Securitized Markets, International Capital Flows, and Global Welfare*

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Abstract

We study the effect of collateralized lending and securitization on the global supply of securitized assets, welfare, and international net and gross capital flows in a two country general equilibrium model with idiosyncratic investment risk. The financial sectors in the two countries, Home and Foreign, differ by the collateral requirement for investment loans, with Home requiring lower margins. In autarky, Home endogenously supplies more assets and enables more risk sharing. Upon financial integration, capital flows from Foreign to Home, leading to lower interest rates and an increase in the global supply of assets. Foreign enjoys substantial welfare gains through better risk sharing and portfolio reallocation, while the welfare experience for Home is ambiguous. Gross capital flows arise when agents face aggregate shocks to the expected payoff to investment projects, but can collapse when shocks concern the variance of returns.

Keywords: collateralized loan obligations, endogenous risk sharing, global imbalances, gross international asset positions.

JEL classification: D52, F32, F36, G11, G15, G23.

1 Introduction

The 2007–2009 global financial crisis has often been attributed to the dissemination of low quality securitized assets originating from the United States. Unlike in traditional banking, where debts were illiquid and creditors held on to their positions until maturity, since the 1990s it has been increasingly common for creditors to pool (create a large portfolio of) debt contracts of similar characteristics through intermediaries and sell off these securitized assets to outside investors. Under this "originate-and-distribute" business model, the insufficiency of "the skin in the game" may lead to moral hazard by the intermediaries and

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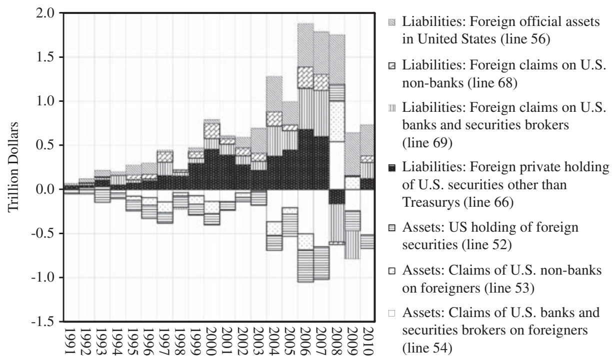
compromise efficiency. However, for that to be the case there have to be buyers of the securitized assets. As Bertaut et al. (2012) argue, those investors were primarily Europeans, which helped the crisis to spread globally. Given that the purchase of U.S. asset-backed securities (ABS) accounted for about one third of aggregate capital inflows at the time (see Figure 1.1), this channel is clearly substantial. However, little theoretical work have been done studying how characteristics of securitized markets affect international capital flows. In this paper we provide a theoretical framework to understand the role of securitization and global financial integration on international capital flows and welfare implications.

Figure 2. U.S. Gross Capital Flows by Category



Source: U.S. Bureau of Economic Analysis. Increase in U.S. liability to foreigners is indicated by positive bar, increase in U.S. claims on foreigners is indicated by negative bar. Only a subset of gross flows is included, so that flows do not sum to zero.

Figure 1.1: Gross capital flows by category: Figure 2 of Shin (2012).

Note: the dark bars labeled “Liabilities: Foreign private holding of U.S. securities other than Treasuries” are capital inflows in the form of purchase of securitized assets.

We consider a general equilibrium model with two countries, collateralized lending, and securitized markets. The two countries, Home (U.S.) and Foreign (Europe), each are populated by a continuum of ex ante identical entrepreneurs and have financial sectors that pool collateralized loans and issue asset-backed securities (ABS). Entrepreneurs have investment projects that are subject to uninsurable idiosyncratic risk, but they effectively share some of the risk by borrowing against their projects and using the proceeds to invest in ABS. The key difference between the Home and Foreign financial systems is that the loan margin, or required down payment, is lower for Home investors. In autarky, Home investors enjoy a greater degree of risk sharing because they are highly leveraged, borrowing against their investment projects to invest in ABS, and thus better able to self-insure against idiosyncratic shocks.

International trade in assets allows Foreign investors to buy Home securities, which offer attractive, relatively safe returns. Capital flows from Foreign to Home, leading to low interest rates. Importantly, we show that as capital flows to Home, the global supply of assets increases, accompanied in Home by less risk sharing and high investment levels. Foreign investors gain substantially in terms of welfare through better risk sharing opportunities driven by improved access to ABS with higher returns and (endogenously) increased state-contingency of domestic loans. On the other hand, the effect of financial integration on the welfare of Home investors is ambiguous. While low interest rates promote investment, they also reduce risk sharing (which occurs through default or loan conversion in our model) as the threshold of loan conversion decreases. The net consequence of these two offsetting effects depends on investors' risk aversion and exposure to idiosyncratic risk.[1]

The key intuition for our results is that the down payments for loans determine the endogenous supply and riskiness of ABS. When loan down payments are low, agents can borrow more against their investment projects, leading to a greater supply of ABS. When loan down payments differ across countries, financial integration leads to net capital flows as the demand for relatively safe assets from the country with more uninsured idiosyncratic risk (high-margin country) is met by the greater supply of "safe-enough" assets by the low-margin country. Financial integration affects the supply of ABS in each country as capital flows to the financial system that can best insure idiosyncratic risk. However, financial integration also affects borrowing rates and therefore default thresholds. As a result, the riskiness of ABS can change after financial integration.

Interestingly, in our model Foreign investors demand Home ABS not because they are safer than the Foreign ones—in fact, with aggregate risk they are generally not. Home securities are attractive because they are *safe enough* compared to entrepreneurial investment risk, and in greater supply than Foreign securities. Thus, Home has an advantage in producing assets because it can produce *more* relatively safe assets, not because it produces *safer* assets.

An important contribution of our analysis is to understand how differences across securitized markets can produce *gross* capital flows. Gross capital flows are an essential and important feature of international finance, but few models studying *net* capital flows also produce gross flows.[2] In our model, risk sharing differentials can produce both net and gross flows. When countries have different margin requirements, the riskiness of ABS created by pooling differs, and the difference depends on shocks. Since investors hold different portfolios, the risk exposures of agents in each country differ by aggregate state. Financial flows arise as agents hold combinations of partially substitutable assets, an effect that would not be present if countries only traded risk-free bonds.

We show that modeling the underlying assets traded, and how they are used to share risk in equilibrium, provides insight into how the nature of aggregate shocks drives gross flows. When agents face aggregate shocks affecting the expected payoff of investment projects (first-moment shocks), trade in relatively safe securities leads to offsetting gross capital flows that are of the same order of magnitude as the net flows. Additionally, net flows decrease when countries' shocks are less correlated. However, when agents face aggregate shocks affecting the variance of investment returns (second-moment shocks), gross flows can collapse entirely, while net flows are not affected by the degree of cross-country correlation. Hence, the risk characteristics of loans underlying securitized assets

[1] When Home is relatively small (perhaps the recipient of a "global saving glut"), it also enjoys a welfare gain, which in some cases even exceeds the complete-markets level (perfect risk sharing). In these cases the benefit from portfolio reallocation greatly outweighs any decline in risk sharing.

[2] One of the reasons is that to generate gross flows, it is necessary to allow for multiple financial assets, but many models suppose that countries only trade in risk-free bonds because numerically solving a portfolio problem is challenging.

can have important implications for gross trade in these financial assets.

Our analysis is consistent with a number of stylized facts and offers testable implications regarding the relationships among these facts. First, there is substantial loan margin heterogeneity across countries, with lower margins in countries with capital inflows.[3] Second, there are significant international flows in financial assets. In particular, foreign acquisitions of U.S. securitized assets increased dramatically before the financial crisis.[4] Third, safe assets primarily originate from the U.S., a low-margin economy, and to a lesser extent from Europe. In addition, the global supply of and demand for safe assets has increased dramatically in recent decades, consistent with increased globalization.[5] Thus, our analysis provides testable implications that capital flows are driven by loan-margin heterogeneity, and that the global supply (and demand) for safe assets increases with globalization.

1.1 Related literature

The basic theoretical framework of our model is the collateral equilibrium introduced by Geanakoplos (1997, 2003) and Geanakoplos and Zame (2014), which has been widely applied in the economics literature.[6] None of these papers consider idiosyncratic investment risk, nor its implication for international capital flows. Fostel et al. (2015) study how different degrees of financial innovation across countries lead to financial flows and increased financial volatility. In their model, financial innovation refers to using new assets as collateral or using existing collateral to make different promises; as a result, countries trade assets as a way of effectively sharing collateral, rather than sharing risk. In contrast, financial innovation in our paper should be interpreted as the degree of downside risk that markets can insure and the ability of the financial sector to create relatively safe assets from risky loans, together with the ability to tranche asset-backed securities.

There is a large theoretical literature studying equilibrium "global imbalances," (see Gourinchas and Rey (2014) for a review) and the mechanisms in

[3] Kalemli-Ozcan et al. (2012) provide evidence that financial intermediary leverage differs across countries, with more leverage in the U.S. Calza et al. (2013) show that there is significant divergence in the structure of mortgage markets across the main industrialized countries, with loan-to-value ("LTV") ratios varying from between 50% in Italy to over 110% in the Netherlands. Nguyen and Qian (2017) use data from the World Bank Enterprise Survey across 43 developing countries to show that there is substantial cross-country variation regarding how frequently collateral is used to borrow and what collateral rate is required. Liberti and Mian (2010) show that financial development and collateralizability are closely related.

[4] Acharya and Schnabl (2010) and Shin (2012) emphasize the importance of securitized markets (specifically asset-backed commercial paper) and global banking flows for understanding the financial crisis. Bertaut et al. (2012) document that foreign acquisitions of private-label ABS before the crisis were primarily by Europeans, which contributed to the decline in their spreads over Treasury yields.

[5] According to the External Wealth of Nation dataset from Lane and Milesi-Ferretti (2007), in 2011, the U.S. net supply of safe assets accounted for roughly two-thirds of the global net supply of safe assets, and the Eurozone accounted for another fifth. The U.S. net supply of safe assets accounted for 9% of world GDP, and the Eurozone accounted for 3%: total global net supply was roughly 14% of world GDP. From 1980–1990, the global net supply of safe assets was between 2 and 3% of world GDP, and has risen to 14% in 2011. The U.S. share was 5% in 2000 and rose to 9% in 2011.

[6] See, for example, Kubler and Schmedders (2003), Fostel and Geanakoplos (2008, 2012a,b, 2015, 2016), Araújo et al. (2012), Simsek (2013), Cao and Nie (2017), and Cao (2017), among others.

our model driving capital flows are closely related to those in the existing literature. However, our model features some important differences regarding the supply of assets, risk sharing, and welfare. Importantly, our mechanism endogenously relates the supply of assets to differences in risk sharing.

As in Caballero et al. (2008), capital flows are driven by differential abilities across countries to supply financial assets. In our model the supply of assets is endogenous (though driven by exogenously different collateral requirements). Crucially, in our model the supply of assets endogenously increases after financial integration, whereas in Caballero et al. (2008) the supply of assets is fixed.[7]

Other papers have emphasized how low levels of risk sharing increases buffer-stock saving, leading to inflows to developed countries. Willen (2004) shows in a constant absolute risk aversion (CARA)-normal framework that incomplete markets (as opposed to complete markets) lead to financial imbalances, which is related to our result that the differences in the degree of market incompleteness leads to imbalances. Mendoza et al. (2009) and Angeletos and Panousi (2011) argue that different levels of financial development (which is defined by decreasing the level of idiosyncratic risk in their model) can lead to sustained global imbalances in Bewley (1983)-type models through the effect on precautionary savings.

In our model savings rates are identical across countries, but low risk sharing has a *portfolio consequence* that increases the demand for assets.[8] Additionally, our modeling strategy has two important differences. First, because we model the ability of countries to insure idiosyncratic risks by the margin required for securitized loans, the degree of risk sharing endogenously responds to capital flows—risk sharing differentials remain but are not fixed. This "endogenous risk sharing" leads to different welfare implications from those in the existing literature, namely Foreign always gains and Home is ambiguous. Second, our mechanism does not require the interpretation that the U.S. is "more developed" in the sense that other countries ought to follow suit. Since we emphasize margin rates and not the development of the legal system, our model can account for sudden changes in the financial system. While the level of development of a financial system is likely to be persistent and improving, margins in securitized markets will almost certainly fluctuate; recent events suggest that these fluctuations can be large and sudden. As margins for securitized loans increased in the U.S., a decline in the current account deficit would naturally follow as a prediction of our model. Indeed, loan margins in developed countries have fluctuated substantially following 2000. Figure 1 in Fostel and Geanakoplos (2012a) (reproduced in Figure 1.2 below) shows that the median down payments on new mortgages for subprime/Alt A borrowers in U.S. decreased from 12% in 2000 to 3% in 2006, and then spiked to 16% in 2007. Since the credit boom primarily affected developed countries, the implication is that cross-country margin differences varied substantially during this time.

[7] In Angeletos and Panousi (2011), the supply of risk-free bonds equals the present value of human capital, which is determined by the wage. Their mechanism is that Home accumulates more capital and thus has a higher wage and can supply more bonds. In contrast, our mechanism is that Home can supply more assets in equilibrium because it can borrow more as a result of its lower collateral requirement and capital inflow.

[8] Maggiori (2017) argues that Home financiers can take on greater financial risk as a result of funding advantages, which leads Home to run persistent current account deficits financed by the risk-premium earned by its financial sector. As a result, Foreign financiers demand safe debt from Home financiers and Home financiers hold leveraged, risky portfolios.

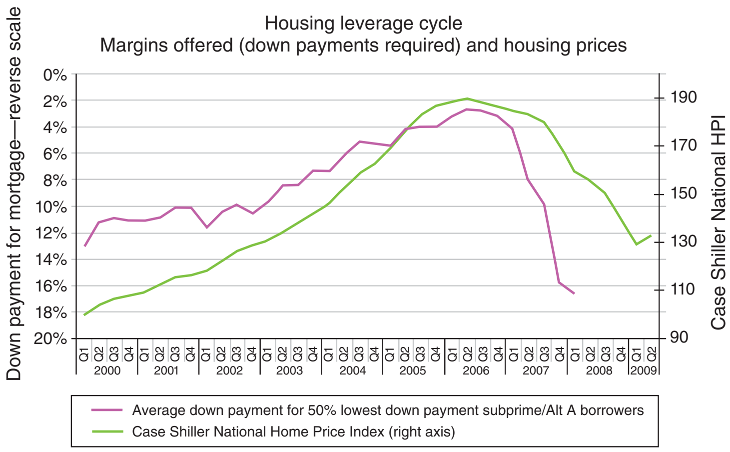


Figure 1.2: Leverage and housing price: Figure 1 of Fostel and Geanakoplos (2012a).

Finally, at a technical level our paper is related to a tractable class of heterogeneous-agent models introduced and studied by Calvet (2001) (idiosyncratic income risk in a CARA-normal framework), Krebs (2006) (idiosyncratic investment risk in a CRRA-Markov framework), Toda (2014) (recursive utility with multiple financial assets), Eisfeldt et al. (2017), among others. Since optimal portfolio problems in heterogeneous-agent models are notoriously difficult to solve numerically, quantitative papers usually consider only one asset or impose strong market participation assumptions. However, in an international finance setting there are necessarily multiple assets. Thus our modeling strategy allows us to provide a clean analysis without compromising the content.

2 Model

In this section we present a stylized two-country, two-period general equilibrium model with collateralized lending and securities created by pooling loans (asset-backed securities, or ABS).[9] The two countries are identical except for the collateral levels required for loans and (potentially) size (aggregate wealth). Throughout the rest of the paper, a subscript refers to time or an asset type; a superscript refers to a country or an agent.

2.1 Description of each economy

There are two countries—Home and Foreign—denoted by H and F. In each country there is a unit continuum of ex ante identical entrepreneurs, indexed by $i \in I = [0, 1]$, operating AK-type investment projects with idiosyncratic

[9] In this section we consider a static model with idiosyncratic risk only. In Section 5 we introduce aggregate risk to analyze the equilibrium riskiness of assets supplied by each country and gross capital flows. In Appendix B we extend the model to infinite horizon to analyze the long-run consequences of securitization and financial integration. Appendix D extends the model to the case with tranching.

risks.[10] Each country has a continuum of risk-neutral, perfectly competitive, profit maximizing financial intermediaries who service loans and issue securities backed by pools of loans. Time is indexed by $t = 0, 1$.

Entrepreneurs Entrepreneurs have identical preferences over consumption in $t = 1$ defined by

$$U(C_1) = \mathrm{E}[u(C_1)], \tag{2.1}$$

where the von Neumann-Morgenstern utility function u is strictly increasing and strictly concave. Entrepreneurs in country $j = H, F$ are endowed with initial capital $W^j > 0$ at $t = 0$. There is no endowment at $t = 1$. The agents have access to a constant-returns-to-scale technology with stochastic, idiosyncratic productivity. Agent i's investments yield A^i (gross return on investment), which realizes at the beginning of time 1. So, if agent i invests k^i in the technology at $t = 0$, he gets $A^i k^i$ at $t = 1$. Idiosyncratic returns A^i are independent and identically distributed across agents; there are no aggregate risks for now.

