

# Shear Design of Reinforced Concrete Beams, Slabs and Walls

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## Abstract

The paper presents the background to the shear design provisions for reinforced concrete beams and slabs used in the Australian practice. Correlation of design equations with experimental results are given. The design provisions are illustrated by examples. The importance of shear strength in the design of structural walls is discussed. A new expression to calculate the shear strength of walls is presented. © 1998 Elsevier Science Ltd. All rights reserved.

**Keywords:** beams, design, reinforced concrete, shear, slabs, standards, walls

## NOTATION

$A_g$	The gross cross-sectional area of a member	$D_s$	The overall depth of a slab or drop panel
$A_l$	The area of vertical steel in a wall on both faces in length $L_w$	$d$	The effective depth of a slab averaged around the critical shear perimeter
$A_{st}$	The cross-sectional area of tension reinforcement	$d_o$	The distance from the extreme compression fibre of the concrete to the centroid of the outermost layer of tensile reinforcement
$A_{sv}$	The cross-sectional area of shear reinforcement	$d_w$	The effective horizontal length of a wall as defined in the text
$A_{sv.min}$	The cross-sectional area of minimum shear reinforcement	$f_{cv}$	The concrete shear strength [eqn (14)]
$A_{sw}$	The cross-sectional area of the bar forming a closed tie	$f'_c$	The compressive strength of concrete
$a$	The width of torsion strip (Fig. 4)	$f_y$	The yield strength of reinforcing steel
$a_v$	The distance from the section at which shear is being considered to the face of the nearest support	$H_w$	The height of a wall
$b_v$	The effective width of a web for shear	$k_3$	The effective concrete strength factor (eqn (24))
$b_w$	The width of the web of a spandrel beam	$L_w$	The overall horizontal length of a wall
$D_b$	The overall depth of a spandrel beam	$M_v^*$	The (factored) design bending moment to be transferred from slab to a support
		$N^*$	The (factored) design axial compressive force
		$p_l$	Vertical reinforcement ratio in a wall given by $A_l/t_w L_w$
		$s$	The centre-to-centre spacing of shear reinforcement measured parallel to the longitudinal axis of a member
		$t_w$	The thickness of a wall
		$u$	The length of the critical shear perimeter
		$V^*$	The (factored) design shear force
		$V_u$	The ultimate shear strength
		$V_{u.max}$	The ultimate shear strength limited by web crushing failure
		$V_{u.min}$	The ultimate shear strength of a beam or a slab provided with minimum shear reinforcement
		$V_{uc}$	The ultimate shear strength excluding shear reinforcement

$V_{uo}$	The ultimate shear strength of a slab with no moment transfer
$y_1$	The larger dimension of a closed rectangular tie
$\theta$	The angle between the compression strut and the longitudinal axis of a member
$\phi$	The strength reduction factor taken as 0.7 for shear design

## INTRODUCTION

Extensive research has been carried out on the behaviour and shear strength of reinforced concrete beams, slabs and walls. The available literature is voluminous and the state-of-the-art reviews are given elsewhere<sup>1-12</sup>. Numerous tests on beams, slabs and walls have been conducted and reported in the literature. Many theories for predicting the shear strength have been advanced. Of these, the strut-and-tie approach and the plasticity approach<sup>12-22</sup> have received wider acceptance by the profession. The shear design provisions given by various codes and standards<sup>23-27</sup> are largely based on these approaches, but empirically modified to fit test trends.

In the Australian Standard<sup>23</sup>, the shear design of reinforced concrete beams and slabs is based on a truss model and empirical data. However, the shear strength of walls is calculated by empirical expressions. The aim of this paper is to describe the shear design provisions for beams and slabs given in this Standard and present correlation with test results. The paper also examines the importance of shear strength of walls and presents a simple rational expression which may be used in lieu of the empirical equations given in the Standard.

## BEAMS

### Design equations

The approach taken in the Australian Standard AS3600<sup>23</sup> is influenced by a re-evaluation of the conventional 45° truss model in the light of test results, and also to some extent by the recent developments<sup>13-16</sup>. It was found that the truss angle was not always 45°, but smaller especially for beams that contain light shear reinforcement. Because most practical beams contain

only light shear reinforcement, it is appropriate to adopt a limited variable angle truss model which takes advantage of the situation. Accordingly, AS3600 retains the approach taken in the previous Standards (AS1480 and AS1481), but the angle  $\theta$  is assumed to vary between 30 and 45°. Instead of a free choice between these limits, the designer is given specific guidance on the selection of  $\theta$ .

