

Segments, Rays, Parallel Lines and Planes

1. Plan

What You'll Learn

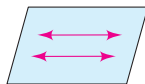
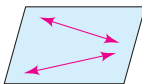
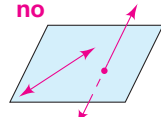
- To identify segments and rays
- To recognize parallel lines

... And Why

To identify compass directions that can be represented by opposite rays, as in Exercise 36

Check Skills You'll Need

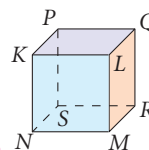
Judging by appearances, will the lines intersect?

-  **no**
-  **yes**
-  **no**

GO for Help Lesson 1-3

Name the plane represented by each surface of the box.

- the bottom **NMR**
- the top **PQL**
- the front **NKL**
- the back **PQR**
- the left side **PKN**
- the right side **LQM**



New Vocabulary

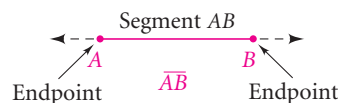
- segment
- ray
- opposite rays
- parallel lines
- skew lines
- parallel planes

1

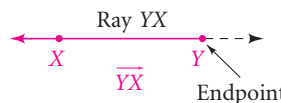
Identifying Segments and Rays



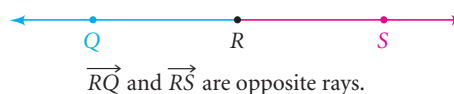
Many geometric figures, such as squares and angles, are formed by parts of lines called segments or rays. A **segment** is the part of a line consisting of two endpoints and all points between them.



A **ray** is the part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.



Opposite rays are two collinear rays with the same endpoint. Opposite rays always form a line.



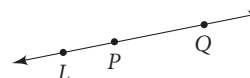
Real-World Connection

A sunbeam models a ray. The sun is its endpoint.

1 EXAMPLE Naming Segments and Rays

Name the segments and rays in the figure at the right.

- The three segments are \overline{LP} , \overline{PQ} , and \overline{LQ} .
- The four rays are \overrightarrow{LP} or \overrightarrow{LQ} , \overrightarrow{PQ} , \overrightarrow{PL} , and \overrightarrow{QP} or \overrightarrow{QL} .



Quick Check

- 1 Critical Thinking** \overrightarrow{LP} and \overrightarrow{PL} form a line. Are they opposite rays? Explain.
No, they do not have the same endpoint.

Lesson 1-4 Segments, Rays, Parallel Lines and Planes

23

Objectives

- To identify segments and rays
- To recognize parallel lines

Examples

- Naming Segments and Rays
- Identifying Parallel and Skew Segments
- Identifying Parallel Planes

Professional Development

Math Background

The undefined terms *point*, *line*, and *plane* form the basis for the definitions of ray, segment, and parallel planes. Together these terms form the beginning vocabulary for the study of geometry. Euclid used this approach in Book 1 of *The Elements*.

More Math Background: p. 2C

Lesson Planning and Resources

See p. 2E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Basic Postulates of Geometry

Lesson 1-3: Examples 3, 4

Extra Skills, Word Problems, Proof Practice, Ch. 1

Differentiated Instruction Solutions for All Learners

Special Needs L1

Draw line AB on the board and ask: Are \overrightarrow{AB} and \overrightarrow{BA} opposite rays? Why or why not? **No, \overrightarrow{AB} and \overrightarrow{BA} are not opposite rays because they do not have only one endpoint in common, they share many points.**

learning style: visual

Below Level L2

Remind students that the different notations for *line*, *line segment*, and *ray* readily identify and distinguish them.

learning style: verbal

2. Teach

Guided Instruction

Teaching Tip

Point out that the first letter naming a ray is always its endpoint. The second letter is any other point on the ray. Emphasize that opposite rays are two *distinct* collinear rays with only their endpoints in common.

