<u>Section 4 – Topic 1</u>
<u>Arithmetic Sequences</u>

Let's look at the following sequence of numbers:
$3, 8, 13, 18, 23, \ldots$.

- The "…" at the end means that this **sequence** goes on forever.

- $3, 8, 13, 18,$ and 23 are the actual **terms** of this sequence.

- There are 5 terms in this sequence so far:

  - 3 is the 1^{st} term
  - 8 is the 2^{nd} term
  - 13 is the $\mathbf{3^{rd}}$ term
  - 18 is the $\mathbf{4^{th}}$ term
  - 23 is the $\mathbf{5^{th}}$ term

This is an example of an ***arithmetic sequence***.

- This is a sequence where each term is the **sum** of the previous term and a common difference, d.

We can represent this sequence in a table:

Term Number	Sequence Term	Term
1	a_1	3
2	a_2	8
3	a_3	13
4	a_4	$\boxed{\mathbf{18}}$
5	a_5	$\boxed{\mathbf{23}}$
$\vdots$	$\vdots$	$\vdots$
n	a_n	$\boxed{a_{n-1} + d}$

New Notation	
$f(1)$	a formula to find the 1st term
$\boxed{f(2)}$	a formula to find the 2nd term
$f(3)$	a formula to find the **3rd** term
$f(4)$	a formula to find the **4th** term
$\boxed{f(5)}$	a formula to find the **5th** term
$\vdots$	$\vdots$
$f(n)$	a formula to find the n^{th} term

How can we find the 9th term of this sequence?

By adding the common difference until you reach the 9th term.

One way is to start by finding the previous term:

Term Number	Sequence Term	Term		Function Notation	
1	a_1	3		$f(1)$	3
2	a_2	$8 = 3 + \mathbf{5}$		$f(2)$	$3 + 5$
3	a_3	$13 = 8 + \mathbf{5}$		$f(3)$	$8 + 5$
4	a_4	$18 = 13 + \mathbf{5}$		$f(4)$	$13 + 5$
5	a_5	$23 = \mathbf{18} + 5$		$f(5)$	$18 + 5$
6	a_6	$\mathbf{28 = 23 + 5}$		$f(6)$	$23 + 5$
7	a_7	$\mathbf{33 = 28 + 5}$		$f(7)$	$28 + 5$
8	a_8	$\mathbf{38 = 33 + 5}$		$f(8)$	$33 + 5$
9	a_9	$\mathbf{43 = 38 + 5}$		$f(9)$	$38 + 5$

Write a general equation that we could use to find *any* term in the sequence.

$a_n = a_{n-1} + 5$, where n is a natural number.

This is a ***recursive formula***.

- In order to solve for a term, you must know the value of its preceding term.

Can you think of a situation where the recursive formula would take a long time to use?

If you were trying to find the 20^{th} term

Let's look at another way to find unknown terms:

Term Number	Sequence Term	Term
1	a_1	3
2	a_2	$8 = 3 + 5$
3	a_3	$13 = 8 + 5 = 3 + 5 + 5$
4	a_4	$18 = 13 + 5 = 3 + 5 + 5 + 5$
5	a_5	$23 = 18 + 5 = 3 + 5 + 5 + 5 + 5$
6	a_6	$28 = 23 + 5$ $= 3 + 5 + 5 + 5 + 5$ $+ 5$
7	a_7	$33 = 28 + 5 =$ $3 + 5 + 5 + 5 + 5 + 5 + 5$
8	a_8	$38 = 33 + 5 =$ $3 + 5 + 5 + 5 + 5 + 5 + 5 + 5$
9	a_9	$43 = 38 + 5 =$ $3 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$

Function Notation	
$f(1)$	3
$f(2)$	$3 + 5(1)$
$f(3)$	$3 + 5(2)$
$f(4)$	$3 + 5(3)$
$f(5)$	$3 + 5(4)$
$f(6)$	$3 + 5(5)$
$f(7)$	$3 + 5(6)$
$f(8)$	$\mathbf{3 + 5(7)}$
$f(9)$	$\mathbf{3 + 5(8)}$

Write a general equation that we could use to find *any* term in the sequence.

$a_n = 3 + 5(n - 1)$, where n is a natural number.

$$\boldsymbol{a_n = a_1 + d(n - 1)}$$

This is an ***explicit formula***.

- To solve for a term, you need to know the first term of the sequence and the difference by which the sequence is increasing or decreasing.

Let's Practice!

1. Consider the sequence $10, 4, -2, -8, \dots$.

   a. Write a recursive formula for the sequence.
   $\boldsymbol{a_n = a_{n-1} - 6}$

   b. Write an explicit formula for the sequence.
   $\boldsymbol{a_n = 10 + (-6)(n - 1)}$

   c. Find the 42^{nd} term of the sequence.
   $\boldsymbol{a_{42} = 10 + (-6)(42 - 1)}$
   $\boldsymbol{a_{42} = 10 + (-6)(41)}$
   $\boldsymbol{a_{42} = 10 - 246}$
   $\boldsymbol{a_{42} = -236}$

Try It!

2. Consider the sequence $7, 17, 27, 37, \dots$.

   a. Find the next three terms of the sequence.
   $\boldsymbol{47, 57, 67}$

   b. Write a recursive formula for the sequence.
   $\boldsymbol{a_n = a_{n-1} + 10}$

   c. Write an explicit formula for the sequence.
   $\boldsymbol{a_n = 7 + (10)(n - 1)}$

   d. Find the 33^{rd} term of the sequence.
   $\boldsymbol{a_n = 7 + (10)(33 - 1)}$
   $\boldsymbol{a_n = 7 + (10)(32)}$
   $\boldsymbol{a_n = 7 + 320}$
   $\boldsymbol{a_n = 327}$

BEAT THE TEST!

1. Yohanna is conditioning all summer to prepare for her high school's varsity soccer team tryouts. She is incorporating walking planks into her daily workout training plan. Every day, she will complete four more walking planks than the day before.

Part A: If she starts with five walking planks on the first day, write an explicit formula that can be used to find the number of walking planks Yohanna completes on any given day.

$$a_n = 5 + (4)(n - 1)$$

Part B: How many walking planks will Yohanna do on the 12^{th} day?

Ⓐ 49
Ⓑ 53
Ⓒ 59
Ⓓ 64

Answer: A

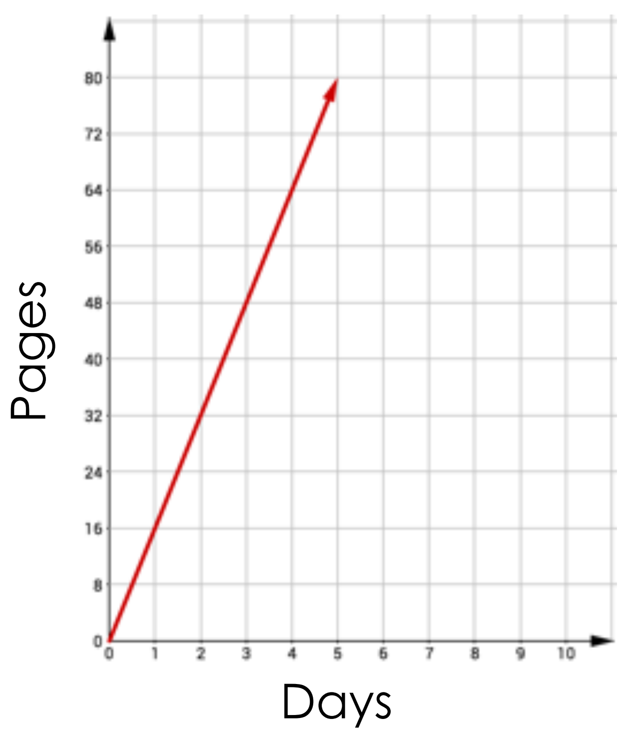
Section 4 – Topic 2
Rate of Change of Linear Functions

Génesis reads 16 pages of *The Fault in Our Stars* every day.

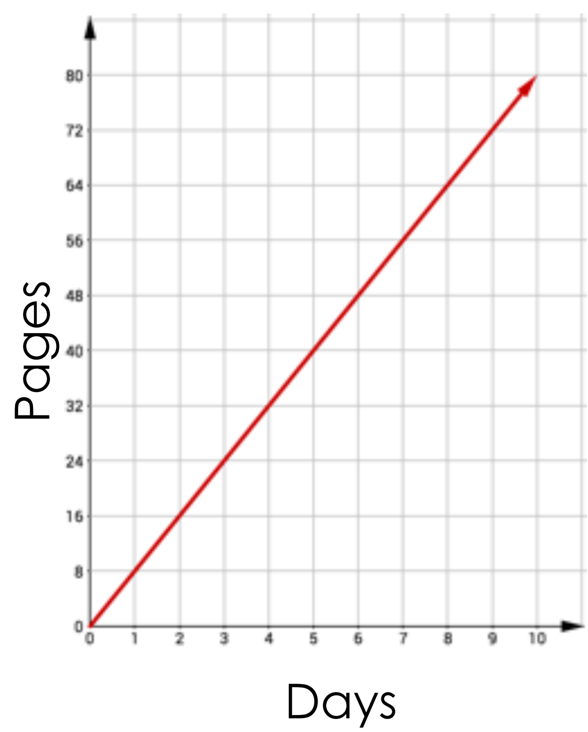
Zully reads 8 pages every day of the same book.

Represent both situations on the graphs below using the same scales for both graphs.

Graph 1: Génesis' Reading Speed



Graph 2: Zully's Reading Speed

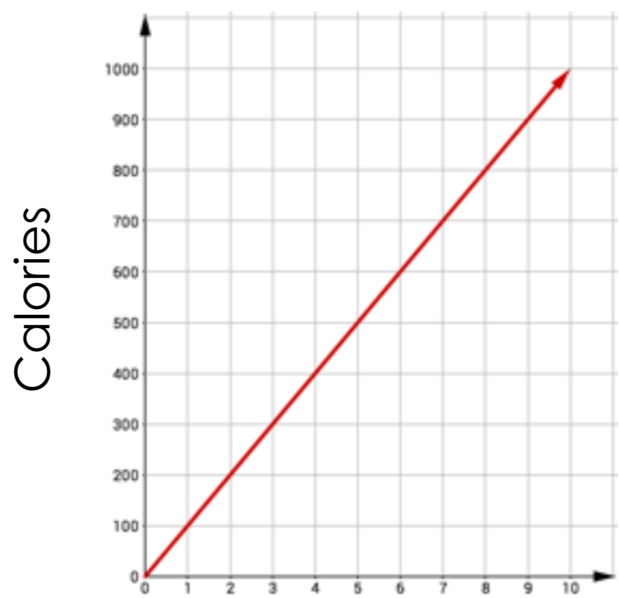


Aaron loves Cherry Coke. Each mini-can contains 100 calories.

Jacobe likes to munch on carrot snack packs. Each snack pack contains 40 calories.

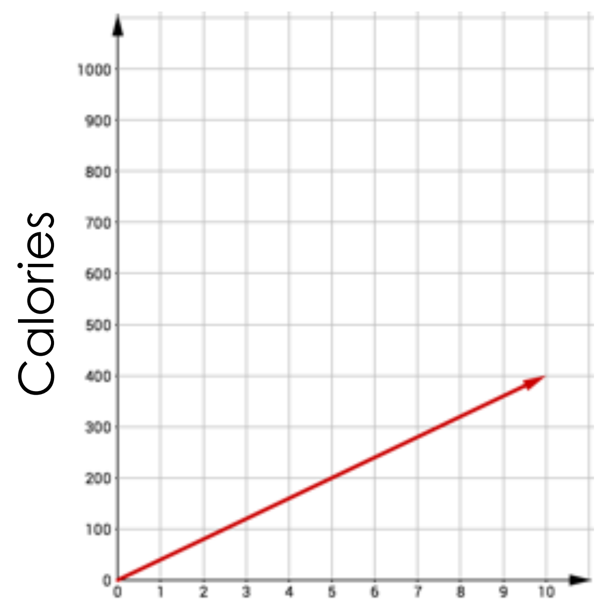
Represent both situations on the graphs below using the same scales for both graphs.

Graph 3: Aaron's Calorie Intake



Mini Coke

Graph 4: Jacobe's Calorie Intake



Carrots

In each of the graphs, we were finding the ***rate of change*** in the given situation.

What is the rate of change for each of the graphs?