Financial structure Markets are incomplete and entrepreneurs cannot directly insure against the idiosyncratic risk, possibly due to moral hazard, costly state verification, or other reasons. However, entrepreneurs can borrow from financial intermediaries by putting up their investments as collateral. Entrepreneurs can only borrow from intermediaries in their country of residence and intermediaries can only make loans in their domestic country. A loan is characterized by an exogenous collateral requirement $c \geq 1$ and an endogenous gross borrowing rate $R_b \in [0, \infty]$. To simplify, and without loss of generality, we assume that each country offers a single exogenous collateral level, denoted by c_j for $j = H, F$.[11] The Home financial sector offers contracts with lower collateral requirements, reflected by $c_H < c_F$. Entrepreneurs in country j borrow at rate R_b^j, which is determined in equilibrium. Throughout the paper we denote loans by j, the country of origin.

For each dollar agent i takes from loan j, she must invest c_j dollars in the investment technology and put up its return (product) $A^i c_j$ as collateral. It is useful to consider the down payment associated with the collateral requirement c_j. If an agent invests 1 in the project, she can borrow $1/c_j$ against the project. Hence, she needs to put up $d_j := 1 - 1/c_j$ as down payment in order to invest 1 unit. Thus, collateral levels define the percent down payment required on a loan.[12]

Following Geanakoplos and Zame (2014), loans are non-recourse, that is, the sole penalty of default is the confiscation of collateral: for each unit taken from loan j at $t = 0$, the entrepreneur has the option of either paying back the

[10]Below, we use "entrepreneurs", "investors", and "agents" interchangeably. AK models are common in the finance literature, for instance Eisfeldt (2004) and He and Krishnamurthy (2012).

[11]Toda (2013) shows that if intermediaries offer arbitrary amount of loans from a finite menu of loan types, then *in equilibrium* agents will borrow exclusively from the loan with the lowest collateral requirement, so the single collateral assumption is without loss of generality. By considering the optimal securitization problem as in Malamud et al. (2013), it may be possible to endogenize the collateral level, but we abstract from that aspect for simplicity.

[12]Nonnegative down payments $1 - 1/c_j \geq 0$ imply $c_j \geq 1$. Toda (2013) shows that $c_j \geq 1$ is a necessary condition for equilibrium existence.

promised interest rate R_b^j or surrendering the collateral $A^i c_j$ at $t = 1$. Therefore, at $t = 1$ she chooses the better option and optimally delivers

$$\min \left\{ A^i c_j, R_b^j \right\} \tag{2.2}$$

to the financial intermediary.[13] Thus, unlike the "popular view" that default is bad and something to be avoided, we adopt the "economic view" that default expands the asset span and is possibly welfare improving (Zame, 1993; Dubey et al., 2005). We can equivalently think of defaultable, non-recourse collateralized loans as convertible bonds. Notice that the payoff of 1 unit of loan with collateral level c_j and interest rate R_b^j is

$$\min \left\{ A^i c_j, R_b^j \right\} = c_j \min \left\{ A^i, R_b^j / c_j \right\},$$

which is equal to the payoff of c_j units of convertible bonds with strike R_b^j / c_j. Yet another interpretation of the contract is a repurchase agreement. The entrepreneur sells the entire project to the lender with the option to buy it back for some pre-agreed strike price. Therefore, we can interchangeably think of default—delivering the project investment instead of the promised payment—as a flexible capital structure that shares investment risk in low-payoff states.

In order to raise capital for lending, each financial intermediary pools all loan contracts (which have the same collateral level) and issues securities backed by the pools of loans.[14] While these securities—which are literally collateralized loan obligations (CLO)—can be interpreted broadly, we refer to these securities as asset-backed securities (ABS). By risk neutrality and perfect competition, the intermediary's profit must be zero. Therefore the total dividend to ABS j is the cross-sectional sum of individual deliveries (2.2),

$$\int_I s_j^i \min \left\{ A^i c_j, R_b^j \right\} \mathrm{d}i,$$

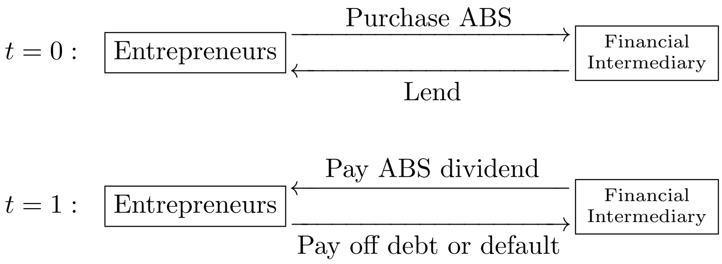
where $s_j^i \geq 0$ is the the size of loan agent i takes from the financial intermediary in country j. Since the amount of collateral is proportional to the loan size, s_j^i does not affect the default decision. Since the productivities are independent across agents, by the law of large numbers we can write the gross return on ABS j as

$$R_{\mathrm{ABS}}^j = \frac{\int_I s_j^i \min \left\{ A^i c_j, R_b^j \right\} \mathrm{d}i}{\int_I s_j^i \, \mathrm{d}i} = \mathrm{E} \left[\min \left\{ A^i c_j, R_b^j \right\} \right], \tag{2.3}$$

which is simply the cross-sectional average of individual deliveries (2.2). The following diagram shows the flow of funds at each point in time.

[13] In this paper we abstract from moral hazard problems and default costs by borrowers by assuming that the distribution of the productivity A^i is exogenously given. Moral hazard and default costs are considered in Toda (2013). It is worth noting that empirically default costs are lower in more financially developed countries.

[14] The modeling of ABS closely follows Toda (2013). See Elul (2005) for more institutional details on securitization.



Budget and portfolio constraints Countries can trade asset-backed securities, and thus investors can hold ABS from either country. In this way, capital can flow between countries as investors buy and sell ABS through international markets.

The portfolio of an investor i in country j consists of the fraction of capital invested in the technology θ^i, the fraction borrowed in (the domestic) collateralized loans ψ^i, and the fraction invested in ABS issued in each country ϕ^i_H and ϕ^i_F. Summing up, the intratemporal budget constraint (accounting) satisfies

$$\theta^i + \phi^i_H + \phi^i_F - \psi^i = 1.$$

The collateral requirement (collateral constraint) is

$$\theta^i \geq c_j \psi^i,$$

that is, the total investment in the technology must exceed the total collateral required. Note that since loans are collateralized, once some part of the investment is used as collateral, it cannot be used again.

The total return on portfolio $\pi^i = (\theta^i, \phi^i_H, \phi^i_F, \psi^i)$ is

$$R^i(\pi^i) = A^i\theta^i + R^H_{\text{ABS}}\phi^i_H + R^F_{\text{ABS}}\phi^i_F - \min\left\{A^i c_j, R^j_b\right\}\psi^i, \tag{2.4}$$

where R^j_{ABS} is the gross return on country-j ABS defined by (2.3) and $\min\left\{A^i c_j, R^j_b\right\}$ is the delivery of investor i to the financial intermediary in j for each unit of loan taken as in (2.2).

Entrepreneur's problem The objective of each investor i in country j is to maximize the utility subject to the budget and portfolio constraints:

$$\begin{aligned}
&\text{maximize} && \mathrm{E}[u(C_1)] \\
&\text{subject to} && C_1 = R^i(\pi^i)W^j, && (2.5\text{a}) \\
& && \theta^i + \phi^i_H + \phi^i_F - \psi^i = 1, && (2.5\text{b}) \\
& && \theta^i \geq c_j\psi^i, && (2.5\text{c})
\end{aligned}$$

where $R^i(\pi^i)$ is the total return on portfolio $\pi^i = (\theta^i, \phi^i_H, \phi^i_F, \psi^i)$ defined by (2.4). (2.5b) is the intratemporal budget constraint (accounting). (2.5c) is the collateral constraint.

Since the productivities $\left\{A^i\right\}_{i\in I}$ are i.i.d. across agents, the optimal portfolio problem (2.5) is common to all residents of a country. Since the objective function is strictly concave, the solution is unique. Therefore if an equilibrium exists, agents in each country choose the same portfolio. Therefore instead of keeping track of individual portfolios, we may focus on country portfolios.

Remark about financial structure It is worth highlighting the difference between our model of collateralized loans and the canonical collateral equilibrium model of Geanakoplos and Zame (2014). Most relevant to our present exercise, in their model the default threshold for a financial contract is exogenous (determined by asset payoffs), but the price of a contract, and thus the downpayment, is endogenous. Conversely, in our model the downpayment is exogenous (the price of a contract is fixed at 1), but the interest rate on a contract is endogenous, and therefore so are the face value and the default threshold. Hence, equilibrium does not determine contract prices but the face value of the debt sold by entrepreneurs, which affects the degree of risk-sharing.

Given the objective of this paper, we see several reasons why our modeling approach is more appropriate. Our model with exogenous margin requirements implicitly allows characteristics besides future payoffs to determine down payments. This is convenient for commercial loans to firms or entrepreneurs, such as considered in our paper, and also relevant for mortgage loans.[15] Jimenéz et al. (2006) show that the presence of collateral in loans depends on banking market structure/concentration and the experience and specialization of lending bank (i.e., not just the payoff of the project). Calomiris et al. (2017) show that loan-to-values of loans collateralized with movable assets are lower in countries with weak collateral laws, relative to immovable assets, and that lending is biased toward the use of immovable assets.

There is substantial evidence that collateral levels are often determined by the institutions supporting collateralization, not necessarily the collateral itself. In particular, differences in legal and regulatory frameworks, reflecting institutional differences, manifest in differing down-payment rates, repayment rates (the rate of equity release), and interest rate schedules. Cerqueiro et al. (2016) document how a legal change in Sweden affected collateral values and equilibrium outcomes, providing evidence that the value of the collateral may depend not only on the value of the secured assets, but also on the legal mechanisms and institutions that define when and how a creditor can seize those assets. Campello and Larrain (2016) show that both legal flexibility and information affect the terms and uses of secured debt transactions; see also Qian and Strahan (2007), Bae and Goyal (2009), and Brown et al. (2017) for more on the importance of legal institutions and enforcement.

Equilibrium A collateral equilibrium with securitization is defined by borrowing rates, consumption choices, and portfolio choices such that (i) agents optimize and (ii) markets clear. Since there are no aggregate shocks, all ABS pools are risk-free (idiosyncratic risks are diversified away).

Definition 2.1 (Collateral Equilibrium)**.** Given the collateral requirements c_H, c_F, the individual consumption and portfolio choice $(C_1^j, \pi^j)_{j=H,F}$ and borrowing rates (R_b^H, R_b^F) constitute a *collateral equilibrium* if

1. Each investor $i \in I$ solves the optimal portfolio problem (2.5).

[15]The Geanakoplos and Zame (2014) framework is the natural approach to model convertible bonds with exogenous strikes, or to model trade in financial securities (such as leveraged investments in MBS). In these cases, the default threshold is determine entirely by the future payoffs of the assets, and the characteristics of lenders and the financial sector have no effect on default thresholds.

2. Asset markets clear: for $j = H, F$, we have

$$W^H \phi_j^H + W^F \phi_j^F = W^j \psi^j. \tag{2.6}$$

Note that the profit maximization by intermediaries is implicit in the definition of the ABS return in (2.3). For the market clearing condition for the asset-backed securities (2.6), the left-hand side is the world demand of the ABS issued by country j, and the right-hand side is the world supply of that ABS, which is supplied only by country j. When asset markets clear, by the budget constraint the capital market automatically clears:

$$W^H \theta^H + W^F \theta^F = W^H + W^F. \tag{2.7}$$

The left-hand side is the total capital invested, and the right-hand side is the total capital available, which must end up in the technology because lending and borrowing through securitization cancel out and there is no consumption at $t = 0$. Toda (2013) proves that an equilibrium exists and that agents leverage to the maximum (so the collateral constraint (2.5c) always binds).

3 Properties of equilibrium: Theoretical results

While it is typically difficult to prove results about properties of equilibria for general equilibrium models with incomplete markets, we are able to theoretically characterize properties of interest rates, leverage decisions, and the creation of safe assets in autarky and with financial integration. The numerical analysis of the next section vividly illustrates these theoretical results (the reader who finds propositions dry may prefer to skip to that section after Proposition 3.2).

Autarky First, we consider when agents cannot trade ABS across countries, and thus countries are in autarky. Equilibrium looks as follows. Since agents choose the same portfolios in equilibrium and leverage to the maximum, investment levels and portfolios are given by

$$\theta^j = 1, \quad \phi^j = \psi^j = \frac{1}{c_j}.$$

Thus, interest rates are determined such that agents invest their entire capital in their projects, borrow to the maximum against the projects, and reinvest the proceeds in ABS in a matched portfolio. As shown by the numerical examples in Toda (2013), multiple equilibria are possible even with CRRA utilities. However, we can focus on the equilibrium with the highest interest rate, which is the most efficient equilibrium. We can state the following proposition as a special case of Theorems 2.6 and 2.7 of Toda (2013).

Proposition 3.1. *Consider an economy with (the minimum) collateral $c > 1$. Then (i) among all equilibria, the one with the highest borrowing rate gives the highest welfare (expected utility in equilibrium), and (ii) the maximum welfare is decreasing in c.*

This result means that lower margins improve welfare by increasing risk sharing through a larger supply of safe assets. Hence, welfare is higher in the Home country because financial intermediaries endogenously supply more safe assets.

Financial integration Next, we consider when countries are integrated and can freely trade ABS. Since there are no aggregate risks, ABS are risk-free and therefore by no arbitrage the ABS in both countries must offer the same return, *i.e.*, $R^H_{\mathrm{ABS}} = R^F_{\mathrm{ABS}} =: R_f$. However, the borrowing rates R^H_b and R^F_b will differ because the collateral rates differ.

As in autarky, investors continue to leverage to the maximum. Letting θ^j and ψ^j be the aggregate technology investment and borrowing by country j, by the maximum leverage property we have

$$\theta^j = c_j \psi^j.$$

Investment in ABS by country j ($\phi^j := \phi^j_H + \phi^j_F$) is determined from the budget constraint, given θ^j: since $\theta^j + \phi^j - \psi^j = 1$, we have

$$\phi^j = 1 - (c_j - 1)\psi^j.$$

A key difference from autarky is that $\theta^j \neq 1$ in general: a country need not invest its entire capital in the domestic technology. Equivalently, we have $\phi^j \neq \psi^j$, so it is no longer the case that investors hold "matched" portfolios, borrowing against their projects to invest in the domestic ABS.

Since safe assets are created by pooling loans, and by maximum leverage we have $\theta^j = c_j \psi^j$, the global supply of safe assets is given by

$$W^H \psi^H + W^F \psi^F = \frac{W^H \theta^H}{c_H} + \frac{W^F \theta^F}{c_F}. \tag{3.1}$$

Although proving properties of the equilibrium is generally challenging due to the possibility of multiple equilibria, we can still make a few definite predictions. Below, superscripts Aut, Int denote variables in autarky and financial integration.

Proposition 3.2. *Consider a country with autarky risk-free rate R^{Aut}_f. If the country faces a higher interest rate $R^{\mathrm{Int}}_f > R^{\mathrm{Aut}}_f$ after financial integration, then the country reduces real investment θ, increases investment in the ABS ϕ, and reduces borrowing ψ. Furthermore, the country gains in terms of welfare.*

The intuition for the welfare result in Proposition 3.2 is as follows. When there is no aggregate risk, letting $R^H_{\mathrm{ABS}} = R^F_{\mathrm{ABS}} = R_f$ be the risk-free rate and $\phi = \phi_H + \phi_F$ be the investment in ABS, the portfolio return (2.4) becomes

$$\begin{aligned}
R(\pi) &= A\theta + R_f \phi - \min\{Ac, R_b\}\psi \\
&= A\theta + R_f(1 - \theta + \psi) - \min\{Ac, R_b\}\psi \\
&= (A + \mathrm{E}[\min\{A, R_b/c\}] - \min\{A, R_b/c\})\theta + R_f(1-\theta),
\end{aligned}$$

where we have used the budget constraint $\theta + \phi - \psi = 1$, the maximum leverage property $\theta = c\psi$, and the definition of the ABS return $R_f = \mathrm{E}[\min\{Ac, R_b\}]$. Letting $k = R_b/c$ be the default threshold, we obtain

$$R(\pi) = (\max\{0, A-k\} + \mathrm{E}[\min\{A, k\}])\theta + R_f(1-\theta). \tag{3.2}$$

Therefore entrepreneurs are investing their wealth in effectively two assets, a risky asset with gross return $\max\{0, A-k\} + \mathrm{E}[\min\{A, k\}]$ and a risk-free asset with gross return R_f, with proportions θ and $1-\theta$, respectively. Noting that

$$\max\{0, A-k\} + \min\{A, k\} \equiv A,$$

the risky asset is just the productivity, except that the down side risk (below threshold k) is pooled across agents.

Thus financial integration has two effects: (i) it alters the default threshold $k = R_b/c$, with more risk sharing with larger k, and (ii) it alters the risk-free rate, which affects the portfolio choice. The first effect (risk sharing effect) is beneficial if and only if the interest rate goes up, in which case there is more risk sharing. On the other hand, the second effect (portfolio effect) is always beneficial. Since agents can choose the autarky portfolio (whose return does not depend on R_f) if they wish, changes in the risk-free rate always improve welfare, as in the classic argument of gains from trade. It follows that high interest rates are always good (both the risk sharing and portfolio effects are positive), while the welfare effect of low interest rates is ambiguous (the portfolio effect is positive but the risk sharing effect is negative).

We can summarize the above intuition in the following proposition.

