SECTION 8 BEVEL GEARING

For intersecting shafts, bevel gears offer a good means of transmitting motion and power. Most transmissions occur at right angles, **Figure 8-1**, but the shaft angle can be any value. Ratios up to 4:1 are common, although higher ratios are possible as well.

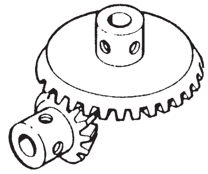
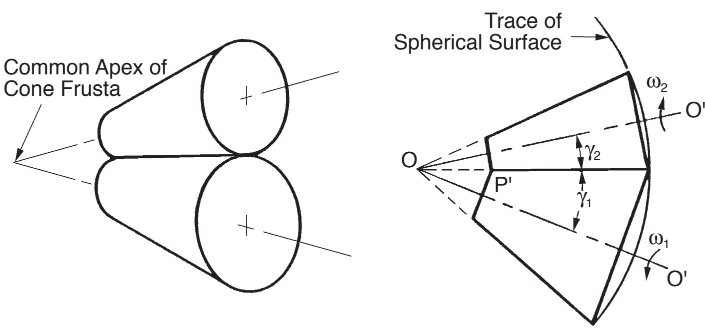


Fig. 8-1 Typical Right Angle Bevel Gear

8.1 Development And Geometry Of Bevel Gears

Bevel gears have tapered elements because they are generated and operate, in theory, on the surface of a sphere. Pitch diameters of mating bevel gears belong to frusta of cones, as shown in **Figure 8-2a**. In the full development on the surface of a sphere, a pair of meshed bevel gears are in conjugate engagement as shown in **Figure 8-2b**.



(a) Pitch Cone Frusta **(b)** Pitch Cones and the Development Sphere

Fig. 8-2 Pitch Cones of Bevel Gears

The crown gear, which is a bevel gear having the largest possible pitch angle (defined in **Figure 8-3**), is analogous to the rack of spur gearing, and is the basic tool for generating bevel gears. However, for practical reasons, the tooth form is not that of a spherical involute, and instead, the crown gear profile assumes a slightly simplified form. Although the deviation from a true spherical involute is minor, it results in a line-of-action having a figure-8 trace in its extreme extension; see **Figure 8-4**. This shape gives rise to the name "octoid" for the tooth form of modern bevel gears.