Graph 1: **16 pages** per **day**

Graph 2: **8 pages** per **day**

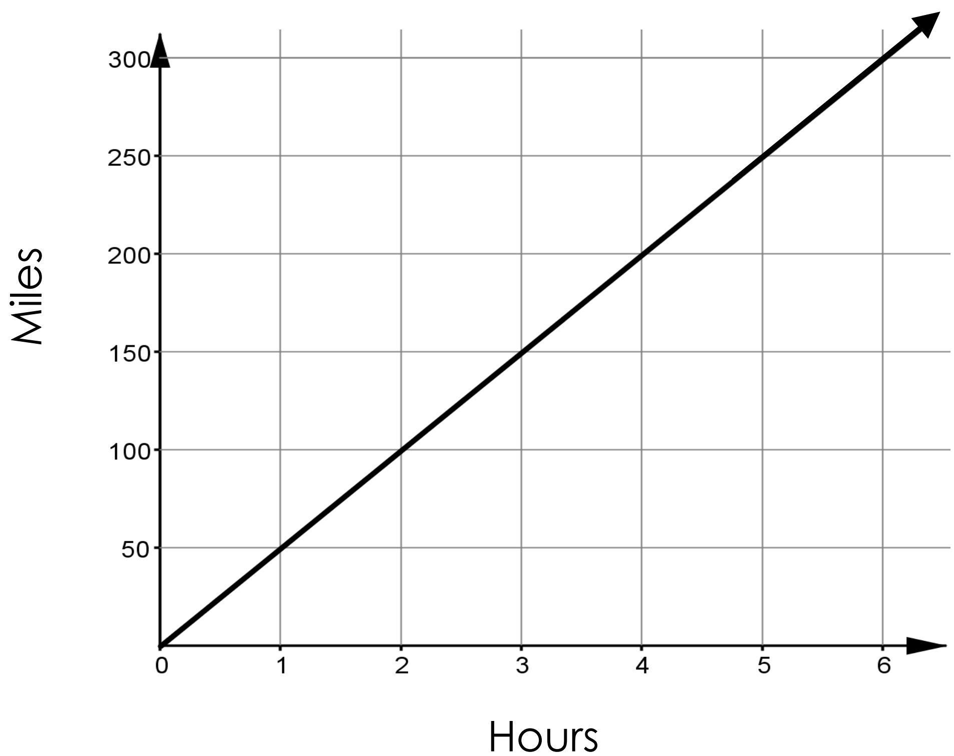
Graph 3: **100 calories** per **can**

Graph 4: **40 calories** per **pack**

This is also called the **slope** of the line.

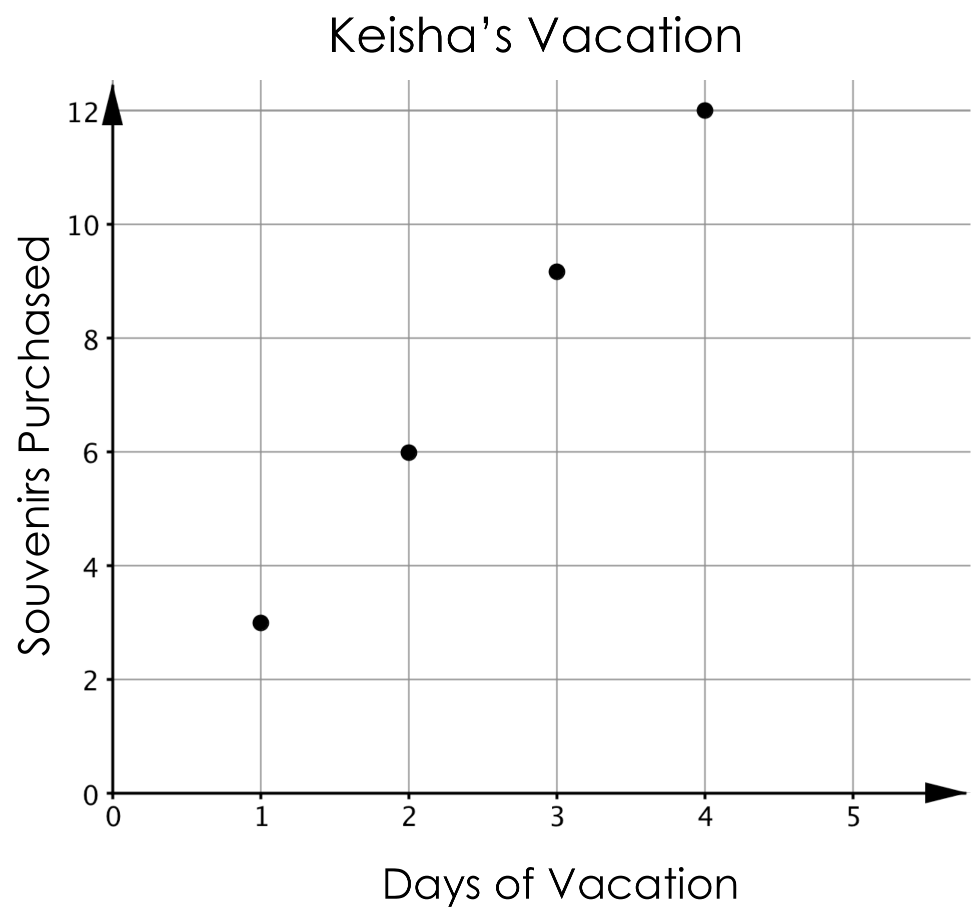
We can also find slope by looking at the $\frac{\textit{change in y}}{\textit{change in x}}$ or $\frac{\textit{rise}}{\textit{run}}$.

What is the slope of the following graph? What does the slope represent? **50, miles per hour.**



Let's Practice!

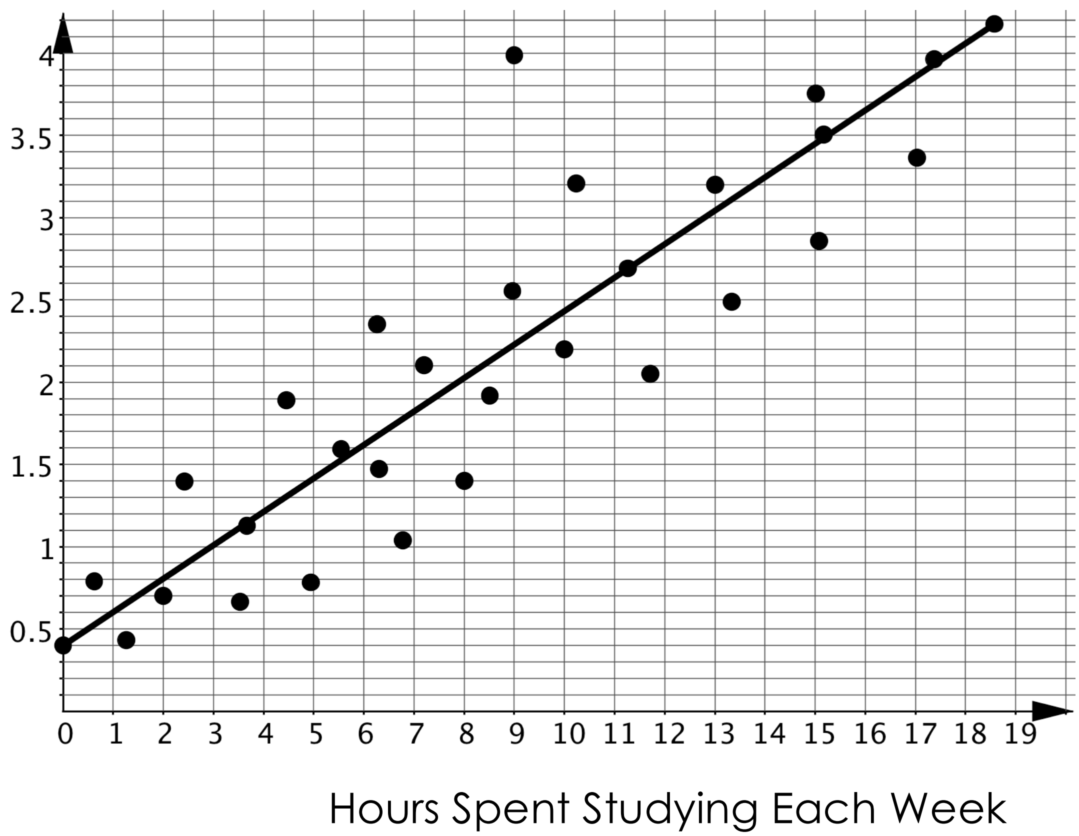
1. Consider the following graph.



a. What is the rate of change of the graph?
3

b. What does the rate of change represent?
Souvenirs purchased per day of vacation

2. Freedom High School collected data on the GPA of various students and the number of hours they spend studying each week. A scatterplot of the data is shown below with the line of best fit.

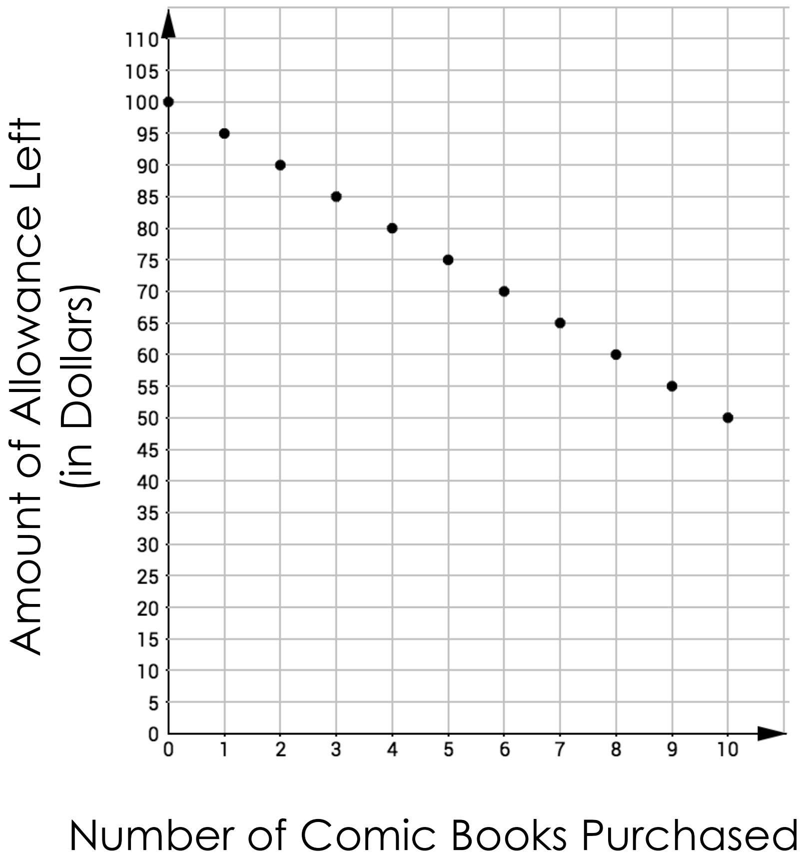


a. What is the slope of the line of best fit?
0.2

b. What does the slope represent?
Change in GPA per hour spent studying each week

Try It!

3. Sarah's parents give her $\$100.00$ allowance at the beginning of each month. Sarah spends her allowance on comic books. The graph below represents the amount of money Sara spent on comic books last month.



a. What is the rate of change?
-5

b. What does the rate of change represent?
$5 spent per comic book

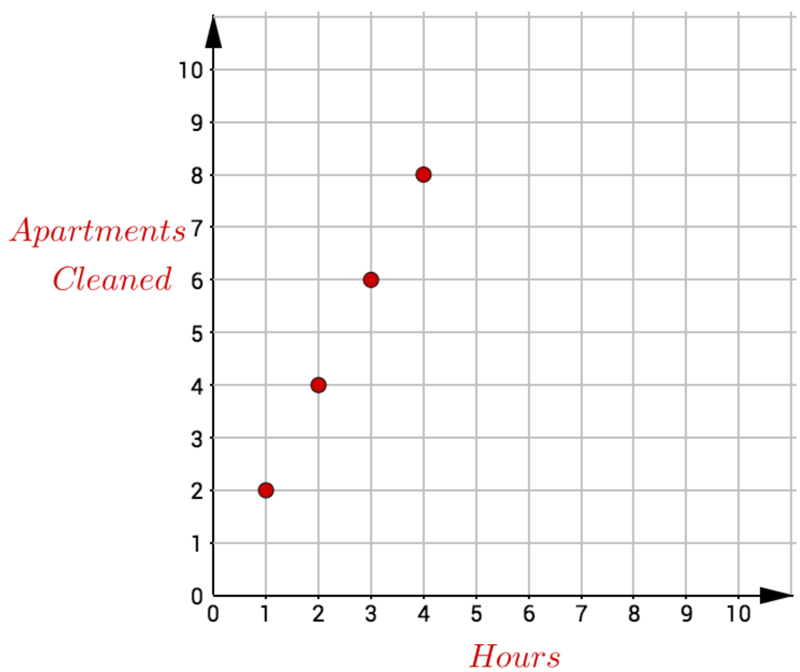
BEAT THE TEST!

1. A cleaning service cleans many apartments each day. The following table shows the number of hours the cleaners spend cleaning and the number of apartments they clean during that time.

Apartment Cleaning

Time (Hours)	1	2	3	4
Apartments Cleaned	2	4	6	8

Part A: Represent the situation on the graph below.



Part B: The data suggest a linear relationship between the number of hours spent cleaning and the number of apartments cleaned. Assuming the relationship is linear, what does the rate of change represent in the context of this relationship?

Ⓐ The number of apartments cleaned after one hour.

Ⓑ The number of hours it took to clean one apartment.

Ⓒ The number of apartments cleaned each hour.

Ⓓ The number of apartments cleaned before the company started cleaning.

Answer: C

Part C: Which equation describes the relationship between the time elapsed and the number of apartments cleaned?

