ASCE NATIONAL CAPITAL SECTION
eNewsletter
National Capital Section
FOUNDED 1916

SEPTEMBER 2017

DR. Z's CORNER

Conquering the FE & PE exams
Problems, Solutions & Applications

This month's topics:

- *NCEES* Calculator Policy for Current FE & PE Exams
- FE CIVIL Exam Topics & Number of Questions
- PE CIVIL Exam Topics & Number of Questions
- Transportation Engineering, *Dr. Bryan Higgs*
- Geotechnical Engineering, *Dr. Lei Wang*
- Structures and Computers, *Dr. Vagelis Plevris*
- Hydraulics, Open Channel Flow (Rectangular & Trapezoidal)
- Hydraulics, Applications of Manning's Equation
- Engineering Economics, Future Worth, Present Worth
- Steel Design, Compression Members
- Mechanics of Materials, Axially Loaded Members
- Mechanics of Materials, Centroids & Moments of Inertia
- Analytical Geometry, Vectors & Matrices
- Structures, Analysis of Determinate Beams & Trusses
- Structures, Deflections of Beams & Trusses
- Structures, Indeterminate Beams
- Reinforced Concrete, Load Factors & Combinations

NCEES CALCULATOR POLICY

FOR CURRENT FE & PE EXAMS

To protect the integrity of its exams, NCEES limits the types of calculators you may bring to the exams. The only calculator models acceptable for use during the FE & PE exams are as follows.

Casio:

All fx-115 models. Any Casio calculator must contain fx-115 in its model name. Examples of acceptable Casio fx-115 models include:

- fx-115 MS
- fx-115 MS Plus
- fx-115 MS SR
- fx-115 ES
- fx-115 ES Plus

Hewlett Packard:

The HP 33s and HP 35s models, but no others.

Texas Instruments:

All TI-30X and TI-36X models. Any Texas Instruments calculator must contain either TI-30X or TI-36X in its model name. Examples of acceptable TI-30X and TI-36X models include (but are not limited to):

- TI-30Xa
- TI-30Xa SOLAR
- TI-30Xa SE
- TI-30XS Multiview
- TI-30X IIB
- TI-30X IIS
- TI-36X II
- TI-36X SOLAR
- TI-36X Pro

PRINCIPLES & PRACTICE OF ENGINEERING

(NCEES - Current Exam Topics)

P.E. EXAMINATION / Transportation & Geotechnical

According to NCEES the new civil P.E. exam is a *breadth* and *depth* examination. This means that examinees work the breadth (AM) exam and one of the five depth (PM) exams. The five areas covered in the civil examination are construction, geotechnical, structural, transportation, and water resources and environmental. The breadth exam contains questions from all five areas of civil engineering. The depth exams focus more closely on a single area of practice in civil engineering.

Examinees work all questions in the morning session and all questions in the afternoon module they have chosen. Depth results are combined with breadth results for final score. The exam is an 8-hour open-book exam. It contains 40 multiple-choice questions in the 4-hour AM session, and 40 multiple-choice questions in the 4-hour PM session. The exam uses both the *International System of Units* (SI) and the *US Customary System* (USCS).

TRANSPORTATION BREADTH EXAM

Exam Topics & Approximate Number of Questions:

I. *Project Planning (4 questions)*
II. *Means and Methods (3 questions)*
III. *Soil Mechanics (6)*
IV. *Structural Mechanics (6)*
V. *Hydraulics and Hydrology (7)*
VI. *Geometrics (3)*
VII. *Materials (6)*
VIII. *Site Development (5)*

TRANSPORTATION DEPTH EXAM

Exam Topics & Approximate Number of Questions:

I. *Traffic Engineering (Capacity Analysis and Transportation Planning) (11 questions)*
II. *Horizontal Design (4 questions)*
III. *Vertical Design 4*
IV. *Intersection Geometry 4*
V. *Roadside and Cross-Section Design 4*
VI. *Signal Design 3*
VII. *Traffic Control Design 3*
VIII. *Geotechnical and Pavement 4*
IX. *Drainage 2*
X. *Alternatives Analysis 1*

PRINCIPLES & PRACTICE OF ENGINEERING
(P.E) EXAM for CIVIL ENGINEERS
(Geotechnical Engineering)

According to NCEES the civil exam is a breadth and depth examination. This means that examinees work the breadth (AM) exam and one of the five depth (PM) exams. The five areas covered in the civil exam are construction, geotechnical, structural, transportation, and water resources and environmental. The breadth exam contains questions from all five areas of civil engineering. The depth exams focus more closely on a single area of practice in civil engineering.

Examinees work all questions in the morning session and all questions in the afternoon module they have chosen. Depth results are combined with breadth results for final score. The exam is an 8-hour open-book exam. It contains 40 multiple-choice questions in the 4-hour AM session, and 40 multiple-choice questions in the 4-hour PM session. The exam uses both the International System of Units (SI) and the US Customary System (USCS).

GEOTECHNICAL BREADTH EXAM

Exam Topics & Approximate Number of Questions:

I. *Project Planning (4 questions)*
II. *Means and Methods (3 questions)*
III. *Soil Mechanics (6)*
IV. *Structural Mechanics (6)*
V. *Hydraulics and Hydrology (7)*
VI. *Geometrics (3)*
VII. *Materials (6)*
VIII. *Site Development (5)*

GEOTECHNICAL DEPTH EXAM

Exam Topics & Approximate Number of Questions:

I. *Site Characterization (5 questions)*
II. *Soil Mechanics, Laboratory Testing, and Analysis (5 questions)*
III. *Field Materials Testing, Methods, and Safety (3)*
IV. *Earthquake Engineering and Dynamic Loads (2)*
V. *Earth Structures 4*
VI. *Groundwater and Seepage 3*
VII. *Problematic Soil and Rock Conditions 3*
VIII. *Earth Retaining Structures (ASD or LRFD) 5*
IX. *Shallow Foundations (ASD or LRFD) 5*
X. *Deep Foundations (ASD or LRFD) 5*

Bryan Higgs, Ph.D.
Assistant Professor
University of the District of Columbia
School of Engineering and Applied Sciences
Department of Civil Engineering
4200 Connecticut Avenue, NW
Washington, D.C. 20008
Tel: 202-274-6600
Email: bryan.higgs@udc.edu

Career Highlights:

1- Ph.D., M.S., B.S. all from Virginia Tech
2- Published 32 papers in peer-reviewed journals and conferences
3- ***Intelligence Community Postdoctoral Research Fellow,*** "Discovering the Vulnerable Physical Routes in a Network,"
4- ***Principal Investigator,*** "Assessment of Sustainability in Transportation Network Design using Game Theory," UDC Faculty Incentive Grant
5- ***Postdoctoral Researcher,*** "I-65 Corridor Feasibility Study," Tennessee Department of Transportation
6- ***Graduate Research Assistant,*** "Driver Behavior in Traffic," Federal Highway Administration – Exploratory Advanced Research
7- ***Member,*** American Society of Civil Engineers
8- ***Member,*** American Society for Engineering Education
9- ***Member,*** Institute of Transportation Engineers
10- ***Friend,*** several Transportation Research Board Committees
11- ***Scholarly Article Reviewer,*** Transportation Research Part C: Emerging Technologies
12- ***Scholarly Article Reviewer,*** Biological Psychology
13- ***Scholarly Article Reviewer,*** IEEE Transactions on Vehicular Technology
14- ***Scholarly Article Reviewer,*** Advances in Mechanical Engineering
15- ***Scholarly Article Reviewer,*** IEEE Intelligent Transportation Systems Conference
16- ***Scholarly Article Reviewer,*** Transportation Research Board Annual Meeting
17- ***Dissertation and Thesis Committee Member,*** multiple occasions
18- **Volunteer,** Choosing Transportation Summit, MATHCOUNTS

PROFESSIONAL ENGINEERING (P.E.)
TRANSPORTATION DISCIPLINE
HORIZONTAL CURVES

PREPARED BY DR. HIGGS

Problem 1
Compute the minimum radius (ft.) of a circular curve for a highway design for 45 mph. Assume that the maximum *superelevation* is 13.8% and the coefficient of friction is 13.9%.

(A) 463
(B) 487.4
(C) 511.7
(D) 536.1

Problem 2
Compute the minimum radius of a circular curve for a highway design for 45 mph. Assume that the maximum superelevation is 10.6% and the coefficient of friction is 10.7%.

(A) 570.4
(B) 602.1
(C) 633.8
(D) 665.5

Problem 3
Compute the superelevation rate for a curve with a design speed of 50 mph, a radius of 1100 ft, and a coefficient of friction of 11.4%.

(A) 0.034
(B) 0.038
(C) 0.041
(D) 0.045

Problem 4
Compute the length of a horizontal curve with a radius of 1250 ft and an intersection angle of 110 degrees.

(A) 2399.8
(B) 2519.8
(C) 2639.8
(D) 2759.8

FUNDAMENTALS OF ENGINEERING

TRANSPORTATION

HORIZONTAL CURVES

SOLUTIONS BY DR. HIGGS

Problem 1

Compute the minimum radius (ft) of a circular curve for a highway design for 45 mph. Assume that the maximum superelevation is 13.8% and the coefficient of friction is 13.9%.

Formulas

$$0.01e + f = \frac{V^2}{15R}$$

Where:

e - superelevation (%)

f - side friction factor

V - design speed (mph)

R - radius of curve (ft)

Solution

$$R = \frac{V^2}{15(0.01e + f)} = \frac{45^2}{15(0.01 * 13.8 + 0.139)} = 487.4$$

Problem 2

Compute the minimum radius (ft) of a circular curve for a highway design for 45 mph. Assume that the maximum superelevation is 10.6% and the coefficient of friction is 10.7%.

Formulas

$$\mathbf{0.01}\boldsymbol{e} + \boldsymbol{f} = \frac{\boldsymbol{V}^{\mathbf{2}}}{\mathbf{15}\boldsymbol{R}}$$

Where:
e - superelevation (%)
f - side friction factor
V - design speed (mph)
R - radius of curve (ft)

Solution

$$R = \frac{V^2}{15(0.01e + f)} = \frac{45^2}{15(0.01 * 10.6 + 0.107)} = 633.8$$

Problem 3

Compute the superelevation rate (%) for a curve with a design speed of 50 mph, a radius of 1100 ft, and a coefficient of friction of 11.4%.

Formulas

$$0.01e + f = \frac{V^2}{15R}$$

Where:
e - superelevation (%)
f - side friction factor
V - design speed (mph)
R - radius of curve (ft)

Solution

$$e = 100\left(\frac{V^2}{15R} - f\right) = 100\left(\frac{50^2}{15 * 1100} - 0.114\right) = 0.04$$

Problem 4

Compute the length of a horizontal curve with a radius of 1250 ft and an intersection angle of 110 degrees.