Proposition 3.3. *Let $V(k, R_f) = \max_{\theta \geq 0} \mathrm{E}[u(R(\pi)W)]$ be the maximum welfare given the initial wealth W, default threshold $k = R_b/c$, risk-free rate R_f, and portfolio return $R(\pi)$ in* (3.2). *Then (i) for any R_f, $V(k, R_f)$ is increasing in k, and (ii) fixing $k = k^{\mathrm{Aut}}$, we have $V(k^{\mathrm{Aut}}, R_f) \geq V(k^{\mathrm{Aut}}, R_f^{\mathrm{Aut}})$ for all R_f.*

As opposed to the welfare change, there is no clear intuition for the portfolio change in Proposition 3.2. If we look at the portfolio return (3.2), increasing the risk-free rate R_f clearly makes the risk-free asset more attractive, but it also improves risk sharing by raising the default threshold $k = R_b/c$, so the risky asset also becomes more attractive.

Using Proposition 3.2, we obtain our main theoretical result: if the interest rate goes up after financial integration relative to the autarky interest rate in the *high*-margin country (Foreign), then all variables move in the natural direction: Home borrows more and invests more in the technology; Foreign borrows less, invests less in the technology, and invests more in ABS; capital flows from Foreign to Home; global supply of assets increases.

Theorem 3.4. *Suppose that $R_f^{\mathrm{Int}} > R_f^{F,\mathrm{Aut}}$. Then $R_f^{\mathrm{Int}} < R_f^{H,\mathrm{Aut}}$,*

1. *Home increases and Foreign decreases real investment:*

$$\theta^{H,\mathrm{Int}} > \theta^{H,\mathrm{Aut}} = 1 = \theta^{F,\mathrm{Aut}} > \theta^{F,\mathrm{Int}},$$

2. *Home decreases and Foreign increases investment in ABS:*

$$\sum_{j=H,F} \phi_j^{H,\mathrm{Int}} < \phi^{H,\mathrm{Aut}}, \quad \sum_{j=H,F} \phi_j^{F,\mathrm{Int}} > \phi^{F,\mathrm{Aut}},$$

3. *Home increases and Foreign decreases borrowing:*

$$\psi^{H,\mathrm{Int}} > \psi^{H,\mathrm{Aut}}, \quad \psi^{F,\mathrm{Int}} < \psi^{F,\mathrm{Aut}},$$

4. *The global supply of safe assets* (3.1) *increases:*

$$\frac{W^H \theta^{H,\mathrm{Int}}}{c_H} + \frac{W^F \theta^{F,\mathrm{Int}}}{c_F} > \frac{W^H \theta^{H,\mathrm{Aut}}}{c_H} + \frac{W^F \theta^{F,\mathrm{Aut}}}{c_F} = \frac{W^H}{c_H} + \frac{W^F}{c_F}.$$

In particular, capital flows from Foreign to Home in the financial integration equilibrium.

Note that since the model so far does not feature aggregate risk, Home and Foreign ABS are both risk free and have the same interest rate in financial integration. Since the Home and Foreign ABS are identical assets, the composition of ABS in country j (ϕ_H^j, ϕ_F^j) is indeterminate, although total ABS investment by country j $(\phi^j = \phi_H^j + \phi_F^j)$ is. Therefore the model is silent about the magnitude of gross capital flows.

Theorem 3.4 is not entirely satisfactory because it imposes an assumption on an endogenous variable (R_f^{Int}). However, in general we cannot omit this assumption since Proposition 3.2 does not say anything when the interest rate decreases.[16] The next theorem shows that if the relative risk aversion is bounded above by 1, then the demand functions for borrowing and investment are downward sloping, and therefore the equilibrium is unique and all variables move in the natural direction.[17]

Theorem 3.5. *Suppose that countries have relative risk aversion bounded above by 1, so* $-\frac{xu''(x)}{u'(x)} \leq 1$ *for all* x. *Then investment* θ *and borrowing* ψ *are decreasing functions of the risk-free rate* R_f, *and ABS investment (lending)* ϕ *is increasing. In particular, the equilibrium is unique and satisfies*

$$R_f^{F,\mathrm{Aut}} < R_f^{\mathrm{Int}} < R_f^{H,\mathrm{Aut}},$$

so the conclusion of Theorem 3.4 holds.

Theorem 3.5 is intuitive. When RRA < 1, the elasticity of substitution (the reciprocal of RRA) is greater than 1. Hence the substitution effect dominates the income effect, and the demand functions are downward sloping. Accordingly, this is a sufficient condition for equilibrium uniqueness.

Combining Proposition 3.2 with Theorems 3.4 and 3.5, it follows that Foreign welfare improves after financial integration. However, the change in Home welfare is ambiguous: according to the numerical examples below, Home may gain or lose from financial integration, depending on the parameters. Additionally, part of the ambiguity arises depending on the *size* of Home relative to Foreign. The next proposition determines when Home gains or loses from financial integration when Home is small (which is intuitive since large countries influence terms of trade through the general equilibrium effect, which can overturn standard welfare results for small economies).

Proposition 3.6. *Suppose that Foreign wealth* W^F *is sufficiently large. Let* $V^{H,\mathrm{Aut}}, V^{H,\mathrm{Int}}$ *be the Home welfare in autarky and after financial integration.*

1. *If Home collateral requirement* $c_H > 1$ *is sufficiently low and* $u(\infty) = \infty$ *(in particular,* u *is CRRA with relative risk aversion* $\gamma \leq 1$*), then*

[16] Unlike in standard GEI (with exogenous asset payoffs), where a positive third derivative is sufficient to generate a positive relationship between measures of market completeness and interest rates (Elul, 1997; Willen, 2004), in our model it is hard to predict the direction of change in interest rates since the asset payoffs are endogenous.

[17] While we do not have general results for $\gamma = \mathrm{RRA} > 1$, we computed the demand for CRRA utility functions with various γ, and found that the downward sloping property fails only if γ is very large (say $\gamma > 50$). Therefore for a plausible range of γ, the conclusion of Theorem 3.4 seems to hold.

Home welfare after financial integration exceeds the complete market level (perfect risk sharing): $V^{H,\mathrm{Int}} > u(\mathrm{E}[A^i]W^H)$.

2. *If u is CRRA with relative risk aversion $\gamma > 1$ sufficiently high, then Home loses from financial integration:* $V^{H,\mathrm{Int}} < V^{H,\mathrm{Aut}}$.

The intuition for Proposition 3.6 is that, as discussed in Proposition 3.3, financial integration has two opposing effects (risk sharing and portfolio effects) on Home welfare. On one hand, the capital inflow allows Home investors to borrow more (at a lower rate) and invest more in the high return technology. On the other hand, the low risk-free rate (and hence the low default threshold) reduces risk sharing. Therefore it is natural that Home loses from financial integration when the risk aversion is high. When Home is small, the general equilibrium consequences for interest rates are predominantly determined by conditions of Foreign, which explains the result.

Our welfare results are in stark contrast with those in the literature. Angeletos and Panousi (2011) find that welfare results vary by the initial wealth of agents and differ in short- and long-run. In the North (Home), the rich lose on impact but gain in the long-run, and the opposite is true for South (Foreign). In both of these models, the degree of risk sharing is not affected by financial integration—the same fraction of idiosyncratic risk is insured in autarky and with financial integration. In our model, the degree of risk sharing depends on margins *and interest rates*, and thus financial integration affects risk sharing.

4 Numerical analysis

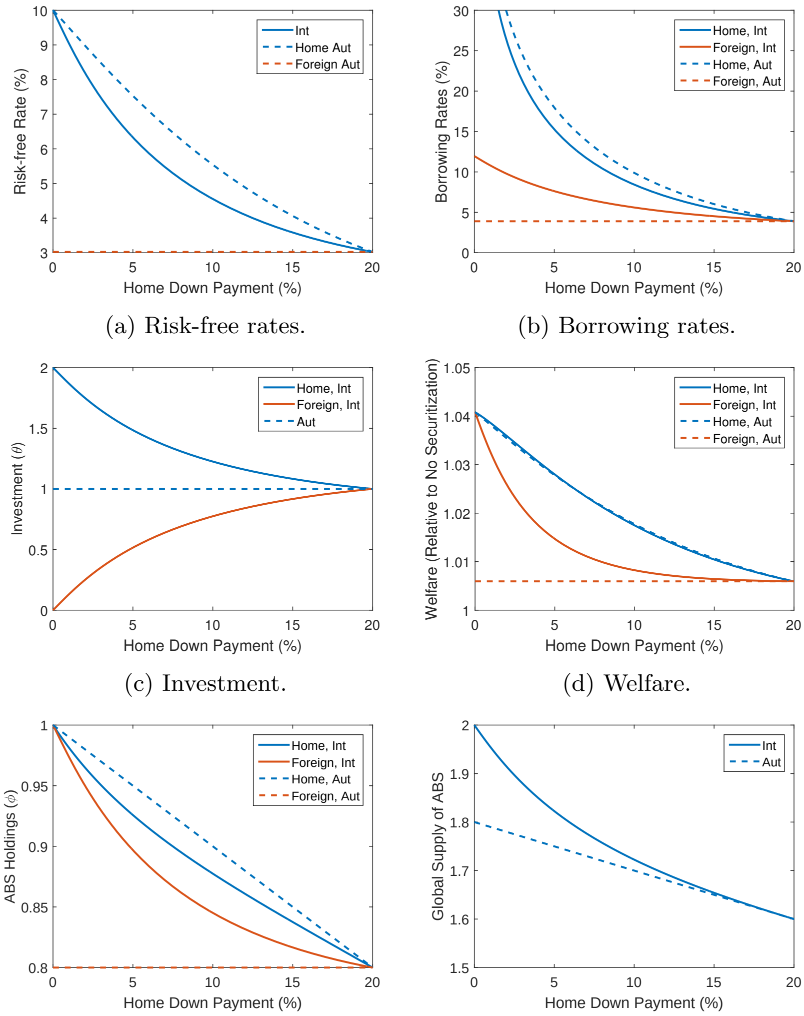
The two country model admits no closed-form solutions and therefore we must resort to a numerical solution with "calibrated" parameters to further characterize the behavior of financial flows and welfare. We assume that agents have constant relative risk aversion (CRRA) utility function $u(C) = \frac{1}{1-\gamma}C^{1-\gamma}$, with coefficient of relative risk aversion $\gamma = 2$, which is within the standard range of estimates. We first normalize the initial wealths be equal with $W^H = W^F = 1$ to focus on effects driven purely by differences in financial structure and not by asymmetric wealth. We later consider changing wealth to consider quantitatively relevant parameters. Productivities are lognormally distributed,

$$\log A^i \sim N(\mu - \sigma^2/2, \sigma^2),$$

where $\mathrm{e}^\mu = 1.1$ and $\sigma = 0.2$.[18] We fix the Foreign collateral level at $c_F = 1.25$ (down payment 20%), which is consistent with the empirical evidence cited earlier. To highlight the effects of differential collateral levels, we vary the collateral level in Home from $c_H = 1$ to 1.25 (down payment 0% to 20%), which

[18] Angeletos and Panousi (2011) argue that a standard deviation of residual risk of 20% is sensible for U.S. entrepreneurs, though they argue that 40% is more sensible for South. We consider robustness to varying σ and γ in Appendix C. $\mathrm{E}[\log A^i] = \mu - \sigma^2/2$ implies that $\mathrm{E}[A^i] = \mathrm{e}^\mu$, so $\mathrm{e}^\mu = 1.1$ means that the expected investment return is 10%. Although there may be disagreements about what are reasonable values for expected investment returns, due to the structure of our model (homotheticity and multiplicative shocks), if μ changes from μ to μ', the portfolio choice remains the same by multiplying all interest rates by $\mathrm{e}^{\mu'-\mu}$. Therefore none of our results qualitatively depend on μ, and even quantitatively it is just a matter of scaling numbers by $\mathrm{e}^{\mu'-\mu}$.

spans the range of down payments found by Fostel and Geanakoplos (2012a). We then solve for investment levels, borrowing rates, and risk-free rates in autarky and with financial integration. Figure 4.1 shows the results.



(a) Risk-free rates.

(b) Borrowing rates.

(c) Investment.

(d) Welfare.

(e) ABS holdings (ϕ).

(f) Global supply of safe assets.

Figure 4.1: Effect of margins and financial integration with $W^F = W^F = 1$.

4.1 Interest rates

Figures 4.1a and 4.1b show how interest rates change as a result of financial integration and changing collateral levels. Figure 4.1a shows the risk-free rates (returns on ABS).[19] After financial integration, the risk-free rate drops in Home and increases in Foreign. Thus, financial integration explains low risk-free rates

[19]In the limit of $c_H \to 1$, Home investors always deliver the full proceeds from their project, so the risk-free rate approaches the expected project return, $\mathrm{E}[A^i]$.

in Home. However, the low risk-free rate is not driven by excessive Foreign savings ("saving glut", Bernanke, 2005)—both countries save the same amount—but by the demand for safe assets.

Figure 4.1b shows the borrowing rates in autarky and after financial integration. Since the borrowing rate is monotonic in the risk-free rate and the risk-free rate in financial integration lies between the autarky rates, the borrowing rate goes down in Home and goes up in Foreign, so the spread narrows.

4.2 Investment and capital flows

Figure 4.1c plots investment levels θ for each country as the Home collateral level c_H varies. In autarky both countries would have investment levels equal to one. As the Home collateral level falls, investment in Home increases. Trivially, Home investors can borrow more against their projects when the collateral rate falls—but this was true in autarky, too. In autarky, as collateral rates drop, investors borrow more against their projects but increase their purchases of ABS, because market clearing requires that investment in the project is constant at $\theta = 1$. With international capital flows, however, Home investors choose to borrow more to invest in their projects. On the other hand, Foreign investors invest less of their own capital in their projects, choosing instead to invest in ABS. These results are consistent with Theorem 3.4. In the limit of $c_H \to 1$, Foreign exclusively invests in Home ABS, so θ^F approaches zero. Since the two countries have equal wealth ($W^H = W^F = 1$), Home investment approaches 2.

Why does Home investment increase when margins decrease and countries can trade? Remember that investors hold a portfolio of ABS and borrow using collateralized loans as a way of insuring idiosyncratic risk. When the collateral rate drops, investors can hold a larger portfolio of ABS, which are risk-free, and thus insure more risk. However, the Foreign collateral rate is fixed, but Foreign investors can buy Home ABS in order to insure their risk. Thus, Foreign investors buy Home ABS as a way of insuring more risk than they can using the Foreign financial sector. Trade allows Foreign investors to "get access" to risk sharing in the Home financial sector. As result, capital flows toward Home.

4.3 Global demand and supply of safe assets

Figure 4.1e plots the ABS holdings ϕ for each country as the Home collateral level c_H varies. Because Home investors increase real investment θ, to satisfy the budget constraint they hold less ABS. On the other hand, Foreign increases ABS holdings, which fuels the capital inflow to Home. Figure 4.1f shows the global supply of safe assets, computed from (3.1). The autarkic supply of assets is higher for low c_H as the Home financial sector can produce more safe assets when margins are low. Consistent with Theorem 3.4, the global supply increases after financial integration, as capital flows to Home. The change in global supply corresponds to an increase in Home assets and a (smaller) decrease in Foreign assets. The supply of Foreign assets decreases since Foreign decreases real investment, but the increase in the supply of Home assets is greater because of Home's greater ability to supply assets from real investments.

Of course, the equilibrium supply of safe assets reflects change in both supply and demand, and naturally financial integration jointly affects both issues.

However, since after financial integration Foreign investors increase their holdings of safe assets while Home investors decrease theirs, we can interpret the demand side of the market as being driven by Foreign demand for safe assets, as Foreign investors can now access safe assets with higher returns (compared to autarky). At the same time, the increased demand for safe assets is met by increasing the Home supply of assets, as the Home financial sector intermediates a greater amount of capital to create more safe assets based on lower margins.

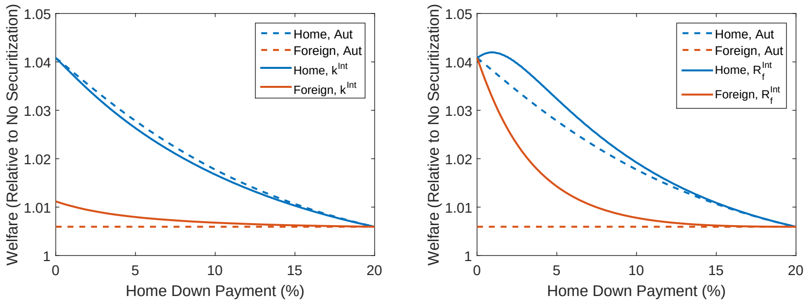
4.4 Welfare

Financial integration has important consequences for welfare in each country. Figure 4.1d plots welfare (in consumption equivalent, so $\mathrm{E}[C_1^{1-\gamma}]^{\frac{1}{1-\gamma}}$) as a function of the home down payment with financial integration and in autarky, relative to the case with no securitization ($c_H = c_F = \infty$). In autarky, lower collateral levels lead to higher welfare for Home investors as risk sharing improves through securitization (Proposition 3.1). Foreign investors benefit from financial integration, both because they can access higher risk-free rates to save and because higher borrowing rates improve risk sharing (Propositions 3.2 and 3.3). However, the welfare implication for Home is ambiguous (Proposition 3.6). While financial integration benefits Home investors by increasing borrowing and investment in the high return technology, it also hurts them by decreasing the borrowing rates and therefore the degree of risk sharing. According to Figure 4.1d, Home investors are slightly hurt by financial integration if down payments are larger than 6%. However, the results may change as we change the parameters. The sensitivity analysis in Appendix C shows that Home loses more as we increase the variance of the idiosyncratic shocks or the relative risk aversion. This result is quite intuitive since financial integration reduces risk sharing in Home through lower interest rates, and hence the welfare losses are larger when investments are riskier or agents are more risk averse.

