CRIMSI Pilot PL Session 8

Welcome to Carnegie Learning's Texas Math Solution

Brandy King, Texas Content Lead
Jennifer Penton, Texas Product Specialist
Alejandra Neira, *Manager of School Partnerships*

Why We’re Here

By the end of today’s live session, participants will be able to...

- Define how the key mathematical concepts flow across this course and how they are connected to other courses.
- Identify the Big Ideas in each Module within this course.
- Connect how this big picture view of each course fits into the topic and lesson level internalization.
- Walk away with concrete tools and processes to use in their instructional planning.

When it comes to the sequence of mathematics concepts in your course:

K KNOW	W WANT TO KNOW	L LEARNED
What do you ***already*** know?	What do you ***want*** to know?	*Leave this blank: we'll come back to it later!*

Think Time

K-W-L Chart

The Story of This Course

Geometry and Algebra II Modules 1 and 2

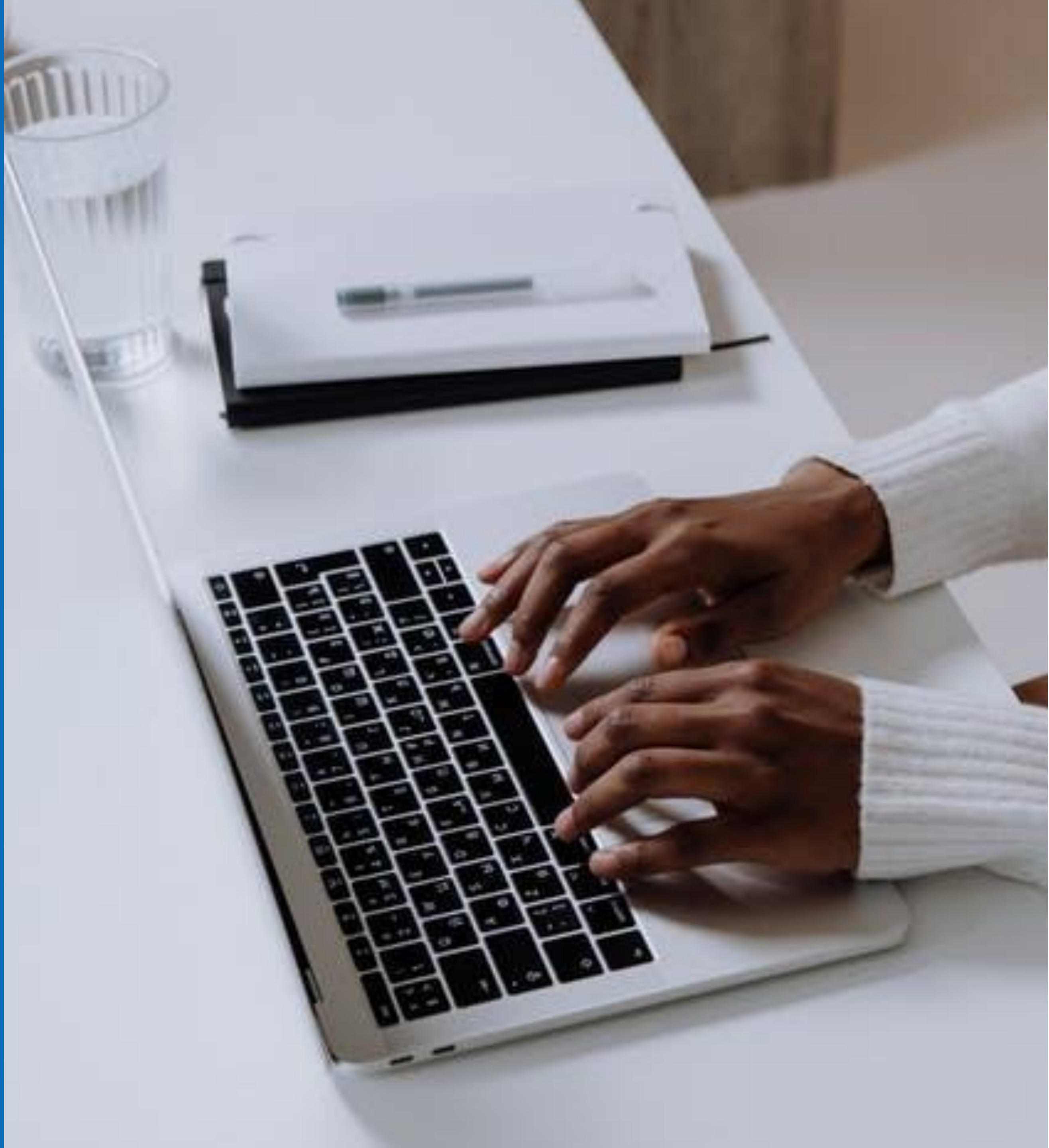
Geometry Module 1 Reasoning with Shapes	Geometry Module 2 Establishing Congruence	Algebra 2 Module 1 Exploring Patterns in Linear and Quadratic Relationships	Algebra 2 Module 2 Analyzing Structure
Topic 1: Using a Rectangular Coordinate System	Topic 1: Composing and Decomposing Shapes	Topic 1: Extending Linear Relationships	Topic 1: Composing and Decomposing Functions
Topic 2: Rigid Motions on a Plane	Topic 2: Justifying Line and Angle Relationships	Topic 2: Exploring and Analyzing Patterns	Topic 2: Characteristics of Polynomial Functions
Topic 3: Congruence Through Transformations	Topic 3: Using Congruence Theorems	Topic 3: Applications of Quadratics	

Using the Zoom Annotation Tools/Stickers, mark on the shared screen your current position (Module/Topic) in the instructional materials. Or use chat to tell us your module/topic location.

Summarizing Module 1

In chat, provide a short summary of the content of module 1 for your course. Type A2 or Geo before your summary.

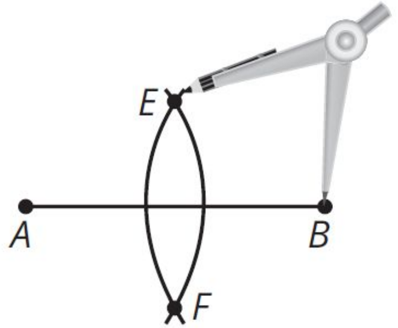
- *Try not to use module or topic names as you summarize*

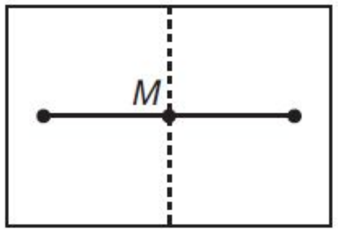


Geometry Module 1: Reasoning with Shapes

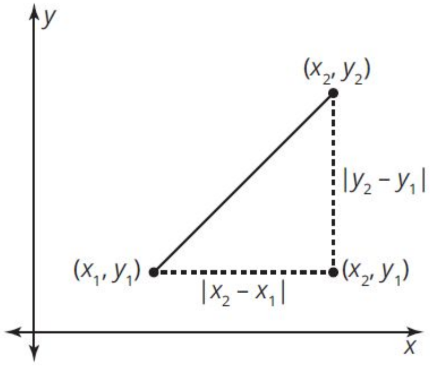
Topic 1: Using the Rectangular Coordinate System

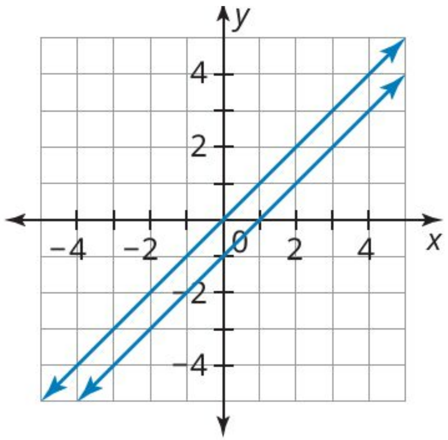
- Informal → Formal Reasoning
- Measurement tools → Constructions
- Connecting Algebra to Geometry through the coordinate plane
- Distance and midpoint formulas to classify shapes
- Proportional and non-proportional changes and the effect on area and perimeter
- First experience with Formal Proofs

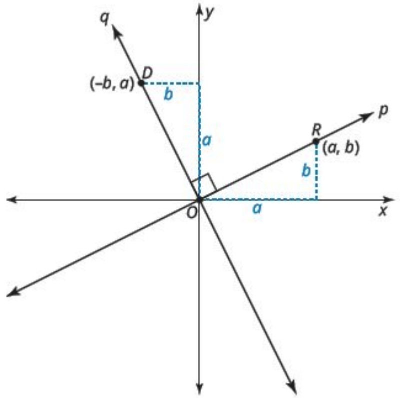


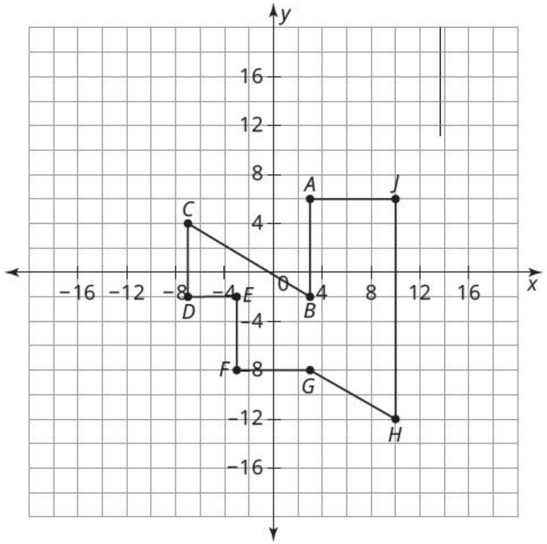


$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$









Geometry Module 1: Reasoning with Shapes

Topic 2: Rigid Motions on a Plane

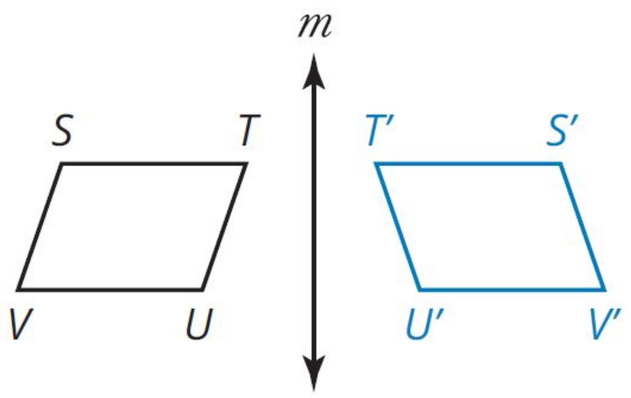
- Rigid motions as functions
- Isometries preserve size and shape
- Rigid motions on the coordinate plane
- Reflectional and Rotational Symmetry

A translation function maps each point of a pre-image an equal distance along parallel lines onto an image.

$$T_{AB}(P) = P'$$
$$T_{AC}(P) = P''$$

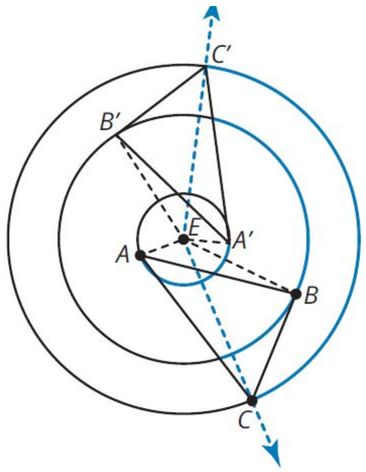
A reflection function maps each point of a pre-image across a line onto the image so that each point is equidistant from the line of reflection.

$$R_m(STUV)$$



A rotation function rotates every point in a pre-image around arcs of concentric circles at a specific rotation angle.