The shear strength  $V_u$  of a beam containing vertical stirrups is given by:

$$V_u = V_{uc} + A_{sv} f_y \frac{d_o}{s} \cot \theta \leq V_{u\max} \quad (1)$$

where the angle  $\theta$  may be taken to vary linearly from 30° when  $V^* = \phi V_{u\min}$  to 45° when  $V^* = \phi V_{u\max}$ . Here  $V^*$  is the (factored) design shear force,  $\phi$  is the strength reduction factor taken as 0.7,  $V_{u\min}$  is the shear strength of a beam provided with minimum shear reinforcement of  $0.35 b_v s / f_y$  and is obtained from eqn (1) as:

$$V_{u\min} = V_{uc} + \left[ \frac{0.35 b_v s}{f_y} \right] f_y \frac{d_o}{s} \cot 30^\circ \quad (2)$$

or

$$V_{u\min} = V_{uc} + 0.6 b_v d_o \quad (3)$$

and  $V_{u\max}$  is the shear strength limited by web crushing and is given by:

$$V_{u\max} = 0.2 f_c' b_v d_o \quad (4)$$

All other symbols are explained in Section 1. In eqns (1) and (3), the shear contributed by the concrete  $V_{uc}$  in the case of a reinforced concrete beam is given by:

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_v d_o \left[ \frac{A_{st}}{b_v d_o} f_c' \right]^{1/3} \quad (5)$$

The factors for determining  $V_{uc}$  in eqn (5) are given by:

$$\beta_1 = 1.1 \left( 1.6 - \frac{d_o}{1000} \right) \geq 1.1 \quad (d_o \text{ in mm}) \quad (6)$$

$$\beta_2 = 1.0 \quad (\text{where no axial load exists}) \quad (7)$$

$$\beta_3 = \frac{2d_o}{a_v} \quad (1.0 \leq \beta_3 \leq 2.0) \quad (8)$$

In AS3600<sup>23</sup> the minimum amount of shear reinforcement is given by

$$A_{svmin} = \frac{0.35b_v s}{f_y} \quad (9)$$

It is proposed to modify eqn (9) with the following expression which is currently used in the Canadian Standard CSA A23.3-94 [26]:

$$A_{svmin} = \frac{0.06\sqrt{f'_c} b_v s}{f_y} \quad (10)$$

so that the shear design method may be applied to concretes with compressive strength up to 100 MPa instead of the 50 MPa upper limit currently imposed by AS3600.

If we take  $A_{svmin}$  as given by eqn (10),  $V_{umin}$  becomes:

$$V_{umin} = V_{uc} + \left[ \frac{0.06\sqrt{f'_c} b_v s}{f_y} \right] f_y \frac{d_o}{s} \cot 30^\circ \quad (11)$$

or

$$V_{umin} = V_{uc} + 0.10\sqrt{f'_c} b_v d_o \quad (12)$$

AS3600 limits the spacing of the stirrups to less than the smaller of one-half the overall beam depth or 300 mm to ensure that the shear reinforcement will intersect the shear crack and effectively provide shear resistance.

### Correlation with test data

The above design equations have shown good correlation with numerous test results, as indicated by Figs 1–3. Figures 1 and 2, taken from Ref. 28, show correlation in the case of normal strength concrete beams. It can be seen that the predictions are conservative. The trend of predicted values follow that of test data with respect to tensile steel reinforcement ratio  $A_{st}/b_v d_o$  and shear reinforcement ratio  $A_{sv} f_y / b_v s$ .

