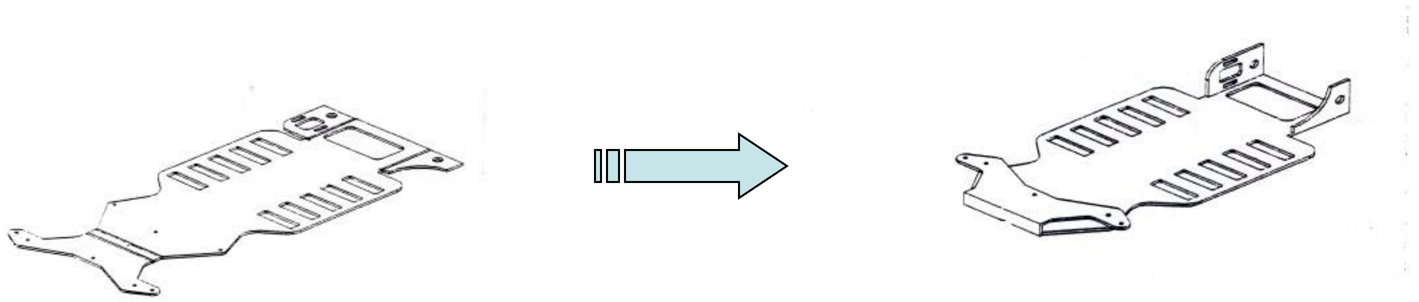


Sheet Metal Forming

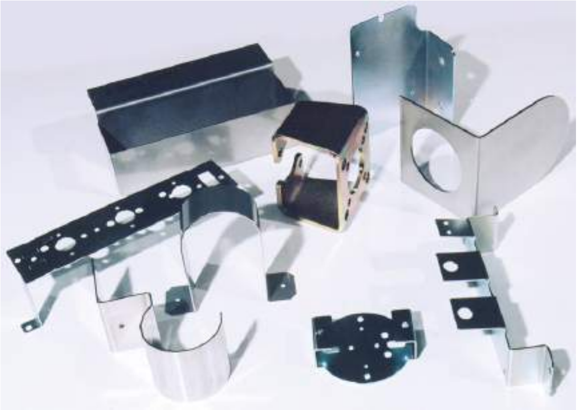
2.810

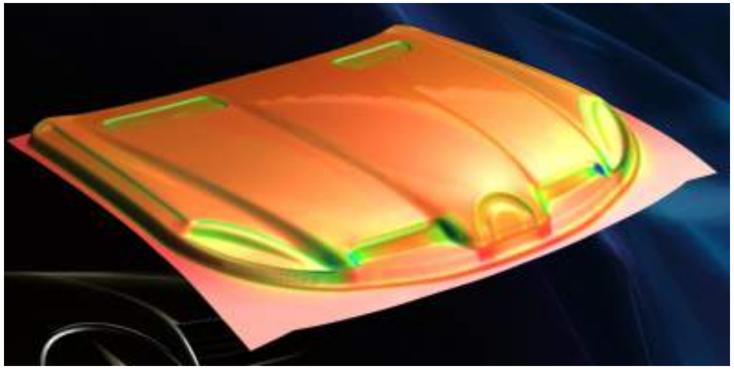
D. Cooper

- “Sheet Metal Forming” Ch. 16 Kalpakjian
- “Design for Sheetmetal Working”,
Ch. 9 Boothroyd, Dewhurst and Knight

Massachusetts
Institute of
Technology

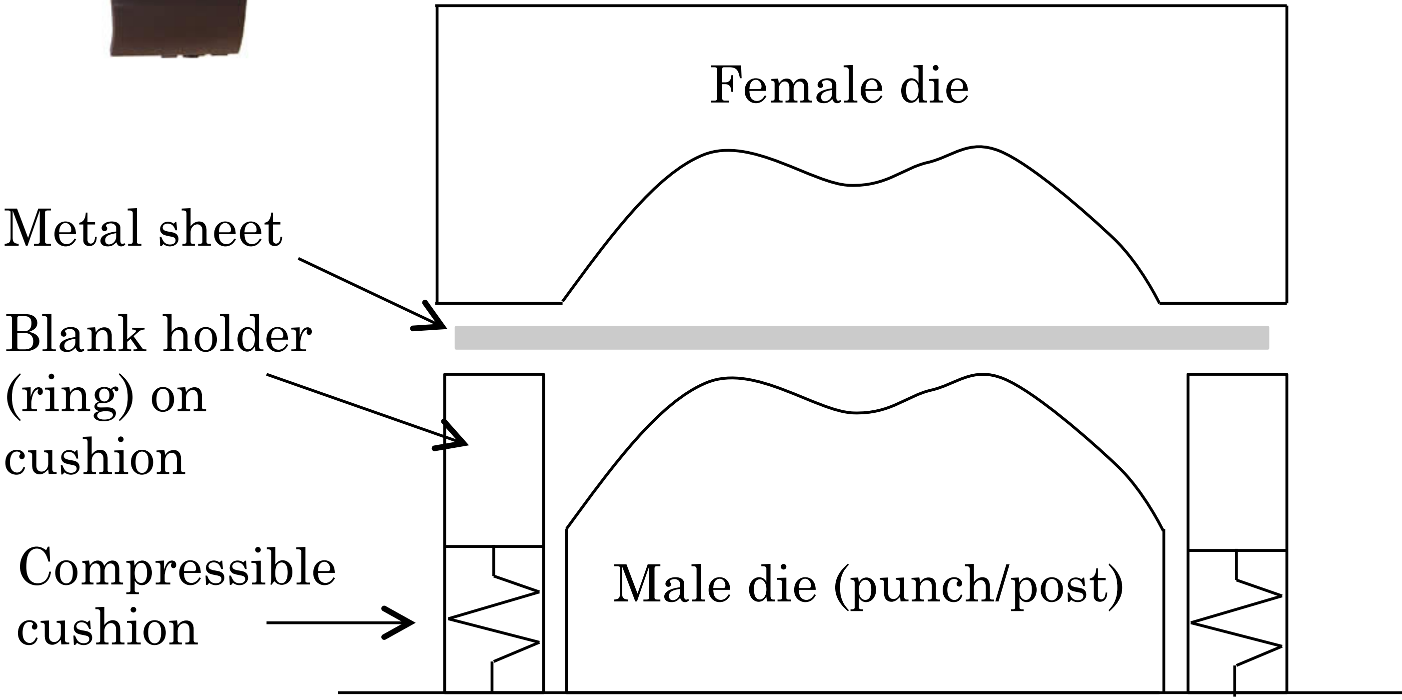
Examples-sheet metal formed





Sheet metal stamping/drawing – car industry

- **90million** cars and commercial vehicles produced worldwide in 2014



Female die

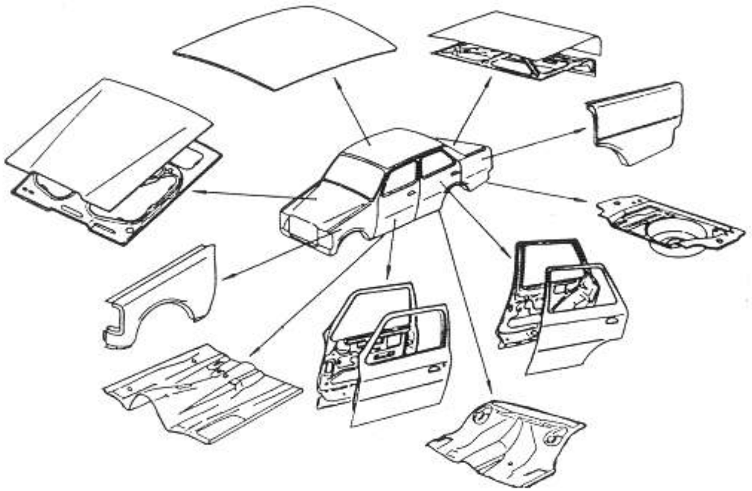
Metal sheet

Blank holder (ring) on cushion

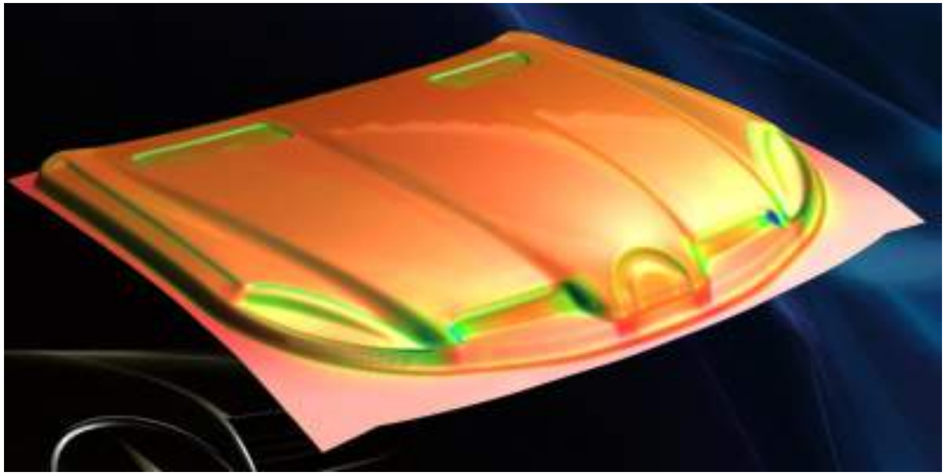
Compressible cushion

Male die (punch/post)

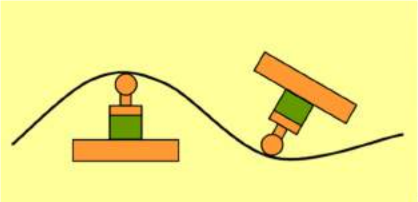
Stamping Auto body panels



- 3 to 5 dies each
- Prototype dies ~ $50,000
- Production dies ~ $0.75-1 mil.



- Forming dies
- Trimming station
- Flanging station



Objectives

By the end of today you should be able to…

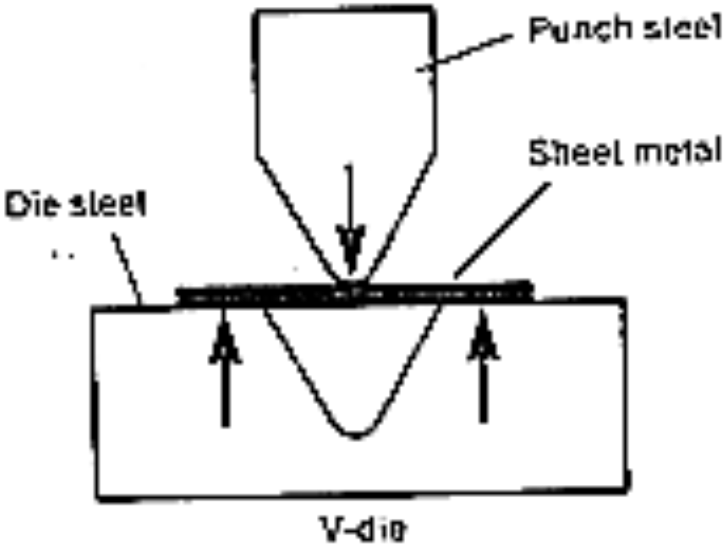
…**describe** different forming processes, when they might be used, and **compare** their production rates, costs and environmental impacts

…**calculate** forming forces, **predict** part defects (tearing, wrinkling, dimensional inaccuracy), and **propose** solutions

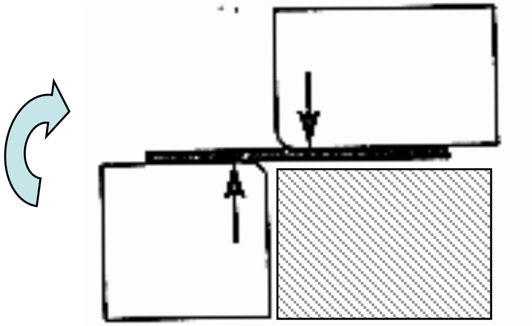
…**explain** current developments: opportunities and challenges

LMP Shop

Brake press



Finger brake



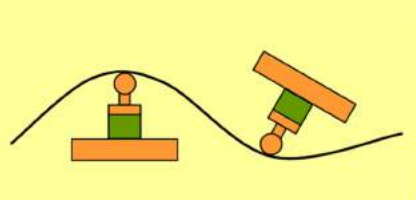
Technology – a brief review

Material drawn into shape

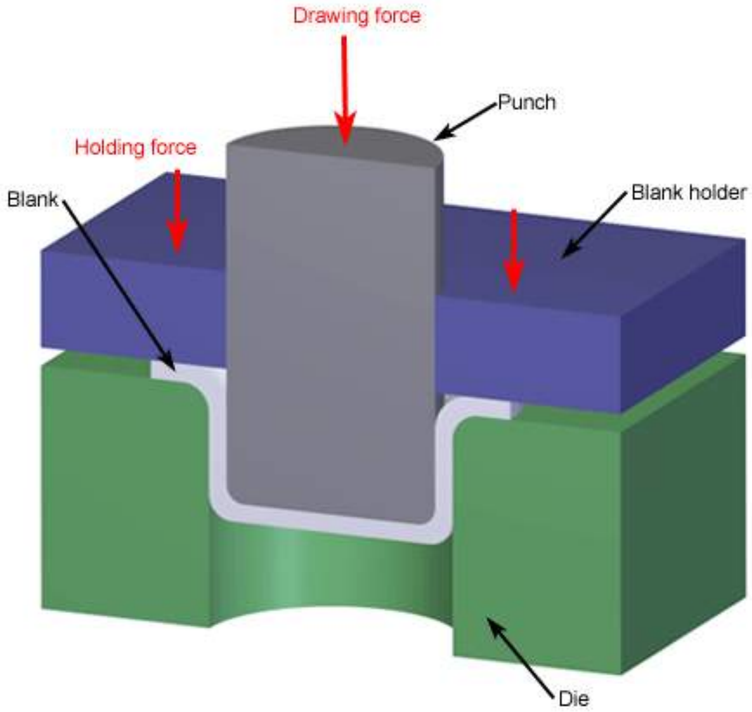
- Conventional drawing/stamping – expensive tooling, no net thinning, quick **20-1000pts/hr**
- Hydro-forming – cheap tooling, no net thinning, slow, high formability **7-13cycles/hr**

Material stretched into shape

- Stretch forming – very cheap tooling, net thinning, slow, low formability **3-8pts/hr**
- Super-plastic forming – cheap tooling, net thinning, expensive sheet metal, slow, very high formability **0.3-4pts/hr**



Drawing – expensive tooling, no net thinning, quick



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Deep-drawing

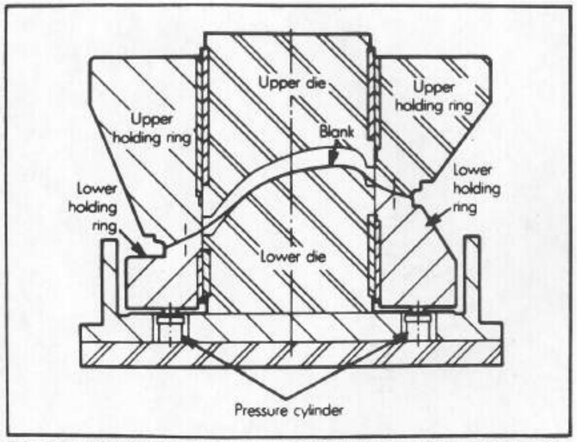
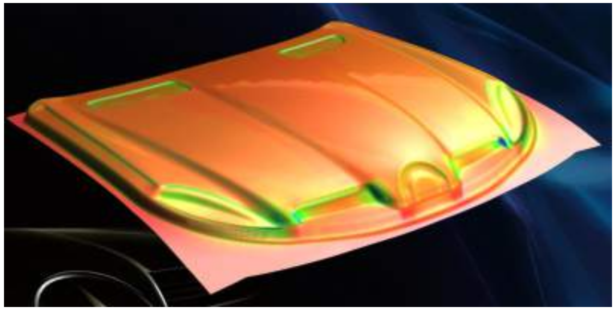
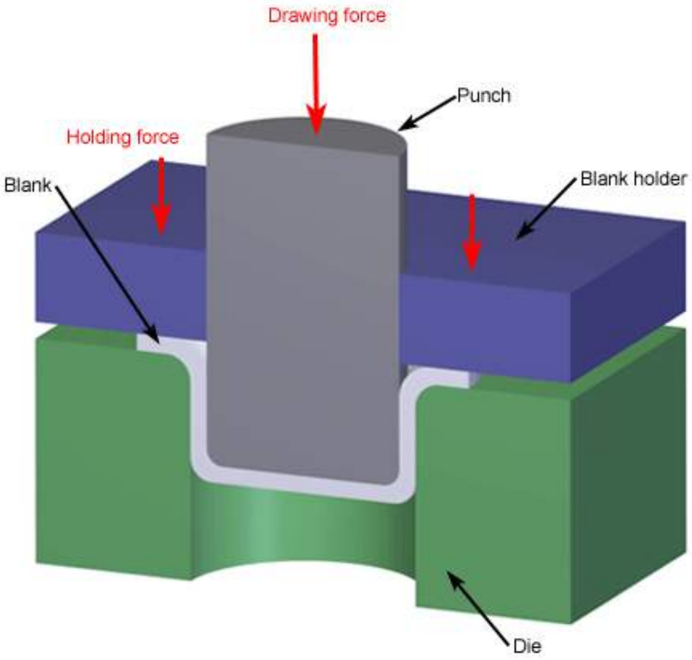


Fig. 7-23 Tooling for stretch-draw forming fenders from steel blanks. (*Oldsmobile Div., General Motors Corp.*)



Shallow-drawing (stamping)

Deep-drawing



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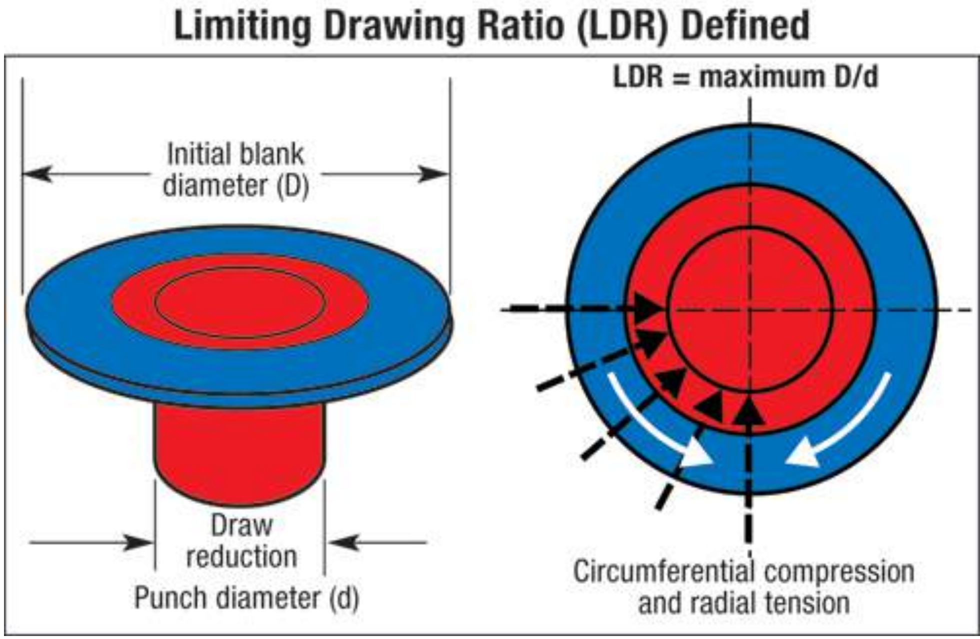
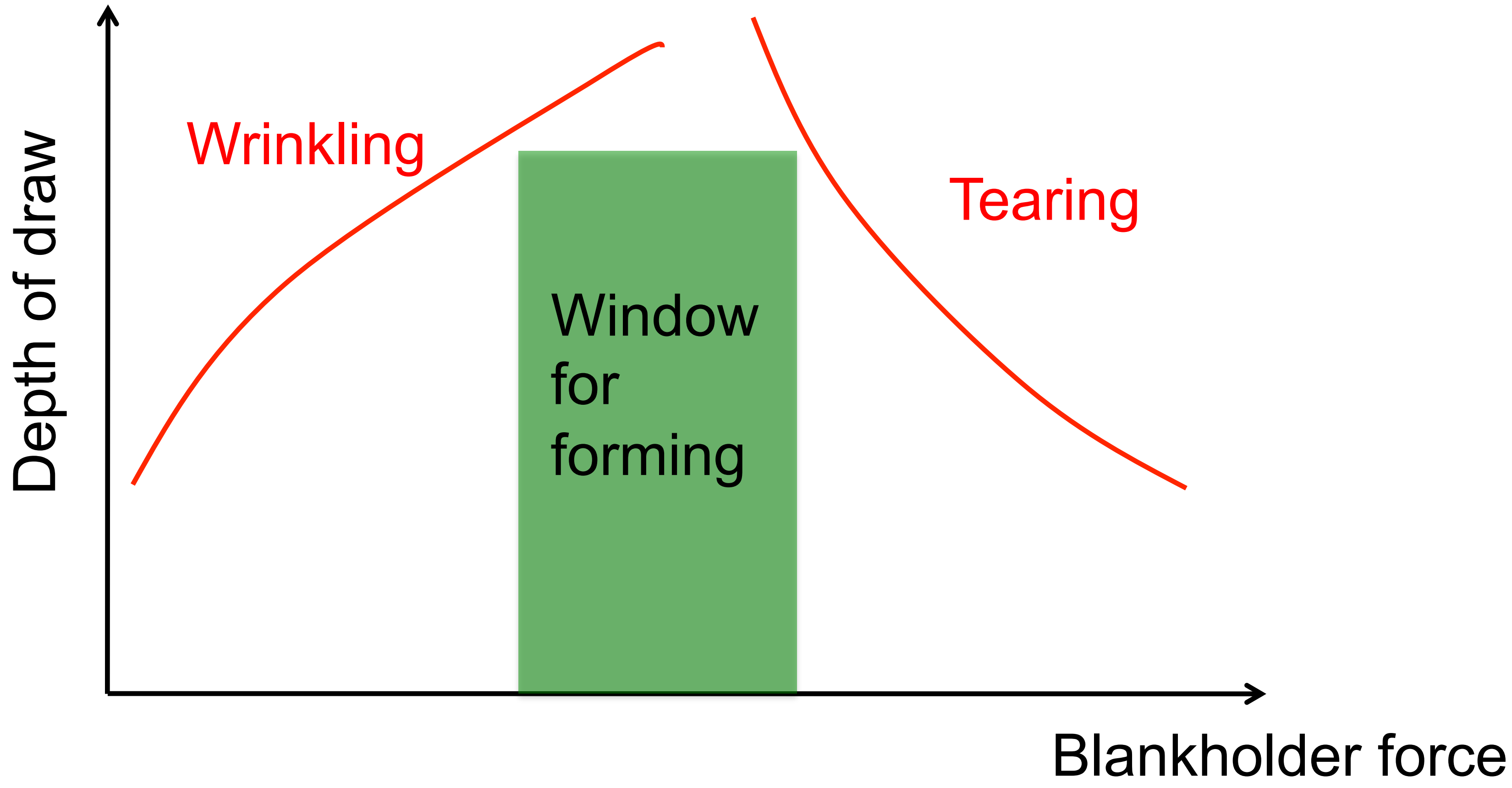


Blank holder helps prevent wrinkling and reduces springback

Blank holder not necessary if blank diameter / blank thickness is less than 25-40. Smaller values for deeper forming.

http://www.thomasnet.com/articles/custom-manufacturing-fabricating/wrinkling-during-deep-drawing

Blank holder force: forming window



Deep Drawing of drinks cans

Hosford and Duncan (can making): http://www.chymist.com/Aluminum%20can.pdf

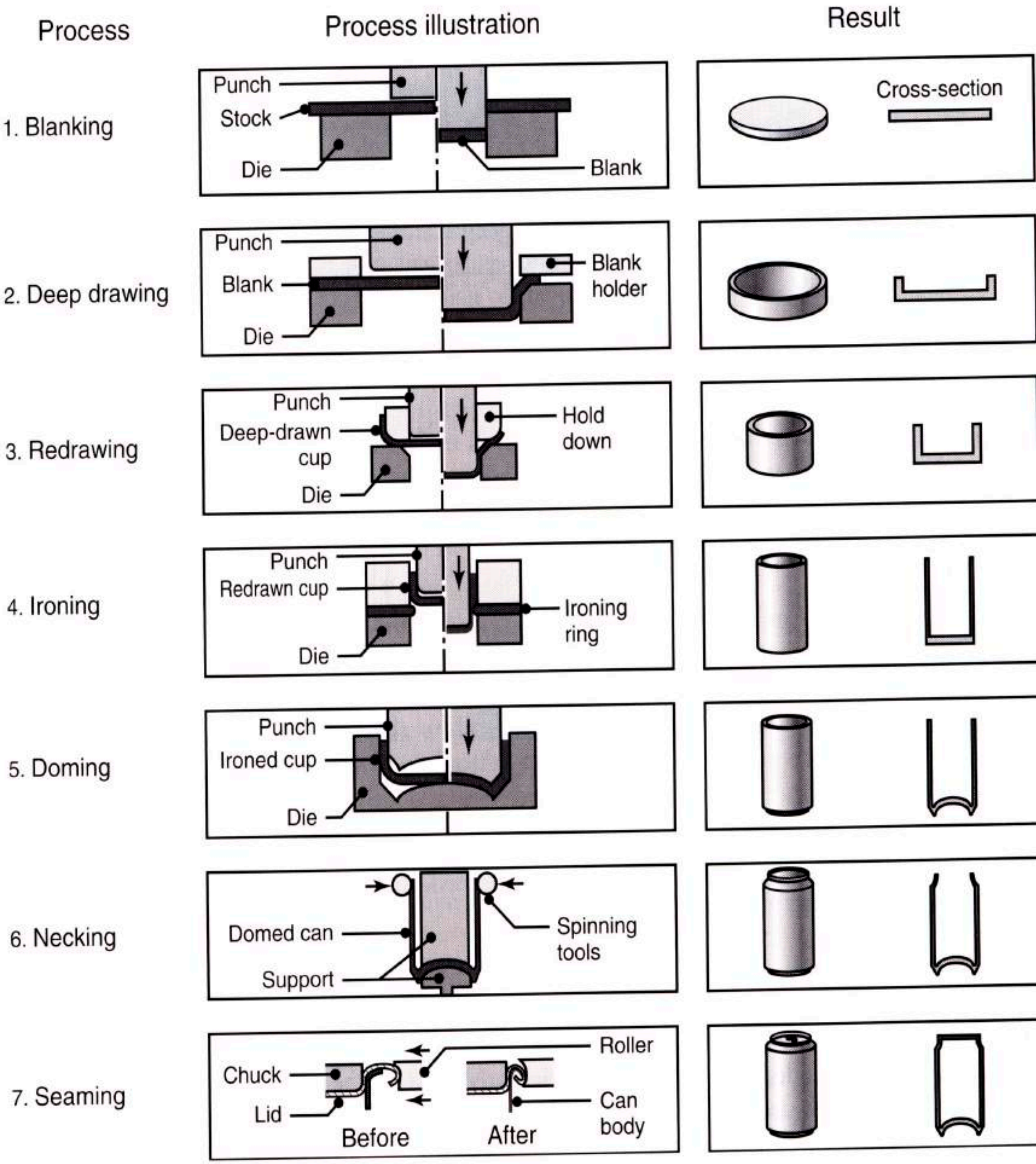
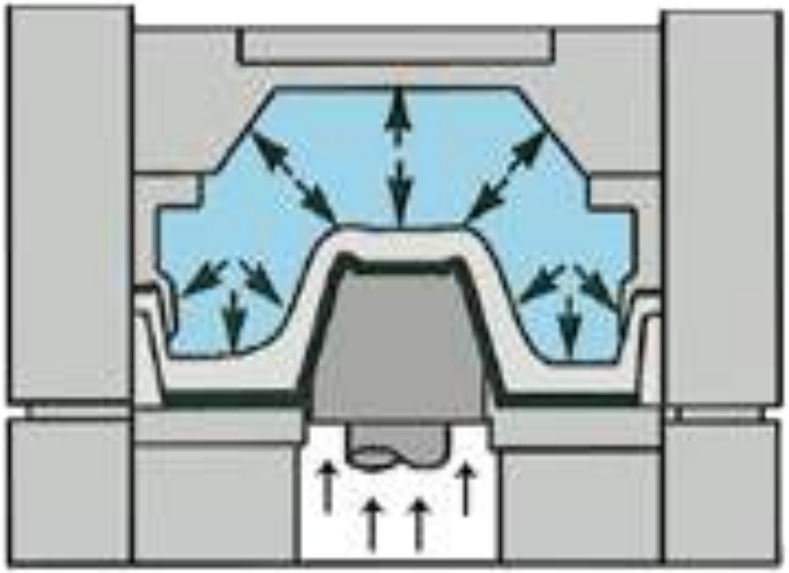


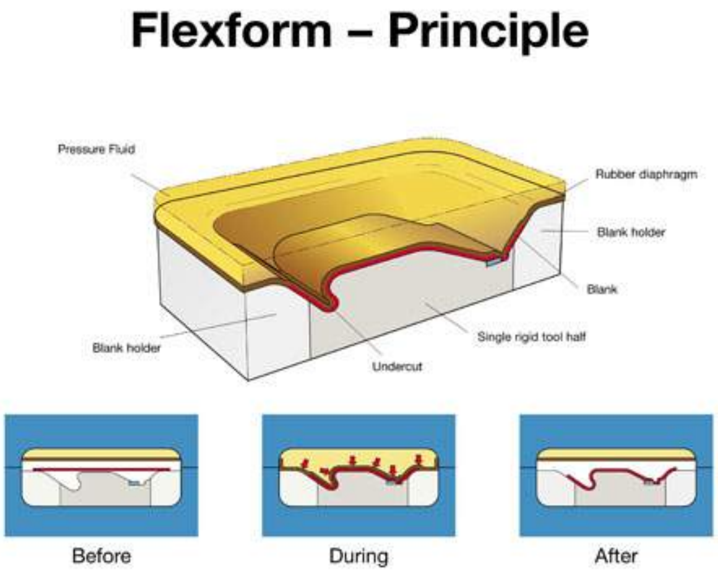
FIGURE 16.31 The metal-forming processes involved in manufacturing a two-piece aluminu beverage can.