Ⓐ $y = x$

Ⓑ $y = x + 2$

Ⓒ $y = 2x$

Ⓓ $y = 2x + 2$

Answer: C

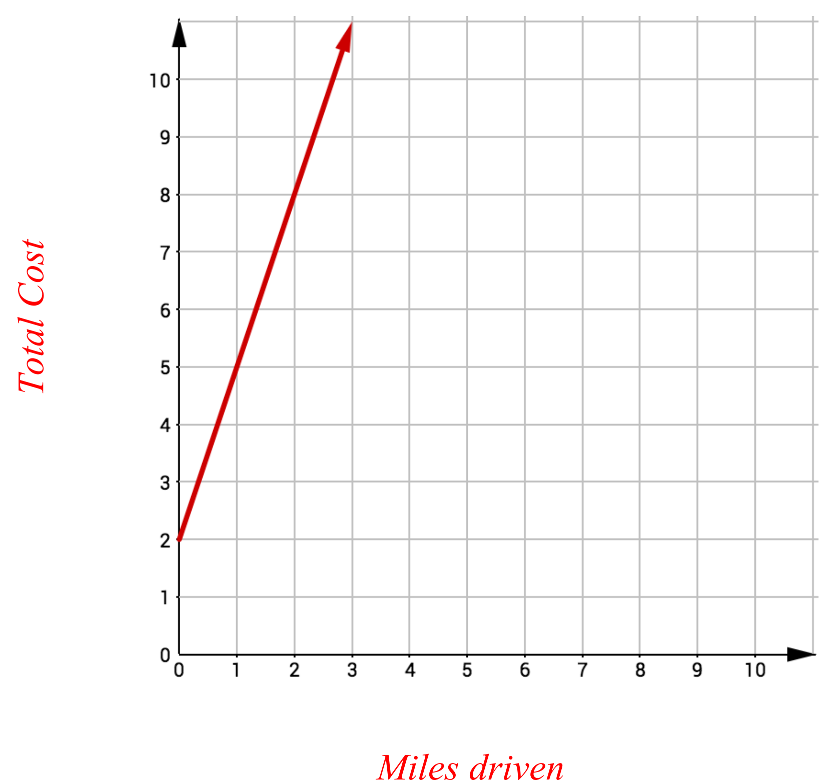
Section 4 – Topic 3
Interpreting Rate of Change and y-Intercept in a Real World Context – Part 1

Cab fare includes an initial fee of $\$2.00$ plus $\$3.00$ for every mile traveled.

Define the variable and write a function that represents this situation.

Let m represent number of miles traveled.
Let $c(m)$ represent the cab fare.
$\boldsymbol{C(m) = 2 + 3m}$

Represent the situation on a graph.



What is the slope of the line? What does the slope represent?
3; cost per mile
At what point does the line intersect the y-axis? What does this point represent?
2; initial or starting cost

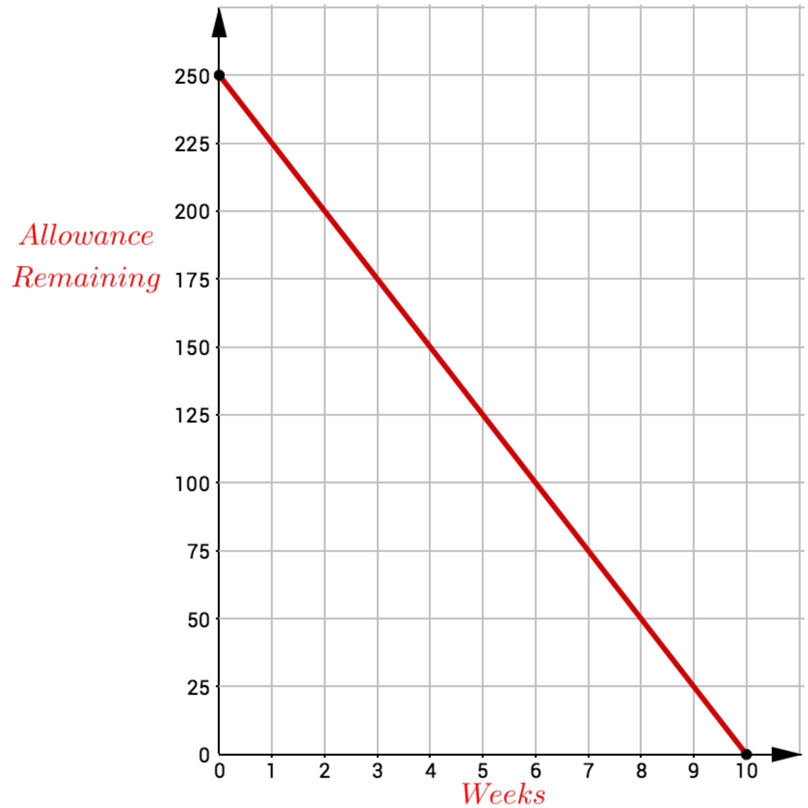
This point is the ***y-intercept*** of a line.

Let's Practice!

1. You saved $250.00 to spend over the summer. You decide to budget $25.00 to spend each week.

   a. Define the variable and write a function that represents this situation.
   Let w represent the number of weeks.
   Let $S(w)$ represent the remaining amount.
   $S(w) = 250 - 25w$

b. Represent the situation on a graph.



c. What is the slope of the line? What does the slope represent?

-25; **money spent per week**

d. What is different about the slope of this line compared to our earlier problem? Why is it different?

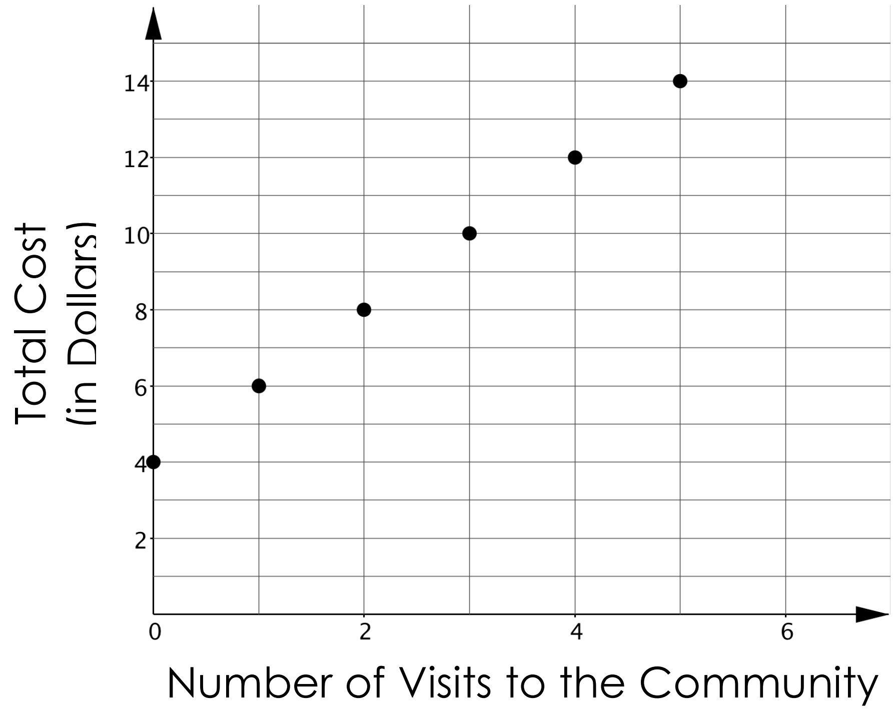
The slope is negative because every time you spend money the initial amount is decreasing.

e. What is the y-intercept? What does this point represent?

$(0, 250)$; **the allowance at the beginning of summer.**

Try It!

2. Consider the following graph:



a. What is the slope of the line? What does the slope represent?

2; Cost per visit

b. What is the y-intercept? What does the y-intercept represent?

$(0, 4)$; Membership fee for using the pool

c. Define the variables and write a function that represents this situation.

Let x represent the number of visits.

Let $g(x)$ represent the total cost.

$g(x) = 4 + 2x$, where x is a whole number.

d. What does each point represent?

The total cost for that number of visits.

Consider the three functions that you wrote regarding the cab ride, summer spending habits, and the community pool membership. What do you notice about the constant term and the coefficient of the x term?

- The constant term is the **y-intercept**.

- The coefficient of the x is the **slope** or **rate of change**.

These functions are written in ***slope-intercept*** form.

We can use slope-intercept form to graph any linear equation.

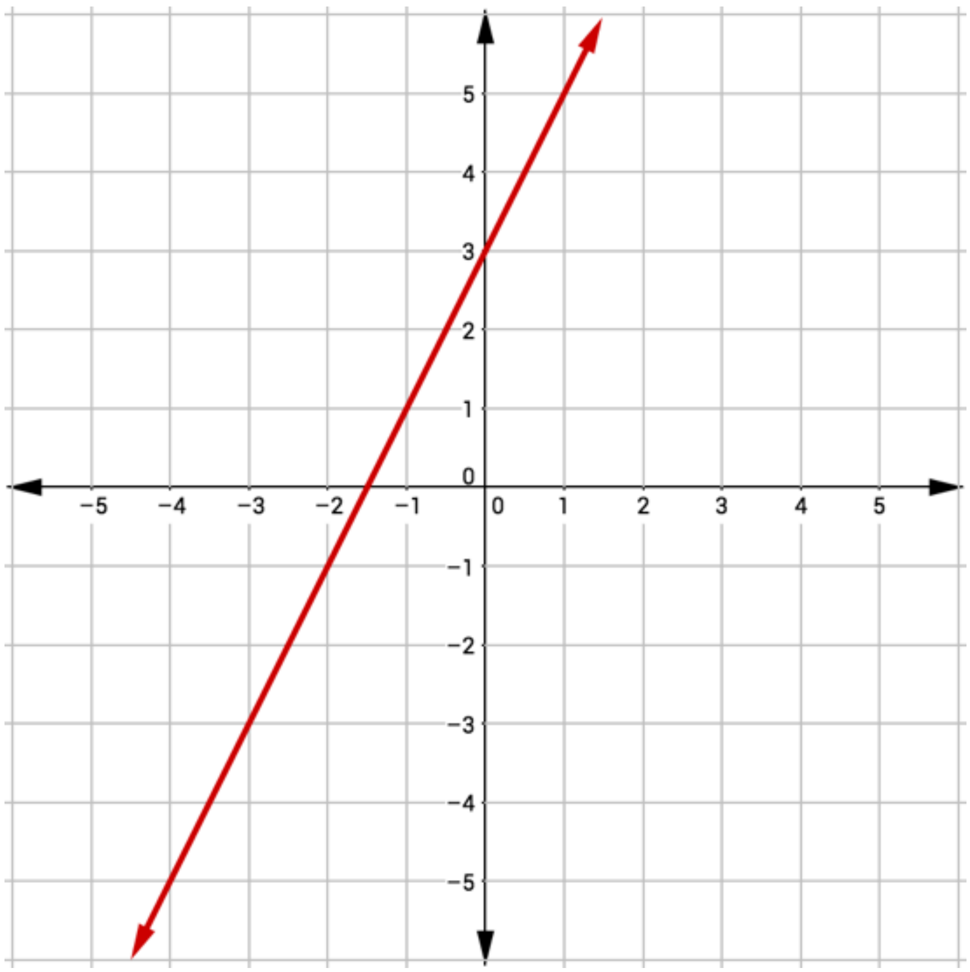
STUDY EDGE TIP

The coefficient of x is the slope and the constant term is the y-intercept ONLY if the equation is in slope-intercept form, $y = mx + b$.

Section 4 – Topic 4
Interpreting Rate of Change and y-Intercept in a Real World Context – Part 2

Let's Practice!

3. Graph $y = 2x + 3$.



4. Consider the equation $2x + 5y = 10$.

a. How does this equation look different from slope-intercept form of an equation?
It is not solved for y.

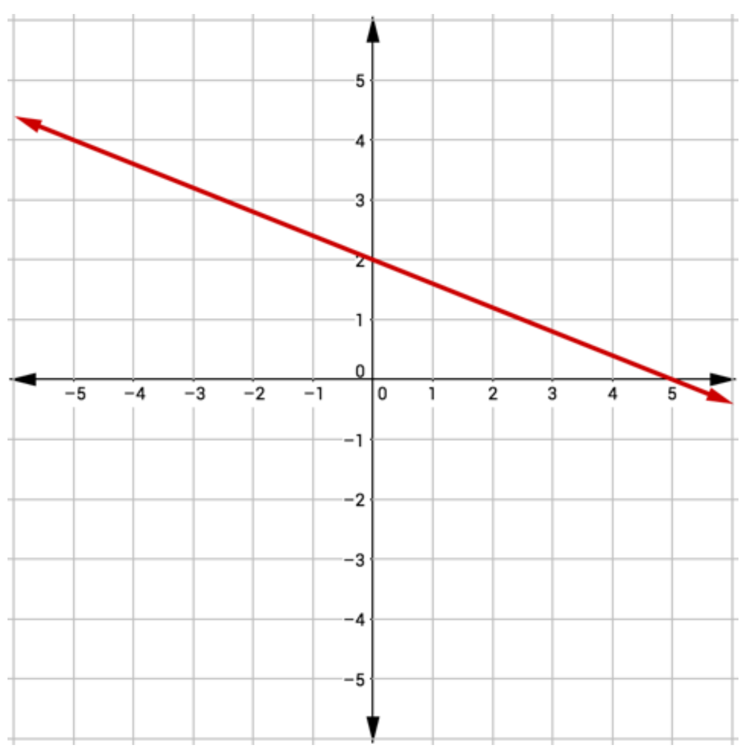
b. Rewrite the equation in slope-intercept form.
$$\mathbf{2x - 2x + 5y = 10 - 2x}$$

$$\frac{5y}{5} = \frac{10}{5} - \frac{2x}{5}$$

$$y = -\frac{2}{5}x + 2$$

c. Identify the slope and y-intercept.
Slope = $-\frac{2}{5}$, y-intercept = 2

d. Graph the equation.