Figure 3, taken from Ref. 29, shows the correlation of test versus predicted shear strengths for high strength concrete beams. The shear strength of test beams showed considerable scatter. The difference in shear strengths of identical pairs of test beams ranged from 15 to 31%. Variations in test shear strengths contributed to the large scatter, and the code predictions were larger than the test shear

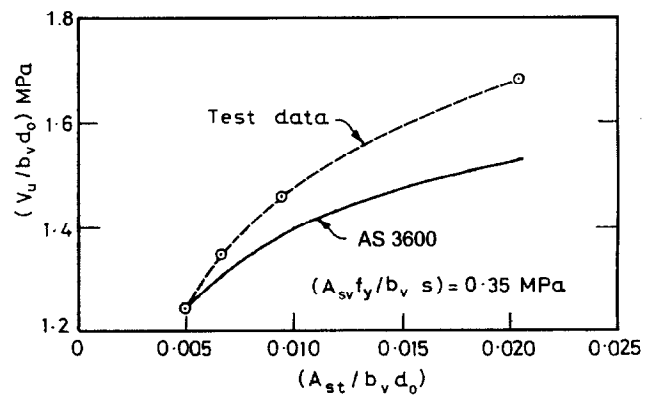
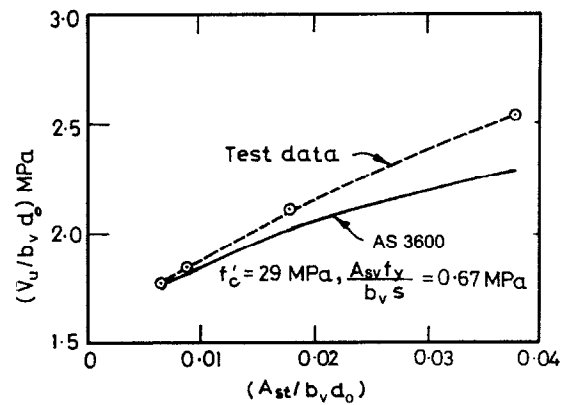


Fig. 1. Correlation of test shear strength versus shear strength predicted by AS3600: normal strength concrete beams<sup>28</sup>.

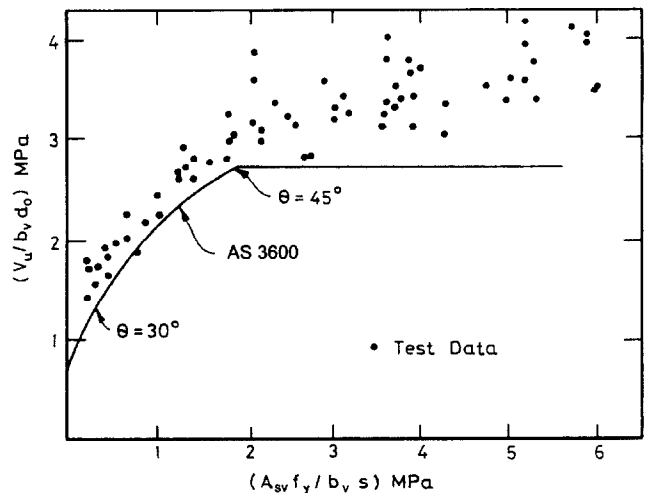


Fig. 2. Correlation of test shear strength versus shear strength predicted by AS3600: normal strength concrete beams<sup>28</sup>.

strengths for about one-third of the test beams studied.

Notwithstanding the above comments, the AS3600 shear provisions appear to be acceptable for concretes with compressive strength up to 100 MPa provided that the minimum shear reinforcement is given by eqn (10).

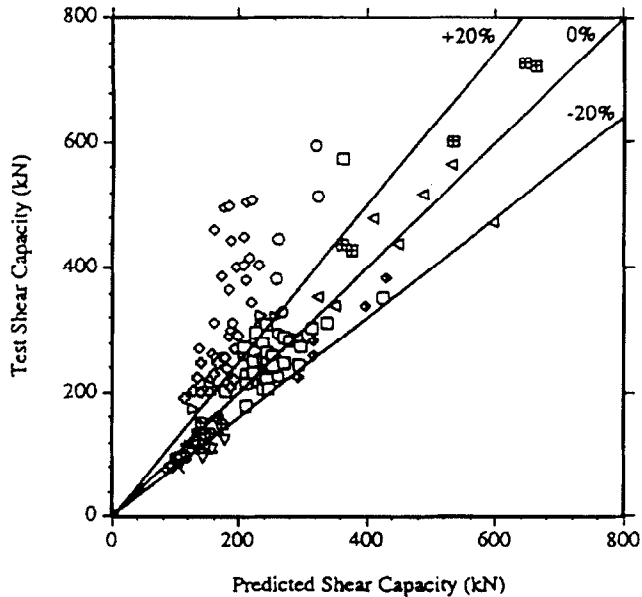


Fig. 3. Correlation of test shear strength versus shear strength predicted by AS3600: high strength concrete beams<sup>29</sup>.

