Section 6.2 Trigonometric Equations PART 1

Linear form: an equation that contains trig functions with each raised no higher than to the 1st power.

Quadratic form: an equation that contains one or more trig functions raised to the 2nd power OR an equation that contains a term with two different trig functions multiplied together.

Helpful Strategies to Solving a Trigonometric Equation

- If the equation is in linear form and only one trigonometric function is present , **use linear equation methods** to get the function on one side and a number on the other side. (EX: 1,2)

- If equation is in quadratic form only one trigonometric function is present throughout, solve the equation for that function.
  - ➢ If only one terms contains a trig function and it is squared, then take the **square root of both sides.** Don't forget $\pm$. (EX 3,4)
  - ➢ If there is a term that contains that trig function squared, and another term that contains that trig function not squared, try rearrangeing the equation so that one side equals 0, then try **factoring**. (EX 5,6)

    If it is not factorable, use the **quadratic formula.** $trig\ function = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (EX 10)

- If more than one trigonometric function is present,
  - ➢ Rearrange the equation so that one side equals 0, then try **factoring.** (EX 7)
  - ➢ Try **using identities to rewrite the equation in terms of a single trigonometric function**. (EX 8, 9) It may be helpful to square both sides first. (Check for extranenous solutions.) (EX 11)

STEPS FOR FINDING SOLUTIONS

The ultimate goal is to eventually get the trigonometric function (raised to the 1st power) on one side by itself and a number on the other side. Once this has been achieved,

Step 1) **Find all angles in the interval $[0°, 360°)$ or $[0, 2\pi)$ that have that trig function value.** If the trig function value is not familiar, use the inverse function on your calculator to find the angle. Use reference angles to find all angles in each quadrant that have that trig ratio.

If asked to find ALL possible solutions, then

Step 2) **add integer multiples of the period of the trig function to the original angles** found in Step 1.

a) For sine, cosine, secant, and cosecant: add the expression "$360°n$" or "$2n\pi$". (Sometimes to describe all of these angles, this actually can be written more concisely by just adding "$180°n$" or "$n\pi$.")

b) For tangent and cotangent: add the expression "$180°n$" or "$n\pi$".

Linear equation methods (ex 1, 2)

Example 1: Solve the equation $3\boxed{\tan\theta} - \sqrt{3} = 0$ a) over the interval $[0°, 360°)$ b) for all solutions.

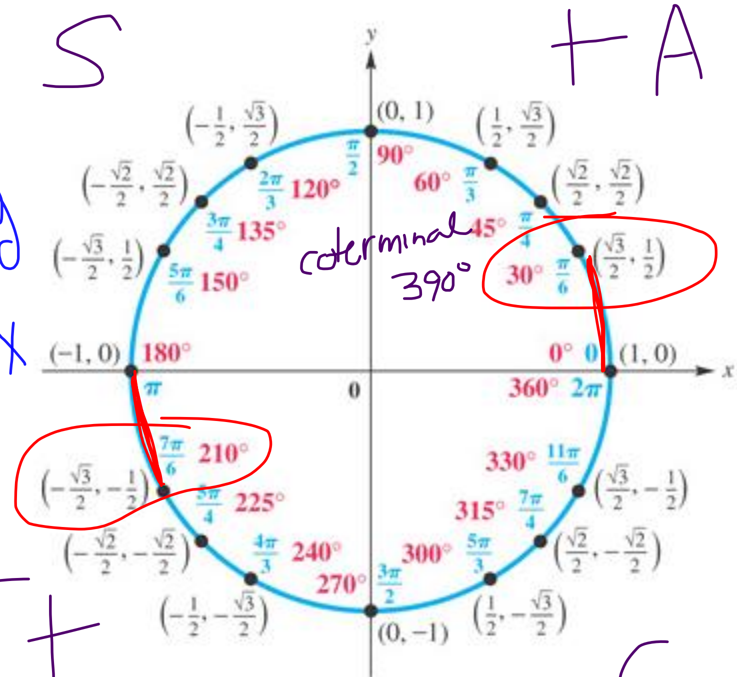
① Isolate trig function $\quad +\sqrt{3} \quad +\sqrt{3}$

$$\frac{3\tan\theta}{3} = \frac{\sqrt{3}}{3}$$

$$\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \quad \frac{y}{x} \quad \frac{\frac{1}{2}\ y}{\frac{\sqrt{3}}{2}\ x}$$