As discussed, there are two competing forces affecting welfare after financial integration: the risk-free rate changes, which allows agents to reoptimize (portfolio effect), and the borrowing rate changes, which affects the default threshold (risk sharing effect). Welfare improves whenever the risk-free rate changes because investors can always choose the autarky portfolio, while welfare is increasing in the default threshold through more risk sharing. We know that for Foreign agents both effects are positive, but for Home investors the two effects go in the opposite direction and hence the total effect can be ambiguous.

To investigate further, as in Proposition 3.3 we isolate each welfare effect by changing either the default threshold $k = R_b/c$ or the risk-free rate R_f to the value in financial integration, while keeping the other parameter at the autarkic level. Specifically, we maximize the investor's expected utility $\mathrm{E}[u(R(\pi)W)]$ over investment θ, where the portfolio return $R(\pi)$ is given by (3.2) for each default threshold k and the risk-free rate R_f.

Figure 4.2 shows the welfare decomposition. In Figure 4.2a, the default threshold $k = R_b/c$ changes to the financial integration level, while the risk-free rate R_f remains at the autarkic level. Since the default threshold goes up in Foreign and down in Home, Foreign welfare increases and Home welfare decreases. In Figure 4.2b, the risk-free rate changes to the financial integration level, while the default threshold remains at the autarkic level. Since the degree of risk sharing is unchanged, both countries gain just because investors can reoptimize facing a new risk-free rate: Home investors can borrow more at lower rates to invest in high return projects, and Foreign agents can save more in safe assets that give higher returns. When the two effects in Figure 4.2 are combined, we exactly get Figure 4.1d, with a positive welfare change for Foreign and an ambiguous effect for Home.



(a) Default thresholds in integration. (b) Risk-free rate in integration.

Figure 4.2: Welfare effect of changing default threshold or risk-free rate.

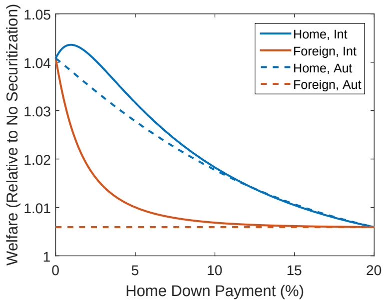
The sensitivity analysis also suggests how incorporating other forms of financing, such as outside equity, would modify the welfare results. Unlike defaultable debt (or convertible bond), which pools only downside risk, outside equity pools all risk. In this case the degree of risk sharing is constant, so Foreign agents would benefit less from financial integration, and Home agents would unambiguously gain from financial integration (as in Figure 4.2b).

4.5 "Saving glut" economy

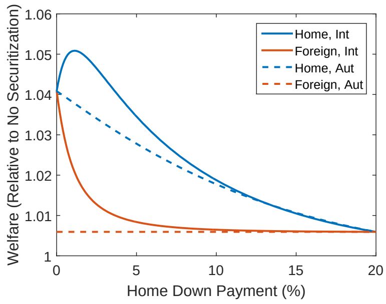
In the above numerical example, we assumed that countries have equal size $W^H = W^F = 1$—and hence equal savings—and found that Home welfare barely changed from autarky to financial integration. However, this may not always be the case. Figure 4.3 shows the result when Foreign is larger than Home: we fix Home wealth at $W^H = 1$ while changing Foreign wealth to $W^F = 3, 5, 7, 9$. We can interpret this case as when Foreign has a "savings glut."

Since Foreign is large, Foreign variables do not change much after financial integration as long as the Home down payment is not too small. On the other hand, Home experiences big changes. Interest rates drop and investment increases sharply, and welfare improves. Consistent with Proposition 3.6, Home welfare may even exceed that of complete markets (perfect risk sharing) when Home down payments are low enough. This result occurs even when Foreign is only 3 times larger than Home.

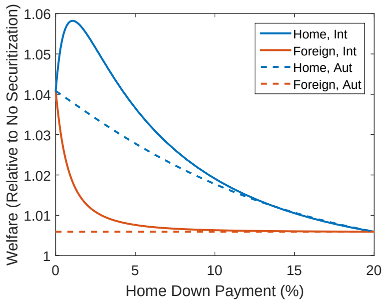
How would we interpret this example? First, the Home country is associated with the global bloc of countries exporting safe assets and running current account deficits. Ignoring important sources of heterogeneity, this result suggests that the mechanisms behind financial integration in our analysis may be beneficial for all countries, both those supplying and demanding safe assets. Second, the policy implication of this numerical example is that, unlike in a closed economy (Proposition 3.1), or even in a very large open economy, in an open economy low margins need not imply high welfare. In fact, in Figure



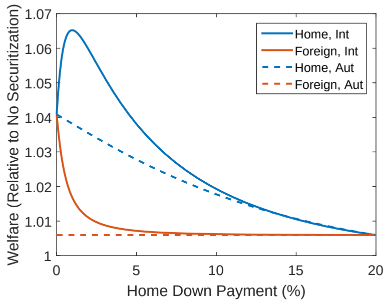
(a) $W^F = 3$.



(b) $W^F = 5$.



(c) $W^F = 7$.



(d) $W^F = 9$.

Figure 4.3: Effect of margins and financial integration on welfare when Foreign is large.

4.3 Home welfare is increasing in margins when the down payments are less than 1%. Thus, even though our model exhibits no pecuniary externalities or dynamic consequences of leverage, macro-prudential policy limiting leverage is quite sensible even in this environment.[20]

5 Aggregate shocks and gross capital flows

In this section we introduce aggregate risk to the model, which allows us to analyze gross versus net capital flows. While, in the early and mid 1990s net and gross flows used to move together, more recently the size and volatility of gross flows have increased while net capital flows have been more stable. As a result, the differentiation between gross inflows and outflows has become more important (Forbes and Warnock, 2012).

With aggregate risk the ABS in each country has risky payoffs and the riskiness of each country's ABS is not the same, which has two important implications. First, the creation of assets will not mean the creation of risk-free assets but rather of assets that are simply "safe enough" compared to agents' own investment projects. In general the Home ABS is riskier than the Foreign ABS because, with lower margins, the Home ABS payoff is more sensitive to the

[20] According to Figure 3 in Obstfeld (2012), the gross foreign assets and liabilities in small countries like Iceland and Ireland reached about 10 times of GDP around 2007, suggesting that these countries may have been inefficiently highly leveraged. Since our example is calibrated to U.S., it is clear that these countries would be dramatically over-leveraged as well.

aggregate state. Nonetheless, financial integration will lead Foreign investors to buy Home ABS because they are safe enough to improve risk sharing.

Second, because the riskiness of ABS differs, agents' demand for Home and Foreign ABS depend on the nature of aggregate shocks. When countries face aggregate risk to expected returns, gross flows are very large, regardless of the correlation of country-level shocks, while net flows decrease the less correlated are the country-level shocks. However, when countries face aggregate risk to the variance of returns, gross flows can collapse, and the correlation of country-level shocks has essentially no effect on net flows.

Additionally, aggregate risk allows for meaningful "tranching" of ABS into different state-contingent payoffs, which increases the asset span and leads to greater risk sharing. The model with tranching is considered in Appendix D.

5.1 Model

Suppose that there are S aggregate states indexed by $s = 1, \ldots, S$. State s occurs with probability p_s. The return to the investment project of an agent in country j is distributed according to the cumulative distribution function $F_s^j(\cdot)$ in state s. As before, agents' productivities are independent and identically distributed in each aggregate state. Countries may potentially have different productivity distributions in a given state (*i.e.*, "country-specific shocks" so that $F_s^H(\cdot) \neq F_s^F(\cdot)$), or the productivity distribution in each country may be the same (*i.e.*, "world-wide shocks").

With aggregate shocks, the returns on ABS are no longer risk-free. In particular, the gross return on country j's ABS in state s is

$$R_{\mathrm{ABS}}^j(s) = \mathrm{E}\left[\min\left\{Ac_j, R_b^j\right\} \,\middle|\, s\right] = \int_0^\infty \min\left\{c_j x, R_b^j\right\} \mathrm{d}F_s^j(x).$$

Other than that, the definition of the equilibrium remains the same, and the maximum leverage property continues to hold. However, unlike the case without aggregate risk, capital need not flow only from Foreign to Home: in general there will be gross flows.

For numerical examples, we consider two forms of aggregate shocks, first- and second-moment shocks. For simplicity, we assume that each country can be in one of two states (call them "good" G and "bad" B), for a total of four aggregate states: $s \in \{GG, GB, BG, BB\}$. First-moment shocks affect the mean project payoff; second-moment shocks affect the variance of payoffs (*i.e.*, idiosyncratic risk). First-moment shocks imply the expected payoff is higher in state G, but the variance of returns are the same in each state: letting $s = (s^H, s^F)$, we have

$$\mathrm{E}\left[A \,\middle|\, s^j = G\right] > \mathrm{E}\left[A \,\middle|\, s^j = B\right], \quad \mathrm{Var}\left[A \,\middle|\, s^j = G\right] = \mathrm{Var}\left[A \,\middle|\, s^j = B\right].$$

Second-moment shocks imply the expected payoff in each state is the same but the variance of returns is higher in state B. Thus,

$$\mathrm{E}\left[A \,\middle|\, s^j = G\right] = \mathrm{E}\left[A \,\middle|\, s^j = B\right], \quad \mathrm{Var}\left[A \,\middle|\, s^j = G\right] < \mathrm{Var}\left[A \,\middle|\, s^j = B\right].$$

With first-moment shocks expected returns are higher in the good state, and with second-moment shocks risk is lower.

Throughout our analysis, the good and bad states are equally likely, but may be correlated across countries. Therefore we parametrize the probability of the four states by

$$\mathbf{p} = (p_s) = \left(\frac{1+\rho}{4}, \frac{1-\rho}{4}, \frac{1-\rho}{4}, \frac{1+\rho}{4} \right),$$

where $\rho \in [-1, 1]$ is the correlation coefficient between Home and Foreign states. $\rho = 1$ corresponds to world-wide shocks, while $\rho = 0$ corresponds to country-specific shocks. If countries' states are not perfectly positively correlated ($\rho < 1$), there would be a natural demand for international diversification even in the absence of idiosyncratic shocks or collateral constraints. Therefore we take perfect correlation ($\rho = 1$) as the benchmark.

5.2 Numerical results with first-moment shocks

We solve for equilibrium with financial integration (and in autarky) when each country can take on lognormally distributed returns with mean $(\mathrm{e}^{\mu_G}, \mathrm{e}^{\mu_B}) = (1.2, 1)$ and log standard deviation $\sigma_G = \sigma_B = 0.2$.

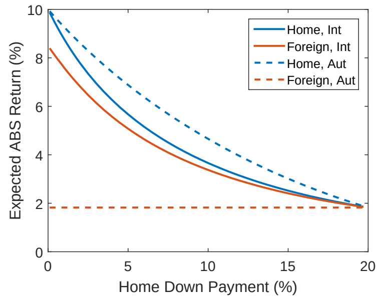
Perfectly correlated shocks The probability distribution of aggregate states with perfect correlation is $\mathbf{p} = (\frac{1}{2}, 0, 0, \frac{1}{2})$. Figure 5.1 shows the results with first-moment shocks.

Figures 5.1a–5.1d show the expected ABS returns $\mathrm{E}[R^j_{\mathrm{ABS}}]$, borrowing rates R^j_b, investment levels θ^j, and welfare $\mathrm{E}[C^{1-\gamma}]^{\frac{1}{1-\gamma}}$, both in autarky and after financial integration. Notice that results are nearly identical to those without aggregate risk (Figure 4.1).

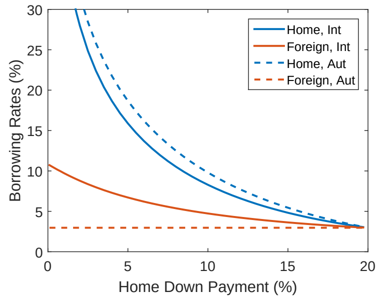
Figure 5.1e shows country j's holdings of ABS issued by Home and Foreign, denoted by ϕ^j_H, ϕ^j_F. When Home collateral levels get lower, capital flows from Foreign to Home through the purchase of Home ABS by Foreign investors. However, unlike the case without aggregate risk, there are now large gross capital flows: when Home collateral levels are relatively high, Home invests in Foreign ABS. Even though shocks are perfectly correlated, the payoffs to ABS differ because the collateral levels, and thus default rates, differ.[21]

Figure 5.1f shows the realized ABS returns after financial integration, state by state. Although the expected returns of the two ABS are quite similar (Figure 5.1a), we can see that Home ABS is riskier. With aggregate risk, the ABS in each country do not have the same riskiness of payoffs because the loan margins in each country differ. In particular, the Home margin is lower, implying a greater default rate in general and a greater sensitivity of defaults to fundamentals. As a result, the Home ABS is riskier than the Foreign ABS. The important result is that even though the Home ABS is riskier and gives only slightly higher returns, Foreign investors still buy Home ABS. This is because Home ABS are safer than investing in the idiosyncratic projects, and since Home margins are low, the Home financial sector can create a greater supply of "relatively safe" or "safe enough" assets, not necessarily assets that are safer than the Foreign safe asset.

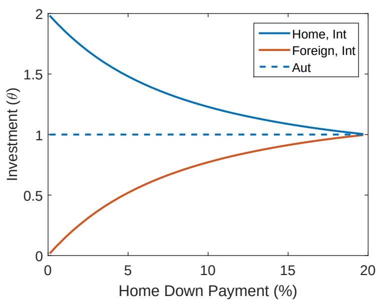
[21] Technically portfolios are indeterminate when $c_H = c_F$.



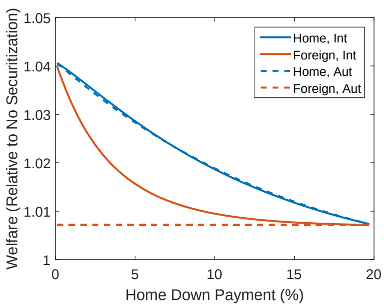
(a) Expected ABS returns.



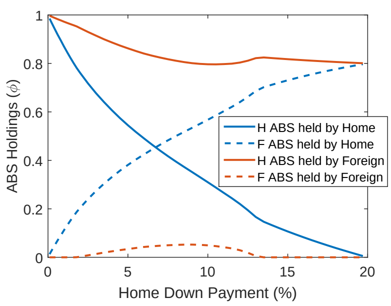
(b) Borrowing rates.



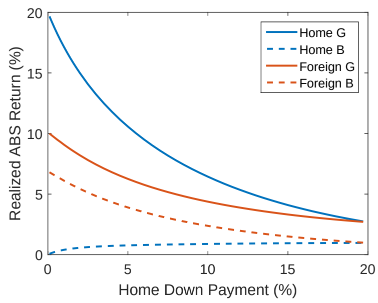
(c) Investment.



(d) Welfare.



(e) ABS holdings.

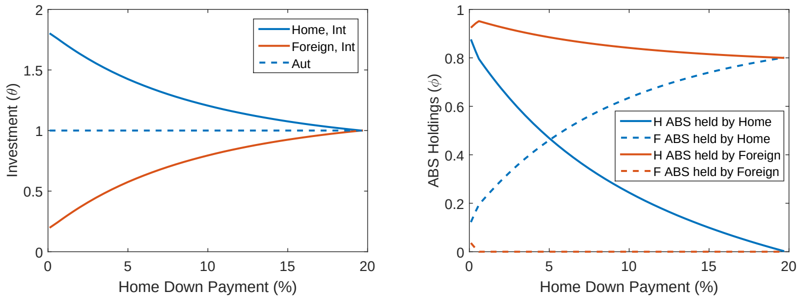


(f) Realized ABS returns in integration.

Figure 5.1: Effect of margins and financial integration with perfectly correlated first-moment shocks.

Independent shocks We next consider the case in which country shocks are independent ($\rho = 0$). Figure 5.2 shows the results.

Changing the correlation of first-moment shocks has almost no effect on prices and returns. One qualitative distinction that arises is that with independent aggregate shocks, Home investment is always lower than in the case with world shocks (notice that θ does not approach 2 at zero down payment in Figure 5.2a). This occurs because Foreign ABS provides an attractive hedge (Foreign shocks are independent). Because international diversification becomes attractive, gross flows generally increases. For example, while in Figure 5.1e Foreign invests some capital in domestic ABS when Home down payments are between 2 and 13%, in Figure 5.2b Foreign invests exclusively in Home ABS (Foreign



(a) Investment.

(b) ABS holdings.

Figure 5.2: Effect of margins and financial integration with independent first-moment shocks.

ABS are entirely held by Home investors).

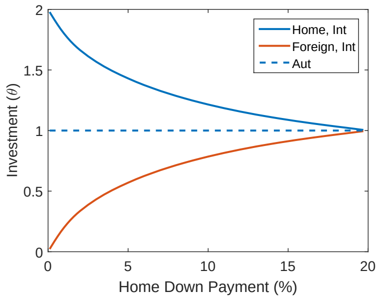
5.3 Numerical results with second-moment shocks

We solve for equilibrium with financial integration (and in autarky) when each country can take on lognormally distributed returns with mean $e^{\mu_G} = e^{\mu_B} = 1.1$ and log standard deviation $(\sigma_G, \sigma_B) = (0.1, 0.3)$. The results on portfolio holdings (gross flows) now depend on correlation. We consider perfect correlation ($\rho = 1$) and independence ($\rho = 0$) as before. Figure 5.3 shows the results with second-moment shocks.