Formulas

$$L = \frac{2\pi RI}{360}$$

Where:
L - length of curve (ft)
R - radius of curve (ft)
I - intersection angle (degrees)

Solution

$$L = \frac{2\pi * 1250 * 110}{360} = 2399.8$$

Lei Wang, Ph.D., P.E.
Assistant Professor of Geotechnical Engineering
University of the District of Columbia
School of Engineering and Applied Sciences
Department of Civil Engineering
4200 Connecticut Avenue, NW
Washington, D.C. 20008
Email: lei.wang@udc.edu

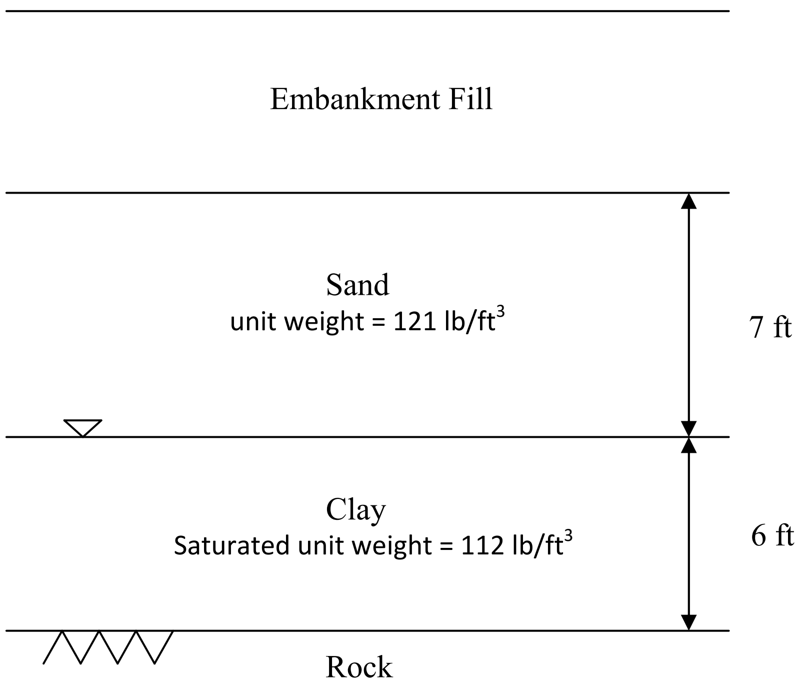
Career Highlights

1. Excellent Paper Award, Journal of GeoEngineering, Taiwan Geotechnical Society, 2015
2. Member of Chi Epsilon, National Civil Engineering Honor Society, 2014
3. Outstanding Graduate Researcher Award, College of Engineering and Science, Clemson University, 2013
4. Best Paper Award, Journal of GeoEngineering, Taiwan Geotechnical Society, 2013
5. Aniket Shrikhande Memorial Annual Graduate Fellowship, Clemson University, 2013
6. Editorial Board Member of four International Journals:
   - *International Journal of Geotechnical Engineering* (Taylor & Francis)
   - *Bulletin of Engineering Geology and the Environment* (Springer)
   - *Marine Georesources & Geotechnology* (Taylor & Francis)
   - *Geoenvironmental Disasters* (Springer)
7. Active in Research; Published 24 Papers in International Refereed Journals, and 16 Papers in Refereed ASCE Geotechnical Special Publications/International Conference Proceedings.
8. Reviewer of 19 International Journals.
9. 469 citations: "Google Scholar", with Author h-index=14.
10. Research Highlights Paper, ASCE Journal of Geotechnical and Geoenvironmental Engineering, January 2014
11. Multiple Top 25 hottest articles in Computers and Geotechnics (Elsevier)
12. Member of International Scientific Committee, Geo-Risk 2017 (6th International Symposium on Geotechnical Safety and Risk), American Society of Civil Engineers
13. Committee Member, Risk Assessment and Management (RAM) Committee, Geo-Institute, American Society of Civil Engineers
14. Served as a Geotechnical Engineer at WSP | Parsons Brinckerhoff, one of the world's leading civil engineering design firms
15. Served as Vice President, Western Branch, Montana Section, American Society of Civil Engineers

GEOTECHNICAL ENGINEERING
SOIL CONSOLIDATION PROBLEMS
PROFESSIONAL ENGINEERING (P.E.)

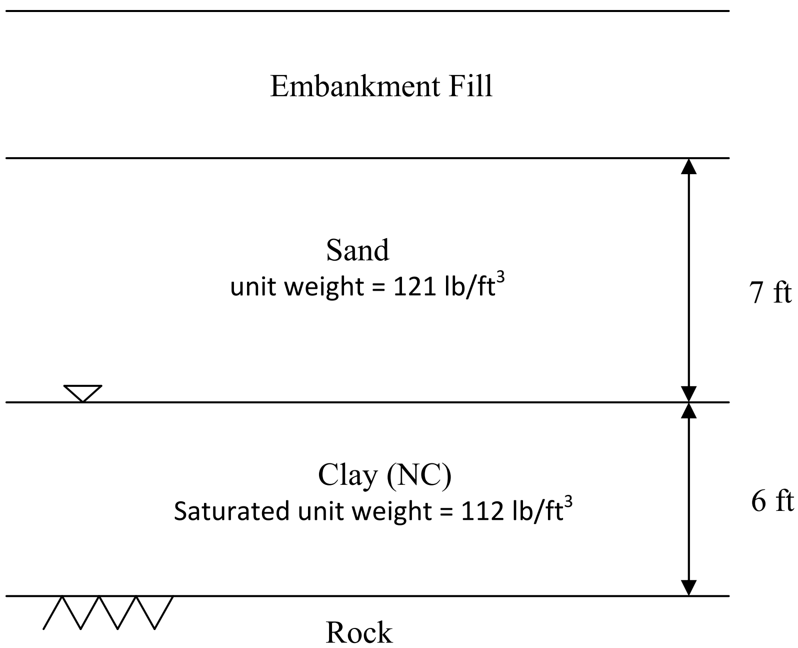
PREPARED BY DR. WANG

(1) A clay layer with thickness of 6 ft is subjected to an increase in vertical stress of 800 psf at the center of the layer caused by a very wide embankment fill. Laboratory tests for the clay layer have indicated the clay has an initial void ratio of 1.0, a preconsolidateion pressure of 1500 psf, a compression index C_c of 0.3 and a swell index C_s of 0.06. What is most nearly the ultimate consolidation settlement for this clay layer?



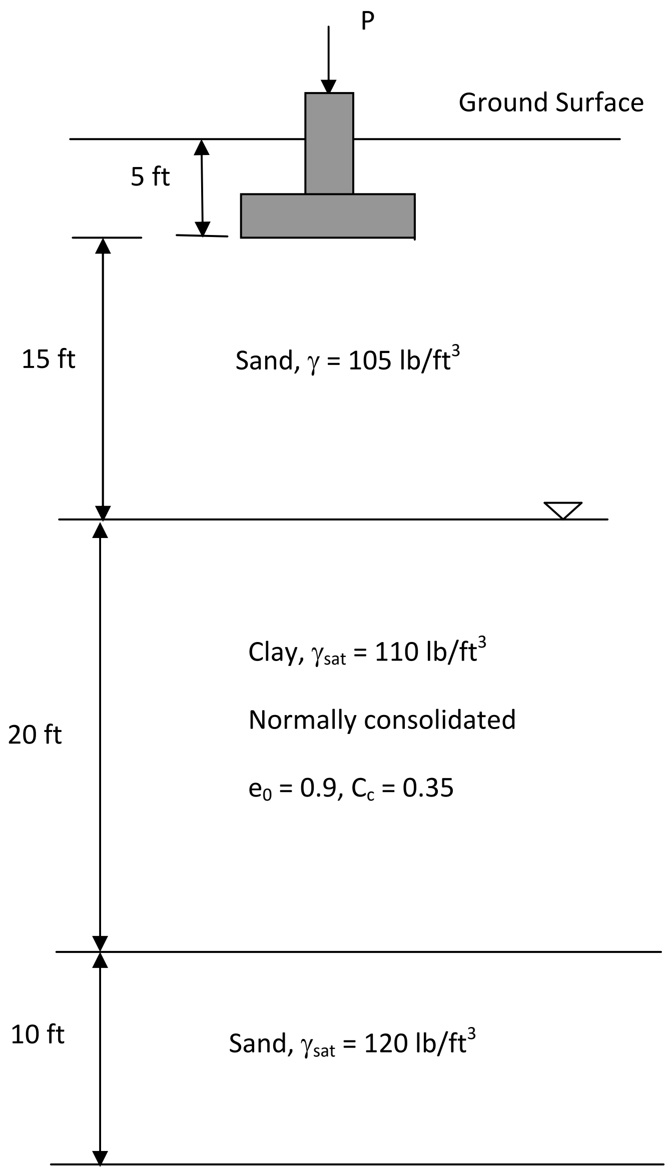
(A) 1.0 in
(B) 1.1 in
(C) 1.2 in
(D) 1.3 in

(2) A clay layer with thickness of 6 ft is subjected to an increase in vertical stress of 600 psf at the center of the layer caused by a very wide embankment fill. Laboratory tests for the clay layer have indicated the clay is a normally consolidated clay, with an initial void ratio of 0.8 and a compression index C_c of 0.35. What is most nearly the ultimate consolidation settlement for this clay layer?



(A) 2.8 in
(B) 2.9 in
(C) 3.0 in
(D) 3.1 in

(3) For the footing problem shown in the following figure, the average stress increase in the clay layer caused by the loads from footing is 800 psf. The clay layer is normally consolidated with a compression index C_c = 0.35 and initial void ratio e_0 = 0.9. The calculated the primary consolidation settlement (at the end of consolidation) for this clay layer is mostly near to:



(A) 5.0 in
(B) 5.1 in
(C) 5.2 in
(D) 5.3 in

SOLUTIONS:

Solution: #1

The initial effective stress at the middle of the clay layer is:

$$\sigma'_0 = (7)(121) + (3)(112 - 62.4) = 995.8 \text{ lb/ft}^2$$

Because $\sigma'_0 < \sigma'_c = 1500 \text{ lb/ft}^2$, so the soil is overconsolidated clay (OC clay)

The effective stress at the end of consolidation is:

$$\sigma'_0 + \Delta\sigma' = 995.8 + 800 = 1795.8 \text{ lb/ft}^2$$

For OC clay and $\sigma_0' + \Delta\sigma' > \sigma_c'$

$$S_c = \frac{C_s H}{1+e_0}\log\left(\frac{\sigma_c'}{\sigma_0'}\right) + \frac{C_c H}{1+e_0}\log\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_c'}\right)$$

$$= \frac{(0.06)(6)}{1+1}\log\left(\frac{1500}{995.8}\right) + \frac{(0.3)(6)}{1+1}\log\left(\frac{1795.8}{1500}\right)$$

$$= 0.102 \text{ ft} = 1.2 \text{ in}$$

The answer is (C)

Solution: #2

The initial effective stress at the middle of the clay layer is:

$$\sigma'_0 = (7)(121) + (3)(112 - 62.4) = 995.8 \text{ lb/ft}^2$$

$$\Delta\sigma' = 400 \text{ lb/ft}^2$$

For NC clay

$$S_c = \frac{C_c H}{1+e_0}\log\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}\right)$$

$$= \frac{(0.35)(6)}{1+0.8}\log\left(\frac{995.8 + 600}{995.8}\right)$$

$$= 0.24 \text{ ft} = 2.87 \text{ in}$$

The answer is (B)

Solution: #3

The initial effective stress at the middle of the clay layer is:

$$\sigma'_0 = (105)(20) + (110 - 62.4)(10) = 2576 \text{ lb/ft}^2$$

$$\Delta\sigma' = 800 \text{ lb/ft}^2$$

For NC clay

$$S_c = \frac{C_c H}{1+e_0} \log\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}\right)$$

$$= \frac{(0.35)(20)}{1+0.9} \log\left(\frac{2576+800}{2576}\right)$$

$$= 0.43 \text{ ft} = 5.2 \text{ in}$$

The answer is (C)

Dr. Vagelis Plevris

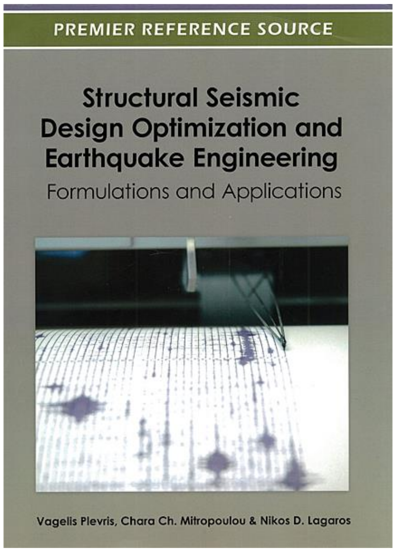
BSc(Eng), MSc, MBA, PhD, P.E., C.Eng, M.ASCE
Associate Professor
Department of Civil Engineering and Energy Technology
Oslo and Akershus University College of Applied Sciences (HiOA)
Oslo, Norway
Email: vagelis.plevris@hioa.no

