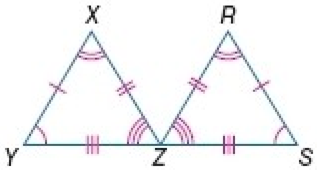
Show that the polygons are congruent by using rigid motions and by identifying all congruent corresponding parts. Then write a congruence statement.

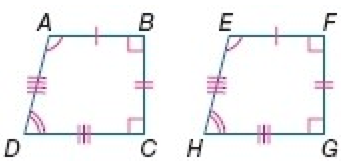
1. 

SOLUTION:

A reflection maps one polygon exactly onto the other
$\angle Y \cong \angle S, \angle X \cong \angle R, \angle XYZ \cong \angle RZS$, $\overline{YX} \cong \overline{SR}$, $\overline{YZ}$
All corresponding parts of the two triangles are congr
$\Delta YXZ \cong \Delta SRZ$.

ANSWER:

Sample answer: A reflection maps one polygon exact
$\angle Y \cong \angle S$, $\angle X \cong \angle R$, $\angle XZY \cong \angle RZS$,
$\overline{YX} \cong \overline{SR}$, $\overline{YZ} \cong \overline{SZ}$, $\overline{XZ} \cong \overline{RZ}$; $\Delta YXZ \cong \Delta SRZ$

2. 

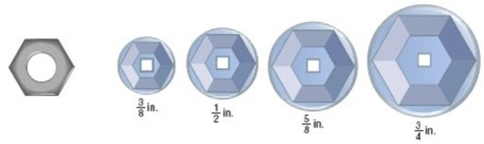
SOLUTION:

A translation maps one polygon exactly onto the other. $\angle A \cong \angle E, \angle B \cong \angle F, \angle C \cong \angle G, \angle D \cong \angle H$, $\overline{AB} \cong \overline{EF}$, $\overline{CD} \cong \overline{GH}$, $\overline{AD} \cong \overline{EH}$, $\overline{BC} \cong \overline{FG}$; All corresponding parts of the two polygons are congruent. Therefore,
polygon $ABCD \cong$ polygon $EFGH$.

ANSWER:

Sample answer: A translation maps one polygon exactly onto the other. $\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, $\angle D \cong \angle H$,
$\overline{AB} \cong \overline{EF}$, $\overline{CD} \cong \overline{GH}$, $\overline{AD} \cong \overline{EH}$, $\overline{BC} \cong \overline{FG}$; polygon
$ABCD \cong$ polygon $EFGH$

3. **TOOLS** Sareeta is changing the tire on her bike and the nut securing the tire looks like the one shown. Which of the sockets below should she use with her wrench to remove the tire? Explain your reasoning.



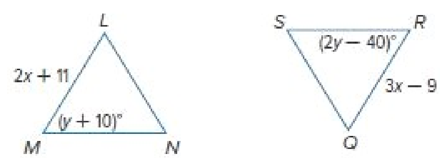
SOLUTION:

$\frac{1}{2}$ in.; The nut is congruent to the opening for the $\frac{1}{2}$ in. socket.

ANSWER:

$\frac{1}{2}$ in.; Sample answer: The nut is congruent to the opening for the $\frac{1}{2}$ in. socket.

In the figure, $\Delta LMN \cong \Delta QRS$.



4. Find x.

SOLUTION:

By CPCTC, $\overline{QR} \cong \overline{LM}$.
By the definition of congruence, $QR = LM$.

Substitute.

$3x - 9 = 2x + 11$	Substitute.
$3x - 9 - 2x = 2x + 11 - 2x$	$-2x$ from each side.
$x - 9 = 11$	Simplify.
$x - 9 + 9 = 11 + 9$	$+9$ to each side.
$x = 20$	Simplify.

ANSWER:

20

5. Find y.

SOLUTION:

By CPCTC, $\angle R \cong \angle M$.
By the definition of congruence, $m\angle R = m\angle M$.

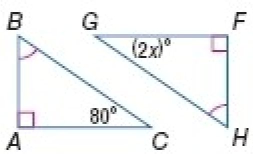
Substitute.

$2y - 40 = y + 10$	Substitute.
$2y - 40 - y = y + 10 - y$	$-y$ from each side.
$y - 40 = 10$	Simplify.
$y - 40 + 40 = 10 + 40$	+40 to each side.
$y = 50$	Simplify.

ANSWER:

50

REGULARITY Find *x*. Explain your reasoning.

6. 

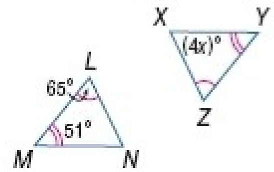
SOLUTION:

Since $\angle A \cong \angle F$ and $\angle B \cong \angle H$, $\angle G$ corresponds to $\angle C$.

$m\angle C = m\angle G$	CPCTC.
$2x = 80$	Subsitution.
$x = 40$	Divide each side by 2.

ANSWER:

40; $\angle G$ corresponds to $\angle C$, so $2x = 80$.

7. 

SOLUTION:

Since $\angle M \cong \angle Y$ and $\angle L \cong \angle Z$, $\angle N$ corresponds to $\angle X$. By the Third Angles Theorem, $m\angle N = 64$.

$m\angle N = m\angle X$	CPCTC.
$4x = 64$	Subsitution.
$x = 16$	Divide each side by 4.

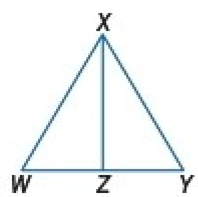
ANSWER:

16; $\angle N$ corresponds to $\angle X$. By the Third Angles Theorem, $m\angle N = 64$, so $4x = 64$.

8. **PROOF** Write a paragraph proof.

Given: $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$, $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$

Prove: $\Delta WXZ \cong \Delta YXZ$



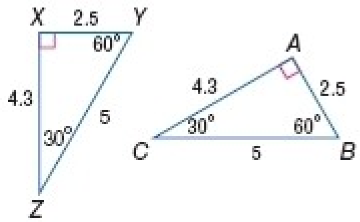
SOLUTION:

We know that $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ} \cong \overline{XZ}$ by the Reflexive Property. We also know $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$ and by the Third Angles Theorem, $\angle W \cong \angle Y$. So, $\Delta WXZ \cong \Delta YXZ$ by the definition of congruent polygons.

ANSWER:

We know that $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ} \cong \overline{XZ}$ by the Reflexive Property. We also know $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$ and by the Third Angles Theorem, $\angle W \cong \angle Y$. So, $\Delta WXZ \cong \Delta YXZ$ by the definition of congruent polygons.

Show that the polygons are congruent by using rigid motions and by identifying all congruent corresponding parts. Then write a congruence statement.

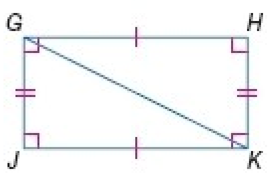
9. 

SOLUTION:

A combination of a rotation and a translation maps one polygon exactly onto the other. $\angle X \cong \angle A$, $\angle Y \cong \angle B$, $\angle Z \cong \angle C$, $\overline{XY} \cong \overline{AB}, \overline{XZ} \cong \overline{AC}, \overline{YZ} \cong \overline{BC}; \Delta XYZ \cong \Delta ABC$; All corresponding parts of the two triangles are congruent.

ANSWER:

Sample answer: A combination of a rotation and a translation maps one polygon exactly onto the other. $\angle X \cong \angle A, \angle Y \cong \angle B, \angle Z \cong \angle C,$ $\overline{XY} \cong \overline{AB}, \overline{XZ} \cong \overline{AC}, \overline{YZ} \cong \overline{BC}; \Delta XYZ \cong \Delta ABC$

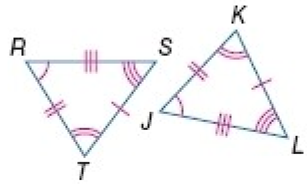
10. 