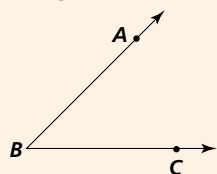
1 EXAMPLE Visual Learners

Remind students to associate the notations for line, segment, and ray with the actual figures.

PowerPoint

Additional Examples

1 Name the segments and rays in the figure.



\overrightarrow{BA} , \overrightarrow{BC} , \overrightarrow{BA} , \overrightarrow{BC}

Guided Instruction

2 EXAMPLE Tactile Learners

Some students may have trouble visualizing skew lines from the figure shown. Provide physical models for these students.

2

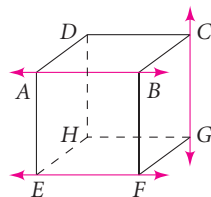
Recognizing Parallel Figures

Lines that do not intersect may or may not be coplanar.

Parallel lines are coplanar lines that do not intersect. **Skew lines** are noncoplanar; therefore, they are not parallel and do not intersect.

Vocabulary Tip

You read $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$ as "line \overleftrightarrow{AB} is parallel to line \overleftrightarrow{EF} ."



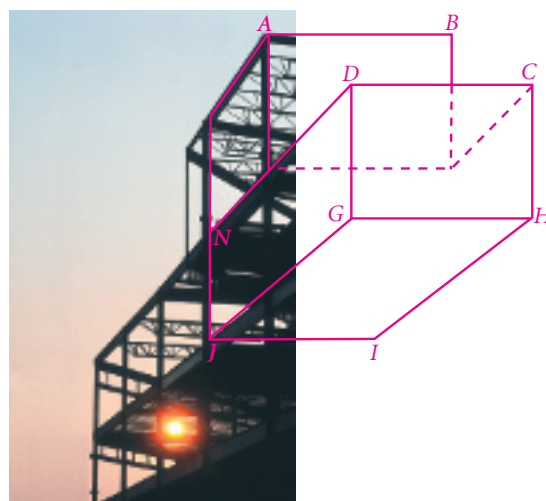
$\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$
 \overleftrightarrow{AB} and \overleftrightarrow{CG} are skew.

Segments or rays are parallel if they lie in parallel lines. They are skew if they lie in skew lines. \overline{AB} and \overline{CG} are skew because \overleftrightarrow{AB} and \overleftrightarrow{CG} are skew.

2 EXAMPLE Identifying Parallel and Skew Segments

a. Name all labeled segments that are parallel to \overline{DC} .
 \overline{AB} , \overline{GH} , and \overline{JI} are parallel to \overline{DC} .

b. Name all labeled segments that are skew to \overline{DC} .
 \overline{NJ} , \overline{GJ} , and \overline{HI} are skew to \overline{DC} .

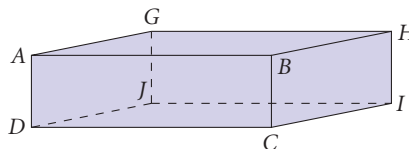


Quick Check

2 Use the diagram in Example 2.

- Name all labeled segments that are parallel to \overline{GJ} . \overline{HI} , \overline{DN}
- Name all labeled segments that are skew to \overline{GJ} . \overline{AB} , \overline{CD} , \overline{CH}
- Name another pair of parallel segments; of skew segments. \overline{DN} , \overline{HI} ; \overline{DN} , \overline{HC}

Parallel planes are planes that do not intersect. A line and a plane that do not intersect are also parallel.



Plane $ABCD \parallel$ Plane $GHJI$.
Plane $ABCD \parallel \overleftrightarrow{GH}$.

Differentiated Instruction Solutions for All Learners

Advanced Learners L4

Have students justify the statement, "Skew lines are noncoplanar; therefore they are not parallel and do not intersect." They may need to reason indirectly.

English Language Learners ELL

Use Exercises 25-33 to reinforce the meaning of the new vocabulary in the lesson as well as the terms *always*, *sometimes*, and *never* in the context of mathematical reasoning.