**Example**

Design suitable vertical stirrups for a reinforced concrete beam given that  $V(\text{dead load}) = 150$  kN,  $V(\text{live load}) = 100$  kN,  $b_v = 300$  mm,  $d_o = 575$  mm,  $D = 650$  mm,  $A_{st} = 3200$  mm<sup>2</sup>,  $f'_c = 25$  MPa and  $f_y = 400$  MPa.

*Design shear force*

$$V^* = 1.25 V(\text{dead}) + 1.50 V(\text{live}) = 1.25(150) + 1.50(100) = 338 \text{ kN}$$

*Check web crushing*

Equation (4):

$$V_{u\text{max}} = 0.2 \times 25 \times 300 \times 575 = 863 \text{ kN}$$

$$\phi V_{u\text{max}} = 0.7 \times 863 = 604 \text{ kN}$$

$\phi V_{u\text{max}} > V^*$ , therefore web crushing is not critical.

*Shear carried by concrete:*

Take

$$\beta_2 = \beta_3 = 1.0$$

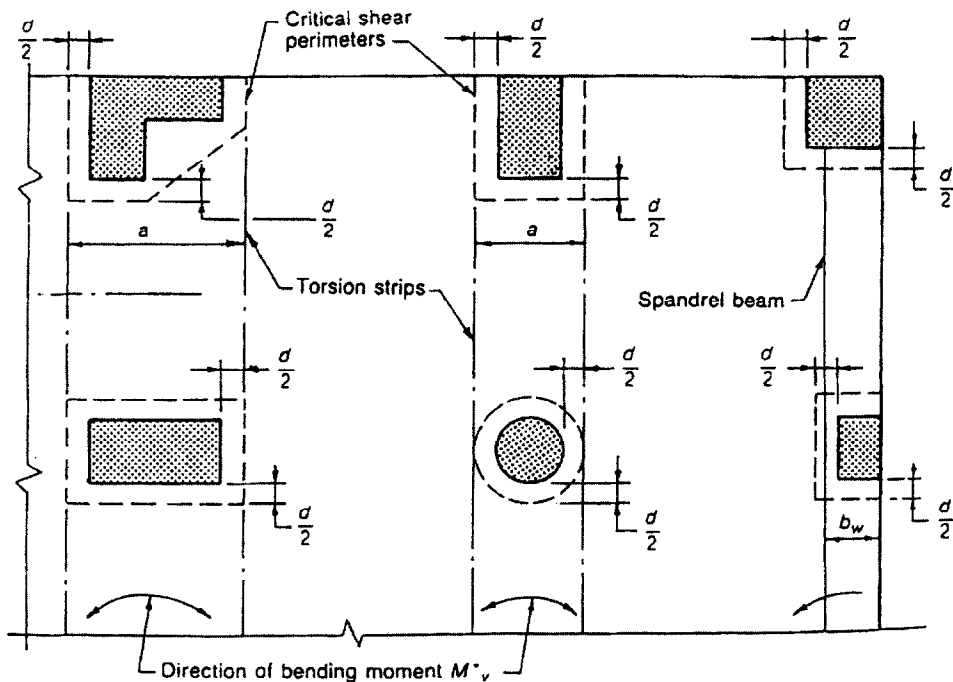


Fig. 4. Torsion strips and spandrel beams<sup>23</sup>.

Equation (6):

$$\beta_1 = 1.1 \left( 1.6 - \frac{575}{1000} \right) = 1.13$$

Equation (5):

$$V_{uc} = 1.13 \times 300 \times 575 \left( \frac{3200}{300 \times 575} \times 25 \right)^{1/3}$$

$$= 151 \text{ kN}$$

Angle  $\theta$

Equation (12):

$$V_{umin} = 151 + 0.1 \sqrt{25} \times 300 \times \frac{575}{1000} = 237 \text{ kN}$$

$$\phi V_{umin} = 0.7 \times 237 = 166 \text{ kN}$$

$$\phi V_{umax} = 604 \text{ kN}$$

Assume a linear variation of  $\theta$  between 30 and 45°:

$$\theta = 30 + (45 - 30) \left( \frac{V^* - \phi V_{umin}}{\phi V_{umax} - \phi V_{umin}} \right) = 30$$

$$+ (45 - 30) \left( \frac{338 - 166}{604 - 166} \right) = 35.9^\circ$$

Shear reinforcement

Design requirement is

$$V^* \leq \phi V_{uc} + \phi A_{sv} f_y \frac{d_o}{s} \cot \theta \quad 338 \times 10^3$$

$$\leq (0.7 \times 151 \times 10^3) + 0.7 \frac{A_{sv}}{s} 400$$

$$\times 575 \cot 35.9^\circ$$

solving  $A_{sv}/s \geq 1.044 \text{ mm}^2 \text{ mm}^{-1}$ .

If we use two-legged vertical stirrups made of 12 mm diameter deformed bars,  $A_{sv} = 2 \times 110 = 220 \text{ mm}^2$  and therefore  $s \leq 211 \text{ mm}$ .

Spacing should not exceed  $D/2$  or 300 mm whichever is less. In this example, 300 mm controls;  $s < 300 \text{ mm}$ , OK.

Use 12 mm diameter two-legged vertical stirrups at 200 mm spacing.

## SLABS

### Design equations

Where shear failure can occur across the width of the slab (i.e. beam shear), the shear strength may be calculated by eqn (5). However, such a failure is generally not critical.

The region of a slab in the vicinity of a column may fail in shear by developing a failure surface in the form of a truncated cone or pyramid. This type of failure, called a punching shear failure, is usually the source of collapse of flat plate and flat slab buildings. Adequate design of this region of slab is therefore of paramount importance.

An extensive review of existing knowledge on punching shear strength of slabs is available in various sources<sup>5,6</sup>. In general, the region of a slab near a column must transfer both shear force and unbalanced bending moment to the column. Numerous tests have been carried out to evaluate the punching shear strength of slabs. Several theories have been put forward to predict the strengths observed in these tests.

The punching shear strength,  $V_{uo}$ , where the moment transfer is zero, may be expressed as:

$$V_{uo} = u d f_{cv} \quad (13)$$

In eqn (13),  $u$  is the length of the critical shear perimeter defined by a line geometrically similar to the boundary of the column or support and located at a distance of  $d/2$  therefrom,  $d$  is the effective depth of the slab averaged around  $u$ , and  $f_{cv}$  is given by AS3600<sup>2,3</sup> as:

$$f_{cv} = 0.17 \left[ 1 + \frac{2}{\beta_c} \right] \sqrt{f'_c} \leq 0.34 \sqrt{f'_c} \quad (14)$$

where  $\beta_c$  is the ratio of the longest overall dimension of the column or support Y to the overall dimension X at right angles to Y, and is the cylinder strength of concrete expressed in terms of MPa.

Based on large-scale tests<sup>6,30</sup>, the author developed a method for the design of slabs to

guard against punching shear failure<sup>31,32</sup>. The method covers flat slab floors with or without spandrel beams as well as slabs with or without closed ties. The punching shear design provisions contained in the Australian Standard<sup>23</sup> are based on this method.

One of the important factors that governs the punching shear strength calculation is how the slab strip (called 'torsion strip') at the side face of a column or support resists the combined effects of torsion and shear acting there. Where the torsion strip contains no closed ties, the torsion and shear must be resisted by slab concrete alone. On the other hand, where the side face includes a spandrel beam provided with closed ties or where the torsion strip contains closed ties, the load-carrying mechanism and, hence, the final strength equations are different.

Accordingly the punching shear strength of a slab  $V_u$  is given by one of the following expressions.

If there are no closed ties in the torsion strip or spandrel beams,  $V_u$  is given by:

$$V_u = \frac{V_{uo}}{1.0 + (uM_v^*/8V^*ad)} \quad (15)$$

If the torsion strip contains the minimum quantity of closed ties,  $V_u$  is taken as  $V_{u\min}$  given by:

$$V_{u\min} = \frac{1.2V_{uo}}{1.0 + (uM_v^*/2V^*a^2)} \quad (16)$$

If there are spandrel beams perpendicular to the direction of  $M_v^*$  which contain the minimum quantity of closed ties,  $V_u$  is taken as  $V_{u\min}$  given by:

$$V_{u\min} = \frac{1.2V_{uo}(D_b/D_s)}{1.0 + (uM_v^*/2V^*ab_w)} \quad (17)$$

If the torsion strip or spandrel beam, contains more than the minimum quantity of closed ties,  $V_u$  is given by:

$$V_u = V_{u\min} \sqrt{\left( \frac{A_{sw}}{s} / \frac{0.2y_1}{f_y} \right)} \quad (18)$$

where  $V_{u\min}$  is calculated by eqn (16) or eqn (17), as appropriate.

