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Signal Field-Strength Measurements: Basics

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Note: These are preliminary notes, intended only for distribution among the participants. Beware of misprints!

Purpose

- to refresh basic concepts related to measurements of physical quantities
  - Radio-wave field-strength
  - Antennas
  - Ect…

Topics for discussion

- Why measurements?
- What is error, uncertainty, accuracy?
- What factors do influence uncertainty?
- How to evaluate errors?
- What is least-square fitting?

- Measurement is essential in scientific research (except mathematics) and in engineering
- Usually, scientific/ engineering projects (calculations, models, reports,) must be supported by experimental evidence get through measurements to make them credible/ reliable
- Experiments/ measurements must be fully documented to make their reproduction possible
    - » Measurement protocols, photographs, etc

Measurements: legal aspects

Spectrum management applications (legal)

- Checking compliance with the regulations, licenses, and standards
- Radio Monitoring
- Checking channel occupancy
- Solving interference problems

- Absolute values required as evidence

Measurements: engineering

- Wanted signal
  - Will my system operate correctly?
    - Producing “local” propagation models for improved predictions (power budget)
    - Where should my antennas be located? On what height? (Optimizing station parameters)
    - Survey/ monitoring of local signal-environment – selection the best channel
  - Does my system operate correctly?
    - Checking the antenna radiation pattern and/ or the station coverage area
    - Required signal intensity/ quality of service/ distance/ area/ volume?, given the geographic region and time period
- Unwanted signals
  - Could my system coexist with other systems? Will my system suffer unacceptable interference? Will it produce such interference to other systems? Degradation of service quality and/ or service range/ area due to potential radio interference?

- **Relative values are often sufficient**

- Legal measurements & Important projects
  - Measurement results must be accompanied by a formal statement of uncertainty (compliance tests)
  - Discrepancies should be clarified
- Comparative measurements
- Qualitative indicators

Indoor

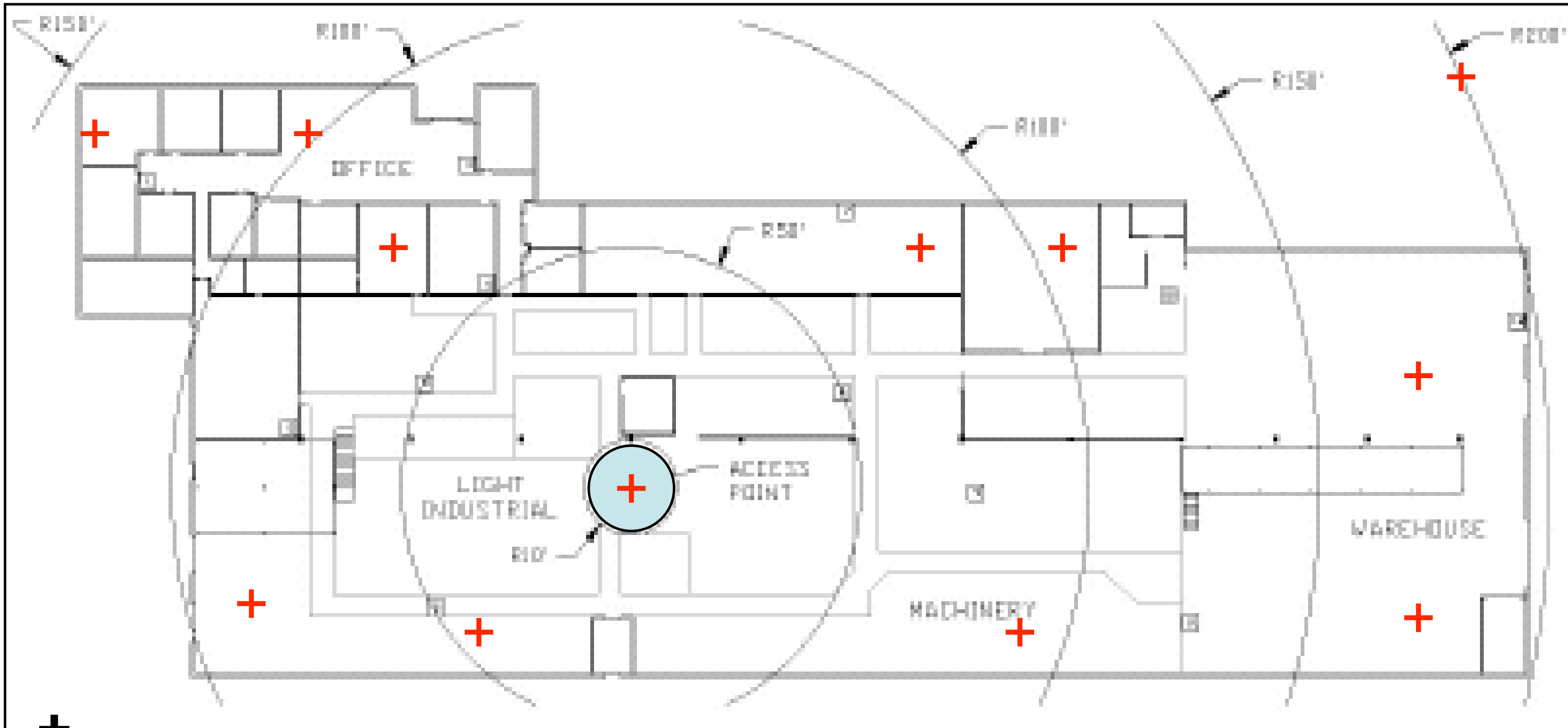
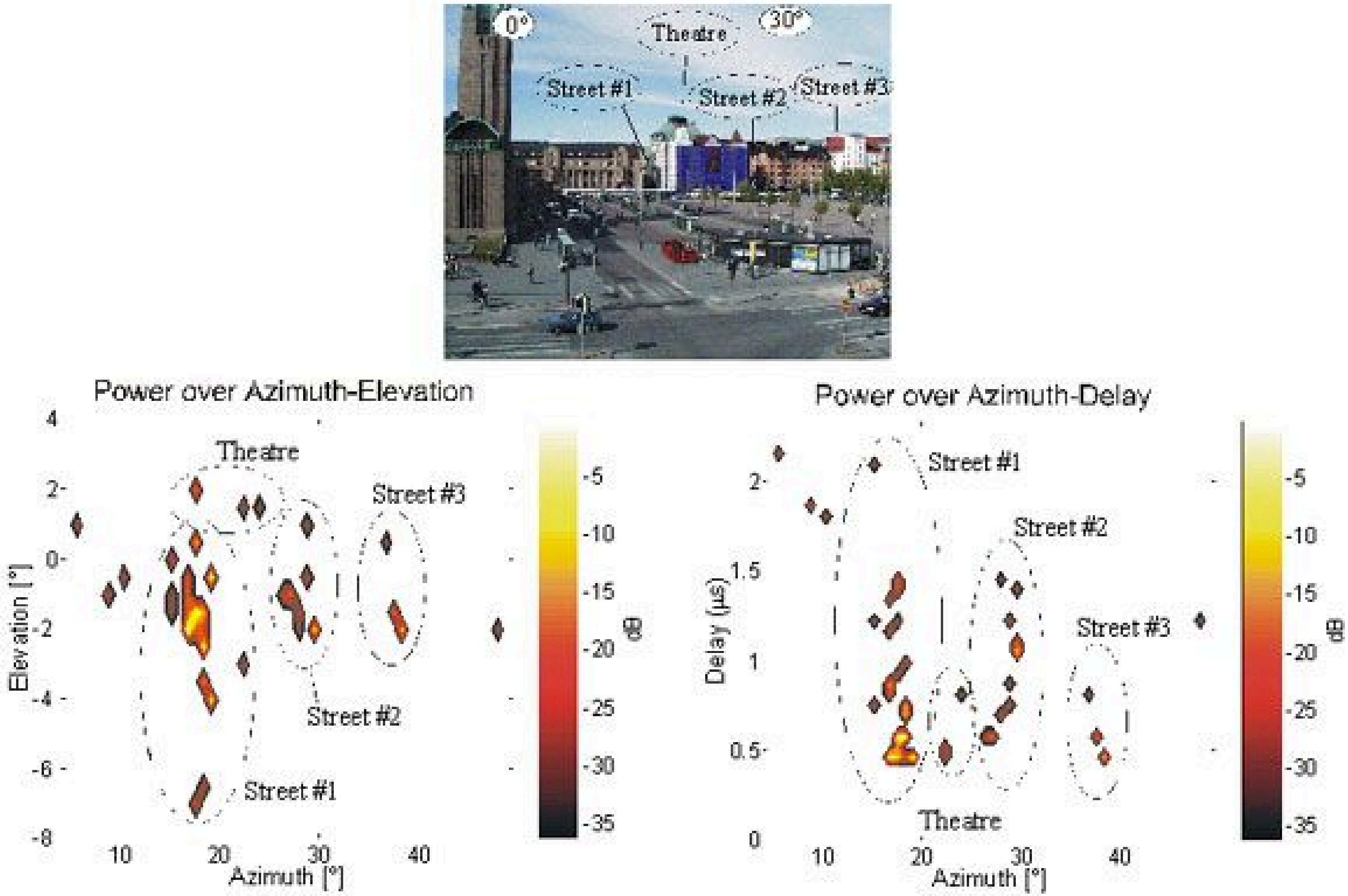