SOLUTION:

A rotation maps one polygon exactly onto the other.
$\angle KGH \cong \angle GKJ,$
two triangles are congruent.

ANSWER:

Sample answer: A rotation maps one polygon exactly
$\cong \angle HKG, \angle KGH \cong \angle GKJ, \overline{GJ} \cong \overline{KH}, \overline{JK} \cong \overline{HG}, \overline{GK}$

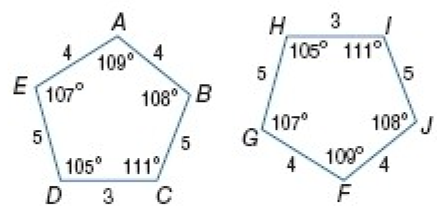
11. 

SOLUTION:

A combination of a rotation, a translation, and a reflection maps one polygon exactly onto the other. $\angle R \cong \angle J, \angle T \cong \angle K, \angle S \cong \angle L$, $\overline{RT} \cong \overline{JK}, \overline{TS} \cong \overline{KL}, \overline{RS} \cong \overline{JL}; \Delta RTS \cong \Delta JKL$; All corresponding parts of the two triangles are congruent.

ANSWER:

Sample answer: A combination of a rotation, a translation, and a reflection maps one polygon exactly onto the other. $\angle R \cong \angle J, \angle T \cong \angle K, \angle S \cong \angle L,$ $\overline{RT} \cong \overline{JK}, \overline{TS} \cong \overline{KL}, \overline{RS} \cong \overline{JL}; \Delta RTS \cong \Delta JKL$

12. 

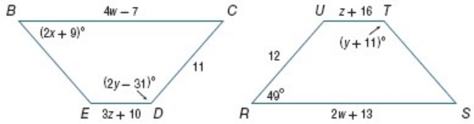
SOLUTION:

A combination of a rotation, a translation, and a reflection maps one polygon exactly onto the other.
$\angle A \cong \angle F, \angle B \cong \angle J, \angle C \cong \angle I, \angle D \cong \angle H, \angle E \cong \angle G,$
$\overline{AB} \cong \overline{FJ}, \overline{BC} \cong \overline{JI}, \overline{CD} \cong \overline{IH}, \overline{DE} \cong \overline{HG}, \overline{AE} \cong \overline{FG};$
All corresponding parts of the two polygons are congruent, so polygon $ABCDE \cong$ polygon $FJIHG$

ANSWER:

Sample answer: A combination of a rotation, a translation, and a reflection maps one polygon exactly onto the other. $\angle A \cong \angle F, \angle B \cong \angle J, \angle C \cong \angle I, \angle D \cong \angle H, \angle E \cong \angle G,$ $\overline{AB} \cong \overline{FJ}, \overline{BC} \cong \overline{JI}, \overline{CD} \cong \overline{IH}, \overline{DE} \cong \overline{HG}, \overline{AE} \cong \overline{FG};$ polygon $ABCDE \cong$ polygon $FJIHG$

Polygon *BCDE* ≅ polygon *RSTU*. Find each value.



13. *x*

SOLUTION:

By CPCTC, $\angle B \cong \angle R$.
By the definition of congruence, $m\angle B = m\angle R$.

Substitute.

$$\begin{aligned} m\angle B &= m\angle R && \text{CPCTC.} \\ 2x + 9 &= 49 && \text{Substitute.} \\ 2x + 9 - 9 &= 49 - 9 && -9 \text{ from each side.} \\ 2x &= 40 && \text{Simplify.} \\ x &= 20 && \div \text{ each side by 2.} \end{aligned}$$

ANSWER:

20

14. *y*

SOLUTION:

By CPCTC, $\angle D \cong \angle T$.
By the definition of congruence, $m\angle D = m\angle T$.

Substitute.

$$\begin{aligned} m\angle D &= m\angle T \\ 2y - 31 &= y + 11 && \text{Substitute.} \\ 2y - 31 + 31 &= y + 11 + 31 && +31 \text{ to each side.} \\ 2y &= y + 42 && \text{Simplify.} \\ 2y - y &= y + 42 - y && -y \text{ from each side.} \\ y &= 42 && \text{Simplify.} \end{aligned}$$

ANSWER:

42

15. *z*

SOLUTION:

By CPCTC, $\overline{ED} \cong \overline{UT}$.
By the definition of congruence, *ED* = *UT*.

Substitute.

$$\begin{aligned} ED &= UT \\ 3z + 10 &= z + 16 && \text{Substitute.} \\ 3z + 10 - 10 &= z + 16 - 10 && -10 \text{ from each side.} \\ 2z &= 6 && \text{Simplify.} \\ z &= 3 && \div \text{ each side by 2.} \end{aligned}$$

ANSWER:

3

16. *w*

SOLUTION:

By CPCTC, $\overline{BC} \cong \overline{RS}$.
By the definition of congruence, *BC* = *RS*.

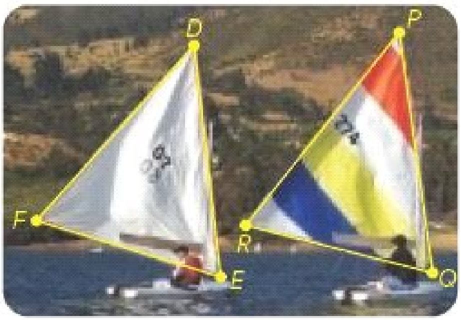
Substitute.

$$\begin{aligned} BC &= RS \\ 4w - 7 &= 2w + 13 && \text{Substitute.} \\ 4w - 7 - 2w &= 2w + 13 - 2w && -2w \text{ from each side.} \\ 2w - 7 &= 13 && \text{Simplify.} \\ 2w - 7 + 7 &= 13 + 7 && +7 \text{ to each side.} \\ 2w &= 20 && \text{Simplify.} \\ w &= 10 && \div \text{ each side by 2.} \end{aligned}$$

ANSWER:

10

17. **SAILING** To ensure that sailboat races are fair, the boats and their sails are required to be the same size and shape.



a. Write a congruence statement relating the triangles in the photo.
b. Name six pairs of congruent segments.
c. Name six pairs of congruent angles.

SOLUTION:
a. $\triangle DEF \cong \triangle PQR$

b. $\overline{DE} \cong \overline{PQ}, \overline{EF} \cong \overline{QR}, \overline{DF} \cong \overline{PR}$

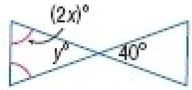
c. $\angle D \cong \angle P, \angle E \cong \angle Q, \angle F \cong \angle R$

ANSWER:
a. $\triangle DEF \cong \triangle PQR$

b. $\overline{DE} \cong \overline{PQ}, \overline{EF} \cong \overline{QR}, \overline{DF} \cong \overline{PR}$

c. $\angle D \cong \angle P, \angle E \cong \angle Q, \angle F \cong \angle R$

Find *x* and *y*.

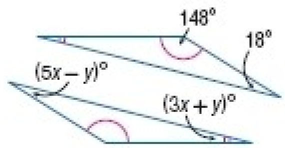
18. 

SOLUTION:
Since vertical angles are congruent, $y = 40$. The sum of the measures of the angles of a triangle is 180. So, $2x + 2x + 40 = 180$.

Solve for x.

$2x + 2x + 40 = 180$	Triangle Angle-Sum Thm.
$4x + 40 = 180$	Addition.
$4x + 40 - 40 = 180 - 40$	−40 from each side.
$4x = 140$	Simplify.
$x = 35$	÷ each side by 4.

ANSWER:
$y = 40$; $x = 35$

19. 

SOLUTION:

Let p be the measure of an unknown angle in the upper triangle. So, $p + 148 + 18 = 180$.
Solve for p.

$p + 148 + 18 = 180$ Triangle Angle-Sum Thm.
$p + 166 = 180$ Simplify.
$p + 166 - 166 = 180 - 166$ −138 from each side.
$p = 14$ Simplify.