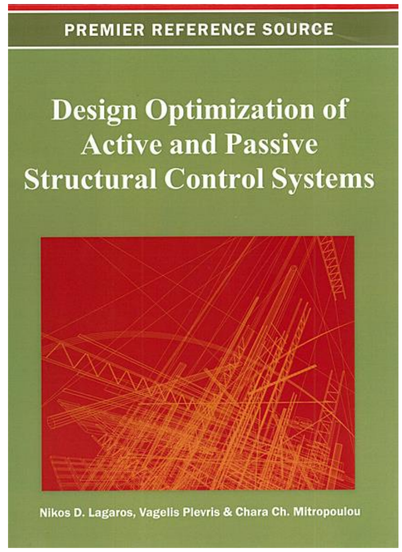
August 28, 2017

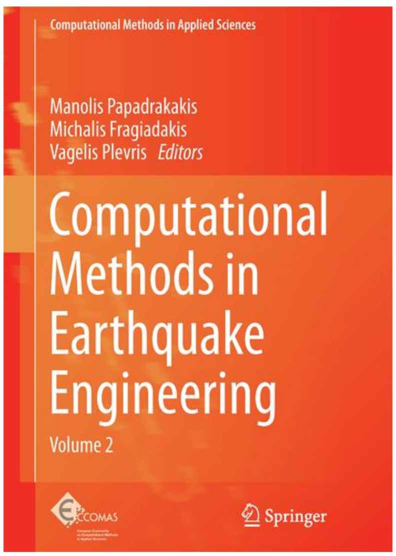
Career Highlights

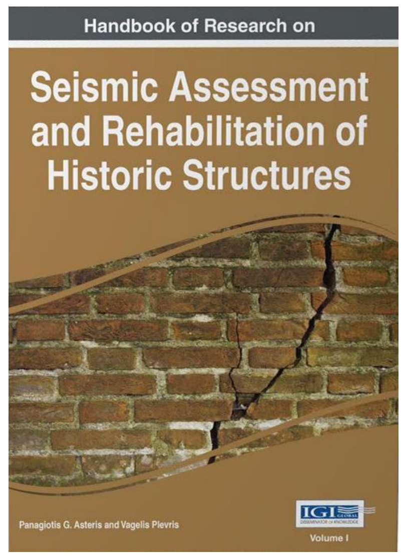
1. *Neurocomputing Strategies for Solving Reliability-Robust Design Optimization Problems*
Winner of the prestigious *Emerald Literati Network Award* – 2011.
2. Developer of a Popular Finite Element Structural Analysis software package, currently being used by scores of Universities and thousands of Engineering students & practicing Engineers worldwide.
3. Principal Investigator (PI) in two Research Grants totalling €400,000.
4. Research Associate (Co-PI) in 5 Research Projects with a total budget of €900,000.
5. Editor of 6 Books:
    - Performance-based Seismic Design of Concrete Structures and Infrastructures
    - *Computational Methods in Earthquake Engineering, Vol. 3* (Springer, 2017).
    - *Seismic Assessment and Rehabilitation of Historic Structures* (IGI Global, 2015).
    - *Computational Methods in Earthquake Engineering, Vol. 2* (Springer, 2013).
    - *Design Optimization of Active and Passive Structural Control Systems* (IGI Global, 2012).
    - *Structural Seismic Design Optimization and Earthquake Engineering: Formulations and Applications* (IGI Global, 2012).
6. Active in Research; Published 17 Papers in International Refereed Journals, 33 Papers in Peer-reviewed International Conference Proceedings and 6 Book Chapters.
7. Editor of 4 International Conference Proceedings volumes.
8. Scholarly Article Reviewer of 40 International Scientific Journals.
9. Associate Editor of 3 International Journals: Frontiers in Built Environment, J. of Computer Science, Oriental J. of Computer Science and Technology
10. Member of the Editorial Board of 16 International Journals and 3 International Conferences.
11. Co-organizer of four International Conferences.
12. 562 citations: "Google Scholar", with Author *h*-index=11.
13. 12 years of university teaching experience, at both Bachelor's and Master's levels.
14. Invited Lecturer in Scientific Workshops, 6 lectures in several European countries.
15. Organizer of 8 Special Sessions in various International Conferences.
16. Working with and mentoring for the elementary and middle school students on STEM topics in rural area public schools.
17. Amateur astronomer, avid star gazer, loves to travel and learn about different cultures & people. Speaks five languages: English, German, Italian, Norwegian and Greek.

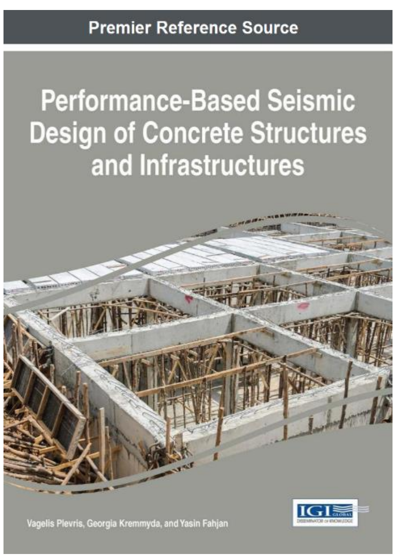
Books By V. Plevris

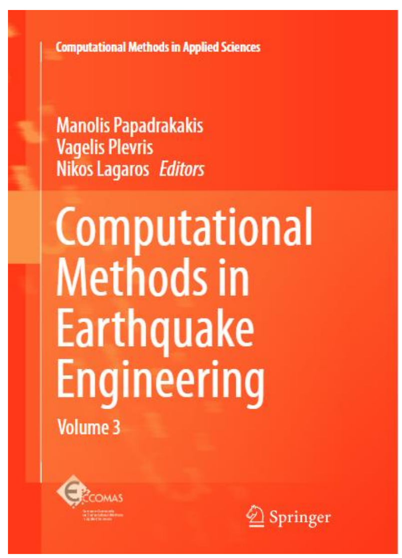












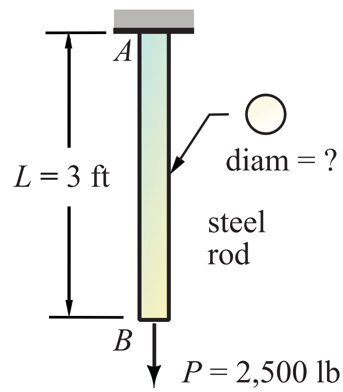
FUNDAMENTALS OF ENGINEERING

STRENGTH OF MATERIALS

AXIALLY LOADED MEMBERS

Problem: (Axial stress)

Figure
Not to scale



$E = 29 \times 10^6$ psi

A 3-ft long circular steel rod is subjected to a tensile force of 2,500 lb as shown in the figure. Knowing that the total elongation of the rod is measured as 0.02 inches, answer the following question:

(1) the diameter (inches) of the rod is most nearly

(A) 0.18
(B) 0.20
(C) 0.44
(D) 0.75

$d = ?$

(2) the max. axial stress (psi) in the rod is most nearly

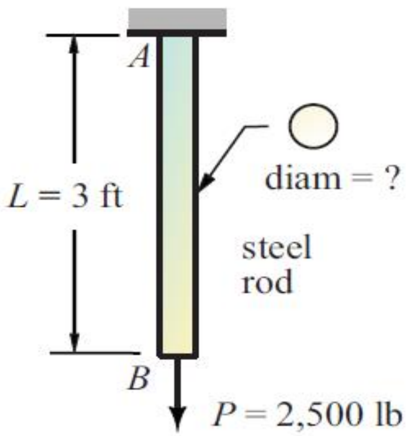
(A) 8,800
(B) 12,300
(C) 16,100
(D) 19,700

$\sigma_{max} = ?$

AXIAL-40
VPLEVRIS
SEPT 2017

FUNDAMENTALS OF ENGINEERING
STRENGTH OF MATERIALS
AXIALLY LOADED MEMBERS

Problem: ***AXIAL-40*** ***Solution in MS Excel***



E =	2.90E+07	psi
P =	2500	lb
L =	3	ft
ΔL =	0.02	in

L = 36 in

$$\sigma = \varepsilon \cdot E = \frac{\Delta L}{L} E \Rightarrow \frac{P}{A} = \frac{\Delta L}{L} E \Rightarrow A = \frac{P \cdot L}{E \cdot \Delta L}$$

A = 0.1552 in^2

$$A = \pi \cdot \frac{d^2}{4} \Rightarrow d = 2\sqrt{\frac{A}{\pi}}$$

***d* = 0.44 in** (C) Answer

***σ* = 16111.1 psi** (C) Answer

$$\sigma = \frac{P}{A}$$

AXIAL-40
VPLEVRIS
SEPT. 2017

MANNING'S EQUATION

The Manning's Equation is an empirical formula estimating the average velocity of water in a conduit that does not completely enclose the liquid like open channel flow. All flow in open channels is driven by gravity.

NCEES Reference Handbook, Version 9.4, page-167:

$$Q = \frac{K}{n} A R_H^{2/3} S^{1/2}$$

Manning's Equation

$$Q = \frac{1}{n} A R_H^{2/3} S^{1/2}$$

SI units (K = 1)

$$Q = \frac{1.486}{n} A R_H^{2/3} S^{1/2}$$

USCS units (K = 1.486)

$$R_H = \frac{A}{P} \qquad V = \frac{Q}{A}$$

Q = Flow rate, quantity of flow, discharge (ft^3/sec or m^3/sec)

K = values are listed above

A = cross-sectional area of flow (ft^2 or m^2)

R_H = hydraulic radius ($R_H = A/P$), (feet or meters)

P = wetted perimeter (feet or meters)

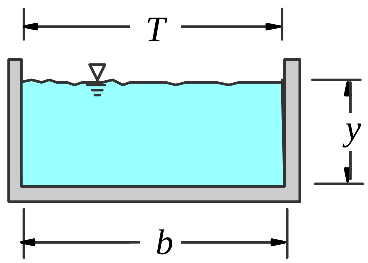
S = gradient, slope of hydraulic surface (ft/ft or m/m)

n = Manning's roughness coefficient, roughness factor

V = Flow velocity (f/sec or m/sec)

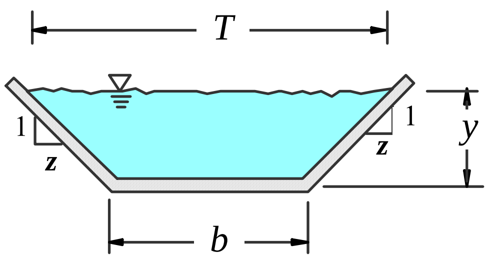
GEOMETRIC ELEMENTS OF CHANNEL SECTIONS
OPEN CHANNEL FLOW

Rectangular Channel



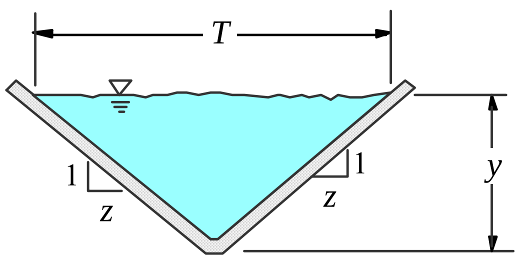
Area
$A = b.y$
Wetted Perimeter
$P = b + 2.\,y$
Top Width
$T = b$