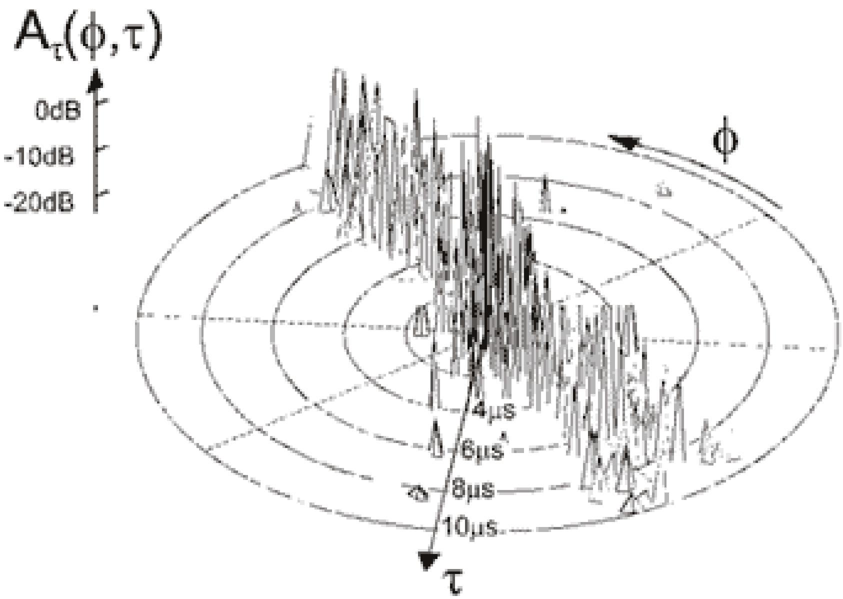
Figure 3. Test facility layout with numbered test locations

Outdoor 1

Revised ERC RECOMMENDATION (00)08 FIELD STRENGTH MEASUREMENTS ALONG A ROUTE WITH GEOGRAPHICAL COORDINATE REGISTRATIONS October 2003

http://www.ero.dk/documentation/docs/doc98/official/pdf/ERCREC0008.PDF

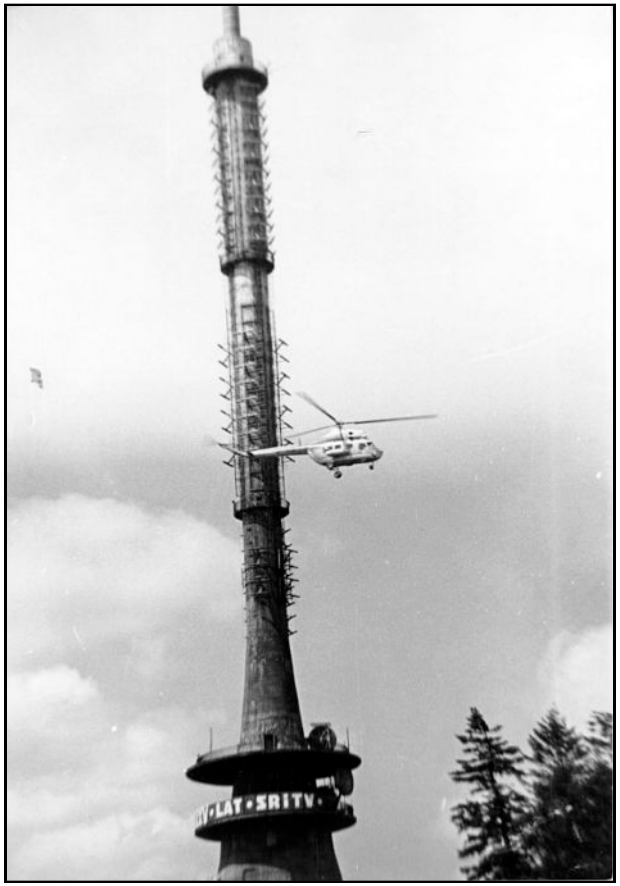

0°
Theatre
30°
Street #1
Street #2
Street #3
Power over Azimuth-Elevation
Theatre
Street #3
Street #2
Street #1
Elevation [°]
Azimuth [°]
dB
Power over Azimuth-Delay
Street #1
Street #2
Street #3
Theatre
Delay (µs)
Azimuth [°]
dB



