides in which Z is a magic number tend to have an above-average number of stable isotopes. There are several *doubly magic* nuclides for which both Z and N are magic, including



All these nuclides have substantially higher binding energy per nucleon than do nuclides with neighboring values of N or Z . They also all have zero nuclear spin. The magic numbers correspond to filled-shell or -subshell configurations of nucleon energy levels with a relatively large jump in energy to the next allowed level.

Test Your Understanding of Section 43.2 Rank the following nuclei in order from largest to smallest value of the binding energy per nucleon. (i) ${}^4_2\text{He}$; (ii) ${}^{52}_{24}\text{Cr}$; (iii) ${}^{152}_{62}\text{Sm}$; (iv) ${}^{200}_{80}\text{Hg}$; (v) ${}^{252}_{92}\text{Cf}$.

43.3 Nuclear Stability and Radioactivity

Among about 2500 known nuclides, fewer than 300 are stable. The others are unstable structures that decay to form other nuclides by emitting particles and electromagnetic radiation, a process called **radioactivity**. The time scale of these decay processes ranges from a small fraction of a microsecond to billions of years. The *stable* nuclides are shown by dots on the graph in Fig. 43.4, where the neutron number N and proton number (or atomic number) Z for each nuclide are plotted. Such a chart is called a *Segrè chart*, after its inventor, the Italian-American physicist Emilio Segrè (1905–1989).

Each blue line perpendicular to the line $N = Z$ represents a specific value of the mass number $A = Z + N$. Most lines of constant A pass through only one or two stable nuclides; that is, there is usually a very narrow range of stability for a given mass number. The lines at $A = 20, A = 40, A = 60,$ and $A = 80$ are examples. In four cases these lines pass through *three* stable nuclides—namely, at $A = 96, 124, 130,$ and 136 .

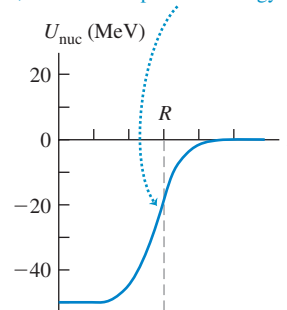
Four stable nuclides have both odd Z and odd N :



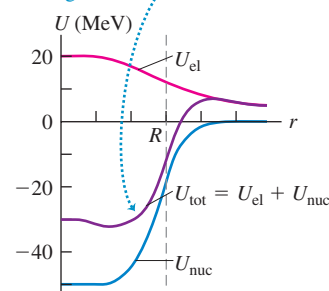
These are called *odd-odd nuclides*. The absence of other odd-odd nuclides shows the influence of pairing. Also, there is *no* stable nuclide with $A = 5$ or $A = 8$. The doubly magic ${}^4_2\text{He}$ nucleus, with a pair of protons and a pair of neutrons, has no interest in accepting a fifth particle into its structure. Collections of eight nucleons decay to smaller nuclides, with a ${}^8_4\text{Be}$ nucleus immediately splitting into two ${}^4_2\text{He}$ nuclei.

43.3 Approximate potential-energy functions for a nucleon in a nucleus. The approximate nuclear radius is R .

(a) The potential energy U_{nuc} due to the nuclear force is the same for protons and neutrons. For neutrons, it is the *total* potential energy.



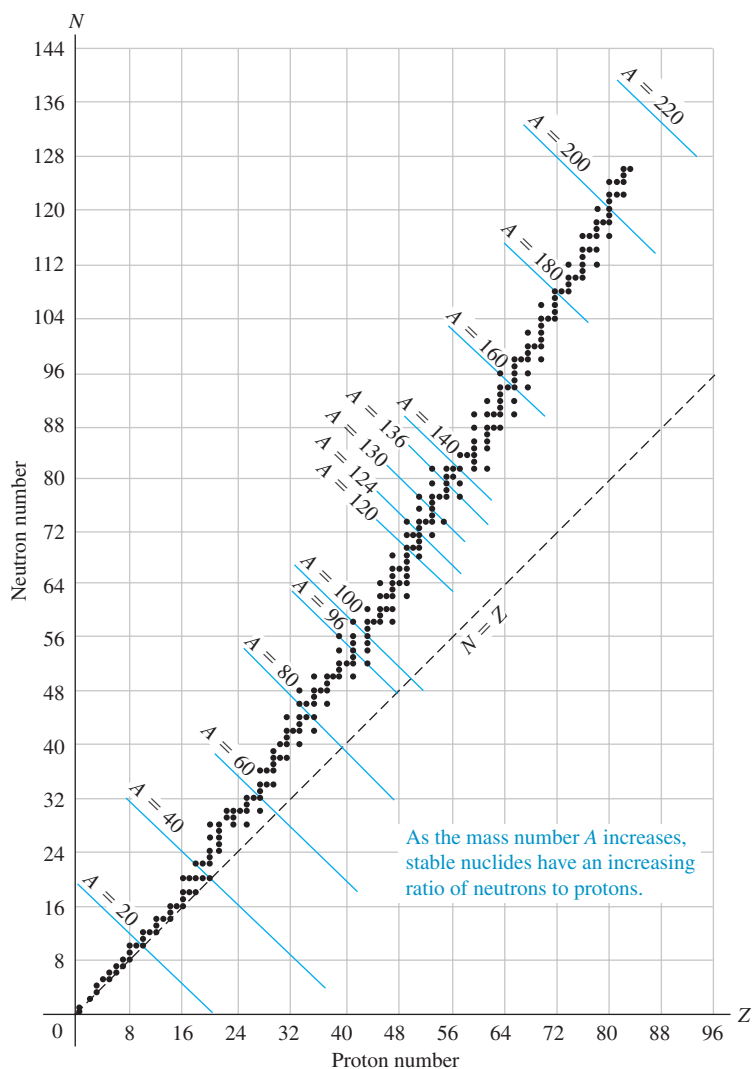
(b) For protons, the total potential energy U_{tot} is the sum of the nuclear (U_{nuc}) and electric (U_{el}) potential energies.



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43.4 Segrè chart showing neutron number and proton number for stable nuclides.

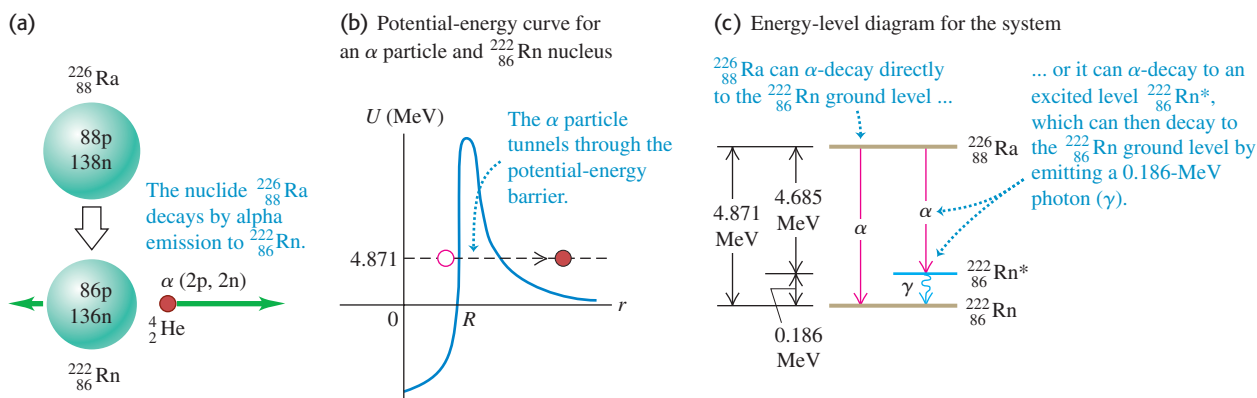


The points on the Segrè chart representing stable nuclides define a rather narrow stability region. For low mass numbers, the numbers of protons and neutrons are approximately equal, $N \approx Z$. The ratio N/Z increases gradually with A , up to about 1.6 at large mass numbers, because of the increasing influence of the electrical repulsion of the protons. Points to the right of the stability region represent nuclides that have too many protons relative to neutrons to be stable. In these cases, repulsion wins, and the nucleus comes apart. To the left are nuclides with too many neutrons relative to protons. In these cases the energy associated with the neutrons is out of balance with that associated with the protons, and the nuclides decay in a process that converts neutrons to protons. The graph also shows that no nuclide with $A > 209$ or $Z > 83$ is stable. A nucleus is unstable if it is too big. Note that there is no stable nuclide with $Z = 43$ (technetium) or 61 (promethium).

Alpha Decay

Nearly 90% of the 2500 known nuclides are *radioactive*; they are not stable but decay into other nuclides. When unstable nuclides decay into different nuclides, they usually emit alpha (α) or beta (β) particles. An **alpha particle** is a ${}^4\text{He}$ nucleus, two protons and two neutrons bound together, with total spin zero. Alpha emission occurs principally with nuclei that are too large to be stable. When a nucleus emits an alpha particle, its N and Z values each decrease by 2 and A decreases by 4, moving it closer to stable territory on the Segrè chart.

43.5 Alpha decay of the unstable radium nuclide $^{226}_{88}\text{Ra}$.



A familiar example of an alpha emitter is radium, $^{226}_{88}\text{Ra}$ (Fig. 43.5a). The speed of the emitted alpha particle, determined from the curvature of its path in a transverse magnetic field, is about 1.52×10^7 m/s. This speed, although large, is only 5% of the speed of light, so we can use the nonrelativistic kinetic-energy expression $K = \frac{1}{2}mv^2$:

$$K = \frac{1}{2}(6.64 \times 10^{-27} \text{ kg})(1.52 \times 10^7 \text{ m/s})^2 = 7.67 \times 10^{-13} \text{ J} = 4.79 \text{ MeV}$$

Alpha particles are always emitted with definite kinetic energies, determined by conservation of momentum and energy. Because of their charge and mass, alpha particles can travel only several centimeters in air, or a few tenths or hundredths of a millimeter through solids, before they are brought to rest by collisions.

Some nuclei can spontaneously decay by emission of α particles because energy is released in their alpha decay. You can use conservation of mass-energy to show that

alpha decay is possible whenever the mass of the original neutral atom is greater than the sum of the masses of the final neutral atom and the neutral helium-4 atom.

In alpha decay, the α particle tunnels through a potential-energy barrier, as Fig. 43.5b shows. You may want to review the discussion of tunneling in Section 40.4.

Example 43.5 Alpha decay of radium

Show that the α -emission process $^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^4_2\text{He}$ is energetically possible, and calculate the kinetic energy of the emitted α particle. The neutral atomic masses are 226.025403 u for $^{226}_{88}\text{Ra}$ and 222.017571 u for $^{222}_{86}\text{Rn}$.

SOLUTION

IDENTIFY and SET UP: Alpha emission is possible if the mass of the $^{226}_{88}\text{Ra}$ atom is greater than the sum of the atomic masses of $^{222}_{86}\text{Rn}$ and ^4_2He . The mass difference between the initial radium atom and the final radon and helium atoms corresponds (through $E = mc^2$) to the energy E released in the decay. Because momentum is conserved as well as energy, both the alpha particle and the $^{222}_{86}\text{Rn}$ atom are in motion after the decay; we will have to account for this in determining the kinetic energy of the alpha particle.

EXECUTE: From Table 43.2, the mass of the ^4_2He atom is 4.002603 u. The difference in mass between the original nucleus and the decay products is

$$226.025403 \text{ u} - (222.017571 \text{ u} + 4.002603 \text{ u}) = +0.005229 \text{ u}$$

Since this is positive, α decay is energetically possible. The energy equivalent of this mass difference is

$$E = (0.005229 \text{ u})(931.5 \text{ MeV/u}) = 4.871 \text{ MeV}$$

Thus we expect the decay products to emerge with total kinetic energy 4.871 MeV. Momentum is also conserved; if the parent $^{226}_{88}\text{Ra}$ nucleus is at rest, the daughter $^{222}_{86}\text{Rn}$ nucleus and the α particle have momenta of equal magnitude p but opposite direction. Kinetic

Continued

energy is $K = \frac{1}{2}mv^2 = p^2/2m$: Since p is the same for the two particles, the kinetic energy divides inversely as their masses. Hence the α particle gets $222/(222 + 4)$ of the total, or 4.78 MeV.

EVALUATE: Experiment shows that ${}^{226}_{88}\text{Ra}$ does undergo alpha decay, and the observed α -particle energy is 4.78 MeV. You can check your results by verifying that the alpha particle and the

${}^{222}_{86}\text{Rn}$ nucleus produced in the decay have the same magnitude of momentum $p = mv$. You can calculate the speed v of each of the decay products from its respective kinetic energy. You'll find that the alpha particle moves at a sprightly $0.0506c = 1.52 \times 10^7$ m/s; if momentum is conserved, you should find that the ${}^{222}_{86}\text{Rn}$ nucleus moves $\frac{4}{222}$ as fast. Does it?

Beta Decay

There are three different simple types of *beta decay*: *beta-minus*, *beta-plus*, and *electron capture*. A **beta-minus particle** (β^-) is an electron. It's not obvious how a nucleus can emit an electron if there aren't any electrons in the nucleus. Emission of a β^- involves *transformation* of a neutron into a proton, an electron, and a third particle called an *antineutrino*. In fact, if you freed a neutron from a nucleus, it would decay into a proton, an electron, and an antineutrino in an average time of about 15 minutes.

Beta particles can be identified and their speeds can be measured with techniques that are similar to the Thomson experiments we described in Section 27.5. The speeds of beta particles range up to 0.9995 of the speed of light, so their motion is highly relativistic. They are emitted with a continuous spectrum of energies. This would not be possible if the only two particles were the β^- and the recoiling nucleus, since energy and momentum conservation would then require a definite speed for the β^- . Thus there must be a *third* particle involved. From conservation of charge, it must be neutral, and from conservation of angular momentum, it must be a spin- $\frac{1}{2}$ particle.

This third particle is an antineutrino, the *antiparticle* of a **neutrino**. The symbol for a neutrino is ν_e (the Greek letter nu). Both the neutrino and the antineutrino have zero charge and zero (or very small) mass and therefore produce very little observable effect when passing through matter. Both evaded detection until 1953, when Frederick Reines and Clyde Cowan succeeded in observing the antineutrino directly. We now know that there are at least three varieties of neutrinos, each with its corresponding antineutrino; one is associated with beta decay and the other two are associated with the decay of two unstable particles, the muon and the tau particle. We'll discuss these particles in more detail in Chapter 44. The antineutrino that is emitted in β^- decay is denoted as $\bar{\nu}_e$. The basic process of β^- decay is



Beta-minus decay usually occurs with nuclides for which the neutron-to-proton ratio N/Z is too large for stability. In β^- decay, N decreases by 1, Z increases by 1, and A doesn't change. You can use conservation of mass-energy to show that

beta-minus decay can occur whenever the mass of the original neutral atom is larger than that of the final atom.

Example 43.6 Why cobalt-60 is a beta-minus emitter

The nuclide ${}^{60}_{27}\text{Co}$, an odd-odd unstable nucleus, is used in medical and industrial applications of radiation. Show that it is unstable relative to β^- decay. The atomic masses you need are 59.933822 u for ${}^{60}_{27}\text{Co}$ and 59.930791 u for ${}^{60}_{28}\text{Ni}$.

SOLUTION

IDENTIFY and SET UP: Beta-minus decay is possible if the mass of the original neutral atom is greater than that of the final atom.

We must first identify the nuclide that will result if ${}^{60}_{27}\text{Co}$ undergoes β^- decay and then compare its neutral atomic mass to that of ${}^{60}_{27}\text{Co}$.

EXECUTE: In the presumed β^- decay of ${}^{60}_{27}\text{Co}$, Z increases by 1 from 27 to 28 and A remains at 60, so the final nuclide is ${}^{60}_{28}\text{Ni}$. The neutral atomic mass of ${}^{60}_{27}\text{Co}$ is greater than that of ${}^{60}_{28}\text{Ni}$ by 0.003031 u, so β^- decay *can* occur.

EVALUATE: With three decay products in β^- decay—the ${}_{28}^{60}\text{Ni}$ nucleus, the electron, and the antineutrino—the energy can be shared in many different ways that are consistent with conservation of energy and momentum. It's impossible to predict precisely how the energy will

be shared for the decay of a particular ${}_{27}^{60}\text{Co}$ nucleus. By contrast, in alpha decay there are just two decay products, and their energies and momenta are determined uniquely (see Example 43.5).

We have noted that β^- decay occurs with nuclides that have too large a neutron-to-proton ratio N/Z . Nuclides for which N/Z is too *small* for stability can emit a *positron*, the electron's antiparticle, which is identical to the electron but with positive charge. (We'll discuss the positron in more detail in Chapter 44.) The basic process, called *beta-plus decay* (β^+), is



where β^+ is a positron and ν_e is the electron neutrino.

Beta-plus decay can occur whenever the mass of the original neutral atom is at least two electron masses larger than that of the final atom.

You can show this using conservation of mass-energy.

The third type of beta decay is *electron capture*. There are a few nuclides for which β^+ emission is not energetically possible but in which an orbital electron (usually in the K shell) can combine with a proton in the nucleus to form a neutron and a neutrino. The neutron remains in the nucleus and the neutrino is emitted. The basic process is



You can use conservation of mass-energy to show that

electron capture can occur whenever the mass of the original neutral atom is larger than that of the final atom.

In all types of beta decay, A remains constant. However, in beta-plus decay and electron capture, N increases by 1 and Z decreases by 1 as the neutron-proton ratio increases toward a more stable value. The reaction of Eq. (43.15) also helps to explain the formation of a neutron star, mentioned in Example 43.1.

CAUTION **Beta decay inside and outside nuclei** The beta-decay reactions given by Eqs. (43.13), (43.14), and (43.15) occur *within* a nucleus. Although the decay of a neutron outside the nucleus proceeds through the reaction of Eq. (43.13), the reaction of Eq. (43.14) is forbidden by conservation of mass-energy for a proton outside the nucleus. The reaction of Eq. (43.15) can occur outside the nucleus only with the addition of some extra energy, as in a collision. **|**

Example 43.7 Why cobalt-57 is not a beta-plus emitter

The nuclide ${}_{27}^{57}\text{Co}$ is an odd-even unstable nucleus. Show that it cannot undergo β^+ decay, but that it *can* decay by electron capture. The atomic masses you need are 56.936296 u for ${}_{27}^{57}\text{Co}$ and 56.935399 u for ${}_{26}^{57}\text{Fe}$.

SOLUTION

IDENTIFY and SET UP: Beta-plus decay is possible if the mass of the original neutral atom is greater than that of the final atom plus two electron masses (0.001097 u). Electron capture is possible if the mass of the original atom is greater than that of the final atom. We must first identify the nuclide that will result if ${}_{27}^{57}\text{Co}$ undergoes β^+ decay or electron capture and then find the corresponding mass difference.

EXECUTE: The original nuclide is ${}_{27}^{57}\text{Co}$. In both the presumed β^+ decay and electron capture, Z decreases by 1 from 27 to 26, and A remains at 57, so the final nuclide is ${}_{26}^{57}\text{Fe}$. Its mass is less than that of ${}_{27}^{57}\text{Co}$ by 0.000897 u, a value smaller than 0.001097 u (two electron masses), so β^+ decay *cannot* occur. However, the mass of the original atom is greater than the mass of the final atom, so electron capture *can* occur.

EVALUATE: In electron capture there are just two decay products, the final nucleus and the emitted neutrino. As in alpha decay (Example 43.5) but unlike in β^- decay (Example 43.6), the decay products of electron capture have unique energies and momenta. In Section 43.4 we'll see how to relate the probability that electron capture will occur to the *half-life* of this nuclide.

Gamma Decay

The energy of internal motion of a nucleus is quantized. A typical nucleus has a set of allowed energy levels, including a *ground state* (state of lowest energy) and several *excited states*. Because of the great strength of nuclear interactions, excitation energies of nuclei are typically of the order of 1 MeV, compared with a few eV for atomic energy levels. In ordinary physical and chemical transformations the nucleus always remains in its ground state. When a nucleus is placed in an excited state, either by bombardment with high-energy particles or by a radioactive transformation, it can decay to the ground state by emission of one or more photons called **gamma rays** or *gamma-ray photons*, with typical energies of 10 keV to 5 MeV. This process is called *gamma (γ) decay*. For example, alpha particles emitted from ^{226}Ra have two possible kinetic energies, either 4.784 MeV or 4.602 MeV. Including the recoil energy of the resulting ^{222}Rn nucleus, these correspond to a total released energy of 4.871 MeV or 4.685 MeV, respectively. When an alpha particle with the smaller energy is emitted, the ^{222}Rn nucleus is left in an excited state. It then decays to its ground state by emitting a gamma-ray photon with energy

$$(4.871 - 4.685) \text{ MeV} = 0.186 \text{ MeV}$$

A photon with this energy is observed during this decay (Fig. 43.5c).

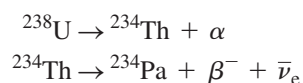
CAUTION γ decay vs. α and β decay In both α and β decay, the Z value of a nucleus changes and the nucleus of one element becomes the nucleus of a different element. In γ decay, the element does *not* change; the nucleus merely goes from an excited state to a less excited state. |

Natural Radioactivity

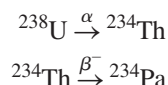
Many radioactive elements occur in nature. For example, you are very slightly radioactive because of unstable nuclides such as carbon-14 and potassium-40 that are present throughout your body. The study of natural radioactivity began in 1896, one year after Röntgen discovered x rays. Henri Becquerel discovered a radiation from uranium salts that seemed similar to x rays. Intensive investigation in the following two decades by Marie and Pierre Curie, Ernest Rutherford, and many others revealed that the emissions consist of positively and negatively charged particles and neutral rays; they were given the names *alpha*, *beta*, and *gamma* because of their differing penetration characteristics.

The decaying nucleus is usually called the *parent nucleus*; the resulting nucleus is the *daughter nucleus*. When a radioactive nucleus decays, the daughter nucleus may also be unstable. In this case a *series* of successive decays occurs until a stable configuration is reached. Several such series are found in nature. The most abundant radioactive nuclide found on earth is the uranium isotope ^{238}U , which undergoes a series of 14 decays, including eight α emissions and six β^- emissions, terminating at a stable isotope of lead, ^{206}Pb (Fig. 43.6).

Radioactive decay series can be represented on a Segrè chart, as in Fig. 43.7. The neutron number N is plotted vertically, and the atomic number Z is plotted horizontally. In alpha emission, both N and Z decrease by 2. In β^- emission, N decreases by 1 and Z increases by 1. The decays can also be represented in equation form; the first two decays in the series are written as

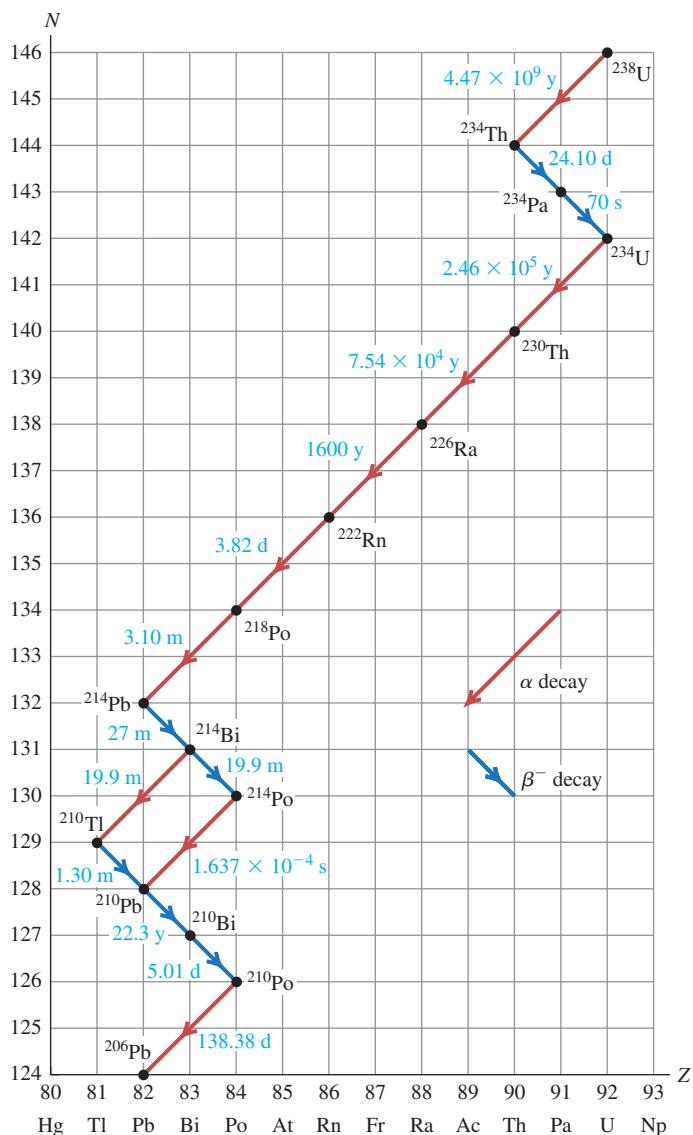


or more briefly as



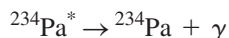
43.6 Earthquakes are caused in part by the radioactive decay of ^{238}U in the earth's interior. These decays release energy that helps to produce convection currents in the earth's interior. Such currents drive the motions of the earth's crust, including the sudden sharp motions that we call earthquakes (like the one that caused this damage).



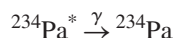


43.7 Segrè chart showing the uranium ^{238}U decay series, terminating with the stable nuclide ^{206}Pb . The times are half-lives (discussed in the next section), given in years (y), days (d), hours (h), minutes (m), or seconds (s).

In the second process, the beta decay leaves the daughter nucleus ^{234}Pa in an excited state, from which it decays to the ground state by emitting a gamma-ray photon. An excited state is denoted by an asterisk, so we can represent the γ emission as



or



An interesting feature of the ^{238}U decay series is the branching that occurs at ^{214}Bi . This nuclide decays to ^{210}Pb by emission of an α and a β^- , which can occur in either order. We also note that the series includes unstable isotopes of several elements that also have stable isotopes, including thallium (Tl), lead (Pb), and bismuth (Bi). The unstable isotopes of these elements that occur in the ^{238}U series all have too many neutrons to be stable.

Many other decay series are known. Two of these occur in nature, one starting with the uncommon isotope ^{235}U and ending with ^{207}Pb , the other starting with thorium (^{232}Th) and ending with ^{208}Pb .

Test Your Understanding of Section 43.3 A nucleus with atomic number Z and neutron number N undergoes two decay processes. The result is a nucleus with atomic number $Z - 3$ and neutron number $N - 1$. Which decay processes may have taken place? (i) two β^- decays; (ii) two β^+ decays; (iii) two α decays; (iv) an α decay and a β^- decay; (v) an α decay and a β^+ decay.



43.4 Activities and Half-Lives

Suppose you need to dispose of some radioactive waste that contains a certain number of nuclei of a particular radioactive nuclide. If no more are produced, that number decreases in a simple manner as the nuclei decay. This decrease is a statistical process; there is no way to predict when any individual nucleus will decay. No change in physical or chemical environment, such as chemical reactions or heating or cooling, greatly affects most decay rates. The rate varies over an extremely wide range for different nuclides.

Radioactive Decay Rates

Let $N(t)$ be the (very large) number of radioactive nuclei in a sample at time t , and let $dN(t)$ be the (negative) change in that number during a short time interval dt . (We'll use $N(t)$ to minimize confusion with the neutron number N .) The number of decays during the interval dt is $-dN(t)$. The rate of change of $N(t)$ is the negative quantity $dN(t)/dt$; thus $-dN(t)/dt$ is called the *decay rate* or the **activity** of the specimen. The larger the number of nuclei in the specimen, the more nuclei decay during any time interval. That is, the activity is directly proportional to $N(t)$; it equals a constant λ multiplied by $N(t)$:

$$-\frac{dN(t)}{dt} = \lambda N(t) \quad (43.16)$$

The constant λ is called the **decay constant**, and it has different values for different nuclides. A large value of λ corresponds to rapid decay; a small value corresponds to slower decay. Solving Eq. (43.16) for λ shows us that λ is the ratio of the number of decays per time to the number of remaining radioactive nuclei; λ can then be interpreted as the *probability per unit time* that any individual nucleus will decay.

The situation is reminiscent of a discharging capacitor, which we studied in Section 26.4. Equation (43.16) has the same form as the negative of Eq. (26.15), with q and $1/RC$ replaced by $N(t)$ and λ . Then we can make the same substitutions in Eq. (26.16), with the initial number of nuclei $N(0) = N_0$, to find the exponential function:

$$N(t) = N_0 e^{-\lambda t} \quad (\text{number of remaining nuclei}) \quad (43.17)$$

43.8 The number of nuclei in a sample of a radioactive element as a function of time. The sample's activity has an exponential decay curve with the same shape.

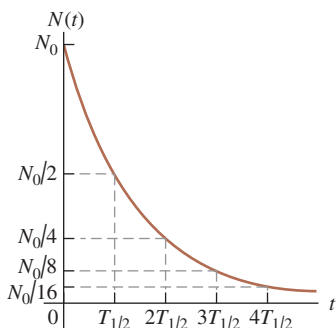


Figure 43.8 is a graph of this function, showing the number of remaining nuclei $N(t)$ as a function of time.

The **half-life** $T_{1/2}$ is the time required for the number of radioactive nuclei to decrease to one-half the original number N_0 . Then half of the remaining radioactive nuclei decay during a second interval $T_{1/2}$, and so on. The numbers remaining after successive half-lives are $N_0/2$, $N_0/4$, $N_0/8$, \dots

To get the relationship between the half-life $T_{1/2}$ and the decay constant λ , we set $N(t)/N_0 = \frac{1}{2}$ and $t = T_{1/2}$ in Eq. (43.17), obtaining

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

We take logarithms of both sides and solve for $T_{1/2}$:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \quad (43.18)$$

The mean lifetime T_{mean} , generally called the *lifetime*, of a nucleus or unstable particle is proportional to the half-life $T_{1/2}$:

$$T_{\text{mean}} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} \quad (\text{lifetime } T_{\text{mean}}, \text{ decay constant } \lambda, \text{ and half-life } T_{1/2}) \quad (43.19)$$

In particle physics the life of an unstable particle is usually described by the lifetime, not the half-life.

Because the activity $-dN(t)/dt$ at any time equals $\lambda N(t)$, Eq. (43.17) tells us that the activity also depends on time as $e^{-\lambda t}$. Thus the graph of activity versus time has the same shape as Fig. 43.8. Also, after successive half-lives, the activity is one-half, one-fourth, one-eighth, and so on of the original activity.

CAUTION A half-life may not be enough It is sometimes implied that any radioactive sample will be safe after a half-life has passed. That's wrong. If your radioactive waste initially has ten times too much activity for safety, it is not safe after one half-life, when it still has five times too much. Even after three half-lives it still has 25% more activity than is safe. The number of radioactive nuclei and the activity approach zero only as t approaches infinity. ■

A common unit of activity is the **curie**, abbreviated Ci, which is defined to be 3.70×10^{10} decays per second. This is approximately equal to the activity of one gram of radium. The SI unit of activity is the *becquerel*, abbreviated Bq. One becquerel is one decay per second, so

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq} = 3.70 \times 10^{10} \text{ decays/s}$$

Example 43.8 Activity of ^{57}Co

The isotope ^{57}Co decays by electron capture to ^{57}Fe with a half-life of 272 d. The ^{57}Fe nucleus is produced in an excited state, and it almost instantaneously emits gamma rays that we can detect. (a) Find the mean lifetime and decay constant for ^{57}Co . (b) If the activity of a ^{57}Co radiation source is now $2.00 \mu\text{Ci}$, how many ^{57}Co nuclei does the source contain? (c) What will be the activity after one year?

SOLUTION

IDENTIFY and SET UP: This problem uses the relationships among decay constant λ , lifetime T_{mean} , and activity $-dN(t)/dt$. In part (a) we use Eq. (43.19) to find λ and T_{mean} from $T_{1/2}$. In part (b), we use Eq. (43.16) to calculate the number of nuclei $N(t)$ from the activity. Finally, in part (c) we use Eqs. (43.16) and (43.17) to find the activity after one year.

EXECUTE: (a) It's convenient to convert the half-life to seconds:

$$T_{1/2} = (272 \text{ d})(86,400 \text{ s/d}) = 2.35 \times 10^7 \text{ s}$$

From Eq. (43.19), the mean lifetime and decay constant are

$$T_{\text{mean}} = \frac{T_{1/2}}{\ln 2} = \frac{2.35 \times 10^7 \text{ s}}{0.693} = 3.39 \times 10^7 \text{ s} = 392 \text{ days}$$

$$\lambda = \frac{1}{T_{\text{mean}}} = 2.95 \times 10^{-8} \text{ s}^{-1}$$

(b) The activity $-dN(t)/dt$ is given as $2.00 \mu\text{Ci}$, so

$$\begin{aligned} -\frac{dN(t)}{dt} &= 2.00 \mu\text{Ci} = (2.00 \times 10^{-6})(3.70 \times 10^{10} \text{ s}^{-1}) \\ &= 7.40 \times 10^4 \text{ decays/s} \end{aligned}$$

From Eq. (43.16) this is equal to $\lambda N(t)$, so we find

$$N(t) = -\frac{dN(t)/dt}{\lambda} = \frac{7.40 \times 10^4 \text{ s}^{-1}}{2.95 \times 10^{-8} \text{ s}^{-1}} = 2.51 \times 10^{12} \text{ nuclei}$$

If you feel we're being too cavalier about the "units" decays and nuclei, you can use decays/(nucleus \cdot s) as the unit for λ .

(c) From Eq. (43.17) the number $N(t)$ of nuclei remaining after one year ($3.156 \times 10^7 \text{ s}$) is


$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-(2.95 \times 10^{-8} \text{ s}^{-1})(3.156 \times 10^7 \text{ s})} = 0.394 N_0$$

The number of nuclei has decreased to 0.394 of the original number. Equation (43.16) says that the activity is proportional to the number of nuclei, so the activity has decreased by this same factor to $(0.394)(2.00 \mu\text{Ci}) = 0.788 \mu\text{Ci}$.

EVALUATE: The number of nuclei found in part (b) is equivalent to $4.17 \times 10^{-12} \text{ mol}$, with a mass of $2.38 \times 10^{-10} \text{ g}$. This is a far smaller mass than even the most sensitive balance can measure.

After one 272-day half-life, the number of ^{57}Co nuclei has decreased to $N_0/2$; after $2(272 \text{ d}) = 544 \text{ d}$, it has decreased to $N_0/2^2 = N_0/4$. This result agrees with our answer to part (c), which says that after 365 d the number of nuclei is between $N_0/2$ and $N_0/4$.

Radioactive Dating

An interesting application of radioactivity is the dating of archaeological and geological specimens by measuring the concentration of radioactive isotopes.  The most familiar example is *carbon dating*. The unstable isotope ^{14}C , produced during nuclear reactions in the atmosphere that result from cosmic-ray bombardment, gives a small proportion of ^{14}C in the CO_2 in the atmosphere. Plants that obtain their carbon from this source contain the same proportion of ^{14}C as the atmosphere. When a plant dies, it stops taking in carbon, and its ^{14}C β^- decays to ^{14}N with a half-life of 5730 years. By measuring the proportion of ^{14}C in the remains, we can determine how long ago the organism died.

One difficulty with radiocarbon dating is that the ^{14}C concentration in the atmosphere changes over long time intervals. Corrections can be made on the basis of other data such as measurements of tree rings that show annual growth cycles. Similar radioactive techniques are used with other isotopes for dating geological specimens. Some rocks, for example, contain the unstable potassium isotope ^{40}K , a beta emitter that decays to the stable nuclide ^{40}Ar with a half-life of 2.4×10^8 y. The age of the rock can be determined by comparing the concentrations of ^{40}K and ^{40}Ar .

Example 43.9 Radiocarbon dating

Before 1900 the activity per unit mass of atmospheric carbon due to the presence of ^{14}C averaged about 0.255 Bq per gram of carbon. (a) What fraction of carbon atoms were ^{14}C ? (b) In analyzing an archaeological specimen containing 500 mg of carbon, you observe 174 decays in one hour. What is the age of the specimen, assuming that its activity per unit mass of carbon when it died was that average value of the air?

SOLUTION

IDENTIFY and SET UP: The key idea is that the present-day activity of a biological sample containing ^{14}C is related to both the elapsed time since it stopped taking in atmospheric carbon and its activity at that time. We use Eqs. (43.16) and (43.17) to solve for the age t of the specimen. In part (a) we determine the number of ^{14}C atoms $N(t)$ from the activity $-dN(t)/dt$ using Eq. (43.16). We find the total number of carbon atoms in 500 mg by using the molar mass of carbon (12.011 g/mol, given in Appendix D), and we use the result to calculate the fraction of carbon atoms that are ^{14}C . The activity decays at the same rate as the number of ^{14}C nuclei; we use this and Eq. (43.17) to solve for the age t of the specimen.

EXECUTE: (a) To use Eq. (43.16), we must first find the decay constant λ from Eq. (43.18):

$$T_{1/2} = 5730 \text{ y} = (5730 \text{ y})(3.156 \times 10^7 \text{ s/y}) = 1.808 \times 10^{11} \text{ s}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{1.808 \times 10^{11} \text{ s}} = 3.83 \times 10^{-12} \text{ s}^{-1}$$

Then, from Eq. (43.16),

$$N(t) = \frac{-dN/dt}{\lambda} = \frac{0.255 \text{ s}^{-1}}{3.83 \times 10^{-12} \text{ s}^{-1}} = 6.65 \times 10^{10} \text{ atoms}$$

The *total* number of C atoms in 1 gram ($1/12.011$ mol) is $(1/12.011)(6.022 \times 10^{23}) = 5.01 \times 10^{22}$. The ratio of ^{14}C atoms to all C atoms is

$$\frac{6.65 \times 10^{10}}{5.01 \times 10^{22}} = 1.33 \times 10^{-12}$$

Only four carbon atoms in every 3×10^{12} are ^{14}C .

(b) Assuming that the activity per gram of carbon in the specimen when it died ($t = 0$) was $0.255 \text{ Bq/g} = (0.255 \text{ s}^{-1} \cdot \text{g}^{-1})(3600 \text{ s/h}) = 918 \text{ h}^{-1} \cdot \text{g}^{-1}$, the activity of 500 mg of carbon then was $(0.500 \text{ g})(918 \text{ h}^{-1} \cdot \text{g}^{-1}) = 459 \text{ h}^{-1}$. The observed activity now, at time t , is 174 h^{-1} . Since the activity is proportional to the number of radioactive nuclei, the activity ratio $174/459 = 0.379$ equals the number ratio $N(t)/N_0$.

Now we solve Eq. (43.17) for t and insert values for $N(t)/N_0$ and λ :

$$t = \frac{\ln(N(t)/N_0)}{-\lambda} = \frac{\ln 0.379}{-3.83 \times 10^{-12} \text{ s}^{-1}} = 2.53 \times 10^{11} \text{ s} = 8020 \text{ y}$$

EVALUATE: After 8020 y the ^{14}C activity has decreased from 459 to 174 decays per hour. The specimen died and stopped taking CO_2 out of the air about 8000 years ago.

Radiation in the Home

A serious health hazard in some areas is the accumulation in houses of ^{222}Rn , an inert, colorless, odorless radioactive gas. Looking at the ^{238}U decay chain in Fig. 43.7, we see that the half-life of ^{222}Rn is 3.82 days. If so, why not just move out of the house for a while and let it decay away? The answer is that ^{222}Rn is continuously being *produced* by the decay of ^{226}Ra , which is found in minute quantities in the rocks and soil on which some houses are built. It's a dynamic

equilibrium situation, in which the rate of production equals the rate of decay. The reason ^{222}Rn is a bigger hazard than the other elements in the ^{238}U decay series is that it's a gas. During its short half-life of 3.82 days it can migrate from the soil into your house. If a ^{222}Rn nucleus decays in your lungs, it emits a damaging α particle and its daughter nucleus ^{218}Po , which is *not* chemically inert and is likely to stay in your lungs until it decays, emits another damaging α particle and so on down the ^{238}U decay series.

How much of a hazard is radon? Although reports indicate values as high as 3500 pCi/L, the average activity per volume in the air inside American homes due to ^{222}Rn is about 1.5 pCi/L (over a thousand decays each second in an average-sized room). *If* your environment has this level of activity, it has been estimated that a lifetime exposure would reduce your life expectancy by about 40 days. For comparison, smoking one pack of cigarettes per day reduces life expectancy by 6 years, and it is estimated that the average emission from all the nuclear power plants in the world reduces life expectancy by anywhere from 0.01 day to 5 days. These figures include catastrophes such as the 1986 nuclear reactor disaster at Chernobyl, for which the *local* effect on life expectancy is much greater.

Test Your Understanding of Section 43.4 Which sample contains a greater number of nuclei: a 5.00- μCi sample of ^{240}Pu (half-life 6560 y) or a 4.45- μCi sample of ^{243}Am (half-life 7370 y)? (i) the ^{240}Pu sample; (ii) the ^{243}Am sample; (iii) both have the same number of nuclei.



I

43.5 Biological Effects of Radiation

The above discussion of radon introduced the interaction of radiation with living organisms, a topic of vital interest and importance. Under *radiation* we include radioactivity (alpha, beta, gamma, and neutrons) and electromagnetic radiation such as x rays. As these particles pass through matter, they lose energy, breaking molecular bonds and creating ions—hence the term *ionizing radiation*. Charged particles interact directly with the electrons in the material. X rays and γ rays interact by the photoelectric effect, in which an electron absorbs a photon and breaks loose from its site, or by Compton scattering (see Section 38.3). Neutrons cause ionization indirectly through collisions with nuclei or absorption by nuclei with subsequent radioactive decay of the resulting nuclei.

These interactions are extremely complex. It is well known that excessive exposure to radiation, including sunlight, x rays, and all the nuclear radiations, can destroy tissues. In mild cases it results in a burn, as with common sunburn. Greater exposure can cause very severe illness or death by a variety of mechanisms, including massive destruction of tissue cells, alterations of genetic material, and destruction of the components in bone marrow that produce red blood cells.

Calculating Radiation Doses

Radiation dosimetry is the quantitative description of the effect of radiation on living tissue. The *absorbed dose* of radiation is defined as the energy delivered to the tissue per unit mass. The SI unit of absorbed dose, the joule per kilogram, is called the *gray* (Gy); 1 Gy = 1 J/kg. Another unit is the *rad*, defined as 0.01 J/kg:

$$1 \text{ rad} = 0.01 \text{ J/kg} = 0.01 \text{ Gy}$$

Absorbed dose by itself is not an adequate measure of biological effect because equal energies of different kinds of radiation cause different extents of biological effect. This variation is described by a numerical factor called the **relative biological effectiveness (RBE)**, also called the *quality factor* (QF), of each specific radiation. X rays with 200 keV of energy are defined to have an

Table 43.3 Relative Biological Effectiveness (RBE) for Several Types of Radiation

Radiation	RBE (Sv/Gy or rem/rad)
X rays and γ rays	1
Electrons	1.0–1.5
Slow neutrons	3–5
Protons	10
α particles	20
Heavy ions	20

RBE of unity, and the effects of other radiations can be compared experimentally. Table 43.3 shows approximate values of RBE for several radiations. All these values depend somewhat on the kind of tissue in which the radiation is absorbed and on the energy of the radiation.

The biological effect is described by the product of the absorbed dose and the RBE of the radiation; this quantity is called the *biologically equivalent dose*, or simply the equivalent dose. The SI unit of equivalent dose for humans is the sievert (Sv):

$$\text{Equivalent dose (Sv)} = \text{RBE} \times \text{Absorbed dose (Gy)} \quad (43.20)$$

A more common unit, corresponding to the rad, is the rem (an abbreviation of *röntgen equivalent for man*):

$$\text{Equivalent dose (rem)} = \text{RBE} \times \text{Absorbed dose (rad)} \quad (43.21)$$

Thus the unit of the RBE is 1 Sv/Gy or 1 rem/rad, and 1 rem = 0.01 Sv.

Example 43.10 Dose from a medical x ray

During a diagnostic x-ray examination a 1.2-kg portion of a broken leg receives an equivalent dose of 0.40 mSv. (a) What is the equivalent dose in mrem? (b) What is the absorbed dose in mrad and in mGy? (c) If the x-ray energy is 50 keV, how many x-ray photons are absorbed?

SOLUTION

IDENTIFY and SET UP: We are asked to relate the equivalent dose (the biological effect of the radiation, measured in sieverts or rems) to the absorbed dose (the energy absorbed per mass, measured in grays or rads). In part (a) we use the conversion factor 1 rem = 0.01 Sv for equivalent dose. Table 43.3 gives the RBE for x rays; we use this value in part (b) to determine the absorbed dose using Eqs. (43.20) and (43.21). Finally, in part (c) we use the mass and the definition of absorbed dose to find the total energy absorbed and the total number of photons absorbed.

EXECUTE: (a) The equivalent dose in mrem is

$$\frac{0.40 \text{ mSv}}{0.01 \text{ Sv/rem}} = 40 \text{ mrem}$$

(b) For x rays, RBE = 1 rem/rad or 1 Sv/Gy, so the absorbed dose is

$$\frac{40 \text{ mrem}}{1 \text{ rem/rad}} = 40 \text{ mrad}$$

$$\frac{0.40 \text{ mSv}}{1 \text{ Sv/Gy}} = 0.40 \text{ mGy} = 4.0 \times 10^{-4} \text{ J/kg}$$

(c) The total energy absorbed is

$$(4.0 \times 10^{-4} \text{ J/kg})(1.2 \text{ kg}) = 4.8 \times 10^{-4} \text{ J} = 3.0 \times 10^{15} \text{ eV}$$

The number of x-ray photons is

$$\frac{3.0 \times 10^{15} \text{ eV}}{5.0 \times 10^4 \text{ eV/photon}} = 6.0 \times 10^{10} \text{ photons}$$

EVALUATE: The absorbed dose is relatively large because x rays have a low RBE. If the ionizing radiation had been a beam of α particles, for which RBE = 20, the absorbed dose needed for an equivalent dose of 0.40 mSv would be only 0.020 mGy, corresponding to a smaller total absorbed energy of $2.4 \times 10^{-5} \text{ J}$.

Radiation Hazards

Here are a few numbers for perspective. To convert from Sv to rem, simply multiply by 100. An ordinary chest x-ray exam delivers about 0.20–0.40 mSv to about 5 kg of tissue. Radiation exposure from cosmic rays and natural radioactivity in soil, building materials, and so on is of the order of 2–3 mSv per year at sea level and twice that at an elevation of 1500 m (5000 ft). A whole-body dose of up to about 0.20 Sv causes no immediately detectable effect. A short-term whole-body dose of 5 Sv or more usually causes death within a few days or weeks. A localized dose of 100 Sv causes complete destruction of the exposed tissues.

The long-term hazards of radiation exposure in causing various cancers and genetic defects have been widely publicized, and the question of whether there is any “safe” level of radiation exposure has been hotly debated. U.S. government regulations are based on a maximum *yearly* exposure, from all except natural resources, of 2 to 5 mSv. Workers with occupational exposure to radiation are permitted 50 mSv per year. Recent studies suggest that these limits are too high and that even extremely small exposures carry hazards, but it is very difficult to

gather reliable statistics on the effects of low doses. It has become clear that any use of x rays for medical diagnosis should be preceded by a very careful estimation of the relationship of risk to possible benefit.

Another sharply debated question is that of radiation hazards from nuclear power plants. The radiation level from these plants is *not* negligible. However, to make a meaningful evaluation of hazards, we must compare these levels with the alternatives, such as coal-powered plants. The health hazards of coal smoke are serious and well documented, and the natural radioactivity in the smoke from a coal-fired power plant is believed to be roughly 100 times as great as that from a properly operating nuclear plant with equal capacity. But the comparison is not this simple; the possibility of a nuclear accident and the very serious problem of safe disposal of radioactive waste from nuclear plants must also be considered. It is clearly impossible to eliminate *all* hazards to health. Our goal should be to try to take a rational approach to the problem of *minimizing* the hazard from all sources. Figure 43.9 shows one estimate of the various sources of radiation exposure for the U.S. population. Ionizing radiation is a two-edged sword; it poses very serious health hazards, yet it also provides many benefits to humanity, including the diagnosis and treatments of disease and a wide variety of analytical techniques.

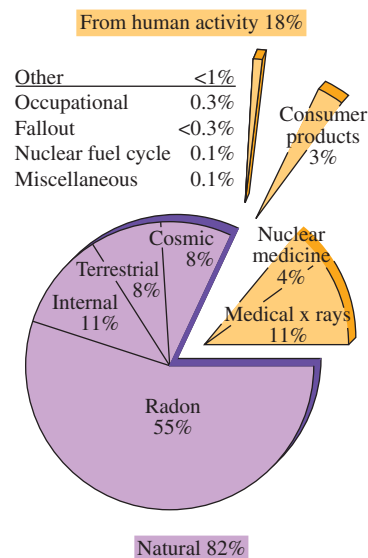
Beneficial Uses of Radiation

Radiation is widely used in medicine for intentional selective destruction of tissue such as tumors. The hazards are considerable, but if the disease would be fatal without treatment, any hazard may be preferable. Artificially produced isotopes are often used as radiation sources. Such isotopes have several advantages over naturally radioactive isotopes. They may have shorter half-lives and correspondingly greater activity. Isotopes can be chosen that emit the type and energy of radiation desired. Some artificial isotopes have been replaced by photon and electron beams from linear accelerators.

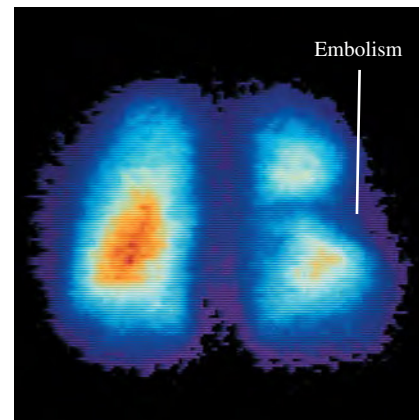
Nuclear medicine is an expanding field of application. Radioactive isotopes have virtually the same electron configurations and resulting chemical behavior as stable isotopes of the same element. But the location and concentration of radioactive isotopes can easily be detected by measurements of the radiation they emit. A familiar example is the use of radioactive iodine for thyroid studies. Nearly all the iodine ingested is either eliminated or stored in the thyroid, and the body's chemical reactions do not discriminate between the unstable isotope ^{131}I and the stable isotope ^{127}I . A minute quantity of ^{131}I is fed or injected into the patient, and the speed with which it becomes concentrated in the thyroid provides a measure of thyroid function. The half-life is 8.02 days, so there are no long-lasting radiation hazards. By use of more sophisticated scanning detectors, one can also obtain a "picture" of the thyroid, which shows enlargement and other abnormalities. This procedure, a type of *autoradiography*, is comparable to photographing the glowing filament of an incandescent light bulb by using the light emitted by the filament itself. If this process discovers cancerous thyroid nodules, they can be destroyed by much larger quantities of ^{131}I .

Another useful nuclide for nuclear medicine is technetium-99 (^{99}Tc), which is formed in an excited state by the β^- decay of molybdenum (^{99}Mo). The technetium then decays to its ground state by emitting a γ -ray photon with energy 143 keV. The half-life is 6.01 hours, unusually long for γ emission. (The ground state of ^{99}Tc is also unstable, with a half-life of 2.11×10^5 y; it decays by β^- emission to the stable ruthenium nuclide ^{99}Ru .) The chemistry of technetium is such that it can readily be attached to organic molecules that are taken up by various organs of the body. A small quantity of such technetium-bearing molecules is injected into a patient, and a scanning detector or *gamma camera* is used to produce an image, or *scintigram*, that reveals which parts of the body take up these γ -emitting molecules. This technique, in which ^{99}Tc acts as a radioactive *tracer*, plays an important role in locating cancers, embolisms, and other pathologies (Fig. 43.10).

43.9 Contribution of various sources to the total average radiation exposure in the U.S. population, expressed as percentages of the total.



43.10 This colored scintigram shows where a chemical containing radioactive ^{99}Tc was taken up by a patient's lungs. The orange color in the lung on the left indicates strong γ -ray emission by the ^{99}Tc , which shows that the chemical was able to pass into this lung through the bloodstream. The lung on the right shows weaker emission, indicating the presence of an embolism (a blood clot or other obstruction in an artery) that is restricting the flow of blood to this lung.



Tracer techniques have many other applications. Tritium (${}^3\text{H}$), a radioactive hydrogen isotope, is used to tag molecules in complex organic reactions; radioactive tags on pesticide molecules, for example, can be used to trace their passage through food chains. In the world of machinery, radioactive iron can be used to study piston-ring wear. Laundry detergent manufacturers have even tested the effectiveness of their products using radioactive dirt.

Many direct effects of radiation are also useful, such as strengthening polymers by cross-linking, sterilizing surgical tools, dispersing of unwanted static electricity in the air, and intentionally ionizing the air in smoke detectors. Gamma rays are also being used to sterilize and preserve some food products.

Test Your Understanding of Section 43.5 Alpha particles have 20 times the relative biological effectiveness of 200-keV x rays. Which would be better to use to radiate tissue deep inside the body? (i) a beam of alpha particles; (ii) a beam of 200-keV x rays; (iii) both are equally effective. I

43.6 Nuclear Reactions

In the preceding sections we studied the decay of unstable nuclei, especially spontaneous emission of an α or β particle, sometimes followed by γ emission. Nothing needs to be done to initiate this decay, and nothing can be done to control it. This section examines some *nuclear reactions*, rearrangements of nuclear components that result from a bombardment by a particle rather than a spontaneous natural process. Rutherford suggested in 1919 that a massive particle with sufficient kinetic energy might be able to penetrate a nucleus. The result would be either a new nucleus with greater atomic number and mass number or a decay of the original nucleus. Rutherford bombarded nitrogen (${}^{14}\text{N}$) with α particles and obtained an oxygen (${}^{17}\text{O}$) nucleus and a proton:



Rutherford used alpha particles from naturally radioactive sources. In Chapter 44 we'll describe some of the particle accelerators that are now used to initiate nuclear reactions.

Nuclear reactions are subject to several *conservation laws*. The classical conservation principles for charge, momentum, angular momentum, and energy (including rest energies) are obeyed in all nuclear reactions. An additional conservation law, not anticipated by classical physics, is conservation of the total number of nucleons. The numbers of protons and neutrons need not be conserved separately; in β decay, neutrons and protons change into one another. We'll study the basis of the conservation of nucleon number in Chapter 44.

When two nuclei interact, charge conservation requires that the sum of the initial atomic numbers must equal the sum of the final atomic numbers. Because of conservation of nucleon number, the sum of the initial mass numbers must also equal the sum of the final mass numbers. In general, these are *not* elastic collisions, and the total initial mass does *not* equal the total final mass.

Reaction Energy

The difference between the masses before and after the reaction corresponds to the **reaction energy**, according to the mass–energy relationship $E = mc^2$. If initial particles A and B interact to produce final particles C and D , the reaction energy Q is defined as

$$Q = (M_A + M_B - M_C - M_D)c^2 \quad (\text{reaction energy}) \quad (43.23)$$

To balance the electrons, we use the neutral atomic masses in Eq. (43.23). That is, we use the mass of ${}^1_1\text{H}$ for a proton, ${}^2_1\text{H}$ for a deuteron, ${}^4_2\text{He}$ for an α particle,

and so on. When Q is positive, the total mass decreases and the total kinetic energy increases. Such a reaction is called an *exoergic reaction*. When Q is negative, the mass increases and the kinetic energy decreases, and the reaction is called an *endoergic reaction*. The terms *exothermal* and *endothermal*, borrowed from chemistry, are also used. In an endoergic reaction the reaction cannot occur at all unless the initial kinetic energy in the center-of-mass reference frame is at least as great as $|Q|$. That is, there is a **threshold energy**, the minimum kinetic energy to make an endoergic reaction go.

Example 43.11 Exoergic and endoergic reactions

(a) When a lithium-7 nucleus is bombarded by a proton, two alpha particles (${}^4\text{He}$) are produced. Find the reaction energy.
 (b) Calculate the reaction energy for the reaction ${}^4_2\text{He} + {}^{14}_7\text{N} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H}$.

SOLUTION

IDENTIFY and SET UP: The reaction energy Q for any nuclear reaction equals c^2 times the difference between the total initial mass and the total final mass, as in Eq. (43.23). Table 43.2 gives the required masses.

EXECUTE: (a) The reaction is ${}^1_1\text{H} + {}^7_3\text{Li} \rightarrow 2({}^4_2\text{He})$. The initial and final masses and their respective sums are

A: ${}^1_1\text{H}$	1.007825 u	C: ${}^4_2\text{He}$	4.002603 u
B: ${}^7_3\text{Li}$	7.016004 u	D: ${}^4_2\text{He}$	4.002603 u
	8.023829 u		8.005206 u

The mass decreases by 0.018623 u. From Eq. (43.23), the reaction energy is

$$Q = (0.018623 \text{ u})(931.5 \text{ MeV/u}) = +17.35 \text{ MeV}$$

(b) The initial and final masses are

A: ${}^4_2\text{He}$	4.002603 u	C: ${}^{17}_8\text{O}$	16.999132 u
B: ${}^{14}_7\text{N}$	14.003074 u	D: ${}^1_1\text{H}$	1.007825 u
	18.005677 u		18.006957 u

The mass increases by 0.001280 u, and the corresponding reaction energy is

$$Q = (-0.001280 \text{ u})(931.5 \text{ MeV/u}) = -1.192 \text{ MeV}$$

EVALUATE: The reaction in part (a) is *exoergic*: The final total kinetic energy of the two separating alpha particles is 17.35 MeV greater than the initial total kinetic energy of the proton and the lithium nucleus. The reaction in part (b) is *endoergic*: In the center-of-mass system—that is, in a head-on collision with zero total momentum—the minimum total initial kinetic energy required for this reaction to occur is 1.192 MeV.

Ordinarily, the endoergic reaction of part (b) of Example 43.11 would be produced by bombarding stationary ${}^{14}\text{N}$ nuclei with alpha particles from an accelerator. In this case an alpha's kinetic energy must be *greater than* 1.192 MeV. If all the alpha's kinetic energy went solely to increasing the rest energy, the final kinetic energy would be zero, and momentum would not be conserved. When a particle with mass m and kinetic energy K collides with a stationary particle with mass M , the total kinetic energy K_{cm} in the center-of-mass coordinate system (the energy available to cause reactions) is

$$K_{\text{cm}} = \frac{M}{M + m} K \quad (43.24)$$

This expression assumes that the kinetic energies of the particles and nuclei are much less than their rest energies. We leave the derivation of Eq. (43.24) to you (see Problem 43.77). In the present example, $K_{\text{cm}} = (14.00/18.01)K$, so K must be at least $(18.01/14.00)(1.192 \text{ MeV}) = 1.533 \text{ MeV}$.

For a charged particle such as a proton or an α particle to penetrate the nucleus of another atom and cause a reaction, it must usually have enough initial kinetic energy to overcome the potential-energy barrier caused by the repulsive electrostatic forces. In the reaction of part (a) of Example 43.11, if we treat the proton and the ${}^7\text{Li}$ nucleus as spherically symmetric charges with radii given by Eq. (43.1), their centers will be $3.5 \times 10^{-15} \text{ m}$ apart when they touch. The repulsive potential

energy of the proton (charge $+e$) and the ${}^7\text{Li}$ nucleus (charge $+3e$) at this separation r is

$$U = \frac{1}{4\pi\epsilon_0} \frac{(e)(3e)}{r} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(3)(1.6 \times 10^{-19} \text{ C})^2}{3.5 \times 10^{-15} \text{ m}}$$

$$= 2.0 \times 10^{-13} \text{ J} = 1.2 \text{ MeV}$$

Even though the reaction is exoergic, the proton must have a minimum kinetic energy of about 1.2 MeV for the reaction to occur, unless the proton *tunnels* through the barrier (see Section 40.4).

Neutron Absorption

Absorption of *neutrons* by nuclei forms an important class of nuclear reactions. Heavy nuclei bombarded by neutrons can undergo a series of neutron absorptions alternating with beta decays, in which the mass number A increases by as much as 25. Some of the *transuranic elements*, elements having Z larger than 92, are produced in this way. These elements have not been found in nature. Many transuranic elements, having Z possibly as high as 118, have been identified.

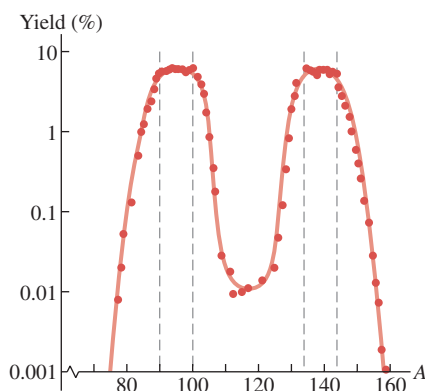
The analytical technique of *neutron activation analysis* uses similar reactions. When bombarded by neutrons, many stable nuclides absorb a neutron to become unstable and then undergo β^- decay. The energies of the β^- and associated γ emissions depend on the unstable nuclide and provide a means of identifying it and the original stable nuclide. Quantities of elements that are far too small for conventional chemical analysis can be detected in this way.

Test Your Understanding of Section 43.6 The reaction described in part (a) of Example 43.11 is exoergic. Can it happen naturally when a sample of solid lithium is placed in a flask of hydrogen gas?

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PhET: Nuclear Fission

43.11 Mass distribution of fission fragments from the fission of ${}^{236}\text{U}^*$ (an excited state of ${}^{236}\text{U}$), which is produced when ${}^{235}\text{U}$ absorbs a neutron. The vertical scale is logarithmic.



43.7 Nuclear Fission

Nuclear fission is a decay process in which an unstable nucleus splits into two fragments of comparable mass. Fission was discovered in 1938 through the experiments of Otto Hahn and Fritz Strassman in Germany. Pursuing earlier work by Fermi, they bombarded uranium ($Z = 92$) with neutrons. The resulting radiation did not coincide with that of any known radioactive nuclide. Urged on by their colleague Lise Meitner, they used meticulous chemical analysis to reach the astonishing but inescapable conclusion that they had found a radioactive isotope of barium ($Z = 56$). Later, radioactive krypton ($Z = 36$) was also found. Meitner and Otto Frisch correctly interpreted these results as showing that uranium nuclei were splitting into two massive fragments called *fission fragments*. Two or three free neutrons usually appear along with the fission fragments and, very occasionally, a light nuclide such as ${}^3\text{H}$.