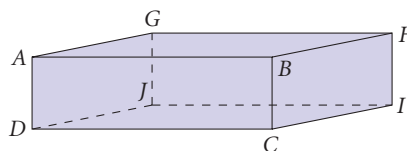
3 EXAMPLE Identifying Parallel Planes

Use the diagram at the right to name the figures.

- a. two pairs of parallel planes
 plane $ABHG \parallel$ plane $DCIJ$
 plane $ADJ \parallel$ plane BCI

- b. a line that is parallel to plane $GHIJ$

\overleftrightarrow{AB} is parallel to $GHIJ$.

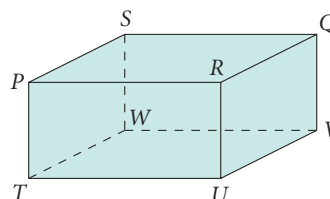


- 3a. $PSWT \parallel RQVU$,
 $PRUT \parallel SQVW$,
 $PSQR \parallel TWVU$

- 3 Name the figures.

- a. three pairs of parallel planes
 b. a line that is parallel to plane $QRUV$

Answers may vary. Sample: \overleftrightarrow{PS}



- 2 Use the figure from Example 3. Name all segments that are parallel to \overline{AD} . Name all that are skew to \overline{AD} . parallel: \overline{GJ} , \overline{HI} , \overline{BC} ; skew: \overline{GH} , \overline{JI} , \overline{BH} , \overline{CI}

- 3 Identify a pair of parallel planes in your classroom.

Sample: floor and ceiling

Resources

- Daily Notetaking Guide 1-4 **L3**
- Daily Notetaking Guide 1-4—Adapted Instruction **L1**

EXERCISES

For more exercises, see *Extra Skill*, *Word Problem*, and *Proof Practice*.

Practice and Problem Solving

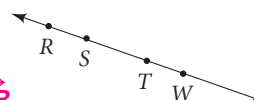
A Practice by Example

Example 1
 (page 23)



Use the figure at the right for Exercises 1–3.

1. Name all the labeled segments. \overline{RS} , \overline{RT} , \overline{RW} , \overline{ST} , \overline{SW} , \overline{TW}
 2. Name all the labeled rays. \overrightarrow{RS} , \overrightarrow{ST} , \overrightarrow{TW} , \overrightarrow{WT} , \overrightarrow{TS} , \overrightarrow{SR}
 3. a. Name a pair of opposite rays with T as an endpoint. \overrightarrow{TS} or \overrightarrow{TR} , \overrightarrow{TW}
 b. Name another pair of opposite rays. \overrightarrow{SR} , \overrightarrow{ST}



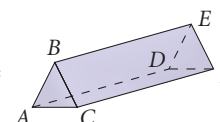
Example 2
 (page 24)

Name all segments shown in the diagram that are parallel to the given segment.

4. \overline{AC} \overline{DF} 5. \overline{EF} \overline{BC} 6. \overline{AD} \overline{BE} , \overline{CF}

Name all segments shown in the diagram that are skew to the given segment.

7. \overline{AC} \overline{DE} , \overline{EF} , \overline{BE} 8. \overline{EF} \overline{AD} , \overline{AB} , \overline{AC} 9. \overline{AD} \overline{BC} , \overline{EF}



Exercises 4–11

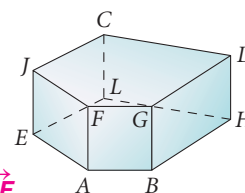
Example 3
 (page 25)

Use the diagram above and name a pair of figures to match each description.

10. parallel planes $ABC \parallel DEF$ 11. a line and a plane that are parallel \overleftrightarrow{BC} , DEF

Use the figure at the right to name the following.