- Andreas F. Molisch, Alexander Kuchar, Juha Laurila, Martin Steinbauer, Martin Toeltsch, and Ernst Bonek: Spatial Channel Measurement and Modeling - http://www.techonline.com/community/ed_resource/feature_article/14707

Figure 1: The figure depicts the azimuth delay power spectrum for a mobile station in a street canyon. The radial axis represents the delay, where the origin

Outdoor 2

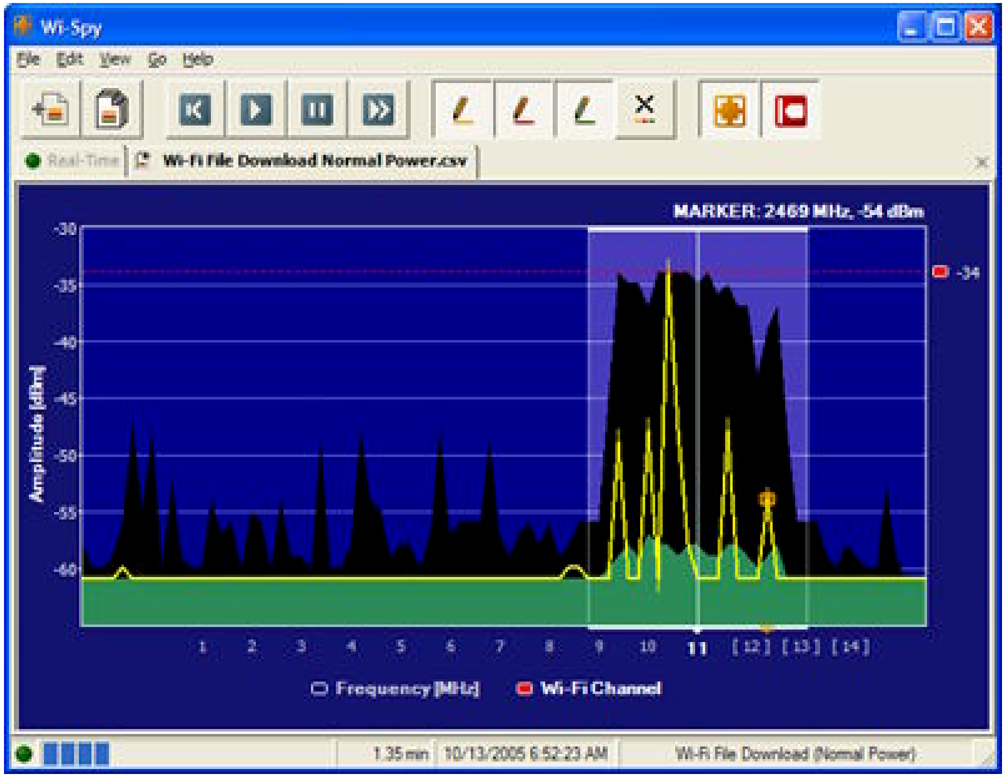


Miniature models can be used

Spectrum analyzer

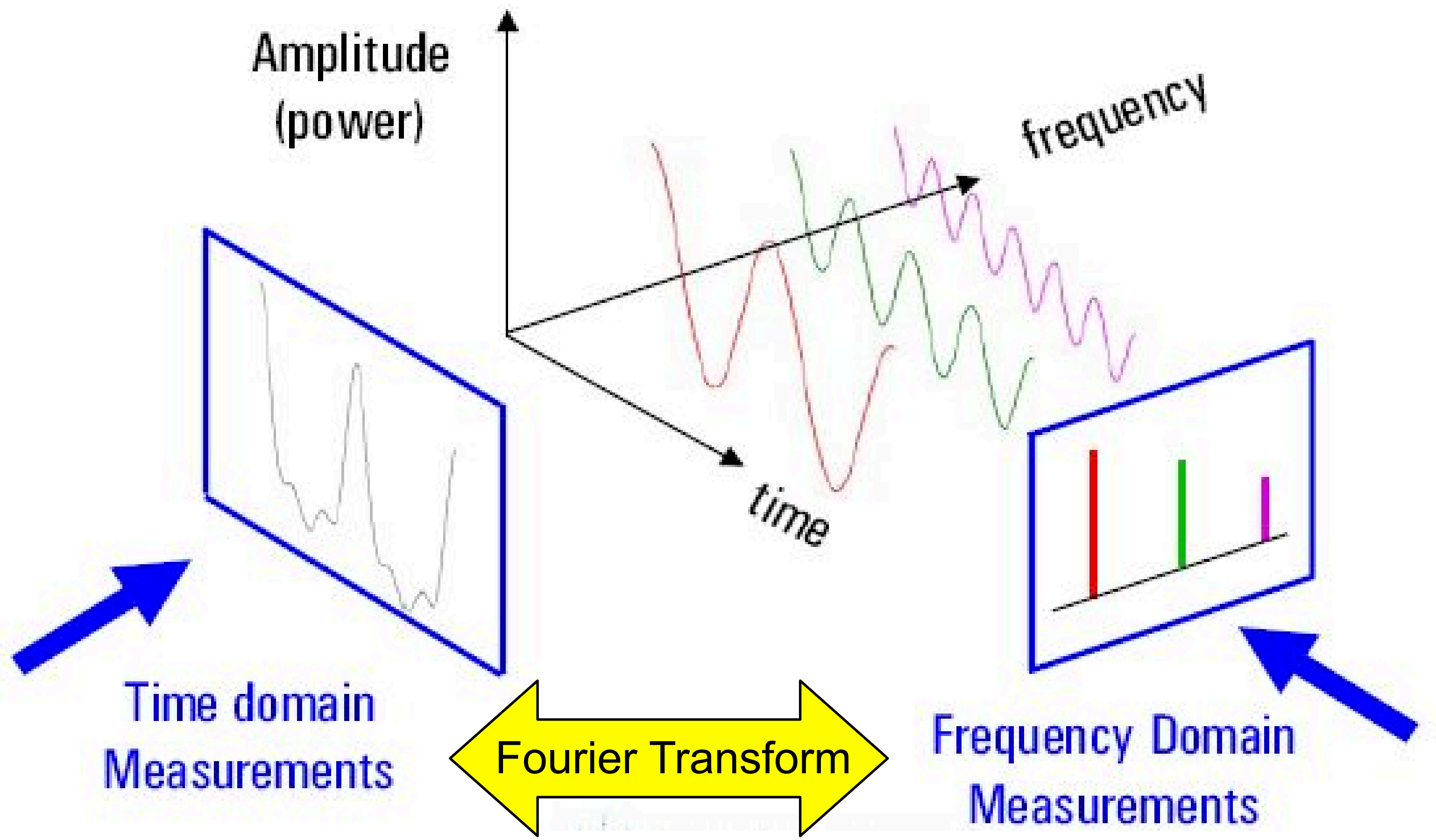
- Measures the signal field-strength, if equipped with an antenna
  - Absolute - if calibrated, otherwise relative values


Wi-Spy


Wi-Spy
File Edit View Go Help
Real-Time
Wi-Fi File Download Normal Power.csv
MARKER: 2469 MHz, -54 dBm
Amplitude [dBm]
-30
-35
-40
-45
-50
-55
-60
-34
1 2 3 4 5 6 7 8 9 10 11 [12] [13] [14]
Frequency [MHz]
Wi-Fi Channel
1.35 min
10/13/2005 6:52:23 AM
Wi-Fi File Download (Normal Power)

Frequency & time domains

•The signal received can be characterized in the **time domain or in** the **frequency** domain.



