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Institute of Industrial Engineers

Chapter 2-3

Short Run SPC

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Consider the Following

- Low production quantity
- One process produces many different items
- Different operators use the same equipment

These are all what we refer to as short run situations and suggest the use of the short run SPC methodology.

Many firms have decided to use smaller production runs or short-runs in their mixed-model production processes to provide greater flexibility in meeting customer demand, improve quality, and reduce unnecessary waste in their production systems (Porteus 1986). Unfortunately, the volatility in lot size and variety of products produced caused by a change to short-runs and mixed-model methods can necessitate other production processes to change. One case in point is the Shewhart Statistical Process Control (SPC) system. Traditionally, Shewhart's SPC techniques are applied in high-volume and repetitive manufacturing systems. The Shewhart SPC methodology is directed at signaling special causes of variation generated during a production run. Such signals are detected using control limits constructed on the basis of within-subgroup variability, measured after the process has been set up, adjusted, and brought into statistical control. In other words, "the process" to be monitored is that which exists after setup and adjustment. However, in a short-run production that is characterized by frequent setups of small production runs, SPC must be applied differently. SPC must incorporate a broader definition of the process, which recognizes ongoing volatility in the sequences of setups and production runs.

From American Journal of Business

Short Run SPC Approaches

- Nominal Short Run SPC
- Target Short Run SPC
- Short Run Short Run SPC

- These work for variables
- These work for attributes

- We will look at the applications for averages and ranges

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Nominal Short Run X Bar and R Chart

The nominal x bar and R chart is used to monitor the behavior of a process running different part numbers and still retain the ability to assess control.

This is done by coding the actual measured readings in a subgroup as a variation from a common reference point, in this case the nominal print specification.

Nominal X Bar and R Chart

The nominal value becomes the zero point on the x bar control chart scale.

Nominal X Bar and R Chart—Constructing the Chart

- Determine the nominal for each part number to be charted.
- Select a part number to begin charting
- Collect several consecutive pieces for the first subgroup and measure each piece
- Subtract the nominal value for the part from each observed measurement in the subgroup.
- These new coded values are now used to calculate the average and range of this first subgroup.
- These values are used on the chart.

Nominal X Bar and R Chart—Constructing the Chart

- Continue for each subgroup for that part.
- Repeat the process when the part number changes.
- (When constructing the graph use a dotted line when part numbers change.)
- Continue until a minimum of 20 subgroups have been measured.
- Calculate the average range.
- Calculate the average of the nominal x bar chart using the coded values.

Nominal X Bar and R Chart—Variation Check

The use of this method assumes the process variation of all part numbers is approximately equal. If one is too large compared to the rest, it cannot be plotted on the same nominal chart.

- Compare the average range for each part (R_s) with the overall average range for all of the parts.
- If the ratio of R_s to the total, R_{total} is greater than 1.3 or less than .77 the part cannot be plotted on the same nominal chart.

Nominal X Bar and R Chart—Control Limit Formulas

$$UCL\bar{x} = \bar{\bar{x}} + A_2\bar{R}$$

$$LCL\bar{x} = \bar{\bar{x}} - A_2\bar{R}$$

$$UCLR = D_4\bar{R}$$

$$LCLR = D_3\bar{R}$$

- Draw the chart
- Analyze the variation

Nominal X Bar and R Chart—Additional Restrictions

- Individual part variation
- Subgroup size constant
- Nominal Specification is the desired target value for the center of the process output.

Example

- The table on the following page shows data for four different parts run on the same machine for the same characteristic.
- The table on the page following that shows some sample data collected.
- Calculate the nominal x bar and R chart control limits.

Information for Example

Part	Nominal
A	40
B	20
C	60
D	10

All parts are to be within ± 5 units of nominal.