S $\qquad$ + A

T + $\qquad$ C



a) over the interval $[0°, 360°)$

$$\boxed{\theta = 30°, 210°}$$

b) for all solutions.

$$\boxed{\begin{aligned} \theta &= 30° + 360°n \\ \theta &= 210° + 360°n \end{aligned}}$$

more consice

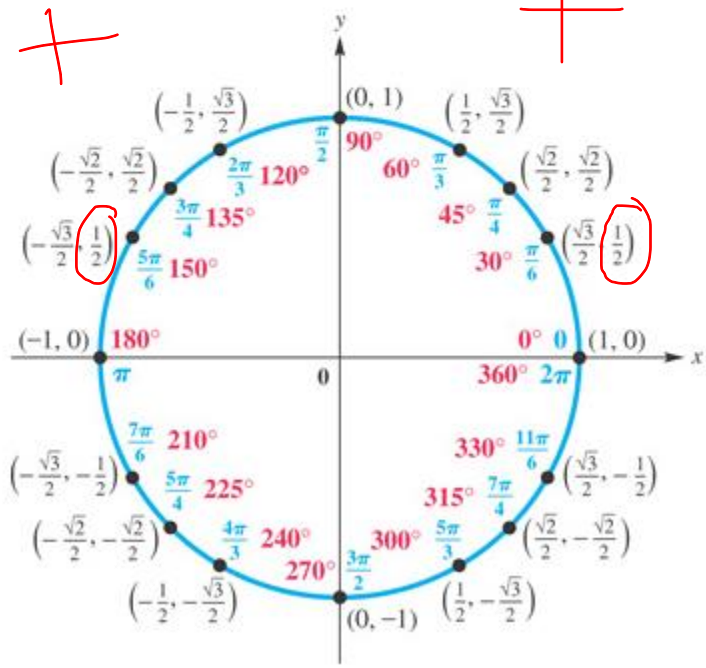
$$\boxed{30° + 180°n}$$

Example 2: Solve the equation $2\boxed{\sin x}+3=4$ a) over the interval $[0, 2\pi)$ b) for all solutions.

$\quad\quad -3 \;\; -3$

$$\frac{2\sin x}{2} = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$



a) over the interval $[0, 2\pi)$

$$\boxed{x = \frac{\pi}{6}, \; \frac{5\pi}{6}}$$

b) for all solutions.

$$\boxed{\begin{aligned} x &= \frac{\pi}{6} + 2\pi n \\ x &= \frac{5\pi}{6} + 2\pi n \end{aligned}}$$

Square root of both sides (ex 3, 4)

Radians

$\frac{\pi}{3}$ family

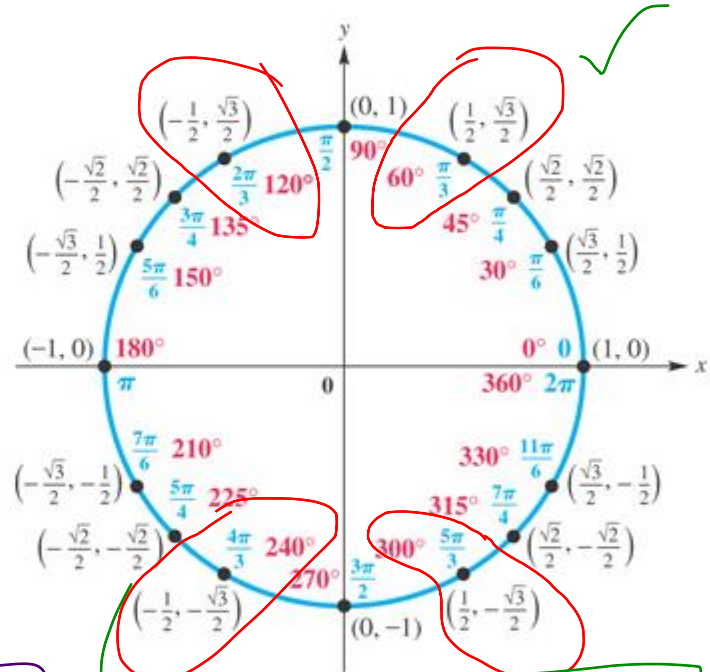
Example 3: Find all solutions of $4\cos^2 x - 1 = 0$.

