THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

School of Engineering
Department of Electrical and Computer Engineering

332:224 Principles of Electrical Engineering II Laboratory

Experiment 1

# *Series and Parallel Resonance*

## 1 Introduction

**Objectives**

- To introduce frequency response by studying the characteristics of two resonant circuits on either side of resonance

**Overview**

In this experiment, the general topic of frequency response is introduced by studying the frequency-selectivity characteristics of two specific circuit structures. The first is referred to as the series-resonant circuit and the second as the parallel-resonant circuit.
The relevant equations and characteristic bell-shaped curves of the frequency response around resonance are given in section 2.
Prelab exercises are designed to enhance understanding of the concepts and calculate anticipated values subsequently measured in the lab.
Current and voltage are then measured in the two resonant circuits as functions of frequency and characteristic frequencies (resonance and 3-dB points) are experimentally determined.


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# 2 Theory

## 2.1 Frequency Domain Analysis

In electrical engineering and elsewhere, *frequency domain analysis* or otherwise known as *Fourier analysis* has been predominantly used ever since the work of French physicist Jean Baptiste Joseph Fourier in the early 19th century. The pioneering work of Fourier led to what are now known as Fourier Series representations of periodic signals and Fourier Transform representations of periodic signals. A periodic signal of interest in engineering can be represented in terms of a Fourier Series[1] which is a weighted linear combination of sinusoids of harmonically related frequencies. Each frequency among harmonically related frequencies is an integer multiple of a particular frequency known as the fundamental frequency. The number of harmonically related sinusoids present in the Fourier Series representation of a periodic signal could be finite or countably infinite. Since a periodic signal can be viewed as being composed of a number of sinusoids, in order to specify a periodic signal, one could equivalently specify the *amplitude and phase* of each sinusoid present in the signal. Such a specification constitutes the frequency domain description of a periodic signal. Similarly, in Fourier Transform representation, under some natural conditions, an aperiodic signal or a signal which is not necessarily periodic can be viewed as being composed of uncountably infinite number of sinusoids or a continuum of sinusoids. In this case, instead of being a weighted *sum* of harmonically related sinusoids, an aperiodic signal is a weighted *integral* of sinusoids of frequencies which are all not harmonically related. Again, instead of specifying an aperiodic signal in terms of the time variable t, one can equivalently specify the *amplitude and phase density* of each sinusoid of frequency $\omega$ contained in the signal. Such a description obviously uses the frequency variable $\omega$ as an independent variable, and thus it is said to be the frequency domain or $\omega$–domain description of the given time domain signal. In this way, a time domain signal is *transformed* to a frequency domain signal. Of course, once the frequency domain description of a signal is known, one can compose all the sinusoids present in the signal to form its time domain description. However, it is important to recognize that the frequency domain description is simply a mathematical tool. In engineering, signals exist in a physically meaningful domain such as time domain. The frequency domain description only serves to help for the better understanding of certain signal characteristics.

[1] A more detailed treatment can be found in the text starting with section 16.1.

## 2.2 Series Resonance

The basic series-resonant circuit is shown in fig. 1. Of interest here in how the steady state amplitude and the phase angle of the current vary with the frequency of the sinusoidal voltage source. As the frequency of the source changes, the maximum amplitude of the source voltage ($V_m$) is held constant.

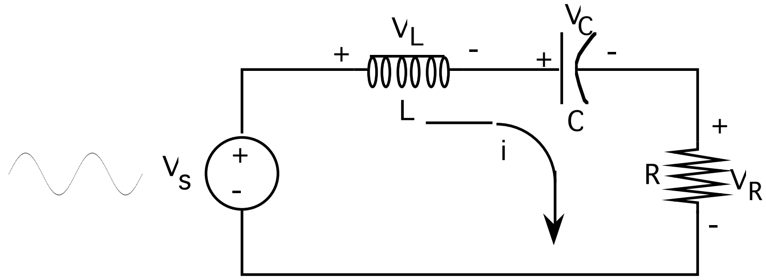


Fig. 1 The Series Resonant Circuit

$$V_s = V_m \cos(\omega t) \qquad\qquad i = I_m \cos(\omega t + \theta)$$

The frequency at which the reactances of the inductance and the capacitance cancel each other is the *resonant frequency* (or the unity power factor frequency) of this circuit. This occurs at

