Section 4.1

Additive, Multiplicative, and Ciphered Systems of Numeration

- *What You Will Learn*
- *Additive, multiplicative, and ciphered systems of numeration*
- *Systems of Numeration*

A ***number*** is a quantity. It answers the question "How many?"

A ***numeral*** is a symbol such as , 10 or used to represent the number (amount).

Systems of Numeration

A *system of numeration* consists of a set of numerals and a scheme or rule for combining the numerals to represent numbers.

Types Of Numeration Systems

Four types of systems used by different cultures will be discussed. They are:

Additive (or repetitive)

Multiplicative

Ciphered

Place-value

Additive Systems:An additive system is one in which the number represented by a set of numerals is simply the *sum* of the values of the numerals.

It is one of the oldest and most primitive types of systems.

Examples: Egyptian hieroglyphics and Roman numerals.

Let's start with Egyptian heiroglyphics

Hindu–Arabic Numerals	Egyptian Numerals	Description
1		Staff (vertical stroke)
10		Heel bone (arch)
100		Scroll (coiled rope)
1000		Lotus flower
10,000		Pointing finger
100,000		Tadpole (or whale)
1,000,000		Astonished person

Example 1: From Egyptian to Hindu-Arabic Numerals

Write the following numeral as a Hindu-Arabic numeral.

Example 2: From Hindu-Arabic to Egyptian Numerals

Write 43,628 as an Egyptian numeral.

Roman Numerals

Two advantages over Egyptian system:

Uses the subtraction principle as well as addition principle

DC = 500 + 100 = 600

CD = 500 – 100 = 400

Uses the multiplication principle for numerals greater than 1000

Example 4: From Roman to Hindu-Arabic Numerals

Write CMLXIV as a Hindu-Arabic numeral.

Solution

It's an additive system so,

= CM + L + X + IV

= (1000 – 100) + 50 + 10 + (5 – 1)

= 900 + 50 + 10 + 4

= 964

Example 5: Writing a Roman Numeral

Write 439 as a Roman numeral.

Multiplicative Systems

Multiplicative systems are more similar to the Hindu-Arabic system which we use today.

Chinese numerals

Written vertically

零	一	二	三	四	五	六
0	1	2	3	4	5	6

七	八	九	十	百	千
7	8	9	10	100	1000

Top numeral from 1 - 9 inclusive

Multiply it by the power of 10 below it.

Example: A Traditional Chinese Numeral

Write 538 as a Chinese numeral.

Ciphered Systems

In this system, there are numerals for numbers up to and including the base and for multiples of the base.

The number (amount) represented by a specific set of numerals is the sum of the values of the numerals.

Ciphered numeration systems require the memorization of many different symbols but have the advantage that numbers can be written in a compact form.

Examples of Ciphered Systems-

We discuss in detail the Ionic Greek system

developed about 3000 B.C.

used letters of Greek alphabet as numerals

Base is 10

An iota, *ι*, placed to the left and above a numeral represents the numeral multiplied by 1000

1	α	alpha	8	η	eta
2	β	beta	9	θ	theta
3	γ	gamma	10	ι	iota
4	δ	delta	20	κ	kappa
5	ε	epsilon	30	λ	lambda
6	ϝ*	digamma	40	μ	mu
7	ζ	zeta	50	ν	nu

60	ξ	xi	400	υ	upsilon
70	ο	omicron	500	φ	phi
80	π	pi	600	χ	chi
90	ϟ*	koppa	700	ψ	psi
100	ρ	rho	800	ω	omega
200	σ	sigma	900	ϡ*	sampi
300	τ	tau			

Examples of Ciphered Systems

Hebrew

Coptic

Hindu

Brahmin

Syrian

Egyptian Hieratic

early Arabic

Ionic Greek System

Example

Example

Section 4.2
Place-Value or Positional-Value Numeration Systems

What You Will Learn

- Place-Value or Position-Value Numeration Systems

Define:Place-Value System
(or Positional-Value System)- A system where the value of the symbol depends on its position in the representation of the number.

It is the most common type of numeration system in the world today.

The most common place-value system is the Hindu-Arabic numeration system.