Hydro-forming – cheap tooling, no net thinning, slow(ish), high formability



Low volume batches

Hydro-forming – cheap tooling, no net thinning, slow(ish), high formability

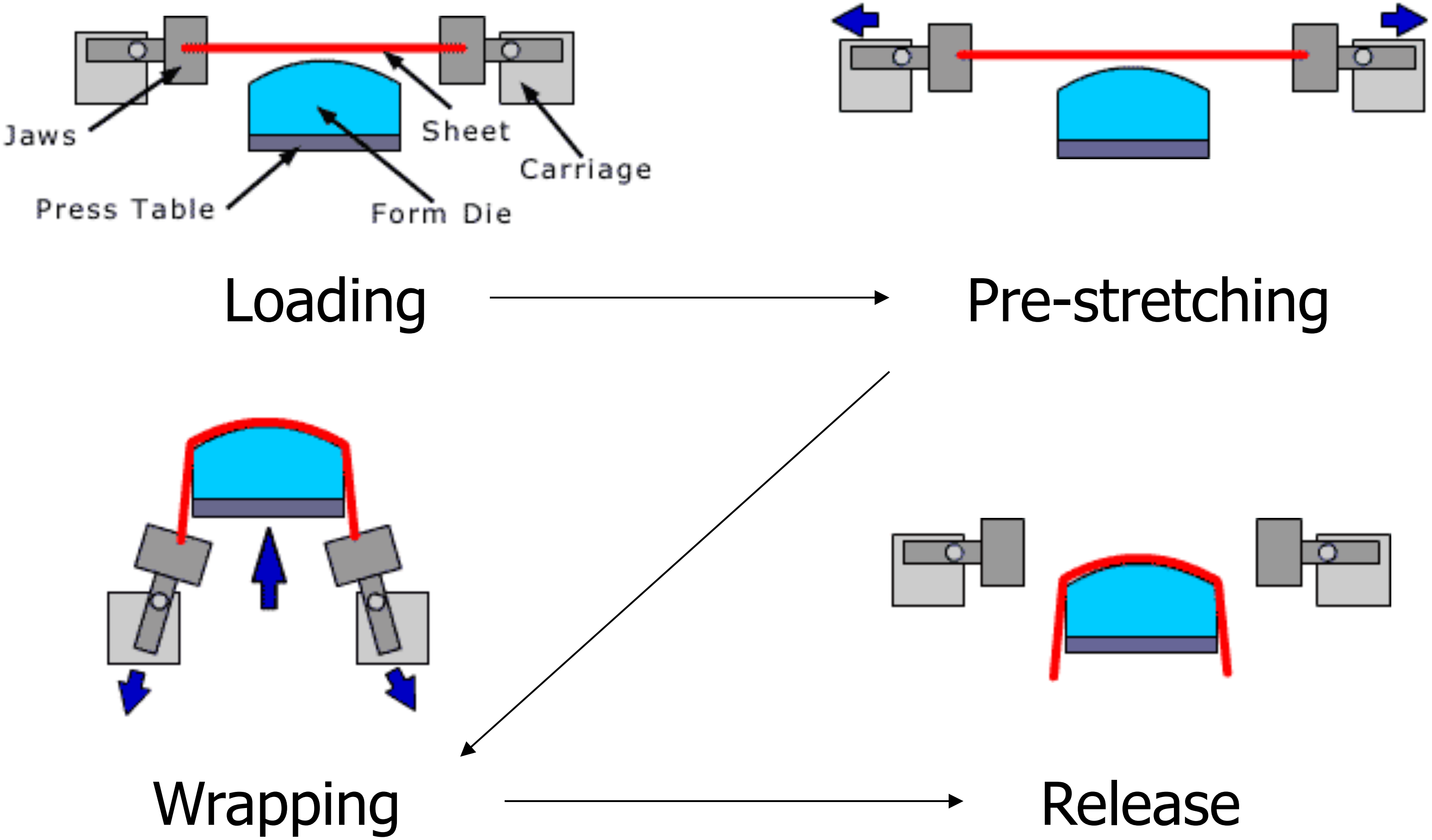


Low volume batches

Hydro-forming – cheap tooling, no net thinning, slow, high formability

Small flexforming tool made by additive manufacturing

Stretch forming – very cheap tooling, net thinning, slow, low formability, sheet metal up to 15mx9m



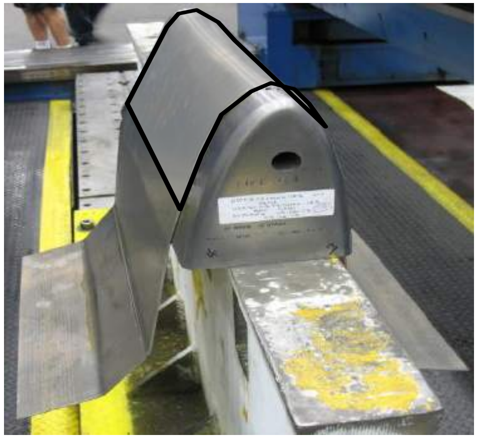
Loading → Pre-stretching

Wrapping → Release

* source: http://www.cyrilbath.com/sheet_process.html

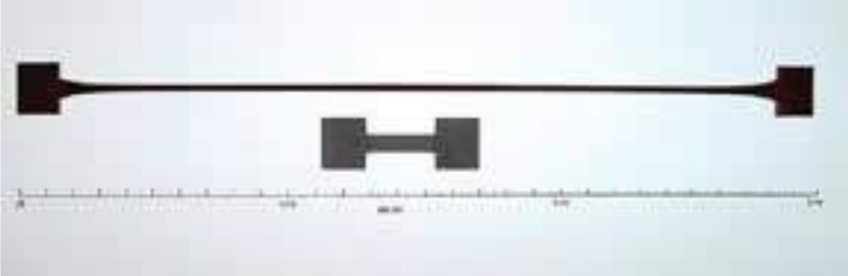
Low volume batches

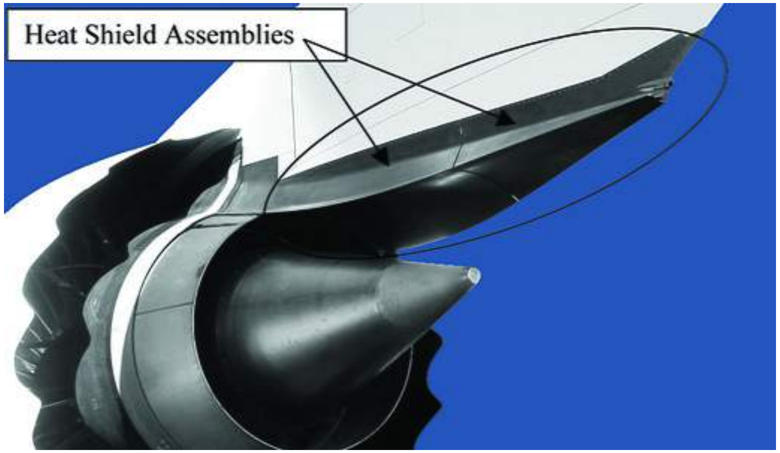
Stretch forming: Example parts

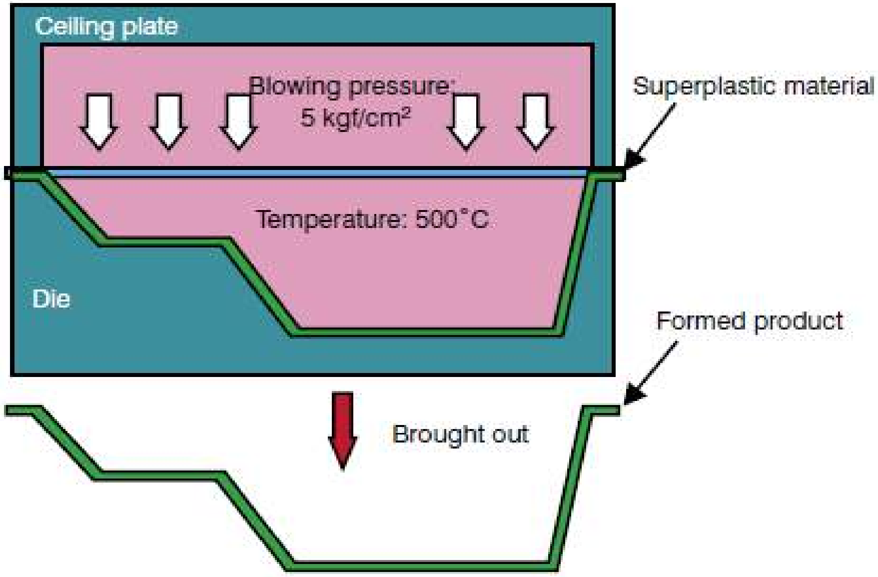


Higher aspect ratio, deeper parts

Super-plastic forming – cheap tooling, net thinning, slow, expensive sheet metal, very high formability





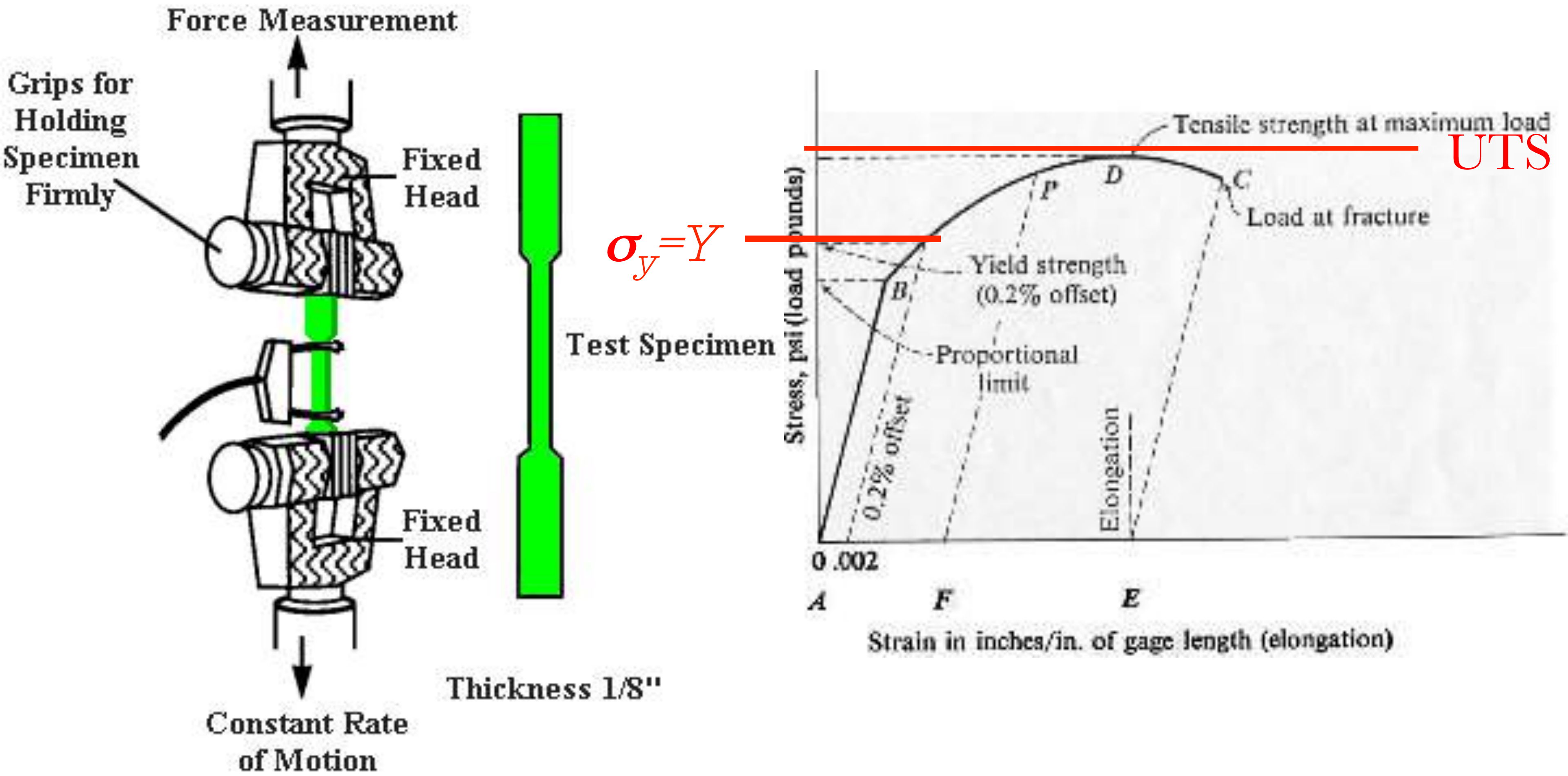


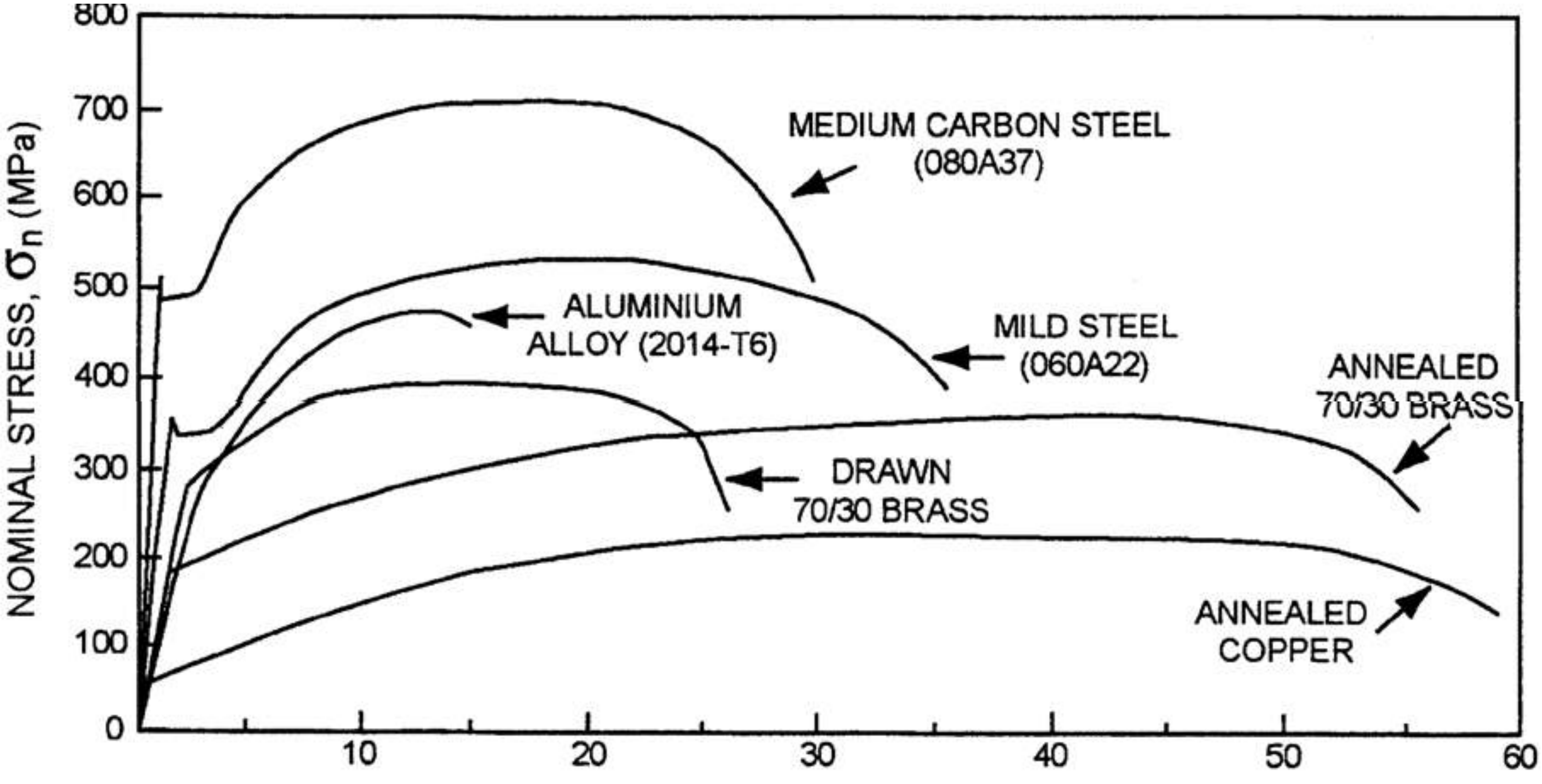
Low volume batches, 0.5-0.75 melting temp

Forming forces and part geometry

Massachusetts
Institute of
Technology

Tensile test – the Stress-strain diagram





Nominal strain

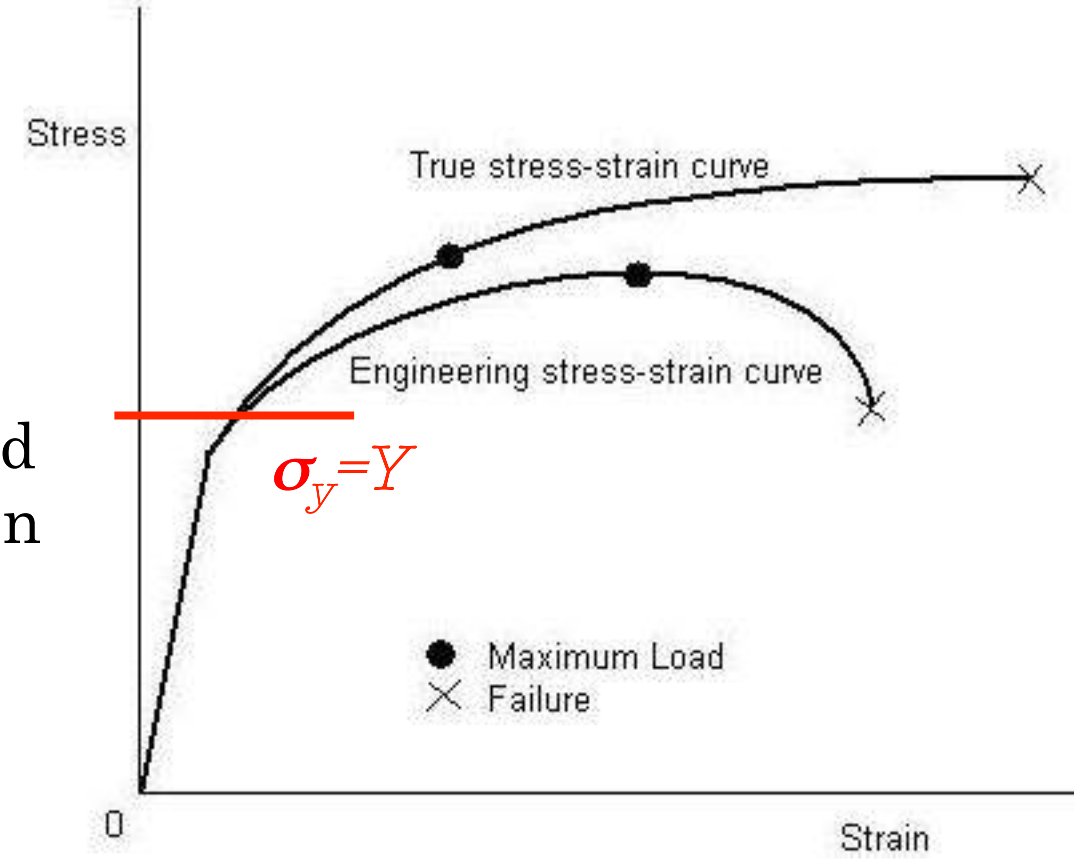
True stress & strain

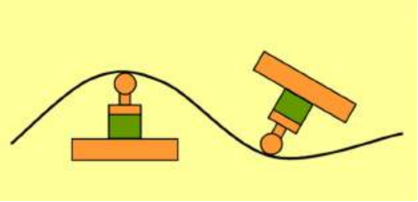
$$\varepsilon_{tr} = \ln(1 + \varepsilon_{en})$$

$$\sigma_{tr} = \sigma_{en}(1 + \varepsilon_{en})$$

True stress can be expressed using a power law (Hollomon equation):

$$\sigma_{tr} = K\varepsilon_{tr}^{n}$$

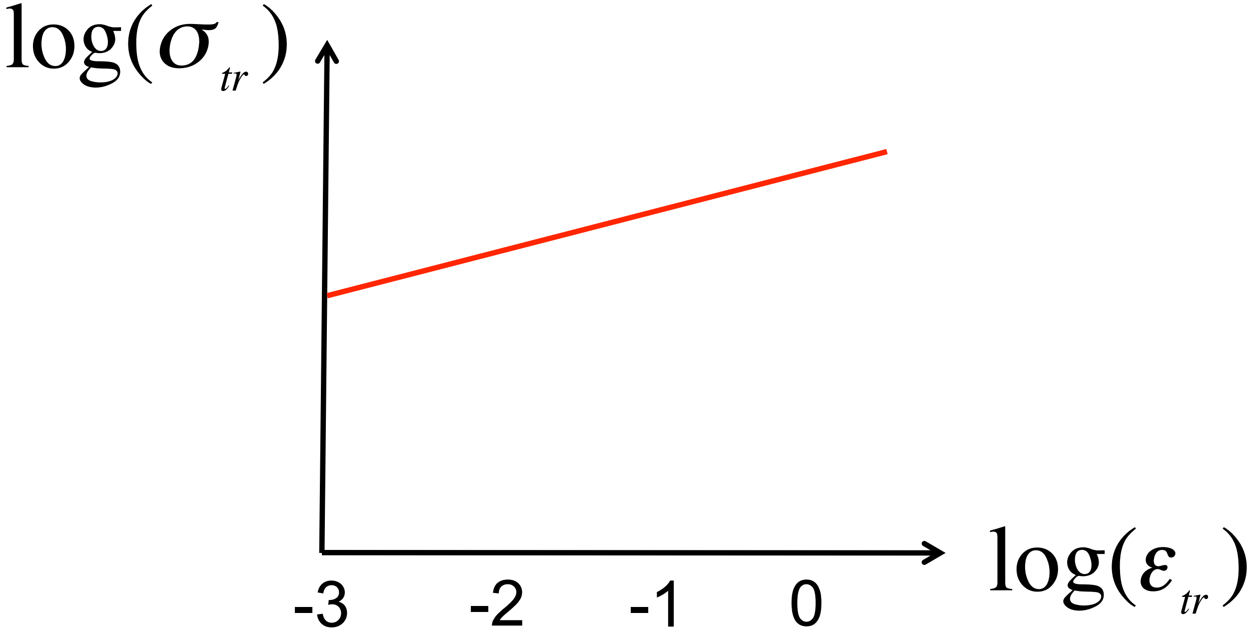




Power-Law Expression (Hollomon equation)

$$\sigma_{tr} = K\varepsilon_{tr}^{n}$$

Can be re-written: $\log(\sigma_{tr}) = n\log(\varepsilon_{tr}) + \log K$



Power-Law Expression (Hollomon equation)

$$\sigma_{tr} = K\varepsilon_{tr}^{n}$$

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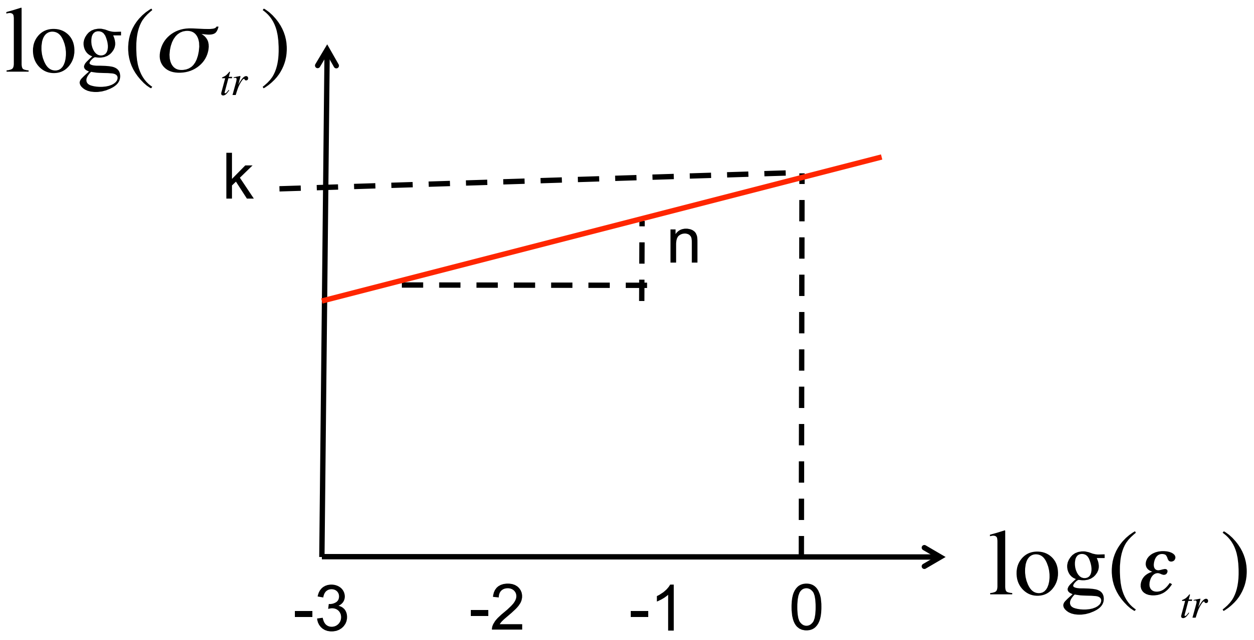


