Section R.3: Polynomials

1. Incorrect; $(mn)^2 = m^2n^2$

3. Incorrect; $\left(\frac{k}{5}\right)^3 = \frac{k^3}{5^3} = \frac{k^3}{125}$

5. $9^3 \cdot 9^5 = 9^{3+5} = 9^8$

7. $(-4x^5)(4x^2) = (-4 \cdot 4)(x^5x^2)$
$= -16x^{5+2}$
$= -16x^7$

9. $(2^2)^5 = 2^{2 \cdot 5} = 2^{10}$

11. $-(4m^3n^0)^2 = -\left[4^2(m^3)^2(n^0)^2\right]$
$= -4^2m^{3 \cdot 2}n^{0 \cdot 2}$
$= -4^2m^6n^0$
$= -(4^2)m^6 \cdot 1$
$= -4^2m^6$ or $-16m^6$

13. $\left(\frac{r^8}{s^2}\right)^3 = \frac{(r^8)^3}{(s^2)^3} = \frac{r^{8 \cdot 3}}{s^{2 \cdot 3}} = \frac{r^{24}}{s^6}$

15. **(a)** $6^0 = 1$; B

(b) $-6^0 = -1$; C

(c) $(-6)^0 = 1$; B

(d) $-(-6)^0 = -1$; C

17. Answers will vary.
$x^2 + x^2 = 2x^2$

19. $-5x^{11}$ is a polynomial. It is a monomial since it has one term. It has degree 11 since 11 is the highest exponent.

21. $18p^5q + 6pq$ is a polynomial. It is a binomial since it has two terms. It has degree 6 because 6 is the sum of the exponents in the term $18p^5q$, and this term has a higher degree than the term $6pq$.

23. $\sqrt{2}x^2 + \sqrt{3}x^6$ is a polynomial. It is a binomial since it has two terms. It has degree 6 since 6 is the highest exponent.

25. $\frac{1}{3}r^2s^2 - \frac{3}{5}r^4s^2 + rs^3$ is a polynomial. It is a trinomial since it has three terms. It has degree 6 because the sum of the exponents in the term $-\frac{3}{5}r^4s^2$ is 6, and this term has the highest degree.

27. $\frac{5}{p} + \frac{2}{p^2} + \frac{5}{p^3}$ is not a polynomial since positive exponents in the denominator are equivalent to negative exponents in the numerator.

29. $(3x^2 - 4x + 5) + (-2x^2 + 3x - 2) = (3-2)x^2 + (-4+3)x + [5 + (-2)] = 1 \cdot x^2 + (-1)x + 3 = x^2 - x + 3$

31.
$$\begin{aligned} 2(12y^2 - 8y + 6) - 4(3y^2 - 4y + 2) &= 2(12y^2) - 2(8y) + 2(6) - 4(3y^2) - 4(-4y) - 4 \cdot 2 \\ &= 24y^2 - 16y + 12 - 12y^2 + 16y - 8 = 12y^2 + 4 \end{aligned}$$

33.
$$\begin{aligned} (6m^4 - 3m^2 + m) - (2m^3 + 5m^2 + 4m) + (m^2 - m) &= 6m^4 - 3m^2 + m - 2m^3 - 5m^2 - 4m + m^2 - m \\ &= 6m^4 - 2m^3 + (-3-5+1)m^2 + (1-4-1)m \\ &= 6m^4 - 2m^3 + (-7)m^2 + (-4)m \\ &= 6m^4 - 2m^3 - 7m^2 - 4m \end{aligned}$$

35. $(4r-1)(7r+2) = 4r(7r) + 4r(2) - 1(7r) - 1(2) = 28r^2 + 8r - 7r - 2 = 28r^2 + r - 2$

37.
$$\begin{aligned} x^2\left(3x - \frac{2}{3}\right)\left(5x + \frac{1}{3}\right) &= x^2\left[\left(3x - \frac{2}{3}\right)\left(5x + \frac{1}{3}\right)\right] = x^2\left[(3x)(5x) + (3x)\left(\frac{1}{3}\right) - \frac{2}{3}(5x) - \frac{2}{3}\left(\frac{1}{3}\right)\right] \\ &= x^2\left(15x^2 + x - \frac{10}{3}x - \frac{2}{9}\right) = x^2\left(15x^2 + \frac{3}{3}x - \frac{10}{3}x - \frac{2}{9}\right) \\ &= x^2\left(15x^2 - \frac{7}{3}x - \frac{2}{9}\right) = 15x^4 - \frac{7}{3}x^3 - \frac{2}{9}x^2 \end{aligned}$$

39. $4x^2(3x^3 + 2x^2 - 5x + 1) = 4x^2(3x^3) + 4x^2(2x^2) - 4x^2(5x) + 4x^2 \cdot 1 = 12x^5 + 8x^4 - 20x^3 + 4x^2$