Trapezoidal Channel



Area
$A = (b + z\,y)\,y$
Wetted Perimeter
$P = b + 2\,y\,(1 + z^2)^{1/2}$
Top Width
$T = b + 2\,z\,y$

Triangular Channel



Area
$A = z\,y^{\,2}$
Wetted Perimeter
$P = 2\,y\,(1 + z^2)^{1/2}$
Top Width
$T = 2\,z\,y$

Hydraulic Radius $\quad R_H = \dfrac{A}{P}$

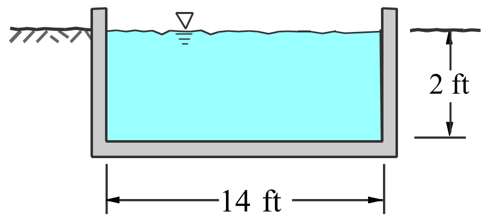
HYDR-222
ZEYTINCI
SEPT 2017

FUNDAMENTALS OF ENGINEERING & PROFESSIONAL ENGINEERING

WATER RESOURCES

OPEN CHANNEL FLOW / MANNING'S EQUATION

Problem: (Open channel flow)



FE / PE EXAM

Gradient (slope) of the channel = 2.5 %
Manning's roughness coeff. (n = 0.013)

A rectangular concrete channel has a bottom width of 14 ft and water depth of 2.0 ft as shown in the figure. Knowing that the channel is on a 2.5 % slope (gradient) answer the following:

(1) The hydraulic radius (ft) of the channel is most nearly:

(A) 1.56
(B) 1.25
(C) 1.00
(D) 0.75

$R_H = ?$

(2) The discharge (ft^3/sec) is most nearly:

(A) 375
(B) 486
(C) 557
(D) 680

$Q = ?$

(3) The flow velocity (ft/sec) is most nearly:

(A) 13.7
(B) 18.8
(C) 24.3
(D) 34.6

$V = ?$

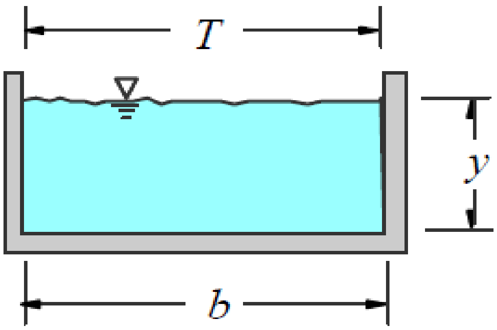
COMPLETE SOLUTION

FUNDAMENTALS OF ENGINEERING & PROFESSIONAL ENGINEERING

WATER RESOURCES

OPEN CHANNEL FLOW / MANNING'S EQUATION

Problem: ***HYDR-224*** ***Solution in MS Excel***



S = 2.5%

n = 0.013

b = 14 ft

y = 2 ft

A = 28 ft^2 (Area) $A = b \cdot y$

Per = 18 ft (Perimeter) $P = b + 2y$

R_H = **1.56** ft

(A) Answer

$$R_H = \frac{Area}{Perimeter}$$

$$Q = \frac{1.486}{n} A \cdot R_H^{2/3} \cdot S^{1/2}$$

USCS Units
(K=1.486)

Q = **679.40** ft^3/sec

(D) Answer

V = **24.3** ft/sec

(C) Answer

$$V = \frac{Q}{A}$$

HYDR-224

VPLEVRIS

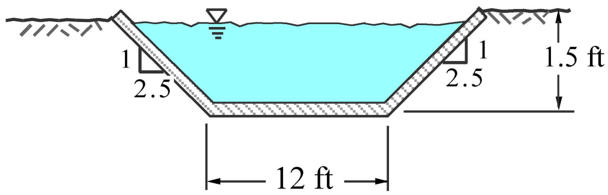
SEPT. 2017

FUNDAMENTALS OF ENGINEERING & PROFESSIONAL ENGINEERING

WATER RESOURCES

OPEN CHANNEL FLOW / MANNING'S EQUATION

Problem:



FE/PE EXAM

> Side slopes of trapezoid 2.5:1 (as shown)
> Gradient (slope) of the channel = 2.5 %
> Manning's roughness coefficient: (n = 0.013)

A concrete trapezoidal channel has a bottom width of 12 ft as shown in the figure. Knowing that the channel is on a 2.5 % slope and is flowing at a depth of 1.5 ft throughout its length, answer the following questions:

(1) The hydraulic radius (ft) of the channel is most nearly:

(A) 1.05
(B) 1.18
(C) 1.47
(D) 2.15

$$R_H = ?$$

(2) The quantity of flow (discharge) (ft^3/sec) is most nearly:

(A) 387
(B) 476
(C) 510
(D) 620

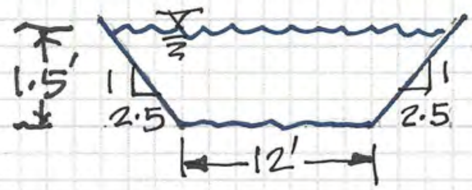
$$Q = ?$$

(3) The flow velocity (ft/sec) is most nearly:

(A) 30
(B) 26
(C) 20
(D) 18

$$V = ?$$

SOLUTION : (HYDR-232)



Side slopes : 2.5 : 1
Gradient : $S = 0.025$
Roughness : $n = 0.013$

Cross-sectional area : (A)

$$A = (b + zy)y = (12 + 2.5 \times 1.5)1.5 = 23.625 \text{ ft}^2$$

Wetted perimeter : (P)

$$P = b + 2y(1 + z^2)^{1/2} = 12 + 2(1.5)(1 + 2.5^2)^{1/2} = 20.08 \text{ ft}$$

$\boxed{P = 20.08 \text{ ft}}$

Hydraulic Radius : (R_H)

$$R_H = A/P = 23.625/20.08 = 1.18 \text{ ft}$$

$\boxed{R_H = 1.18 \text{ ft}}$

Quantity of flow (discharge) : Q

$$Q = \frac{1.486}{n} A R^{2/3} S^{1/2}$$

$$= \frac{1.486}{0.013}(23.625)(1.18)^{2/3}(0.025)^{1/2}$$

$\boxed{Q = 476 \text{ ft}^3/\text{sec}}$

Flow velocity : (V)

$$V = Q/A = 476/23.625 = 20.1 \text{ ft/sec}$$

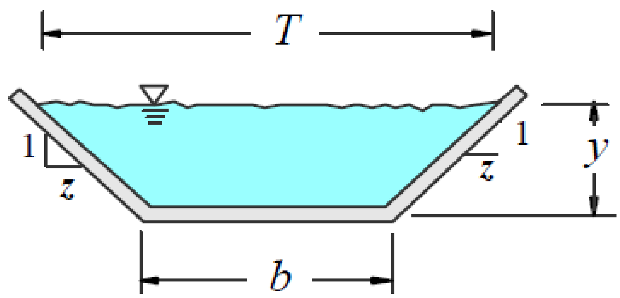
$\boxed{V = 20.1 \text{ ft/sec}}$

FUNDAMENTALS OF ENGINEERING & PROFESSIONAL ENGINEERING

WATER RESOURCES

OPEN CHANNEL FLOW / MANNING'S EQUATION

Problem: ***HYDR-232*** ***Solution in MS Excel***



S =	2.5%	
n =	0.013	
b =	12	ft
y =	1.5	ft
Side slopes z =	2.5	:1

$$P = b + 2y\left(1 + z^2\right)^{1/2}$$

P = 20.08 ft (Wetted Perimeter)

Area = 23.63 ft^2

$$A = (b + zy)\cdot y$$

R_H = **1.18** ft

(B) Answer

$$R_H = \frac{Area}{Perimeter}$$

$$Q = \frac{1.486}{n} A \cdot R_H^{2/3} \cdot S^{1/2}$$

USCS Units
(K=1.486)

Q = **475.91** ft^3/sec

(B) Answer

V = **20.1** ft/sec

(C) Answer

$$V = \frac{Q}{A}$$

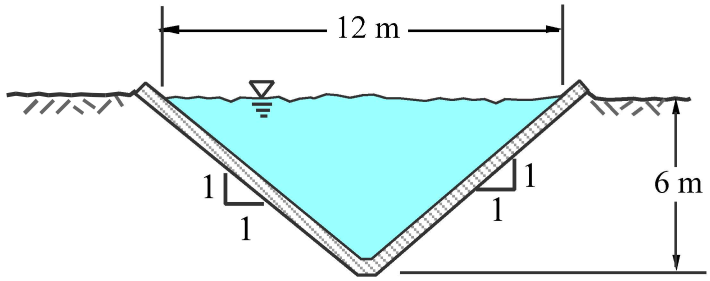
HYDR-232
VPLEVRIS
SEPT. 2017

PROFESSIONAL ENGINEERING (P.E.)

WATER RESOURCES

OPEN CHANNEL FLOW / MANNING'S EQUATION

Problem:



FE/PE EXAM

WATER RESOURCES

Side slopes of triangle 1:1 (as shown)
Gradient (slope) of the channel = 2.5 %
Manning's roughness coefficient: (n = 0.013)

A concrete triangular channel is shown in the figure. Knowing that the channel is on a 2.5 % slope and is flowing at a depth of 6.0 m throughout its length, answer the following questions:

(1) The hydraulic radius (m) of the channel is most nearly:

(A) 1.05
(B) 1.18
(C) 1.47
(D) 2.12

$R_H = ?$

(2) The quantity of flow (discharge) (m^3/sec) is most nearly:

(A) 387
(B) 526
(C) 610
(D) 723

$Q = ?$

(3) The flow velocity (m/sec) is most nearly:

(A) 30
(B) 26
(C) 20
(D) 18

$V = ?$

COMPLETE SOLUTION

A.Z.

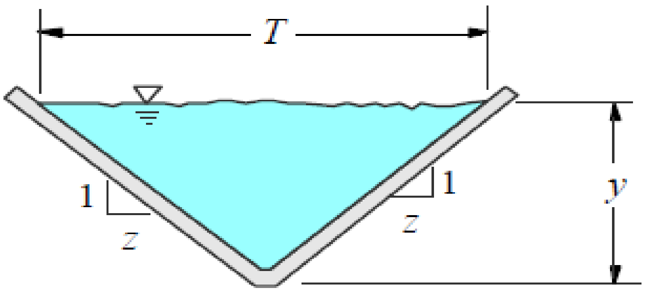
HYDR-420
ZEYTINCI
SEPT 2017

FUNDAMENTALS OF ENGINEERING & PROFESSIONAL ENGINEERING

WATER RESOURCES

OPEN CHANNEL FLOW / MANNING'S EQUATION

Problem: ***HYDR-420*** ***Solution in MS Excel***



S =	2.5%	
n =	0.013	
y =	6	m
Side slopes z =	1	:1

$$P = 2y\left(1+z^2\right)^{1/2}$$

P = 16.97 m (Wetted Perimeter)

Area = 36.00 m^2

$$A = z \cdot y^2$$

R_H = **2.12** m

(D) Answer

$$R_H = \frac{Area}{Perimeter}$$

$$Q = \frac{1}{n} A \cdot R_H^{2/3} \cdot S^{1/2}$$

SI Units
(K=1)

Q = **722.88** ft^3/sec

(D) Answer

V = **20.1** ft/sec

(C) Answer

$$V = \frac{Q}{A}$$

HYDR-420

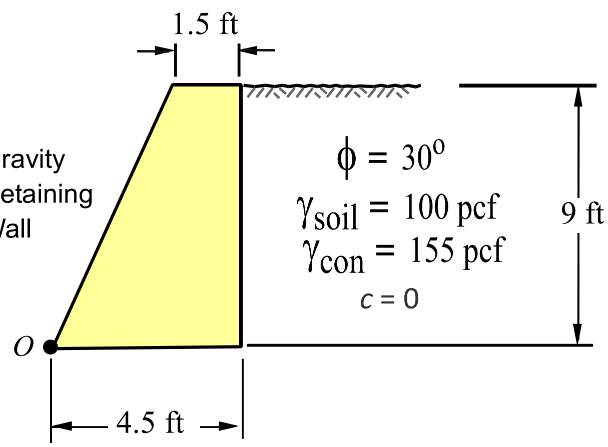
VPLEVRIS

SEPT. 2017

GRAVITY RETAINING WALLS

FUNDAMENTALS OF ENGINEERING (F.E.)
PROFESSIONAL ENGINEERING (P.E.)

