Shear Forces and Bending Moments in Beams

Bending Stress:

$$\sigma = \frac{My}{I}$$

Moment of Inertia:

$$I_x = \int_A y^2 dA$$

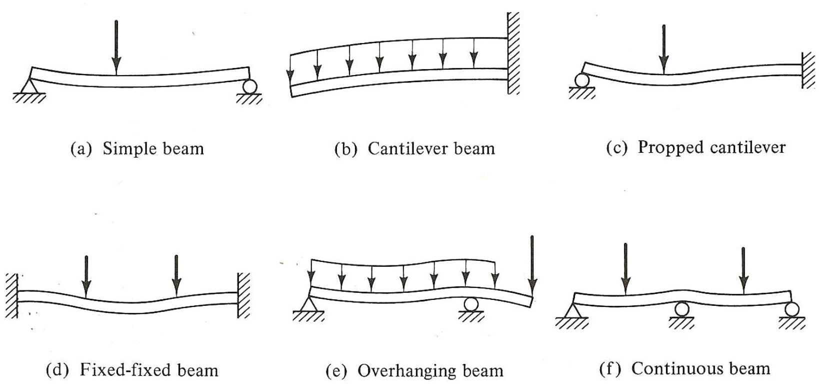
$$I_y = \int_A x^2 dA$$

Parallel Axis Theorem:

$$I_x = I_{xc} + Ad^2$$

$$I_y = I_{yc} + Ad^2$$

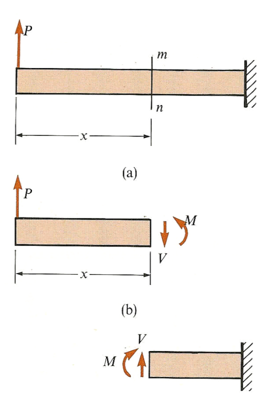
Beam Classifications:

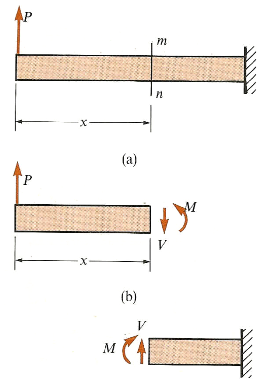


Beams are also classified according to the shape of their cross sections.

Shear Force and Bending Moment:

When a beam is loaded by forces or couples, internal stresses and strains are created. To determine these stresses and strains, we first must find the internal forces and couples that act on cross sections of the beam. Consider the following example.



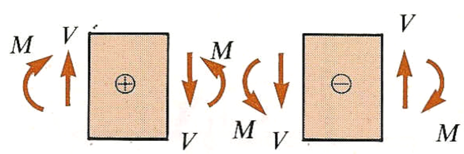


It is convenient to reduce the resultant to a shear force, V, and a bending moment, M. Because shear forces and bending moments are the resultants of stresses distributed over the cross section, they are known as **stress resultants** and in statically determinate beams can be calculated from the equations of static equilibrium.

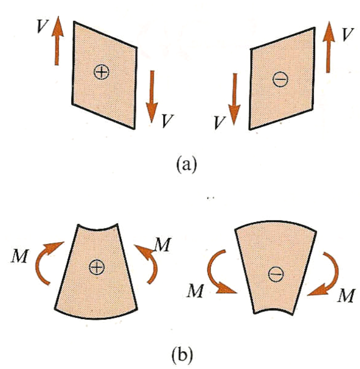
<u>Deformation Sign Conventions:</u>

As can be seen in the previous diagram (left hand section vs. right hand section), we recognize that the algebraic sign does not depend on its direction in space, such as upward or downward or clockwise or counterclockwise. The sign depends on the direction of the stress resultant with respect to the material against which it acts.

Shear and Moment Sign Convention



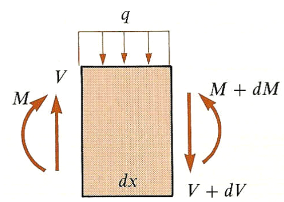
Deformations highly exaggerated



Positive shear forces always deform right hand face downward with respect to the left hand face. Positive bending moments always elongate the lower section of the beam.

Load, Shear Force and Bending Moment Relationships:

Consider the following beam segment with a uniformly distributed load with load intensity q. Note that distributed loads are positive when acting downward and negative when acting upward.