$$\omega_o = \frac{1}{\sqrt{LC}} \tag{1}$$

Since $i = V_R/R$, then the current $i$ can be studied by studying the voltage across the resistor. The current i has the expression

$$i = I_m \cos(\omega t + \theta)$$

where

$$I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{2A}$$

and

$$\theta = -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \tag{2B}$$

The bandwidth of the series circuit is defined as the range of frequencies in which the amplitude of the current is equal to or greater than $\left(1/\sqrt{2} = \sqrt{2}/2\right)$ times its maximum amplitude, as shown in fig. 2. This yields the bandwidth $B = \omega_2 - \omega_1 = R/L$

Where $$\omega_{2,1} = \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \pm \frac{R}{2L} \qquad (3)$$

$\omega_{2,1}$ are called the *half power frequencies* or the *3 dB frequencies*, i.e the frequencies at which the value of $I_m$ equals the maximum possible value divided by $\sqrt{2} = 1.414$ **.**

The quality factor $$Q = \frac{\omega_o}{B} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad (4)$$

Then the maximum value of :

1- $\boldsymbol{V_R}$ occurs at $$\omega = \omega_o \qquad (5A)$$

2- $V_L$ occurs at $$\frac{\omega_o}{\sqrt{1 - \frac{R^2C}{2L}}} \qquad (5B)$$

3- $V_C$ occurs at $$\omega_o\sqrt{1 - \frac{R^2C}{2L}} \qquad (5C)$$

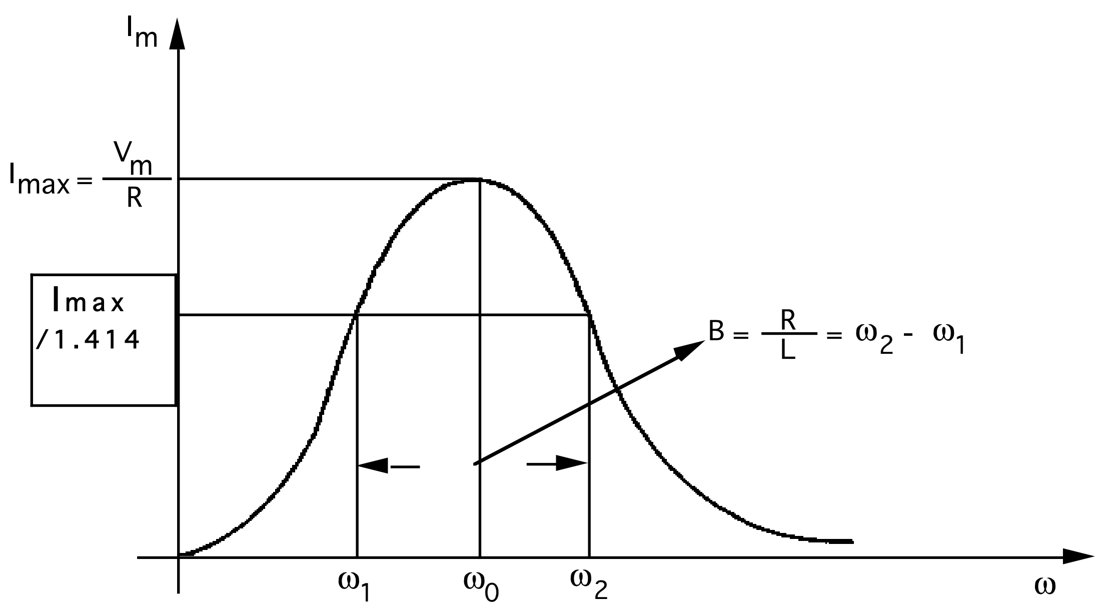


Fig. 2 Frequency Response of a Series - Resonant Circuit

## 2.3 Parallel Resonance

The basic parallel-resonant circuit is shown in fig. 3. Of interest here in how the steady state amplitude and the phase angle of the output voltage $V_0$ vary with the frequency of the sinusoidal voltage source.