Data Set 2-3-1

Product	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6
A	38.5	39.0	39.5	40.0	40.5	41.0
A	40.5	41.0	41.5	41.5	42.5	43.0
A	39.5	40.5	40.5	41.0	43.5	43.0
A	38.0	37.1	37.5	41.5	42.0	42.5
B	19.5	19.8	20.0	20.3	20.5	20.8
B	17.0	22.3	21.3	19.0	19.3	19.5
B	18.3	18.5	18.8	23.0	23.3	23.5
B	19.3	19.5	19.8	20.0	20.3	20.5
C	58.3	57.7	57.8	62.3	61.1	60.8
C	57.8	58.5	59.3	60.0	60.8	61.5
C	63.0	59.0	60.0	60.0	60.4	60.6
D	9.8	12.2	10.1	10.0	10.1	8.9
D	12.2	10.0	10.1	9.2	10.0	9.9
D	10.7	9.0	8.0	10.1	10.1	9.8
D	8.7	10.2	11.2	9.7	9.9	10.0
A	39.5	40.0	40.5	41.0	43.0	42.5
A	44.0	41.5	41.5	43.0	43.0	42.0
C	58.3	57.0	57.8	58.5	59.3	61.0
C	60.0	62.3	63.0	63.8	59.0	62.0
B	18.3	18.5	18.8	19.0	18.5	22.0
B	19.5	19.8	20.0	20.3	20.5	20.0
B	19.5	20.0	20.0	20.3	20.5	21.5
D	8.0	10.9	10.1	9.8	9.8	9.9
D	10.2	12.0	10.0	9.8	10.0	9.9
A	36.5	37.0	37.5	38.0	41.3	41.0
A	41.0	39.6	40.0	39.1	38.9	42.0
B	19.0	19.3	21.0	22.0	21.8	22.3
B	18.5	19.5	20.5	21.5	20.8	18.6
C	61.4	59.6	61.7	58.1	62.2	60.0
C	59.2	57.0	61.5	59.3	57.8	60.0

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Nominal X Bar and R Chart—Example Calculations

$$UCL\bar{x} = \bar{\bar{x}} + A_2 \bar{R}$$

$$LCL\bar{x} = \bar{\bar{x}} - A_2 \bar{R}$$

$$UCLR = D_4 \bar{R}$$

$$LCLR = D_3 \bar{R}$$

Target X Bar and R Chart

Sometimes…

- **Process should not be centered at nominal.**
- **There is a unilateral spec, either max or min but not both**

In That Case…

- Use the historical process average of a part number as the target process mean and measure the variation of future pieces from this target.
- Use the following formulas for control limits.

Target Chart Formulas

$$UCL\bar{x} = \bar{\bar{x}} + A_2 \bar{R}$$

$$LCL\bar{x} = \bar{\bar{x}} - A_2 \bar{R}$$

$$UCLR = D_4 \bar{R}$$

$$LCLR = D_3 \bar{R}$$

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Historical Average or Nominal?

Use the nominal unless the absolute value of the difference between the nominal and historical value is greater than the critical value of f_1 times the sample standard deviation.

$$\text{If } \left|\text{Nominal - Historical Average}\right| \leq f_1 s \text{ then use Nominal}$$

Values of f_1 are on the next page.

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Values of f_1

Number of Historical Values	f_1
5	1.24
6	1.05
7	.92
8	.84
9	.77
10	.72
11	.67
12	.64
14	.58
15	.55
16	.53
18	.50
20	.47
25	.41
30	.37

Example

Suppose the historical average based on 10 measurements taken from the last time a given part number was run is 20.8 and the sample standard deviation was 1.07. Determine if the nominal of 20.0 or the historical average of 20.4 should be used.

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Solution

- Calculate the difference
- Multiply the f_1 value times the standard deviation
- Compare the difference and the product

Short Run X Bar and R Chart

- Average ranges for different products produced on the same process are significantly different
- Data is transformed
  - Ranges
  - Averages

Short Run X Bar and R Chart Plot Point Calculations

$$\text{R Plot Point} = \frac{R}{Trgt\overline{R}}$$

$$\overline{\text{X}}\text{ Plot Point} = \frac{Subgroup\,\overline{X} - Trgt\,\overline{X}}{Trgt\overline{R}}$$

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Control Limits

- Short Run Range Chart
  - UCL = D_4
  - LCL = D_3
- Short Run Average Chart
  - UCL = $+A_2$
  - LCL = $-A_2$

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6σ ANALYZE

Determining Target Values
Average and Range

1) Prior charts for this part number
2) Historical data
   - Target average is average of historical data
   - Target range is $f_2 s$

   (Values found on following page)

Values of f_2

Total Number of Measurements	Sample Size 2	3	4	5	6
5	1.20	1.80	2.19	2.41	2.70
10	1.16	1.74	2.12	2.39	2.60
15	1.15	1.72	2.10	2.37	2.58
20	1.14	1.72	2.09	2.36	2.57
25	1.14	1.71	2.08	2.35	2.56
30	1.13	1.69	2.06	2.33	2.53

Determining Target Values
Average and Range

3) Prior experience on similar part numbers. New averages equal old averages.

4) Specification limit.

Target average = nominal print specification

Target average range =

$$\text{Target } \overline{R} = \frac{(d_2)(USL - LSL)}{6Cpk}$$

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Example

Construct short run x bar and R chart control limits for the data on the following page.