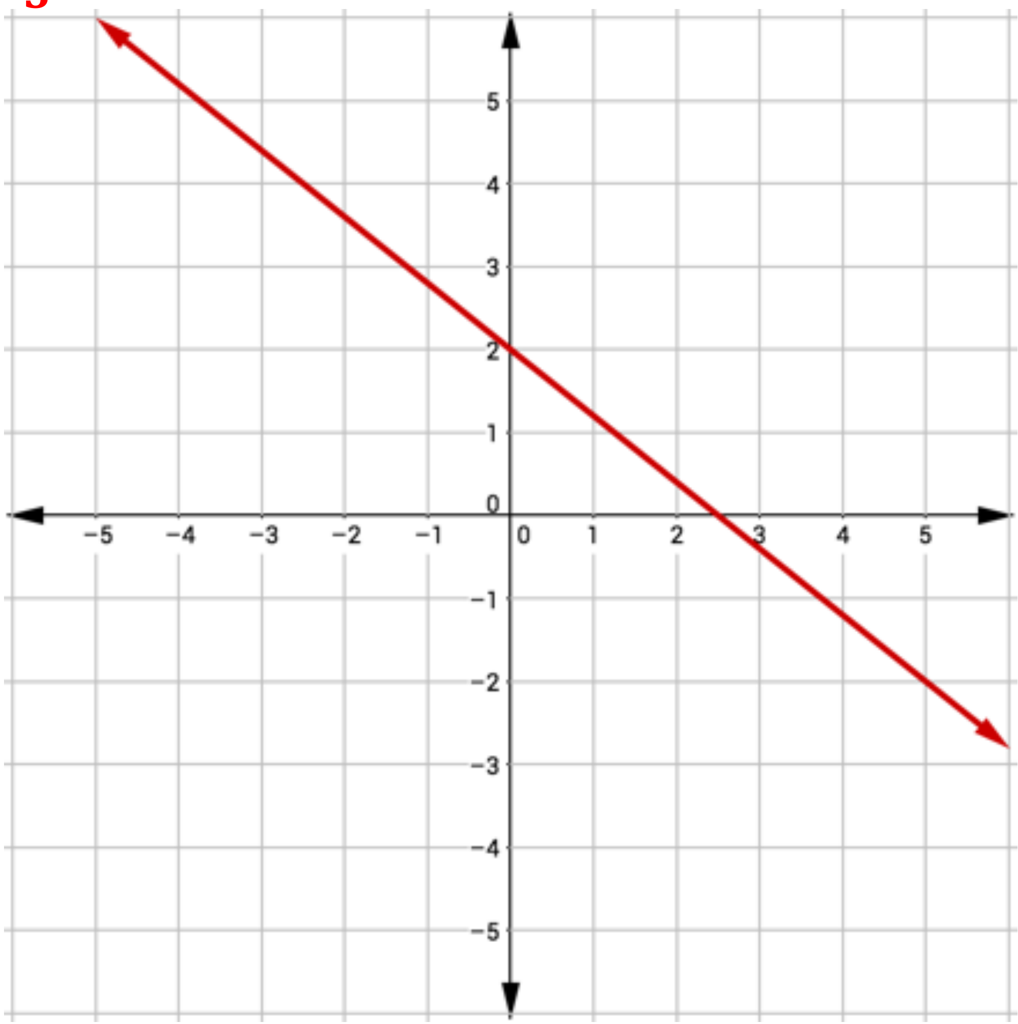
Try It!

5. Graph the equation $-4x - 5y = -10$.

$$-4x + 4x - 5y = -10 + 4x$$

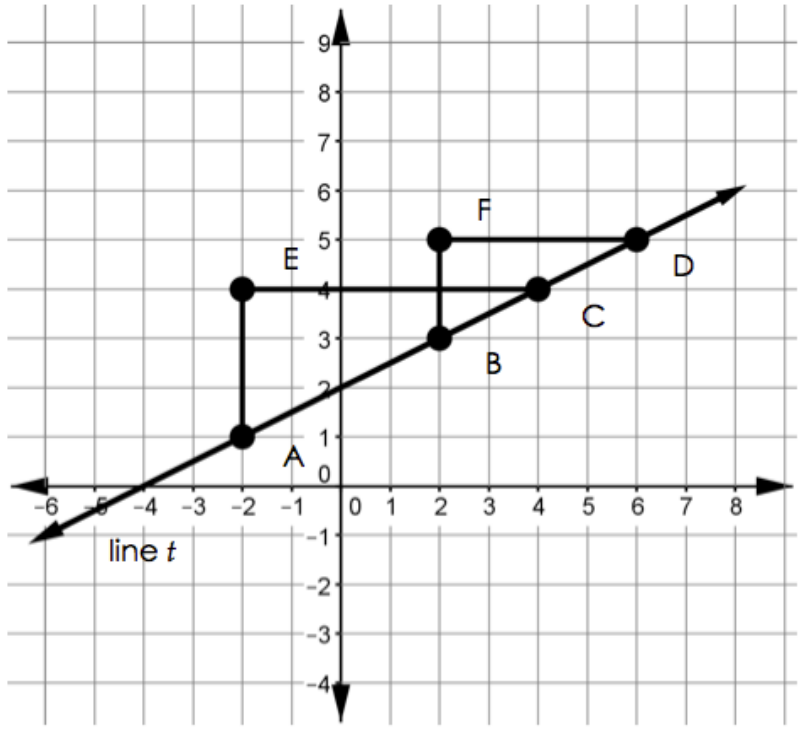
$$\frac{-5y}{-5} = \frac{-10}{-5} + \frac{4x}{-5}$$

$$y = -\frac{4}{5}x + 2$$



BEAT THE TEST!

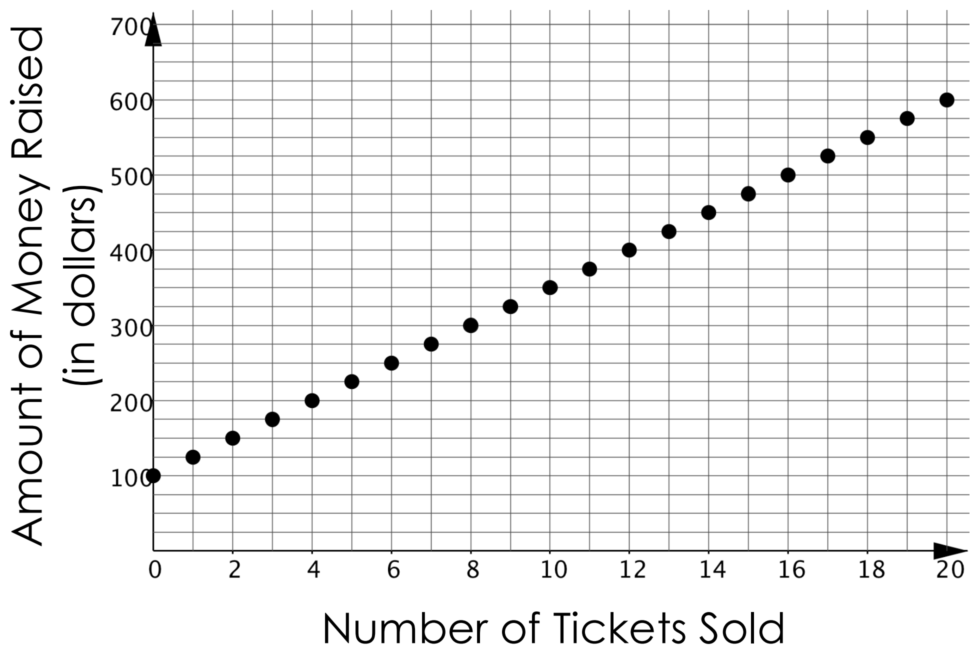
1. Line t, ΔECA, and ΔFDB are shown on the coordinate grid below.



Which of the following statements are true? Select all that apply.

- ☒ The slope of $\overline{AC}$ is equal to the slope of $\overline{BD}$.
- ☒ The slope of $\overline{AC}$ is equal to the slope of line t.
- ☐ The slope of line t is equal to $\frac{EC}{AE}$.
- ☒ The slope of line t is equal to $\frac{BF}{FD}$.
- ☒ The y-intercept of line t is 2.
- ☐ Line t represents a discrete function.

2. The senior class at Elizabeth High School was selling tickets to raise money for prom. The graph below represents the situation.



Part A: How much does one ticket cost?

$25

Part B: How much money did the senior class have at the start of the fundraiser?

$100

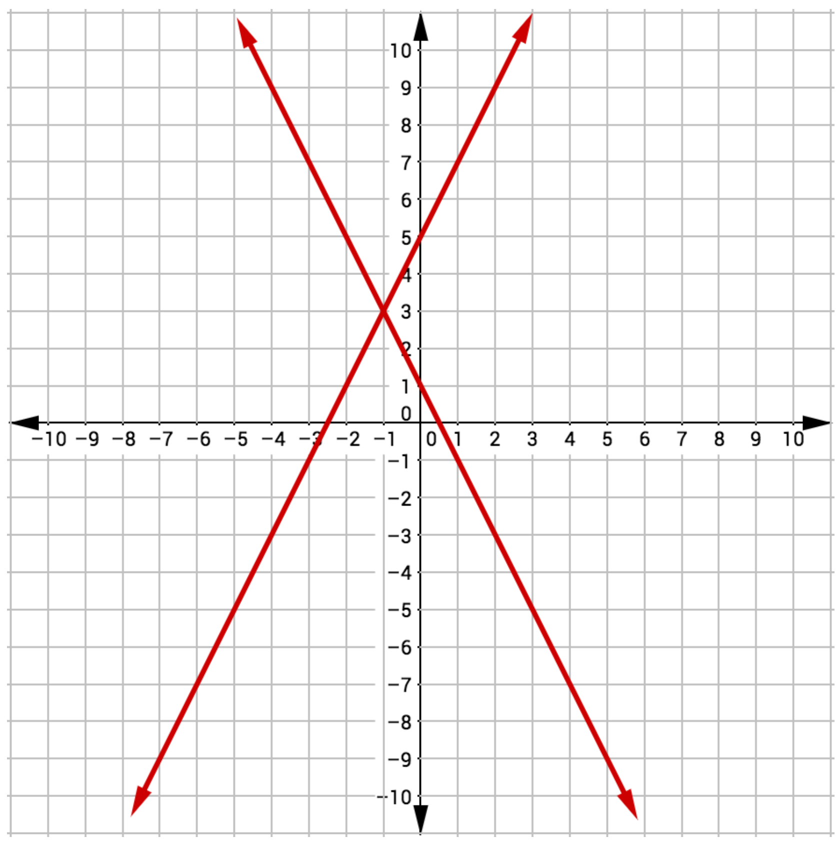
Section 4 – Topic 5
Introduction to Systems of Equations

A system of equations is a set of 2 or more equations.

Consider the following systems of equations.

Line 1: $2x - y = -5$
Line 2: $2x + y = 1$

Graph the system of equations on the coordinate plane below.



Recall that a solution to a linear equation is any ordered pair that makes that equation a true statement.

What do you notice about the point $(-2,5)$?
It falls on the Line 2.

What do you notice about the point $(1,7)$?
It falls on the Line 1.

What do you notice about the point $(-1,3)$?
It falls on both lines.

What do you notice about the point $(1,1)$?
It does not fall on either line.

Let's Practice!

1. Consider the following system of equations made up of Line 1 and Line 2.

$$\text{Line 1: } 5x + 2y = 8$$
$$\text{Line 2: } -3x - 2y = -4$$

Complete the following sentences.

a. The ordered pair $(-2,5)$ is a solution to

- ○ Line 1
- ● Line 2
- ○ The system of equations

.

b. The ordered pair $(2,-1)$ is a solution to

- ● Line 1
- ● Line 2
- ● The system of equations

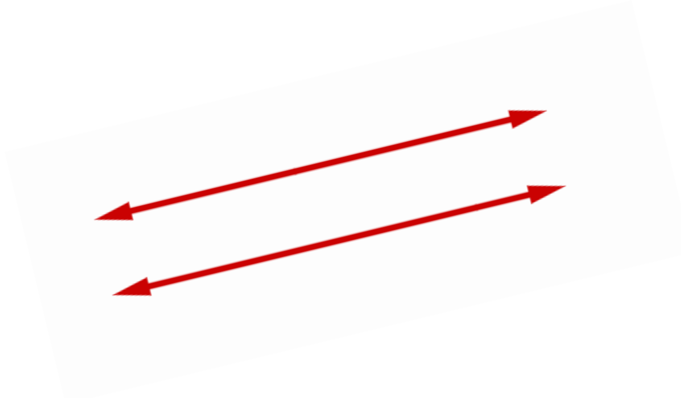
.

c. The ordered pair $(0,4)$ is a solution to

- ● Line 1
- ○ Line 2
- ○ The system of equations

.