$+1 \quad +1$

$$\frac{4\cos^2 x}{4} = \frac{1}{4}$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{4}}$$

$$\cos x = \pm\frac{1}{2}$$



$$\boxed{\begin{array}{ll} x = \frac{\pi}{3} + 2\pi n & x = \frac{4\pi}{3} + 2\pi n \\ x = \frac{2\pi}{3} + 2\pi n & x = \frac{5\pi}{3} + 2\pi n \end{array}}$$

more concise

$$x = \frac{\pi}{3} + \pi n$$

$$x = \frac{2\pi}{3} + \pi n$$

Example 4: Solve $\tan^2 x + 3 = 0$ over the interval $[0, 2\pi)$.

$-3 \quad -3$

$$\sqrt{\tan^2 x} = \sqrt{-3}$$

$$\tan x = \pm\sqrt{-3}$$ ← imaginary solution

$$\tan x = \pm i\sqrt{3}$$ ← not possible

$$\boxed{\text{no solution}}$$

Factoring (ex 5, 6, 7)

Radians

Example 5: Solve $-2\sin^2 x = 3\sin x + 1$ over the interval $[0, 2\pi)$.

$+2\sin^2 x \quad +2\sin^2 x$

$0 = 2\sin^2 x + 3\sin x + 1$

$0 = (2\sin x + 1)(\sin x + 1)$

$2\sin x + 1 = 0$

$-1 \quad -1$

$\frac{2\sin x}{2} = \frac{-1}{2}$

$\sin x = -\frac{1}{2}$

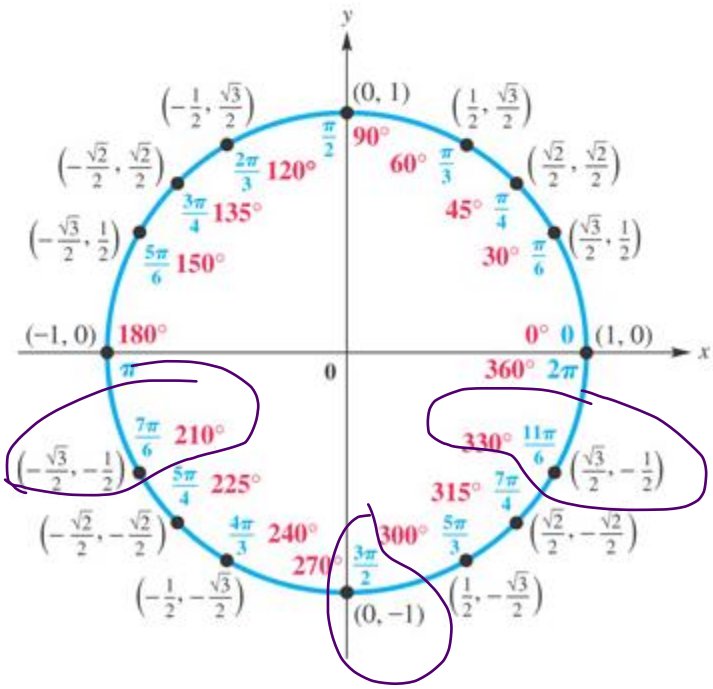
$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin x + 1 = 0$

$-1 \quad -1$

$\sin x = -1$

$x = \frac{3\pi}{2}$



Radians

Example 6: Solve $\tan^2 x + \tan x - 2 = 0$ a) over the interval $[0, 2\pi)$ b) all possible solutions.