Summing forces vertically we find:

$$\frac{dV}{dx} = -q$$

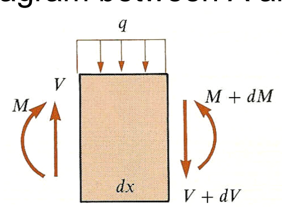
Thus, the shear force varies with the distance x, and the rate of change (slope) with respect to x is equal to –q. Also, if q = 0, then the shear force is constant.

Rearranging and integrating between two points A and B on the beam we have:

$$dV = -qdx$$

$$\int_A^B dV = -\int_A^B qdx$$

$V_B - V_A$ = Area of load intensity diagram between A and B



Summing moments and discarding products of differentials because they are negligible compared to other terms, we have:

$$\frac{dM}{dx} = V$$

The above shows that the rate of change of moment with respect to x is equal to the shear force. Also, the above equation applies only in regions where distributed loads act on the beam. At a point where a concentrated load acts, a sudden change in shear force results and the derivative dm/dx is undefined.

Rearranging and integrating between two points A and B on the beam we have:

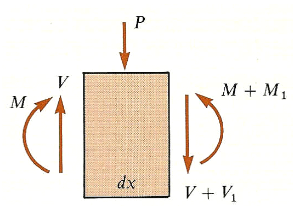
$$dM = Vdx$$

$$\int_A^B dM = \int_A^B V dx$$

$M_B – M_A$ = Area of shear force diagram between A and B

The above can be used even when concentrated loads are acting on the beam between points A and B. However, it is not valid if a couple acts between points A and B.

Now consider the following beam segment with a concentrated load, P. Again, concentrated loads are positive when acting downward and negative when acting upward.



Summing forces vertically we find:

$$V_1 = -P$$

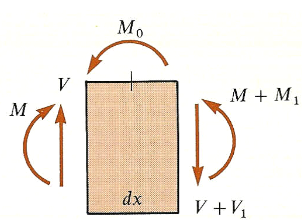
Thus, an abrupt change occurs in the shear force at a point where a concentrated load acts. As one moves from left to right through a point of load application, the shear force decreases by an amount equal to the magnitude of the downward load.

Summing moments we find:

$$M_1 = P(dx/2) + Vdx + V_1dx$$

Since dx is infinitesimally small, we see that M_1 is infinitesimally small. Therefore, we conclude that the bending moment does not change as we move through a point of concentrated load application. Recall that dm/dx = V. Since the shear force changes at the point of load application, we conclude that the rate of change dm/dx decreases abruptly by an amount equal to P.

Finally, we consider the following beam segment with a concentrated couple, M_0. Counterclockwise couples are considered to be positive and clockwise couples are considered to be negative.



Summing moments and disregarding terms that contain differentials, we have:

$$M_1 = -M_0$$

The previous equation shows that there is an abrupt decrease in the bending moment in the beam due to the applied couple, M_0, as we move from left to right through the point of load application.

In summary:

Distributed loads

Shear force slope (dV/dx) = -q
$V_B - V_A$ = Area of load intensity diagram between A and B
Moment slope (dM/dx) = V
$M_B - M_A$ = Area of shear force diagram between A and B

Concentrated loads

Shear force slope (dV/dx) = 0
At load application $V_1 = -P$
Moment slope (dM/dx) = V (not valid at load application)

Concentrated loads

Moment slope (dM/dx) decreases by P
$M_B - M_A$ = Area of shear force diagram between A and B

Concentrated Couples

Causes an abrupt change in bending moment

Shear and Bending Moment Diagrams:

The loading on most beams is such that the stress resultant on planes perpendicular to the axis of the beam consists of a shear force, V, and a bending moment, M. In determining beam responses, it is very convenient, if not essential, to first determine the ***shear*** and ***bending moment diagrams***.

The basic procedure for determining the shear and moment diagrams is to determine the values of V and M at various locations along the beam and plotting the results.

In doing so, we will determine critical sections within the beam. A critical section is one where a critical or maximum stress occurs. **Section of Maximum Shear** – Since the shear, V, at any transverse section of the beam is the algebraic sum of the transverse forces to the left of the section, the shear, in most cases, can be evaluated at a glance.

Section of Maximum Moment – It can be shown mathematically, that when the shear force is zero or changes sign; the bending moment will be either a maximum or relative maximum.