2. Is there ever a time when a system of equations will not have a solution? If so, sketch an example.

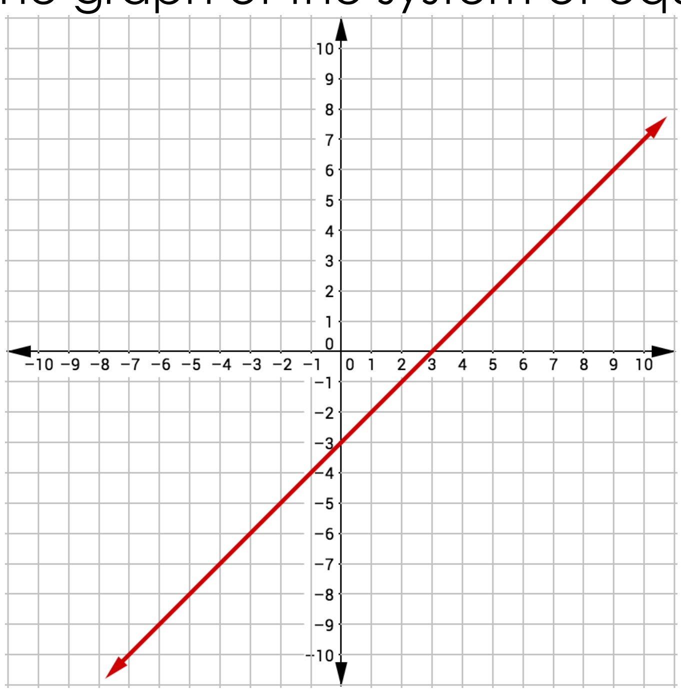


Try It!

3. Consider the following system of equations.

$$x - y = 3$$
$$-2x + 2y = -6$$

a. Sketch the graph of the system of equations.

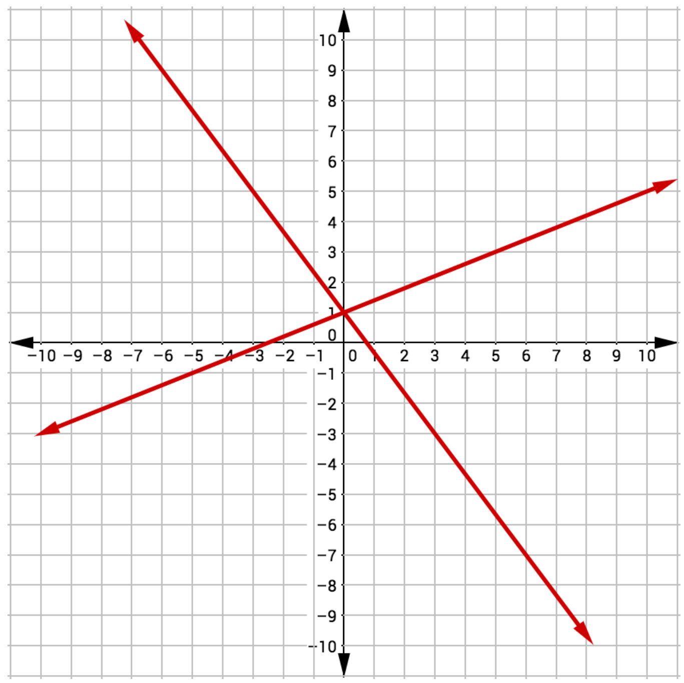


b. What can be said about the solution to this system of equations? **The solution is all real numbers.**

4. Consider the following system of equations.

$$4x + 3y = 3$$
$$2x - 5y = -5$$

a. Graph the system of equations.



b. What is the solution to the system?

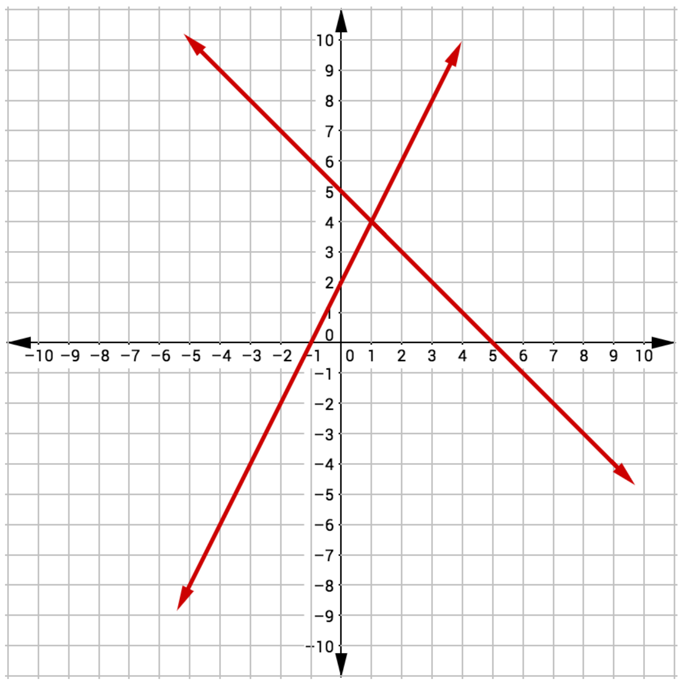
$\mathbf{(0, 1)}$

BEAT THE TEST!

1. Consider the following system of equations.

$$x + y = 5$$
$$2x - y = -2$$

Part A: Sketch the graph of the system of equations.



Part B: Determine the solution to the system of equations.

$(1, 4)$

Part C: Create a third equation that could be added to the system so that the solution does not change. Graph the line on the coordinate plane above.
Answers vary. Sample Answer: Add the two equations together to get a new equation $3x = 3$

Section 4 – Topic 6
Finding Solution Sets to Systems of Equations Using Substitution and Graphing

There are many times that we are able to use systems of equations to solve real world problems.

One method of solving systems of equations is by graphing like we did in the previous video.

Let's Practice!

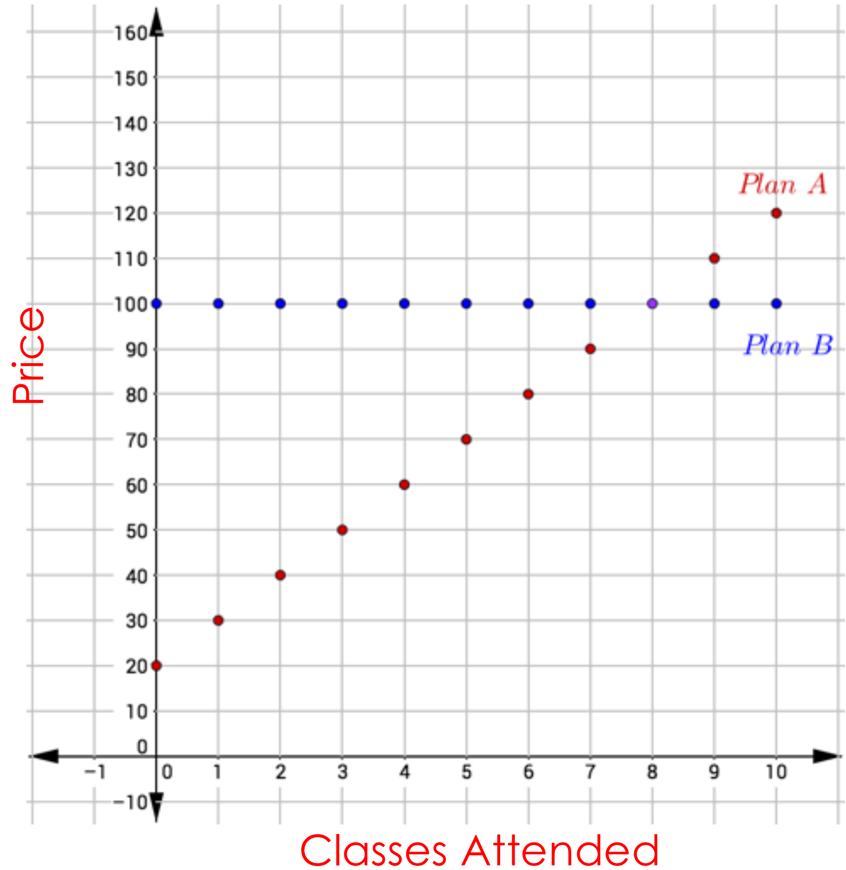
1. Brianna's lacrosse coach suggested that she practices yoga to improve her flexibility. "Yoga-ta Try This!" Yoga Studio has two membership plans. Plan A costs $\$20.00$ per month plus $\$10.00$ per class. Plan B costs $\$100.00$ per month for unlimited classes.

   a. Define a variable and write two functions to represent the monthly cost of each plan.

      Let c represent the number of monthly classes attended and $f(c)$ represent monthly cost.

      Plan A: $f(c) = 20 + 10c$
      Plan B: $f(c) = 100$

b. Represent the two situations on the graph below.



c. What is the rate of change for each plan?

Plan A: 10

Plan B: 0

d. What does the rate of change represent in this situation?

The cost per class

e. What do the y-intercepts of the graphs represent?

The initial cost

2. Brianna is trying to determine which plan is more appropriate for the number of classes she wants to attend.

   a. When will the two plans cost exactly the same?
   When she goes to 8 classes

   b. When is Plan A the better deal?
   If she goes to less than 8 classes

   c. When is plan B the better deal?
   When she goes to more than 8 classes

We can also help Brianna determine the best plan for her without graphing. Consider our two equations again.
$\boldsymbol{f(c) = 20 + 10c}$ **and** $\boldsymbol{f(c) = 100}$

We simply want to know when the total costs would be equal.

- Set the two plans equal to each other and solve for the number of visits.
  $\boldsymbol{100 = 20 + 10c}$
  $\boldsymbol{10c = 80}$
  $\boldsymbol{c = 8}$

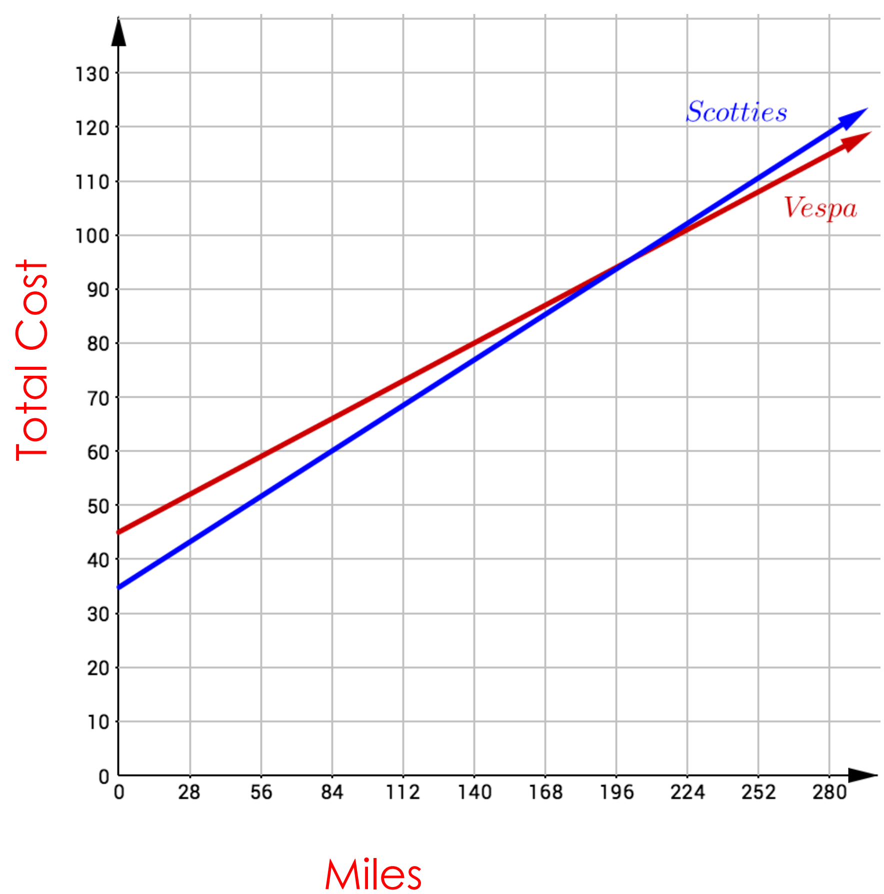
- This method is called solving by **substitution**.

Try It!

3. Vespa Scooter Rental rents scooters for $\$45.00$ and $\$0.25$ per mile. Scottie's Scooter Rental rents scooters for $\$35.00$ and $\$0.30$ per mile.

   a. Define a variable and write two functions to represent the situation.
   Let m represent the number of miles driven.
   Vespa: $g(m) = 45 + 0.25m$
   Scotties: $g(m) = 35 + 0.3m$

   b. Represent the two situations on the graph below.



c. What is the rate of change of each line? What do they represent?