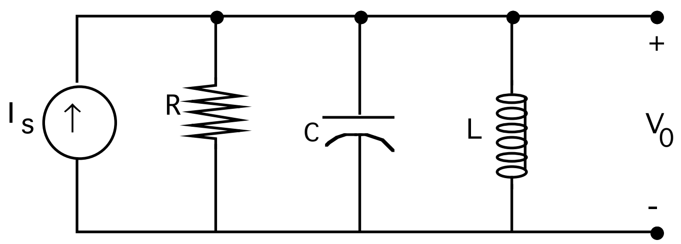


Fig. 3 The Parallel Resonant circuit

$I_s = I_m\cos(\omega t)$ $V_o = V_m\cos(\omega t+\theta)$

If $I_s = I_m\cos(\omega t)$, then $V_o = V_m\cos(\omega t+\theta)$ where

$$V_m = \frac{I_m}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}} \tag{6A}$$

and

$$\theta = -\tan^{-1}\left(R\left(\omega C - \frac{1}{\omega L}\right)\right) \tag{6B}$$

The resonant frequency is $\omega_o = \frac{1}{\sqrt{LC}}$

The 3 dB frequencies are: $$\omega_{2,1} = \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \pm \frac{1}{2RC} \tag{7}$$

The bandwidth $B = \omega_2 - \omega_1 = 1/RC.$

The quality factor $$Q = \frac{\omega_o}{B} = R\sqrt{\frac{C}{L}} \tag{8}$$

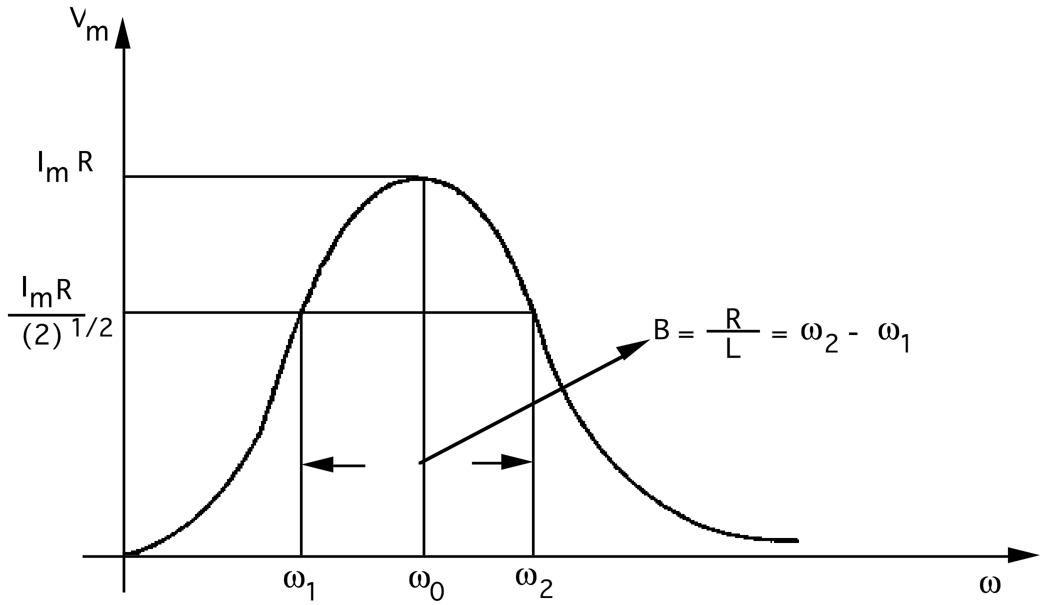

Fig. 4 Frequency Response of the Parallel - Resonant Circuit

## 2.4 A More Realistic Parallel Resonance Circuit

A more realistic parallel-resonant circuit is shown in fig. 5. It is a more realistic model because it accounts for the losses in the inductor through its d.c. resistance $R_L$.