41. $(2z-1)(-z^2+3z-4)=(2z-1)(-z^2)+(2z-1)(3z)-(2z-1)(4)$

$$=2z(-z^2)-1(-z^2)+2z(3z)-1(3z)-(2z)(4)-(-1)(4)$$

$$=-2z^3+z^2+6z^2-3z-8z-(-4)=-2z^3+7z^2-11z+4$$

We may also multiply vertically.

$$\begin{array}{rl} -z^2+3z\ -4 & \\ \underline{2z\ -1} & \\ z^2-3z\ +4 & \leftarrow -1(-z^2+3z\ -4) \\ \underline{-2z^3+6z^2-8z\qquad} & \leftarrow 2z(-z^2+3z\ -4) \\ -2z^3+7z^2-11z+4 & \end{array}$$

43. $(m-n+k)(m+2n-3k)=(m-n+k)(m)+(m-n+k)(2n)-(m-n+k)(3k)$

$$=m^2-mn+km+2mn-2n^2+2kn-3km+3kn-3k^2$$

$$=m^2+mn-2n^2-2km+5kn-3k^2$$

We may also multiply vertically.

$$\begin{array}{rl} m-\ n+\ k & \\ \underline{m+2n\ -3k} & \\ -3km+3kn-3k^2 & \leftarrow -3k(m-\ n+\ k) \\ 2mn-2n^2\qquad +2kn\qquad & \leftarrow 2n(m-\ n+\ k) \\ \underline{m^2-mn\qquad +km\qquad\qquad} & \leftarrow m(m-\ n+\ k) \\ m^2+mn-2n^2-2km+5kn-3k^2 & \end{array}$$

45. $(2m+3)(2m-3)=(2m)^2-3^2$

$$=4m^2-9$$

47. $(4x^2-5y)(4x^2+5y)=(4x^2)^2-(5y)^2$

$$=16x^4-25y^2$$

49. $(4m+2n)^2=(4m)^2+2(4m)(2n)+(2n)^2$

$$=16m^2+16mn+4n^2$$

51. $(5r-3t^2)^2=(5r)^2-2(5r)(3t^2)+(3t^2)^2$

$$=25r^2-30rt^2+9t^4$$

53. $[(2p-3)+q]^2=(2p-3)^2+2(2p-3)(q)+q^2=(2p)^2-2(2p)(3)+(3)^2+4pq-6q+q^2$

$$=4p^2-12p+9+4pq-6q+q^2$$

55. $[(3q+5)-p][(3q+5)+p]=(3q+5)^2-p^2=[(3q)^2+2(3q)(5)+5^2]-p^2=9q^2+30q+25-p^2$

57. $[(3a+b)-1]^2=(3a+b)^2-2(3a+b)(1)+1^2$

$$=(9a^2+6ab+b^2)-2(3a+b)+1=9a^2+6ab+b^2-6a-2b+1$$

59. $(y+2)^3=(y+2)^2(y+2)=(y^2+4y+4)(y+2)=y^3+4y^2+4y+2y^2+8y+8=y^3+6y^2+12y+8$

61. $(q-2)^4=(q-2)^2(q-2)^2=\left(q^2-4q+4\right)\left(q^2-4q+4\right)$
$=q^4-4q^3+4q^2-4q^3+16q^2-16q+4q^2-16q+16=q^4-8q^3+24q^2-32q+16$

63. $\left(p^3-4p^2+p\right)-\left(3p^2+2p+7\right)=p^3-4p^2+p-3p^2-2p-7=p^3-7p^2-p-7$

65. $(7m+2n)(7m-2n)=(7m)^2-(2n)^2=49m^2-4n^2$

67. $-3(4q^2-3q+2)+2(-q^2+q-4)=-12q^2+9q-6-2q^2+2q-8=-14q^2+11q-14$

69. $p(4p-6)+2(3p-8)=4p^2-6p+6p-16=4p^2-16$

71. $-y(y^2-4)+6y^2(2y-3)=-y^3+4y+12y^3-18y^2=11y^3-18y^2+4y$

73.
$$\begin{array}{r|l} & 2x^5+7x^4-5x^2+7 \\ \hline -2x^2 & -4x^7-14x^6+10x^4-14x^2 \\ & \underline{-4x^7} \\ & \quad -14x^6 \\ & \quad \underline{-14x^6} \\ & \qquad 10x^4 \\ & \qquad \underline{10x^4} \\ & \qquad\quad -14x^2 \\ & \qquad\quad \underline{-14x^2} \\ & \qquad\qquad 0 \end{array}$$

$$\frac{-4x^7-14x^6+10x^4-14x^2}{-2x^2}=2x^5+7x^4-5x^2+7$$

75.
$$\begin{array}{r|l} & -5x^2+8 \\ \hline -2x^6 & 10x^8-16x^6-4x^4 \\ & \underline{10x^8} \\ & \quad -16x^6 \\ & \quad \underline{-16x^6} \\ & \qquad -4x^4 \end{array}$$

$$\frac{10x^8-16x^6-4x^4}{-2x^6}=-5x^2+8-\frac{4x^4}{-2x^6}$$
$$=-5x^2+8+\frac{2}{x^2}$$