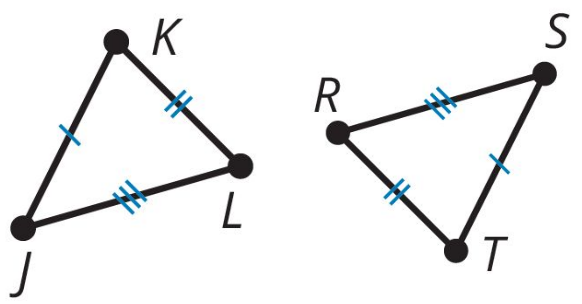
$$R_{E,\,150}(\triangle ABC)$$



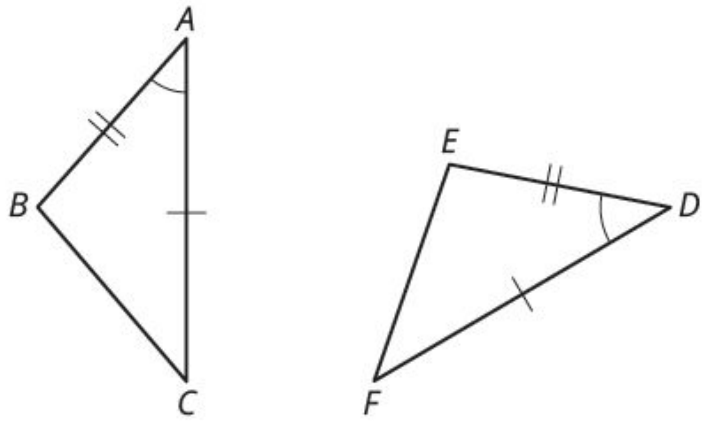
Geometry Module 1: Reasoning with Shapes

Topic 3: Congruence Through Transformations

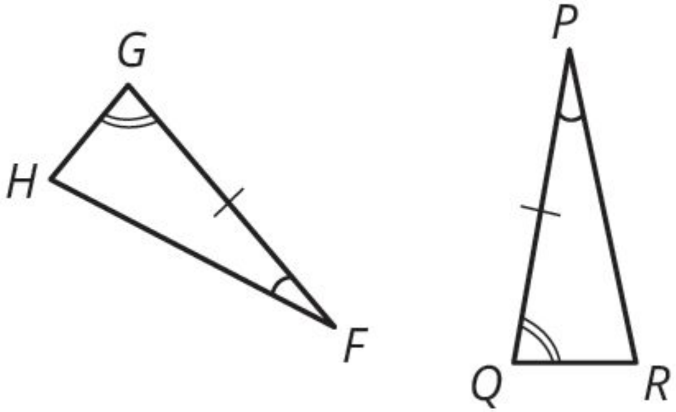
- Students use formal reasoning to prove geometric theorems.
- Triangle congruence theorems are proved using constructions.
- Students use the triangle congruence theorems to determine whether triangles are congruent.
- The Distance Formula is used to apply the congruence theorems to triangles with given measurements on the plane.



Side-Side-Side Congruence Theorem (SSS)



Side-Angle-Side Congruence Theorem (SAS)



Angle-Side-Angle Congruence Theorem (ASA)

Algebra II Module 1: Exploring Patterns in Linear and Quadratic Relationships

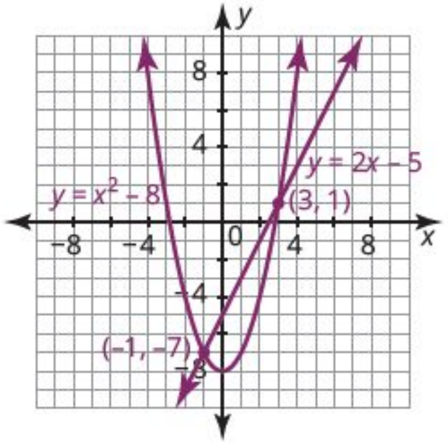
Topic 1: Extending Linear Relationships

Extending Linear Relationships advances students' ability to solve systems of equations and introduces students to absolute value functions.

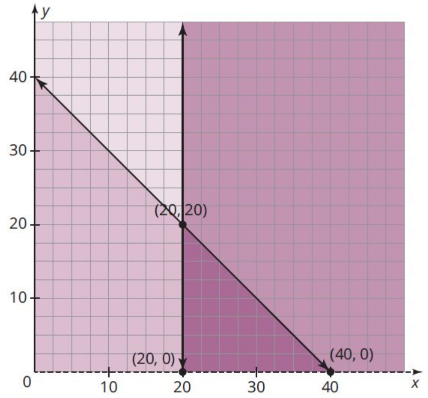
$$\begin{bmatrix} 2 & 8 & 3 \\ 7 & -5 & 1 \\ -6 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -21 \\ 72 \\ 17 \end{bmatrix}$$

$$A \cdot X = B$$

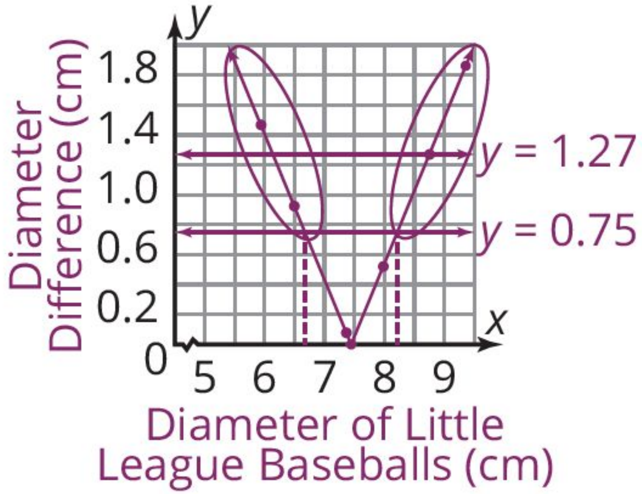
Students use Gaussian elimination and matrices to solve systems of linear equations in three variables.



Students solve systems consisting of a linear and a quadratic equation.



Student explore linear programming, where they use the vertices of the solution region to determine maximum or minimum values.

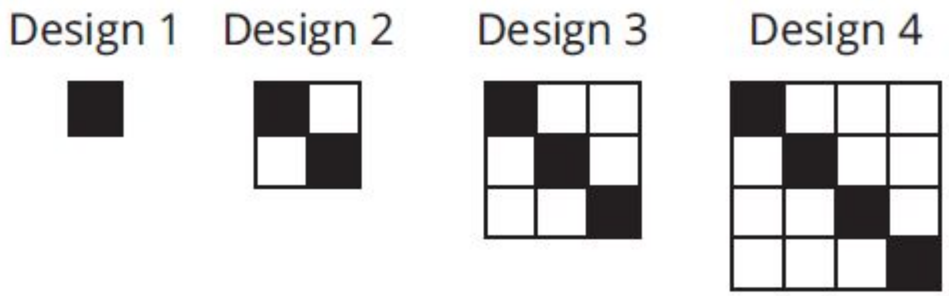


They graph, transform and then solve absolute value equations and inequalities.

Algebra II Module 1: Exploring Patterns in Linear and Quadratic Relationships

Topic 2: Exploring and Analyzing Patterns

Exploring and Analyzing Patterns provides opportunities for students to analyze and describe various patterns. Lessons provide opportunities for students to review linear, exponential, and quadratic functions using multiple representations.



Quadratic functions can be written in different forms.

Standard form: $f(x) = ax^2 + bx + c$, where a does not equal 0.

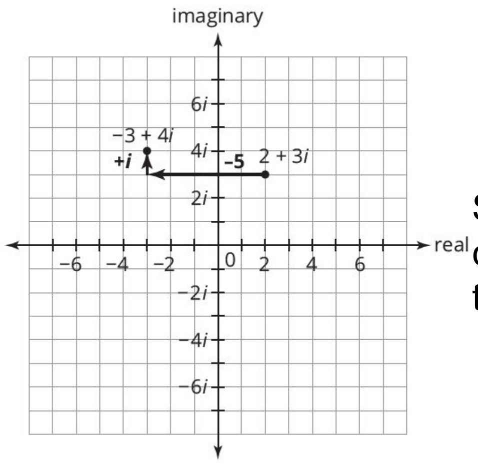
Factored form: $f(x) = a(x - r_1)(x - r_2)$, where a does not equal 0.

Vertex form: $f(x) = a(x - h)^2 + k$, where a does not equal 0.

$$x^2 - 4x = -3$$
$$x^2 - 4x + 3 = -3 + 3$$
$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$

$(x - 3) = 0$	and	$(x - 1) = 0$
$x - 3 + 3 = 0 + 3$	and	$x - 1 + 1 = 0 + 1$
$x = 3$	and	$x = 1$

Students use factoring, completing the square and the quadratic formula to solve quadratic equations.



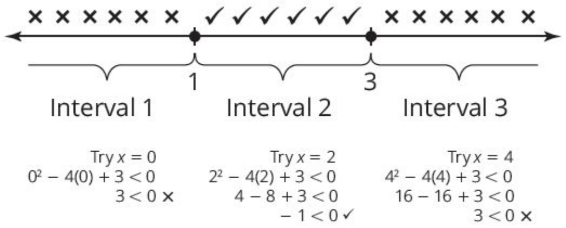
Students are introduced to complex number operations through the complex plane.

Algebra II Module 1: Exploring Patterns in Linear and Quadratic Relationships

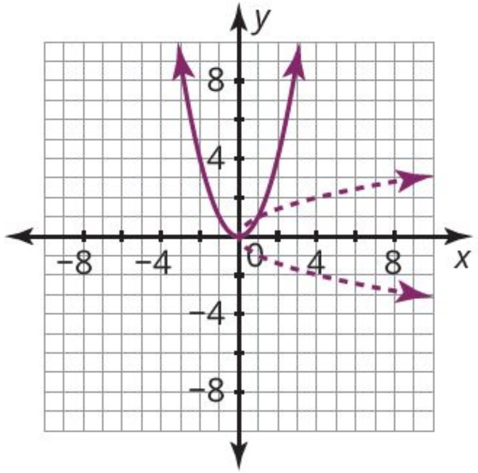
Topic 3: Applications of Quadratics

Applications of Quadratics provides students with an opportunity to review what they have learned about quadratics in Algebra 1 by modeling and solving problems for situations involving quadratics.