Vespa: 0.25

Scotties: 0.3

They represent the cost per mile driven.

d. What do the y-intercepts of each line represent?

The initial cost

It's difficult to find the solution by looking at the graph. In such cases, it's better to use substitution to solve the problem.

4. Use the substitution method to help the renter determine when the two scooter rentals will cost the same amount.

a. When will renting a scooter from Vespa Scooter Rental cost the same as renting a scooter from Scottie's Scooter Rental?

$\mathbf{45 + 0.25}\boldsymbol{m} \mathbf{= 35 + 0.3}\boldsymbol{m}$

$\mathbf{45 - 45 + 0.25}\boldsymbol{m} \mathbf{= 35 - 45 + 0.3}\boldsymbol{m}$

$\mathbf{0.25}\boldsymbol{m} \mathbf{= -10 + 0.3}\boldsymbol{m}$

$\mathbf{0.25}\boldsymbol{m} \mathbf{- 0.3}\boldsymbol{m} \mathbf{= -10 + 0.3}\boldsymbol{m} \mathbf{- 0.3}\boldsymbol{m}$

$\frac{-0.05m}{-0.05} = \frac{-10}{-0.05}$

$\boldsymbol{m} \mathbf{= 200}$

Driving 200 miles the cost will be the same.

b. Describe a situation when renting from Vespa Scooter Rental would be a better deal than renting from Scottie's Scooter Rental.

Vespa will be a better deal if you drive more than 200 miles.

BEAT THE TEST!

1. Lyle and Shaun open a savings account at the same time. Lyle deposits $100 initially and adds $20 per week. Shaun deposits $500 initially and adds $10 per week. Lyle wants to know when he will have the same amount in his savings account as Shaun.

   Part A: Write two equations to represent the amount of money Lyle and Shaun have in their accounts.

   Let x represent the number of weeks they make deposits.

   Lyle: $y = 100 + 20x$

   Shaun: $y = 500 + 10x$

   Part B: Which method would you use to solve the problem, substitution or graphing? Explain your answer.

   Answers vary. Sample answer: I would use substitution since graphing large numbers might be more difficult.

   Part C: After how many weeks of making the additional deposits will Lyle have the same amount of money as Shaun?

   $$100 + 20x = 500 + 10x$$
   $$100 - 100 + 20x = 500 - 100 + 10x$$
   $$20x - 10x = 400 + 10x - 10x$$
   $$\frac{10x}{10} = \frac{400}{10}$$
   $$x = 40$$

   At 40 weeks of deposits, they will have the same amount of money.

Section 4 – Topic 7
Using Equivalent Systems of Equations

An ordered pair that satisfies all equations in a system is called the **solution** to that system.

If two systems of equations have the same solution, they are called **equivalent** systems.

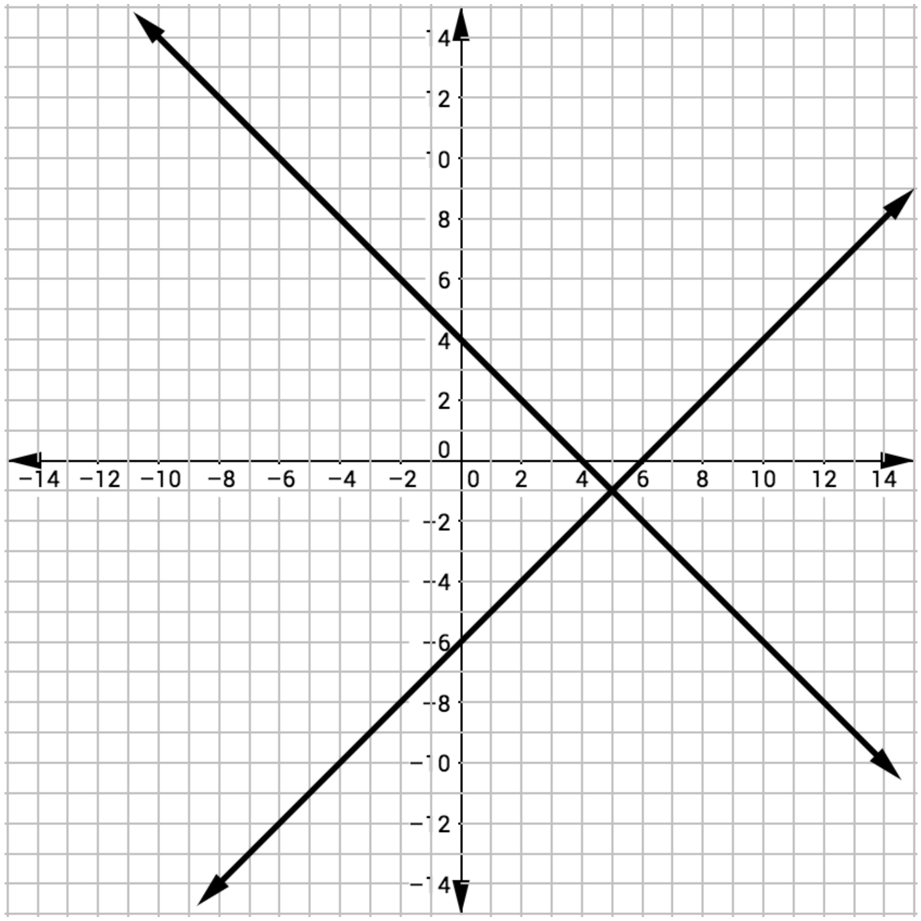
Let's explore how to write equivalent systems of equations.

Consider the following system of equations:

$$x + y = 4$$

$$x - y = 6$$

The solution to this system is $(5, -1)$. We can also see this when we graph the lines.



Describe the result when we multiply either of the equations by some factor.

The resulting equation would be the same line.

Use this process to write an equivalent system.

Multiply the first equation by 4.

$4x + 4y = 16$

$x - y = 6$

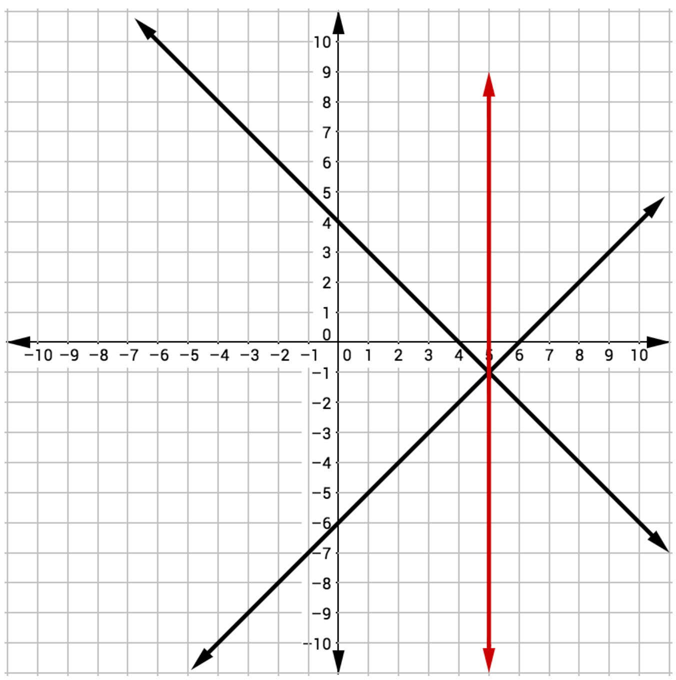
Consider the original system of equations again.

$$x + y = 4$$
$$x - y = 6$$

What is the resulting equation when we add the two equations in the system together?

$\mathbf{2x = 10}$ **or** $\mathbf{x = 5}$

Graph the new equation on the same coordinate plane with our original system.



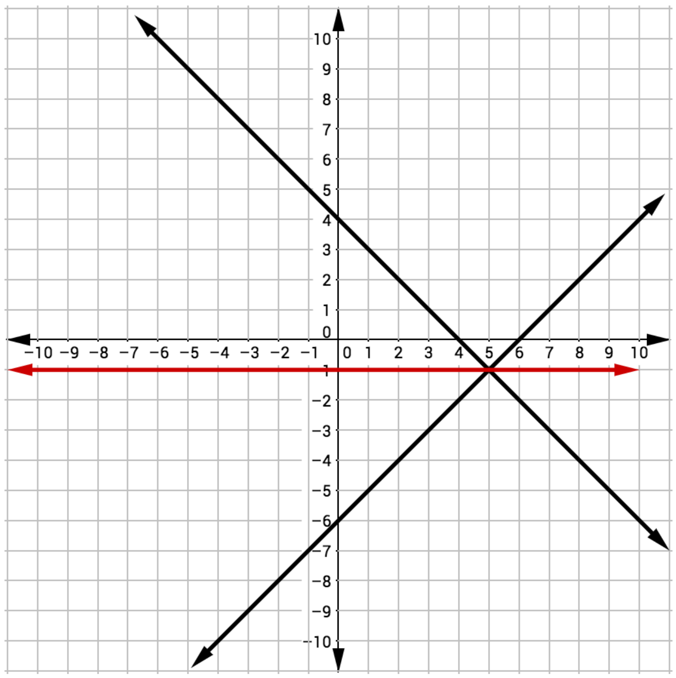
Algebraically, show that $(5, -1)$ is also a solution to the sum of the two lines.

$\mathbf{2x = 10,\ 2(5) = 10}$

What is the resulting equation when we subtract the second equation from the first equation?

$2y = -2$ **or** $y = -1$

Graph the new equation on the same coordinate plane with our original system.



Algebraically, show that $(5, -1)$ is also a solution to the difference of the two lines.

$2y = -2$ **or** $2(-1) = -2$

Let's revisit the original system:

Equation 1: $x + y = 4$
Equation 2: $x - y = 6$

Complete the following steps to show that replacing one equation by the sum of that equation and a multiple of the other equation produces a system with the same solutions.

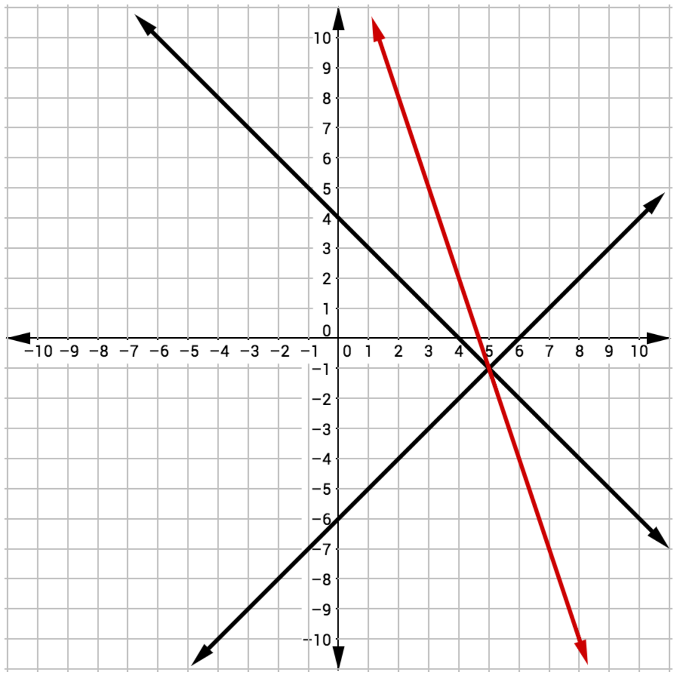
Create a third equation by multiplying Equation 1 by two.

Equation 3: $\mathbf{2x + 2y = 8}$

Create a fourth equation by finding the sum of the third equation and Equation 2.

$$\begin{array}{l} \mathbf{2x + 2y = 8} \\ \underline{\ \ \mathbf{x - \ \ y = 6}} \\ \mathbf{3x + \ \ y = 14} \end{array}$$

Graph the fourth equation on the same coordinate plane with our original system.



Algebraically, show that $(5, -1)$ is also a solution to the difference of the two lines.

$3x + y = 14$ or $3(5) - 1 = 14$

Let's Practice!

1. Consider the following system, which has a solution of $(2,5)$ and $M, N, P, R, S,$ and T are non-zero real numbers:

$$Mx + Ny = P$$
$$Rx + Sy = T$$

Write two new equations that could be used to create an equivalent system of equations.

Answers vary. Sample answer:

$(M + R)x + (N + S)y = P + T$

$2Rx + 2Sy = 2T$

Try It!

2. List three ways that we can write new equations that can be used to create equivalent systems.

Answers vary. Sample answer:

Multiply the equations by some factor.

Add the two equations together.

Multiply one equation by some factor and add to the other equation.

BEAT THE TEST!

1. The system $\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$ has the solution $(1, -3)$, where $A, B, C, D, E,$ and F are non-zero real numbers. Select all the systems of equations with the same solution.