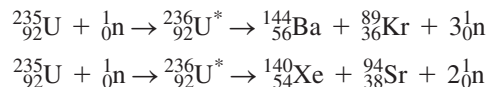
Both the common isotope (99.3%) ${}^{238}\text{U}$ and the uncommon isotope (0.7%) ${}^{235}\text{U}$ (as well as several other nuclides) can be easily split by neutron bombardment: ${}^{235}\text{U}$ by slow neutrons (kinetic energy less than 1 eV) but ${}^{238}\text{U}$ only by fast neutrons with a minimum of about 1 MeV of kinetic energy. Fission resulting from neutron absorption is called *induced fission*. Some nuclides can also undergo *spontaneous fission* without initial neutron absorption, but this is quite rare. When ${}^{235}\text{U}$ absorbs a neutron, the resulting nuclide ${}^{236}\text{U}^*$ is in a highly excited state and splits into two fragments almost instantaneously. Strictly speaking, it is ${}^{236}\text{U}^*$, not ${}^{235}\text{U}$, that undergoes fission, but it's usual to speak of the fission of ${}^{235}\text{U}$.

Over 100 different nuclides, representing more than 20 different elements, have been found among the fission products. Figure 43.11 shows the distribution of mass numbers for fission fragments from the fission of ${}^{235}\text{U}$. Most of the

fragments have mass numbers from 90 to 100 and from 135 to 145; fission into two fragments with nearly equal mass is unlikely.

Fission Reactions

You should check the following two typical fission reactions for conservation of nucleon number and charge:



The total kinetic energy of the fission fragments is enormous, about 200 MeV (compared to typical α and β energies of a few MeV). The reason for this is that nuclides at the high end of the mass spectrum (near $A = 240$) are less tightly bound than those nearer the middle ($A = 90$ to 145). Referring to Fig. 43.2, we see that the average binding energy per nucleon is about 7.6 MeV at $A = 240$ but about 8.5 MeV at $A = 120$. Therefore a rough estimate of the expected *increase* in binding energy during fission is about $8.5 \text{ MeV} - 7.6 \text{ MeV} = 0.9 \text{ MeV}$ per nucleon, or a total of $(235)(0.9 \text{ MeV}) \approx 200 \text{ MeV}$.

CAUTION **Binding energy and rest energy** It may seem to be a violation of conservation of energy to have an increase in both the binding energy and the kinetic energy during a fission reaction. But relative to the total rest energy E_0 of the separated nucleons, the rest energy of the nucleus is E_0 minus E_B . Thus an *increase* in binding energy corresponds to a *decrease* in rest energy as rest energy is converted to the kinetic energy of the fission fragments. ■

Fission fragments always have too many neutrons to be stable. We noted in Section 43.3 that the neutron–proton ratio (N/Z) for stable nuclides is about 1 for light nuclides but almost 1.6 for the heaviest nuclides because of the increasing influence of the electrical repulsion of the protons. The N/Z value for stable nuclides is about 1.3 at $A = 100$ and 1.4 at $A = 150$. The fragments have about the same N/Z as ${}^{235}\text{U}$, about 1.55. They usually respond to this surplus of neutrons by undergoing a series of β^- decays (each of which increases Z by 1 and decreases N by 1) until a stable value of N/Z is reached. A typical example is

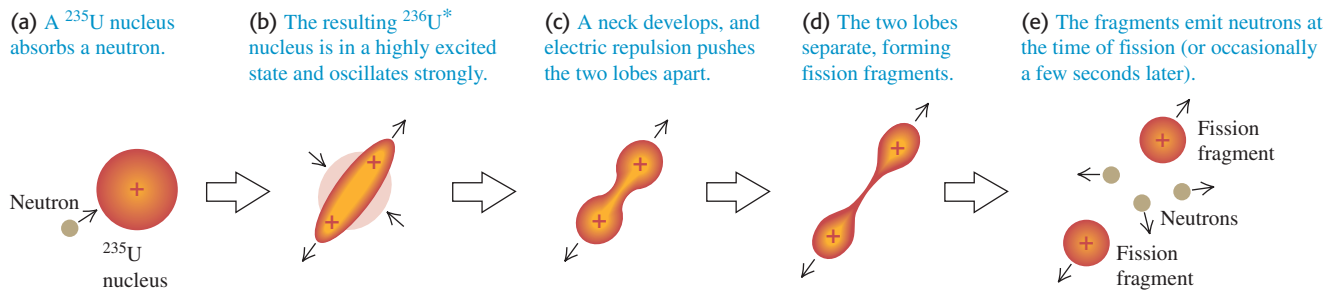


The nuclide ${}^{140}\text{Ce}$ is stable. This series of β^- decays produces, on average, about 15 MeV of additional kinetic energy. The neutron excess of fission fragments also explains why two or three free neutrons are released during the fission.

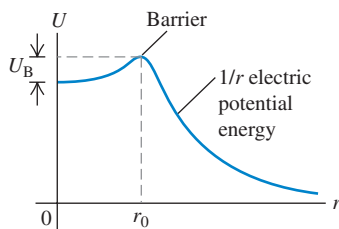
Fission appears to set an upper limit on the production of transuranic nuclei, mentioned in Section 43.6, that are relatively stable. There are theoretical reasons to expect that nuclei near $Z = 114$, $N = 184$ or 196 , might be stable with respect to spontaneous fission. In the shell model (see Section 43.2), these numbers correspond to filled shells and subshells in the nuclear energy-level structure. Such *superheavy nuclei* would still be unstable with respect to alpha emission. In 2009 it was confirmed that there are at least four isotopes with $Z = 114$, the longest-lived of which has a half-life due to alpha decay of about 2.6 s.

Liquid-Drop Model

We can understand fission qualitatively on the basis of the liquid-drop model of the nucleus (see Section 43.2). The process is shown in Fig. 43.12 in terms of an electrically charged liquid drop. These sketches shouldn't be taken too literally, but they may help to develop your intuition about fission. A ${}^{235}\text{U}$ nucleus absorbs a neutron (Fig. 43.12a), becoming a ${}^{236}\text{U}^*$ nucleus with excess energy (Fig. 43.12b). This excess energy causes violent oscillations, during which a neck between two lobes develops (Fig. 43.12c). The electric repulsion of these two lobes stretches the neck farther (Fig. 43.12d), and finally two smaller fragments are formed (Fig. 43.12e) that move rapidly apart.

43.12 A liquid-drop model of fission.

43.13 Hypothetical potential-energy function for two fission fragments in a fissionable nucleus. At distances r beyond the range of the nuclear force, the potential energy varies approximately as $1/r$. Fission occurs if there is an excitation energy greater than U_B or an appreciable probability for tunneling through the potential-energy barrier.

**Application Making Radioactive Isotopes for Medicine**

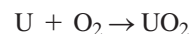
The fragments that result from nuclear fission are typically unstable, neutron-rich isotopes. A number of these are useful for medical diagnosis and cancer radiotherapy (see Section 43.5). This photograph shows a nuclear fission reactor used for producing such isotopes. The uranium fuel is kept in a large tank of water for cooling. Some of the neutron-rich fission fragments undergo beta decay and emit electrons that move faster than the speed of light in water (about $0.75c$). Like an airplane that produces an intense sonic boom when it flies faster than sound (see Section 16.9), these ultrafast electrons produce a “light boom” called *Čerenkov radiation* that has a characteristic blue color.



This qualitative picture has been developed into a more quantitative theory to explain why some nuclei undergo fission and others don't. Figure 43.13 shows a hypothetical potential-energy function for two possible fission fragments. If neutron absorption results in an excitation energy greater than the energy barrier height U_B , fission occurs immediately. Even when there isn't quite enough energy to surmount the barrier, fission can take place by quantum-mechanical *tunneling*, discussed in Section 40.4. In principle, many stable heavy nuclei can fission by tunneling. But the probability depends very critically on the height and width of the barrier. For most nuclei this process is so unlikely that it is never observed.

Chain Reactions

Fission of a uranium nucleus, triggered by neutron bombardment, releases other neutrons that can trigger more fissions, suggesting the possibility of a **chain reaction** (Fig. 43.14). The chain reaction may be made to proceed slowly and in a controlled manner in a nuclear reactor or explosively in a bomb. The energy release in a nuclear chain reaction is enormous, far greater than that in any chemical reaction. (In a sense, *fire* is a chemical chain reaction.) For example, when uranium is “burned” to uranium dioxide in the chemical reaction



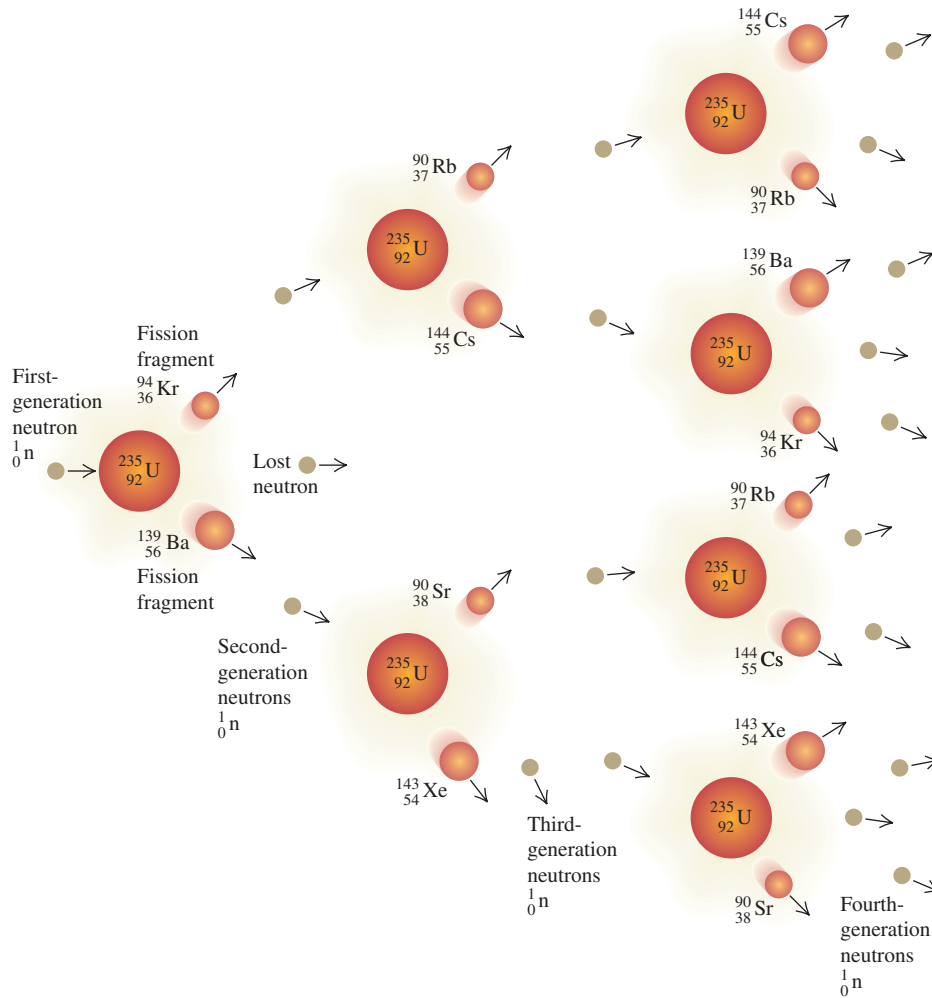
the heat of combustion is about 4500 J/g . Expressed as energy per atom, this is about 11 eV per atom. By contrast, fission liberates about 200 MeV per atom, nearly 20 million times as much energy.

Nuclear Reactors

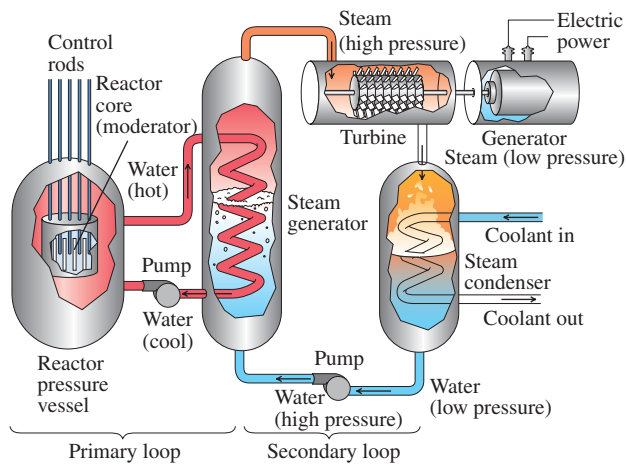
A *nuclear reactor* is a system in which a controlled nuclear chain reaction is used to liberate energy. In a nuclear power plant, this energy is used to generate steam, which operates a turbine and turns an electrical generator.

On average, each fission of a ^{235}U nucleus produces about 2.5 free neutrons, so 40% of the neutrons are needed to sustain a chain reaction. A ^{235}U nucleus is much more likely to absorb a low-energy neutron (less than 1 eV) than one of the higher-energy neutrons (1 MeV or so) that are liberated during fission. In a nuclear reactor the higher-energy neutrons are slowed down by collisions with nuclei in the surrounding material, called the *moderator*, so they are much more likely to cause further fissions. In nuclear power plants, the moderator is often water, occasionally graphite. The *rate* of the reaction is controlled by inserting or withdrawing *control rods* made of elements (such as boron or cadmium) whose nuclei *absorb* neutrons without undergoing any additional reaction. The isotope ^{238}U can also absorb neutrons, leading to $^{239}\text{U}^*$, but not with high enough probability for it to sustain a chain reaction by itself. Thus uranium that is used in reactors is often “enriched” by increasing the proportion of ^{235}U above the natural value of 0.7%, typically to 3% or so, by isotope-separation processing.

43.14 Schematic diagram of a nuclear fission chain reaction.



The most familiar application of nuclear reactors is for the generation of electric power. As was noted above, the fission energy appears as kinetic energy of the fission fragments, and its immediate result is to increase the internal energy of the fuel elements and the surrounding moderator. This increase in internal energy is transferred as heat to generate steam to drive turbines, which spin the electrical generators. Figure 43.15 is a schematic diagram of a nuclear power plant.



43.15 Schematic diagram of a nuclear power plant.

The energetic fission fragments heat the water surrounding the reactor core. The steam generator is a heat exchanger that takes heat from this highly radioactive water and generates nonradioactive steam to run the turbines.

A typical nuclear plant has an electric-generating capacity of 1000 MW (or 10^9 W). The turbines are heat engines and are subject to the efficiency limitations imposed by the second law of thermodynamics, discussed in Chapter 20. In modern nuclear plants the overall efficiency is about one-third, so 3000 MW of thermal power from the fission reaction is needed to generate 1000 MW of electrical power.

Example 43.12 Uranium consumption in a nuclear reactor

What mass of ^{235}U must undergo fission each day to provide 3000 MW of thermal power?

SOLUTION

IDENTIFY and SET UP: Fission of ^{235}U liberates about 200 MeV per atom. We use this and the mass of the ^{235}U atom to determine the required amount of uranium.

EXECUTE: Each second, we need 3000 MJ or 3000×10^6 J. Each fission provides 200 MeV, or

$$(200 \text{ MeV/fission})(1.6 \times 10^{-13} \text{ J/MeV}) = 3.2 \times 10^{-11} \text{ J/fission}$$

The number of fissions needed each second is

$$\frac{3000 \times 10^6 \text{ J}}{3.2 \times 10^{-11} \text{ J/fission}} = 9.4 \times 10^{19} \text{ fissions}$$

Each ^{235}U atom has a mass of $(235 \text{ u})(1.66 \times 10^{-27} \text{ kg/u}) = 3.9 \times 10^{-25} \text{ kg}$, so the mass of ^{235}U that undergoes fission each second is

$$(9.4 \times 10^{19})(3.9 \times 10^{-25} \text{ kg}) = 3.7 \times 10^{-5} \text{ kg} = 37 \mu\text{g}$$

In one day (86,400 s), the total consumption of ^{235}U is

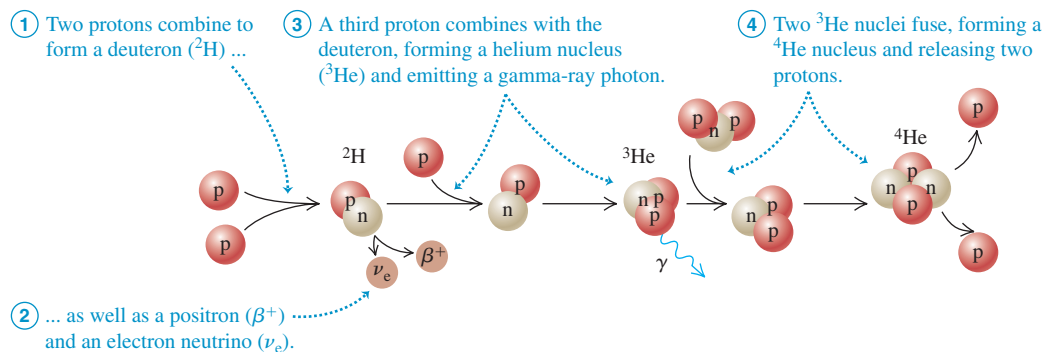
$$(3.7 \times 10^{-5} \text{ kg/s})(86,400 \text{ s}) = 3.2 \text{ kg}$$

EVALUATE: For comparison, a 1000-MW coal-fired power plant burns 10,600 tons (about 10 million kg) of coal per day!

We mentioned above that about 15 MeV of the energy released after fission of a ^{235}U nucleus comes from the β^- decays of the fission fragments. This fact poses a serious problem with respect to control and safety of reactors. Even after the chain reaction has been completely stopped by insertion of control rods into the core, heat continues to be evolved by the β^- decays, which cannot be stopped. For a 3000-MW reactor this heat power is initially very large, about 200 MW. In the event of total loss of cooling water, this power is more than enough to cause a catastrophic meltdown of the reactor core and possible penetration of the containment vessel. The difficulty in achieving a “cold shutdown” following an accident at the Three Mile Island nuclear power plant in Pennsylvania in March 1979 was a result of the continued evolution of heat due to β^- decays.

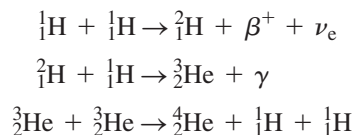
The catastrophe of April 26, 1986, at Chernobyl reactor No. 4 in Ukraine resulted from a combination of an inherently unstable design and several human errors committed during a test of the emergency core cooling system. Too many control rods were withdrawn to compensate for a decrease in power caused by a buildup of neutron absorbers such as ^{135}Xe . The power level rose from 1% of normal to 100 times normal in 4 seconds; a steam explosion ruptured pipes in the core cooling system and blew the heavy concrete cover off the reactor. The graphite moderator caught fire and burned for several days, and there was a meltdown of the core. The total activity of the radioactive material released into the atmosphere has been estimated as about 10^8 Ci.

Test Your Understanding of Section 43.7 The fission of ^{235}U can be triggered by the absorption of a slow neutron by a nucleus. Can a slow *proton* be used to trigger ^{235}U fission?

43.16 The proton-proton chain.**43.8 Nuclear Fusion**

In a **nuclear fusion** reaction, two or more small light nuclei come together, or *fuse*, to form a larger nucleus. Fusion reactions release energy for the same reason as fission reactions: The binding energy per nucleon after the reaction is greater than before. Referring to Fig. 43.2, we see that the binding energy per nucleon increases with A up to about $A = 60$, so fusion of nearly any two light nuclei to make a nucleus with A less than 60 is likely to be an exoergic reaction. In comparison to fission, we are moving toward the peak of this curve from the opposite side. Another way to express the energy relationships is that the total mass of the products is less than that of the initial particles.

Here are three examples of energy-liberating fusion reactions, written in terms of the neutral atoms:



In the first reaction, two protons combine to form a deuteron (${}^2\text{H}$), with the emission of a positron (β^+) and an electron neutrino. In the second, a proton and a deuteron combine to form the nucleus of the light isotope of helium, ${}^3\text{He}$, with the emission of a gamma ray. Now double the first two reactions to provide the two ${}^3\text{He}$ nuclei that fuse in the third reaction to form an alpha particle (${}^4\text{He}$) and two protons. Together the reactions make up the process called the *proton-proton chain* (Fig. 43.16).

The net effect of the chain is the conversion of four protons into one α particle, two positrons, two electron neutrinos, and two γ 's. We can calculate the energy release from this part of the process: The mass of an α particle plus two positrons is the mass of neutral ${}^4\text{He}$, the neutrinos have zero (or negligible) mass, and the gammas have zero mass.

Mass of four protons	4.029106 u
Mass of ${}^4\text{He}$	4.002603 u
Mass difference and energy release	0.026503 u and 24.69 MeV

The two positrons that are produced during the first step of the proton-proton chain collide with two electrons; mutual annihilation of the four particles takes place, and their rest energy is converted into $4(0.511 \text{ MeV}) = 2.044 \text{ MeV}$ of gamma radiation. Thus the total energy released is $(24.69 + 2.044) \text{ MeV} = 26.73 \text{ MeV}$. The proton-proton chain takes place in the interior of the sun and other stars (Fig. 43.17). Each gram of the sun's mass contains about 4.5×10^{23} protons. If all of these protons were fused into helium, the energy released would be about 130,000 kWh. If the sun were to continue to radiate at its present rate, it would take about 75×10^9 years to exhaust its supply of protons. As we will see below, fusion reactions can

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43.17 The energy released as starlight comes from fusion reactions deep within a star's interior. When a star is first formed and for most of its life, it converts the hydrogen in its core into helium. As a star ages, the core temperature can become high enough for additional fusion reactions that convert helium into carbon, oxygen, and other elements.



occur only at extremely high temperatures; in the sun, these temperatures are found only deep within the interior. Hence the sun cannot fuse *all* of its protons, and can sustain fusion for a total of only about 10×10^9 years in total. The present age of the solar system (including the sun) is 4.54×10^9 years, so the sun is about halfway through its available store of protons.

Example 43.13 A fusion reaction

Two deuterons fuse to form a *triton* (a nucleus of tritium, or ${}^3\text{H}$) and a proton. How much energy is liberated?

SOLUTION

IDENTIFY and SET UP: This is a nuclear reaction of the type discussed in Section 43.6. We find the energy released using Eq. (43.23).

EXECUTE: Adding one electron to each nucleus makes each a neutral atom; we find their masses in Table 43.2. Substituting into Eq. (43.23), we find

$$Q = [2(2.014102 \text{ u}) - 3.016049 \text{ u} - 1.007825 \text{ u}] \times (931.5 \text{ MeV/u}) = 4.03 \text{ MeV}$$

EVALUATE: Thus 4.03 MeV is released in the reaction; the triton and proton together have 4.03 MeV more kinetic energy than the two deuterons had together.

Achieving Fusion

For two nuclei to undergo fusion, they must come together to within the range of the nuclear force, typically of the order of 2×10^{-15} m. To do this, they must overcome the electrical repulsion of their positive charges. For two protons at this distance, the corresponding potential energy is about 1.2×10^{-13} J or 0.7 MeV; this represents the total initial *kinetic* energy that the fusion nuclei must have—for example, 0.6×10^{-13} J each in a head-on collision.

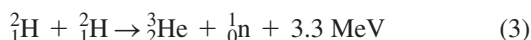
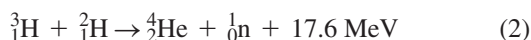
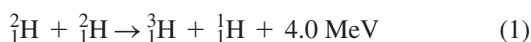
Atoms have this much energy only at extremely high temperatures. The discussion of Section 18.3 showed that the average translational kinetic energy of a gas molecule at temperature T is $\frac{3}{2}kT$, where k is Boltzmann's constant. The temperature at which this is equal to $E = 0.6 \times 10^{-13}$ J is determined by the relationship

$$E = \frac{3}{2}kT$$

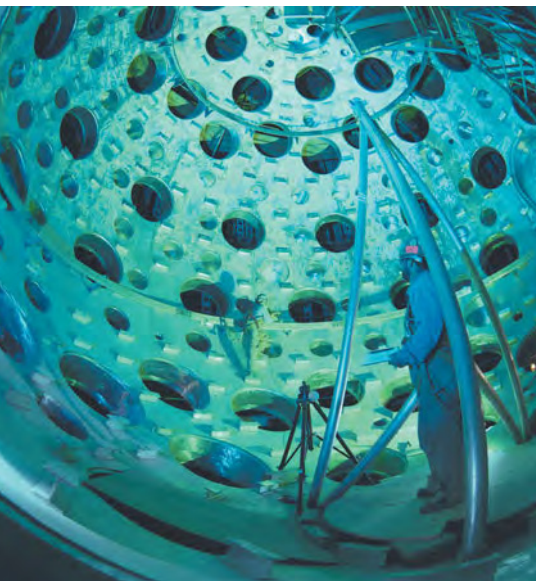
$$T = \frac{2E}{3k} = \frac{2(0.6 \times 10^{-13} \text{ J})}{3(1.38 \times 10^{-23} \text{ J/K})} = 3 \times 10^9 \text{ K}$$

Fusion reactions are possible at lower temperatures because the Maxwell–Boltzmann distribution function (see Section 18.5) gives a small fraction of protons with kinetic energies much higher than the average value. The proton–proton reaction occurs at “only” 1.5×10^7 K at the center of the sun, making it an extremely low-probability process; but that’s why the sun is expected to last so long. At these temperatures the fusion reactions are called *thermonuclear* reactions.

Intensive efforts are under way to achieve controlled fusion reactions, which potentially represent an enormous new resource of energy (see Fig. 24.11). At the temperatures mentioned, light atoms are fully ionized, and the resulting state of matter is called a *plasma*. In one kind of experiment using *magnetic confinement*, a plasma is heated to extremely high temperature by an electrical discharge, while being contained by appropriately shaped magnetic fields. In another, using *inertial confinement*, pellets of the material to be fused are heated by a high-intensity laser beam (see Fig. 43.18). Some of the reactions being studied are



43.18 This target chamber at the National Ignition Facility in California has apertures for 192 powerful laser beams. The lasers deliver 5×10^{14} W of power for a few nanoseconds to a millimeter-sized pellet of deuterium and tritium at the center of the chamber, thus triggering thermonuclear fusion.



We considered reaction (1) in Example 43.13; two deuterons fuse to form a triton and a proton. In reaction (2) a triton combines with another deuteron to form an alpha particle and a neutron. The result of both of these reactions together is the conversion of three deuterons into an alpha particle, a proton, and a neutron, with the liberation of 21.6 MeV of energy. Reactions (3) and (4) together achieve the same conversion. In a plasma that contains deuterons, the two pairs of reactions occur with roughly equal probability. As yet, no one has succeeded in producing these reactions under controlled conditions in such a way as to yield a net surplus of usable energy.

Methods of achieving fusion that don't require high temperatures are also being studied; these are called *cold fusion*. One scheme that does work uses an unusual hydrogen molecule ion. The usual H_2^+ ion consists of two protons bound by one shared electron; the nuclear spacing is about 0.1 nm. If the protons are replaced by a deuteron (^2H) and a triton (^3H) and the electron by a *muon*, which is 208 times as massive as the electron, the spacing is made smaller by a factor of 208. The probability then becomes appreciable for the two nuclei to tunnel through the narrow repulsive potential-energy barrier and fuse in reaction (2) above. The prospect of making this process, called *muon-catalyzed fusion*, into a practical energy source is still distant.

Test Your Understanding of Section 43.8 Are *all* fusion reactions exoergic? |

CHAPTER 43 SUMMARY

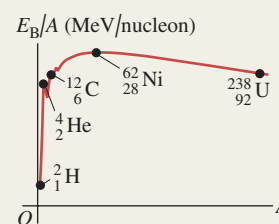
Nuclear properties: A nucleus is composed of A nucleons (Z protons and N neutrons). All nuclei have about the same density. The radius of a nucleus with mass number A is given approximately by Eq. (43.1). A single nuclear species of a given Z and N is called a nuclide. Isotopes are nuclides of the same element (same Z) that have different numbers of neutrons. Nuclear masses are measured in atomic mass units. Nucleons have angular momentum and a magnetic moment. (See Examples 43.1 and 43.2.)

$$R = R_0 A^{1/3} \quad (43.1)$$

$$(R_0 = 1.2 \times 10^{-15} \text{ m})$$

Nuclear binding and structure: The mass of a nucleus is always less than the mass of the protons and neutrons within it. The mass difference multiplied by c^2 gives the binding energy E_B . The binding energy for a given nuclide is determined by the nuclear force, which is short range and favors pairs of particles, and by the electric repulsion between protons. A nucleus is unstable if A or Z is too large or if the ratio N/Z is wrong. Two widely used models of the nucleus are the liquid-drop model and the shell model; the latter is analogous to the central-field approximation for atomic structure. (See Examples 43.3 and 43.4.)

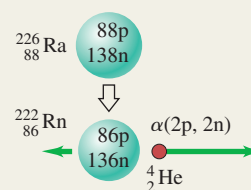
$$E_B = (ZM_H + Nm_n - \frac{A}{2}M)c^2 \quad (43.10)$$



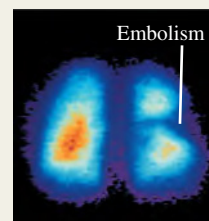
Radioactive decay: Unstable nuclides usually emit an alpha particle (a ${}^4_2\text{He}$ nucleus) or a beta particle (an electron) in the process of changing to another nuclide, sometimes followed by a gamma-ray photon. The rate of decay of an unstable nucleus is described by the decay constant λ , the half-life $T_{1/2}$, or the lifetime T_{mean} . If the number of nuclei at time $t = 0$ is N_0 and no more are produced, the number at time t is given by Eq. (43.17). (See Examples 43.5–43.9.)

$$N(t) = N_0 e^{-\lambda t} \quad (43.17)$$

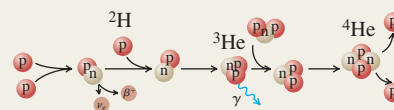
$$T_{\text{mean}} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} \quad (43.19)$$



Biological effects of radiation: The biological effect of any radiation depends on the product of the energy absorbed per unit mass and the relative biological effectiveness (RBE), which is different for different radiations. (See Example 43.10.)



Nuclear reactions: In a nuclear reaction, two nuclei or particles collide to produce two new nuclei or particles. Reactions can be exoergic or endoergic. Several conservation laws, including charge, energy, momentum, angular momentum, and nucleon number, are obeyed. Energy is released by the fission of a heavy nucleus into two lighter, always unstable, nuclei. Energy is also released by the fusion of two light nuclei into a heavier nucleus. (See Examples 43.11–43.13.)



BRIDGING PROBLEM

Saturation of ^{128}I Production

In an experiment, the iodine isotope ^{128}I is created by irradiating a sample of ^{127}I with a beam of neutrons, yielding 1.50×10^6 ^{128}I nuclei per second. Initially no ^{128}I nuclei are present. A ^{128}I nucleus decays by β^- emission with a half-life of 25.0 min. (a) To what nuclide does ^{128}I decay? (b) Could that nuclide decay back to ^{128}I by β^+ emission? Why or why not? (c) After the sample has been irradiated for a long time, what is the maximum number of ^{128}I atoms that can be present in the sample? What is the maximum activity that can be produced? (This steady-state situation is called *saturation*.) (d) Find an expression for the number of ^{128}I atoms present in the sample as a function of time.

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.



IDENTIFY and SET UP

1. What happens to the values of Z , N , and A in β^- decay? What must be true for β^- decay to be possible? For β^+ decay to be possible?
2. You'll need to write an equation for the rate of change dN/dt of the number N of ^{128}I atoms in the sample, taking account of

both the creation of ^{128}I by the neutron irradiation and the decay of any ^{128}I present. In the steady state, how do the rates of these two processes compare?

3. List the unknown quantities for each part of the problem and identify your target variables.

EXECUTE

4. Find the values of Z and N of the nuclide produced by the decay of ^{128}I . What element is this?
5. Decide whether this nuclide can decay back to ^{128}I .
6. Inspect your equation for dN/dt . What is the value of dN/dt in the steady state? Use this to solve for the steady-state values of N and the activity.
7. Solve your dN/dt equation for the function $N(t)$. (*Hint*: See Section 26.4.)

EVALUATE

8. Your result from step 6 tells you the value of N after a long time (that is, for large values of t). Is this consistent with your result from step 7? What would constitute a “long time” under these conditions?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q43.1 BIO Neutrons have a magnetic dipole moment and can undergo spin flips by absorbing electromagnetic radiation. Why, then, are protons rather than neutrons used in MRI of body tissues? (See Fig. 43.1.)

Q43.2 In Eq. (43.11), as the total number of nucleons becomes larger, the importance of the second term in the equation decreases relative to that of the first term. Does this make physical sense? Explain.

Q43.3 Why aren't the masses of all nuclei integer multiples of the mass of a single nucleon?

Q43.4 Can you tell from the value of the mass number A whether to use a plus value, a minus value, or zero for the fifth term of Eq. (43.11)? Explain.

Q43.5 What are the six known elements for which Z is a magic number? Discuss what properties these elements have as a consequence of their special values of Z .

Q43.6 The binding energy per nucleon for most nuclides doesn't vary much (see Fig. 43.2). Is there similar consistency in the *atomic* energy of atoms, on an “energy per electron” basis? If so, why? If not, why not?

Q43.7 Heavy, unstable nuclei usually decay by emitting an α or β particle. Why don't they usually emit a single proton or neutron?

Q43.8 The only two stable nuclides with more protons than neutrons are ^1_1H and ^3_2He . Why is $Z > N$ so uncommon?

Q43.9 Since lead is a stable element, why doesn't the ^{238}U decay series shown in Fig. 43.7 stop at lead, ^{214}Pb ?

Q43.10 In the ^{238}U decay series shown in Fig. 43.7, some nuclides in the series are found much more abundantly in nature than others, even though every ^{238}U nucleus goes through every step in the series before finally becoming ^{206}Pb . Why don't the intermediate nuclides all have the same abundance?

Q43.11 Compared to α particles with the same energy, β particles can much more easily penetrate through matter. Why is this?

Q43.12 If $^A_Z\text{E}_i$ represents the initial nuclide, what is the decay process or processes if the final nuclide is (a) $^A_{Z+1}\text{E}_f$; (b) $^A_{Z-2}\text{E}_f$; (c) $^A_{Z-1}\text{E}_f$?

Q43.13 In a nuclear decay equation, why can we represent an electron as $^0_{-1}\beta^-$? What are the equivalent representations for a positron, a neutrino, and an antineutrino?

Q43.14 Why is the alpha, beta, or gamma decay of an unstable nucleus unaffected by the *chemical* situation of the atom, such as the nature of the molecule or solid in which it is bound? The chemical situation of the atom can, however, have an effect on the half-life in electron capture. Why is this?

Q43.15 In the process of *internal conversion*, a nucleus decays from an excited state to a ground state by giving the excitation energy directly to an atomic electron rather than emitting a gamma-ray photon. Why can this process also produce x-ray photons?

Q43.16 In Example 43.9 (Section 43.4), the activity of atmospheric carbon *before* 1900 was given. Discuss why this activity may have changed since 1900.

Q43.17 BIO One problem in radiocarbon dating of biological samples, especially very old ones, is that they can easily be contaminated with modern biological material during the measurement process. What effect would such contamination have on the estimated age? Why is such contamination a more serious problem for samples of older material than for samples of younger material?

Q43.18 The most common radium isotope found on earth, ^{226}Ra , has a half-life of about 1600 years. If the earth was formed well over 10^9 years ago, why is there any radium left now?

Q43.19 Fission reactions occur only for nuclei with large nucleon numbers, while exoergic fusion reactions occur only for nuclei with small nucleon numbers. Why is this?

Q43.20 When a large nucleus splits during nuclear fission, the daughter nuclei of the fission fly apart with enormous kinetic energy. Why does this happen?

Q43.21 As stars age, they use up their supply of hydrogen and eventually begin producing energy by a reaction that involves the fusion of three helium nuclei to form a carbon nucleus. Would you expect the interiors of these old stars to be hotter or cooler than the interiors of younger stars? Explain.

EXERCISES

Section 43.1 Properties of Nuclei

43.1 • How many protons and how many neutrons are there in a nucleus of the most common isotope of (a) silicon, $^{28}_{14}\text{Si}$; (b) rubidium, $^{85}_{37}\text{Rb}$; (c) thallium, $^{205}_{81}\text{Tl}$?

43.2 •• **CP** Hydrogen atoms are placed in an external 1.65-T magnetic field. (a) The *protons* can make transitions between states where the nuclear spin component is parallel and antiparallel to the field by absorbing or emitting a photon. Which state has lower energy: the state with the nuclear spin component parallel or antiparallel to the field? What are the frequency and wavelength of the photon? In which region of the electromagnetic spectrum does it lie? (b) The *electrons* can make transitions between states where the electron spin component is parallel and antiparallel to the field by absorbing or emitting a photon. Which state has lower energy: the state with the electron spin component parallel or antiparallel to the field? What are the frequency and wavelength of the photon? In which region of the electromagnetic spectrum does it lie?

43.3 • Hydrogen atoms are placed in an external magnetic field. The protons can make transitions between states in which the nuclear spin component is parallel and antiparallel to the field by absorbing or emitting a photon. What magnetic-field magnitude is required for this transition to be induced by photons with frequency 22.7 MHz?

43.4 •• Neutrons are placed in a magnetic field with magnitude 2.30 T. (a) What is the energy difference between the states with the nuclear spin angular momentum components parallel and antiparallel to the field? Which state is lower in energy: the one with its spin component parallel to the field or the one with its spin component antiparallel to the field? How do your results compare with the energy states for a proton in the same field (see Example 43.2)? (b) The neutrons can make transitions from one of these states to the other by emitting or absorbing a photon with energy equal to the energy difference of the two states. Find the frequency and wavelength of such a photon.

Section 43.2 Nuclear Binding and Nuclear Structure

43.5 • The most common isotope of boron is $^{11}_5\text{B}$. (a) Determine the total binding energy of $^{11}_5\text{B}$ from Table 43.2 in Section 43.1. (b) Calculate this binding energy from Eq. (43.11). (Why is the fifth

term zero?) Compare to the result you obtained in part (a). What is the percent difference? Compare the accuracy of Eq. (43.11) for $^{11}_5\text{B}$ to its accuracy for $^{62}_{28}\text{Ni}$ (see Example 43.4).

43.6 • The most common isotope of uranium, $^{238}_{92}\text{U}$, has atomic mass 238.050783 u. Calculate (a) the mass defect; (b) the binding energy (in MeV); (c) the binding energy per nucleon.

43.7 • **CP** What is the maximum wavelength of a γ ray that could break a deuteron into a proton and a neutron? (This process is called photodisintegration.)

43.8 • Calculate (a) the total binding energy and (b) the binding energy per nucleon of $^{12}_6\text{C}$. (c) What percent of the rest mass of this nucleus is its total binding energy?

43.9 • **CP** A photon with a wavelength of 3.50×10^{-13} m strikes a deuteron, splitting it into a proton and a neutron. (a) Calculate the kinetic energy released in this interaction. (b) Assuming the two particles share the energy equally, and taking their masses to be 1.00 u, calculate their speeds after the photodisintegration.

43.10 • Calculate the mass defect, the binding energy (in MeV), and the binding energy per nucleon of (a) the nitrogen nucleus, $^{14}_7\text{N}$, and (b) the helium nucleus, ^4_2He . (c) How does the binding energy per nucleon compare for these two nuclei?

43.11 • Use Eq. (43.11) to calculate the binding energy per nucleon for the nuclei $^{86}_{36}\text{Kr}$ and $^{180}_{73}\text{Ta}$. Do your results confirm what is shown in Fig. 43.2—that for A greater than 62 the binding energy per nucleon decreases as A increases?

Section 43.3 Nuclear Stability and Radioactivity

43.12 • (a) Is the decay $n \rightarrow p + \beta^- + \bar{\nu}_e$ energetically possible? If not, explain why not. If so, calculate the total energy released. (b) Is the decay $p \rightarrow n + \beta^+ + \nu_e$ energetically possible? If not, explain why not. If so, calculate the total energy released.

43.13 • What nuclide is produced in the following radioactive decays? (a) α decay of $^{239}_{94}\text{Pu}$; (b) β^- decay of $^{24}_{11}\text{Na}$; (c) β^+ decay of $^{15}_8\text{O}$.

43.14 •• **CP** ^{238}U decays spontaneously by α emission to ^{234}Th . Calculate (a) the total energy released by this process and (b) the recoil velocity of the ^{234}Th nucleus. The atomic masses are 238.050788 u for ^{238}U and 234.043601 u for ^{234}Th .

43.15 •• The atomic mass of ^{14}C is 14.003242 u. Show that the β^- decay of ^{14}C is energetically possible, and calculate the energy released in the decay.

43.16 • What particle (α particle, electron, or positron) is emitted in the following radioactive decays? (a) $^{27}_{14}\text{Si} \rightarrow ^{27}_{13}\text{Al}$; (b) $^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th}$; (c) $^{74}_{33}\text{As} \rightarrow ^{74}_{34}\text{Se}$.

43.17 •• (a) Calculate the energy released by the electron-capture decay of $^{57}_{27}\text{Co}$ (see Example 43.7). (b) A negligible amount of this energy goes to the resulting $^{57}_{26}\text{Fe}$ atom as kinetic energy. About 90% of the time, the $^{57}_{26}\text{Fe}$ nucleus emits two successive gamma-ray photons after the electron-capture process, of energies 0.122 MeV and 0.014 MeV, respectively, in decaying to its ground state. What is the energy of the neutrino emitted in this case?

43.18 • Tritium (^3_1H) is an unstable isotope of hydrogen; its mass, including one electron, is 3.016049 u. (a) Show that tritium must be unstable with respect to beta decay because the decay products (^3_2He plus an emitted electron) have less total mass than the tritium. (b) Determine the total kinetic energy (in MeV) of the decay products, taking care to account for the electron masses correctly.

Section 43.4 Activities and Half-Lives

43.19 • If a 6.13-g sample of an isotope having a mass number of 124 decays at a rate of 0.350 Ci, what is its half-life?

43.20 • BIO Radioactive isotopes used in cancer therapy have a “shelf-life,” like pharmaceuticals used in chemotherapy. Just after it has been manufactured in a nuclear reactor, the activity of a sample of ^{60}Co is 5000 Ci. When its activity falls below 3500 Ci, it is considered too weak a source to use in treatment. You work in the radiology department of a large hospital. One of these ^{60}Co sources in your inventory was manufactured on October 6, 2004. It is now April 6, 2007. Is the source still usable? The half-life of ^{60}Co is 5.271 years.

43.21 •• The common isotope of uranium, ^{238}U , has a half-life of 4.47×10^9 years, decaying to ^{234}Th by alpha emission. (a) What is the decay constant? (b) What mass of uranium is required for an activity of 1.00 curie? (c) How many alpha particles are emitted per second by 10.0 g of uranium?

43.22 •• BIO Radiation Treatment of Prostate Cancer. In many cases, prostate cancer is treated by implanting 60 to 100 small seeds of radioactive material into the tumor. The energy released from the decays kills the tumor. One isotope that is used (there are others) is palladium (^{103}Pd), with a half-life of 17 days. If a typical grain contains 0.250 g of ^{103}Pd , (a) what is its initial activity rate in Bq, and (b) what is the rate 68 days later?

43.23 •• A 12.0-g sample of carbon from living matter decays at the rate of 180.0 decays/min due to the radioactive ^{14}C in it. What will be the decay rate of this sample in (a) 1000 years and (b) 50,000 years?

43.24 •• BIO Radioactive Tracers. Radioactive isotopes are often introduced into the body through the bloodstream. Their spread through the body can then be monitored by detecting the appearance of radiation in different organs. ^{131}I , a β^- emitter with a half-life of 8.0 d, is one such tracer. Suppose a scientist introduces a sample with an activity of 375 Bq and watches it spread to the organs. (a) Assuming that the sample all went to the thyroid gland, what will be the decay rate in that gland 24 d (about $3\frac{1}{2}$ weeks) later? (b) If the decay rate in the thyroid 24 d later is actually measured to be 17.0 Bq, what percentage of the tracer went to that gland? (c) What isotope remains after the I-131 decays?

43.25 •• The unstable isotope ^{40}K is used for dating rock samples. Its half-life is 1.28×10^9 y. (a) How many decays occur per second in a sample containing 1.63×10^{-6} g of ^{40}K ? (b) What is the activity of the sample in curies?

43.26 • As a health physicist, you are being consulted about a spill in a radiochemistry lab. The isotope spilled was $500 \mu\text{Ci}$ of ^{131}Ba , which has a half-life of 12 days. (a) What mass of ^{131}Ba was spilled? (b) Your recommendation is to clear the lab until the radiation level has fallen $1.00 \mu\text{Ci}$. How long will the lab have to be closed?

43.27 • Measurements on a certain isotope tell you that the decay rate decreases from 8318 decays/min to 3091 decays/min in 4.00 days. What is the half-life of this isotope?

43.28 • The isotope ^{226}Ra undergoes α decay with a half-life of 1620 years. What is the activity of 1.00 g of ^{226}Ra ? Express your answer in Bq and in Ci.

43.29 • The radioactive nuclide ^{199}Pt has a half-life of 30.8 minutes. A sample is prepared that has an initial activity of 7.56×10^{11} Bq. (a) How many ^{199}Pt nuclei are initially present in the sample? (b) How many are present after 30.8 minutes? What is the activity at this time? (c) Repeat part (b) for a time 92.4 minutes after the sample is first prepared.

43.30 •• Radiocarbon Dating. A sample from timbers at an archeological site containing 500 g of carbon provides 3070 decays/min. What is the age of the sample?

Section 43.5 Biological Effects of Radiation

43.31 •• BIO (a) If a chest x ray delivers 0.25 mSv to 5.0 kg of tissue, how many *total* joules of energy does this tissue receive? (b) Natural radiation and cosmic rays deliver about 0.10 mSv per year at sea level. Assuming an RBE of 1, how many rem and rads is this dose, and how many joules of energy does a 75-kg person receive in a year? (c) How many chest x rays like the one in part (a) would it take to deliver the same *total* amount of energy to a 75-kg person as she receives from natural radiation in a year at sea level, as described in part (b)?

43.32 •• BIO A person exposed to fast neutrons receives a radiation dose of 200 rem on part of his hand, affecting 25 g of tissue. The RBE of these neutrons is 10. (a) How many rad did he receive? (b) How many joules of energy did this person receive? (c) Suppose the person received the same rad dosage, but from beta rays with an RBE of 1.0 instead of neutrons. How many rem would he have received?

43.33 •• BIO A nuclear chemist receives an accidental radiation dose of 5.0 Gy from slow neutrons (RBE = 4.0). What does she receive in rad, rem, and J/kg?

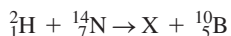
43.34 • BIO To Scan or Not to Scan? It has become popular for some people to have yearly whole-body scans (CT scans, formerly called CAT scans) using x rays, just to see if they detect anything suspicious. A number of medical people have recently questioned the advisability of such scans, due in part to the radiation they impart. Typically, one such scan gives a dose of 12 mSv, applied to the *whole body*. By contrast, a chest x ray typically administers 0.20 mSv to only 5.0 kg of tissue. How many chest x rays would deliver the same *total* amount of energy to the body of a 75-kg person as one whole-body scan?

43.35 • BIO Food Irradiation. Food is often irradiated with either x rays or electron beams to help prevent spoilage. A low dose of 5–75 kilorads (krad) helps to reduce and kill inactive parasites, a medium dose of 100–400 krad kills microorganisms and pathogens such as salmonella, and a high dose of 2300–5700 krad sterilizes food so that it can be stored without refrigeration. (a) A dose of 175 krad kills spoilage microorganisms in fish. If x rays are used, what would be the dose in Gy, Sv, and rem, and how much energy would a 220-g portion of fish absorb? (See Table 43.3.) (b) Repeat part (a) if electrons of RBE 1.50 are used instead of x rays.

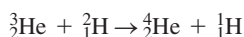
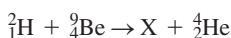
43.36 • BIO In an industrial accident a 65-kg person receives a lethal whole-body equivalent dose of 5.4 Sv from x rays. (a) What is the equivalent dose in rem? (b) What is the absorbed dose in rad? (c) What is the total energy absorbed by the person’s body? How does this amount of energy compare to the amount of energy required to raise the temperature of 65 kg of water 0.010 C° ?

43.37 •• BIO A 67-kg person accidentally ingests 0.35 Ci of tritium. (a) Assume that the tritium spreads uniformly throughout the body and that each decay leads on the average to the absorption of 5.0 keV of energy from the electrons emitted in the decay. The half-life of tritium is 12.3 y, and the RBE of the electrons is 1.0. Calculate the absorbed dose in rad and the equivalent dose in rem during one week. (b) The β^- decay of tritium releases more than 5.0 keV of energy. Why is the average energy absorbed less than the total energy released in the decay?

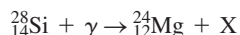
43.38 •• CP BIO In a diagnostic x-ray procedure, 5.00×10^{10} photons are absorbed by tissue with a mass of 0.600 kg. The x-ray wavelength is 0.0200 nm. (a) What is the total energy absorbed by the tissue? (b) What is the equivalent dose in rem?

Section 43.6 Nuclear Reactions, Section 43.7 Nuclear Fission, and Section 43.8 Nuclear Fusion**43.39** • Consider the nuclear reaction

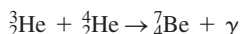
where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Calculate the reaction energy Q (in MeV). (c) If the ${}^2_1\text{H}$ nucleus is incident on a stationary ${}^{14}_7\text{N}$ nucleus, what minimum kinetic energy must it have for the reaction to occur?

43.40 • **Energy from Nuclear Fusion.** Calculate the energy released in the fusion reaction**43.41** • Consider the nuclear reaction

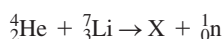
where X is a nuclide. (a) What are the values of Z and A for the nuclide X? (b) How much energy is liberated? (c) Estimate the threshold energy for this reaction.

43.42 • The United States uses 1.0×10^{20} J of electrical energy per year. If all this energy came from the fission of ${}^{235}\text{U}$, which releases 200 MeV per fission event, (a) how many kilograms of ${}^{235}\text{U}$ would be used per year and (b) how many kilograms of uranium would have to be mined per year to provide that much ${}^{235}\text{U}$? (Recall that only 0.70% of naturally occurring uranium is ${}^{235}\text{U}$.)**43.43** •• At the beginning of Section 43.7 the equation of a fission process is given in which ${}^{235}\text{U}$ is struck by a neutron and undergoes fission to produce ${}^{144}\text{Ba}$, ${}^{89}\text{Kr}$, and three neutrons. The measured masses of these isotopes are 235.043930 u (${}^{235}\text{U}$), 143.922953 u (${}^{144}\text{Ba}$), 88.917630 u (${}^{89}\text{Kr}$), and 1.0086649 u (neutron). (a) Calculate the energy (in MeV) released by each fission reaction. (b) Calculate the energy released per gram of ${}^{235}\text{U}$, in MeV/g.**43.44** •• Consider the nuclear reaction

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Ignoring the effects of recoil, what minimum energy must the photon have for this reaction to occur? The mass of a ${}^{28}_{14}\text{Si}$ atom is 27.976927 u, and the mass of a ${}^{24}_{12}\text{Mg}$ atom is 23.985042 u.

43.45 • The second reaction in the proton-proton chain (see Fig. 43.16) produces a ${}^3_2\text{He}$ nucleus. A ${}^3_2\text{He}$ nucleus produced in this way can combine with a ${}^4_2\text{He}$ nucleus:

Calculate the energy liberated in this process. (This is shared between the energy of the photon and the recoil kinetic energy of the beryllium nucleus.) The mass of a ${}^7_4\text{Be}$ atom is 7.016929 u.

43.46 • Consider the nuclear reaction

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Is energy absorbed or liberated? How much?

43.47 •• **CP** In a 100.0-cm³ sample of water, 0.015% of the molecules are D₂O. Compute the energy in joules that is liberated if all the deuterium nuclei in the sample undergo the fusion reaction of Example 43.13.**PROBLEMS****43.48** • **Comparison of Energy Released per Gram of Fuel.** (a) When gasoline is burned, it releases 1.3×10^8 J of energy per

gallon (3.788 L). Given that the density of gasoline is 737 kg/m³, express the quantity of energy released in J/g of fuel. (b) During fission, when a neutron is absorbed by a ${}^{235}\text{U}$ nucleus, about 200 MeV of energy is released for each nucleus that undergoes fission. Express this quantity in J/g of fuel. (c) In the proton-proton chain that takes place in stars like our sun, the overall fusion reaction can be summarized as six protons fusing to form one ${}^4_2\text{He}$ nucleus with two leftover protons and the liberation of 26.7 MeV of energy. The fuel is the six protons. Express the energy produced here in units of J/g of fuel. Notice the huge difference between the two forms of nuclear energy, on the one hand, and the chemical energy from gasoline, on the other. (d) Our sun produces energy at a measured rate of 3.86×10^{26} W. If its mass of 1.99×10^{30} kg were all gasoline, how long could it last before consuming all its fuel? (*Historical note:* Before the discovery of nuclear fusion and the vast amounts of energy it releases, scientists were confused. They knew that the earth was at least many millions of years old, but could not explain how the sun could survive that long if its energy came from chemical burning.)

43.49 •• Use conservation of mass-energy to show that the energy released in alpha decay is positive whenever the mass of the original neutral atom is greater than the sum of the masses of the final neutral atom and the neutral ${}^4_2\text{He}$ atom. (*Hint:* Let the parent nucleus have atomic number Z and nucleon number A. First write the reaction in terms of the nuclei and particles involved, and then add Z electron masses to both sides of the reaction and allot them as needed to arrive at neutral atoms.)**43.50** •• Use conservation of mass-energy to show that the energy released in β^- decay is positive whenever the neutral atomic mass of the original atom is greater than that of the final atom. (See the hint in Problem 43.49.)**43.51** •• Use conservation of mass-energy to show that the energy released in β^+ decay is positive whenever the neutral atomic mass of the original atom is at least two electron masses greater than that of the final atom. (See the hint in Problem 43.49.)**43.52** •• (a) Calculate the minimum energy required to remove one proton from the nucleus ${}^{12}_6\text{C}$. This is called the proton-removal energy. (*Hint:* Find the difference between the mass of a ${}^{12}_6\text{C}$ nucleus and the mass of a proton plus the mass of the nucleus formed when a proton is removed from ${}^{12}_6\text{C}$.) (b) How does the proton-removal energy for ${}^{12}_6\text{C}$ compare to the binding energy per nucleon for ${}^{12}_6\text{C}$, calculated using Eq. (43.10)?**43.53** •• (a) Calculate the minimum energy required to remove one neutron from the nucleus ${}^{17}_8\text{O}$. This is called the neutron-removal energy. (See Problem 43.52.) (b) How does the neutron-removal energy for ${}^{17}_8\text{O}$ compare to the binding energy per nucleon for ${}^{17}_8\text{O}$, calculated using Eq. (43.10)?**43.54** •• The neutral atomic mass of ${}^{14}_6\text{C}$ is 14.003242 u. Calculate the proton removal energy and the neutron removal energy for ${}^{15}_7\text{N}$. (See Problems 43.52 and 43.53.) What is the percentage difference between these two energies, and which is larger?**43.55** • **BIO Radioactive Fallout.** One of the problems of in-air testing of nuclear weapons (or, even worse, the use of such weapons!) is the danger of radioactive fallout. One of the most problematic nuclides in such fallout is strontium-90 (${}^{90}\text{Sr}$), which breaks down by β^- decay with a half-life of 28 years. It is chemically similar to calcium and therefore can be incorporated into bones and teeth, where, due to its rather long half-life, it remains for years as an internal source of radiation. (a) What is the daughter nucleus of the ${}^{90}\text{Sr}$ decay? (b) What percentage of the original level of ${}^{90}\text{Sr}$ is left after 56 years? (c) How long would you have to wait for the original level to be reduced to 6.25% of its original value?

43.56 •• CP Thorium $^{230}_{90}\text{Th}$ decays to radium $^{226}_{88}\text{Ra}$ by α emission. The masses of the neutral atoms are 230.033127 u for $^{230}_{90}\text{Th}$ and 226.025403 u for $^{226}_{88}\text{Ra}$. If the parent thorium nucleus is at rest, what is the kinetic energy of the emitted α particle? (Be sure to account for the recoil of the daughter nucleus.)

43.57 •• The atomic mass of $^{25}_{12}\text{Mg}$ is 24.985837 u, and the atomic mass of $^{25}_{13}\text{Al}$ is 24.990429 u. (a) Which of these nuclei will decay into the other? (b) What type of decay will occur? Explain how you determined this. (c) How much energy (in MeV) is released in the decay?

43.58 •• The polonium isotope $^{210}_{84}\text{Po}$ has atomic mass 209.982857 u. Other atomic masses are $^{206}_{82}\text{Pb}$, 205.974449 u; $^{209}_{83}\text{Bi}$, 208.980383 u; $^{210}_{83}\text{Bi}$, 209.984105 u; $^{209}_{84}\text{Po}$, 208.982416 u; and $^{210}_{85}\text{At}$, 209.987131 u. (a) Show that the alpha decay of $^{210}_{84}\text{Po}$ is energetically possible, and find the energy of the emitted α particle. (b) Is $^{210}_{84}\text{Po}$ energetically stable with respect to emission of a proton? Why or why not? (c) Is $^{210}_{84}\text{Po}$ energetically stable with respect to emission of a neutron? Why or why not? (d) Is $^{210}_{84}\text{Po}$ energetically stable with respect to β^- decay? Why or why not? (e) Is $^{210}_{84}\text{Po}$ energetically stable with respect to β^+ decay? Why or why not?

43.59 •• BIO Irradiating Ourselves! The radiocarbon in our bodies is one of the naturally occurring sources of radiation. Let's see how large a dose we receive. ^{14}C decays via β^- emission, and 18% of our body's mass is carbon. (a) Write out the decay scheme of carbon-14 and show the end product. (A neutrino is also produced.) (b) Neglecting the effects of the neutrino, how much kinetic energy (in MeV) is released per decay? The atomic mass of ^{14}C is 14.003242 u. (c) How many grams of carbon are there in a 75-kg person? How many decays per second does this carbon produce? (*Hint:* Use data from Example 43.9.) (d) Assuming that all the energy released in these decays is absorbed by the body, how many MeV/s and J/s does the ^{14}C release in this person's body? (e) Consult Table 43.3 and use the largest appropriate RBE for the particles involved. What radiation dose does the person give himself in a year, in Gy, rad, Sv, and rem?

43.60 •• BIO Pion Radiation Therapy. A neutral pion (π^0) has a mass of 264 times the electron mass and decays with a lifetime of 8.4×10^{-17} s to two photons. Such pions are used in the radiation treatment of some cancers. (a) Find the energy and wavelength of these photons. In which part of the electromagnetic spectrum do they lie? What is the RBE for these photons? (b) If you want to deliver a dose of 200 rem (which is typical) in a single treatment to 25 g of tumor tissue, how many π^0 mesons are needed?

43.61 • Gold, $^{198}_{79}\text{Au}$, undergoes β^- decay to an excited state of $^{198}_{80}\text{Hg}$. If the excited state decays by emission of a γ photon with energy 0.412 MeV, what is the maximum kinetic energy of the electron emitted in the decay? This maximum occurs when the antineutrino has negligible energy. (The recoil energy of the $^{198}_{80}\text{Hg}$ nucleus can be ignored. The masses of the neutral atoms in their ground states are 197.968225 u for $^{198}_{79}\text{Au}$ and 197.966752 u for $^{198}_{80}\text{Hg}$.)

43.62 •• Calculate the mass defect for the β^+ decay of $^{11}_{6}\text{C}$. Is this decay energetically possible? Why or why not? The atomic mass of $^{11}_{6}\text{C}$ is 11.011434 u.

43.63 •• Calculate the mass defect for the β^+ decay of $^{13}_{7}\text{N}$. Is this decay energetically possible? Why or why not? The atomic mass of $^{13}_{7}\text{N}$ is 13.005739 u.

43.64 •• The results of activity measurements on a radioactive sample are given in the table. (a) Find the half-life. (b) How many radioactive nuclei were present in the sample at $t = 0$? (c) How many were present after 7.0 h?

Time (h)	Decays/s
0	20,000
0.5	14,800
1.0	11,000
1.5	8,130
2.0	6,020
2.5	4,460
3.0	3,300
4.0	1,810
5.0	1,000
6.0	550
7.0	300

43.65 •• BIO A person ingests an amount of a radioactive source with a very long lifetime and activity $0.63 \mu\text{Ci}$. The radioactive material lodges in the lungs, where all of the 4.0-MeV α particles emitted are absorbed within a 0.50-kg mass of tissue. Calculate the absorbed dose and the equivalent dose for one year.

43.66 •• Measuring Very Long Half-Lives. Some radioisotopes such as samarium (^{149}Sm) and gadolinium (^{152}Gd) have half-lives that are much longer than the age of the universe, so we can't measure their half-lives by watching their decay rate decrease. Luckily, there is another way of calculating the half-life, using Eq. (43.16). Suppose a 12.0-g sample of ^{149}Sm is observed to decay at a rate of 2.65 Bq. Calculate the half-life of the sample in years. (*Hint:* How many nuclei are there in the 12.0-g sample?)

43.67 • We Are Stardust. In 1952 spectral lines of the element technetium-99 (^{99}Tc) were discovered in a red giant star. Red giants are very old stars, often around 10 billion years old, and near the end of their lives. Technetium has *no* stable isotopes, and the half-life of ^{99}Tc is 200,000 years. (a) For how many half-lives has the ^{99}Tc been in the red-giant star if its age is 10 billion years? (b) What fraction of the original ^{99}Tc would be left at the end of that time? This discovery was extremely important because it provided convincing evidence for the theory (now essentially known to be true) that most of the atoms heavier than hydrogen and helium were made inside of stars by thermonuclear fusion and other nuclear processes. If the ^{99}Tc had been part of the star since it was born, the amount remaining after 10 billion years would have been so minute that it would not have been detectable. This knowledge is what led the late astronomer Carl Sagan to proclaim that "we are stardust."

43.68 • BIO A 70.0-kg person experiences a whole-body exposure to α radiation with energy 4.77 MeV. A total of 6.25×10^{12} α particles are absorbed. (a) What is the absorbed dose in rad? (b) What is the equivalent dose in rem? (c) If the source is 0.0320 g of ^{226}Ra (half-life 1600 y) somewhere in the body, what is the activity of this source? (d) If all the alpha particles produced are absorbed, what time is required for this dose to be delivered?

43.69 •• Measurements indicate that 27.83% of all rubidium atoms currently on the earth are the radioactive ^{87}Rb isotope. The rest are the stable ^{85}Rb isotope. The half-life of ^{87}Rb is 4.75×10^{10} y. Assuming that no rubidium atoms have been formed since, what percentage of rubidium atoms were ^{87}Rb when our solar system was formed 4.6×10^9 y ago?

43.70 •• A $^{186}_{76}\text{Os}$ nucleus at rest decays by the emission of a 2.76-MeV α particle. Calculate the atomic mass of the daughter

nuclide produced by this decay, assuming that it is produced in its ground state. The atomic mass of ^{186}Os is 185.953838 u.

43.71 •• BIO A ^{60}Co source with activity 2.6×10^{-4} Ci is embedded in a tumor that has mass 0.200 kg. The source emits γ photons with average energy 1.25 MeV. Half the photons are absorbed in the tumor, and half escape. (a) What energy is delivered to the tumor per second? (b) What absorbed dose (in rad) is delivered per second? (c) What equivalent dose (in rem) is delivered per second if the RBE for these γ rays is 0.70? (d) What exposure time is required for an equivalent dose of 200 rem?

