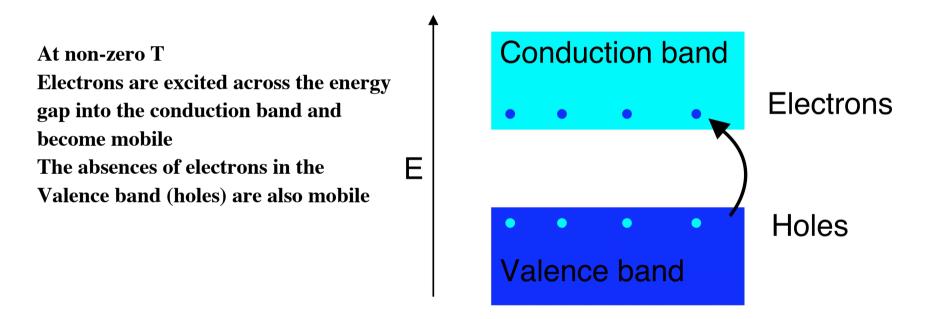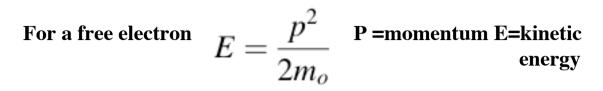# Semiconductor Physics

Lecture 2

Introduction

Recall from last lecture that semiconductors are band insulators at 0K
>   The electron wavefunctions are delocalised but there is a completely full valence
>   band (VB) separated by an energy gap from an empty conduction band (CB)

No mobile carriers at 0K

At non-zero T

Electrons are excited across the energy
gap into the conduction band and
become mobile

The absences of electrons in the
Valence band (holes) are also mobile

Today we will look in detail at how temperature and chemical changes
>   produce mobile carriers
>   by injecting carriers into the CB (electrons)
>   removing electrons from the VB to create holes

Revision —effective mass

For a free electron
$$E = \frac{p^2}{2m_o}$$
P =momentum E=kinetic energy

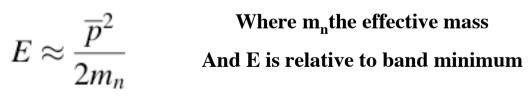The periodic potential means we have to replace momentum with crystal momentum

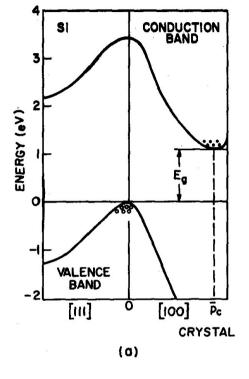$$\overline{p} = \hbar\overline{k} \ , \ \overline{k} = \frac{2\pi}{\lambda}$$

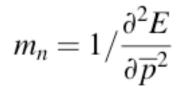Clearly bands in semiconductors are not in general parabolic

But because the gap is large wrt to $k_b T$
   we only need to consider the top of the VB and the bottom of the CB

Hence we can make an approximation that the bands are locally parabolic

$$E \approx \frac{\overline{p}^2}{2m_n}$$

Where m_n the effective mass

And E is relative to band minimum

$$m_n = 1 / \frac{\partial^2 E}{\partial \overline{p}^2}$$

For the purposes of modelling semi conductors

can parameterise bandstructure in the effective mass (curvature of band)

Revision —effective mass II

For silicon we have an indirect band gap
Minimum of CB is not at zone centre
For electrons in the CB the KE is given by

$$E_n = \frac{(\overline{p} - \overline{p}_c)^2}{2m_n}$$

For holes in the VB band curvature is -ve
And

So

$$m_p = -1 \Big/ \frac{\partial^2 E}{\partial \overline{p}^2}$$

$$E_p \approx \frac{\overline{p}^2}{2m_p}$$

E relative to top of VB

Electron energy

Hole energy

Revision —Density of States

For a crystal which has length L
Where n is an integer

$$\lambda = \frac{L}{n}$$
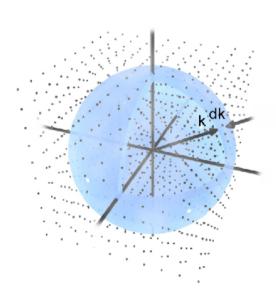
Given

$$\bar{p} = \hbar\bar{k} = \frac{h}{\lambda}$$

Then

$$L\bar{p} = hn$$

And the smallest increment in momentum is

$$\partial p = \frac{h}{L}$$

Allowed values of P form a lattice in momentum space

Spaced at intervals of

$$\partial p = \frac{h}{L}$$
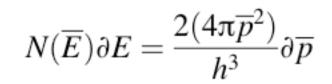
With a volume/point of

$$v = \left(\frac{h}{L}\right)^3$$
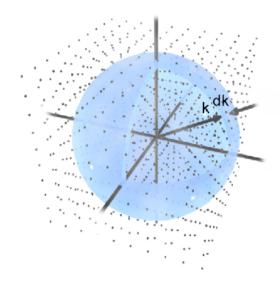
Revision —Density of States