Since the corresponding angles are congruent, the triangles are congruent.

$5x - y = 18$

$3x + y = 14$

Add the above equations.

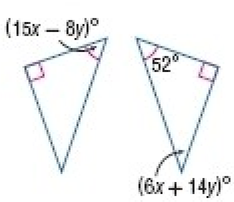
$5x - y = 18$ Equation 1
$(+)\ 3x + y = 14$ Equation 2
$8x = 32$ Addition.
$x = 4$ ÷ each side by 8.

Substitute $x = 4$ in $5x - y = 18$.

$5(4) - y = 18$ Substitute.
$20 - y = 18$ Simplify.
$y = 2$ Simplify.

ANSWER:

$x = 4; y = 2$

20. 

SOLUTION:

The given triangles are similar by AA, so $15x - 8y = 52$.

Consider the triangle at right. In that triangle, by the Triangle Angle-Sum Theorem, $52 + 6x + 14y + 90 = 180$.

Simplify.

$52 + 6x + 14y + 90 = 180$ Triangle Angle-Sum Thm.
$6x + 14y + 142 = 180$ Simplify.
$6x + 14y + 142 - 142 = 180 - 140$ −142 from each side.
$6x + 14y = 38$ Simplify.

Solve the equation $15x - 8y = 52$ for y.

$52 = 15x - 8y$ CPCTC.
$8y + 52 = 15x$ +8y to each side.
$8y = 15x - 52$ −52 from each side.
$y = \frac{15x - 52}{8}$ ÷ each side by 8.

To solve for x, substitute $y = \frac{15x - 52}{8}$ in $6x + 14y = 38$.

$6x + 14\left(\frac{15x - 52}{8}\right) = 38$ Substitute.
$6x + 7\left(\frac{15x - 52}{4}\right) = 38$ Simplify.
$6x + \frac{105x - 364}{4} = 38$ Distributive Property
$6x + \frac{105x}{4} - 91 = 38$ Simplify.
$6x + \frac{105x}{4} = 129$ +91 to each side.
$\frac{129x}{4} = 129$ Combineliketerms.
$129x = (129)(4)$ Multiply each side by 4.
$x = 4$ Simplify.

To solve for y, substitute $x = 4$ in $15x - 8y = 52$.

$15(4) - 8y = 52$ Substitute.
$60 - 8y = 52$ Simplify.
$8y = 8$ Simplify.
$y = 1$ Simplify.

ANSWER:

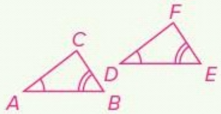
$x = 4; y = 1$

21. **PROOF** Write a two-column proof of Theorem 4.3.

SOLUTION:

Given: $\angle A \cong \angle D$
$\angle B \cong \angle E$

Prove: $\angle C \cong \angle F$



Proof:

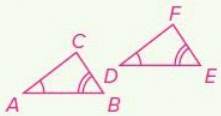
Statements (Reasons)

1. $\angle A \cong \angle D$, $\angle B \cong \angle E$ (Given)
2. $m\angle A = m\angle D$, $m\angle B = m\angle E$ (Def. of $\cong$ ∠s)
3. $m\angle A + m\angle B + m\angle C = 180$, $m\angle D + m\angle E + m\angle F = 180$ (∠ Sum Theorem)
4. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$ (Trans. Prop.)
5. $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$ (Subst.)
6. $m\angle C = m\angle F$ (Subt. Prop.)
7. $\angle C \cong \angle F$ (Def. of $\cong$ ∠s)

ANSWER:

Given: $\angle A \cong \angle D$
$\angle B \cong \angle E$

Prove: $\angle C \cong \angle F$



Proof:

Statements (Reasons)

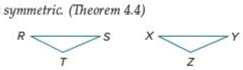
1. $\angle A \cong \angle D$, $\angle B \cong \angle E$ (Given)
2. $m\angle A = m\angle D$, $m\angle B = m\angle E$ (Def. of $\cong$ ∠s)
3. $m\angle A + m\angle B + m\angle C = 180$, $m\angle D + m\angle E + m\angle F = 180$ (∠ Sum Theorem)
4. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$ (Trans. Prop.)
5. $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$ (Subst.)
6. $m\angle C = m\angle F$ (Subt. Prop.)
7. $\angle C \cong \angle F$ (Def. of $\cong$ ∠s)

22. **PROOF** Put the statements used to prove the theorem below in the correct order. Provide the reasons for each statement.

Congruence of triangles is symmetric. (Theorem 4.4)

Given: $\triangle RST \cong \triangle XYZ$

Prove: $\triangle XYZ \cong \triangle RST$



Proof:

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$	$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$	$\triangle RST \cong \triangle XYZ$	$\triangle XYZ \cong \triangle RST$
?	?	?	?

SOLUTION:

Proof:

$\triangle RST \cong \triangle XYZ$
Given

↓

$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$
CPCTC

↓

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$
Congruence of ∠ and segments is symmetric.

↓

$\triangle XYZ \cong \triangle RST$
Def. of $\cong$ △

ANSWER:

Proof:

$\triangle RST \cong \triangle XYZ$
Given

↓

$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$
CPCTC

↓

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$
Congruence of ∠ and segments is symmetric.

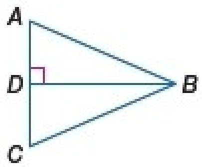
↓

$\triangle XYZ \cong \triangle RST$
Def. of $\cong$ △

CONSTRUCT ARGUMENTS Write a two-column proof.

23. **Given:** $\overline{BD}$ bisects $\angle B$.
$\overline{BD} \perp \overline{AC}$

Prove: $\angle A \cong \angle C$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{BD}$ bisects $\angle B$, $\overline{BD} \perp \overline{AC}$. (Given)
2. $\angle ABD \cong \angle DBC$ (Def. of angle bisector)
3. $\angle ADB$ and $\angle BDC$ are right angles. ($\perp$ lines form rt. $\angle$s.)
4. $\angle ADB \cong \angle BDC$ (All rt. $\angle$s are $\cong$.)
5. $\angle A \cong \angle C$ (Third $\angle$s Thm.)

ANSWER:

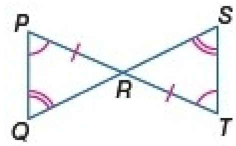
Proof:

Statements (Reasons)

1. $\overline{BD}$ bisects $\angle B$, $\overline{BD} \perp \overline{AC}$. (Given)
2. $\angle ABD \cong \angle DBC$ (Def. of angle bisector)
3. $\angle ADB$ and $\angle BDC$ are right angles. ($\perp$ lines form rt. $\angle$s.)
4. $\angle ADB \cong \angle BDC$ (All rt. $\angle$s are $\cong$.)
5. $\angle A \cong \angle C$ (Third $\angle$s Thm.)