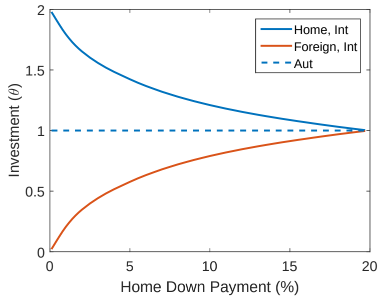
The interest rates and welfare are similar to the case with first-moment shocks, and therefore we omit the figures. However, the portfolios and ABS returns are quite different. Figures 5.3c and 5.3d show the ABS holdings of each country for correlated and independent shocks. With high Home down payments, there are gross capital flows. With intermediate Home down payments, Home does not invest in Foreign ABS but capital flows from Foreign to Home, as in the case without aggregate risk. The results are qualitatively similar across correlations, although gross flows do provide slightly better diversification when aggregate shocks are independent, and so gross flows do not collapse as quickly.

With very low Home down payments ($< 1\%$), however, the portfolios change discontinuously. This is because at around 1% down payment, according to Figure 5.3e the riskiness of the two ABS reverses and Home chooses to invest in the riskier Foreign ABS. Note that for very low margins, the borrowing rates are so high that default becomes common, which makes the Home ABS nearly risk-free because the expected project return is the same in the two states. In general the Home ABS is riskier than the Foreign ABS, but for low Home margins the Home ABS can in fact be safer. However, compared to first-moment shocks, Home ABS are only marginally riskier than the Foreign ABS.

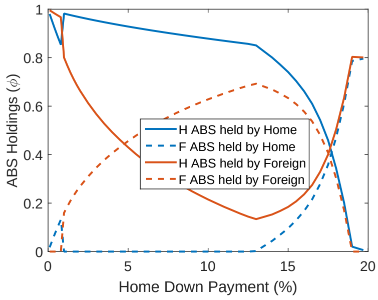
Given how ABS payoffs depend on the aggregate states, cross-country investment in safe assets do not provide as good of diversification (hence why gross flows decrease and may collapse). As a result, in contrast to when countries face first-moment aggregate risk, net flows are hardly affected by the dependence of shocks across countries. Figures 5.3a and 5.3b plots investment for perfectly correlated and independent shocks. In both cases, investment levels are quantitatively very similar, in both cases approaching Home investment of 2 when



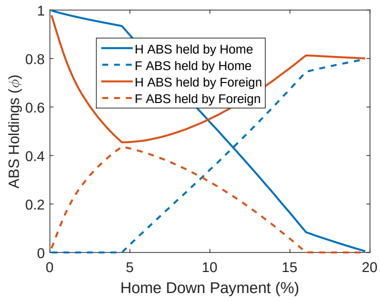
(a) Investment (correlated shocks).



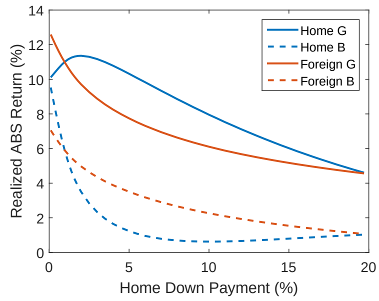
(b) Investment (independent shocks).



(c) ABS holdings (correlated shocks).



(d) ABS holdings (independent shocks).



(e) Realized ABS returns (correlated shocks).

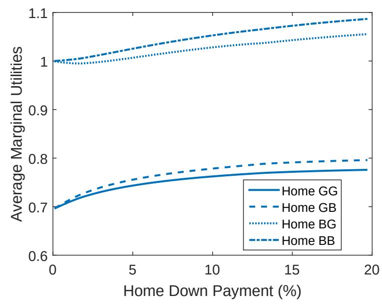
Figure 5.3: Effect of margins and financial integration with second-moment shocks.

Home down payments approach zero.

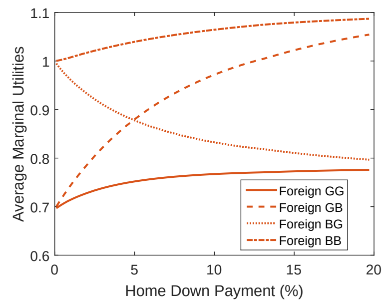
Our results provide an explanation for why gross flows collapse when global risk increases, consistent with the evidence in Forbes and Warnock (2012). The standard puzzle is why risky countries would invest less in safer countries when global risk increases. Our model provides at least a partial explanation for why gross flows would decrease in the presence of second-moment shocks. As well, there is ample evidence that macro and micro uncertainty increased during the 2007–2009 financial crisis (see for example Bloom (2014)), which would cause a collapse in gross flows in our model.

How can we understand the different implications of first- and second-moment shocks on gross flows? Figure 5.4 plots the average marginal utilities for agents in each country in each aggregate state, for perfectly correlated first- and second-moment shocks (results are qualitatively similar with correlated shocks). With first-moment shocks, agents have strong desires to hedge aggregate risks: marginal utilities in good and bad states are on average quite different (about .75 compared to about 1.05). Since most investment goes through Home when collateral levels are low, Foreign agents get payoffs (through ABS investments), very close to the Home average payoff. Thus, Home and Foreign marginal utilities converge as more capital flows to Home—but the marginal utility differences across states remain large.

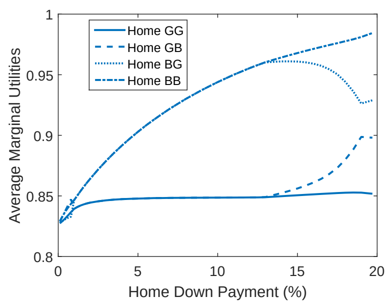
In contrast, marginal utilities with second-moment shocks are much closer on average (between .82 and 1), and the difference converges as Home down payments decrease. Thus, aggregate-state hedging needs are extremely small for low Home collateral levels; if anything Home agents would like to short Foreign ABS to hedge, which of course they cannot do.



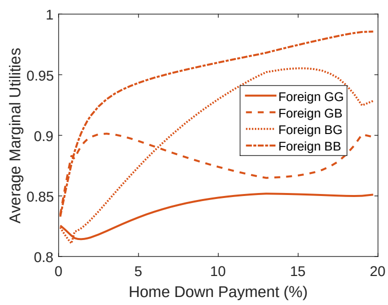
(a) First-moment shocks, Home.



(b) First-moment shocks, Foreign.



(c) Second-moment shocks, Home.



(d) Second-moment shocks, Foreign.

Figure 5.4: Average marginal utility by aggregate state (independent shocks).

The reason for these differences is not surprising: first-moment shocks affect the average payoff and thus the average level of consumption—the effect on marginal utilities is large—but second-moment shocks affect average payoffs only through changes in default rates, and much of this channel is mitigated by risk sharing across countries.

6 Conclusion

This paper provides a unified theoretical framework for understanding the implications of cross-country differences in risk sharing, the supply of relatively safe assets, and credit conditions for international capital flows and global welfare. We show that financial integration can lead to large international financial flows when countries have different collateral requirements for loans that are securitized into asset-backed securities, and these flows affect the global supply of safe assets. Capital flows from countries with low financial development (represented by high margins)—and thus low risk sharing and high demand for safe assets—to countries with high financial development (represented by low margins), with high interest rates. Gross financial flows arise when agents face aggregate shocks to the expected payoff to investment projects (first-moment), but gross flows can collapse when shocks concern the variance of returns (second-moment). Additionally, the correlation of shocks across countries matters for net flows when countries face first-moment shocks, but net flows are unaffected by correlations when countries face second-moment shocks.

Financial integration endogenously affects the global supply of safe assets and the amount of risk sharing that occurs in each country. The equilibrium supply of safe assets increases from a combination of higher Foreign demand for safe assets and increased intermediation of capital by the Home financial sector resulting from capital flows. Integration improves welfare for the country with high margin requirements but has smaller, ambiguous effects on the country with low margin requirements, typically hurting welfare for the country with low margin requirements if idiosyncratic risk or agents' risk aversion are high. However, if the high-margin country is sufficiently large compared to the low-margin country, then financial integration improves welfare for both countries. Tranching can have important consequences for gross flows but generally has modest effects on welfare.

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A Proofs

Proof of Proposition 3.2. Since the economy does not have aggregate risk, the asset-backed securities are risk-free. Therefore with financial integration, the Home and Foreign ABS must have the same risk-free rate R_f. Since Home and Foreign ABS are redundant assets, let $\phi^j = \phi^j_H + \phi^j_F$ be the total ABS holdings of country j. The budget constraint of country j is then

$$\theta^j + \phi^j - \psi^j = 1.$$

Since the maximum leverage property holds regardless of autarky or financial integration, without loss of generality we may assume $\theta^j = c_j\psi^j$, where $c_j > 1$ is the collateral requirement in country j.

Given the risk-free rate (ABS return) R_f, country j's optimal portfolio problem is

$$\begin{aligned} &\text{maximize} && \mathrm{E}[u(R(\pi)W)] \\ &\text{subject to} && \theta = c\psi,\ \phi = 1-(c-1)\psi, \end{aligned}$$

where we have suppressed the j super/subscript, $\pi = (\theta, \phi, \psi)$, and $R(\pi)$ is the portfolio return (2.4). Given R_f, since θ, ϕ are determined by the maximum leverage property and the budget constraint once we fix ψ, we can regard the portfolio return $R(\pi)$ as a linear function of ψ. With a slight abuse of notation, write it as $R(\psi)$. Then by (2.4) we have

$$R(\psi) = Ac\psi + (1-(c-1)\psi)R_f - \min\{Ac, R_b\}\psi.$$

Note that the borrowing rate R_b satisfies $R_f = \mathrm{E}[\min\{Ac, R_b\}]$ by the definition of the ABS return, so R_b is an increasing function of R_f. Since u is strictly concave, the optimal ψ is unique and depends only on R_f. Write this as $\psi = \psi(R_f)$. By the maximum theorem, ψ is continuous.

Step 1. $\psi(R_f) = 0$ *if* $R_f > \mathrm{E}[A]$.

To see this, let $f(\psi) = \mathrm{E}[u(R(\psi)W)]$. Then

$$f'(\psi) = \mathrm{E}[u'(R(\psi)W)(Ac - (c-1)R_f - \min\{Ac, R_b\})W].$$

Since $R(0) = R_f$ is a constant, if $R_f > \mathrm{E}[A]$, then we have

$$\begin{aligned} f'(0) &= u'(R_f W)(\mathrm{E}[A]c - (c-1)R_f - \mathrm{E}[\min\{Ac, R_b\}])W \\ &= u'(R_f W)c(\mathrm{E}[A] - R_f)W < 0, \end{aligned}$$

where we have used $R_f = \mathrm{E}[\min\{Ac, R_b\}]$. Since f is concave, the optimal ψ is $\psi = 0$.

Step 2. If $R_f > R_f^{\mathrm{Aut}}$, *then* $\psi(R_f) \le \psi(R_f^{\mathrm{Aut}})$.

Suppose on the contrary that $\psi(R_f) > \psi(R_f^{\mathrm{Aut}})$. Since $\psi(x)$ is continuous and $\psi(x) = 0$ if $x > \mathrm{E}[A]$, by the intermediate value theorem there exists $R_f^* > R_f$ such that $\psi(R_f^*) = \psi(R_f^{\mathrm{Aut}}) = 1/c$. This equation shows that $R_f^* > R_f^{\mathrm{Aut}}$ also gives an autarky equilibrium risk-free rate. However, this is a contradiction because by convention we always select the autarky equilibrium that has the maximum interest rate (Proposition 3.1). Therefore $\psi(R_f) \le \psi(R_f^{\mathrm{Aut}})$.

Since $\theta = c\psi$ by the maximum leverage property and $\phi = 1-(c-1)\psi$ by the budget constraint (and $c > 1$), it follows that θ decreases and ϕ increases if the risk-free rate goes up.

Step 3. If $R_f^{\mathrm{Int}} > R_f^{\mathrm{Aut}}$, *then the country gains from financial integration.*

Let $V(\pi, R_b) = \mathrm{E}[u(R(\pi)W)]$ be the welfare criterion given portfolio π and borrowing rate R_b. Letting π^{Int} be the portfolio in financial integration equilibrium, and $\pi^{\mathrm{Aut}} = (1, 1/c, 1/c)$ be the autarky portfolio, we have

$$V(\pi^{\mathrm{Int}}, R_b^{\mathrm{Int}}) = \max_\pi V(\pi, R_b^{\mathrm{Int}}) \ge V(\pi^{\mathrm{Aut}}, R_b^{\mathrm{Int}}). \tag{A.1}$$

Consider the welfare of the autarky portfolio $\pi^{\text{Aut}} = (1, 1/c, 1/c)$ at a general borrowing rate R_b. Since the portfolio return

$$\begin{aligned} R(\pi^{\text{Aut}}) &= A + \mathrm{E}[\min\{Ac, R_b\}]\frac{1}{c} - \min\{Ac, R_b\}\frac{1}{c} \\ &= A + \mathrm{E}[\min\{A, k\}] - \min\{A, k\} \end{aligned}$$

depends only on the default threshold $k = R_b/c$, so does the the welfare $V(\pi^{\text{Aut}}, R_b) = \mathrm{E}[R(\pi^{\text{Aut}})W]$. Furthermore, by the welfare monotonicity theorem (Theorem 2.6 in Toda (2013) or Proposition 3.1), $V(\pi^{\text{Aut}}, R_b)$ is increasing in $k = R_b/c$. If $R_f^{\text{Int}} > R_f^{\text{Aut}}$, then since R_b is an increasing function of R_f, we have $R_b^{\text{Int}} > R_b^{\text{Aut}}$. Since the collateral requirement c is fixed, the state contingency in financial integration goes up, so

$$V(\pi^{\text{Aut}}, R_b^{\text{Int}}) > V(\pi^{\text{Aut}}, R_b^{\text{Aut}}). \tag{A.2}$$

Combining (A.1) and (A.2), we obtain $V(\pi^{\text{Int}}, R_b^{\text{Int}}) > V(\pi^{\text{Aut}}, R_b^{\text{Aut}})$. □

Proof of Proposition 3.3. Without loss of generality we may assume $W = 1$.

To show the first claim, let $R(\theta; k, R_f)$ be the portfolio return (3.2) and $k_1 < k_2$ be two default thresholds. Fixing θ and R_f, using the same argument as in Toda (2013), we obtain

$$\mathrm{E}[u(R(\theta; k_2, R_f))] \geq \mathrm{E}[u(R(\theta; k_1, R_f))].$$

Maximizing both sides with respect to θ, we obtain

$$V(k_2, R_f) = \max_\theta \mathrm{E}[u(R(\theta; k_2, R_f))] \geq \max_\theta \mathrm{E}[u(R(\theta; k_1, R_f))] = V(k_1, R_f).$$

To show the second claim, let $\pi^{\text{Aut}} = (1, 1/c, 1/c)$ be the autarky portfolio. By (3.2),

$$R(\pi^{\text{Aut}}) = R(1; k^{\text{Aut}}, R_f) = \max\left\{0, A - k^{\text{Aut}}\right\} + \mathrm{E}[\min\left\{A, k^{\text{Aut}}\right\}]$$

does not depend on the risk-free rate. Therefore,

$$V(k^{\text{Aut}}, R_f) = \max_\theta \mathrm{E}[u(R(\pi))] \geq \mathrm{E}[u(R(\pi^{\text{Aut}}))] = V(k^{\text{Aut}}, R_f^{\text{Aut}}). \quad \square$$

Proof of Theorem 3.4. First let us show that the risk-free rate after financial integration is lower than the autarky interest rate in at least one country. Suppose on the contrary that $R_f^{\text{Int}} > R_f^{j,\text{Aut}}$ for all j. Then by Proposition 3.2, we have $\theta^{j,\text{Int}} < \theta^{j,\text{Aut}} = 1$. Then $W^H\theta^{H,\text{Int}} + W^F\theta^{F,\text{Int}} < W^H + W^F$, which contradicts the market clearing condition for capital (2.7).

If $R_f^{\text{Int}} > R_f^{F,\text{Aut}}$, since the risk-free rate must fall in at least one country, we have $R_f^{\text{Int}} < R_f^{H,\text{Aut}}$. By $R_f^{\text{Int}} > R_f^{F,\text{Aut}}$ and Proposition 3.2, we have $\theta^{F,\text{Int}} < \theta^{F,\text{Aut}} = 1$, $\phi^{F,\text{Int}} > \phi^{F,\text{Aut}} = 1/c_F$, and $\psi^{F,\text{Int}} < \psi^{F,\text{Aut}} = 1/c_F$. By market clearing for capital, we have

$$W^H\theta^{H,\text{Int}} + W^F\theta^{F,\text{Int}} = W^H + W^F,$$

so it must be $\theta^{H,\text{Int}} > 1 = \theta^{H,\text{Aut}}$. By the maximum leverage property, we have $\theta^H = c_H\psi^H$ always, so $\psi^{H,\text{Int}} > \psi^{H,\text{Aut}}$. By the budget constraint, we have

$$\theta^H + \phi^H - \psi^H = 1 \iff \phi^H = 1 - (c_H - 1)\psi^H$$

always, so $\phi^{H,\mathrm{Int}} < \phi^{F,\mathrm{Aut}}$. Finally, let us show that the global supply of safe assets increases. Using the market clearing condition for capital, the global supply of safe assets (3.1) is

$$\begin{aligned}\frac{W^H\theta^H}{c_H}+\frac{W^F\theta^F}{c_F}&=\frac{W^H\theta^H}{c_H}+\frac{W^H+W^F-W^H\theta^H}{c_F}\\&=\left(\frac{1}{c_H}-\frac{1}{c_F}\right)W^H\theta^H+\frac{W^H+W^F}{c_F}.\end{aligned}$$

Since $c_H < c_F$, this is an increasing function of θ^H. Since θ^H increases after financial integration, so does the global supply of safe assets. $\square$

Proof of Theorem 3.5. By the maximum leverage property, we have $\theta = c\psi$. By the budget constraint, $\phi = 1-\theta+\psi = 1-\left(1-\frac{1}{c}\right)\theta = 1-d\theta$, where $d = 1-1/c$ is down payment. With a slight abuse of notation, let $R^i(\theta)$ be the portfolio return, given θ. Then

$$\begin{aligned}R^i(\theta)&=A^i\theta+R_f\phi-\min\left\{A^ic,R_b\right\}\psi\\&=R_f+\left(\max\left\{A^i-R_b/c,0\right\}-dR_f\right)\theta=:R_f+X^i\theta.\end{aligned}$$

By redefining the utility function if necessary, without loss of generality we may assume that initial wealth is 1. Suppressing the i superscript, by the first-order condition we have

$$\mathrm{E}[u'(R(\theta))X]=0.$$

Let $F(\theta, R_f)$ be the left-hand side. By the implicit function theorem, we have $\mathrm{d}\theta/\mathrm{d}R_f = -F_{R_f}/F_\theta$.

