Slide 1 / 252

Slide 2 / 252

NEW JERSEY CENTER
FOR TEACHING & LEARNING

Geometry

Similar Triangles & Trigonometry

2015-10-22

www.njctl.org

Slide 3 / 252

Table of Contents

click on the topic to go to that section

Slide 4 / 252

Throughout this unit, the Standards for Mathematical Practice are used.

MP1: Making sense of problems & persevere in solving them.
MP2: Reason abstractly & quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for & make use of structure.
MP8: Look for & express regularity in repeated reasoning.

Additional questions are included on the slides using the "Math Practice" Pull-tabs (e.g. a blank one is shown to the right on this slide) with a reference to the standards used.

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.

Slide 5 / 252

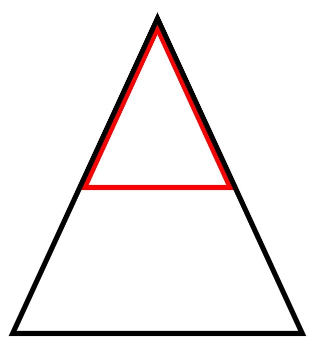
Problem Solving with Similar Triangles and Right Triangles

Three basic approaches to real world problem solving include:

- Similar Triangles
- Trigonometry
- Pythagorean Theorem

Slide 6 / 252

Problem Solving with Similar Triangles



Return to the Table of Contents

Slide 7 / 252

Shadows and Similar Triangles

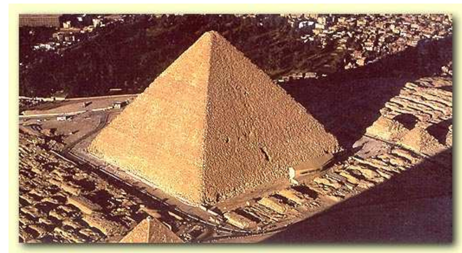
One of the oldest math problems was solved using similar right triangles.

About 2600 years ago, Thales of Miletus, perhaps the first Greek mathematician, was visiting Egypt and wondered what the height was of one of the Great Pyramid of Giza.

Due to the shape of the pyramid, he couldn't directly measure its height.

Slide 8 / 252

Shadows and Similar Triangles



http://www.metrolic.com/travel-guides-the-great-pyramid-of-giza-147358/

When Thales visited the Great Pyramid of Giza 2600 years ago, it was already 2000 years old.

He wanted to know its height.

Slide 9 / 252

Shadows and Similar Triangles

He noticed that the pyramid cast a shadow, which could be measured on the ground using a measuring rod.

And he realized that the measuring rod standing vertically also cast a shadow.

Based on those two observations, can you think of a way he could measure the height of the pyramid?

Discuss this at your table for a minute or two.

Slide 10 / 252

Shadows and Similar Triangles

What 2 facts can you recall from our study of similar triangles? Fill in the blanks below.

Their angles are all ______________
click

Their corresponding sides are in ________ to one another.
click

Slide 11 / 252

Shadows and Similar Right Triangles

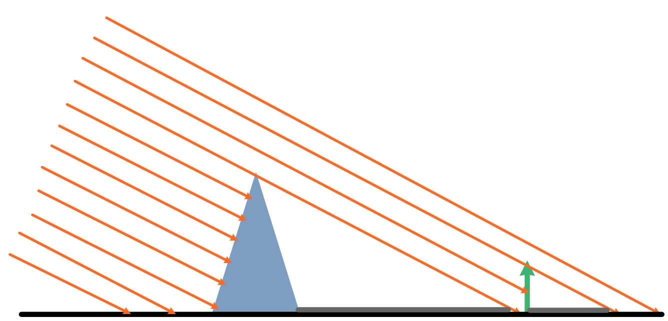
Draw a sketch of the pyramid being measured and its shadow....and the measuring rod and its shadow.

Represent the pyramid and rod as vertical lines, with the rod being much shorter than the pyramid.

You won't be able to draw them to scale, since the rod is so small compared to the pyramid, but that won't affect our thinking.

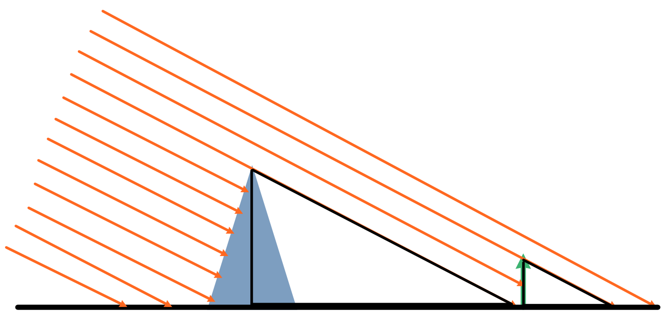
Slide 12 / 252

Shadows and Similar Right Triangles



Slide 13 / 252

Shadows and Similar Right Triangles

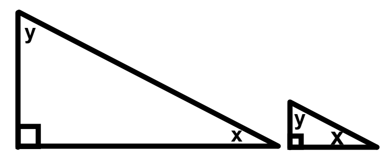


Slide 14 / 252

Similar Right Triangles

Taking away the objects and just leaving the triangles created by the height of the object, the sunlight and the shadow on the ground, we can see these are similar triangles.

All the angles are equal, so the sides must be in proportion.

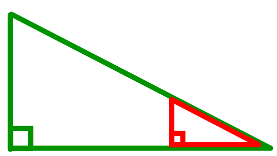


Slide 15 / 252

Similar Right Triangles

By putting one triangle atop the other it's easy to see that they are similar.

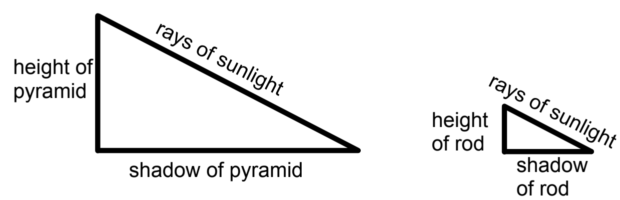
Using the 2 ideas we came up with before, you know the angles are all the same, and the sides are in proportion.



Slide 16 / 252

Similar Right Triangles

Which means that the length of each shadow is in proportion to the height of each object.



Slide 17 / 252

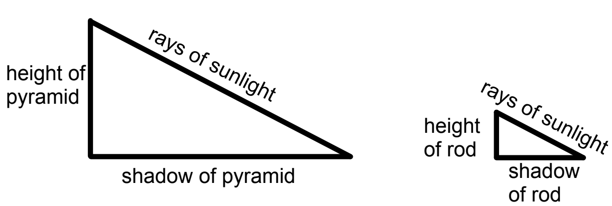
Shadows and Similar Right Triangles

If the shadow of the rod was 2 meters long.

And the shadow of the pyramid was 120 meters long.

And the height of the rod was 1 meter.

How tall is the pyramid?



Slide 18 / 252

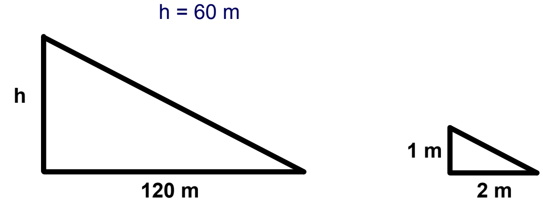
Shadows and Similar Right Triangles

$$\frac{\text{height of pyramid}}{\text{pyramid's shadow}} = \frac{\text{height of rod}}{\text{rod's shadow}}$$

$$\text{height of pyramid} = \frac{\text{height of rod}}{\text{rod's shadow}} \times \text{pyramid's shadow}$$

$$h = \frac{1\text{ m}}{2\text{ m}}(120\text{ m})$$

$$h = 60\text{ m}$$



Slide 19 / 252

Shadows and Similar Right Triangles

This approach can be used to measure the height of a lot of objects which cast a shadow.

And, a convenient measuring device is then your height, and the length of the shadow you cast.

Try doing this on the next sunny day you can get outside.

Measure the height of any object which is casting a shadow by comparing the length of its shadow to the length of your own.

Lab - Indirect Measurement

Reminder - Mirrors also create indirect measurement if you are doing this lab on a cloudy day.

Slide 20 / 252

1 A lamppost casts a 9 ft shadow at the same time a person 6 ft tall casts a 4 ft shadow. Find the height of the lamppost.

- A 6 ft
- B 2.7 ft
- C 13.5 ft
- D 15 ft

Slide 21 / 252

2 You're 6 feet tall and you notice that your shadow at one time is 3 feet long. The shadow of a nearby building at that same moment is 20 feet long

How tall is the building?

Slide 22 / 252

3 You're 1.5 m tall and you notice that your shadow at one time is 4.8 m long. The shadow of a nearby tree at that same moment is 35 m long

How tall is the tree?

Slide 23 / 252

4 Two buildings are side by side. The 35 m tall building casts a 21 m shadow.

How long will the shadow of the 8 m tall building be at the same time?

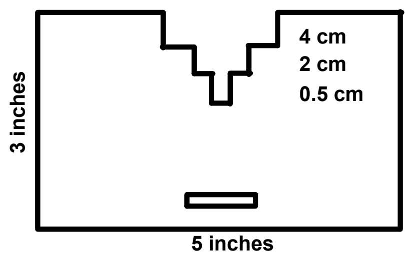
Slide 24 / 252

Similar Triangle Measuring Device

Similar Triangle Measuring Device

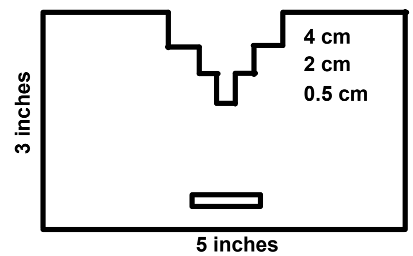
We can also make a device to set up similar triangles in order to make measurements.

Take a piece 3" x 5" card and cut it as shown below:



Similar Triangle Measuring Device

Now slide a meter stick through the slot in the bottom of the card. In that way, you can move the card a specific distance from one end of the stick.

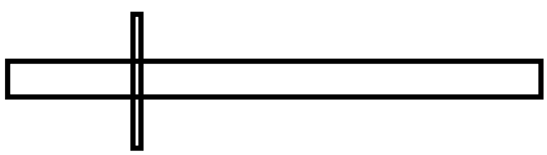


Similar Triangle Measuring Device

By looking along the meter stick, you can then move the card so that a distant object fills either the 0.5 cm, 2 cm or 4 cm slot.

You can then measure how far the card is from your eye, along the meter stick.

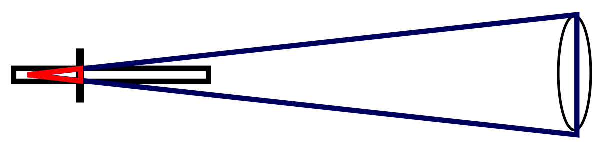
This creates a similar triangle that allows you to find how far away an object of known size is, or the size of an object of known distance away.



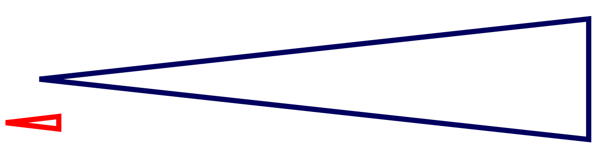
Similar Triangle Measuring Device

This shows how by lining up a distant object to fill a slot on the device two similar triangles are created, the small red one and the larger blue one.

All the angles are equal and the sides are in proportion. Also, the base and altitude of each isosceles triangle will be in proportion.



Similar Triangle Measuring Device



The altitude and base of the small isosceles triangle can be directly measured, which means that the ratio of those on the larger triangle is know. Given the size or the distance to the object, the other can be determined.