24. **Given:** $\angle P \cong \angle T$, $\angle S \cong \angle Q$
$\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$,
$\overline{RT} \cong \overline{RS}$
$\overline{PQ} \cong \overline{TS}$

Prove: $\Delta PRQ \cong \Delta TRS$



SOLUTION:

Proof:

Statements (Reasons)

1. $\angle P \cong \angle T$, $\angle S \cong \angle Q$, $\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$, $\overline{RT} \cong \overline{RS}$, $\overline{PQ} \cong \overline{TS}$ (Given)
2. $\overline{PR} \cong \overline{QR}$, $\overline{TR} \cong \overline{SR}$ (Symm. Prop.)
3. $\overline{TR} \cong \overline{QR}$ (Trans. Prop)
4. $\overline{QR} \cong \overline{TR}$ (Symm. Prop.)
5. $\overline{QR} \cong \overline{SR}$ (Trans. Prop.)
6. $\angle PRQ \cong \angle TRS$ (Vert. $\angle$s are $\cong$.)
7. $\triangle PRQ \cong \triangle TRS$ (Def. of $\cong$ $\triangle$s)

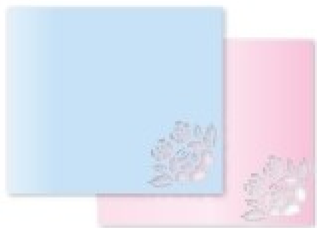
ANSWER:

Proof:

Statements (Reasons)

1. $\angle P \cong \angle T$, $\angle S \cong \angle Q$, $\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$, $\overline{RT} \cong \overline{RS}$, $\overline{PQ} \cong \overline{TS}$ (Given)
2. $\overline{PR} \cong \overline{QR}$, $\overline{TR} \cong \overline{SR}$ (Symm. Prop.)
3. $\overline{TR} \cong \overline{QR}$ (Trans. Prop)
4. $\overline{QR} \cong \overline{TR}$ (Symm. Prop.)
5. $\overline{QR} \cong \overline{SR}$ (Trans. Prop.)
6. $\angle PRQ \cong \angle TRS$ (Vert. $\angle$s are $\cong$.)
7. $\triangle PRQ \cong \triangle TRS$ (Def. of $\cong$ $\triangle$s)

25. **SCRAPBOOKING** Lanie is using a flower-shaped corner decoration punch for a scrapbook she is working on. If she punches the corners of two pages as shown, what property guarantees that the punched designs are congruent? Explain.



SOLUTION:

Both of the punched flowers are congruent to the flower on the stamp, because it was used to create the images. According to the Transitive Property of Polygon Congruence, the two stamped images are congruent to each other because they are both congruent to the flowers on the punch.

ANSWER:

Sample answer: Both of the punched flowers are congruent to the flower on the stamp, because it was used to create the images. According to the Transitive Property of Polygon Congruence, the two stamped images are congruent to each other because they are both congruent to the flowers on the punch.

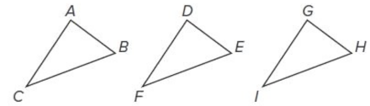
PROOF Write the specified type of proof of the indicated part of Theorem 4.4.

26. Congruence of triangles is transitive. (paragraph proof)

SOLUTION:

Given: $\triangle ABC \cong \triangle DEF$, $\triangle DEF \cong \triangle GHI$

Prove: $\triangle ABC \cong \triangle GHI$



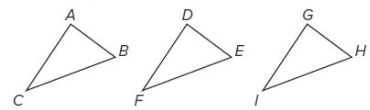
Proof:

We know that $\triangle ABC \cong \triangle DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$. We also know that $\triangle DEF \cong \triangle GHI$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, $\overline{DF} \cong \overline{GI}$, by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\triangle ABC \cong \triangle GHI$ by the definition of congruent triangles.

ANSWER:

Given: $\triangle ABC \cong \triangle DEF$, $\triangle DEF \cong \triangle GHI$

Prove: $\triangle ABC \cong \triangle GHI$

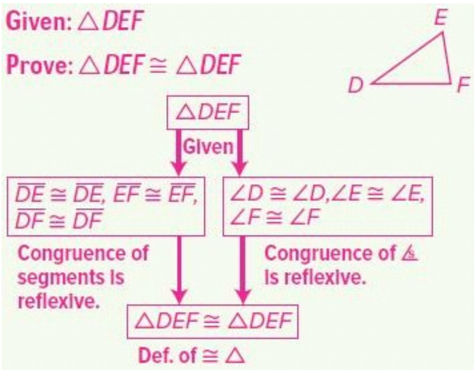


Proof:

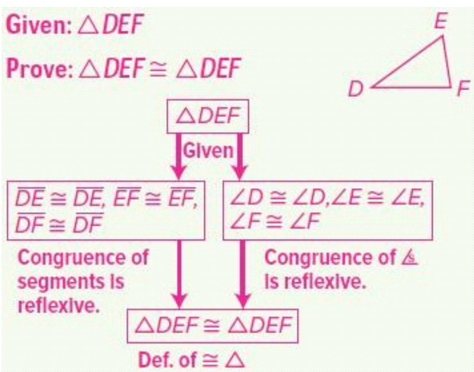
We know that $\triangle ABC \cong \triangle DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$. We also know that $\triangle DEF \cong \triangle GHI$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, $\overline{DF} \cong \overline{GI}$, by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\triangle ABC \cong \triangle GHI$ by the definition of congruent triangles.

27. Congruence of triangles is reflexive. (flow proof)

SOLUTION:



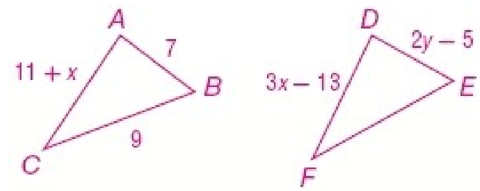
ANSWER:



ALGEBRA Draw and label a figure to represent the congruent triangles. Then find *x* and *y*.

28. $\Delta ABC \cong \Delta DEF$, $AB = 7$, $BC = 9$, $AC = 11 + x$, $DF = 3x - 13$, and $DE = 2y - 5$

SOLUTION:



Since the triangles are congruent, the corresponding sides are congruent.

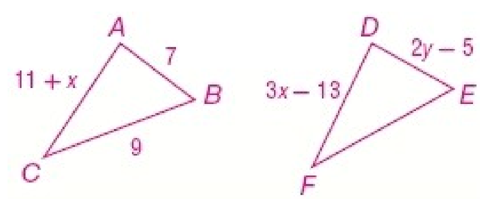
$AC = DF$	CPCTC.
$11 + x = 3x - 13$	Substitution.
$11 + x - 3x = 3x - 3x - 13$	$-3x$ from each side.
$11 - 2x = -13$	Simplfy.
$11 - 11 - 2x = -13 - 11$	-11 from each side.
$-2x = -24$	Simplify.
$x = 12$	$\div$ each side by -2.

Similarly, $AB = DE$.

$AB = DE$	CPCTC.
$7 = 2y - 5$	Substitution.
$7 + 5 = 2y - 5 + 5$	$+5$ to each side.
$12 = 2y$	Simplfy.
$6 = y$	$\div$ each side by 2.

That is, $y = 6$.

ANSWER:

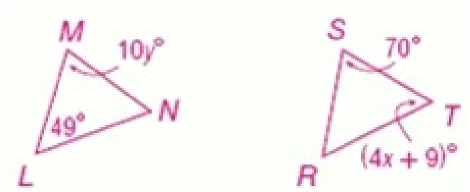


$x = 12$; $y = 6$

4-2 Congruent Triangles

29. $\Delta LMN \cong \Delta RST$, $m\angle L = 49$, $m\angle M = 10y$, $m\angle S = 70$, and $m\angle T = 4x + 9$

SOLUTION:



Since the triangles are congruent, the corresponding angles are congruent.
$\angle M \cong \angle S$ and $\angle N \cong \angle T$.
By the definition of congruence $m\angle M = m\angle S$.

Substitute.

$$\begin{aligned} m\angle M &= m\angle S && \text{Def. of congruence.} \\ 10y &= 70 && \text{Substitute.} \\ y &= 7 && \div \text{ each side by 10.} \end{aligned}$$

So, $m\angle M \cong 70$.

Use the Triangle Angle Sum Theorem in $\triangle LMN$.

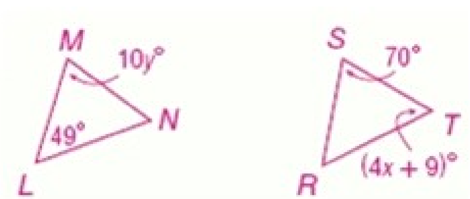
$$\begin{aligned} m\angle L + m\angle M + m\angle N &= 180 && \text{Triangle Angle-Sum Thm.} \\ 49 + 70 + m\angle N &= 180 && \text{Substitute.} \\ 119 + m\angle N &= 180 && \text{Simplify.} \\ m\angle N &= 61 && -119 \text{ from each side.} \end{aligned}$$

By the definition of congruence $m\angle N \cong m\angle T$.
Substitute.

$$\begin{aligned} m\angle N &= m\angle N && \text{Def. of congruence.} \\ 61 &= 4x + 9 && \text{Substitute.} \\ 61 - 9 &= 4x + 9 - 9 && -9 \text{ from each side.} \\ 52 &= 4x && \text{Simplify.} \\ 13 &= x && \div \text{ each side by 4.} \end{aligned}$$

That is, $x = 13$.