12. all lines that are parallel to \overleftrightarrow{AB} \overleftrightarrow{FG}
 13. two lines that are skew to \overleftrightarrow{EJ}
 14. all lines that are parallel to plane $JFAE$
 15. the intersection of plane FAB and plane FAE \overleftrightarrow{AF}



13. Answers may vary.
 Sample: \overleftrightarrow{CD} , \overleftrightarrow{AB}
 14. \overleftrightarrow{BG} , \overleftrightarrow{DH} , \overleftrightarrow{CL}

3. Practice

Assignment Guide

1 A B	1-3, 34-36
2 A B	4-33, 37-39
C Challenge	40-45
Test Prep	46-50
Mixed Review	51-66

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 2, 6, 22, 35, 38.

Error Prevention!

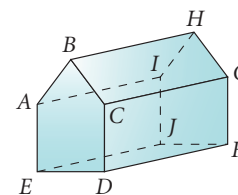
Exercise 2 Students may think that \overrightarrow{TR} and \overrightarrow{SW} are opposite rays. Ask: *How many points do opposite rays have in common?* **exactly 1** *What is it?* **the endpoint of both rays**

B Apply Your Skills

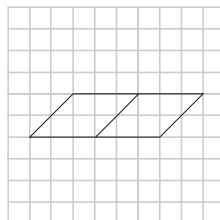
21. **False; they intersect above pt. A.**

In Exercises 16–23, describe the statement as true or false. If false, explain.

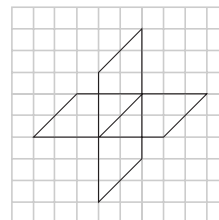
16. $\overleftrightarrow{CB} \parallel \overleftrightarrow{HG}$ **true**
17. $\overleftrightarrow{ED} \parallel \overleftrightarrow{HG}$ **False; they are skew.**
18. plane $AED \parallel$ plane FGH **true**
19. plane $ABH \parallel$ plane CDF **False; they intersect above \overleftrightarrow{CG} .**
20. \overleftrightarrow{AB} and \overleftrightarrow{HG} are skew lines. **true**
21. \overleftrightarrow{AE} and \overleftrightarrow{BC} are skew lines. **See left.**
22. \overleftrightarrow{CG} and \overleftrightarrow{AI} are skew lines. **False; they are \parallel .**
23. \overleftrightarrow{CF} and \overleftrightarrow{AJ} are skew lines. **False; they are \parallel .**
24. The following steps show how to draw planes A and B intersecting in \overleftrightarrow{FG} .



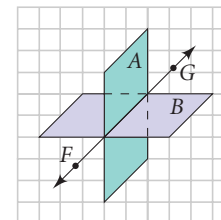
Step 1



Step 2



Step 3



Use similar steps to draw plane DFE and plane DFJ intersecting in \overleftrightarrow{DF} . **See margin.**

Vocabulary Tip

Always, sometimes, and never refer to all possible cases, not to intervals of time.

Complete Exercises 25–33 with *always*, *sometimes*, or *never* to make a true statement.

25. Two parallel lines are always coplanar.
26. Two skew lines are never coplanar.
27. Two opposite rays always form a line.
28. \overleftrightarrow{TQ} and \overleftrightarrow{QT} are always the same line.
29. \overleftrightarrow{GH} and \overleftrightarrow{HG} are never the same ray.
30. \overleftrightarrow{JK} and \overleftrightarrow{LJ} are sometimes the same ray.
31. Two planes that do not intersect are always parallel.
32. Two lines that lie in parallel planes are sometimes parallel.
33. Two lines in intersecting planes are sometimes skew.
34. **Multiple Choice** \overleftrightarrow{FG} has endpoints $F(-3, 3)$ and $G(3, 1)$. Which point is also on \overleftrightarrow{FG} ? **C**
 (A) $(-6, 4)$ (B) $(-1, 2)$ (C) $(0, 2)$ (D) $(6, 0)$

