Section 4.6: Elastic Potential Energy and Simple Harmonic Motion

Mini Investigation: Spring Force, page 193

Answers may vary. Sample answers:

A. The relationship between F_g and Δx is linear.

B. The slope of the best fit line of my graph is 50. This line represents the relationship between F_g and Δx, where the slope is the spring constant.

C. For the equipment used in this investigation, where k is the slope of the line of best fit, the equation is $F_g = 50\Delta x$.

Tutorial 1 Practice, page 195

1. (a) Given: $m = 0.65$ kg; $\Delta x = 0.44$ m; $g = 9.8$ m / s^2

Required: k

Analysis: The force of gravity on the mass points down. The restorative spring force on the mass points up because the spring is stretched down. To calculate the total force, subtract the magnitudes:

$$\vec{F}_g = mg \text{ [down]} = -mg \text{ [up]};\ \vec{F}_x = -k\Delta\vec{x} = k\Delta x \text{ [up]}$$

Since the mass is not accelerating, $\Sigma\vec{F} = 0$ according to Newton's second law.

Solution:

$$\begin{aligned}
\Sigma\vec{F} &= 0 \\
k\Delta x - mg &= 0 \\
k &= \frac{mg}{\Delta x} \\
&= \frac{(0.65 \text{ kg})(9.8 \text{ m/s}^2)}{0.44 \text{ m}} \\
k &= 14.5 \text{ N/m (one extra digit carried)}
\end{aligned}$$

Statement: The spring constant is 14 N/m.

(b) Given: $k = 14.5$ N / m; $\Delta x = 0.74$ m; $g = 9.8$ m / s^2

Required: m

Analysis: Use the equation $k\Delta x - mg = 0$ from (a).

Solution:

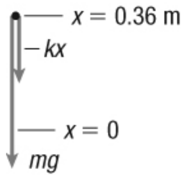
$$\begin{aligned}
k\Delta x - mg &= 0 \\
k\Delta x &= mg \\
m &= \frac{k\Delta x}{g} \\
&= \frac{(14.5 \text{ N/m})(0.74 \text{ m})}{9.8 \text{ m/s}^2} \\
m &= 1.1 \text{ kg}
\end{aligned}$$

Statement: The new mass is 1.1 kg.

2. Given: $m = 5.3$ kg; $k = 720$ N / m; $\Delta x = 0.36$ m

Required: $\vec{F}_{net}$; $\vec{a}$

Analysis: The free-body diagram for the mass is shown.



The force of gravity on the mass points down. The spring force on the mass points down because the spring is compressed upward:

$$\vec{F}_g = mg \text{ [down]},\ \vec{F}_x = -k\Delta\vec{x} = k\Delta x \text{ [down]}$$

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_x$$

Use the equation $\vec{F}_{net} = m\vec{a}$ to find the acceleration.

Solution:
$$\begin{aligned}\vec{F}_{net} &= \vec{F}_g + \vec{F}_x \\ &= mg \text{ [down]} + k\Delta x \text{ [down]} \\ &= (5.3 \text{ kg})(9.8 \text{ m/s}^2) \text{ [down]} + (720 \text{ N/m})(0.36 \text{ m}) \text{ [down]} \\ \vec{F}_{net} &= 311 \text{ N (one extra digit carried)}\end{aligned}$$

$$\begin{aligned}\vec{F}_{net} &= m\vec{a} \\ \vec{a} &= \frac{\vec{F}_{net}}{m} \\ &= \frac{311 \text{ N [down]}}{5.3 \text{ kg}} \\ \vec{a} &= 59 \text{ m/s}^2 \text{ [down]}\end{aligned}$$

Statement: The force on the mass is 310 N [down], and the acceleration is 59 m/s^2 [down].

Tutorial 2 Practice, page 196

1. Given: $m_b = 2m$; $k = 2.29 \times 10^3 \text{ N/m}$; $g = 9.8 \text{ m/s}^2$

Required: ratio of $E_{eb} : E_e$

Analysis: $E = \frac{1}{2}k(\Delta x)^2$

To find E_{eb}, we need Δx_b. Use the same equation as in Sample Problem 1, $k\Delta x \text{ [up]} - mg \text{ [down]} = 0$.