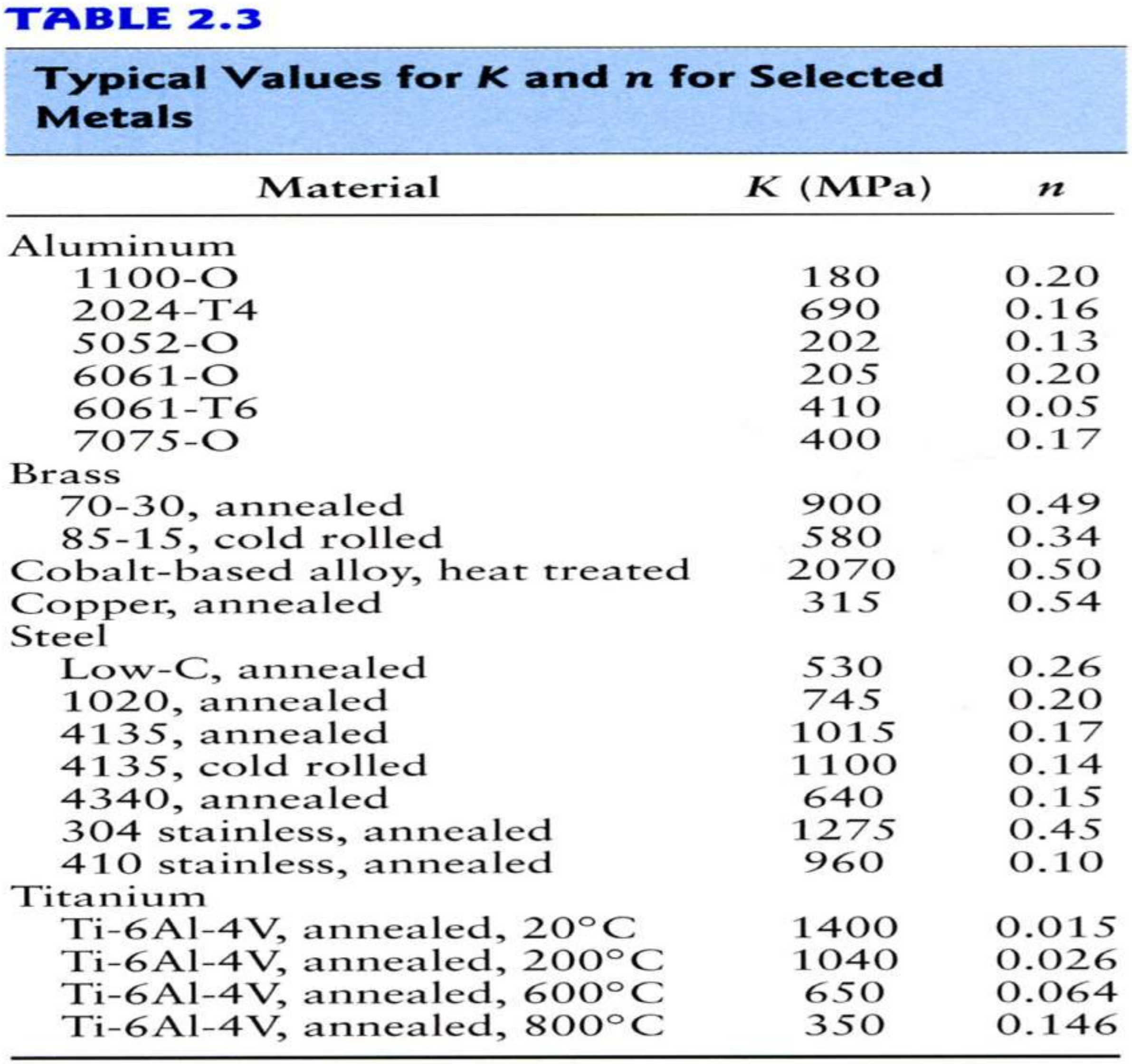
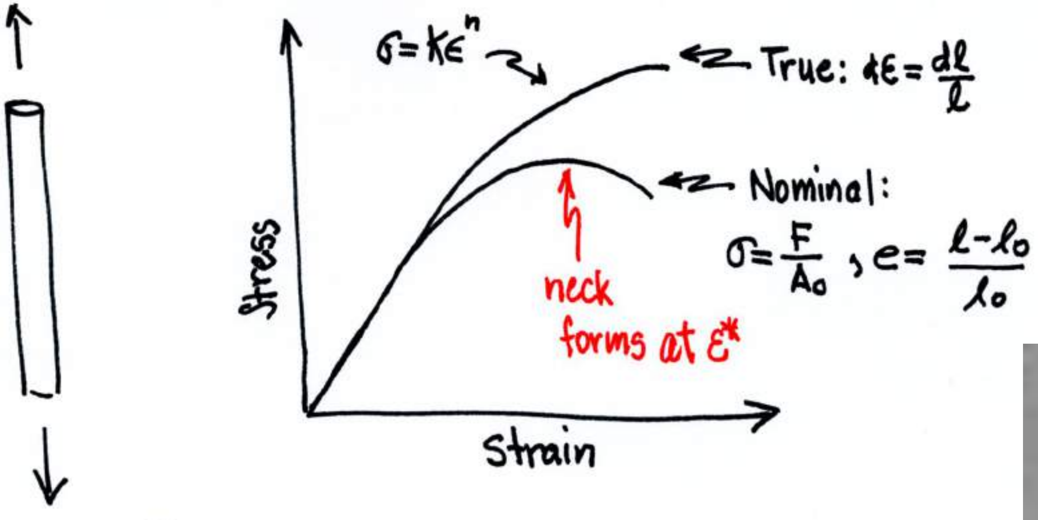


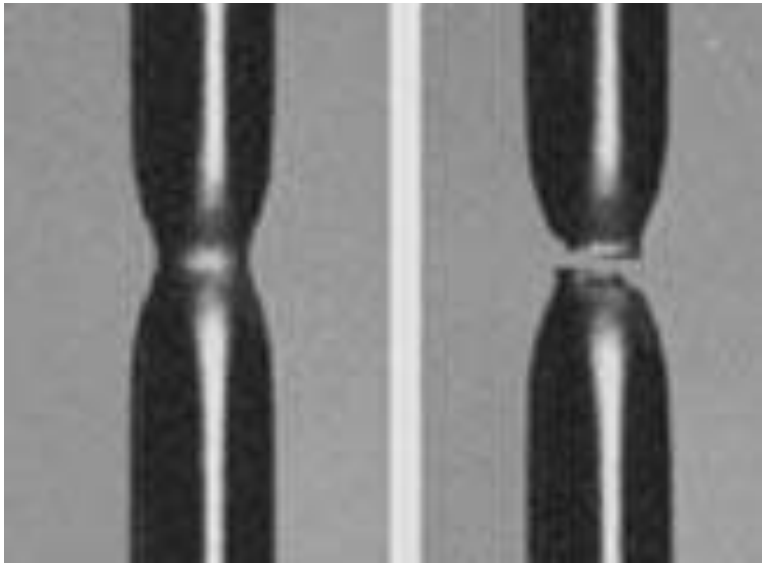
TABLE 2.3

Typical Values for *K* and *n* for Selected Metals

Material	*K* (MPa)	*n*
Aluminum		
1100-O	180	0.20
2024-T4	690	0.16
5052-O	202	0.13
6061-O	205	0.20
6061-T6	410	0.05
7075-O	400	0.17
Brass		
70-30, annealed	900	0.49
85-15, cold rolled	580	0.34
Cobalt-based alloy, heat treated	2070	0.50
Copper, annealed	315	0.54
Steel		
Low-C, annealed	530	0.26
1020, annealed	745	0.20
4135, annealed	1015	0.17
4135, cold rolled	1100	0.14
4340, annealed	640	0.15
304 stainless, annealed	1275	0.45
410 stainless, annealed	960	0.10
Titanium		
Ti-6Al-4V, annealed, 20°C	1400	0.015
Ti-6Al-4V, annealed, 200°C	1040	0.026
Ti-6Al-4V, annealed, 600°C	650	0.064
Ti-6Al-4V, annealed, 800°C	350	0.146

Tensile instability - necking





Tensile instability (1-D)

$F = \sigma A$; so $dF = \sigma dA + A d\sigma = 0$ at max load

$$\frac{d\sigma}{\sigma} = -\frac{dA}{A} = d\varepsilon$$

$$\frac{d\sigma}{d\varepsilon} = \sigma$$

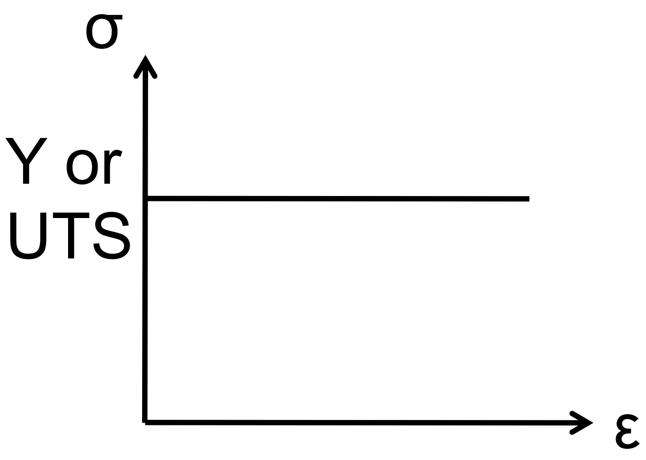
With $\sigma = K\varepsilon^n$: $\frac{d\sigma}{d\varepsilon} = nK\varepsilon^{n-1} = \sigma = K\varepsilon^n$

$$\Rightarrow \boxed{\varepsilon^* = n}$$

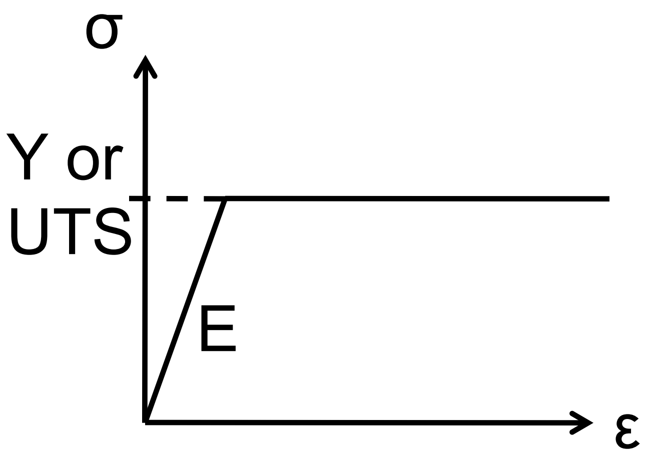
Useful assumptions

Only interested in plastic effects:
Perfectly plastic material

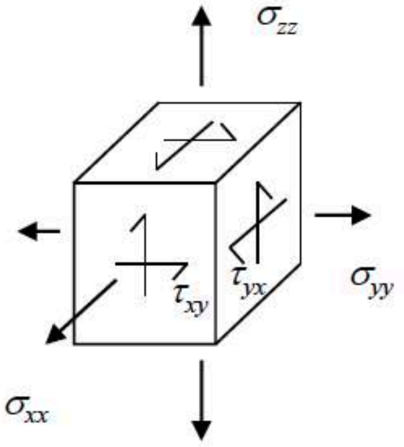
At Y, material defoms (‘flows’) in compression and fails in tension



Interested in elastic and plastic effects:
Elastic-perfectly plastic material

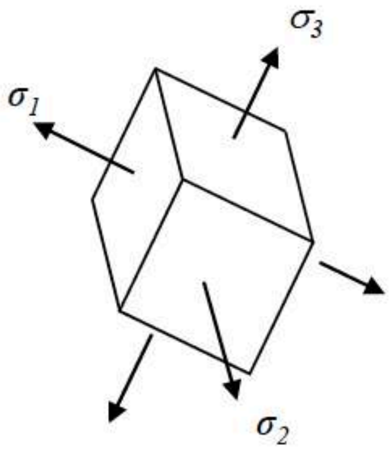


3D Problems



general stress state

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

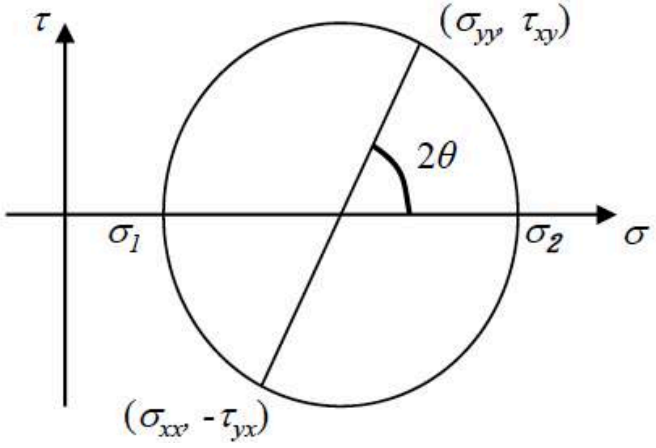


in terms of principal stresses

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

For any general stress state we can find a set of *principal axes*. The stress tensor for these axes contains no off-diagonal (shear) terms – only three principal stresses along the three axes.

Mohr's circle allows rotation of axes in two dimensions about one principal axis



In 1-D, $\sigma = K\varepsilon^n$ assuming perfectly plastic, yielding at: $\sigma = Y$

In 3-D, $\sigma_{eff} = K\varepsilon_{eff}^n$ assuming perfectly plastic, yielding at:

$$\sigma_{eff} = Y$$

3D Yield Criteria

Tresca: Yielding occurs at a maximum shear stress

Effective stress (in principal directions):

$$\sigma_{eff} = \left[\sigma_i - \sigma_j\right]_{\substack{\max,\\ i\neq j}}$$

Yield criterion:

$$\sigma_{eff} = Y$$

$$\tau_{\max} = k = \frac{Y}{2}$$

Effective strain:

$$\varepsilon_{eff} = \left(\varepsilon_i\right)_{\max}$$

Von Mises: Yielding at maximum distortion strain energy

Effective stress (in principal directions):

$$\sigma_{eff} = \sqrt{\frac{1}{2} \times \begin{bmatrix} (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \\ +(\sigma_1 - \sigma_2)^2 \end{bmatrix}}$$

Yield criterion:

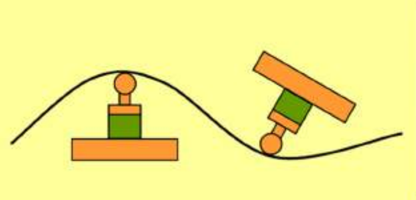
$$\sigma_{eff} = Y$$

$$Y = \sqrt{3}k$$

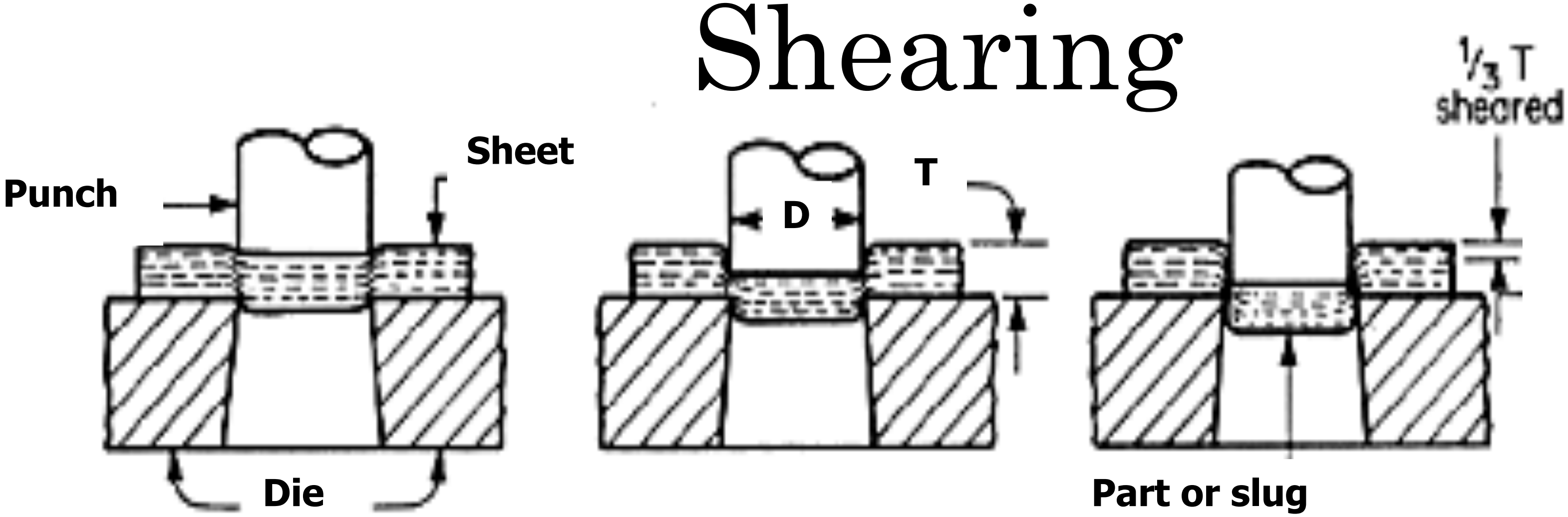
Effective strain:

$$\varepsilon_{eff} = \sqrt{\left(\frac{2}{3}\right) \times \left(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2\right)}$$

Massachusetts Institute of Technology



Shearing

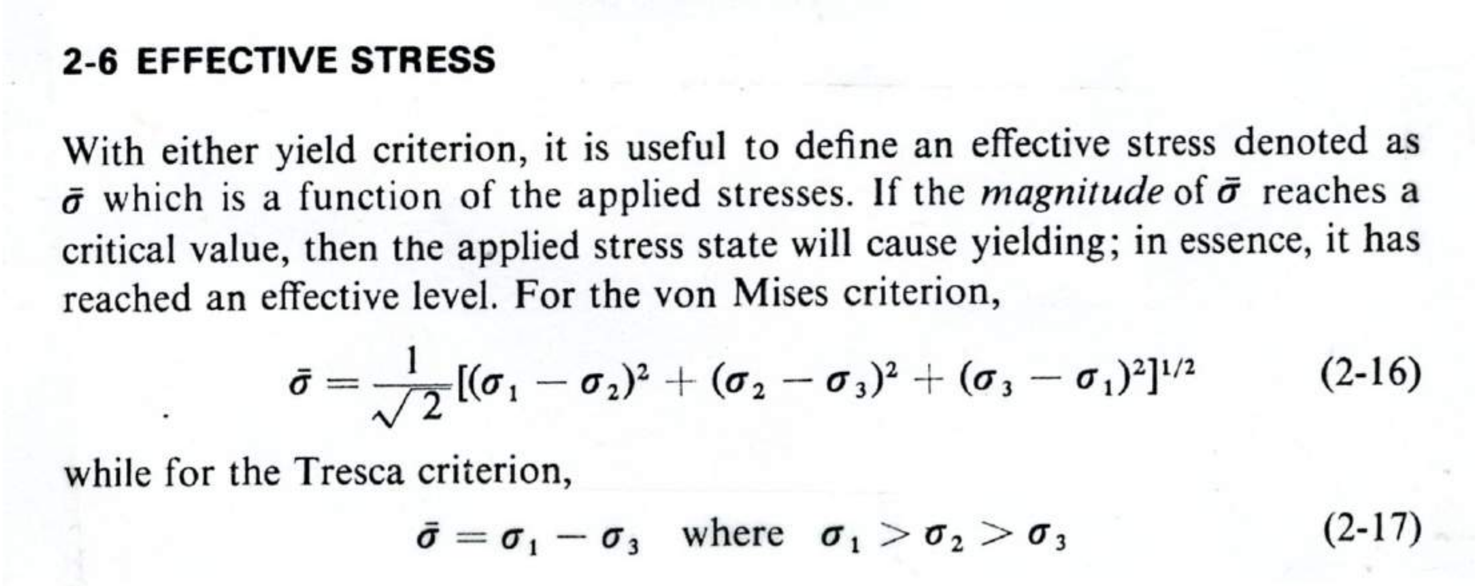


$$F = 0.7\ T\ L\ (UTS)$$

T = Sheet Thickness
L = Total length Sheared
UTS = Ultimate Tensile Strength of material

Shear press - LMP Shop

Side Note: For a general state of stress use "effective stress"



2-6 EFFECTIVE STRESS

With either yield criterion, it is useful to define an effective stress denoted as $\bar{\sigma}$ which is a function of the applied stresses. If the *magnitude* of $\bar{\sigma}$ reaches a critical value, then the applied stress state will cause yielding; in essence, it has reached an effective level. For the von Mises criterion,

$$\bar{\sigma} = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \qquad (2\text{-}16)$$

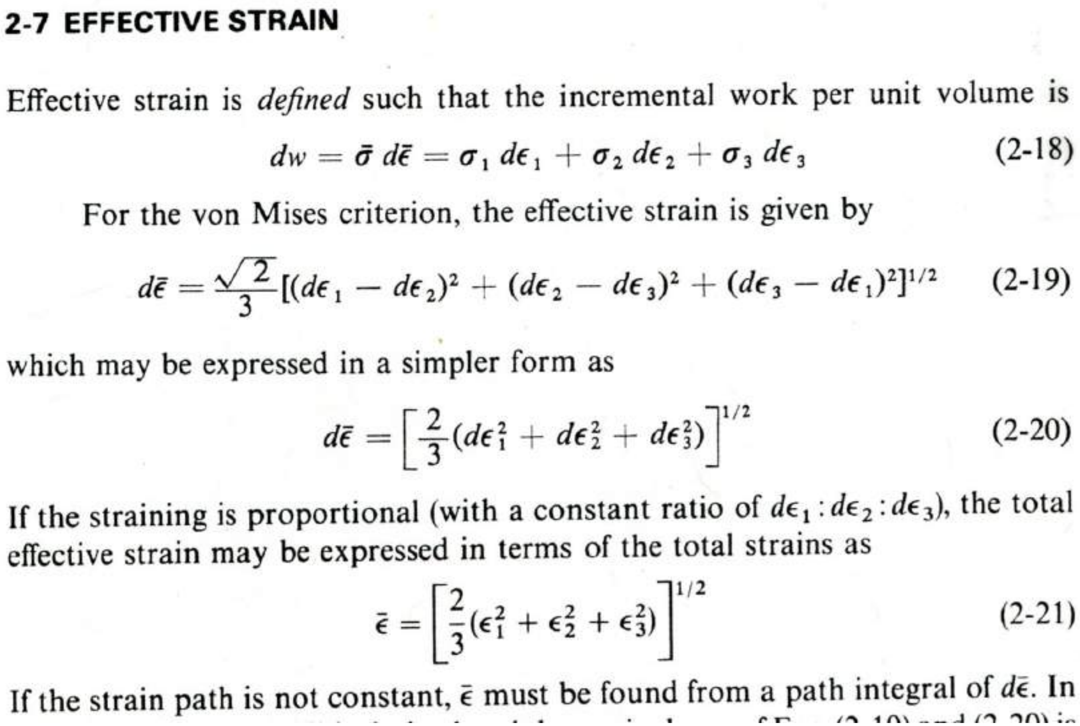
while for the Tresca criterion,

$$\bar{\sigma} = \sigma_1 - \sigma_3 \quad \text{where} \quad \sigma_1 > \sigma_2 > \sigma_3 \qquad (2\text{-}17)$$

Yielding occurs when $\sigma_{\text{effective}} = Y$

Material taken from ***Metal Forming***, by Hosford and Caddell

Origin of effective strain



2-7 EFFECTIVE STRAIN

Effective strain is *defined* such that the incremental work per unit volume is

$$dw = \bar{\sigma}\, d\bar{\epsilon} = \sigma_1\, d\epsilon_1 + \sigma_2\, d\epsilon_2 + \sigma_3\, d\epsilon_3 \tag{2-18}$$

For the von Mises criterion, the effective strain is given by

$$d\bar{\epsilon} = \frac{\sqrt{2}}{3}[(d\epsilon_1 - d\epsilon_2)^2 + (d\epsilon_2 - d\epsilon_3)^2 + (d\epsilon_3 - d\epsilon_1)^2]^{1/2} \tag{2-19}$$

which may be expressed in a simpler form as

$$d\bar{\epsilon} = \left[\frac{2}{3}(d\epsilon_1^2 + d\epsilon_2^2 + d\epsilon_3^2)\right]^{1/2} \tag{2-20}$$

If the straining is proportional (with a constant ratio of $d\epsilon_1 : d\epsilon_2 : d\epsilon_3$), the total effective strain may be expressed in terms of the total strains as