Types of spectrum analyzers

- Analogue or Swept-Spectrum
  - A tunable measuring receiver (analogue band pass analogue filter), whose mid-frequency is automatically swept through the range of frequencies of interest
  - Usually offers only amplitude information in the frequency domain
- Digital
  - A combination of a fast A/D converter and specialized computer that implements the Fast Fourier Transform (FFT)
  - Can offer amplitude and phase information in the frequency domain

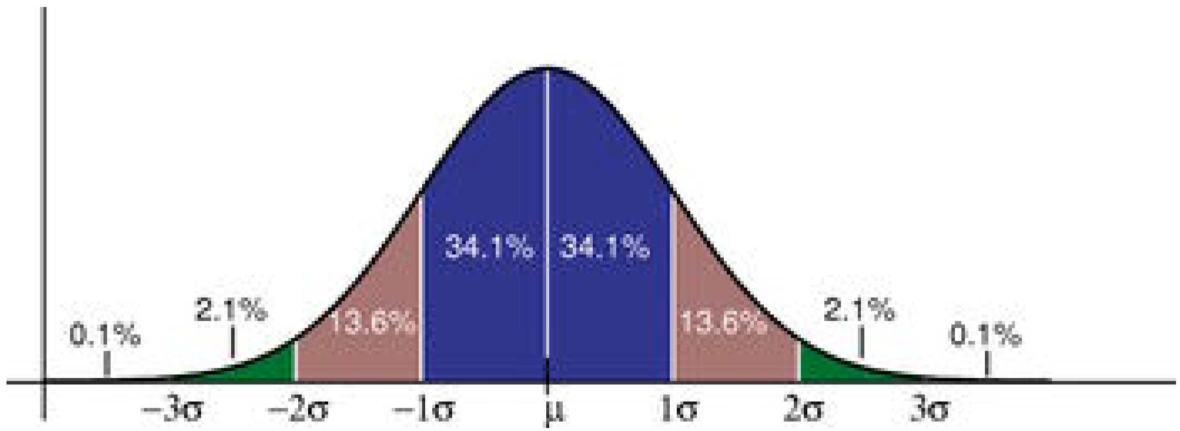
What is error?

- An **error** is
  - a difference between a computed, estimated, or measured value and the true, specified, or theoretically correct value
  - a bound on the precision and accuracy of the result of a measurement
- Errors can be classified into two types: **statistical** and **systematic**.

Systematic vs. random errors

- Statistical (random) error
  - Unpredictable
  - Due to random causes (fluctuations)
  - Can be reduced by repeating measurements many times and their statistical analysis
- Systematic error
  - caused by a non-random influence
    - If the cause of the systematic error can be identified, then it can usually be eliminated.

Standard deviation



Measurement results follow often the normal distribution: $f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

Arithmetic mean is the expected (most probable) result: $\mu = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

The standard deviation σ_x is a measure of how widely the measured values are dispersed from their average value: $\sigma_x = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$

Uncertainty & accuracy

- Since the true value of measured quantity is unknown (the measured values are only its *estimates*), measurements are associated with *uncertainty*.
- The absolute uncertainty is an interval into which the true value falls with a given probability; it is expressed in the same unit as the measurement result.
- The relative uncertainty is the quotient of the absolute uncertainty and the best possible estimate of the true value.
- The lower the uncertainty the higher is the accuracy with which a measurement is made.

Confidence

Confidence interval (for a population mean) shows the interval within which the true mean value is

Conf. interval = 100 $(1-\alpha)$%;
e.g. $\alpha=0.05$ indicates a 95% confidence level

α = the significance level
σ = standard deviation
n = size of the sample
= no. of measurements

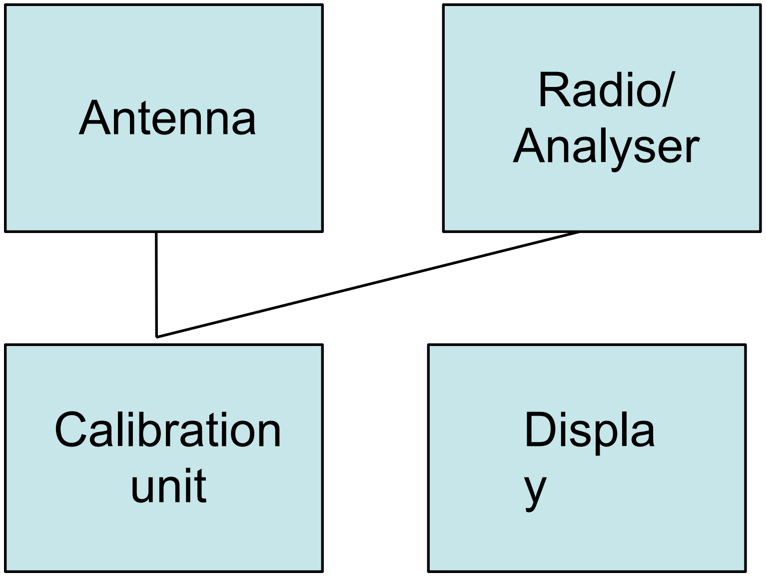
E.g. for $\alpha = 0.05$, we need to calculate the area under the standard normal curve that equals $(1-\alpha)$ or 95%. This value is ± 1.96. The corresponding confidence interval is therefore $\bar{x} \pm 1.96\sigma$

Factors influencing uncertainty

- Field strength & power flux density measurements at microwaves depend on local environment
  - Errors due to interfering & reflected signals
    - Simulation: http://www.educatorscorner.com/index.cgi?CONTENT_ID=2490
  - Reading errors
  - antenna calibration factor
  - attenuation of the connections between antenna and receiver
  - receiver sine-wave voltage accuracy