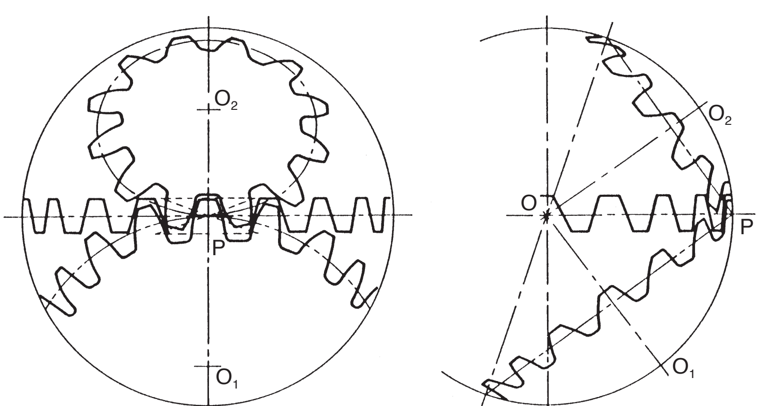


Fig. 8-3 Meshing Bevel Gear Pair with Conjugate Crown Gear

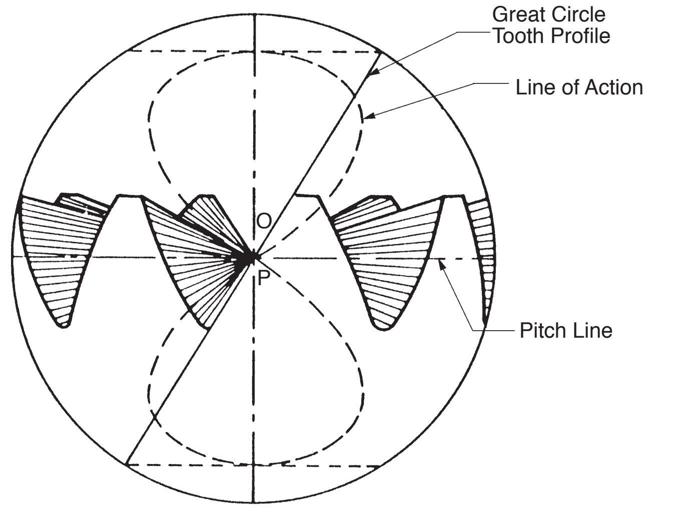


Fig. 8-4 Spherical Basis of Octoid Bevel Crown Gear

8.2 Bevel Gear Tooth Proportions

Bevel gear teeth are proportioned in accordance with the standard system of tooth proportions used for spur gears. However, the pressure angle of all standard design bevel gears is limited to 20°. Pinions with a small number of teeth are enlarged automatically when the design follows the Gleason system.

Since bevel-tooth elements are tapered, tooth dimensions and pitch diameter are referenced to the outer end (heel). Since the narrow end of the teeth (toe) vanishes at the pitch apex (center of reference generating sphere), there is a practical limit to the length (face) of a bevel gear. The geometry and identification of bevel gear parts is given in **Figure 8-5**.

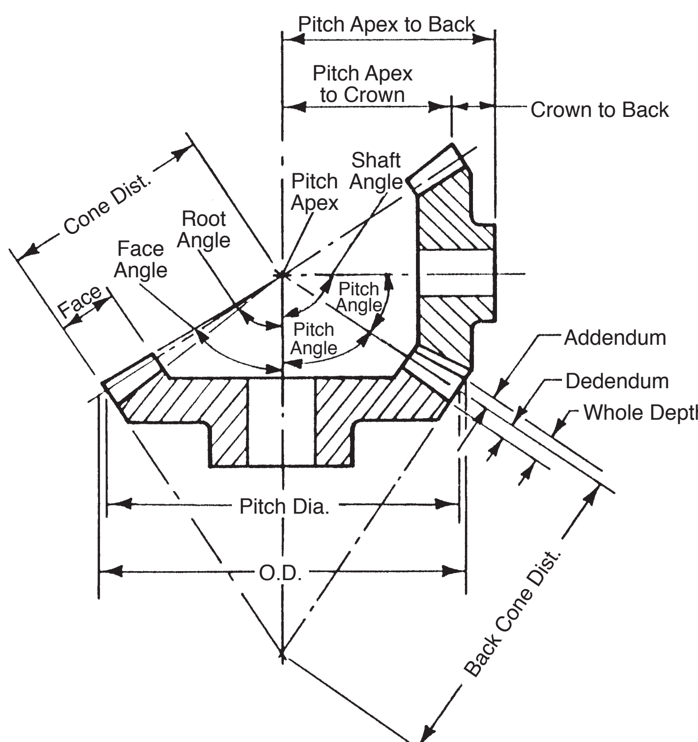


Fig. 8-5 Bevel Gear Pair Design Parameters

8.3 Velocity Ratio

The velocity ratio, i , can be derived from the ratio of several parameters:

$$i = \frac{z_1}{z_2} = \frac{d_1}{d_2} = \frac{\sin\delta_1}{\sin\delta_2} \qquad (8\text{-}1)$$

where: δ = pitch angle (see **Figure 8-5**)

8.4 Forms Of Bevel Teeth *

In the simplest design, the tooth elements are straight radial, converging at the cone apex. However, it is possible to have the teeth curve along a spiral as they converge on the cone apex, resulting in greater tooth overlap, analogous to the overlapping action of helical teeth. The result is a spiral bevel tooth. In addition, there are other possible variations. One is the zerol bevel, which is a curved tooth having elements that start and end on the same radial line.

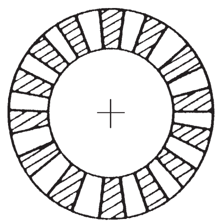
Straight bevel gears come in two variations depending upon the fabrication equipment. All current Gleason straight bevel generators are of the Coniflex form which gives an almost imperceptible convexity to the tooth surfaces. Older machines produce true straight elements. See **Figure 8-6a**.

