SECTION 5.1: INTRO TO POLYNOMIALS.

I. Some Terminologies:
- **Polynomial:** combination of one or more terms, and the exponents of each variable is a nonnegative whole number.

  Example: $-6x^5 + 7x^4 - x^2 + 9x - 3,$ $x,$ $-2x^2,$ 5

  Note: Graphs of polynomials are smooth, continuous curves. Domain of a polynomial is all real numbers, i.e, polynomials are defined everywhere**.**

  Examples of some graphs of polynomials functions:

  These are NOT polynomials: $|x|,$ $\sqrt{x},$ $\frac{2}{x},$ $\frac{x^2+3}{x-1},$ $-3x^3 + 2x^2 - \frac{5}{x}$

  Degree of a term: sum of the exponents of the variables in that term
  Example: $7x^3y^2$ *has degree 5 (since the sum of the exponents of x and y is 5).*

  Degree of a polynomial: the greatest degree of any term.
  Example: $-6x^2 + 5x - 11x^3 - 3x^9 + 2.$ This polynomial has degree ___.

  What is the degree of the polynomial below?

  $x^5 + 3x^2y^4 + 7xy + 9x - 2$

  Leading Coefficient: coefficient of the term with the highest degree. For example, the leading coefficient of $x^5 + 3x^2 + 9x - 2$ is **1.** The leading coefficient of $-6x^2 + 5x - 11x^3 - 3x^9 + 2$ is _____

SECTION 5.2: MULTIPLICATION OF POLYNOMIALS

I. Multiplying polynomials:

   Examples: Multiply
1. $(x + 2)(x - 2)(x^2 + 3)$

2. $(3x - 2y)^2$

1

<u>Square Binomial</u> (Short Cut):

$$(A + B)^2 = A^2 + 2AB + B^2 \ \ and \ \ (A - B)^2 = A^2 - 2AB + B^2,$$ where A and B can be any expression.

Example: $(4xy^2 - 3xy)^2$

II. <u>Product of Sum and Difference of Two Terms (Short Cut):</u>

$$(A + B)(A - B) =$$

***Examples:** Multiply* $(5x + 7y)(5x - 7y)$

SECTION 5.3: GREATEST COMMON FACTOR AND FACTOR BY GROUPING

I. <u>Factor Greatest Common Factor:</u>

Examples: Factor out the greatest common factor.

1. $20x^2 + 30x$ *2.* $2x^3y + 10x^2y^2 - 6xy^3$

3. $x(x - 6) - 7(x - 6)$

II. <u>Factor by Grouping</u> (4 terms only):

Examples: Factor the followings.

4. $x^3 + 2x^2 - x - 2$ *5.* $10x^2 - 12xy + 35xy - 42y^2$

6. $2x^3 - 10 + 4x^2 - 5x$

SECTION 5.4: FACTORING TRINOMIALS

Examples: Factor completely. **(NOTE: ALWAYS FACTOR OUT THE GREATEST COMMON FACTOR FIRST IF POSSIBLE).**

a. $x^2 - 3x - 28$

c. $2x^3 - 14x + 24x$

d. $24x^2 + 3xy - 27y^2$

e. $8x^2 - 18x + 9$

f. $6x^2 + 14x + 3$

Factor the perfect square trinomials (if possible)

h. $x^2 - 16x + 64$

i. $4x^2 + 20x + 25$

j. $9x^2 - 5x + 1$

Perfect Square Trinomials: $A^2 \pm 2AB + B^2 = (A \pm B)(A \pm B) = (A \pm B)^2$

SECTION 5.5: FACTORING SPECAL FORMS:

I. Difference of Two Square: $(A + B)(A - B) =$

Examples: Factor completely.

1. $x^2 - 16$

2. $9x^4 - 25y^2$

3. $x^2 + 25$

4. $2x^3y - 32xy$

II. Sum and Difference of Two Cubes:

$A^3 + B^3 =$

$A^3 - B^3 =$

Examples: Factor completely.

1. $x^3 - 8$

2. $1 - 27x^3y^3$

3. $2x^4 + 128x$

4. $x^6 + 8y^3$

More Example:

Factor: $(3x - 5)^2 + (3x - 5) - 6$

General Factoring Strategy:

1. Always factor out the greatest common factor (if possible).
2. Recognize the problem as:
   - *Four terms:* factor by grouping.
   - *Trinomial:* factor by guess and check, or turn the trinomial into four terms and factor by grouping
   - *Difference of 2 squares:* $a^2 - b^2 = (a + b)(a - b)$
   - *Sum of 2 cubes:* $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
   - *Difference of 2 cubes:* $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

SECTION 5.7: POLYNOMIAL EQUATIONS AND THEIR APPLICATIONS

*To solve polynomial equations by factoring, use the *zero product* property.

Zero Product property: *If AB = 0, then A =*_____ *or B =* _____ .

Example: solve: $(x + 5)(x - 2) = 0$

Solve each of the following by factoring
1. $3x(x - 4)(x + 1) = 0$

2. $x^2 = 2x + 8$

3. $x^3 = 36x$

4. $x^3 = \frac{13}{2}x^2 + \frac{7}{2}x$

5. An Alpha III model rocket is launched from the ground with an A8-3 engine. Without a parachute, the height of the rocket h at time t seconds is approximated by the equation
$$h = -16t^2 + 144t$$

A. What is the height of the rocket after 1 second?

B. How long will it take for the rocket to return to the ground?

6. A rectangular floor has an area of 24 square meters. The length of the floor is 2 meters less than twice the width. What is the length of the floor?

7. There is a pool with a sidewalk around it. The pool measures 6m by 10m, and the total area of the pool and walk is 96 square meters. What is the width of the sidewalk?