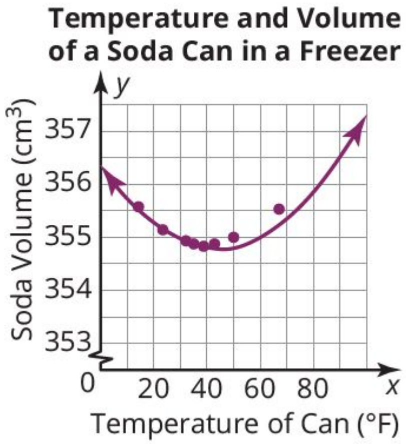
$$x^2 - 4x + 3 < 0.$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$(x-3) = 0 \quad \text{or} \quad (x-1) = 0$$
$$x = 3 \quad \text{or} \quad x = 1$$



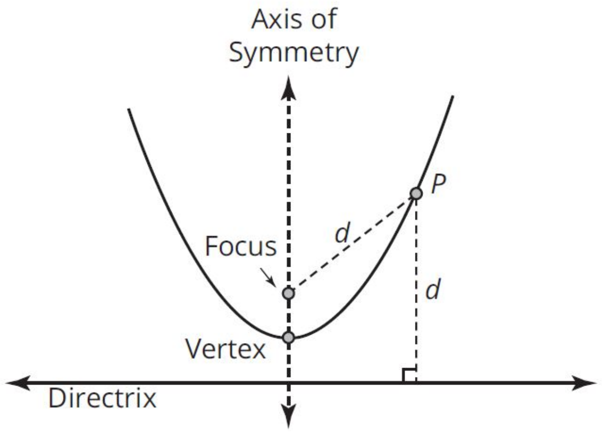
Quadratic inequalities are solved graphically and algebraically.



Students explore inverse functions and determine inverses from a graph, table, and equation.

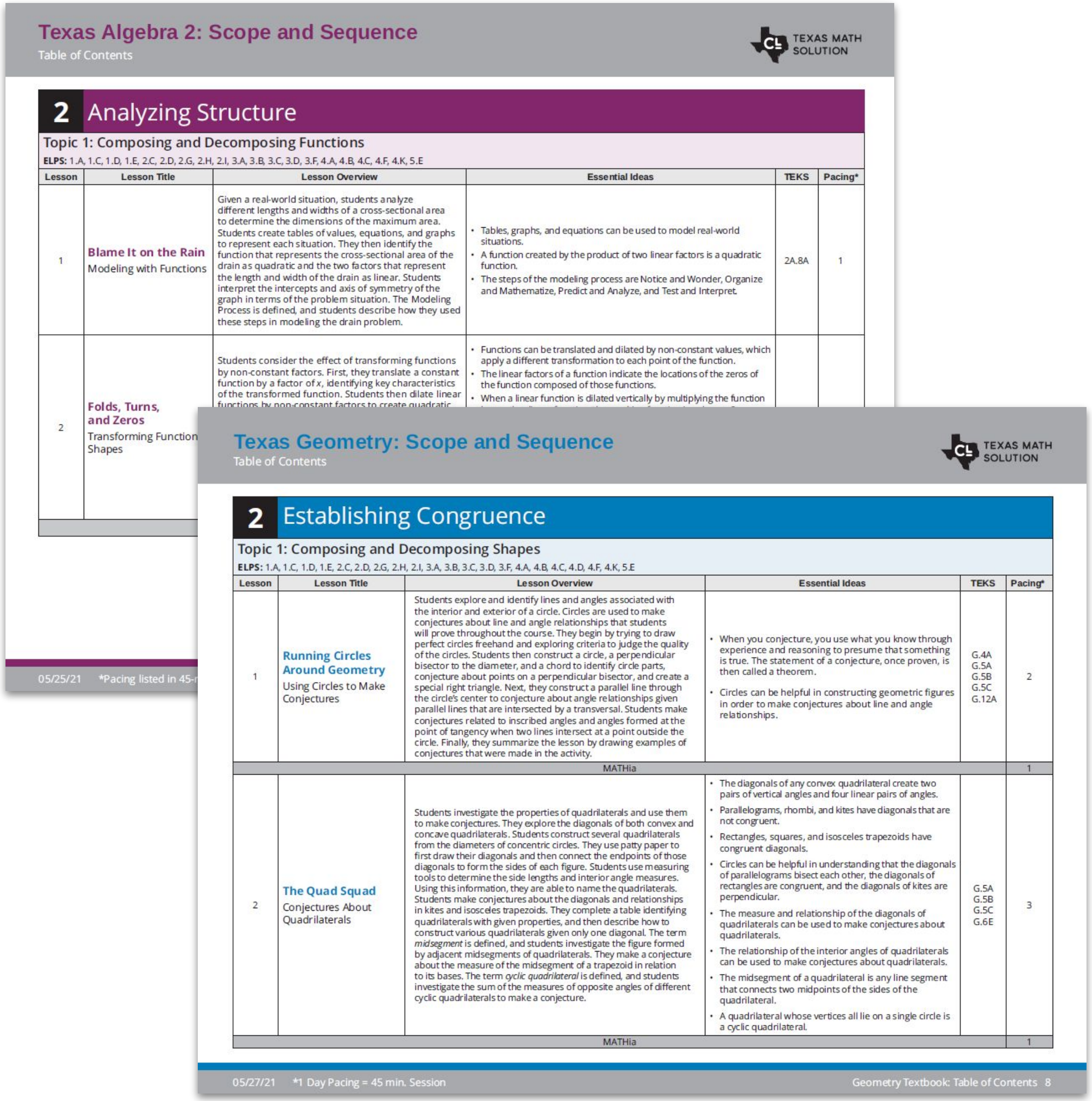


Students determine a quadratic regression equation to model a set of data and use the regression equation to make predictions.



Students explore parabolas as a conic section and write the general and standard equations.

Summarizing Module 2



Texas Algebra 2: Scope and Sequence
Table of Contents

TEXAS MATH SOLUTION

2 Analyzing Structure

Topic 1: Composing and Decomposing Functions

ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E

Lesson	Lesson Title	Lesson Overview	Essential Ideas	TEKS	Pacing*
1	**Blame It on the Rain** Modeling with Functions	Given a real-world situation, students analyze different lengths and widths of a cross-sectional area to determine the dimensions of the maximum area. Students create tables of values, equations, and graphs to represent each situation. They then identify the function that represents the cross-sectional area of the drain as quadratic and the two factors that represent the length and width of the drain as linear. Students interpret the intercepts and axis of symmetry of the graph in terms of the problem situation. The Modeling Process is defined, and students describe how they used these steps in modeling the drain problem.	• Tables, graphs, and equations can be used to model real-world situations. • A function created by the product of two linear factors is a quadratic function. • The steps of the modeling process are Notice and Wonder, Organize and Mathematize, Predict and Analyze, and Test and Interpret.	2A.8A	1
2	**Folds, Turns, and Zeros** Transforming Function[…] Shapes	Students consider the effect of transforming functions by non-constant factors. First, they translate a constant function by a factor of *x*, identifying key characteristics of the transformed function. Students then dilate linear functions by non-constant factors to create quadratic […]	• Functions can be translated and dilated by non-constant values, which apply a different transformation to each point of the function. • The linear factors of a function indicate the locations of the zeros of the function composed of those functions. • When a linear function is dilated vertically by multiplying the function […]		

05/25/21 *Pacing listed in 45-[…]

Texas Geometry: Scope and Sequence
Table of Contents

TEXAS MATH SOLUTION

2 Establishing Congruence

Topic 1: Composing and Decomposing Shapes

ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.F, 4.K, 5.E

Lesson	Lesson Title	Lesson Overview	Essential Ideas	TEKS	Pacing*
1	**Running Circles Around Geometry** Using Circles to Make Conjectures	Students explore and identify lines and angles associated with the interior and exterior of a circle. Circles are used to make conjectures about line and angle relationships that students will prove throughout the course. They begin by trying to draw perfect circles freehand and exploring criteria to judge the quality of the circles. Students then construct a circle, a perpendicular bisector to the diameter, and a chord to identify circle parts, conjecture about points on a perpendicular bisector, and create a special right triangle. Next, they construct a parallel line through the circle's center to conjecture about angle relationships given parallel lines that are intersected by a transversal. Students make conjectures related to inscribed angles and angles formed at the point of tangency when two lines intersect at a point outside the circle. Finally, they summarize the lesson by drawing examples of conjectures that were made in the activity.	• When you conjecture, you use what you know through experience and reasoning to presume that something is true. The statement of a conjecture, once proven, is then called a theorem. • Circles can be helpful in constructing geometric figures in order to make conjectures about line and angle relationships.	G.4A G.5A G.5B G.5C G.12A	2
MATHia					1
2	**The Quad Squad** Conjectures About Quadrilaterals	Students investigate the properties of quadrilaterals and use them to make conjectures. They explore the diagonals of both convex and concave quadrilaterals. Students construct several quadrilaterals from the diameters of concentric circles. They use patty paper to first draw their diagonals and then connect the endpoints of those diagonals to form the sides of each figure. Students use measuring tools to determine the side lengths and interior angle measures. Using this information, they are able to name the quadrilaterals. Students make conjectures about the diagonals and relationships in kites and isosceles trapezoids. They complete a table identifying quadrilaterals with given properties, and then describe how to construct various quadrilaterals given only one diagonal. The term *midsegment* is defined, and students investigate the figure formed by adjacent midsegments of quadrilaterals. They make a conjecture about the measure of the midsegment of a trapezoid in relation to its bases. The term *cyclic quadrilateral* is defined, and students investigate the sum of the measures of opposite angles of different cyclic quadrilaterals to make a conjecture.	• The diagonals of any convex quadrilateral create two pairs of vertical angles and four linear pairs of angles. • Parallelograms, rhombi, and kites have diagonals that are not congruent. • Rectangles, squares, and isosceles trapezoids have congruent diagonals. • Circles can be helpful in understanding that the diagonals of parallelograms bisect each other, the diagonals of rectangles are congruent, and the diagonals of kites are perpendicular. • The measure and relationship of the diagonals of quadrilaterals can be used to make conjectures about quadrilaterals. • The relationship of the interior angles of quadrilaterals can be used to make conjectures about quadrilaterals. • The midsegment of a quadrilateral is any line segment that connects two midpoints of the sides of the quadrilateral. • A quadrilateral whose vertices all lie on a single circle is a cyclic quadrilateral.	G.5A G.5B G.5C G.6E	3
MATHia					1

05/27/21 *1 Day Pacing = 45 min. Session Geometry Textbook: Table of Contents 8

Use the Course Scope and Sequence document to get an overview of the content of Module 2.

- What are the key concepts?
- What connections exist within this module or back to Module 1?

Be prepared to provide a summary statement in chat when directed.