Straight bevel gears are the simplest and most widely used type of bevel gears for the transmission of power and/or motion between intersecting shafts. Straight bevel gears are recommended:

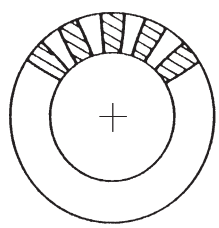
1. When speeds are less than 300 meters/min (1000 feet/min) – at higher speeds, straight bevel gears may be noisy.
2. When loads are light, or for high static loads when surface wear is not a critical factor.
3. When space, gear weight, and mountings are a premium. This includes planetary gear sets, where space does not permit the inclusion of rolling-element bearings.

Other forms of bevel gearing include the following:

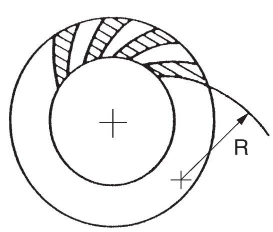
- Coniflex gears (**Figure 8-6b**) are produced by current Gleason straight bevel gear generating machines that crown the sides of the teeth in their lengthwise direction. The teeth, therefore, tolerate small amounts of misalignment in the assembly of the gears and some displacement of the gears under load without concentrating the tooth contact at the ends of the teeth. Thus, for the operating conditions, Coniflex gears are capable of transmitting larger loads than the predecessor Gleason straight bevel gears.
- Spiral bevels (**Figure 8-6c**) have curved oblique teeth which contact each other gradually and smoothly from one end to the other. Imagine cutting a straight bevel into an infinite number of short face width sections, angularly displace one relative to the other, and one has a spiral bevel gear. Well-designed spiral bevels have two or more teeth in contact at all times. The overlapping tooth action transmits motion more smoothly and quietly than with straight bevel gears.



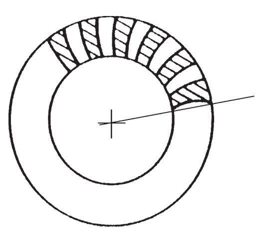
(a) Straight Teeth



(b) Coniflex Teeth
(Exaggerated Tooth Curving)



(c) Spiral Teeth



(d) Zerol Teeth

Fig. 8-6 Forms of Bevel Gear Teeth

- Zerol bevels (**Figure 8-6d**) have curved teeth similar to those of the spiral bevels, but with zero spiral angle at the middle of the face width; and they have little end thrust.

Both spiral and Zerol gears can be cut on the same machines with the same circular face-mill cutters or ground on the same grinding machines. Both are produced with localized tooth contact which can be controlled for length, width, and shape.

Functionally, however, Zerol bevels are similar to the straight bevels and thus carry the same ratings. In fact, Zerols can be used in the place of straight bevels without mounting changes.

Zerol bevels are widely employed in the aircraft industry, where ground-tooth precision gears are generally required. Most hypoid cutting machines can cut spiral bevel, Zerol or hypoid gears.

8.5 Bevel Gear Calculations

Let z_1 and z_2 be pinion and gear tooth numbers; shaft angle Σ; and pitch cone angles δ_1 and δ_2; then:

$$\left.\begin{aligned} \tan\delta_1 &= \frac{\sin\Sigma}{\dfrac{z_2}{z_1} + \cos\Sigma} \\ \tan\delta_2 &= \frac{\sin\Sigma}{\dfrac{z_1}{z_2} + \cos\Sigma} \end{aligned}\right\} \qquad (8\text{-}2)$$

Generally, shaft angle $\Sigma = 90°$ is most used. Other angles (**Figure 8-7**) are sometimes used. Then, it is called "bevel gear in nonright angle drive". The 90° case is called "bevel gear in right angle drive".

When $\Sigma = 90°$, **Equation (8-2)** becomes:

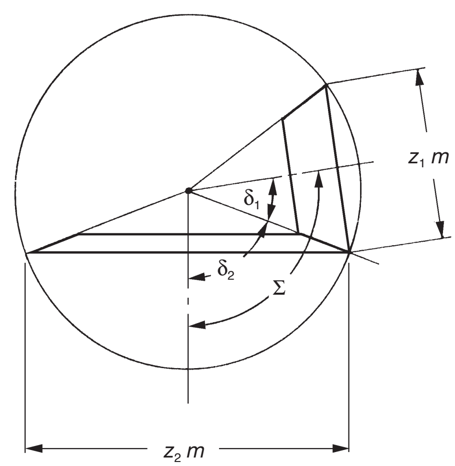


Fig. 8-7 The Pitch Cone Angle of Bevel Gear

$$\left.\begin{aligned} \delta_1 &= \tan^{-1}\left(\frac{z_1}{z_2}\right) \\ \delta_2 &= \tan^{-1}\left(\frac{z_2}{z_1}\right) \end{aligned}\right\} \qquad (8\text{-}3)$$

Miter gears are bevel gears with $\Sigma = 90°$ and $z_1 = z_2$. Their speed ratio $z_1 / z_2 = 1$. They only change the direction of the shaft, but do not change the speed.