In no case should  $V_u$  be taken greater than  $V_{u\max}$  given by:

$$V_{u\max} = 3V_{u\min} \sqrt{\frac{x}{y}} \quad (19)$$

where  $x$  and  $y$  are the shorter and longer dimensions respectively of the cross-section of the torsion strip or spandrel beam.

The minimum cross-sectional area of the reinforcement forming the closed ties should satisfy the following inequality:

$$\frac{A_{sw}}{s} \geq \frac{0.2y_1}{f_y} \quad (20)$$

Reinforcement for slab shear in torsion strips and spandrel beams must be in the form of closed ties arranged and detailed in accordance with Clause 9.2.6 in the Standard. The spacing of the ties should not exceed the greater of 300 mm and,  $D_b$  or  $D_s$ , as applicable. At least one longitudinal bar should be provided at each corner of the tie.

In all cases,  $V_u$  should not be taken greater than  $V_{uo}$  to prevent failure of the slab at the front face (and the back face if any) in shear.

The definition of torsion strips and other symbols used in the above expressions are explained in Fig. 4 taken from AS3600<sup>23</sup>.

### Correlation with test results

The punching shear strength predictions by the above design equations show good correlation with available test results<sup>32,33</sup>. The average ratio of test strength to calculated strength of 117 results is 1.59 with a coefficient of variation of 25%.

### Example

A reinforced concrete flat-plate floor has the following details: slab thickness = 250 mm, 500 mm square columns and spandrel beam size = 500 mm wide and 400 mm deep.

Take  $M_v^* = 60$  kNm and  $V^* = 550$  kN at an interior column, and  $M_v^* = 150$  kNm in the direction normal to the free edge and  $V^* = 280$  kN at an edge column.

Assume  $f'_c = 25$  MPa,  $f_y = 250$  MPa and clear cover of 25 mm.

Check the adequacy of the slab to guard against punching shear failure.

At an interior column, assume 16 mm diameter bars are used as the slab steel. Then average  $d = 250 - 25 - 16 = 209$  mm,  $u = 4(500 + 209) = 2836$  mm and width of torsion strip  $a = 500 + 209 = 709$  mm.

From eqns (13) and (14):

$$V_{uo} = 2836 \times 209 \times 0.34\sqrt{25} = 1008 \text{ kN}$$

Because there are no closed ties provided in the torsion strip, the punching shear strength is given by eqn (15):

$$V_u = \frac{1008}{1 + \frac{2836 \times 60}{8 \times 550 \times 709 \times 0.209}} = 799 \text{ kN} < V_{uo}$$

$$\phi V_u = 0.7 \times 799 = 560 > V^*$$

Therefore, punching shear strength is adequate.

At an edge column  $u = (2 \times 604.5) + 709 = 1918$  mm and width of torsion strip  $a = 500 + 209/2 = 604.5$  mm.

From eqns (13) and (14):

$$V_{uo} = 1918 \times 209 \times 0.34\sqrt{25} = 681 \text{ kN}$$

For the spandrel beam,  $D_b = 400$  mm and  $b_w = 500$  mm. To check whether the spandrel beam requires more than the minimum closed ties, calculate  $V_{umin}$  by eqn (17):

$$V_{umin} = \frac{\left(1.2 \times 681 \times \frac{400}{250}\right)}{1 + \left(\frac{1918 \times 150}{2 \times 280 \times 604.5 \times 0.5}\right)} = 484 \text{ kN} < V_{uo}, \text{ OK.}$$

$$\phi V_{umin} = 0.7 \times 484 = 339 \text{ kN} > V^*$$

Therefore, we require only minimum closed ties in the spandrel beam. Assume we use 12 mm diameter bars as closed ties. The larger dimensions of the tie is  $y_1 = 500 - (2 \times 25) - 12 = 438$  mm.