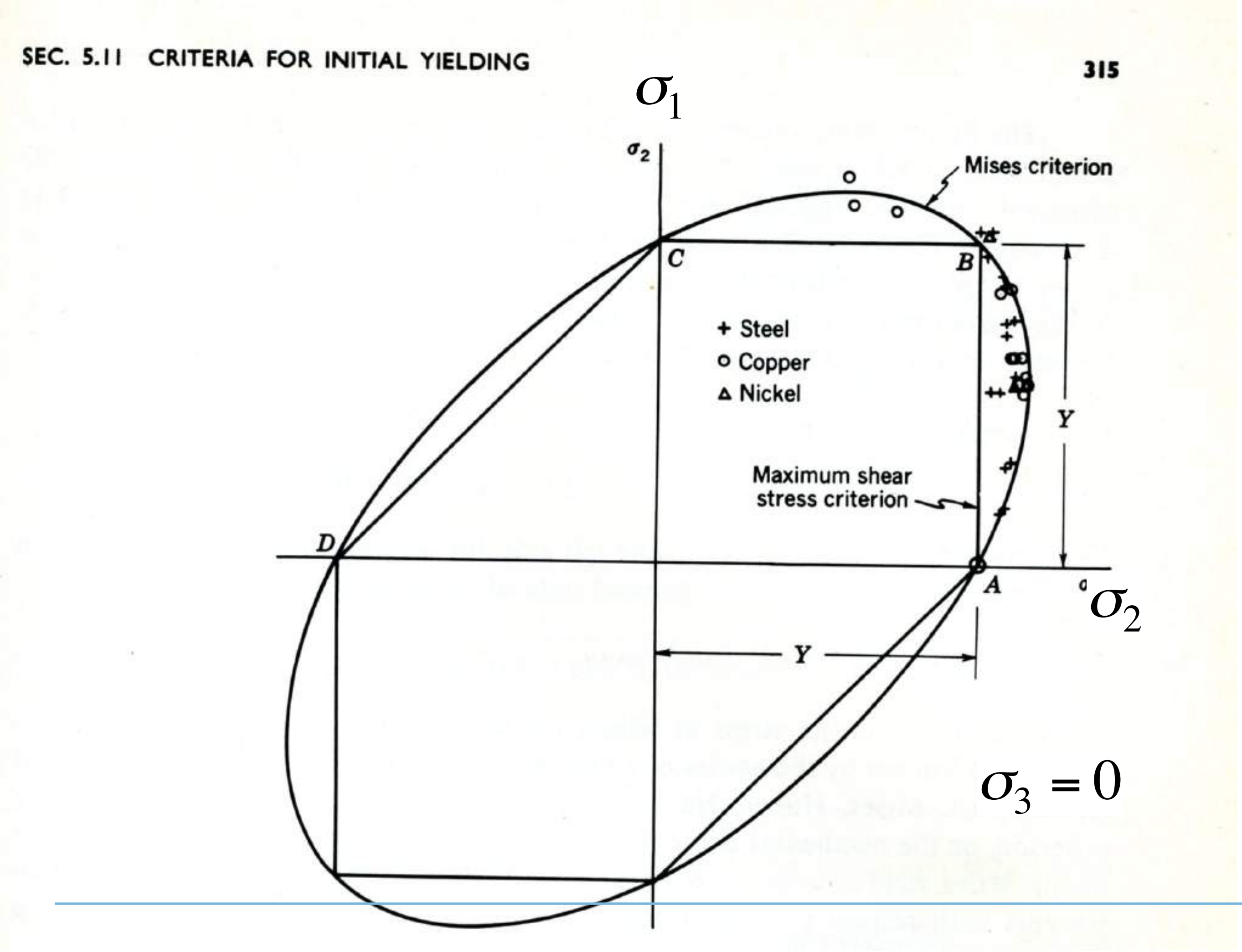
$$\bar{\epsilon} = \left[\frac{2}{3}(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)\right]^{1/2} \tag{2-21}$$

If the strain path is not constant, $\bar{\epsilon}$ must be found from a path integral of $d\bar{\epsilon}$. In

$$\bar{\sigma} = K\bar{\varepsilon}^n$$

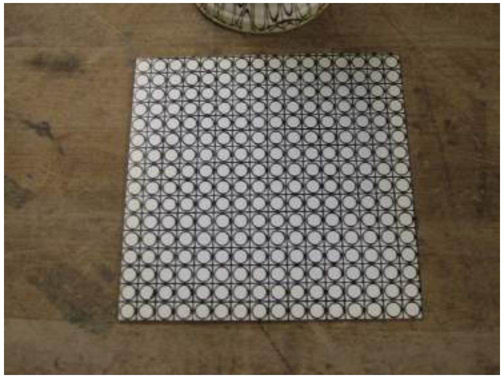
Material taken from ***Metal Forming***, by Hosford and Caddell

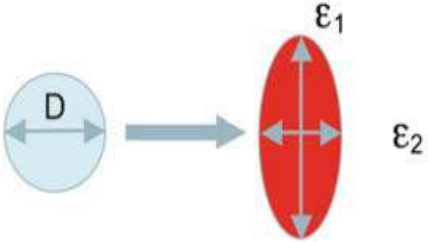
3D Yield Effective stress

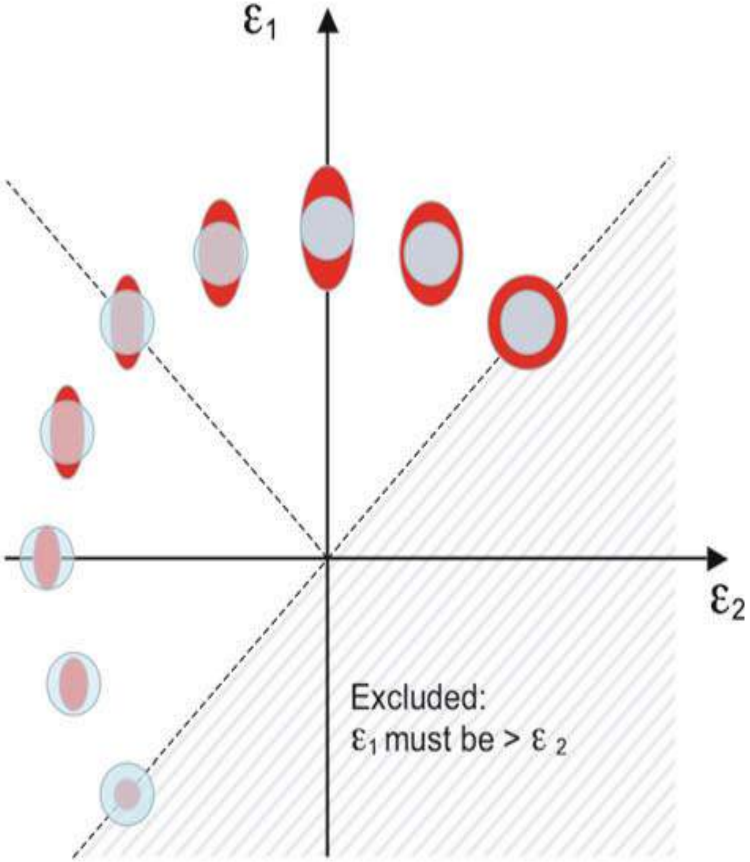


Tresca predicts ‘flow’ for lower stresses than von Mises

Forming Limit Diagrams







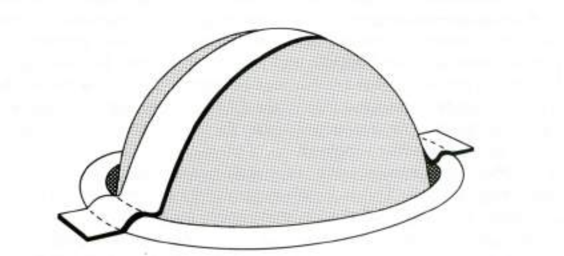
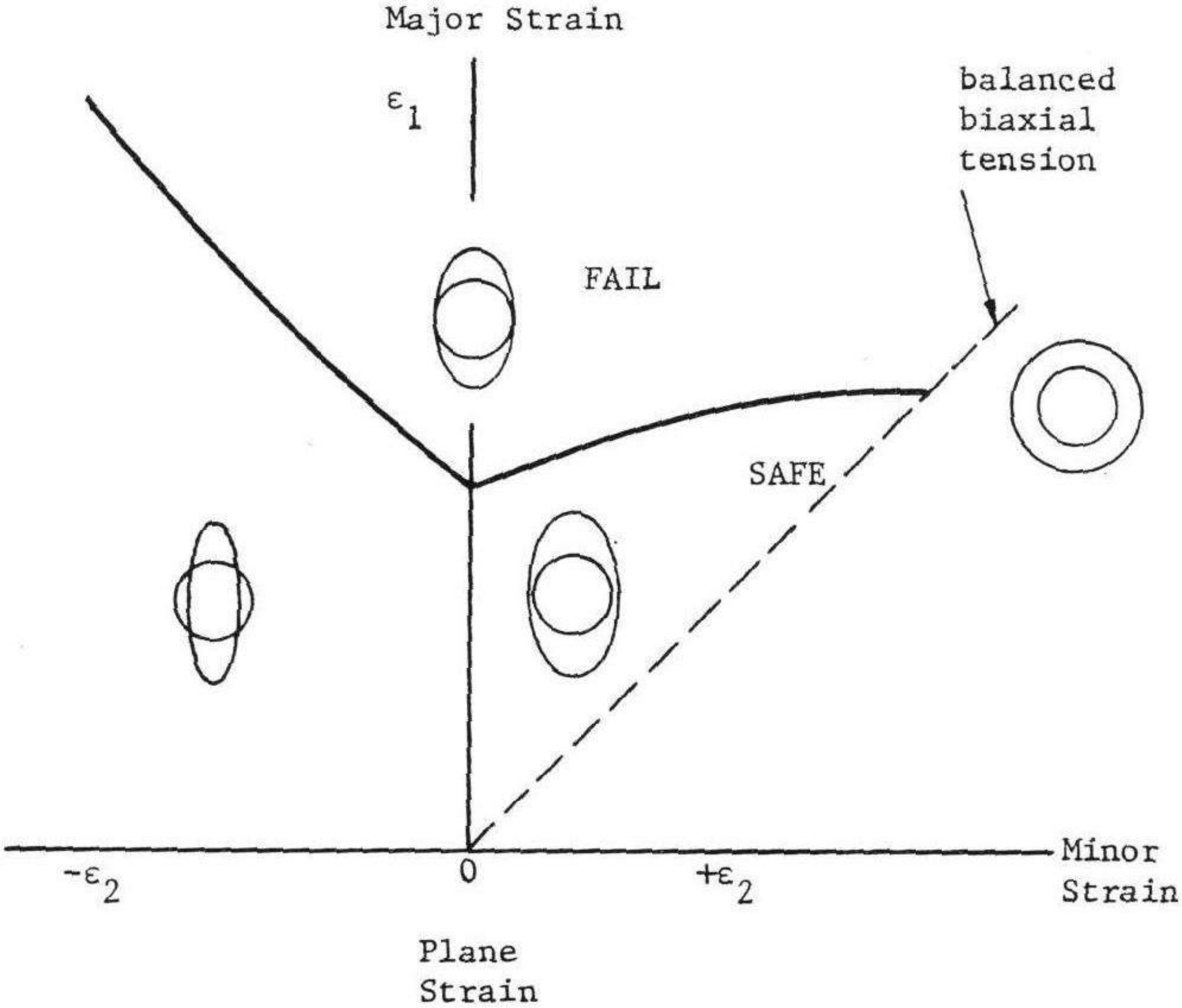
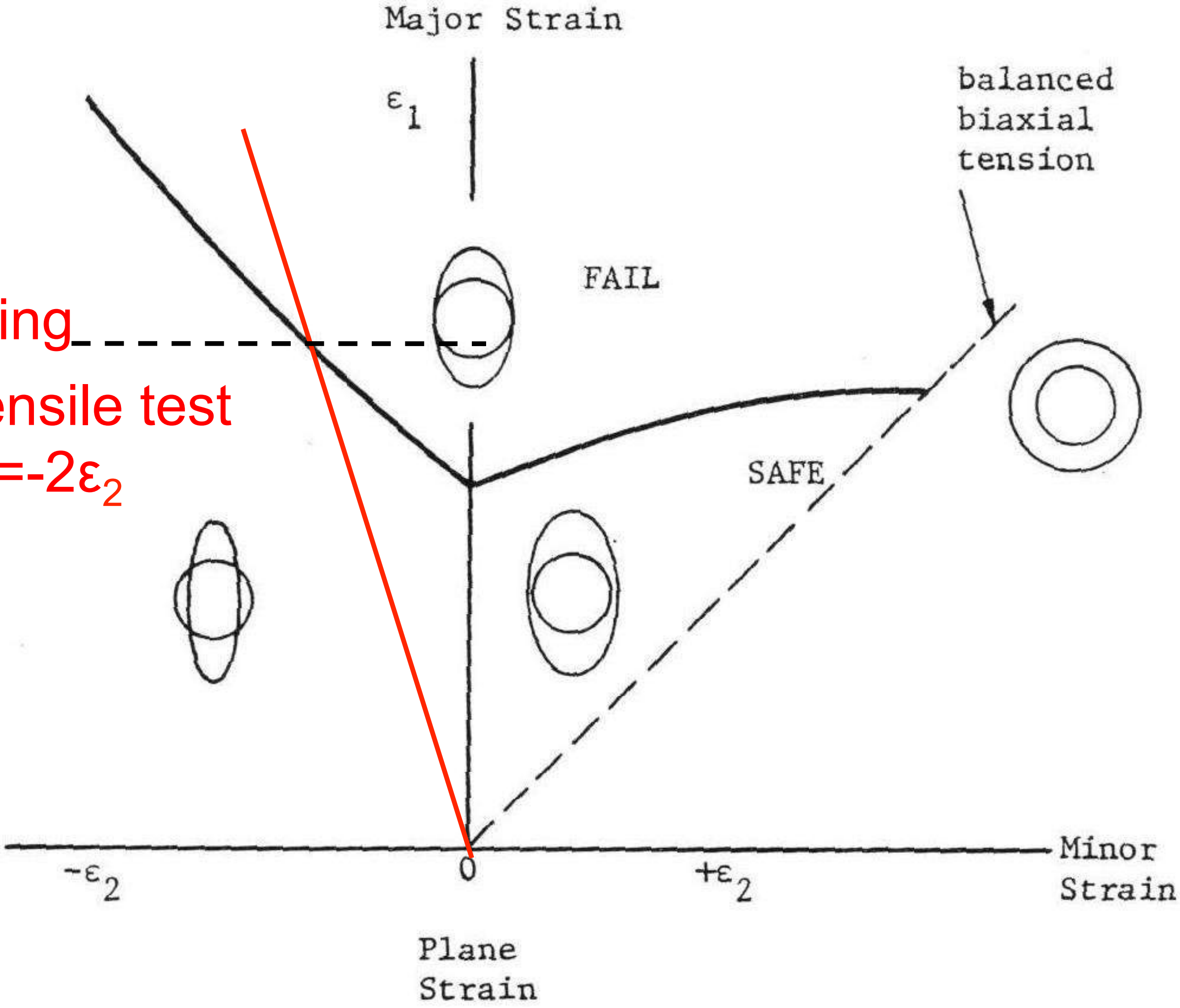


Figure 15-8 Strips of varying width are stretched to obtain different e_2/e_1 ratios.

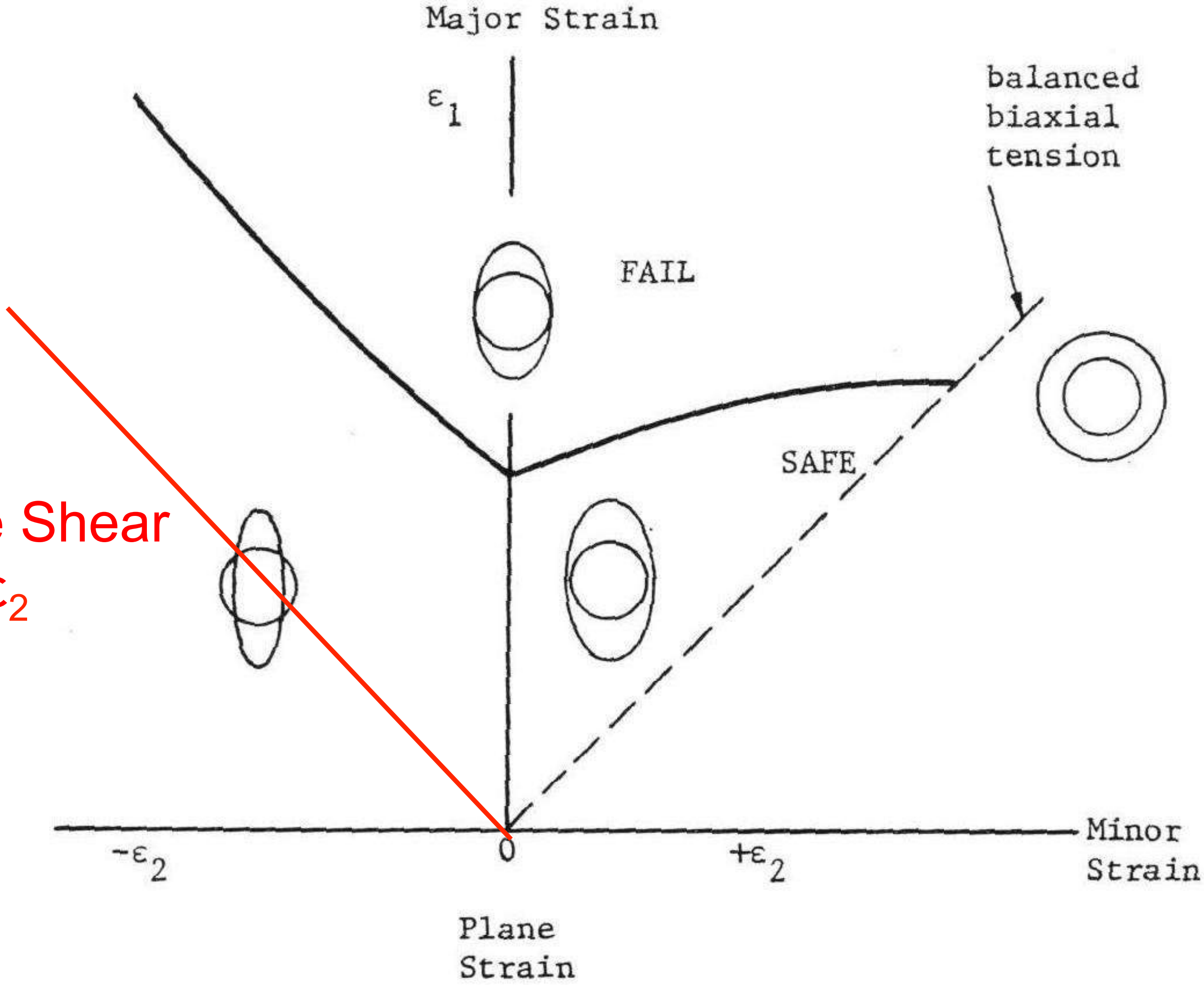

Major Strain
ε_1
balanced
biaxial
tension
FAIL
SAFE
$-\varepsilon_2$
0
$+\varepsilon_2$
Minor
Strain
Plane
Strain



$\varepsilon_1 = n =$necking

Tensile test

$\varepsilon_1 = -2\varepsilon_2$



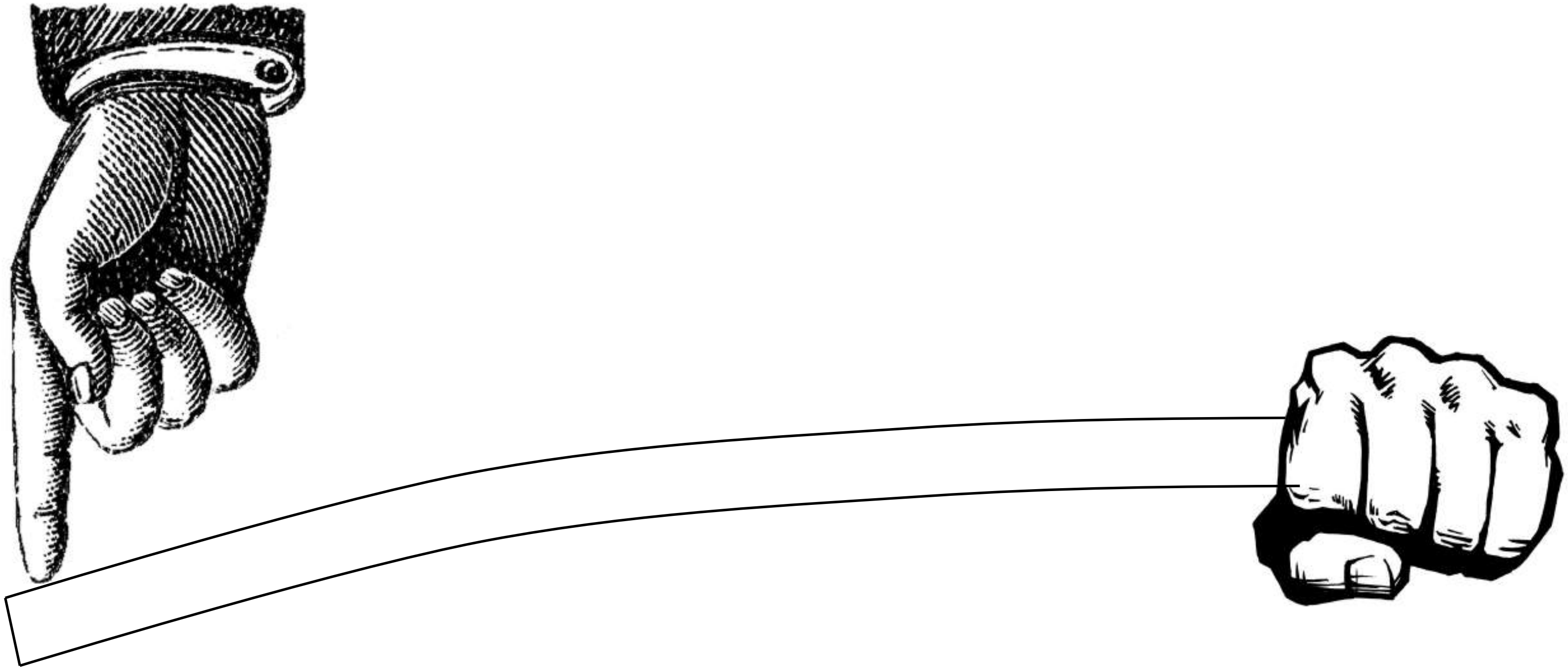
Pure Shear
$\varepsilon_1 = -\varepsilon_2$

Stretch forming: Forming force

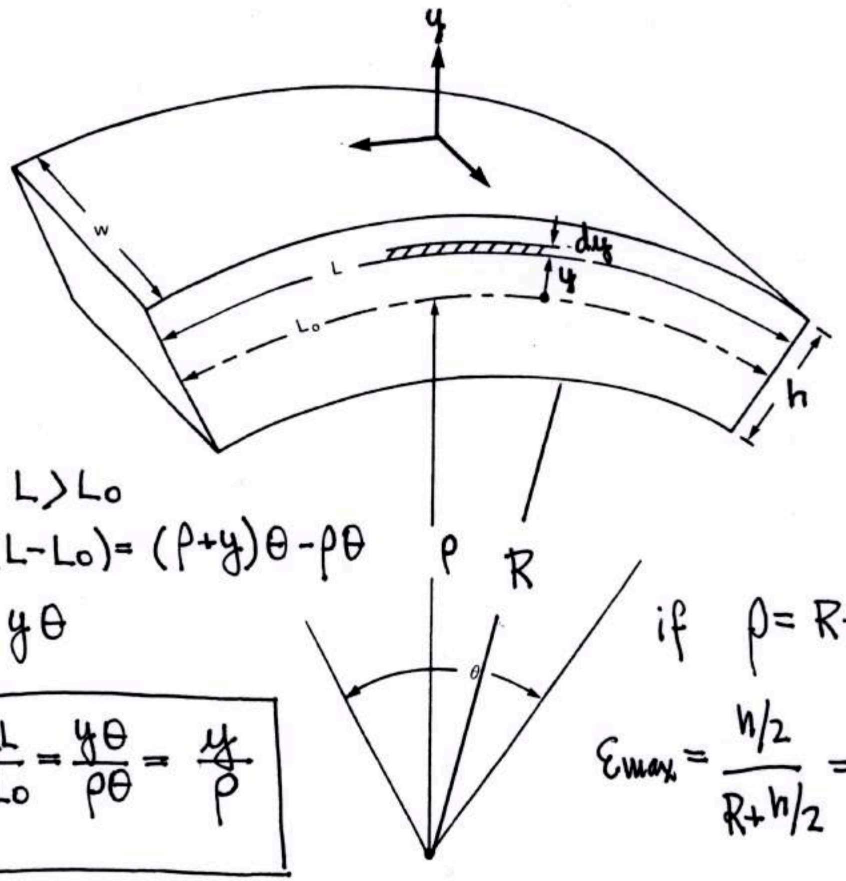
$$F = (Y_S + UTS)/2 * A$$

F = stretch forming force (lbs)
Y_S = material yield strength (psi)
UTS = ultimate tensile strength of the material (psi)
A = Cross-sectional area of the workpiece (in2)

Forces needed to bend sheet metal



Bending



note: $L > L_o$

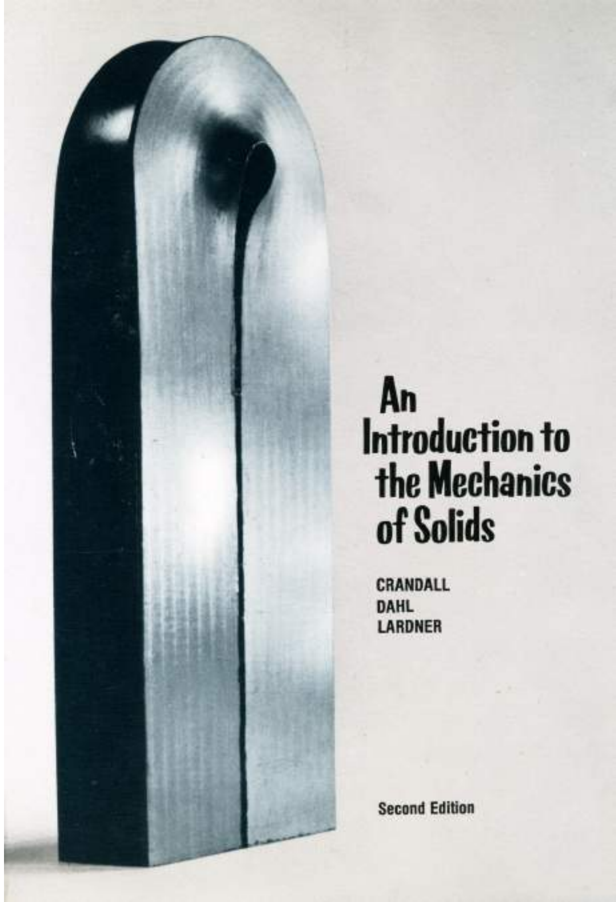
$$\Delta L = (L - L_o) = (\rho + y)\theta - \rho\theta$$
$$= y\theta$$

$$\boxed{\varepsilon = \frac{\Delta L}{L_o} = \frac{y\theta}{\rho\theta} = \frac{y}{\rho}}$$

if $\rho = R + \frac{h}{2}$

$$\varepsilon_{max} = \frac{h/2}{R + h/2} = \frac{1}{\frac{2R}{h} + 1}$$

Figure Coordinate system for analysis of bending.