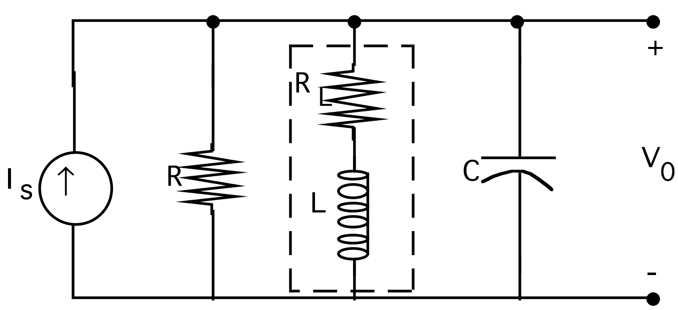

Fig. 5 A More Realistic Parallel - Resonant Circuit

In this case : $$\omega_o = \sqrt{\frac{1}{LC} - \left(\frac{R_L}{L}\right)^2} \qquad (9)$$

$$Z(\omega_o) = \frac{RL}{R_L RC + L} \qquad (10)$$

and

$$|V_o(\omega_o)| = |I_s(\omega_o)| \frac{RL}{R_L RC + L} \qquad (11)$$

An analysis of the amplitude of the output voltage as a function of frequency reveals that the amplitude is not maximum at $\omega_0$. It can be derived that $|V_0|$ is maximum when

$$\omega = \omega_m = (x - y)^{1/2} \qquad (12)$$

where $x = (a + b)^{1/2}$

$$a = \frac{1}{(LC)^2}\left(1 + \frac{2R_L}{R}\right) \qquad b = \left(\frac{R_L}{L}\right)^2 \frac{2}{LC} \quad \text{and} \quad y = \left(\frac{R_L}{L}\right)^2$$

This analysis can be followed by first expressing $V_o$ as a function of ω, differentiating this expression with respect to ω and then finding the value of ω that makes the derivative zero.

## 3 Prelab Exercises

3.1 Derive equations 1, 2, 3, and 4 for the series-resonant circuit in fig. 1.

3.2 Derive equations: 5A, 5B, and 5C for the series-resonant circuit in fig. 1.
HINT: $|V_R| = |I| R$ where $I = I_m$ is given by equation 2A. So $V_R$ is maximum when $I_m R$ is maximum i.e., $I_m$ is maximum (since R is constant). Similarly solve for $V_L = |I| Z_L$ and $V_C = |I| Z_C$.

3.3 For the series-resonant circuit shown in fig. 6, use equations: 5A, 5B, and 5C to determine the frequencies at which $V_R$, $V_C$, and $V_{L+RL}$ are maximum.

# 4 Experiments

Suggested Equipment:

Tektronix FG 501A 2MHz Function Generator[2]
Tektronix 504A Counter - Timer
HP 54600A or Agilent 54622A Oscilloscope
Protek Model B-845 Digital Multimeter
LS-400A Inductance Substituter Box
620 Ω Resistor
0.1 μF Capacitor
Breadboard
Other circuit elements to be determined by the students.

## 4.1 Series Resonance

Any function generator used has internal resistance. Also, the inductor has internal resistance. Both need to be determined since all resistances affect the behavior of the circuit.

### Function generator resistance

The internal resistance of the function generator will affect the damping of an RLC circuit to which it is connected. Check the resistance in the following way:

**a**- With a sine wave output, set the open circuit voltage to some convenient value, say 1V.

**b**- Connect a pure variable resistance load (potentiometer) thus forming a voltage divider. Adjust R until the terminal voltage falls to one-half the open circuit value. At this point the two resistances of the voltage divider have to be equal. Therefore, the resistance of the potentiometer should now be equal to the internal resistance of the function generator. Disconnect the potentiometer from the circuit and measure its resistance.

### Inductor internal resistance

Use the digital ohmmeter to measure the internal resistance of the inductor used.

Measure $R_s$ and $R_L$.

$R_s$ =_____________Ω. $R_L$=_____________Ω.

[2] NOTE: The oscillator is designed to work for a very wide range of frequencies but may not be stable at very low frequencies, say in the order of 100 Hz or 200Hz. To start with it is a good idea to have the circuit working at some mid-range frequency, say in the order of 1K Hz or 2K Hz, and then change the frequency slowly as needed.