Problem:



FE/PE EXAMS

REINFORCED CONCRETE DESIGN

The dimensions of a masonry retaining wall is given as shown. Knowing that *Rankine active earth pressure* will be used in the computations and the wall friction will be disregarded, answer the following:

(1) The *overturning moment* (ft.kip/ft) about the toe at *O* is most nearly:

(A) 4.05
(B) 5.25
(C) 6.32
(D) 7.25

$M_{ot} = ?$

(2) The *stabilizing moment* (ft.kip/ft) about the toe at *O* is most nearly:

(A) 16.44
(B) 15.49
(C) 12.03
(D) 11.75

$M_{stab} = ?$

(3) The *factor of safety* against overturning about the toe at *O* is most nearly:

(A) 1.90
(B) 2.20
(C) 2.97
(D) 3.25

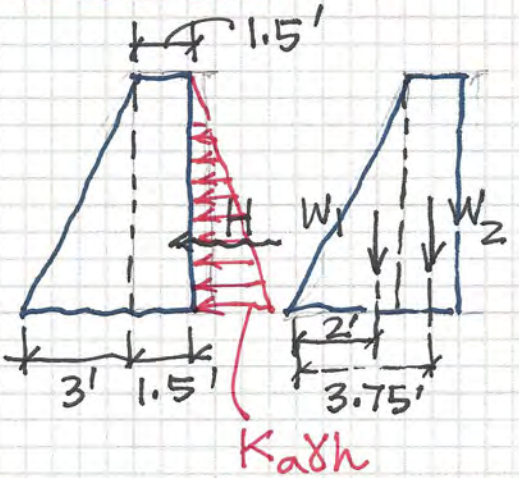
$F.S. = ?$

COMPLETE SOLUTION

A.Z.

GRW-266
ZEYTINCI
SEPT-2017

SOLUTION: (GRW-266)



K_a: Rankine active soil pressure coefficient

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = 1/3$$

H = Total Lateral force

$$H = \tfrac{1}{2} K_a \gamma_{soil} \cdot h^2$$

$$= \tfrac{1}{2}(1/3)(100)(9^2) = \underline{1350\ lb}$$

Overturning moment: (M_{OT})

$$M_{OT} = H(h/3) = 1350(9/3) = 4050\ ft \cdot lb$$

Vertical loads: (W_i)

$$W_1 = \tfrac{1}{2}(3)(9')(155)(1) = 2092\ lb$$

$$W_2 = (1.5)(9')(155)(1) = 2092\ lb$$

Stabilizing moment: (M_S)

$$M_S = W_1(2') + W_2(3.75') = (2092)(2') + (2092)(3.75')$$

$$M_S = 12{,}029\ ft\text{-}lb$$

Factor of safety against overturning

$$F.S. = M_S / M_{OT} = 12{,}029/4050 = \underline{2.97}$$

Dr. Z

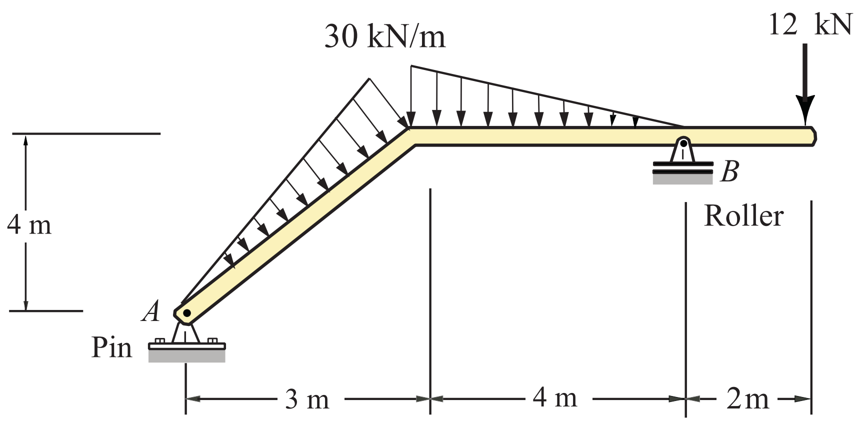
ANSWERS
$M_{OT} = 4050$ ft-lb
$M_S = 12{,}029$ ft·lb
F.S = 2.97

FUNDAMENTALS OF ENGINEERING
PROFESSIONAL ENGINEERING

STRUCTURAL ANALYSIS

DETERMINATE FRAMES

Problem:



FE/PE EXAM

STRUCTURES

Support A : Hinge
Support B : Roller

A determinate frame is loaded as shown in the figure. Knowing that support A is a pin and support B is a roller, answer the following questions:

(1) The magnitude of the horizontal support reaction (kN) at A

(A) 60.0
(B) 54.2
(C) 34.5
(D) 20.0

$A_x = ?$

(2) The magnitude of the vertical support reaction (kN) at B

(A) 65.1
(B) 78.4
(C) 88.3
(D) 92.7

$B_y = ?$

(3) The magnitude of the vertical support reaction (kN) at A

(A) 18.5
(B) 28.7
(C) 35.8
(D) 46.9

$A_y = ?$

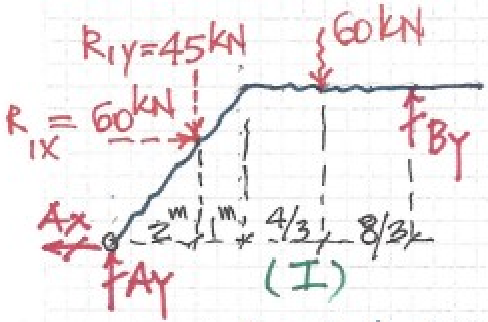
COMPLETE SOLUTION

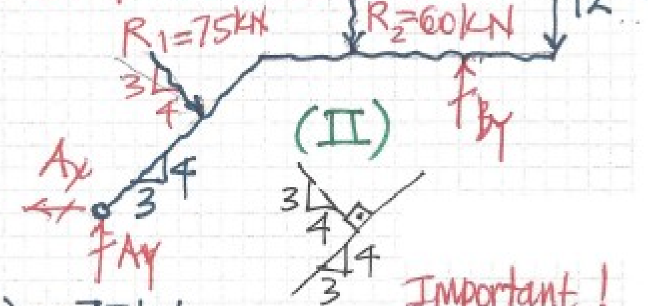
A.Z.

FR-152
ZEYTINCI
SEPT 2017

SOLUTION: (FR-152)

First find the resultants:





$$R_1 = \frac{1}{2}(5^m)(30) = 75\,kN$$
$$R_2 = \frac{1}{2}(4^m)(30) = 60\,kN$$

Important !
When two lines are ⊥ slopes will be as shown

Horizontal equilibrium equation:

$\overset{+}{\leftarrow}\Sigma F_x = 0$ $\quad A_x - 60^{kN} = 0 \rightarrow \quad \boxed{A_x = 60^{kN}} \leftarrow$

Taking moment about point (A)

$\curvearrowleft + \Sigma M_A = 0$ (using fig. II would be better)

$$(75kN)(\tfrac{2}{3}\times 5) + (60kN)(3 + 4/3) + (12kN)(9^m) = 7B_Y$$

$$7B_Y = 250 + 260 + 108$$

$\boxed{B_Y = 88.29\,kN} \uparrow \qquad \boxed{B_Y = 88.29^{kN}} \uparrow$

Taking moment about B

$\curvearrowleft + \Sigma M_B = 0$

$$(-12kN)(2^m) + 60(\tfrac{2}{3}\times 4) + 45(1+4) + 60(4/3) - 4A_x - 7A_Y = 0$$

$$-24 + 160 + 225 + 80 - 4(60) - 7A_Y = 0$$

CHECK

$\boxed{A_Y = 28.71\,kN} \uparrow$

$+\uparrow \Sigma F_Y \overset{?}{=} 0$

$$A_Y + B_Y \overset{?}{=} 45^{kN} + 60^{kN} + 12^{kN}$$

$117^{kN} = 117^{kN}$ O.K.

COMPRESSION MEMBERS

NCEES-Reference Handbook, Page 157

$$P_u \leq \phi_c P_n$$

$$P_u \leq \phi_c P_n$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$\sqrt{\frac{E}{F_y}} = \sqrt{(29{,}000/50)} = \sqrt{580} = 28.03$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2}$$

$KL/r_{\min} =$ slenderness ratio

$$\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$$

$$F_{cr} = 0.658^{\left(\frac{F_y}{F_e}\right)} F_y$$

$$\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$$

$$F_{cr} = 0.877\, F_e$$

K = effective length factor (AISC)
L = column length
KL = effective column length
KL/r = slenderness ratio
r = radius of gyration
F_y = yield stress of steel
F_e = Euler stress
F_{cr} = critical stress
E = modulus of elasticity of steel

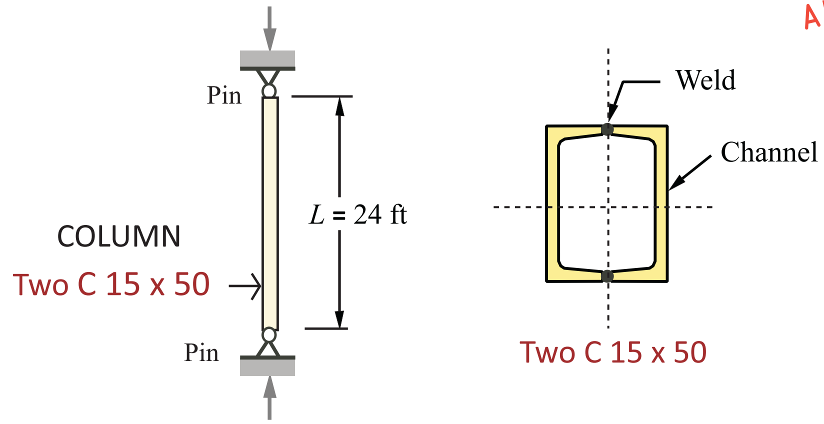
DESIGN OF STEEL STRUCTURES

COMPRESSION MEMBERS

Problem: Steel Column (W)

STRUCTURAL ANALYSIS

FE/PE EXAMS



Two C-Channels are welded together at flanges

Two C 15 x 50 channels are welded together to form a column as shown in the figure. Using the data for the cross section and the column length, answer the following questions:

(1) The min. area moment of inertia (in^4) is most nearly:

(A) 211.20
(B) 272.85
(C) 312.53
(D) 355.10

$I_{\text{min}} = ?$

(2) The minimum radius of gyration (in.) is most nearly:

(A) 4.60
(B) 4.08
(C) 3.60
(D) 3.05

$r_{\text{min}} = ?$

(3) The slenderness ratio of the column is most nearly:

(A) 98.50
(B) 94.43
(C) 85.24
(D) 82.42

$KL/r_{\text{min}} = ?$

LOAD FACTORS & COMBINATIONS (LRFD)

FUNDAMENTALS OF ENGINEERING (F.E.)

PROFESSIONAL ENGINEERING (P.E.)