Solution: $k\Delta x_b \text{ [up]} - m_b g \text{ [down]} = 0$

$$\begin{aligned}k\Delta x_b &= m_b g \\ \Delta x_b &= \frac{m_b g}{k} \\ \Delta x_b &= \frac{2mg}{k}\end{aligned}$$

$$E_{\mathrm{e\,b}} = \frac{1}{2}k(\Delta x_{\mathrm{b}})^2 \qquad E_{\mathrm{e}} = \frac{1}{2}k(\Delta x)^2$$

$$= \frac{1}{2}k\left(\frac{2mg}{k}\right)^2 \qquad = \frac{1}{2}k\left(\frac{mg}{k}\right)^2$$

$$= \frac{1}{\cancel{2}}\cancel{k}\left(\frac{\overset{2}{\cancel{4}}\, m^2g^2}{k^{\cancel{2}}}\right) \qquad = \frac{1}{2}\cancel{k}\left(\frac{m^2g^2}{k^{\cancel{2}}}\right)$$

$$E_{\mathrm{e\,b}} = \frac{2m^2g^2}{k} \qquad E_{\mathrm{e}} = \frac{m^2g^2}{2k}$$

$$\frac{E_{\mathrm{e\,b}}}{E_{\mathrm{e}}} = \frac{\dfrac{2\cancel{m^2}\,\cancel{g^2}}{\cancel{k}}}{\dfrac{\cancel{m^2}\,\cancel{g^2}}{2\cancel{k}}}$$

$$\frac{E_{\mathrm{e\,b}}}{E_{\mathrm{e}}} = \frac{4}{1}$$

Statement: The ratio of the elastic potential energy, $E_{\mathrm{eb}} : E_{\mathrm{e}}$ is 4:1.

2. Given: $\vec{F}_x = 220$ N; $\Delta x = 0.14$ m

Required: E_{e}

Analysis: $\vec{F}_x = -k\Delta\vec{x} = k\Delta x$; $E_{\mathrm{e}} = \frac{1}{2}k(\Delta x)^2$

Solution: $\vec{F}_x = k\Delta x$

$$k = \frac{\vec{F}_x}{\Delta x}$$

$$= \frac{220\ \text{N}}{0.14\ \text{m}}$$

$k = 1570$ N/m (one extra digit carried)

$$E_{\mathrm{e}} = \frac{1}{2}k(\Delta x)^2$$

$$= \frac{1}{2}(1570\ \text{N/m})(0.14\ \text{m})^2$$

$$E_{\mathrm{e}} = 15\ \text{J}$$

Statement: The elastic potential energy of the toy is 15 J.

Tutorial 3 Practice, page 199

1. Given: $m = 105\text{ kg};\ k = 8.1\times 10^3\text{ N / m}$

Required: *f*, *T*

Analysis: Use the equations for simple harmonic motion period and frequency:

$T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{T}$

Solution: $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{105\text{ kg}}{7.6\times 10^3\text{ N/m}}}$$

$$T = 0.74\text{ s}$$

$$f = \frac{1}{T}$$

$$= \frac{1}{0.74\text{ s}}$$

$$f = 1.4\text{ Hz}$$

Statement: The period of the vibrations is 0.74 s, and the frequency is 1.4 Hz.

2. The frequency of oscillations will change if passengers are added to the car because when the mass increases, the period increases. This happens because mass is in the numerator of the equation for the period. If the period increases, the frequency decreases, because frequency is the reciprocal of period.

Section 4.6 Questions, page 200

1. Spring A is more difficult to stretch because it has a greater spring constant.

2. Given: $\vec{F} = 5\text{ N};\ \Delta x = 10\text{ mm } = 0.01\text{ m}$

Required: *k*

Analysis: The spring force opposes the applied force, so $\vec{F}_x = -5\text{ N}$. Rearrange the formula $\vec{F}_x = -k\Delta x$ to solve for *k*.

Solution: $\vec{F}_x = -k\Delta x$

$$k = -\frac{\vec{F}_x}{\Delta x}$$

$$= -\frac{(-5\text{ N})}{0.01\text{ m}}$$

$$k = 500\text{ N/m}$$

Statement: The spring constant is 500 N/m.