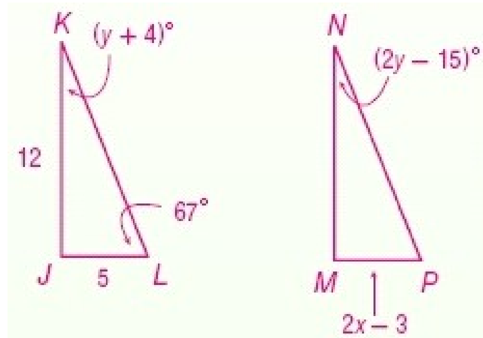
ANSWER:



$x = 13; y = 7$

30. $\Delta JKL \cong \Delta MNP$, $JK = 12$, $LJ = 5$, $PM = 2x - 3$, $m\angle L = 67$, $m\angle K = y + 4$ and $m\angle N = 2y - 15$

SOLUTION:

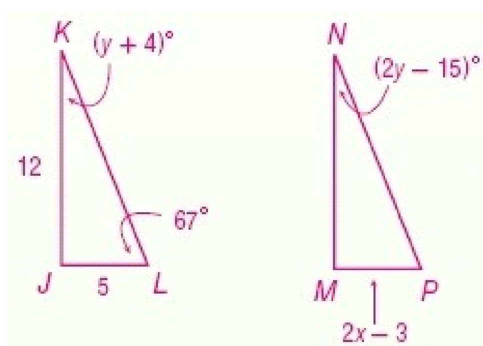


Since the triangles are congruent, the corresponding angles and corresponding sides are congruent.

$$\begin{aligned} \angle K &\cong \angle N && \text{CPCTC} \\ m\angle K &= m\angle N && \text{Def.of congruence} \\ y + 4 &= 2y - 15 && \text{Substitute.} \\ y + 4 - 2y &= 2y - 15 - 2y && -2y \text{ from each side.} \\ -y + 4 &= -15 && \text{Simplify.} \\ -y &= -19 && -4 \text{ from each side.} \\ y &= 19 && \times \text{ each side by } -1. \end{aligned}$$

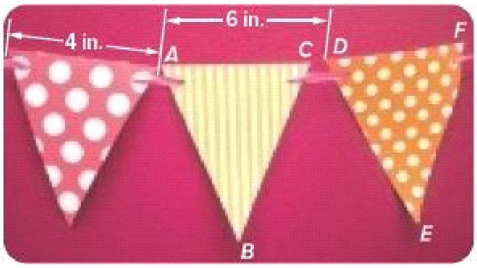
$$\begin{aligned} \overline{MP} &\cong \overline{JL} && \text{CPCTC} \\ MP &= JL && \text{Def.of congruence} \\ 2x - 3 &= 5 && \text{Substitute.} \\ 2x - 3 + 3 &= 5 + 3 && +3 \text{ to each side.} \\ 2x &= 8 && \text{Simplify.} \\ x &= 4 && \div \text{ each side by 2.} \end{aligned}$$

ANSWER:



$x = 4; y = 19$

31. **PENNANTS** Marren is decorating an area of 100 the pep rally. She is using a string of pennants that a isosceles triangles.



a. List seven pairs of congruent segments in the phot
b. If the area decorated is a square, how long will the need to be?
c. How many pennants will be on the string?

SOLUTION:

a. $\overline{AB} \cong \overline{CB}, \overline{AB} \cong \overline{DE}, \overline{AB} \cong \overline{FE}, \overline{CB} \cong \overline{DE}, \overline{CB} \cong \overline{FE}, \overline{DE} \cong \overline{FE}, \overline{AC} \cong \overline{DF}$

b. Substitute the area value in the formula for area of solve for its side.

$A = s^2$ Area formula

$100 = s^2$ Area is 100

$10 = s$ take the square root

The length of each side is 10 ft. Since the perimeter the perimeter is 4(10) or 40. So, 40 ft of pennant strin

c. Each pennant is 4 inches wide and they are place So, there are 2 pennants for each foot of rope. So, 4 pennants per foot means that 80 pennants will fit on t

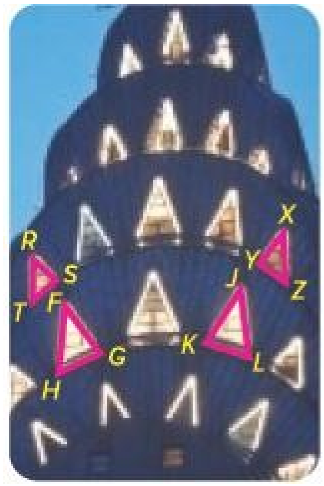
ANSWER:

a. $\overline{AB} \cong \overline{CB}, \overline{AB} \cong \overline{DE}, \overline{AB} \cong \overline{FE}, \overline{CB} \cong \overline{DE}, \overline{CB} \cong \overline{FE}, \overline{DE} \cong \overline{FE}, \overline{AC} \cong \overline{DF}$

b. 40 ft

c. 80

32. **SENSE-MAKING** In the photo of New York City's Chrysler Building, $\overline{TS} \cong \overline{ZY}$, $\overline{XY} \cong \overline{RS}$, $\overline{TR} \cong \overline{ZX}$, $\angle X \cong \angle R$, $\angle T \cong \angle Z$, $\angle Y \cong \angle S$, and $\Delta HGF \cong \Delta LKJ$.



a. Which triangle, if any, is congruent to ΔYXZ? Explain your reasoning.

b. Which side(s) are congruent to $\overline{JL}$? Explain your reasoning.

c. Which angle(s) are congruent to $\angle G$? Explain your reasoning.

SOLUTION:

a. ΔSRT is congruent to ΔYXZ. The corresponding parts of the triangles are congruent, therefore the triangles are congruent.

b. $\overline{FH}$ is congruent to $\overline{JL}$. We are given that $\Delta HGF \cong \Delta LKJ$, and $\overline{JL}$ corresponds with $\overline{FH}$. Since corresponding parts of congruent triangles are congruent, $\overline{JL} \cong \overline{FH}$.

c. $\angle K$ is congruent to $\angle G$. We are given that $\Delta HGF \cong \Delta LKJ$, and $\angle G$ corresponds with $\angle K$. Since corresponding parts of congruent triangles are congruent, $\angle G \cong \angle K$.

ANSWER:

a. ΔSRT; The corresponding parts of the triangles are congruent, therefore the triangles are congruent.

b. $\overline{FH}$; We are given that $\Delta HGF \cong \Delta LKJ$, and $\overline{JL}$ corresponds with $\overline{FH}$. Since corresponding parts of congruent triangles are congruent, $\overline{JL} \cong \overline{FH}$.

c. $\angle K$; We are given that $\Delta HGF \cong \Delta LKJ$, and $\angle G$ corresponds with $\angle K$. Since corresponding parts of congruent triangles are congruent, $\angle G \cong \angle K$.

33. **MULTIPLE REPRESENTATIONS** In this problem, you will explore criteria for triangle

congruence. Select a tool such as compass and straightedge or dynamic geometry software.

a. Geometric Draw two triangles that meet each of the following criteria.