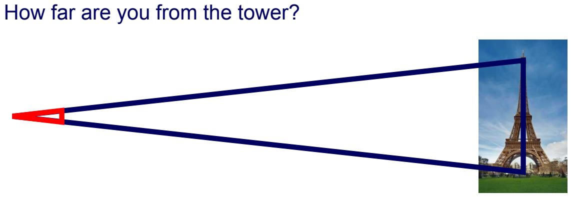
Similar Triangle Measuring Device

You are visiting Paris and have your similar triangle measuring device with you.

You know that the Eiffel Tower is 324 meters tall.

You adjust your device so that turned sidewise the height of the tower fills the 2 cm slot when the card is 20 cm from your eye.

How far are you from the tower?



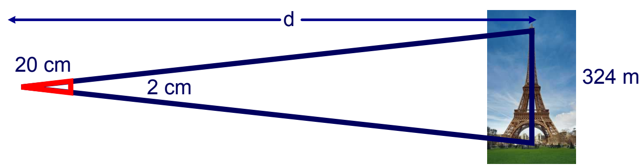
Shadows and Similar Right Triangles

$$\frac{\text{distance to tower}}{\text{height of tower}} = \frac{\text{distance to card}}{\text{width of slot}}$$

$$\text{distance to tower} = \frac{\text{distance to card}}{\text{width of slot}} \times \text{height of tower}$$

$$d = \frac{20\text{ cm}}{2\text{ cm}}(324\text{ m})$$

$$d = 3240\text{ m}$$



5 You move to another location and the Eiffel Tower (324 m tall) now fills the 4 cm slot when the card is 48 cm from your eye. How far are you from the Eiffel Tower now?

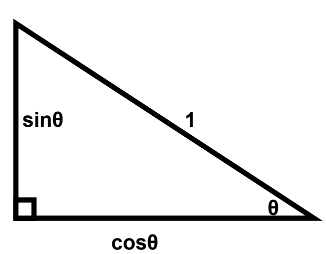
6 The tallest building in the world, the Burj Kalifah in Dubai, is 830 m tall. You turn your device so that it fills the 4 cm slot when it is 29.4 cm from your eye. How far are you from the building?

7 The width of a storage tank fills the 2 cm slot when the card is 48 cm from your eye. You know that the tank is 680 m away. What is its width?

8 The moon has a diameter of 3480 km. You measure it one night to about fill the 0.5 cm slot when the card is 54 cm from your eye.

What is the distance to the moon?

Similar Triangles and Trigonometry

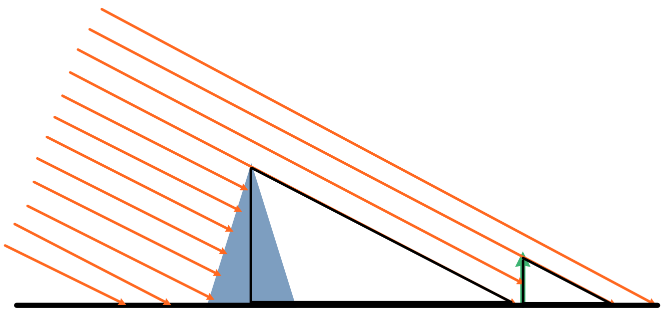


Return to the
Table of Contents

Slide 37 / 252

Problem Solving

Recall that Thales found the height of the pyramid by using similar triangles created by the shadow of the pyramid and a rod of known length.



Slide 38 / 252

Problem Solving

But what if he were trying to solve this problem and there wasn't a shadow to use.

Or you are trying to solve other types of problems which don't allow you to set up a similar triangle so easily.

Trigonometry provides the needed similar triangle for any circumstance, which is why it is a powerful tool.

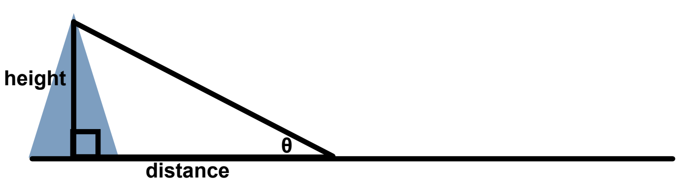
Slide 39 / 252

Problem Solving with Trigonometry

So, if Thales used trig to solve his problem, he'd have considered this right triangle.

First he'd measure theta, the angle between the ground and the top of the pyramid, when at a certain distance away on the ground.

Then he'd imagine a similar triangle with the same angle.

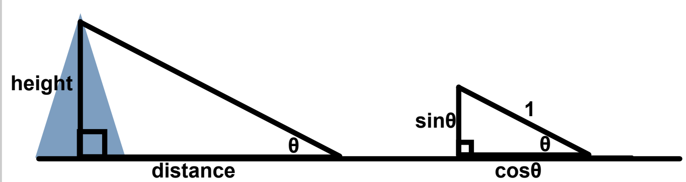


Slide 40 / 252

Problem Solving with Trigonometry

He has a ready-made right triangle, thanks to mathematicians who calculated all the possible right triangles that could be created with a hypotenuse of 1 and put their measurements in a table, a trigonometry table.

The side opposite the angle is named sine θ, or sinθ for short, and the side adjacent to the angle is called cosine θ, or cosθ for short.



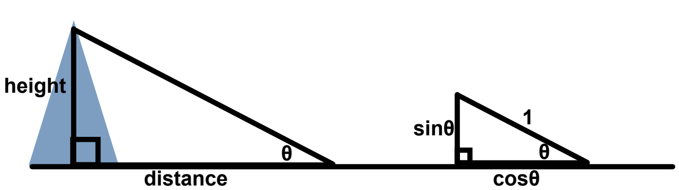
Slide 41 / 252

Problem Solving with Trigonometry

We know all the angles are equal since both triangles include a right angle and the angle theta, so those two angles are the same in both.

And, since all the angles of a triangle total to 180º, all three angles must be equal.

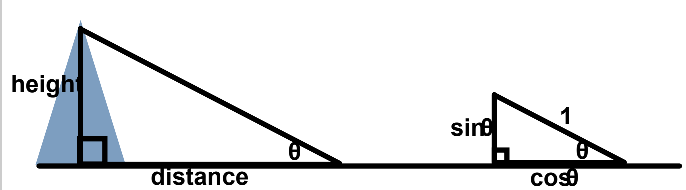
Since all the angles are equal, these triangles are similar.



Slide 42 / 252

Problem Solving with Trigonometry

Since all the angles are equal, the sides are in proportion, so what would this ratio be equal to in the triangle to the right?



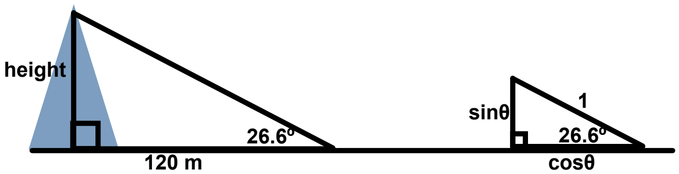
Slide 43 / 252

Problem Solving with Trigonometry

When we did the problem earlier we used the rod's height of 1 m and it's shadow's length of 2 m. That would mean that the angle between the rays of sunlight and the ground would have been 26.6º.

And the length of the pyramid's shadow was 120 m.

Let's use that angle and distance and see if we get the same answer.



Slide 44 / 252

Problem Solving with Trigonometry

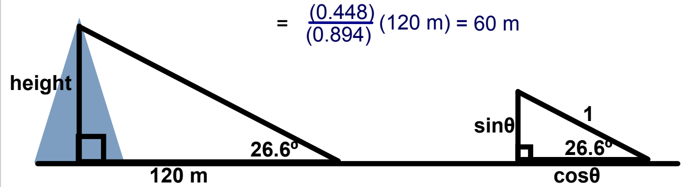
If the distance was 120 m, and the angle was 26.6º, you find the height by solving for it and then using your calculator to look up the values for sin and cos.

$$\frac{\text{height}}{\text{distance}} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{\text{height}}{120\text{ m}} = \frac{\sin(26.6^\circ)}{\cos(26.6^\circ)}$$

$$\text{height} = \frac{\sin(26.6^\circ)}{\cos(26.6^\circ)}(120\text{ m})$$

$$= \frac{(0.448)}{(0.894)}(120\text{ m}) = 60\text{ m}$$



Slide 45 / 252

Tangentθ

Early in the last problem we found that: $\frac{\text{height}}{\text{distance}} = \frac{\sin\theta}{\cos\theta}$

This ratio of sine to cosine is used very often, and has its own name: Tangentθ, or tanθ for short.

Tangentθ is defined as Sine θ divided by Cosine θ.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Slide 46 / 252

Using Calculators with Trigonometry

The last step of that problem required finding the values of the sine and the cosine of 26.6º.

When working with trigonometry, you'll need to find the values of sine, cosine and other trig functions when given an angle.

This used to involve using tables, but now it's pretty simple to use a basic scientific calculator.

Slide 47 / 252

Using Calculators with Trigonometry

Basic scientific calculators are available on computers, tablets and smart phones.

They can also be a separate device, similar to the inexpensive calculator shown here. It can do everything you'll need for this course.

Slide 48 / 252

Using Calculators with Trigonometry

The trig functions we're going to be using right now are sine, cosine and tangent.

Those are marked in the box on the calculator.

On most calculators, they are noted by buttons which say

SIN

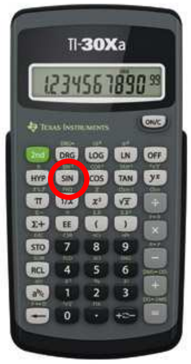
COS

TAN

Slide 49 / 252

Using Calculators with Trigonometry

This is for finding the sine of an angle.



Slide 50 / 252

Using Calculators with Trigonometry

This is for finding the cosine of an angle.

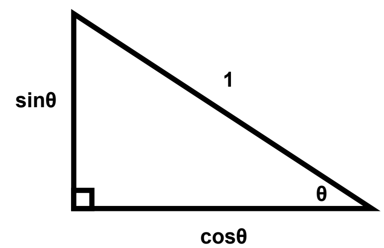
Slide 51 / 252

Using Calculators with Trigonometry

This is for finding the tangent of an angle.

Slide 52 / 252

Problem Solving with Trigonometry



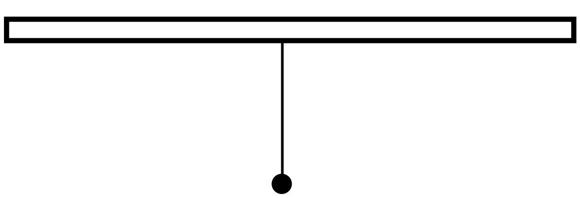
Slide 53 / 252

Inclinometer

In practice, we often have to measure angles of elevation or depression in order to solve problems.

There are very accurate ways of doing that which are used by surveyors, navigators and others.

But you can make a simple device, called an inclinometer, to accomplish the same thing, and then solve problems on your own.

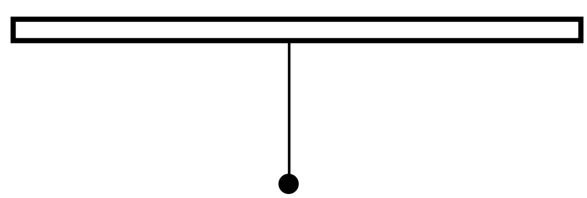


Slide 54 / 252

Inclinometer

Just tape a protractor to a meter stick and hang a small weight from the hole in the protractor.

Set it up so that when the meter stick is horizontal, the string goes straight down.

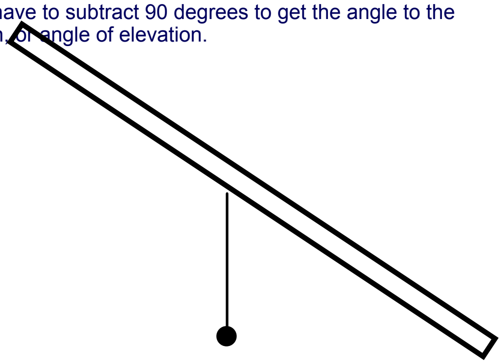


Slide 55 / 252

Inclinometer

Then, if you look along the meter stick, you can hold the string where it touches the protractor and read the angle.