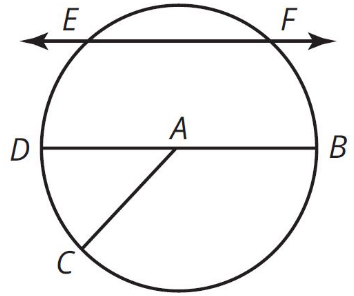
Geometry Course Overview

Module 1 Reasoning with Shapes	Module 2 Establishing Congruence	Module 3 Investigating Proportionality	Module 4 Connecting Geometric and Algebraic Descriptions	Module 5 Making Informed Decisions
Topic 1: Using a Rectangular Coordinate System	Topic 1: Composing and Decomposing Shapes	Topic 1: Similarity	Topic 1: Circles and Volume	Topic 1: Independence and Conditional Probability
Topic 2: Rigid Motions on a Plane	Topic 2: Justifying Line and Angle Relationships	Topic 2: Trigonometry	Topic 2: Circles and Cross Sections	Topic 2: Computing Probabilities
Topic 3: Congruence Through Transformations	Topic 3: Using Congruence Theorems			

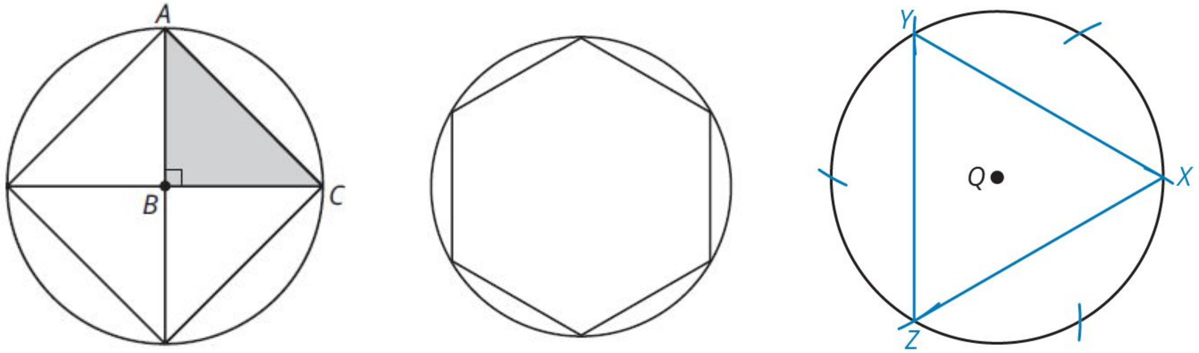
Geometry Module 2: Establishing Congruence

Topic 1: Composing and Decomposing Shapes

- In this topic we are moving toward abstraction.
- The content builds from shapes students are already familiar with, squares, circles, and quadrilaterals.
- Conjecturing in this topic prepares students to write formal proofs in the next topic.
- Circles are used to conjecture about line and angle relationships as well as quadrilaterals.



introduction to circle vocabulary: central angle, major arc, minor arc, secant, chord, inscribed angle, intercepted arc, circumscribed angle, tangent



three major constructions: inscribing a square, an equilateral triangle, and a regular hexagon in a circle

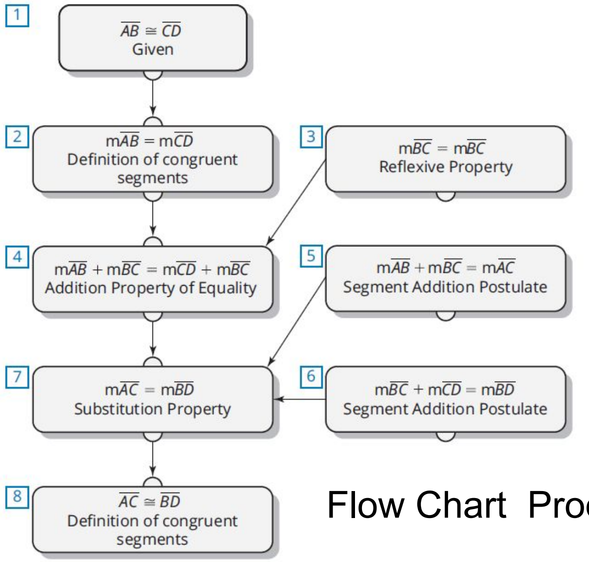
	Acute Triangle	Obtuse Triangle	Right Triangle
Circumcenter	Interior	Exterior	On Hypotenuse
Incenter	Interior	Interior	Interior
Centroid	Interior	Interior	Interior
Orthocenter	Interior	Exterior	At Vertex of Right Angle

Students investigate points of concurrency through constructions

Geometry Module 2: Establishing Congruence

Topic 2: Justifying Line and Angle Relationships

- This topic moves from the conjectures made in Composing and Decomposing Shapes to formal proofs.
- Students have the opportunity to experience proofs before having to write them entirely on their own.
- Have students develop proof plans before they write formal proofs.



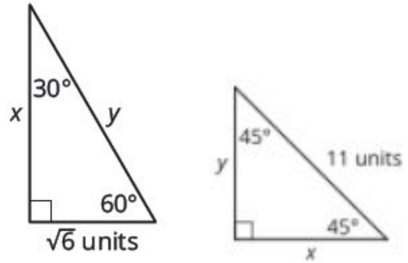
Flow Chart Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $m\overline{AB} = m\overline{CD}$	2. Definition of congruent segments
3. $m\overline{BC} = m\overline{BC}$	3. Reflexive Property
4. $m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$	4. Addition Property of Equality
5. $m\overline{AB} + m\overline{BC} = m\overline{AC}$	5. Segment Addition Postulate
6. $m\overline{BC} + m\overline{CD} = m\overline{BD}$	6. Segment Addition Postulate
7. $m\overline{AC} = m\overline{BD}$	7. Substitution Property
8. $\overline{AC} \cong \overline{BD}$	8. Definition of congruent segments

Two-column Proof

Proof plan

Abelina	Madison
I'll show that alternate interior angles 4 and 5 are congruent. It's given that the two lines are parallel and are cut by a transversal, so I know that angles 4 and 6 are supplementary, because they are same-side interior angles. And angles 5 and 6 are supplementary, because they are a linear pair. So, $m\angle 4 + m\angle 6 = 180°$. But $m\angle 5 + m\angle 6$ is also equal to 180°. That means that angles 4 and 5 have to have the same measure.	I need to prove that angles 3 and 6 are congruent, which means they have the same measure. The Given states that the lines are parallel and cut by a transversal, so angles 2 and 6 are congruent, because they are corresponding. But angles 2 and 3 are congruent too, because they are vertical angles. If $\angle 3$ and $\angle 6$ are both congruent to the same angle $\angle 2$, then they must be congruent to each other.

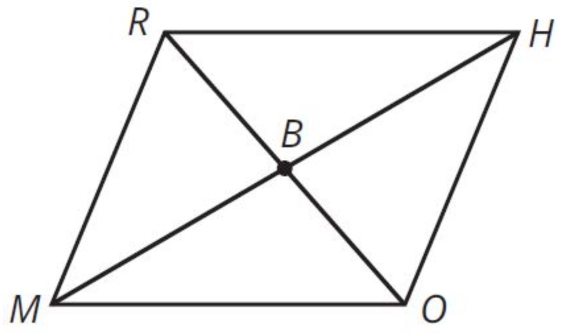


Students are reminded that corresponding parts of congruent triangles are congruent (CPCTC) and then learn how to use CPCTC as a reason in proofs. They consider special types of right triangles—45°-45°-90° and 30°-60°-90° triangles—and think about the relationship between their sides and angles in preparation for trigonometry.

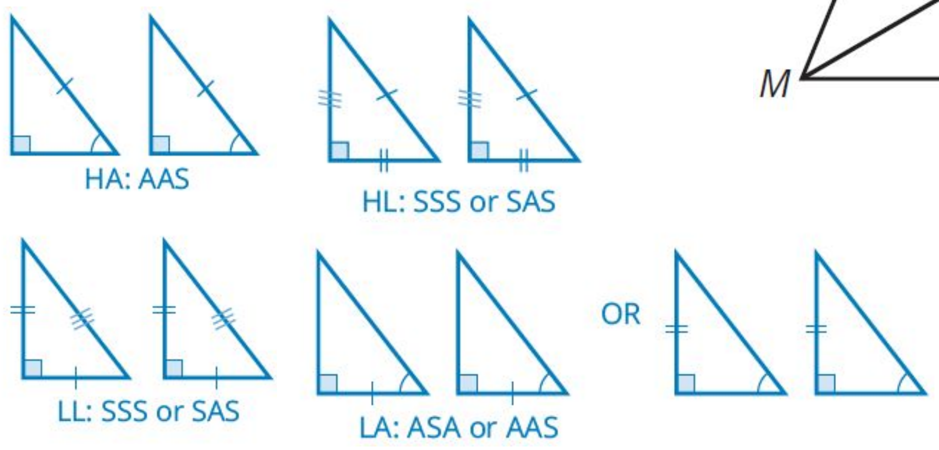
Geometry Module 2: Establishing Congruence

Topic 3: Using Congruence Theorems

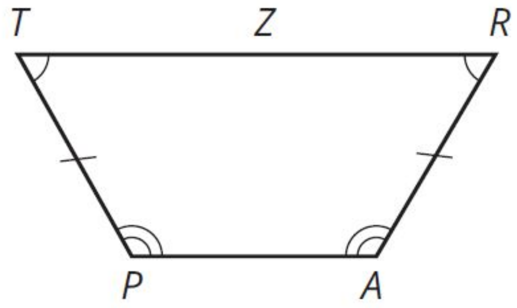
Students use the theorems that they have proven to prove new theorems about triangles, quadrilaterals, and relationships between chords.



Students use triangle congruence theorems to verify properties of parallelograms.



Students use the four triangle congruence theorems to prove three additional right triangle congruence theorems: the Hypotenuse-Leg Congruence Theorem, the Leg-Leg Congruence Theorem, and the Leg-Angle Congruence Theorem.



Note: This text uses the inclusive definition of a trapezoid: a trapezoid is a quadrilateral with at least one pair of parallel sides.