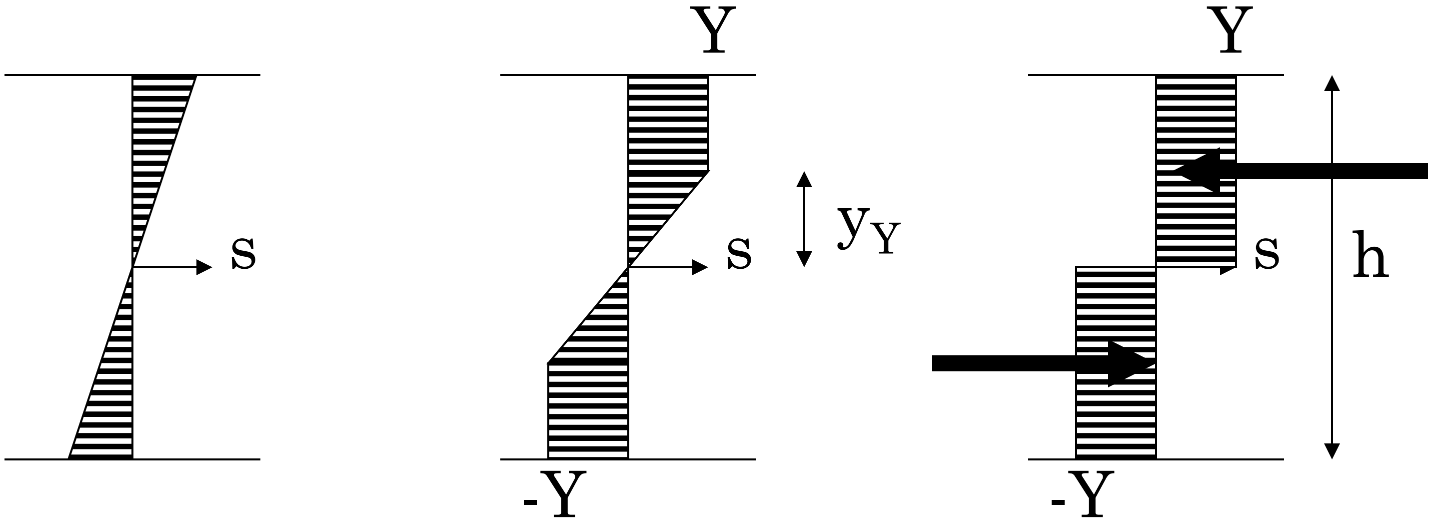
An Introduction to the Mechanics of Solids

CRANDALL
DAHL
LARDNER

Second Edition

Massachusetts Institute of Technology

Stress distribution through the thickness of the part

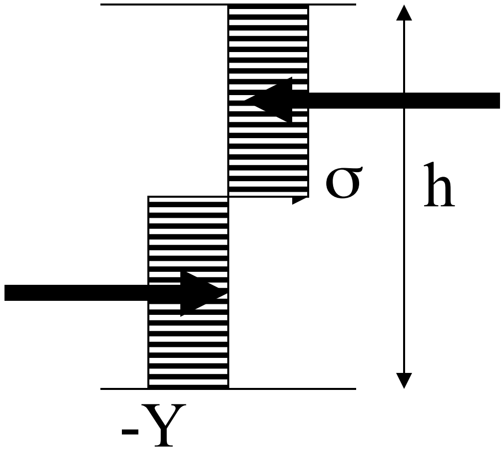


Elastic Elastic-plastic Fully plastic

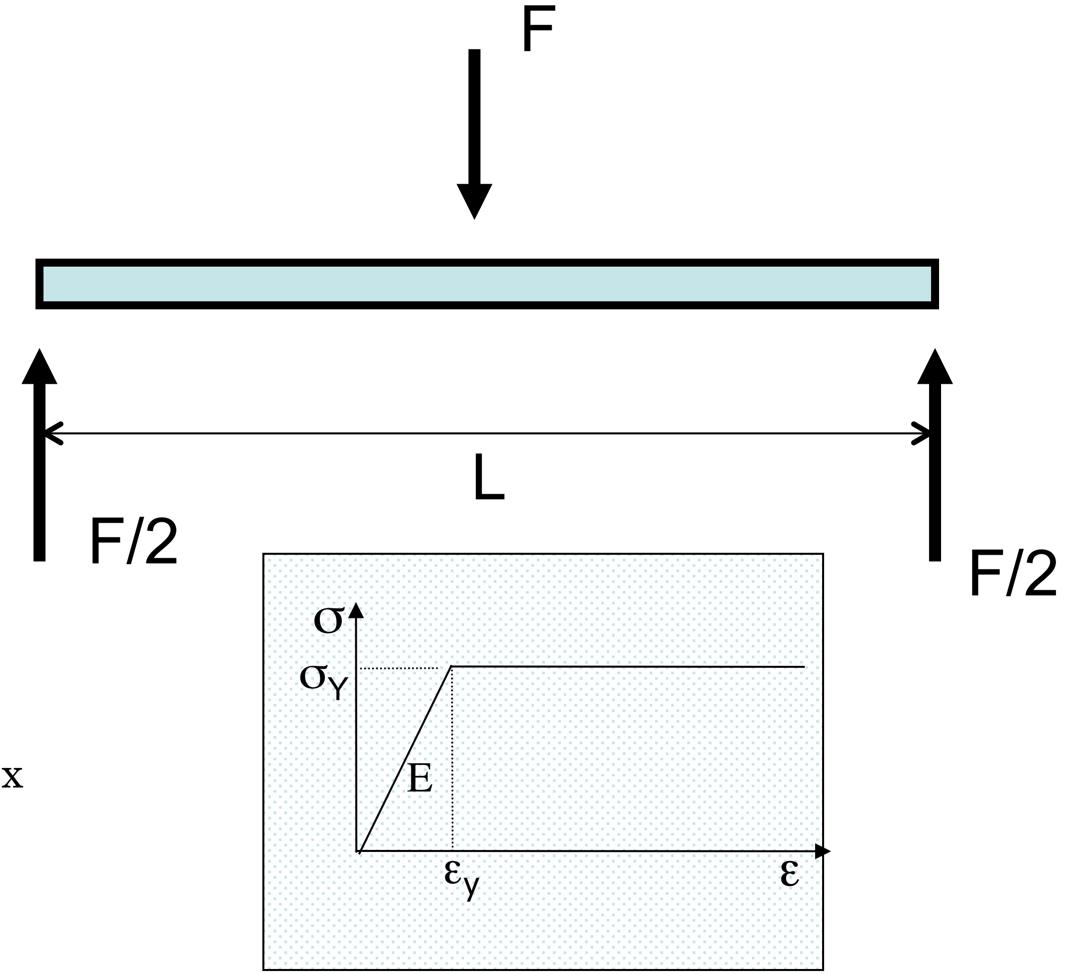
Fully Plastic Moment, $M = Y\ (b\ h/2)\ h/2 = Ybh^2/4$

Massachusetts Institute of Technology

Balance external and internal moments



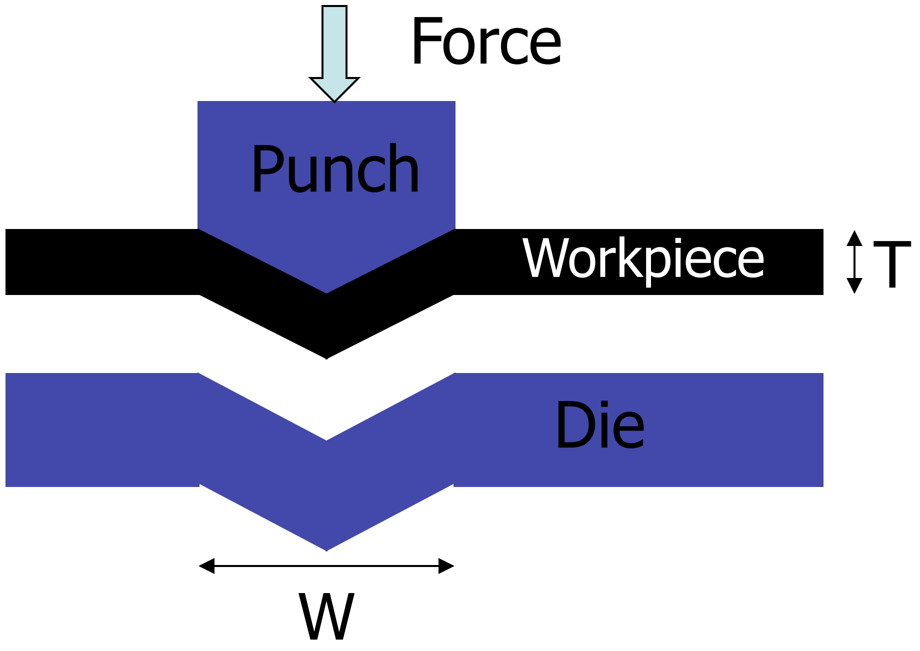
Fully plastic



$$Ybh^2/4 = FL/4 = M_{max}$$

$$F = bh^2Y/L$$

Bending Force Requirement



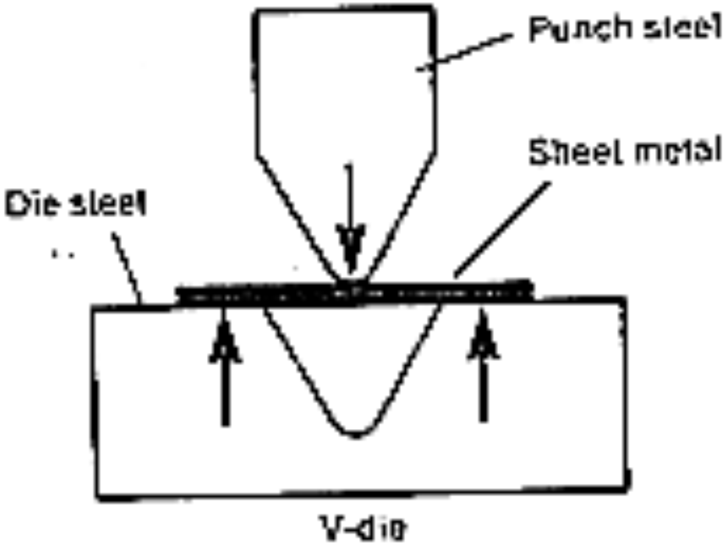
$$F = \frac{LT^2}{W}(UTS)$$

T = Sheet Thickness
W = Width of Die Opening
L = Total length of bend (into the page)
UTS = Ultimate Tensile Strength of material

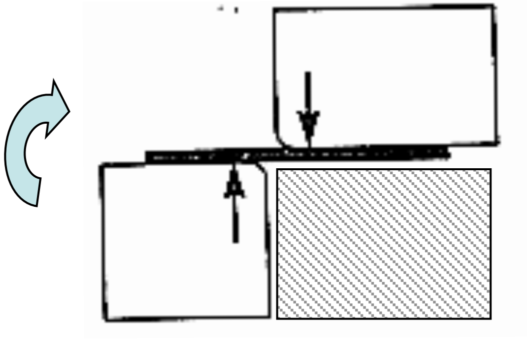
Note: the notation used in the text (L, W) differs from that used in the previous development (b, L).

LMP Shop

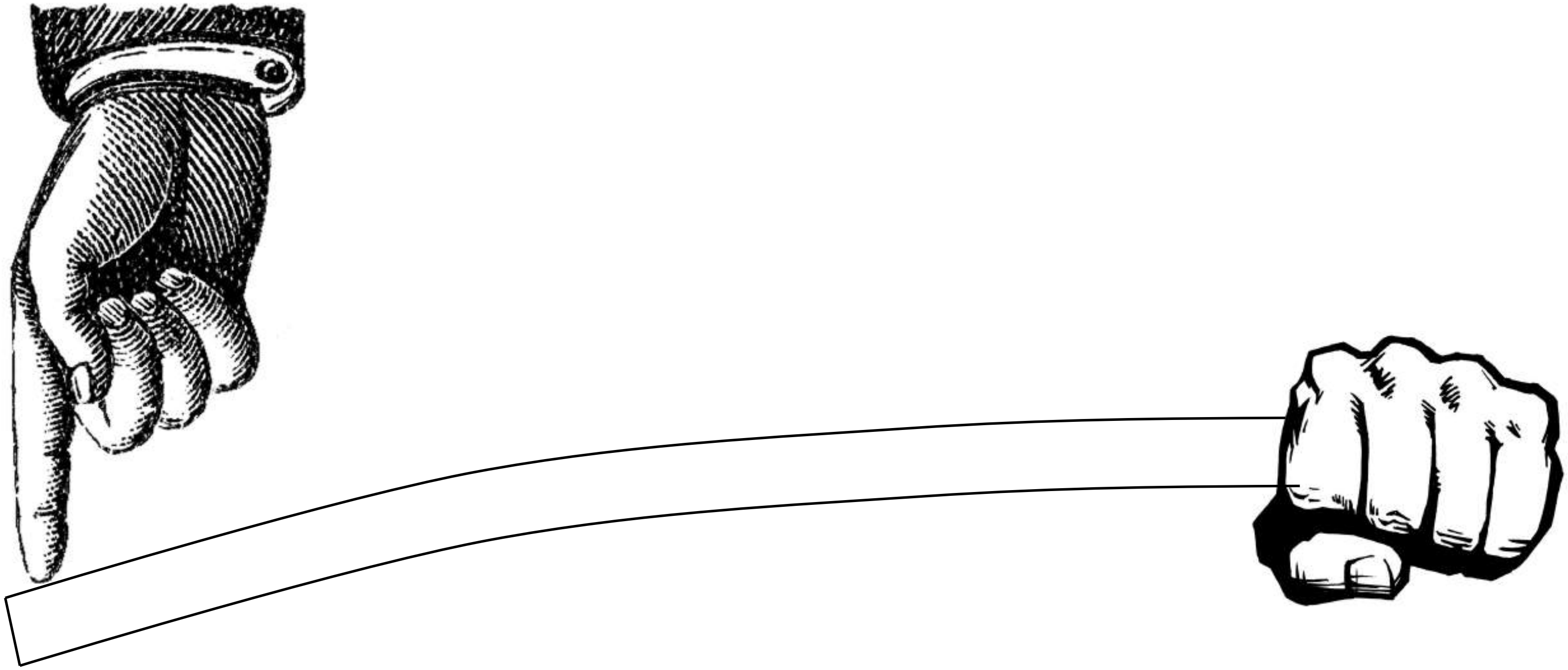
Brake press



Finger brake

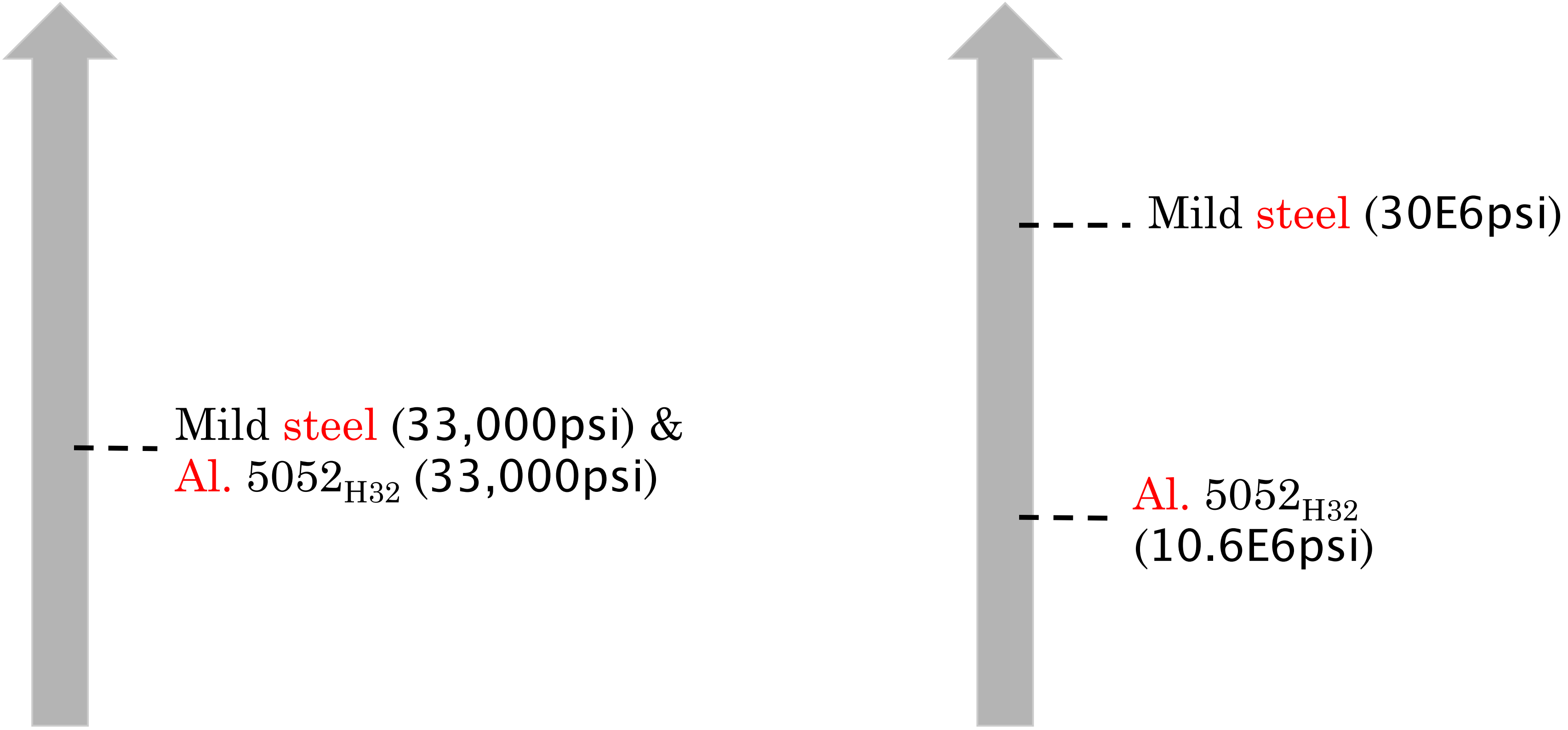


What shape have we created?



Steel versus aluminum…

Strength (σ_y) versus Stiffness (E)



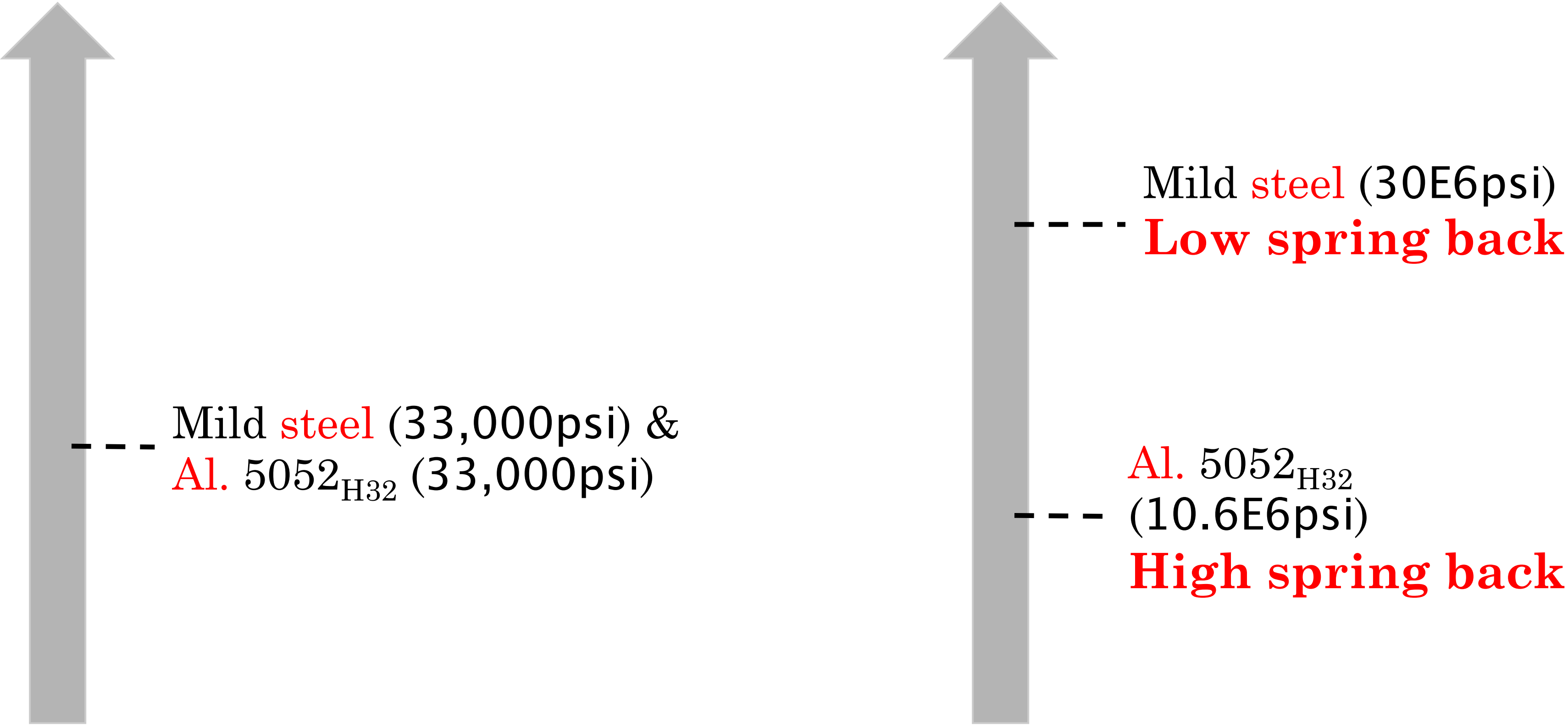
Mild steel (30E6psi)

Mild steel (33,000psi) &
Al. 5052_{H32} (33,000psi)

Al. 5052_{H32}
(10.6E6psi)

Steel versus aluminum…

Strength (σ_y) versus Stiffness (E)



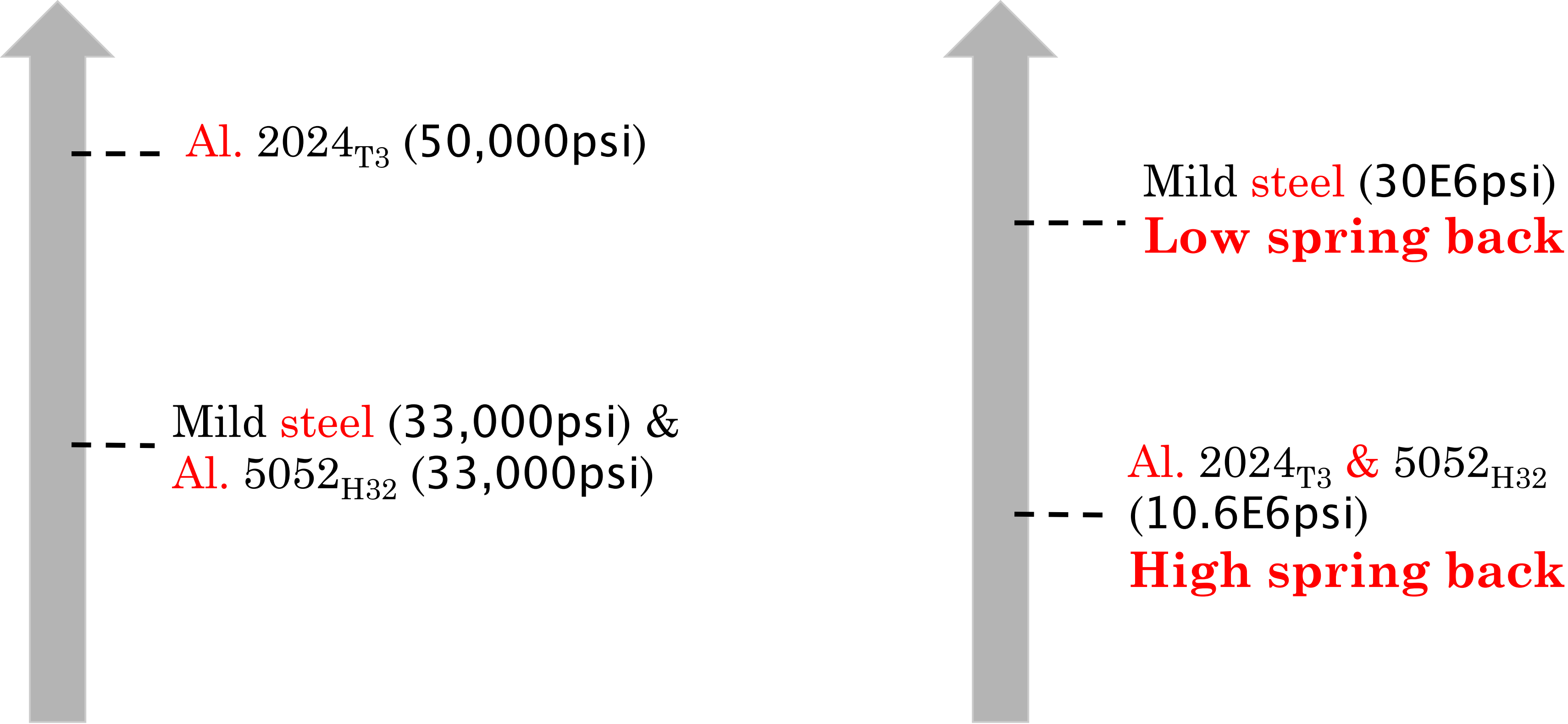
Mild steel (33,000psi) &
Al. 5052_{H32} (33,000psi)

Mild steel (30E6psi)
Low spring back

Al. 5052_{H32}
(10.6E6psi)
High spring back

Steel versus aluminum…

Strength (σ_y) versus Stiffness (E)



Al. 2024_{T3} (50,000psi)

Mild steel (33,000psi) &
Al. 5052_{H32} (33,000psi)

Mild steel (30E6psi)
Low spring back

Al. 2024_{T3} & 5052_{H32}
(10.6E6psi)
High spring back

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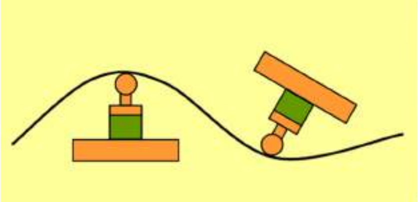


Steel versus aluminum…

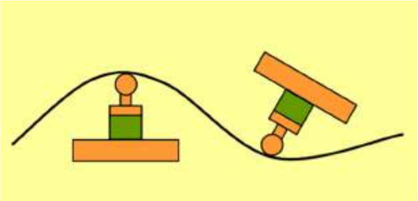
Strength (σ_y) versus Stiffness (E)

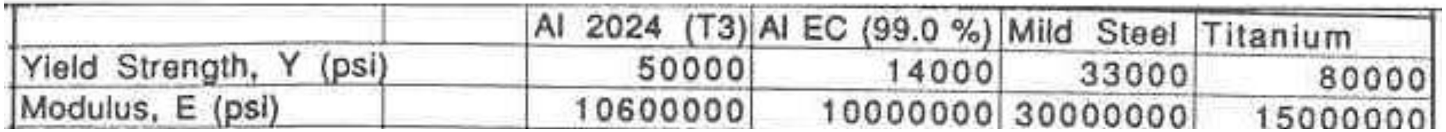
Al. 2024_{T3} (50,000psi)
High spring back