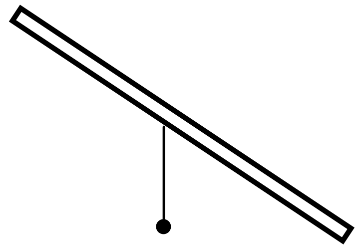
You'll have to subtract 90 degrees to get the angle to the horizon, or angle of elevation.



Slide 56 / 252

Inclinometer

You are standing on the ground and look along your inclinometer to see the top of a building to be at an angle of 30º. You then measure the distance to the base of the building to be 30 m. Find the height of the building, remembering to add in the height your eye is above the ground.



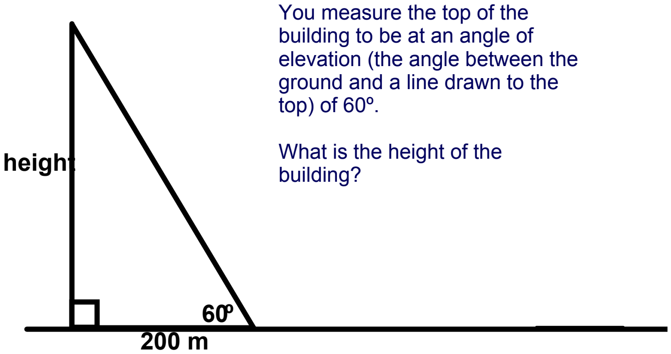
Slide 57 / 252

Example

You are standing 200 m away from the base of a building.

You measure the top of the building to be at an angle of elevation (the angle between the ground and a line drawn to the top) of 60º.

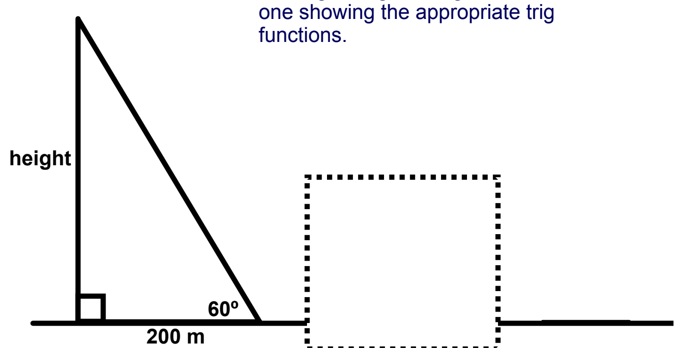
What is the height of the building?



Slide 58 / 252

Example

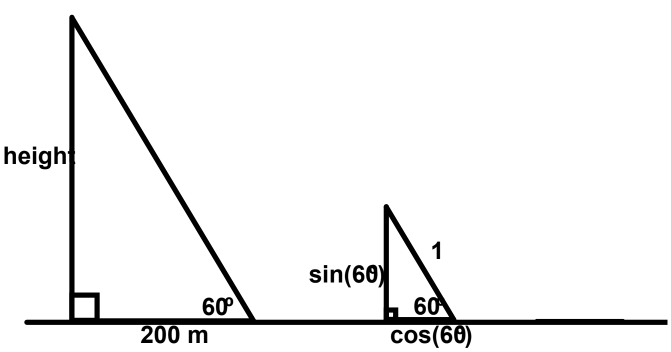
Make a quick sketch showing the original right triangle and one showing the appropriate trig functions.



Slide 59 / 252

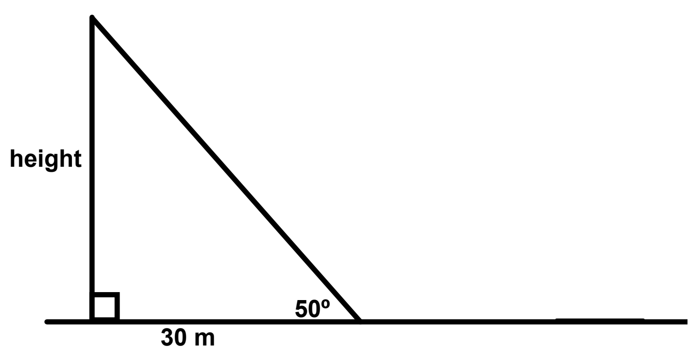
Example

Then set up the ratios, substitute the values and solve.



Slide 60 / 252

9 You are standing 30 m away from the base of a building. The top of the building lies at an angle of elevation (the angle between the ground and the hypotenuse) of 50º. What is the height of the building?



10 You are standing 50 m away from the base of a building. The building creates an angle of elevation with the ground measuring 80º. What is the height of the building?

11 Use the tanθ function of your calculator to determine the height of a flagpole if it is 30 m away and it's angle of elevation with the ground measures 70º.

12 Use the tanθ function of your calculator to determine the height of a building if its base is 50 m away and it's angle of elevation with the ground measures 20º.

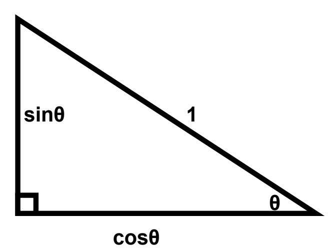
13 You are on top of a building and look down to see someone who standing the ground. The angle of depression (the angle below the horizontal to an object) is 30º and they are 90 m from the base of the building. How high is the building? (Neglect the heights of you and the other person.)

Make sure to draw a sketch!

14 Determine the distance an object lies from the base of a 45 m tall building if the angle of depression to it is 40º.

Trigonometric Ratios

When solving problems with trig, you find a right triangle which is similar to the one below. Then you find the solution by setting up the ratios of proportion. But, since the hypotenuse is 1, often it's forgotten that these are ratios.



Slide 67 / 252

Trigonometric Ratios

Return to the
Table of Contents

Slide 68 / 252

Trigonometric Ratios

Fill in the fundamental trig ratios below:

________________ called "sin" for short
click

________________ called "cos" for short
click

________________ called "tan" for short
click

Slide 69 / 252

Trigonometric Ratios

The name of the angle usually follows the trig function. If the angle is named θ (theta) the names become:

- sinθ
- cosθ
- tanθ

If the angle is named α (alpha) the functions become:

- sinα
- cosα
- tanα

Slide 70 / 252

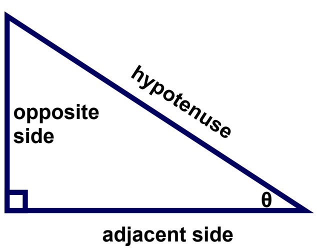
Trigonometric Ratios

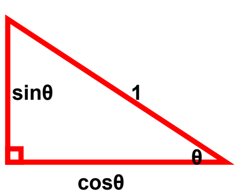
If you have the sides, trig ratios let you find the angles.

But if you have a side and an angle, trig ratios also let you find the other sides.

Slide 71 / 252

Trigonometric Ratios





These ratios depend on which angle you are calling θ; never the right angle.

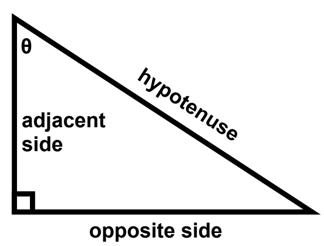
You know that the side opposite the right angle is called the hypotenuse.

The leg opposite θ is called the opposite side.

The leg that touches θ is called the adjacent side.

Slide 72 / 252

Trigonometric Ratios



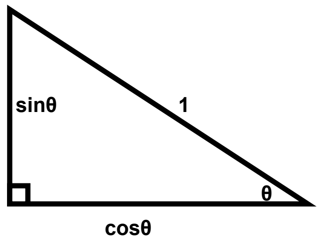
There are two possible angles that can be called #.

Once you choose which angle is #, the names of the sides are defined.

You can change later, but then the names of the sides also change.

Slide 73 / 252

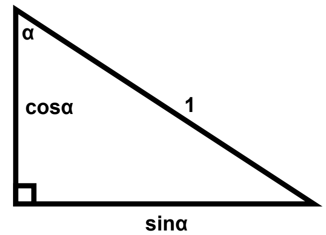
Trigonometric Ratios



With this theta, these become the sides.

Slide 74 / 252

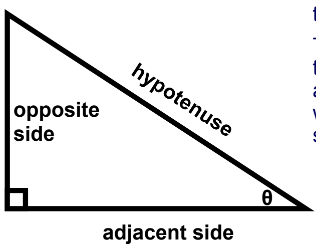
Trigonometric Ratios



If you use the other angle, named α here, the names change accordingly.

Slide 75 / 252

Trigonometric Ratios

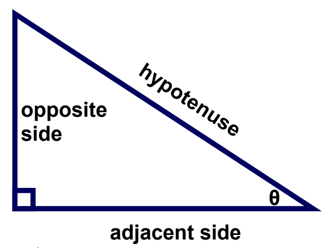


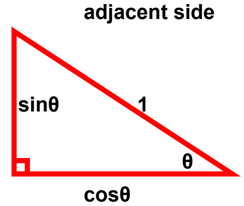
Let's say I'm solving a problem that involves this right triangle.

To use trig, I'd find a right triangle with hypotenuse of 1 and legs of sinθ and cosθ which has the same angle θ so, it's similar.

Slide 76 / 252

Trigonometric Ratios





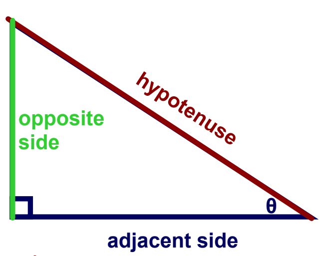
Then set up the ratios.

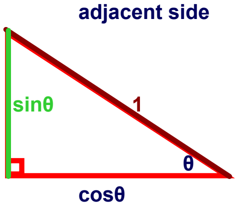
There are basic ratios relating the sides of these two triangles.

Since they are similar triangles, the ratio of any two sides in one triangle is equal to that ratio of sides in the other.

Slide 77 / 252

Trigonometric Ratios

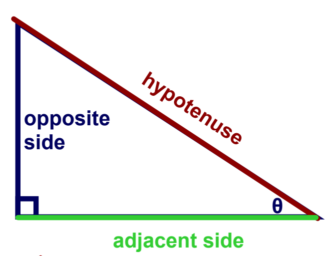


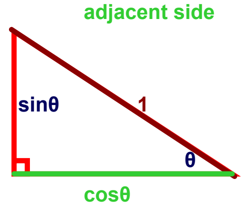


$$\frac{\sin\theta}{1} = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

Slide 78 / 252

Trigonometric Ratios

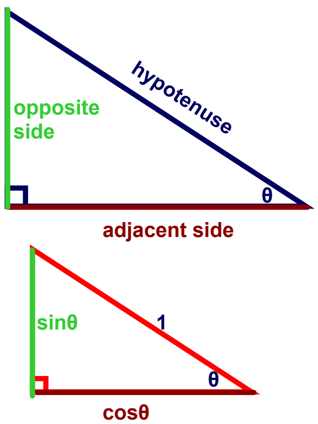




$$\frac{\cos\theta}{1} = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$$

Slide 79 / 252

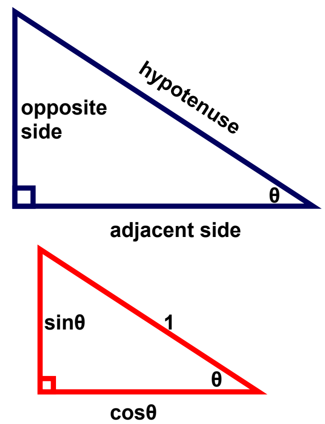
Trigonometric Ratios



$$\frac{\sin\theta}{\cos\theta} = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opp}}{\text{adj}}$$

Slide 80 / 252

Trigonometric Ratios



$$\frac{\sin\theta}{1} = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

$$\frac{\cos\theta}{1} = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opp}}{\text{adj}}$$

Slide 81 / 252

Trigonometric Ratios

But these can be simplified since:

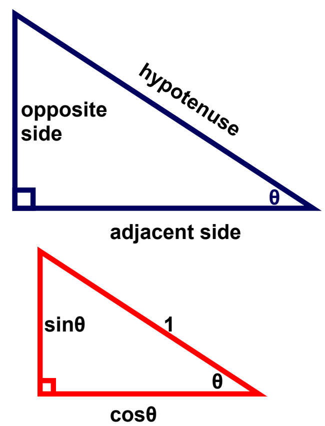
$$\frac{\sin\theta}{1} = \sin\theta \qquad \frac{\sin\theta}{1} = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

$$\frac{\cos\theta}{1} = \cos\theta \qquad \frac{\cos\theta}{1} = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \frac{\sin\theta}{\cos\theta} = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opp}}{\text{adj}}$$

Slide 82 / 252

Trigonometric Ratios



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opp}}{\text{adj}}$$

Slide 83 / 252

Trigonometric Ratios

These trig ratios are used so often that they are memorized with the expression "SOH CAH TOA."