43.72 • The nucleus ^{15}O has a half-life of 122.2 s; ^{19}O has a half-life of 26.9 s. If at some time a sample contains equal amounts of ^{15}O and ^{19}O , what is the ratio of ^{15}O to ^{19}O (a) after 4.0 minutes and (b) after 15.0 minutes?

43.73 • A bone fragment found in a cave believed to have been inhabited by early humans contains 0.29 times as much ^{14}C as an equal amount of carbon in the atmosphere when the organism containing the bone died. (See Example 43.9 in Section 43.4.) Find the approximate age of the fragment.

43.74 •• An Oceanographic Tracer. Nuclear weapons tests in the 1950s and 1960s released significant amounts of radioactive tritium (^3H , half-life 12.3 years) into the atmosphere. The tritium atoms were quickly bound into water molecules and rained out of the air, most of them ending up in the ocean. For any of this tritium-tagged water that sinks below the surface, the amount of time during which it has been isolated from the surface can be calculated by measuring the ratio of the decay product, ^3He , to the remaining tritium in the water. For example, if the ratio of ^3He to ^3H in a sample of water is 1:1, the water has been below the surface for one half-life, or approximately 12 years. This method has provided oceanographers with a convenient way to trace the movements of subsurface currents in parts of the ocean. Suppose that in a particular sample of water, the ratio of ^3He to ^3H is 4.3 to 1.0. How many years ago did this water sink below the surface?

43.75 •• Consider the fusion reaction $^2_1\text{H} + ^2_1\text{H} \rightarrow ^3_2\text{He} + ^1_0\text{n}$. (a) Estimate the barrier energy by calculating the repulsive electrostatic potential energy of the two ^2_1H nuclei when they touch. (b) Compute the energy liberated in this reaction in MeV and in joules. (c) Compute the energy liberated *per mole* of deuterium, remembering that the gas is diatomic, and compare with the heat of combustion of hydrogen, about 2.9×10^5 J/mol.

43.76 •• BIO In the 1986 disaster at the Chernobyl reactor in the Soviet Union (now Ukraine), about $\frac{1}{8}$ of the ^{137}Cs present in the reactor was released. The isotope ^{137}Cs has a half-life for β decay of 30.07 y and decays with the emission of a total of 1.17 MeV of energy per decay. Of this, 0.51 MeV goes to the emitted electron and the remaining 0.66 MeV to a γ ray. The radioactive ^{137}Cs is absorbed by plants, which are eaten by livestock and humans. How many ^{137}Cs atoms would need to be present in each kilogram of body tissue if an equivalent dose for one week is 3.5 Sv? Assume

that all of the energy from the decay is deposited in that 1.0 kg of tissue and that the RBE of the electrons is 1.5.

43.77 •• CP (a) Prove that when a particle with mass m and kinetic energy K collides with a stationary particle with mass M , the total kinetic energy K_{cm} in the center-of-mass coordinate system (the energy available to cause reactions) is

$$K_{\text{cm}} = \frac{M}{M+m}K$$

Assume that the kinetic energies of the particles and nuclei are much lower than their rest energies. (b) If K_{th} is the minimum, or threshold, kinetic energy to cause an endoergic reaction to occur in the situation of part (a), show that

$$K_{\text{th}} = -\frac{M+m}{M}Q$$

43.78 • Calculate the energy released in the fission reaction $^{235}_{92}\text{U} + ^1_0\text{n} \rightarrow ^{140}_{54}\text{Xe} + ^{94}_{38}\text{Sr} + 2^1_0\text{n}$. You can ignore the initial kinetic energy of the absorbed neutron. The atomic masses are $^{235}_{92}\text{U}$, 235.043923 u; $^{140}_{54}\text{Xe}$, 139.921636 u; and $^{94}_{38}\text{Sr}$, 93.915360 u.

CHALLENGE PROBLEMS

43.79 ••• The results of activity measurements on a mixed sample of radioactive elements are given in the table. (a) How many different nuclides are present in the mixture? (b) What are their half-lives? (c) How many nuclei of each type are initially present in the sample? (d) How many of each type are present at $t = 5.0$ h?

Time (h)	Decays/s
0	7500
0.5	4120
1.0	2570
1.5	1790
2.0	1350
2.5	1070
3.0	872
4.0	596
5.0	414
6.0	288
7.0	201
8.0	140
9.0	98
10.0	68
12.0	33

43.80 ••• Industrial Radioactivity. Radioisotopes are used in a variety of manufacturing and testing techniques. Wear measurements can be made using the following method. An automobile engine is produced using piston rings with a total mass of 100 g, which includes $9.4 \mu\text{Ci}$ of ^{59}Fe whose half-life is 45 days. The engine is test-run for 1000 hours, after which the oil is drained and its activity is measured. If the activity of the engine oil is 84 decays/s, how much mass was worn from the piston rings per hour of operation?

Answers

Chapter Opening Question ?

When an organism dies, it stops taking in carbon from atmospheric CO_2 . Some of this carbon is radioactive ^{14}C , which decays with a half-life of 5730 years. By measuring the proportion of ^{14}C that remains in the specimen, scientists can determine how long ago the organism died. (See Section 43.4.)

Test Your Understanding Questions

43.1 Answers: (a) (iii), (b) (v) The radius R is proportional to the cube root of the mass number A , while the volume is proportional to R^3 and hence to $(A^{1/3})^3 = A$. Therefore, doubling the volume requires increasing the mass number by a factor of 2; doubling the radius implies increasing both the volume and the mass number by a factor of $2^3 = 8$.

43.2 Answer: (ii), (iii), (iv), (v), (i) You can find the answers by inspecting Fig. 43.2. The binding energy per nucleon is lowest for very light nuclei such as ${}^4_2\text{He}$, is greatest around $A = 60$, and then decreases with increasing A .

43.3 Answer: (v) Two protons and two neutrons are lost in an α decay, so Z and N each decrease by 2. A β^+ decay changes a proton to a neutron, so Z decreases by 1 and N increases by 1. The net result is that Z decreases by 3 and N decreases by 1.

43.4 Answer: (iii) The activity $-dN(t)/dt$ of a sample is the product of the number of nuclei in the sample $N(t)$ and the decay constant $\lambda = (\ln 2)/T_{1/2}$. Hence $N(t) = (-dN(t)/dt)T_{1/2}/(\ln 2)$. Taking the ratio of this expression for ${}^{240}\text{Pu}$ to this same expression for ${}^{243}\text{Am}$, the factors of $\ln 2$ cancel and we get

$$\frac{N_{\text{Pu}}}{N_{\text{Am}}} = \frac{(-dN_{\text{Pu}}/dt)T_{1/2-\text{Pu}}}{(-dN_{\text{Am}}/dt)T_{1/2-\text{Am}}} = \frac{(5.00 \mu\text{Ci})(6560 \text{ y})}{(4.45 \mu\text{Ci})(7370 \text{ y})} = 1.00$$

The two samples contain *equal* numbers of nuclei. The ${}^{243}\text{Am}$ sample has a longer half-life and hence a slower decay rate, so it has a lower activity than the ${}^{240}\text{Pu}$ sample.

43.5 Answer: (ii) We saw in Section 43.3 that alpha particles can travel only a very short distance before they are stopped. By contrast, x-ray photons are very penetrating, so they can easily pass into the body.

43.6 Answer: no The reaction ${}^1_1\text{H} + {}^7_3\text{Li} \rightarrow {}^4_2\text{He} + {}^4_2\text{He}$ is a *nuclear* reaction, which can take place only if a proton (a hydrogen nucleus) comes into contact with a lithium nucleus. If the hydrogen is in atomic form, the interaction between its electron cloud and the electron cloud of a lithium atom keeps the two nuclei from getting close to each other. Even if isolated protons are used, they must be fired at

the lithium atoms with enough kinetic energy to overcome the electric repulsion between the protons and the lithium nuclei. The statement that the reaction is exoergic means that more energy is released by the reaction than had to be put in to make the reaction occur.

43.7 Answer: no Because the neutron has no electric charge, it experiences no electric repulsion from a ${}^{235}\text{U}$ nucleus. Hence a slow-moving neutron can approach and enter a ${}^{235}\text{U}$ nucleus, thereby providing the excitation needed to trigger fission. By contrast, a slow-moving *proton* (charge $+e$) feels a strong electric repulsion from a ${}^{235}\text{U}$ nucleus (charge $+92e$). It never gets close to the nucleus, so it cannot trigger fission.

43.8 Answer: no Fusion reactions between sufficiently light nuclei are exoergic because the binding energy per nucleon E_B/A increases. If the nuclei are too massive, however, E_B/A decreases and fusion is *endoergic* (i.e., it takes in energy rather than releasing it). As an example, imagine fusing together two nuclei of $A = 100$ to make a single nucleus with $A = 200$. From Fig. 43.2, E_B/A is more than 8.5 MeV for the $A = 100$ nuclei but is less than 8 MeV for the $A = 200$ nucleus. Such a fusion reaction is possible, but requires a substantial input of energy.

Bridging Problem

Answers: (a) ${}^{128}\text{Xe}$

(b) no; β^+ emission would be endoergic

(c) 3.25×10^9 atoms, 1.50×10^6 Bq

(d) $N(t) = (3.25 \times 10^9 \text{ atoms})(1 - e^{-(4.62 \times 10^{-4} \text{ s}^{-1})t})$

44

PARTICLE PHYSICS AND COSMOLOGY

LEARNING GOALS

By studying this chapter, you will learn:

- The key varieties of fundamental subatomic particles and how they were discovered.
- How physicists use accelerators and detectors to probe the properties of subatomic particles.
- The four ways in which subatomic particles interact with each other.
- How the structure of protons, neutrons, and other particles can be explained in terms of quarks.
- How physicists probe the limits of the standard model of particles and interactions.
- The evidence that the universe is expanding and that the expansion is speeding up.
- The history of the first 380,000 years after the Big Bang.



? Images made using infrared and x-ray wavelengths were combined to produce this view of the dynamic center of our Milky Way galaxy. The image shows atoms in various states: as isolated atoms in glowing, diffuse clouds of gas (shown in blue), as clumps of atoms and molecules in immense, cold dust clouds (shown in red), and in dense accumulations that we call stars. What fraction of the mass and energy in the universe is composed of “normal” matter—that is, atoms and their constituents?

What is the world made of? What are the most fundamental constituents of matter? Philosophers and scientists have been asking these questions for at least 2500 years. We still don’t have the final answer, but as we’ll see in this chapter, we’ve come a long way.

The chapter title, “Particle Physics and Cosmology,” may seem strange. Fundamental particles are the *smallest* things in the universe, and cosmology deals with the *biggest* thing there is—the universe itself. Nonetheless, we’ll see in this chapter that physics on the most microscopic scale plays an essential role in determining the nature of the universe on the largest scale.

Fundamental particles, we’ll find, are not permanent entities; they can be created and destroyed. The development of high-energy particle accelerators and associated detectors has been crucial in our emerging understanding of particles. We can classify particles and their interactions in several ways in terms of conservation laws and symmetries, some of which are absolute and others of which are obeyed only in certain kinds of interactions. We’ll conclude by discussing our present understanding of the nature and evolution of the universe as a whole.

44.1 Fundamental Particles—A History

The idea that the world is made of fundamental particles has a long history. In about 400 B.C. the Greek philosophers Democritus and Leucippus suggested that matter is made of indivisible particles that they called *atoms*, a word derived from *a-* (not) and *tomos* (cut or divided). This idea lay dormant until about 1804, when the English scientist John Dalton (1766–1844), often called the father of

modern chemistry, discovered that many chemical phenomena could be explained if atoms of each element are the basic, indivisible building blocks of matter.

The Electron and the Proton

Toward the end of the 19th century it became clear that atoms are *not* indivisible. The existence of characteristic atomic spectra of elements suggested that atoms have internal structure, and J. J. Thomson's discovery of the negatively charged *electron* in 1897 showed that atoms could be taken apart into charged particles. Rutherford's experiments in 1910–11 (see Section 39.2) revealed that an atom's positive charge resides in a small, dense nucleus. In 1919 Rutherford made an additional discovery: When alpha particles are fired into nitrogen, one of the products is hydrogen gas. He reasoned that the hydrogen nucleus is a constituent of the nuclei of heavier atoms such as nitrogen, and that a collision with a fast-moving alpha particle can dislodge one of those hydrogen nuclei. Thus the hydrogen nucleus is an elementary particle, to which Rutherford gave the name *proton*. The following decade saw the blossoming of quantum mechanics, including the Schrödinger equation. Physicists were on their way to understanding the principles that underlie atomic structure.

The Photon

Einstein explained the photoelectric effect in 1905 by assuming that the energy of electromagnetic waves is quantized; that is, it comes in little bundles called *photons* with energy $E = hf$. Atoms and nuclei can emit (create) and absorb (destroy) photons (see Section 38.1). Considered as particles, photons have zero charge and zero rest mass. (Note that any discussions of a particle's mass in this chapter will refer to its rest mass.) In particle physics, a photon is denoted by the symbol γ (the Greek letter gamma).

The Neutron

In 1930 the German physicists Walther Bothe and Herbert Becker observed that when beryllium, boron, or lithium was bombarded by alpha particles, the target material emitted a radiation that had much greater penetrating power than the original alpha particles. Experiments by the English physicist James Chadwick in 1932 showed that the emitted particles were electrically neutral, with mass approximately equal to that of the proton. Chadwick christened these particles *neutrons* (symbol n or ${}_0^1n$). A typical reaction of the type studied by Bothe and Becker, using a beryllium target, is



Elementary particles are usually detected by their electromagnetic effects—for instance, by the ionization that they cause when they pass through matter. (This is the principle of the cloud chamber, described below.) Because neutrons have no charge, they interact hardly at all with electrons and produce little ionization when they pass through matter and so are difficult to detect directly. However, neutrons can be slowed down by scattering from nuclei, and they can penetrate a nucleus. Hence slow neutrons can be detected by means of a nuclear reaction in which a neutron is absorbed and an alpha particle is emitted. An example is



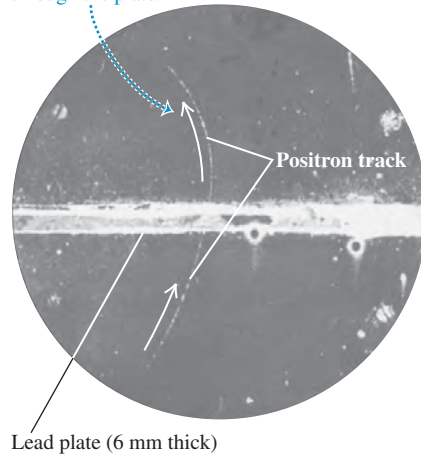
The ejected alpha particle is easy to detect because it is charged. Later experiments showed that neutrons, like protons and electrons, are spin- $\frac{1}{2}$ particles (see Section 43.1).

The discovery of the neutron cleared up a mystery about the composition of the nucleus. Before 1930 the mass of a nucleus was thought to be due only to protons, but no one understood why the charge-to-mass ratio was not the same for all nuclides. It soon became clear that all nuclides (except ${}_1^1\text{H}$) contain both protons

44.1 Photograph of the cloud-chamber track made by the first positron ever identified. The photograph was made by Carl D. Anderson in 1932.

The positron follows a curved path owing to the presence of a magnetic field.

The track is more strongly curved above the lead plate, showing that the positron was traveling upward and lost energy and speed as it passed through the plate.



and neutrons. Hence the proton, the neutron, and the electron are the building blocks of atoms. One might think that would be the end of the story. On the contrary, it is barely the beginning. These are not the only particles, and they can do more than build atoms.

The Positron

The positive electron, or positron, was discovered by the American physicist Carl D. Anderson in 1932, during an investigation of particles bombarding the earth from space. Figure 44.1 shows a historic photograph made with a *cloud chamber*, an instrument used to visualize the tracks of charged particles. The chamber contained a supercooled vapor; ions created by the passage of charged particles through the vapor served as nucleation centers, and liquid droplets formed around them, making a visible track.

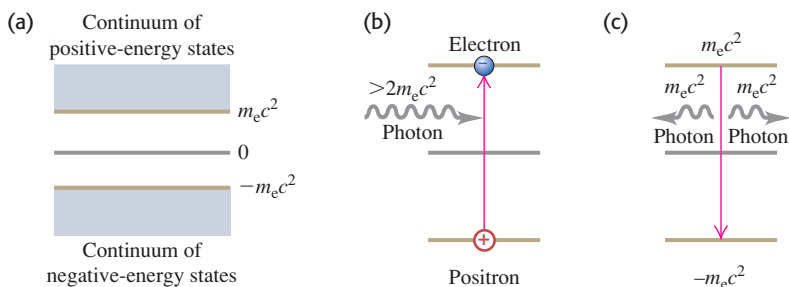
The cloud chamber in Fig. 44.1 is in a magnetic field directed into the plane of the photograph. The particle has passed through a thin lead plate (which extends from left to right in the figure) that lies within the chamber. The track is more tightly curved above the plate than below it, showing that the speed was less above the plate than below it. Therefore the particle had to be moving upward; it could not have gained energy passing through the lead. The thickness and curvature of the track suggested that its mass and the magnitude of its charge equaled those of the electron. But the directions of the magnetic field and the velocity in the magnetic force equation $\vec{F} = q\vec{v} \times \vec{B}$ showed that the particle had *positive* charge. Anderson christened this particle the *positron*.

To theorists, the appearance of the positron was a welcome development. In 1928 the English physicist Paul Dirac had developed a relativistic generalization of the Schrödinger equation for the electron. In Section 41.5 we discussed how Dirac's ideas helped explain the spin magnetic moment of the electron.

One of the puzzling features of the Dirac equation was that for a *free* electron it predicted not only a continuum of energy states greater than its rest energy $m_e c^2$, as should be expected, but also a continuum of *negative* energy states *less than* $-m_e c^2$ (Fig. 44.2a). That posed a problem. What was to prevent an electron from emitting a photon with energy $2m_e c^2$ or greater and hopping from a positive state to a negative state? It wasn't clear what these negative-energy states meant, and there was no obvious way to get rid of them. Dirac's ingenious interpretation was that all the negative-energy states were filled with electrons, and that these electrons were for some reason unobservable. The exclusion principle (see Section 41.6) would then forbid a transition to a state that was already occupied.

A vacancy in a negative-energy state would act like a positive charge, just as a hole in the valence band of a semiconductor (see Section 42.6) acts like a positive charge. Initially, Dirac tried to argue that such vacancies were protons. But after

44.2 (a) Energy states for a free electron predicted by the Dirac equation. (b) Raising an electron from an $E < 0$ state to an $E > 0$ state corresponds to electron–positron pair production. (c) An electron dropping from an $E > 0$ state to a vacant $E < 0$ state corresponds to electron–positron pair annihilation.



Anderson's discovery it became clear that the vacancies were observed physically as *positrons*. Furthermore, the Dirac energy-state picture provides a mechanism for the *creation* of positrons. When an electron in a negative-energy state absorbs a photon with energy greater than $2m_e c^2$, it goes to a positive state (Fig. 44.2b), in which it becomes observable. The vacancy that it leaves behind is observed as a positron; the result is the creation of an electron–positron pair. Similarly, when an electron in a positive-energy state falls into a vacancy, both the electron and the vacancy (that is, the positron) disappear, and photons are emitted (Fig. 44.2c). Thus the Dirac theory leads naturally to the conclusion that, like photons, *electrons can be created and destroyed*. While photons can be created and destroyed singly, electrons can be produced or destroyed only in electron–positron pairs or in association with other particles. (Creating or destroying an electron alone would mean creating or destroying an amount of charge $-e$, which would violate the conservation of electric charge.)

In 1949 the American physicist Richard Feynman showed that a positron could be described mathematically as an electron traveling backward in time. His reformulation of the Dirac theory eliminated difficult calculations involving the infinite sea of negative-energy states and put electrons and positrons on the same footing. But the creation and destruction of electron–positron pairs remain. The Dirac theory provides the beginning of a theoretical framework for creation and destruction of all fundamental particles.

Experiment and theory tell us that the masses of the positron and electron are identical, and that their charges are equal in magnitude but opposite in sign. The positron's spin angular momentum \vec{S} and magnetic moment $\vec{\mu}$ are parallel; they are opposite for the electron. However, \vec{S} and $\vec{\mu}$ have the same magnitude for both particles because they have the same spin. We use the term **antiparticle** for a particle that is related to another particle as the positron is to the electron. Each kind of particle has a corresponding antiparticle. For a few kinds of particles (necessarily all neutral) the particle and antiparticle are identical, and we can say that they are their own antiparticles. The photon is an example; there is no way to distinguish a photon from an antiphoton. We'll use the standard symbols e^- for the electron and e^+ for the positron, and the generic term “electron” will often include both electrons and positrons. Other antiparticles are often denoted by a bar over the particle's symbol; for example, an antiproton is \bar{p} . We'll see several other examples of antiparticles later.

Positrons do not occur in ordinary matter. Electron–positron pairs are produced during high-energy collisions of charged particles or γ rays with matter. This process is called *e^+e^- pair production* (Fig. 44.3). Enough energy E must be available to account for the rest energy $2m_e c^2$ of the two particles. The minimum energy for electron–positron pair production is

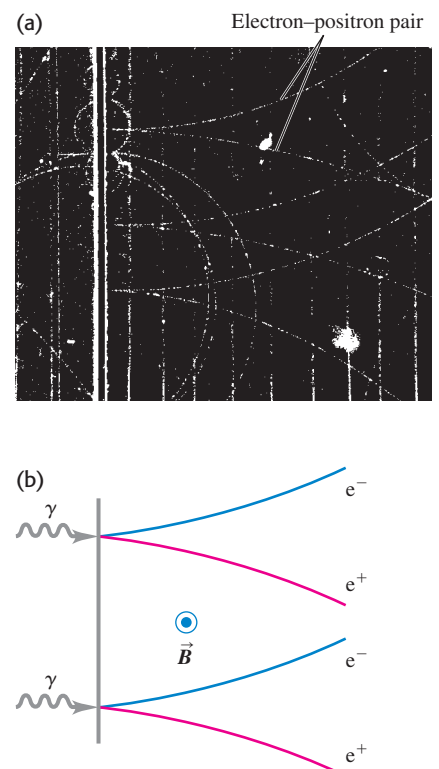
$$\begin{aligned} E_{\min} &= 2m_e c^2 = 2(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 \\ &= 1.637 \times 10^{-13} \text{ J} = 1.022 \text{ MeV} \end{aligned}$$

The inverse process, *e^+e^- pair annihilation*, occurs when a positron and an electron collide (see Example 38.6 in Section 38.3). Both particles disappear, and two (or occasionally three) photons can appear, with total energy of at least $2m_e c^2 = 1.022 \text{ MeV}$. Decay into a *single* photon is impossible: Such a process could not conserve both energy and momentum.

Positrons also occur in the decay of some unstable nuclei, in which they are called beta-plus particles (β^+). We discussed β^+ decay in Section 43.3.

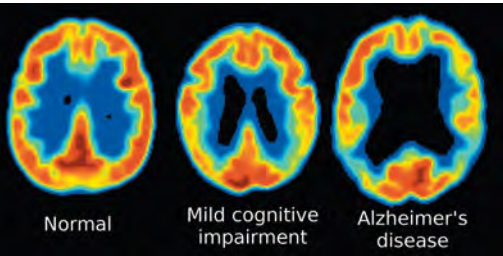
It's often convenient to represent particle masses in terms of the equivalent rest energy using $m = E/c^2$. Then typical mass units are MeV/c^2 ; for example, $m = 0.511 \text{ MeV}/c^2$ for an electron or positron. We'll use these units frequently in this chapter.

44.3 (a) Photograph of bubble-chamber tracks of electron–positron pairs that are produced when 300-MeV photons strike a lead sheet. A magnetic field directed out of the photograph made the electrons and positrons curve in opposite directions. (b) Diagram showing the pair-production process for two of the photons.



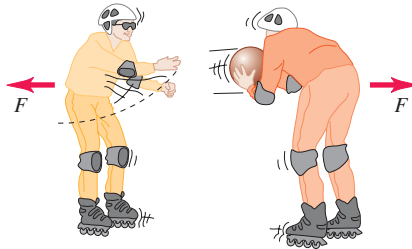
Application Pair Annihilation in Medical Diagnosis

A technique called positron emission tomography (PET) can be used to identify the early stages of Alzheimer’s disease. A patient is administered a glucose-like compound called FDG in which one of the oxygen atoms is replaced by radioactive ^{18}F . FDG accumulates in active areas of the brain, where glucose metabolism is high. The ^{18}F undergoes β^+ decay (positron emission) with a half-life of 110 minutes, and the emitted positron immediately annihilates with an atomic electron to produce two gamma-ray photons. A scanner detects both photons, then calculates where the annihilation took place and hence the site of FDG accumulation. These PET images—which show areas of strongest emission, and hence greatest glucose metabolism, in red—reveal changes in the brains of patients with mild cognitive impairment and with Alzheimer’s disease.

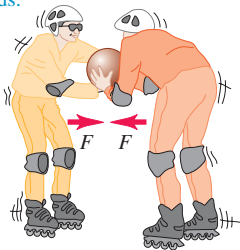


44.4 An analogy for how particles act as force mediators.

(a) Two skaters exert repulsive forces on each other by tossing a ball back and forth.



(b) Two skaters exert attractive forces on each other when one tries to grab the ball out of the other’s hands.



Particles As Force Mediators

In classical physics we describe the interaction of charged particles in terms of electric and magnetic forces. In quantum mechanics we can describe this interaction in terms of emission and absorption of photons. Two electrons repel each other as one emits a photon and the other absorbs it, just as two skaters can push each other apart by tossing a heavy ball back and forth between them (Fig. 44.4a). For an electron and a proton, in which the charges are opposite and the force is attractive, we imagine the skaters trying to grab the ball away from each other (Fig. 44.4b). The electromagnetic interaction between two charged particles is *mediated* or transmitted by photons.

If charged-particle interactions are mediated by photons, where does the energy to create the photons come from? Recall from our discussion of the uncertainty principle (see Sections 38.4 and 39.6) that a state that exists for a short time Δt has an uncertainty ΔE in its energy such that

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{44.3}$$

This uncertainty permits the creation of a photon with energy ΔE , provided that it lives no longer than the time Δt given by Eq. (44.3). A photon that can exist for a short time because of this energy uncertainty is called a *virtual photon*. It’s as though there were an energy bank; you can borrow energy, provided that you pay it back within the time limit. According to Eq. (44.3), the more you borrow, the sooner you have to pay it back.

Mesons

Is there a particle that mediates the *nuclear* force? By the mid-1930s the nuclear force between two nucleons (neutrons or protons) appeared to be described by a potential energy $U(r)$ with the general form

$$U(r) = -f^2 \left(\frac{e^{-r/r_0}}{r} \right) \quad (\text{nuclear potential energy}) \tag{44.4}$$

The constant f characterizes the strength of the interaction, and r_0 describes its range. Figure 44.5 shows a graph of the absolute value of this function and compares it with the function f^2/r , which would be analogous to the *electric* interaction of two protons:

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad (\text{electric potential energy}) \tag{44.5}$$

In 1935 the Japanese physicist Hideki Yukawa suggested that a hypothetical particle that he called a **meson** might mediate the nuclear force. He showed that the range of the force was related to the mass of the particle. Yukawa argued that the particle must live for a time Δt long enough to travel a distance comparable to the range r_0 of the nuclear force. This range was known from the sizes of nuclei and other information to be about $1.5 \times 10^{-15} \text{ m} = 1.5 \text{ fm}$. If we assume that an average particle’s speed is comparable to c and travels about half the range, its lifetime Δt must be about

$$\Delta t = \frac{r_0}{2c} = \frac{1.5 \times 10^{-15} \text{ m}}{2(3.0 \times 10^8 \text{ m/s})} = 2.5 \times 10^{-24} \text{ s}$$

From Eq. (44.3), the minimum necessary uncertainty ΔE in energy is

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{1.05 \times 10^{-34} \text{ J}\cdot\text{s}}{2(2.5 \times 10^{-24} \text{ s})} = 2.1 \times 10^{-11} \text{ J} = 130 \text{ MeV}$$

The mass equivalent Δm of this energy is

$$\Delta m = \frac{\Delta E}{c^2} = \frac{2.1 \times 10^{-11} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = 2.3 \times 10^{-28} \text{ kg} = 130 \text{ MeV}/c^2$$

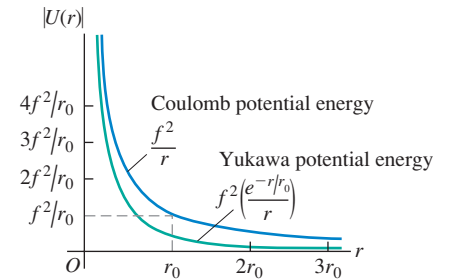
This is about 250 times the electron mass, and Yukawa postulated that an as yet undiscovered particle with this mass serves as the messenger for the nuclear force.

A year later, Carl Anderson and his colleague Seth Neddermeyer discovered in cosmic radiation two new particles, now called **muons**. The μ^- has charge equal to that of the electron, and its antiparticle the μ^+ has a positive charge with equal magnitude. The two particles have equal mass, about 207 times the electron mass. But it soon became clear that muons were *not* Yukawa's particles because they interacted with nuclei only very weakly.

In 1947 a family of three particles, called π mesons or **pions**, were discovered. Their charges are $+e$, $-e$, and zero, and their masses are about 270 times the electron mass. The pions interact strongly with nuclei, and they *are* the particles predicted by Yukawa. Other, heavier mesons, the ω and ρ , evidently also act as shorter-range messengers of the nuclear force. The complexity of this explanation suggests that it has simpler underpinnings; these involve the quarks and gluons that we'll discuss in Section 44.4. Before discussing mesons further, we'll describe some particle accelerators and detectors to see how mesons and other particles are created in a controlled fashion and observed.

Test Your Understanding of Section 44.1 Each of the following particles can be exchanged between two protons, two neutrons, or a neutron and a proton as part of the nuclear force. Rank the particles in order of the range of the interaction that they mediate, from largest to smallest range. (i) the π^+ (pi-plus) meson of mass $140 \text{ MeV}/c^2$; (ii) the ρ^+ (rho-plus) meson of mass $776 \text{ MeV}/c^2$; (iii) the η^0 (eta-zero) meson of mass $548 \text{ MeV}/c^2$; (iv) the ω^0 (omega-zero) meson of mass $783 \text{ MeV}/c^2$.

44.5 Graph of the magnitude of the Yukawa potential-energy function for nuclear forces, $|U(r)| = f^2 e^{-r/r_0}/r$. The function $U(r) = f^2/r$, proportional to the potential energy for Coulomb's law, is also shown. The two functions are similar at small r , but the Yukawa potential energy drops off much more quickly at large r .



44.2 Particle Accelerators and Detectors

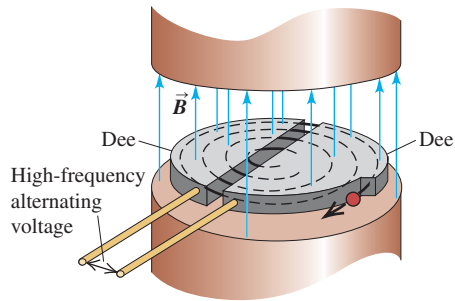
Early nuclear physicists used alpha and beta particles from naturally occurring radioactive elements for their experiments, but they were restricted in energy to the few MeV that are available in such random decays. Present-day particle accelerators can produce precisely controlled beams of particles, from electrons and positrons up to heavy ions, with a wide range of energies. These beams have three main uses. First, high-energy particles can collide to produce new particles, just as a collision of an electron and a positron can produce photons. Second, a high-energy particle has a short de Broglie wavelength and so can probe the small-scale interior structure of other particles, just as electron microscopes (see Section 39.1) can give better resolution than optical microscopes. Third, they can be used to produce nuclear reactions of scientific or medical use.

Linear Accelerators

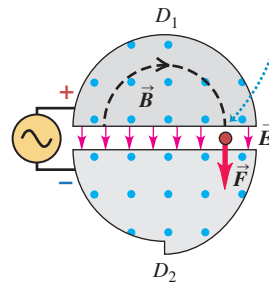
Particle accelerators use electric and magnetic fields to accelerate and guide beams of charged particles. A *linear accelerator* (linac) accelerates particles in a straight line. J. J. Thomson's cathode-ray tubes were early examples of linacs. Modern linacs use a series of electrodes with gaps to give the particles a series of boosts. Most present-day high-energy linear accelerators use a traveling electromagnetic wave; the charged particles "ride" the wave in more or less the way that a surfer rides an incoming ocean wave. In the highest-energy linac in the world today, at the SLAC National Accelerator Laboratory, electrons and positrons can be accelerated to 50 GeV in a tube 3 km long. At this energy their de Broglie wavelengths are 0.025 fm, much smaller than the size of a proton or a neutron.

44.6 Layout and operation of a cyclotron.

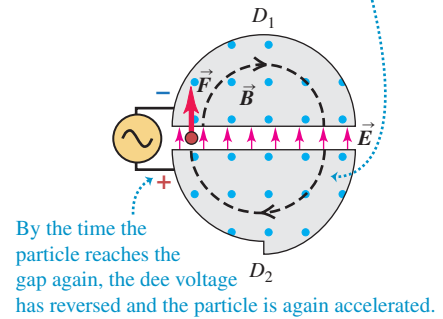
(a) Schematic diagram of a cyclotron



(b) As the positive particle reaches the gap, it is accelerated by the electric-field force ...



(c) ... and the next semicircular orbit has a larger radius.



The Cyclotron

Many accelerators use magnets to deflect the charged particles into circular paths. The first of these was the *cyclotron*, invented in 1931 by E. O. Lawrence and M. Stanley Livingston at the University of California (Fig. 44.6a). Particles with mass m and charge q move inside a vacuum chamber in a uniform magnetic field \vec{B} that is perpendicular to the plane of their paths. In Section 27.4 we showed that in such a field, a particle with speed v moves in a circular path with radius r given by

$$r = \frac{mv}{|q|B} \quad (44.6)$$

and with angular speed (angular frequency) ω given by

$$\omega = \frac{v}{r} = \frac{|q|B}{m} \quad (44.7)$$

An alternating potential difference is applied between the two hollow electrodes D_1 and D_2 (called *dees*), creating an electric field in the gap between them. The polarity of the potential difference and electric field is changed precisely twice each revolution (Figs. 44.6b and 44.6c), so that the particles get a push each time they cross the gap. The pushes increase their speed and kinetic energy, boosting them into paths of larger radius. The maximum speed v_{\max} and kinetic energy K_{\max} are determined by the radius R of the largest possible path. Solving Eq. (44.6) for v , we find $v = |q|BR/m$ and $v_{\max} = |q|BR/m$. Assuming nonrelativistic speeds, we have

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{q^2B^2R^2}{2m} \quad (44.8)$$

Example 44.1 Frequency and energy in a proton cyclotron

One cyclotron built during the 1930s has a path of maximum radius 0.500 m and a magnetic field of magnitude 1.50 T. If it is used to accelerate protons, find (a) the frequency of the alternating voltage applied to the dees and (b) the maximum particle energy.

SOLUTION

IDENTIFY and SET UP: The frequency f of the applied voltage must equal the frequency of the proton orbital motion. Equation (44.7) gives the *angular* frequency ω of the proton orbital motion; we find f using $f = \omega/2\pi$. The proton reaches its maximum energy

K_{\max} , given by Eq. (44.8), when the radius of its orbit equals the radius of the dees.

EXECUTE: (a) For protons, $q = 1.60 \times 10^{-19}$ C and $m = 1.67 \times 10^{-27}$ kg. From Eq. (44.7),

$$\begin{aligned} f &= \frac{\omega}{2\pi} = \frac{|q|B}{2\pi m} = \frac{(1.60 \times 10^{-19} \text{ C})(1.50 \text{ T})}{2\pi(1.67 \times 10^{-27} \text{ kg})} \\ &= 2.3 \times 10^7 \text{ Hz} = 23 \text{ MHz} \end{aligned}$$

(b) From Eq. (44.8) the maximum kinetic energy is

$$K_{\max} = \frac{(1.60 \times 10^{-19} \text{ C})^2 (1.50 \text{ T})^2 (0.50 \text{ m})^2}{2(1.67 \times 10^{-27} \text{ kg})}$$

$$= 4.3 \times 10^{-12} \text{ J} = 2.7 \times 10^7 \text{ eV} = 27 \text{ MeV}$$

This proton kinetic energy is much larger than that available from natural radioactive sources.

EVALUATE: From Eq. (44.6) or Eq. (44.7), the proton speed is $v = 7.2 \times 10^7 \text{ m/s}$, which is about 25% of the speed of light. At such speeds, relativistic effects are beginning to become important. Since we ignored these effects in our calculation, the above results for f and K_{\max} are in error by a few percent; this is why we kept only two significant figures.

The maximum energy that can be attained with a cyclotron is limited by relativistic effects. The relativistic version of Eq. (44.7) is

$$\omega = \frac{|q|B}{m} \sqrt{1 - v^2/c^2}$$

As the particles speed up, their angular frequency ω decreases, and their motion gets out of phase with the alternating dee voltage. In the *synchrocyclotron* the particles are accelerated in bursts. For each burst, the frequency of the alternating voltage is decreased as the particles speed up, maintaining the correct phase relationship with the particles' motion.

Another limitation of the cyclotron is the difficulty of building very large electromagnets. The largest synchrocyclotron ever built has a vacuum chamber that is about 8 m in diameter and accelerates protons to energies of about 600 MeV.

The Synchrotron

To attain higher energies, another type of machine, called the *synchrotron*, is more practical. Particles move in a vacuum chamber in the form of a thin doughnut, called the *accelerating ring*. The particle beam is bent to follow the ring by a series of electromagnets placed around the ring. As the particles speed up, the magnetic field is increased so that the particles retrace the same trajectory over and over. The Large Hadron Collider (LHC) near Geneva, Switzerland, is the highest-energy accelerator in the world (Fig. 44.7). It is designed to accelerate protons to a maximum energy of 7 TeV, or $7 \times 10^{12} \text{ eV}$. (As we'll discuss in Section 44.3, *hadrons* are a class of elementary particles that includes protons and neutrons.)

As we pointed out in Section 32.1, accelerated charges radiate electromagnetic energy. In an accelerator in which the particles move in curved paths, this radiation is often called *synchrotron radiation*. High-energy accelerators are typically constructed underground to provide protection from this radiation. From the accelerator standpoint, synchrotron radiation is undesirable, since the energy given to an accelerated particle is radiated right back out. It can be minimized by making the accelerator radius r large so that the centripetal acceleration v^2/r is small. On the positive side, synchrotron radiation is used as a source of well-controlled high-frequency electromagnetic waves.

Available Energy

When a beam of high-energy particles collides with a stationary target, not all the kinetic energy of the incident particles is *available* to form new particle states. Because momentum must be conserved, the particles emerging from the collision must have some net motion and thus some kinetic energy. The discussion following Example 43.11 (Section 43.6) presented a nonrelativistic example of this principle. The maximum available energy is the kinetic energy in the frame of reference in which the total momentum is zero. We call this the *center-of-momentum system*; it is the relativistic generalization of the center-of-mass system that we discussed in Section 8.5. In this system the total kinetic energy after the collision can be zero, so that the maximum amount of the initial kinetic energy becomes available to cause the reaction being studied.

44.7 (a) The Large Hadron Collider at the European Organization for Nuclear Research (CERN). The underground accelerating ring (shown by the red circle) is 100 m underground and 8.5 km in diameter, so large that it spans the border between Switzerland and France. (Note the Alps in the background.) When accelerated to 7 TeV, protons travel around the ring more than 11,000 times per second. (b) An engineer working on one of the 9593 superconducting electromagnets around the LHC ring.

(a)



(b)



Consider the *laboratory system*, in which a target particle with mass M is initially at rest and is bombarded by a particle with mass m and total energy (including rest energy) E_m . The total available energy E_a in the center-of-momentum system (including rest energies of all the particles) can be shown to be given by

$$E_a^2 = 2Mc^2E_m + (Mc^2)^2 + (mc^2)^2 \quad (\text{available energy}) \quad (44.9)$$

When the masses of the target and projectile particles are equal, this can be simplified to

$$E_a^2 = 2mc^2(E_m + mc^2) \quad (\text{available energy, equal masses}) \quad (44.10)$$

If in addition E_m is much greater than mc^2 , we can neglect the second term in the parentheses in Eq (44.10). Then E_a is

$$E_a = \sqrt{2mc^2E_m} \quad (\text{available energy, equal masses, } E_m \gg mc^2) \quad (44.11)$$

The square root in Eq. (44.11) is a disappointing result for an accelerator designer: Doubling the energy E_m of the bombarding particle increases the available energy E_a by only a factor of $\sqrt{2} = 1.414$. Examples 44.2 and 44.3 explore the limitations of having a stationary target particle.

Example 44.2 Threshold energy for pion production

A proton (rest energy 938 MeV) with kinetic energy K collides with a proton at rest. Both protons survive the collision, and a neutral pion (π^0 , rest energy 135 MeV) is produced. What is the threshold energy (minimum value of K) for this process?

SOLUTION

IDENTIFY and SET UP: The final state includes the two original protons (mass m) and the pion (mass m_π). The threshold energy corresponds to the minimum-energy case in which all three particles are at rest in the center-of-momentum system. The total available energy E_a in that system must be at least the total rest energy, $2mc^2 + m_\pi c^2$. We use this to solve Eq. (44.10) for the total energy E_m of the bombarding proton; the kinetic energy K (our target variable) is then E_m minus the proton rest energy mc^2 .

EXECUTE: We substitute $E_a = 2mc^2 + m_\pi c^2$ into Eq. (44.10), simplify, and solve for E_m :

$$4m^2c^4 + 4mm_\pi c^4 + m_\pi^2 c^4 = 2mc^2E_m + 2(mc^2)^2$$

$$E_m = mc^2 + m_\pi c^2 \left(2 + \frac{m_\pi}{2m} \right) = mc^2 + K$$

$$K = m_\pi c^2 \left(2 + \frac{m_\pi}{2m} \right)$$

We see that the bombarding proton's kinetic energy K must be somewhat greater than twice the pion rest energy $m_\pi c^2$. With $mc^2 = 938$ MeV and $m_\pi c^2 = 135$ MeV, we have $m_\pi/2m = 0.072$ and

$$K = (135 \text{ MeV})(2 + 0.072) = 280 \text{ MeV}$$

EVALUATE: Compare this result with the result of Example 37.11 (Section 37.8), where we found that a pion can be produced in a head-on collision of two protons, each with only 67.5 MeV of kinetic energy. We discuss the energy advantage of such collisions in the next subsection.

Example 44.3 Increasing the available energy

The Fermilab accelerator in Illinois was designed to bombard stationary targets with 800-GeV protons. (a) What is the available energy E_a in a proton-proton collision? (b) What is E_a if the beam energy is increased to 980 GeV?

SOLUTION

IDENTIFY and SET UP: Our target variable is the available energy E_a in a stationary-target collision between identical particles. In both parts (a) and (b) the beam energy E_m is much larger than the proton rest energy $mc^2 = 938$ MeV = 0.938 GeV, so we can safely use the approximation of Eq. (44.11).

EXECUTE: (a) For $E_m = 800$ GeV, Eq. (44.11) gives

$$E_a = \sqrt{2(0.938 \text{ GeV})(800 \text{ GeV})} = 38.7 \text{ GeV}$$

(b) For $E_m = 980$ GeV,

$$E_a = \sqrt{2(0.938 \text{ GeV})(980 \text{ GeV})} = 42.9 \text{ GeV}$$

EVALUATE: With a stationary-proton target, increasing the proton beam energy by 180 GeV increases the available energy by only 4.2 GeV! This shows a major limitation of experiments in which one of the colliding particles is initially at rest. Below we describe how physicists can overcome this limitation.

Colliding Beams

The limitation illustrated by Example 44.3 is circumvented in *colliding-beam* experiments. In these experiments there is no stationary target; instead, beams of particles moving in opposite directions are tightly focused onto one another so that head-on collisions can occur. Usually the two colliding particles have momenta of equal magnitude and opposite direction, so the total momentum is zero. Hence the laboratory system is also the center-of-momentum system, and the available energy is maximized.

The highest-energy colliding beams available are those at the Large Hadron Collider (see Fig. 44.7). In operation, 2808 bunches of 7-TeV protons circulate around the ring, half in one direction and half in the opposite direction. Each bunch contains about 10^{11} protons. Magnets steer the oppositely moving bunches to collide at interaction points. The available energy E_a in the resulting head-on collisions is the *total* energy of the two colliding particles: $E_a = 2 \times 7 \text{ TeV} = 14 \text{ TeV}$. (Strictly, E_a is 14 TeV minus the rest energy of the two colliding protons. But this rest energy is only $2mc^2 = 2(938 \text{ MeV}) = 1.876 \times 10^{-3} \text{ TeV}$, which is so small compared to 14 TeV that it can be ignored.) Physicists expect that the very large available energy at the Large Hadron Collider will make it possible to produce particles that have never been seen before.

Detectors

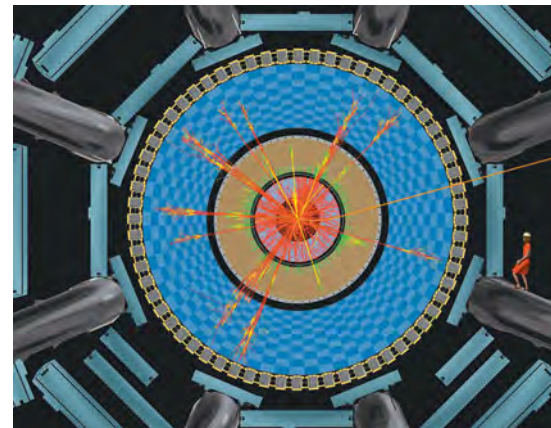
Ordinarily, we can't see or feel individual subatomic particles or photons. How, then, do we measure their properties? A wide variety of devices have been designed. Many detectors use the ionization caused by charged particles as they move through a gas, liquid, or solid. The ions along the particle's path act as nucleation centers for droplets of liquid in the supersaturated vapor of a cloud chamber (Fig. 44.1) or cause small volumes of vapor in the superheated liquid of a bubble chamber (Fig. 44.3a). In a semiconducting solid the ionization can take the form of electron-hole pairs. We discussed their detection in Section 42.7. *Wire chambers* contain arrays of closely spaced wires that detect the ions. The charge collected and time information from each wire are processed using computers to reconstruct the particle trajectories. The detectors at the Large Hadron Collider use an array of devices to follow the tracks of particles produced by collisions between protons (Fig. 44.8). The giant solenoid in the photo that opens Chapter 28 is at the heart of one these detector arrays. The intense magnetic field of the solenoid helps identify newly produced particles, which curve in different directions and along paths of different radii depending on their charge and energy.

Cosmic-Ray Experiments

Large numbers of particles called *cosmic rays* continually bombard the earth from sources both within and beyond our galaxy. These particles consist mostly of neutrinos, protons, and heavier nuclei, with energies ranging from less than 1 MeV to more than 10^{20} eV. The earth's atmosphere and magnetic field protect us from much of this radiation. This means that cosmic-ray experimentation often must be carried out above all or most of the atmosphere by means of rockets or high-altitude balloons.

In contrast, neutrino detectors are buried below the earth's surface in tunnels or mines or submerged deep in the ocean. This is done to screen out all other types of particles so that only neutrinos, which interact only very weakly with matter, reach the detector. It would take a light-year's thickness of lead to absorb a sizable fraction of a beam of neutrinos. Thus neutrino detectors consist of huge amounts of matter: The Super-Kamiokande detector looks for flashes of light produced when a neutrino interacts in a tank containing 5×10^7 kg of water (see Section 44.5).

44.8 This computer-generated image shows the result of a simulated collision between two protons (not shown) in one of the interaction regions at the Large Hadron Collider. The view is along the beampipe. The different color tracks show different types of particles emerging from the collision. A variety of different detectors surround the collision region. (Note the drawing of woman in a red dress, shown for scale.)



Cosmic rays were important in early particle physics, and their study currently brings us important information about the rest of the universe. Although cosmic rays provide a source of high-energy particles that does not depend on expensive accelerators, most particle physicists use accelerators because the high-energy cosmic-ray particles they want are too few and too random.

Test Your Understanding of Section 44.2 In a colliding-beam experiment, a 90-GeV electron collides head-on with a 90-GeV positron. The electron and the positron annihilate each other, forming a single virtual photon that then transforms into other particles. Does the virtual photon obey the same relationship $E = pc$ as real photons do? **I**

44.3 Particles and Interactions

We have mentioned the array of subatomic particles that were known as of 1947: photons, electrons, positrons, protons, neutrons, muons, and pions. Since then, literally hundreds of additional particles have been discovered in accelerator experiments. The vast majority of known particles are *unstable* and decay spontaneously into other particles. Particles of all kinds, whether stable or unstable, can be created or destroyed in interactions between particles. Each such interaction involves the exchange of virtual particles, which exist on borrowed energy allowed by the uncertainty principle.

Although the world of subatomic particles and their interactions is complex, some key results bring order and simplicity to the seeming chaos. One key simplification is that there are only four fundamental types of interactions, each mediated or transmitted by the exchange of certain characteristic virtual particles. Furthermore, not all particles respond to all four kinds of interaction. In this section we will examine the fundamental interactions more closely and see how physicists classify particles in terms of the ways in which they interact.

Four Forces and Their Mediating Particles

In Section 5.5 we first described the four fundamental types of forces or interactions (Fig. 44.9). They are, in order of decreasing strength:

1. The strong interaction
2. The electromagnetic interaction
3. The weak interaction
4. The gravitational interaction

The *electromagnetic* and *gravitational* interactions are familiar from classical physics. Both are characterized by a $1/r^2$ dependence on distance. In this scheme, the mediating particles for both interactions have mass zero and are stable as ordinary particles. The mediating particle for the electromagnetic interaction is the familiar photon, which has spin 1. (That means its spin quantum number is $s = 1$, so the magnitude of its spin angular momentum is $S = \sqrt{s(s + 1)}\hbar = \sqrt{2}\hbar$.) That particle for the gravitational force is the spin-2 *graviton* ($s = 2$, $S = \sqrt{s(s + 1)}\hbar = \sqrt{6}\hbar$). The graviton has not yet been observed experimentally because the gravitational force is very much weaker than the electromagnetic force. For example, the gravitational attraction of two protons is smaller than their electrical repulsion by a factor of about 10^{36} . The gravitational force is of primary importance in the structure of stars and the large-scale behavior of the universe, but it is not believed to play a significant role in particle interactions at the energies that are currently attainable.

The other two forces are less familiar. One, usually called the *strong interaction*, is responsible for the nuclear force and also for the production of pions and several other particles in high-energy collisions. At the most fundamental level, the mediating particle for the strong interaction is called a *gluon*. However, the force between nucleons is more easily described in terms of mesons as the mediating particles. We'll discuss the spin-1, massless gluon in Section 44.4.

44.9 The ties that bind us together originate in the fundamental interactions of nature. The nuclei within our bodies are held together by the strong interaction. The electromagnetic interaction binds nuclei and electrons together to form atoms, binds atoms together to form molecules, and binds molecules together to form us.





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Equation (44.4) is a possible potential-energy function for the nuclear force. The strength of the interaction is described by the constant f^2 , which has units of energy times distance. A better basis for comparison with other forces is the dimensionless ratio $f^2/\hbar c$, called the *coupling constant* for the interaction. (We invite you to verify that this ratio is a pure number and so must have the same value in all systems of units.) The observed behavior of nuclear forces suggests that $f^2/\hbar c \approx 1$. The dimensionless coupling constant for *electromagnetic* interactions is

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} = 7.297 \times 10^{-3} = \frac{1}{137.0} \quad (44.12)$$

Thus the strong interaction is roughly 100 times as strong as the electromagnetic interaction; however, it drops off with distance more quickly than $1/r^2$.

The fourth interaction is called the *weak* interaction. It is responsible for beta decay, such as the conversion of a neutron into a proton, an electron, and an anti-neutrino. It is also responsible for the decay of many unstable particles (pions into muons, muons into electrons, and so on). Its mediating particles are the short-lived particles W^+ , W^- , and Z^0 . The existence of these particles was confirmed in 1983 in experiments at CERN, for which Carlo Rubbia and Simon van der Meer were awarded the Nobel Prize in 1984. The W^\pm and Z^0 have spin 1 like the photon and the gluon, but they are *not* massless. In fact, they have enormous masses, $80.4 \text{ GeV}/c^2$ for the W 's and $91.2 \text{ GeV}/c^2$ for the Z^0 . With such massive mediating particles the weak interaction has a much shorter range than the strong interaction. It also lives up to its name by being weaker than the strong interaction by a factor of about 10^9 .

Table 44.1 compares the main features of these four fundamental interactions.

More Particles

In Section 44.1 we mentioned the discoveries of muons in 1937 and of pions in 1947. The electric charges of the muons and the charged pions have the same magnitude e as the electron charge. The positive muon μ^+ is the antiparticle of the negative muon μ^- . Each has spin $\frac{1}{2}$, like the electron, and a mass of about $207m_e = 106 \text{ MeV}/c^2$. Muons are unstable; each decays with a lifetime of $2.2 \times 10^{-6} \text{ s}$ into an electron of the same sign, a neutrino, and an antineutrino.

There are three kinds of pions, all with spin 0; they have *no* spin angular momentum. The π^+ and π^- have masses of $273m_e = 140 \text{ MeV}/c^2$. They are unstable; each π^\pm decays with a lifetime of $2.6 \times 10^{-8} \text{ s}$ into a muon of the same sign along with a neutrino for the π^+ and an antineutrino for the π^- . The π^0 is somewhat less massive, $264m_e = 135 \text{ MeV}/c^2$, and it decays with a lifetime of $8.4 \times 10^{-17} \text{ s}$ into two photons. The π^+ and π^- are antiparticles of one another, while the π^0 is its own antiparticle. (That is, there is no distinction between particle and antiparticle for the π^0 .)

The existence of the *antiproton* \bar{p} had been suspected ever since the discovery of the positron. The \bar{p} was found in 1955, when proton–antiproton ($p\bar{p}$) pairs were created by use of a beam of 6-GeV protons from the Bevatron at the University of

Table 44.1 Four Fundamental Interactions

Interaction	Relative Strength	Range	Mediating Particle			
			Name	Mass	Charge	Spin
Strong	1	Short ($\sim 1 \text{ fm}$)	Gluon	0	0	1
Electromagnetic	$\frac{1}{137}$	Long ($1/r^2$)	Photon	0	0	1
Weak	10^{-9}	Short ($\sim 0.001 \text{ fm}$)	W^\pm, Z^0	80.4, 91.2 GeV/c^2	$\pm e, 0$	1
Gravitational	10^{-38}	Long ($1/r^2$)	Graviton	0	0	2

California, Berkeley. The *antineutron* \bar{n} was found soon afterward. After 1960, as higher-energy accelerators and more sophisticated detectors were developed, a veritable blizzard of new unstable particles were identified. To describe and classify them, we need a small blizzard of new terms.

Initially, particles were classified by mass into three categories: (1) leptons (“light ones” such as electrons); (2) mesons (“intermediate ones” such as pions); and (3) baryons (“heavy ones” such as nucleons and more massive particles). But this scheme has been superseded by a more useful one in which particles are classified in terms of their *interactions*. For instance, *hadrons* (which include mesons and baryons) have strong interactions, and *leptons* do not.

In the following discussion we will also distinguish between **fermions**, which have half-integer spins, and **bosons**, which have zero or integer spins. Fermions obey the exclusion principle, on which the Fermi-Dirac distribution function (see Section 42.5) is based. Bosons do not obey the exclusion principle and have a different distribution function, the Bose-Einstein distribution.

Leptons

The **leptons**, which do not have strong interactions, include six particles; the electron (e^-) and its neutrino (ν_e), the muon (μ^-) and its neutrino (ν_μ), and the tau particle (τ^-) and its neutrino (ν_τ). Each of the six particles has a distinct antiparticle. All leptons have spin $\frac{1}{2}$ and thus are fermions. The family of leptons is shown in Table 44.2. The taus have mass $3478m_e = 1777 \text{ MeV}/c^2$. Taus and muons are unstable; a τ^- decays into a μ^- plus a tau neutrino and a muon antineutrino, or an electron plus a tau neutrino and an electron antineutrino. A μ^- decays into an electron plus a muon neutrino and an electron antineutrino. They have relatively long lifetimes because their decays are mediated by the weak interaction. Despite their zero charge, a neutrino is distinct from an antineutrino; the spin angular momentum of a neutrino has a component that is opposite its linear momentum, while for an antineutrino that component is parallel to its linear momentum. Because neutrinos are so elusive, physicists have only been able to place upper limits on the rest masses of the ν_e , the ν_μ , and the ν_τ . Until recently, it was thought that the rest masses of the neutrinos were zero; compelling evidence now indicates that they have small but nonzero masses. We’ll return to this point and its implications later.

Leptons obey a *conservation principle*. Corresponding to the three pairs of leptons are three lepton numbers L_e , L_μ , and L_τ . The electron e^- and the electron neutrino ν_e are assigned $L_e = 1$, and their antiparticles e^+ and $\bar{\nu}_e$ are given $L_e = -1$. Corresponding assignments of L_μ and L_τ are made for the μ and τ particles and their neutrinos. **In all interactions, each lepton number is separately conserved.** For example, in the decay of the μ^- , the lepton numbers are

$$\begin{array}{ccccccc} \mu^- & \rightarrow & e^- & + & \bar{\nu}_e & + & \nu_\mu \\ L_\mu = 1 & & L_e = 1 & & L_e = -1 & & L_\mu = 1 \end{array}$$

These conservation principles have no counterpart in classical physics.

Table 44.2 The Six Leptons

Particle Name	Symbol	Anti-particle	Mass (MeV/c ²)	L_e	L_μ	L_τ	Lifetime (s)	Principal Decay Modes
Electron	e^-	e^+	0.511	+1	0	0	Stable	
Electron neutrino	ν_e	$\bar{\nu}_e$	$<3 \times 10^{-6}$	+1	0	0	Stable	
Muon	μ^-	μ^+	105.7	0	+1	0	2.20×10^{-6}	$e^- \bar{\nu}_e \nu_\mu$
Muon neutrino	ν_μ	$\bar{\nu}_\mu$	<0.19	0	+1	0	Stable	
Tau	τ^-	τ^+	1777	0	0	+1	2.9×10^{-13}	$\mu^- \bar{\nu}_\mu \nu_\tau$
Tau neutrino	ν_τ	$\bar{\nu}_\tau$	<18.2	0	0	+1	Stable	or $e^- \bar{\nu}_e \nu_\tau$

Example 44.4 Lepton number conservation

Check conservation of lepton numbers for these decay schemes:

$$(a) \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

$$(b) \pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$(c) \pi^0 \rightarrow \mu^- + e^+ + \nu_e$$

$$(a) \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

$$L_e: 0 = -1 + 1 + 0$$

$$L_\mu: -1 = 0 + 0 + (-1)$$

$$(b) \pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$L_e: 0 = 0 + 0$$

$$L_\mu: 0 = 1 + (-1)$$

$$(c) \pi^0 \rightarrow \mu^- + e^+ + \nu_e$$

$$L_e: 0 = 0 + (-1) + 1$$

$$L_\mu: 0 \neq 1 + 0 + 0$$

SOLUTION

IDENTIFY and SET UP: Lepton number conservation requires that L_e , L_μ , and L_τ (given in Table 44.2) separately have the same sums after the decay as before.

EXECUTE: We tabulate L_e and L_μ for each decay scheme. An antiparticle has the opposite lepton number from its corresponding particle listed in Table 44.2. No τ particles or τ neutrinos appear in any of the schemes, so $L_\tau = 0$ both before and after each decay and L_τ is conserved.

EVALUATE: Decays (a) and (b) are consistent with lepton number conservation and are observed. Decay (c) violates the conservation of L_μ and has *never* been observed. Physicists used these and other experimental results to deduce the principle that all three lepton numbers must separately be conserved.

Hadrons

Hadrons, the strongly interacting particles, are a more complex family than leptons. Each hadron has an antiparticle, often denoted with an overbar, as with the antiproton \bar{p} . There are two subclasses of hadrons: *mesons* and *baryons*. Table 44.3 shows some of the many hadrons that are currently known. (We'll explain *strangeness* and *quark content* later in this section and in the next one.)

Mesons include the pions that have already been mentioned, K mesons or *kaons*, η mesons, and others that we will discuss later. Mesons have spin 0 or 1 and therefore are all bosons. There are no stable mesons; all can and do decay to less massive particles, obeying all the conservation laws for such decays.

Table 44.3 Some Hadrons and Their Properties

Particle	Mass (MeV/c ²)	Charge Ratio, Q/e	Spin	Baryon Number, B	Strangeness, S	Mean Lifetime (s)	Typical Decay Modes	Quark Content
<i>Mesons</i>								
π^0	135.0	0	0	0	0	8.4×10^{-17}	$\gamma \gamma$	$u\bar{u}, d\bar{d}$
π^+	139.6	+1	0	0	0	2.60×10^{-8}	$\mu^+ \nu_\mu$	$u\bar{d}$
π^-	139.6	-1	0	0	0	2.60×10^{-8}	$\mu^- \bar{\nu}_\mu$	$\bar{u}d$
K^+	493.7	+1	0	0	+1	1.24×10^{-8}	$\mu^+ \nu_\mu$	$u\bar{s}$
K^-	493.7	-1	0	0	-1	1.24×10^{-8}	$\mu^- \bar{\nu}_\mu$	$\bar{u}s$
η^0	547.3	0	0	0	0	$\approx 10^{-18}$	$\gamma \gamma$	$u\bar{u}, d\bar{d}, s\bar{s}$
<i>Baryons</i>								
p	938.3	+1	$\frac{1}{2}$	1	0	Stable	—	uud
n	939.6	0	$\frac{1}{2}$	1	0	886	$p e^- \bar{\nu}_e$	udd
Λ^0	1116	0	$\frac{1}{2}$	1	-1	2.63×10^{-10}	$p\pi^-$ or $n\pi^0$	uds
Σ^+	1189	+1	$\frac{1}{2}$	1	-1	8.02×10^{-11}	$p\pi^0$ or $n\pi^+$	uus
Σ^0	1193	0	$\frac{1}{2}$	1	-1	7.4×10^{-20}	$\Lambda^0 \gamma$	uds
Σ^-	1197	-1	$\frac{1}{2}$	1	-1	1.48×10^{-10}	$n\pi^-$	dds
Ξ^0	1315	0	$\frac{1}{2}$	1	-2	2.90×10^{-10}	$\Lambda^0 \pi^0$	uss
Ξ^-	1321	-1	$\frac{1}{2}$	1	-2	1.64×10^{-10}	$\Lambda^0 \pi^-$	dss
Δ^{++}	1232	+2	$\frac{3}{2}$	1	0	$\approx 10^{-23}$	$p\pi^+$	uuu
Ω^-	1672	-1	$\frac{3}{2}$	1	-3	8.2×10^{-11}	$\Lambda^0 K^-$	sss
Λ_c^+	2285	+1	$\frac{1}{2}$	1	0	2.0×10^{-13}	$pK^-\pi^+$	udc

Baryons include the nucleons and several particles called *hyperons*, including the Λ , Σ , Ξ , and Ω . These resemble the nucleons but are more massive. Baryons have half-integer spin, and therefore all are fermions. The only stable baryon is the proton; a free neutron decays to a proton, and the hyperons decay to other hyperons or to nucleons by various processes. Baryons obey the principle of *conservation of baryon number*, analogous to conservation of lepton numbers, again with no counterpart in classical physics. We assign a baryon number $B = 1$ to each baryon (p , n , Λ , Σ , and so on) and $B = -1$ to each antibaryon (\bar{p} , \bar{n} , $\bar{\Lambda}$, $\bar{\Sigma}$, and so on).