- [x] $(A - D)x + (B - E)y = C - F$
  $Dx + Ey = F$
- [ ] $(2A + D)x + (2B + E)y = C + 2F$
  $Dx + Ey = F$
- [x] $Ax + By = C$
  $-3Dx - 3Ey = -3F$
- [x] $(A - 5D)x + (B - 5E)y = C - 5F$
  $Dx + Ey = F$
- [ ] $Ax + (B + E)y = C$
  $(A + D)x + Ey = C + F$

Section 4 – Topic 8
Finding Solution Sets to Systems of Equations Using Elimination

Consider the following system of equations:

$$2x + y = 8$$
$$x - 2y = -1$$

Write an equivalent system that will eliminate one of the variables when you add the equations.

Answers vary. Sample answer:

Multiply the first equation by 2.

$$\mathbf{4x + 2y = 16}$$
$$\mathbf{x - 2y = -1}$$

Determine the solution to the system of equations.

$$\begin{aligned} 4x + 2y &= 16 \\ \underline{x - 2y} &\underline{= -1} \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 2(3) + y &= 8 \\ 6 + y &= 8 \\ 6 - 6 + y &= 8 - 6 \\ y &= 2 \end{aligned}$$

The solution to the system is $(3, 2)$.

Describe what the graph of the two systems would look like.

The lines intersect at the point $(3, 2)$.

This method of solving a system is called **elimination**.

Let's Practice!

1. Ruxin and Andre were invited to a Super Bowl party. They were asked to bring pizzas and sodas. Ruxin brought three pizzas and four bottles of soda and spent $\$48.05$. Andre brought five pizzas and two bottles of soda and spent $\$67.25$.

   a. Write a system of equations to represent the situation.

   Let x represent the cost of one pizza.
   Let y represent the cost of one soda.

$$\begin{aligned} 3x + 4y &= 48.05 \\ 5x + 2y &= 67.25 \end{aligned}$$

   b. Write an equivalent system that will eliminate one of the variables when you add the equations.

$$\begin{aligned} 3x + 4y &= 48.05 \\ -10x - 4y &= -134.50 \end{aligned}$$

c. Solve the system to determine the cost of one pizza and one soda.

$$\begin{aligned} 3x + 4y &= 48.05 \\ \underline{-10x - 4y} &\underline{= -134.50} \\ -7x &= -86.45 \\ x &= 12.35 \end{aligned}$$

$$3x + 4y = 48.05$$
$$3(12.35) + 4y = 48.05$$
$$37.05 + 4y = 48.05$$
$$37.05 - 37.05 + 4y = 48.05 - 37.05$$
$$4y = 11$$
$$y = 2.75$$

The cost of one pizza is $\$12.35$ and the cost of one soda is $\$2.75$.

Try It!

2. Jazmin and Justine went shopping for back to school clothes. Jazmin purchased three shirts and one pair of shorts and spent $\$38.00$. Justine bought four shirts and three pairs of shorts and spent $\$71.50$.

   a. Assuming all the shirts cost the same amount and all the shorts cost the same amount, write a system of equations to represent each girl's shopping spree.
   Let x represent the cost of one shirt.
   Let y represent the cost of one pair of shorts.

$$\begin{aligned} 3x + \quad y &= 38 \\ 4x + 3y &= 71.50 \end{aligned}$$

   b. Use the elimination method to solve for the price of shorts.

$$-9x - 3y = -114$$
$$\underline{\quad 4x + 3y = 71.50}$$
$$-5x = -42.50$$
$$x = 8.5$$

$$3x + y = 38$$
$$3(8.5) + y = 38$$
$$25.5 + y = 38$$
$$25.5 - 25.5 + y = 38 - 25.5$$
$$y = 12.5$$

The cost of one shirt is $\$8.50$. The cost of one pair of shorts is $\$12.50$.

BEAT THE TEST!

1. Complete the following table.

Solve by Elimination: $\begin{cases} 2x - 3y = 8 \\ 3x + 4y = 46 \end{cases}$

Operations	Equations	Labels
	$2x - 3y = 8$ $3x + 4y = 46$	Equation 1 Equation 2
Multiply Equation 1 by -3	$-6x + 9y = -24$	New equation 1
Multiply equation 2 by 2.	$\mathbf{6x + 8y = 92}$	New equation 2
Add the equations together.	$-6x + 9y = -24$ $6x + 8y = 92$ $17y = 68$	
Divide by 17.	$\frac{17y}{17} = \frac{68}{17}$ $y = 4$	
Solve for x.	$2x - 3(4) = 8$ $x = 10$	
Write x and y as coordinates.	$(\boxed{10}, \boxed{4})$	Solution to the system

2. Which of the systems of equations below could not be used to solve the following system for x and y?

$$\begin{aligned} 6x + 4y &= 24 \\ -2x + 4y &= -10 \end{aligned}$$

Ⓐ $\begin{aligned} 6x + 4y &= 24 \\ 2x - 4y &= 10 \end{aligned}$

Ⓑ $\begin{aligned} 6x + 4y &= 24 \\ -4x + 8y &= -20 \end{aligned}$

Ⓒ $\begin{aligned} 18x + 12y &= 72 \\ -6x + 12y &= -30 \end{aligned}$

Ⓓ $\begin{aligned} 12x + 8y &= 48 \\ -4x + 8y &= -10 \end{aligned}$

Answer: D

Section 4 – Topic 9
Solution Sets to Inequalities with Two Variables

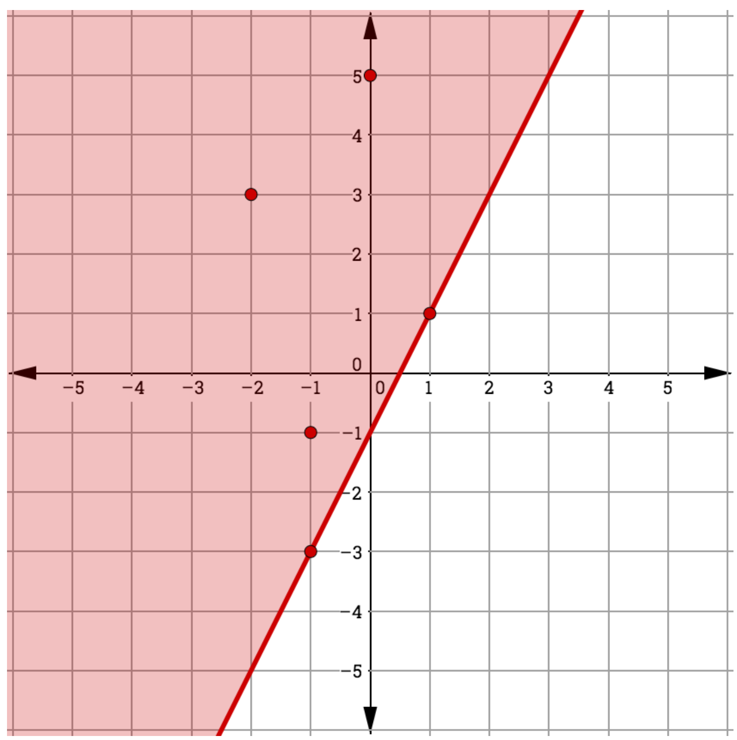
Consider the following linear inequality.

$$y \geq 2x - 1$$

Underline each ordered pair (x, y) that is a solution to the above inequality.

<u>**$(0, 5)$**</u> $(4, 1)$ <u>**$(-1, -1)$**</u> <u>**$(1, 1)$**</u> $(3, 0)$ <u>**$(-2, 3)$**</u> $(4, 3)$ <u>**$(-1, -3)$**</u>

Plot each solution as a point (x, y) in the coordinate plane.



Graph the line $y = 2x - 1$ in the same coordinate plane. What do you notice about the solutions to the inequality $y \geq 2x - 1$ and the graph of the line $y = 2x - 1$?

The solutions to $y = 2x - 1$ are also solutions to $y \geq 2x - 1$.

Let's Practice!

1. The senior class is raising money for Grad Bash. The students' parents are donating cakes. The students plan to sell entire cakes for $\$20.00$ each and slices of cake for $\$3.00$ each. If they want to raise at least $\$500.00$, how many of each could they sell?

   a. List two possibilities for the number of whole cakes and cake slices students could sell to reach their goal of raising at least $\$500.00$.

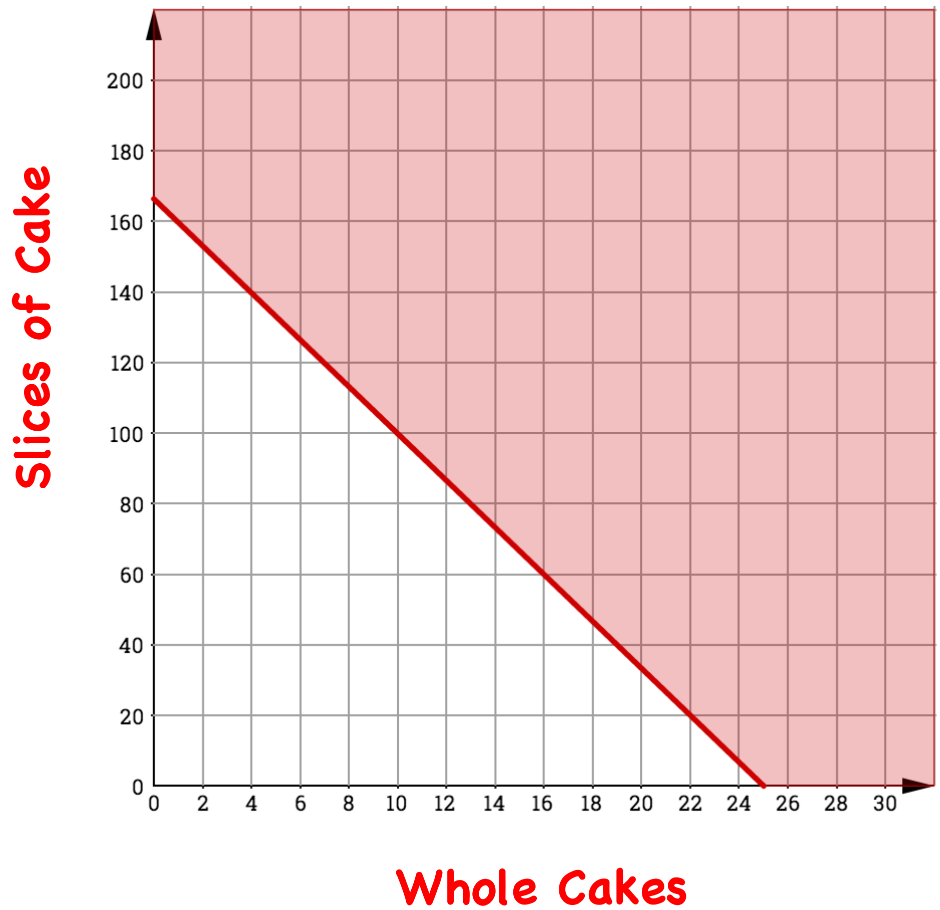
      Answers vary. Sample answers:
      10 whole cakes and 100 slices
      20 whole cakes and 50 slices

   b. Write an inequality to represent the situation.

      Let x represent the number of whole cakes.
      Let y represent the number of slices.

      $$20x + 3y \geq 500$$

c. Graph the region where the solutions to the inequality would lie.



d. What is the difference between the ordered pairs that fall on the line and the ones that fall in the shaded area?

The ones on the lines will raise exactly $\$500$. The ones in the shaded area will raise more than $\$500$.

e. What does the x – intercept represent?

The x – intercept would represent selling only whole cakes and no slices.

Try It!

2. The freshman class wants to include at least 120 people in the pep rally. Each skit will have 15 people, and the dance routines will feature 12 people.

   a. List two possible combinations of skits and dance routines.

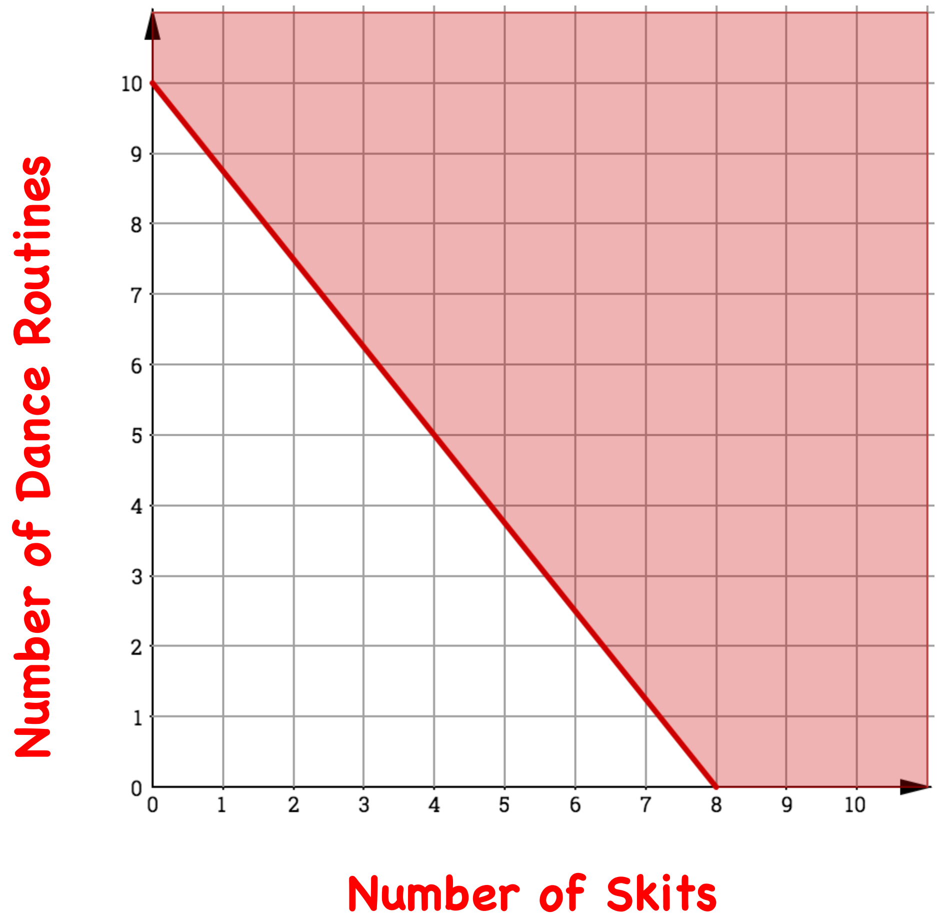
      Answers vary. Sample answer:
      4 skits and 5 dance routines
      2 skits and 8 dance routines

   b. Write an inequality to represent the situation.