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}} \quad \textbf{SOH}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}} \quad \textbf{CAH}$$

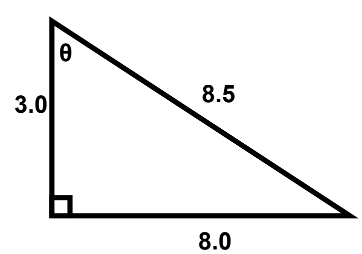
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opp}}{\text{adj}} \quad \textbf{TOA}$$

If you get confused w/ the vowel sounds in SOH CAH TOA, you could also try the mnemonic sentence below.

Some Old Horse
Caught Another Horse
Taking Oats Away.

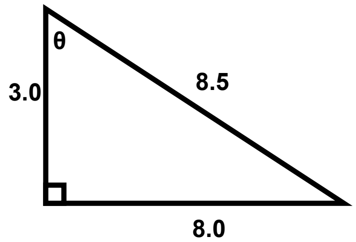
Slide 84 / 252

15



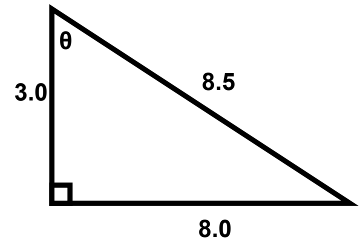
Slide 85 / 252

16 Find the cosθ. Round your answer to the nearest hundredth.



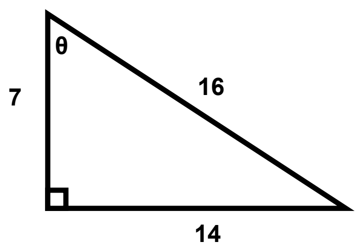
Slide 86 / 252

17 Find the tanθ. Round your answer to the nearest hundredth.



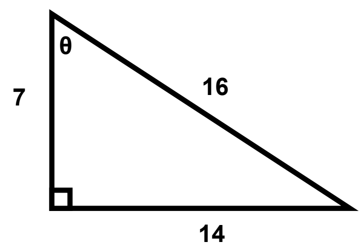
Slide 87 / 252

18 Find the tanθ. Round your answer to the nearest hundredth.



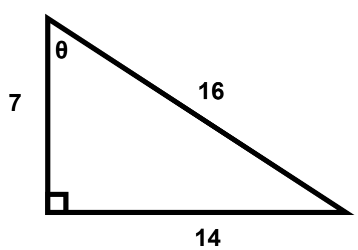
Slide 88 / 252

19 Find the sinθ. Round your answer to the nearest hundredth.



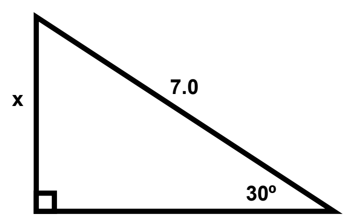
Slide 89 / 252

20 Find the cosθ. Round your answer to the nearest hundredth.



Slide 90 / 252

Trigonometric Ratios



For instance, let's find the length of side x.

The side we're looking for is opposite the given angle;

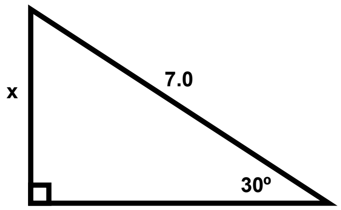
and the given length is the hypotenuse;

so we'll use the trig function that relates these three:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

Slide 91 / 252

Trigonometric Ratios



$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$

opp = (hyp) (sinθ)

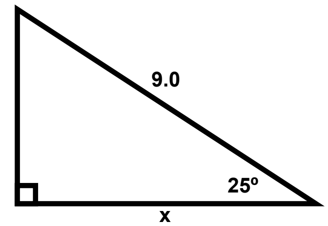
x = (7.0)(sin(30°))

x = (7.0)(0.50)

x = 3.5

Slide 92 / 252

Trigonometric Ratios



Now, let's find the length of side x in this case.

The side we're looking for is adjacent the given angle;

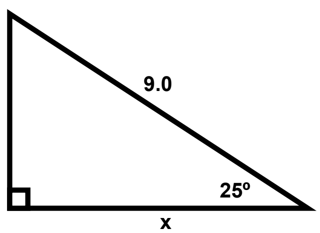
and the given length is the hypotenuse;

so we'll use the trig function that relates these three:

$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$

Slide 93 / 252

Trigonometric Ratios



$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

adj = (hyp)(cosθ)

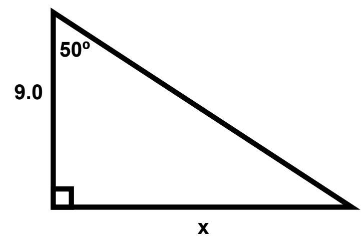
x = (9.0)(cos(25°))

x = (9.0)(0.91)

x = 8.2

Slide 94 / 252

Trigonometric Ratios



Now, let's find the length of side x in this case.

The side we're looking for is adjacent the given angle;

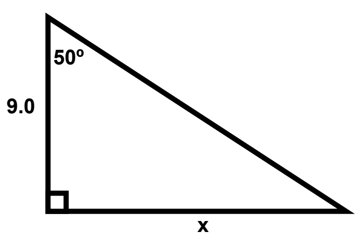
and the given length is the opposite the given angle;

so we'll use the trig function that relates these three:

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opp}}{\text{adj}}$

Slide 95 / 252

Trigonometric Ratios



$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opp}}{\text{adj}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

opp = (adj)(tanθ)

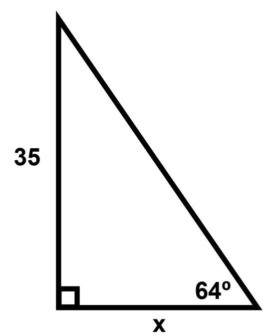
x = (9.0)(tan(50°))

x = (9.0)(1.2)

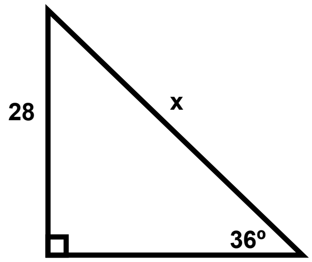
x = 10.8

Slide 96 / 252

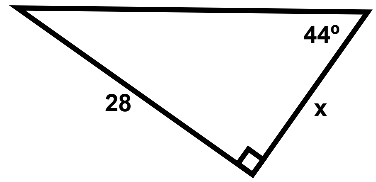
21 Find the value of x. Round your answer to the nearest tenth.



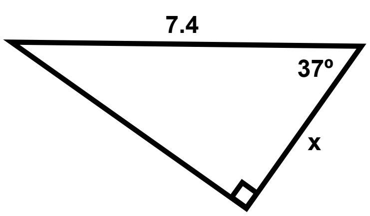
22 Find the value of x. Round your answer to the nearest tenth.



23 Find the value of x. Round your answer to the nearest tenth.



24 Find the value of x. Round your answer to the nearest tenth.



Applications of Trigonometric Ratios

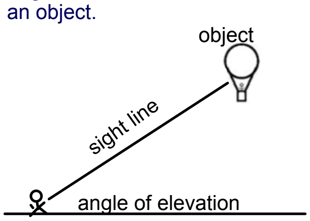
Most of the time, trigonometric ratios are used to solve real-world problems, as you saw at the beginning of this unit.

Now that you are familiar with the derivation of the three trigonometric ratios (sine, cosine, and tangent), you are ready to apply your knowledge and practice solving these problems.

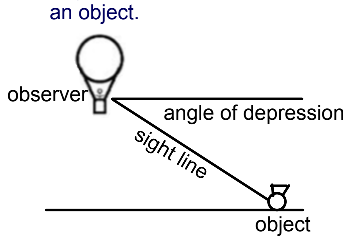
Before we begin, let's review some key vocabulary that you will see in these word problems.

Applications of Trigonometric Ratios

The angle of elevation is the angle above the horizontal to an object.

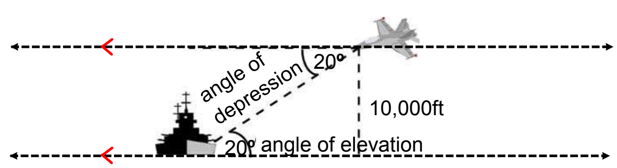


The angle of depression is the angle below the horizontal to an object.



Applications of Trigonometric Ratios

The angle of elevation and the angle of depression are both measured relative to parallel horizontal lines, so they are equal in measure.



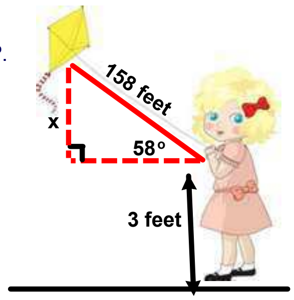
Applications of Trigonometric Ratios

Example

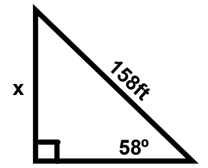
Amy is flying a kite at an angle of 58°.

The kite's string is 158 feet long and Amy's arm is 3 feet off the ground.

How high is the kite off the ground?



Applications of Trigonometric Ratios



$\sin\theta = \frac{x}{158}$

$\sin 58 = \frac{x}{158}$

$.8480 = \frac{x}{158}$

$x = 134$

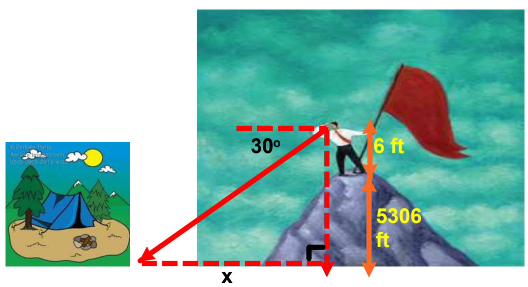
Now, we must add in Amy's arm height.
134 + 3 = 137

The kite is about 137 feet off the ground.

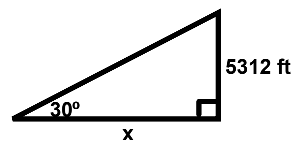
Applications of Trigonometric Ratios

Example

You are standing on a mountain that is 5306 feet high. You look down at your campsite at angle of 30°. If you are 6 feet tall, how far is the base of the mountain from the campsite?



Applications of Trigonometric Ratios



$\tan 30 = \frac{5312}{x}$

$.5774 = \frac{5312}{x}$

$.5774x = 5312$

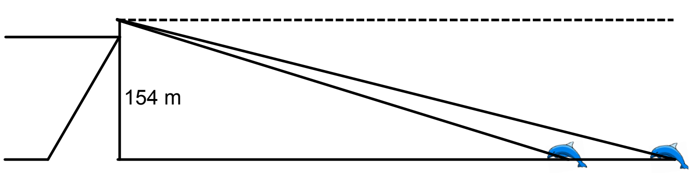
$x \approx 9{,}200$ ft

The campsite is about 9,200 ft from the base of the mountain.