In all interactions, the total baryon number is conserved.

This principle is the reason the mass number A was conserved in all of the nuclear reactions that we studied in Chapter 43.

Example 44.5 Baryon number conservation

Check conservation of baryon number for these reactions:

- (a) $n + p \rightarrow n + p + p + \bar{p}$
 (b) $n + p \rightarrow n + p + \bar{n}$

SOLUTION

IDENTIFY and SET UP: This example is similar to Example 44.4. We compare the total baryon number before and after each reaction, using data from Table 44.3.

EXECUTE: We tabulate the baryon numbers, noting that a baryon has $B = 1$ and an antibaryon has $B = -1$:

- (a) $n + p \rightarrow n + p + p + \bar{p}$: $1 + 1 = 1 + 1 + 1 + (-1)$
 (b) $n + p \rightarrow n + p + \bar{n}$: $1 + 1 \neq 1 + 1 + (-1)$

EVALUATE: Reaction (a) is consistent with baryon number conservation. It can occur if enough energy is available in the $n + p$ collision. Reaction (b) violates baryon number conservation and has never been observed.

Example 44.6 Antiproton creation

What is the minimum proton energy required to produce an antiproton in a collision with a stationary proton?

SOLUTION

IDENTIFY and SET UP: The reaction must conserve baryon number, charge, and energy. Since the target and bombarding protons are of equal mass and the target is at rest, we determine the minimum energy E_m of the bombarding proton using Eq. (44.10).

EXECUTE: Conservation of charge and conservation of baryon number forbid the creation of an antiproton by itself; it must be created as part of a proton–antiproton pair. The complete reaction is



For this reaction to occur, the minimum available energy E_a in Eq. (44.10) is the final rest energy $4mc^2$ of three protons and an antiproton. Equation (44.10) then gives

$$(4mc^2)^2 = 2mc^2(E_m + mc^2)$$

$$E_m = 7mc^2$$

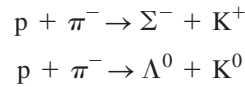
EVALUATE: The energy E_m of the bombarding proton includes its rest energy mc^2 , so its minimum *kinetic* energy must be $6mc^2 = 6(938 \text{ MeV}) = 5.63 \text{ GeV}$.

The search for the antiproton was a principal reason for the construction of the Bevatron at the University of California, Berkeley, with beam energy of 6 GeV. The search succeeded in 1955, and Emilio Segrè and Owen Chamberlain were later awarded the Nobel Prize for this discovery.

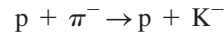
Strangeness

The K mesons and the Λ and Σ hyperons were discovered during the late 1950s. Because of their unusual behavior they were called *strange particles*. They were produced in high-energy collisions such as $\pi^- + p$, and a K meson and a hyperon were always produced *together*. The relatively high rate of production of these particles suggested that it was a *strong*-interaction process, but their relatively long lifetimes suggested that their decay was a *weak*-interaction process. The K^0 appeared to have *two* lifetimes, one about $9 \times 10^{-11} \text{ s}$ and another nearly 600 times longer. Were the K mesons strongly interacting hadrons or not?

The search for the answer to this question led physicists to introduce a new quantity called **strangeness**. The hyperons Λ^0 and $\Sigma^{\pm,0}$ were assigned a strangeness quantum number $S = -1$, and the associated K^0 and K^+ mesons were assigned $S = +1$. The corresponding antiparticles had opposite strangeness, $S = +1$ for $\bar{\Lambda}^0$ and $\bar{\Sigma}^{\pm,0}$ and $S = -1$ for \bar{K}^0 and K^- . Then strangeness was *conserved* in production processes such as

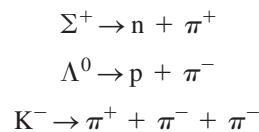


The process



does not conserve strangeness and it does not occur.

When strange particles decay individually, strangeness is usually *not* conserved. Typical processes include



In each of these decays, the initial strangeness is 1 or -1 , and the final value is zero. All observations of these particles are consistent with the conclusion that *strangeness is conserved in strong interactions but it can change by zero or one unit in weak interactions*. There is no counterpart to the strangeness quantum number in classical physics.

CAUTION **Strangeness vs. spin** Take care not to confuse the symbol S for strangeness with the identical symbol for the magnitude of the spin angular momentum. **|**

Conservation Laws

The decay of strange particles provides our first example of a *conditional conservation law*, one that is obeyed in some interactions and not in others. By contrast, several conservation laws are obeyed in *all* interactions. These include the familiar conservation laws; energy, momentum, angular momentum, and electric charge. These are called *absolute conservation laws*. Baryon number and the three lepton numbers are also conserved in all interactions. Strangeness is conserved in strong and electromagnetic interactions but *not* in all weak interactions.

Two other quantities, which are conserved in some but not all interactions, are useful in classifying particles and their interactions. One is *isospin*, a quantity that is used to describe the charge independence of the strong interactions. The other is *parity*, which describes the comparative behavior of two systems that are mirror images of each other. Isospin is conserved in strong interactions, which are charge independent, but not in electromagnetic or weak interactions. (The electromagnetic interaction is certainly *not* charge independent.) Parity is conserved in strong and electromagnetic interactions but not in weak ones. The Chinese-American physicists T. D. Lee and C. N. Yang received the Nobel Prize in 1957 for laying the theoretical foundations for nonconservation of parity in weak interactions.

This discussion shows that conservation laws provide another basis for classifying particles and their interactions. Each conservation law is also associated with a *symmetry* property of the system. A familiar example is angular momentum. If a system is in an environment that has spherical symmetry, there can be no torque acting on it because the direction of the torque would violate the symmetry. In such a system, total angular momentum is *conserved*. When a conservation law is violated, the interaction is often described as a *symmetry-breaking interaction*.

Test Your Understanding of Section 44.3 From conservation of energy, a particle of mass m and rest energy mc^2 can decay only if the decay products have a total mass less than m . (The remaining energy goes into the kinetic energy of the decay products.) Can a proton decay into less massive mesons?

44.4 Quarks and the Eightfold Way

The leptons form a fairly neat package: three particles and three neutrinos, each with its antiparticle, and a conservation law relating their numbers. Physicists believe that leptons are genuinely fundamental particles. The hadron family, by comparison, is a mess. Table 44.3 contains only a sample of well over 100 hadrons that have been discovered since 1960, and it has become clear that these particles *do not* represent the most fundamental level of the structure of matter.

Our present understanding of the structure of hadrons is based on a proposal made initially in 1964 by the American physicist Murray Gell-Mann and his collaborators. In this proposal, hadrons are not fundamental particles but are composite structures whose constituents are spin- $\frac{1}{2}$ fermions called **quarks**. (The name is found in the line “Three quarks for Muster Mark!” from *Finnegans Wake* by James Joyce.) Each baryon is composed of three quarks (qqq), each antibaryon of three antiquarks ($\bar{q}\bar{q}\bar{q}$), and each meson of a quark–antiquark pair ($q\bar{q}$). Table 44.3 of the preceding section gives the quark content of many hadrons. No other compositions seem to be necessary. This scheme requires that quarks have electric charges with magnitudes $\frac{1}{3}$ and $\frac{2}{3}$ of the electron charge e , which had previously been thought to be the smallest unit of charge. Each quark also has a fractional value $\frac{1}{3}$ for its baryon number B , and each antiquark has a baryon-number value $-\frac{1}{3}$. In a meson, a quark and antiquark combine with net baryon number 0 and can have their spin angular momentum components parallel to form a spin-1 meson or antiparallel to form a spin-0 meson. Similarly, the three quarks in a baryon combine with net baryon number 1 and can form a spin- $\frac{1}{2}$ baryon or a spin- $\frac{3}{2}$ baryon.

The Three Original Quarks

The first (1964) quark theory included three types (called *flavors*) of quarks, labeled u (up), d (down), and s (strange). Their principal properties are listed in Table 44.4. The corresponding antiquarks \bar{u} , \bar{d} , and \bar{s} have opposite values of charge Q , B , and S . Protons, neutrons, π and K mesons, and several hyperons can be constructed from these three quarks. For example, the proton quark content is uud . Checking Table 44.4, we see that the values of Q/e add to 1 and that the values of the baryon number B also add to 1, as we should expect. The neutron is udd , with total $Q = 0$ and $B = 1$. The π^+ meson is $u\bar{d}$, with $Q/e = 1$ and $B = 0$, and the K^+ meson is $u\bar{s}$. Checking the values of S for the quark content, we see that the proton, neutron, and π^+ have strangeness 0 and that the K^+ has strangeness 1, in agreement with Table 44.3. The antiproton is $\bar{p} = \bar{u}\bar{u}\bar{d}$, the negative pion is $\pi^- = \bar{u}\bar{d}$, and so on. The quark content can also be used to explain hadron excited states and magnetic moments. Figure 44.10 shows the quark content of two baryons and two mesons.

44.10 Quark content of four different hadrons. The various color combinations that are needed for color neutrality are not shown.

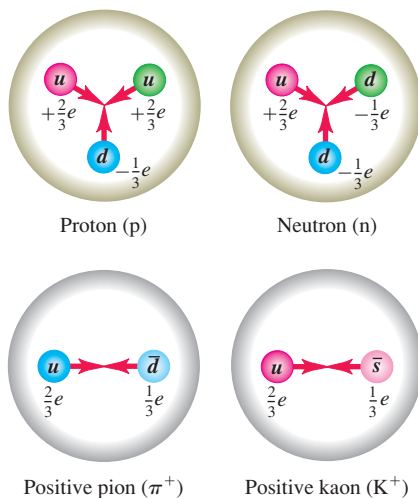


Table 44.4 Properties of the Three Original Quarks

Symbol	Q/e	Spin	Baryon Number, B	Strangeness, S	Charm, C	Bottomness, B'	Topness, T
u	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0
d	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0
s	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	-1	0	0	0

Example 44.7 Determining the quark content of baryons

Given that they contain only $u, d, s, \bar{u}, \bar{d}$, and/or \bar{s} , find the quark content of (a) Σ^+ and (b) $\bar{\Lambda}^0$. The Σ^+ and Λ^0 (the antiparticle of the $\bar{\Lambda}^0$) are both baryons with strangeness $S = -1$.

SOLUTION

IDENTIFY and SET UP: We use the idea that the total charge of each baryon is the sum of the individual quark charges, and similarly for the baryon number and strangeness. We use the quark properties given in Table 44.4.

EXECUTE: Baryons contain three quarks. If $S = -1$, exactly *one* of the three must be an s quark, which has $S = -1$ and $Q/e = -\frac{1}{3}$.

(a) The Σ^+ has $Q/e = +1$, so the other two quarks must both be u quarks (each of which has $Q/e = +\frac{2}{3}$). Hence the quark content of Σ^+ is uus .

(b) First we find the quark content of the Λ^0 . To yield zero total charge, the other two quarks must be u ($Q/e = +\frac{2}{3}$) and d ($Q/e = -\frac{1}{3}$), so the quark content of the Λ^0 is uds . The quark content of the $\bar{\Lambda}^0$ is therefore $\bar{u}\bar{d}\bar{s}$.

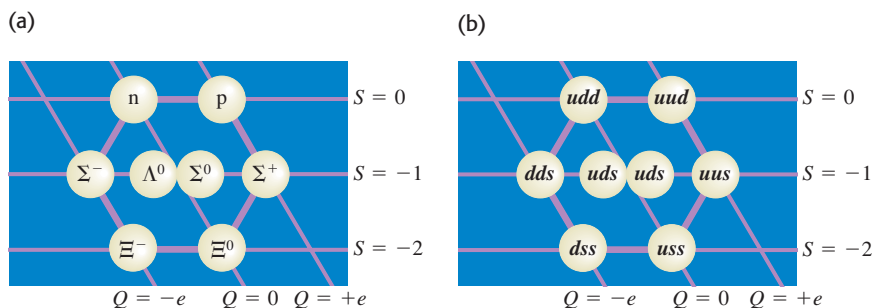
EVALUATE: Although the Λ^0 and $\bar{\Lambda}^0$ are both electrically neutral and have the same mass, they are different particles: Λ^0 has $B = 1$ and $S = -1$, while $\bar{\Lambda}^0$ has $B = -1$ and $S = 1$.

Motivating the Quark Model

What caused physicists to suspect that hadrons were made up of something smaller? The magnetic moment of the neutron (see Section 43.1) was one of the first reasons. In Section 27.7 we learned that a magnetic moment results from a circulating current (a motion of electric charge). But the neutron has *no* charge, or, to be more accurate, no *total* charge. It could be made up of smaller particles whose charges add to zero. The quantum motion of these particles within the neutron would then give its surprising nonzero magnetic moment. To verify this hypothesis by “seeing” inside a neutron, we need a probe with a wavelength that is much less than the neutron’s size of about a femtometer. This probe should not be affected by the strong interaction, so that it won’t interact with the neutron as a whole but will penetrate into it and interact electromagnetically with these supposed smaller charged particles. A probe with these properties is an electron with energy above 10 GeV. In experiments carried out at SLAC, such electrons were scattered from neutrons and protons to help show that nucleons are indeed made up of fractionally charged, spin- $\frac{1}{2}$ pointlike particles.

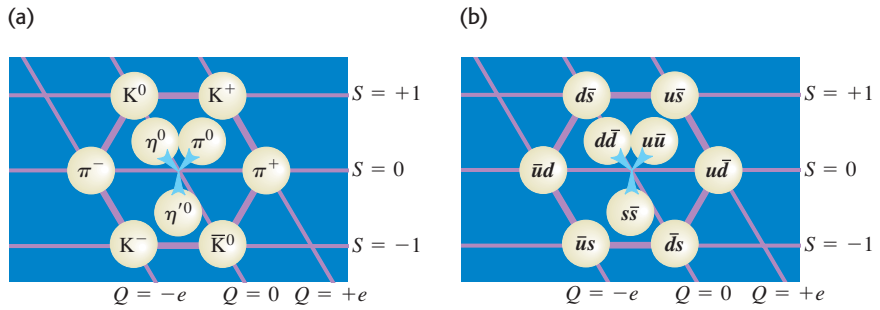
The Eightfold Way

Symmetry considerations play a very prominent role in particle theory. Here are two examples. Consider the eight spin- $\frac{1}{2}$ baryons we’ve mentioned: the familiar p and n ; the strange $\Lambda^0, \Sigma^+, \Sigma^0$, and Σ^- ; and the doubly strange Ξ^0 and Ξ^- . For each we plot the value of strangeness S versus the value of charge Q in Fig. 44.11. The result is a hexagonal pattern. A similar plot for the nine spin-0 mesons (six shown in Table 44.3 plus three others not included in that table) is shown in Fig. 44.12; the particles fall in exactly the same hexagonal pattern! In each plot, all the particles have masses that are within about $\pm 200 \text{ MeV}/c^2$ of the median mass value of that plot, with variations due to differences in quark masses and internal potential energies.



44.11 (a) Plot of S and Q values for spin- $\frac{1}{2}$ baryons, showing the symmetry pattern of the eightfold way. (b) Quark content of each spin- $\frac{1}{2}$ baryon. The quark contents of the Σ^0 and Λ^0 are the same; the Σ^0 is an excited state of the Λ^0 and can decay into it by photon emission.

44.12 (a) Plot of S and Q values for nine spin-0 mesons, showing the symmetry pattern of the eightfold way. Each particle is on the opposite side of the hexagon from its antiparticle; each of the three particles in the center is its own antiparticle. (b) Quark content of each spin-0 meson. The particles in the center are different mixtures of the three quark–antiquark pairs shown.



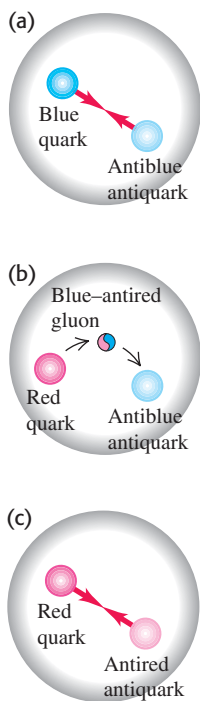
The symmetries that lead to these and similar patterns are collectively called the **eightfold way**. They were discovered in 1961 by Murray Gell-Mann and independently by Yu'val Ne'emán. (The name is a slightly irreverent reference to the Noble Eightfold Path, a set of principles for right living in Buddhism.) A similar pattern for the spin- $\frac{3}{2}$ baryons contains *ten* particles, arranged in a triangular pattern like pins in a bowling alley. When this pattern was first discovered, one of the particles was missing. But Gell-Mann gave it a name anyway (Ω^-), predicted the properties it should have, and told experimenters what they should look for. Three years later, the particle was found during an experiment at Brookhaven National Laboratory, a spectacular success for Gell-Mann's theory. The whole series of events is reminiscent of the way in which Mendeleev used gaps in the periodic table of the elements to predict properties of undiscovered elements and to guide chemists in their search for these elements.

44.13 (a) A pion containing a blue quark and an antiblue antiquark. (b) The blue quark emits a blue–antired gluon, changing to a red quark. (c) The gluon is absorbed by the antiblue antiquark, which becomes an antired antiquark. The pion now consists of a red–antired quark–antiquark pair. The actual quantum state of the pion is an equal superposition of red–antired, green–antigreen, and blue–antiblue pairs.

What binds quarks to one another? The attractive interactions among quarks are mediated by massless spin-1 bosons called **gluons** in much the same way that photons mediate the electromagnetic interaction or that pions mediated the nucleon–nucleon force in the old Yukawa theory.

Color

Quarks, having spin $\frac{1}{2}$, are fermions and so are subject to the exclusion principle. This would seem to forbid a baryon having two or three quarks with the same flavor and same spin component. To avoid this difficulty, it is assumed that each quark comes in three varieties, which are whimsically called *colors*. Red, green, and blue are the usual choices. The exclusion principle applies separately to each color. A baryon always contains one red, one green, and one blue quark, so the baryon itself has no net color. Each gluon has a color–anticolor combination (for example, blue–antired) that allows it to transmit color when exchanged, and color is conserved during emission and absorption of a gluon by a quark. The gluon-exchange process changes the colors of the quarks in such a way that there is always one quark of each color in every baryon. The color of an individual quark changes continually as gluons are exchanged.



Similar processes occur in mesons such as pions. The quark–antiquark pairs of mesons have canceling color and anticolor (for example, blue and antiblue), so mesons also have no net color. Suppose a pion initially consists of a blue quark and an antiblue antiquark. The blue quark can become a red quark by emitting a blue–antired virtual gluon. The gluon is then absorbed by the antiblue antiquark, converting it to an antired antiquark (Fig. 44.13). Color is conserved in each emission and absorption, but a blue–antiblue pair has become a red–antired pair. Such changes occur continually, so we have to think of a pion as a superposition of three quantum states: blue–antiblue, green–antigreen, and red–antired. On a larger scale, the strong interaction between nucleons was described in Section 44.3 as due to the exchange of virtual mesons. In terms of quarks and gluons, these mediating virtual mesons are quark–antiquark systems bound together by the exchange of gluons.

The theory of strong interactions is known as *quantum chromodynamics* (QCD). No one has been able to isolate an individual quark, and indeed QCD predicts that quarks are bound in such a way that it is impossible to obtain a free quark. An impressive body of experimental evidence supports the correctness of the quark model and the idea that quantum chromodynamics is the key to understanding the strong interactions.

Three More Quarks

Before the tau particles were discovered, there were four known leptons. This fact, together with some puzzling decay rates, led to the speculation that there might be a fourth quark flavor. This quark is labeled c (the *charmed* quark); it has $Q/e = \frac{2}{3}$, $B = \frac{1}{3}$, $S = 0$, and a new quantum number **charm** $C = +1$. This was confirmed in 1974 by the observation at both SLAC and the Brookhaven National Laboratory of a meson, now named ψ , with mass $3097 \text{ MeV}/c^2$. This meson was found to have several decay modes, decaying into e^+e^- , $\mu^+\mu^-$, or hadrons. The mean lifetime was found to be about 10^{-20} s. These results are consistent with ψ being a spin-1 $c\bar{c}$ system. Almost immediately after this, similar mesons of greater mass were observed and identified as excited states of the $c\bar{c}$ system. A few years later, individual mesons with a nonzero net charm quantum number, D^0 ($c\bar{u}$) and D^+ ($c\bar{d}$), and a charmed baryon, Λ_c^+ (udc), were also observed.

In 1977 a meson with mass $9460 \text{ MeV}/c^2$, called *upsilon* (Y), was discovered at Brookhaven. Because it had properties similar to ψ , it was conjectured that the meson was really the bound system of a new quark, b (the *bottom* quark), and its antiquark, \bar{b} . The bottom quark has the value 1 of a new quantum number B' (not to be confused with baryon number B) called *bottomness*. Excited states of the Y were soon observed, as were the B^+ ($\bar{b}u$) and B^0 ($\bar{b}d$) mesons.

With the five flavors of quarks (u , d , s , c , and b) and the six flavors of leptons (e , μ , τ , ν_e , ν_μ , and ν_τ) it was an appealing conjecture that nature is symmetric in its building blocks and that therefore there should be a *sixth* quark. This quark, labeled t (top), would have $Q/e = \frac{2}{3}$, $B = \frac{1}{3}$, and a new quantum number, $T = 1$. In 1995, groups using two different detectors at Fermilab's Tevatron announced the discovery of the top quark. The groups collided 0.9-TeV protons with 0.9-TeV antiprotons, but even with 1.8 TeV of available energy, a top–antitop ($t\bar{t}$) pair was detected in fewer than two of every 10^{11} collisions! Table 44.5 lists some properties of the six quarks. Each has a corresponding antiquark with opposite values of Q , B , S , C , B' , and T .

Table 44.5 Properties of the Six Quarks

Symbol	Q/e	Spin	Baryon Number, B	Strangeness, S	Charm, C	Bottomness, B'	Topness, T
u	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0
d	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0
s	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	-1	0	0	0
c	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	+1	0	0
b	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	+1	0
t	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	0	+1

Test Your Understanding of Section 44.4 Is it possible to have a baryon with charge $Q = +e$ and strangeness $S = -2$? I

44.5 The Standard Model and Beyond

The particles and interactions that we've discussed in this chapter provide a reasonably comprehensive picture of the fundamental building blocks of nature. There is enough confidence in the basic correctness of this picture that it is called the **standard model**.

The standard model includes three families of particles: (1) the six leptons, which have no strong interactions; (2) the six quarks, from which all hadrons are made; and (3) the particles that mediate the various interactions. These mediators are gluons for the strong interaction among quarks, photons for the electromagnetic interaction, the W^\pm and Z^0 particles for the weak interaction, and the graviton for the gravitational interaction.

Electroweak Unification

Theoretical physicists have long dreamed of combining all the interactions of nature into a single unified theory. As a first step, Einstein spent much of his later life trying to develop a field theory that would unify gravitation and electromagnetism. He was only partly successful.

Between 1961 and 1967, Sheldon Glashow, Abdus Salam, and Steven Weinberg developed a theory that unifies the weak and electromagnetic forces. One outcome of their **electroweak theory** is a prediction of the weak-force mediator particles, the Z^0 and W^\pm bosons, including their masses. The basic idea is that the mass difference between photons (zero mass) and the weak bosons ($\approx 100 \text{ GeV}/c^2$) makes the electromagnetic and weak interactions behave quite differently at low energies. At sufficiently high energies (well above 100 GeV), however, the distinction disappears, and the two merge into a single interaction. This prediction was verified in 1983 in experiments with proton-antiproton collisions at CERN. The weak bosons were found, again with the help provided by the theoretical description, and their observed masses agreed with the predictions of the electroweak theory, a wonderful convergence of theory and experiment. The electroweak theory and quantum chromodynamics form the backbone of the standard model. Glashow, Salam, and Weinberg received the Nobel Prize in 1979.

A remaining difficulty in the electroweak theory is that photons are massless but the weak bosons are very massive. To account for the broken symmetry among these interaction mediators, a particle called the Higgs boson has been proposed. Its mass is expected to be less than $1 \text{ TeV}/c^2$, but to produce it in the laboratory may require a much greater available energy. The search for the Higgs boson is an important mission of the Large Hadron Collider at CERN.

Grand Unified Theories

Perhaps at sufficiently high energies the strong interaction and the electroweak interaction have a convergence similar to that between the electromagnetic and weak interactions. If so, they can be unified to give a comprehensive theory of strong, weak, and electromagnetic interactions. Such schemes, called **grand unified theories** (GUTs), are still speculative.

One interesting feature of some grand unified theories is that they predict the decay of the proton (in violation of conservation of baryon number), with an estimated lifetime of more than 10^{28} years. (For comparison the age of the universe is known to be 1.37×10^{10} years.) With a lifetime of 10^{28} years, six metric tons of protons would be expected to have only one decay per day, so huge amounts of material must be examined. Some of the neutrino detectors that we mentioned in Section 44.2 originally looked for, and failed to find, evidence of proton decay. Nevertheless, experimental work continues, with current estimates setting the proton lifetime well over 10^{33} years. Some GUTs also predict the existence of magnetic monopoles, which we mentioned in Chapter 27. At present there is no confirmed experimental evidence that magnetic monopoles exist.

In the standard model, the neutrinos have zero mass. Nonzero values are controversial because experiments to determine neutrino masses are difficult both to perform and to analyze. In most GUTs the neutrinos *must* have nonzero masses. If neutrinos do have mass, transitions called *neutrino oscillations* can occur, in which one type of neutrino (ν_e , ν_μ , or ν_τ) changes into another type.


In 1998, scientists using the Super-Kamiokande neutrino detector in Japan (Fig. 44.14) reported the discovery of oscillations between muon neutrinos and tau neutrinos. Subsequent measurements at the Sudbury Neutrino Observatory in Canada have confirmed the existence of neutrino oscillations. This discovery is evidence for exciting physics beyond that predicted by the standard model.

The discovery of neutrino oscillations has cleared up a long-standing mystery about the sun. Since the 1960s, physicists have been using sensitive detectors to look for electron neutrinos produced by nuclear fusion reactions in the sun's core (see Section 43.8). However, the observed flux of solar electron neutrinos is only one-third of the predicted value. The explanation was provided in 2002 by the Sudbury Neutrino Observatory, which can detect neutrinos of all three flavors. The results showed that the combined flux of solar neutrinos of *all* flavors is equal to the theoretical prediction for the flux of *electron* neutrinos. The explanation is that the sun is indeed producing electron neutrinos at the rate predicted by theory, but that two-thirds of these electron neutrinos are transformed into muon or tau neutrinos during their flight from the sun's core to a detector on earth.

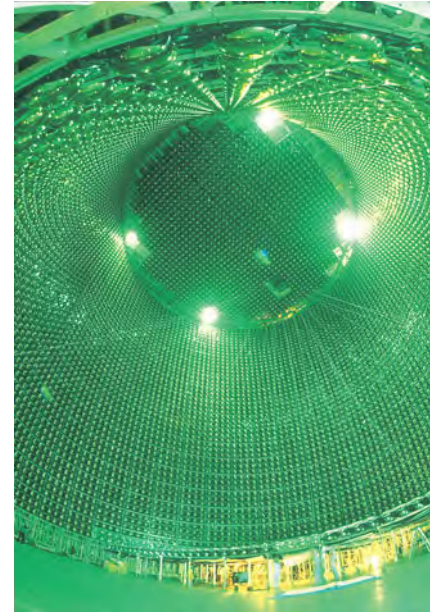
Supersymmetric Theories and TOEs

The ultimate dream of theorists is to unify all four fundamental interactions, adding gravitation to the strong and electroweak interactions that are included in GUTs. Such a unified theory is whimsically called a Theory of Everything (TOE). It turns out that an essential ingredient of such theories is a space-time continuum with more than four dimensions. The additional dimensions are “rolled up” into extremely tiny structures that we ordinarily do not notice. Depending on the scale of these structures, it may be possible for the next generation of particle accelerators to reveal the presence of extra dimensions.

Another ingredient of many theories is *supersymmetry*, which gives every boson and fermion a “superpartner” of the other spin type. For example, the proposed supersymmetric partner of the spin- $\frac{1}{2}$ electron is a spin-0 particle called the *selectron*, and that of the spin-1 photon is a spin- $\frac{1}{2}$ *photino*. As yet, no superpartner particles have been discovered, perhaps because they are too massive to be produced by the present generation of accelerators. Within a few years, new data from the Large Hadron Collider and other accelerators will help us decide whether these intriguing theories have merit.

Test Your Understanding of Section 44.5 One aspect of the standard model is that a *d* quark can transform into a *u* quark, an electron, and an antineutrino by means of the weak interaction. If this happens to a *d* quark inside a neutron, what kind of particle remains afterward in addition to the electron and antineutrino? (i) a proton; (ii) a Σ^- ; (iii) a Σ^+ ; (iv) a Λ^0 or a Σ^0 ; (v) any of these. 

44.14 This photo shows the interior of the Super-Kamiokande neutrino detector in Japan. When in operation, the detector is filled with 5×10^7 kg of water. A neutrino passing through the detector can produce a faint flash of light, which is detected by the 13,000 photomultiplier tubes lining the detector walls. Data from this detector were the first to indicate that neutrinos have mass.



44.6 The Expanding Universe

In the last two sections of this chapter we'll explore briefly the connections between the early history of the universe and the interactions of fundamental particles. It is remarkable that there are such close ties between physics on the smallest scale that we've explored experimentally (the range of the weak interaction, of the order of 10^{-18} m) and physics on the largest scale (the universe itself, of the order of at least 10^{26} m).

Gravitational interactions play an essential role in the large-scale behavior of the universe. One of the great achievements of Newtonian mechanics, including the law of gravitation, was the understanding it brought to the motion of planets in the solar system. Astronomical evidence shows that gravitational

44.15 (a) The galaxy M101 is a larger version of the Milky Way galaxy of which our solar system is a part. Like all galaxies, M101 is held together by the mutual gravitational attraction of its stars, gas, dust, and other matter, all of which orbit around the galaxy's center of mass. M101 is 25 million light-years away. (b) This image shows part of the Coma cluster, an immense grouping of over 1000 galaxies that lies 300 million light-years from us. The galaxies within the cluster are all in motion. Gravitational forces between the member galaxies of this cluster prevent them from escaping.

(a)



(b)



forces also dominate in larger systems such as galaxies and clusters of galaxies (Fig. 44.15).

Until early in the 20th century it was usually assumed that the universe was *static*; stars might move relative to each other, but there was not thought to be any overall expansion or contraction. But if everything is initially sitting still in the universe, why doesn't gravity just pull it all together into one big clump? Newton himself recognized the seriousness of this troubling question.

Measurements that were begun in 1912 by Vesto Slipher at Lowell Observatory in Arizona, and continued in the 1920s by Edwin Hubble with the help of Milton Humason at Mount Wilson in California, indicated that the universe is *not* static. The motions of galaxies relative to the earth can be measured by observing the shifts in the wavelengths of their spectra. For distant galaxies these shifts are always toward longer wavelength, so they appear to be receding from us and from each other. Astronomers first assumed that these were Doppler shifts and used a relationship between the wavelength λ_0 of light measured now from a source receding at speed v and the wavelength λ_S measured in the rest frame of the source when it was emitted. We can derive this relationship by inverting Eq. (37.25) for the Doppler effect, making subscript changes, and using $\lambda = c/f$; the result is

$$\lambda_0 = \lambda_S \sqrt{\frac{c+v}{c-v}} \quad (44.13)$$

Wavelengths from receding sources are always shifted toward longer wavelengths; this increase in λ is called the **redshift**. We can solve Eq. (44.13) for v ; the result is

$$v = \frac{(\lambda_0/\lambda_S)^2 - 1}{(\lambda_0/\lambda_S)^2 + 1} c \quad (44.14)$$

CAUTION Redshift, not Doppler shift Equations (44.13) and (44.14) are from the *special* theory of relativity and refer to the Doppler effect. As we'll see, the redshift from *distant* galaxies is caused by an effect that is explained by the *general* theory of relativity and is *not* a Doppler shift. However, as the ratio v/c and the fractional wavelength change $(\lambda_0 - \lambda_S)/\lambda_S$ become small, the general theory's equations approach Eqs. (44.13) and (44.14), and those equations may be used. **|**

Example 44.8 Recession speed of a galaxy

The spectral lines of various elements are detected in light from a galaxy in the constellation Ursa Major. An ultraviolet line from singly ionized calcium ($\lambda_S = 393 \text{ nm}$) is observed at wavelength $\lambda_0 = 414 \text{ nm}$, redshifted into the visible portion of the spectrum. At what speed is this galaxy receding from us?

SOLUTION

IDENTIFY and SET UP: This example uses the relationship between redshift and recession speed for a distant galaxy. We can use the wavelengths λ_S at which the light is emitted and λ_0 that we detect on earth in Eq. (44.14) to determine the galaxy's recession speed v if the fractional wavelength shift is not too great.

EXECUTE: The fractional redshift is $\lambda_0/\lambda_S = (414 \text{ nm})/(393 \text{ nm}) = 1.053$. This is only a 5.3% increase, so we can use Eq. (44.14) with reasonable accuracy:

$$v = \frac{(1.053)^2 - 1}{(1.053)^2 + 1} c = 0.0516c = 1.55 \times 10^7 \text{ m/s}$$

EVALUATE: The galaxy is receding from the earth at 5.16% of the speed of light. Rather than going through this calculation, astronomers often just state the *redshift* $z = (\lambda_0 - \lambda_S)/\lambda_S = (\lambda_0/\lambda_S) - 1$. This galaxy has redshift $z = 0.053$.

The Hubble Law

Analysis of redshifts from many distant galaxies led Edwin Hubble to a remarkable conclusion: The speed of recession v of a galaxy is proportional to its distance r from us (Fig. 44.16). This relationship is now called the **Hubble law**; expressed as an equation,

$$v = H_0 r \quad (44.15)$$

where H_0 is an experimental quantity commonly called the *Hubble constant*, since at any given time it is constant over space. Determining H_0 has been a key goal of the Hubble Space Telescope, which can measure distances to galaxies with unprecedented accuracy. The current best value is $2.3 \times 10^{-18} \text{ s}^{-1}$, with an uncertainty of 5%.

Astronomical distances are often measured in *parsecs* (pc); one parsec is the distance at which there is a one-arcsecond ($1/3600^\circ$) angular separation between two objects $1.50 \times 10^{11} \text{ m}$ apart (the average distance from the earth to the sun). A distance of 1 pc is equal to 3.26 *light-years* (ly), where $1 \text{ ly} = 9.46 \times 10^{12} \text{ km}$ is the distance that light travels in one year. The Hubble constant is then commonly expressed in the mixed units (km/s)/Mpc (kilometers per second per megaparsec), where $1 \text{ Mpc} = 10^6 \text{ pc}$:

$$H_0 = (2.3 \times 10^{-18} \text{ s}^{-1}) \left(\frac{9.46 \times 10^{12} \text{ km}}{1 \text{ ly}} \right) \left(\frac{3.26 \text{ ly}}{1 \text{ pc}} \right) \left(\frac{10^6 \text{ pc}}{1 \text{ Mpc}} \right) = 71 \frac{\text{km/s}}{\text{Mpc}}$$

Example 44.9 Determining distance with the Hubble law

Use the Hubble law to find the distance from earth to the galaxy in Ursa Major described in Example 44.8.

SOLUTION

IDENTIFY and SET UP: The Hubble law relates the redshift of a distant galaxy to its distance r from earth. We solve Eq. (44.15) for r and substitute the recession speed v from Example 44.8.

EXECUTE: Using $H_0 = 71 \text{ (km/s)/Mpc} = 7.1 \times 10^4 \text{ (m/s)/Mpc}$,

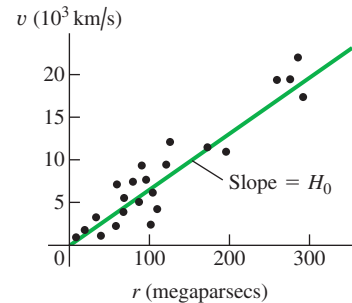
$$\begin{aligned} r &= \frac{v}{H_0} = \frac{1.55 \times 10^7 \text{ m/s}}{7.1 \times 10^4 \text{ (m/s)/Mpc}} = 220 \text{ Mpc} \\ &= 2.2 \times 10^8 \text{ pc} = 7.1 \times 10^8 \text{ ly} = 6.7 \times 10^{24} \text{ m} \end{aligned}$$

Another aspect of Hubble's observations was that, *in all directions*, distant galaxies appeared to be receding from us. There is no particular reason to think that our galaxy is at the very center of the universe; if we lived in some other galaxy, every distant galaxy would still seem to be moving away. That is, at any given time, *the universe looks more or less the same, no matter where in the universe we are*. This important idea is called the **cosmological principle**. There are local fluctuations in density, but on average, the universe looks the same from all locations. Thus the Hubble constant is constant in space although not necessarily constant in time, and the laws of physics are the same everywhere.

The Big Bang

The Hubble law suggests that at some time in the past, all the matter in the universe was far more concentrated than it is today. It was then blown apart in an immense explosion called the **Big Bang**, giving all observable matter more or less the velocities that we observe today. When did this happen? According to the

44.16 Graph of recession speed versus distance for several galaxies. The best-fit straight line illustrates Hubble's law. The slope of the line is the Hubble constant, H_0 .



Hubble law, matter at a distance r away from us is traveling with speed $v = H_0 r$. The time t needed to travel a distance r is

$$t = \frac{r}{v} = \frac{r}{H_0 r} = \frac{1}{H_0} = 4.3 \times 10^{17} \text{ s} = 1.4 \times 10^{10} \text{ y}$$

By this hypothesis the Big Bang occurred about 14 billion years ago. It assumes that all speeds are *constant* after the Big Bang; that is, it neglects any change in the expansion rate due to gravitational attraction or other effects. We'll return to this point later. For now, however, notice that the age of the earth determined from radioactive dating (see Section 43.4) is 4.54 billion (4.54×10^9) years. It's encouraging that our hypothesis tells us that the universe is older than the earth!

Expanding Space

The general theory of relativity takes a radically different view of the expansion just described. According to this theory, the increased wavelength is *not* caused by a Doppler shift as the universe expands into a previously empty void. Rather, the increase comes from *the expansion of space itself* and everything in intergalactic space, including the wavelengths of light traveling to us from distant sources. This is not an easy concept to grasp, and if you haven't encountered it before, it may sound like doubletalk.

Here's an analogy that may help to develop some intuition on this point. Imagine we are all bugs crawling around on a horizontal surface. We can't leave the surface, and we can see in any direction along the surface, but not up or down. We are then living in a two-dimensional world; some writers have called such a world *flatland*. If the surface is a plane, we can locate our position with two Cartesian coordinates (x, y) . If the plane extends indefinitely in both the x - and y -directions, we described our space as having *infinite* extent, or as being *unbounded*. No matter how far we go, we never reach an edge or a boundary.

An alternative habitat for us bugs would be the surface of a sphere with radius R . The space would still seem infinite in the sense that we could crawl forever and never reach an edge or a boundary. Yet in this case the space is *finite* or *bounded*. To describe the location of a point in this space, we could still use two coordinates: latitude and longitude, or the spherical coordinates θ and ϕ shown in Fig. 41.5.

Now suppose the spherical surface is that of a balloon (Fig. 44.17). As we inflate the balloon more and more, increasing the radius R , the coordinates of a point don't change, yet the distance between any two points gets larger and larger. Furthermore, as R increases, the *rate of change* of distance between two points (their recession speed) is proportional to their distance apart. *The recession speed is proportional to the distance*, just as with the Hubble law. For example, the distance from Pittsburgh to Miami is twice as great as the distance from Pittsburgh to Boston. If the earth were to begin to swell, Miami would recede from Pittsburgh twice as fast as Boston would.

We see that although the quantity R isn't one of the two coordinates giving the position of a point on the balloon's surface, it nevertheless plays an essential role in any discussion of distance. It is the radius of curvature of our two-dimensional space, and it is also a varying *scale factor* that changes as this two-dimensional universe expands.

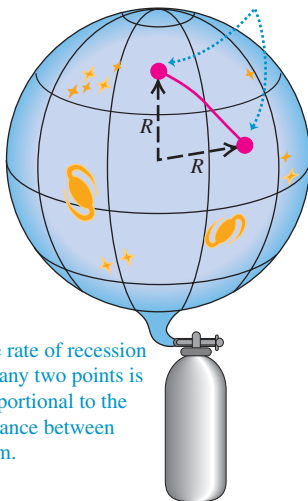
Generalizing this picture to three dimensions isn't so easy. We have to think of our three-dimensional space as being embedded in a space with four or more dimensions, just as we visualized the two-dimensional spherical flatland as being embedded in a three-dimensional Cartesian space. Our real three-space is *not Cartesian*; to describe its characteristics in any small region requires at least one additional parameter, the curvature of space, which is analogous to the radius of the sphere. In a sense, this scale factor, which we'll continue to call R , describes the *size* of the universe, just as the radius of the sphere described the size of our

44.17 An inflating balloon as an analogy for an expanding universe.

(a) Points (representing galaxies) on the surface of a balloon are described by their latitude and longitude coordinates.



(b) The radius R of the balloon has increased. The coordinates of the points are the same, but the distance between them has increased.



The rate of recession for any two points is proportional to the distance between them.

two-dimensional spherical universe. We'll return later to the question of whether the universe is bounded or unbounded.

Any length that is measured in intergalactic space is proportional to R , so the wavelength of light traveling to us from a distant galaxy increases along with every other dimension as the universe expands. That is,

$$\frac{\lambda_0}{\lambda} = \frac{R_0}{R} \quad (44.16)$$

The zero subscripts refer to the values of the wavelength and scale factor *now*, just as H_0 is the current value of the Hubble constant. The quantities λ and R without subscripts are the values at *any* time—past, present, or future. In the situation described in Example 44.8, we have $\lambda_0 = 414$ nm and $\lambda = \lambda_S = 393$ nm, so Eq. (44.16) gives $R_0/R = 1.053$. That is, the scale factor *now* (R_0) is 5.3% larger than it was 710 million years ago when the light was emitted from that galaxy in Ursa Major. This increase of wavelength with time as the scale factor increases in our expanding universe is called the *cosmological redshift*. The farther away an object is, the longer its light takes to get to us and the greater the change in R and λ . The current largest measured wavelength ratio for galaxies is about 7, meaning that the volume of space itself is about $7^3 \approx 340$ times larger than it was when the light was emitted. Do *not* attempt to substitute $\lambda_0/\lambda_S = 7$ into Eq. (44.14) to find the recession speed; that equation is accurate only for small cosmological redshifts and $v \ll c$. The actual value of v depends on the density of the universe, the value of H_0 , and the expansion history of the universe.

Here's a surprise for you: If the distance from us in the Hubble law is large enough, then the speed of recession will be greater than the speed of light! This does *not* violate the special theory of relativity because the recession speed is *not* caused by the motion of the astronomical object relative to some coordinates in its region of space. Rather, we can have $v > c$ when two sets of coordinates move apart fast enough as space itself expands. In other words, there are objects whose coordinates have been moving away from our coordinates so fast that light from them hasn't had enough time in the entire history of the universe to reach us. What we see is just the *observable* universe; we have no direct evidence about what lies beyond its horizon.

CAUTION **The universe isn't expanding into emptiness** The balloon shown in Fig. 44.17 is expanding into the empty space around it. It's a common misconception to picture the universe in the same way as a large but finite collection of galaxies that's expanding into unoccupied space. The reality is quite different! All the accumulated evidence shows that our universe is *infinite*: It has no edges, so there is nothing "outside" it and it isn't "expanding into" anything. The expansion of the universe simply means that the scale factor of the universe is increasing. A good two-dimensional analogy is to think of the universe as a flat, infinitely large rubber sheet that's stretching and expanding much like the surface of the balloon in Fig. 44.17. In a sense, the infinite universe is simply becoming more infinite! **|**

Critical Density

We've mentioned that the law of gravitation isn't consistent with a static universe. We need to look at the role of gravity in an *expanding* universe. Gravitational attractions should slow the initial expansion, but by how much? If these attractions are strong enough, the universe should expand more and more slowly, eventually stop, and then begin to contract, perhaps all the way down to what's been called a *Big Crunch*. On the other hand, if gravitational forces are much weaker, they slow the expansion only a little, and the universe should continue to expand forever.

The situation is analogous to the problem of escape speed of a projectile launched from the earth. We studied this problem in Example 13.5 (Section 13.3); now would be an excellent time to review that discussion. The total energy

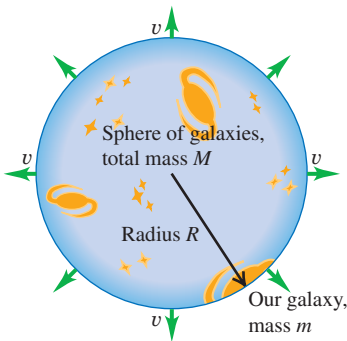
$E = K + U$ when a projectile of mass m and speed v is at a distance r from the center of the earth (mass m_E) is

$$E = \frac{1}{2}mv^2 - \frac{Gmm_E}{r}$$

If E is positive, the projectile has enough kinetic energy to move infinitely far from the earth ($r \rightarrow \infty$) and have some kinetic energy left over. If E is negative, the kinetic energy $K = \frac{1}{2}mv^2$ becomes zero and the projectile stops when $r = -Gmm_E/E$. In that case, no greater value of r is possible, and the projectile can't escape the earth's gravity.

We can carry out a similar analysis for the universe. Whether the universe continues to expand indefinitely should depend on the average *density* of matter. If matter is relatively dense, there is a lot of gravitational attraction to slow and eventually stop the expansion and make the universe contract again. If not, the expansion should continue indefinitely. We can derive an expression for the *critical density* ρ_c needed to just barely stop the expansion.

44.18 An imaginary sphere of galaxies. The net gravitational force exerted on our galaxy (at the surface of the sphere) by the other galaxies is the same as if all of their mass were concentrated at the center of the sphere. (Since the universe is infinite, there's also an infinity of galaxies outside this sphere.)



Here's a calculation based on Newtonian mechanics; it isn't relativistically correct, but it illustrates the idea. Consider a large sphere with radius R , containing many galaxies (Fig. 44.18), with total mass M . Suppose our own galaxy has mass m and is located at the surface of this sphere. According to the cosmological principle, the average distribution of matter within the sphere is uniform. The total gravitational force on our galaxy is just the force due to the mass M inside the sphere. The force on our galaxy and potential energy U due to this spherically symmetric distribution are the same as though m and M were both points, so $U = -GmM/R$, just as in Section 13.3. The net force from all the uniform distribution of mass *outside* the sphere is zero, so we'll ignore it.

The total energy E (kinetic plus potential) for our galaxy is

$$E = \frac{1}{2}mv^2 - \frac{GmM}{R} \quad (44.17)$$

If E is *positive*, our galaxy has enough energy to escape from the gravitational attraction of the mass M inside the sphere; in this case the universe should keep expanding forever. If E is *negative*, our galaxy cannot escape and the universe should eventually pull back together. The crossover between these two cases occurs when $E = 0$, so that

$$\frac{1}{2}mv^2 = \frac{GmM}{R} \quad (44.18)$$

The total mass M inside the sphere is the volume $4\pi R^3/3$ times the density ρ_c :

$$M = \frac{4}{3}\pi R^3 \rho_c$$

We'll assume that the speed v of our galaxy relative to the center of the sphere is given by the Hubble law: $v = H_0 R$. Substituting these expressions for m and v into Eq. (44.18), we get

$$\begin{aligned} \frac{1}{2}m(H_0 R)^2 &= \frac{Gm}{R} \left(\frac{4}{3}\pi R^3 \rho_c \right) \quad \text{or} \\ \rho_c &= \frac{3H_0^2}{8\pi G} \quad (\text{critical density of the universe}) \end{aligned} \quad (44.19)$$

This is the *critical density*. If the average density is less than ρ_c , the universe should continue to expand indefinitely; if it is greater, the universe should eventually stop expanding and begin to contract.

Putting numbers into Eq. (44.19), we find

$$\rho_c = \frac{3(2.3 \times 10^{-18} \text{ s}^{-1})^2}{8\pi(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 9.5 \times 10^{-27} \text{ kg/m}^3$$

The mass of a hydrogen atom is 1.67×10^{-27} kg, so this density is equivalent to about six hydrogen atoms per cubic meter.

Dark Matter, Dark Energy, and the Accelerating Universe

Astronomers have made extensive studies of the average density of matter in the universe. One way to do so is to count the number of galaxies in a patch of sky. Based on the mass of an average star and the number of stars in an average galaxy, this effort gives an estimate of the average density of *luminous* matter in the universe—that is, matter that emits electromagnetic radiation. (You are made of luminous matter because you emit infrared radiation as a consequence of your temperature; see Sections 17.7 and 39.5.) It's also necessary to take into account other luminous matter within a galaxy, including the tenuous gas and dust between the stars.

Another technique is to study the motions of galaxies within clusters of galaxies (Fig. 44.19; see also Fig. 44.15b). The motions are so slow that we can't actually see galaxies changing positions within a cluster. However, observations show that different galaxies within a cluster have somewhat different redshifts, which indicates that the galaxies are moving relative to the center of mass of the cluster. The speeds of these motions are related to the gravitational force exerted on each galaxy by the other members of the cluster, which in turn depends on the total mass of the cluster. By measuring these speeds, astronomers can determine the average density of *all* kinds of matter within the cluster, whether or not the matter emits electromagnetic radiation.

Observations using these and other techniques show that the average density of *all* matter in the universe is 27.4% of the critical density, but the average density of *luminous* matter is only 4.6% of the critical density. In other words, most of the matter in the universe is not luminous: It does not emit electromagnetic radiation of *any* kind. At present, the nature of this **dark matter** remains an outstanding mystery. Some proposed candidates for dark matter are WIMPs (weakly interacting massive particles, which are hypothetical subatomic particles far more massive than those produced in accelerator experiments) and MACHOs (massive compact halo objects, which include objects such as black holes that might form “halos” around galaxies). Whatever the true nature of dark matter, it is by far the dominant form of matter in the universe. For every kilogram of the ordinary matter that has been our subject for most of this book—including electrons, protons, atoms, molecules, blocks on inclined planes, planets, and stars—there are *five* kilograms of dark matter.

Since the average density of matter in the universe is less than the critical density, it might seem fair to conclude that the universe will continue to expand indefinitely, and that gravitational attraction between matter in different parts of the universe should slow the expansion down (albeit not enough to stop it). One way to test this prediction is to examine the redshifts of extremely distant objects. When astronomers look at a galaxy 10^9 light-years away, the light they receive has been in transit for 10^9 years, so they are seeing 10^9 years into the past. If the expansion of the universe has been slowing down, the expansion must have been more rapid in the distant past. Thus we would expect very distant galaxies to have *greater* redshifts than predicted by the Hubble law, Eq. (44.15).

Only since the 1990s has it become possible to accurately measure both the distances and the redshifts of extremely distant galaxies. The results have been totally surprising: Very distant galaxies actually have *smaller* redshifts than predicted by the Hubble law! The implication is that the expansion of the universe was slower in the past than it is now, so the expansion has been *speeding up* rather than slowing down.

If gravitational attraction should make the expansion slow down, why is it speeding up instead? The explanation generally accepted by astronomers and physicists is that space is suffused with a kind of energy that has no gravitational

44.19 The bright spots in this image are not stars, but entire galaxies. They are part of a cluster of galaxies about 10.2 billion ly (3.13 billion pc, or 3130 Mpc) away. (The blue glow is x-ray emission from hot gas within the cluster.) When the galaxies emitted the light used to make this image, the scale factor of the universe was only about 35% as large as it is now. By comparison, we see the relatively nearby Coma cluster (see Fig. 44.15b) as it was 300 million years ago, when the scale factor was 98% of the present-day value.



Application A Fossil Both Ancient and Recent

This fossil trilobite is an example of a group of marine arthropods that flourished in earth's oceans from 540 to 250 million years ago. (By comparison, the first dinosaurs did not appear until 230 million years ago.) From our perspective, this makes trilobites almost unfathomably ancient. But compared to the time that has elapsed since the Big Bang, 13.7 billion years, even trilobites are a very recent phenomenon: They first appeared when the universe was already 96% of its present age.



effect and emits no electromagnetic radiation, but rather acts as a kind of “anti-gravity” that produces a universal *repulsion*. This invisible, immaterial energy is called **dark energy**. As the name suggests, the nature of dark energy is poorly understood but is the subject of very active research.

Observations show that the *energy* density of dark energy (measured in, say, joules per cubic meter) is 72.6% of the critical density times c^2 ; that is, it is equal to $0.726\rho_c c^2$. As described above, the average density of matter of all kinds is 27.4% of the critical density. From the Einstein relationship $E = mc^2$, the average *energy* density of matter in the universe is therefore $0.274\rho_c c^2$. Because the energy density of dark energy is nearly three times greater than that of matter, the expansion of the universe will continue to accelerate. This expansion will never stop, and the universe will never contract.

If we account for energy of *all* kinds, the average energy density of the **?** universe is equal to $0.726\rho_c c^2 + 0.274\rho_c c^2 = 1.00\rho_c c^2$. Of this, 72.6% is the mysterious dark energy, 22.8% is the no less mysterious dark matter, and a mere 4.6% is well-understood conventional matter. How little we know about the contents of our universe! When we take account of the density of matter in the universe (which tends to slow the expansion of space) and the density of dark energy (which tends to speed up the expansion), the age of the universe turns out to be 13.7 billion (1.37×10^{10}) years.

What is the significance of the result that within observational error, the average energy density of the universe is equal to $\rho_c c^2$? It tells us that the universe is infinite and unbounded, but just barely so. If the average energy density were even slightly larger than $\rho_c c^2$, the universe would be finite like the surface of the balloon depicted in Fig. 44.17. As of this writing, the observational error in the average energy density is still large enough (about 1%) that we can't be totally sure that the universe *is* unbounded. Improving these measurements will be an important task for physicists and astronomers in the years ahead.

Test Your Understanding of Section 44.6 Is it accurate to say that your body is made of “ordinary” matter? **I**

44.7 The Beginning of Time

What an odd title for the very last section of a book! We will describe in general terms some of the current theories about the very early history of the universe and their relationship to fundamental particle interactions. We'll find that an astonishing amount happened in the very first second. A lot of loose ends will be left untied, and many questions will be left unanswered. This is, after all, one of the frontiers of physics.

Temperatures

The early universe was extremely dense and extremely hot, and the average particle energies were extremely large, all many orders of magnitude beyond anything that exists in the present universe. We can compare particle energy E and absolute temperature T using the equipartition principle (see Section 18.4):

$$E = \frac{3}{2}kT \quad (44.20)$$

In this equation k is Boltzmann's constant, which we'll often express in eV/K:

$$k = 8.617 \times 10^{-5} \text{ eV/K}$$

Thus we can replace Eq. (44.20) by $E \approx (10^{-4} \text{ eV/K})T = (10^{-13} \text{ GeV/K})T$ when we're discussing orders of magnitude.

Example 44.10 Temperature and energy

(a) What is the average kinetic energy E (in eV) of particles at room temperature ($T = 290$ K) and at the surface of the sun ($T = 5800$ K)? (b) What approximate temperature corresponds to the ionization energy of the hydrogen atom and to the rest energies of the electron and the proton?

(b) The ionization energy of hydrogen is 13.6 eV. Using the approximation $E \approx (10^{-4} \text{ eV/K})T$, we have

$$T \approx \frac{E}{10^{-4} \text{ eV/K}} = \frac{13.6 \text{ eV}}{10^{-4} \text{ eV/K}} \approx 10^5 \text{ K}$$

Repeating this calculation for the rest energies of the electron ($E = 0.511$ MeV) and proton ($E = 938$ MeV) gives temperatures of 10^{10} K and 10^{13} K, respectively.

EVALUATE: Temperatures in excess of 10^5 K are found in the sun's interior, so most of the hydrogen there is ionized. Temperatures of 10^{10} K or 10^{13} K are not found anywhere in the solar system; as we will see, temperatures were this high in the very early universe.

SOLUTION

IDENTIFY and SET UP: In this example we are to apply the equipartition principle. We use Eq. (44.20) to relate the target variables E and T .

EXECUTE: (a) At room temperature, from Eq. (44.20),

$$E = \frac{3}{2}kT = \frac{3}{2}(8.617 \times 10^{-5} \text{ eV/K})(290 \text{ K}) = 0.0375 \text{ eV}$$

The temperature at the sun's surface is higher than room temperature by a factor of $(5800 \text{ K})/(290 \text{ K}) = 20$, so the average kinetic energy there is $20(0.0375 \text{ eV}) = 0.75 \text{ eV}$.

Uncoupling of Interactions

We've characterized the expansion of the universe by a continual increase of the scale factor R , which we can think of very roughly as characterizing the *size* of the universe, and by a corresponding decrease in average density. As the total gravitational potential energy increased during expansion, there were corresponding *decreases* in temperature and average particle energy. As this happened, the basic interactions became progressively uncoupled.

To understand the uncouplings, recall that the unification of the electromagnetic and weak interactions occurs at energies that are large enough that the differences in mass among the various spin-1 bosons that mediate the interactions become insignificant by comparison. The electromagnetic interaction is mediated by the massless photon, and the weak interaction is mediated by the weak bosons W^\pm and Z^0 with masses of the order of $100 \text{ GeV}/c^2$. At energies much *less* than 100 GeV the two interactions seem quite different, but at energies much *greater* than 100 GeV they become part of a single interaction.

The grand unified theories (GUTs) provide a similar behavior for the strong interaction. It becomes unified with the electroweak interaction at energies of the order of 10^{14} GeV , but at lower energies the two appear quite distinct. One of the reasons GUTs are still very speculative is that there is no way to do controlled experiments in this energy range, which is larger by a factor of 10^{11} than energies available with any current accelerator.

Finally, at sufficiently high energies and short distances, it is assumed that gravitation becomes unified with the other three interactions. The distance at which this happens is thought to be of the order of 10^{-35} m . This distance, called the *Planck length* l_P , is determined by the speed of light c and the fundamental constants of quantum mechanics and gravitation, \hbar and G , respectively. The Planck length l_P is defined as

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} \text{ m} \quad (44.21)$$

You should verify that this combination of constants does indeed have units of length. The *Planck time* $t_P = l_P/c$ is the time required for light to travel a distance l_P :

$$t_P = \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}} = 0.539 \times 10^{-43} \text{ s} \quad (44.22)$$

If we mentally go backward in time, we have to stop when we reach $t = 10^{-43}$ s because we have no adequate theory that unifies all four interactions. So as yet we have no way of knowing what happened or how the universe behaved at times earlier than the Planck time or when its size was less than the Planck length.

The Standard Model of the History of the Universe

The description that follows is called the *standard model* of the history of the universe. The title indicates that there are substantial areas of theory that rest on solid experimental foundations and are quite generally accepted. The figure on pages 1512–1513 is a graphical description of this history, with the characteristic sizes, particle energies, and temperatures at various times. Referring to this figure frequently will help you to understand the following discussion.

In this standard model, the temperature of the universe at time $t = 10^{-43}$ s (the Planck time) was about 10^{32} K, and the average energy per particle was approximately

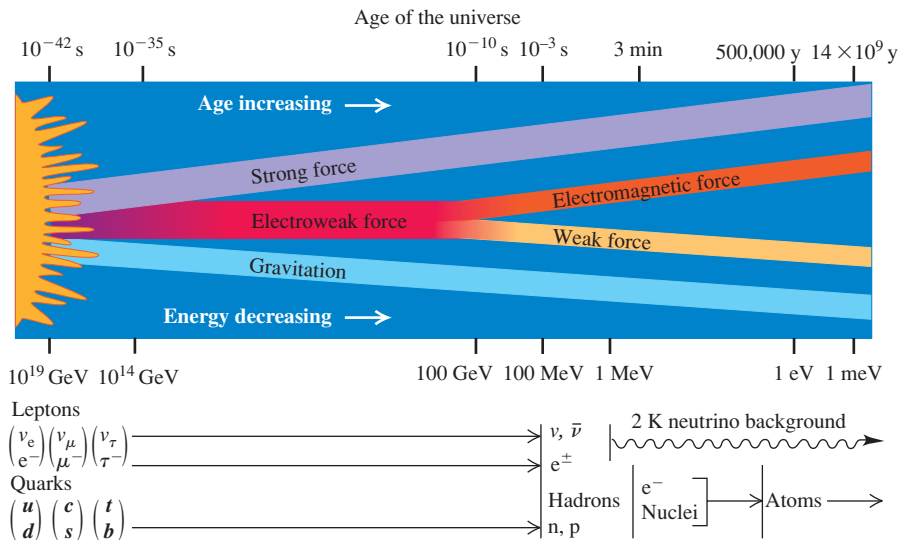
$$E \approx (10^{-13} \text{ GeV/K})(10^{32} \text{ K}) = 10^{19} \text{ GeV}$$

In a totally unified theory this is about the energy below which gravity begins to behave as a separate interaction. This time therefore marked the transition from any proposed TOE to the GUT period.

During the GUT period, roughly $t = 10^{-43}$ to 10^{-35} s, the strong and electroweak forces were still unified, and the universe consisted of a soup of quarks and leptons transforming into each other so freely that there was no distinction between the two families of particles. Other, much more massive particles may also have been freely created and destroyed. One important characteristic of GUTs is that at sufficiently high energies, baryon number is not conserved. (We mentioned earlier the proposed decay of the proton, which has not yet been observed.) Thus by the end of the GUT period the numbers of quarks and anti-quarks may have been unequal. This point has important implications; we'll return to it at the end of the section.

By $t = 10^{-35}$ s the temperature had decreased to about 10^{27} K and the average energy to about 10^{14} GeV. At this energy the strong force separated from the electroweak force (Fig. 44.20), and baryon number and lepton numbers began to be separately conserved. This separation of the strong force was analogous to a phase

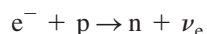
44.20 Schematic diagram showing the times and energies at which the various interactions are thought to have uncoupled. The energy scale is backward because the average energy decreased as the age of the universe increased.



change such as boiling a liquid, with an associated heat of vaporization. Think of it as being similar to boiling a heavy nucleus, pulling the particles apart beyond the short range of the nuclear force. As a result, the universe underwent a dramatic expansion (far more rapid than the present-day expansion rate) called *cosmic inflation*. In one model, the scale factor R increased by a factor of 10^{50} in 10^{-32} s.