Build the circuit shown in fig. 6 using R = 620 Ω, L = 100 mH, and C = 0.1 μF.
Apply a sinusoidal input to the circuit and display both input and output on the screen of the oscilloscope.

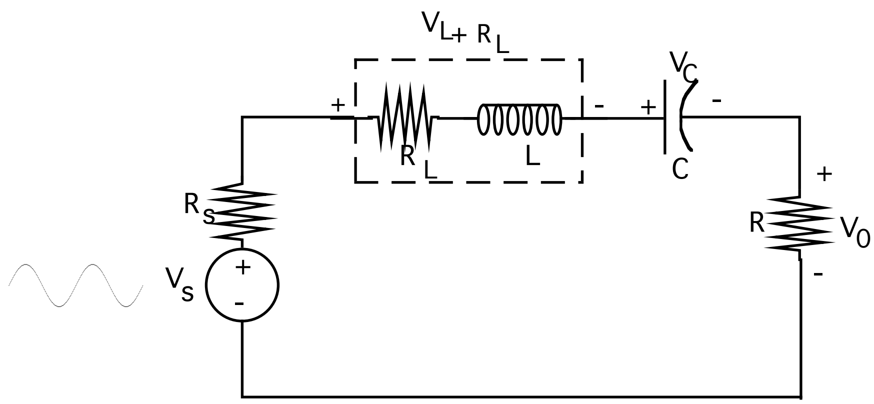


Fig. 6 A Series - Resonant Circuit

With the frequency varied from 600 Hz to 2,500 Hz in increments of 100 Hz (using the frequency counter), measure the rms values of $V_R$, $V_{L+RL}$, and $V_C$ using the DVM and the phase angle from the scope (take the phase angle of $V_s$ as the reference). Download the scope trace for your report.

The phase angle between two sinusoidal signals of the same frequency can be determined as follows: Trace both signals on two different channels with the same horizontal sensitivities (the same horizontal scale). To calibrate the horizontal scale in terms of degrees, one can use the fact that the angular difference between the two successive zero crossing points of a sinusoidal signal is 180 degrees. Thus, by measuring the distance between the successive zero crossing points of either sinusoidal signal, one can calibrate the horizontal scale in terms of degrees. To determine the phase difference between the two sinusoidal signals, determine the distance between the zero crossing point of one signal to a similar zero crossing point of another signal and convert it into degrees.

Also, to save tedious calculations later, set the rms values of $V_s$ to 1.00 volt before each reading. Make sure that you use the frequency counter for all frequency measurements, and to note the exact frequencies at which $V_R$, $V_C$, and $V_{L+RL}$ are maximum.

Once the maximum output voltage ($V_0 = V_R$) is known, vary the frequency and find the 3 dB (the half power) frequencies, $f_{1,2}$.

Before dismantling the equipment, check your results against those obtained from the theoretical relationships in eqs 3 & 5. (Make sure to account for the internal resistance of the function generator and the d.c. resistance $R_L$ of the inductor L in all calculations.)

| f nominal | f (Hz) | $V_R$ | $V_{L+RL}$ | $V_C$ | $\phi$ |
|---|---|---|---|---|---|
| 600 | | | | | |
| 700 | | | | | |
| 800 | | | | | |
| 900 | | | | | |
| 1,000 | | | | | |
| 1,100 | | | | | |
| 1,200 | | | | | |
| 1,300 | | | | | |
| 1,400 | | | | | |
| 1,500 | | | | | |
| 1,600 | | | | | |
| 1,700 | | | | | |
| 1,800 | | | | | |
| 1,900 | | | | | |
| 2,000 | | | | | |
| 2,100 | | | | | |
| 2,200 | | | | | |
| 2,300 | | | | | |
| 2,400 | | | | | |
| 2,500 | | | | | |

## 4.2 Parallel Resonance

Using source transformation, the parallel - resonant circuit in fig. 5 can be represented as shown in fig. 7 where $R_s$ is the internal resistance of the function generator.