Step 1. $F_\theta < 0$.

Since $F_\theta = \mathrm{E}[u''(R(\theta))X^2]$ and $u'' < 0$, it suffices to show that $X \neq 0$ with positive probability. Since by definition $X = \max\{A - R_b/c, 0\} - dR_f$ and $d, R_f > 0$, we have

$$X=0 \iff A-R_b/c-dR_f=0 \iff A=R_b/c+dR_f.$$

But since A is a random variable, $A \neq R_b/c + dR_f$ with positive probability, so $X \neq 0$ with positive probability.

Step 2. $F_{R_f} \leq 0$, with strict inequality if agents do not always default.

Let

$$Y=\frac{\mathrm{d}X}{\mathrm{d}R_f}=-\frac{1}{c}\frac{\mathrm{d}R_b}{\mathrm{d}R_f}1\left\{A>R_b/c\right\}-d.$$

Since $R_f = \mathrm{E}[\min\{Ac, R_b\}]$, R_b is an increasing function of R_f, so $\mathrm{d}R_b/\mathrm{d}R_f \geq 0$. Therefore $Y \leq -d < 0$. Letting $R = R(\theta)$, by simple algebra we have

$$\begin{aligned}F_{R_f}&=\mathrm{E}[u''(R)(1+\theta Y)X+u'(R)Y]\\&=\mathrm{E}[Y(u''(R)R+u'(R))+u''(R)((1+\theta Y)X-RY)]\\&=\mathrm{E}[Yu'(R)(1-\gamma(R))]+\mathrm{E}[u''(R)(X-R_fY)],\end{aligned}$$

where $\gamma(x) = -xu''(x)/u'(x)$ is the relative risk aversion and the last line uses $R = R_f + \theta X$. Since $Y \le -d < 0$ and $\gamma(x) \le 1$, the first term in the right-hand side is negative, so

$$\begin{aligned} F_{R_f} &\le \mathrm{E}[u''(R)(X - R_f Y)] \\ &= \mathrm{E}[u''(R)(\max\{A - R_b/c, 0\} - R_f(Y + d))]. \end{aligned}$$

Since $u''(R) < 0$, $\max\{A - R_b/c, 0\} \ge 0$, $R_f > 0$, and $Y \le -d < 0 \implies Y + d \le 0$, it follows that $F_{R_f} \le 0$. If agents do not always default, then $\max\{A - R_b/c, 0\} > 0$ with positive probability, so the inequality is strict.

Step 3. The financial integration equilibrium is unique and $R_f^{F,\mathrm{Aut}} < R_f^{\mathrm{Int}} < R_f^{H,\mathrm{Aut}}$.

Since $F_\theta < 0$ and $F_{R_f} < 0$, we have $\mathrm{d}\theta/\,\mathrm{d}R_f = -F_{R_f}/F_\theta < 0$, so the demand of investment is downward sloping. Therefore the excess demand for capital, $W^H\theta^H + W^F\theta^F - (W^H + W^F)$, is also downward sloping, so the equilibrium is unique. Since $\theta^j = 1$ at $R_f^{j,\mathrm{Aut}}$ and $R_f^{F,\mathrm{Aut}} < R_f^{H,\mathrm{Aut}}$, it must be $R_f^{F,\mathrm{Aut}} < R_f^{\mathrm{Int}} < R_f^{H,\mathrm{Aut}}$. $\square$

Proof of Proposition 3.6. Without loss of generality we may assume $W^H = 1$. It suffices to prove when Foreign is infinitely large ($W^F = \infty$), because other cases follow by continuity. Since Home is negligible in this case, interest rates are set by Foreign: $R_f^{\mathrm{Int}} = R_f^{F,\mathrm{Aut}} < R_f^{H,\mathrm{Aut}}$. Let us suppress the H and Int superscripts. Since $\theta + \phi - \psi = 1$ by the budget constraint and $\theta = c\psi$ by maximum leverage, we have $\psi = \frac{1-\phi}{c-1}$ and $\theta = c\frac{1-\phi}{c-1}$. Then the portfolio return (2.4) is

$$\begin{aligned} R^i(\pi) &= A^i c\frac{1-\phi}{c-1} + R_f\phi - \min\left\{A^i c, R_b\right\}\frac{1-\phi}{c-1} \\ &= \max\left\{A^i - R_b/c, 0\right\}\frac{1-\phi}{d} + R_f\phi, \end{aligned} \tag{A.3}$$

where $d = 1 - 1/c$ is the down payment.

Case 1: $u(\infty) = \infty$. Fix $\phi < 1$. Since the Home collateral requirement is low, letting $c \to 1$ ($d \to 0$) in (A.3) we obtain

$$\lim_{c\to 1} R^i(\pi) = \begin{cases} R_f\phi, & (A^i \le R_b) \\ \infty. & (A^i > R_b) \end{cases}$$

Therefore when c approaches 1,

$$\lim_{c\to 1} \mathrm{E}[u(R^i(\pi))] = \Pr(A^i \le R_b)u(R_f\phi) + \Pr(A^i > R_b)u(\infty) = \infty.$$

Since $V^{H,\mathrm{Int}} \ge \mathrm{E}[R^i(\pi)]$ for any fixed portfolio π, by continuity, $V^{H,\mathrm{Int}} > u(\mathrm{E}[A^i])$ for sufficiently small $c > 1$.

Case 2: $u(C) = \frac{1}{1-\gamma}C^{1-\gamma}$ with $\gamma > 1$. Let π be the Home equilibrium portfolio in financial integration. Since $\psi = \frac{1-\phi}{c-1} \ge 0$, we have $\phi \le 1$. Hence by (A.3) we obtain

$$R^i(\pi) \le \infty \times 1\left\{A^i > k\right\} + R_f^{\mathrm{Int}},$$

where $k = R_b^{H,\text{Int}}/c$ is the default threshold. Applying u and taking expectations, since $u(\infty) = 0$ we obtain

$$V^{H,\text{Int}} = \mathrm{E}[u(R^i(\pi))] \leq \Pr(A^i \leq k)\frac{1}{1-\gamma}(R_f^{\text{Int}})^{1-\gamma}.$$

Since agents can always choose to invest exclusively in ABS, in autarky we have

$$V^{H,\text{Aut}} \geq u(R_f^{H,\text{Aut}}) = \frac{1}{1-\gamma}(R_f^{H,\text{Aut}})^{1-\gamma}.$$

Therefore Home gains from financial integration only if

$$\begin{aligned} &\Pr(A^i \leq k)\frac{1}{1-\gamma}(R_f^{\text{Int}})^{1-\gamma} \geq V^{H,\text{Int}} > V^{H,\text{Aut}} \geq \frac{1}{1-\gamma}(R_f^{H,\text{Aut}})^{1-\gamma} \\ \iff &\Pr(A^i \leq k) < \left(R_f^{\text{Int}}/R_f^{H,\text{Aut}}\right)^{\gamma-1}. \end{aligned} \tag{A.4}$$

However, since $R_f^{\text{Int}} = R_f^{F,\text{Aut}} < R_f^{H,\text{Aut}}$, we have $R_f^{\text{Int}}/R_f^{H,\text{Aut}} < 1$, so for large enough γ the inequality (A.4) is violated. $\square$

B Long-run effects of financial integration

Thus far, we have considered static models and therefore analyzed only the short-term impact of financial integration. However, our model provides a rich enough framework to consider how capital flows affect economic outcomes in the long run. In this section, we study an infinite-horizon model and address the effect of financial integration on economic growth and inequality. For tractability, we consider a model without aggregate shocks.

B.1 Infinite-horizon model

In order to separate the portfolio decision (which reflects risk aversion) from the consumption/savings decision (which reflects the elasticity of intertemporal substitution), we use recursive utility. Namely, entrepreneurs have identical Epstein-Zin constant relative risk aversion, constant elasticity of intertemporal subtitution (CRRA/CEIS) utility function defined by

$$U_t = \left((1-\beta)C_t^{1-1/\varepsilon} + \beta\,\mathrm{E}[U_{t+1}^{1-\gamma}]^{\frac{1-1/\varepsilon}{1-\gamma}}\right)^{\frac{1}{1-1/\varepsilon}}, \tag{B.1}$$

where U_t is the continuation utility at time t, C_t is consumption, $0 < \beta < 1$ is the discount factor, $\gamma > 0$ is the coefficient of relative risk aversion, and $\varepsilon > 0$ is the elasticity of intertemporal substitution.[22]

[22] If $\gamma = 1$, $\mathrm{E}[U^{1-\gamma}]^{\frac{1}{1-\gamma}}$ should be replaced by $\exp(\mathrm{E}[\log U])$. If $\varepsilon = 1$, (B.1) becomes

$$U_t = \exp\left((1-\beta)\log C_t + \beta \log\left(\mathrm{E}[U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}\right)\right).$$

B.1.1 Individual decision

The individual problem is the same as in the earlier two period model, except that agents maximize the recursive utility (B.1) and the intertemporal budget constraint is

$$W_{t+1} = R_{t+1}^i(\pi)(W_t - C_t), \tag{B.2}$$

where $R_{t+1}^i(\pi)$ is the total return on portfolio $\pi = (\theta, \phi, \psi)$ defined by (2.4).

Suppose that the productivities A^i are i.i.d. across agents and time. Then the portfolio returns $R_{t+1}^i(\pi)$ for a given portfolio π is also i.i.d. across agents and time. For simplicity let us focus on the steady state (we consider the entire the transitional dynamics later). Then the only state variable for an agent is his own wealth, and by homotheticity the value function takes the form $V(W) = bW$, where $b > 0$ is a constant and W is wealth. Substituting the budget constraint (B.2) into the definition of the recursive utility (B.1), the Bellman equation becomes

$$V(W) = \max_{C,\pi} \left((1-\beta)C^{1-1/\varepsilon} + \beta\, \mathrm{E}[V(R^i(\pi)(W-C))^{1-\gamma}]^{\frac{1-1/\varepsilon}{1-\gamma}} \right)^{\frac{1}{1-1/\varepsilon}}. \tag{B.3}$$

Substituting $V(W) = bW$, by homotheticity we can separate the portfolio decision from the consumption-savings decision. The optimal portfolio problem is to maximize

$$\mathrm{E}[R^i(\pi)^{1-\gamma}]^{\frac{1}{1-\gamma}} \tag{B.4}$$

subject to the portfolio constraints (2.5b) and (2.5c). Since productivities A^i are i.i.d. across agents and time, so are portfolio returns $R_{t+1}^i(\pi)$ for a given portfolio π, and hence the maximum of (B.4) takes a common value. Let

$$\rho = \max_{\pi} \mathrm{E}[R^i(\pi)^{1-\gamma}]^{\frac{1}{1-\gamma}} \tag{B.5}$$

be the maximum value. The remaining consumption-savings problem is straightforward to solve by calculus. The optimal consumption rule is

$$C(W) = (1 - \beta^\varepsilon \rho^{\varepsilon-1})W, \tag{B.6}$$

and the coefficient of the value function is

$$b = \begin{cases} (1-\beta)^{\frac{1}{1-1/\varepsilon}}(1-\beta^\varepsilon\rho^{\varepsilon-1})^{\frac{1}{1-\varepsilon}}, & (\varepsilon \neq 1) \\ (1-\beta)(\beta\rho)^{\frac{\beta}{1-\beta}}, & (\varepsilon = 1) \end{cases} \tag{B.7}$$

which is continuous in ε and increasing in ρ. Using (B.6), individual wealth evolves according to $W' = \beta^\varepsilon\rho^{\varepsilon-1}R^i(\pi)W$. Since consumption is proportional to wealth, the growth rate of individual consumption is

$$\frac{C_{t+1}^i}{C_t^i} = \beta^\varepsilon \rho^{\varepsilon-1} R_{t+1}^i(\pi). \tag{B.8}$$

B.1.2 Equilibrium

Next we study the equilibrium. Since there are two countries and the consumption growth rate (B.8) will generically differ across the two countries, if agents are infinitely lived, then one country will dominate in the long run. Therefore in order to obtain non-degenerate wealth distributions, we assume that the entrepreneurs go bankrupt and are replaced by new ones at small probability δ each period (Yaari-Blanchard model).[23]

The formal model is as follows. A newborn entrepreneur in country j is endowed with human capital W_0^j, immediately starts her own private business, and converts her human capital to physical capital one-for-one.[24] Then the entrepreneur becomes "mature", and operates the technology as before until bankruptcy. Bankruptcy arrives with probability δ each period; when bankrupt, the entrepreneur's wealth is wiped out, the entrepreneur exits from the economy, and a new entrepreneur is born.

Since agents disappear and are reborn at constant probability δ, using (B.8), the steady state aggregate wealth W of a country must satisfy

$$\begin{aligned} W &= (1-\delta)\beta^{\varepsilon}\rho^{\varepsilon-1}\,\mathrm{E}[R^i(\pi)]W + \delta W_0 \\ \iff W &= \frac{\delta}{1-(1-\delta)\beta^{\varepsilon}\rho^{\varepsilon-1}\,\mathrm{E}[R^i(\pi)]}W_0, \end{aligned} \tag{B.9}$$

where W_0 is the initial capital of newborn agents.

The definition of equilibrium with financial integration remains the same—borrowing rates and consumption-portfolio choices such that markets clear and agents optimize—with the modification that market clearing in capital and asset markets reflects consumption decisions. Let C^j be the steady state aggregate consumption in country j, given by (B.6) with $W = W^j$. Since country j has remaining capital $W^j - C^j$ after consumption, the market clearing condition for the asset-backed securities are

$$(W^H - C^H)\phi_j^H + (W^F - C^F)\phi_j^F = (W^j - C^j)\psi^j$$

for $j = H, F$, where ϕ_j^H, ϕ_j^F denote the Home and Foreign portfolio share of ABS issued by country j. The left-hand side is the world demand of the ABS issued by country j. The right-hand side is the world supply of that ABS, which is supplied only by country j. The market clearing condition for capital is

$$(W^H - C^H)\theta^H + (W^F - C^F)\theta^F = (W^H - C^H) + (W^F - C^F), \tag{B.10}$$

where θ^H, θ^F denote the Home and Foreign portfolio share of the investment in the technology. The left-hand side is the total capital invested. The right-hand side is the total capital available, which must end up in the technology because lending and borrowing through securitization cancel out.

Since the optimal portfolio problem (B.4) is the same as in the two period model and the coefficient of the value function (B.7) is monotonic in ρ, Proposition 3.2 continues to hold. Parallel to Theorem 3.4, we obtain the following result.

[23]Introducing stochastic birth/death is a common trick to obtain nondegenerate stationary distributions. See, for example, Angeletos and Panousi (2011), Toda (2014), Gârleanu and Panageas (2015), and Toda and Walsh (2015).

[24]One may object to this "capital birth" assumption, but it is necessary for obtaining a nondegenerate distribution. If newborn agents inherit physical capital instead, then the aggregate growth rate of the two countries will generally differ, leading to a degenerate wealth distribution. Alternatively, one can consider a finite-horizon model, but the results are similar. To obtain balanced growth paths, the capital endowment need not be constant but may grow exponentially with time. If initial capital is proportional to $\mathrm{e}^{\eta t}$, then all calculations below are valid by replacing $\mathrm{E}[R^i(\pi)]$ with $\mathrm{E}[R^i(\pi)]\mathrm{e}^{-\eta}$.

Corollary B.1. *Suppose that $R_f^{\mathrm{Int}} > R_f^{F,\mathrm{Aut}}$. Then (i) $R_f^{\mathrm{Int}} < R_f^{H,\mathrm{Aut}}$, (ii) Home increases and Foreign decreases real investment, (iii) Home decreases and Foreign increases investment in ABS, and (iv) Home increases and Foreign decreases borrowing.*

Proof. Essentially identical to Theorem 3.4. □

B.1.3 Growth and inequality

By (B.8), the average growth rate of consumption (as well as wealth) for a surviving agent is

$$g = \mathrm{E}[C_{t+1}^i/C_t^i] = \beta^\varepsilon \rho^{\varepsilon-1}\,\mathrm{E}[R^i(\pi)]. \tag{B.11}$$

Therefore growth is governed by four factors: (i) patience (β), (ii) risk-adjusted portfolio return ($\rho = \mathrm{E}[R^i(\pi)^{1-\gamma}]^{\frac{1}{1-\gamma}}$, which is the same as welfare), (iii) elasticity of intertemporal substitution ε, and (iv) expected portfolio return $\mathrm{E}[R^i(\pi)]$. The first three determine the saving rate $\beta^\varepsilon\rho^{\varepsilon-1}$. Furthermore, by (B.9) the steady state aggregate wealth is $W = \frac{\delta}{1-(1-\delta)g}W_0$, which is monotonic in g.