REINFORCED CONCRETE

$U = 1.2\,D + 1.6\,L$
D = Dead Load
L = Live Load

ACI = American Concrete Institute
IBC = International Building Code
ASCE = American Society of Civil Engineers
NCEES = Ref. Handbook (Page-153), v. 9.4

(1) $U = 1.4\,(D + F)$

(2) $U = 1.2\,(D + F + T) + 1.6\,(L + H) + 0.5\,(L_r \text{ or } S \text{ or } R)$

(3) $U = 1.2\,D + 1.6\,(L_r \text{ or } S \text{ or } R) + (1.0\,L \text{ or } 0.8\,W)$

(4) $U = 1.2\,D + 1.6\,W + 1.0\,L + 0.5\,(L_r \text{ or } S \text{ or } R)$

(5) $U = 1.2\,D + 1.0\,E + 1.0\,L + 0.2\,S$

(6) $U = 0.9\,D + 1.6\,W + 1.6\,H$

(7) $U = 0.9\,D + 1.0\,E + 1.6\,H$

The max. value will be used.

NCEES, Reference Handbook (Page-153), v. 9.4

D = Dead loads
L = Live loads (floor)
L_r = Live loads (roof)
E = Earthquake loads
R = Rain load
S = Snow load
W = Wind load

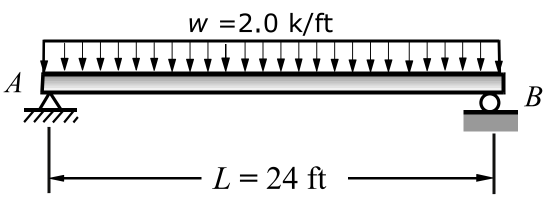
LFCTR-222
ZEYTINCI
SEPT 2017

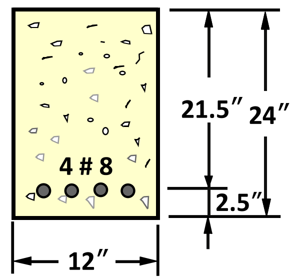
DESIGN OF CONCRETE STRUCTURES

FUNDAMENTALS OF ENGINEERING (F.E.)

PROFESSIONAL ENGINEERING (P.E.)

Problem: (Transformed Section Method)





Section

$n = 10$

Beam weight is included in the uniform load

The dimensions of a R/C beam section is given as shown. Knowing that the sections have **cracked** and the"Transformed Section Method" will be used, answer the following:

(1) Flexural stress in concrete (psi) is most nearly:

- (A) 1620
- (B) 1730
- (C) 1850
- (D) 1980

$f_{con} = ?$

(2) Flexural stress (psi) in steel is most nearly:

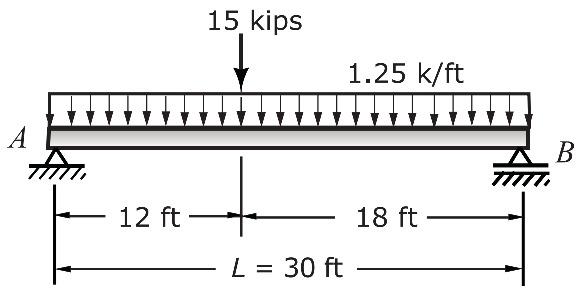
- (A) 24,650
- (B) 27,470
- (C) 29,380
- (D) 31,120

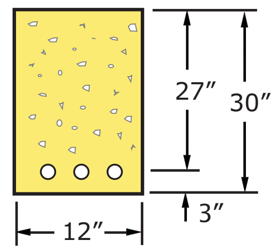
$f_{steel} = ?$

RC-334
ZEYTINCI
SEPT 2017

FACTORED LOADS / FACTORED MOMENTS

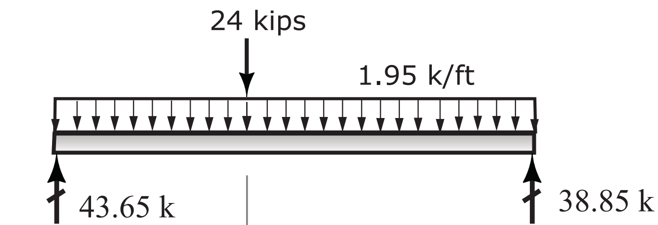
Service Loads





Section

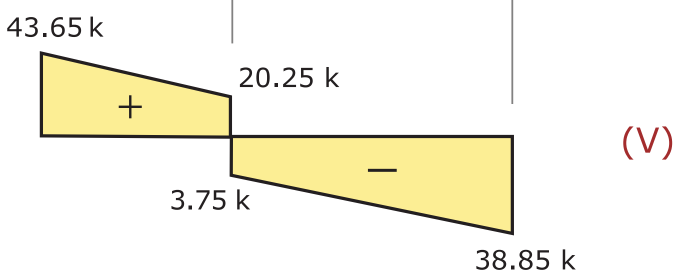
Factored Loads



Beam Weight

BM = 375.0 lb/ft

Shear Diagram



Beam Weight

$$BM = \frac{(12)(30)}{144}\ (150) = 375 \text{ lb/ft} = 0.375 \text{ k/ft}$$

Factored Dead Load

$$w_u = 1.2\ (0.375 + 1.25) = 1.2\ (1.625) = 1.95 \text{ k/ft}$$

Factored Live Load

$$w_u = 1.6\ (15 \text{ kips}) = 24 \text{ kips}$$

Maximum Factored Moment

$$M_u = 0.5\ (12)\ (43.65 + 20.25) = 383.4 \text{ ft-kips}$$

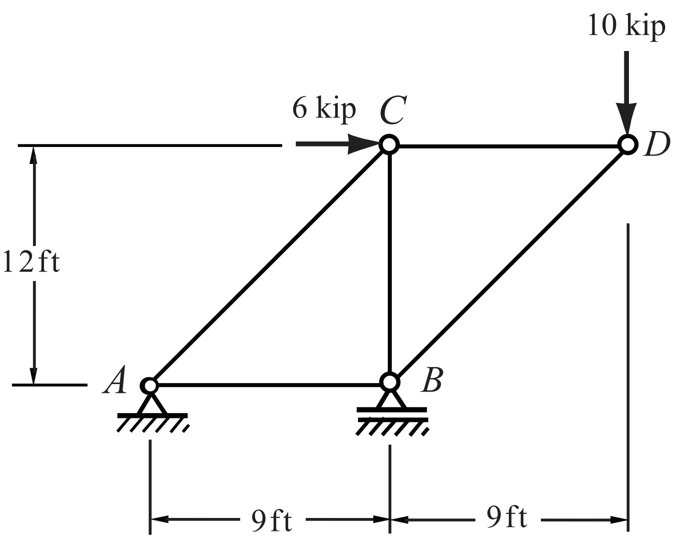
$$\boxed{M_u = 383.4 \text{ ft-kips}}$$

FCTRL-154 S
ZEYTINCI
SEPT 2017

DEFLECTIONS OF DETERMINATE TRUSSES

FUNDAMENTALS OF ENGINEERING (F.E.)

PROFESSIONAL ENGINEERING (P.E.)



STRUCTURAL ANALYSIS

Support A : Hinge
Support B : Roller

$E = 29 \times 10^6$ psi

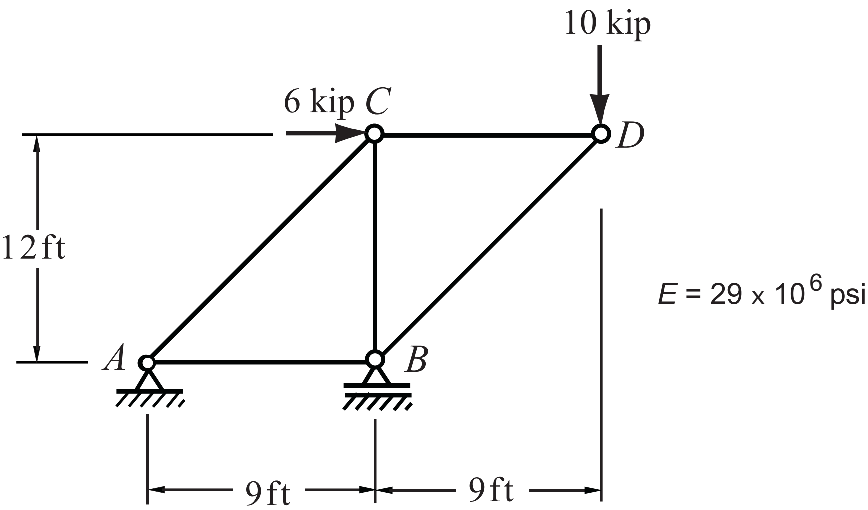
Member	AC	CD	BD	AB	BC
Area (in^2)	3.00	2.00	2.00	1.50	2.50

A determinate plane truss is given as shown in the figure. Using the listed cross-sectional areas and material constants answer the following questions:

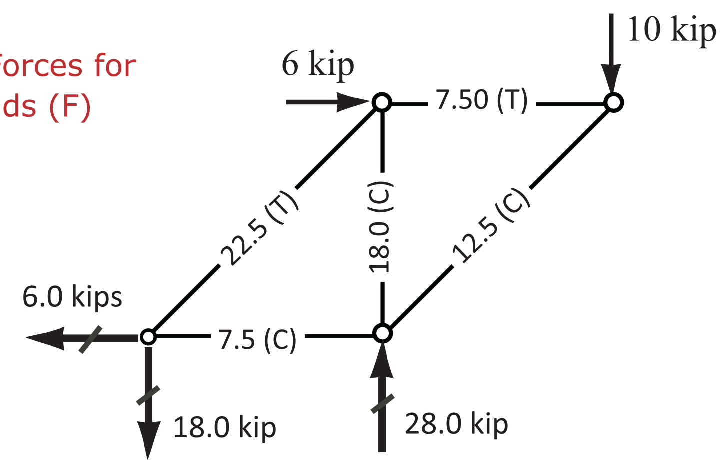
(1) Determine the vertical deflection of joint D
(2) Determine the horizontal displacement of joint D

TRSS-135
ZEYTINCI
SEPT 2017

MEMBER FORCES (SUMMARY)



Member Forces for
Given Loads (F)

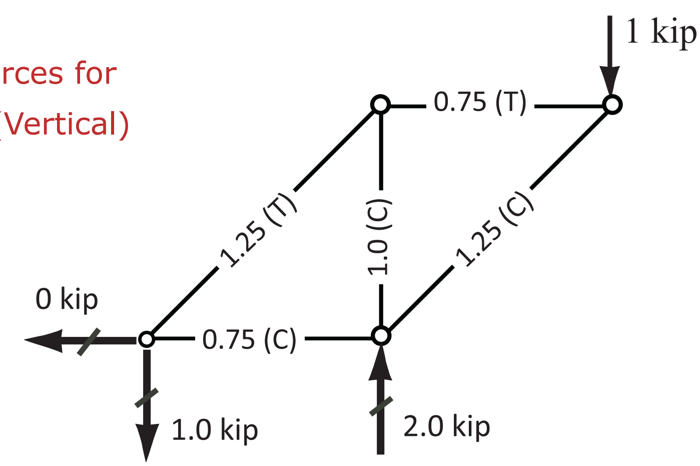


(F)

Given Load

Member Forces for

Unit Load (Vertical)

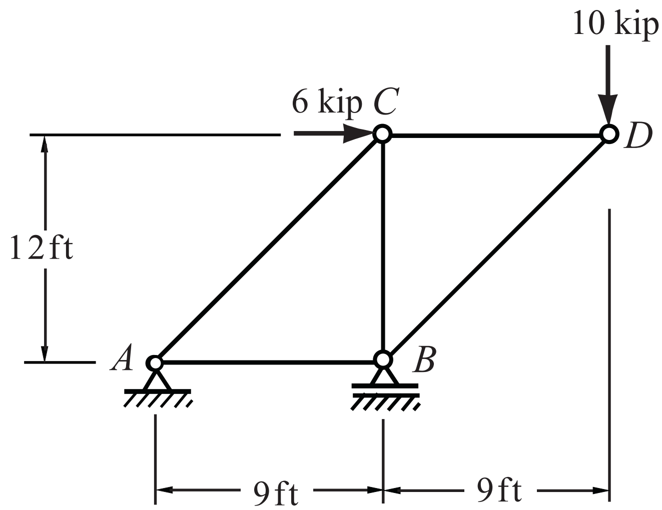


(f)