Figure 8-8 depicts the meshing of bevel gears. The meshing must be considered in pairs. It is because the pitch cone angles δ_1 and δ_2 are restricted by the gear ratio z_1 / z_2. In the facial view, which is normal to the contact line of pitch cones, the meshing of bevel gears appears to be similar to the meshing of spur gears.

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8.5.1 Gleason Straight Bevel Gears

The straight bevel gear has straight teeth flanks which are along the surface of the pitch cone from the bottom to the apex. Straight bevel gears can be grouped into the Gleason type and the standard type.

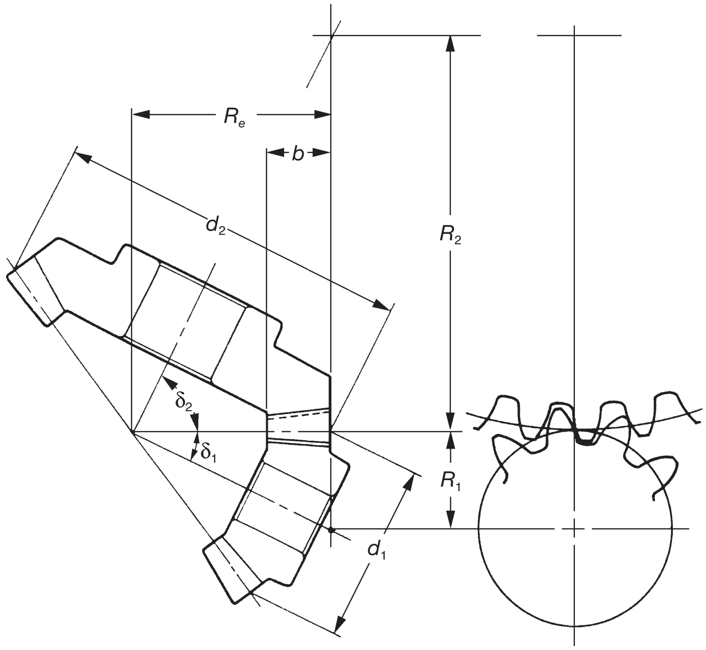

Fig. 8-8 The Meshing of Bevel Gears

In this section, we discuss the Gleason straight bevel gear. The Gleason Company defined the tooth profile as: whole depth h =2.188m; top clearance c_a = 0.188m; and working depth h_w = 2.000m.

The characteristics are:

- **Design specified profile shifted gears:**

In the Gleason system, the pinion is positive shifted and the gear is negative shifted. The reason is to distribute the proper strength between the two gears. Miter gears, thus, do not need any shifted tooth profile.

- **The top clearance is designed to be parallel**

The outer cone elements of two paired bevel gears are parallel. That is to ensure that the top clearance along the whole tooth is the same. For the standard bevel gears, top clearance is variable. It is smaller at the toe and bigger at the heel.

Table 8-1 shows the minimum number of teeth to prevent undercut in the Gleason system at the shaft angle $\Sigma = 90°$.

Table 8-2 presents equations for designing straight bevel gears in the Gleason system. The meanings of the dimensions and angles are shown in **Figure 8-9**. All the equations in **Table 8-2** can also be applied to bevel gears with any shaft angle.

The straight bevel gear with crowning in the Gleason system is called a Coniflex gear. It is manufactured by a special Gleason "Coniflex" machine. It can successfully eliminate poor tooth wear due to improper mounting and assembly.

The first characteristic of a Gleason straight bevel gear is its profile shifted tooth. From **Figure 8-10**, we can see the positive tooth profile shift in the pinion. The tooth thickness at the root diameter of a Gleason pinion is larger than that of a standard straight bevel gear.

