MTH95 Day 14 Sections 6.7, 7.7

Section 6.7 Variation and Problem Solving

Really are not hard, and you deserve some easier problems! You just need to pay attention to the "code" words.

Let's do a little exploration first. Say you have a job where you are paid $10 an hour. What is the relationship between the total you are paid and the number of hours you work?

You also have a second job, and you agreed to complete the job for a total of $100. What is the relationship between how much you ended up earning per hour, and the total number of hours you took to complete the job?

Solving problems involving direct variation

For the first job the amount you are paid is 10 times the number of hours you work: $P = 10h$. This is an example of direct variation; one variable is a constant multiple of another variable:

$$y = kx$$

We can say this in different ways:

- P varies directly with s
- P varies directly as s
- P is directly proportional to s

Any direct variation problem has the variables related by: $\boldsymbol{y = kx}$.

This is simply our slope-intercept formula, with an intercept at the origin (0, 0).

But rather than calling the constant "k" the slope, we give it the name of **constant of variation**, or **constant of proportionality**, or **proportionality constant**.

Example: Suppose y varies directly to x. If y is 24 when x is 8, find the constant of proportionality and the direct variation equation.

The key to the code**: y varies directly to x** → $\boldsymbol{y = kx}$

We know $y = 24$ when $x = 8$, so:

Constant of proportionality, $k =$

Direct variation equation:

Once we have the equation we could then calculate values of y given values of x.

It is hard to find simpler equations than these!

Solving problems using inverse variation

With direct variation, as one variable gets larger the other variable varies directly with it and gets larger also. When two variables are related inversely, if one variable increases the other variable varies inversely and decreases.

For the second job, the amount you earn per hour is 100 divided by the number of hours it takes you to complete the job. $R = \frac{100}{h}$. This is an example of inverse variation, $y = \frac{k}{x}$, where k is again the **constant of variation**, or **constant of proportionality**, or **proportionality constant**.

Example: Suppose y varies inversely to x. If y is 6 when x is 3, find the constant of variation and the inverse variation equation.

The key to the code**: y varies inversely to x →** $y = \frac{k}{x}$

We know y = 6 when x = 3, so:

Constant of proportionality, $k =$

Inverse variation equation:

Once we have the equation we could then calculate values of y given values of x.

These are still very simple equations!

Solving problems using joint variation

This is just direct variation using more variables: $y = kxz$

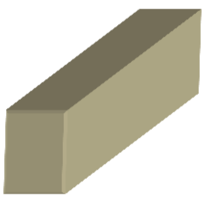
The area of a triangle is $A =$

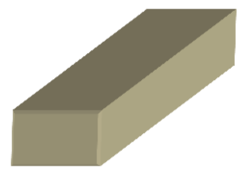
Solving problems involving combined variation

…combining everything all together… Remember: you have to have a k; anything direct goes on top with the k; anything inverse goes in the denominator.

Example: the maximum load a rectangular beam can support varies jointly as its width and the square of its height, and inversely with its length.

If a beam 1/3 foot wide, 1 foot high and 10 feet long can support 3 tons. How much weight can a similar beam support if it is 1 foot wide, 1/3 foot high and 9 feet long?





Section 7.7 Complex Numbers

Writing numbers in the form bi

It is time to deal with numbers that are not real numbers, these "imaginary" numbers do not exist on a real number line, but they do still have real world applications.

Definition: The imaginary unit: i, is defined as $i^2 = -1$ and $i = \sqrt{-1}$

All of our other non-real numbers can be written as multiples of i.

$\sqrt{-4} =$

$\sqrt{-3} =$

Short cut: lose the − and tack on an i

i is just a symbol. It allows us to get in and do algebraic manipulations with these non-real numbers.

$\sqrt{-6} \cdot \sqrt{-6} =$

Avoid this:
$\sqrt{-6} \cdot \sqrt{-6} = \sqrt{-6 \cdot -6} =$

The product rule **does not** apply to non-real numbers.

To multiply square roots of negative numbers, first rewrite each number using i. Then simplify.

$\sqrt{27} \cdot \sqrt{-3} =$

$\sqrt{-10} \cdot \sqrt{-2}$

This works for division also: $\frac{\sqrt{-8}}{\sqrt{2}} =$

We have all types of numbers: Integers, Rational, Irrational, Real and now these: Non-Real. When we combine all of the Real and Non-Real numbers together, we get the set of Complex Numbers.

A complex number is any number that can be written as: $a + bi$, where a and b are real numbers.

Note: if $b = 0$, then $a + bi = a$, which would be a real number…so all real numbers are also complex numbers.
But not all complex numbers are real numbers, if $a = 0$, then $a + bi = b$i which is called an **imaginary number**.

Add or subtract complex numbers & Multiply complex numbers

This is actually pretty easy…similar to working with like terms and foiling binomials.
Remember that $i^2 = -1$.

$(5 + 2i) + (3 - 4i)$

$(5 + 2i) - (3 - 4i)$

$(5 + 2i)(3 - 4i)$

$3i + (2 - i)$

$3i - (2 - i)$

$3i(2 - i)$

Omit Objectives 4 and 5, dividing complex number and finding powers of i.
(Ignore examples 5 and 6)