If we make the crystal of unit length L=1

In 3D the number of states between p and p+dp is

$$N(\overline{E})\partial E = \frac{2(4\pi\overline{p}^2)}{h^3}\partial\overline{p}$$

Recalling

$$\overline{p}^2 = 2m_n E$$

$$\overline{p} = \sqrt{2m_n E}$$

$$\partial\overline{p} = (2m_n E)^{\frac{-1}{2}}\partial E$$

Then

$$N(E)\partial E = 4\pi(\frac{2m_n}{h^2})^{\frac{3}{2}}E^{\frac{1}{2}}\partial E$$
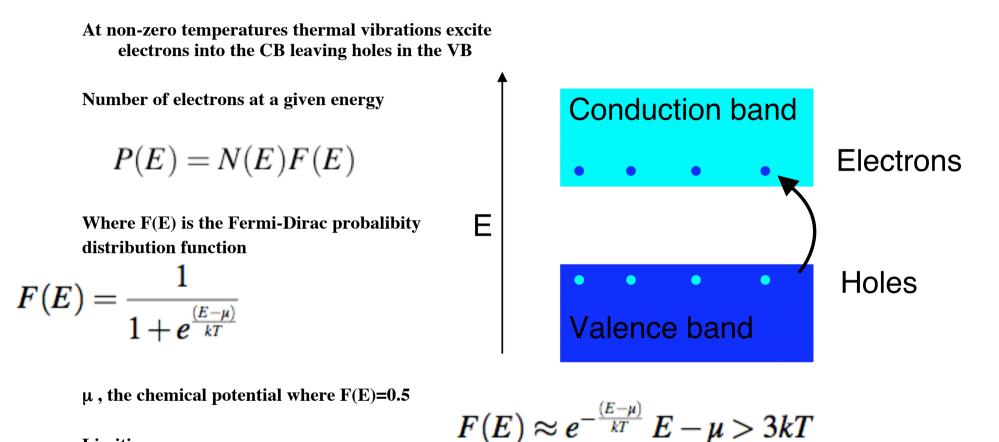

Same as for free electrons except we are using the effective mass

As before the assumption of locally parabolic bands near the bottom of the CB allows us to simply incorporate the band structure as an effective mass

Thermal excitation of carriers

At non-zero temperatures thermal vibrations excite electrons into the CB leaving holes in the VB

Number of electrons at a given energy

$$P(E) = N(E)F(E)$$

Where F(E) is the Fermi-Dirac probalibity distribution function

$$F(E) = \frac{1}{1 + e^{\frac{(E-\mu)}{kT}}}$$

μ , the chemical potential where F(E)=0.5

Limiting cases

Recall band gap is ~1 eV
kT=0.025 eV at RT

Conduction band

Electrons

E

Holes

Valence band

$$F(E) \approx e^{-\frac{(E-\mu)}{kT}} \quad E - \mu > 3kT$$

$$F(E) \approx 1 - e^{-\frac{(E-\mu)}{kT}} \quad E - \mu < 3kT$$

Carrier density

n the number of electrons/unit volume

$$n = \int_0^{E_{top}} N(E)F(E)\partial E$$

Where we integrate across the whole conduction band

Because the gap energy is larger than kT we can set $E_{top} = \infty$
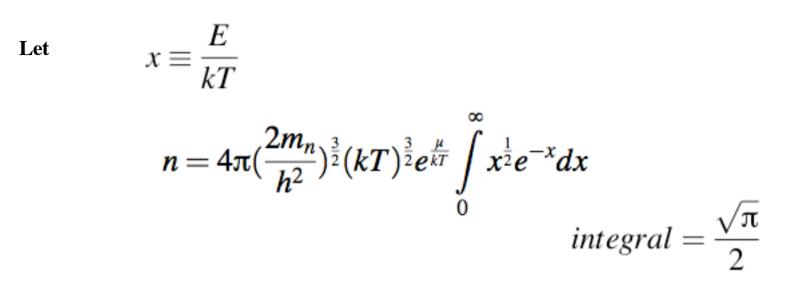
E

