## Key Solution HW 12

**8–3.** The thin-walled cylinder can be supported in one of two ways as shown. Determine the state of stress in the wall of the cylinder for both cases if the piston P causes the internal pressure to be 0.5 MPa. The wall has a thickness of 6 mm. and the inner diameter of the cylinder is 200 mm.



Case a;

$$\sigma_1 = \frac{pr}{t}; \quad \sigma_1 = \frac{0.5 * 100}{6} = 8.33 MPa.$$
 Ans.

$$\sigma_2 = 0$$
 Ans.

Case b

$$\sigma_1 = \frac{pr}{t}; \quad \sigma_1 = \frac{0.5 * 100}{6} = 8.33 MPa.$$
 Ans

$$\sigma_2 = \frac{pr}{2t} = \frac{0.5 * 100}{2 * 6} = 4.167 MPa.$$
 Ans.

### Key Solution HW 12

# **8–4.** The tank of the air compressor is subjected to an internal pressure of 0.63 MPa. If the internal diameter of the tank is 550 mm, and the wall thickness is 6 mm, determine the stress components acting at point *A*. Draw a volume element of the material at this point, and show the results on the

Hoop Stress for Cylindrical Vessels:  $r = \frac{550}{2} = 275;$ 

Since  $\frac{r}{t} = \frac{275}{6} = 45.83 > 10$ , then *thin wall* 

analysis can be used. Applying Eq. 8-1

$$\sigma_1 = \frac{pr}{t} = 0.63 * \frac{275}{6} = 28.875 MPa.$$
 Ans

*Longitudinal Stress for Cylindrical Vessels:* Applying Eq. 8–2

$$\sigma_2 = \frac{pr}{2t} = 0.63 * \frac{275}{2*6}$$

= 14.4375 MPa. Ans









#### Key Solution HW 12

CE 203

**8–6.** The spherical gas tank is fabricated by bolting together two hemispherical thin shells. If the 8-m inner diameter tank is to be designed to withstand a gauge pressure of 2 MPa, determine the minimum wall thickness of the tank and the minimum number of 25-mm diameter bolts that must be used to seal it. The tank and the bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively.

Normal Stress: For the spherical tank's wall,

$$\sigma_{\text{allow}} = \frac{pr}{2t}$$

$$150(10^6) = \frac{2(10^6)(4)}{2t}$$

$$t = 0.02667 \text{ m} = 26.7 \text{ mm}$$

Since  $\frac{r}{t} = \frac{4}{0.02667} = 150 > 10$ , thin-wall analysis is valid.

Referring to the free-body diagram shown in Fig. *a*,  $P = pA = 2(10^6) \left[ \frac{\pi}{4} (8^2) \right] = 32\pi (10^6)$  N. Thus,

+ 
$$\uparrow \Sigma F_y = 0;$$
  $32\pi (10^6) - \frac{n}{2} (P_b)_{allow} - \frac{n}{2} (P_b)_{allow} = 0$   
 $n = \frac{32\pi (10^6)}{(P_b)_{allow}}$  (1)

The allowable tensile force for each bolt is

$$(P_b)_{\text{allow}} = \sigma_{\text{allow}} A_b = 250(10^6) \left[ \frac{\pi}{4} (0.025^2) \right] = 39.0625(10^3) \pi \text{N}$$

Substituting this result into Eq. (1),

$$n = \frac{32\pi (10^6)}{39.0625\pi (10^3)} = 819.2 = 820$$
 Ans.





Ans.

### Key Solution HW 12

**8–35.** The wide-flange beam is subjected to the loading shown. Determine the stress components at points A and B and show the results on a volume element at each of these points. Use the shear formula to compute the shear stress.





$$Q_B = \sum \overline{y'}A' = 81 * 100 * 12 + 50 * 50 * 12 = 127200 \ mm^3$$

 $Q_A = 0;$ 

$$\sigma_A = -\frac{MC}{I} = -14.375 * 10^6 * \frac{87}{19150200} = -65.31 \, MPa \quad Ans.$$

$$\tau_A = 0$$
 Ans



$$\sigma_B = \frac{MY}{I} = 14.375 * 10^6 * \frac{25}{19150200} = \mathbf{18.76} \, \mathbf{MPa} \quad \mathbf{Ans}.$$
  
$$\tau_B = \frac{VQ_B}{It} = 13.125 * 10^3 * \frac{127200}{19150200 * 12} = \mathbf{7.265} \, \mathbf{MPa} \quad \mathbf{Ans}.$$

CE 203

#### Key Solution HW 12

**8-63** The uniform sign has a weight of 7.5 KN and is supported by the pipe *AB*, which has an inner radius of 68 mm. and an outer radius of 75 mm. If the face of the sign is subjected to a uniform wind pressure of  $p=8 \text{ KN/m}^2$  determine the state of stress at points *C* and *D*. Show the results on a differential volume element located at each of these points. Neglect the thickness of the sign, and assume that it is supported along the outside edge of the pipe.