FACTOR

$(\tan x - 1)(\tan x + 2) = 0$

$\tan x - 1 = 0$

$+1 \quad +1$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

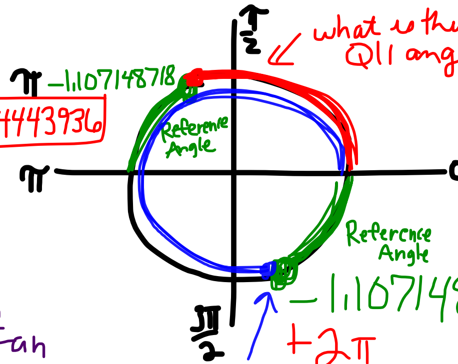
$\tan x + 2 = 0$

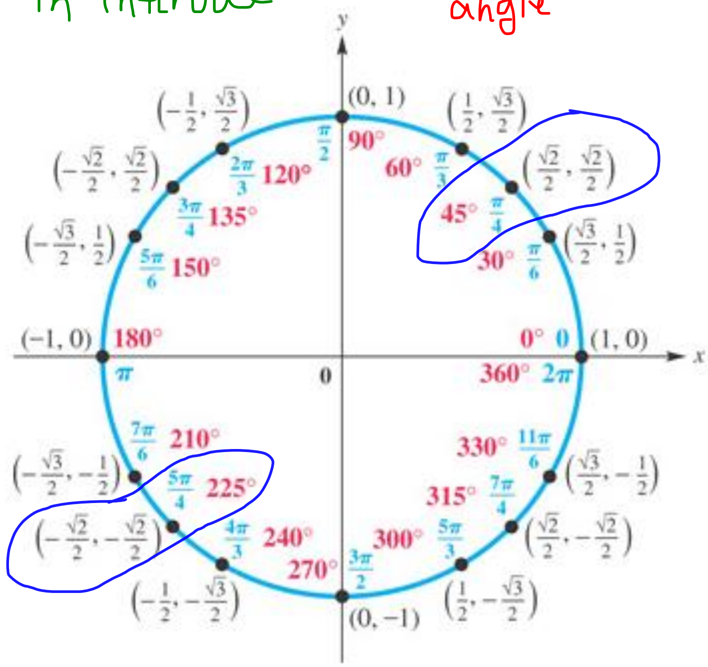
$\tan x = -2$ QIV, QII have negative tan values

$\tan^{-1}(-2) = x$

$x = -1.107148718$ not in interval

add 2π to find coterminal angle





a) over the interval $[0, 2\pi)$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}, 5.176036589, 2.034443936$$

b) all possible solutions.

$$x = \frac{\pi}{4} + \pi n$$

$$x = 2.034443936 + \pi n$$

Example 7: Solve $\sec^2\theta \tan\theta = 2\tan\theta$ over the interval $[0°, 360°)$.

$$\sec^2\theta \underline{\tan\theta} - 2\underline{\tan\theta} = 0$$

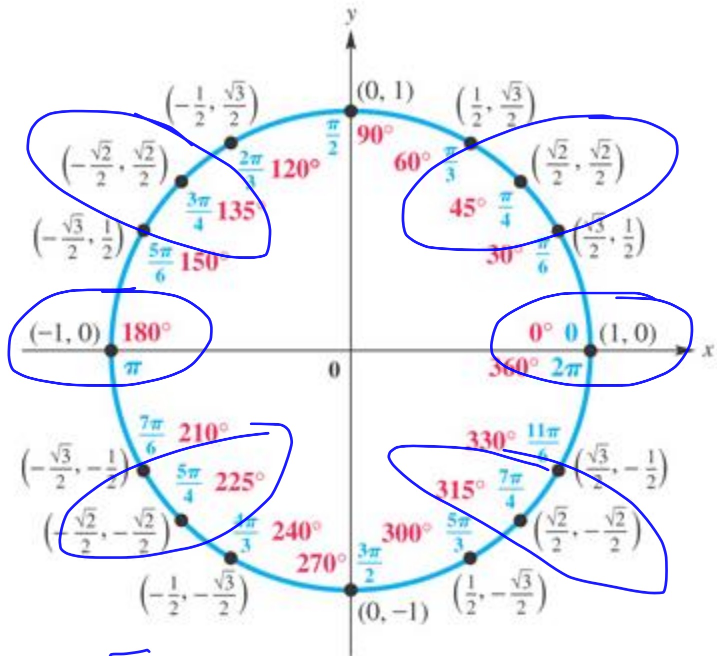
$$\tan\theta(\sec^2\theta - 2) = 0$$

$$\tan\theta = 0 \qquad \frac{y}{x}\frac{0}{x}$$

$$\boxed{\theta = 0°, 180°}$$

$$\sec^2\theta - 2 = 0$$
$$+2 \quad +2$$
$$\sqrt{\sec^2\theta} = \sqrt{2}$$
$$\sec\theta = \pm\sqrt{2}$$

$$\cos\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$



$$\boxed{\theta = 45°, 135°, 225°, 315°}$$

Using Identities and then Factoring (ex 8, 9)