This is used in the United States.

A true positional-value system requires a base and a set of symbols, including a symbol for zero and one for each counting number less than the base.

The most common place-value system is the base 10 system.

It is called the decimal number system.

Hindu-Arabic System(what we use in the U.S.A)

Digits: In the Hindu-Arabic system, the *digits* are

0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

Positions: In the Hindu-Arabic system, the positional values or place values are

… 10^5, 10^4, 10^3, 10^2, 10, 1

Expanded Form

To evaluate a numeral in this system, multiply the first digit on the right by 1.

Multiply the second digit from the right by base 10.

Multiply the third digit from the right by base 10^2 or 100, and so on.

In general, we multiply the digit *n* places from the right by 10^{n-1} to show a number in expanded form.

Expanded Form

In expanded form, 1234 is written

$1234 = (1 \times 10^3) + (2 \times 10^2)$
$+ (3 \times 10) + (4 \times 1)$

or

$= (1 \times 1000) + (2 \times 100)$
$+ (3 \times 10) + 4$

Babylonian Numerals(draw them here)

Babylonian numerals	𒐕	𒌋
Hindu–Arabic numerals	1	10

Oldest known numeration system that resembled a place-value system

Developed in about 2500 B.C.

Resembled a place-value system with a base of 60, a sexagesimal system

Not a true place-value system because it lacked a symbol for zero

The lack of a symbol for zero led to a great deal of ambiguity and confusion

The positional values in the Babylonian system are

$\ldots, (60)^3, (60)^2, 60, 1$

A gap is left between characters to distinguish place values.

From right to left, the sum of the first group of numerals is multiplied by 1.

The sum of the second group is multiplied by 60.

The sum of the third group is multiplied by 60^2, and so on.

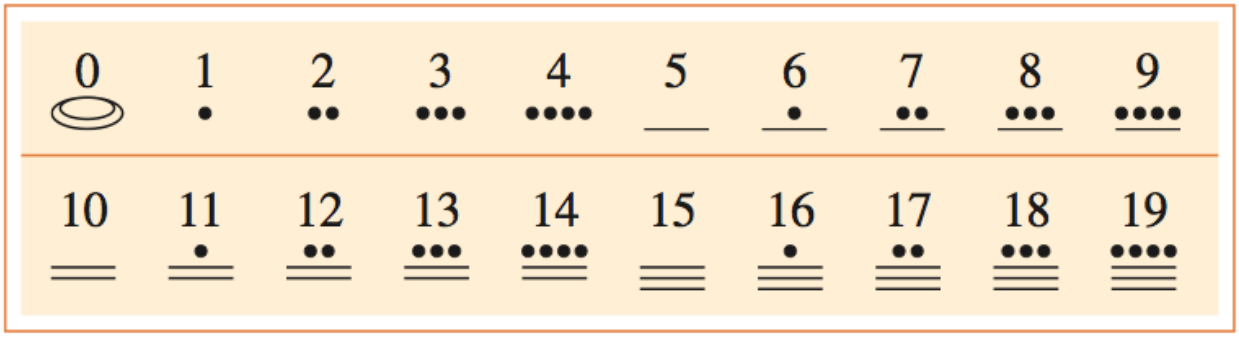
Example : The Babylonian System: A Positional-Value System

Write as a Hindu-Arabic numeral.(work through together)

Example : A Babylonian Numeral with a Blank Space

Write 7223 as a Babylonian numeral.

Mayan Numerals



Numerals are written vertically.

Units position is on the bottom.

Numeral in bottom row is multiplied by
1.

Numeral in second row is multiplied by 20.

Numeral in third row is multiplied by
18×20, or 360.

Numeral in fourth row is multiplied by
18×20^2, or 7200, and so on.

The positional values in the Mayan system are

$\ldots, 18 \times (20)^3, 18 \times (20)^2, 20, 1$

or …, 144,000, 7200, 20, 1

Example : From Mayan to Hindu-Arabic Numerals

Write as a Hindu-Arabic numeral.(do with class)

Example : From Hindu-Arabic to Mayan Numerals

Write 4025 as a Mayan numeral.do with class