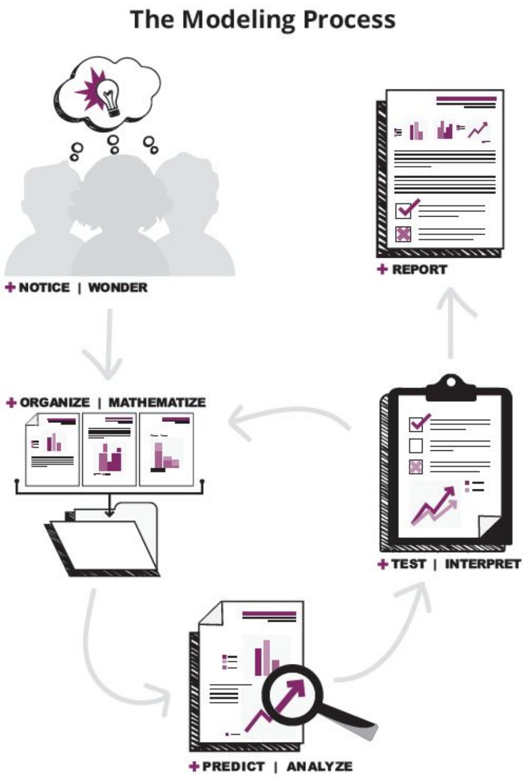
Algebra II Course Overview

Module 1 Exploring patterns in Linear and Quadratic Relationships	Module 2 Analyzing Structure	Module 3 Developing Structural Similarities	Module 4 Extending Beyond Polynomials	Module 5 Inverting Functions
Topic 1: Extending Linear Relationships	Topic 1: Composing and Decomposing Functions	Topic 1: Relating Factors and Zeros	Topic 1: Rational Functions	Topic 1: Exponential and Logarithmic Functions
Topic 2: Exploring and Analyzing Patterns	Topic 2: Characteristics of Polynomial Functions	Topic 2: Polynomial Models	Topic 2: Radical Functions	Topic 2: Exponential and Logarithmic Equations
Topic 3: Applications of Quadratics				Topic 3: Applications of Exponential Functions

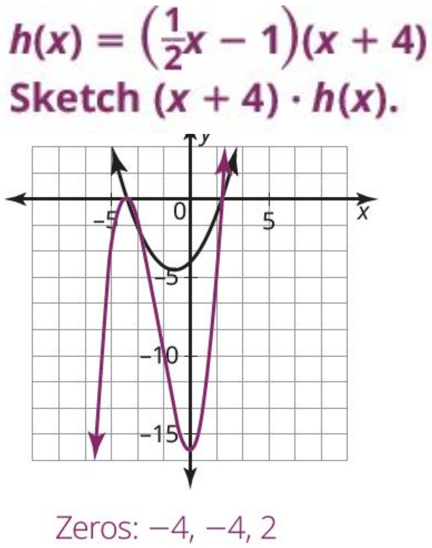
Algebra II Module 2: Analyzing Structure

Topic 1: Composing and Decomposing Functions

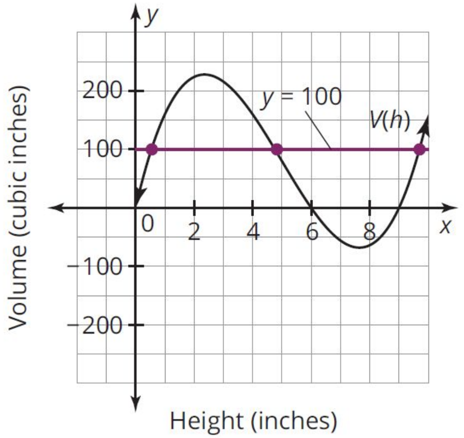
Composing and Decomposing Functions introduces students to the concept of building new functions on the coordinate plane by operating on or transforming functions.



The modeling process is reviewed as students build a quadratic function from two linear factors to model a situation that represents the cross-sectional area of a drain.



Students then use transformations to build a degree-3 polynomial and explore how the zeros of the function are related to the linear factors that were multiplied to build the function.



Students continue to build cubic functions, using what they know about the degree-3 formulas that are used to calculate the volume of prisms and cylinders.

Algebra II Module 2: Analyzing Structure

Topic 2: Characteristics of Polynomial Functions

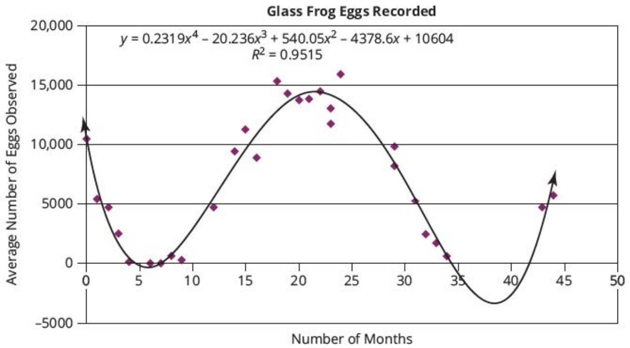
Students investigate the key characteristics of a polynomial function.

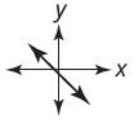
$$g(x) = Af(B(x - C)) + D$$

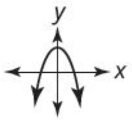
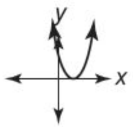
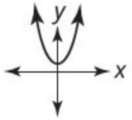
Type of Transformation Performed on $f(x)$	Coordinates of $f(x) \rightarrow$ Coordinates of $g(x)$
Vertical Dilation by a Factor of A	$(x, y) \rightarrow (x, Ay)$
Horizontal Dilation by a Factor of B	$(x, y) \rightarrow \left(\left(\frac{1}{B}\right)x, y\right)$
Horizontal Translation of C units	$(x, y) \rightarrow (x + C, y)$
Vertical Translation of D units	$(x, y) \rightarrow (x, y + D)$
All four transformations: A, B, C, and D	$(x, y) \rightarrow \left(\left(\frac{1}{B}\right)x + C, Ay + D\right)$

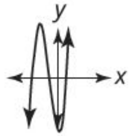
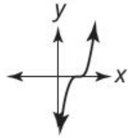
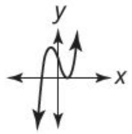
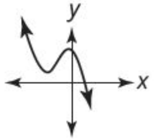
Students transform polynomial functions.

Students analyze a polynomial regression that models a problem situation.



Linear Function	
Zeros	**Graph**
no zeros	

Quadratic Function	
Zeros	**Graph**
2 real distinct	
1 real (multiplicity 2)	
2 imaginary	

Cubic Function	
Zeros	**Graph**
3 real distinct	
1 real (multiplicity 3)	
1 real (multiplicity 1) 1 real (multiplicity 2)	
1 real 2 imaginary	

They extend their understanding of zeros to include cubic polynomials. With a clear understanding of these characteristics, students are now able to sketch graphs of polynomial functions.

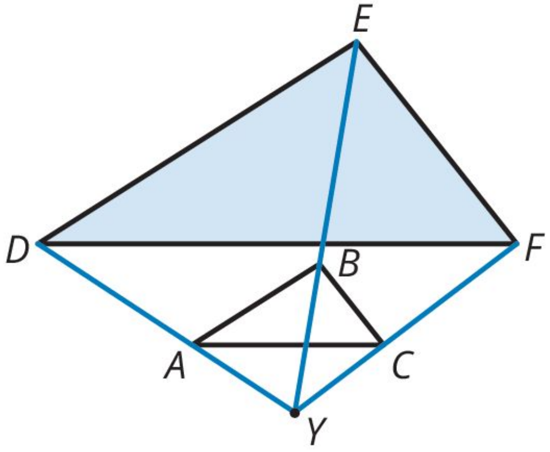
Geometry Course Overview

Module 1 Reasoning with Shapes	Module 2 Establishing Congruence	Module 3 Investigating Proportionality	Module 4 Connecting Geometric and Algebraic Descriptions	Module 5 Making Informed Decisions
Topic 1: Using a Rectangular Coordinate System	Topic 1: Composing and Decomposing Shapes	Topic 1: Similarity	Topic 1: Circles and Volume	Topic 1: Independence and Conditional Probability
Topic 2: Rigid Motions on a Plane	Topic 2: Justifying Line and Angle Relationships	Topic 2: Trigonometry	Topic 2: Circles and Cross Sections	Topic 2: Computing Probabilities
Topic 3: Congruence Through Transformations	Topic 3: Using Congruence Theorems			

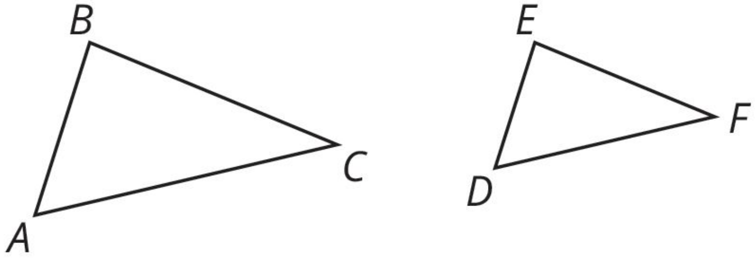
Geometry Module 3: Investigating Proportionality

Topic 1: Similarity

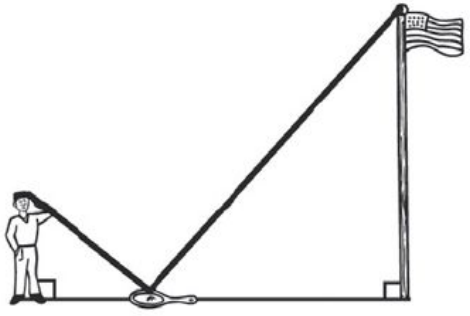
- Student use proof by construction to prove theorems related to similar triangles.
- They investigate specific cases of proportionality within similar triangles.
- The geometric mean is defined and used to solve problems and to prove the Pythagorean Theorem with similar triangles.
- Students solve indirect measurement problems using similarity and right triangles.
- Students use triangle proportionality theorems to verify that they partitioned a line segment by a specified ratio.



Dilations are non-rigid motion transformations that create similar figures



Students prove the Angle-Angle Similarity Theorem, the Side-Side-Side Similarity Theorem, and the Side-Angle-Side Similarity Theorem.



Students go outside and apply what they have learned about similarity to measure the height of objects such as flagpoles, tops of trees, telephone poles, or buildings.

Geometry Module 3: Investigating Proportionality

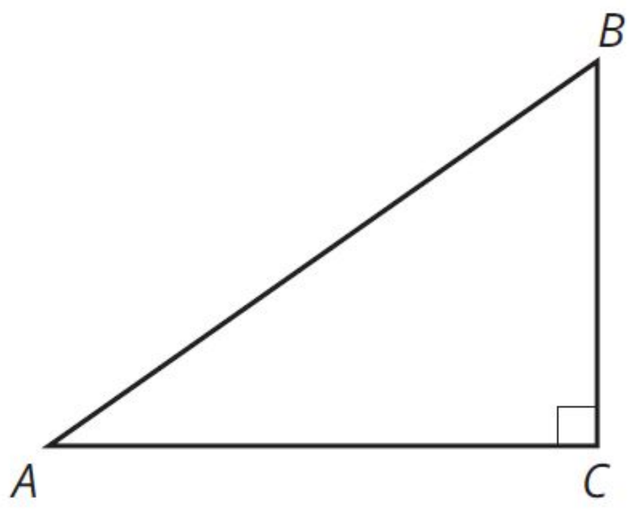
Topic 2: Trigonometry

- Through an exploration of 45°-45°-90° and 30°-60°-90° triangles, students learn that there are common ratios that exist between the side lengths within a special right triangle.
- Students gain understanding through measurement and then generalize based on investigations of multiple triangles.
- Tangent is formally introduced first by relating Tangent to slope. Sine and Cosine follow.

$$\tan A = \frac{\text{length of side opposite of } \angle A}{\text{length of side adjacent to } \angle A} = \frac{BC}{AC}$$

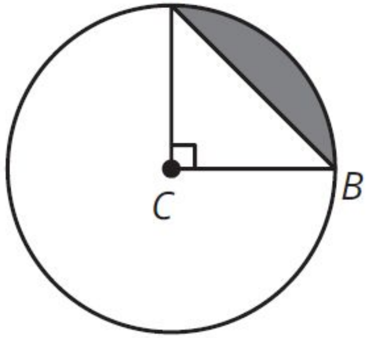
$$\sin A = \frac{\text{length of side opposite of } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AC}{AB}$$



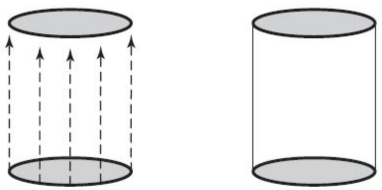
Geometry Module 4: Connecting Geometric and Algebraic Descriptions

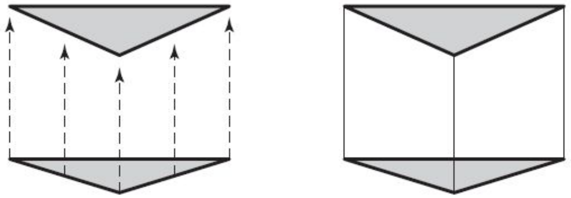
Area of segment $= \frac{m}{360°}(\pi r^2) - \frac{1}{2}bh$

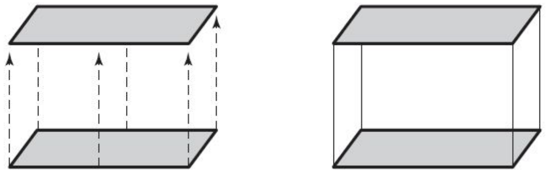


Topic 1: Circles and Volume

- Circles and Volume begins by using translation and dilation to establish the similarity of all circles.
- Students recognize arc lengths as proportions of the circumference
- They define a sector as proportional to the area of a circle. They then use what they know about the areas of triangles along with the areas of sectors to determine the areas of segments of circles.
- Using transformations, two-dimensional figures are used to build three-dimensional solids.
- Students develop the formulas for surface area and volume of prisms, cylinders, cones and spheres.