At $t = 10^{-32}$ s the universe was a mixture of quarks, leptons, and the mediating bosons (gluons, photons, and the weak bosons W^\pm and Z^0). It continued to expand and cool from the inflationary period to $t = 10^{-6}$ s, when the temperature was about 10^{13} K and typical energies were about 1 GeV (comparable to the rest energy of a nucleon; see Example 44.11). At this time the quarks began to bind together to form nucleons and antinucleons. Also there were still enough photons of sufficient energy to produce nucleon–antinucleon pairs to balance the process of nucleon–antinucleon annihilation. However, by about $t = 10^{-2}$ s, most photon energies fell well below the threshold energy for such pair production. There was a slight excess of nucleons over antinucleons; as a result, virtually all of the antinucleons and most of the nucleons annihilated one another. A similar equilibrium occurred later between the production of electron–positron pairs from photons and the annihilation of such pairs. At about $t = 14$ s the average energy dropped to around 1 MeV, below the threshold for e^+e^- pair production. After pair production ceased, virtually all of the remaining positrons were annihilated, leaving the universe with many more protons and electrons than the antiparticles of each.

Up until about $t = 1$ s, neutrons and neutrinos could be produced in the endoergic reaction



After this time, most electrons no longer had enough energy for this reaction. The average neutrino energy also decreased, and as the universe expanded, equilibrium reactions that involved *absorption* of neutrinos (which occurred with decreasing probability) became inoperative. At this time, in effect, the flux of neutrinos and antineutrinos throughout the universe uncoupled from the rest of the universe. Because of the extraordinarily low probability for neutrino absorption, most of this flux is still present today, although cooled greatly by expansion. The standard model of the universe predicts a present neutrino temperature of about 2 K, but no experiment has yet been able to test this prediction.

Nucleosynthesis

At about $t = 1$ s, the ratio of protons to neutrons was determined by the Boltzmann distribution factor $e^{-\Delta E/kT}$, where ΔE is the difference between the neutron and proton rest energies: $\Delta E = 1.294$ MeV. At a temperature of about 10^{10} K, this distribution factor gives about 4.5 times as many protons as neutrons. However, as we have discussed, free neutrons (with a half-life of 887 s) decay spontaneously to protons. This decay caused the proton–neutron ratio to increase until about $t = 225$ s. At this time, the temperature was about 10^9 K, and the average energy was well below 2 MeV.

This energy distribution was critical because the binding energy of the *deuteron* (a neutron and a proton bound together) is 2.22 MeV (see Section 43.2). A neutron bound in a deuteron does not decay spontaneously. As the average energy decreased, a proton and a neutron could combine to form a deuteron, and there were fewer and fewer photons with 2.22 MeV or more of energy to dissociate the deuterons again. Therefore the combining of protons and neutrons into deuterons halted the decay of free neutrons.

The formation of deuterons starting at about $t = 225$ s marked the beginning of the period of formation of nuclei, or *nucleosynthesis*. At this time, there were about seven protons for each neutron. The deuteron (${}^2\text{H}$) can absorb a neutron and form a triton (${}^3\text{H}$), or it can absorb a proton and form ${}^3\text{He}$. Then ${}^3\text{H}$ can absorb a proton, and ${}^3\text{He}$ can absorb a neutron, each yielding ${}^4\text{He}$ (the alpha particle).

AGE OF QUARKS AND GLUONS (GUT Period)

Dense concentration of matter and antimatter; gravity a separate force; more quarks than antiquarks.

Inflationary period (10^{-35} s): rapid expansion, strong force separates from electroweak force.

AGE OF LEPTONS

Leptons distinct from quarks; W^\pm and Z^0 bosons mediate weak force (10^{-12} s).

AGE OF NUCLEONS AND ANTINUCLEONS

Quarks bind together to form nucleons and antinucleons; energy too low for nucleon-antinucleon pair production at 10^{-2} s.

AGE OF NUCLEOSYNTHESIS

Stable deuterons; matter 74% H, 25% He, 1% heavier nuclei.

BIG BANG

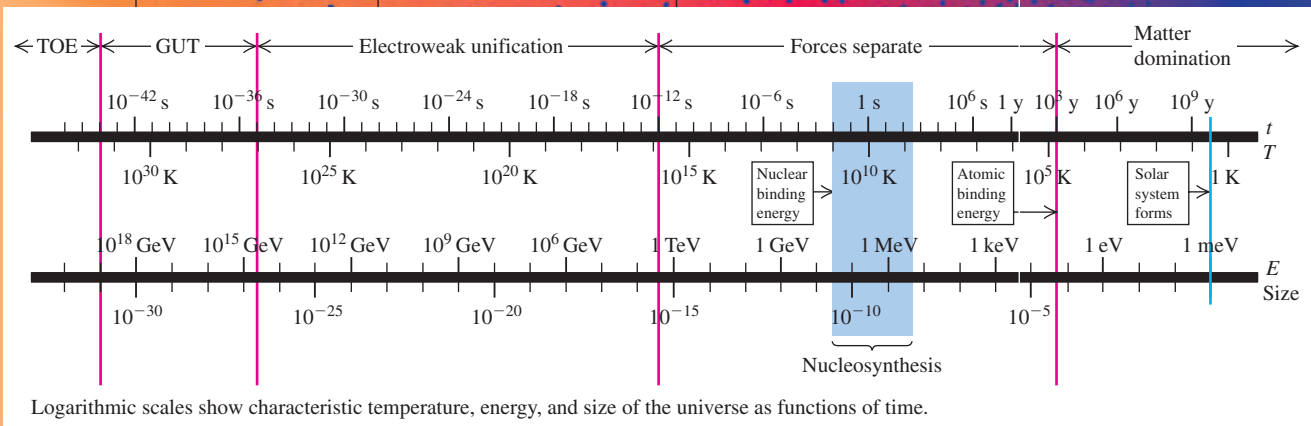
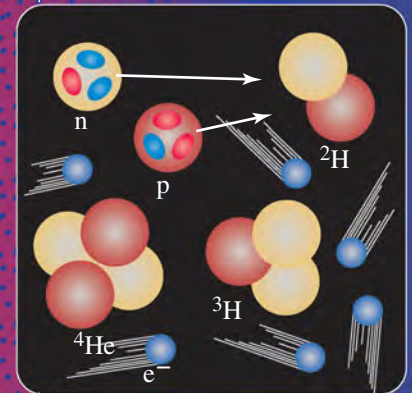
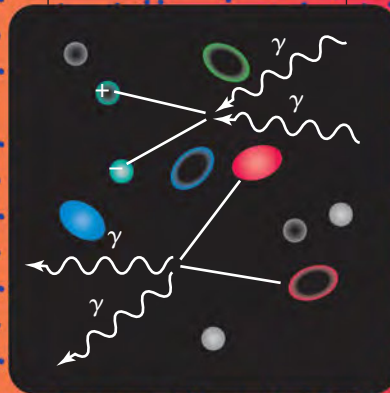
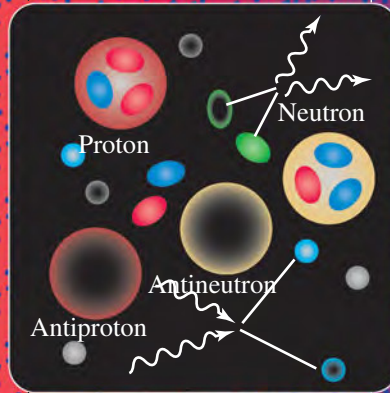
10^{-43} s

10^{-32} s

10^{-6} s

225 s

10^3 s



Logarithmic scales show characteristic temperature, energy, and size of the universe as functions of time.

A Brief History of the Universe

AGE OF IONS
Expanding, cooling gas of ionized H and He.

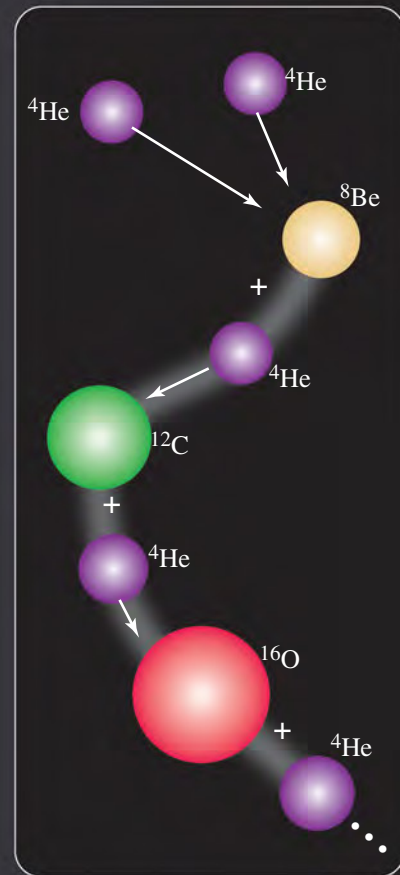
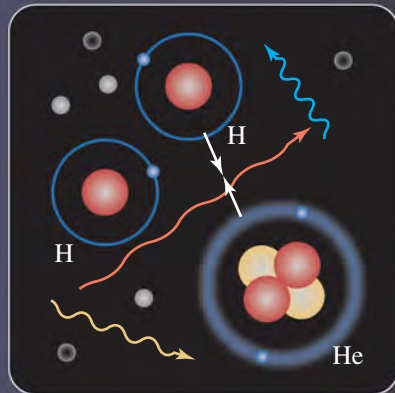
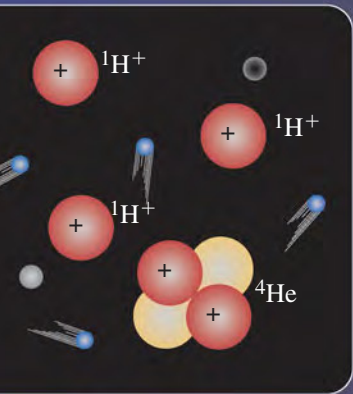
AGE OF ATOMS
Neutral atoms form; universe becomes transparent to most light.

AGE OF STARS AND GALAXIES
Thermonuclear fusion begins in stars, forming heavier nuclei.

10^{13} s

10^{15} s

NOW



A few ${}^7\text{Li}$ nuclei may also have been formed by fusion of ${}^3\text{H}$ and ${}^4\text{He}$ nuclei. According to the theory, essentially all the ${}^1\text{H}$ and ${}^4\text{He}$ in the present universe was formed at this time. But then the building of nuclei almost ground to a halt. The reason is that *no* nuclide with mass number $A = 5$ has a half-life greater than 10^{-21} s. Alpha particles simply do not permanently absorb neutrons or protons. The nuclide ${}^8\text{Be}$ that is formed by fusion of two ${}^4\text{He}$ nuclei is unstable, with an extremely short half-life, about 7×10^{-17} s. Note also that at this time, the average energy was still much too large for electrons to be bound to nuclei; there were not yet any atoms.

Conceptual Example 44.11 The relative abundance of hydrogen and helium in the universe

Nearly all of the protons and neutrons in the seven-to-one ratio at $t = 225$ s either formed ${}^4\text{He}$ or remained as ${}^1\text{H}$. After this time, what was the resulting relative abundance of ${}^1\text{H}$ and ${}^4\text{He}$, by mass?

SOLUTION

The ${}^4\text{He}$ nucleus contains two protons and two neutrons. For every two neutrons present at $t = 225$ s there were 14 protons. The two neutrons and two of the 14 protons make up one ${}^4\text{He}$ nucleus,

leaving 12 protons (${}^1\text{H}$ nuclei). So there were eventually 12 ${}^1\text{H}$ nuclei for every ${}^4\text{He}$ nucleus. The masses of ${}^1\text{H}$ and ${}^4\text{He}$ are about 1 u and 4 u, respectively, so there were 12 u of ${}^1\text{H}$ for every 4 u of ${}^4\text{He}$. Therefore the relative abundance, by mass, was 75% ${}^1\text{H}$ and 25% ${}^4\text{He}$. This result agrees very well with estimates of the present H–He ratio in the universe, an important confirmation of this part of the theory.

44.21 The Veil Nebula in the constellation Cygnus is a remnant of a supernova explosion that occurred more than 20,000 years ago. The gas ejected from the supernova is still moving very rapidly. Collisions between this fast-moving gas and the tenuous material of interstellar space excite the gas and cause it to glow. The portion of the nebula shown here is about 40 ly (12 pc) in length.



Further nucleosynthesis did not occur until very much later, well after $t = 10^{13}$ s (about 380,000 y). At that time, the temperature was about 3000 K, and the average energy was a few tenths of an electron volt. Because the ionization energies of hydrogen and helium atoms are 13.6 eV and 24.5 eV, respectively, almost all the hydrogen and helium was electrically neutral (not ionized). With the electrical repulsions of the nuclei canceled out, gravitational attraction could slowly pull the neutral atoms together to form clouds of gas and eventually stars. Thermonuclear reactions in stars then produced all of the more massive nuclei. In Section 43.8 we discussed one cycle of thermonuclear reactions in which ${}^1\text{H}$ becomes ${}^4\text{He}$.

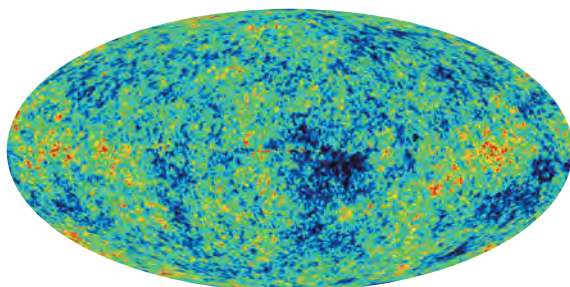
For stars whose mass is 40% of the sun's mass or greater, as the hydrogen is consumed the star's core begins to contract as the inward gravitational pressure exceeds the outward gas and radiation pressure. The gravitational potential energy decreases as the core contracts, so the kinetic energy of nuclei in the core increases. Eventually the core temperature becomes high enough to begin another process, *helium fusion*. First two ${}^4\text{He}$ nuclei fuse to form ${}^8\text{Be}$, which is highly unstable. But because a star's core is so dense and collisions among nuclei are so frequent, there is a nonzero probability that a third ${}^4\text{He}$ nucleus will fuse with the ${}^8\text{Be}$ nucleus before it can decay. The result is the stable nuclide ${}^{12}\text{C}$. This is called the *triple-alpha process*, since three ${}^4\text{He}$ nuclei (that is, alpha particles) fuse to form one carbon nucleus. Then successive fusions with ${}^4\text{He}$ give ${}^{16}\text{O}$, ${}^{20}\text{Ne}$, and ${}^{24}\text{Mg}$. All these reactions are exoergic. They release energy to heat up the star, and ${}^{12}\text{C}$ and ${}^{16}\text{O}$ can fuse to form elements with higher and higher atomic number.

For nuclides that can be created in this manner, the binding energy per nucleon peaks at mass number $A = 56$ with the nuclide ${}^{56}\text{Fe}$, so exoergic fusion reactions stop with Fe. But successive neutron captures followed by beta decays can continue the synthesis of more massive nuclei. If the star is massive enough, it may eventually explode as a *supernova*, sending out into space the heavy elements that were produced by the earlier processes (Fig. 44.21; see also Fig. 37.7). In space, the debris and other interstellar matter can gravitationally bunch together to form a new generation of stars and planets. Our own sun is one such "second-generation" star. This means that the sun's planets and everything on them (including you) contain matter that was long ago blasted into space by an exploding supernova.

Background Radiation

In 1965 Arno Penzias and Robert Wilson, working at Bell Telephone Laboratories in New Jersey on satellite communications, turned a microwave antenna skyward and found a background signal that had no apparent preferred direction. (This signal produces about 1% of the “hash” you see on a TV screen when you turn to an unused channel.) Further research has shown that the radiation that is received has a frequency spectrum that fits Planck’s blackbody radiation law, Eq. (39.24) (Section 39.5). The wavelength of peak intensity is 1.063 mm (in the microwave region of the spectrum), with a corresponding absolute temperature $T = 2.725$ K. Penzias and Wilson contacted physicists at nearby Princeton University who had begun the design of an antenna to search for radiation that was a remnant from the early evolution of the universe. We mentioned above that neutral atoms began to form at about $t = 380,000$ years when the temperature was 3000 K. With far fewer charged particles present than previously, the universe became transparent at this time to electromagnetic radiation of long wavelength. The 3000-K blackbody radiation therefore survived, cooling to its present 2.725-K temperature as the universe expanded. The *cosmic background radiation* is among the most clear-cut experimental confirmations of the Big Bang theory. Figure 44.22 shows a modern map of the cosmic background radiation.

44.22 This false-color map shows microwave radiation from the entire sky mapped onto an oval. When this radiation was emitted 380,000 years after the Big Bang, the regions shown in blue were slightly cooler and denser than average. Within these cool, dense regions formed galaxies, including the Milky Way galaxy of which our solar system, our earth, and our selves are part.



Example 44.12 Expansion of the universe

By approximately what factor has the universe expanded since $t = 380,000$ y?

SOLUTION

IDENTIFY and SET UP: We use the idea that as the universe has expanded, all intergalactic wavelengths have expanded with it. The Wien displacement law, Eq. (39.21), relates the peak wavelength λ_m in blackbody radiation to the temperature T . Given the temperatures of the cosmic background radiation today (2.725 K) and at $t = 380,000$ y (3000 K) we can determine the factor by which wavelengths have changed and hence determine the factor by which the universe has expanded.

EXECUTE: We rewrite Eq. (39.21) as

$$\lambda_m = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}$$

Hence the peak wavelength λ_m is inversely proportional to T . As the universe expands, all intergalactic wavelengths (including λ_m) increase in proportion to the scale factor R . The temperature has decreased by the factor $(3000 \text{ K})/(2.725 \text{ K}) \approx 1100$, so λ_m and the scale factor must both have *increased* by this factor. Thus, between $t = 380,000$ y and the present, the universe has expanded by a factor of about 1100.

EVALUATE: Our results show that since $t = 380,000$ y, any particular intergalactic *volume* has increased by a factor of about $(1100)^3 = 1.3 \times 10^9$. They also show that when the cosmic background radiation was emitted, its peak wavelength was $\frac{1}{1100}$ of the present-day value of 1.063 mm, or 967 nm. This is in the infrared region of the spectrum.

Matter and Antimatter

One of the most remarkable features of our universe is the asymmetry between matter and antimatter. One might think that the universe should have equal numbers of protons and antiprotons and of electrons and positrons, but this doesn't appear to be the case. Theories of the early universe must explain this imbalance.

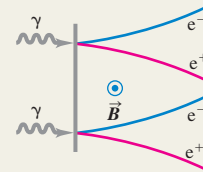
We've mentioned that most GUTs include violation of conservation of baryon number at energies at which the strong and electroweak interactions have converged. If particle–antiparticle symmetry is also violated, we have a mechanism for making more quarks than antiquarks, more leptons than antileptons, and eventually more matter than antimatter. One serious problem is that any asymmetry that is created in this way during the GUT era might be wiped out by the electroweak interaction after the end of the GUT era. If so, there must be some mechanism that creates particle–antiparticle asymmetry at a much *later* time. The problem of the matter–antimatter asymmetry is still very much an open one.

There are still many unanswered questions at the intersection of particle physics and cosmology. Is the energy density of the universe precisely equal to $\rho_c c^2$, or are there small but important differences? What is dark energy? Has the density of dark energy remained constant over the history of the universe, or has the density changed? What is dark matter? What happened during the first 10^{-43} s after the Big Bang? Can we see evidence that the strong and electroweak interactions undergo a grand unification at high energies? The search for the answers to these and many other questions about our physical world continues to be one of the most exciting adventures of the human mind.

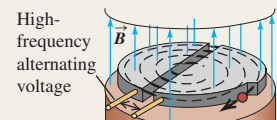
Test Your Understanding of Section 44.7 Given a sufficiently powerful telescope, could we detect photons emitted earlier than $t = 380,000$ y? **|**

Fundamental particles: Each particle has an antiparticle; some particles are their own antiparticles. Particles can be created and destroyed, some of them (including electrons and positrons) only in pairs or in conjunction with other particles and antiparticles.

Particles serve as mediators for the fundamental interactions. The photon is the mediator of the electromagnetic interaction. Yukawa proposed the existence of mesons to mediate the nuclear interaction. Mediating particles that can exist only because of the uncertainty principle for energy are called virtual particles.



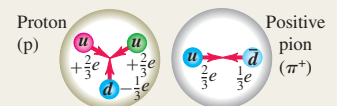
Particle accelerators and detectors: Cyclotrons, synchrotrons, and linear accelerators are used to accelerate charged particles to high energies for experiments with particle interactions. Only part of the beam energy is available to cause reactions with targets at rest. This problem is avoided in colliding-beam experiments. (See Examples 44.1–44.3.)



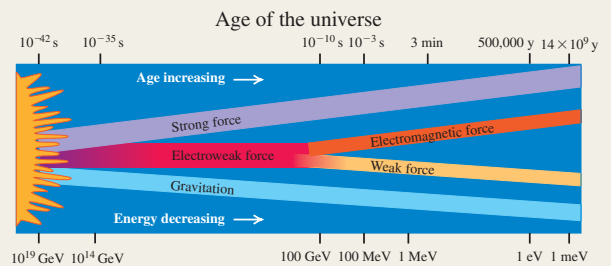
Particles and interactions: Four fundamental interactions are found in nature: the strong, electromagnetic, weak, and gravitational interactions. Particles can be described in terms of their interactions and of quantities that are conserved in all or some of the interactions.

Fermions have half-integer spins; bosons have integer spins. Leptons, which are fermions, have no strong interactions. Strongly interacting particles are called hadrons. They include mesons, which are always bosons, and baryons, which are always fermions. There are conservation laws for three different lepton numbers and for baryon number. Additional quantum numbers, including strangeness and charm, are conserved in some interactions and not in others. (See Examples 44.4–44.6.)

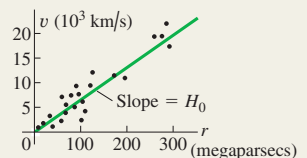
Quarks: Hadrons are composed of quarks. There are thought to be six types of quarks. The interaction between quarks is mediated by gluons. Quarks and gluons have an additional attribute called color. (See Example 44.7.)



Symmetry and the unification of interactions: Symmetry considerations play a central role in all fundamental-particle theories. The electromagnetic and weak interactions become unified at high energies into the electroweak interaction. In grand unified theories the strong interaction is also unified with these interactions, but at much higher energies.



The expanding universe and its composition: The Hubble law shows that galaxies are receding from each other and that the universe is expanding. Observations show that the rate of expansion is accelerating due to the presence of dark energy, which makes up 72.6% of the energy in the universe. Only 4.6% of the energy in the universe is in the form of ordinary matter; the remaining 22.8% is dark matter, whose nature is poorly understood. (See Examples 44.8 and 44.9.)



The history of the universe: In the standard model of the universe, a Big Bang gave rise to the first fundamental particles. They eventually formed into the lightest atoms as the universe expanded and cooled. The cosmic background radiation is a relic of the time when these atoms formed. The heavier elements were manufactured much later by fusion reactions inside stars. (See Examples 44.10–44.12.)



BRIDGING PROBLEM

Hyperons, Pions, and the Expanding Universe

A Λ^0 hyperon at rest decays into a neutron and a π^0 . (a) Find the kinetic energies of the decay products. (b) What fraction of the total kinetic energy is carried off by each particle? (c) A physicist on earth detects one of the two photons that was emitted in the decay of a π^0 . The π^0 was at rest in the cluster shown in Fig. 44.19 before it decayed. What is the energy of the photon that is detected on earth?

SOLUTION GUIDE

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IDENTIFY and SET UP

1. Which quantities are conserved in the Λ^0 decay? In the π^0 decay?
2. The universe expanded during the time that the photon traveled from the cluster to earth. How does this affect the wavelength and energy of the photon that the physicist detects?
3. List the unknown quantities for each part of the problem and identify the target variables.
4. Select the equations that will allow you to solve for the target variables.

EXECUTE

5. Write the conservation equations for the decay of the Λ^0 . (*Hint:* It's useful to write the energy E of a particle in terms of its momentum p and mass m using $E = (p^2c^2 + m^2c^4)^{1/2}$.)
6. Solve the conservation equations for the energy of one of the decay products. (*Hint:* Rearrange the energy conservation equation so that one of the $(p^2c^2 + m^2c^4)^{1/2}$ terms is on one side of the equation. Then square both sides.) Then use $K = E - mc^2$.
7. Find the fraction of the total kinetic energy that goes into the neutron and into the pion.
8. Write the conservation equations for the decay of the π^0 at rest and find the energy of each emitted photon. By what factor does the wavelength of this photon change as it travels from the galaxy cluster to earth? By what factor does the photon energy change? (*Hint:* See Fig. 44.19.)

EVALUATE

9. Which of the Λ^0 decay products should have the greater kinetic energy? Should the detected π^0 decay photon have more or less energy than when it was emitted?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

- Q44.1** Is it possible that some parts of the universe contain antimatter whose atoms have nuclei made of antiprotons and antineutrons, surrounded by positrons? How could we detect this condition without actually going there? Can we detect these antiatoms by identifying the light they emit as composed of antiphotons? Explain. What problems might arise if we actually *did* go there?
- Q44.2** Given the Heisenberg uncertainty principle, is it possible to create particle–antiparticle pairs that exist for extremely short periods of time before annihilating? Does this mean that empty space is really empty?
- Q44.3** When they were first discovered during the 1930s and 1940s, there was confusion as to the identities of pions and muons. What are the similarities and most significant differences?
- Q44.4** The gravitational force between two electrons is weaker than the electrical force by the order of 10^{-40} . Yet the gravitational interactions of matter were observed and analyzed long before electrical interactions were understood. Why?
- Q44.5** When a π^0 decays to two photons, what happens to the quarks of which it was made?
- Q44.6** Why can't an electron decay to two photons? To two neutrinos?
- Q44.7** According to the standard model of the fundamental particles, what are the similarities between baryons and leptons? What are the most important differences?
- Q44.8** According to the standard model of the fundamental particles, what are the similarities between quarks and leptons? What are the most important differences?

Q44.9 What are the main advantages of colliding-beam accelerators compared with those using stationary targets? What are the main disadvantages?

Q44.10 Does the universe have a center? Explain.

Q44.11 Does it make sense to ask, “If the universe is expanding, what is it expanding into?”

Q44.12 Assume that the universe has an edge. Placing yourself at that edge in a thought experiment, explain why this assumption violates the cosmological principle.

Q44.13 Explain why the cosmological principle requires that H_0 must have the same value everywhere in space, but does not require that it be constant in time.

EXERCISES

Section 44.1 Fundamental Particles—A History

- 44.1 •** A neutral pion at rest decays into two photons. Find the energy, frequency, and wavelength of each photon. In which part of the electromagnetic spectrum does each photon lie? (Use the pion mass given in terms of the electron mass in Section 44.1.)
- 44.2 •• CP** Two equal-energy photons collide head-on and annihilate each other, producing a $\mu^+\mu^-$ pair. The muon mass is given in terms of the electron mass in Section 44.1. (a) Calculate the maximum wavelength of the photons for this to occur. If the photons have this wavelength, describe the motion of the μ^+ and μ^- immediately after they are produced. (b) If the wavelength of each photon is half the value calculated in part (a), what is the speed of each muon after they have moved apart? Use correct relativistic expressions for momentum and energy.

44.3 •• A positive pion at rest decays into a positive muon and a neutrino. (a) Approximately how much energy is released in the decay? (Assume the neutrino has zero rest mass. Use the muon and pion masses given in terms of the electron mass in Section 44.1.) (b) Why can't a positive muon decay into a positive pion?

44.4 • A proton and an antiproton annihilate, producing two photons. Find the energy, frequency, and wavelength of each photon (a) if the p and \bar{p} are initially at rest and (b) if the p and \bar{p} collide head-on, each with an initial kinetic energy of 830 MeV.

44.5 •• CP For the nuclear reaction given in Eq. (44.2) assume that the initial kinetic energy and momentum of the reacting particles are negligible. Calculate the speed of the α particle immediately after it leaves the reaction region.

44.6 • Estimate the range of the force mediated by an ω^0 meson that has mass $783 \text{ MeV}/c^2$.

44.7 • The starship *Enterprise*, of television and movie fame, is powered by combining matter and antimatter. If the entire 400-kg antimatter fuel supply of the *Enterprise* combines with matter, how much energy is released? How does this compare to the U.S. yearly energy use, which is roughly $1.0 \times 10^{20} \text{ J}$?

Section 44.2 Particle Accelerators and Detectors

44.8 • An electron with a total energy of 20.0 GeV collides with a stationary positron. (a) What is the available energy? (b) If the electron and positron are accelerated in a collider, what total energy corresponds to the same available energy as in part (a)?

44.9 • Deuterons in a cyclotron travel in a circle with radius 32.0 cm just before emerging from the dees. The frequency of the applied alternating voltage is 9.00 MHz. Find (a) the magnetic field and (b) the kinetic energy and speed of the deuterons upon emergence.

44.10 • The magnetic field in a cyclotron that accelerates protons is 1.30 T. (a) How many times per second should the potential across the dees reverse? (This is twice the frequency of the circulating protons.) (b) The maximum radius of the cyclotron is 0.250 m. What is the maximum speed of the proton? (c) Through what potential difference would the proton have to be accelerated from rest to give it the same speed as calculated in part (b)?

44.11 • (a) A high-energy beam of alpha particles collides with a stationary helium gas target. What must the total energy of a beam particle be if the available energy in the collision is 16.0 GeV? (b) If the alpha particles instead interact in a colliding-beam experiment, what must the energy of each beam be to produce the same available energy?

44.12 •• (a) What is the speed of a proton that has total energy 1000 GeV? (b) What is the angular frequency ω of a proton with the speed calculated in part (a) in a magnetic field of 4.00 T? Use both the nonrelativistic Eq. (44.7) and the correct relativistic expression, and compare the results.

44.13 • In Example 44.3 it was shown that a proton beam with an 800-GeV beam energy gives an available energy of 38.7 GeV for collisions with a stationary proton target. (a) You are asked to design an upgrade of the accelerator that will double the available energy in stationary-target collisions. What beam energy is required? (b) In a colliding-beam experiment, what total energy of each beam is needed to give an available energy of $2(38.7 \text{ GeV}) = 77.4 \text{ GeV}$?

44.14 •• Calculate the minimum beam energy in a proton-proton collider to initiate the $p + p \rightarrow p + p + \eta^0$ reaction. The rest energy of the η^0 is 547.3 MeV (see Table 44.3).

Section 44.3 Particles and Interactions

44.15 • A K^+ meson at rest decays into two π mesons. (a) What are the allowed combinations of π^0 , π^+ , and π^- as decay products? (b) Find the total kinetic energy of the π mesons.

44.16 • How much energy is released when a μ^- muon at rest decays into an electron and two neutrinos? Neglect the small masses of the neutrinos.

44.17 • What is the mass (in kg) of the Z^0 ? What is the ratio of the mass of the Z^0 to the mass of the proton?

44.18 • Table 44.3 shows that a Σ^0 decays into a Λ^0 and a photon. (a) Calculate the energy of the photon emitted in this decay, if the Λ^0 is at rest. (b) What is the magnitude of the momentum of the photon? Is it reasonable to ignore the final momentum and kinetic energy of the Λ^0 ? Explain.

44.19 • If a Σ^+ at rest decays into a proton and a π^0 , what is the total kinetic energy of the decay products?

44.20 • The discovery of the Ω^- particle helped confirm Gell-Mann's eightfold way. If an Ω^- decays into a Λ^0 and a K^- , what is the total kinetic energy of the decay products?

44.21 • In which of the following decays are the three lepton numbers conserved? In each case, explain your reasoning. (a) $\mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu$; (b) $\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau$; (c) $\pi^+ \rightarrow e^+ + \gamma$; (d) $n \rightarrow p + e^- + \bar{\nu}_e$.

44.22 • Which of the following reactions obey the conservation of baryon number? (a) $p + p \rightarrow p + e^+$; (b) $p + n \rightarrow 2e^+ + e^-$; (c) $p \rightarrow n + e^- + \bar{\nu}_e$; (d) $p + \bar{p} \rightarrow 2\gamma$.

44.23 • In which of the following reactions or decays is strangeness conserved? In each case, explain your reasoning. (a) $K^+ \rightarrow \mu^+ + \nu_\mu$; (b) $n + K^+ \rightarrow p + \pi^0$; (c) $K^+ + K^- \rightarrow \pi^0 + \pi^0$; (d) $p + K^- \rightarrow \Lambda^0 + \pi^0$.

44.24 •• CP (a) Show that the coupling constant for the electromagnetic interaction, $e^2/4\pi\epsilon_0\hbar c$, is dimensionless and has the numerical value $1/137.0$. (b) Show that in the Bohr model the orbital speed of an electron in the $n = 1$ orbit is equal to c times the coupling constant $e^2/4\pi\epsilon_0\hbar c$.

44.25 • Show that the nuclear force coupling constant $f^2/\hbar c$ is dimensionless.

Section 44.4 Quarks and the Eightfold Way

44.26 • Nine of the spin- $\frac{3}{2}$ baryons are four Δ particles, each with mass $1232 \text{ MeV}/c^2$, strangeness 0, and charges $+2e$, $+e$, 0 , and $-e$; three Σ^* particles, each with mass $1385 \text{ MeV}/c^2$, strangeness -1 , and charges $+e$, 0 , and $-e$; and two Ξ^* particles, each with mass $1530 \text{ MeV}/c^2$, strangeness -2 , and charges 0 and $-e$. (a) Place these particles on a plot of S versus Q . Deduce the Q and S values of the tenth spin- $\frac{3}{2}$ baryon, the Ω^- particle, and place it on your diagram. Also label the particles with their masses. The mass of the Ω^- is $1672 \text{ MeV}/c^2$; is this value consistent with your diagram? (b) Deduce the three-quark combinations (of u , d , and s) that make up each of these ten particles. Redraw the plot of S versus Q from part (a) with each particle labeled by its quark content. What regularities do you see?

44.27 • Determine the electric charge, baryon number, strangeness quantum number, and charm quantum number for the following quark combinations: (a) uds ; (b) $c\bar{u}$; (c) ddd ; and (d) $d\bar{c}$. Explain your reasoning.

44.28 •• Determine the electric charge, baryon number, strangeness quantum number, and charm quantum number for the following quark combinations: (a) uus , (b) $c\bar{s}$, (c) $\bar{d}d\bar{u}$, and (d) $\bar{c}b$.

44.29 • The weak force may change quark flavor in an interaction. Explain how β^+ decay changes quark flavor. If a proton undergoes β^+ decay, determine the decay reaction.

44.30 • What is the total kinetic energy of the decay products when an upsilon particle at rest decays to $\tau^+ + \tau^-$?

44.31 • The quark content of the neutron is udd . (a) What is the quark content of the antineutron? Explain your reasoning. (b) Is the neutron its own antiparticle? Why or why not? (c) The quark

content of the ψ is $c\bar{c}$. Is the ψ its own antiparticle? Explain your reasoning.

44.32 • Given that each particle contains only combinations of u , d , s , \bar{u} , \bar{d} , and \bar{s} , use the method of Example 44.7 to deduce the quark content of (a) a particle with charge $+e$, baryon number 0, and strangeness $+1$; (b) a particle with charge $+e$, baryon number -1 , and strangeness $+1$; (c) a particle with charge 0, baryon number $+1$, and strangeness -2 .

Section 44.6 The Expanding Universe

44.33 • The spectrum of the sodium atom is detected in the light from a distant galaxy. (a) If the 590.0-nm line is redshifted to 658.5 nm, at what speed is the galaxy receding from the earth? (b) Use the Hubble law to calculate the distance of the galaxy from the earth.

44.34 • **Redshift Parameter.** The definition of the redshift parameter z is given in Example 44.8. (a) Show that Eq. (44.13) may be written as $1 + z = ([1 + \beta]/[1 - \beta])^{1/2}$, where $\beta = v/c$. (b) The observed redshift parameter for a certain galaxy is $z = 0.500$. Find the speed of the galaxy relative to the earth, if the redshift is due to the Doppler shift. (c) Use the Hubble law to find the distance of this galaxy from the earth.

44.35 • A galaxy in the constellation Pisces is 5210 Mly from the earth. (a) Use the Hubble law to calculate the speed at which this galaxy is receding from earth. (b) What redshifted ratio λ_0/λ_S is expected for light from this galaxy?

44.36 •• (a) According to the Hubble law, what is the distance r from us for galaxies that are receding from us with a speed c ? (b) Explain why the distance calculated in part (a) is the size of our observable universe (ignoring any change in the expansion rate of the universe due to gravitational attraction or dark energy).

44.37 •• The critical density of the universe is $9.5 \times 10^{-27} \text{ kg/m}^3$. (a) Assuming that the universe is all hydrogen, express the critical density in the number of H atoms per cubic meter. (b) If the density of the universe is equal to the critical density, how many atoms, on the average, would you expect to find in a room of dimensions $4 \text{ m} \times 7 \text{ m} \times 3 \text{ m}$? (c) Compare your answer in part (b) with the number of atoms you would find in the same room under normal conditions on the earth.

Section 44.7 The Beginning of Time

44.38 • (a) Show that the expression for the Planck length, $\sqrt{\hbar G/c^3}$, has dimensions of length. (b) Evaluate the numerical value of $\sqrt{\hbar G/c^3}$, and verify the value given in Eq. (44.21).

44.39 • Calculate the energy released in each reaction: (a) $p + {}^2\text{H} \rightarrow {}^3\text{He}$; (b) $n + {}^3\text{He} \rightarrow {}^4\text{He}$.

44.40 • Calculate the energy (in MeV) released in the triple-alpha process $3 {}^4\text{He} \rightarrow {}^{12}\text{C}$.

44.41 • Calculate the reaction energy Q (in MeV) for the reaction $e^- + p \rightarrow n + \nu_e$. Is this reaction endoergic or exoergic?

44.42 • Calculate the reaction energy Q (in MeV) for the nucleosynthesis reaction



Is this reaction endoergic or exoergic?

44.43 • **CP** The 2.728-K blackbody radiation has its peak wavelength at 1.062 mm. What was the peak wavelength at $t = 700,000 \text{ y}$ when the temperature was 3000 K?

PROBLEMS

44.44 • **CP** A positronium atom consists of an electron and a positron. In the Bohr model the two particles orbit around their

common center of mass. In the Bohr model, what is the ionization energy for a positronium atom when it is in its ground state?

44.45 • In the LHC, each proton will be accelerated to a kinetic energy of 7.0 TeV. (a) In the colliding beams, what is the available energy E_a in a collision? (b) In a fixed-target experiment in which a beam of protons is incident on a stationary proton target, what must the total energy (in TeV) of the particles in the beam be to produce the same available energy as in part (a)?

44.46 •• A proton and an antiproton collide head-on with equal kinetic energies. Two γ rays with wavelengths of 0.780 fm are produced. Calculate the kinetic energy of the incident proton.

44.47 •• **CP BIO Radiation Therapy with π^- Mesons.** Beams of π^- mesons are used in radiation therapy for certain cancers. The energy comes from the complete decay of the π^- to stable particles. (a) Write out the complete decay of a π^- meson to stable particles. What are these particles? (b) How much energy is released from the complete decay of a single π^- meson to stable particles? (You can ignore the very small masses of the neutrinos.) (c) How many π^- mesons need to decay to give a dose of 50.0 Gy to 10.0 g of tissue? (d) What would be the equivalent dose in part (c) in Sv and in rem? Consult Table 43.3 and use the largest appropriate RBE for the particles involved in this decay.

44.48 •• Calculate the threshold kinetic energy for the reaction $\pi^- + p \rightarrow \Sigma^0 + K^0$ if a π^- beam is incident on a stationary proton target. The K^0 has a mass of $497.7 \text{ MeV}/c^2$.

44.49 •• Calculate the threshold kinetic energy for the reaction $p + p \rightarrow p + p + K^+ + K^-$ if a proton beam is incident on a stationary proton target.

44.50 •• An η^0 meson at rest decays into three π mesons. (a) What are the allowed combinations of π^0 , π^+ , and π^- as decay products? (b) Find the total kinetic energy of the π mesons.

44.51 • Each of the following reactions is missing a single particle. Calculate the baryon number, charge, strangeness, and the three lepton numbers (where appropriate) of the missing particle, and from this identify the particle. (a) $p + p \rightarrow p + \Lambda^0 + ?$; (b) $K^- + n \rightarrow \Lambda^0 + ?$; (c) $p + \bar{p} \rightarrow n + ?$; (d) $\bar{\nu}_\mu + p \rightarrow n + ?$

44.52 • Estimate the energy width (energy uncertainty) of the ψ if its mean lifetime is $7.6 \times 10^{-21} \text{ s}$. What fraction is this of its rest energy?

44.53 • The ϕ meson has mass $1019.4 \text{ MeV}/c^2$ and a measured energy width of $4.4 \text{ MeV}/c^2$. Using the uncertainty principle, estimate the lifetime of the ϕ meson.

44.54 •• A ϕ meson (see Problem 44.53) at rest decays via $\phi \rightarrow K^+ + K^-$. It has strangeness 0. (a) Find the kinetic energy of the K^+ meson. (Assume that the two decay products share kinetic energy equally, since their masses are equal.) (b) Suggest a reason the decay $\phi \rightarrow K^+ + K^- + \pi^0$ has not been observed. (c) Suggest reasons the decays $\phi \rightarrow K^+ + \pi^-$ and $\phi \rightarrow K^+ + \mu^-$ have not been observed.

44.55 •• **CP BIO** One proposed proton decay is $p^+ \rightarrow e^+ + \pi^0$, which violates both baryon and lepton number conservation, so the proton lifetime is expected to be very long. Suppose the proton half-life were $1.0 \times 10^{18} \text{ y}$. (a) Calculate the energy deposited per kilogram of body tissue (in rad) due to the decay of the protons in your body in one year. Model your body as consisting entirely of water. Only the two protons in the hydrogen atoms in each H_2O molecule would decay in the manner shown; do you see why? Assume that the π^0 decays to two γ rays, that the positron annihilates with an electron, and that all the energy produced in the primary decay and these secondary decays remains in your body. (b) Calculate the equivalent dose (in rem) assuming an RBE of 1.0 for all the radiation products, and compare with the 0.1 rem due to the natural background and the 5.0-rem guideline

for industrial workers. Based on your calculation, can the proton lifetime be as short as 1.0×10^{18} y?

44.56 ••• CP A Ξ^- particle at rest decays to a Λ^0 and a π^- . (a) Find the total kinetic energy of the decay products. (b) What fraction of the energy is carried off by each particle? (Use relativistic expressions for momentum and energy.)

44.57 •• CALC Consider the spherical balloon model of a two-dimensional expanding universe (see Fig. 44.17 in Section 44.6). The shortest distance between two points on the surface, measured along the surface, is the arc length r , where $r = R\theta$. As the balloon expands, its radius R increases, but the angle θ between the two points remains constant. (a) Explain why, at any given time, $(dR/dt)/R$ is the same for all points on the balloon. (b) Show that $v = dr/dt$ is directly proportional to r at any instant. (c) From your answer to part (b), what is the expression for the Hubble constant H_0 in terms of R and dR/dt ? (d) The expression for H_0 you found in part (c) is constant in space. How would R have to depend on time for H_0 to be constant in time? (e) Is your answer to part (d) consistent with the observed rate of expansion of the universe?

44.58 ••• CALC Suppose all the conditions are the same as in Problem 44.57, except that $v = dr/dt$ is constant for a given θ , rather than H_0 being constant in time. Show that the Hubble constant is $H_0 = 1/t$ and, hence, that the current value is $1/T$, where T is the age of the universe.

44.59 •• Cosmic Jerk. The densities of ordinary matter and dark matter have decreased as the universe has expanded, since the same amount of mass occupies an ever-increasing volume. Yet observations suggest that the density of dark energy has remained constant over the entire history of the universe. (a) Explain why the expansion of the universe actually slowed down in its early history but is speeding up today. “Jerk” is the term for a change in acceleration, so the change in cosmic expansion from slowing

down to speeding up is called *cosmic jerk*. (b) Calculations show that the change in acceleration took place when the combined density of matter of all kinds was equal to twice the density of dark energy. Compared to today’s value of the scale factor, what was the scale factor at that time? (c) We see the galaxy clusters in Figs. 44.15b and 44.19 as they were 300 million years ago and 10.2 billion years ago. Was the expansion of the universe slowing down or speeding up at these times? (*Hint:* See the caption for Fig. 44.19.)

44.60 ••• CP The K^0 meson has rest energy 497.7 MeV. A K^0 meson moving in the $+x$ -direction with kinetic energy 225 MeV decays into a π^+ and a π^- , which move off at equal angles above and below the $+x$ -axis. Calculate the kinetic energy of the π^+ and the angle it makes with the $+x$ -axis. Use relativistic expressions for energy and momentum.

44.61 ••• CP A Σ^- particle moving in the $+x$ -direction with kinetic energy 180 MeV decays into a π^- and a neutron. The π^- moves in the $+y$ -direction. What is the kinetic energy of the neutron, and what is the direction of its velocity? Use relativistic expressions for energy and momentum.

CHALLENGE PROBLEM

44.62 ••• CP Consider a collision in which a stationary particle with mass M is bombarded by a particle with mass m , speed v_0 , and total energy (including rest energy) E_m . (a) Use the Lorentz transformation to write the velocities v_m and v_M of particles m and M in terms of the speed v_{cm} of the center of momentum. (b) Use the fact that the total momentum in the center-of-momentum frame is zero to obtain an expression for v_{cm} in terms of m , M , and v_0 . (c) Combine the results of parts (a) and (b) to obtain Eq. (44.9) for the total energy in the center-of-momentum frame.

Answers

Chapter Opening Question ?

Only 4.6% of the mass and energy of the universe is in the form of “normal” matter. Of the rest, 22.8% is poorly understood dark matter and 72.6% is even more mysterious dark energy.

Test Your Understanding Questions

44.1 Answer: (i), (iii), (ii), (iv) The more massive the virtual particle, the shorter its lifetime and the shorter the distance that it can travel during its lifetime.

44.2 Answer: no In a head-on collision between an electron and a positron of equal energy, the net momentum is zero. Since both momentum and energy are conserved in the collision, the virtual photon also has momentum $p = 0$ but has energy $E = 90 \text{ GeV} + 90 \text{ GeV} = 180 \text{ GeV}$. Hence the relationship $E = pc$ is definitely *not* true for this virtual photon.

44.3 Answer: no Mesons all have baryon number $B = 0$, while a proton has $B = 1$. The decay of a proton into one or more mesons would require that baryon number *not* be conserved. No violation of this conservation principle has ever been observed, so the proposed decay is impossible.

44.4 Answer: no Only the s quark, with $S = -1$, has nonzero strangeness. For a baryon to have $S = -2$, it must have two s quarks and one quark of a different flavor. Since each s quark has

charge $-\frac{1}{3}e$, the nonstrange quark must have charge $+\frac{5}{3}e$ to make the net charge equal to $+e$. But *no* quark has charge $+\frac{5}{3}e$, so the proposed baryon is impossible.

44.5 Answer: (i) If a d quark in a neutron (quark content udd) undergoes the process $d \rightarrow u + e^- + \bar{\nu}_e$, the remaining baryon has quark content uud and hence is a proton (see Fig. 44.11). An electron is the same as a β^- particle, so the net result is beta-minus decay: $n \rightarrow p + \beta^- + \bar{\nu}_e$.

44.6 Answer: yes . . . and no The material of which your body is made is ordinary to us on earth. But from a cosmic perspective your material is quite *extraordinary*: Only 4.6% of the mass and energy in the universe is in the form of atoms.

44.7 Answer: no Prior to $t = 380,000$ y the temperature was so high that atoms could not form, so free electrons and protons were plentiful. These charged particles are very effective at scattering photons, so light could not propagate very far and the universe was opaque. The oldest photons that we can detect date from the time $t = 380,000$ y when atoms formed and the universe became transparent.

Bridging Problem

Answers: (a) 5.78 MeV for the neutron, 35.62 MeV for the pion
(b) 0.140 for the neutron, 0.860 for the pion
(c) 24 MeV

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APPENDIX A

THE INTERNATIONAL SYSTEM OF UNITS

The Système International d'Unités, abbreviated SI, is the system developed by the General Conference on Weights and Measures and adopted by nearly all the industrial nations of the world. The following material is adapted from the National Institute of Standards and Technology (<http://physics.nist.gov/cuu>).

Quantity	Name of unit	Symbol	
SI base units			
length	meter	m	
mass	kilogram	kg	
time	second	s	
electric current	ampere	A	
thermodynamic temperature	kelvin	K	
amount of substance	mole	mol	
luminous intensity	candela	cd	
SI derived units			Equivalent units
area	square meter	m ²	
volume	cubic meter	m ³	
frequency	hertz	Hz	s ⁻¹
mass density (density)	kilogram per cubic meter	kg/m ³	
speed, velocity	meter per second	m/s	
angular velocity	radian per second	rad/s	
acceleration	meter per second squared	m/s ²	
angular acceleration	radian per second squared	rad/s ²	
force	newton	N	kg · m/s ²
pressure (mechanical stress)	pascal	Pa	N/m ²
kinematic viscosity	square meter per second	m ² /s	
dynamic viscosity	newton-second per square meter	N · s/m ²	
work, energy, quantity of heat	joule	J	N · m
power	watt	W	J/s
quantity of electricity	coulomb	C	A · s
potential difference, electromotive force	volt	V	J/C, W/A
electric field strength	volt per meter	V/m	N/C
electric resistance	ohm	Ω	V/A
capacitance	farad	F	A · s/V
magnetic flux	weber	Wb	V · s
inductance	henry	H	V · s/A
magnetic flux density	tesla	T	Wb/m ²
magnetic field strength	ampere per meter	A/m	
magnetomotive force	ampere	A	
luminous flux	lumen	lm	cd · sr
luminance	candela per square meter	cd/m ²	
illuminance	lux	lx	lm/m ²
wave number	1 per meter	m ⁻¹	
entropy	joule per kelvin	J/K	
specific heat capacity	joule per kilogram-kelvin	J/kg · K	
thermal conductivity	watt per meter-kelvin	W/m · K	

Quantity	Name of unit	Symbol	Equivalent units
radiant intensity	watt per steradian	W/sr	
activity (of a radioactive source)	becquerel	Bq	s ⁻¹
radiation dose	gray	Gy	J/kg
radiation dose equivalent	sievert	Sv	J/kg
SI supplementary units			
plane angle	radian	rad	
solid angle	steradian	sr	

Definitions of SI Units

meter (m) The *meter* is the length equal to the distance traveled by light, in vacuum, in a time of 1/299,792,458 second.

kilogram (kg) The *kilogram* is the unit of mass; it is equal to the mass of the international prototype of the kilogram. (The international prototype of the kilogram is a particular cylinder of platinum-iridium alloy that is preserved in a vault at Sèvres, France, by the International Bureau of Weights and Measures.)

second (s) The *second* is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

ampere (A) The *ampere* is that constant current that, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.

kelvin (K) The *kelvin*, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

ohm (Ω) The *ohm* is the electric resistance between two points of a conductor when a constant difference of potential of 1 volt, applied between these two points, produces in this conductor a current of 1 ampere, this conductor not being the source of any electromotive force.

coulomb (C) The *coulomb* is the quantity of electricity transported in 1 second by a current of 1 ampere.

candela (cd) The *candela* is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

mole (mol) The *mole* is the amount of substance of a system that contains as many elementary entities as there are carbon atoms in 0.012 kg of carbon 12. The elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

newton (N) The *newton* is that force that gives to a mass of 1 kilogram an acceleration of 1 meter per second per second.

joule (J) The *joule* is the work done when the point of application of a constant force of 1 newton is displaced a distance of 1 meter in the direction of the force.

watt (W) The *watt* is the power that gives rise to the production of energy at the rate of 1 joule per second.

volt (V) The *volt* is the difference of electric potential between two points of a conducting wire carrying a constant current of 1 ampere, when the power dissipated between these points is equal to 1 watt.

weber (Wb) The *weber* is the magnetic flux that, linking a circuit of one turn, produces in it an electromotive force of 1 volt as it is reduced to zero at a uniform rate in 1 second.

lumen (lm) The *lumen* is the luminous flux emitted in a solid angle of 1 steradian by a uniform point source having an intensity of 1 candela.

farad (F) The *farad* is the capacitance of a capacitor between the plates of which there appears a difference of potential of 1 volt when it is charged by a quantity of electricity equal to 1 coulomb.

henry (H) The *henry* is the inductance of a closed circuit in which an electromotive force of 1 volt is produced when the electric current in the circuit varies uniformly at a rate of 1 ampere per second.

radian (rad) The *radian* is the plane angle between two radii of a circle that cut off on the circumference an arc equal in length to the radius.

steradian (sr) The *steradian* is the solid angle that, having its vertex in the center of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

SI Prefixes To form the names of multiples and submultiples of SI units, apply the prefixes listed in Appendix F.

APPENDIX B

USEFUL MATHEMATICAL RELATIONS

Algebra

$$a^{-x} = \frac{1}{a^x} \quad a^{(x+y)} = a^x a^y \quad a^{(x-y)} = \frac{a^x}{a^y}$$

Logarithms: If $\log a = x$, then $a = 10^x$. $\log a + \log b = \log(ab)$ $\log a - \log b = \log(a/b)$ $\log(a^n) = n \log a$

If $\ln a = x$, then $a = e^x$. $\ln a + \ln b = \ln(ab)$ $\ln a - \ln b = \ln(a/b)$ $\ln(a^n) = n \ln a$

Quadratic formula: If $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Binomial Theorem

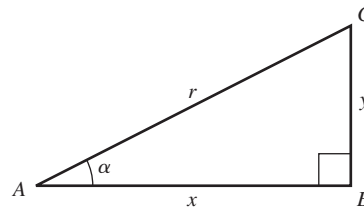
$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \dots$$

Trigonometry

In the right triangle ABC , $x^2 + y^2 = r^2$.

Definitions of the trigonometric functions:

$$\sin \alpha = y/r \quad \cos \alpha = x/r \quad \tan \alpha = y/x$$



Identities: $\sin^2 \alpha + \cos^2 \alpha = 1$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$$

$$\sin \frac{1}{2} \alpha = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{1}{2} \alpha = \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\sin(-\alpha) = -\sin \alpha$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(-\alpha) = \cos \alpha$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin(\alpha \pm \pi/2) = \pm \cos \alpha$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

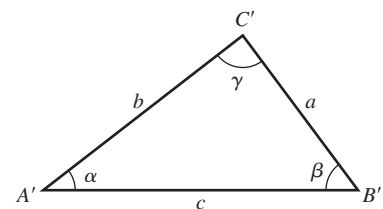
$$\cos(\alpha \pm \pi/2) = \mp \sin \alpha$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

For any triangle $A'B'C'$ (not necessarily a right triangle) with sides a , b , and c and angles α , β , and γ :

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$

Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos \gamma$



Geometry

Circumference of circle of radius r :

$$C = 2\pi r$$

Surface area of sphere of radius r :

$$A = 4\pi r^2$$

Area of circle of radius r :

$$A = \pi r^2$$

Volume of cylinder of radius r and height h :

$$V = \pi r^2 h$$

Volume of sphere of radius r :

$$V = \frac{4\pi r^3}{3}$$

Calculus

Derivatives:

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\ln ax = \frac{1}{x}$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}\sin ax = a \cos ax$$

$$\frac{d}{dx}\cos ax = -a \sin ax$$

Integrals:

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$\int \frac{dx}{x} = \ln x$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax dx = \frac{1}{a}\sin ax$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan \frac{x}{a}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Power series (convergent for range of x shown):

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)}{3!}x^3$$

$$+ \dots \quad (|x| < 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad (\text{all } x)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots \quad (|x| < \pi/2)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \quad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad (|x| < 1)$$

APPENDIX C

THE GREEK ALPHABET

Name	Capital	Lowercase	Name	Capital	Lowercase	Name	Capital	Lowercase
Alpha	A	α	Iota	I	ι	Rho	P	ρ
Beta	B	β	Kappa	K	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	T	τ
Delta	Δ	δ	Mu	M	μ	Upsilon	Y	υ
Epsilon	E	ϵ	Nu	N	ν	Phi	Φ	ϕ
Zeta	Z	ζ	Xi	Ξ	ξ	Chi	X	χ
Eta	H	η	Omicron	O	o	Psi	Ψ	ψ
Theta	Θ	θ	Pi	Π	π	Omega	Ω	ω

APPENDIX D

PERIODIC TABLE OF THE ELEMENTS

Group 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
 Period

1																	2	
1	H 1.008																	He 4.003
2	Li 6.941	Be 9.012											B 10.811	C 12.011	N 14.007	O 15.999	F 18.998	Ne 20.180
3	Na 22.990	Mg 24.305											Al 26.982	Si 28.086	P 30.974	S 32.065	Cl 35.453	Ar 39.948
4	K 39.098	Ca 40.078	Sc 44.956	Ti 47.867	V 50.942	Cr 51.996	Mn 54.938	Fe 55.845	Co 58.933	Ni 58.693	Cu 63.546	Zn 65.409	Ga 69.723	Ge 72.64	As 74.922	Se 78.96	Br 79.904	Kr 83.798
5	Rb 85.468	Sr 87.62	Y 88.906	Zr 91.224	Nb 92.906	Mo 95.94	Tc (98)	Ru 101.07	Rh 102.906	Pd 106.42	Ag 107.868	Cd 112.411	In 114.818	Sn 118.710	Sb 121.760	Te 127.60	I 126.904	Xe 131.293
6	Cs 132.905	Ba 137.327	Lu 174.967	Hf 178.49	Ta 180.948	W 183.84	Re 186.207	Os 190.23	Ir 192.217	Pt 195.078	Au 196.967	Hg 200.59	Tl 204.383	Pb 207.2	Bi 208.980	Po (209)	At (210)	Rn (222)
7	Fr (223)	Ra (226)	Lr (262)	Rf (261)	Db (262)	Sg (266)	Bh (264)	Hs (269)	Mt (268)	Ds (271)	Rg (272)	Uub (285)	Uut (284)	Uuq (289)	Uup (288)	Uuh (292)	Uus (294)	Uuo

Lanthanoids	57 La 138.905	58 Ce 140.116	59 Pr 140.908	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.964	64 Gd 157.25	65 Tb 158.925	66 Dy 162.500	67 Ho 164.930	68 Er 167.259	69 Tm 168.934	70 Yb 173.04
Actinoids	89 Ac (227)	90 Th (232)	91 Pa (231)	92 U (238)	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)

For each element the average atomic mass of the mixture of isotopes occurring in nature is shown. For elements having no stable isotope, the approximate atomic mass of the longest-lived isotope is shown in parentheses. For elements that have been predicted but not yet confirmed, no atomic mass is given. All atomic masses are expressed in atomic mass units ($1 \text{ u} = 1.660538782(83) \times 10^{-27} \text{ kg}$), equivalent to grams per mole (g/mol).

APPENDIX E

UNIT CONVERSION FACTORS

Length

$$1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm} = 10^6 \mu\text{m} = 10^9 \text{ nm}$$

$$1 \text{ km} = 1000 \text{ m} = 0.6214 \text{ mi}$$

$$1 \text{ m} = 3.281 \text{ ft} = 39.37 \text{ in.}$$

$$1 \text{ cm} = 0.3937 \text{ in.}$$

$$1 \text{ in.} = 2.540 \text{ cm}$$

$$1 \text{ ft} = 30.48 \text{ cm}$$

$$1 \text{ yd} = 91.44 \text{ cm}$$

$$1 \text{ mi} = 5280 \text{ ft} = 1.609 \text{ km}$$

$$1 \text{ \AA} = 10^{-10} \text{ m} = 10^{-8} \text{ cm} = 10^{-1} \text{ nm}$$

$$1 \text{ nautical mile} = 6080 \text{ ft}$$

$$1 \text{ light year} = 9.461 \times 10^{15} \text{ m}$$

Area

$$1 \text{ cm}^2 = 0.155 \text{ in.}^2$$

$$1 \text{ m}^2 = 10^4 \text{ cm}^2 = 10.76 \text{ ft}^2$$

$$1 \text{ in.}^2 = 6.452 \text{ cm}^2$$

$$1 \text{ ft}^2 = 144 \text{ in.}^2 = 0.0929 \text{ m}^2$$

Volume

$$1 \text{ liter} = 1000 \text{ cm}^3 = 10^{-3} \text{ m}^3 = 0.03531 \text{ ft}^3 = 61.02 \text{ in.}^3$$

$$1 \text{ ft}^3 = 0.02832 \text{ m}^3 = 28.32 \text{ liters} = 7.477 \text{ gallons}$$

$$1 \text{ gallon} = 3.788 \text{ liters}$$

Time

$$1 \text{ min} = 60 \text{ s}$$

$$1 \text{ h} = 3600 \text{ s}$$

$$1 \text{ d} = 86,400 \text{ s}$$

$$1 \text{ y} = 365.24 \text{ d} = 3.156 \times 10^7 \text{ s}$$

Angle

$$1 \text{ rad} = 57.30^\circ = 180^\circ/\pi$$

$$1^\circ = 0.01745 \text{ rad} = \pi/180 \text{ rad}$$

$$1 \text{ revolution} = 360^\circ = 2\pi \text{ rad}$$

$$1 \text{ rev/min (rpm)} = 0.1047 \text{ rad/s}$$

Speed

$$1 \text{ m/s} = 3.281 \text{ ft/s}$$

$$1 \text{ ft/s} = 0.3048 \text{ m/s}$$

$$1 \text{ mi/min} = 60 \text{ mi/h} = 88 \text{ ft/s}$$

$$1 \text{ km/h} = 0.2778 \text{ m/s} = 0.6214 \text{ mi/h}$$

$$1 \text{ mi/h} = 1.466 \text{ ft/s} = 0.4470 \text{ m/s} = 1.609 \text{ km/h}$$

$$1 \text{ furlong/fortnight} = 1.662 \times 10^{-4} \text{ m/s}$$

Acceleration

$$1 \text{ m/s}^2 = 100 \text{ cm/s}^2 = 3.281 \text{ ft/s}^2$$

$$1 \text{ cm/s}^2 = 0.01 \text{ m/s}^2 = 0.03281 \text{ ft/s}^2$$

$$1 \text{ ft/s}^2 = 0.3048 \text{ m/s}^2 = 30.48 \text{ cm/s}^2$$

$$1 \text{ mi/h} \cdot \text{s} = 1.467 \text{ ft/s}^2$$

Mass

$$1 \text{ kg} = 10^3 \text{ g} = 0.0685 \text{ slug}$$

$$1 \text{ g} = 6.85 \times 10^{-5} \text{ slug}$$

$$1 \text{ slug} = 14.59 \text{ kg}$$

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$$

$$1 \text{ kg has a weight of } 2.205 \text{ lb when } g = 9.80 \text{ m/s}^2$$

Force

$$1 \text{ N} = 10^5 \text{ dyn} = 0.2248 \text{ lb}$$

$$1 \text{ lb} = 4.448 \text{ N} = 4.448 \times 10^5 \text{ dyn}$$

Pressure

$$1 \text{ Pa} = 1 \text{ N/m}^2 = 1.450 \times 10^{-4} \text{ lb/in.}^2 = 0.209 \text{ lb/ft}^2$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

$$1 \text{ lb/in.}^2 = 6895 \text{ Pa}$$

$$1 \text{ lb/ft}^2 = 47.88 \text{ Pa}$$

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 1.013 \text{ bar}$$

$$= 14.7 \text{ lb/in.}^2 = 2117 \text{ lb/ft}^2$$

$$1 \text{ mm Hg} = 1 \text{ torr} = 133.3 \text{ Pa}$$

Energy

$$1 \text{ J} = 10^7 \text{ ergs} = 0.239 \text{ cal}$$

$$1 \text{ cal} = 4.186 \text{ J (based on } 15^\circ \text{ calorie)}$$

$$1 \text{ ft} \cdot \text{lb} = 1.356 \text{ J}$$

$$1 \text{ Btu} = 1055 \text{ J} = 252 \text{ cal} = 778 \text{ ft} \cdot \text{lb}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$1 \text{ kWh} = 3.600 \times 10^6 \text{ J}$$

Mass–Energy Equivalence

$$1 \text{ kg} \leftrightarrow 8.988 \times 10^{16} \text{ J}$$

$$1 \text{ u} \leftrightarrow 931.5 \text{ MeV}$$

$$1 \text{ eV} \leftrightarrow 1.074 \times 10^{-9} \text{ u}$$

Power

$$1 \text{ W} = 1 \text{ J/s}$$

$$1 \text{ hp} = 746 \text{ W} = 550 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ Btu/h} = 0.293 \text{ W}$$

APPENDIX F

NUMERICAL CONSTANTS

Fundamental Physical Constants*

Name	Symbol	Value
Speed of light in vacuum	c	2.99792458×10^8 m/s
Magnitude of charge of electron	e	$1.602176487(40) \times 10^{-19}$ C
Gravitational constant	G	$6.67428(67) \times 10^{-11}$ N·m ² /kg ²
Planck's constant	h	$6.62606896(33) \times 10^{-34}$ J·s
Boltzmann constant	k	$1.3806504(24) \times 10^{-23}$ J/K
Avogadro's number	N_A	$6.02214179(30) \times 10^{23}$ molecules/mol
Gas constant	R	8.314472(15) J/mol·K
Mass of electron	m_e	$9.10938215(45) \times 10^{-31}$ kg
Mass of proton	m_p	$1.672621637(83) \times 10^{-27}$ kg
Mass of neutron	m_n	$1.674927211(84) \times 10^{-27}$ kg
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Wb/A·m
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854187817 \dots \times 10^{-12}$ C ² /N·m ²
	$1/4\pi\epsilon_0$	$8.987551787 \dots \times 10^9$ N·m ² /C ²

Other Useful Constants*

Mechanical equivalent of heat		4.186 J/cal (15° calorie)
Standard atmospheric pressure	1 atm	1.01325×10^5 Pa
Absolute zero	0 K	-273.15°C
Electron volt	1 eV	$1.602176487(40) \times 10^{-19}$ J
Atomic mass unit	1 u	$1.660538782(83) \times 10^{-27}$ kg
Electron rest energy	$m_e c^2$	0.510998910(13) MeV
Volume of ideal gas (0°C and 1 atm)		22.413996(39) liter/mol
Acceleration due to gravity (standard)	g	9.80665 m/s ²

*Source: National Institute of Standards and Technology (<http://physics.nist.gov/cuu>). Numbers in parentheses show the uncertainty in the final digits of the main number; for example, the number 1.6454(21) means 1.6454 ± 0.0021 . Values shown without uncertainties are exact.