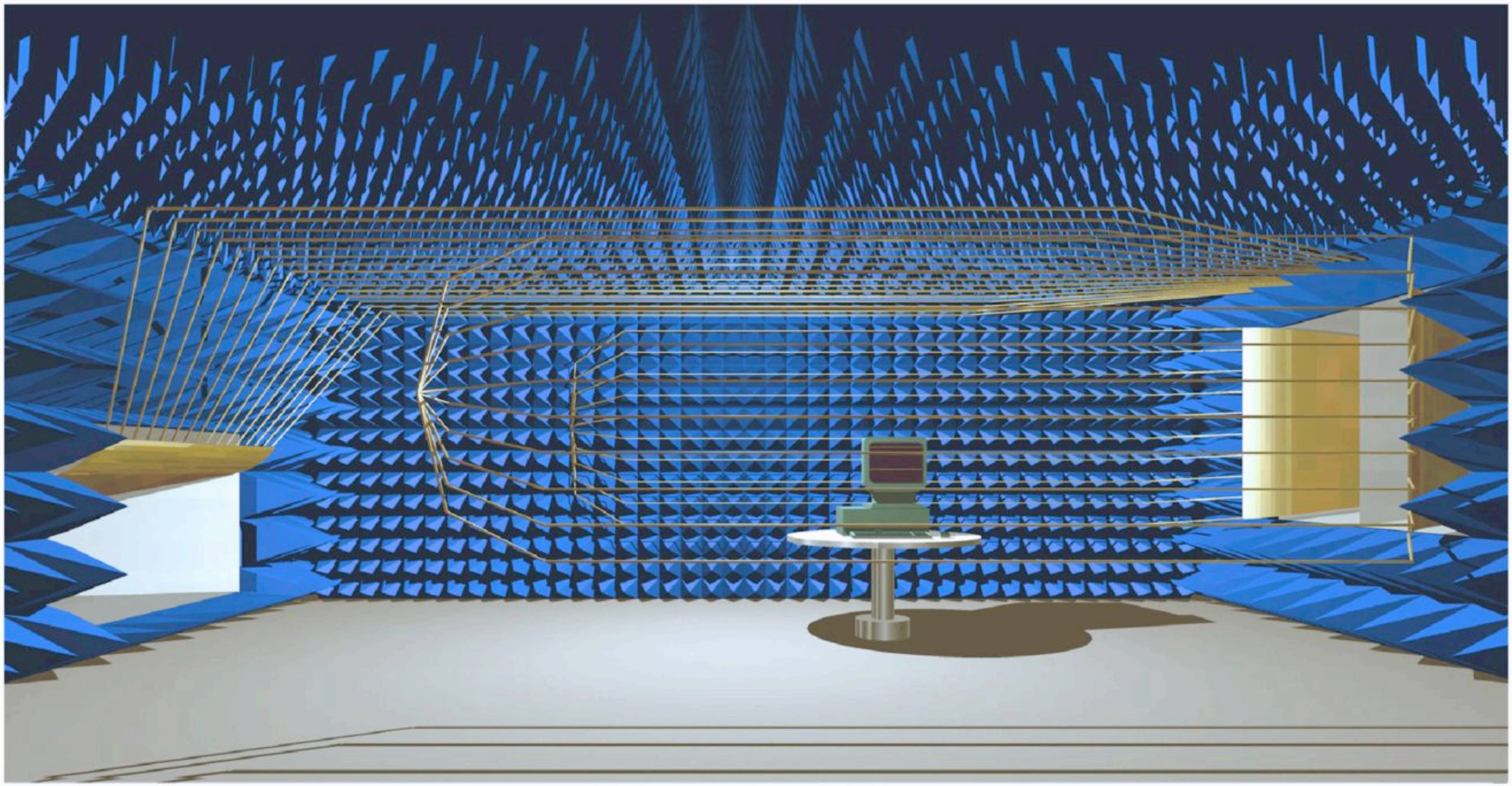
- shadowing due to obstacles
- device selectivity relative to occupied bandwidth
- device noise floor
- antenna factor frequency interpolation
- antenna factor variation with height above ground and other mutual coupling effects

- Antenna impedance mismatch (between antenna port and the input)
- antenna balance mismatch
- antenna directivity mismatch
- antenna cross-polarisation response
- Errors due to spectrum analyzers:
  - Hewlett – Packard: Spectrum Analysis Basics
  - Rauscher C: Fundamentals of Spectrum Analysis

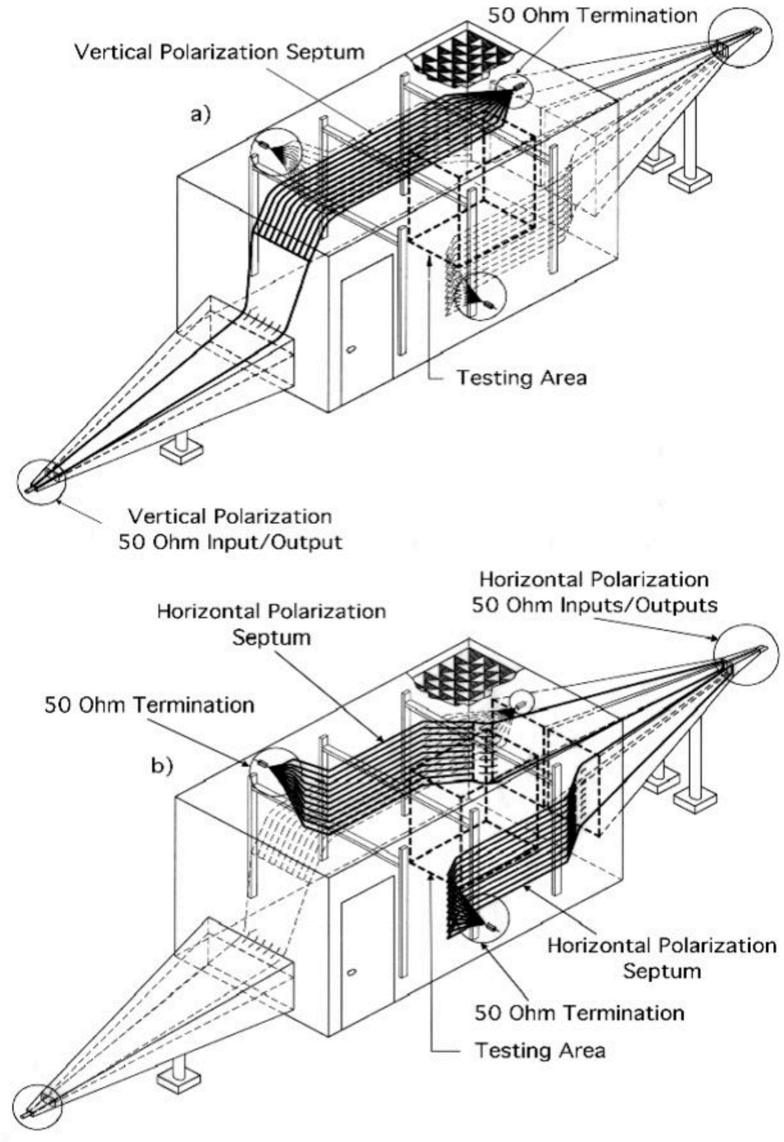


- Calibration: setting the response of a measuring system within specified accuracy/ precision
- Traceability: relating an instrument's accuracy to the master reference standard s
  - http://en.wikipedia.org/wiki/Metrology