Mild steel (33,000psi) &
Al. 5052_{H32} (33,000psi)
Low spring back

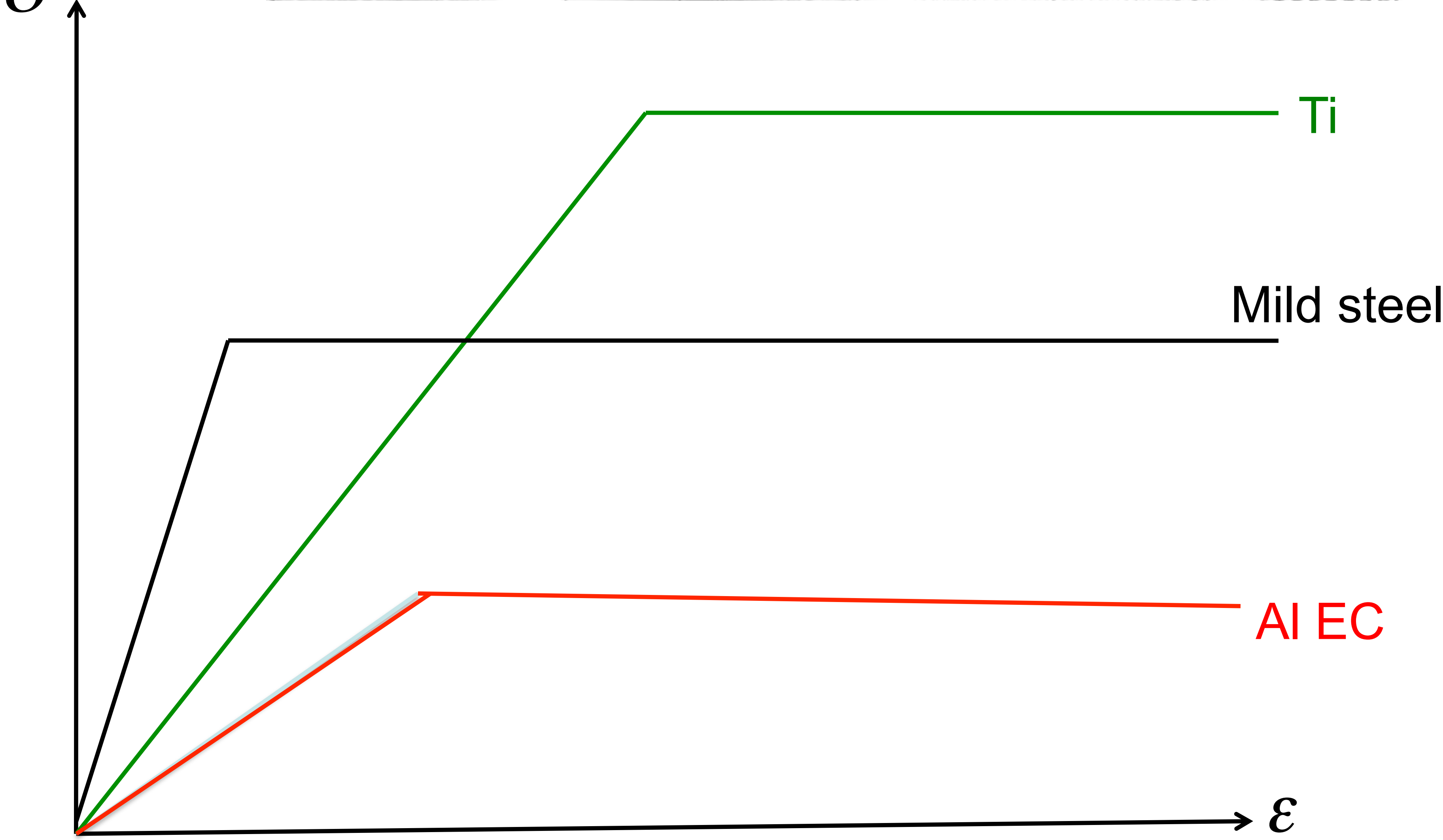
Mild steel (30E6psi)
Low spring back

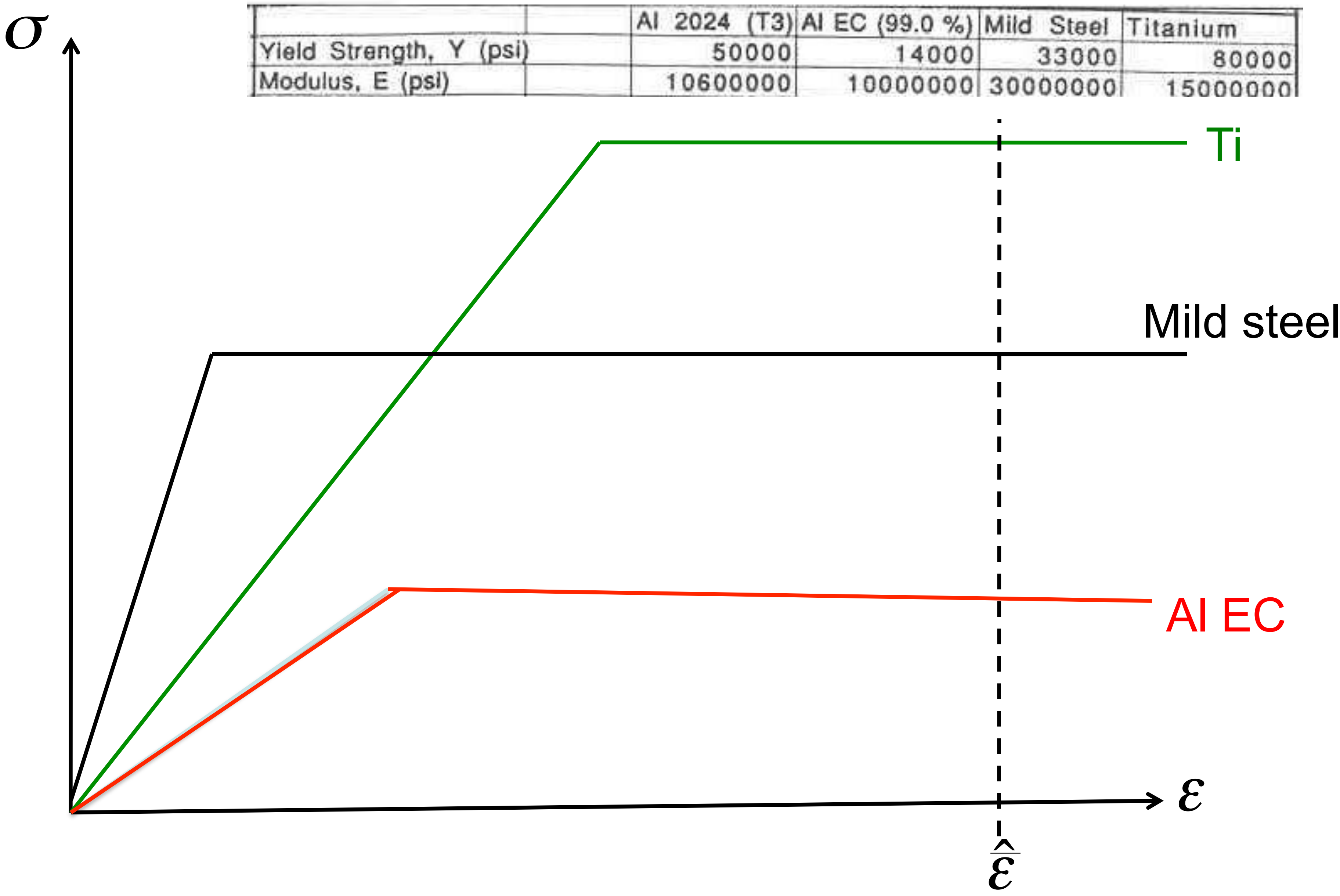
Al. 2024_{T3} & 5052_{H32}
(10.6E6psi)
High spring back





	Al 2024 (T3)	Al EC (99.0 %)	Mild Steel	Titanium
Yield Strength, Y (psi)	50000	14000	33000	80000
Modulus, E (psi)	10600000	10000000	30000000	15000000





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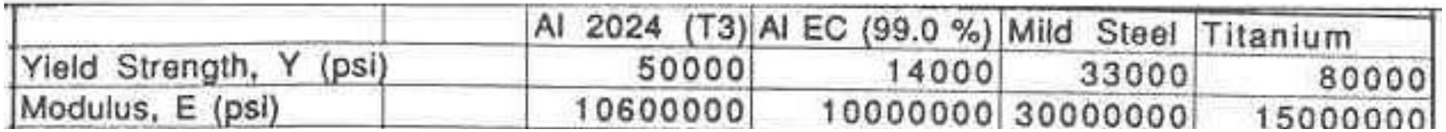

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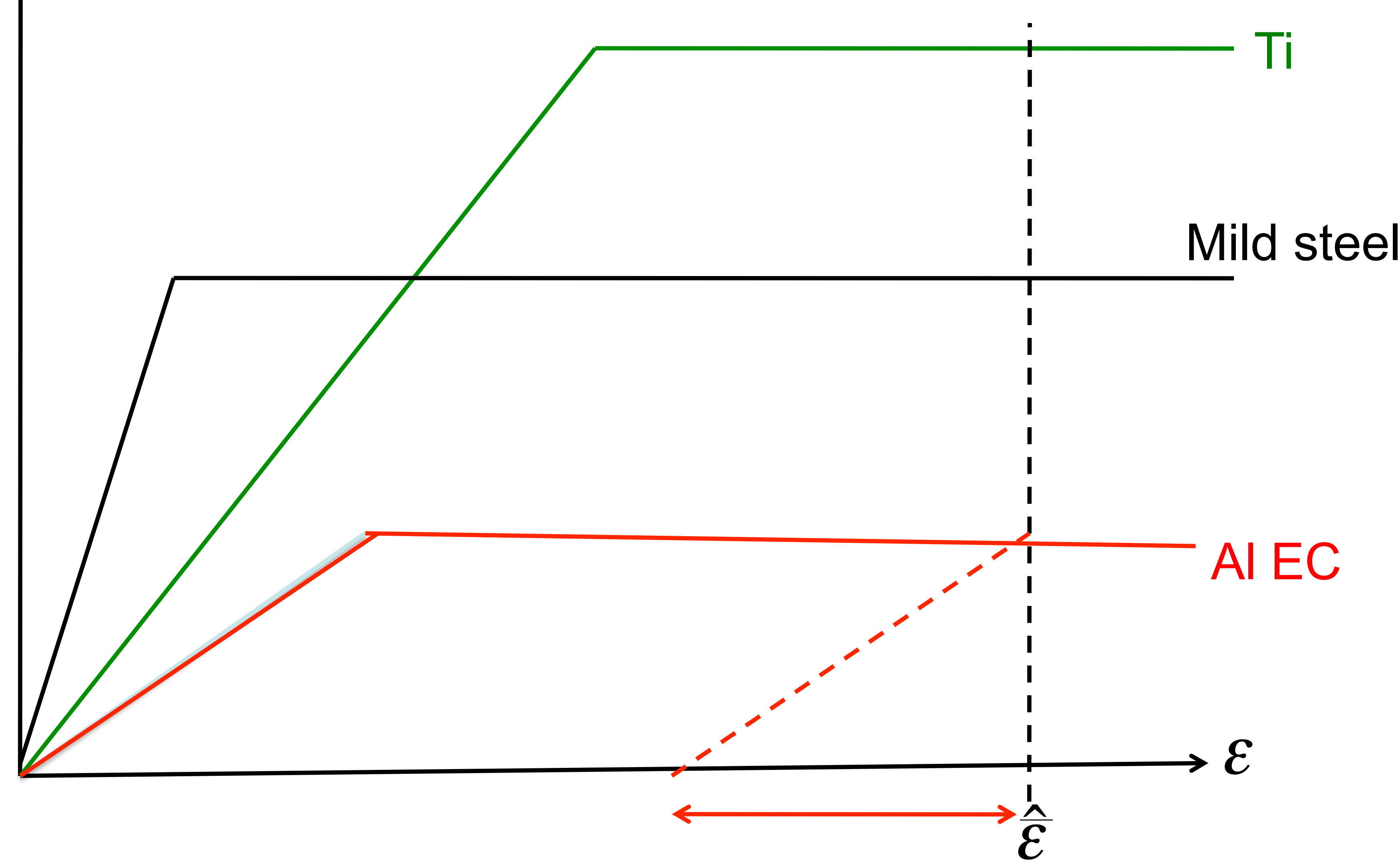

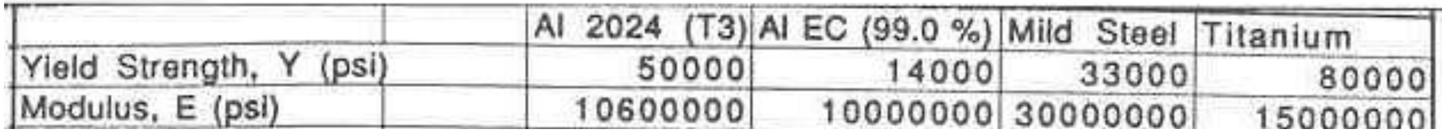

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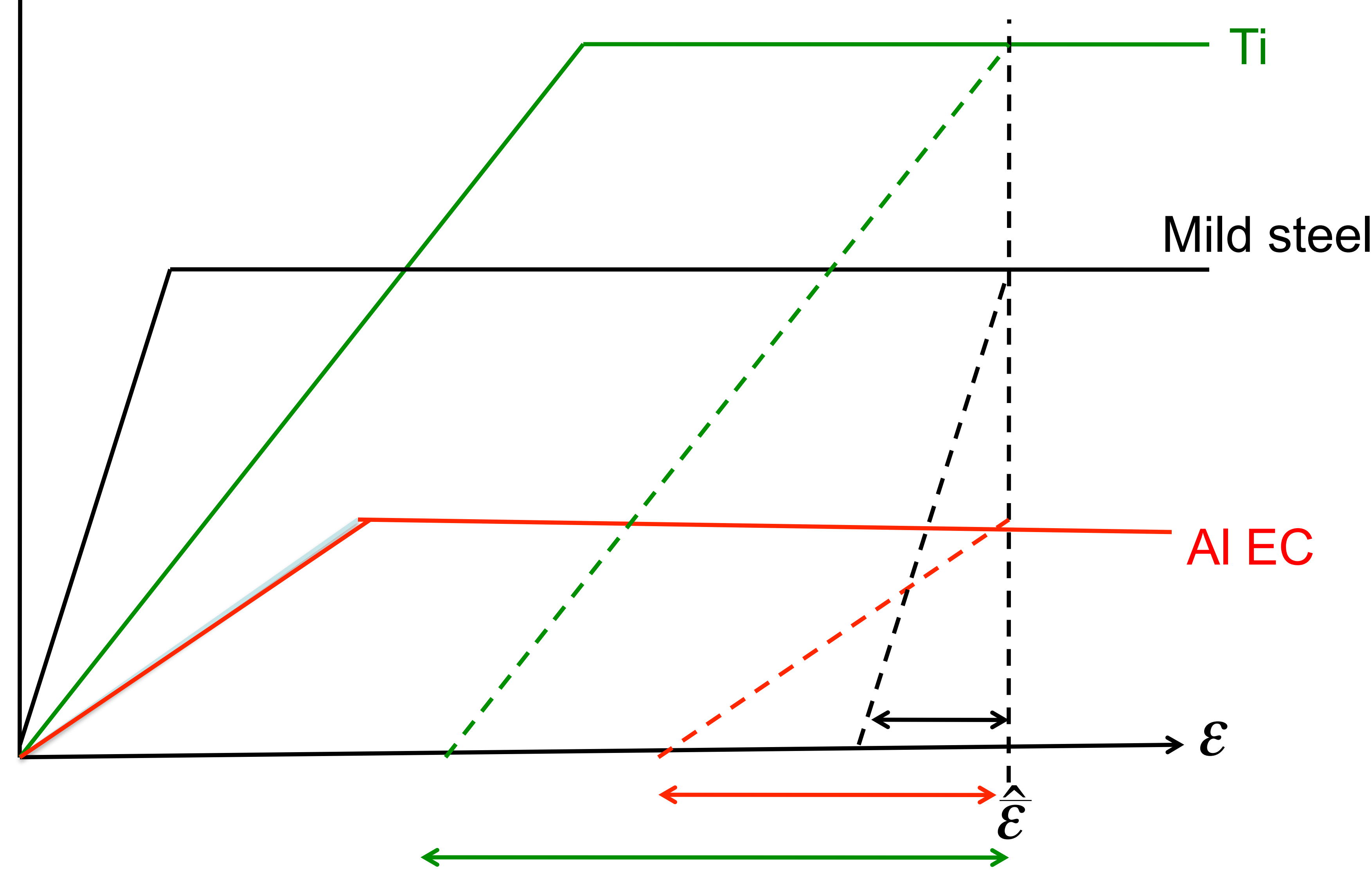
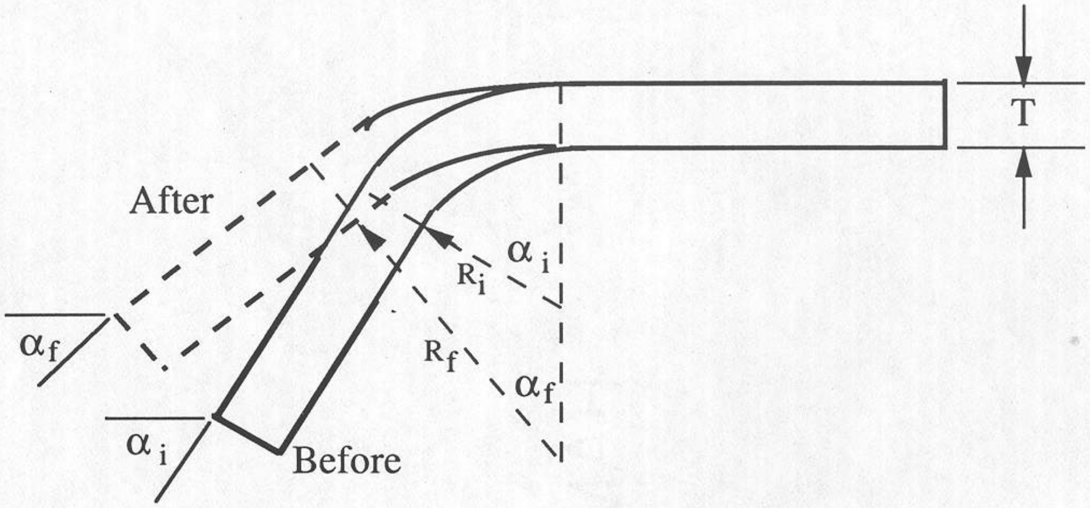


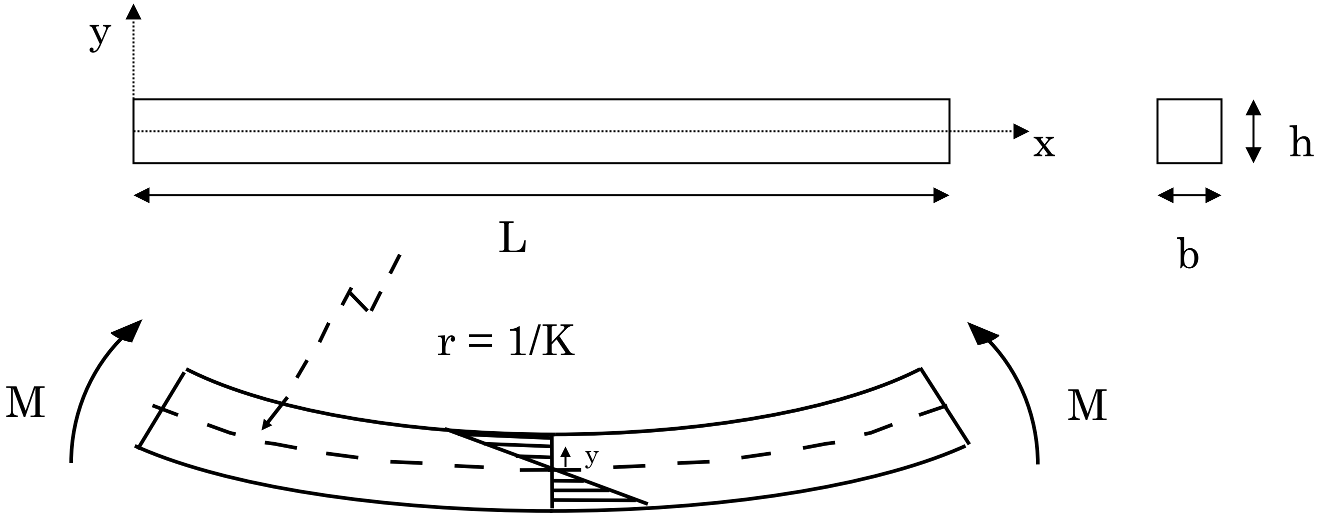
Springback note R in the figure below is mislabeled, should go to the centerline of the sheet



Springback:

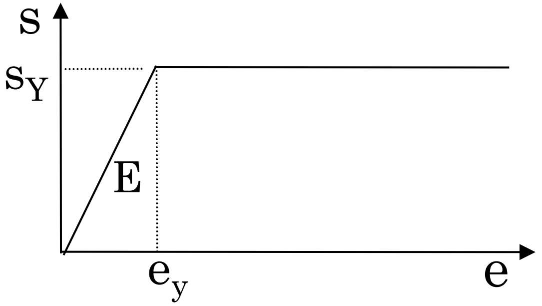
$$\frac{R_i}{R_f} = 4\left(\frac{R_i Y}{E T}\right)^3 - 3\left(\frac{R_i Y}{E T}\right) + 1$$

Elastic Springback Analysis



1. Assume plane sections remain plane:
   $e_y = -\, y/r$ (1)

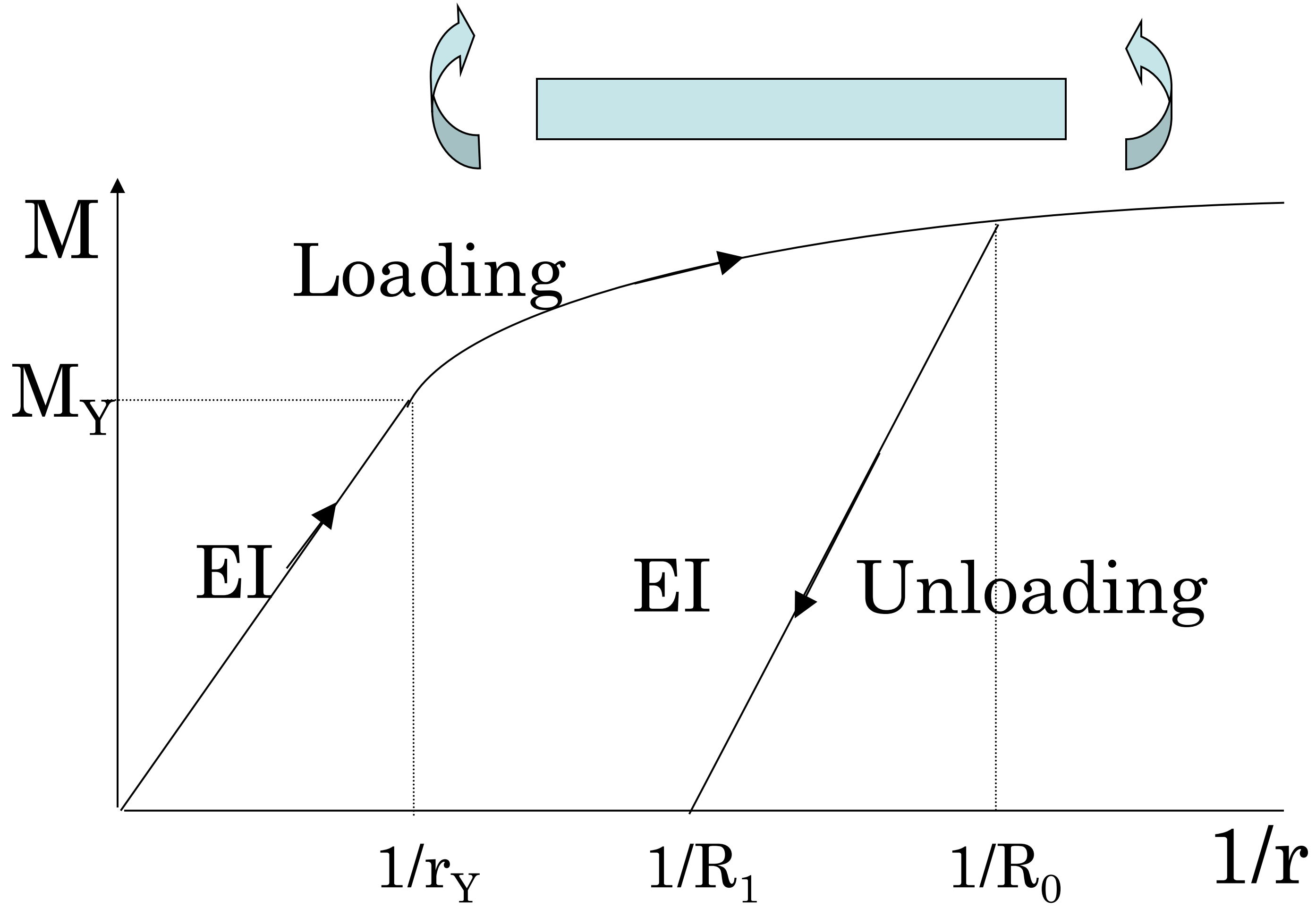
2. Assume elastic-plastic behavior for material