77.
$$\begin{array}{r|l} & 2m^2+m-2 \\ \hline 3m+2 & 6m^3+7m^2-4m+2 \\ & \underline{6m^3+4m^2} \\ & \qquad 3m^2-4m \\ & \qquad \underline{3m^2+2m} \\ & \qquad\quad -6m+2 \\ & \qquad\quad \underline{-6m-4} \\ & \qquad\qquad 6 \end{array}$$

$$\frac{6m^3+7m^2-4m+2}{3m+2}=2m^2+m-2+\frac{6}{3m+2}$$

79.
$$\begin{array}{r} x^3 - x^2 - x + 4 \\ 3x+3 \overline{\smash{\big)}\, 3x^4 - 0x^3 - 6x^2 + 9x - 5} \\ \underline{3x^4 + 3x^3} \\ -3x^3 - 6x^2 \\ \underline{-3x^3 - 3x^2} \\ -3x^2 + 9x \\ \underline{-3x^2 - 3x} \\ 12x - 5 \\ \underline{12x + 12} \\ -17 \end{array}$$

Thus, $\dfrac{3x^4 - 6x^2 + 9x - 5}{3x+3} = x^3 - x^2 - x + 4 + \dfrac{-17}{3x+3}$ or $x^3 - x^2 - x + 4 - \dfrac{17}{3x+3}$.

81. $99 \times 101 = (100-1)(100+1) = 100^2 - 1^2$
$= 10{,}000 - 1 = 9999$

82. $63 \times 57 = (60+3)(60-3) = 60^2 - 3^2$
$= 3600 - 9 = 3591$

83. $102^2 = (100+2)^2 = 100^2 + 2(100)(2) + 2^2$
$= 10{,}000 + 400 + 4 = 10{,}404$

84. $71^2 = (70+1)^2 = 70^2 + 2(70)(1) + 1^2$
$= 4900 + 140 + 1 = 5041$

85. **(a)** The area of the largest square is $s^2 = (x+y)^2$.

(b) The areas of the two squares are x^2 and y^2. The area of each rectangle is xy. Therefore, the area of the largest square can be written as $x^2 + 2xy + y^2$.

(c) Answers will vary. The total area must equal the sum of the four parts.

(d) It reinforces the special product for squaring a binomial: $(x+y)^2 = x^2 + 2xy + y^2$.

87. **(a)** The volume is
$$V = \frac{1}{3}h(a^2 + ab + b^2) = \frac{1}{3}(200)(314^2 + 314 \times 756 + 756^2) \approx 60{,}501{,}000\,\text{ft}^3.$$

(b) The shape becomes a rectangular box with a square base. Its volume is given by length × width × height or b^2h.

(c) If we let $a = b$, then $\frac{1}{3}h(a^2 + ab + b^2)$ becomes $\frac{1}{3}h(b^2 + bb + b^2)$, which simplifies to hb^2. Yes, the Egyptian formula gives the same result.

89. $x = 1940$
$0.000020591075(1940)^3 - 0.1201456829(1940)^2 + 233.5530856(1940) - 151{,}249.8184 \approx 6.2$
The formula is 0.1 high.

91. $x = 1978$
$0.000020591075(1978)^3 - 0.1201456829(1978)^2 + 233.5530856(1978) - 151{,}249.8184 \approx 2.3$
The formula is exact.

93. $(0.25^3)(400^3) = [(0.25)(400)]^3$
$= 100^3 = 1{,}000{,}000$

95. $\dfrac{4.2^5}{2.1^5} = \left(\dfrac{4.2}{2.1}\right)^5 = 2^5 = 32$

97. $(y-x)^3 = (y-x)^2(y-x) = (y^2 - 2yx + x^2)(y-x) = y^3 - 2y^2x + x^2y - y^2x + 2yx^2 - x^3$
$= y^3 - 3y^2x + 3x^2y - x^3$

$-(x-y)^3 = -(x-y)^2(x-y) = -(x^2 - 2xy + y^2)(x-y) = -(x^3 - 2x^2y + y^2x - x^2y + 2xy^2 - y^3)$
$= -(x^3 - 3x^2y + 3xy^2 - y^3) = -x^3 + 3x^2y - 3xy^2 + y^3$

Both expressions are equal to $-x^3 + 3x^2y - 3xy^2 + y^3$.