TEM cell



Source: A Podgorski

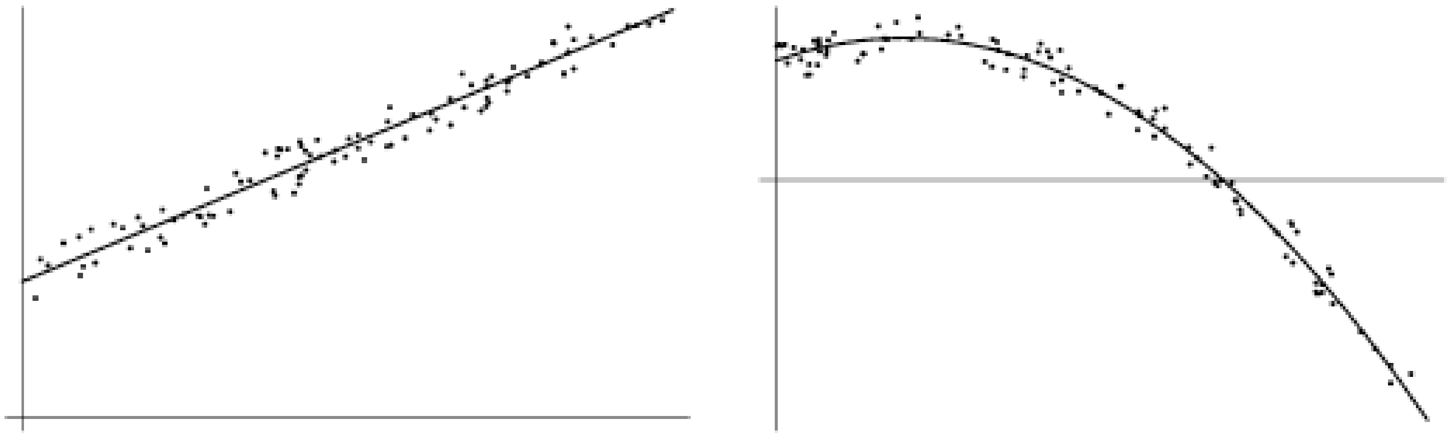


Source: A Podgorski

Field-strength (1 point in space)

- Simple simulation of the field-strength in a single point MeasurSimul1.xls
  - Vienna agreement
- What with field-strength distence dependence (2 variables)?

Least Squares Fitting (LSF)



- Most popular approach: statistical analysis under assumption that measurement errors are random (normally distributed)
- LSF = a mathematical procedure for finding the best-fitting curve to a given set of points
- Minimizing the sum of the squares of the offsets ("the residuals") of the points from the curve.

Linear least squares

- Provides solution to the problem of finding the best fitting *straight* line through a set of points
    - The simplest and most commonly applied form of linear regression
- Applicable to linear models (and models that can be linearized)

Theoretical background

- Vertical least squares fitting proceeds by finding such a straight line $y = a + bx$ that minimizes the sum of the *squares* of the *vertical* deviations of the data points (x_i, y_i)
- The square deviations from each point are summed, and the resulting residual (correlation factor) is then minimized to find the best-fit line.

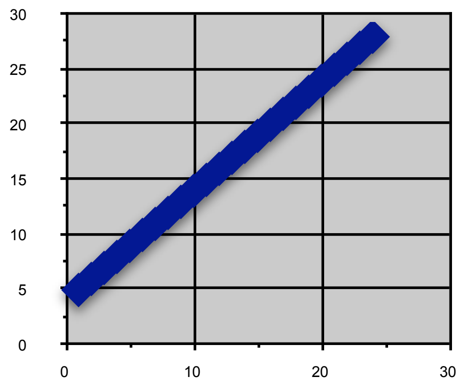
$$R^2(a, b, x_i) \equiv \sum_{i=1}^{n} \left[y_i - (a + bx_i) \right]^2$$

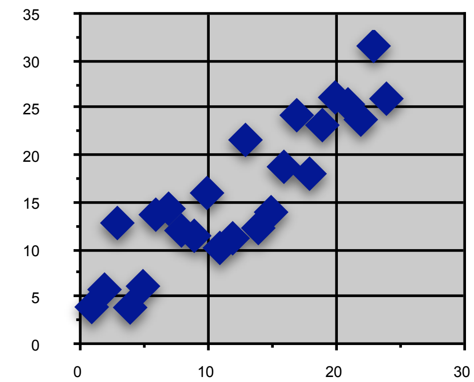
- The square deviations from each point are summed, and the resulting residual is then minimized to find the best fit line.

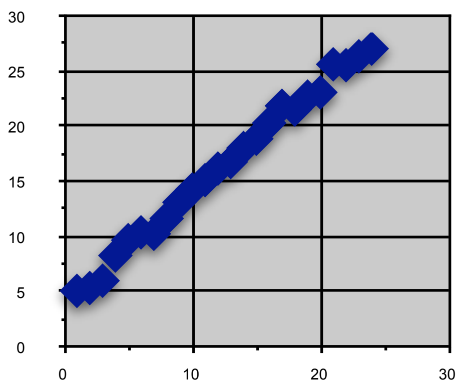
$$\frac{\partial(R^2)}{\partial a} = -2 \sum_{i=1}^{n} [y_i - (a + b\, x_i)] = 0$$

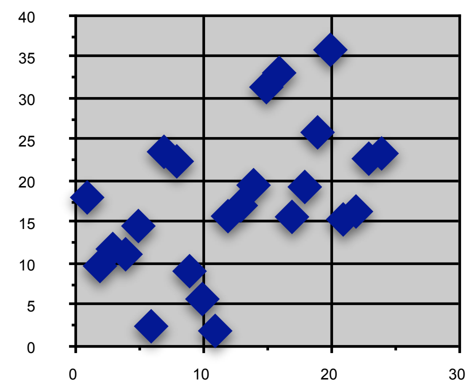
$$\frac{\partial(R^2)}{\partial b} = -2 \sum_{i=1}^{n} [y_i - (a + b\, x_i)]\, x_i = 0.$$