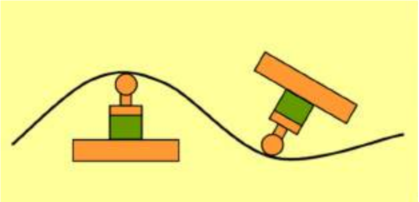


$\sigma = E\, e$ $\qquad e < e_{\psi}$

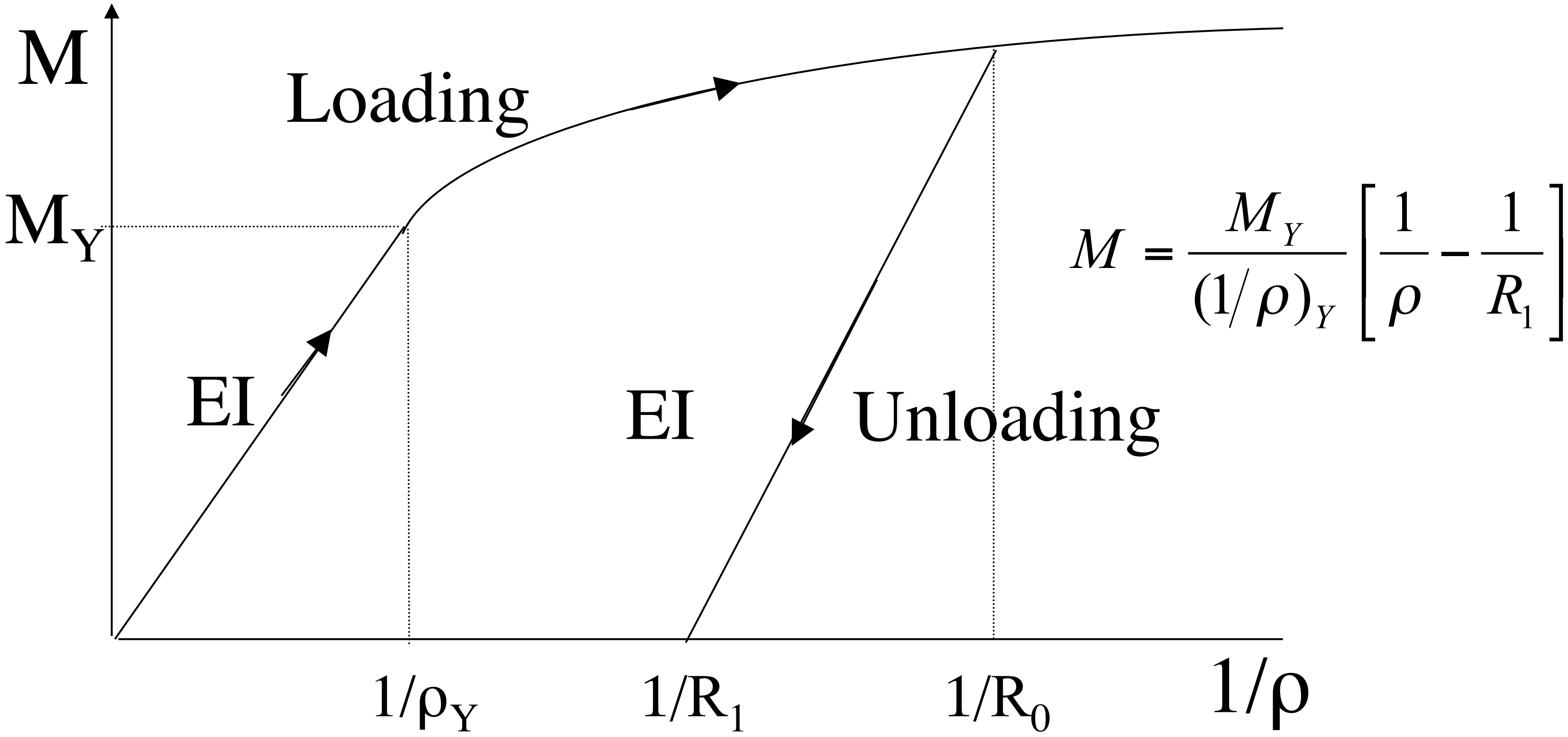
$\sigma = \sigma_Y$ $\qquad e \geq e$

Bending Moment – Curvature





3. We want to construct the following Bending Moment "M" vs. curvature "1/ρ" curve

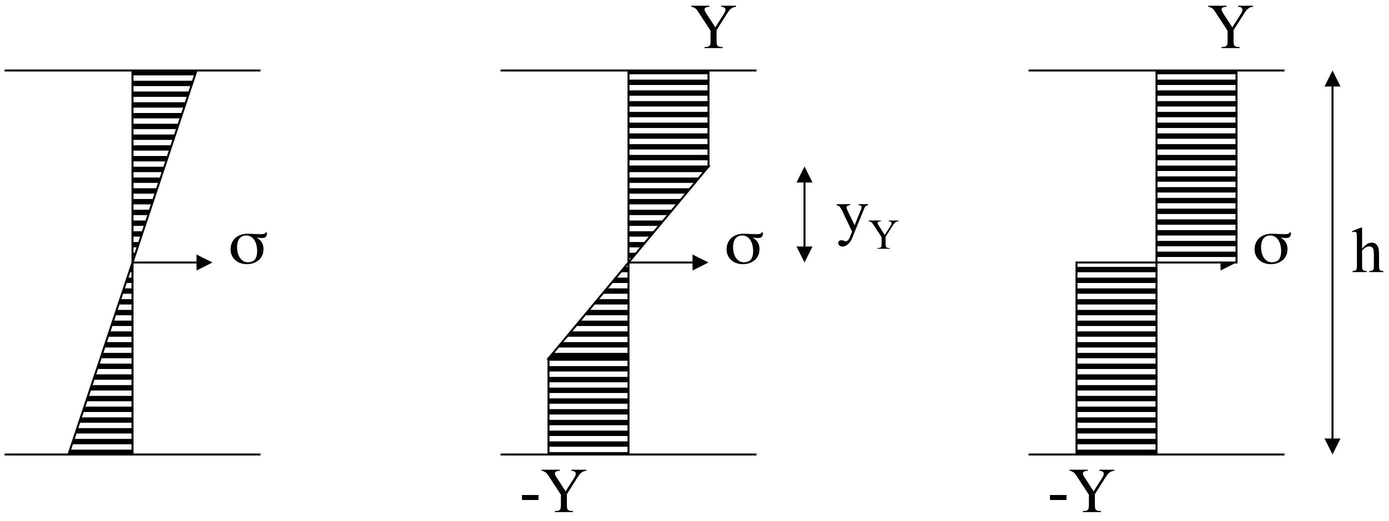


$$M = \frac{M_Y}{(1/\rho)_Y}\left[\frac{1}{\rho} - \frac{1}{R_1}\right]$$

Springback is measured as $1/R_0 - 1/R_1$ (2)

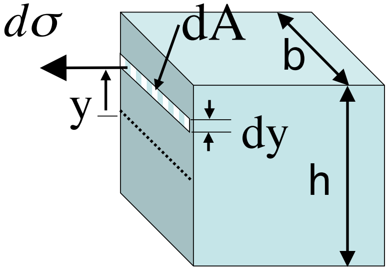
Permanent set is $1/R_1$

4. Stress distribution through the thickness of the beam



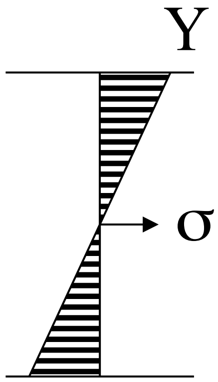
Elastic Elastic-plastic Fully plastic

5. $M = \int_A \sigma\, y\, dA$



Elastic region

$$M = \int \sigma y dA = -E\int \frac{y^2}{\rho} dA = -\frac{EI}{\rho} \qquad (3)$$



At the onset of plastic behavior

$$\sigma = -\, y/\rho\, E = -\, h/2\rho\, E = -Y \qquad (4)$$

This occurs at

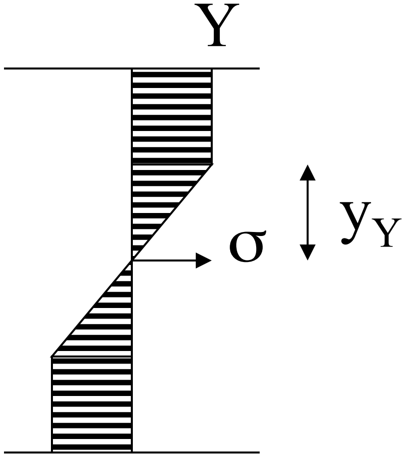
$$1/\rho = 2Y / hE = 1/\rho_Y \qquad (5)$$

Substitution into eqn (3) gives us the moment at on-set of yield, M_Y

$$M_Y = -\, EI/\rho_Y = EI\, 2Y / hE = 2IY/h \qquad (6)$$

After this point, the M vs 1/r curve starts to “bend over.” Note from M=0 to M=M_Y the curve is linear.

In the elastic – plastic region



$$M = \int \sigma y b dy = 2\int_{y_Y}^{h/2} Ybydy + 2\int_0^{y_Y} \frac{y}{y_Y} Ybydy$$

$$= 2Yb\frac{y^2}{2}\Bigg|_{y_Y}^{h/2} + 2\frac{Y}{y_Y}b\frac{y^3}{3}\Bigg|_0^{y_Y}$$

$$= Yb(\frac{h^2}{4} - y_Y^2) + \frac{2}{3}y_Y^2 Yb$$

$$M = \frac{bh^2}{4}Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{h/2}\right)^2\right] \qquad (7)$$

Note at $y_Y=h/2$, you get on-set at yield, $M = M_Y$
And at $y_Y=0$, you get fully plastic moment, $M = 3/2\ M_Y$

To write this in terms of M vs $1/\rho$ rather than M vs y_Y, note that the yield curvature $(1/\rho)_Y$ can be written as (see eqn (1))

$$\frac{1}{\rho_Y} = \frac{\varepsilon_Y}{h/2} \tag{8}$$

Where ε_Y is the strain at yield. Also since the strain at y_Y is $-\varepsilon_Y$, we can write

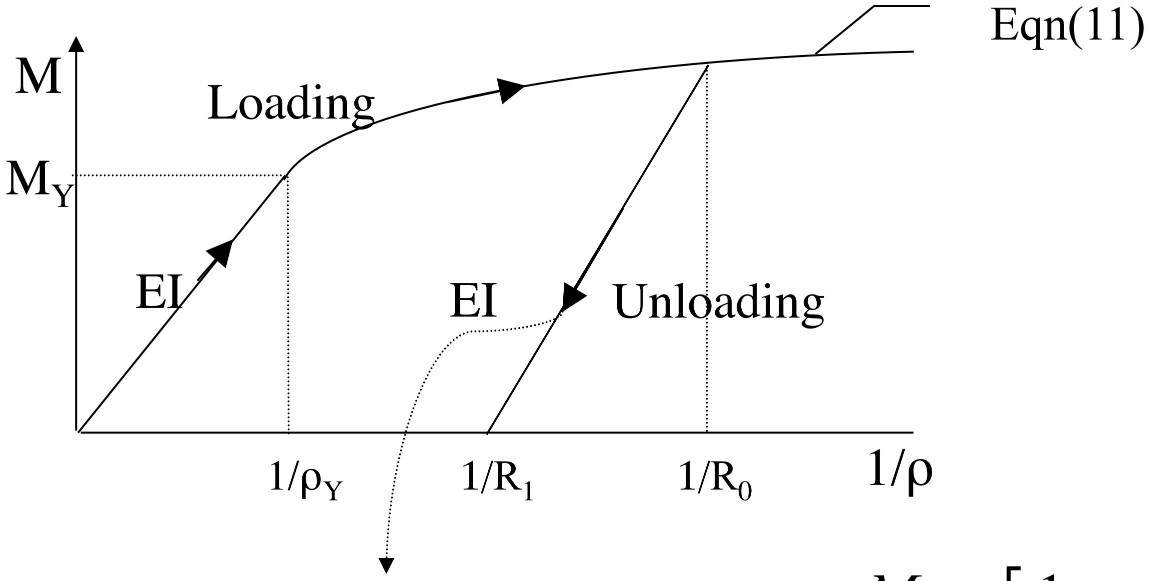
$$\frac{1}{\rho} = \frac{\varepsilon_Y}{y_Y} \tag{9}$$

Combining (8) and (9) gives

$$\frac{y_Y}{h/2} = \frac{(1/\rho)_Y}{1/\rho} \tag{10}$$

Substitution into (7) gives the result we seek:

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3} \left(\frac{(1/\rho)_Y}{1/\rho} \right)^2 \right] \qquad (11)$$



Elastic unloading curve

$$M = \frac{M_Y}{(1/\rho)_Y} \left[\frac{1}{\rho} - \frac{1}{R_1} \right] \qquad (12)$$

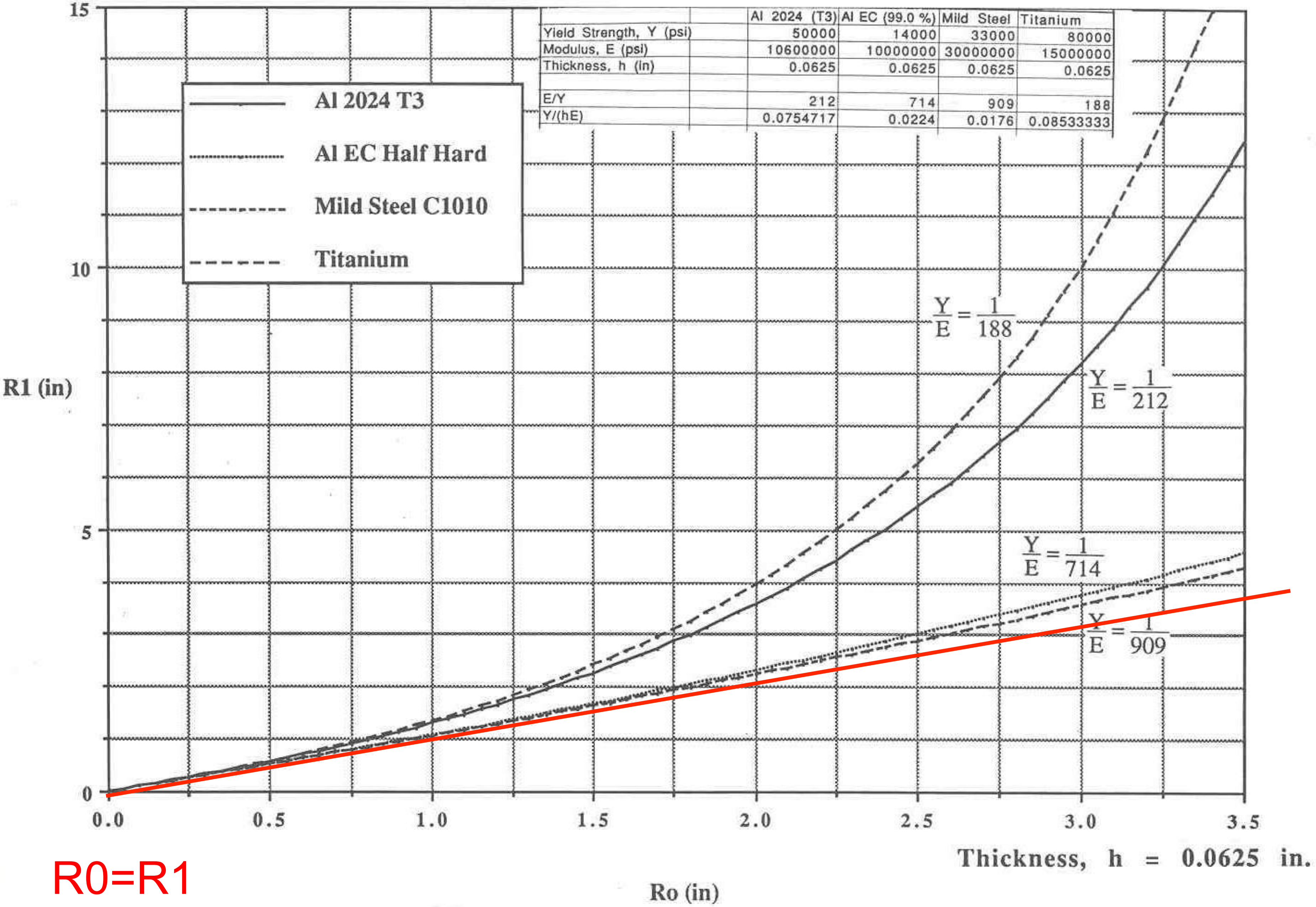
Now, eqn's (11) and (12) intersect at $1/\rho = 1/R_0$

Hence,

$$\frac{M_Y}{(1/\rho)_Y}\left[\frac{1}{R_0}-\frac{1}{R_1}\right]=\frac{3}{2}M_Y\left[1-\frac{1}{3}\left(\frac{(1/\rho)_Y}{1/R_0}\right)^2\right]$$

Rewriting and using $(1/\rho)_Y = 2Y/hE$ *(from a few slides back)*, we get

$$\boxed{\left[\frac{1}{R_0}-\frac{1}{R_1}\right]=3\frac{Y}{hE}-4R_0^2\left(\frac{Y}{hE}\right)^3} \qquad (13)$$



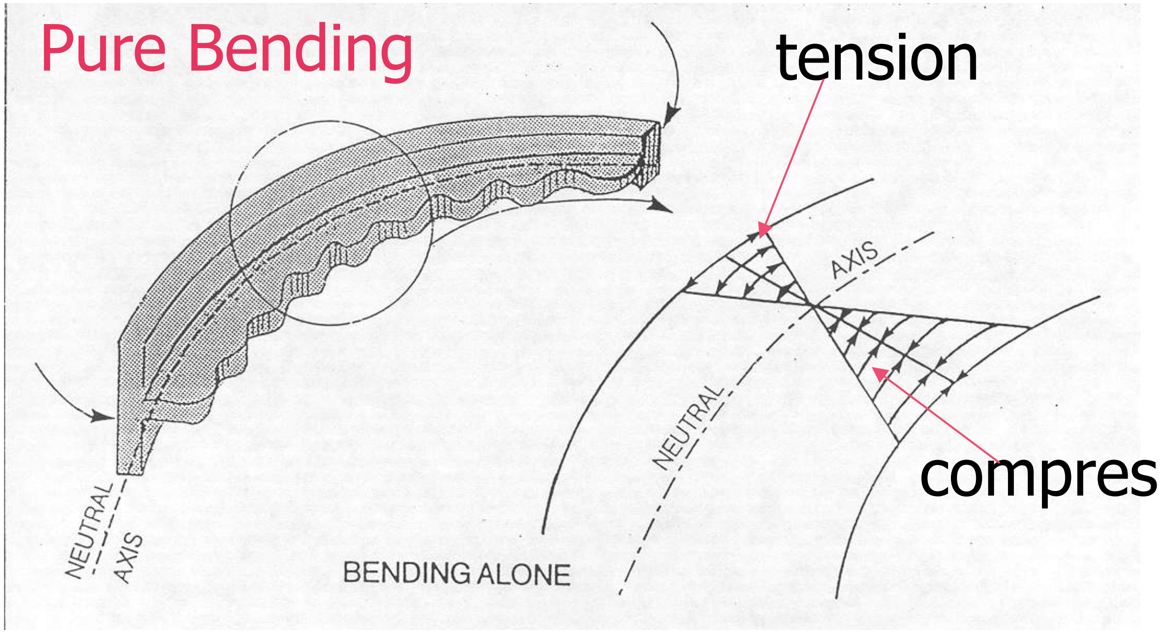
	Al 2024 (T3)	Al EC (99.0 %)	Mild Steel	Titanium
Yield Strength, Y (psi)	50000	14000	33000	80000
Modulus, E (psi)	10600000	10000000	30000000	15000000
Thickness, h (in)	0.0625	0.0625	0.0625	0.0625
E/Y	212	714	909	188
Y/(hE)	0.0754717	0.0224	0.0176	0.08533333

$$\frac{1}{R_0}-\frac{1}{R_1}=3\frac{Y}{hE}-4R_0^2\left(\frac{Y}{hE}\right)^3$$

Methods to reduce springback

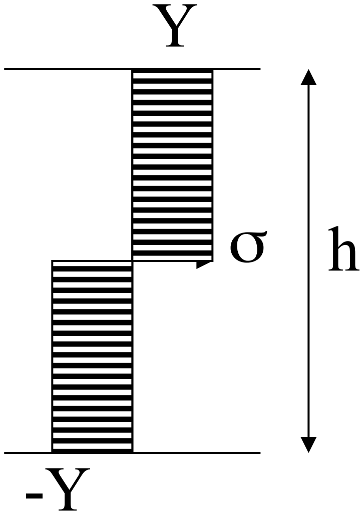
- Smaller Y/E
- Larger thickness
- Over-bending
- Stretch forming
- “coining” or bottoming the punch

Pure Bending

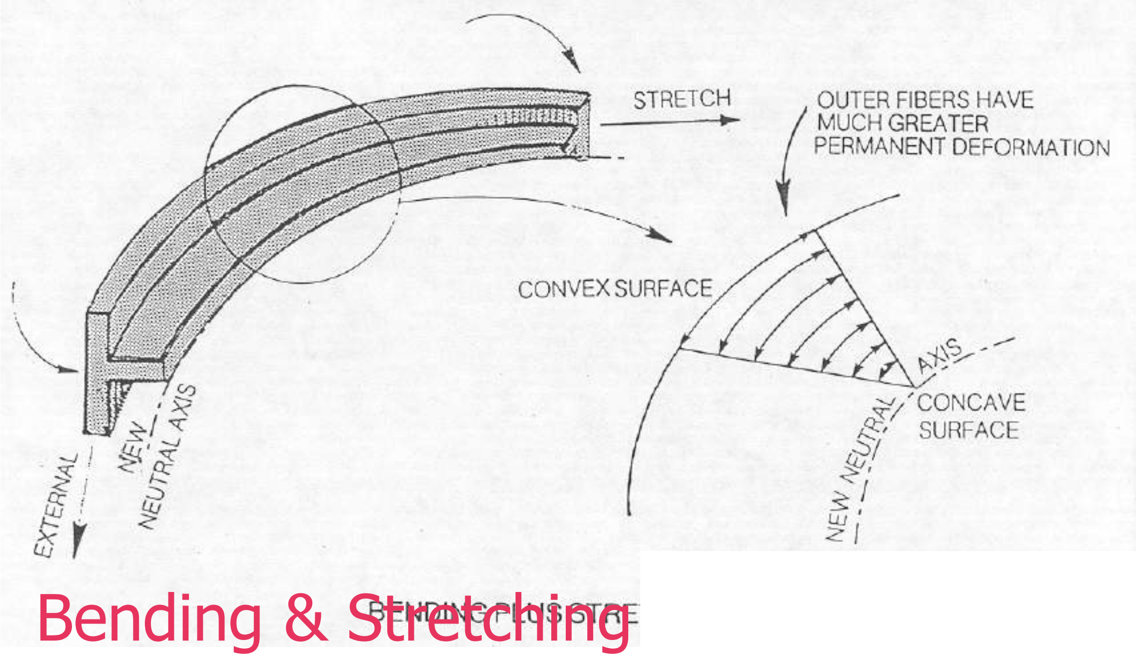


tension

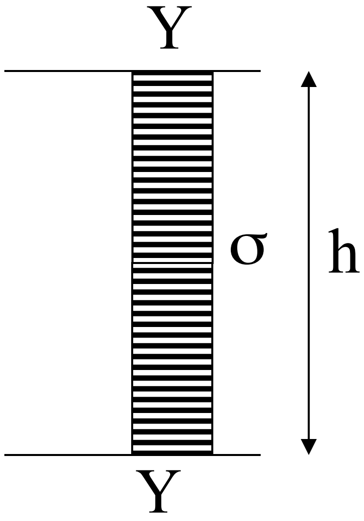
compression



Fully plastic



Bending & Stretching



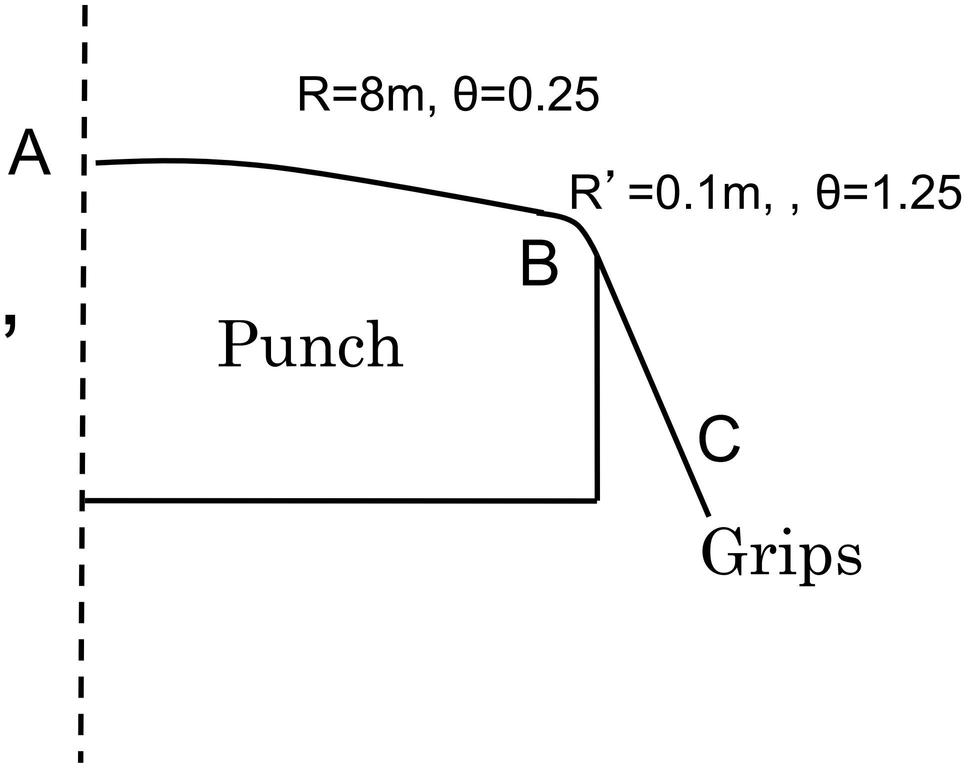
Fully plastic

Stretch forming: can we achieve a strain of 0.035 at A?

Sheet thickness 1mm, $\mu=0.1$

Material:

$\boldsymbol{\sigma=520\varepsilon^{0.18}}$**MPa**



Can we achieve a strain of 0.035 at A?