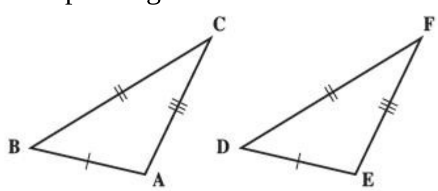
- the sides of the first triangle are congruent to the corresponding sides of the second triangle
- two of the sides and the included angle are congruent
- two angles and the included side are congruent
- two angles and a nonincluded side are congruent
- two sides and a nonincluded angle are congruent
- the angles of one are congruent to the corresponding angles of the second triangle

b. Verbal Analyze the triangles drawn in part **a** by drawing additional triangles or transforming those you drew. Which criteria resulted in congruent triangles? Which criteria did not result in congruent triangles?

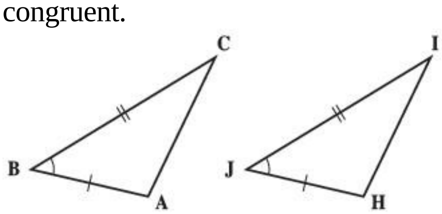
c. Verbal Make a conjecture about the criteria to determine whether two triangles are congruent.

SOLUTION:

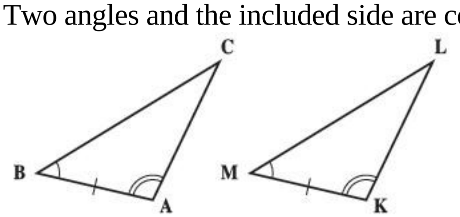
a. The sides of the first triangle are congruent to the corresponding sides of the second triangle.



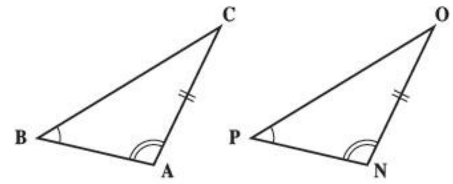
Two of the sides and the included angle are congruent.



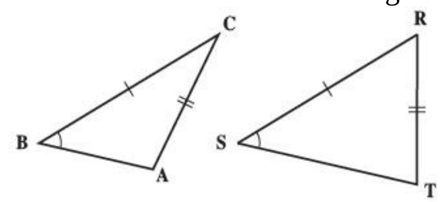
Two angles and the included side are congruent.



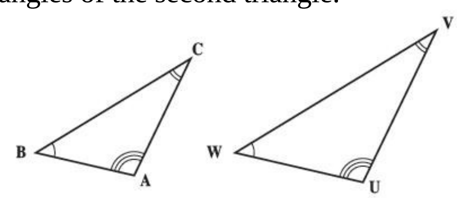
Two angles and a non-included side are congruent.



Two sides and a non-included angle are congruent.



The angles of one are congruent to the corresponding angles of the second triangle.

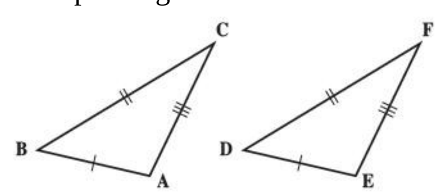


b. Congruent triangles resulted from these criteria: corresponding sides of each triangle are congruent, two sides and included angle are congruent, and two angles and nonincluded side are congruent. These criteria did not result in congruent triangles: two sides and a nonincluded angle are congruent and corresponding angles are congruent.

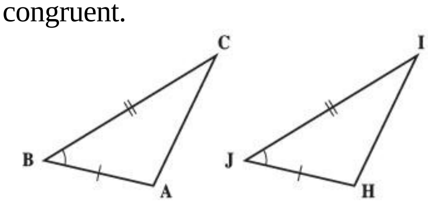
c. Two triangles are congruent if all three sides are congruent, two sides and an including angle, two angles and the including side, and two angles and a nonincluded side.

ANSWER:

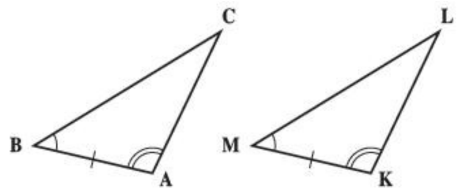
a. The sides of the first triangle are congruent to the corresponding sides of the second triangle.



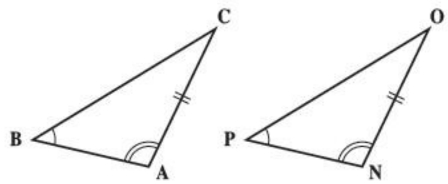
Two of the sides and the included angle are congruent.



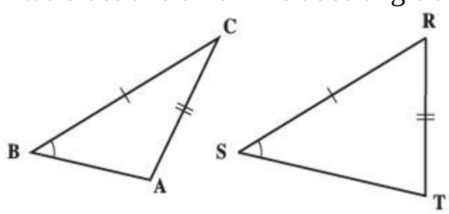
Two angles and the included side are congruent.



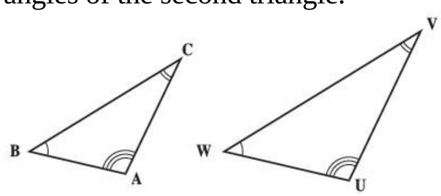
Two angles and a non-included side are congruent.



Two sides and a non-included angle are congruent.



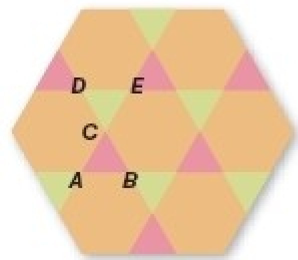
The angles of one are congruent to the corresponding angles of the second triangle.



b. Congruent triangles resulted from these criteria: corresponding sides of each triangle are congruent, two sides and included angle are congruent, and two angles and nonincluded side are congruent. These criteria did not result in congruent triangles: two sides and a nonincluded angle are congruent and corresponding angles are congruent.

c. Two triangles are congruent if all three sides are congruent, two sides and an including angle, two angles and the including side, and two angles and a nonincluded side.

34. **PATTERNS** The pattern shown is created using regular polygons.



a. What two polygons are used to create the pattern?

b. Name a pair of congruent triangles.

c. Name a pair of corresponding angles.

d. If *CB* = 2 inches, what is *AE*? Explain.

e. What is the measure of $\angle D$? Explain.

SOLUTION:

a. Hexagons and triangles are used to create the pattern.

b. $\Delta ABC \cong \Delta DEC$

c. $\angle B$ and $\angle E$ are corresponding angles.

d. *AE* is 4 in.; Because the polygons that make the pattern are regular, all of the sides of the triangles must be equal, so the triangles are equilateral. That means that *CB* is equal to *AC* and *CE*, so *AE* is 2 (*CB*), or 4 inches.

e. $m\angle D$ is 60°. Because the triangles are regular, they must be equilateral, and all of the angles of an equilateral triangle are 60°.

ANSWER:

a. hexagons and triangles

b. Sample answer: $\Delta ABC \cong \Delta DEC$

c. Sample answer: $\angle B$ and $\angle E$

d. 4 in.; Sample answer: Because the polygons that make the pattern are regular, all of the sides of the triangles must be equal, so the triangles are equilateral. That means that *CB* is equal to *AC* and *CE*, so *AE* is 2(*CB*), or 4 inches.

e. 60°; Sample answer: Because the triangles are regular, they must be equilateral, and all of the angles of an equilateral triangle are 60°.

35. **FITNESS** A fitness instructor is starting a new aerobics class using fitness hoops. She wants to confirm that all of the hoops are the same size. What measure(s) can she use to prove that all of the hoops are congruent? Explain your reasoning.

SOLUTION:

To prove that all of the hoops are congruent, use the diameter, radius, or circumference; Two circles are the same size if they have the same diameter, radius, or circumference, so she can determine if the hoops are congruent if she measures any of them.

ANSWER:

diameter, radius, or circumference; Sample answer: Two circles are the same size if they have the same diameter, radius, or circumference, so she can determine if the hoops are congruent if she measures any of them.

36. **WRITING IN MATH** Explain why the order of the vertices is important when naming congruent triangles. Give an example to support your answer.