3. The elastic potential energy stored in a spring is the same whether it is stretched by 1.5 cm or compressed by 1.5 cm. The spring constant is exactly the same whether the spring is stretched or compressed, so the elastic potential energy must also be the same.

4. (a) Given: $k = 5.5\times10^3\text{ N/m};\ \Delta x = 2.0\text{ cm} = 0.020\text{ m}$
Required: E_e
Analysis: $E_e = \dfrac{1}{2}k(\Delta x)^2$
Solution:
$$\begin{aligned} E_e &= \frac{1}{2}k(\Delta x)^2 \\ &= \frac{1}{2}(5.5\times10^3\text{ N/m})(0.020\text{ m})^2 \\ E_e &= 1.1\text{ J} \end{aligned}$$
Statement: The elastic potential energy of the spring when it stretches 2.0 cm is 1.1 J.
(b) Given: $k = 5.5\times10^3\text{ N/m};\ \Delta x = -3.0\text{ cm} = -0.030\text{ m}$
Required: E_e
Analysis: $E_e = \dfrac{1}{2}k(\Delta x)^2$
Solution:
$$\begin{aligned} E_e &= \frac{1}{2}k(\Delta x)^2 \\ &= \frac{1}{2}(5.5\times10^3\text{ N/m})(-0.030\text{ m})^2 \\ E_e &= 2.5\text{ J} \end{aligned}$$
Statement: The elastic potential energy of the spring when it compresses 3.0 cm is 2.5 J.
5. (a) Given: $m = 0.63\text{ kg}; k = 65\text{ N/m}; g = 9.8\text{ m/s}^2$
Required: Δx
Analysis: The force of gravity on the mass points down. The restorative force on the mass points up since the spring is compressed.
$\vec{F}_g = mg\text{ [down]} = -mg\text{ [up]};\ \vec{F}_x = -k\Delta\vec{x} = k\Delta x\text{ [up]}$
Solution: Since the mass is at rest, $\Sigma\vec{F} = 0$.
$$\begin{aligned} \Sigma\vec{F} &= 0 \\ k\Delta x - mg &= 0 \\ \Delta x &= \frac{mg}{k} \\ &= \frac{(0.63\text{ kg})(9.8\text{ m/s}^2)}{65\text{ N/m}} \\ \Delta x &= 0.095\text{ m} \end{aligned}$$
Statement: The spring is compressed 0.095 m from its equilibrium position.
(b) Given: $m = 0.63\text{ kg};\ k = 65\text{ N/m};\ g = 9.8\text{ m/s}^2$
Required: $\vec{a}$
Analysis: The force of gravity on the mass points down. The spring force on the mass points up because the mass is being compressed downward.
$\vec{F}_g = mg\text{ [down]};\ \vec{F}_x = -k\Delta\vec{x} = k\Delta x\text{ [up]}$

$\vec{F}_{\text{net}} = \vec{F}_{\text{g}} + \vec{F}_x$; $\vec{F}_{\text{net}} = m\vec{a}$

Solution:

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_{\text{g}} + \vec{F}_x \\ &= mg \text{ [down]} + k\Delta x \text{ [up]} \\ &= (0.63 \text{ kg})(9.8 \text{ m/s}^2) \text{ [down]} + (65 \text{ N/m})(0.041 \text{ cm}) \text{ [up]} \\ \vec{F}_{\text{net}} &= 3.5 \text{ N [down]}\end{aligned}$$

$$\begin{aligned}\vec{F}_{\text{net}} &= m\vec{a} \\ \vec{a} &= \frac{\vec{F}_{\text{net}}}{m} \\ &= \frac{3.5 \text{ N [down]}}{0.63 \text{ kg}} \\ \vec{a} &= 5.6 \text{ m/s}^2 \text{ [down]}\end{aligned}$$

Statement: The acceleration of the mass after it falls 4.1 cm is 5.6 m/s^2 [down].