Internal forces can be found from the FBD above as follows:

$$\sum F_x = 0; 7500 - N_x = 0 \rightarrow N_x = 7500 N. ; \sum F_y = 0; -51840 + V_y = 0 \rightarrow V_y = 51840 N. ;$$

$$\sum F_z = 0; V_z = 0; \sum M_x = 0; -51840 * 1.8 + T_x = 0 \rightarrow T_x = 93312 N. m. ;$$

$$\sum M_y = 0; -518407500 * 1.8 + M_y = 0 \rightarrow M_y = 13500 N. m. ;$$

$$\sum M_z = 0; 51840 * 1.8 + M_z = 0 \rightarrow M_z = -93312 N. m.$$

$$A = \pi (75^2 - 68^2) = 1001 \,\pi \,\mathrm{mm^2}; \ I_y = I_z = \frac{\pi}{4} (75^4 - 68^4) = 8057595.32 \,\mathrm{mm^4};$$
  
$$(Q_c)_z = (Q_D)_y = 0; \ (Q_c)_y = (Q_D)_z = 4 * \frac{75}{3 * \pi} (\frac{\pi}{2} * 75^2) - 4 * \frac{68}{3 * \pi} (\frac{\pi}{2} * 68^2) = 71628.67 \,\mathrm{mm^3};$$
  
$$J = \frac{\pi}{2} (75^4 - 68^4) = 16115190.64 \,\mathrm{mm^4};$$

Normal stresses as follows:

 $\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$ 

CE 203

# Key Solution HW 12

$$\sigma_{c} = \frac{-8 * 10^{-3}}{1001\pi} - \frac{-9331 * 1000 * 0}{8057595.32} + \frac{13500 * 1000 * 68}{8057595.32} = 109.4 MPa(T) \quad Ans.$$
  
$$\sigma_{D} = \frac{-8 * 10^{-3}}{1001\pi} - \frac{-9331 * 1000 * 75}{8057595.32} + \frac{13500 * 1000 * 0}{8057595.32} = 885.7MPa(T) \quad Ans.$$

**Shear stresses** 

$$(\tau_{xz})_D = \tau_{twist} = \frac{93312 * 1000 * 75}{16115190.64} = 434.27$$
 MPa. Ans

$$(\mathbf{\tau}_{xy})_{\mathbf{D}} = \mathbf{\tau}_{Vy} = \mathbf{0}$$
 Ans.

$$(\tau_{xy})_c = \tau_{Vy} - \tau_{twist} = \frac{51840 * 71628.67}{8057595.32 * 2 * 7} - \frac{93312 * 1000 * 68}{16115190.64} = -375 \text{ MPa.} \text{ Ans}$$

$$(\mathbf{\tau}_{\mathbf{x}\mathbf{z}})_{\mathbf{c}} = \mathbf{\tau}_{\mathbf{V}\mathbf{z}} = \mathbf{0}$$
 Ans.

#### Key Solution HW 12

**8–79.** If the cross section of the femur at section a-a can be approximated as a circular tube as shown, determine the maximum normal stress developed on the cross section at section a-a due to the load of 375 N.



**Internal Loadings:** Considering the equilibrium for the free-body diagram of the femur's upper segment, Fig. *a*,

+  $\sum F_y = 0;$  N-375=0; N = 375  $\zeta + \sum M_O = 0;$  M-375 \*50 = 0; M= 18750 N.mm

Section Properties: The cross-sectional area, the moment of inertia about the centroidal axis of the femur's cross section are

$$A = \pi (24^2 - 12^2) = 432 \,\pi \,\mathrm{mm}^2; \quad \mathrm{I} = \frac{\pi}{4} (24^4 - 12^4) = 77760 \,\pi \,\mathrm{mm}^4;$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress is in compression.

$$\sigma_{max} = \frac{-375}{432 \pi} - \frac{18750 * 24}{77760 \pi} = -2.12 MPa = 2.12 MPa (c) \qquad Ans.$$