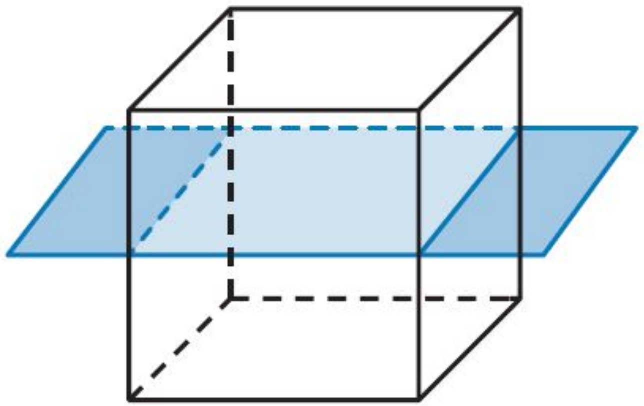


Prisms	Pyramids	Cylinders	Cones	Spheres
$V =$ (area of base) x height $V = Bh$	$V = \frac{1}{3}Bh$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$

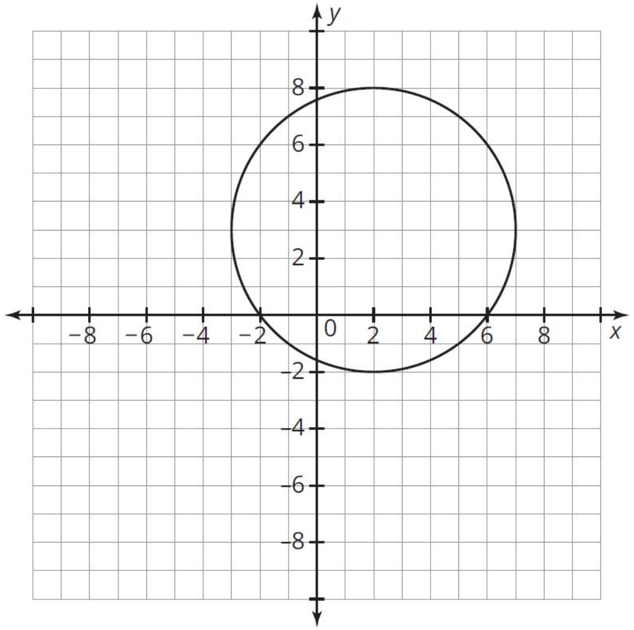
Geometry Module 4: Connecting Geometric and Algebraic Descriptions

Topic 2: Circles and Cross Sections

The Distance Formula is used to derive equations for a circle centered at the origin and a circle centered at point (h, k).



Students begin by reasoning about cross-sections formed when a plane intersects a geometric solid.



The equation of a circle centered at the origin is $x^2 + y^2 = r^2$, where r is the radius of the circle.

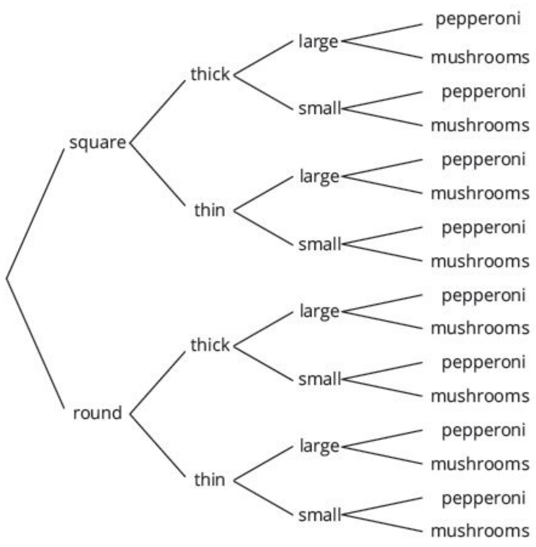
The standard form of the equation of a circle centered at (h, k) with radius r can be expressed as $(x - h)^2 + (y - k)^2 = r^2$.

Students use the Pythagorean Theorem and the Distance Formula to determine whether a point lies on a circle.

Geometry Module 5: Making Informed Decisions

Topic 1: Independence and Conditional Probability

- Students formalize their intuitive understanding of probability.
- This topic emphasizes modeling and analyzing sample spaces to determine rules for calculating probabilities in different situations.
- Students are presented with a variety of scenarios which they must categorize as independent or dependent events and calculate the compound probability of an intersection or union of events.



Students learn strategies for determining the sample space of compound events including tree diagrams and organized lists.

The **Rule of Compound Probability involving *and*** states: "If Event *A* and Event *B* are independent events, then the probability that Event *A* happens and Event *B* happens is the product of the probability that Event *A* happens and the probability that Event *B* happens, given that Event *A* has happened."

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

The **Addition Rule for Probability** states: "The probability that Event *A* occurs or Event *B* occurs is the probability that Event *A* occurs plus the probability that Event *B* occurs minus the probability that both *A* and *B* occur."

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Geometry Module 5: Making Informed Decisions

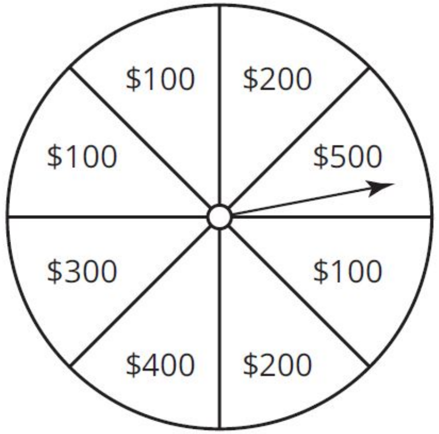
Topic 2: Computing Probabilities

- This topic builds on the learning from the previous topic by addressing more compound probability concepts and counting strategies.
- Compound probability concepts are presented using two-way frequency tables, conditional probability, and independent trials. The counting strategies include permutations, permutations with repetition, circular permutations, and combinations.

Sports Participation

	Individual	Team	Does Not Play	Total
Left	$\frac{3}{63} \approx 4.8\%$	$\frac{13}{63} \approx 25.4\%$	$\frac{8}{63} \approx 12.7\%$	$\frac{24}{63} \approx 38.1\%$
Right	$\frac{6}{63} \approx 9.5\%$	$\frac{23}{63} \approx 36.5\%$	$\frac{4}{63} \approx 6.3\%$	$\frac{33}{63} \approx 52.4\%$
Mixed	$\frac{1}{63} \approx 1.6\%$	$\frac{3}{63} \approx 4.8\%$	$\frac{2}{63} \approx 3.2\%$	$\frac{6}{63} \approx 9.5\%$
Total	$\frac{10}{63} \approx 15.9\%$	$\frac{39}{63} \approx 61.9\%$	$\frac{14}{63} \approx 22.2\%$	$\frac{63}{63} = 100\%$

You can use a two-way relative frequency table to calculate the probability of events occuring.



Students consider probabilities using geometric shapes. They calculate expected values in gaming situations and make decisions as to whether the game is worth the risk.

Algebra 2 Course Overview

Module 1 Exploring patterns in Linear and Quadratic Relationships	Module 2 Analyzing Structure	Module 3 Developing Structural Similarities	Module 4 Extending Beyond Polynomials	Module 5 Inverting Functions
Topic 1: Extending Linear Relationships Topic 2: Exploring and Analyzing Patterns Topic 3: Applications of Quadratics	Topic 1: Composing and Decomposing Functions Topic 2: Characteristics of Polynomial Functions	Topic 1: Relating Factors and Zeros Topic 2: Polynomial Models	Topic 1: Rational Functions Topic 2: Radical Functions	Topic 1: Exponential and Logarithmic Functions Topic 2: Exponential and Logarithmic Equations Topic 3: Applications of Exponential Functions

Module Overview: A Resource for Planning

- Why is this module named what it is?
- What is the mathematics of this module?
- How is this module connected to prior learning?
- When will students use knowledge from this module in future learning?

Module 2 Overview
Analyzing Structure

"Although mathematics derives much of its power from its ab
understanding of the nature of mathematics can be a slow pr
ideas, teachers sometimes leap to the abstract symbolic repr
and reducing the flexibility of their understanding. A rigid und
students' transfer or application of knowledge from one dom
and analogy can be important aids in developing a flexible un
Mathematical Connections in Grades 9-12, NCTM, p. 50.)

Why is the Module named Analyzing Structure?

The ability to look for and make use of structure is important in the study of functions. In this course, students transition from the simpler functions of earlier courses—linear, quadratic, and exponential—to the complexity of polynomial, rational, radical, and logarithmic functions. Knowing the key and defining characteristics of each function type helps students to build a systematic understanding of algebraic functions and their relationships. In **Analyzing Structure**, students begin by recalling the structure of quadratic functions—degree-2 polynomials—a familiar function type from previous courses. They expand on this structure to build and analyze higher-order polynomials.

What is the mathematics of Analyzing Structure?

Analyzing Structure contains two topics: *Composing and Decomposing Functions* and *Characteristics of Polynomial Functions*. In this

Module 2 Overview
Establishing Congruence

"Proofs are stories that convince suitably qualified others that a certain statement is true. If I present you with a proof, and you have the appropriate background knowledge and ability, you can—usually with some time and effort—as a result of reading my story, become convinced that what I claim is true. But if you take that as your working definition of proof, you have to acknowledge it is fundamentally about communication, not truth." (Keith Devlin, "What is a proof, really?" 2014)

Why is the Module named Establishing Congruence?