35. **Coordinate Geometry** \overleftrightarrow{AB} has endpoint $A(2, 3)$ and contains $B(4, 6)$. Give possible coordinates for point C so that \overleftrightarrow{AB} and \overleftrightarrow{AC} are opposite rays. Graph your answer. **Answers may vary. Sample: $(0, 0)$; check students' graphs.**
36. **Directional Compass** On a directional compass, the directions north and south can be represented by opposite rays.
 - a. Name two other compass directions that can be represented by opposite rays.
 - b. What other pairs of opposite directions, if any, can you find? **a-b. See margin.**
37. **Open-Ended** Summarize the three ways in which two lines may be related. Give examples from the real world that illustrate the relationships. **See margin.**



Differentiated Instruction Resources

GPS Guided Problem Solving L3

Enrichment L4

Reteaching L2

Adapted Practice L1

Practice L3

Practice 1-4 Measuring Segments and Angles

1. x

2. GH

3. HJ

4. Find FD if the coordinate of F is -7 and the coordinate of D is -1 .

5. Find SE if the coordinate of S is 17 and the coordinate of E is -5 .

6. Find the coordinate of B if $AB = 9$ and the coordinate of A is -2 .

7. Find the coordinate of X if $XY = 1$ and the coordinate of Y is 0 .

8. Name the angle at the right in three different ways.

9. y

10. AQ

11. QX

Find the measure of each angle.

12. $\angle EBF$ 13. $\angle EBA$

14. $\angle DBE$ 15. $\angle DBC$

16. $\angle ABF$ 17. $\angle DBF$

18. Name all acute angles in the figure.

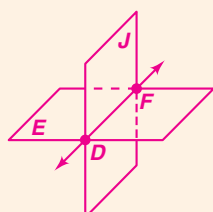
19. Name all obtuse angles in the figure.

20. Name all right angles in the figure.

21. If $\angle AC = 62$, find the value of x . Then find AB and BC .

22. If $\angle AC = 206$, find the value of x . Then find AB and BC .

24.



26 Chapter 1 Tools of Geometry

36. a. Answers may vary. Sample: northeast and southwest
- b. Answers may vary. Sample: northwest and southeast, east and west

37. Two lines can be parallel, skew, or intersecting in one point. Samples: Train tracks—parallel; vapor trail of a northbound jet and an eastbound jet at different altitudes—

skew; streets that cross—intersecting

39. b. Examples may vary: Sample: The floor and ceiling are parallel. A wall intersects both. The lines of intersection are parallel.

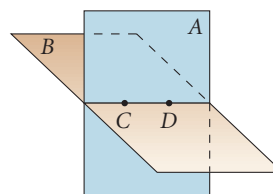
38. Answers may vary.
Sample: Skew lines cannot be contained in one plane. Therefore, they have "escaped" a plane.

Challenge

38. **Writing** The term *skew* is a Middle English word meaning "to escape." Explain how this meaning might be appropriate for skew lines. **See left.**
39. **Critical Thinking** Suppose two parallel planes A and B are each intersected by a third plane C .
- Make a conjecture about the intersection of planes A and C and the intersection of planes B and C . **The lines of intersection are parallel.**
 - Find examples in your classroom. **See margin.**
40. a. Draw a line. Draw points E and F on the line. How many different segments do points E and F determine? Name the segments. **See margin.**
- b. Draw another line. Draw points E, F , and G on the line. How many segments do points E, F , and G determine? Name them. **See margin.**
- c. Continue to draw lines, labeling one more point each time. Make a table showing the number of points and the number of segments determined. Look for and describe a pattern in the data. **See margin.**
- d. Use your pattern to find how many segments are determined if you label 10 points on a line. **45 segments**
- e. If you label n points on a line, how many segments can you name? $\frac{n(n-1)}{2}$

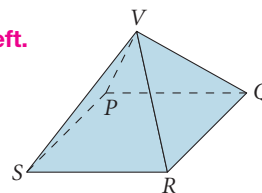
Use the figure at the right for Exercises 41 and 42.