Table 8-1 The Minimum Numbers of Teeth to Prevent Undercut

Pressure Angle	Combination of Numbers of Teeth $\frac{z_1}{z_2}$					
(14.5°)	29 / Over 29	28 / Over 29	27 / Over 31	26 / Over 35	25 / Over 40	24 / Over 57
20°	16 / Over 16	15 / Over 17	14 / Over 20	13 / Over 30	—	—
(25°)	13 / Over 13	—	—	—	—	—

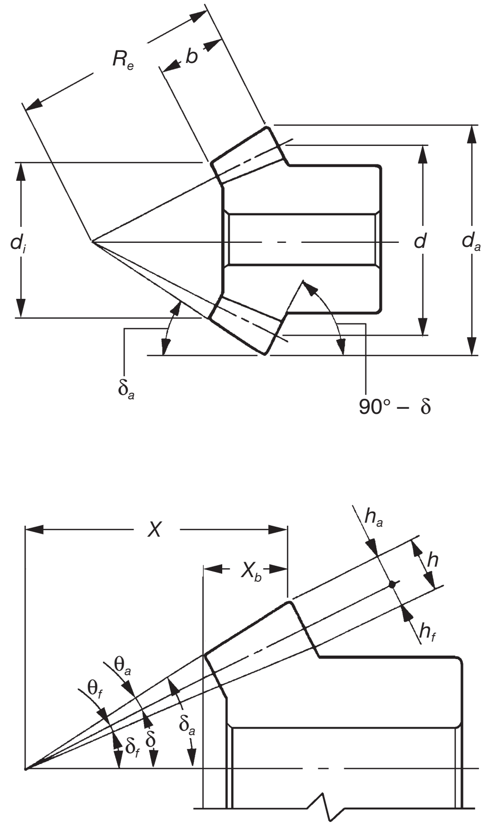

Fig. 8-9 Dimensions and Angles of Bevel Gears

8.5.2. Standard Straight Bevel Gears

A bevel gear with no profile shifted tooth is a standard straight bevel gear. The applicable equations are in **Table 8-3**.

These equations can also be applied to bevel gear sets with other than 90° shaft angle.

8.5.3 Gleason Spiral Bevel Gears

A spiral bevel gear is one with a spiral tooth flank as in **Figure 8-11**. The spiral is generally consistent with the curve of a cutter with the diameter d_c. The spiral angle β is the angle between a generatrix element of the pitch cone and the tooth flank. The spiral angle just at the tooth flank center is called central spiral angle β_m. In practice, spiral angle means central spiral angle.

All equations in **Table 8-6** are dedicated for the manufacturing method of Spread Blade or of Single Side from Gleason. If a gear is not cut per the Gleason system, the equations will be different from these.

The tooth profile of a Gleason spiral bevel gear shown here has the whole depth h = 1.888m; top clearance c_a = 0.188m; and working depth h_w = 1.700m. These Gleason spiral bevel gears belong to a stub gear system. This is applicable to gears with modules $m > 2.1$.

Table 8-4 shows the minimum number of teeth to avoid undercut in the Gleason system with shaft angle $\Sigma = 90°$ and pressure angle $\alpha_n = 20°$.

If the number of teeth is less than 12, **Table 8-5** is used to determine the gear sizes.

All equations in **Table 8-6** are also applicable to Gleason bevel gears with any shaft angle. A spiral bevel gear set requires matching of hands; left-hand and right-hand as a pair.

Table 8-2 The Calculations of Straight Bevel Gears of the Gleason System

No.	Item	Symbol	Formula	Example: Pinion	Example: Gear
1	Shaft Angle	Σ		90°	
2	Module	m		3	
3	Pressure Angle	α		20°	
4	Number of Teeth	z_1, z_2		20	40
5	Pitch Diameter	d	zm	60	120
6	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma - \delta_1$	26.56505°	63.43495°
7	Cone Distance	R_e	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Face Width	b	It should be less than R_e / 3 or 10m	22	
9	Addendum	h_{a1} h_{a2}	$2.000m - h_{a2}$ $0.540m + \dfrac{0.460m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	4.035	1.965
10	Dedendum	h_f	$2.188m - h_a$	2.529	4.599
11	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	2.15903°	3.92194°
12	Addendum Angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194°	2.15903°
13	Outer Cone Angle	δ_a	$\delta + \theta_a$	30.48699°	65.59398°
14	Root Cone Angle	δ_f	$\delta - \theta_f$	24.40602°	59.51301°
15	Outside Diameter	d_a	$d + 2h_a\cos\delta$	67.2180	121.7575
16	Pitch Apex to Crown	X	$R_e\cos\delta - h_a\sin\delta$	58.1955	28.2425
17	Axial Face Width	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.0029	9.0969
18	Inner Outside Diameter	d_i	$d_a - \dfrac{2b\sin\delta_a}{\cos\theta_a}$	44.8425	81.6609