The van Hiele Model of geometric thinking defines the five levels through which students progress as they reason geometrically: visualization, analysis, abstraction, deduction, and rigor. Throughout their elementary school math education, students visualize geometric shapes and relationships. As they transition to middle school, students begin analyzing geometric shapes—recognizing names and identifying properties. Throughout the first two modules of this course, the goal is for students to move from analysis to abstraction, where they define properties and relationships precisely and construct informal arguments as to whether these properties and relationships are true in all cases. As students work through **Establishing Congruence**, their reasoning formalizes, and they learn to construct formal proofs. Throughout the rest of this course, students will use the reasoning developed in this module to discover new congruence relationships and use deduction to prove whether these congruence relationships are true in all cases.

What is the mathematics of Establishing Congruence?

Establishing Congruence contains three topics: *Composing and Decomposing Shapes*, *Justifying Line and Angle Relationships*, and *Using Congruence Theorems*. The module addresses the standards in the category, *Proofs and Congruence*. Building an intuitive understanding of the geometric relationships in the first topic mediates the cognitive demand of writing a proof; when the time comes, students can focus on the process of writing a tightly knit deductive proof instead of trying to determine whether they believe the relationship to be true in the first place. This module also addresses applying theorems about circles. Finally, using radicals to determine the side lengths of special right triangles gives students an opportunity to practice rewriting expressions involving radicals.

In *Composing and Decomposing Shapes*, students investigate geometric relationships and make conjectures. They construct circles and use diameters, radii, and chords to construct other figures and relationships within circles. They first

Module Overview: Big Picture of Planning

Lesson Internalization Process Overview

0. Read the Topic Overview
1. Read to Understand
2. DO the MATH
3. Read the Facilitator Notes for Engage, Develop, and Demonstrate
4. Chunk, Pace, and Strategize
5. Identify or Create Opening and Closing Activities

5 Minute Quick Hit

Best Practice Tip(s)

- Utilize the plethora of resources on the Texas Help Center as you explore connections across other grade level courses.
- Use your common planning, department meetings, or PLC time (with other grade levels) to share your module/course overviews in order to make connections across courses.

When it comes to the sequence of mathematics concepts in your course:

K	KNOW	W	WANT TO KNOW	L	LEARNED
	complete		*complete*		What have you learned during this session?

Think Time
K-W-L Chart

Revisiting Why We're Here

After today's live session, participants are now able to...

- Define how the key mathematical concepts flow across this course and how they are connected to other courses.
- Identify the Big Ideas in each Module within this course.
- Connect how this big picture view of each course fits into the topic and lesson level internalization.
- Walk away with concrete tools and processes to use in their instructional planning.

We’re here to help!

Contact Us

texas-partner@carnegielearning.com

Session Survey: We'd love your feedback!

The facilitation of today's session was strong. *

- Strongly Disagree
- Disagree
- Neutral
- Agree
- Strongly Agree

The logistics and technology for today's session were smooth. *

- Strongly Disagree
- Disagree
- Neutral
- Agree
- Strongly Agree

The pacing of today's session was... *

- Too Slow
- Just Right
- Too Fast

Please pay close attention to the wording when selecting your answer. Positive and critical responses are in different places.

What about today's learning was valuable?

What could we do differently to improve your learning experience?

Please leave actionable feedback. Tell us what specific experience was good or bad **and** why. This helps us keep specifics that work and eliminate those that don't.

Next Steps

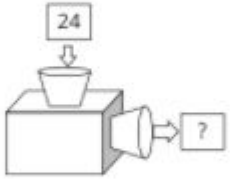
Before signing off, please take the survey on your experience in today’s session (link in chat).

The post-work and exit ticket are posted in TEALearn. Please complete in the next ten days.

What questions might you have about what’s next for you in terms of the professional learning tasks?

Algebra II Module 3: Developing Structural Similarities

Topic 1: Relating Factors and Zeros



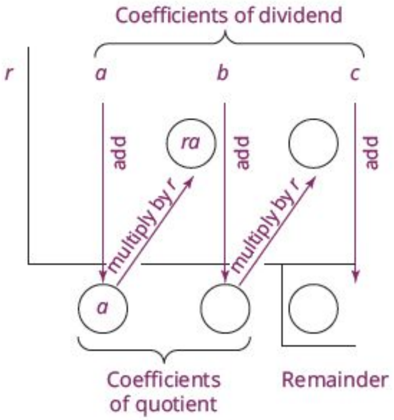
This warm up sets the stage for the entire topic.

Now that students are familiar with the key characteristics of polynomial functions, this topic presents them with opportunities to analyze, factor, solve, and sketch polynomial functions given algebraic representations.

$$\begin{aligned} 9x^2 + 21x + 10 &= (3x)^2 + 7(3x) + 10 \\ &= z^2 + 7z + 10 \\ &= (z + 5)(z + 2) \\ &= (3x + 5)(3x + 2) \end{aligned}$$

Students investigate methods to factor polynomial expressions, such as factoring out the greatest common factor, chunking, grouping, and using quadratic form.

$$\begin{array}{r} x^2 + 0x + 3 \\ x + 1 \overline{) x^3 + x^2 + 3x + 3} \\ \underline{-(x^3 + x^2)} \\ 0x^2 + 3x \\ \underline{-(0x^2 + 0x)} \\ 3x + 3 \\ \underline{-(3x + 3)} \\ 0 \end{array}$$



Students divide polynomials when one factor is known, using long division and synthetic division to write polynomials in factored form.

	Integer Example	Polynomial Example
Addition	400 + 30 + 7 + 20 + 5 400 + 50 + 12	$4x^2 + 3x + 7$ $+ \quad 2x + 5$ $4x^2 + 5x + 12$
Subtraction	400 + 30 + 7 − (20 + 5) 400 + 10 + 2	$4x^2 + 3x + 7$ $- \quad (2x + 5)$ $4x^2 + x + 2$
Multiplication	400 + 30 + 7 × 20 + 5 2000 + 150 + 35 8000 + 600 + 140 8000 + 2600 + 290 + 35	$4x^2 + 3x + 7$ $\times \quad 2x + 5$ $20x^2 + 15x + 35$ $8x^3 + 6x^2 + 14x$ $8x^3 + 26x^2 + 29x + 35$
Division	$\frac{437}{25} = 17$ R12	$\frac{4x^2 + 3x + 7}{2x + 5} = (2x - 3)$ R$(-x + 22)$

Polynomial operations are compared to integer operations throughout.

Algebra II Module 3: Developing Structural Similarities

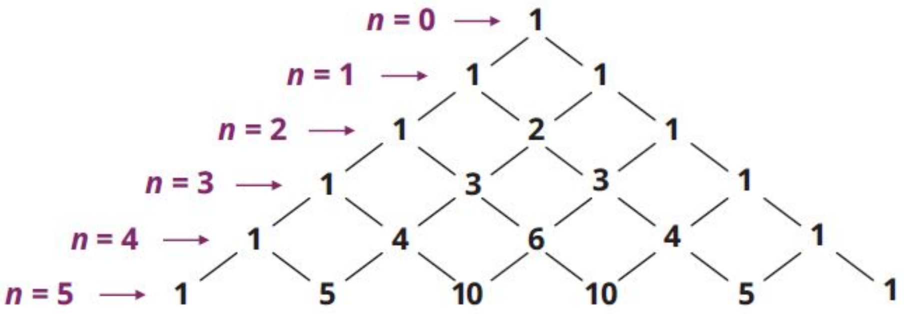
Topic 2: Polynomial Models

In this topic, students use the concept of equality to represent mathematical relationships in different ways.

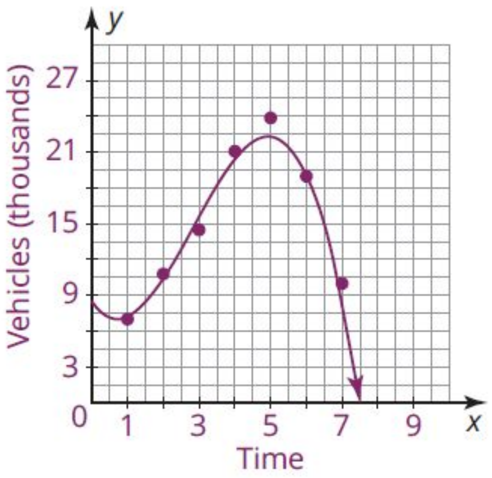
$$
\begin{aligned}
46^2 &= (40 + 6)^2 \\
&= 40^2 + 2(40)(6) + 6^2 \\
&= 1600 + 2(40)(6) + 36 \\
&= 1600 + 480 + 36 \\
&= 2116
\end{aligned}
$$

The value of 46^2 is 2116.

Students use polynomial identities to perform calculations, verify Euclid's Formula, and generate Pythagorean triples.



Students then explore patterns in Pascal's Triangle and use the values in Pascal's Triangle to expand powers of binomials. Next, they apply the Binomial Theorem and its combinatorics as an alternative method to expand powers of binomials.

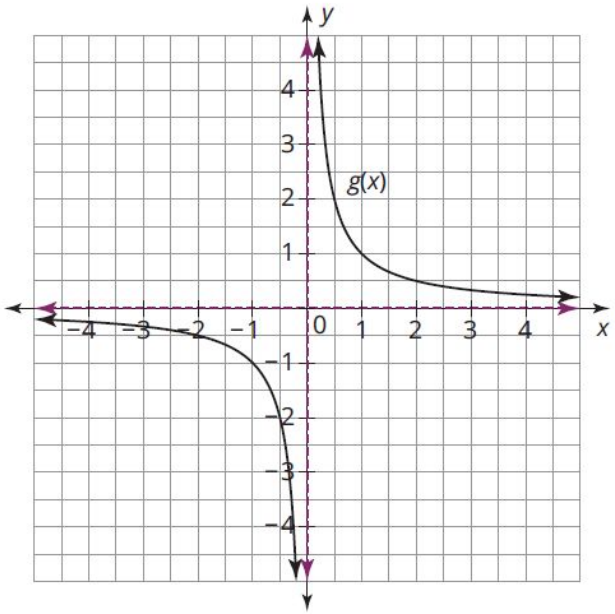


They determine the degree of the polynomial that is most appropriate for a given data set and use technology to write a regression equation.

Algebra II Module 4: Extending Beyond Polynomials

Topic 1: Rational Functions

In Rational Functions, student analyze, graph, transform, and solve rational functions.



Students are introduced to and graph rational functions.

	Domain	Range
Inequalities	$x > 4$ or $x < 4$	$g(x) > 4$ or $g(x) < 4$
Interval notation	$(-\infty, 4) \cup (4, \infty)$	$(-\infty, 4) \cup (4, \infty)$
Set notation	$\{x \mid x \neq 4\}$	$\{g(x) \mid g(x) \neq 4\}$

Students write the domain and range as an inequality, in interval notation, and using set notation.

$$r(x) = A\left(\frac{1}{B(x - C)}\right) + D$$

Students apply what they know about transformation form to rational functions.