From eqn (20):

$$(A_{sw}/s)_{min} = 0.2 \times \frac{438}{250} = 0.350 \text{ mm}^2 \text{ mm}^{-1}$$

For  $A_{sw} = 110 \text{ mm}^2$  and  $s = 110/0.350 = 314$  mm. But the spacing should not exceed 300 mm in this case.

Therefore use 12 mm diameter closed ties at 300 mm spacing.

## WALLS

### Background

Reinforced concrete structural walls are often critical structural members of buildings. Walls are used to resist the lateral loads due to wind and seismic effects on buildings and the vertical loads due to dead loads and live loads from the floors of the buildings. Therefore, walls are subjected to axial force, bending moment and shear force.

The flexural behaviour of structural walls is well documented. The available literature is extensive as listed in a recent book by Paulay and Priestley<sup>9</sup>. In that book the state-of-the-art of structural concrete walls is very well summarized. The superiority of structural walls to resist lateral forces especially due to seismic effect is documented by Fintel<sup>7</sup>. These and other extensive number of publications (which are not included here due to space limitations) clearly point out that the ductility of structural concrete walls is of paramount importance. To design a structural wall to behave in a ductile manner, two issues are critical. First, the wall and other members in a building including the joints must be appropriately detailed. After extensive research, Paulay and others<sup>9-11</sup> have proposed detailing rules which are now incorporated in many building codes<sup>2,3-27</sup>.

The other issue in the design of ductile structural walls is that flexural yielding should control the strength. In other words, to design a structural concrete wall to behave in a ductile manner requires that its strength be governed by flexure rather than by shear. Because a shear failure is significantly less ductile it should not be permitted to occur. To achieve this, the shear capacity of a wall must be known and be larger than the shear corresponding to its moment capacity.

Extensive information is available with regard to design of walls. For instance, Paulay and Priestley<sup>9</sup> describe a capacity–design method for seismic regions which has been used in New

Zealand since the 1970s. In this method and in other similar methods, the flexural strength and the ductility capacity may be calculated using the conventional analysis of reinforced concrete sections subjected to axial force and bending moment<sup>10,34</sup>.

Methods are available to calculate the shear force a wall must resist. Paulay and Priestley<sup>9</sup> recommend that the shear force obtained by a static load analysis must be multiplied by a magnifying factor to account for inelastic dynamic effects. In design, the design shear strength of a wall must be greater than the design shear force.

A large number of structural concrete wall tests have been reported in the literature. Due to space limitations, the details of these tests and a complete list of references are not given, but these are available elsewhere<sup>8-12</sup>. As far as analytical work is concerned, attempts have been made in the past to predict the shear behaviour of walls. Although solutions for a certain type of wall have been reached, a complete solution is yet to be obtained. For instance, Hsu and Mo<sup>20</sup> assumed that the wall is infinitely stiff in the transverse direction and predicted the strength of certain low-rise walls. Recently, Liu<sup>12</sup> has used the plasticity theory to calculate the shear strength of walls.

In building codes<sup>23,24</sup>, the shear strength of a wall is calculated by summing the shear resisted by the concrete and the shear carried by the shear reinforcement. In the American Concrete Institute Code ACI318<sup>24</sup>, the horizontal reinforcement is considered as the shear reinforcement and the effect of vertical reinforcements is ignored. In the Australian Standard AS3600<sup>23</sup>, the shear reinforcement is either the horizontal reinforcement or the vertical reinforcement depending on the height-to-length ratio of walls. In both ACI318 and AS3600, the shear resisted by the concrete is a very significant component of the calculated shear strength.

The above code methods are empirical extrapolations of similar methods used in the calculation of shear strength of slender beams and has very little relevance to the actual behaviour of a structural wall. The effects of reinforcements and the concrete strength are therefore not realistically represented by the code expressions.

Tests and analytical results available in the literature<sup>8,12,21,22</sup> show that:

1. the horizontal reinforcement is not efficient as shear reinforcement;
2. the contribution of concrete to shear strengths is severely over-estimated by the code expressions; and
3. the vertical reinforcement contributes significantly to the shear strength.

For these reasons, the shear strengths calculated by the code expressions usually show a large scatter with respect to test data.