E_c • • • •

E_v

Conduction band

Electrons

Holes

Valence band

$$N(E)\partial E = 4\pi \left(\frac{2m_n}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} \partial E \qquad F(E) \approx e^{-\frac{(E-\mu)}{kT}} \quad E - \mu > 3kT$$

$$n = 4\pi \left(\frac{2m_n}{h^2}\right)^{\frac{3}{2}} \int_0^\infty E^{\frac{1}{2}} e^{-\frac{E-\mu}{kT}} dE$$

Carrier density II

Let
$$x \equiv \frac{E}{kT}$$

$$n = 4\pi \left(\frac{2m_n}{h^2}\right)^{\frac{3}{2}} (kT)^{\frac{3}{2}} e^{\frac{\mu}{kT}} \int_0^\infty x^{\frac{1}{2}} e^{-x} dx$$

$$integral = \frac{\sqrt{\pi}}{2}$$

Electron density
$$n = 2\left(\frac{2\pi m_n kT}{h^2}\right)^{\frac{3}{2}} e^{\frac{\mu}{kT}}$$

Refer to bottom of CB as E_c rather than E=0

$$n = 2\left(\frac{2\pi m_n kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_c - \mu}{kT}}$$

Carrier density III

Rewrite

$$n = 2\left(\frac{2\pi m_n kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_c - \mu}{kT}}$$

as

$$n = N_c e^{-\frac{E_c - \mu}{kT}}$$

$$N_c = 2\left(\frac{2\pi m_n kT}{h^2}\right)^{\frac{3}{2}}$$

N_c **is the effective density of states for the conduction band**

Similarly for holes in the valence band

$$p = 2\left(\frac{2\pi m_p kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{\mu - E_v}{kT}} \qquad\qquad p = N_v e^{-\frac{\mu - E_v}{kT}}$$

$$N_v = 2\left(\frac{2\pi m_p kT}{h^2}\right)^{\frac{3}{2}}$$

Fermi level

But we don't know what μ is

$$p = N_v e^{-\frac{\mu - E_v}{kT}} \qquad\qquad n = N_c e^{-\frac{E_c - \mu}{kT}}$$

For no doping n=p intrinsic semiconductor

$$\mu = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln \frac{N_v}{N_c}$$

$$N_v = 2\left(\frac{2\pi m_p kT}{h^2}\right)^{\frac{3}{2}} \qquad\qquad N_c = 2\left(\frac{2\pi m_n kT}{h^2}\right)^{\frac{3}{2}}$$

$$\mu = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln \frac{m_p}{m_n}$$

Chemical potential is T dependent if valence and conduction bands have different curvature

Intrinsic carrier density

Intrinsic carrier density

$$np = n_i^2 = N_c N_v e^{-(E_c - E_v)} = N_c N_v e^{\frac{-E_g}{kT}}$$

$$n_i = \sqrt{N_c N_v} \, exp^{\frac{-E_g}{2kT}}$$

Law of mass action

Valid also if we add an impurity which either donates extra electrons or holes the number of carriers remains constant for a given temperature
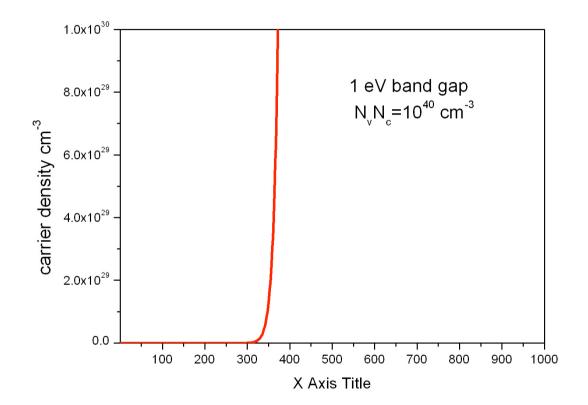
Note also that the number of carriers is very low

At RT n_i=1.45x10^{10} cm^{-3} in Si and 1.79x10^6 cm^{-3} in GaAs

Since the number of atoms is of the order of 10^{22} only a small fraction of atoms contribute free carriers

Also since the number of carriers is dependent on an exponential term in temperature both it and the electrical conductivity will be strongly temperature dependent.