Applications of Trigonometric Ratios

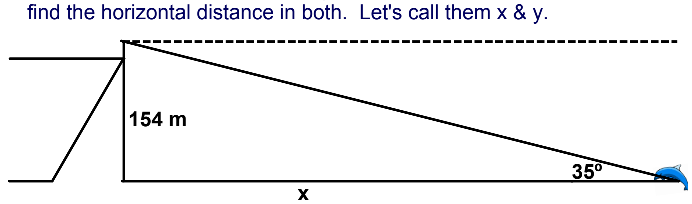
Example:

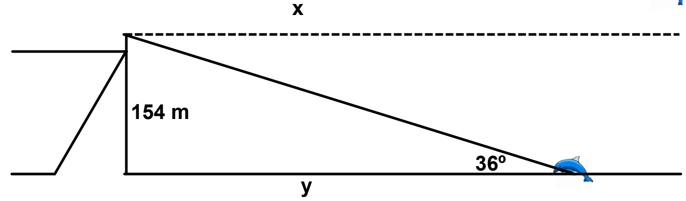
Vernon is on the top deck of a cruise ship and observes 2 dolphins following each other directly away from the ship in a straight line. Veron's position is 154 m above sea level, and the angles of depression to the 2 dolphins to the ship are 35° and 36°, respectively. Find the distance between the 2 dolphins to the nearest hundredth of a meter.



Applications of Trigonometric Ratios

The first step is to divide the diagram into two separate ones. Then, find the horizontal distance in both. Let's call them x & y.

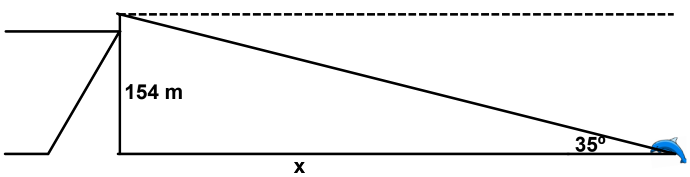




Then, use your trigonometric ratios to find these values.

Slide 109 / 252

Applications of Trigonometric Ratios



$\tan 35 = \frac{154}{x}$

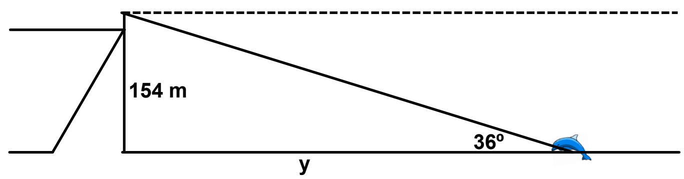
$0.7002 = \frac{154}{x}$

$0.7002x = 154$

$x = 219.94$ m

Slide 110 / 252

Applications of Trigonometric Ratios



$\tan 36 = \frac{154}{y}$

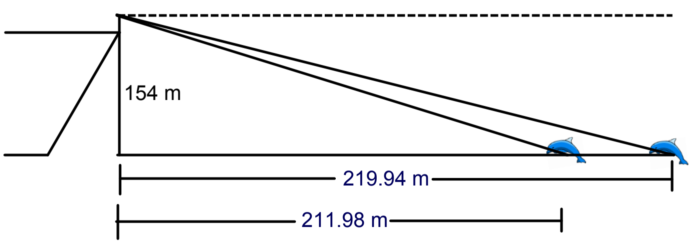
$0.7265 = \frac{154}{y}$

$0.7265y = 154$

$y = 211.98$ m

Slide 111 / 252

Applications of Trigonometric Ratios



Now, if we subtract these measurements, then we will find the distance between the 2 dolphins.

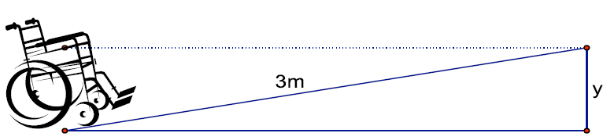
219.94 - 211.98 = 7.96 m

Slide 112 / 252

25 You are looking at the top of a tree. The angle of elevation is 55º. The distance from the top of the tree to your position (line of sight) is 84 feet. If you are 5.5 feet tall, how far are you from the base of the tree?

Slide 113 / 252

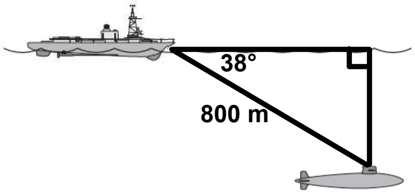
26 A wheelchair ramp is 3 meters long and inclines at 6º. Find the height of the ramp to the nearest hundredth of a centimeter.



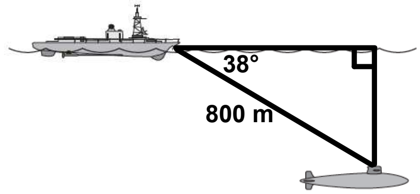
Slide 114 / 252

27 John wants to find the height of a building which is casting a shadow of 175 ft at an angle of 73.75º. Find the height of the building to the nearest foot.

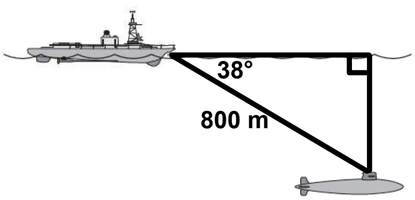
28 A sonar operator on a ship detects a submarine that is located 800 meters away from the ship at an angle of depression of 38°. How deep is the submarine?



29 A sonar operator on a ship detects a submarine that is located 800 meters away from the ship at an angle of depression of 38°. If the submarine stays in the same position, then how far would the ship need to travel to be directly above the submarine?



30 The ship is traveling at a speed of 32 meters per second, in the direction towards the submarine. From its current position, how many minutes, to the nearest tenth of a minute, will it take the ship to be directly over the submarine.



Inverse Trigonometric Ratios

Return to the
Table of Contents

Inverse Trigonometric Ratios

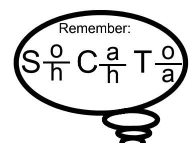
So far, you have used the sine, cosine, and tangent ratios when given the measurement of the acute angle θ in a right triangle to find the measurements of the missing sides.

What can you use when you need to find the measurements of the acute angles?

We have what are called the inverse sine, inverse cosine and inverse tangent ratios that will help us answer the question above. If you know the measures of 2 sides of a triangle, then you can find the measurement of the angle with these ratios.

Inverse Trigonometric Ratios

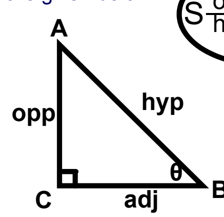
The Inverse Trigonometric Ratios are given below



If $\sin\theta = \frac{opp}{hyp}$, $\theta = \sin^{-1}\left(\frac{opp}{hyp}\right)$

If $\cos\theta = \frac{adj}{hyp}$, $\theta = \cos^{-1}\left(\frac{adj}{hyp}\right)$

If $\tan\theta = \frac{opp}{adj}$, $\theta = \tan^{-1}\left(\frac{opp}{adj}\right)$



Slide 121 / 252

Using Calculators with Inverse Trigonometry

The inverse trig functions are located just above the sine, cosine and tangent buttons.

They are marked in the box on the calculator.

On most calculators, they are noted by text which says

SIN^{-1}

COS^{-1}

TAN^{-1}

In most cases, they can be used by pressing the 2nd, or shift, button (arrow pointing to it) & the sine, cosine, or tangent button.

Slide 122 / 252

31 Find $\sin^{-1}(0.8)$ Round the angle measure to the nearest hundredth.

Slide 123 / 252

32 Find $\tan^{-1}(2.3)$. Round the angle measure to the nearest hundredth.

Slide 124 / 252

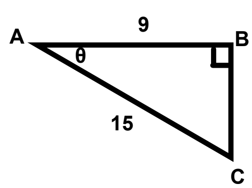
33 Find $\cos^{-1}(0.45)$. Round the angle measurement to the nearest hundredth.

Slide 125 / 252

Inverse Trigonometric Ratios

To find an unknown angle measure in a right triangle,you need to identify the correct trig function that will find the missing value. Use "SOH CAH TOA" to help.

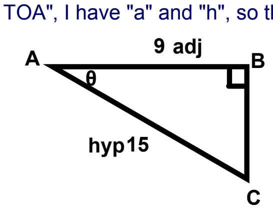
∠A is your angle of reference.
Label the two given sides
of your triangle opp, adj, or hyp.
Identify the trig funtion that uses ∠A, and the two sides.



Slide 126 / 252

Inverse Trigonometric Ratios

Using "SOH CAH TOA", I have "a" and "h", so the ratio is a/h which is cosine.



$\cos A = \frac{9}{15}$

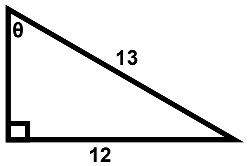
now you can solve for m∠A, the missing angle using the inverse trig function.

$m\angle A = \cos^{-1}\left(\frac{9}{15}\right)$

$m\angle A = 53.13°$

Once you find m∠A, you can easily find m∠C, using the Triangle Sum Theorem.

Inverse Trigonometric Ratios



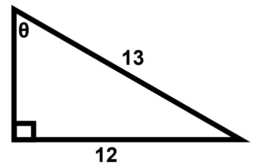
Now, let's find the measurement of the angle θ in this case.

The sides that we are given are the opposite side & the hypotenuse;

so we'll use the trig function that relates these two sides with our angle:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

Inverse Trigonometric Ratios

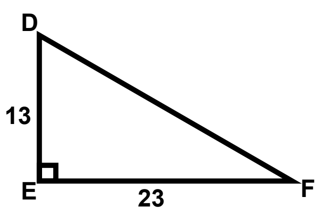


$$\sin\theta = \frac{12}{13}$$

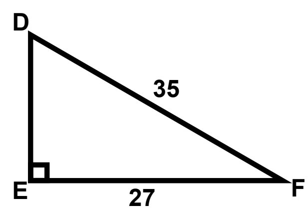
$$\theta = \sin^{-1}\left(\frac{12}{13}\right)$$

$$\theta = 67.38°$$

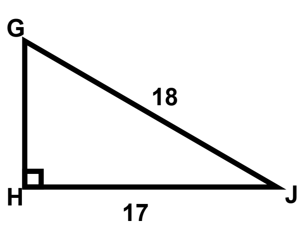
34 Find the m∠D in the figure below.



35 Find the m∠F in the figure below.



36 Find the m∠G in the figure below.



Applications of Inverse Trigonometric Ratios

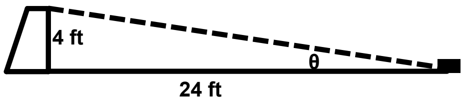
As we discussed earlier in this unit, trigonometric ratios and the inverse trigonometric ratios are used to solve real-world problems.

Now that you are familiar with the three inverse trigonometric ratios (inverse sine, inverse cosine, and inverse tangent), you are ready to apply your knowledge and practice solving these problems.

Slide 133 / 252

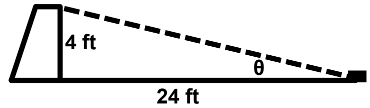
Applications of Inverse Trigonometric Ratios

A hockey player is 24 feet from the goal line. He shoots the puck directly at the goal. The height of the goal is 4 feet. What is the maximum angle of elevation at which the player can shoot the puck and still score a goal?



Slide 134 / 252

Applications of Inverse Trigonometric Ratios



$\tan\theta = \frac{4}{24}$

$\theta = \tan^{-1}\left(\frac{4}{24}\right)$

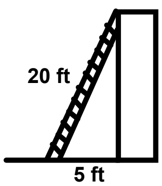
$\theta = 9.46°$

The angle of elevation that the player can shoot the puck is a maximum of 9.46°.

Slide 135 / 252

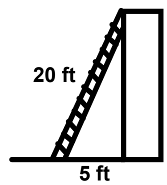
Applications of Inverse Trigonometric Ratios

You lean a 20 foot ladder up against a wall. The base of the ladder is 5 feet from the edge of the wall. What is the angle of elevation is created by the ladder & the ground.