      Let x represent the number of skits.
      Let y represent the number of dance routines.

      $\mathbf{15x + 12y \geq 120}$

c. Graph the region where the solutions to the inequality would lie.

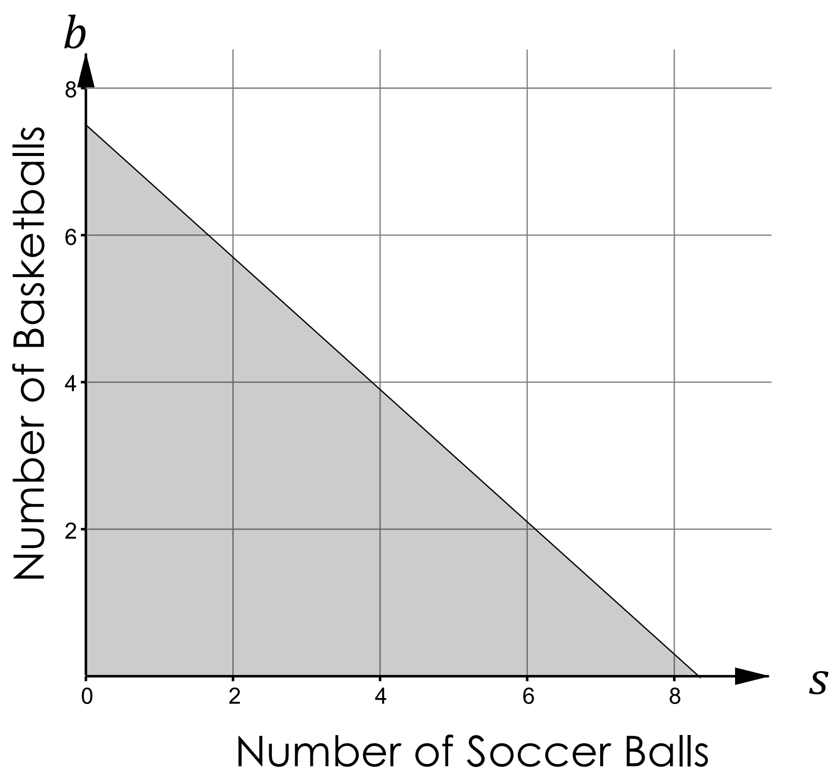


d. What does the y-intercept represent?

The $y-$ intercept would represent having only dance routines and no skits.

BEAT THE TEST!

1. Coach De Leon purchases sports equipment. Basketballs cost $\$20.00$ each, and soccer balls cost $\$18.00$ each. He had a budget of $\$150.00$. The graph shown below represents the number of basketballs and soccer balls he can buy given his budget constraint.



Part A: Write an inequality to represent the situation.

Let s represent the number of soccer balls.
Let b represent the number of basketballs.

$\mathbf{18s + 20b \leq 150}$

Part B: Determine whether these combinations of basketballs, b, and soccer balls, s, can be purchased.

	$b = 5$ $s = 3$	$b = 2$ $s = 4$	$b = 7$ $s = 3$	$b = 0$ $s = 8$	$b = 8$ $s = 0$	$b = 6$ $s = 3$	$b = 4$ $s = 7$
Yes	○	●	○	●	○	○	○
No	●	○	●	○	●	●	●

Section 4 – Topic 10
Finding Solution Sets to Systems of Linear Inequalities

Juan must purchase car insurance. He needs to earn at least $\$50.00$ a week to cover the payments. The most he can work each week is 8 hours because of football practice. Juan can earn $\$10.00$ per hour mowing yards and $\$12.00$ per hour washing cars.

The system $\begin{cases} 10x + 12y \geq 50 \\ x + y \leq 8 \end{cases}$ represents Juan's situation.

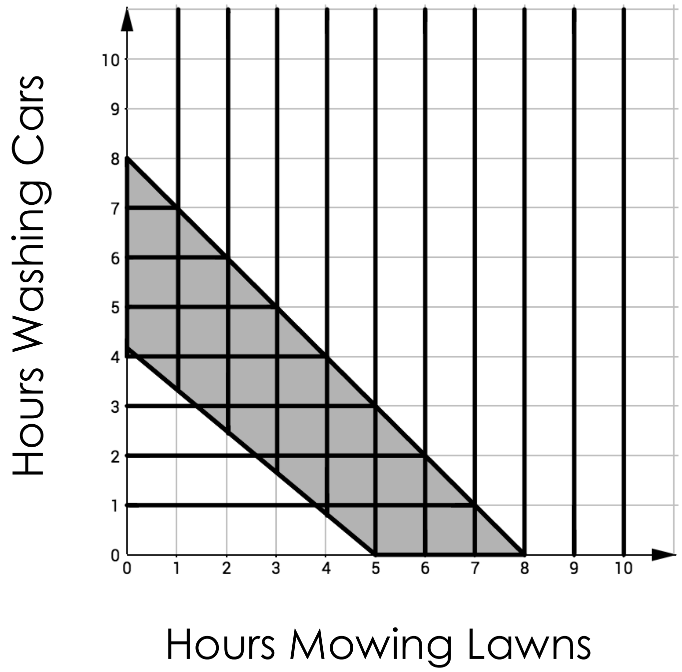
Define the variables.

x represents hours spent mowing lawns.

y represents hours spent washing cars.

The graph below depicts Juan's situation. Interpret the graph and identify two different solutions for Juan's situation.

The shaded region shows the combinations of jobs that Juan could work to meet his goal. 5 hours mowing lawns and 2 hours washing cars. 3 hours mowing lawns and 3 hours washing cars.


Hours Washing Cars
Hours Mowing Lawns
0
1
2
3
4
5
6
7
8
9
10

Let's Practice!

1. Bristol is having a party and has invited 24 friends. She plans to purchase sodas that cost \$5.00 for a 12-pack and chips that cost \$3.00 per bag. She wants each friend to have at least two sodas. Bristol's budget is \$35.00.

   a. Write a system of inequalities to represent the situation.

   Let x represent the number of 12-packs of sodas.
   Let y represent the number of bags of chips.
   $5x + 3y \leq 35$
   $x \geq 4$

   b. Graph the region where the solutions to the inequality would lie.

c. Name two difference solutions for Bristols's situation.
Answers Vary.
Sample Answer:
5 cases of soda and 1 bag of chips.
4 cases of soda and 4 bags of chips.

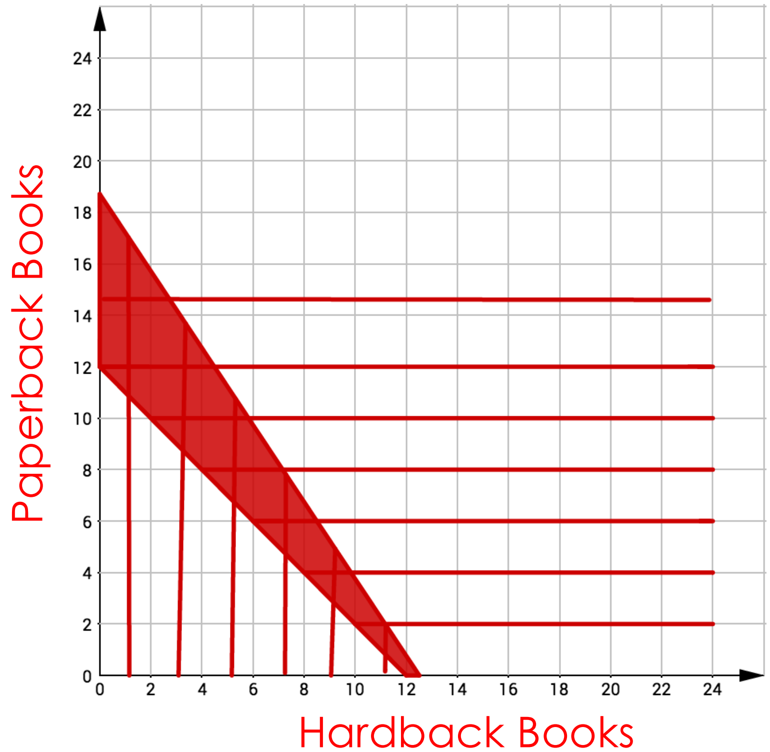
Try It!

2. Anna is an avid reader. Her generous grandparents gave her money for her birthday, and she decided to spend at most $\$150.00$ on books. *Reading Spot* is running a special: all paperback books are $\$8.00$ and hardback books are $\$12.00$. Anna wants to purchase at least 12 books.

   a. Write a system of inequalities to represent the situation.
   Let x represent the number of hardback books
   Let y represent the number of paperback books

   $\mathbf{12x + 8y \leq 150}$
   $\mathbf{x + y \geq 12}$

b. Graph the region where the solutions to the inequality would lie.



c. Name two different solutions for Anna's situation.

Answers Vary.

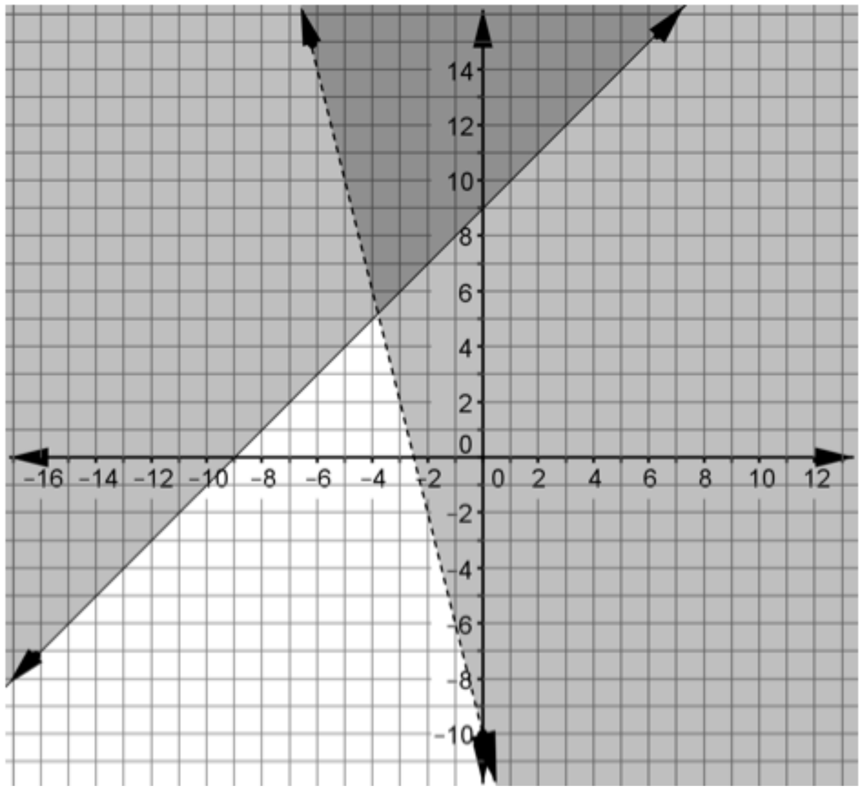
Sample Answer:

7 hardbacks and 6 paperbacks

2 hardbacks and 12 paperbacks

<u>BEAT THE TEST!</u>

1. Tatiana is reviewing for the Algebra 1 Final exam. She made this graph representing a system of inequalities:



Part A: Underline the ordered pairs below that represent solutions to the system of inequalities.

$(-8,3)$ <u>$\mathbf{(-3,8)}$</u> <u>$\mathbf{(-1,9)}$</u> <u>$\mathbf{(-4,9)}$</u> $(9,6)$ <u>$\mathbf{(0,9)}$</u>

$(5,5)$ $(-5,10)$ $(-9,1)$ <u>$\mathbf{(-2,7)}$</u> $(1,6)$ $(0,0)$

Part B: Derive the system of inequalities that describes the region of the graph Tatiana drew.

$$\boldsymbol{y \geq x+9}$$
$$\boldsymbol{y > -4x-10}$$