Table 8-3 Calculation of a Standard Straight Bevel Gears

No.	Item	Symbol	Formula	Example: Pinion	Example: Gear
1	Shaft Angle	Σ		90°	
2	Module	m		3	
3	Pressure Angle	α		20°	
4	Number of Teeth	z_1, z_2		20	40
5	Pitch Diameter	d	zm	60	120
6	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma - \delta_1$	26.56505°	63.43495°
7	Cone Distance	R_e	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Face Width	b	It should be less than R_e/3 or 10m	22	
9	Addendum	h_a	1.00 m	3.00	
10	Dedendum	h_f	1.25 m	3.75	
11	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	3.19960°	
12	Addendum Angle	θ_a	$\tan^{-1}(h_a/R_e)$	2.56064°	
13	Outer Cone Angle	δ_a	$\delta + \theta_a$	29.12569°	65.99559°
14	Root Cone Angle	δ_f	$\delta - \theta_f$	23.36545°	60.23535°
15	Outside Diameter	d_a	$d + 2h_a\cos\delta$	65.3666	122.6833
16	Pitch Apex to Crown	X	$R_e\cos\delta - h_a\sin\delta$	58.6584	27.3167
17	Axial Face Width	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.2374	8.9587
18	Inner Outside Diameter	d_i	$d_a - \dfrac{2b\sin\delta_a}{\cos\theta_a}$	43.9292	82.4485

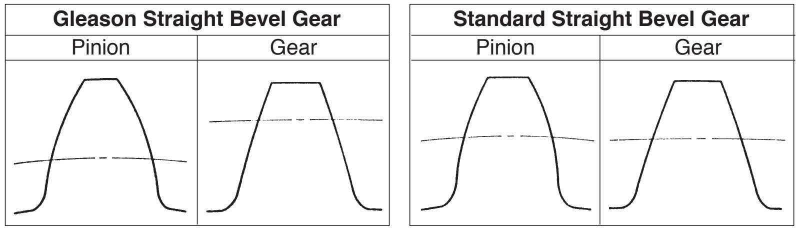

Fig. 8-10 The Tooth Profile of Straight Bevel Gears

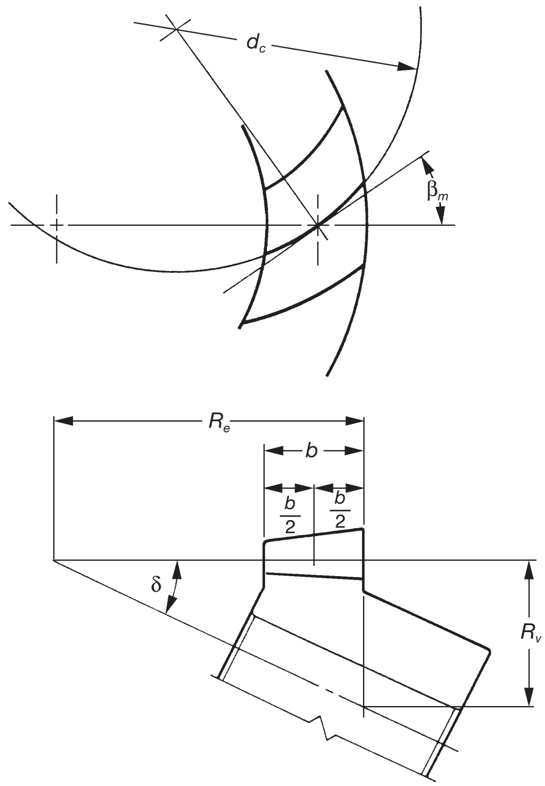

Fig. 8-11 Spiral Bevel Gear (Left-Hand)

Table 8-4 The Minimum Numbers of Teeth to Prevent Undercut $\beta_m = 35°$

Pressure Angle	Combination of Numbers of Teeth $\frac{z_1}{z_2}$					
20°	17 / Over 17	16 / Over 18	15 / Over 19	14 / Over 20	13 / Over 22	12 / Over 26