### Design equations

Earlier, the author has carried out research into the shear behaviour of reinforced concrete structural walls<sup>8,21,22</sup>. A theoretical method was developed to study the behaviour and the shear capacity of structural concrete walls based on a strut-and-tie model. The analytical results were simplified to yield a design expression which showed good correlation with test results<sup>22</sup>.

Accordingly, the shear strength  $V_u$  of a wall is given by:

$$V_u = t_w d_w \left( p_1 f_y + \frac{N^*}{A_g} \right) \tan \theta \quad (21)$$

but not greater than  $V_{u\max}$  given by

$$V_{u\max} = \frac{k_3 f'_c t_w d_w \sin \theta \cos \theta}{(1.14 + 0.68 \cot^2 \theta)} \quad (22)$$

In eqn (21)  $p_1 = A_1/t_w L_w$ ,  $A_1$  is the area of vertical steel in the wall on both faces in length  $L_w$ ,  $t_w$  is the wall thickness and  $d_w$  is the effective horizontal length of the wall, taken as the horizontal length of the wall between centres of end elements. In the absence of end elements,  $d_w$  is assumed to be equal to 0.8 times the wall length  $L_w$ . Also,  $N^*$  is the design (factored) axial compressive force on the wall cross-section and  $A_g$  is the gross concrete area of the wall cross-section.

The strut angle  $\theta$  is given by

$$\tan \theta = \frac{d_w}{H_w} \quad (23)$$

within the limits  $30^\circ \leq \theta \leq 60^\circ$ , where  $H_w$  is the height of the wall. In eqn (22), the factor  $k_3$  accounts for the difference between the compressive strength of *in situ* concrete in the wall



and the cylinder compressive strength of concrete, and is taken as<sup>3,5</sup>:

$$k_3 = \left( 0.6 + \frac{10}{f'_c} \right) \leq 0.85 \quad (24)$$

Note that eqn (21) is in terms of the vertical steel in the wall. Both analytical and experimental results showed that the vertical steel is an important parameter in the evaluation of shear strength of a wall, provided that the wall contained a minimum amount of horizontal steel<sup>8,21,22</sup>.

### Correlation with test results

The flexural strength of a structural wall under inplane horizontal and vertical forces may be calculated using the conventional flexure theory of reinforced concrete sections subjected to bending moment and axial compression<sup>34</sup>. The shear strength of a wall is given by eqn (21). The smaller of the calculated values is the predicted strength. In Ref.<sup>22</sup>, the correlation of test and calculated strengths of 67 test walls is given. Of the 67 test walls, 29 walls are predicted to have failed in shear mode. The mean value of test/calculated shear strengths is 1.09 with a coefficient of variation of 12.4%. The remaining 38 walls are predicted to have failed in flexure mode. The mean value of test/calculated flexure strengths of these walls is 1.17 with a coefficient of variation of 13.5%.

The calculated strengths are generally conservative. The test parameters covered a wide range as follows:  $f'_c$ : 20–140 MPa,  $H_w/L_w$ : 0.25–2.4;  $N^*/A_g f'_c$ : 0–0.3; and  $p_l f_y / f'_c$ : 0.1–0.5. In all, eqn (21) correlated well with test results and is considered to be suitable for use in design calculations.

The design of walls incorporating eqn (21) has been illustrated by a numerical example elsewhere<sup>22</sup>.

### CONCLUSIONS

The paper presented a summary of shear design provisions for beams and slabs used in the Australian practice. The variable angle truss model, modified empirically to include the contribution of concrete [eqn (1)], has shown good correlation

with test data (Figs 1–3). This approach may be used for concretes with compressive strength up to 100 MPa provided that the minimum shear reinforcement and  $V_{\text{umin}}$  are calculated by eqns (10) and (12), respectively.

Expressions to calculate the punching shear strength of slabs in the vicinity of a column or support have been presented [eqns (15)–(18)]. These expressions show good correlation with test data and cover various situations ranging from slabs with no spandrel beams and no shear reinforcement to slabs with spandrel beams and closed ties.

The paper also discussed the importance of shear strength in the design of walls. An expression [eqn (21)] based on a strut-tie-model has been proposed to calculate the shear strength of walls. It is believed that this expression provides a rational basis for the design of walls to guard against non-ductile shear failure.

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