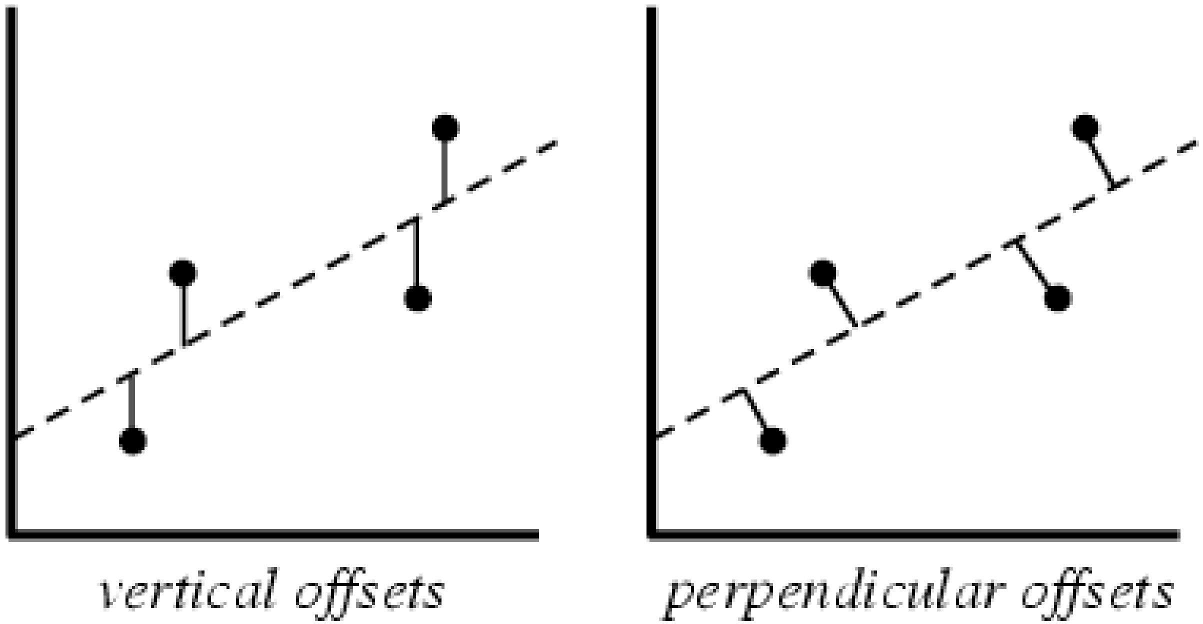
R^2 Interpretation











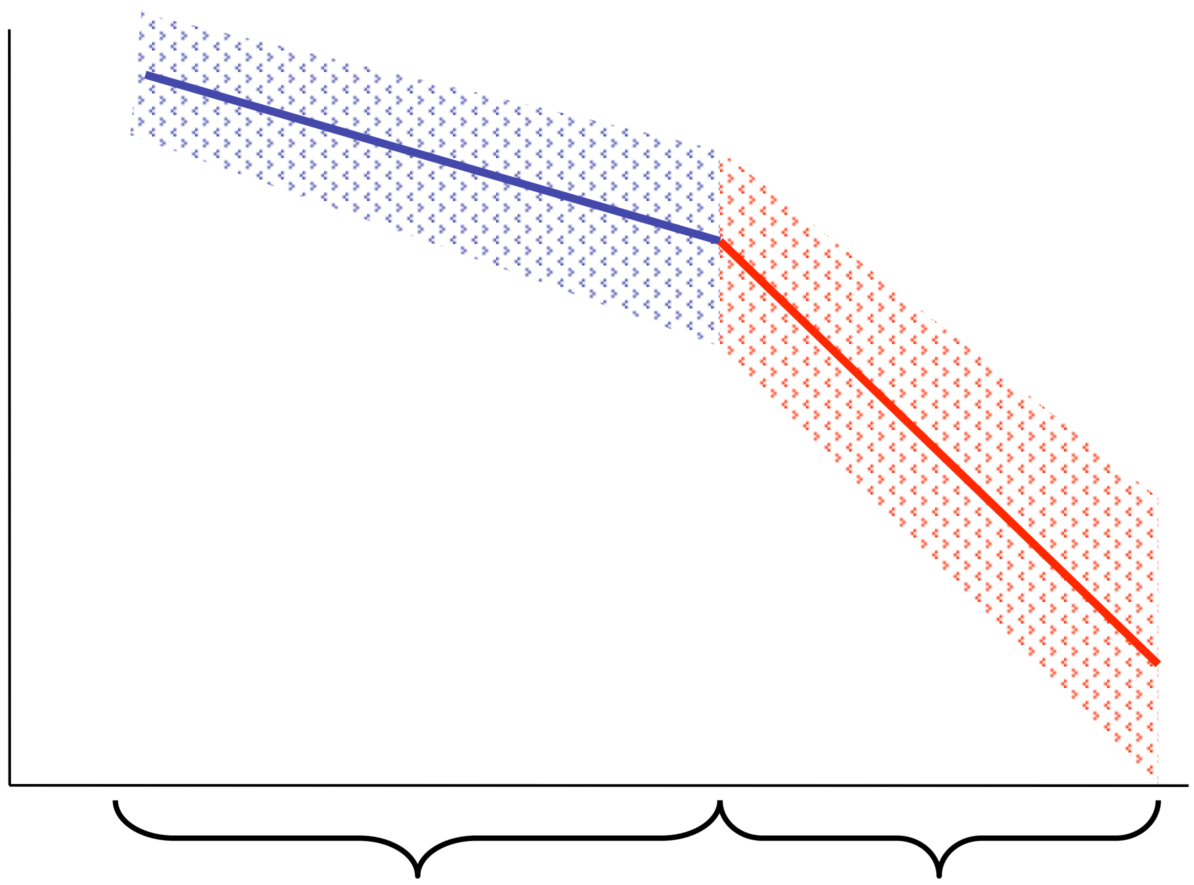
- For simplicity, the *vertical* offsets from a line (surface, etc.) are usually minimized instead of the perpendicular offsets
  - Least Squares Fitting--Perpendicular Offsets,

Example: Linear least squares

- Simple simulation of distance-dependence of the field-strength measurements: MeasurSimul2

What with non-linear?