In autarky, since $\theta = 1$ and $\phi = \psi = 1/c$, we have $\mathrm{E}[R^i(\pi)] = \mathrm{E}[A^i]$, so the expected portfolio return is constant. In this case the growth rate g is increasing or decreasing in ρ according as $\varepsilon > 1$ or $\varepsilon < 1$. Since more securitization increases ρ (Proposition 3.1), it follows that with securitization the economy grows faster if the elasticity of intertemporal substitution (EIS) ε is greater than 1 and slower if $\varepsilon < 1$. This dependency is not surprising. In fact, Schmidt and Toda (2015) show in a general setting that there is a precautionary saving motive if and only if EIS < 1. Since securitization offers risk sharing, it reduces the precautionary saving motive if EIS < 1.

What happens to growth and aggregate wealth after financial integration? The following proposition shows that Foreign will typically experience a slower economic growth and lower aggregate wealth.

Proposition B.2. *Suppose that $\varepsilon \le 1$. If a country faces a higher risk-free rate after financial integration, then the growth rate of individual wealth goes down and the steady state capital stock becomes lower than autarky.*

Proof. Since $x \mapsto \frac{1}{1-\gamma}x^{1-\gamma}$ is monotonic, the optimal portfolio problem (B.4) is the same as (2.5) with $u(C) = \frac{1}{1-\gamma}C^{1-\gamma}$. If $R_f^{\mathrm{Int}} > R_f^{\mathrm{Aut}}$, then by Proposition 3.2 ρ goes up and $\theta^{\mathrm{Int}} < \theta^{\mathrm{Aut}} = 1$. Since $\varepsilon \le 1$, the saving rate $\beta^\varepsilon\rho^{\varepsilon-1}$ goes down. Furthermore, since $R_f = \mathrm{E}[\min\left\{A^i c, R_b\right\}]$, we have

$$\mathrm{E}[R^i(\pi)] = \mathrm{E}[A^i]\theta + R_f\phi - R_f\psi = R_f + (\mathrm{E}[A^i] - R_f)\theta,$$

so the expected portfolio return $\mathrm{E}[R^i(\pi)]$ goes down because $R_f < \mathrm{E}[A^i]$ and $\theta^{\mathrm{Int}} < 1$. Therefore by (B.8), the consumption growth rate g goes down. □

This result is in sharp contrast to Angeletos and Panousi (2011), who find that South (Foreign) has a higher steady state aggregate wealth. The reason for this difference is that the degree of risk sharing is endogenous in our setting, which affects the portfolio choice (Home agents allocate more capital to the high return technology).

What does the stationary distribution of wealth within each country look like? Toda (2014) shows in a general setting that with multiplicative shocks and constant probability of birth/death, the stationary distribution is approximately *double Pareto* (Reed, 2001), which has density

$$f(x) = \begin{cases} \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} W_0^{\alpha_1} x^{-\alpha_1-1}, & (x \geq W_0) \\ \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} W_0^{-\alpha_2} x^{\alpha_2-1}, & (0 \leq x < W_0) \end{cases}$$

where $\alpha_1, \alpha_2 > 0$ are Pareto exponents of the upper and lower tails. Letting

$$\begin{aligned} \mu &= \log(\beta^\varepsilon \rho^{\varepsilon-1}) + \mathrm{E}[\log R^i(\pi)], \\ \sigma^2 &= \mathrm{Var}[\log R^i(\pi)] \end{aligned}$$

be the mean and variance of individual log wealth growth, by Theorem 16 of Toda (2014), $\zeta = -\alpha_1, \alpha_2$ are solutions to the quadratic equation

$$\frac{\sigma^2}{2}\zeta^2 - \mu\zeta - \delta = 0.$$

Note that

$$\alpha_1 = \frac{\sqrt{\mu^2 + 2\sigma^2\delta} - \mu}{\sigma^2} = \frac{2\delta}{\sqrt{\mu^2 + 2\sigma^2\delta} + \mu}, \tag{B.12}$$

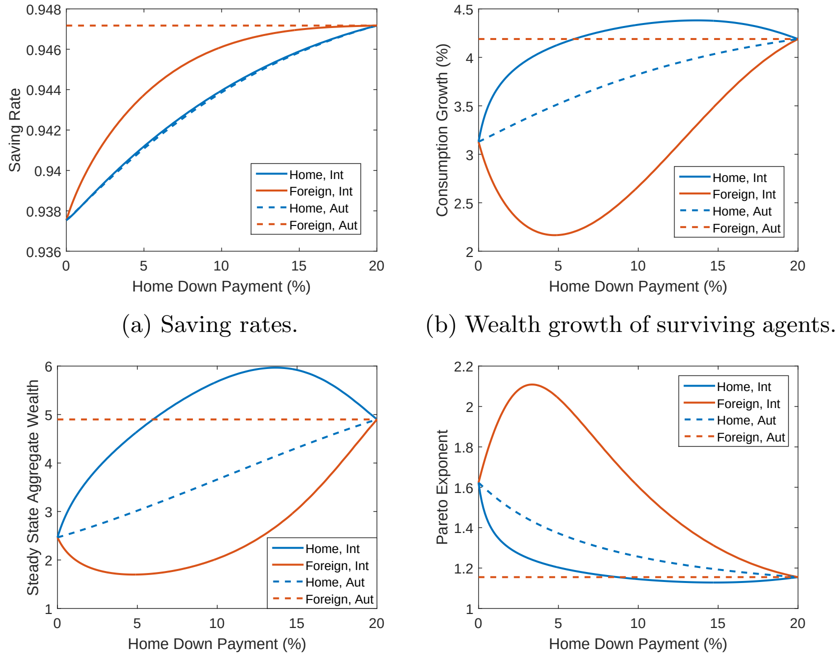
so increasing growth (μ) or variance (σ^2) makes the Pareto exponent α_1 smaller, and hence increases inequality.

B.2 Numerical example

We compute a numerical example with the same parameter values as in Section 2. In addition, we set $\varepsilon = 0.7$, $\beta = 0.95$, and $\delta = 0.05$ (so private businesses operate for an average of $1/0.05 = 20$ years). Figure B.1 shows the results. The interest rates, investment, portfolio, and welfare are similar to the static model, and therefore we omit the figures.

Figure B.1a shows the saving rate. Since the risk-free rate goes up in Foreign after financial integration, consistent with Proposition B.2, the Foreign saving rate falls. This is because Home ABS offers insurance, which reduces the precautionary saving motive. Note that the saving rates range from 0.938 to 0.947, so the magnitude of change is economically insignificant.

Figures B.1b and B.1c show the consumption growth rate defined by (B.8) and the steady state aggregate wealth defined by (B.9). Consistent with Proposition B.2, Foreign growth slows down after financial integration and the steady state aggregate wealth shrinks. Since the saving rate hardly change, the reduction in the growth rate and wealth is almost entirely due to the portfolio effect: Foreign reduces high-risk, high return investment in the production technology and increases risk-free investments in ABS. The effect of financial integration on growth for Home is ambiguous. With moderate collateral requirement, since risk-free rates are relatively low, Home enjoys high growth by borrowing at low rates. However, with low collateral requirement, the risk premium on the production technology shrinks, and growth goes down because savings goes down. This result is in contrast to the case in autarky, where consumption growth always decreases after securitization. The reason for this difference is that in an



(a) Saving rates.

(b) Wealth growth of surviving agents.

(c) Steady state aggregate wealth.

(d) Pareto exponent.

Figure B.1: Effect of margins and financial integration in an infinite-horizon model.

open economy the country with low margins can borrow from the other country to invest in the high return production technology.

Figure B.1d shows the Pareto exponent of the upper tail computed by (B.12). Since financial integration increases (decreases) risk sharing in Foreign (Home), the Pareto exponent goes up (down) in Foreign (Home). Therefore financial integration decreases (increases) inequality in Foreign (Home). This result is consistent with the empirical finding that wealth inequality has increased in U.S. since the late 1980s, which is about the time securitization has been introduced.

B.3 Transitional dynamics

Although we have so far only considered the steady states, it is possible to compute the entire transitional dynamics as explained in Appendix E.4. In this section we consider the following transitions. The economy is initially in the steady state of financial autarky with Home and Foreign collateral requirement (c_H, c_F). At $t = 0$, the economy unexpectedly shifts to financial integration. At $t = 60$, Home collateral unexpectedly drops to $c'_H < c_H$. At $t = 80$, Home collateral unexpectedly reverts to c_H.

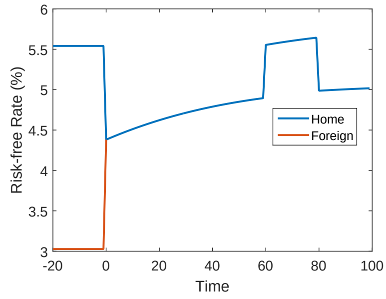
In the numerical examples below, we set $(c_H, c_F) = (1/0.9, 1/0.8)$ (down payments 10% and 20%, respectively), $c'_H = 1/0.92$ (down payment 8%), and all other parameters are as before. We compute the dynamics for 100 years after the transition.

Figure B.2 shows the transitional dynamics. After financial integration, the risk-free rates become identical in the two countries.[25] Since interest rates go up in Foreign and down in Home, capital flows from Foreign to Home, and Foreign gains and Home loses in terms of welfare.[26] Since Home invests in the high return technology by borrowing from Foreign, Home enjoys an economic growth. As Home gets bigger, it cannot borrow as much, and hence investment decreases. Interestingly, as Home gets bigger, the interest rates rise even more, which makes the ABS even more attractive for Foreign investors. Therefore Foreign invests less in the technology and more in ABS over time. The convergence to the new steady state is extremely slow. This is because the interest rate is pinned down by the market clearing condition, but since the market clearing condition involves aggregate wealth (a stock variable, which slowly adjusts), the interest rate also adjusts slowly.

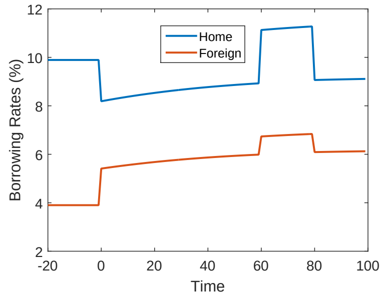
When Home down payment unexpectedly decreases at $t = 60$, since there is more default, the interest rates go up. Since the low down payment allows Home to borrow more, investment goes up in Home and down in Foreign. Since there is more risk sharing, welfare improves and saving rates go down in both countries. Therefore the economy grows slower and aggregate wealth goes down. When Home down payment reverts to the normal level, exactly the opposite happens.

[25] In reality integration is a process that occurs slowly, so the sudden convergence in interest rates and spikes in investment should be understood in that context.

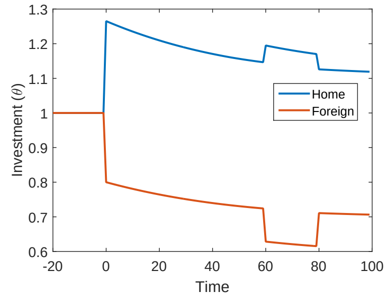
[26] Welfare at time t is the coefficient of the value function at time t, which is b_t in Appendix E.4.



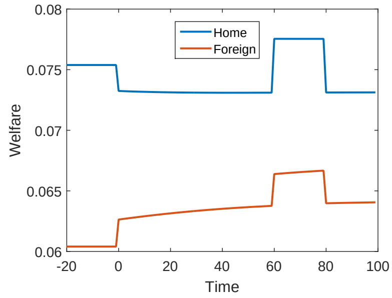
(a) Risk-free rate.



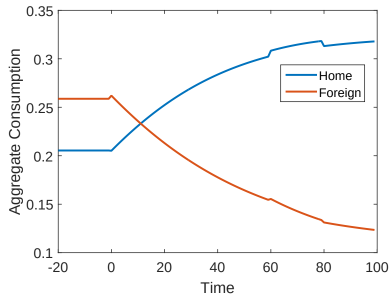
(b) Borrowing rates.



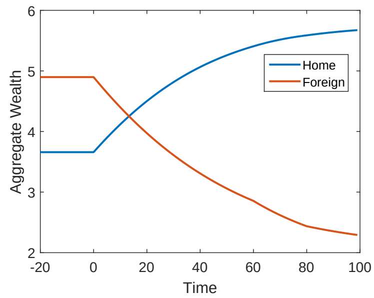
(c) Investment.



(d) Welfare.



(e) Aggregate consumption.



(f) Aggregate wealth.

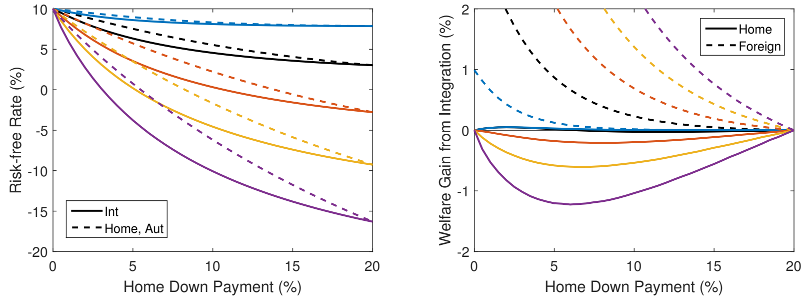
Figure B.2: Transitional dynamics. At $t = 0$, the economy shifts from financial autarky to financial integration. At $t = 60$, Home down payment unexpectedly drops from 10% to 8%. At $t = 80$, Home collateral unexpectedly reverts to 10%.

Online Appendix (not for publication)

C Sensitivity analysis over parameter values

In this appendix we solve the two country model in Section 2 with different parameter values. We consider changing the standard deviation of technology σ and the relative risk aversion coefficient γ to show how the welfare consequences for Home, driven by risk sharing, depend on underlying risk.

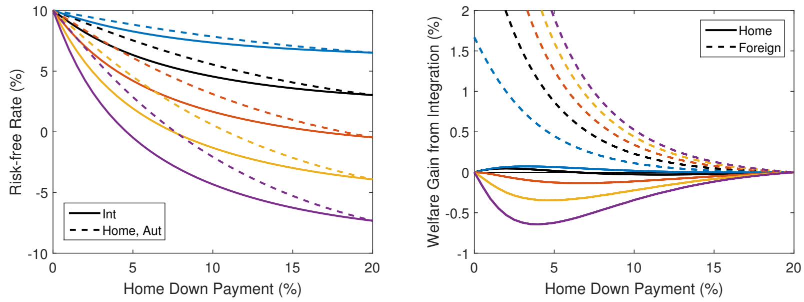
Figure C.1 shows the risk-free rate and the welfare gain (relative to autarky) when σ ranges over $\sigma = 0.1, 0.2, \dots, 0.5$. The baseline case ($\sigma = 0.2$) is shown in black. The larger σ, the lower the risk-free rate and the larger the welfare gain for Foreign, which is natural because Foreign gains from more risk sharing the riskier the technology is. As before, the welfare implication for Home is ambiguous. Home loses more from financial integration with higher σ because the interest rate drops by more, which reduces risk sharing.



(a) Risk-free rate. (b) Welfare gain.

Figure C.1: The effect of changing standard deviation σ. Black: baseline case ($\sigma = 0.2$); Blue: low σ; Purple: high σ.

Figure C.2 shows the risk-free rate and the welfare gain when γ ranges over $\gamma = 1, 2, \dots, 5$. The baseline case ($\gamma = 2$) is shown in black. The result is quantitatively similar to changing σ. However, with low risk aversion ($\gamma = 1$), Home also uniformly gains from financial integration for all collateral levels.



(a) Risk-free rate. (b) Welfare gain.

Figure C.2: The effect of changing relative risk aversion γ. Black: baseline case ($\gamma = 2$); Blue: low γ; Purple: high γ.

D Tranching

One of the most important recent financial innovations has been the "tranching" of assets or collateral. In tranched securitization, the dividend payments are divided into state-contingent payoffs, which better caters the needs of different investors.[27]

It is worth understanding why tranching would be meaningful in this situation, and why it would change results. The result of our paper so far has been that because collateral rates differ across countries, agents in each country have different exposures to risk, and international trade transpires precisely because agents face different levels of risk. Trade allows agents to hold portfolios that share risk more effectively than the autarkic portfolios. With ex ante identical agents, tranching has no effect on the autarky equilibrium because investors hold identical portfolios and thus have identical exposure to aggregate risks. However, when agents in each country are subject to different collateral rates, investors in different countries hold different portfolios and are thus subject to different aggregate risk exposure. Because collateral levels differ across countries, the payoffs to ABS are linearly independent. However, investors cannot sell short ABS, and thus the ABS need not span the aggregate states. Tranching, by design, allows agents to isolate aggregate risks, and thus tranching will lead to greater risk sharing.