Slide 136 / 252

Applications of Inverse Trigonometric Ratios



$\cos\theta = \frac{5}{20}$

$\theta = \cos^{-1}\left(\frac{5}{20}\right)$

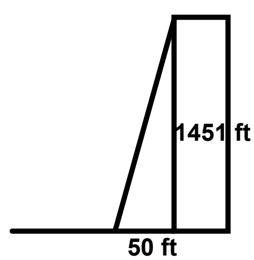
$\theta = 75.52°$

Slide 137 / 252

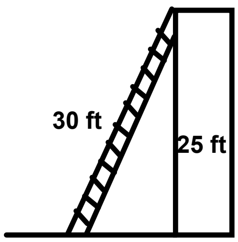
37 Katherine looks down out of the crown of the statue of liberty to an incoming ferry about 345 feet. The distance from crown to the ground is about 250 feet. What is the angle of depression?

Slide 138 / 252

38 The Sear's Tower in Chicago, Illinois is 1451 feet tall. The sun is casting a 50 foot shadow on the ground. What is the angle of elevation created by the tip of the shadow and the ground?



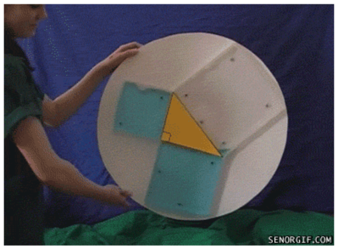
39 You lean a 30 foot ladder up against the side of your home to get into a bedroom on the second floor. The height of the window is 25 feet. What angle of elevation must you set the ladder at in order to reach the window?



40 You are looking out your bedroom window towards the tip of the shadow made by your home. Your friend measures the length of the shadow to be 10 feet long. If you are20 feet off the ground, what is the angle of depression needed to see the tip of your home's shadow.

41 You return to view your home's shadow 3 hours later. Your friend measures the length of the shadow to be 25 feet long. If you are20 feet off the ground, what is the angle of depression needed to see the tip of your home's shadow.

Review of the Pythagorean Theorem



Return to the
Table of Contents

Review of Pythagorean Theorem

$c^2 = a^2 + b^2$

"c" is the hypotenuse

"a" and "b" are the two legs;

which leg is "a" and which is "b" doesn't matter.

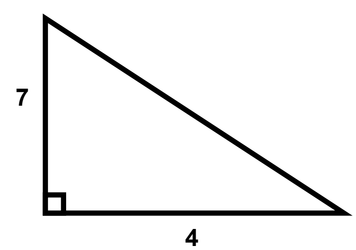
42 The legs of a right triangle are 7.0m and 3.0m, what is the length of the hypotenuse?

43 The legs of a right triangle are 2.0m and 12m, what is the length of the hypotenuse?

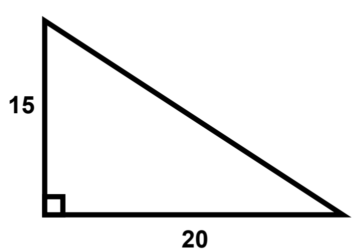
44 The hypotenuse of a right triangle has a length of 4.0m and one of its legs has a length of 2.5m. What is the length of the other leg?

45 The hypotenuse of a right triangle has a length of 9.0m and one of its legs has a length of 4.5m. What is the length of the other leg?

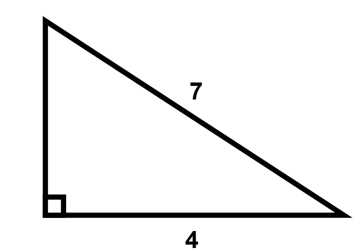
46 What is the length of the third side?



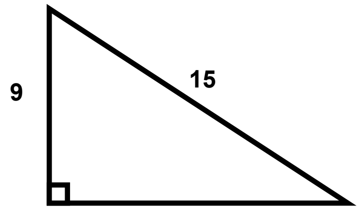
47 What is the length of the third side?



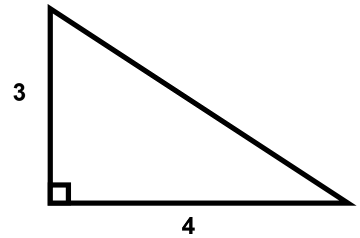
48 What is the length of the third side?



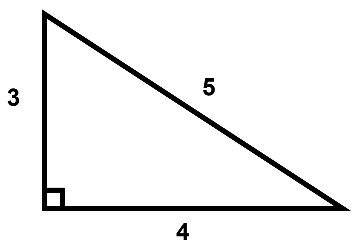
49 What is the length of the third side?



50 What is the length of the third side?



Pythagorean Triples

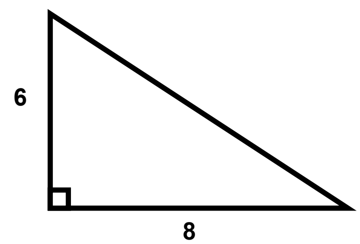


Triples are integer solutions of the Pythagorean Theorem.

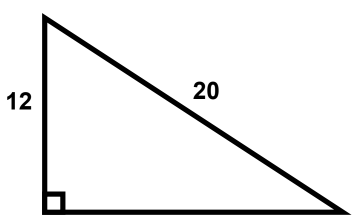
3-4-5 is the most famous of the triples:

You don't need a calculator if you recognize the sides are in this ratio.

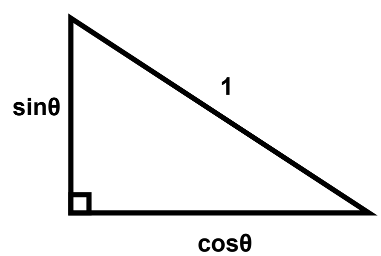
51 What is the length of the third side?



52 What is the length of the third side?



53 $(\sin\theta)^2 + (\cos\theta)^2 = ?$



54 Katherine looks down out of the crown of the statue of liberty to an incoming ferry about 345 feet. The distance from crown to the ground is about 250 feet. What is the distance from the ferry to the base of the statue?

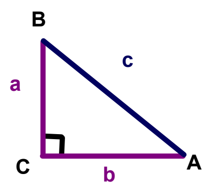
Converse of the Pythagorean Theorem

Return to the Table of Contents

Converse of the Pythagorean Theorem

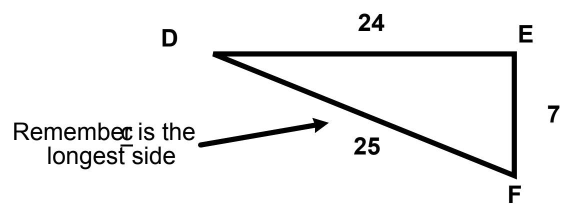
If the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

If $c^2 = a^2 + b^2$, then ΔABC is a right triangle.



Example

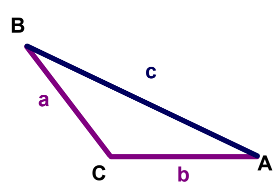
Tell whether the triangle is a right triangle
Explain your reasoning.



Theorem

If the square of the longest side of a triangle is greater than the sum of the squares of the other two sides, then the triangle is obtuse.

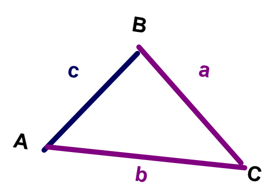
If $c^2 > a^2 + b^2$, then ΔABC is obtuse.



Theorem

If the square of the longest side of a triangle is less than the sum of the squares of the other two sides, then the triangle is acute.

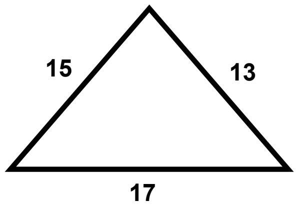
If $c^2 < a^2 + b^2$, then ΔABC is acute.



Slide 163 / 252

Example

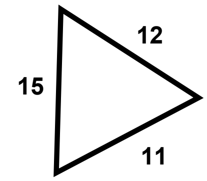
Classify the triangle as acute, right, or obtuse.
Explain your reasoning.



Slide 164 / 252

55 Classify the triangle as acute, right, obtuse, or not a triangle.

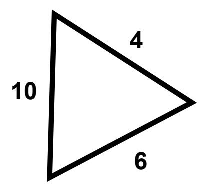
- A acute
- B right
- C obtuse
- D not a triangle



Slide 165 / 252

56 Classify the triangle as acute, right, obtuse, or not a triangle.

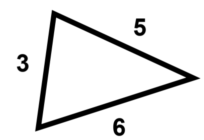
- A acute
- B right
- C obtuse
- D not a triangle



Slide 166 / 252

57 Classify the triangle as acute, right, obtuse, or not a triangle.

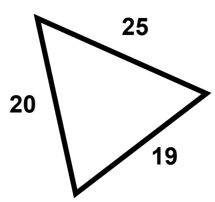
- A acute
- B right
- C obtuse
- D not a triangle



Slide 167 / 252

58 Classify the triangle as acute, right, obtuse, or not a triangle.

- A acute
- B right
- C obtuse
- D not a triangle



Slide 168 / 252

59 Tell whether the lengths 35, 65, and 56 represent the sides of an acute, right, or obtuse triangle.

- A acute
- B right
- C obtuse

Slide 169 / 252

60 Tell whether the lengths represent the sides of an acute, right, or obtuse triangle.

$$\sqrt{3}, 2, 3$$

- A acute triangle
- B right triangle
- C obtuse triangle

Slide 170 / 252

Review

If $c^2 = a^2 + b^2$, then triangle is right.

If $c^2 > a^2 + b^2$, then triangle is obtuse.

If $c^2 < a^2 + b^2$, then triangle is acute.

Slide 171 / 252

Special Right Triangles

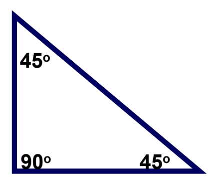
Return to the Table of Contents

Slide 172 / 252

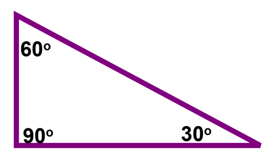
Special Right Triangles

In this section you will learn about the properties of the two special right triangles.

45-45-90



30-60-90

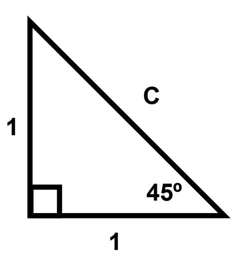


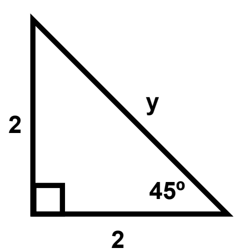
Slide 173 / 252

Investigation: 45-45-90 Triangle Theorem

Find the missing side lengths in the triangles.

Leave answers in simplified radical/fractional form...NO DECIMALS!



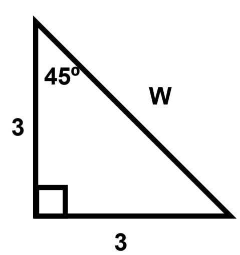


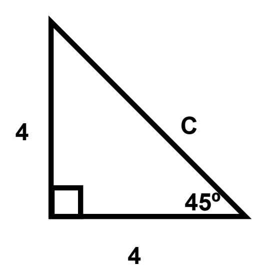
Slide 174 / 252

Investigation: 45-45-90 Triangle Theorem

Find the missing side lengths in the triangles.

Leave answers in simplified radical/fractional form...NO DECIMALS!

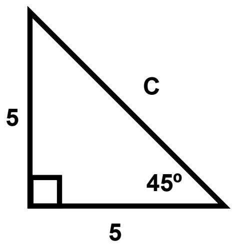


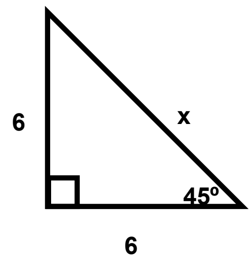


Investigation: 45-45-90 Triangle Theorem

Find the missing side lengths in the triangles.