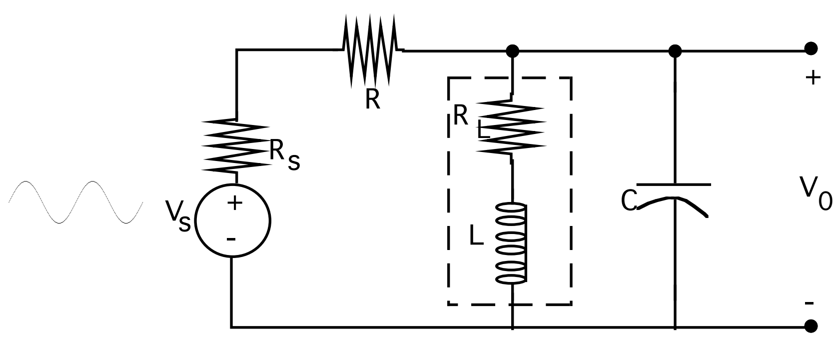


Fig. 7 A Parallel - Resonant Circuit

Build the circuit of fig. 7 using R = 620 Ω, L = 100 mH, and C = 0.1 µF.

Apply a sinusoidal input to the circuit and display both input and output on the scope. Set the rms value of $V_s$ = 1.00 volts.

With the frequency of the source varied from 600 Hz to 2,500 Hz in increments of 100 Hz (using the frequency counter), measure $V_0$ using the DVM, and the phase angle using the scope. Download the scope trace for your report.

Make sure to note the exact frequency, $f_m$, at which $V_0$ is maximum.

Once the maximum output voltage is known,, increase the frequency from 200 Hz and find the 3 dB frequencies, $f_{1,2}$.

Before dismantling the equipment, check the measured $f_m$ against the theoretical one obtained from eq. 12.

| f nominal | f (Hz) | $V_o$ | $\phi$ |
|---|---|---|---|
| 600 | | | |
| 700 | | | |
| 800 | | | |
| 900 | | | |
| 1,000 | | | |
| 1,100 | | | |
| 1,200 | | | |
| 1,300 | | | |
| 1,400 | | | |
| 1,500 | | | |
| 1,600 | | | |
| 1,700 | | | |
| 1,800 | | | |
| 1,900 | | | |
| 2,000 | | | |
| 2,100 | | | |
| 2,200 | | | |
| 2,300 | | | |
| 2,400 | | | |
| 2,500 | | | |

## 5 Report

5.1 In pre-lab exercise 3.3, by using equations: 5A, 5B, and 5C, the frequencies were determined at which $V_R$, $V_C$, and $V_{L+RL}$ are maximum. Compare them with those experimentally observed.

5.2 Tabulate the frequency f, $V_R$, $V_C$, and $V_{L+RL}$ and the phase angle measured in Section 4.1. Print out the scope trace and show how the phase angle was measured.

5.3 Plot $V_R$, $V_C$, $V_{L+RL}$ vs frequency on the same graph paper with rectangular coordinates. Circle, on the plot, the resonant frequency and the 3 dB frequencies.

5.4 Use eqs. 1 & 3 to determine the theoretical resonant frequency, the 3 dB frequencies, and the bandwidth. Compare with the experimental ones.

5.5 Tabulate f, $V_0$ and the phase angle measure in Section 4.2. Print out the scope trace and show how the phase angle was measured.

5.6 Using eqs. 9 & 12, determine the theoretical $f_0$, and $f_m$ for the resonant circuit shown in fig. 7. Compare with the experimental ones.

5.7 Plot $V_0$ vs f on a graph paper with rectangular coordinates. Circle, on the plot, $f_0$, $f_m$, and the 3 dB frequencies, $f_{1,2}$.

5.8 Simulate the series-resonant circuit of fig. 6 in PSpice, and plot $V_R$, $V_C$, and $V_{L+RL}$ vs frequency. Vary the frequency from 600 Hz to 2,500 Hz in increments of 100 Hz. Compare with the experimental plot.

5.9 Simulate the parallel-resonant circuit of fig. 7 in PSpice, and plot $V_0$ vs frequency. Vary the frequency from 600 Hz to 2,500 Hz in increments of 100 Hz. Compare with the experimental plot.

5.10 Prepare a summary.