Table 8-5 Dimensions for Pinions with Numbers of Teeth Less than 12

Number of Teeth in Pinion	z_1	6	7	8	9	10	11
Number of Teeth in Gear	z_2	Over 34	Over 33	Over 32	Over 31	Over 30	Over 29
Working Depth	h_w	1.500	1.560	1.610	1.650	1.680	1.695
Whole Depth	h	1.666	1.733	1.788	1.832	1.865	1.882
Gear Addendum	h_{a2}	0.215	0.270	0.325	0.380	0.435	0.490
Pinion Addendum	h_{a1}	1.285	1.290	1.285	1.270	1.245	1.205
Circular Tooth Thickness of Gear s_2	30	0.911	0.957	0.975	0.997	1.023	1.053
	40	0.803	0.818	0.837	0.860	0.888	0.948
	50	—	0.757	0.777	0.828	0.884	0.946
	60	—	—	0.777	0.828	0.883	0.945
Pressure Angle	α_n	20°					
Spiral Angle	β_m	35°... 40°					
Shaft Angle	Σ	90°					

NOTE: All values in the table are based on m = 1.

8.5.4 Gleason Zerol Spiral Bevel Gears

When the spiral angle β_m = 0, the bevel gear is called a Zerol bevel gear. The calculation equations of **Table 8-2** for Gleason straight bevel gears are applicable. They also should take care again of the rule of hands; left and right of a pair must be matched. **Figure 8-12** is a left-hand Zerol bevel gear.

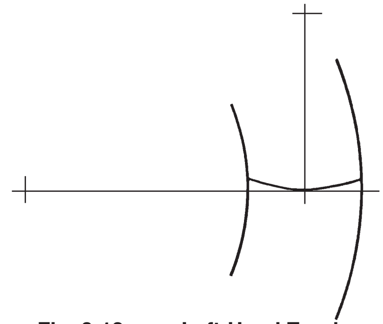

Fig. 8-12 Left-Hand Zerol Bevel Gear

Table 8-6 The Calculations of Spiral Bevel Gears of the Gleason System

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Shaft Angle	Σ		90°	
2	Outside Radial Module	m		3	
3	Normal Pressure Angle	α_n		20°	
4	Spiral Angle	β_m		35°	
5	Number of Teeth and Spiral Hand	z_1, z_2		20 (L)	40 (R)
6	Radial Pressure Angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta_m}\right)$	23.95680	
7	Pitch Diameter	d	zm	60	120
8	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\frac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
9	Cone Distance	R_e	$\frac{d_2}{2\sin\delta_2}$	67.08204	
10	Face Width	b	It should be less than $R_e/3$ or $10m$	20	
11	Addendum	h_{a1} h_{a2}	$1.700m - h_{a2}$ $0.460m + \frac{0.390m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	3.4275	1.6725
12	Dedendum	h_f	$1.888m - h_a$	2.2365	3.9915
13	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	1.90952°	3.40519°
14	Addendum Angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.40519°	1.90952°
15	Outer Cone Angle	δ_a	$\delta+\theta_a$	29.97024°	65.34447°
16	Root Cone Angle	δ_f	$\delta-\theta_f$	24.65553°	60.02976°
17	Outside Diameter	d_a	$d+2h_a\cos\delta$	66.1313	121.4959
18	Pitch Apex to Crown	X	$R_e\cos\delta - h_a\sin\delta$	58.4672	28.5041
19	Axial Face Width	X_b	$\frac{b\cos\delta_a}{\cos\theta_a}$	17.3563	8.3479
20	Inner Outside Diameter	d_i	$d_a - \frac{2b\sin\delta_a}{\cos\theta_a}$	46.1140	85.1224

SECTION 9 WORM MESH

The worm mesh is another gear type used for connecting skew shafts, usually 90°. See **Figure 9-1**. Worm meshes are characterized by high velocity ratios. Also, they offer the advantage of higher load capacity associated with their line contact in contrast to the point contact of the crossed-helical mesh.

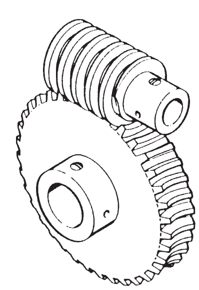

Fig. 9-1 Typical Worm Mesh