Data Set 2-3-2

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Example

	A	A	M	A	A	A	M	M	A	A	A	M	A
	0.50	0.00	4.50	-1.00	-1.00	-0.50	6.00	5.50	2.00	0.00	1.00	5.00	1.50
	1.50	-0.50	5.00	1.00	-0.50	1.50	6.00	6.00	-1.00	1.00	0.50	5.50	2.00
	1.00	1.00	4.00	1.50	0.50	0.50	5.25	5.00	2.50	0.50	-0.50	3.50	1.50
Target Avg	0.70		5.20										
Average													
Target Range	1.4		0.6										
Range													

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Another Example

Part No.	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Target Avg	Average	Target Rng	Range	Avg Plt Pt	Rng Plt Pt
A	7	7	9	6	8	**8.50**		**1.1**			
A	8	7	8	6	8	**8.50**		**1.1**			
A	6	8	7	9	7	**8.50**		**1.1**			
B	200	202	204	201	201	**202.00**		**4.3**			
B	202	201	202	200	199	**202.00**		**4.3**			
B	205	204	204	200	203	**202.00**		**4.3**			
B	203	203	199	206	205	**202.00**		**4.3**			
C	23	25	24	23	24	**22.75**		**2.6**			
C	25	22	25	24	23	**22.75**		**2.6**			
C	26	23	24	24	25	**22.75**		**2.6**			
C	24	27	26	24	24	**22.75**		**2.6**			
B	198	202	201	206	202	**202.00**		**4.3**			
B	199	200	206	204	202	**202.00**		**4.3**			
B	203	201	198	205	204	**202.00**		**4.3**			
A	8	8	6	9	7	**8.50**		**1.1**			
A	7	7	9	5	7	**8.50**		**1.1**			
A	8	7	6	9	8	**8.50**		**1.1**			
A	6	7	8	9	10	**8.50**		**1.1**			
C	24	22	23	24	24	**22.75**		**2.6**			
C	23	25	24	23	24	**22.75**		**2.6**			

Data Set 2-3-31

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Short Run Individual and Moving Range Chart Formulas

$$UCLX = \overline{X} + (2.66)\overline{MR}$$

$$LCLX = \overline{X} - (2.66)\overline{MR}$$

$$UCLMR = (3.27)\overline{MR}$$

$$LCLMR = 0$$

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Nominal Individual and Moving Range Chart

$$\text{X Plot Point} = x - \text{nominal}$$

$$\text{MR Plot Point} = \left| (\text{X Plot Point})_i - (\text{X Plot Point})_{i-1} \right|$$

Target Individual and Moving Range Chart

$$\text{X Plot Point} = \text{x} - \text{target average}$$

$$\text{MR Plot Point} = \left|(\text{X Plot Point})_{i} - (\text{X Plot Point})_{i-1}\right|$$

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Short Run Individual and Moving Range Chart

$$\text{X Plot Point} = \frac{\text{x} - \text{target average}}{\text{target average range}}$$

$$\text{MR Plot Point} = \left| (\text{X Plot Point})_{\text{i}} - (\text{X Plot Point})_{\text{i}-1} \right|$$

$$UCL_X = +2.66$$

$$LCL_X = -2.66$$

$$UCL_{\overline{MR}} = 3.27$$

$$LCL_{\overline{MR}} = 0$$

Practice

Use the following data to calculate short run individuals and moving range control limits using the nominal method.

Data Set 2-3-3

Sample	1	2	3	4	5	6	7	8	9	10	11	12
Value	6	9	8	13	13	12	12	7	8	9	8	11
Nominal	8	8	8	12	12	12	12	8	8	8	8	12
MR												

One More Practice Problem

Data Set 2-3-4

Attribute Charts

Chart	Target	Plot Point
np	$n\bar{p}$	$\frac{\text{Actual np} - \text{Target } n\bar{p}}{\sqrt{n\bar{p}(1-\bar{p})}}$
p	$\bar{p}$	$\frac{\text{Actual p} - \text{Target } \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}}$
c	$\bar{c}$	$\frac{\text{Actual c} - \text{Target } \bar{c}}{\sqrt{\bar{c}}}$
u	$\bar{u}$	$\frac{\text{Actual u} - \text{Target } \bar{u}}{\sqrt{\frac{\bar{u}}{n}}}$

Review

Control limits describe the representative nature of a stable process. Specifically, control limits identify the expected limits of normal, random, or chance variation that is present in the process being monitored.