41. Do planes A and B have other lines in common that are parallel to \overleftrightarrow{CD} ? Explain. **See margin.**
42. **Visualization** Are there planes that intersect planes A and B in lines parallel to \overleftrightarrow{CD} ? Draw a sketch to support your answer. **See margin.**



The figure at the right is a pyramid.

43. Name three lines that intersect at one point. **See left.**
44. What line could be parallel to \overleftrightarrow{PS} ? **\overleftrightarrow{QR}**
45. **Visualization** Consider a plane through V that is parallel to plane $PQRS$. Can a line in that plane be parallel to \overleftrightarrow{SR} ? Can it intersect \overleftrightarrow{SR} ? Can it be skew to \overleftrightarrow{SR} ? Explain each answer. **Yes; no; yes; explanations may vary.**



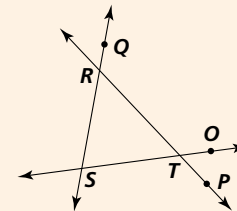
43. Answers may vary.
Sample: \overleftrightarrow{VR} , \overleftrightarrow{QR} , \overleftrightarrow{SR}

4. Assess & Reteach



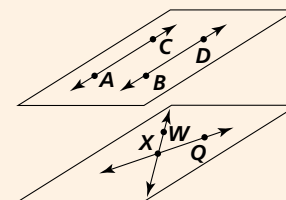
Lesson Quiz

Use the figure below for Exercises 1–3.



- Name the segments that form the triangle.
 \overline{RS} , \overline{TR} , \overline{ST}
- Name the rays that have point T as their endpoint.
 \overrightarrow{TO} , \overrightarrow{TP} , \overrightarrow{TR} , \overrightarrow{TS}
- Explain how you can tell that no lines in the figure are parallel or skew.
The three pairs of lines intersect, so they cannot be parallel or skew.

Use the figure below for Exercises 4 and 5.



- Name a pair of parallel planes.
plane $ABCD \parallel$ plane XWQ
- Name a line that is skew to \overleftrightarrow{XW} . **\overleftrightarrow{AC} or \overleftrightarrow{BD}**

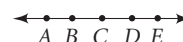
Alternative Assessment

Provide each student with a model of a rectangular solid, such as an empty cereal box. Have students describe how to find each of the following on the model: intersecting lines, parallel lines, skew lines, parallel planes, and intersecting planes.

Test Prep

Multiple Choice

- Use the figure at the right for Exercises 46–49.
46. How many labeled segments are in the figure? **D**
- A. 1 B. 4 C. 6 D. 10
47. Which ray is opposite \overrightarrow{BC} ? **H**
- F. \overrightarrow{BE} G. \overrightarrow{BD} H. \overrightarrow{BA} J. \overrightarrow{AB}
48. What is another name for \overleftrightarrow{CA} ? **B**
- A. \overleftrightarrow{AC} B. \overleftrightarrow{CB} C. \overleftrightarrow{CE} D. \overleftrightarrow{DC}
49. Which figure could be the intersection of two planes? **F**
- F. line G. ray H. point J. segment



40. a. one segment, \overline{EF}
- b. 3 segments, \overline{EF} , \overline{EG} , \overline{FG}

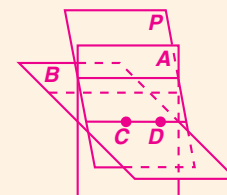
Number of points	Number of segments
2	1
3	3
4	6
5	10
6	15

Answers may vary.

Sample: For each "new" point, the number of new segments equals the number of "old" points.

41. No; two different planes cannot intersect in more than one line.

42. yes; plane P , for example



Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 75
- Test-Taking Strategies, p. 70
- Test-Taking Strategies with Transparencies

Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 1-1 through 1-4.