Temperature dependence

Carrier concentration is strongly T-dependent

1 eV band gap

$N_v N_c = 10^{40} \ cm^{-3}$

carrier density cm^{-3}

X Axis Title

Not good for a device

Doping

The carrier concentration of an intrinsic semiconductor is strongly T dependent

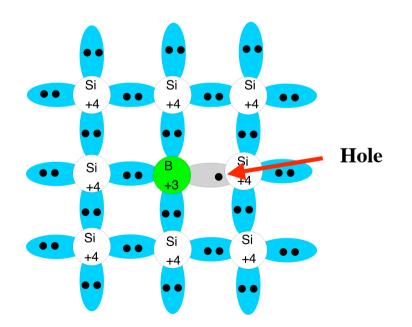In silicon it increases by 2 orders of magnitude from RT to 100 C

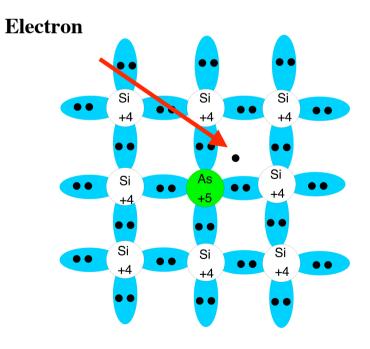Generally semiconductors are doped with a small amount of impurity

If we put a a boron atom with 3 valence electrons into silicon (4 valence electrons) then we have 1 electron too few and create a hole

boron is said to be an acceptor and the silicon is then p-type

Similarly arsenic (5 valent) produces an electron to many and is said to be a donor and
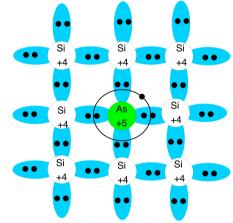
the silicon is n-type

Doping levels

The interaction of an electron (charge -e) with the doner atom (charge +e) is approximately similar to a hydrogen atom. Similarly we can model the positive hole around the negative acceptor atom

Ionisation energy of an hydrogen atom is

$$E_H = \frac{m_0 e^4}{8\varepsilon_0^2 h^2} = 13.6 \ eV$$

We can estimate the ionisation energy by replacing the electron mass by the effective mass and the permitivity of free space by that of the material

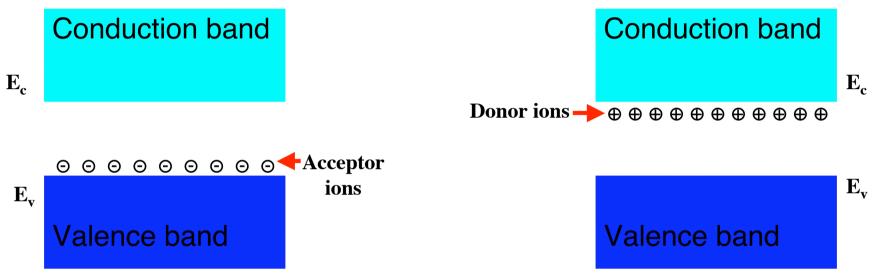$$E_D = (\frac{\varepsilon_0}{\varepsilon_s})^2 (\frac{m_0}{m_h}) E_H$$

Gives 0.025 eV donors in silicon and 0.007 eV in GaAs

0.05 eV for acceptors in both materials

Reasonable agreement with measurements in for many types of dopants

Doping levels 2

Donor levels lie just below conduction band and acceptor levels just above the valence band

Conduction band

E_c

$\ominus$ $\ominus$ $\ominus$ $\ominus$ $\ominus$ $\ominus$ $\ominus$ $\ominus$ $\ominus$ ← **Acceptor ions**

E_v

Valence band

Conduction band

E_c

Donor ions → $\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$

E_v

Valence band

Donor levels lie E_D below the bottom of the conduction band and acceptor levels E_A above the valence band

Since E_a and E_d are smaller than kT at room temperature the dopant sites are fully ionised so the number of majority carrier (those provided by doping, electrons in the case of n-type, holes in the case of p-type) is simply the number of dopant atoms