6. Given: $m = 5.2$ kg; $T = 1.2$ s

Required: k

Analysis: $T = 2\pi\sqrt{\frac{m}{k}}$

Solution:

$$\begin{aligned}T &= 2\pi\sqrt{\frac{m}{k}} \\ \frac{T}{2\pi} &= \sqrt{\frac{m}{k}} \\ \left(\frac{T}{2\pi}\right)^2 &= \frac{m}{k} \\ k &= \frac{m}{\left(\frac{T}{2\pi}\right)^2} \\ &= \frac{5.2 \text{ kg}}{\left(\frac{1.2 \text{ s}}{2\pi}\right)^2} \\ k &= 140 \text{ N/m}\end{aligned}$$

Statement: The spring constant is 140 N/m.

7. Given: $k = 1.5 \times 10^3 \text{ N / m}$; $E_{\text{e}} = 80.0$ J

Required: Δx

Analysis: $E_{\text{e}} = \frac{1}{2}k(\Delta x)^2$

Solution: $E_e = \frac{1}{2}k(\Delta x)^2$

$$\frac{2E_e}{k} = (\Delta x)^2$$

$$\Delta x = \sqrt{\frac{2E_e}{k}}$$

$$= \sqrt{\frac{2(80.0\text{ J})}{1.5\times 10^3\text{ N/m}}}$$

$$\Delta x = 0.33\text{ m}$$

Statement: The spring should be stretched 0.33 m to store 80.0 J of energy.

8. Given: $\Delta x = 15\text{ mm} = 0.015\text{ m};\ k = 400.0\text{ N / m}$

Required: W

Analysis: The work done is equal to the elastic potential energy:

$$W = E_e;\ E_e = \frac{1}{2}k(\Delta x)^2$$

Solution: $E_e = \frac{1}{2}k(\Delta x)^2$

$$= \frac{1}{2}(400.0\text{ N/m})(0.015\text{ m})^2$$

$$E_e = 0.045\text{ J}$$

Statement: The work done by the spring force acting on the spring is 4.5×10^{-2} J.

9. Given: $E_e = 7.50\text{ J};\ m = 0.20\text{ kg};\ k = 240\text{ N / m}$

Required: f; Δx

Analysis: $T = 2\pi\sqrt{\frac{m}{k}};\ f = \frac{1}{T};\ E_e = \frac{1}{2}k(\Delta x)^2$

Solution: $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{0.20\text{ kg}}{240\text{ N/m}}}$$

$$T = 0.181\text{ s (one extra digit carried)}$$

$$f = \frac{1}{T}$$

$$= \frac{1}{0.181\text{ s}}$$

$$f = 5.5\text{ Hz}$$

$$E_e = \frac{1}{2}k(\Delta x)^2$$

$$\frac{2E_e}{k} = (\Delta x)^2$$

$$\Delta x = \sqrt{\frac{2E_e}{k}}$$

$$= \sqrt{\frac{2(7.50\text{ J})}{240\text{ N/m}}}$$

$$\Delta x = 0.25\text{ m}$$

Statement: The frequency of oscillation is 5.5 Hz and the amplitude of oscillation is 0.25 m.

10. Given: $m = 5.5 \times 10^2\text{ kg}$; six cycles in 4.4 s

Required: k

Analysis: Divide 4.4 s by 6 to get T. Use the formula for the period, $T = 2\pi\sqrt{\frac{m}{k}}$, to calculate k.

Solution: $T = \frac{4.4\text{ s}}{6}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$$

$$\frac{T^2}{4\pi^2} = \frac{m}{k}$$

$$k = \frac{4\pi^2 m}{T^2}$$

$$= \frac{4\pi^2(5.5\times 10^2\text{ kg})}{\left(\frac{4.4\text{ s}}{6}\right)^2}$$

$$k = 4.0\times 10^4\text{ N/m}$$

Statement: The spring constant of either spring is 4.0×10^4 N/m.

11. Answers may vary. Sample answer: Pyon pyon shoes strap onto the outside of your regular shoes. They are made of two curved springy pieces of material joined together in a shape similar to that of a football. When you jump, the springy material increases the height you can attain. When your mass presses down on the springs, the springs press back up, causing you to jump up in the air much higher than normal.