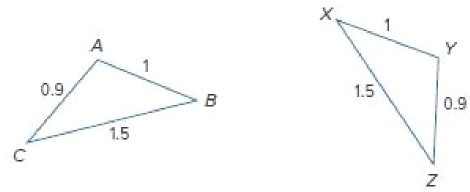
SOLUTION:

When naming congruent triangles, it is important that the corresponding vertices be in the same location for both triangles because the location indicates congruence. For example if ΔABC is congruent to ΔDEF, then $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

ANSWER:

Sample answer: When naming congruent triangles, it is important that the corresponding vertices be in the same location for both triangles because the location indicates congruence. For example if ΔABC is congruent to ΔDEF, then $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

37. **ERROR ANALYSIS** Jasmine and Will are evaluating the congruent figures below. Jasmine says that $\Delta CAB \cong \Delta ZYX$ and Will says that $\Delta ABC \cong \Delta YXZ$. Is either of them correct? Explain.



SOLUTION:

Both Jasmine and Will are correct. $\angle A$ corresponds with $\angle Y$, $\angle B$ corresponds with $\angle X$, and $\angle C$ corresponds with $\angle Z$. ΔCAB is the same triangle as ΔABC and ΔZXY is the same triangle as ΔXYZ.

ANSWER:

Both; Sample answer: $\angle A$ corresponds with $\angle Y$, $\angle B$ corresponds with $\angle X$, and $\angle C$ corresponds with $\angle Z$. ΔCAB is the same triangle as ΔABC and ΔZXY is the same triangle as ΔXYZ.

38. **WRITE A QUESTION** A classmate is using the Third Angles Theorem to show that if two corresponding pairs of the angles of two triangles are congruent, then the third pair is also congruent. Write a question to help him decide if he can use the same strategy for quadrilaterals.

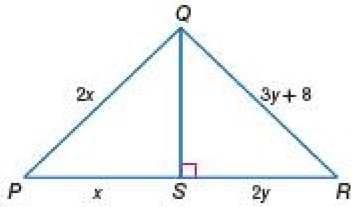
SOLUTION:

Do you think that the sum of the angles of a quadrilateral is constant? If so, do you think that the final pair of corresponding angles will be congruent if three other pairs of corresponding angles are congruent for a pair of quadrilaterals?

ANSWER:

Sample answer: Do you think that the sum of the angles of a quadrilateral is constant? If so, do you think that the final pair of corresponding angles will be congruent if three other pairs of corresponding angles are congruent for a pair of quadrilaterals?

39. **PERSEVERENCE** Find x and y if $\Delta PQS \cong \Delta RQS$.



SOLUTION:

If two triangles are congruent, then their corresponding sides are congruent.

$PQ = QR$
$PS = SR$

Substitute.

$PG = QR$ CPCTC
$2x = 3y + 8$ Substitute.
$PS = SR$ CPCTC
$x = 2y$ Substitute.

Substitute $x = 2y$ in $2x = 3y + 8$.

$2x = 3y + 8$ CPCTC
$2(2y) = 3y + 8$ Substitute.
$4y = 3y + 8$ Multiply.
$4y - 3y = 3y + 8 - 3y$ $-3y$from each side.
$y = 8$ Simplify.

Substitute $y = 8$ in $x = 2y$.

$x = 2y$
$x = 2(8)$
$x = 16$

ANSWER:

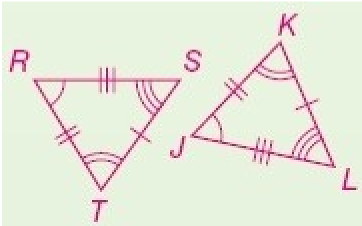
$x = 16, y = 8$

CRITIQUE ARGUMENTS Determine whether each statement is ***true*** or ***false***. If false, give a counterexample. If true, explain your reasoning.

40. Two triangles with two pairs of congruent corresponding angles and three pairs of congruent corresponding sides are congruent.

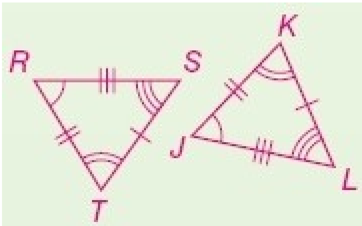
SOLUTION:

The statement is true. Using the Third Angles Theorem, the third pair of angles is also congruent and all corresponding sides are congruent, so since CPCTC, the triangles are congruent.



ANSWER:

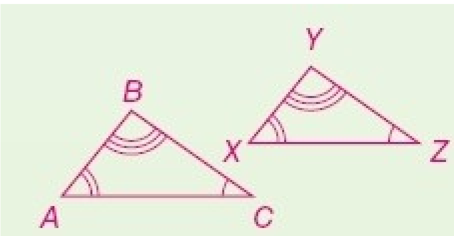
True; Sample answer: Using the Third Angles Theorem, the third pair of angles is also congruent and all corresponding sides are congruent, so since CPCTC, the triangles are congruent.



41. Two triangles with three pairs of corresponding congruent angles are congruent.

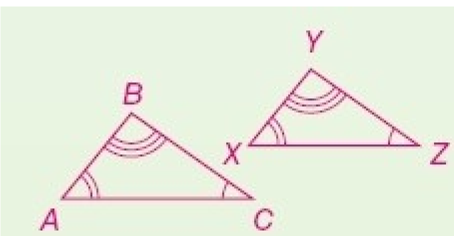
SOLUTION:

False; a pair of triangles can have corresponding angles congruent with the sides of one triangle longer than the sides of the other triangle; for example; $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, but corresponding sides are not congruent.

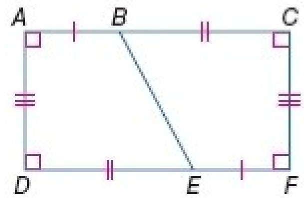


ANSWER:

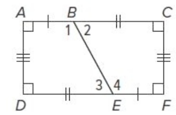
False; $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, but corresponding sides are not congruent.



42. **SENSE-MAKING** Write a paragraph proof to prove polygon *ABED* $\cong$ polygon *FEBC*.

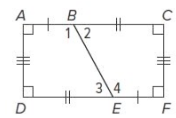


SOLUTION:



We know that $\overline{AB} \cong \overline{FE}$, $\overline{ED} \cong \overline{BC}$, and $\overline{AD} \cong \overline{FC}$. By the reflexive property, $\overline{BE} \cong \overline{EB}$. $\angle A \cong \angle F$ and $\angle D \cong \angle C$ since all right angles are congruent. Because $\overline{AC}$ and $\overline{DF}$ are both perpendicular to $\overline{CF}$, $\overline{AC} \parallel \overline{DF}$ (Theorem 3.8). $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ because alternate interior angles are congruent to each other. Because all corresponding parts are congruent, polygon *ABED* $\cong$ polygon *FEBC*.

ANSWER:



We know that $\overline{AB} \cong \overline{FE}$, $\overline{ED} \cong \overline{BC}$, and $\overline{AD} \cong \overline{FC}$. By the reflexive property, $\overline{BE} \cong \overline{EB}$. $\angle A \cong \angle F$ and $\angle D \cong \angle C$ since all right angles are congruent. Because $\overline{AC}$ and $\overline{DF}$ are both perpendicular to $\overline{CF}$, $\overline{AC} \parallel \overline{DF}$ (Theorem 3.8). $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ because alternate interior angles are congruent to each other. Because all corresponding parts are congruent, polygon *ABED* $\cong$ polygon *FEBC*.

43. **WRITING IN MATH** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

Equilateral triangles are congruent.

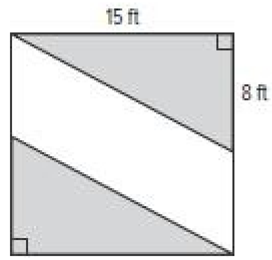
SOLUTION:

The statement is sometimes true. While equilateral triangles are equiangular, the corresponding sides may not be congruent. Equilateral triangles will be congruent if one pair of corresponding sides are congruent.