Chemical Potential
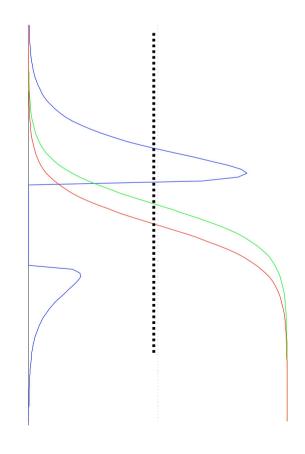
For an n-type semiconductor $n = N_D$

Recall carrier density is $n = N_c e^{-\frac{E_c - \mu}{kT}}$

But we have already specified n so we can combine these two and rearrange

$$E_c - \mu = kT \ln\left(\frac{N_c}{N_D}\right)$$

Might seem strange but what it is saying is that
the overall numbers of carriers remains constant
so the chemical potential (which is defined as
where the Fermi-Dirac function=0.5)
adjusts to keep this.

Fermi energy II

$$E_c - \mu = kT \ln\left(\frac{N_c}{N_D}\right)$$

If N_D is larger than N_c (or N_A is larger than $N_{V)}$ then Fermi level lies within the conduction (valence) band

We then have a degenerate semiconductor

Approximations we have made no longer apply

$$F(E) \approx e^{-\frac{(E-\mu)}{kT}} \quad E - \mu > 3kT$$

Carrier concentrations

Useful to express concentrations with respect to the intrinsic carrier concentration n_i and μ_I (chemical potential for intrinsic (undoped) semi-conductor)

$$n = N_c e^{-\frac{E_c - \mu}{kT}}$$

$$= N_c \exp\frac{-(E_c - \mu_i)}{kT} \exp\frac{\mu - \mu_i}{kT}$$

The first part is just n_i

$$= n_i \exp\frac{\mu - \mu_i}{kT}$$

Similarly the hole concentration is

$$p = N_v \exp\frac{-(\mu - E_v)}{kT} = n_i \exp\frac{\mu_i - \mu}{kT}$$

Mixed Semiconductors

For a system which has both donors and acceptors

$$n + N_A = p + N_D \text{ and } pn = n_i^2 \quad n_i = \sqrt{N_c N_v} \, exp^{\frac{-E_g}{2kT}}$$

Charge neutrality and the law of mass action

Solving gives

$$n_n = \frac{1}{2}(N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2} \text{ and } p_n = \frac{n_i}{n_n}$$

For a system where electrons dominate
And

$$p_p = \frac{1}{2}(N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2} \text{ and } n_p = \frac{n_i}{p_p}$$

Temperature dependence

Recalling the expression for the Fermi energy

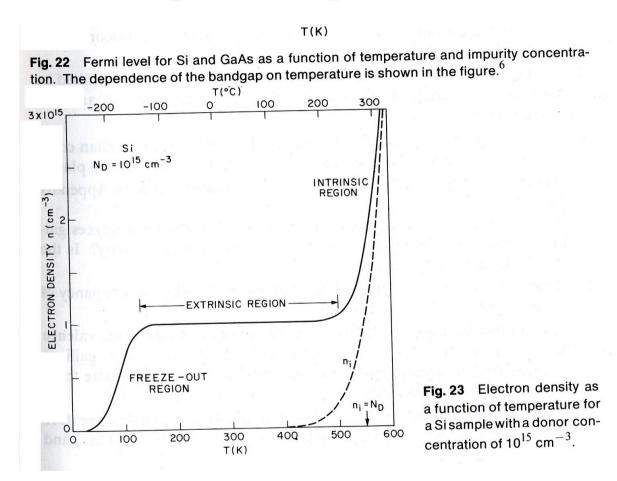$$n_n = \frac{1}{2}\left(N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2}\right) \text{ and } p_n = \frac{n_i}{n_n}$$

And the law of mass action

$$n_i = \sqrt{N_c N_v} \, exp^{\frac{-E_g}{2kT}}$$

Eventually as the temperature increase n_i becomes much larger than N_D and the semiconductor becomes intrinsic again

Temperature dependence II

T(K)

Fig. 22 Fermi level for Si and GaAs as a function of temperature and impurity concentration. The dependence of the bandgap on temperature is shown in the figure.[6]

Fig. 23 Electron density as a function of temperature for a Si sample with a donor concentration of 10^{15} cm^{-3}.

Initial rise as dopant sites become fully ionised

Flat region where majority of carriers are from the dopant (good for devices)

Final rise where intrinsic behaviour dominates

Round up

We can model the band behaviour with parabolic bands provided we use the appropriate effective mass

The Fermi energy is in general temperature dependent as a consequence of the band behaviour