D.1 Model

We model tranching as the ability of financial intermediaries to divide the payments from a pool of collateral into different bonds that pay in different states. To simplify, we consider perfectly correlated ("world shocks") so that there are only two aggregate states. To allow for the cleanest form of tranching, we consider splitting the Home pool of collateral into Arrow securities that pay in each state: the s-th tranche pays the value of the Home collateral in state s, and zero otherwise.[28] Furthermore, we suppose that only the Home financial sector has the ability to tranche assets. We call these tranches "Home-s", denoted by Hs. The payoff of this tranche in state s' per unit of capital investment is

$$D^{Hs}(s') = \begin{cases} R^H_{\text{ABS}}(s), & (s' = s) \\ 0. & (s' \neq s) \end{cases}$$

Let q_s be the price of one share of the Hs tranche. Since holding 1 share of all tranches is the same as holding the entire ABS, which has price 1, we have

$$\sum_{s=1}^{S} q_s = 1.$$

The budget constraint is modified as follows. Let $\phi_{Hs}, \phi_F \geq 0$ be the fraction of wealth invested in the ABS tranches of each country, and let $\psi \geq 0$ be the fraction borrowed. By accounting, the budget constraint of country j with tranching becomes

$$\theta^j + \sum_{s=1}^{S} \phi^j_{Hs} + \phi^j_F - \psi^j = 1.$$

[27] See Fostel and Geanakoplos (2012a) for details about tranching.

[28] While considering Arrow securities is an extreme case, the equilibrium asset holdings with two states are not so far from "balanced" so that we could instead consider two tranches, one safer and one slightly riskier, and still get similar quantitative results.

The collateral constraint remains the same. Since the return on tranche s is D^{Hs}/q_s, the return on portfolio $\pi = (\theta, (\phi_{Hs}), \phi_F, \psi)$ of agent i in country j is

$$R^i(\pi) = A^i\theta + \sum_{s=1}^{S} \frac{D^{Hs}}{q_s}\phi_{Hs} + R^F_{\mathrm{ABS}}\phi_F - \min\left\{A^i c_j, R^j_b\right\}\psi.$$

Equilibrium is modified to include market clearing in tranches. Financial intermediaries create as many tranches as are backed by collateral, so

$$W^H\phi^H_{Hs} + W^F\phi^F_{Hs} = q_s W^H\psi^H$$

for all $s = 1, \dots, S$. The left-hand side is the world investment in tranche Hs. The right-hand side is the total value of tranche Hs, which equals price q_s times the number of shares $W^H\psi^H$. The market clearing condition for Foreign ABS is unchanged. As before, borrowing rates and tranche prices are determined such that agents optimize and asset markets clear.

One caveat with tranching is that the Foreign ABS is a redundant asset since there are as many tranches as aggregate states, which implies that equilibrium portfolios are indeterminate. To overcome this issue, we assume that investors are home-biased. Thus Foreign investors hold Foreign ABS as much as possible.

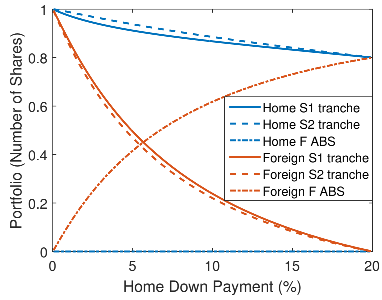
D.2 Numerical results with tranching

Figure D.1 shows the results with tranching. The figures in the left are for first-moment shocks, whereas those in the right are for second-moment shocks. Since the interest rates, investment, and welfare hardly change from the case without tranching, we omit the figures.

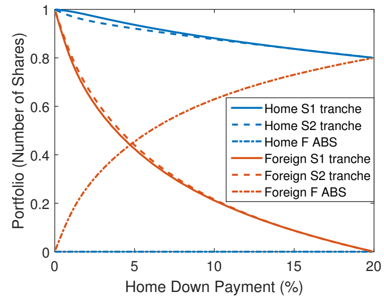
Tranching has a large impact on international capital flows. Without tranching, gross flows arise because countries try to insure against aggregate risk using the two ABS (Figures 5.1e and 5.3c). With tranching, since Home tranches are essentially Arrow securities, Foreign ABS becomes a redundant asset. Since Foreign borrows less after financial integration and therefore the market capitalization of Foreign ABS is small even compared to Foreign capital, Foreign can absorb all Foreign ABS. Thus capital flows only from Foreign to Home through the purchase of tranches (Figures D.1a and D.1b), just as in the case without aggregate risk (Theorem 3.4).[29]

According to Figures D.1a and D.1b, each country holds roughly "balanced" portfolios of tranches. However, while with first-moment shocks Home holds more shares of the $H2$ tranche (corresponding to low investment returns), Home holds more shares of the $H1$ tranche (corresponding to low variance) with second-moment shocks. This result may appear surprising, because state $s = 1$ corresponds to either high return or low variance, which is the "good" state. Since Home is relatively insured against idiosyncratic risks, we would expect that Home will demand larger shares of the asset that pays in the "bad" aggregate state, which is $s = 2$. The reason why Home holds more shares of the $H1$ tranche with second-moment shocks is that the high variance state is indeed "good" for Home because the high idiosyncratic variance makes default more likely and enables more risk sharing.

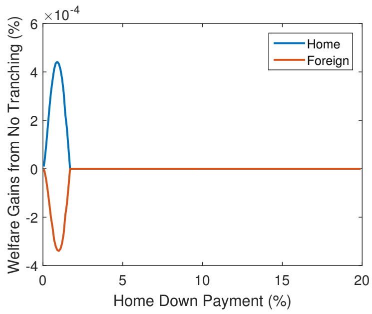
[29] If "risk-off" is interpreted as a home-bias, then tranching would make global flows more sensitive to changes in risk-on/risk-off.



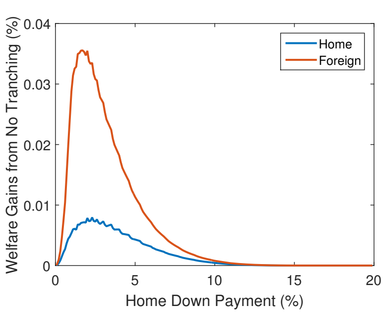
(a) Portfolio.



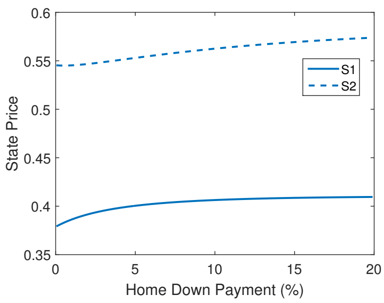
(b) Portfolio.



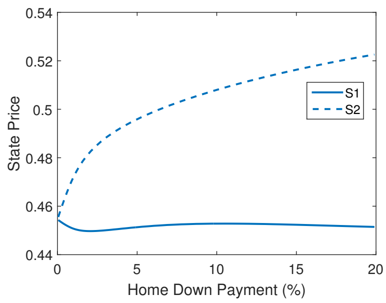
(c) Welfare gains from no tranching.



(d) Welfare gains from no tranching.



(e) State prices.

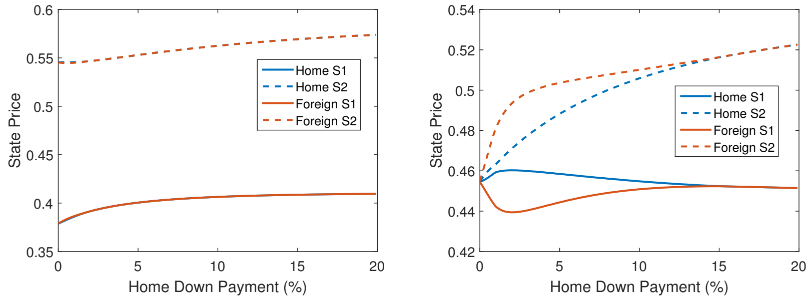


(f) State prices.

Figure D.1: Effect of tranching with first-moment shocks (left) and second-moment shocks (right).

Since there are two aggregate states and two ABS, tranching would change welfare only when the two ABS do not fully span the aggregate states, *i.e.*, when the no-shortselling constraint binds for at least one country. This is the case when Home down payments are less than 5% with first-moment shocks or less than 15% with second-moment shocks. In those cases, tranching affects welfare but only slightly: with first-moment shocks, Foreign loses after tranching (Figure D.1c); with second-moment shocks, both countries gain (Figure D.1d).

Since tranching completes the market with respect to aggregate states, the state prices become the same across countries. Figure D.2 plots state prices without tranching. With first-moment shocks (Figure D.1e), the state prices hardly change from the case without tranching because the prices were nearly identical across countries (Figure D.2a). With second-moment shocks, the state prices without tranching are quite different across countries (Figure D.2b), and the state prices after tranching lie in between (Figure D.1f).



(a) First-moment Shocks.

(b) Second-moment shocks.

Figure D.2: State prices without tranching.

E Solution Algorithm

E.1 Model without aggregate risk

We solve for the equilibrium by reducing the equilibrium conditions to one equation in one unknown, as follows. Given the risk-free rate R_f, the borrowing rates in each country are determined by $R_f = \mathrm{E}\left[\min\left\{A^i c_j, R_b^j\right\}\right]$. These borrowing rates completely determine the portfolio return. Since $\theta^j + \phi^j - \psi^j = 1$ by the budget constraint and $\theta^j = c_j \psi^j$ by the maximum leverage property, we have $\phi^j = 1 - (c_j - 1)\psi^j$. Therefore we can express the portfolio return as a function of ψ^j alone, and numerically solve the portfolio problem. Thus all allocations are functions of R_f, and we can invoke the market clearing condition for capital to pin down the equilibrium risk-free rate R_f.

E.2 Model with aggregate risk but no tranching

It is numerically more efficient to start from allocations and then to solve for allocations and borrowing rates jointly. Given (ϕ_H^j, ϕ_F^j), we can compute θ^j, ψ^j by the budget constraint and the maximum leverage property. There are thus 6 unknowns, $(\phi_H^j, \phi_F^j, R_b^j)$ for $j = H, F$. We compute them by solving 6 nonlinear equations, the two first-order conditions (Home and Foreign ABS) for each country, and the 2 market clearing conditions for ABS.

Finding a solution requires a fairly accurate initial guess of the solution. When country shocks are perfectly correlated ($\rho = 1$), we use the solution of the case with no aggregate shocks as an initial guess. When $\rho < 1$, we start from $\rho = 1$ and iteratively solve for the case with slightly smaller ρ using the previous solution as an initial guess until we reach the desired ρ.

E.3 Model with tranching

Since Home tranches are essentially Arrow securities, Foreign ABS is a redundant asset. Therefore we solve for the equilibrium with Arrow securities first and then replicate them by Home tranches and Foreign ABS. Let n_s^H be the number of shares of state s Arrow security (which pays dividend 1 in state s and 0 otherwise) held by Home per unit of wealth, and let $n^H = \left\{n_s^H\right\}_{s=1}^S$. There are $S+3$ unknown variables, $n^H, \psi^H, R_b^H, R_b^F$. Given ψ^H, we obtain $\theta^H = c_H \psi^H$ by maximum leverage. Letting $\pi^H = (\theta^H, n^H, \psi^H)$ be the Home portfolio, we obtain the Home portfolio return

$$R(\pi^H) = A\theta^H + \sum_{s=1}^S D_s n_s^H - \min\left\{Ac_H, R_b^H\right\}\psi^H,$$

where D_s is the dividend of state s Arrow security. By the first-order condition, the state s price is

$$\tilde{q}_s = \frac{\mathrm{E}[R(\pi^H)^{-\gamma} D_s]}{\mathrm{E}[R(\pi^H)^{1-\gamma}]}.$$

We obtain θ^F by capital market clearing, and also $\psi^F = \theta^F / c_F$ by maximum leverage. We can then compute the Foreign holdings of Arrow securities, n^F, using the market clearing condition

$$W^H n_s^H + W^F n_s^F = R_{\mathrm{ABS}}^H(s) W^H \psi^H + R_{\mathrm{ABS}}^F(s) W^F \psi^F,$$

where $s = 1, \dots, S$. Since the Foreign portfolio is determined, we obtain the Foreign portfolio return $R(\pi^F)$. Finally, we compute the $S+3$ unknown variables $n^H, \psi^H, R_b^H, R_b^F$ by solving $S+3$ nonlinear equations. The first S equations are the Foreign first-order conditions

$$\tilde{q}_s = \frac{\mathrm{E}[R(\pi^F)^{-\gamma} D_s]}{\mathrm{E}[R(\pi^F)^{1-\gamma}]},$$

where $s = 1, \dots, S$. The other three are Foreign budget constraint

$$\theta^F + \sum_{s=1}^S \tilde{q}_s n_s^F - \psi^F = 1$$

and the no-arbitrage pricing of Home and Foreign ABS,

$$\sum_{s=1}^S \tilde{q}_s R_{\mathrm{ABS}}^j(s) = 1,$$

where $j = H, F$.

E.4 Transitional dynamics in infinite-horizon model

To solve for the transitional dynamics, we fix some large T. Assume the risk-free rate path $\{R_{f,t}\}_{t=0}^T$ is given. We can then compute the borrowing rate $R_{b,t}$ for each country by $R_{f,t} = \min\left\{A^i c, R_{b,t}\right\}$, where $c \geq 1$ is collateral requirement. The Bellman equation (B.3) still holds, except that the value functions are indexed by time. By homotheticity, we have $V_t(W) = b_t W$, where $b_t > 0$. Since there is no aggregate risk, $\{b_t\}_{t=0}^T$ is deterministic. Since shocks are i.i.d. over time, the portfolio problem is myopic, which is (B.4). Let

$$\rho_{t+1} = \max_\pi \mathrm{E}[R_{t+1}^i(\pi)^{1-\gamma}]^{\frac{1}{1-\gamma}}. \tag{E.1}$$

Substituting into the Bellman equation, we obtain

$$b_t = \max_{0 \le x \le 1} \left((1-\beta) x^{1-1/\varepsilon} + \beta (1-x)^{1-1/\varepsilon} (b_{t+1}\rho_{t+1})^{1-1/\varepsilon} \right)^{\frac{1}{1-1/\varepsilon}},$$

where $x = C/W$ is the consumption rate. Solving the first-order condition, the optimal consumption rate is

$$x = \frac{C_t}{W_t} = \frac{(1-\beta)^\varepsilon}{(1-\beta)^\varepsilon + \beta^\varepsilon (b_{t+1}\rho_{t+1})^{\varepsilon-1}}. \tag{E.2}$$

Substituting into the Bellman equation and letting $a_t = b_t^{\varepsilon-1}$, we obtain

$$a_t = (1-\beta)^\varepsilon + \beta^\varepsilon \rho_{t+1}^{\varepsilon-1} a_{t+1}. \tag{E.3}$$

Substituting into (E.2), the optimal consumption rate simplifies to $x = C_t/W_t = (1-\beta)^\varepsilon / a_t$ and the saving rate out of wealth is $1-x = \beta^\varepsilon \rho_{t+1}^{\varepsilon-1} \frac{a_{t+1}}{a_t}$. Adding individual budget constraints and noting that agents go bankrupt with probability $\delta > 0$, the aggregate wealth satisfies

$$W_{t+1} = (1-\delta)\beta^\varepsilon \rho_{t+1}^{\varepsilon-1} \frac{a_{t+1}}{a_t} \mathrm{E}[R_{t+1}(\pi_t)] W_t + \delta k_0, \tag{E.4}$$

where π_t is the optimal portfolio at time t and k_0 is the initial capital of newborn agents.

Therefore, given a guess of risk-free rates $\{R_{f,t}\}_{t=0}^T$, we can compute the aggregate excess demand of capital as follows.

1. Solve for the equilibrium in steady state as before.
2. For each time t and country j, solve the optimal portfolio problem (E.1). Let $\pi_t^j = (\theta_t^j, \phi_t^j, \psi_t^j)$ be the optimal portfolio and ρ_{t+1}^j be the risk-adjusted portfolio return (E.1).
3. For each country, iterate (E.3) backward from $t = T-1$ to $t = 0$ to obtain $\{a_t\}_{t=0}^T$, where a_T is computed from the steady-state value.
4. Given any aggregate initial wealth W_0^j of country j, iterate (E.4) forward from $t = 0$ to $t = T-1$ to obtain aggregate wealth $\left\{W_t^j\right\}_{t=0}^T$.
5. The aggregate excess demand of capital at time t is

$$z_t := \sum_{j=H,F} (W_t^j - C_t^j)(\theta_t^j - 1) = \sum_{j=H,F} \beta^\varepsilon (\rho_{t+1}^j)^{\varepsilon-1} \frac{a_{t+1}^j}{a_t^j} W_t^j (\theta_t^j - 1).$$

In order to solve for the equilibrium, we need to find a risk-free rate path $\{R_{f,t}\}_{t=0}^T$ that makes the excess demand path $\{z_t\}_{t=0}^T$ equal to zero. This problem is computational intractable since there are many unknowns. To make the problem operational, we conjecture that $R_{f,t} = r(t/T)$, where $r : [0,1] \to \mathbb{R}$ is a smooth function. Dividing $[0,1]$ in even-spaced intervals, we approximate r by a cubic spline. If there are M subintervals, then the cubic spline is completely determined by the values $r_m := r(m/M)$, where $m = 0, 1, \dots, M$. We can compute the squared sum of aggregate excess demand $\sum_{t=0}^{T-1} z_t^2$ and minimize it over $\{r_0, r_1, \dots, r_M\}$.

In practice, we start from small M (say $M = 1$ with $r_0 = r_1 = R_{f,\infty}$ as an initial guess) and increase M (say $M = 2, 4, 8, \dots$), using previous values as the initial guess for the next M. We find that setting $T = 300$ and $M = 4$ is quite accurate.