ANSWER:

Sometimes; Equilateral triangles will be congruent if one pair of corresponding sides are congruent.

44. A cement path is placed as shown in a square region of a park. If the triangular grassy areas along both sides of the path are congruent, what is the perimeter of the path?



A 40 ft
B 48 ft
C 60 ft
D 105 ft

SOLUTION:

Since the triangles are right triangles, $a^2 + b^2 = c^2$. So, $8^2 + 15^2 = c^2$.

$c^2 = 289$
$c = 17$

So, the long sides of the path are 17 ft long.

To find the short sides of the path, subtract 15 – 8, or 7.

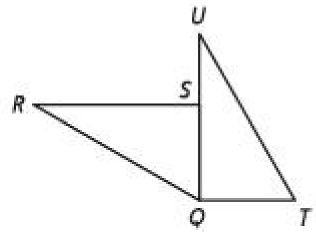
Now, to find the perimeter of the path, add the side lengths.

17 + 7 + 17 + 7 = 48.

The perimeter of the path is 48 ft. So, the correct answer is choice B.

ANSWER:
B

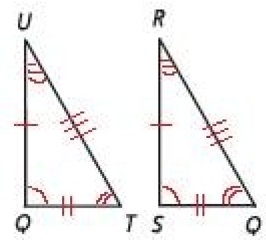
45. $\triangle QRS \cong \triangle TUQ$



Which statement is not necessarily true?

A $\overline{RS} \cong \overline{UQ}$
B $\overline{SQ} \cong \overline{QT}$
C $\angle T \cong \angle R$
D $\angle RSQ \cong \angle UQT$
E $\angle RQS \cong \angle UTQ$

SOLUTION:



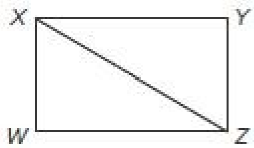
Consider each statement

A $\overline{RS} \cong \overline{UQ}$ is true.
B $\overline{SQ} \cong \overline{QT}$ is true.
C $\angle T \cong \angle R$ cannot be assumed with the given information.
D $\angle RSQ \cong \angle UQT$ is true.
E $\angle RQS \cong \angle UTQ$ is true.

So, the correct answer is choice C.

ANSWER:
C

46. The opposite sides of quadrilateral *WXYZ* are congruent. Prove that $\triangle WXZ \cong \triangle YZX$.



SOLUTION:
Because the opposite sides of quadrilateral *WXYZ* are congruent, $\overline{WX} \cong \overline{YZ}$ and $\overline{WZ} \cong \overline{YX}$. $\overline{XZ} \cong \overline{ZX}$ by the reflexive property of congruence. Therefore $\triangle WXZ \cong \triangle YZX$ by SSS.

ANSWER:
Because the opposite sides of quadrilateral *WXYZ* are congruent, $\overline{WX} \cong \overline{YZ}$ and $\overline{WZ} \cong \overline{YX}$. $\overline{XZ} \cong \overline{ZX}$ by the reflexive property of congruence. Therefore $\triangle WXZ \cong \triangle YZX$ by SSS.

47. Use the following information to find the values of x and y. $\triangle TUV \cong \triangle HJK$, $TU = 14$, $UV = 18$, $TV = 4y + 1$, $JK = 2x - 4$, and $HK = 6y - 5$.

SOLUTION:
Because the triangles are congruent, we can set corresponding sides equal to one another to find x and y.

To find x, set $JK = UV$.

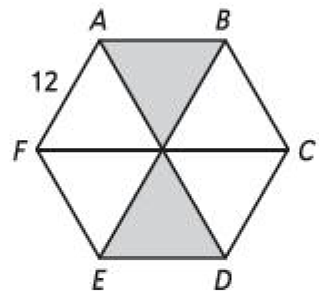
$$\begin{aligned} 2x - 4 &= 18 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

To find y, set $HK = TV$.

$$\begin{aligned} 6y - 5 &= 4y + 1 \\ 6y &= 4y + 6 \\ 2y &= 6 \\ y &= 3 \end{aligned}$$

ANSWER:
$x = 11, y = 3$

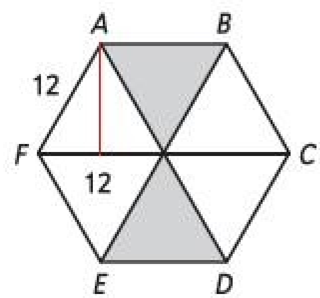
48. A regular hexagon is divided into six congruent triangles.



Which of the following best approximates the area of the shaded region?

A 62.35 square units
B 124.71 square units
C 249.41 square units
D 374.12 square units

SOLUTION:



A regular hexagon will have all sides equal and all triangles are equilateral. Find the area of one triangle and multiply by 2.

First, find the height of the triangle.

$$12^2 = 6^2 + b^2$$

The height is $\sqrt{108}$ or $6\sqrt{3}$.

Then, the area of one triangle is

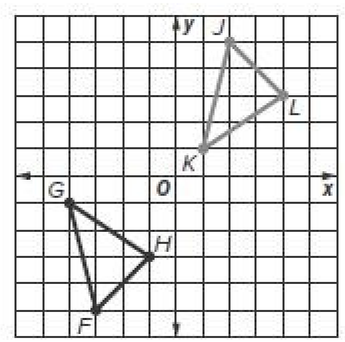
$$\begin{aligned} A &= \frac{1}{2}bh \\ A &= \frac{1}{2}(12)\left(6\sqrt{3}\right) \\ A &\approx 62.35 \end{aligned}$$

The area of the two shaded triangles would be $62.35 \cdot 2 = 124.7$.

So, the correct answer is choice B.

ANSWER:
B

49. **MULTI-STEP** The graph shows $\triangle FGH$ and $\triangle JKL$.



a. Describe a set of rigid motions that could be performed on $\triangle FGH$ to prove that it is congruent to $\triangle JKL$.

b. Perform the set of rigid motions, and list the coordinates of the vertices of the image of $\triangle FGH$ at each stage.

c. Given that $m\angle F = 59°$ and $m\angle G = 42°$, what is the measure of $\angle L$? Explain your reasoning.

SOLUTION:
a. A reflection in the x-axis and a translation 5 units right could be performed to prove the triangles are congruent.

b. after reflection: $F'(-3, 5)$, $G'(-4, 1)$, $H'(-1, 3)$; after translation: $F''(2, 5)$, $G''(1, 1)$, $H''(4, 3)$

c. 79°; $m\angle H = 180° - 59° - 42° = 79°$; because $\angle H \cong \angle L$, $m\angle L = 79°$.

ANSWER:
a. a reflection in the x-axis and a translation 5 units right

b. after reflection: $F'(-3, 5)$, $G'(-4, 1)$, $H'(-1, 3)$; after translation: $F''(2, 5)$, $G''(1, 1)$, $H''(4, 3)$

c. 79°; $m\angle H = 180° - 59° - 42° = 79°$; because $\angle H \cong \angle L$, $m\angle L = 79°$.

50. $\triangle ABC \cong \triangle DEF$. $AB = 4x - 2$, $BC = 10$, $DE = 18$, $EF = 3y + 1$, and $DF = 2x + 4$.

a. Find the values of x and y.

b. Find DF.

SOLUTION:
a. Because the triangles are congruent that means congruent sides are equal.

To find x, set $AB = DE$.

$$\begin{aligned} 4x - 2 &= 18 \\ 4x &= 20 \\ x &= 5 \end{aligned}$$

To find y, set $EF = BC$.

$$\begin{aligned} 3y + 1 &= 10 \\ 3y &= 9 \\ y &= 3 \end{aligned}$$

b. Substitute 5 for x to find DF.

$$\begin{aligned} DF &= 2(5) + 4 \\ DF &= 10 + 4 \\ DF &= 14 \end{aligned}$$

ANSWER:
a. $x = 5, y = 3$
b. 14