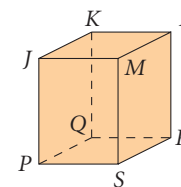
Resources

Grab & Go

- Checkpoint Quiz 1

Short Response

50. a. Use the diagram to explain how parallel lines and skew lines are alike and how parallel lines and skew lines are different. **a-b. See margin.**
b. Does the diagram suggest other lines that are parallel to \overleftrightarrow{JM} , besides \overleftrightarrow{KL} , \overleftrightarrow{QR} , and \overleftrightarrow{PS} ? Explain.



Mixed Review

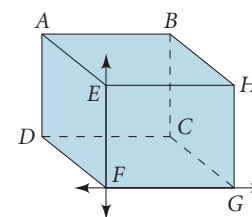


Lesson 1-3

51–58. Answers may vary. Samples are given.

Use the diagram for Exercises 51–58 and name each geometric figure.

51. a line \overleftrightarrow{EF} 52. a point **A**
53. the intersection of \overleftrightarrow{DC} and \overleftrightarrow{CG} **C**
54. two planes that intersect in \overleftrightarrow{EF} **AEF and HEF**
55. the plane represented by the top of the box **ABH**
56. the plane represented by the front of the box **EHG**
57. the intersection of planes EFG and DFG **\overleftrightarrow{FG}**
58. another point in plane CGH **B**



Draw the following. 59–61. See margin.

59. \overleftrightarrow{TR} 60. \overline{PQ} 61. \overrightarrow{NV}

Lesson 1-1

Find the next two terms in each sequence.

62. 1, 1.08, 1.16, 1.24, 1.32, ... **1.4, 1.48** 63. -1, -2, -4, -7, -11, -16, ... **-22, -29**
64. AB, BC, CD, DE, EF, ... **FG, GH** 65. A, D, G, J, M, ... **P, S**
66. **Reasoning** Raven conjectured: "If you subtract a number from a given number, the result is always less than the given number." Is her conjecture true? Explain. **No; whenever you subtract a negative number, the answer is greater than the given number. Also, if you subtract 0, the answer stays the same.**



Checkpoint Quiz 1

Lessons 1-1 through 1-4

Find the next two terms in each sequence.

1. 19, 21.5, 24, 26.5, ... **29, 31.5** 2. 3.4, 3.45, 3.456, 3.4567, ... **3.45678, 3.456789**

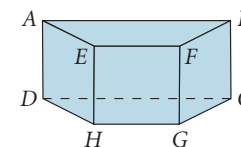
3. For 1: Add 2.5. For 2: Extend the decimal to one more place with a digit that is 1 more than the one to its left.



3. **Writing** Describe the pattern of each sequence in Exercises 1 and 2. **See left.**

Use the diagram for Exercises 4–10. In Exercises 4–7, do the points appear to be coplanar? If yes, name the plane. If no, explain. 4–7. See margin.

4. Points A, E, F, and B 5. Points D, C, E, and F
6. Points H, G, F, and B 7. Points A, E, B, and C
8. Name all the segments parallel to \overline{HG} . **\overline{CD} , \overline{AB} , \overline{EF}**
9. Name a pair of skew lines. **See left.**
10. Draw a net for the figure. **See margin.**



50. [2] a. Alike: They do not intersect. Different: Parallel lines are coplanar and skew lines lie in different planes.

- b. No; of the 8 other lines shown, 4 intersect \overleftrightarrow{JM} and 4 are skew to \overleftrightarrow{JM} .

- [1] one likeness, one difference

9. Answers may vary. Sample: \overleftrightarrow{AE} and \overleftrightarrow{BC}

28 Chapter 1 Tools of Geometry



Checkpoint Quiz 1

4. yes, plane **AEF**
5. yes, plane **DCEF**
6. No; H, G, and F are in the front plane, B is not.

7. No; A, E, and B are in the top plane, C is not.

10.