Leave answers in simplified radical/fractional form...NO DECIMALS!





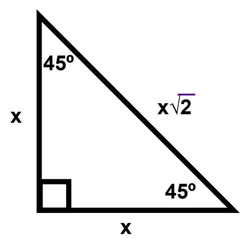
45-45-90 Triangle Theorem

Using the side lengths that you found in the Investigation, can you figure out the rule, or formula, for the 45-45-90 Triangle Theorem?

45-45-90 Triangle Theorem

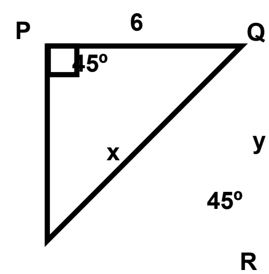
This theorem can be proved algebraically using Pythagorean Theorem.

$a^2 + b^2 = c^2$
$x^2 + x^2 = c^2$
$2x^2 = c^2$
$x\sqrt{2} = c$



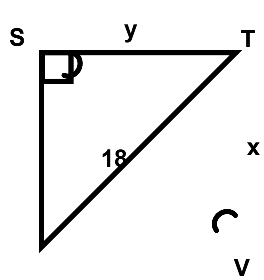
45-45-90 Example

Find the length of the missing sides.
Write the answer in simplest radical form.



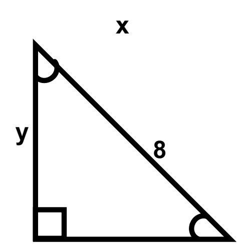
45-45-90 Example

Find the length of the missing sides.
Write the answer in simplest radical form.



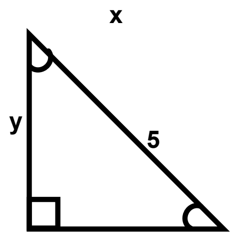
45-45-90 Example

Find the length of the missing sides.
Write the answer in simplest radical form.



61 Find the value of x.

- A 5
- B $5\sqrt{2}$
- C



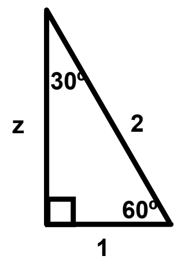
62 What is the length of the hypotenuse of an isosceles right triangle if the length of the legs is $8\sqrt{2}$ inches.

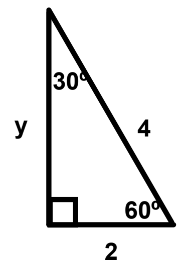
63 What is the length of each leg of an isosceles, if the length of the hypotenuse is 20 cm.

Investigation: 30-60-90 Triangle Theorem

Find the missing side lengths in the triangles.

Leave answers in simplified radical/fractional form...NO DECIMALS!

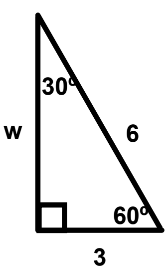


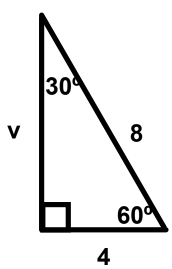


Investigation: 30-60-90 Triangle Theorem

Find the missing side lengths in the triangles.

Leave answers in simplified radical/fractional form...NO DECIMALS!

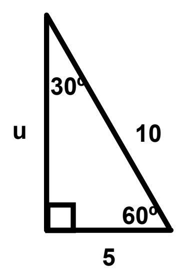


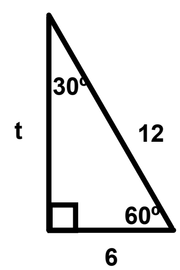


Investigation: 30-60-90 Triangle Theorem

Find the missing side lengths in the triangles.

Leave answers in simplified radical/fractional form...NO DECIMALS!





Slide 187 / 252

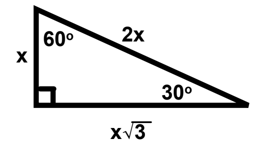
30-60-90 Triangle Theorem

Using the side lengths that you found in the Investigation, can you figure out the rule, or formula, for the 30-60-90 Triangle Theorem?

Slide 188 / 252

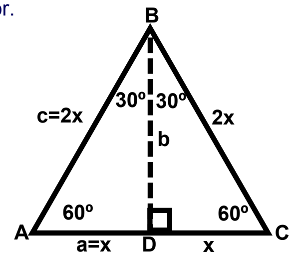
30-60-90 Triangle Theorem

This theorem can be proved using an equilateral triangle and Pythagorean Theorem.



For right triangle ABD,
$\overline{BD}$ is a perpendicular bisector.
let a = x, c = 2x and b = BD

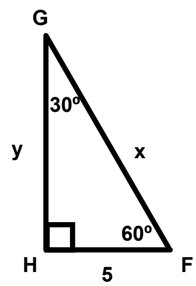
$a^2 + b^2 = c^2$
$x^2 + b^2 = (2x)^2$
$x^2 + b^2 = 4x^2$
$b^2 = 3x^2$
$b = x\sqrt{3}$



Slide 189 / 252

30-60-90 Example

Example:
Find the length of the missing sides of the right triangle.

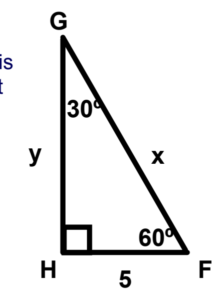


Slide 190 / 252

30-60-90 Example

Recall triangle inequality, the shortest side is opposite the smallest angle and the longest side is opposite the largest angle.

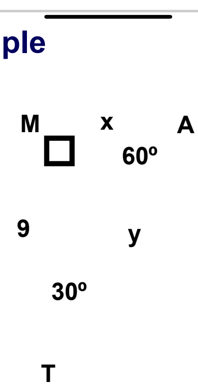
$\overline{HF}$ is the shortest side
$\overline{GF}$ is the longest side (hypotenuse)
$\overline{GH}$ is the 2nd longest side
HF < GH < GF



Slide 191 / 252

30-60-90 Example

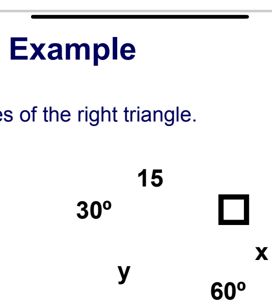
Example:
Find the length of the missing sides of the right triangle.



Slide 192 / 252

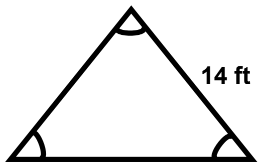
30-60-90 Example

Example:
Find the length of the missing sides of the right triangle.



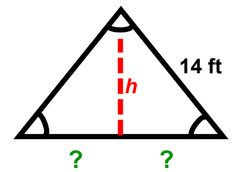
30-60-90 Example

Example:
Find the area of the triangle.



30-60-90 Example

The altitude (or height) divides the triangle into two 30°-60°-90° triangles.



The length of the shorter leg is 7 ft.

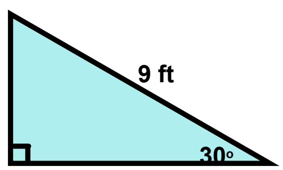
The length of the longer leg is $7\sqrt{3}$ ft.

$A = \frac{1}{2} b(h) = \frac{1}{2} 14(7\sqrt{3})$

$A = 49\sqrt{3}$ square ft ≈ 84.87 square ft

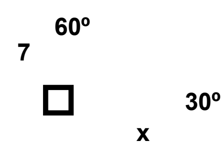
30-60-90 Example

Example:
Find the area of the triangle.



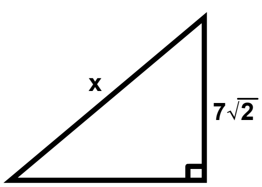
64 Find the value of x.

- A 7
- B $7\sqrt{3}$
- C $\frac{7\sqrt{2}}{2}$
- D 14



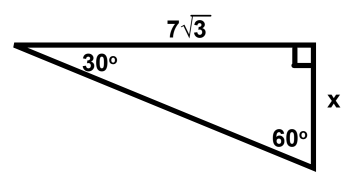
65 Find the value of x.

- A 7
- B $7\sqrt{3}$
- C $\frac{7\sqrt{2}}{2}$
- D 14



66 Find the value of x.

- A 7
- B $7\sqrt{3}$
- C $\frac{7\sqrt{2}}{2}$
- D 14



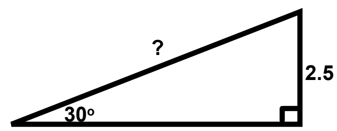
67 The hypotenuse of a 30º-60º-90º triangle is 13 cm. What is the length of the shorter leg?

68 The length the longer leg of a 30º-60º-90º triangle is 7 cm. What is the length of the hypotenuse?

Real World Example

The wheelchair ramp at your school has a height of 2.5 feet and rises at angle of 30º. What is the length of the ramp?

Real World Example

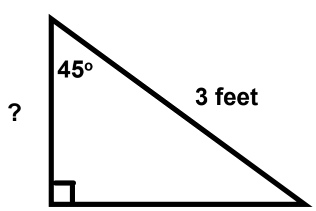


The triangle formed by the ramp is a 30º-60º-90º right triangle. The length of the ramp is the hypotenuse.

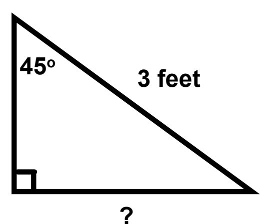
hypotenuse = 2(shorter leg)
hypotenuse = 2(2.5)
hypotenuse = 5

The ramp is 5 feet long.

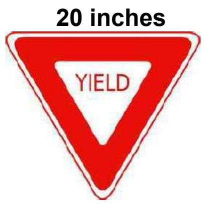
69 A skateboarder constructs a ramp using plywood. The length of the plywood is 3 feet long and falls at an angle of 45º. What is the height of the ramp? Round to the nearest hundredth.



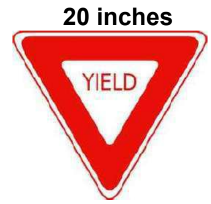
70 What is the length of the base of the ramp? Round to the nearest hundredth.



71 The yield sign is shaped like an equilateral triangle. Find the length of the altitude.



72 The yield sign is shaped like an equilateral triangle. Find the area of the sign.



PARCC Sample Questions

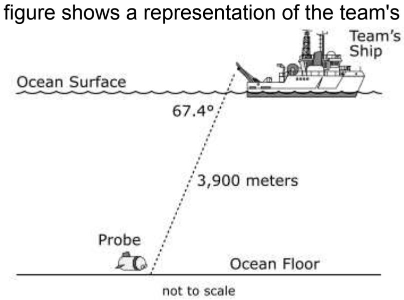
The remaining slides in this presentation contain questions from the PARCC Sample Test. After finishing this unit, you should be able to answer these questions.

Good Luck!

Return to Table of Contents

Question 10/25 **Topic: Trigonometric Ratios**

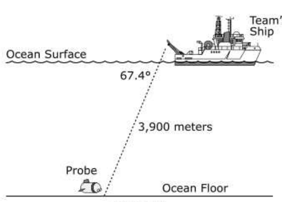
An archaeological team is excavating artifacts from a sunken merchant vessel on the ocean floor. To help with teh exploration the team uses a robotic probe. The probe travels approximately 3,900 meters at an angle of depression of 67.4 degrees from the team's ship on the ocean surface down to the sunken vessel on the ocean floor. The figure shows a representation of the team's sip and the probe.



PARCC Released Question (EOY)

Question 10/25 **Topic: Trigonometric Ratios**

73 When the probe reaches the ocean floor, the probe will be approximately __________ meters below the ocean surface.

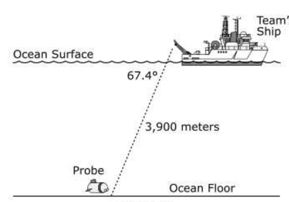


- A 1,247
- B 1,500
- C 1,623
- D 3,377
- E 3,600

PARCC Released Question (EOY)

Question 10/25 **Topic: Trigonometric Ratios**

74 When the probe reaches the ocean floor, the horizontal distance of the probe behind the team's ship on the ocean surface will be approximately ___________ meters.