Unit Load at D
For Vertical
Deflection

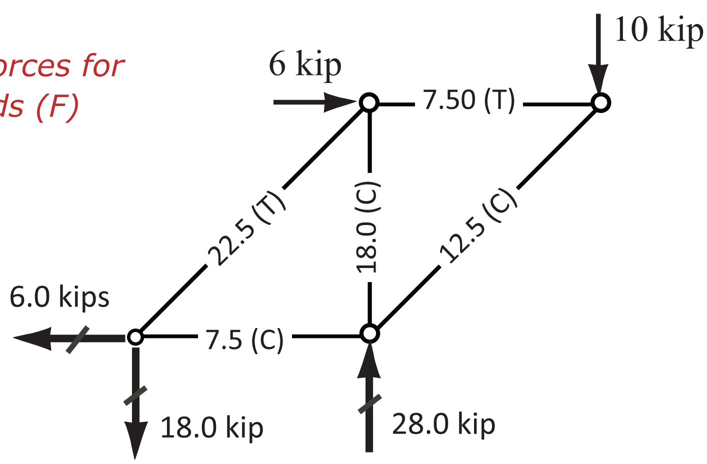
TRUSS-135-2
ZEYTINCI
SEPT-2017

MEMBER FORCES (SUMMARY)



$E = 29 \times 10^6$ psi

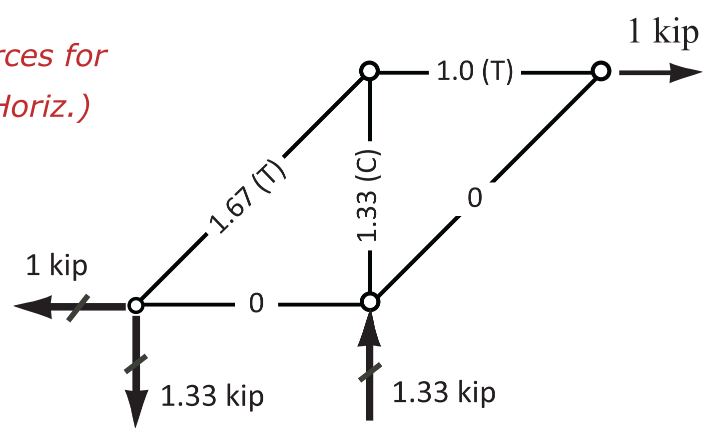
Member Forces for Given Loads (F)



(F)

Given Load

Member Forces for Unit Load (Horiz.)

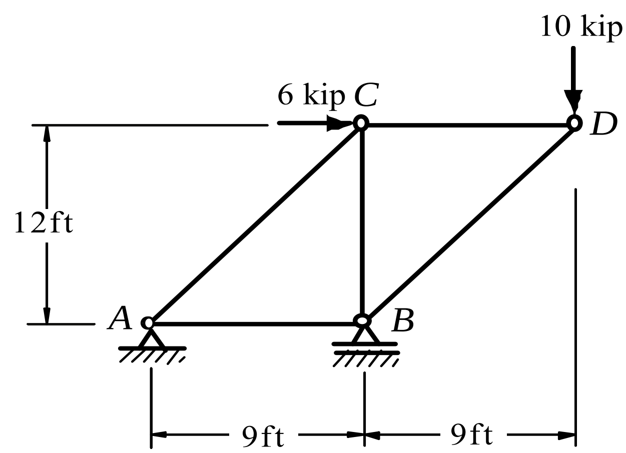


(f)

Unit Load at *D*
For Horizontal
Deflection

TRUSS-135-3
ZEYTINCI
SEPT-2017

Deflections of Determinate Trusses



Member	L	L	A	L/A	F	f	$F\,f\,.\,L/A$
	(ft)	(in)	(in^2)		(kip)		
AC	15.00	180.00	3.00	60.00	22.50	1.25	1,688
CD	9.00	108.00	2.00	54.00	7.50	0.75	304
BD	15.00	180.00	2.00	90.00	-12.50	-1.25	1,406
AB	9.00	108.00	1.50	72.00	-7.50	-0.75	405
BC	12.00	144.00	2.50	57.60	-18.00	-1.00	1,037

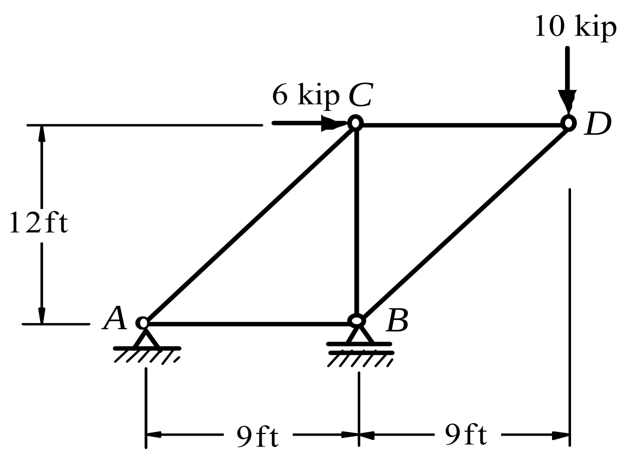
$\Sigma =$ **4,839**

Vertical deflection of point *D*

$\delta_D = \Sigma\ Ff.L\,/AE$ = (4,839).(1000) / 29,000,000 = 0.1669 in. ⇣

TRSS-135-Y
ZEYTINCI
SEPT 2017

Deflections of Determinate Trusses



Member	L	L	A	L/A	F	f	Ff.L/A
	(ft)	(in)	(in^2)		(kip)		
AC	15.00	180.00	3.00	60.00	22.50	1.67	2,255
CD	9.00	108.00	2.00	54.00	7.50	1.00	405
BD	15.00	180.00	2.00	90.00	-12.50	0.00	0
AB	9.00	108.00	1.50	72.00	-7.50	0.00	0
BC	12.00	144.00	2.50	57.60	-18.00	-1.33	1,379

$\Sigma =$ **4,038**

Horizontal deflection of point *D*

$\delta_{D\,x} = \Sigma\ Ff.L\ /AE$ = (4,038).(1000) / 29,000,000 = 0.1392 in. ←

TRSS-135-X
ZEYTINCI
SEPT 2017

DETERMINATE BEAMS / SUPPORT REACTIONS

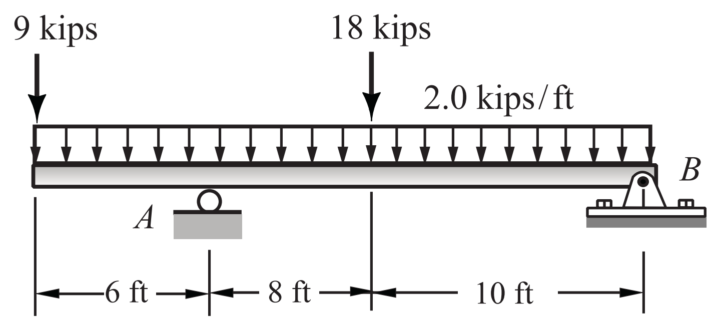
FUNDAMENTALS OF ENGINEERING (F.E.)

PROFESSIONAL ENGINEERING (P.E.)

STRUCTURAL ANALYSIS

FE/PE EXAMS

Problem:



An overhanging beam is loaded as shown. Using the given dimensions and support conditions, answer the following:

(1) The vertical support reaction (kips) at B is most nearly:

(A) 18
(B) 21
(C) 27
(D) 34

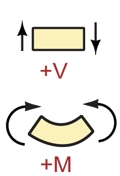
$B_y = ?$

(2) The vertical support reaction (kips) at A is most nearly:

(A) 21
(B) 31
(C) 46
(D) 54

$A_y = ?$

(3) The absolute maximum bending moment (ft.kips) in the beam is most nearly:



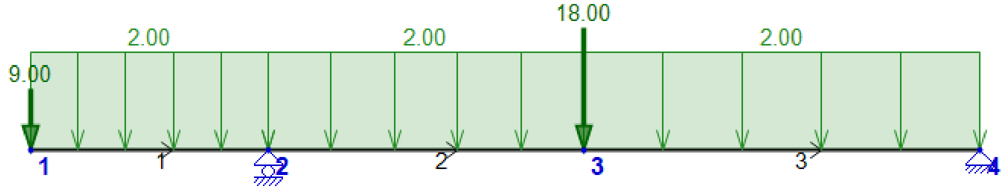
(A) 98
(B) 110
(C) 135
(D) 146

$M_{max} = ?$

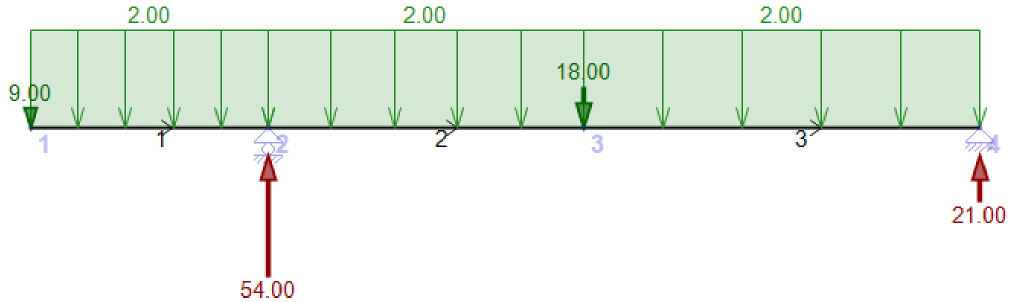
STUDENT'S SOLUTION

Solution by Dr. Vagelis Plevris

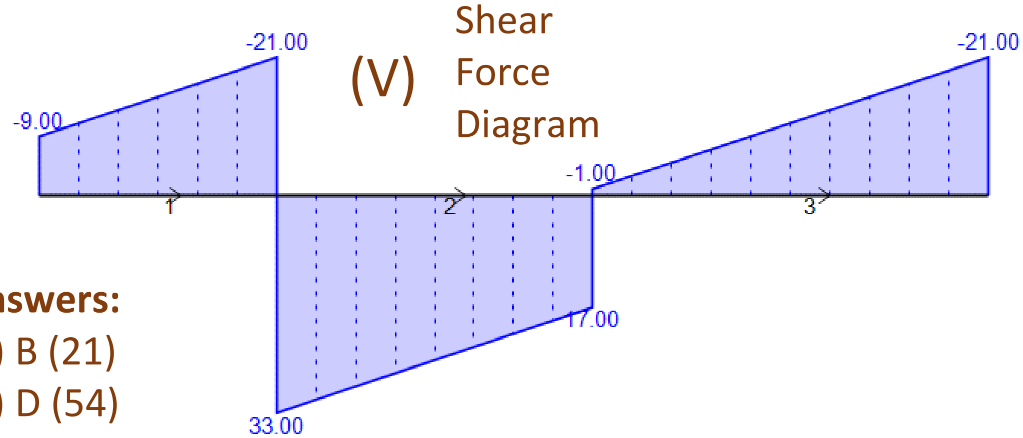
Model



Support Reactions



(V) Shear Force Diagram



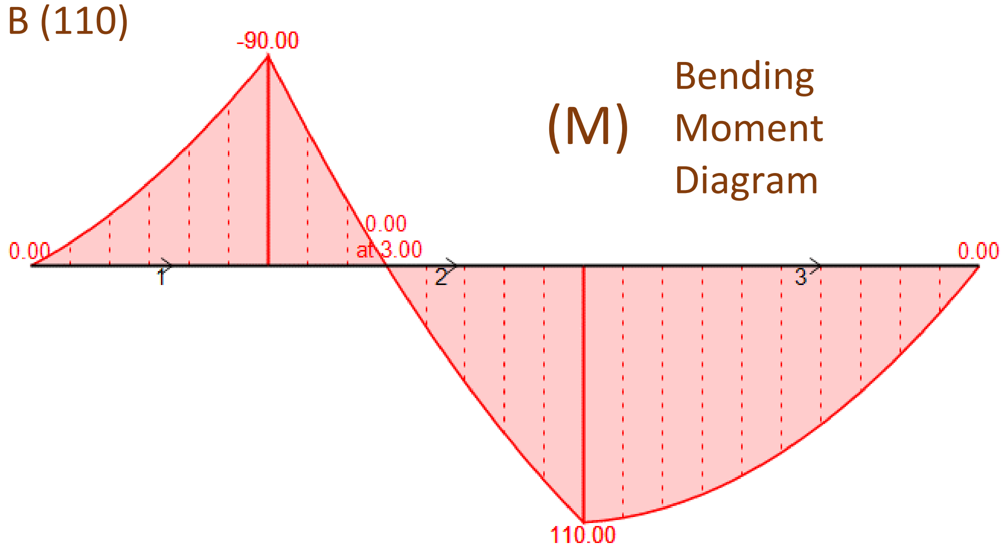
Answers:

(1) B (21)

(2) D (54)

(3) B (110)

(M) Bending Moment Diagram



BM-472
VPLEVRIS
SEPT. 2017

Software BEAM.2D by ENGILAB

www.engilab.com

MATHEMATICS

VECTORS

Dot Product of Two Vectors:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$$

The result of the dot product is a scalar.