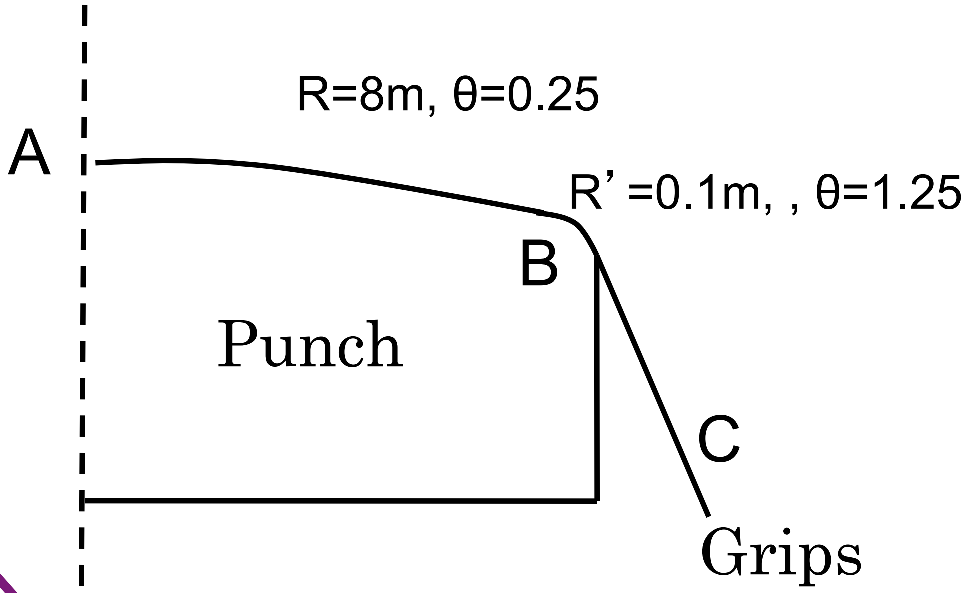
Sheet thickness 1mm,
$\mu=0.1$
Material: $\boldsymbol{\sigma=520\varepsilon^{0.18}}$**MPa**

$F_A = 0.001*520*(0.035)^{0.18}=284$kN/m

$F_B=F_A*\exp(0.1*0.25)=292$kN/m

$F_C=F_B*\exp(0.1*1.05)=$**323kN/m**

Max allowable force
$= 0.001*520*(0.18)^{0.18}=$**381kN/m**



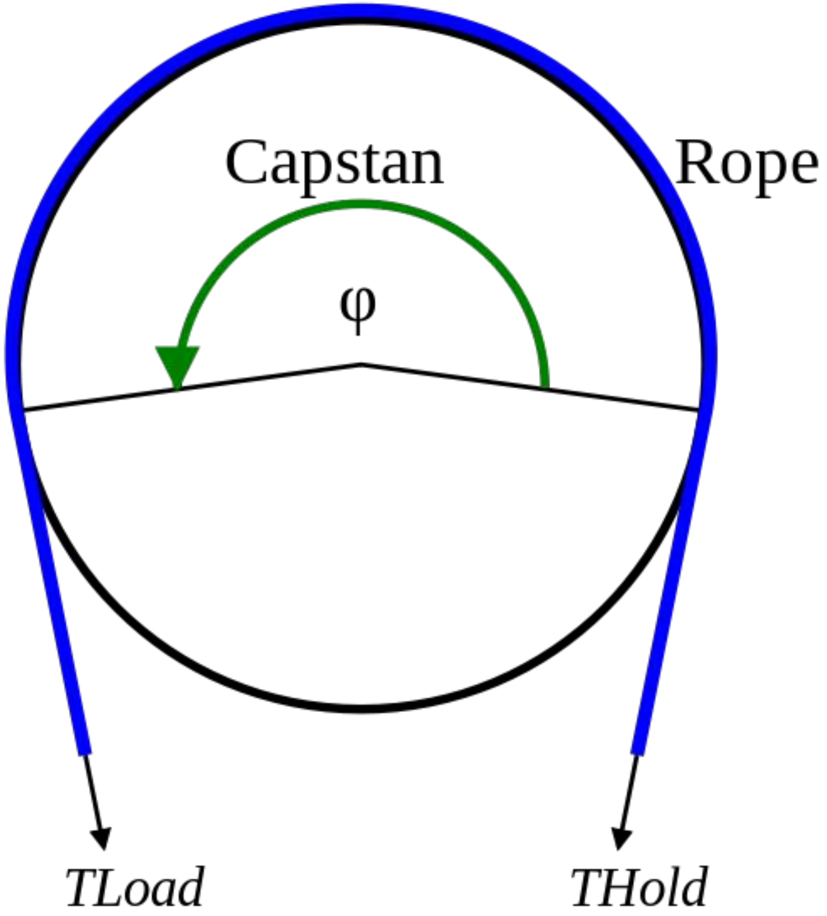
Capstan equation

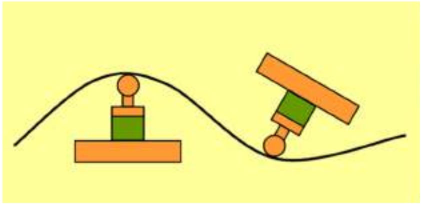
Friction and the capstan equation

Typical stamping lubricants:

- Oil-based lubricants
- Aqueous lubricants
- Soaps and greases
- Solid films

$$T_{load} = T_{hold} \times \exp(\mu\theta)$$





Research opportunities and challenges: reducing cost and environmental impacts

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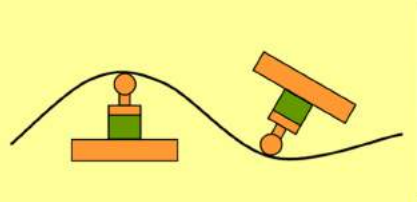
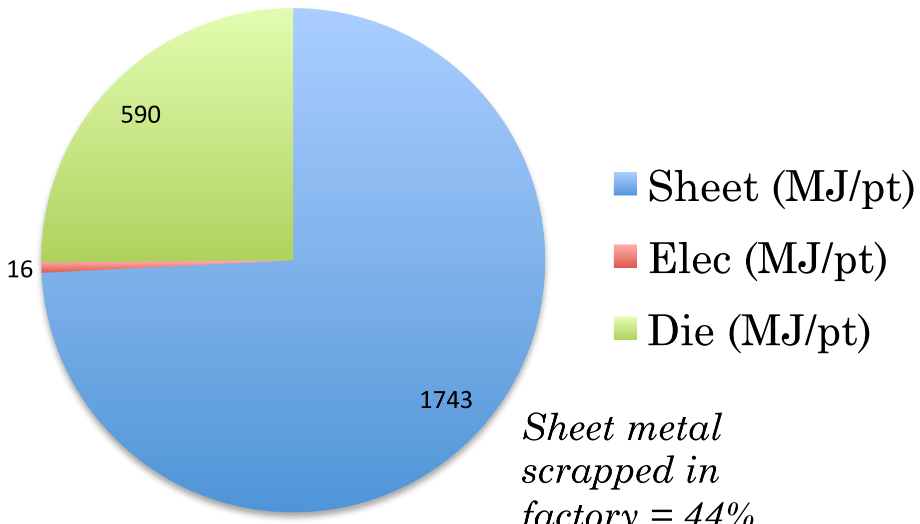


Energy & cost: Stamping alum car hoods

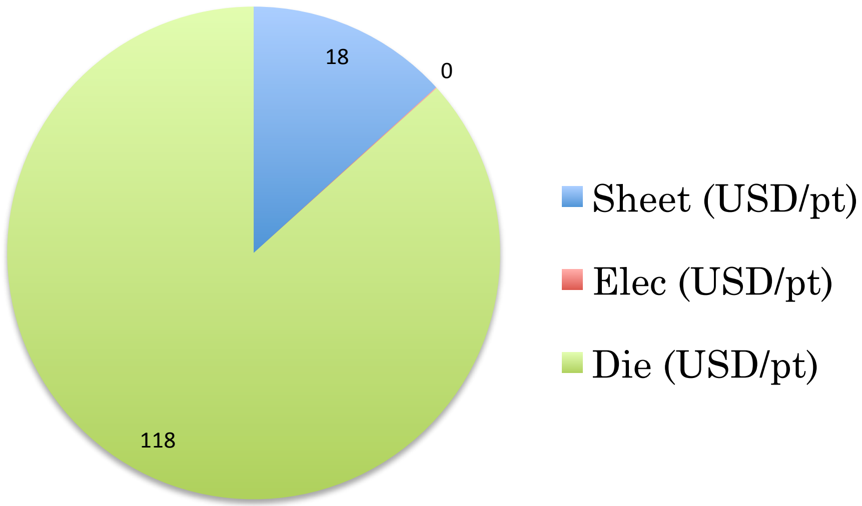
- Final part = 5.4kgs
- Total number of parts made = 400
- Die material: cast and machined zinc alloy

Energy. 2.3GJ/pt. Stamping alum. car hoods. 5.4kgO/P. (400pts)



Sheet metal scrapped in factory = 44%

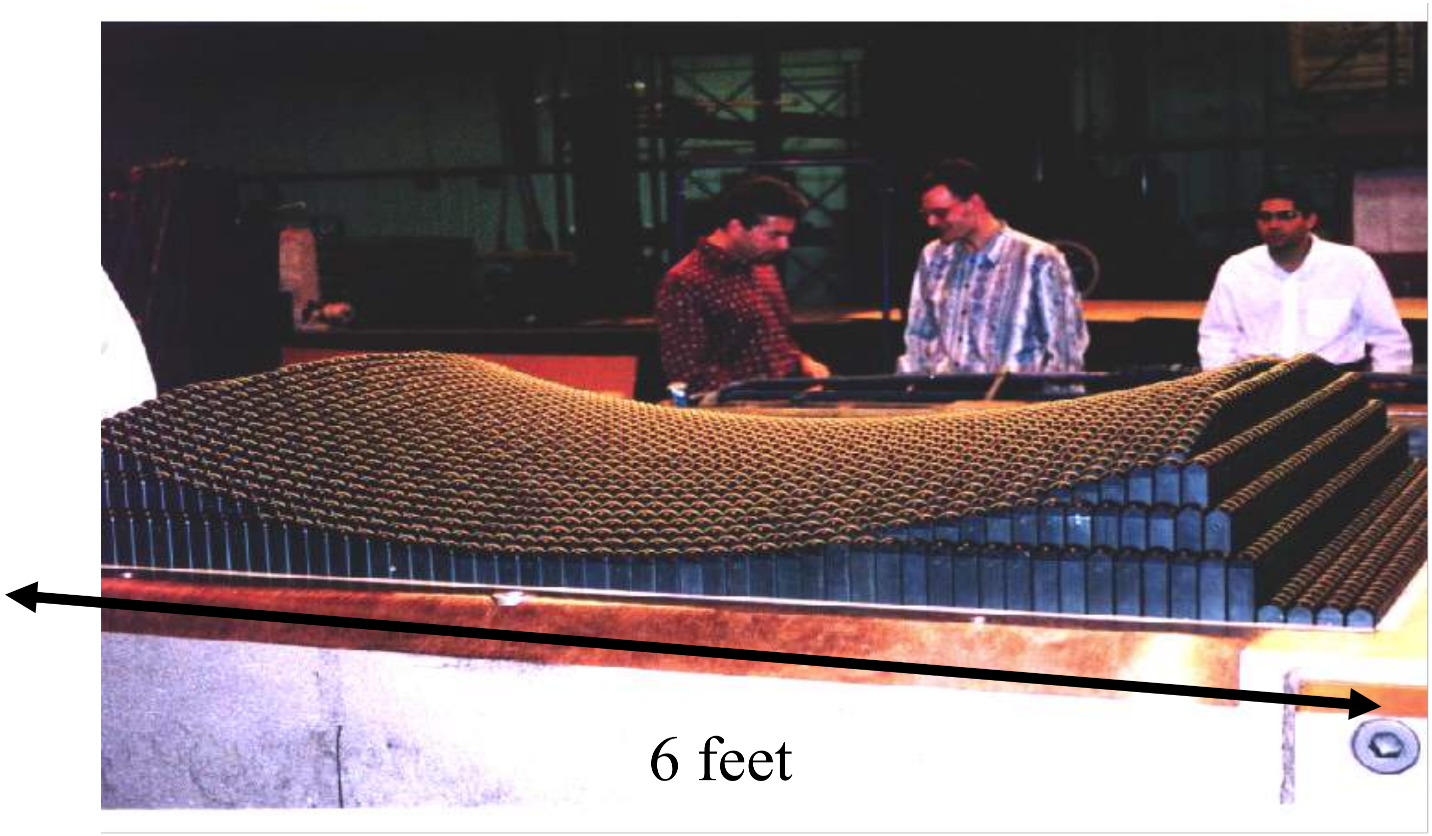
Cost. 136USD/pt

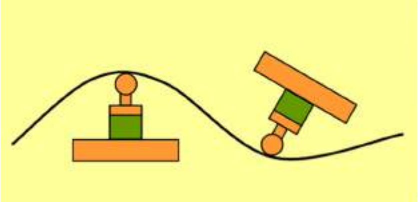


Source: Unpublished work: Cooper, Rossie, Gutowski (2015)

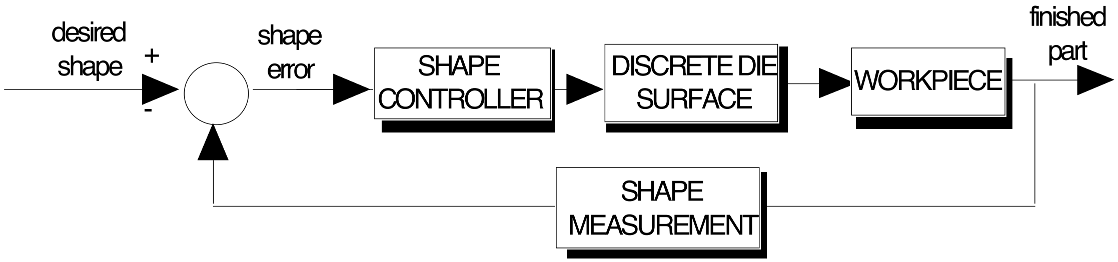
Excludes equipment depreciation and labor during forming

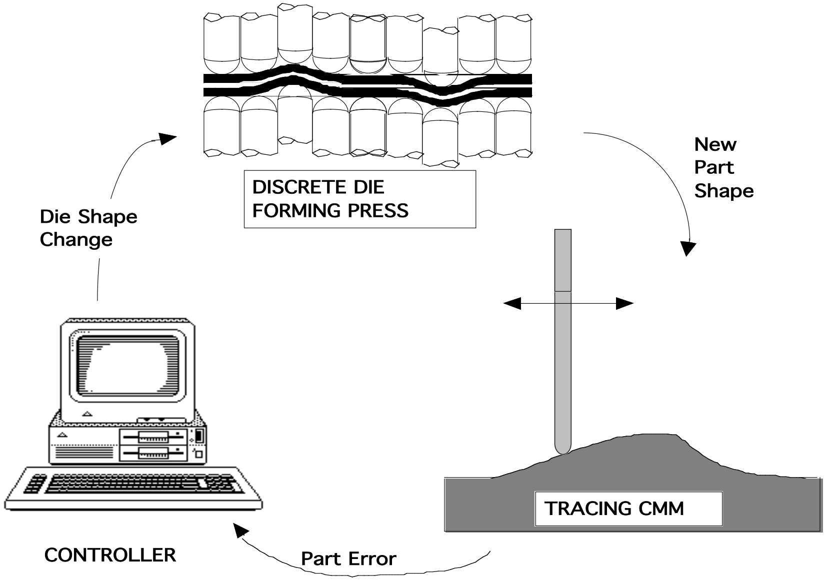
60 Ton Discrete Die Press (LMP - Hardt)





The Shape Control Concept



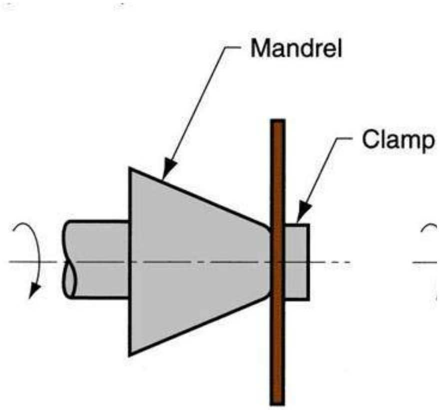


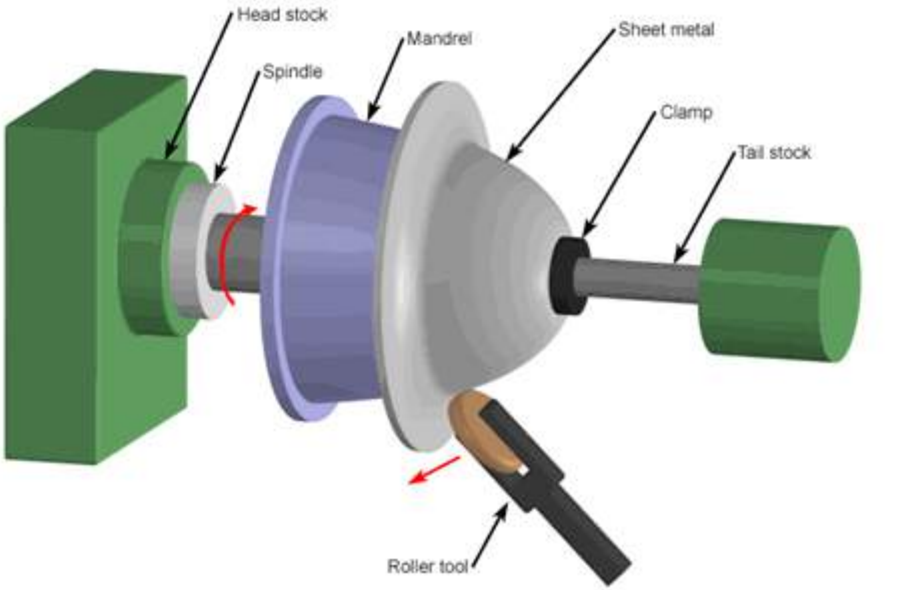
Stretch Forming with Reconfigurable Tool @ Northrop Grumman

Flexible Forming at Ford

Massachusetts
Institute of
Technology

Conventional Spinning





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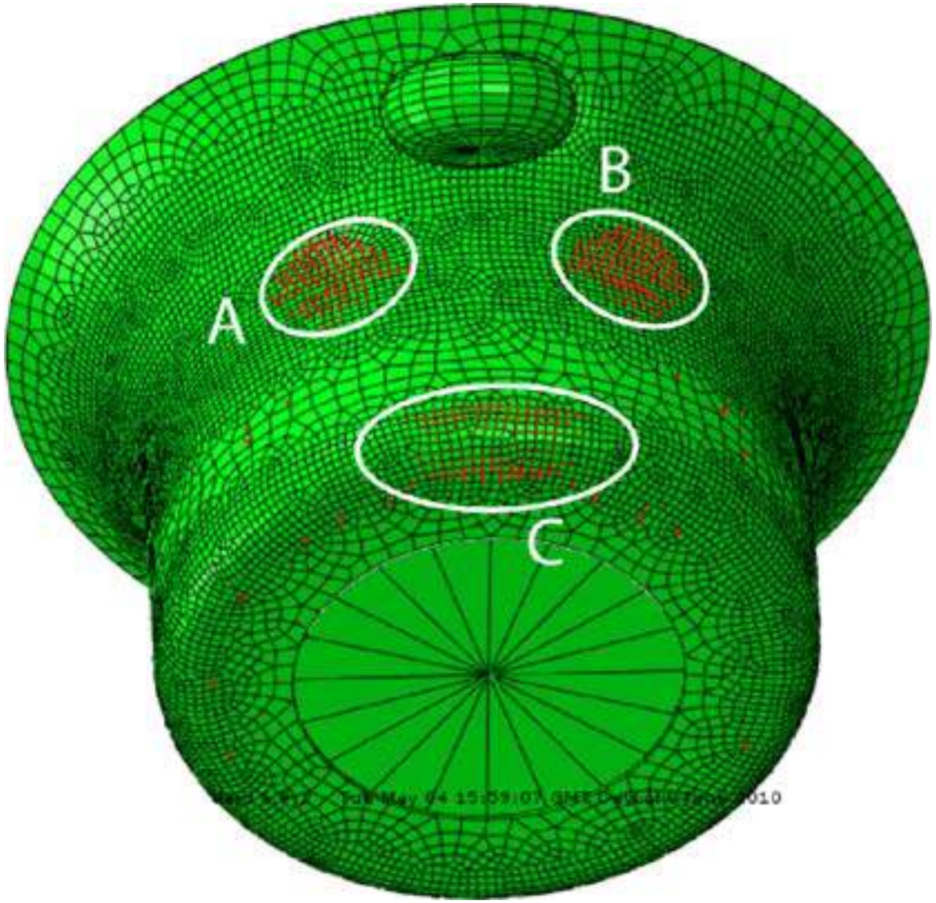
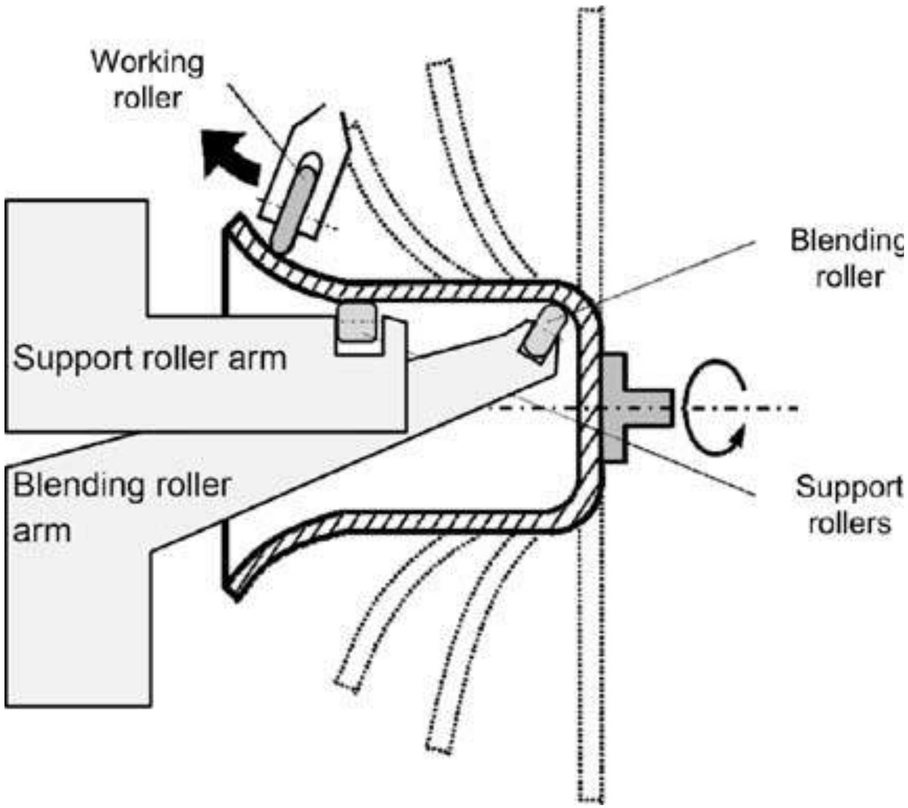
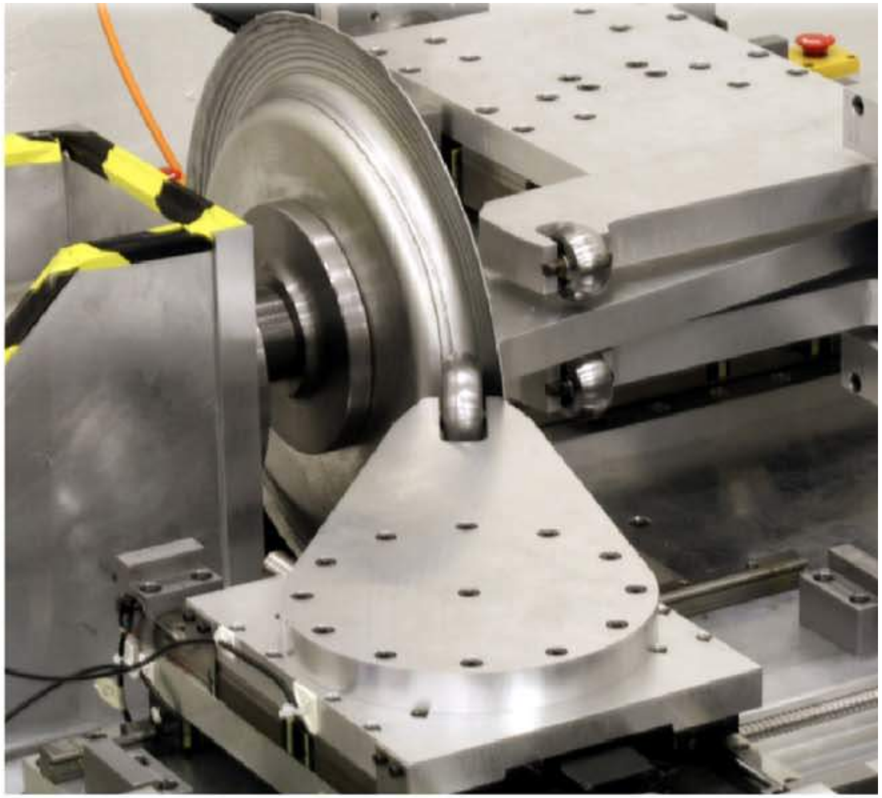


http://www.custompartnet.com/wu/sheet-metal-forming

Flexible Spinning





(b) Machine in operation

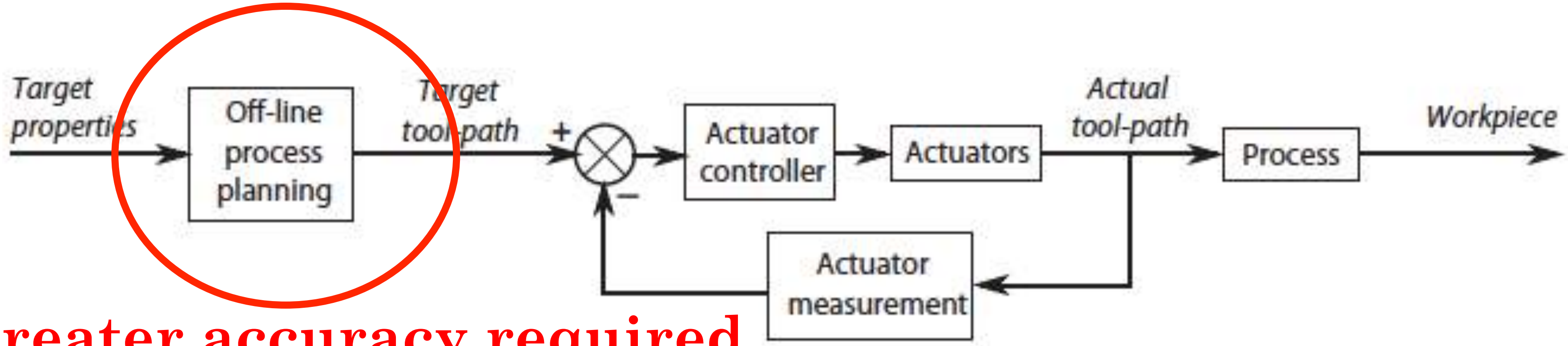
Circular cup

Elliptical cup

Rectangular cup

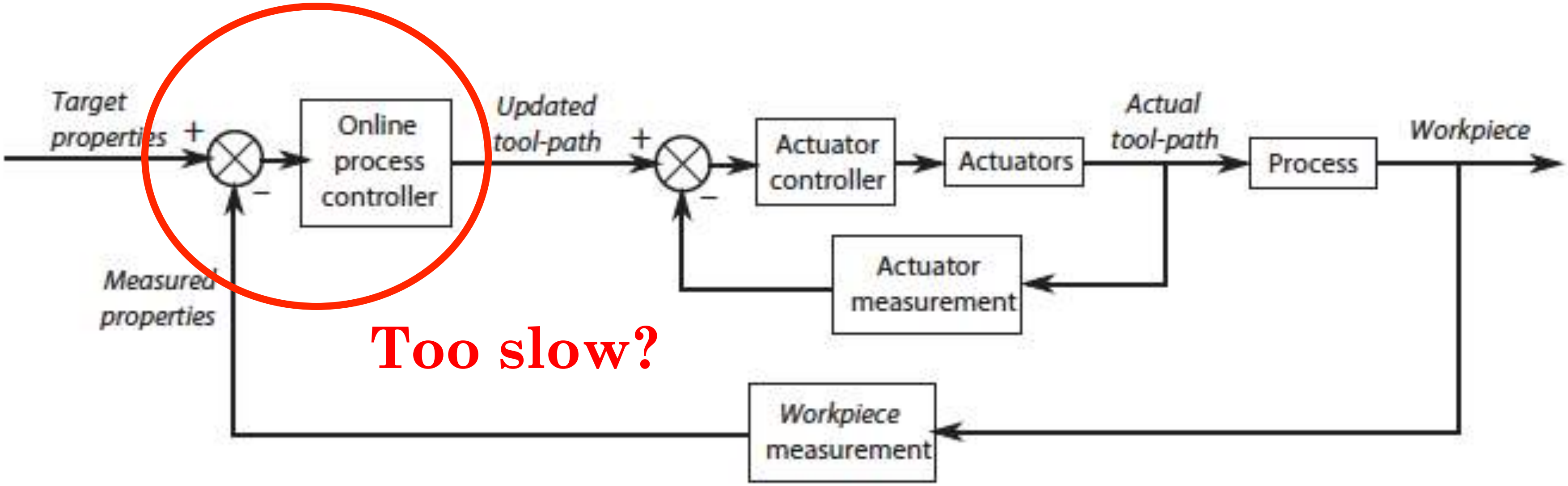
Kidney bean

Music, O., & Allwood, J. M. (2011). Flexible asymmetric spinning. *CIRP Annals - Manufacturing Technology*, *60*(1), 319–322. doi:10.1016/j.cirp.2011.03.136



Greater accuracy required

Fig. 1. A system diagram for open-loop control of metal forming.

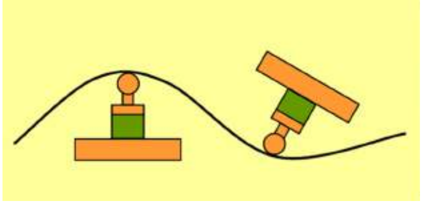


Too slow?

Fig. 2. A system diagram for closed-loop control of metal forming.

Massachusetts Institute of Technology

Polyblank, J. a., Allwood, J. M., & Duncan, S. R. (2014). Closed-loop control of product properties in metal forming: A review and prospectus. *Journal of Materials Processing Technology*, *214*(11), 2333–2348. doi:10.1016/j.jmatprotec.2014.04.014



Thank you

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