	Rational Numbers	Rational Expressions Involving Variables	
Example 1	$\frac{1}{5} \div \frac{3}{10} = \frac{1}{5} \cdot \frac{10}{3} = \frac{2}{3}$	$\frac{xy}{5z} \div \frac{3xy}{10z} = \frac{xy}{5z} \cdot \frac{10z}{3xy} = \frac{2}{3}$	$\frac{xy^2}{5z} \div \frac{3xy}{10z^2} = \frac{xy^2}{5z} \cdot \frac{10z^2}{3xy} = \frac{2yz}{3}; x, y, z \neq 0$
Example 2	$\frac{6}{7} \div 4 = \frac{6}{7} \cdot \frac{1}{4} = \frac{3}{14}$	$\frac{6a}{7b} \div 4a = \frac{6a}{7b} \cdot \frac{1}{4a} = \frac{3}{14b}$	$\frac{6a^3}{7b} \div 4a = \frac{6a^3}{7b} \cdot \frac{1}{4a} = \frac{3a^2}{14b}; a, b, \neq 0$

Operations with Rational functions are introduced by comparing them to operations with rational numbers.

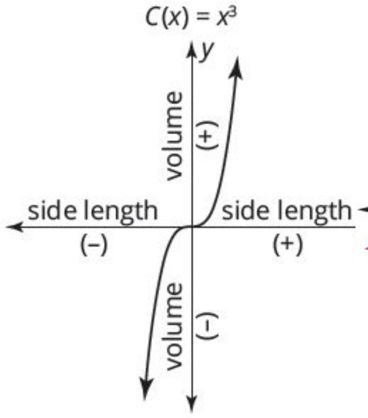
	Portion of the Garden Watered	Time Spent Watering	Rate of Watering
	Gardens	Minutes	Gardens/Minute
Maureen	$40\left(\frac{1}{90}\right) = \frac{4}{9}$	40	$\frac{1}{90}$
Sandra Jane	$40\left(\frac{1}{x}\right) = \frac{40}{x}$	40	$\frac{1}{x}$
Entire Job, or 1 Garden	$\frac{4}{9} + \frac{40}{x}$	40	

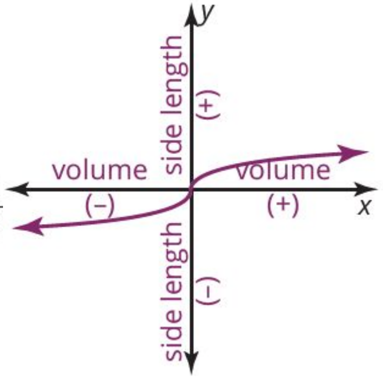
Students apply rational equations to solve work, distance, cost and mixture problems.

Algebra II Module 4: Extending Beyond Polynomials

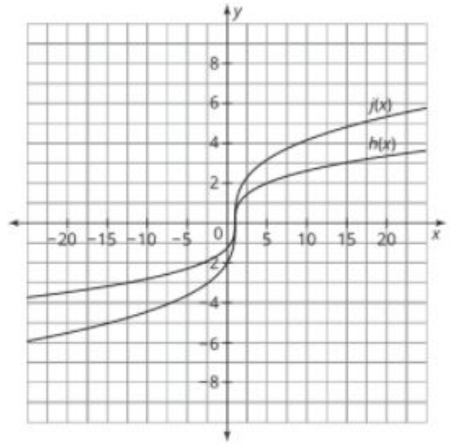
Topic 2: Radical Functions

In Rational Functions, student analyze, graph, transform, and solve rational functions.





Students are introduced to rational functions by investigating inverses of quadratic and cubic functions.



Students graph radical functions, write their equations, and determine their key characteristics.

They transform radical functions and determine what effect transformations have on the inverse function.

Transformation of Quadratic Function, $f(x)$	Transformation of Inverse Function, $f^{-1}(x)$
translation up *D* units	translation right *D* units
translation down *D* units	translation left *D* units
translation right *C* units	translation up *C* units
translation left *C* units	translation down *C* units

Using Radicals

$$\sqrt[3]{8x^6} = \sqrt[3]{2^3 \cdot x^6}$$
$$= \sqrt[3]{2^3 \cdot (x^2)^3}$$
$$= \sqrt[3]{2^3} \cdot \sqrt[3]{(x^2)^3}$$
$$= 2x^2$$

Using Powers

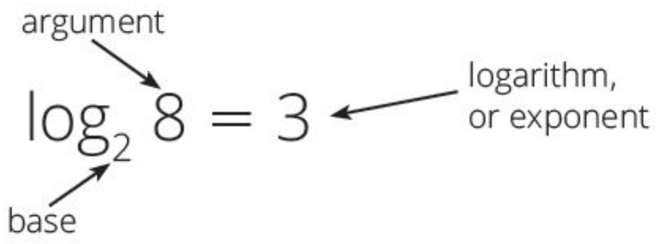
$$\sqrt[3]{8x^6} = (8x^6)^{\frac{1}{3}}$$
$$= (2^3 \cdot x^6)^{\frac{1}{3}}$$
$$= 2^{\frac{3}{3}} \cdot x^{\frac{6}{3}}$$
$$= 2^1 \cdot x^2$$
$$= 2x^2$$

Students rewrite radical expressions using radicals and powers. They also solve radical equations in and out of context.

Algebra II Module 5: Inverting Functions

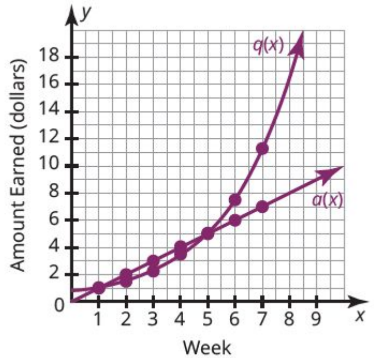
Topic 1: Exponentials and Logarithmic Functions

In Exponential and Logarithmic Functions, students build of their knowledge of exponential functions to analyze, graph, and transform exponential functions and their inverses, logarithmic functions.



$$g(x) = A \cdot \log(B(x - C)) + D.$$

Students apply transformation form to logarithmic and exponential functions.

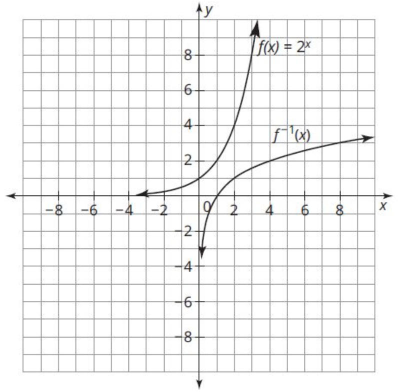


Students use exponential functions to model real-world situations. They use tables and graphs to compare linear and exponential growth.

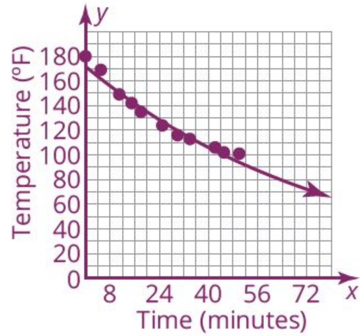
$$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

$$e \approx 2.718281828459045\ldots$$

They use the compound interest formula to derive the value of the constant e.



Students then integrate their knowledge of exponential functions and inverses to define a new function: the logarithm.



Students use technology to generate exponential regressions, and they use these equations to make predictions.

Algebra II Module 5: Inverting Functions

Topic 2: Exponential and Logarithmic Equations

Students use their understanding of exponential and logarithmic functions to solve exponential and logarithmic equations.
NOTE: In this topic, some of the content goes beyond the scope of the course standards. The content is included to enhance students' understanding of mathematics and provide opportunities for extension.

Argument Is Unknown	Exponent Is Unknown	Base Is Unknown
$\log_4 y = 3$	$\log_4 64 = x$	$\log_b 64 = 3$
$4^3 = y$	$4^x = 64$	$b^3 = 64$
$64 = y$	$4^x = 4^3$	$b^3 = 4^3$
	$x = 3$	$b = 4$

Students convert between exponential and logarithmic forms of an equation, and they then use this relationship to solve for an unknown base, exponent, or argument in a logarithmic equation.

$$\log_7\left(\frac{y^4}{x^3}\right)$$

$$\log_7(y^4) - \log_7(x^3)$$

$$4\log_7(y) - 3\log_7 x$$

They use properties of logarithms to write a single logarithms in expanded form and vice versa.

$$\log_b c = \frac{\log_a c}{\log_a b}$$

They learn the Change of Base Formula and learn to solve exponential equations by taking the logarithms of both sides.

They solve for the base, argument, or exponent of logarithmic equations, using the properties of logarithms.

$$2.2 = -\log H^+$$

$$10^{-2.2} = H^+$$

$$H^+ \approx 0.0063$$

The concentration of hydrogen ions in vinegar is about 0.0063 mole per cubic liter.

Students write and solve exponential and logarithmic equations arising from real-world contexts.

Algebra II Module 5: Inverting Functions

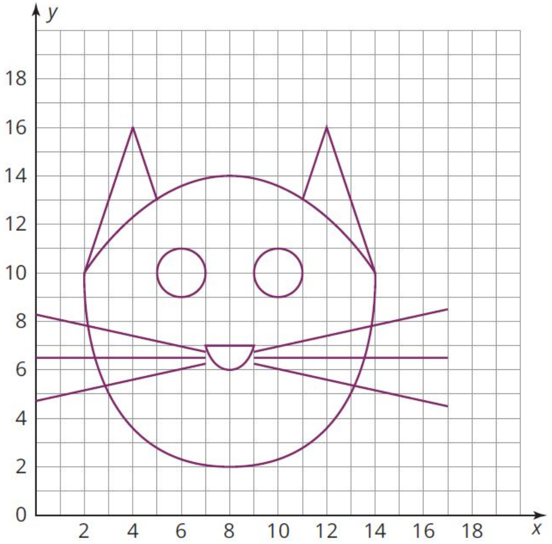
Topic 3: Applications of Exponential Functions

Students explore various real-world and mathematical situations that are modeled by exponential functions. **NOTE:** In this topic, some of the content goes beyond the scope of the course standards. The content is included to enhance students' understanding of mathematics and provide opportunities for extension.

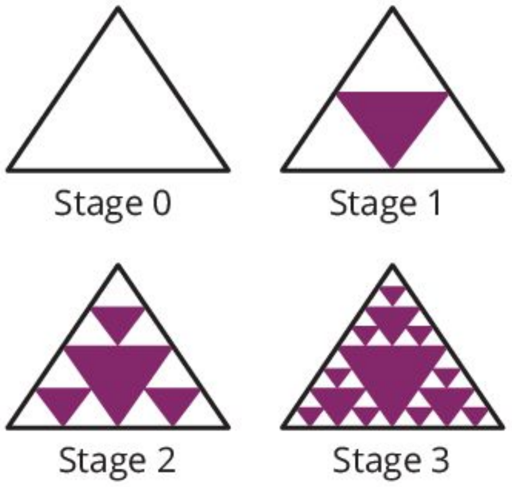
$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_n = \frac{g_1(r^n - 1)}{r - 1}$$

Students develop two formulas to determine the sum of the terms of a geometric sequence or geometric series. form. They also apply the formulas in the real-world context of credit card debt.



Students use transformed functions to draw and interpret graphics, paying attention to the domain restrictions required by the image.



They investigate the Sierpinski Triangle, the Menger Sponge, the Koch Snowflake, and the Sierpinski Carpet as examples of fractals. Students describe the growth patterns of the fractals in terms of geometric sequences.