- If the general form of the functional relationship between the two quantities being graphed is non-linear, we can apply
  - functional transformation of the variables in such a way that the resulting line *is* a straight line
  - Apply more complex fitting, e.g.
    - Least Squares Fitting--Exponential,
    - Least Squares Fitting--Logarithmic,
    - Least Squares Fitting--Polynomial,
    - Least Squares Fitting--Power Law,



Example: local propagation model

- Assume a simple propagation model of the form $P = Cd^{-n}$ where P is the signal power in W, d is distance in m, and C and n are constants to be determined from measurements
- We take logarithm (base 10): log(P[W]) = log(C) – n*log(d[m])
- We substitute for new variables: y = log(P/1W); x = log(d/1m) and for new constants: a = log(C) and b = n
- The propagation model is linear in new variables: y = a + bx
- New constants a and b can be determined using the Linear Least Square Fit and then the original constants are determined
- Note: It means that the P-axis is *linear* if P is expressed in dBW, and the d-axis in [m] is *logarithmic*

Formulas

$$y = a + bx$$

$$b = \frac{ss_{xy}}{ss_{xx}}$$

$$a = \overline{y} - b\overline{x}$$

$$r^2 = \frac{ss_{xy}^2}{ss_{xx} ss_{yy}}$$

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \; ; \qquad \overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \; ;$$

$$ss_{xx} = \left(\sum_{i=1}^{n} x_i^2\right) - n\left(\overline{x}\right)^2$$

$$ss_{yy} = \left(\sum_{i=1}^{n} y_i^2\right) - n\left(\overline{y}\right)^2$$

$$ss_{xy} = \left(\sum_{i=1}^{n} x_i y_i\right) - n\overline{x}\,\overline{y}$$

$$s = \sqrt{\frac{ss_{yy} - \frac{ss_{xy}^2}{ss_{xx}}}{n-2}}$$

$$SE(a) = s\sqrt{\frac{1}{n} + \frac{\overline{x}^2}{ss_{xx}}}$$

$$SE(b) = \frac{s}{\sqrt{ss_{xx}}}$$

Mathematics of Linear least squares

$$y = a + bx$$

$$\{x_i, y_i\}, i = 1, 2, \ldots n;$$

$$R^2(x_i, a, b) = \sum_{i=1}^{n} \left[y_i - (a + bx_i) \right]^2$$

$$\frac{\partial (R^2)}{\partial a} = -2\sum_{i=1}^{n} \left[y_i - (a + bx_i) \right] = 0$$

$$\frac{\partial (R^2)}{\partial b} = -2\sum_{i=1}^{n} \left[y_i - (a + bx_i) \right] x_i = 0$$

$$na + b\sum_{1}^{n} x_i = \sum_{1}^{n} y_i$$

$$a\sum_{1}^{n} x_i + b\sum_{1}^{n} x_i^2 = \sum_{1}^{n} x_i y_i$$

$$\begin{bmatrix} n & \sum_{1}^{n} x_i \\ \sum_{1}^{n} x_i & \sum_{1}^{n} x_i^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{1}^{n} y_i \\ \sum_{1}^{n} x_i y_i \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} n & \sum_1^n x_i \\ \sum_1^n x_i & \sum_1^n x_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_1^n y_i \\ \sum_1^n x_i y_i \end{bmatrix} = \frac{1}{n\sum_1^n x_i^2 - \left(\sum_1^n x_i\right)^2} \begin{bmatrix} \sum_1^n y_i \sum_1^n x_i^2 - \sum_1^n x_i \sum_1^n x_i y_i \\ n\sum_1^n x_i y_i - \sum_1^n x_i \sum_1^n y_i \end{bmatrix}$$

$$a = \frac{\sum_1^n y_i \sum_1^n x_i^2 - \sum_1^n x_i \sum_1^n x_i y_i}{n\sum_1^n x_i^2 - \left(\sum_1^n x_i\right)^2} = \frac{\overline{y}\sum_1^n x_i^2 - \overline{x}\sum_1^n x_i y_i}{\sum_1^n x_i^2 - n\left(\overline{x}\right)^2}$$

$$b = \frac{n\sum_1^n x_i y_i - \sum_1^n x_i \sum_1^n y_i}{n\sum_1^n x_i^2 - \left(\sum_1^n x_i\right)^2} = \frac{\left(\sum_1^n x_i y_i\right) - n\overline{x}\,\overline{y}}{\sum_1^n x_i^2 - n\left(\overline{x}\right)^2}$$

$$ss_{xx} = \sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 = \left(\sum_{i=1}^{n} x_i^2\right) - n\left(\overline{x}\right)^2$$

$$ss_{yy} = \sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2 = \left(\sum_{i=1}^{n} y_i^2\right) - n\left(\overline{y}\right)^2$$

$$ss_{xy} = \sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right) = \left(\sum_{i=1}^{n} x_i y_i\right) - n\overline{x}\,\overline{y}$$

$$\sigma_x^2 = \frac{ss_{xx}}{n}$$

$$\sigma_y^2 = \frac{ss_{yy}}{n}$$

$$\mathrm{cov}(x, y) = \frac{ss_{Xy}}{n}$$

$$b = \frac{\operatorname{cov}(x, y)}{\sigma_x^2} = \frac{ss_{xy}}{ss_{xx}}$$

$$a = \overline{y} - b\overline{x}$$

$$r^2 = \frac{ss_{xy}^2}{ss_{xx} ss_{yy}}$$

- r2 is correlation coefficient that gives the proportion of ssyy which is accounted for by the regression

- Let yi* be the vertical coordinate of the best-fit line at coordinate xi, and ei be its distance to the actual measurement point yi. Then

$$y_i^* \equiv a + bx_i$$

$$e_i \equiv y_i - y_i^*$$

$$s^2 = \sum_{i=1}^{n} \frac{e_i^2}{n-2}$$

$$s = \sqrt{\frac{ss_{yy} - bss_{xy}}{n-2}} = \sqrt{\frac{ss_{yy} - \frac{ss_{xy}^2}{ss_{xx}}}{n-2}}$$

Standard errors

$$SE(a) = s\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{ss_{xx}}}$$

$$SE(b) = \frac{s}{\sqrt{ss_{xx}}}$$

Summary

- We have reviewed basic issues that should be taken into account when measuring the signal field strength
- There are numerous programs that facilitate the processing of measurement results
  - But they cannot be used blindly
  - They should be used with full understanding -- they cannot replace common sense

References

- Alevy A M: In-Building Propagation Measurements at 2.4 GHz
- Taylor B N, Kuyatt C E: Guidelines for Evaluating and Expressing the Uncertainty of Measurement Results http://physics.nist.gov/Pubs/guidelines/appa.html
- Weisstein E W: "Least Squares Fitting." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/LeastSquaresFitting.html
- Wysocki TA, Zepernick HJ: Characterization of the indoor radio propagation channel at 2.4 GHz; Journal of Telecommunications and Information Technology Nr. 3-4/2000, p. 84-90

Links

- ANOVA,
- Correlation Coefficient,
- Interpolation,
- MANOVA,
- Matrix 1-Inverse,
- Moore-Penrose Matrix Inverse,
- Nonlinear Least Squares Fitting,
- Pseudoinverse,
- Regression Coefficient,
- Residual,
- Spline.

Thank you