$|\mathbf{a}|$: the magnitude (length) of vector **a**
$|\mathbf{b}|$: the magnitude (length) of vector **b**
θ : the angle between **a** and **b**

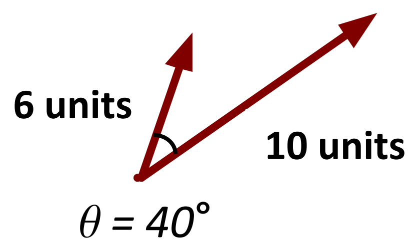
Example: (Using Magnitudes / Angle)

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$

$\mathbf{a} \cdot \mathbf{b} = 6 \times 10 \times \cos(40°)$

$\mathbf{a} \cdot \mathbf{b} = 6 \times 10 \times 0.766$

$\mathbf{a} \cdot \mathbf{b} = 45.96$ units



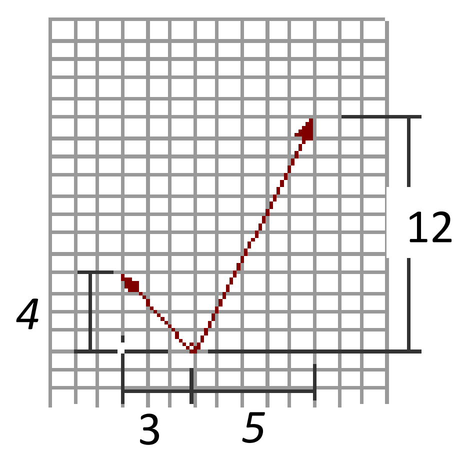
Example: (Using Components)

$\mathbf{a} \cdot \mathbf{b} = a_x \times b_x + a_y \times b_y$

$\mathbf{a} \cdot \mathbf{b} = -3 \times 5 + 4 \times 12$

$\mathbf{a} \cdot \mathbf{b} = -15 + 48$

$\mathbf{a} \cdot \mathbf{b} = 33$ units



MATH-266
ZEYTINCI
SEPT 2017

VECTOR COMPUTATIONS

CROSS PRODUCT

Problem:

NCEES
REF. HANDBOOK
PAGE-34

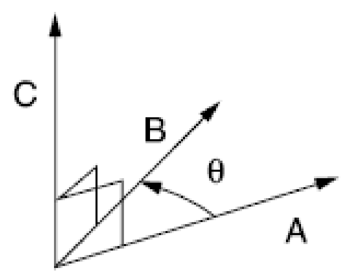
$$\mathbf{A} = a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k}$$
$$\mathbf{B} = b_x\,\mathbf{i} + b_y\,\mathbf{j} + b_z\,\mathbf{k}$$

The cross product is a vector product of magnitude

|B| x |A| sin θ

$|\mathbf{B}|\,|\mathbf{A}| \sin\theta$ which is perpendicular to the plane containing **A** and **B**. The product is

$$\mathrm{A} \times \mathrm{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = -\ \mathrm{B} \times \mathrm{A}$$



The sense of $\mathbf{A} \times \mathbf{B}$ is determined by the right-hand rule.

$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}|\,\mathbf{n} \sin\theta, \text{ where}$$

$\mathbf{n}$ = unit vector perpendicular to the plane of **A** and **B**.

FUNDAMENTALS OF ENGINEERING

MATHEMATICS

VECTORS

FE EXAM

Dot product of two vectors:

(1) $\boldsymbol{a} = \{ -2, 3, 1 \}$, $\boldsymbol{b} = \{ -1, 4, 3 \}$ — $\boldsymbol{a} \cdot \boldsymbol{b} = 17$

(2) $\boldsymbol{a} = \{ 1, 3, 2 \}$, $\boldsymbol{b} = \{ 4, -2, 5 \}$ — $\boldsymbol{a} \cdot \boldsymbol{b} = 8$

(3) $\boldsymbol{a} = \{ -2, 4, 1 \}$, $\boldsymbol{b} = \{ -3, -2, 3 \}$ — $\boldsymbol{a} \cdot \boldsymbol{b} = 1$

(4) $\boldsymbol{a} = \{ -2, 5, 4 \}$, $\boldsymbol{b} = \{ -3, 1, 2 \}$ — $\boldsymbol{a} \cdot \boldsymbol{b} = ?$

(5) $\boldsymbol{a} = \{ 3, 4, 2 \}$, $\boldsymbol{b} = \{ 1, 3, -2 \}$ — $\boldsymbol{a} \cdot \boldsymbol{b} = ?$

Cross product of two vectors:

(1) $\boldsymbol{a} = \{ 2, -3, 1 \}$, $\boldsymbol{b} = \{ 3, 2, 5 \}$ — $\boldsymbol{a} \times \boldsymbol{b} = \{ -17, -7, 13 \}$

(2) $\boldsymbol{a} = \{ 1, -2, 5 \}$, $\boldsymbol{b} = \{ 4, -2, 3 \}$ — $\boldsymbol{a} \times \boldsymbol{b} = \{ 4, 17, 6 \}$

(3) $\boldsymbol{a} = \{ 2, -4, 7 \}$, $\boldsymbol{b} = \{ 5, 3, 1 \}$ — $\boldsymbol{a} \times \boldsymbol{b} = \{ -25, 33, 26 \}$

(4) $\boldsymbol{a} = \{ -2, 3, 1 \}$, $\boldsymbol{b} = \{ 4, -3, 5 \}$ — $\boldsymbol{a} \times \boldsymbol{b} = \{ 18, 14, -6 \}$

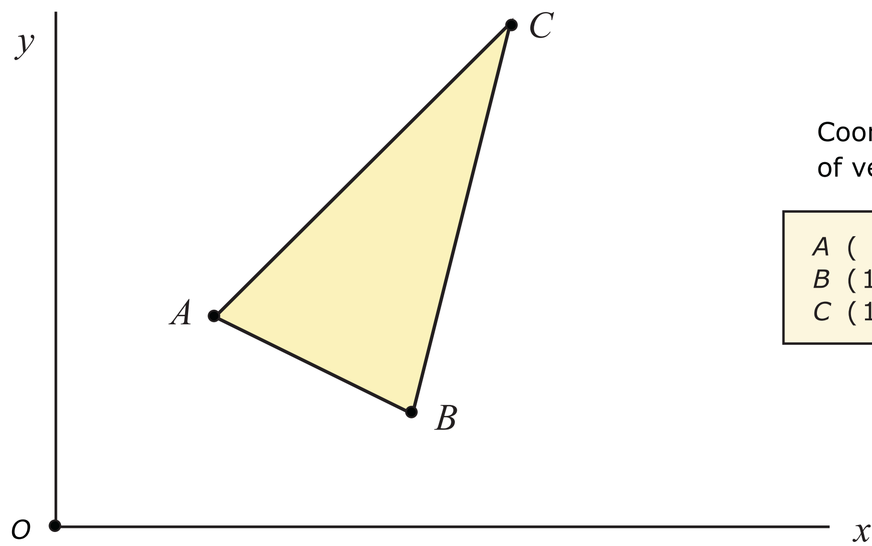
(5) $\boldsymbol{a} = \{ 2, 3, 4 \}$, $\boldsymbol{b} = \{ 1, 3, -2 \}$ — $\boldsymbol{a} \times \boldsymbol{b} = \{ -18, 8, 3 \}$

MATH-710
ZEYTINCI
SEPT. 2017

FUNDAMENTALS OF ENGINEERING (F.E.)

MATHEMATICS

AREAS OF TRIANGLES



Coordinates of vertices:

A	(8, 10)
B	(12, 8)
C	(14, 16)

A plane triangle is given as shown in the figure. Using the listed coordinates of the vertices A, B, and C answer the following:

The area (unit ^{2}) of this triangle is most nearly:

(A) 12
(B) 15
(C) 18
(D) 38

$A = ?$

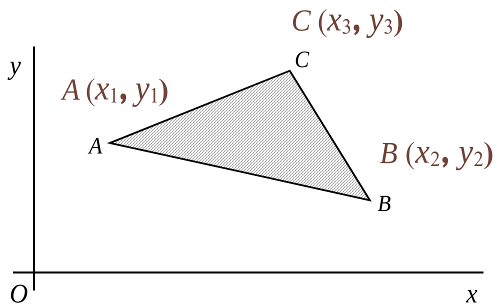
COMPLETE SOLUTION

MATH-12
ZEYTINCI
SEPT 2017

APPLICATIONS OF DETERMINANTS

Determinants may also be used in simplifying notation of complex mathematical expressions

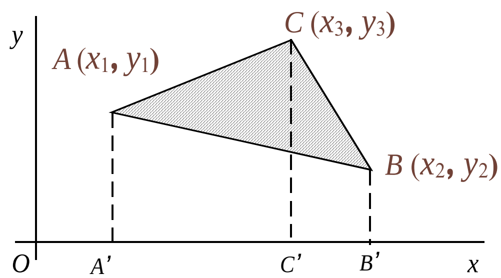
Example:



A triangle *ABC* is given as shown in the figure. Using the listed coordinates of *A*, *B*, and *C* determine the area of the triangle.

Solution:

Draw the vertical dashed lines from points *A*, *B*, and *C*.



Area of the Trapezoid $AA'CC' = \left(\dfrac{x_3 - x_1}{2}\right)(y_1 + y_3)$

Area of the Trapezoid $BB'CC' = \left(\dfrac{x_2 - x_3}{2}\right)(y_2 + y_3)$

Area of the Trapezoid $AA'BB' = \left(\dfrac{x_2 - x_1}{2}\right)(y_2 + y_1)$

Area of *ABC* = Area *AA'CC'*
+ Area *BB'CC'*
- Area *AA' BB'*

This will yield a fairly complex relationship for the area. But if we use determinant relationship, the required area may be calculated from the following simple determinant and applying the given data:

$$\text{Area} = ½ \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = ½ \begin{vmatrix} 8 & 10 & 1 \\ 12 & 8 & 1 \\ 14 & 16 & 1 \end{vmatrix} = 36/2 = 18$$

Correct Answer is (C)