Astronomical Data[†]

Body	Mass (kg)	Radius (m)	Orbit radius (m)	Orbit period
Sun	1.99×10^{30}	6.96×10^8	—	—
Moon	7.35×10^{22}	1.74×10^6	3.84×10^8	27.3 d
Mercury	3.30×10^{23}	2.44×10^6	5.79×10^{10}	88.0 d
Venus	4.87×10^{24}	6.05×10^6	1.08×10^{11}	224.7 d
Earth	5.97×10^{24}	6.38×10^6	1.50×10^{11}	365.3 d
Mars	6.42×10^{23}	3.40×10^6	2.28×10^{11}	687.0 d
Jupiter	1.90×10^{27}	6.91×10^7	7.78×10^{11}	11.86 y
Saturn	5.68×10^{26}	6.03×10^7	1.43×10^{12}	29.45 y
Uranus	8.68×10^{25}	2.56×10^7	2.87×10^{12}	84.02 y
Neptune	1.02×10^{26}	2.48×10^7	4.50×10^{12}	164.8 y
Pluto [‡]	1.31×10^{22}	1.15×10^6	5.91×10^{12}	247.9 y

[†]Source: NASA Jet Propulsion Laboratory Solar System Dynamics Group (<http://ssd.jpl.nasa.gov>), and P. Kenneth Seidelmann, ed., *Explanatory Supplement to the Astronomical Almanac* (University Science Books, Mill Valley, CA, 1992), pp. 704–706. For each body, “radius” is its radius at its equator and “orbit radius” is its average distance from the sun or (for the moon) from the earth.

[‡]In August 2006, the International Astronomical Union reclassified Pluto and other small objects that orbit the sun as “dwarf planets.”

Prefixes for Powers of 10

Power of ten	Prefix	Abbreviation	Pronunciation
10^{-24}	yocto-	y	yoc-toe
10^{-21}	zepto-	z	zep-toe
10^{-18}	atto-	a	at-toe
10^{-15}	femto-	f	fem-toe
10^{-12}	pico-	p	pee-koe
10^{-9}	nano-	n	nan-oe
10^{-6}	micro-	μ	my-crow
10^{-3}	milli-	m	mil-i
10^{-2}	centi-	c	cen-ti
10^3	kilo-	k	kil-oe
10^6	mega-	M	meg-a
10^9	giga-	G	jig-a or gig-a
10^{12}	tera-	T	ter-a
10^{15}	peta-	P	pet-a
10^{18}	exa-	E	ex-a
10^{21}	zetta-	Z	zet-a
10^{24}	yotta-	Y	yot-a

Examples:

1 femtometer = 1 fm = 10^{-15} m

1 picosecond = 1 ps = 10^{-12} s

1 nanocoulomb = 1 nC = 10^{-9} C

1 microkelvin = $1\mu\text{K}$ = 10^{-6} K

1 millivolt = 1 mV = 10^{-3} V

1 kilopascal = 1 kPa = 10^3 Pa

1 megawatt = 1 MW = 10^6 W

1 gigahertz = 1 GHz = 10^9 Hz

ANSWERS TO ODD-NUMBERED PROBLEMS

Chapter 1

- 1.1 a) 1.61 km b) 3.28×10^3 ft
 1.3 1.02 ns
 1.5 5.36 L
 1.7 31.7 y
 1.9 a) 23.4 km/L b) 1.4 tanks
 1.11 9.0 cm
 1.13 a) $1.1 \times 10^{-3}\%$ b) no
 1.15 0.45%
 1.17 a) no b) no c) no d) no e) no
 1.19 $\approx 10^6$
 1.21 $\approx 4 \times 10^8$
 1.23 $\approx \$70$ million
 1.25 $\approx 10^4$
 1.27 7.8 km, 38° north of east
 1.29 144 m, 41° south of west
 1.31 $A_x = 0, A_y = -8.00$ m, $B_x = 7.50$ m, $B_y = 13.0$ m, $C_x = -10.9$ m, $C_y = -5.07$ m, $D_x = -7.99$ m, $D_y = 6.02$ m
 1.33 a) -8.12 m b) 15.3 m
 1.35 a) 9.01 m, 33.8° b) 9.01 m, 33.7°
 c) 22.3 m, 250° d) 22.3 m, 70.3°
 1.37 3.39 km, 31.1° north of west
 1.39 a) 2.48 cm, 18.4° b) 4.09 cm, 83.7°
 c) 4.09 cm, 264°
 1.41 $\vec{A} = -(8.00 \text{ m})\hat{j}$,
 $\vec{B} = (7.50 \text{ m})\hat{i} + (13.0 \text{ m})\hat{j}$,
 $\vec{C} = (-10.9 \text{ m})\hat{i} + (-5.07 \text{ m})\hat{j}$,
 $\vec{D} = (-7.99 \text{ m})\hat{i} + (6.02 \text{ m})\hat{j}$
 1.43 a) $\vec{A} = (1.23 \text{ m})\hat{i} + (3.38 \text{ m})\hat{j}$,
 $\vec{B} = (-2.08 \text{ m})\hat{i} + (-1.20 \text{ m})\hat{j}$
 b) $\vec{C} = (12.0 \text{ m})\hat{i} + (14.9 \text{ m})\hat{j}$
 c) 19.2 m, 51.2°
 1.45 a) -104 m^2 b) -148 m^2 c) 40.6 m^2
 1.47 a) 165° b) 28° c) 90°
 1.49 a) $-63.9 \text{ m}^2\hat{k}$ b) $63.9 \text{ m}^2\hat{k}$
 1.51 a) -6.62 m^2 b) $5.55 \text{ m}^2\hat{k}$
 1.53 a) $A = 5.38, B = 4.36$
 b) $-5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}$ c) 8.83, yes
 1.55 a) 1.64×10^4 km b) $2.57r_E$
 1.57 a) 2200 g b) 2.1 m
 1.59 a) $(2.8 \pm 0.3) \text{ cm}^3$ b) 170 ± 20
 1.61 $\approx 6 \times 10^{27}$
 1.63 $\$9 \times 10^{14}, \3×10^6 per person
 1.65 196 N, 392 N, 57.7° east of north; 360 N, 720 N, 57.7° east of south
 1.67 b) $A_x = 3.03$ cm, $A_y = 8.10$ cm c) 8.65 cm, 69.5°
 1.69 144 m, 41° south of west
 1.71 954 N, 16.8° above the forward direction
 1.73 3.30 N
 1.75 a) 45.5 N b) 139°
 1.77 a) (87, 258) b) 136, 25° below straight left
 1.79 160 N, 13° below horizontal
 1.81 911 m, 8.9° west of south
 1.83 29.6 m, 18.6° east of south
 1.85 26.2 m, 34.2° east of south
 1.87 124°
 1.89 170 m^2
 1.91 a) 54.7° b) 35.3°
 1.93 28.0 m
 1.95 $C_x = 8.0, C_y = 6.1$
 1.97 b) 72.2
 1.99 38.5 yd, 24.6° to the right of downfield
 1.101 a) 76.2 ly b) 129°

Chapter 2

- 2.1 25.0 m
 2.3 1 hr 10 min
 2.5 a) 0.312 m/s b) 1.56 m/s
 2.7 a) 12.0 m/s
 b) (i) 0 (ii) 15.0 m/s (iii) 12.0 m/s
 c) 13.3 m/s
 2.9 a) 2.33 m/s, 2.33 m/s
 b) 2.33 m/s, 0.33 m/s

- 2.11 6.7 m/s, 6.7 m/s, 0, -40.0 m/s , -40.0 m/s , -40.0 m/s , 0
 2.13 a) no
 b) (i) 12.8 m/s^2 (ii) 3.50 m/s^2
 (iii) 0.718 m/s^2
 2.15 a) 2.00 cm/s, 50.0 cm, -0.125 cm/s^2
 b) 16.0 s c) 32.0 s
 d) 6.20 s, 1.23 cm/s; 25.8 s, -1.23 cm/s ; 36.4 s, -2.55 cm/s
 2.17 a) 0.500 m/s^2 b) 0, 1.00 m/s^2
 2.19 a) 5.0 m/s b) 1.43 m/s^2
 2.21 a) 675 m/s^2 b) 0.0667 s
 2.23 1.70 m
 2.25 38 cm
 2.27 a) $3.1 \times 10^6 \text{ m/s}^2 = 3.2g$
 b) 1.6 ms c) no
 2.29 a) (i) 5.59 m/s^2 (ii) 7.74 m/s^2
 b) (i) 179 m (ii) 1.28×10^4 m
 2.31 a) 0, 6.3 m/s 2 , -11.2 m/s^2
 b) 100 m, 230 m, 320 m
 2.33 a) 20.5 m/s^2 upward, 3.8 m/s^2 upward, 53.0 m/s^2 upward
 b) 722 km
 2.35 a) 2.94 m/s b) 0.600 s
 2.37 1.67 s
 2.39 a) 33.5 m b) 15.8 m/s
 2.41 a) $t = \sqrt{2d/g}$ b) 0.190 s
 2.43 a) 646 m b) 16.4 s, 112 m/s
 2.45 a) 249 m/s^2 b) 25.4
 c) 101 m d) no (if a is constant)
 2.47 0.0868 m/s^2
 2.49 a) 3.3 s b) 9H
 2.51 a) 467 m b) 110 m/s
 2.53 a) $v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$,
 $x = (0.25 \text{ m/s}^3)t^3 - (0.010 \text{ m/s}^4)t^4$
 b) 39.1 m/s
 2.55 a) 10.0 m
 b) (i) 8.33 m/s (ii) 9.09 m/s (iii) 9.52 m/s
 2.57 b) 0.627 s, 1.59 s
 c) negative at 0.627 s, positive at 1.59 s
 d) 1.11 s e) 2.45 m f) 2.00 s, 0
 2.59 250 km
 2.61 a) 197 m/s b) 169 m/s
 2.63 a) 82 km/h b) 31 km/h
 2.65 a) 3.5 m/s^2 b) 0 c) 1.5 m/s^2
 2.67 a) 92.0 m b) 92.0 m
 2.69 50.0 m
 2.71 4.6 m/s^2
 2.73 a) 6.17 s b) 24.8 m
 c) auto: 21.0 m/s, truck: 13.0 m/s
 2.75 a) 7.85 cm/s b) 5.00 cm/s
 2.77 a) 15.9 s b) 393 m c) 29.5 m/s
 2.79 a) -4.00 m/s b) 12.0 m/s
 2.81 a) 2.64H b) 2.64T
 2.83 a) no
 b) yes, 14.4 m/s, not physically attainable
 2.85 a) 6.69 m/s b) 4.49 m c) 1.42 s
 2.87 a) 7.7 m/s b) 0.78 s c) 0.59 s d) 1.3 m
 2.89 a) 380 m b) 184 m
 2.91 a) 20.5 m/s b) yes
 2.93 a) 945 m b) 393 m
 2.95 a) car A b) 2.27 s, 5.73 s
 c) 1.00 s, 4.33 s d) 2.67 s
 2.97 a) 9.55 s, 47.8 m
 b) 1.62 m/s d) 8.38 m/s
 e) no
 f) 3.69 m/s, 21.7 s, 80.0 m
 2.99 a) 8.18 m/s b) (i) 0.411 m (ii) 1.15 km
 c) 9.80 m/s d) 4.90 m/s

Chapter 3

- 3.1 a) 1.4 m/s, -1.3 m/s
 b) 1.9 m/s, 317°
 3.3 a) 7.1 cm/s, 45°
 b) 5.0 cm/s, 90° ; 7.1 cm/s, 45° ;
 11 cm/s, 27°

- 3.5 b) $-8.67 \text{ m/s}^2, -2.33 \text{ m/s}^2$
 c) $8.98 \text{ m/s}^2, 195^\circ$
 3.7 b) $\vec{v} = \alpha\hat{i} - 2\beta\hat{j}$, $\vec{a} = -2\beta\hat{j}$
 c) 5.4 m/s, 297° ; 2.4 m/s 2 , 270°
 d) speeding up and turning right
 3.9 a) 0.600 m b) 0.385 m
 c) $v_x = 1.10 \text{ m/s}, v_y = -3.43 \text{ m/s}$, 3.60 m/s, 72.2° below the horizontal
 3.11 3.32 m
 3.13 a) 30.6 m/s b) 36.3 m/s
 3.15 1.28 m/s 2
 3.17 a) 0.683 s, 2.99 s
 b) 24.0 m/s, 11.3 m/s; 24.0 m/s, -11.3 m/s
 c) 30.0 m/s, 36.9° below the horizontal
 3.19 a) 1.5 m b) -0.89 m/s
 3.21 a) 13.6 m b) 34.6 m/s c) 103 m
 3.23 a) 296 m b) 176 m c) 198 m
 d) (i) $v_x = 15.0 \text{ m/s}, v_y = -58.8 \text{ m/s}$
 (ii) $v_x = 15.0 \text{ m/s}, v_y = -78.8 \text{ m/s}$
 3.25 a) $0.034 \text{ m/s}^2 = 0.0034g$ b) 1.4 h
 3.27 140 m/s = 310 mph
 3.29 a) 3.50 m/s^2 upward
 b) 3.50 m/s^2 downward
 c) 12.6 s
 3.31 a) 14 s b) 70 s
 3.33 0.36 m/s, 52.5° south of west
 3.35 a) 4.7 m/s, 25° south of east
 b) 190 s c) 380 m
 3.37 b) -7.1 m/s , -42 m/s
 c) 43 m/s, 9.6° west of south
 3.39 a) 24° west of south b) 5.5 h
 3.41 a) $A = 0, B = 2.00 \text{ m/s}^2, C = 50.0 \text{ m}, D = 0.500 \text{ m/s}^3$
 b) $\vec{v} = 0, \vec{a} = (4.00 \text{ m/s}^2)\hat{i}$
 c) $v_x = 40.0 \text{ m/s}, v_y = 150 \text{ m/s}$, 155 m/s
 d) $\vec{r} = (200 \text{ m})\hat{i} + (550 \text{ m})\hat{j}$
 3.43 $2b/3c$
 3.45 4.41 s
 3.47 a) 123 m
 b) 280 m
 3.49 22 m/s
 3.51 31 m/s
 3.53 274 m
 3.55 795 m
 3.57 33.7 m
 3.59 a) 42.8 m/s b) 42.0 m
 3.61 a) $\sqrt{2gh}$ b) 30.0° c) 6.93h
 3.63 a) 1.50 m/s
 b) 4.66 m
 3.65 a) 6.91 m c) no
 3.67 a) 17.8 m/s
 b) in the river, 28.4 m horizontally from his launch point
 3.69 a) 81.6 m b) in the cart
 c) 245 m d) 53.1°
 3.71 a) 49.5 m/s b) 50 m
 3.73 a) 2000 m b) 2180 m
 3.75 $\pm 25.4^\circ$
 3.77 61.2 km/h, 140 km/h
 3.79 b) $v_x = R\omega(1 - \cos\omega t)$,
 $v_y = R\omega\sin\omega t$; $a_x = R\omega^2\sin\omega t$,
 $a_y = R\omega^2\cos\omega t$ c) $t = 0, 2\pi/\omega, 4\pi/\omega, \dots$;
 $x = 0, 2\pi R, 4\pi R, \dots$;
 $y = 0$; $a = R\omega^2$ in the +y-direction d) no
 3.81 a) 44.7 km/h, 26.6° west of south
 b) 10.5° north of west
 3.83 7.39 m/s, 12.4° north of east
 3.85 a) 0.659 s b) (i) 9.09 m/s (ii) 6.46 m/s
 c) 3.00 m, 2.13 m
 3.87 a) $49.3^\circ, 17.5^\circ$ for level ground b) -17.0°
 3.89 a) 1.5 km/h b) 3.5 km/h

Chapter 4

- 4.1 a) 0° b) 90° c) 180°
 4.3 3.15 N
 4.5 494 N, 31.8°

A-10 Answers to Odd-Numbered Problems

- 4.7 46.7 N, opposite to the motion of the skater
 4.9 16.0 kg
 4.11 a) 3.12 m, 3.12 m/s
 b) 21.9 m, 6.24 m/s
 4.13 a) 45.0 N, between 2.0 s and 4.0 s
 b) between 2.0 s and 4.0 s
 c) 0 s, 6.0 s
 4.15 a) $A = 100 \text{ N}$, $B = 12.5 \text{ N/s}^2$
 b) (i) 21.6 N, 2.70 m/s² (ii) 134 N, 16.8 m/s²
 c) 26.6 m/s²
 4.17 2940 N
 4.19 a) 4.49 kg b) 4.49 kg, 8.13 N
 4.21 825 N, blocks
 4.23 20 N
 4.25 $7.4 \times 10^{-23} \text{ m/s}^2$
 4.27 b) yes
 4.29 a) yes b) no
 4.31 b) 142 N
 4.33 2.03 s
 4.35 1840 N, 135°
 4.37 a) 17 N, 90° clockwise from the +x-axis
 b) 840 N
 4.39 a) 4.85 m/s
 b) 16.2 m/s² upward
 c) 1470 N upward (on him), 2360 N downward (on ground)
 4.41 a) 153 N
 4.43 a) 2.50 m/s² b) 10.0 N
 c) to the right, $F > T$
 d) 25.0 N
 4.45 a) 4.4 m b) 300 m/s
 c) (i) $2.7 \times 10^3 \text{ N}$ (ii) $9.0 \times 10^3 \text{ N}$
 4.47 a) $T > mg$
 b) 79.6 N
 4.49 b) 0.049 N c) 410mg
 4.51 a) 7.79 m/s
 b) 50.6 m/s² upward
 c) 4530 N upward, 6.16mg
 4.53 a) w b) 0 c) w/2
 4.55 b) 1395 N
 4.57 a) 4.34 kg
 b) 5.30 kg
 4.59 $F_x(t) = -6mBt$
 4.61 7.78 m

Chapter 5

- 5.1 a) 25.0 N b) 50.0 N
 5.3 a) 990 N, 735 N
 b) 926 N
 5.5 48°
 5.7 a) $T_A = 0.732w$, $T_B = 0.897w$, $T_C = w$
 b) $T_A = 2.73w$, $T_B = 3.35w$, $T_C = w$
 5.9 a) 337 N b) 343 N
 5.11 a) $1.10 \times 10^8 \text{ N}$ b) 5w
 c) 8.4 s
 5.13 a) 4610 m/s² = 470g
 b) $9.70 \times 10^5 \text{ N} = 471w$
 c) 0.0187 s
 5.15 b) 2.96 m/s² c) 191 N
 5.17 b) 2.50 m/s² c) 1.37 kg
 d) 0.75mg
 5.19 a) 0.832 m/s² b) 17.3 s
 5.21 a) 3.4 m/s (c) 2.2w
 5.23 a) 14.0 m
 b) 18.0 m/s
 5.25 50°
 5.27 a) 22 N b) 3.1 m
 5.29 a) 0.710, 0.472 b) 258 N
 c) (i) 51.8 N (ii) 4.97 m/s²
 5.31 a) 57.1 N
 b) 146 N up the ramp
 5.33 a) 54.0 m b) 16.3 m/s
 5.35 a) $\mu_k(m_A + m_B)g$ b) $\mu_k m_A g$
 5.37 a) 0.218 m/s
 b) 11.7 N
 5.39 a) $\frac{\mu_k mg}{\cos\theta - \mu_k \sin\theta}$
 b) $1/\tan\theta$
 5.41 a) 0.44 kg/m b) 42 m/s
 5.43 a) 3.61 m/s b) bottom c) 3.33 m/s
 5.45 a) 21.0°, no b) 11,800 N; 23,600 N
 5.47 1410 N, 8370 N

- 5.49 a) 1.5 rev/min
 b) 0.92 rev/min
 5.51 a) 38.3 m/s = 138 km/h
 b) 3580 N
 5.53 2.42 m/s
 5.55 a) 1.73 m/s²
 c) 0.0115 N upward
 d) 0.0098 N
 5.57 a) rope making 60° angle b) 6400 N
 5.59 a) 470 N b) 163 N
 5.61 762 N
 5.63 a) (i) -3.80 m/s (ii) 24.6 m/s
 b) 4.36 m c) 2.45 s
 5.65 a) 11.4 N b) 2.57 kg
 5.67 10.4 kg
 5.69 0.0259 (low pressure), 0.00505 (high pressure)
 5.71 a) $m_1(\sin\alpha + \mu_k \cos\alpha)$
 b) $m_1(\sin\alpha - \mu_k \cos\alpha)$
 c) $m_1(\sin\alpha - \mu_k \cos\alpha) \leq m_2 \leq m_1(\sin\alpha + \mu_k \cos\alpha)$
 5.73 a) 1.80 N b) 2.52 N
 5.75 a) $1.3 \times 10^{-4} \text{ N} = 62mg$
 b) $2.9 \times 10^{-4} \text{ N}$, at $t = 1.2 \text{ ms}$
 c) 1.2 m/s
 5.77 920 N
 5.79 a) 11.5 m/s b) 7.54 m/s
 5.81 0.40
 5.83 a) $g \frac{m_B + m_{\text{rope}}(d/L)}{m_A + m_B + m_{\text{rope}}}$, increase
 b) 0.63 m
 c) will not move for any value of d
 5.85 a) 88.0 N northward
 b) 78 N southward
 5.87 a) 294 N, 152 N, 152 N b) 40.0 N
 5.89 3.0 N
 5.91 a) 12.9 kg
 b) $T_{AB} = 47.2 \text{ N}$, $T_{BC} = 101 \text{ N}$
 5.93 $a_1 = \frac{2m_2g}{4m_1 + m_2}$, $a_2 = \frac{m_2g}{4m_1 + m_2}$
 5.95 1.46 m above the floor
 5.97 g/μ_s
 5.99 b) 0.452
 5.101 0.34
 5.103 b) 8.8 N (c) 31.0 N
 d) 1.54 m/s²
 5.105 a) moves up
 b) remains constant
 c) remains constant
 d) slows down at the same rate as the monkey
 5.107 a) 6.00 m/s²
 b) 3.80 m/s²
 c) 7.36 m/s
 d) 8.18 m/s
 e) 7.78 m, 6.29 m/s, 1.38 m/s²
 f) 3.14 s
 5.109 a) 0.015, 0.36 N · s²/m²
 b) 29 m/s
 c) $v/v_i = \sqrt{\sin\beta - (0.015)\cos\beta}$
 5.111 a) $v_x(t) = v_0 e^{-kt/m} + v_i(1 - e^{-kt/m})$
 5.113 a) 120 N
 b) 3.79 m/s
 5.115 b) 0.28 c) no
 5.117 a) right b) 120 m
 5.119 a) 81.1° b) no
 c) The bead rides at the bottom of the hoop.
 5.121 a) $F = \frac{\mu_k w}{\cos\theta + \mu_k \sin\theta}$
 b) $\theta = \tan^{-1}(\mu_k)$, 14.0°
 5.123 $F = (M + m)g \tan\alpha$
 5.125 a) $g \frac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$
 b) $a_B = -a_3$
 c) $g \frac{4m_1m_2 - 3m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$
 d) $g \frac{4m_1m_2 - 3m_1m_3 + m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$
 e) $g \frac{4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$

- f) $g \frac{8m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$
 g) All the accelerations are zero, $T_A = m_2g$,
 $T_C = 2m_2g$.
 5.127 $\cos^2\beta$

Chapter 6

- 6.1 a) 3.60 J b) -0.900 J
 c) 0 d) 0
 e) 2.70 J
 6.3 a) 74 N b) 333 J
 c) -330 J d) 0, 0
 e) 0
 6.5 a) -1750 J b) no
 6.7 a) (i) 9.00 J (ii) -9.00 J
 b) (i) 0 (ii) 9.00 J (iii) -9.00 J (iv) 0
 c) zero for each block
 6.9 a) (i) 0 (ii) 0
 b) (i) 0 (ii) -25.1 J
 6.11 a) 324 J b) -324 J
 c) 0
 6.13 a) 36,000 J b) 4
 6.15 a) $1.0 \times 10^{16} \text{ J}$ b) 2.4
 6.17 a) 7.50 N b) (i) 9.00 J (ii) 5.40 J
 c) 14.4 J, same d) 2.97 m/s
 6.19 a) 43.2 m/s b) 101 m/s
 c) 5.80 m d) 3.53 m/s
 e) 7.35 m
 6.21 $\sqrt{2gh(1 + \mu_k \tan\alpha)}$
 6.23 32.0 N
 6.25 a) 4.48 m/s b) 3.61 m/s
 6.27 a) 4.96 m/s b) 1.43 m/s², 4.96 m/s
 6.29 a) $\frac{v_0^2}{2\mu_k g}$ b) (i) 1/2 (ii) 4 (iii) 2
 6.31 a) 40.0 N/m b) 0.456 N
 6.33 b) 14.4 cm, 13.6 cm, 12.8 cm
 6.35 a) 2.83 m/s b) 3.46 m/s
 6.37 8.5 cm
 6.39 a) 1.76 b) 0.666 m/s
 6.41 a) 4.0 J b) 0
 c) -1.0 J d) 3.0 J
 e) -1.0 J
 6.43 a) 2.83 m/s b) 2.40 m/s
 6.45 a) 0.0565 m b) 0.57 J, no
 6.47 8.17 m/s
 6.49 a) 360,000 J b) 100 m/s
 6.51 $3.9 \times 10^{13} \text{ P}$
 6.53 745 W \approx 1 hp
 6.55 a) 84.6/min b) 22.7/min
 6.57 29.6 kW
 6.59 0.20 W
 6.61 877 J
 6.63 a) 532 J b) -315 J
 c) 0 d) -202 J
 e) 15 J f) 1.2 m/s
 6.65 a) 987 J b) 3.02 s
 6.67 a) $2.59 \times 10^{12} \text{ J}$ b) 4800 J
 6.69 a) 1.8 m/s = 4.0 mph
 b) 180 m/s² \approx 18g, 900 N
 6.71 a) $k\left(\frac{1}{x_2} - \frac{1}{x_1}\right)$, negative
 b) $k\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$, positive
 c) same magnitude but opposite signs because the net work is zero
 6.73 a) 5.11 m b) 0.304
 c) 10.3 m
 6.75 a) 0.11 N b) 7.1 N c) 0.33 J
 6.77 a) 2.56 m/s b) 3.52 N c) 13.1 J
 6.79 $6.3 \times 10^4 \text{ N/m}$
 6.81 1.1 m
 6.83 a) $1.02 \times 10^4 \text{ N/m}$, 8.16 m
 6.85 a) 0.600 m b) 1.50 m/s
 6.87 0.786
 6.89 1.3 m
 6.91 a) $1.10 \times 10^5 \text{ J}$ b) $1.30 \times 10^5 \text{ J}$
 c) 3.99 kW
 6.93 3.6 h
 6.95 a) $1.26 \times 10^5 \text{ J}$ b) 1.46 W
 6.97 a) 2.4 MW b) 61 MW c) 6.0 MW
 6.99 a) 513 W b) 354 W c) 52.1 W

- 6.101 a) 358 N b) 47.1 hp c) 4.06 hp
d) 2.03%
6.103 a) $\frac{Mu^2}{6}$ b) 6.1 m/s
c) 3.9 m/s d) 0.40 J, 0.60 J

Chapter 7

- 7.1 a) 6.6×10^5 J b) -7.7×10^5 J
7.3 a) 820 N b) (i) 0 (ii) 740 J
7.5 a) 24.0 m/s b) 24.0 m/s c) part (b)
7.7 a) 2.0 m/s b) 9.8×10^{-7} J, 2.0 J/kg
c) 200 m, 63 m/s d) 5.9 J/kg
7.9 a) (i) 0 (ii) 0.98 J b) 2.8 m/s
c) only gravity is constant d) 5.1 N
7.11 -5400 J
7.13 a) 880 J b) -157 J c) 470 J d) 253 J
e) 3.16 m/s², 7.11 m/s, 253 J
7.15 a) 80.0 J b) 5.0 J
7.17 a) (i) $4U_0$ (ii) $U_0/4$
b) (i) $x_0\sqrt{2}$ (ii) $x_0/\sqrt{2}$
7.19 a) 6.32 cm b) 12 cm
7.21 ± 0.092 m
7.23 a) 3.03 m/s, as it leaves the spring
b) 95.9 m/s², when the spring has its maximum compression
7.25 a) 4.46×10^3 N/m b) 0.128 m
7.27 a) -308 J b) -616 J
c) nonconservative
7.29 a) -3.6 J b) -3.6 J
c) -7.2 J d) nonconservative
7.31 a) -59 J b) -42 J
c) -59 J d) nonconservative
7.33 a) 8.41 m/s b) 638 J
7.35 2.46 N, +x-direction
7.37 130 m/s², 132° counterclockwise from the +x-axis
7.39 a) $F(r) = (12a/r^{13}) - (6b/r^7)$
b) $(2a/b)^{1/6}$, stable c) $b^2/4a$
d) $a = 6.67 \times 10^{-138}$ J \cdot m¹²,
 $b = 6.41 \times 10^{-78}$ J \cdot m⁶
7.41 a) zero, 637 N b) 2.99 m/s
7.43 0.41
7.45 a) 16.0 m/s b) 11,500 N
7.47 a) 20.0 m along the rough bottom
b) -78.4 J
7.49 a) 22.2 m/s b) 16.4 m c) no
7.51 0.602 m
7.53 15.5 m/s
7.55 4.4 m/s
7.57 a) no b) yes, \$150
7.59 a) 7.00 m/s b) 8.82 N
7.61 a) $mg(1-h/d)$ b) 441 N
c) $\sqrt{2gh(1-y/d)}$
7.63 48.2°
7.65 a) 0.392 b) -0.83 J
7.67 a) $U(x) = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3$ b) 7.85 m/s
7.69 7.01 m/s
7.71 a) $\frac{m(g+a)^2}{2gh}$ b) $\frac{2gh}{g+a}$
7.73 a) 0.480 m/s b) 0.566 m/s
7.75 a) 3.87 m/s b) 0.10 m
7.77 0.456 N
7.79 a) 4.4×10^{12} J
b) 2.7×10^3 m³, 9.0×10^{-4} m
7.81 119 J
7.83 a) -50.6 J b) -67.5 J c) nonconservative
7.85 b) 0, 3.38 J, 0, 0; 3.38 J c) nonconservative
7.87 b) $v(x) = \sqrt{\frac{2\alpha}{mx_0^2} \left[\frac{x_0}{x} - \left(\frac{x_0}{x} \right)^2 \right]}$
c) $x = 2x_0$, $v = \sqrt{\frac{\alpha}{2mx_0^2}}$ d) 0
e) $v(x) = \sqrt{\frac{2\alpha}{mx_0^2} \left[\frac{x_0}{x} - \left(\frac{x_0}{x} \right)^2 - \frac{2}{9} \right]}$
f) first case: x_0, ∞ ; second case: $3x_0/2, 3x_0$

Chapter 8

- 8.1 a) 1.20×10^5 kg \cdot m/s
b) (i) 60.0 m/s (ii) 26.8 m/s
8.3 b) 0.526, baseball c) 0.641, woman
8.5 a) 22.5 kg \cdot m/s, to the left
b) 838 J
8.7 562 N, not significant
8.9 a) 10.8 m/s, to the right
b) 0.750 m/s, to the left
8.11 a) 500 N/s² b) 5810 N \cdot s
c) 2.70 m/s
8.13 a) 2.50 N \cdot s, in the direction of the force
b) (i) 6.25 m/s, to the right (ii) 3.75 m/s, to the right
8.15 0.593 kg \cdot m/s
8.17 0.87 kg \cdot m/s, in the same direction as the bullet is traveling
8.19 a) 6.79 m/s b) 55.2 J
8.21 a) 0.790 m/s b) -0.0023 J
8.23 1.53 m/s for both
8.25 a) 0.0559 m/s b) 0.0313 m/s
8.27 a) 7.20 m/s, 38.0° from Rebecca's original direction b) -680 J
8.29 a) 3.56 m/s
8.31 a) 29.3 m/s, 20.7 m/s b) 19.6%
8.33 a) 0.846 m/s b) 2.10 J
8.35 a) -1.4×10^{-6} km/h, no
b) -6.7×10^{-8} km/h, no
8.37 5.9 m/s, 58° north of east
8.39 a) Both cars have the same magnitude momentum change, but the lighter car has a greater velocity change.
b) $2.50\Delta v$ c) occupants of small car
8.41 19.5 m/s, 21.9 m/s
8.43 a) 2.93 cm b) 866 J c) 1.73 J
8.45 186 N
8.47 a) 3.33 J, 0.333 m/s b) 1.33 m/s, 0.667 m/s
8.49 a) $v_i/3$ b) $K_i/9$ c) 10
8.51 (0.0444 m, 0.0556 m)
8.53 2520 km
8.55 0.700 m to the right and 0.700 m upward
8.57 0.47 m/s
8.59 $F_x = -(1.50 \text{ N/s})t$, $F_y = 0.25$ N, $F_z = 0$
8.61 a) 0.053 kg b) 5.19 N
8.63 a) 0.442 b) 800 m/s c) 530 m/s
8.65 45.2
8.67 a) 0.474 kg \cdot m/s, upward
b) 237 N, upward
8.69 a) -1.14 N \cdot s, 0.330 N \cdot s
b) 0.04 m/s, 1.8 m/s
8.71 2.40 m/s, 3.12 m/s
8.73 a) 1.75 m/s, 0.260 m/s b) -0.092 J
8.75 3.65×10^3 m/s
8.77 0.946 m
8.79 1.8 m
8.81 12 m/s, 21 m/s
8.83 a) 2.60 m/s b) 325 m/s
8.85 a) 5.3 m/s b) 5.7 m
8.87 53.7°
8.89 102 N
8.91 a) 0.125 b) 248 J c) 0.441 J
8.93 b) $M = m$ c) zero
8.95 a) 9.35 m/s b) 3.29 m/s
8.97 a) 3.56 m/s b) 5.22 m/s c) 4.66 m/s
8.99 13.6 m/s, 6.34 m/s, 65.0°
8.101 0.0544%
8.103 1.61×10^{-22} kg \cdot m/s, to the left
8.105 1.33 m
8.107 0.400 m/s
8.109 a) 71.6 m/s, 14.3 m/s b) 347 m
8.111 a) yes b) decreases by 4800 J
8.113 a) $1.37v_{ex}$ b) $1.18v_{ex}$
c) $2.38v_{ex}$ d) 2.94 km/s
8.115 b) $2L/3$

Chapter 9

- 9.1 a) 34.4° b) 6.27 cm c) 1.05 m
9.3 a) rad/s, rad/s³ b) (i) 0 (ii) 15.0 rad/s²
c) 9.50 rad
9.5 $\omega_z = \gamma + 3\beta t^2$ b) 0.400 rad/s
c) 1.30 rad/s, 0.700 rad/s

- 9.7 a) $\pi/4$ rad, 2.00 rad/s, -0.139 rad/s³ b) 0
c) 19.5 rad, 9.36 rad/s
9.9 a) 2.25 rad/s b) 4.69 rad
9.11 a) 24.0 s b) 68.8 rev
9.13 10.5 rad/s
9.15 a) 300 rpm b) 75.0 s, 312 rev
9.17 9.00 rev
9.19 a) 1.99×10^{-7} rad/s b) 7.27×10^{-5} rad/s
c) 2.98×10^4 m/s d) 464 m/s
e) 0.0337 m/s², 0
9.21 a) 15.1 m/s² b) 15.1 m/s²
9.23 a) 0.180 m/s², 0, 0.180 m/s²
b) 0.180 m/s², 0.377 m/s², 0.418 m/s²
c) 0.180 m/s², 0.754 m/s², 0.775 m/s²
9.25 0.107 m, no
9.27 a) 0.831 m/s b) 109 m/s²
9.29 a) 2.29 b) 1.51 c) 15.7 m/s, 108g
9.31 a) (i) 0.469 kg \cdot m² (ii) 0.117 kg \cdot m²
(iii) 0
b) (i) 0.0433 kg \cdot m² (ii) 0.0722 kg \cdot m²
c) (i) 0.0288 kg \cdot m² (ii) 0.0144 kg \cdot m²
9.33 a) 2.33 kg \cdot m² b) 7.33 kg \cdot m²
c) 0 d) 1.25 kg \cdot m²
9.35 0.193 kg \cdot m²
9.37 8.52 kg \cdot m²
9.39 5.61 m/s
9.41 a) 3.15×10^{23} J b) 158 y, no
9.43 0.600 kg \cdot m²
9.45 7.35×10^4 J
9.47 a) 0.673 m b) 45.5%
9.49 46.5 kg
9.51 a) f^2 b) 6.37×10^8 J
9.53 an axis that is parallel to a diameter and is 0.516R from the center
9.55 $\frac{1}{3}M(a^2 + b^2)$
9.57 a) $ML^2/12$ b) $ML^2/12$
9.59 $\frac{1}{2}MR^2$
9.61 a) 14.2 rad/s b) 59.6 rad
9.63 9.41 m
9.65 a) 0.600 m/s³ b) $\alpha = (2.40 \text{ rad/s}^3)t$
c) 3.54 s d) 17.7 rad
9.67 a) 0.0333 rad/s² b) 0.200 rad/s
c) 2.40 m/s² e) 3.12 m/s², 3.87 kN
f) 50.2°
9.69 a) 1.70 m/s b) 94.2 rad/s
9.71 2.99 cm
9.73 b) 1.50 m/s² d) 0.208 kg \cdot m²
9.75 a) 7.36 m b) 327 m/s²
9.77 a) 2.14×10^{29} J b) 2.66×10^{33} J
9.79 a) $Mb^2/6$ b) 182 J
9.81 a) -0.882 J
b) 5.42 rad/s
c) 5.42 m/s
d) 5.42 m/s compared to 4.43 m/s
9.83 $\sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$
9.85 $\sqrt{g(1 - \cos \beta)}/R$
9.87 a) 2.25×10^{-3} kg \cdot m²
b) 3.40 m/s c) 4.95 m/s
9.89 13.9 m
9.91 a) $\frac{247}{512}MR^2$ b) $\frac{383}{512}MR^2$
9.93 a) 1.05 rad/s b) 5.0 J c) 78.5 J d) 6.4%
9.95 $\frac{1}{4}M(R_1^2 + R_2^2)$
9.97 a) $\frac{3}{5}MR^2$ b) larger
9.99 a) 55.3 kg b) 0.804 kg \cdot m²
9.101 a) $s(\theta) = r_0\theta + \frac{\beta}{2}\theta^2$
b) $\theta(t) = \frac{1}{\beta}(\sqrt{r_0^2 + 2\beta vt} - r_0)$
c) $\omega_z(t) = \frac{v}{\sqrt{r_0^2 + 2\beta vt}}$
 $\alpha_z(t) = -\frac{\beta v^2}{(r_0^2 + 2\beta vt)^{3/2}}$, no
d) 25.0 mm, 0.247 μ m/rad, 2.13×10^4 rev

Chapter 10

- 10.1 a) 40.0 N·m, out of the page
 b) 34.6 N·m, out of the page
 c) 20.0 N·m, out of the page
 d) 17.3 N·m, into the page
 e) 0 f) 0
- 10.3 2.50 N·m, out of the page
- 10.5 b) $-\hat{k}$ c) $(-1.05 \text{ N}\cdot\text{m})\hat{k}$
- 10.7 a) 8.7 N·m counterclockwise, 0, 5.0 N·m clockwise, 10.0 N·m clockwise
 b) 6.3 N·m clockwise
- 10.9 13.1 N·m
- 10.11 a) 14.8 rad/s² b) 1.52 s
- 10.13 a) 7.5 N, 18.2 N b) 0.016 kg·m²
- 10.15 0.255 kg·m²
- 10.17 a) 32.6 N, 35.4 N b) 2.72 m/s²
 c) 32.6 N, 55.0 N
- 10.19 a) 1.80 m/s b) 7.13 J
 c) (i) 3.60 m/s to the right (ii) 0
 (iii) 2.55 m/s at 45° below the horizontal
 d) (i) 1.80 m/s to the right (ii) 1.80 m/s to the left (iii) 1.80 m/s downward
- 10.21 a) 1/3 b) 2/7 c) 2/5 d) 5/13
- 10.23 a) 0.613 b) no c) no slipping
- 10.25 11.7 m
- 10.27 a) 3.76 m b) 8.58 m/s
- 10.29 a) 67.9 rad/s b) 8.35 J
- 10.31 a) 0.309 rad/s b) 100 J c) 6.67 W
- 10.33 a) 0.377 N·m b) 157 rad
 c) 59.2 J d) 59.2 J
- 10.35 a) 358 N·m b) 1790 N c) 83.8 m/s
- 10.37 a) 115 kg·m²/s into the page
 b) 125 kg·m²/s² out of the page
- 10.39 $4.71 \times 10^{-6} \text{ kg}\cdot\text{m}^2/\text{s}$
- 10.41 4600 rad/s
- 10.43 1.14 rev/s
- 10.45 a) 1.38 rad/s b) 1080 J, 495 J
- 10.47 a) 0.120 rad/s b) $3.20 \times 10^{-4} \text{ J}$
 c) work done by bug
- 10.49 a) 5.88 rad/s
- 10.51 a) 1.71 rad/s
- 10.53 a) 1.62 N b) 1800 rev/min
- 10.55 a) halved b) doubled
 c) halved d) doubled e) unchanged
- 10.57 a) 67.6 N b) 62.9 N c) 3.27 s
- 10.59 0.483
- 10.61 7.47 N
- 10.63 a) 16.3 rad/s² b) decreases c) 5.70 rad/s
- 10.65 a) FR b) FR c) $\sqrt{4F/MR}$
 d) $2F/M$ e) $4F/M$
- 10.67 0.730 m/s², 6.08 rad/s², 36.3 N, 21.1 N
- 10.69 a) 293 N b) 16.2 rad/s²
- 10.71 a) 2.88 m/s² b) 6.13 m/s²
- 10.73 270 N

$$10.75 \quad a = \frac{2g}{2 + (R/b)^2}, \quad \alpha = \frac{2g}{2b + R^2/b}$$

$$T = \frac{2mg}{2(b/R)^2 + 1}$$

- 10.77 a) 1.41 s, 70.5 m/s b) t larger, v smaller
- 10.79 $\frac{3}{5}H_0$
- 10.81 29.0 m/s
- 10.83 a) 26.0 m/s b) unchanged
- 10.85 a) $\sqrt{20hy/7}$ b) no
 c) rolling friction d) $\sqrt{8hy/3}$
- 10.87 $g/3$
- 10.89 1.87 m
- 10.91 a) $\frac{6}{19}v/L$ b) 3/19
- 10.93 a) 5.46 rad/s b) 3.17 cm c) 1010 m/s
- 10.95 a) 2.00 rad/s b) 6.58 rad/s
- 10.97 0.30 rad/s clockwise
- 10.99 0.710 m

10.101 a) $a = \mu_k g, \alpha = \frac{2\mu_k g}{R}$
 b) $\frac{R^2\omega_0^2}{18\mu_k g}$ c) $-\frac{MR^2\omega_0^2}{6}$

10.103 a) $mv_1^2 r_1^2 / r^3$
 b) $\frac{mv_1^2}{2} r_1^2 \left(\frac{1}{r_2^2} - \frac{1}{r_1^2} \right)$
 c) same

Chapter 11

- 11.1 29.8 cm
- 11.3 1.35 m
- 11.5 5.45 kN
- 11.7 a) 1000 N, 0.800 m from the end where the 600-N force is applied
 b) 800 N, 0.75 m from the end where the 600-N force is applied
- 11.9 a) 550 N
 b) 0.614 m from A
- 11.11 a) 1920 N b) 1140 N
- 11.13 a) $T = 2.60w; 3.28w, 37.6^\circ$
 b) $T = 4.10w; 5.39w, 48.8^\circ$
- 11.15 272 N on each hand, 130 N on each foot
- 11.17 246 N, 0.34 m from the front feet
- 11.19 270 N, 303 N, 40°
- 11.21 a) 0.800 m b) clockwise
 c) 0.800 m, clockwise
- 11.23 b) 208 N
- 11.25 1.4 mm
- 11.27 $2.0 \times 10^{11} \text{ Pa}$
- 11.29 a) 3.1×10^{-3} (upper), 2.0×10^{-3} (lower)
 b) 1.6 mm (upper), 1.0 mm (lower)
- 11.31 a) 150 atm b) 1.5 km, no
- 11.33 8.6°
- 11.35 $4.8 \times 10^9 \text{ Pa}, 2.1 \times 10^{-10} \text{ Pa}^{-1}$
- 11.37 b) $6.6 \times 10^5 \text{ N}$ c) 1.8 mm
- 11.39 $3.41 \times 10^7 \text{ Pa}$
- 11.41 10.2 m/s²
- 11.43 20.0 kg
- 11.45 a) 525 N b) 222 N, 328 N c) 1.48
- 11.47 tail: 600 N down, wing: 7300 N up
- 11.49 a) 140 N b) 6 cm to the right
- 11.51 a) 379 N b) 141 N
- 11.53 160 N to the right, 213 N upward
- 11.55 49.9 cm
- 11.57 a) 370 N b) when he starts to raise his leg c) no
- 11.59 a) $V = mg + w, H = T = \left(w + \frac{mg}{4} \right) \cot \theta$
 b) 926 N c) 6.00°
- 11.61 4900 N
- 11.63 b) 2000 N = 2.72mg c) 4.4 mm
- 11.65 a) 4.90 m b) 60 N
- 11.67 a) 175 N at each hand, 200 N at each foot
 b) 91 N at each hand and at each foot
- 11.69 a) 1150 N b) 1940 N
 c) 918 N d) 0.473
- 11.71 590 N (person above), 1370 N (person below); person above
- 11.73 a) $\frac{T_{\max} h D}{L\sqrt{h^2 + D^2}}$
 b) positive, $\frac{T_{\max} h}{L\sqrt{h^2 + D^2}} \left(1 - \frac{D^2}{h^2 + D^2} \right)$
- 11.75 a) 7140 N, tall walls b) 7900 N
- 11.77 a) 268 N b) 232 N
 c) 366 N
- 11.79 a) 0.424 N (A), 1.47 N (B), 0.424 N (C)
 b) 0.848 N
- 11.81 a) 27° to tip, 31° to slip, tips first
 b) 27° to tip, 22° to slip, slips first
- 11.83 a) 80 N (A), 870 N (B) b) 1.92 m
- 11.85 a) $T = 3700 \text{ N}, 2000 \text{ N}$ upward
- 11.87 a) 0.36 mm b) 0.045 mm
 c) 0.33 mm
- 11.89 a) 0.54 cm b) 0.42 cm
- 11.91 a) 0.70 m from A b) 0.60 m from A
- 11.93 a) 1.63 m
 b) brass: $2.00 \times 10^8 \text{ Pa}$, nickel: $4.00 \times 10^8 \text{ Pa}$
 c) brass: 2.22×10^{-3} , nickel: 1.90×10^{-3}
- 11.95 0.0542 L
- 11.97 a) 600 N b) 13.5 kN

c) $F = \frac{\mu_s w}{\sin \theta - \mu_s \cos \theta}$ (to slide),
 $F = \frac{w}{(1/9) \cos \theta + 2 \sin \theta}$ (to tip), 66°

- 11.99 $h^2/L + L/2; L$ if $h > L/\sqrt{2}$
- 11.101 a) 0.66 mm b) 0.022 J c) $8.35 \times 10^{-3} \text{ J}$
 d) $-3.04 \times 10^{-2} \text{ J}$ e) $3.04 \times 10^{-2} \text{ J}$

Chapter 12

- 12.1 41.8 N, no
- 12.3 7020 kg/m³, yes
- 12.5 1.6
- 12.7 61.6 N
- 12.9 a) $1.86 \times 10^6 \text{ Pa}$ b) 184 m
- 12.11 0.581 m
- 12.13 a) $1.90 \times 10^4 \text{ Pa}$
 b) causes additional force on the walls of the blood vessels
- 12.15 2.8 m
- 12.17 $6.0 \times 10^4 \text{ Pa}$
- 12.19 $2.27 \times 10^5 \text{ N}$
- 12.21 a) 636 Pa b) (i) 1170 Pa (ii) 1170 Pa
- 12.23 10.9
- 12.25 0.107 m
- 12.27 $6.43 \times 10^{-4} \text{ m}^3, 2.78 \times 10^3 \text{ kg/m}^3$
- 12.29 a) $\rho < \rho_{\text{fluid}}$
 c) above: $1 - \frac{\rho}{\rho_{\text{fluid}}}$, submerged: $\frac{\rho}{\rho_{\text{fluid}}}$
 d) 32%
- 12.31 a) 116 Pa b) 921 Pa c) 0.822 kg, 822 kg/m³
- 12.33 1910 kg/m³
- 12.35 9.6 m/s
- 12.37 a) 17.0 m/s b) 0.317 m
- 12.39 0.956 m
- 12.41 28.4 m/s
- 12.43 $1.47 \times 10^5 \text{ Pa}$
- 12.45 $2.03 \times 10^4 \text{ Pa}$
- 12.47 $2.25 \times 10^5 \text{ Pa}$
- 12.49 1.19D
- 12.51 a) $(p_0 - p)\pi \frac{D^2}{4}$ b) 776 N
- 12.53 a) $5.9 \times 10^5 \text{ N}$ b) $1.8 \times 10^5 \text{ N}$
- 12.55 c) independent of surface area
- 12.57 0.964 cm, rises
- 12.59 a) 1470 Pa b) 13.9 cm
- 12.61 $9.8 \times 10^6 \text{ kg}$, yes
- 12.63 a) 0.30 b) 0.70
- 12.65 $3.50 \times 10^{-4} \text{ m}^3, 3.95 \text{ kg}$
- 12.67 a) $8.27 \times 10^3 \text{ m}^3$ b) 83.8 kN
- 12.69 2.05 m
- 12.71 a) $H/2$ b) H
- 12.73 0.116 kg
- 12.75 33.4 N
- 12.77 b) 12.2 N c) 11.8 N
- 12.79 b) $2.52 \times 10^{-4} \text{ m}^3, 0.124$
- 12.81 $5.57 \times 10^{-4} \text{ m}$
- 12.83 a) $1 - \frac{\rho_B}{\rho_L}$ b) $\left(\frac{\rho_L - \rho_B}{\rho_L - \rho_w} \right) L$ c) 4.60 cm
- 12.85 a) al/g b) $\omega^2 l^2 / 2g$
- 12.89 a) $2\sqrt{h(H-h)}$ b) h
- 12.91 a) 0.200 m/s³ b) $6.97 \times 10^4 \text{ Pa}$
- 12.93 $3h_1$
- 12.95 a) $r = \frac{r_0 \sqrt{v_0}}{(v_0^2 + 2gy)^{1/4}}$ b) 1.10 m
- 12.97 a) 80.4 N

Chapter 13

- 13.1 2.18
- 13.3 a) $1.2 \times 10^{-11} \text{ m/s}^2$ b) 15 days
 c) increase
- 13.5 $2.1 \times 10^{-9} \text{ m/s}^2$, downward
- 13.7 a) $2.4 \times 10^{-3} \text{ N}$
 b) $F_{\text{moon}}/F_{\text{earth}} = 3.5 \times 10^{-6}$
- 13.9 a) 0.634 m from $3m$
 b) (i) unstable (ii) stable

- 13.11 1.38×10^7 m
 13.13 a) 0.37 m/s^2 b) 1700 kg/m^3
 13.15 610 N , 735 N (on earth)
 13.17 a) 5020 m/s b) $60,600 \text{ m/s}$
 13.19 a) 7460 m/s b) 1.68 h
 13.21 6200 m/s
 13.23 a) $4.7 \text{ m/s} = 11 \text{ mph}$, easy to achieve
 b) 2.23 h
 13.25 a) $82,700 \text{ m/s}$ b) 14.5 days
 13.27 b) Pluto: $4.45 \times 10^{12} \text{ m}$,
 Neptune: $4.55 \times 10^{12} \text{ m}$ c) 248 y
 13.29 $2.3 \times 10^{30} \text{ kg} = 1.2M_{\text{S}}$
 13.31 a) (i) $5.31 \times 10^{-9} \text{ N}$ (ii) $2.67 \times 10^{-9} \text{ N}$
 13.33 a) $-\frac{GmM}{\sqrt{x^2 + a^2}}$ b) $-GmM/x$
 c) $\frac{GmMx}{(x^2 + a^2)^{3/2}}$, attractive d) GmM/x^2
 e) $U = -GmM/a$, $F_x = 0$
 13.35 a) 53 N b) 52 N
 13.37 a) $4.3 \times 10^{37} \text{ kg} = 2.1 \times 10^7 M_{\text{S}}$
 b) no c) $6.32 \times 10^{10} \text{ m}$, yes
 13.39 a) $4.64 \times 10^{11} \text{ m}$
 b) $6.26 \times 10^{36} \text{ kg} = 3.15 \times 10^6 M_{\text{S}}$
 c) $9.28 \times 10^9 \text{ m}$
 13.41 $9.16 \times 10^{13} \text{ N}$
 13.43 a) $9.67 \times 10^{-12} \text{ N}$, at 45° above the $+x$ -axis
 b) $3.02 \times 10^{-5} \text{ m/s}$
 13.45 a) 1.62 m/s^2 b) 0.69 N c) 4.2 N
 13.47 a) $2.9 \times 10^{15} \text{ kg}$, 0.0077 m/s^2 b) 6.2 m/s
 13.49 b) (i) $1.49 \times 10^{-5} \text{ m/s}$, $7.46 \times 10^{-6} \text{ m/s}$
 (ii) $2.24 \times 10^{-5} \text{ m/s}$ c) 26.4 m
 13.51 a) $3.59 \times 10^7 \text{ m}$
 13.53 177 m/s
 13.55 a) $1.39 \times 10^7 \text{ m}$ b) $3.59 \times 10^7 \text{ m}$
 13.57 $(0.01)R_{\text{E}} = 6.4 \times 10^4 \text{ m}$
 13.59 $1.83 \times 10^{27} \text{ kg}$
 13.61 0.28%
 13.63 6060 km/h
 13.65 $v_2 = \sqrt{\frac{2Gm_{\text{E}}h}{R_{\text{E}}(R_{\text{E}} + h)}}$
 13.67 a) $13,700 \text{ m/s}$ b) $13,300 \text{ m/s}$
 c) $13,200 \text{ m/s}$
 13.69 a) (i) 2.84 y (ii) 6.11 y
 b) $4.90 \times 10^{11} \text{ m}$ c) $4.22 \times 10^{11} \text{ m}$
 13.71 a) $GM^2/4R^2$
 b) $v = \sqrt{GM/4R}$, $T = 4\pi\sqrt{R^3/GM}$
 c) $GM^2/4R$
 13.73 $6.8 \times 10^4 \text{ m/s}$
 13.75 a) $12,700 \text{ kg/m}^3$ (at $r = 0$), 3150 kg/m^3
 (at $r = R$)
 13.77 a) 7910 s b) 1.53 c c) 5510 m/s (apogee),
 8430 m/s (perigee) d) 2410 m/s (perigee),
 3250 m/s (apogee); perigee
 13.79 $5.36 \times 10^9 \text{ J}$
 13.81 9.36 m/s^2
 13.83 $GmMx/(a^2 + x^2)^{3/2}$
 13.85 a) $U(r) = \frac{Gm_{\text{E}}m}{2R_{\text{E}}^3}r^2$ b) $7.90 \times 10^3 \text{ m/s}$
 13.87 a) against the direction of motion in both cases
 b) 259 days c) 44.1°
 13.89 $\frac{2GmM}{a^2} \left(1 - \frac{x}{\sqrt{a^2 + x^2}}\right)$

Chapter 14

- 14.1 a) 2.15 ms , 2930 rad/s
 b) $2.00 \times 10^4 \text{ Hz}$, $1.26 \times 10^5 \text{ rad/s}$
 c) $4.3 \times 10^{14} \text{ Hz} \leq f \leq 7.5 \times 10^{14} \text{ Hz}$;
 $1.3 \times 10^{-15} \text{ s} \leq T \leq 2.3 \times 10^{-15} \text{ s}$
 d) $2.0 \times 10^{-7} \text{ s}$, $3.1 \times 10^7 \text{ rad/s}$
 14.3 5530 rad/s , 1.14 ms

- 14.5 0.0500 s
 14.7 a) 0.167 s b) 37.7 rad/s c) 0.0844 kg
 14.9 a) 0.150 s b) 0.0750 s
 14.11 a) 0.98 m b) $\pi/2 \text{ rad}$
 c) $x = (-0.98 \text{ m}) \sin [(12.2 \text{ rad/s})t]$
 14.13 a) -2.71 m/s^2
 b) $x = (1.46 \text{ cm}) \cos [(15.7 \text{ rad/s})t + 0.715 \text{ rad}]$,
 $v_x = (-22.9 \text{ cm/s}) \sin [(15.7 \text{ rad/s})t + 0.715 \text{ rad}]$,
 $a_x = (-359 \text{ cm/s}^2) \cos [(15.7 \text{ rad/s})t + 0.715 \text{ rad}]$
 14.15 120 kg
 14.17 a) 0.253 kg b) 1.21 cm
 c) 3.03 N
 14.19 a) 1.51 s b) 26.0 N/m
 c) 30.8 cm/s d) 1.92 N
 e) -0.0125 m , 30.4 cm/s , 0.216 m/s^2
 f) 0.324 N
 14.21 a) $x = (0.0030 \text{ m}) \cos [(2760 \text{ rad/s})t]$
 b) 8.3 m/s , $2.3 \times 10^4 \text{ m/s}^2$
 c) $da_x/dt = (6.3 \times 10^7 \text{ m/s}^3) \times \sin [(2760 \text{ rad/s})t]$, $6.3 \times 10^7 \text{ m/s}^3$
 14.23 127 m/s^2
 14.25 a) 1.48 m/s b) $2.96 \times 10^{-5} \text{ J}$
 14.27 a) 1.20 m/s b) 1.11 m/s
 c) 36 m/s^2 d) 13.5 m/s^2 e) 0.36 J
 14.29 $3M^{3/4}$
 14.31 0.240 m
 14.33 $A/\sqrt{2}$
 14.35 a) 0.0778 m b) 1.28 Hz c) 0.624 m/s
 14.37 a) 4.06 cm b) 1.21 m/s c) 29.8 rad/s
 14.39 b) 23.9 cm , 1.45 Hz
 14.41 a) $2.7 \times 10^{-9} \text{ kg} \cdot \text{m}^2$
 b) $4.3 \times 10^{-6} \text{ N} \cdot \text{m/rad}$
 14.43 $0.0512 \text{ kg} \cdot \text{m}^2$
 14.45 a) 0.25 s b) 0.25 s
 14.47 0.407 swings per second
 14.49 10.7 m/s^2
 14.51 a) 2.84 s b) 2.89 s
 c) 2.89 s , -2%
 14.53 $0.129 \text{ kg} \cdot \text{m}^2$
 14.55 A: $2\pi\sqrt{\frac{L}{g}}$, B: $\frac{2\sqrt{2}}{3} \left(2\pi\sqrt{\frac{L}{g}}\right)$, pendulum A
 14.57 A: $2\pi\sqrt{\frac{L}{g}}$, B: $\sqrt{\frac{11}{10}} \left(2\pi\sqrt{\frac{L}{g}}\right)$, pendulum B
 14.59 a) 0.393 Hz b) 1.73 kg/s
 14.61 a) A b) $-Ab/2m$
 c) $A \left(\frac{b^2}{2m^2} - \frac{k}{m}\right)$; negative if $b < \sqrt{2km}$,
 zero if $b = \sqrt{2km}$, positive if $b > \sqrt{2km}$
 14.63 a) kg/s
 c) (i) $5.0 \frac{F_{\text{max}}}{k}$ (ii) $2.5 \frac{F_{\text{max}}}{k}$
 14.65 0.353 m
 14.67 a) $1.11 \times 10^4 \text{ m/s}^2$ b) $5.00 \times 10^3 \text{ N}$
 c) 23.6 m/s , 125 J d) 37.5 kW
 e) $1.21 \times 10^4 \text{ N}$, 36.7 m/s , 302 J , 141 kW
 14.69 a) none of them change
 b) $1/4$ as great c) $1/2$ as great
 d) $1/\sqrt{5}$ as great
 e) potential energy is the same, kinetic energy is $1/5$ as great
 14.71 a) 24.4 cm b) 0.221 s c) 1.19 m/s
 14.73 a) 0.373 Hz , 0.426 m , 2.68 s b) 1.34 s
 14.75 2.00 m
 14.77 a) 0.107 m b) 2.42 s
 14.79 $(0.921) \left(\frac{1}{2\pi\sqrt{L/g}}\right)$
 14.81 a) 1.49 s b) $-2.12 \times 10^{-4} \text{ s}$ per s, shorter
 c) 0.795 s