Example 8: Find all solutions of $\quad 2\sin\theta - 1 = \csc\theta$

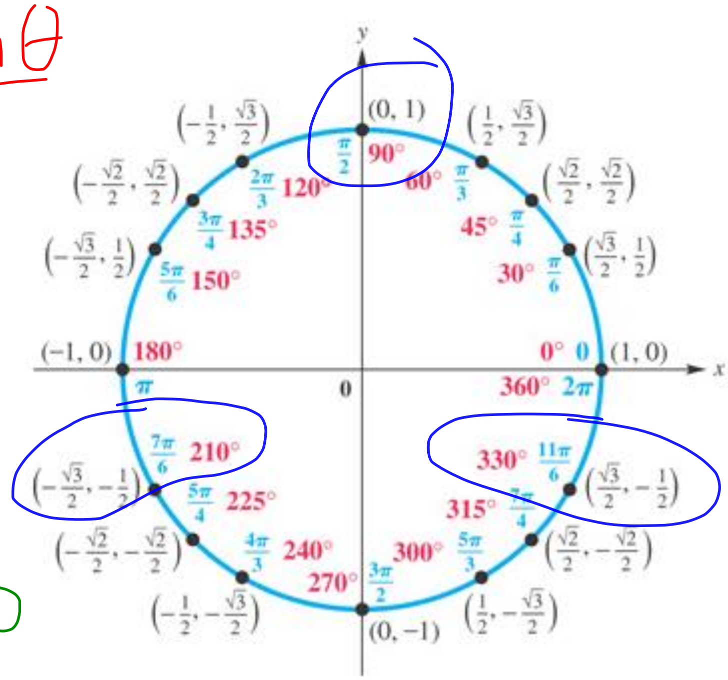
$$\left(2\sin\theta - 1 = \frac{1}{\sin\theta}\right)\frac{\sin\theta}{1}$$

$$2\sin^2\theta - \sin\theta = 1$$

$$2\sin^2\theta - \sin\theta - 1 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$2\sin\theta + 1 = 0$	$\sin\theta - 1 = 0$
$\sin\theta = -\frac{1}{2}$	$\sin\theta = 1$



$$\boxed{\begin{array}{l}\theta = 210°,\ 330°,\ 90° \\ \quad +360°n \quad +360n \quad +360n\end{array}}$$

Degrees

Example 9: Find all solutions of $5 + 5\tan^2\theta = 6\sec\theta$.

use Pythag I,D

$\tan^2\theta + 1 = \sec^2\theta$

$-1 \quad -1$

$\tan^2\theta = \sec^2\theta - 1$

$5 + 5(\sec^2\theta - 1) = 6\sec\theta$

$\cancel{5} + 5\sec^2\theta - \cancel{5} = 6\sec\theta$

$5\sec^2\theta - 6\sec\theta = 0$

$\sec\theta(5\sec\theta - 6) = 0$

output for $\sec\theta$ $(-\infty, -1] \cup [1, \infty)$

$\sec\theta = \frac{0}{1}$

$\cos\theta = \frac{1}{0}$

undefined

no solution

$5\sec\theta - 6 = 0$

$+6 \quad +6$

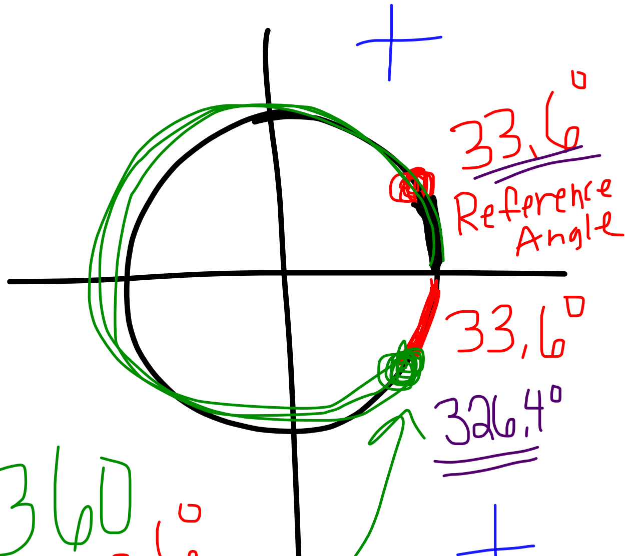
$\frac{5\sec\theta}{5} = \frac{6}{5}$

$\sec\theta = \frac{6}{5}$

$\cos\theta = \frac{5}{6}$

QI, QIV has positive cosine values

$\cos^{-1}\left(\frac{5}{6}\right) = \theta$



360
$-33.6°$

What is thin QIV angle?

$$\boxed{\begin{aligned}\theta &= 33.6° + 360°n\\ \theta &= 326.4° + 360°n\end{aligned}}$$

Quadratic Formula (ex 10)

Example 10: $9\sin^2\theta - 6\sin\theta = 1$ over the interval $[0°, 360°)$.

Squaring both sides and then using Identities (ex 11) **CHECK YOUR ANSWER!**

Example 11: Solve $\tan x + \sqrt{3} = \sec x$ over the interval $[0, 2\pi)$.