- 14.83 a) 0.150 m/s b) 0.112 m/s^2 downward
 c) 0.700 s d) 4.38 m
 14.85 a) 2.6 m/s b) 0.21 m c) 0.49 s
 14.87 $9.08 \times 10^{24} \text{ kg}$
 14.89 1.17 s
 14.91 0.505 s
 14.93 c) $-7.57 \times 10^{-19} \text{ J}$ e) $8.39 \times 10^{12} \text{ Hz}$
 14.95 0.705 Hz , 14.5°
 14.97 $2\pi\sqrt{\frac{M}{3k}}$
 14.99 $\frac{1}{4\pi}\sqrt{\frac{6g}{\sqrt{2L}}}$
 14.101 a) $k_1 + k_2$ b) $k_1 + k_2$
 c) $\frac{k_1k_2}{k_1 + k_2}$ d) $\sqrt{2}$
 14.103 a) $Mv^2/6$ c) $\omega = \sqrt{\frac{3k}{M}}$, $M' = M/3$

Chapter 15

- 15.1 a) 0.439 m , 1.28 ms
 b) 0.219 m
 15.3 $220 \text{ m/s} = 800 \text{ km/h}$
 15.5 a) 1.7 cm to 17 m
 b) $4.3 \times 10^{14} \text{ Hz}$ to $7.5 \times 10^{14} \text{ Hz}$
 c) 1.5 cm d) 6.4 cm
 15.7 a) 25.0 Hz , 0.0400 s , 19.6 rad/m
 b) $y(x, t) = (0.0700 \text{ m}) \cos [(19.6 \text{ m}^{-1})x + (157 \text{ rad/s})t]$ c) 4.95 cm
 d) 0.0050 s
 15.9 a) yes b) yes c) no
 d) $v_y = \omega A \cos (kx + \omega t)$,
 $a_y = -\omega^2 A \sin (kx + \omega t)$
 15.11 a) 4 mm b) 0.040 s c) 0.14 m , 3.5 m/s
 d) 0.24 m , 6.0 m/s e) no
 15.13 b) $+x$ -direction
 15.15 a) 16.3 m/s b) 0.136 m
 c) both increase by a factor of $\sqrt{2}$
 15.17 0.337 kg
 15.19 a) 18.6 N b) 29.1 m/s
 15.21 a) 10.0 m/s b) 0.250 m
 c) $y(x, t) = (3.00 \text{ cm}) \cos [\pi(8.00 \text{ rad/m})x - (80.0\pi \text{ rad/s})t]$ d) 1890 m/s^2 e) yes
 15.23 4.51 mm
 15.25 a) 95 km b) $0.25 \mu\text{W/m}^2$
 c) 110 kW
 15.27 a) 0.050 W/m^2 b) 22 kJ
 15.29 $9.48 \times 10^{27} \text{ W}$
 15.37 a) $(1.33 \text{ m})n$, $n = 0, 1, 2, \dots$
 b) $(1.33 \text{ m})(n + 1/2)$, $n = 0, 1, 2, \dots$
 15.41 a) 96.0 m/s b) 461 N
 c) 1.13 m/s , 426 m/s^2
 15.43 a) 2.80 cm c) 277 cm
 d) 185 cm , 7.96 Hz , 0.126 s , 1470 cm/s
 e) 280 cm/s
 f) $y(x, t) = (5.60 \text{ cm}) \times \sin [(0.0906 \text{ rad/cm})x] \sin [(133 \text{ rad/s})t]$
 15.45 a) $y(x, t) = (4.60 \text{ mm}) \times \sin [(6.98 \text{ rad/m})x] \sin [(742 \text{ rad/s})t]$
 b) 3^{rd} harmonic c) 39.4 Hz
 15.47 a) 45.0 cm b) no
 15.49 a) 311 m/s b) 246 Hz
 c) 245 Hz , 1.40 m
 15.51 a) 20.0 Hz , 126 rad/s , 3.49 rad/m
 b) $y(x, t) = (2.50 \times 10^{-3} \text{ m}) \times \cos [(3.49 \text{ rad/m})x - (126 \text{ rad/s})t]$
 c) $y(0, t) = (2.50 \times 10^{-3} \text{ m}) \times \cos [(126 \text{ rad/s})t]$
 d) $y(1.35 \text{ m}, t) = (2.50 \times 10^{-3} \text{ m}) \times \cos [(126 \text{ rad/s})t - 3\pi/2 \text{ rad}]$
 e) 0.315 m/s
 f) $-2.50 \times 10^{-3} \text{ m}$, 0
 15.53 a) $\frac{7L}{2}\sqrt{\frac{\mu_1}{F}}$ b) no

A-14 Answers to Odd-Numbered Problems

- 15.55 a) $\frac{2\pi A}{\lambda} \sqrt{\frac{FL}{M}}$ b) increase F by a factor of 4
- 15.57 a) $\frac{4\pi^2 F \Delta x}{\lambda^2}$
- 15.59 32.4 Hz
- 15.61 1.83 m
- 15.63 330 Hz (copper), 447 Hz (aluminum)
- 15.65 c) C/B
- 15.67 b) ω must be decreased by a factor of $1/\sqrt{2}$, k must be decreased by a factor of $1/\sqrt{8}$
- 15.69 a) 7.07 cm b) 0.400 kW
- 15.71 d) $P(x, t) = -Fk\omega A^2 \sin^2(kx + \omega t)$
- 15.73 (0.800 Hz) n , $n = 1, 2, 3, \dots$
- 15.75 c) $2A, 2A\omega, 2A\omega^2$
- 15.77 233 N
- 15.79 a) 0, L b) 0, $L/2, L$ d) no
- 15.81 1780 kg/m³
- 15.83 a) $r = 0.640$ mm, $L = 0.40$ m
b) 380 Hz
- 15.85 b) $u_k = \frac{1}{2} \mu \omega^2 A^2 \sin^2(kx - \omega t)$
c) $u_p = \frac{1}{2} Fk^2 A^2 \sin^2(kx - \omega t)$

Chapter 16

- 16.1 a) 0.344 m b) 1.2×10^{-5} m
c) 6.9 m, 50 Hz
- 16.3 a) 7.78 Pa b) 77.8 Pa c) 778 Pa
- 16.5 a) 90 m b) 102 kHz c) 1.4 cm
d) 4.4 mm to 8.8 mm e) 6.2 MHz
- 16.7 90.8 m
- 16.9 81.4°C
- 16.11 0.208 s
- 16.13 a) 5.5×10^{-15} J b) 0.074 mm/s
- 16.15 a) 9.44×10^{-11} m, 0.434 m
b) 5.66×10^{-9} m, 0.100 m
- 16.17 a) 1.95 Pa b) 4.58×10^{-3} W/m²
c) 96.6 dB
- 16.19 a) 4.4×10^{-12} W/m² b) 6.4 dB
c) 5.8×10^{-11} m
- 16.21 14.0 dB
- 16.23 a) 2.0×10^{-7} W/m² b) 6.0 m c) 290 m
- 16.25 a) *fundamental*: displacement node at 0.60 m, pressure nodes at 0 and 1.20 m; *first overtone*: displacement nodes at 0.30 m and 0.90 m, pressure nodes at 0, 0.60 m, 1.20 m; *second overtone*: displacement nodes at 0.20 m, 0.60 m, 1.00 m, pressure nodes at 0, 0.40 m, 0.80 m, 1.20 m
b) *fundamental*: displacement node at 0, pressure node at 1.20 m; *first overtone*: displacement nodes at 0 and 0.80 m, pressure nodes at 0.40 m and 1.20 m; *second overtone*: displacement nodes at 0, 0.48 m, 0.96 m, pressure nodes at 0.24 m, 0.72 m, 1.20 m
- 16.27 506 Hz, 1517 Hz, 2529 Hz
- 16.29 a) 767 Hz b) no
- 16.31 a) 614 Hz b) 1230 Hz
- 16.33 a) 172 Hz b) 86 Hz
- 16.35 0.125 m
- 16.37 destructive
- 16.39 a) 433 Hz b) loosen
- 16.41 1.3 Hz
- 16.43 780 m/s
- 16.45 a) 375 Hz b) 371 Hz c) 4 Hz
- 16.47 a) 0.25 m/s b) 0.91 m
- 16.49 19.8 m/s
- 16.51 a) 1910 Hz b) 0.188 m
- 16.53 0.0950c, toward us
- 16.55 a) 36.0° b) 2.23 s
- 16.57 b) 0.68%
- 16.59 a) 1.00 b) 8.00
c) 4.73×10^{-8} m = 47.3 nm
- 16.61 b) $3f_0$
- 16.63 flute harmonic $3N$ resonates with string harmonic $4N$, $N = 1, 3, 5, \dots$
- 16.65 a) stopped b) 7th and 9th c) 0.439 m
- 16.67 a) 375 m/s b) 1.39 c) 0.8 cm

- 16.69 1.27
- 16.71 a) 548 Hz b) 652 Hz
- 16.73 a) 2186 Hz, 0.157 m b) 2920 Hz, 0.118 m
c) 734 Hz
- 16.75 a) 0.0674 m b) 147 Hz
- 16.77 b) 2.0 m/s
- 16.79 a) 1.2×10^6 m/s
b) 3.6×10^{16} m = 3.8 ly
c) 5200 ly, about 4100 BCE
- 16.81 a) $f_0 \left(\frac{2v_w}{v - v_w} \right)$ b) $f_0 \left(\frac{2v_w}{v + v_w} \right)$
- 16.83 d) 9.69 cm/s, 667 m/s²

Chapter 17

- 17.1 a) -81.0°F b) 134.1°F c) 88.0°F
- 17.3 a) 27.2°C b) -55.6°C
- 17.5 a) -18.0°F b) -10.0°C
- 17.7 0.964 atm
- 17.9 a) -282°C b) 47,600 Pa, no
- 17.11 0.39 m
- 17.13 Death Valley: 1.9014 cm, Greenland: 1.8964 cm
- 17.15 0.26 mm
- 17.17 49.4°C
- 17.19 1.7×10^{-5} (C°)⁻¹
- 17.21 a) 1.431 cm² b) 1.436 cm²
- 17.23 a) 3.2×10^{-5} (C°)⁻¹ b) 2.6×10^9 Pa
- 17.25 a) 5.0 mm b) -8.4×10^7 Pa
- 17.27 5.79×10^5 J
- 17.29 240 J/kg · K
- 17.31 23 min
- 17.33 a) -1.54 kJ b) 0.0121 C°
- 17.35 45.2°C
- 17.37 0.0613 C°
- 17.39 a) 215 J/kg · K b) water c) too small
- 17.41 27.5°C
- 17.43 a) 5.9°C b) yes
- 17.45 150°C
- 17.47 7.6 min
- 17.49 36.4 kJ, 8.70 kcal, 34.5 Btu
- 17.51 357 m/s
- 17.53 3.45 L
- 17.55 5.05×10^{15} kg
- 17.57 0.0674 kg
- 17.59 2.10 kg
- 17.61 190 g
- 17.63 a) 222 K/m b) 10.7 W c) 73.3°C
- 17.65 a) -5.8°C b) 11 W/m²
- 17.67 4.0×10^{-3} W/m · C°
- 17.69 105.5°C
- 17.71 a) 21 kW b) 6.4 kW
- 17.73 2.1 cm²
- 17.75 a) 1.61×10^{11} m b) 5.43×10^6 m
- 17.77 a) 35.1°C b) 39.6°C
- 17.79 53.3°C
- 17.81 35.0°C
- 17.83 23.0 cm, 7.0 cm
- 17.85 b) 1.9×10^8 Pa
- 17.87 a) 99.4 N c) -4.2 Hz, falls
- 17.89 a) 87°C b) -80°C
- 17.91 20.2°C
- 17.93 a) 54.3
- 17.95 a) 83.6 J b) 1.86 J/mol · K
c) 5.60 J/mol · K
- 17.97 a) 2.70×10^7 J b) 6.89 C° c) 19.3 C°
- 17.99 2.5 cm
- 17.101 a) 86.1°C b) no ice, no steam, 0.130 kg liquid water
- 17.103 a) 100°C b) 0.0214 kg steam, 0.219 kg liquid water
- 17.105 1.743 kg
- 17.107 a) 93.9 W b) 1.35
- 17.109 2.9
- 17.111 c) 170 h d) 1.5×10^{10} s \approx 500 y, no
- 17.113 0.106 W/m · K
- 17.115 5.82 g
- 17.117 a) 1.04 kW b) 87.1 W c) 1.13 kW
d) 28 g e) 1.1 bottles
- 17.119 a) 69.6°C
- 17.121 1.76 C°
- 17.123 b) 0°C d) 3140 C°/m e) 121 W f) zero

- g) 1.1×10^{-4} m²/s h) -11 C°/s
i) 9.17 s j) decrease k) -7.71 C°/s
- 17.125 a) 103°C b) 27 W
- 17.127 a) (i) 280 W (ii) 0.248 W (iii) 2.10 kW
(iv) 116 W; radiation from the sun
b) 3.72 L/h c) 1.4 L/h

Chapter 18

- 18.1 a) 0.122 mol b) 14,700 Pa, 0.145 atm
- 18.3 0.100 atm
- 18.5 a) 0.0136 kg/m³ (Mars), 67.6 kg/m³ (Venus), 5.39 kg/m³ (Titan)
- 18.7 503°C
- 18.9 16.8 kPa
- 18.11 0.159 L
- 18.13 0.0508 V
- 18.15 a) 70.2°C b) yes
- 18.17 850 m
- 18.19 a) 6.95×10^{-16} kg b) 2.32×10^{-13} kg/m³
- 18.21 22.8 kPa
- 18.23 a) \$8720 b) 3.88 cm
- 18.25 a) 8.2×10^{-17} atm b) no
- 18.27 55.6 mol, 3.35×10^{25} molecules
- 18.29 a) 9.00×10^{-5} m³ b) 3.1×10^{-10} m
c) about the same
- 18.31 b) 1.004
- 18.33 (d) must be true, the others could be true
- 18.35 a) 1.93×10^6 m/s, no b) 7.3×10^{10} K
- 18.37 a) 6.21×10^{-21} J b) 2.34×10^5 m²/s²
c) 484 m/s d) 2.57×10^{-23} kg · m/s
e) 1.24×10^{-19} N f) 1.24×10^{-17} Pa
g) 8.17×10^{21} molecules
h) 2.45×10^{22} molecules
- 18.39 3800°C
- 18.41 a) 2600 J b) 1560 J
- 18.43 a) 741 J/kg · K, $c_w = 5.65c_{N_2}$
b) 5.65 kg c) 4.85 m³
- 18.45 a) 923 J/kg · K
b) The value calculated is too large by about 1.4%.
- 18.47 a) 337 m/s b) 380 m/s c) 412 m/s
- 18.49 a) 610 Pa b) 22.12 MPa
- 18.51 no, no
- 18.53 a) 11.8 kPa b) 0.566 L
- 18.55 272°C
- 18.57 0.213 kg
- 18.59 a) -179°C b) 1.2×10^{26} molecules/m³
c) The atmosphere of Titan is 4.8 times denser than that of the earth.
- 18.61 1.92 atm
- 18.63 a) 30.7 cylinders b) 8420 N c) 7800 N
- 18.65 a) 26.2 m/s b) 16.1 m/s, 5.44 m/s
c) 1.74 m
- 18.67 $\approx 5 \times 10^{27}$ atoms
- 18.69 a) A b) B c) 4250°C d) B
- 18.71 a) 4.65×10^{-26} kg b) 6.11×10^{-21} J
c) 2.04×10^{24} molecules d) 12.5 kJ
- 18.73 b) r_2 c) $r_1 = \frac{R_0}{2^{1/6}}$, $r_2 = R_0 \cdot 2^{-1/6}$ d) U_0
- 18.75 a) 517 m/s b) 298 m/s
- 18.77 b) 1.40×10^5 K (N), 1.01×10^4 K (H)
c) 6370 K (N), 459 K (H)
- 18.79 a) 1.24×10^{-14} kg
b) 4.16×10^{11} molecules
c) 2.95 μm, no
- 18.81 a) $2R = 16.6$ J/mol · K b) less
- 18.83 CO₂: 20.79 J/mol · K, 27%; SO₂: 24.94 J/mol · K, 21%; H₂S: 24.94 J/mol · K, 3.9%
- 18.85 $3kT/m$
- 18.87 b) 0.0421 N c) 2.94×10^{-21} N
d) 0.0297 N, 2.08×10^{-21} N
e) 0.0595 N, 4.15×10^{-21} N
- 18.89 42.6%
- 18.91 a) 4.5×10^{11} m
b) 703 m/s, 6.4×10^8 s (≈ 20 y)
c) 1.4×10^{-14} Pa d) 650 m/s, evaporate
f) 2×10^5 K, >3 times the temperature of the sun, no

Chapter 19

- 19.1 b) 1330 J
 19.3 b) -6180 J
 19.5 a) 0.942 atm
 19.7 a) $(p_1 - p_2)(V_2 - V_1)$ b) Negative of work done in reverse direction
 19.9 a) 34.7 kJ b) 80.4 kJ c) no
 19.11 a) 278 K b) 0, 162 J c) 53 J
 19.13 a) 16.4 min b) 139 m/s = 501 km/h
 19.15 a) 0 b) $T_b = 2T_a$ c) $U_b = U_a + 700$ J
 19.17 a) positive b) $W_1 > 0, W_{II} < 0$ c) into the system d) into the system for loop I, out of the system for loop II
 19.19 b) 208 J c) on the piston d) 712 J e) 920 J f) 208 J
 19.21 a) 948 K b) 900 K
 19.23 $2/5$
 19.25 a) 25.0 K b) 17.9 K c) higher for (a)
 19.27 a) -605 J b) 0 c) liberates 605 J
 19.29 a) 476 kPa b) -10.6 kJ c) 1.59, heated
 19.31 5.05 kJ, internal energy and temperature both increase
 19.33 b) 224 J c) -224 J
 19.35 11.6°C
 19.37 a) 600 J out of the gas b) -1500 J, decreases
 19.39 a) increases b) 4800 J
 19.41 a) 45.0 J b) liberates 65.0 J c) 23.0 J, 22.0 J
 19.43 a) the same b) absorbs 4.0 kJ c) 8.0 kJ
 19.45 b) -2460 J
 19.47 a) 0.80 L b) 305 K, 1220 K, 1220 K c) ab : 76 J, into the gas; ca : -107 J, out of the gas bc : 56 J, into the gas d) ab : 76 J, increase bc : 0, no change ca : -76 J, decrease
 19.49 a) 3.00 kJ, into the gas b) 2.00 kJ, into the gas c) $Q_a > Q_b$
 19.51 a) 899°C b) 12.2 kJ c) 42.6 kJ d) 45.6 kJ
 19.53 -0.226 m³
 19.55 a) 4.32×10^{-4} m³ b) 648 J c) 715 kJ d) 715 kJ e) no substantial difference
 19.57 3.4×10^5 J/kg
 19.59 b) 11.9°C
 19.61 a) 0.173 m b) 207°C c) 74.7 kJ
 19.63 a) $Q = 300$ J, $\Delta U = 0$ b) $Q = 0, \Delta U = -300$ J c) $Q = 750$ J, $\Delta U = 450$ J
 19.65 a) $W = 738$ J, $Q = 2590$ J, $\Delta U = 1850$ J b) $W = 0, Q = -1850$ J, $\Delta U = -1850$ J c) $\Delta U = 0$
 19.67 a) $W = -187$ J, $Q = -654$ J, $\Delta U = -467$ J b) $W = 113$ J, $Q = 0, \Delta U = -113$ J c) $W = 0, Q = 580$ J, $\Delta U = 580$ J
 19.69 a) $p_0 + \frac{mg}{\pi r^2}$ b) $-\left(\frac{y}{h}\right)(p_0 \pi r^2 + mg)$ c) $\frac{1}{2\pi} \sqrt{\frac{g}{h} \left(1 + \frac{p_0 \pi r^2}{mg}\right)}$, no

Chapter 20

- 20.1 a) 6500 J b) 34%
 20.3 a) 23% b) 12,400 J c) 0.350 g d) 222 kW = 298 hp
 20.5 a) 12.3 atm b) 5470 J (ca) c) 3723 J (bc) d) 1747 J e) 31.9%
 20.7 a) 58% b) 1.4%
 20.9 a) 16.2 kJ b) 50.2 kJ
 20.11 1.7 h
 20.13 a) 215 J b) 378 K c) 39.0%
 20.15 a) 42.4 kJ b) 441°C
 20.17 a) 492 J b) 212 W c) 5.4
 20.19 4.5 kJ
 20.21 37.1 hp

- 20.23 a) 429 J/K b) -393 J/K c) 36 J/K
 20.25 a) irreversible b) 1250 J/K
 20.27 -6.31 J/K
 20.29 a) 6.05 kJ/K b) about five times greater for vaporization
 20.31 a) 33.3 J/K b) irreversible
 20.33 a) no b) 18.3 J/K c) 18.3 J/K
 20.35 a) 121 J b) 3800 cycles
 20.37 a) 33 J b) 117 J c) 45°C d) 0 e) 96.2 g
 20.39 -5.8 J/K, decrease
 20.41 b) absorbed during bc , rejected during ab and ca c) $T_a = T_b = 241$ K, $T_c = 481$ K d) 610 J, 610 J e) 8.7%
 20.43 a) 21.0 kJ (enters), 16.6 kJ (leaves) b) 4.4 kJ, 21% c) 67%
 20.45 a) 7.0% b) 3.0 MW, 2.8 MW c) 6×10^5 kg/h = 6×10^5 L/h
 20.47 a) 2.00 atm, 4.00 L; 2.00 atm, 6.00 L; 1.11 atm, 6.00 L; 1.67 atm, 4.00 L b) $1 \rightarrow 2$: 1422 J, 405 J; $2 \rightarrow 3$: -1355 J, 0; $3 \rightarrow 4$: -274 J, -274 J; $4 \rightarrow 1$: 339 J, 0 c) 131 J d) 7.44%, 44.4%
 20.49 a) 2.26% b) 29.4 J (gravitational), 1.30 kJ c) 1.11×10^{-3} candy bars
 20.51 a) ab : 225 kJ, 90 kJ, 135 kJ; bc : -240 kJ, 0, -240 kJ; ca : 45 kJ, -60 kJ, 105 kJ b) 30 kJ, 30 kJ, 0 c) 11.1%
 20.53 $1 - T_C/T_H$
 20.55 a) 122 J, -78 J b) 5.10×10^{-4} m³ c) at b : 2.32 MPa, 4.81×10^{-5} m³, 771 K; at c : 4.01 MPa, 4.81×10^{-5} m³, 1332 K; at d : 0.147 MPa, 5.10×10^{-4} m³, 518 K d) 61.1%, 77.5%
 20.57 b) 357 kJ, 62 kJ c) 385 kJ, 34 kJ
 20.59 a) $nC_V \ln(T_C/T_B), nC_V \ln(T_D/T_C)$ b) zero
 20.61 a) -107 J/K b) 147 J/K c) 0 d) 39.4 J/K

Chapter 21

- 21.1 a) 2.00×10^{10} b) 8.59×10^{-13}
 21.3 $2.1 \times 10^{28}, -3.35 \times 10^9$ C
 21.5 1.3 nC
 21.7 3.7 km
 21.9 a) 0.742 μ C on each b) 0.371 μ C, 1.48 μ C
 21.11 1.43×10^{13} , away from each other
 21.13 a) 2.21×10^4 m/s² b) 0.750 nC
 21.17 1.8×10^{-4} N, in the $+x$ -direction
 21.19 $x = -0.144$ m
 21.21 2.58 μ N, in the $-y$ -direction
 21.23 a) 8.80×10^{-9} N, attractive b) 8.22×10^{-8} N, about 10 times larger than the bonding force
 21.25 a) 4.40×10^{-16} N b) 2.63×10^{11} m/s² c) 2.63×10^5 m/s
 21.27 a) 3.30×10^6 N/C, to the left b) 14.2 ns c) 1.80×10^3 N/C, to the right
 21.29 a) -21.9 μ C b) 1.02×10^{-7} N/C
 21.31 a) 8740 N/C, to the right b) 6540 N/C, to the right c) 1.40×10^{-15} N, to the right
 21.33 a) 364 N/C b) no, 2.73 μ m, downward
 21.35 1.79×10^6 m/s
 21.37 1.73×10^{-8} N, toward a point midway between the two electrons
 21.39 a) $-\hat{j}$ b) $\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}$ c) $-0.39\hat{i} + 0.92\hat{j}$
 21.41 a) 633 km/s b) 15.9 km/s
 21.43 a) 0 b) for $|x| < a$: $E_x = -\frac{q}{\pi\epsilon_0} \frac{ax}{(x^2 - a^2)^2}$; for $x > a$: $E_x = \frac{q}{2\pi\epsilon_0} \frac{x^2 + a^2}{(x^2 - a^2)^2}$

$$\text{for } x < -a: E_x = -\frac{q}{2\pi\epsilon_0} \frac{x^2 + a^2}{(x^2 - a^2)^2}$$

- 21.45 a) (i) 574 N/C, $+x$ -direction (ii) 268 N/C, $-x$ -direction (iii) 404 N/C, $-x$ -direction b) (i) 9.20×10^{-17} N, $-x$ -direction (ii) 4.30×10^{-17} N, $+x$ -direction (iii) 6.48×10^{-17} N, $+x$ -direction
 21.47 1.04×10^7 N/C, toward the -2.00 - μ C charge
 21.49 a) $E_x = E_y = E = 0$ b) $E_x = 2660$ N/C, $E_y = 0, E = 2660$ N/C, $+x$ -direction c) $E_x = 129$ N/C, $E_y = 510$ N/C, $E = 526$ N/C, 284° counterclockwise from the $+x$ -axis d) $E_x = 0, E_y = 1380$ N/C, $E = 1380$ N/C, $+y$ -direction
 21.51 a) $E_x = -4790$ N/C, $E_y = 0, E = 4790$ N/C, $-x$ -direction b) $E_x = 2130$ N/C, $E_y = 0, E = 2130$ N/C, $+x$ -direction c) $E_x = -129$ N/C, $E_y = -164$ N/C, $E = 209$ N/C, 232° counterclockwise from the $+x$ -axis d) $E_x = -1040$ N/C, $E_y = 0, E = 1040$ N/C, $-x$ -direction
 21.53 a) $(7.0$ N/C) \hat{i} b) $(1.75 \times 10^{-5}$ N) \hat{i}
 21.55 a) 1.14×10^5 N/C, toward the center of the disk b) 8.92×10^4 N/C, toward the center of the disk c) 1.46×10^5 N/C, toward the charge
 21.57 a) 1.4×10^{-11} C \cdot m, from q_1 toward q_2 b) 860 N/C
 21.59 a) \vec{p} is aligned in either the same or the opposite direction as \vec{E} . b) \vec{p} is aligned in the same direction as \vec{E} .
 21.61 a) 1680 N, from the $+5.00$ - μ C charge toward the -5.00 - μ C charge b) 22.3 N \cdot m, clockwise
 21.63 b) $\frac{Q^2}{8\pi\epsilon_0 L^2} (1 + 2\sqrt{2})$, away from the center of the square
 21.65 a) 3.17 nC b) $+x$ -direction c) $x = -1.76$ m
 21.67 a) $\frac{1}{2\pi} \sqrt{\frac{qQ}{\pi\epsilon_0 ma^3}}$ b) accelerates away from the origin along the y -axis
 21.69 b) 2.80 μ C c) 39.5°
 21.71 a) 2.09×10^{21} N b) 5.90×10^{23} m/s² c) no
 21.73 3.41×10^4 N/C, to the left
 21.75 between the charges, 0.24 m from the 0.500-nC charge
 21.77 a) $\frac{6q^2}{4\pi\epsilon_0 L^2}$, away from the vacant corner b) $\frac{3q^2}{4\pi\epsilon_0 L^2} \left(\sqrt{2} + \frac{1}{2}\right)$, toward the center of the square
 21.79 a) 6.0×10^{23} b) 510 kN (electric), 4.1×10^{-31} N (gravitation) c) yes for electric force, no for gravitation
 21.81 a) ≈ 480 C b) 8.3×10^{13} N, repulsive
 21.83 2190 km/s
 21.85 a) 3.5×10^{20} b) 1.6 C, 2.3×10^{10} N
 21.87 a) $\frac{mv_0^2 \sin^2 \alpha}{2eE}$ b) $\frac{mv_0^2 \sin 2\alpha}{eE}$ d) 0.418 m, 2.89 m
 21.89 a) $E_x = \frac{Q}{4\pi\epsilon_0 a} \left(\frac{1}{x-a} - \frac{1}{x}\right), E_y = 0$ b) $\frac{qQ}{4\pi\epsilon_0 a} \left(\frac{1}{r} - \frac{1}{r+a}\right) \hat{i}$
 21.91 a) $(-6110$ N/C) \hat{i} b) smaller c) 0.177 m

A-16 Answers to Odd-Numbered Problems

- 21.93 a) 1.56 N/C, +x-direction c) smaller
d) 4.7%
- 21.95 a) $-\frac{qQ}{2\pi\epsilon_0 a} \left(\frac{1}{y} - \frac{1}{\sqrt{a^2 + y^2}} \right) \hat{i}$
b) $\frac{qQ}{4\pi\epsilon_0 a} \left(\frac{1}{x-a} + \frac{1}{x+a} - \frac{2}{x} \right) \hat{i}$
- 21.97 $E_x = E_y = \frac{Q}{2\pi^2\epsilon_0 a^2}$
- 21.99 a) 6.25×10^4 N/C, 225° counterclockwise from an axis pointing to the right at the point P
b) 1.00×10^{-14} N, opposite the electric field direction
- 21.101 a) 1.19×10^6 N/C, to the left
b) 1.19×10^5 N/C, to the left
c) 1.19×10^5 N/C, to the right
- 21.103 $\frac{\sigma}{2\epsilon_0} \left(-\frac{x}{|x|} \hat{i} + \frac{z}{|z|} \hat{k} \right)$
- 21.105 b) $q_1 < 0, q_2 > 0$ c) 0.843 μC d) 56.2 N
- 21.107 a) $\frac{Q}{2\pi\epsilon_0 L} \left(\frac{1}{2x+a} - \frac{1}{2L+2x+a} \right)$

Chapter 22

- 22.1 a) 1.8 N·m²/C b) no c) (i) 0° (ii) 90°
- 22.3 a) 3.53×10^5 N·m²/C b) 3.13 μC
- 22.5 $\pi r^2 E$
- 22.7 0.977 N·m²/C, inward
- 22.9 a) 0 b) 8.74×10^7 N/C
c) 2.60×10^7 N/C
- 22.11 a) 1.17×10^5 N·m²/C b) no change
- 22.13 a) 4.50×10^4 N/C b) 918 N/C
- 22.15 0.0810 N
- 22.17 2.04×10^{10}
- 22.19 a) 6.47×10^5 N/C, +y-direction
b) 7.2×10^4 N/C, -y-direction
- 22.21 a) 5.73 $\mu\text{C}/\text{m}^2$ b) 6.47×10^5 N/C
c) -5.65×10^4 N·m²/C
- 22.23 a) 0.260 $\mu\text{C}/\text{m}^3$ b) 1960 N/C
- 22.25 1.16 km/s
- 22.27 23.6 μJ
- 22.29 0 (outside the plates), σ/ϵ_0 (between the plates)
- 22.31 a) $2\pi R\sigma$ b) $\sigma R/\epsilon_0 r$ c) $\lambda/2\pi\epsilon_0 r$
- 22.33 a) yes, + Q b) no c) yes d) no, no
e) no, yes, no
- 22.35 a) 750 N·m²/C b) 0
c) 577 N/C, +x-direction d) charges both within and outside
- 22.37 a) -0.598 nC b) charges both within and outside
- 22.39 a) $\lambda/2\pi\epsilon_0 r$, radially outward b) $\lambda/2\pi\epsilon_0 r$, radially outward
d) $-\lambda$ (inner surface), $+\lambda$ (outer surface)
- 22.41 a) (i) $\frac{\alpha}{2\pi\epsilon_0 r}$, radially outward (ii) 0 (iii) 0
b) (i) $-\alpha$ (ii) 0
- 22.43 10.2°
- 22.45 a) 0 ($r < R$); $\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$ ($R < r < 2R$),
radially outward; $\frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$ ($r > 2R$),
radially outward
- 22.47 a) (i) 0 (ii) 0 (iii) $\frac{q}{2\pi\epsilon_0 r^2}$, radially outward
(iv) 0 (v) $\frac{3q}{2\pi\epsilon_0 r^2}$, radially outward
b) (i) 0 (ii) +2 q (iii) -2 q (iv) +6 q
- 22.49 a) (i) 0 (ii) 0 (iii) $\frac{q}{2\pi\epsilon_0 r^2}$, radially outward
(iv) 0 (v) $\frac{q}{2\pi\epsilon_0 r^2}$, radially inward

- b) (i) 0 (ii) +2 q (iii) -2 q (iv) -2 q
- 22.51 a) $\frac{qQ}{4\pi\epsilon_0 r^2}$, toward the center of the shell
b) 0
- 22.53 a) $\frac{\alpha}{2\epsilon_0} \left(1 - \frac{a^2}{r^2} \right)$
b) $q = 2\pi\alpha a^2, E = \frac{\alpha}{2\epsilon_0}$
- 22.55 $R/2$
- 22.57 $|x| > d$ (outside the slab):
 $\frac{\rho_0 d}{3\epsilon_0} \frac{x}{|x|} \hat{i}; |x| < d$ (inside the slab): $\frac{\rho_0 x^3}{3\epsilon_0 d^2} \hat{i}$
- 22.61 b) $\rho\vec{b}/3\epsilon_0$
- 22.63 a) $-\frac{Q}{16\pi\epsilon_0 R^2} \hat{i}$ b) $\frac{Q}{72\pi\epsilon_0 R^2} \hat{i}$
c) 0 d) $\frac{5Q}{18\pi\epsilon_0 R^2} \hat{i}$
- 22.65 c) $\frac{Qr}{4\pi\epsilon_0 R^3} \left(4 - \frac{3r}{R} \right)$ e) $2R/3, \frac{Q}{3\pi\epsilon_0 R^2}$
- 22.67 a) $\frac{480Q}{233\pi R^3}$
b) $r \leq R/2: E = \frac{180Qr^2}{233\pi\epsilon_0 R^4}; R/2 \leq r \leq R:$
 $E = \frac{480Q}{233\pi\epsilon_0 r^2} \left[\frac{1}{3} \left(\frac{r}{R} \right)^3 - \frac{1}{5} \left(\frac{r}{R} \right)^5 - \frac{23}{1920} \right];$
 $r \geq R: E = \frac{Q}{4\pi\epsilon_0 r^2}$ c) 0.807
d) $E = \frac{180}{233} \frac{Q}{4\pi\epsilon_0 R^2}$

Chapter 23

- 23.1 -0.356 J
- 23.3 3.46×10^{-13} J = 2.16 MeV
- 23.5 a) 12.5 m/s b) 0.323 m
- 23.7 -1.42×10^{-18} J
- 23.9 a) 13.6 km/s b) 2.45×10^{17} m/s²
- 23.11 - $q/2$
- 23.13 7.42 m/s, faster
- 23.15 a) 0 b) 0.750 mJ c) -2.06 mJ
- 23.17 a) 0 b) -175 kV c) -0.875 J
- 23.19 a) -737 V b) -704 V c) 8.2×10^{-8} J
- 23.21 b) $V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{|x|} - \frac{2}{|x-a|} \right)$
c) $x = -a, a/3$
- 23.23 a) 156 V b) -182 V
- 23.25 a) point b b) 800 V/m c) -48.0 μJ
- 23.27 a) (i) 180 V (ii) -270 V (iii) -450 V
b) 719 V, inner shell
- 23.29 a) oscillatory b) 1.67×10^7 m/s
- 23.31 a) 94.9 nC/m b) less c) zero
- 23.33 a) 78.2 kV b) zero
- 23.35 0.474 J
- 23.37 a) 9.3×10^6 V/m, inward
b) outer surface
- 23.39 a) 8.00 kV/m b) 19.2 μN c) 0.864 μJ
d) -0.864 μJ
- 23.41 b) -20 nC c) no
- 23.43 -760 V
- 23.45 a) $E_x = -Ay + 2Bx, E_y = -Ax - C, E_z = 0$
b) $x = -C/A, y = -2BC/A^2$, any value of z
- 23.47 a) (i) $V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$
(ii) $V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{r_b} \right)$ (iii) $V = 0$
d) 0 e) $E = \frac{q-Q}{4\pi\epsilon_0 r^2}$

- 23.49 a) cylinders coaxial with the given cylinder
b) 2.90 cm, 4.20 cm, 6.08 cm
c) get farther apart
- 23.51 a) -0.360 μJ b) $x = 0.074$ m
- 23.53 a) 7.66×10^{-13} J b) 5.17×10^{-14} m
- 23.55 a) -21.5 μJ b) -2.83 kV c) 35.4 kV/m
- 23.57 a) 7.85×10^4 V/m^{4/3}
b) $E_x = -(1.05 \times 10^5 \text{ V/m}^{4/3})x^{1/3}$
c) 3.13×10^{-15} N, toward the positive anode
- 23.59 a) $-\frac{1.46q^2}{\pi\epsilon_0 d}$
- 23.61 a) -8.60×10^{-18} J = -53.7 eV
b) 2.88×10^{-11} m
- 23.63 a) (i) $V = (\lambda/2\pi\epsilon_0) \ln(b/r)$ (ii) $V = 0$
(iii) $V = 0$
d) $(\lambda/2\pi\epsilon_0) \ln(b/a)$
- 23.65 a) 1.76×10^{-16} N, downward
b) 1.93×10^{14} m/s², downward
c) 0.822 cm d) 15.3° e) 3.29 cm
- 23.67 a) 97.1 kV/m b) 30.3 pC
- 23.69 a) $r > R: V_r = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{R}\right);$
 $r < R: V_r = \frac{\lambda}{4\pi\epsilon_0} \left[1 - \left(\frac{r}{R}\right)^2 \right]$
- 23.71 $\frac{3}{5} \left(\frac{Q^2}{4\pi\epsilon_0 R} \right)$
- 23.73 360 kV
- 23.75 b) yes c) no, no
- 23.77 $\frac{Q}{8\pi\epsilon_0 R}$
- 23.79 a) $\frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x+a}{x}\right)$
b) $\frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right)$
c) part (a): $\frac{Q}{4\pi\epsilon_0 x}$, part (b): $\frac{Q}{4\pi\epsilon_0 y}$
- 23.81 a) 1/3 b) 3
- 23.83 a) $E = \frac{Q_1}{4\pi\epsilon_0 R_1^2}, V = \frac{Q_1}{4\pi\epsilon_0 R_1}$
b) $q_1 = \frac{Q_1 R_1}{R_1 + R_2}, q_2 = \frac{Q_1 R_2}{R_1 + R_2}$
c) $V = \frac{Q_1}{4\pi\epsilon_0 (R_1 + R_2)}$ for either sphere
d) $E_1 = \frac{Q_1}{4\pi\epsilon_0 R_1 (R_1 + R_2)},$
 $E_2 = \frac{Q_1}{4\pi\epsilon_0 R_2 (R_1 + R_2)}$
- 23.85 a) 7580 km/s b) 7260 km/s
c) 2.3×10^9 K (protons), 6.4×10^9 K (helium)
- 23.87 a) 5.9×10^{-15} m b) 20.7 pJ
c) 1.06×10^{15} J = 253 kilotons of TNT
- 23.89 1.01×10^{-12} m, 1.11×10^{-13} m,
 2.54×10^{-14} m
- 23.91 c) 3, 0.507 μm

Chapter 24

- 24.1 a) 10.0 kV b) 22.6 cm² c) 8.00 pF
- 24.3 a) 604 V b) 90.8 cm² c) 1840 kV/m
d) 16.3 $\mu\text{C}/\text{m}^2$
- 24.5 a) 120 μC b) 60 μC c) 480 μC
- 24.7 2.8 mm, yes
- 24.9 a) 1.05 mm b) 84.0 V
- 24.11 a) 4.35 pF b) 2.30 V
- 24.13 a) 15.0 pF b) 3.09 cm c) 31.2 kN/C
- 24.15 a) series b) 5000
- 24.17 a) $Q_1 = Q_2 = 22.4 \mu\text{C}, Q_3 = 44.8 \mu\text{C},$
 $Q_4 = 67.2 \mu\text{C}$

- b) $V_1 = V_2 = 5.6 \text{ V}$, $V_3 = 11.2 \text{ V}$, $V_4 = 16.8 \text{ V}$
 c) 11.2 V
 24.19 a) $Q_1 = 156 \mu\text{C}$, $Q_2 = 260 \mu\text{C}$
 b) $V_1 = V_2 = 52.0 \text{ V}$
 24.21 a) 19.3 nF b) 482 nC c) 162 nC
 d) 25 V
 24.23 $57 \mu\text{F}$
 24.25 0.0283 J/m^3
 24.27 a) $\frac{xQ^2}{2\epsilon_0 A}$ b) $\left(\frac{Q^2}{2\epsilon_0 A}\right) dx$ c) $\frac{Q^2}{2\epsilon_0 A}$
 24.29 a) $U_p = 4U_s$ b) $Q_p = 2Q_s$ c) $E_p = 2E_s$
 24.31 a) $24.2 \mu\text{C}$ b) $Q_{35} = 7.7 \mu\text{C}$, $Q_{75} = 16.5 \mu\text{C}$
 c) 2.66 mJ d) $U_{35} = 0.85 \text{ mJ}$, $U_{75} = 1.81 \text{ mJ}$
 e) 220 V for each capacitor
 24.33 a) 1.60 nC b) 8.05
 24.35 a) 3.60 mJ (before), 13.5 mJ (after)
 b) increased by 9.9 mJ
 24.37 a) $0.620 \mu\text{C/m}^2$ b) 1.28
 24.39 0.0135 m^2
 24.41 a) $2.31 \times 10^{-11} \text{ C}^2/\text{N} \cdot \text{m}^2$
 b) $4.0 \times 10^4 \text{ V}$
 c) $\sigma = 4.6 \times 10^{-4} \text{ C/m}^2$,
 $\sigma_i = 2.8 \times 10^{-4} \text{ C/m}^2$
 24.43 a) 10.1 V b) 2.25
 24.45 a) $\frac{Q}{\epsilon_0 AK}$ b) $\frac{Qd}{\epsilon_0 AK}$ c) $K \frac{\epsilon_0 A}{d} = KC_0$
 24.47 a) 421 J b) 0.054 F
 24.49 a) 31 pF b) 0.37 nC c) 1.6 kV/m
 d) 2.2 nJ
 24.51 133 kV/m
 24.53 a) 0.0160 C b) 533 V c) 4.26 J d) 2.14 J
 24.55 a) $158 \mu\text{J}$ b) $72.1 \mu\text{J}$
 24.57 a) $2.5 \mu\text{F}$
 b) $Q_1 = 550 \mu\text{C}$, $Q_2 = 370 \mu\text{C}$, $Q_3 = Q_4 = 180 \mu\text{C}$, $Q_5 = 550 \mu\text{C}$; $V_1 = 65 \text{ V}$, $V_2 = 87 \text{ V}$,
 $V_3 = V_4 = 43 \text{ V}$, $V_5 = 65 \text{ V}$
 24.59 $C_2 = 6.00 \mu\text{F}$, $C_3 = 4.50 \mu\text{F}$
 24.61 a) $76 \mu\text{C}$ b) 1.4 mJ
 c) 11 V across each capacitor d) 1.3 mJ
 24.63 a) $2.3 \mu\text{F}$ b) $Q_1 = 970 \mu\text{C}$, $Q_2 = 640 \mu\text{C}$
 c) 47 V
 24.65 a) 3.91 b) 22.8 V
 24.67 a) $C = 4\pi\epsilon_0 R$ b) $710 \mu\text{F}$
 24.69 a) 0.065 F b) $23,000 \text{ C}$
 c) $4.0 \times 10^9 \text{ J}$
 24.71 $48.3 \mu\text{F}$
 24.73 $0.185 \mu\text{J}$
 24.75 b) 2.38 nF
 24.77 a) $\frac{\epsilon_0 L}{D}[L + (K - 1)x]$
 c) $Q = \frac{\epsilon_0 LV}{D}[L + (K - 1)x]$

Chapter 25

- 25.1 1.0 C
 25.3 a) 3.12×10^{19} b) $1.51 \times 10^6 \text{ A/m}^2$
 c) 0.111 mm/s d) both J and v_d would decrease
 25.5 a) 110 min b) 440 min c) $v_d \propto 1/d^2$
 25.7 a) 330 C b) 41 A
 25.9 $9.0 \mu\text{A}$
 25.11 a) $1.06 \times 10^{-5} \Omega \cdot \text{m}$
 b) $0.00105 (\text{C}^\circ)^{-1}$
 25.13 a) 0.206 mV b) 0.176 mV
 25.15 a) 1.21 V/m b) 0.0145Ω c) 0.182 V
 25.17 0.125Ω
 25.19 15 g
 25.21 $1.53 \times 10^{-8} \Omega$
 25.23 a) 11 A b) 3.1 V c) 0.28Ω
 25.25 a) 99.54Ω b) 0.0158Ω
 25.27 a) $4.67 \times 10^{-8} \Omega$ b) $6.72 \times 10^{-4} \Omega$
 25.29 a) 27.4 V b) 12.3 MJ
 25.31 a) zero b) 5.0 V c) 5.0 V
 25.33 3.08 V , 0.067Ω , 1.80Ω
 25.35 a) 1.41 A , clockwise b) 13.7 V c) -1.0 V
 25.37 b) yes c) 3.88Ω

- 25.39 a) 144Ω b) 240Ω c) 0.833 A , 0.500 A
 25.41 a) 29.8 W b) 0.248 A
 25.43 a) EJ b) ρJ^2 c) E^2/ρ
 25.45 a) 300 W b) 0.90 J
 25.47 a) 2.6 MJ b) 0.063 L c) 1.6 h
 25.49 12.3%
 25.51 a) 24.0 W b) 4.0 W c) 20.0 W
 25.53 a) 26.7Ω b) 4.50 A c) 453 W d) larger
 25.55 a) $3.65 \times 10^{-8} \Omega \cdot \text{m}$ b) 172 A
 c) 2.58 mm/s
 25.57 0.060Ω
 25.59 a) 2.5 mA b) $2.14 \times 10^{-5} \text{ V/m}$
 c) $8.55 \times 10^{-5} \text{ V/m}$ d) 0.180 mV
 25.61 42.0 s
 25.63 a) 80 C° b) no
 25.65 a) $\frac{\rho h}{\pi r_1 r_2}$
 25.67 a) 0.057Ω b) $3.34 \times 10^{-8} \Omega \cdot \text{m}$
 c) 0.86 mm d) 0.00240Ω
 e) $0.0011 (\text{C}^\circ)^{-1}$
 25.69 a) 0.20Ω b) 8.7 V
 25.71 a) $1.0 \text{ k}\Omega$ b) 100 V c) 10 W
 25.73 1.42 A
 25.75 a) $I_A \left(1 + \frac{R_A}{r + R}\right)$ b) 0.0429Ω
 25.77 b) 8-gauge c) 106 W d) $\$19.25$
 25.79 a) 0.40 A b) 1.6 W
 c) 4.8 W , in the 12.0-V battery
 d) 3.2 W , in the 8.0-V battery
 25.81 a) 14.4 V b) 2.59 MJ c) 0.432 MJ
 d) 0.96Ω e) 1.73 MJ f) 0.432 MJ
 25.83 6.67 V
 25.85 a) a/E b) aL/V_{bc} c) point c
 d) $3.5 \times 10^8 \text{ m/s}^2$
 25.87 a) $R = \frac{\rho_0 L}{A} \left(1 - \frac{1}{e}\right)$, $I = \frac{V_0 A}{\rho_0 L \left(1 - \frac{1}{e}\right)}$
 b) $E(x) = \frac{V_0 e^{-x/L}}{L \left(1 - \frac{1}{e}\right)}$,
 $V_0 \left(\frac{e^{-x/L}}{1 - \frac{1}{e}} - \frac{1}{e}\right)$
 c) $V(x) = \frac{1}{1 - \frac{1}{e}}$

Chapter 26

- 26.1 $3R/4$
 26.3 22.5 W
 26.5 a) 3.50 A b) 4.50 A c) 3.15 A d) 3.25 A
 26.7 0.769 A
 26.9 a) 8.80Ω b) 3.18 A c) 3.18 A d) 5.09 V ,
 7.63 V , 15.3 V e) 16.2 W , 24.3 W , 48.5 W
 f) resistor with greater resistance
 26.11 a) 8.00 A , 12.0 A b) 84.0 V
 26.13 5.00Ω ; $I_3 = 8.00 \text{ A}$, $I_4 = 9.00 \text{ A}$,
 $I_6 = 4.00 \text{ A}$, $I_{12} = 3.00 \text{ A}$
 26.15 a) $I_1 = 1.50 \text{ A}$, $I_2 = I_3 = I_4 = 0.500 \text{ A}$
 b) $P_1 = 10.1 \text{ W}$, $P_2 = P_3 = P_4 = 1.12 \text{ W}$; R_1
 glows brightest c) $I_1 = 1.33 \text{ A}$, $I_2 = I_3 = 0.667 \text{ A}$ d) $P_1 = 8.00 \text{ W}$, $P_2 = P_3 = 2.00 \text{ W}$
 e) R_1 glows less brightly; R_2 and R_3 glow brighter
 26.17 18.0 V , 3.00 A
 26.19 a) 2 resistors in parallel and that combination in series with 3 more resistors b) 10 resistors in parallel c) 3 resistors in parallel d) 2 resistors in parallel and that combination in series with 4 resistors in parallel
 26.21 a) 0.100 A b) 4.0 W , 8.0 W c) 12.0 W
 d) 0.300 A , 0.150 A e) 36.0 W , 18.0 W
 f) 54.0 W g) in series, the $800\text{-}\Omega$ bulb is brighter; in parallel, the $400\text{-}\Omega$ bulb is brighter
 h) parallel connection
 26.23 1010 s
 26.25 a) 2.00 A b) 5.00Ω c) 42.0 V
 d) 3.50 A
 26.27 a) 8.00 A b) $\epsilon_1 = 36.0 \text{ V}$, $\epsilon_2 = 54.0 \text{ V}$
 c) 9.00Ω
 26.29 a) 1.60 A , 1.40 A , 0.20 A b) 10.4 V

- 26.31 a) 36.4 V b) 0.500 A
 26.33 a) 2.14 V , a is higher
 b) 0.050 A , downward; 0
 26.35 a) 0.641Ω b) 975Ω
 26.37 a) 17.9 V b) 22.7 V c) 21.4%
 26.39 a) 543Ω b) 1.88 mA
 c) 1824Ω , 608Ω , 203Ω
 26.41 a) $0.849 \mu\text{F}$ b) 2.89 s
 26.43 a) 4.21 ms b) 0.125 A
 26.45 $192 \mu\text{C}$
 26.47 13.6 A
 26.49 a) 0.937 A b) 0.606 A
 26.51 a) $133 \mu\text{C}$ b) 8.87 V , 9.13 V
 c) 8.87 V for both d) $67.4 \mu\text{C}$
 26.53 900 W
 26.55 12.1Ω
 26.57 a) $13.6 \mu\Omega$ b) $2.14 \times 10^{-8} \Omega \cdot \text{m}$
 26.59 a) 2.2 A , 4.4 V , 9.7 W b) 16.3 W , brighter
 26.61 a) 18.7Ω b) 7.5Ω
 26.63 $I_1 = 0.848 \text{ A}$, $I_2 = 2.14 \text{ A}$, $I_3 = 0.171 \text{ A}$
 26.65 $I_2 = 5.21 \text{ A}$, $I_4 = 1.11 \text{ A}$, $I_5 = 6.32 \text{ A}$
 26.67 a) $+0.22 \text{ V}$ b) 0.464 A
 26.69 0.447Ω (end-to-end), 0.423Ω (in parallel)
 26.71 a) 186 V , upper terminal positive
 b) 3.00 A , upward c) 20.0Ω
 26.73 a) $P_1 + P_2$ b) $\frac{P_1 P_2}{P_1 + P_2}$
 26.75 a) 1.35 W b) 8.31 ms c) 0.337 W
 26.77 a) -12.0 V b) 1.71 A c) 4.21Ω
 26.79 a) 114 V b) 263 V c) 266 V
 26.81 b) 1897Ω
 26.83 a) 24.8 V , 65.2 V b) 3840Ω c) 62.6 V
 d) no
 26.85 $1.7 \text{ M}\Omega$, $3.1 \mu\text{F}$
 26.87 a) 19.4 s , 31.4 time constants b) yes
 26.89 a) $P = \frac{Q_0^2}{RC^2} e^{-2t/RC}$
 26.93 b) 4 segments c) $3.2 \text{ M}\Omega$, 4.0×10^{-3}
 d) 3.4×10^{-4} e) 0.88

Chapter 27

- 27.1 a) $(-6.68 \times 10^{-4} \text{ N})\hat{k}$
 b) $(6.68 \times 10^{-4} \text{ N})\hat{i} + (7.27 \times 10^{-4} \text{ N})\hat{j}$
 27.3 a) positive b) 0.0505 N
 27.5 9490 km/s
 27.7 a) $B_x = -0.175 \text{ T}$, $B_z = -0.256 \text{ T}$
 b) B_y is not determined c) $0, 90^\circ$
 27.9 a) 1.46 T , in the xz -plane at 40° from the $+x$ -axis toward the $-z$ -axis b) $7.47 \times 10^{-16} \text{ N}$, in the xz -plane at 50° from the $-x$ -axis toward the $-z$ -axis.
 27.11 a) 3.05 mWb b) 1.83 mWb c) 0
 27.13 -0.78 mWb
 27.15 a) 0.160 mT , into the page b) $0.111 \mu\text{s}$
 27.17 $7.93 \times 10^{-10} \text{ N}$, toward the south
 27.19 a) $4.94 \times 10^{-21} \text{ kg} \cdot \text{m/s}$
 b) $2.31 \times 10^{-23} \text{ kg} \cdot \text{m}^2/\text{s}$
 27.21 a) 835 km/s b) 26.2 ns c) 7.27 kV
 27.23 a) 107 T b) no
 27.25 0.838 mT
 27.27 a) $(1.60 \times 10^{-14} \text{ N})\hat{j}$ b) yes
 c) helix, no d) 1.40 cm
 27.29 a) 4.81 kN/C c) yes
 27.31 0.0445 T , out of the page
 27.33 a) 4.92 km/s b) $9.96 \times 10^{-26} \text{ kg}$
 27.35 2.0 cm
 27.37 a) 13.4 kA , no
 27.39 0.724 N , 63.4° below the direction of the current in the upper wire segment
 27.41 a) 817 V b) 113 m/s^2
 27.43 a) $-ILB\hat{j}$ b) yes
 27.45 b) $F_{cd} = 1.20 \text{ N}$ b) $0.420 \text{ N} \cdot \text{m}$
 27.47 a) A_2 b) 290 rad/s^2
 27.49 -2.42 J
 27.51 a) 1.13 A b) 3.69 A c) 98.2 V
 d) 362 W
 27.53 a) 4.7 mm/s b) $+4.5 \times 10^{-3} \text{ V/m}$, $+z$ -direction c) $53 \mu\text{V}$
 27.55 a) $-\frac{F_2}{qv_1}\hat{j}$ b) $F_2/\sqrt{2}$

A-18 Answers to Odd-Numbered Problems

- 27.57 a) 8.3×10^6 m/s b) 0.14 T
 27.59 3.45 T, perpendicular to the initial velocity of the coin
 27.61 a) 8.9×10^{-13} J = 5.5 MeV b) 77 ns
 c) 1.2 T d) same as in (a)
 27.63 a) $-3.89 \mu\text{C}$
 b) $(7.60 \times 10^{14} \text{ m/s}^2)\hat{i} + (5.70 \times 10^{14} \text{ m/s}^2)\hat{j}$
 c) 2.90 cm d) 2.88×10^7 Hz
 e) (0.0290 m, 0, 0.874 m)
 27.65 9τ
 27.67 1.6 mm
 $\frac{Mg \tan \theta}{LB}$, right to left
 27.71 $B_r(r) = -\beta r/2$
 27.73 a) 8.46 mT b) 27.2 cm c) 2.2 cm, yes
 27.75 1.80 N, to the left
 27.77 0.024 T, +y-direction
 27.79 a) 0.0442 N·m, clockwise b) stretched
 c) 7.98 mJ
 27.81 2.39 A
 27.83 $-(0.444 \text{ N})\hat{j}$
 27.85 b) (0, 0) to (0, L): $\frac{1}{2}B_0LI\hat{i}$
 (0, L) to (L, L): $-IB_0L\hat{j}$
 (L, L) to (L, 0): $-\frac{1}{2}B_0LI\hat{i}$
 (L, 0) to (0, 0): 0
 c) $-IB_0L\hat{j}$
 27.87 a) 2.52 m/s b) 7.58 A c) 0.198 Ω
 27.89 a) 5.14 m b) 1.72 μs c) 6.08 mm
 d) 3.05 cm

Chapter 28

- 28.1 a) $-(19.2 \mu\text{T})\hat{k}$ b) 0 c) $(19.2 \mu\text{T})\hat{i}$
 d) $(6.79 \mu\text{T})\hat{i}$
 28.3 a) 60.0 nT, out of the page, at A and B
 b) 0.120 μT , out of the page c) 0
 28.5 a) 0 b) $-(1.31 \mu\text{T})\hat{k}$ c) $-(0.462 \mu\text{T})\hat{k}$
 d) $(1.31 \mu\text{T})\hat{j}$
 28.7 a) (i) $\frac{\mu_0 qv}{8\pi d^2}$, into the page (ii) 0
 (iii) $\frac{\mu_0 qv}{4\pi d^2}$, out of the page
 b) $\frac{\mu_0 q^2 v'v}{16\pi d^2}$, attractive c) 1.00×10^{-6}
 28.9 $(97.5 \text{ nT})\hat{k}$
 28.11 a) 0.440 μT , out of the page
 b) 16.7 nT, out of the page c) 0
 28.13 a) $(5.00 \times 10^{-11} \text{ T})\hat{j}$
 b) $-(5.00 \times 10^{-11} \text{ T})\hat{i}$
 c) $(-1.77 \times 10^{-11} \text{ T})(\hat{i} - \hat{j})$ d) 0
 28.15 17.6 μT , into the page
 28.17 a) 0.8 mT b) 40 μT , 20 times larger
 28.19 25 μA
 28.21 a) 10.0 A
 b) at all points directly above the wire
 c) at all points directly east of the wire
 28.23 a) $-(0.10 \mu\text{T})\hat{i}$
 b) 2.19 μT , at 46.8° from the x-axis to the z-axis c) $(7.9 \mu\text{T})\hat{i}$
 28.25 a) 0 b) 6.67 μT , toward the top of the page
 c) 7.54 μT , to the left
 28.27 a) 0 b) 0 c) 0.40 mT, to the left
 28.29 a) at P: 41 μT , into the page; at Q: 25 μT , out of the page b) at P: 9.0 μT , out of the page; at Q: 9.0 μT , into the page
 28.31 a) 6.00 μN , repulsive b) 24.0 μN
 28.33 46 $\mu\text{N/m}$, repulsive, no
 28.35 0.38 μA
 28.37 $\frac{\mu_0 |I_1 - I_2|}{4R}$, 0
 28.39 a) 9.42 mT b) 0.134 mT
 28.41 18.0 A, counterclockwise
 28.43 a) 305 A b) $-3.83 \times 10^{-4} \text{ T} \cdot \text{m}$
 28.45 a) $\mu_0 I/2\pi r$ b) 0

- 28.47 $r = R/2$ and $2R$
 28.49 a) 1790 turns/m b) 63.0 m
 28.51 a) 3.72 MA b) 124 kA c) 237 A
 28.53 1.11 mT
 28.55 a) 72.5 mA b) 19.5 mA
 28.57 a) (i) 1.13 mT (ii) 4.68 MA/m (iii) 5.88 T
 28.59 a) 1.00 μT , into the page
 b) $(7.49 \times 10^{-8} \text{ N})\hat{j}$
 28.61 a) in the plane of the wires, between them, 0.300 m from the 75.0-A wire
 b) in the plane of the wires, 0.200 m from the 25.0-A wire and 0.600 m from the 75.0-A wire
 28.63 a) $5.7 \times 10^{12} \text{ m/s}^2$, away from the wire
 b) 32.5 N/C, away from the wire c) no
 28.65 $5.59 \times 10^{-18} \text{ N}$
 28.67 a) 2.00 A, out of the page
 b) 2.13 μT , to the right c) 2.06 μT
 28.69 a) 11.1 $\mu\text{N/m}$, -y-direction
 b) 11.1 $\mu\text{N/m}$, +y-direction
 28.71 23.2 A
 28.73 $2.21 \times 10^{-3} \text{ N} \cdot \text{m}$
 28.75 a) $\frac{\mu_0 NIa^2}{2} \left\{ \frac{1}{[(x+a/2)^2 + a^2]^{3/2}} + \frac{1}{[(x-a/2)^2 + a^2]^{3/2}} \right\}$
 c) $\left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 NI}{a}$ d) 20.2 mT e) 0, 0
 28.77 a) $\frac{3I}{2\pi R^3}$ b) (i) $B = \frac{\mu_0 I r^2}{2\pi R^3}$ (ii) $B = \frac{\mu_0 I}{2\pi r}$
 28.79 a) 0 b) $B = \frac{\mu_0 I}{2\pi r} \frac{r^2 - a^2}{b^2 - a^2}$ c) $B = \frac{\mu_0 I}{2\pi r}$
 28.81 b) $B = \frac{\mu_0 I_0}{2\pi r}$ c) $\frac{I_0 r^2}{a^2} \left(2 - \frac{r^2}{a^2}\right)$
 d) $B = \frac{\mu_0 I_0 r}{2\pi a^2} \left(2 - \frac{r^2}{a^2}\right)$
 28.83 a) $B = \frac{1}{2} \mu_0 I n$, +x-direction
 b) $B = \frac{1}{2} \mu_0 I n$, -x-direction
 28.85 $7.73 \times 10^{-25} \text{ A} \cdot \text{m}^2 = 0.0834 \mu\text{B}$
 28.87 b) $\frac{1}{2g} \left(\frac{\mu_0 Q_0^2}{4\pi \lambda R C d}\right)^2$

Chapter 29

- 29.1 a) 17.1 mV b) 28.5 mA
 29.3 a) $Q = NBA/R$ b) no
 29.5 a) 34 V b) counterclockwise
 29.7 a) $\mu_0 i/2\pi r$, into the page b) $\frac{\mu_0 i}{2\pi r} Ldr$
 c) $\frac{\mu_0 iL}{2\pi} \ln(b/a)$ d) $\frac{\mu_0 L}{2\pi} \ln(b/a) \frac{di}{dt}$
 e) 0.506 μV
 29.9 a) 5.44 mV b) clockwise
 29.11 a) bAv b) clockwise
 c) bAv , counterclockwise
 29.13 10.4 rad/s
 29.15 a) counterclockwise b) clockwise
 c) no induced current
 29.17 a) a to b b) b to a c) b to a
 29.19 a) clockwise b) no induced current
 c) counterclockwise
 29.21 13.2 mA, counterclockwise
 29.23 a) 0.675 V b) point b c) 2.25 V/m, b to a
 d) point b e) (i) 0 (ii) 0
 29.25 46.2 m/s = 103 mph, no
 29.27 a) 3.00 V b) b to a c) 0.800 N, to the right
 d) 6.00 W for each
 29.29 a) counterclockwise b) 42.4 mW
 29.31 35.0 m/s, to the right
 29.33 a) 2.55 V, point a b) 3.38 V, point a c) 0
 d) 4.23 V
 29.35 a) $\pi r \int \frac{dB}{dt}$ b) $\frac{r_1}{2} \frac{dB}{dt}$ c) $\frac{R^2}{2r_2} \frac{dB}{dt}$

- e) $\frac{\pi R^2}{4} \frac{dB}{dt}$ f) $\pi R^2 \frac{dB}{dt}$ g) $\pi R^2 \frac{dB}{dt}$
 29.37 9.21 A/s
 29.39 0.950 mV
 29.41 2.34
 29.43 a) 0.599 nC b) 6.00 mA c) 6.00 mA
 29.45 a) 0.15 V/m b) 38 V/m·s
 c) $3.4 \times 10^{-10} \text{ A/m}^2$ d) 53.3 μT
 29.47 a) $B = 0$, $\vec{M} = -(4.38 \times 10^4 \text{ A/m})\hat{i}$
 b) $\vec{B} = (15.0 \text{ T})\hat{i}$, $M = 0$
 29.49 a) 3.7 A b) 1.33 mA c) counterclockwise
 29.51 16.2 μV
 29.53 a) $\frac{\mu_0 Iabv}{2\pi r(r+a)}$ b) clockwise
 29.55 190 rpm
 29.57 a) 17.9 mV b) a to b
 29.59 b) $\frac{RF}{B^2 L^2}$
 29.61 1.2 V
 29.63 $\mu_0 W/4\pi$
 29.65 a) $\frac{\mu_0 I v}{2\pi} \ln(1 + L/d)$ b) point a c) 0
 29.67 a) 0.165 V b) 0.165 V c) 0, 0.0412 V
 29.69 a) $B^2 a^2 v/R$
 29.71 point a: $\frac{qr}{2} \frac{dB}{dt}$, to the left; point b: $\frac{qr}{2} \frac{dB}{dt}$, toward the top of the page; point c: 0
 29.73 5.0 s
 29.75 a) 196 $\mu\text{A/m}^2$ b) 3.00 nA/m² c) 7.82 MHz
 d) 90°
 29.77 a) a to b b) $\frac{Rmg \tan \phi}{L^2 B^2 \cos \phi}$ c) $\frac{mg \tan \phi}{LB}$
 d) $\frac{Rm^2 g^2 \tan^2 \phi}{L^2 B^2}$ e) $\frac{Rm^2 g^2 \tan^2 \phi}{L^2 B^2}$

Chapter 30

- 30.1 a) 0.270 V, yes b) 0.270 V
 30.3 6.32 μH
 30.5 a) 1.96 H b) 7.11 mWb
 30.7 a) 1940 b) 800 A/s
 30.9 a) 0.250 H b) 0.450 mWb
 30.11 a) 4.68 mV b) point a
 30.13 a) 1000 b) 2.09 Ω
 30.15 b) 0.111 μH
 30.17 2850
 30.19 a) 0.161 T b) 10.3 kJ/m³ c) 0.129 J
 d) 40.2 μH
 30.21 91.7 J
 30.23 a) 2.40 A/s b) 0.800 A/s c) 0.413 A
 d) 0.750 A
 30.25 a) 17.3 μs b) 30.7 μs
 30.27 a) 0.250 A b) 0.137 A c) 32.9 V, point c
 d) 0.462 ms
 30.29 a) $P = (4.50 \text{ W})(1 - e^{-(3.20 \text{ s}^{-1})t})$
 b) $P_R = (4.50 \text{ W})(1 - e^{-(3.20 \text{ s}^{-1})t})^2$
 c) $P_L = (4.50 \text{ W})(e^{-(3.20 \text{ s}^{-1})t} - e^{-(6.40 \text{ s}^{-1})t})$
 30.33 a) 25.0 mH b) 90.0 nC c) 0.540 μJ
 d) 6.58 mA
 30.35 a) 105 rad/s, 0.0596 s b) 0.720 mC
 c) 4.32 mJ d) -0.542 mC
 e) -0.050 A, counterclockwise
 f) $U_c = 2.45 \text{ mJ}$, $U_L = 1.87 \text{ mJ}$
 30.37 a) 7.50 μC b) 15.9 kHz c) 0.0212 J
 30.39 a) 298 rad/s b) 83.8 Ω
 30.43 a) 0.288 μH b) 14.2 μV
 30.47 20 km/s, about 30 times smaller
 30.49 a) $\frac{\mu_0 i}{2\pi r}$ b) $\frac{\mu_0 i^2 l}{4\pi r} dr$ c) $\frac{\mu_0 i^2 l}{4\pi} \ln(b/a)$
 d) $\frac{\mu_0 I}{2\pi} \ln(b/a)$
 30.51 a) 5.00 H b) 31.7 m, no
 30.53 a) 0.281 J b) 0.517 J c) 0.236 J
 30.57 222 μF , 9.31 μH
 30.59 13.0 mA, 184 A/s
 30.61 a) 0 A, 20.0 V b) 0.267 A, 0 V
 c) 0.147 A, 9.0 V

- 30.63 a) solenoid c) 50 V d) 3.5 A
e) 4.3 Ω, 43 mH
- 30.65 a) $A_1 = A_4 = 0.800$ A, $A_2 = A_3 = 0$;
 $V_1 = 40.0$ V, $V_2 = V_3 = V_4 = V_5 = 0$
b) $A_1 = 0.480$ A, $A_2 = 0.160$ A, $A_3 = 0.320$ A,
 $A_4 = 0$; $V_1 = 24.0$ V, $V_2 = 0$, $V_3 = V_4 = V_5 =$
 16.0 V c) 192 μC
- 30.67 a) $A_1 = A_4 = 0.455$ A, $A_2 = A_3 = 0$
b) $A_1 = 0.585$ A, $A_2 = 0.320$ A,
 $A_3 = 0.160$ A, $A_4 = 0.107$ A
- 30.69 a) 60.0 V b) point a c) 60.0 V d) point c
e) -96.0 V f) point b g) 156 V h) point d
- 30.71 a) 0; $v_{ac} = 0$, $v_{cb} = 36.0$ V
b) 0.180 A; $v_{ac} = 9.0$ V, $v_{cb} = 27.0$ V
c) $i_0 = (0.180 \text{ A})(1 - e^{-t/(0.020 \text{ s})})$,
 $v_{ac} = (9.0 \text{ V})(1 - e^{-t/(0.020 \text{ s})})$,
 $v_{cb} = (9.0 \text{ V})(3.00 + e^{-t/(0.020 \text{ s})})$
- 30.75 a) $i_1 = \frac{\mathcal{E}}{R_1}$, $i_2 = \frac{\mathcal{E}}{R_2} (1 - e^{-R_2 t/L})$
b) $i_1 = \frac{\mathcal{E}}{R_1}$, $i_2 = \frac{\mathcal{E}}{R_2}$ c) $i = \frac{\mathcal{E}}{R_2} e^{-(R_1+R_2)t/L}$
d) 21.2 Ω, 12.7 V e) 35.3 mA
- 30.77 a) $d = D \left(\frac{L - L_0}{L_f - L_0} \right)$
b) 0.63024 H, 0.63048 H, 0.63072 H, 0.63096 H
c) 0.63000 H, 0.62999 H, 0.62999 H, 0.62998 H
d) oxygen
- 30.79 a) $i_1 = \frac{\mathcal{E}}{R_1} (1 - e^{-R_1 t/L})$, $i_2 = \frac{\mathcal{E}}{R_2} e^{-t/R_2 C}$,
 $q_2 = \mathcal{E}C (1 - e^{-t/R_2 C})$
b) $i_1 = 0$, $i_2 = 9.60$ mA c) $i_1 = 1.92$ A,
 $i_2 = 0$ d) 1.6 mA e) 9.4 mA f) 0.22 s

Chapter 31

- 31.1 1.06 A
31.3 a) 31.8 V b) 0
31.5 a) 90°, leads b) 193 Hz
31.7 13.3 μF
31.9 a) 1510 Ω b) 0.239 H c) 497 Ω
d) 16.6 μF
- 31.11 a) (12.5 V) cos[(480 rad/s)t] b) 7.17 V
- 31.13 a) $i = (0.0253 \text{ A}) \cos[(720 \text{ rad/s})t]$
b) 180 Ω
c) $v_L = -(4.56 \text{ V}) \sin[(720 \text{ rad/s})t]$
- 31.15 a) 601 Ω b) 0.0499 A c) -70.6°, lags
d) 9.98 V, 4.99 V, 33.3 V
- 31.17 50.0 V
31.19 a) 40.0 W b) 0.167 A c) 720 Ω
31.21 b) 76.7 V
31.23 a) 45.8°, 0.697 b) 344 Ω c) 155 V
d) 48.6 W e) 48.6 W f) 0 g) 0
- 31.25 a) 0.302 b) 0.370 W c) 0.370 W, 0, 0
31.27 a) 113 Hz, 15.0 mA b) 7.61 mA, lag
31.29 a) 150 V b) 150 V, 1290 V, 1290 V
c) 37.5 W
- 31.31 a) 1.00 b) 75.0 W c) 75.0 W
31.33 a) 115 Ω b) 146 Ω c) 146 Ω
31.35 a) 10 b) 2.40 A c) 28.8 W d) 500 Ω
31.37 a) use a step-down transformer with $N_2/N_1 = 1/2$
b) 6.67 A b) 36.0 Ω
- 31.39 0.124 H
31.41 3.59×10^7 rad/s
31.43 230 Ω
31.45 a) inductor b) 0.133 H
31.47 a) 0.831 b) 161 V
31.49 a) inductor: 1.15 A, 31.6 V; resistor: 1.15 A,
57.5 V; capacitor: 1.15 A, 14.6 V
b) all change; inductor: 0.860 A, 47.3 V; resistor:
0.860 A, 43.0 V; capacitor: 0.860 A, 5.46 V
- 31.51 $\frac{V_{\text{out}}}{V_s} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$
- 31.55 a) 102 Ω b) 0.882 A c) 270 V
31.57 b) yes c) 0 e) no
31.59 a) $I_R = V/R$, $I_C = V\omega C$, $I_L = V/\omega L$
d) 159 Hz e) 0.50 A
f) 0.50 A, 0.050 A, 0.050 A

- 31.61 a) L and C b) $1/2$ for each
31.63 a) 0.750 A b) 160 Ω c) 341 Ω, 619 Ω
d) 341 Ω
31.65 0, $I_0/\sqrt{3}$
31.67 a) decreases by $1/2$ b) increases by 2
c) decreases by $1/2$ d) no
- 31.69 a) $\frac{V}{\sqrt{R^2 + \frac{9L}{4C}}}$ b) $\sqrt{\frac{L}{C}} \frac{2V}{\sqrt{R^2 + \frac{9L}{4C}}}$
c) $\sqrt{\frac{L}{C}} \frac{V/2}{\sqrt{R^2 + \frac{9L}{4C}}}$ d) $\frac{2LV^2}{R^2 + \frac{9L}{4C}}$
e) $\frac{1}{2} \frac{LV^2}{R^2 + \frac{9L}{4C}}$
- 31.71 a) 75.8 V b) 13.6 Ω
31.73 a) $V_R/2$ b) 0 c) 0
31.75 a) 0.400 A b) 36.9° c) (400 - 300i) Ω,
500 Ω d) (0.320 + 0.240i) A, 0.400 A
e) 36.9° f) $V_{R\text{-cpx}} = (128 + 96i)$ V,
 $V_{L\text{-cpx}} = (-120 + 160i)$ V,
 $V_{C\text{-cpx}} = (192 - 256i)$ V

Chapter 32

- 32.1 a) 1.28 s b) 8.15×10^{13} km
32.3 13.3 nT, +y-direction
32.5 3.0×10^{18} Hz, 3.3×10^{-19} s,
 6.3×10^{10} rad/m
- 32.7 a) 6.94×10^{14} Hz b) 375 V/m c) $E(x, t) =$
(375 V/m) cos [(1.45 × 10⁷ rad/m)x -
(4.36 × 10¹⁵ rad/s)t], $B(x, t) =$
(1.25 μT) cos [(1.45 × 10⁷ rad/m)x -
(4.36 × 10¹⁵ rad/s)t]
- 32.9 a) (i) 60 kHz (ii) 6.0 × 10¹³ Hz
(iii) 6.0 × 10¹⁶ Hz b) (i) 4.62×10^{-14} m =
 4.62×10^{-5} nm
(ii) 508 m = 5.08 × 10¹¹ nm
- 32.11 a) +y-direction b) 0.149 mm c) $\vec{B} =$
(1.03 mT) \hat{i} cos [(4.22 × 10⁴ rad/m)y -
(1.265 × 10¹³ rad/s)t]
- 32.13 a) 361 m b) 0.0174 rad/m
c) 5.22×10^6 rad/s d) 0.0144 V/m
- 32.15 a) 0.381 μm b) 0.526 μm c) 1.38
d) 1.90
- 32.17 a) 330 W/m² b) 500 V/m, 1.7 μT
32.19 a) 11 μW/m² b) 0.30 nT c) 840 W
32.21 2.5×10^{25} W
32.23 12.0 V/m, 40.0 nT
32.25 850 kW
32.27 a) 0.18 mW b) 274 V/m, 0.913 μT
c) 0.18 mJ/s d) 0.010 W/cm²
- 32.29 a) 637 W/m² b) 693 V/m, 2.31 μT
c) 2.12 μJ/m³
- 32.31 a) 30.5 cm b) 2.46 GHz c) 2.11 GHz
32.33 a) 7.10 mm b) 3.55 mm
c) 1.56×10^8 m/s
32.35 a) 4.38 mm b) 4.38 mm c) 4.38 mm
32.37 a) 0.375 mJ b) 4.08 mPa
c) 604 nm, 3.70×10^{14} Hz
d) 30.3 kV/m, 101 μT
- 32.39 a) 71.6 MW/m² b) 232 kV/m, 774 μT
c) 35.8 MW/m² d) 637 W/m²
- 32.41 a) 0.00602 W/m² b) 2.13 V/m, 7.10 nT
c) 1.20 pN, no
32.43 a) 840 V/m, 2.80 μT
b) 1.56 μJ/m³ for each c) 15.3 pJ
32.45 a) at $r = R$: 64 MW/m², 0.21 Pa; at $r = R/2$:
260 MW/m², 0.85 Pa b) no
32.47 3.89×10^{-13} rad/s²
32.49 a) $\rho I/\pi a^2$, in the direction of the current
b) $\mu_0 I/2\pi a$, counterclockwise if the current is
out of the page
c) $\frac{\rho I^2}{2\pi^2 a^3}$, radially inward

- d) $\frac{\rho I^2}{\pi a^2} = I^2 R$
- 32.51 a) 23.6 h b) throw it
32.53 a) 9.75×10^{-15} W/m²
b) 2.71 μV/m, 9.03×10^{-15} T, 67.3 ms
c) 3.25×10^{-23} Pa
d) 0.190 m
- 32.55 a) $\frac{4\rho G\pi MR^3}{3r^2}$ b) $\frac{LR^2}{4cr^2}$ c) 0.19 μm, no
32.57 b) 1.39×10^{-11} c) 2.54×10^{-8}

Chapter 33

- 33.1 39.4°
33.3 a) 2.04×10^8 m/s
b) 442 nm
33.5 a) 1.55 b) 550 nm
33.7 a) 47.5° b) 66.0°
33.9 2.51×10^8 m/s
33.11 a) 2.34 b) 82°
33.13 a) $f, n\lambda, nv$
b) $f, (n/n')\lambda, (n/n')v$
- 33.15 71.8°
33.17 a) 51.3° b) 33.6°
33.19 a) 58.1° b) 22.8°
33.21 1.77
33.23 462 nm
33.25 0.6°
33.27 0.375I₀
33.29 a) I₀/2, 0.125I₀, 0.0938I₀ b) 0
33.31 a) 1.40 b) 35.5°
33.33 $\arccos\left(\frac{\cos\theta}{\sqrt{2}}\right)$
- 33.35 6.38 W/m²
33.37 a) I₀/2, 0.250I₀, 0.125I₀, linearly polarized
along the axis of each filter b) I₀/2 linearly
polarized along the axis of the filter; 0
- 33.41 a) 46.7° b) 13.4°
33.43 72.1°
33.45 1.28
33.47 1.53
33.49 1.84
33.51 a) 48.6° b) 48.6°
33.53 39.1°
33.55 0.23°, about the same
33.59 b) 38.9° c) 5.0°
33.61 a) 35° b) I_p = 19.9 W/m², I₀ = 10.1 W/m²
33.63 23.3°
33.67 a) $\Delta = 2\theta_a^A - 6 \arcsin\left(\frac{\sin\theta_a^A}{n}\right) + 2\pi$
b) $\cos^2\theta_2 = \frac{n^2 - 1}{8}$
c) violet: $\theta_2 = 71.55^\circ$, $\Delta = 233.2^\circ$;
red: $\theta_2 = 71.94^\circ$, $\Delta = 230.1^\circ$

Chapter 34

- 34.1 39.2 cm to the right of the mirror, 4.85 cm
34.3 9.0 cm, tip of the lead
34.5 b) 33.0 cm to the left of the vertex, 1.20 cm,
inverted, real
34.7 0.213 mm
34.9 18.0 cm from the vertex, 0.50 cm, erect, virtual
34.11 b) 28.0 cm, inverted c) 8.00 cm
34.13 a) concave b) 2.50 cm, 5.00 cm
34.15 2.67 cm
34.17 3.30 m
34.19 a) at the center of the bowl, 1.33 b) no
34.21 39.5 cm
34.23 8.35 cm to the left of the vertex, 0.326 mm,
erect
34.25 a) 107 cm to the right of the lens, 17.8 mm,
real, inverted b) same as part (a)
34.27 a) 71.2 cm to the right of the lens b) -2.97
34.29 3.69 cm, 2.82 cm to the left of the lens
34.31 1.67
34.33 a) 18.6 mm b) 19 mm from the cornea
c) 0.61 mm, real, inverted
34.35 a) 36.0 cm to the right of the lens
b) 180 cm to the left of the lens

A-20 Answers to Odd-Numbered Problems

- c) 7.20 cm to the left of the lens
d) 13.8 cm to the left of the lens
- 34.37 26.3 cm from the lens, 12.4 mm, erect, same side
- 34.39 a) 200 cm to the right of the first lens, 4.80 cm
b) 150 cm to the right of the second lens, 7.20 cm
- 34.41 a) 53.0 cm b) real c) 2.50 mm, inverted
- 34.43 10.2 m
- 34.45 a) 1.4×10^{-4} b) 5.3×10^{-4}
c) 1.5×10^{-3}
- 34.47 a) 85 mm b) 135 mm
- 34.49 a) $f/11$ b) $1/480$ s = 2.1 ms
- 34.51 a) convex b) 50 mm to 56 mm
- 34.53 a) 80.0 cm b) 76.9 cm
- 34.55 49.4 cm, 2.02 diopters
- 34.57 -1.37 diopters
- 34.59 a) 6.06 cm b) 4.12 mm
- 34.61 a) 640 b) 43
- 34.63 a) 8.37 mm b) 21.4 c) -297
- 34.65 a) -6.33 b) 1.90 cm c) 0.127 rad
- 34.67 a) 0.661 m b) 59.1
- 34.69 7.20 m/s
- 34.71 $h/2$
- 34.73 a) 20.0 cm b) 39.0 cm
- 34.75 a) 46 cm from the mirror, on the opposite side of the mirror, virtual b) 29 mm, erect c) no
- 34.77 51 m/s
- 34.79 b) 2.4 cm, -0.133
- 34.81 2.00
- 34.83 a) -3.3 cm b) virtual
c) 1.9 cm to the right of the vertex at the right-hand end of the rod d) real, inverted, 1.06 mm
- 34.85 a) 58.7 cm, converging b) 4.48 mm, virtual
- 34.87 a) 6.48 mm b) no, behind the retina
c) 19.3 mm from the cornea, in front of the retina
- 34.89 50.3 cm
- 34.91 10.6 cm
- 34.93 a) 0.24 m b) 0.24 m
- 34.95 72.1 cm to the right of the surface vertex
- 34.97 0.80 cm
- 34.99 -26.7 cm
- 34.101 1.24 cm above the page
- 34.103 a) 46.7 m b) 35.0 m
- 34.105 134 cm to the left of the object
- 34.107 a) 3.5 cm b) 7.0 cm c) 100 cm
d) 57 e) no
- 34.109 4.17 diopters
- 34.111 a) 30.9 cm b) 29.2 cm
- 34.113 d) 36.0 cm, 21.6 cm, $d = 1.2$ cm
- 34.115 a) 552 b) 25.8 cm
- 34.117 a) 4f
- 34.119 b) 1.35 cm

Chapter 35

- 35.1 a) 14 cm, 48 cm, 82 cm, 116 cm, 150 cm
b) 31 cm, 65 cm, 99 cm, 133 cm
- 35.3 b) 427 Hz b) 0.796 m
- 35.5 0.75 m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m, 8.25 m
- 35.7 a) 2.0 m b) constructively
c) 1.0 m, destructively
- 35.9 1.14 mm
- 35.11 0.83 mm
- 35.13 a) 39 b) $\pm 73.3^\circ$
- 35.15 12.6 cm
- 35.17 1200 nm
- 35.19 a) $0.750I_0$ b) 80 nm
- 35.21 1670 rad
- 35.23 71.4 m
- 35.25 114 nm
- 35.27 0.0234°
- 35.29 a) 55.6 nm b) (i) 2180 nm (ii) 11.0 wavelengths
- 35.31 a) 514 nm, green b) 603 nm, orange
- 35.33 $0.11 \mu\text{m}$
- 35.35 0.570 mm
- 35.37 1.54 mm
- 35.39 a) 96.0 nm b) no, no
- 35.41 a) 1.58 mm (green), 1.72 mm (orange)
b) 3.45 mm (violet), 4.74 mm (green), 5.16 mm (orange) c) $9.57 \mu\text{m}$
- 35.43 1.730
- 35.45 $0^\circ, 27.3^\circ, 66.5^\circ$

- 35.47 1.57
- 35.49 above centerline: $3.14^\circ, 15.9^\circ, 29.5^\circ, 45.4^\circ, 68.6^\circ$; below centerline: $9.45^\circ, 22.5^\circ, 37.0^\circ, 55.2^\circ$
- 35.51 $6.8 \times 10^{-5} (\text{C}^\circ)^{-1}$
- 35.53 $\lambda/2d$, independent of m
- 35.55 b) 0.72 m
- 35.57 1.42
- 35.59 b) $I = I_0 \cos^2 \left\{ \frac{\pi [d \sin \theta + L(n-1)]}{\lambda_0} \right\}$
c) $\sin \theta = \frac{m\lambda_0 - L(n-1)}{d}$
- 35.61 14.0

Chapter 36

- 36.1 506 nm
- 36.3 a) 226 b) $\pm 83.0^\circ$
- 36.5 9.07 m
- 36.7 a) 63.8 cm
b) $\pm 22.1^\circ, \pm 34.3^\circ, \pm 48.8^\circ, \pm 70.1^\circ$
 $\pm 16.0^\circ, \pm 33.4^\circ, \pm 55.6^\circ$
- 36.9 0.920 μm
- 36.11 0.920 μm
- 36.13 a) 580 nm b) 0.128
- 36.15 a) 6.75 mm b) $2.43 \mu\text{W}/\text{m}^2$
- 36.17 a) 668 nm b) $9.36 \times 10^{-5} I_0$
- 36.19 a) $\pm 13.0^\circ, \pm 26.7^\circ, \pm 42.4^\circ, \pm 64.1^\circ$
b) $2.08 \text{ W}/\text{m}^2$
- 36.21 a) 3 b) 2
- 36.23 a) 0.0627, 0.125° b) $0.249I_0, 0.0256I_0$
- 36.25 a) 1 & 3, 2 & 4 b) 1 & 2, 3 & 4
c) 1 & 3, 2 & 4
- 36.27 15.0 μm (width), 45.0 μm (separation)
- 36.29 a) 4790 slits/cm b) $19.1^\circ, 40.8^\circ$ c) no
- 36.31 a) yes b) 13.3 nm
- 36.33 a) 4830 lines/cm b) $\pm 37.7^\circ, \pm 66.5^\circ$
- 36.35 $10.5^\circ, 21.3^\circ, 33.1^\circ$
- 36.37 a) 17,500 b) yes
c) (i) 587.8170 nm (ii) 587.7834 nm
(iii) $587.7834 \text{ nm} < \lambda < 587.8170 \text{ nm}$
- 36.39 0.232 nm
- 36.41 0.559 nm
- 36.43 1.88 m
- 36.45 92 cm
- 36.47 1.45 m
- 36.49 a) 77 m (Hubble), 1100 km (Arecibo)
b) 1500 km
- 36.51 30.2 μm
- 36.53 a) 78 b) $\pm 80.8^\circ$ c) $555 \mu\text{W}/\text{m}^2$
- 36.55 1.68
- 36.57 a) 1.80 mm b) 0.796 mm
- 36.61 b) for $3\pi/2$: any two slits separated by one other slit; for the other cases: any two slits separated by three other slits
- 36.63 360 nm
- 36.65 second
- 36.67 c) ± 2.61 rad
- 36.69 387 km
- 36.71 1.40

Chapter 37

- 37.1 bolt A
- 37.3 $0.867c$, no
- 37.5 a) $0.998c$ b) 126 m
- 37.7 1.12 h, in the spacecraft
- 37.9 92.5 m
- 37.11 a) 0.66 km b) 49 μs , 15 km c) 0.45 km
- 37.13 a) 3570 m b) 90.0 μs c) 89.2 μs
- 37.15 a) 0.806c b) 0.974c c) 0.997c
- 37.17 a) toward b) 0.385c
- 37.19 0.784c
- 37.21 0.611c
- 37.23 0.837c, away from
- 37.25 a) 0.159c b) 172 million dollars
- 37.27 $3.06p_0$
- 37.29 a) 0.866c b) 0.608c
- 37.31 a) $5.49 \times 10^{15} \text{ m/s}^2$
b) $9.26 \times 10^{14} \text{ m/s}^2$
- 37.33 a) 0.866c b) 0.986c
- 37.35 a) 0.450 nJ b) $1.94 \times 10^{-18} \text{ kg} \cdot \text{m/s}$
c) 0.968c

- 37.37 a) $3.3 \times 10^{-13}\%$ b) 4.0×10^{-13} g, increases, no
- 37.39 a) $1.11 \times 10^3 \text{ kg}$ b) 52.1 cm
- 37.41 a) 0.867 nJ b) 0.270 nJ c) 0.452
- 37.43 a) 5.34 pJ (nonrel), 5.65 pJ (rel), 1.06
b) 67.8 pJ (nonrel), 331 pJ (rel), 4.88
- 37.45 a) 2.06 MV
b) 0.330 pJ = 2.06 MeV
- 37.47 a) $4.2 \times 10^9 \text{ kg} = 4.6 \times 10^6$ tons
b) 1.5×10^{13} y
- 37.49 a) $\Delta = 8.42 \times 10^{-6}$ b) 34.0 GeV
- 37.51 0.700c
- 37.53 a) 0.995c b) 1%
- 37.55 a) $\Delta = 9 \times 10^{-9}$ b) 7000m
- 37.57 0.168 MeV
- 37.59 a) $4c/5$ b) c c) (i) 145 MeV (ii) 625 MeV
d) (i) 117 MeV (ii) 469 MeV
- 37.65 b) $\Delta x' = \sqrt{(\Delta x)^2 - c^2(\Delta t)^2}$ c) 14.4 ns
- 37.67 0.357c, receding
- 37.69 a) 1000 y, 866 y, 1.4×10^{19} J, 14%
b) 505 y, 71 y, 5.5×10^{20} J, 550%
c) 501 y, 7.1 y, 6.3×10^{21} J, 6300%
- 37.71 2.04×10^{-13} N
- 37.75 a) toward us at 13.1 km/s, 39.4 km/s
b) 5.96×10^9 m, about 0.040 times the earth-sun distance; $5.55 \times 10^{29} \text{ kg} = 0.279 m_{\text{sun}}$
- 37.77 a) 2494 MeV b) 2.526 c) 987.4 MeV

Chapter 38

- 38.1 a) $K_2 = 4K_1$ b) $E_2 = 2E_1$
- 38.3 $5.77 \times 10^{14} \text{ Hz}$, $1.28 \times 10^{-27} \text{ kg} \cdot \text{m/s}$,
 $3.84 \times 10^{-19} \text{ J} = 2.40 \text{ eV}$
- 38.5 a) $5.00 \times 10^{14} \text{ Hz}$ b) 1.13×10^{19} photons/s
c) no
- 38.7 a) 4.8 eV b) $6.1 \times 10^{-34} \text{ J} \cdot \text{s}$
d) f_m , ϕ , and the horizontal-axis intercept are different, the slope is the same
- 38.9 249 km/s
- 38.11 2.14 eV
- 38.13 a) 264 nm b) 4.70 eV
- 38.15 0.311 nm, same
- 38.17 1.13 keV
- 38.19 0.0714 nm, 180°
- 38.21 a) $4.39 \times 10^{-4} \text{ nm}$ b) 0.04294 nm
c) 300 eV, loss d) 300 eV
- 38.23 51.0°
- 38.25 a) $1.27 \times 10^{-14} \text{ J}$ b) $9.46 \times 10^{-14} \text{ J}$
c) 2.10 pm, less
- 38.27 $1.19 \times 10^{-27} \text{ kg} \cdot \text{m/s}$, $1.96 \times 10^{-29} \text{ kg} \cdot \text{m/s}$
- 38.29 a) 1.04 eV b) 1200 nm c) $2.50 \times 10^{14} \text{ Hz}$
d) $4.14 \times 10^{-7} \text{ eV}$
- 38.31 a) $4.56 \times 10^{14} \text{ Hz}$ b) 658 nm
c) 1.89 eV d) $6.58 \times 10^{-34} \text{ J} \cdot \text{s}$
- 38.33 a) 5.07 mJ b) 11.3 W
c) 1.49×10^{16} photons/s
- 38.35 a) $6.99 \times 10^{-24} \text{ kg} \cdot \text{m/s}$ b) 705 eV
- 38.37 $6.28 \times 10^{-24} \text{ kg} \cdot \text{m/s}$, 59.4°
- 38.39 a) $5 \times 10^{-33} \text{ m}$ b) 4×10^{-9} deg
c) 0.1 mm
- 38.41 a) 319 eV, $1.06 \times 10^7 \text{ m/s}$ b) 3.89 nm
- 38.43 a) 4.85 pm b) 0.256 MeV

Chapter 39

- 39.1 a) 0.155 nm b) $8.46 \times 10^{-14} \text{ m}$
- 39.3 a) $2.37 \times 10^{-24} \text{ kg} \cdot \text{m/s}$
b) $3.08 \times 10^{-18} \text{ J} = 19.3 \text{ eV}$
- 39.5 a) 0.332 nm, equals the circumference of the orbit
b) 1.33 nm, $1/4$ the circumference of the orbit
- 39.7 a) $8.8 \times 10^{-36} \text{ m}$ b) no
- 39.9 a) 62.0 nm (photon), 0.274 nm (electron)
b) 4.96 eV (photon), $2.41 \times 10^{-5} \text{ eV}$ (electron) c) ≈ 250 nm, electron
- 39.11 $3.90 \times 10^{-34} \text{ m}$, no
- 39.13 a) 0.0607 V b) 248 eV c) 20.5 μm

- 39.15 a) 7.3×10^6 m/s b) 150 eV
 c) 12 keV d) electron
 39.17 0.432 eV
 39.19 a) $2.07^\circ, 4.14^\circ$ b) 1.81 cm
 39.21 a) 8260 b) electron
 39.23 a) 3.63 MeV b) 3.63 MeV
 c) 1.32×10^7 m/s
 39.25 3.16×10^{-34} kg·m²/s
 39.27 a) -218 eV, 16 times b) 218 eV, 16 times
 c) 7.60 nm d) $1/4$ hydrogen radius
 39.29 a) 2.18×10^6 m/s, 1.09×10^6 m/s,
 7.27×10^5 m/s
 b) 1.53×10^{-16} s, 1.22×10^{-15} s,
 4.13×10^{-15} s c) 8.2×10^6
 39.31 a) -17.50 eV, -4.38 eV, -1.95 eV, -1.10 eV,
 -0.71 eV b) 378 nm
 39.33 a) -5.08 eV b) -5.64 eV
 39.35 5.32×10^{21} photons/s
 39.37 4.00×10^{17} photons/s
 39.39 a) 1.2×10^{-33} b) 3.5×10^{-17}
 c) 5.9×10^{-9}
 39.41 a) 2060 K b) 1410 nm
 39.43 1.06 mm, microwave
 39.45 a) 1.7T b) 0.58
 39.47 a) 97 nm, no b) 8.2×10^9 m, $12R_{\text{sun}}$ c) no
 39.49 a) 4.40×10^{-32} m/s b) no
 39.51 not valid
 39.53 6.34×10^{-14} eV
 39.55 a) 1.69×10^{-28} kg b) -2.53 keV
 c) 0.655 nm
 39.57 a) 12.1 eV b) 3; 103 nm, 122 nm, 657 nm
 39.59 a) 0.90 eV
 39.61 a) 5×10^{49} photons/s b) 30,000
 39.63 29,800 K
 39.65 a) $h/2mc$
 b) 6.61×10^{-16} m, independent of n
 39.67 a) $I(f) = \frac{2\pi hf^5}{c^3(e^{hf/kT} - 1)}$
 c) $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴
 39.69 a) 12 eV b) 0.15 mV, 7300 m/s
 c) 8.2×10^{-8} V, 4.0 m/s
 39.71 a) no b) 2.52 V
 39.73 a) $E = c\sqrt{2mK}$ b) photon
 39.75 1.66×10^{-17} m, no
 39.77 b) $\Delta = \frac{m^2c^2\lambda^2}{2h^2}$ c) $\Delta = 8.50 \times 10^{-8}$
 39.79 a) $\frac{h}{mc\sqrt{15}}$ b) (i) 1.53 MeV, 6.26×10^{-13} m
 (ii) 2810 MeV, 3.41×10^{-16} m
 39.81 a) 1.1×10^{-20} kg·m/s b) 19 MeV
 c) $U_{\text{cool}} = -0.29$ MeV, no
 39.83 7.0×10^{-36} kg, 2.9×10^{-8}
 39.85 a) 1.1×10^{-35} m/s b) 2.3×10^{27} y, no
 39.87 a) no b) 1.51 V c) 1.51 eV, about $1/4$ the
 potential energy of the NaCl crystal
 d) 1240 eV, yes
 39.89 a) $2d \sin \theta = m\lambda$ b) 53.3° c) smaller
 39.91 a) 248 eV b) 0.0603 eV
 39.93 a) $F = -\frac{A|x|}{x}, x \neq 0$ b) $E = \frac{3}{2} \left(\frac{h^2 A^2}{m} \right)^{1/3}$

Chapter 40

- 40.1 $\Psi(x, t) = Ae^{-i(4.27 \times 10^{10} \text{ m}^{-1})x} e^{-i(1.05 \times 10^{17} \text{ s}^{-1})t}$
 40.3 a) $8\pi/k$ b) $4\omega/k$, yes
 40.5 a) $\lambda/4, 3\lambda/4, 5\lambda/4, \dots$ b) 0, $\lambda/2, \lambda, 3\lambda/2, \dots$
 40.7 no
 40.11 a) 1.6×10^{-67} J b) 1.3×10^{-33} m/s,
 1.0×10^{33} s c) 4.9×10^{-67} J d) no
 40.13 1.66×10^{-10} m
 40.15 0.61 nm
 40.19 a) 0, $L/2, L$ b) $L/4, 3L/4$ c) yes

- 40.23 a) 6.0×10^{-10} m (twice the width of the box),
 1.1×10^{-24} kg·m/s b) 3.0×10^{-10} m
 (same as the width of the box),
 2.2×10^{-24} kg·m/s
 c) 2.0×10^{-10} m ($2/3$ the width of the box),
 3.3×10^{-24} kg·m/s
 40.25 b) yes
 40.27 3.43×10^{-10} m
 40.29 $-A \left(\frac{2mE}{\hbar^2} \right) \sin \left(\frac{\sqrt{2mE}}{\hbar} x \right) -$
 $B \left(\frac{2mE}{\hbar^2} \right) \cos \left(\frac{\sqrt{2mE}}{\hbar} x \right)$
 40.31 22 fm
 40.33 a) 4.4×10^{-8} b) 4.2×10^{-4}
 40.35 $1/\sqrt{2}$
 40.37 a) 0.0013 b) 10^{-143}
 40.39 1.11×10^{-33} J = 6.93×10^{-15} eV,
 2.22×10^{-33} J = 1.39×10^{-14} eV
 40.41 a) 0.21 eV b) 5900 N/m
 40.43 $(2n + 1) \frac{\hbar}{2}$, increases with n
 40.45 a) 5.89×10^{-3} eV b) 106 μm c) 0.0118 eV
 40.47 a) $|\Psi(x, t)|^2 = \frac{2}{L} \left[1 - \cos \left(\frac{4\pi^2 \hbar t}{mL^2} \right) \right]$
 b) $\frac{4\pi^2 \hbar}{mL^2}$
 40.49 a) $\psi(x) = \frac{\sin k_0 x}{k_0 x}$
 b) 0, $\pm \pi/k_0$; $w_x = 2\pi/k_0, L$ c) $2L$
 d) h , which is greater than $\hbar/2$
 40.51 $B = \left(\frac{k_1 - k_2}{k_1 + k_2} \right) A, C = \left(\frac{2k_2}{k_1 + k_2} \right) A$
 40.53 a) 19.2 μm b) 11.5 μm
 40.55 a) 0.818 b) 0.500 c) yes
 40.57 a) $(2/L) dx$ b) 0 c) $(2/L) dx$
 40.61 a) $A = C, B \sin kL + A \cos kL = De^{-\kappa L}$,
 where $k = \frac{\sqrt{2mE}}{\hbar}$ b) $kB = \kappa C$,
 $kB \cos kL - kA \sin kL = -\kappa De^{-\kappa L}$
 40.65 6.63×10^{-34} J = 4.14×10^{-15} eV;
 1.33×10^{-33} J = 8.30×10^{-15} eV
 40.69 b) 134 eV
 40.71 a) $E_n = \frac{(2n)^2 \hbar^2}{8mL^2}, n = 1, 2, \dots$
 b) $E_n = \frac{(2n + 1)^2 \hbar^2}{8mL^2}, n = 0, 1, 2, \dots$
 c) same
 d) part (a)'s are odd, part (b)'s are even
 40.73 a) $x = \pm \sqrt{2E/K'}$
 c) underestimates

Chapter 41

- 41.1 a) 1 b) 3
 41.3 3.51 nm
 41.5 (2, 2, 1): $x = L/2, y = L/2$; (2, 1, 1): $x = L/2$;
 (1, 1, 1): none
 41.7 a) 0 b) 3.65×10^{-34} kg·m²/s
 c) 3.16×10^{-34} kg·m²/s
 d) 5.27×10^{-35} kg·m²/s e) $1/6$
 41.9 4
 41.11 $1.414\hbar, 19.49\hbar, 199.5\hbar$; as n increases, the
 maximum L gets closer to $n\hbar$.
 41.13 a) 18 b) $m_l = -4, 153.4^\circ$
 c) $m_l = +4, 26.6^\circ$
 41.17 a) 0.468 T b) 3
 41.19 a) 9 b) 3.47×10^{-5} eV
 c) 2.78×10^{-4} eV
 41.21 a) 2.5×10^{30} rad/s b) 2.5×10^{13} m/s, not
 valid since $v > c$

- 41.23 1.68×10^{-4} eV, $m_s = +1/2$
 41.25 g
 41.27 $n = 1, l = 0, m_l = 0, m_s = \pm 1/2$; 2 states;
 $n = 2, l = 0, m_l = 0, m_s = \pm 1/2$; 2 states;
 $n = 2, l = 1, m_l = \pm 1, m_s = \pm 1/2$; 6 states
 41.29 a) $1s^2 2s^2$ b) $1s^2 2s^2 2p^6 3s^2$, magnesium
 c) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$, calcium
 41.31 4.18 eV
 41.33 a) $1s^2 2s^2 2p$ b) -30.6 eV c) $1s^2 2s^2 2p^6 3s^2 3p$
 d) -13.6 eV
 41.35 a) -13.6 eV b) -3.4 eV
 41.37 a) 8.95×10^{17} Hz, 3.71 keV, 3.35×10^{-10} m
 b) 1.68×10^{18} Hz, 6.96 keV, 1.79×10^{-10} m
 c) 5.48×10^{18} Hz, 22.7 keV, 5.47×10^{-11} m
 41.39 $3E_{1,1,1}$
 41.41 a) $1/64 = 0.0156$ b) 7.50×10^{-4}
 c) 2.06×10^{-3}
 41.43 a) $4\pi A^2 r^2 e^{-2ar^2} dr$ b) $1/\sqrt{2\alpha}$, no
 41.45 a) $E = \hbar \left[(n_x + n_y + 1) \omega_1^2 + \left(n_z + \frac{1}{2} \right) \omega_2^2 \right]$,
 with n_x, n_y , and n_z all nonnegative integers
 b) $\hbar \left(\omega_1^2 + \frac{1}{2} \omega_2^2 \right), \hbar \left(\omega_1^2 + \frac{3}{2} \omega_2^2 \right)$, c) one
 41.47 b) $n = 5$ shell
 41.49 a) 0, $\sqrt{2\hbar}, \sqrt{6\hbar}, \sqrt{12\hbar}, \sqrt{20\hbar}$
 b) 7470 nm, no
 41.51 a) 1.51e, 9.49 electrons
 b) (i) 1.8 (ii) -2.75 eV
 41.53 a) $2a$ b) 0.238
 41.55 b) 0.176
 41.57 b) $(\theta_L)_{\text{max}} = \arccos(-\sqrt{1 - 1/n})$
 41.59 $4a$, same
 41.61 $2 \rightarrow 1, 1 \rightarrow 0, 0 \rightarrow -1, \frac{e\hbar B}{2m}$;
 $1 \rightarrow 1, 0 \rightarrow 0, -1 \rightarrow -1, 0$;
 $0 \rightarrow 1, -1 \rightarrow 0, -2 \rightarrow -1, -\frac{e\hbar B}{2m}$
 41.63 3.00 T
 41.65 a) 0.99999978 = $1 - 2.2 \times 10^{-7}$ b) 0.9978
 c) 0.978
 41.67 a) 4, 20 b) $1s^4 2s^4 2p^3$
 41.69 a) 122 nm b) 1.52 pm, increase
 41.71 a) 0.188 nm, 0.250 nm
 b) 0.0471 nm, 0.0624 nm
 41.75 a) $2a/z$ b) 0.238, independent of Z

Chapter 42

- 42.1 277 nm, ultraviolet
 42.3 a) 6.1 K b) 34,600 K c) He₂ no, H₂ yes
 42.5 40.8 μm
 42.7 5.65×10^{-13} m
 42.9 a) 0.0644 nm (carbon), 0.0484 nm (oxygen)
 b) 1.45×10^{-46} kg·m², yes
 42.11 a) 1.03×10^{12} rad/s
 b) 66.3 m/s (carbon), 49.8 m/s (oxygen)
 c) 6.10×10^{-12} s
 42.13 a) 7.49×10^{-3} eV b) 166 μm
 42.15 b) $\frac{\hbar h}{2\pi I}$
 42.17 2170 kg/m³
 42.19 a) 1.12 eV
 42.21 1.20×10^6
 42.23 1.5×10^{22} states/eV
 42.25 a) 0.0233R b) 0.767%
 c) no, motion of the ions
 42.27 31.2%
 42.29 0.20 eV below the bottom of the conduction band
 42.31 a) (i) 0.0204 mA (ii) -0.0196 mA
 (iii) 26.8 mA (iv) -0.491 mA
 b) good for V between ± 1.0 mV, otherwise no
 42.33 a) 5.56 mA b) -5.18 mA, -3.77 mA
 42.35 a) 977 N/m b) 1.25×10^{14} Hz
 42.37 a) 3.8×10^{-29} C·m b) 1.3×10^{-19} C
 c) 0.81 d) 0.058
 42.39 a) 0.96 nm b) 1.8 nm
 42.41 a) 0.129 nm b) 8, 7, 6, 5, 4 c) 485 μm
 d) 118 μm , 134 μm , 156 μm , 188 μm , 234 μm

A-22 Answers to Odd-Numbered Problems

- 42.43 b) (i) 2.95 (ii) 4.73 (iii) 7.57 (iv) 0.838
(v) 5.69×10^{-9}
- 42.45 a) 1.146 cm, 2.291 cm
b) 1.171 cm, 2.341 cm; 0.025 cm ($2 \rightarrow 1$),
0.050 cm ($1 \rightarrow 0$)
- 42.47 0.274 eV, much less
- 42.49 a) $4.24 \times 10^{-47} \text{ kg} \cdot \text{m}^2$ b) (i) 4.30 μm
(ii) 4.28 μm (iii) 4.40 μm
- 42.51 2.03 eV
- 42.53 a) $4.66 \times 10^{28} \text{ atoms/m}^3$ b) 4.7 eV
- 42.55 b) $3.81 \times 10^{10} \text{ Pa} = 3.76 \times 10^5 \text{ atm}$
- 42.57 a) $1.67 \times 10^{33} \text{ m}^{-3}$ b) yes
c) $6.66 \times 10^{35} \text{ m}^{-3}$ d) no
- 42.59 a) $\frac{-2p^2}{4\pi\epsilon_0 r^3}$, attractive b) $\frac{+2p^2}{4\pi\epsilon_0 r^3}$, repulsive

Chapter 43

- 43.1 a) 14 proton, 14 neutrons
b) 37 protons, 48 neutrons
c) 81 protons, 124 neutrons
- 43.3 0.533 T
- 43.5 a) 76.21 MeV b) 76.68 MeV, 0.6%
- 43.7 0.5575 pm
- 43.9 a) 1.32 MeV b) $1.13 \times 10^7 \text{ m/s}$
- 43.11 $^{86}_{36}\text{Kr}$: 8.73 MeV/nucleon,
 $^{180}_{73}\text{Ta}$: 8.08 MeV/nucleon
- 43.13 a) $^{235}_{92}\text{U}$ b) ^{24}Mg c) ^{15}N
- 43.15 156 keV
- 43.17 a) 0.836 MeV b) 0.700 MeV
- 43.19 $5.01 \times 10^4 \text{ y}$
- 43.21 a) $4.92 \times 10^{-18} \text{ s}^{-1}$ b) 2990 kg
c) $1.24 \times 10^3 \text{ decays/s}$
- 43.23 a) 159 decays/min b) 0.43 decays/min
- 43.25 a) 0.421 decays/s b) 11.4 pCi
- 43.27 2.80 days
- 43.29 a) 2.02×10^{15}
b) 1.01×10^{15} , $3.78 \times 10^{11} \text{ decays/s}$
c) 2.53×10^{14} , $9.45 \times 10^{10} \text{ decays/s}$
- 43.31 a) 1.2 mJ b) 10 mrad, 10 mrem, 7.5 mJ
c) 6.2

- 43.33 500 rad, 2000 rem, 5.0 J/kg
- 43.35 a) 1.75 kGy, 175 krem, 1.75 kSv, 385 J
b) 1.75 kGy, 2.625 kSv, 262.5 krem, 385 J
- 43.37 a) 9.32 rad, 9.32 rem
- 43.39 a) $Z = 3, A = 6$ b) -10.14 MeV
c) 11.59 MeV
- 43.41 a) $Z = 3, A = 7$ b) 7.152 MeV c) 1.4 MeV
- 43.43 a) 173.3 MeV b) $4.42 \times 10^{23} \text{ MeV/g}$
- 43.45 1.586 MeV
- 43.47 324 MJ
- 43.53 a) 4.14 MeV b) 7.75 MeV/nucleon
- 43.55 a) ^{99}Y b) 25% c) 112 y
- 43.57 a) ^{25}Al will decay into ^{25}Mg b) β^+ or
electron capture
c) 3.255 MeV, 4.277 MeV
- 43.59 a) $^{14}\text{C} \rightarrow e^- + ^{14}\text{N} + \bar{\nu}_e$ b) 0.156 MeV
c) 13.5 kg, 3400 decays/s
d) $530 \text{ MeV/s} = 8.5 \times 10^{-11} \text{ J/s}$
e) $36 \mu\text{Gy}$, 3.6 mrad, $36 \mu\text{Sv}$, 3.6 mrem
- 43.61 0.960 MeV
- 43.63 0.001286 u, yes
- 43.65 94.3 rad, 1900 rem
- 43.67 a) 5.0×10^4 b) $10^{-15,000}$
- 43.69 29.2%
- 43.71 a) $0.96 \mu\text{J/s}$ b) 0.48 mrad/s c) 0.34 mrem
d) 6.9 days
- 43.73 $1.0 \times 10^4 \text{ y}$
- 43.75 a) 0.48 MeV
b) 3.270 MeV = $5.239 \times 10^{-13} \text{ J}$
c) $3.155 \times 10^{11} \text{ J/mol}$, more than a million
times larger
- 43.79 a) two b) 0.400 h, 1.92 h
c) 1.04×10^7 (short-lived), 2.49×10^7
(long-lived)
d) 1800 (short-lived), 4.10×10^6 (long-lived)

Chapter 44

- 44.1 69 MeV, $1.7 \times 10^{22} \text{ Hz}$, 18 fm, gamma ray
- 44.3 a) 32 MeV
- 44.5 $9.26 \times 10^6 \text{ m/s}$
- 44.7 $7.2 \times 10^{19} \text{ J}$, 70%

- 44.9 a) 1.18 T b) 3.42 MeV, $1.81 \times 10^7 \text{ m/s}$
- 44.11 a) 30.6 GeV b) 8.0 GeV
- 44.13 a) 3200 GeV b) 38.7 GeV
- 44.15 a) π^0, π^+ b) 219.1 MeV
- 44.17 $1.63 \times 10^{-25} \text{ kg}$, 97.2
- 44.19 116 MeV
- 44.21 (b) and (d)
- 44.23 (c) and (d)
- 44.27 a) 0, 1, -1, 0 b) 0, 0, 0, 1
c) $-e, 1, 0, 0$ d) $-e, 0, 0, -1$
- 44.29 $p \rightarrow e^+ + n + \nu_e$
- 44.31 a) $\bar{u} \bar{d} \bar{d}$ b) no c) yes
- 44.33 a) $3.28 \times 10^7 \text{ m/s}$ b) 1510 Mly
- 44.35 a) $1.1 \times 10^5 \text{ km/s}$ b) 1.5
- 44.37 a) 3.8 atoms/m^3 b) 320 c) 2.0×10^{27}
- 44.39 a) 5.494 MeV b) 20.58 MeV
- 44.41 -0.783 MeV , endoergic
- 44.43 966 nm
- 44.45 a) 14.0 TeV b) $1.0 \times 10^5 \text{ TeV}$
- 44.47 a) $\pi^- \rightarrow \mu^- + \text{neutrino} \rightarrow e^- + 3 \text{ neutrinos}$,
an electron and neutrinos b) 139 MeV
c) 2.24×10^{10} d) 50 Sv, 5.0 krem
- 44.49 2.494 GeV
- 44.51 a) 0, +e, 1, all lepton numbers are 0, K^+
b) 0, -e, 0, all lepton numbers are 0, π^-
c) -1, 0, 0, all lepton numbers are 0,
antineutron (\bar{n})
d) 0, +e, 0, muonic lepton number is -1, all
other lepton numbers are 0, μ^+
- 44.53 $7.5 \times 10^{-23} \text{ s}$
- 44.55 a) 0.70 rad b) 0.70 rem, no
- 44.57 c) $H_0 = \frac{dR/dt}{R}$ d) $R(t) = R_0 e^{H_0 t}$ e) no
- 44.59 b) $R/R_0 = 0.574$
c) speeding up at 300 My, slowing down at
10.2 Gy
- 44.61 230 MeV, 12.5° below the +x-axis

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NUMERICAL CONSTANTS

Fundamental Physical Constants*

Name	Symbol	Value
Speed of light in vacuum	c	2.99792458×10^8 m/s
Magnitude of charge of electron	e	$1.602176487(40) \times 10^{-19}$ C
Gravitational constant	G	$6.67428(67) \times 10^{-11}$ N·m ² /kg ²
Planck's constant	h	$6.62606896(33) \times 10^{-34}$ J·s
Boltzmann constant	k	$1.3806504(24) \times 10^{-23}$ J/K
Avogadro's number	N_A	$6.02214179(30) \times 10^{23}$ molecules/mol
Gas constant	R	8.314472(15) J/mol·K
Mass of electron	m_e	$9.10938215(45) \times 10^{-31}$ kg
Mass of proton	m_p	$1.672621637(83) \times 10^{-27}$ kg
Mass of neutron	m_n	$1.674927211(84) \times 10^{-27}$ kg
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Wb/A·m
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$ $1/4\pi\epsilon_0$	$8.854187817 \dots \times 10^{-12}$ C ² /N·m ² $8.987551787 \dots \times 10^9$ N·m ² /C ²

Other Useful Constants*

Mechanical equivalent of heat		4.186 J/cal (15° calorie)
Standard atmospheric pressure	1 atm	1.01325×10^5 Pa
Absolute zero	0 K	−273.15°C
Electron volt	1 eV	$1.602176487(40) \times 10^{-19}$ J
Atomic mass unit	1 u	$1.660538782(83) \times 10^{-27}$ kg
Electron rest energy	$m_e c^2$	0.510998910(13) MeV
Volume of ideal gas (0°C and 1 atm)		22.413996(39) liter/mol
Acceleration due to gravity (standard)	g	9.80665 m/s ²

*Source: National Institute of Standards and Technology (<http://physics.nist.gov/cuu>). Numbers in parentheses show the uncertainty in the final digits of the main number; for example, the number 1.6454(21) means 1.6454 ± 0.0021 . Values shown without uncertainties are exact.

Astronomical Data†

Body	Mass (kg)	Radius (m)	Orbit radius (m)	Orbit period
Sun	1.99×10^{30}	6.96×10^8	—	—
Moon	7.35×10^{22}	1.74×10^6	3.84×10^8	27.3 d
Mercury	3.30×10^{23}	2.44×10^6	5.79×10^{10}	88.0 d
Venus	4.87×10^{24}	6.05×10^6	1.08×10^{11}	224.7 d
Earth	5.97×10^{24}	6.38×10^6	1.50×10^{11}	365.3 d
Mars	6.42×10^{23}	3.40×10^6	2.28×10^{11}	687.0 d
Jupiter	1.90×10^{27}	6.91×10^7	7.78×10^{11}	11.86 y
Saturn	5.68×10^{26}	6.03×10^7	1.43×10^{12}	29.45 y
Uranus	8.68×10^{25}	2.56×10^7	2.87×10^{12}	84.02 y
Neptune	1.02×10^{26}	2.48×10^7	4.50×10^{12}	164.8 y
Pluto‡	1.31×10^{22}	1.15×10^6	5.91×10^{12}	247.9 y

†Source: NASA Jet Propulsion Laboratory Solar System Dynamics Group (<http://ssd.jpl.nasa.gov>), and P. Kenneth Seidelmann, ed., *Explanatory Supplement to the Astronomical Almanac* (University Science Books, Mill Valley, CA, 1992), pp. 704–706. For each body, “radius” is its radius at its equator and “orbit radius” is its average distance from the sun or (for the moon) from the earth.

‡In August 2006, the International Astronomical Union reclassified Pluto and other small objects that orbit the sun as “dwarf planets.”

UNIT CONVERSION FACTORS

Length

$$1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm} = 10^6 \mu\text{m} = 10^9 \text{ nm}$$

$$1 \text{ km} = 1000 \text{ m} = 0.6214 \text{ mi}$$

$$1 \text{ m} = 3.281 \text{ ft} = 39.37 \text{ in.}$$

$$1 \text{ cm} = 0.3937 \text{ in.}$$

$$1 \text{ in.} = 2.540 \text{ cm}$$

$$1 \text{ ft} = 30.48 \text{ cm}$$

$$1 \text{ yd} = 91.44 \text{ cm}$$

$$1 \text{ mi} = 5280 \text{ ft} = 1.609 \text{ km}$$

$$1 \text{ \AA} = 10^{-10} \text{ m} = 10^{-8} \text{ cm} = 10^{-1} \text{ nm}$$

$$1 \text{ nautical mile} = 6080 \text{ ft}$$

$$1 \text{ light year} = 9.461 \times 10^{15} \text{ m}$$

Area

$$1 \text{ cm}^2 = 0.155 \text{ in.}^2$$

$$1 \text{ m}^2 = 10^4 \text{ cm}^2 = 10.76 \text{ ft}^2$$

$$1 \text{ in.}^2 = 6.452 \text{ cm}^2$$

$$1 \text{ ft}^2 = 144 \text{ in.}^2 = 0.0929 \text{ m}^2$$

Volume

$$1 \text{ liter} = 1000 \text{ cm}^3 = 10^{-3} \text{ m}^3 = 0.03531 \text{ ft}^3 = 61.02 \text{ in.}^3$$

$$1 \text{ ft}^3 = 0.02832 \text{ m}^3 = 28.32 \text{ liters} = 7.477 \text{ gallons}$$

$$1 \text{ gallon} = 3.788 \text{ liters}$$

Time

$$1 \text{ min} = 60 \text{ s}$$

$$1 \text{ h} = 3600 \text{ s}$$

$$1 \text{ d} = 86,400 \text{ s}$$

$$1 \text{ y} = 365.24 \text{ d} = 3.156 \times 10^7 \text{ s}$$

Angle

$$1 \text{ rad} = 57.30^\circ = 180^\circ/\pi$$

$$1^\circ = 0.01745 \text{ rad} = \pi/180 \text{ rad}$$

$$1 \text{ revolution} = 360^\circ = 2\pi \text{ rad}$$

$$1 \text{ rev/min (rpm)} = 0.1047 \text{ rad/s}$$

Speed

$$1 \text{ m/s} = 3.281 \text{ ft/s}$$

$$1 \text{ ft/s} = 0.3048 \text{ m/s}$$

$$1 \text{ mi/min} = 60 \text{ mi/h} = 88 \text{ ft/s}$$

$$1 \text{ km/h} = 0.2778 \text{ m/s} = 0.6214 \text{ mi/h}$$

$$1 \text{ mi/h} = 1.466 \text{ ft/s} = 0.4470 \text{ m/s} = 1.609 \text{ km/h}$$

$$1 \text{ furlong/fortnight} = 1.662 \times 10^{-4} \text{ m/s}$$

Acceleration

$$1 \text{ m/s}^2 = 100 \text{ cm/s}^2 = 3.281 \text{ ft/s}^2$$

$$1 \text{ cm/s}^2 = 0.01 \text{ m/s}^2 = 0.03281 \text{ ft/s}^2$$

$$1 \text{ ft/s}^2 = 0.3048 \text{ m/s}^2 = 30.48 \text{ cm/s}^2$$

$$1 \text{ mi/h} \cdot \text{s} = 1.467 \text{ ft/s}^2$$

Mass

$$1 \text{ kg} = 10^3 \text{ g} = 0.0685 \text{ slug}$$

$$1 \text{ g} = 6.85 \times 10^{-5} \text{ slug}$$

$$1 \text{ slug} = 14.59 \text{ kg}$$

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$$

$$1 \text{ kg has a weight of } 2.205 \text{ lb when } g = 9.80 \text{ m/s}^2$$

Force

$$1 \text{ N} = 10^5 \text{ dyn} = 0.2248 \text{ lb}$$

$$1 \text{ lb} = 4.448 \text{ N} = 4.448 \times 10^5 \text{ dyn}$$

Pressure

$$1 \text{ Pa} = 1 \text{ N/m}^2 = 1.450 \times 10^{-4} \text{ lb/in.}^2 = 0.209 \text{ lb/ft}^2$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

$$1 \text{ lb/in.}^2 = 6895 \text{ Pa}$$

$$1 \text{ lb/ft}^2 = 47.88 \text{ Pa}$$

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 1.013 \text{ bar}$$

$$= 14.7 \text{ lb/in.}^2 = 2117 \text{ lb/ft}^2$$

$$1 \text{ mm Hg} = 1 \text{ torr} = 133.3 \text{ Pa}$$

Energy

$$1 \text{ J} = 10^7 \text{ ergs} = 0.239 \text{ cal}$$

$$1 \text{ cal} = 4.186 \text{ J (based on } 15^\circ \text{ calorie)}$$

$$1 \text{ ft} \cdot \text{lb} = 1.356 \text{ J}$$

$$1 \text{ Btu} = 1055 \text{ J} = 252 \text{ cal} = 778 \text{ ft} \cdot \text{lb}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$1 \text{ kWh} = 3.600 \times 10^6 \text{ J}$$

Mass–Energy Equivalence

$$1 \text{ kg} \leftrightarrow 8.988 \times 10^{16} \text{ J}$$

$$1 \text{ u} \leftrightarrow 931.5 \text{ MeV}$$

$$1 \text{ eV} \leftrightarrow 1.074 \times 10^{-9} \text{ u}$$

Power

$$1 \text{ W} = 1 \text{ J/s}$$

$$1 \text{ hp} = 746 \text{ W} = 550 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ Btu/h} = 0.293 \text{ W}$$