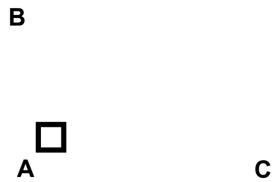
- F 1,247
- G 1,500
- H 1,623
- I 3,377
- J 3,600

PARCC Released Question (EOY)

Question 3/25 **Topic: Trigonometric Ratios**

75 In right triangle ABC, $m\angle B \neq m\angle C$. Let $\sin B = r$ and $\cos B = s$. What is $\sin C - \cos C$?

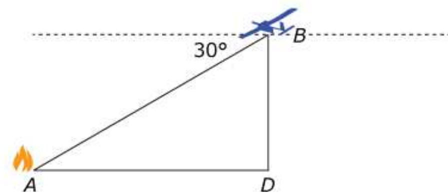
- A $r + s$
- B $r - s$
- C $s - r$
- D $\frac{r}{s}$



PARCC Released Question (EOY)

Question 16/25 **Topic: Trigonometric Ratios**

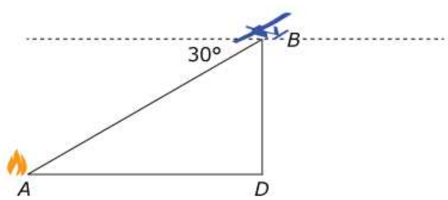
An unmanned aerial vehicle (UAV) is equipped with cameras used to monitor forest fires. The figure represents a moment in time at which a UAV, at point *B*, flying at an altitude of 1,000 meters (m) is directly above point *D* on the forest floor. Point *A* represents the location of a small fire on the forest floor.



PARCC Released Question (EOY)

Question 16/25 Part A **Topic: Trigonometric Ratios**

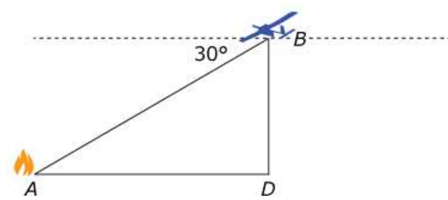
76 At the moment in time represented by the figure, the angle of depression from the UAV to the fire has a measure of 30°. At the moment in time represented by the figure, what is the distance from the UAV to the fire?



PARCC Released Question (EOY)

Question 16/25 Part B **Topic: Trigonometric Ratios**

77 What is the distance, to the nearest meter, from the fire to point *D*?

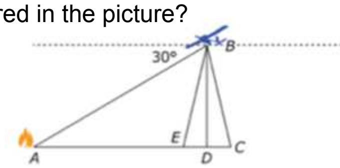


PARCC Released Question (EOY)

Question 16/25 Part C **Topic: Trigonometric Ratios**

78 Points *C* and *E* represent the linear range of view of the camera when it is pointed directly down at point *D*. The field of view of the camera is 20° and is represented in the figure by $\angle CBE$. The camera takes a picture directly over point *D*, what is the approximate width of the forest floor that will be captured in the picture?

- A 170 meters
- B 353 meters
- C 364 meters
- D 728 meters

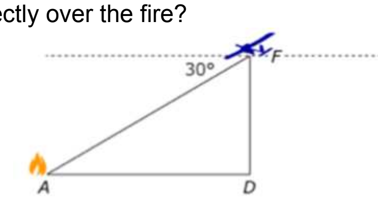


PARCC Released Question (EOY)

Question 16/25 Part D **Topic: Trigonometric Ratios**

79 The UAV is flying at a speed of 13 meters per second in the direction toward the fire. Suppose the altitude of the UAV is now 800 meters. The new position is represented at *F* in the figure. From its position at point *F*, how many minutes, to the nearest tenth of a minute, will it take the UAV to be directly over the fire?

- A 0.6
- B 1.2
- C 1.8
- D 2.0

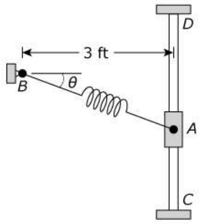


PARCC Released Question (EOY)

Slide 217 / 252

Question 20/25 Part A — **Topic: Trigonometric Ratios**

A spring is attached at one end to support B and at the other end to collar A, as represented in the figure. Collar A slides along the vertical bar between points C and D. In the figure, the angle θ is the angle created as the collar moves between points C and D.



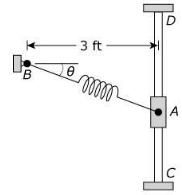
80 When θ = 28°, what is the distance from point A to point B to the nearest tenth of a foot?

PARCC Released Question (EOY)

Slide 218 / 252

Question 20/25 Part B — **Topic: Inverse Trigonometric Ratios**

A spring is attached at one end to support B and at the other end to collar A, as represented in the figure. Collar A slides along the vertical bar between points C and D. In the figure, the angle θ is the angle created as the collar moves between points C and D.



81 When the spring is stretched and the distance from A to B is 5.2 feet, what is the value of θ to the nearest tenth of a degree?

- ○ 35.2°
- ○ 54.8°
- ○ 45.1°
- ○ 60.0°

PARCC Released Question (EOY)

Slide 219 / 252

Question 4/7 — **Topic: Trigonometric Ratios**

82 Right triangle WXY is similar to triangle DEF. The following are measurements in right triangle DEF.

m∠F = 90º
DE = $\sqrt{113}$
DF = 7
EF = 8

Write an expression that represents cos W.

Which number represents the **numerator** of the fraction?

- ○ 90
- ○ $\sqrt{113}$
- ○ 7
- ○ 8

$\cos W = \frac{\square}{\square}$

PARCC Released Question (PBA)

Slide 220 / 252

Question 4/7 — **Topic: Trigonometric Ratios**

83 Right triangle WXY is similar to triangle DEF. The following are measurements in right triangle DEF.

m∠F = 90º
DE = $\sqrt{113}$
DF = 7
EF = 8

Write an expression that represents cos W.

Which number represents the **denominator** of the fraction?

- ○ 90
- ○ $\sqrt{113}$
- ○ 7
- ○ 8

$\cos W = \frac{\square}{\square}$

PARCC Released Question (PBA)

Slide 221 / 252

Question 6/7 — **Topic: Trigonometric Ratios**

84 The degree measure of an angle in a right triangle is x, and sin x = 1/3. Which of these expressions are also equal to 1/3? Select **all** that apply.

- ☐ cos(x)
- ☐ cos(x - 45°)
- ☐ cos(45° - x)
- ☐ cos(60° - x)
- ☐ cos(90° - x)

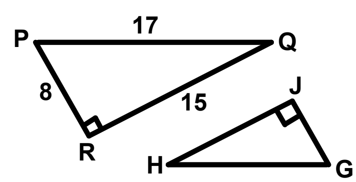
PARCC Released Question (PBA)

Slide 222 / 252

Question 7/7 — **Topic: Trigonometric Ratios**

85 In this figure, triangle GHJ is similar to triangle PQR. Based on this information, which ratio represents tan H?

- ○ A $\frac{8}{15}$
- ○ B $\frac{8}{17}$
- ○ C $\frac{15}{8}$
- ○ D $\frac{17}{8}$

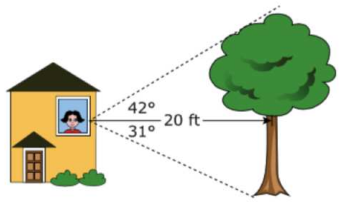


PARCC Released Question (PBA)

Slide 223 / 252

Question 5/11 **Topic: Trigonometric Ratios**

86 Mariela is standing in a building and looking out of a window at a tree. The tree is 20 feet away from Mariela. Mariela's line of sight to the top of the tree creates a 42° angle of elevation, and her line of sight to the base of the tree creates a 31° angle of depression.



What is the height, in feet, of the tree? Type in your answer.

PARCC Released Question (PBA)

Slide 224 / 252

Released PARCC Exam Question

The following question from the released PARCC - PBA exam uses what we just learned and combines it with what we learned earlier to create a good question.

Please try it on your own.

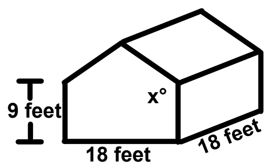
Then we'll go through the processes that we can use to solve it.

PARCC Released Question (PBA)

Slide 225 / 252

Question 1/11 **Topic: Trigonometric Ratios**

The figure shows the design of a shed that will be built. Use the figure to answer all parts of the task.



The base of the shed will be a square measuring 18 feet by 18 feet. The height of the rectangular sides will be 9 feet. The measure of the ansgle made by the roof with the side of the shed can vary and is labeled as *x*°. Different roof angles create different surface areas of the roof. The surface area of the roof will determine the number of roofing shingles needed in constructing the shed. To meet drainage requirements, the roof angles must be at least 117°.

Slide 226 / 252

Part A

The builder of the shed is considering using an angle that measures 125°. Determine the surface area of the roof if 125° angle is used. Explain or show your process.

Part B

Without changing the measurements of the base of the shed, the builder is also considering using a roof angle that will create a roof surface area that is 10% less than the area obtained in Part A. Less surface area will require less roofing shingles. Will such an angle meet the specified drainage requirements. Explain how you came to your conclusion.

Part C

The roofing shingles cost $27.75 for a bundle. Each bundle can cover approximately 35 square feet. Shingles must be purchased in full bundles. The builder has a budget of $325 for shingles.

What is the greatest angle the builder can use and stay within budget? Explain or show your process.

Slide 227 / 252

Question 1/11 Part A **Topic: Trigonometric Ratios**

The builder of the shed is considering using an angle that measures 125°. Determine the surface area of the roof if 125° angle is used. Explain or show your process.

87 What concepts could we use to solve this problem?

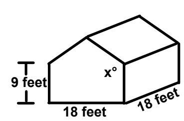
- Area of a rectangle
- Right Triangle Trigonometry
- Angle Addition Postulate
- All of the above

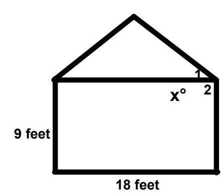
Slide 228 / 252

Question 1/11 Part A **Topic: Trigonometric Ratios**

88 If the value of x is 125°, what would be the m∠1?

- 90°
- 25°
- 35°
- 160°



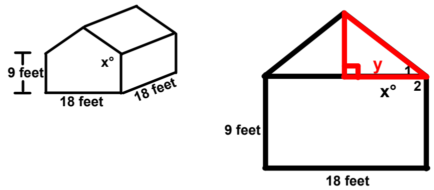


Front view of the shed

Question 1/11 Part A **Topic: Trigonometric Ratios**

89 What would be the value of y in the figure to the right?

- 6 ft
- 9 ft
- 12 ft
- 18 ft

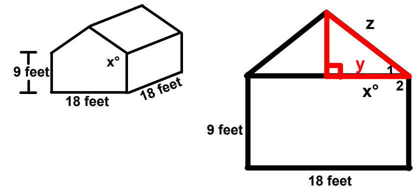


Front view of the shed

Question 1/11 Part A **Topic: Trigonometric Ratios**

90 What ratio would we use to find the value of z in the figure below?

- $\sin(35) = \frac{z}{18}$
- $\tan(35) = \frac{9}{z}$
- $\cos(35) = \frac{9}{z}$
- $\tan(35) = \frac{z}{9}$

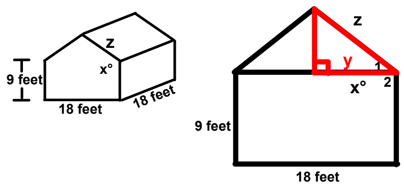


Front view of the shed

Question 1/11 Part A **Topic: Trigonometric Ratios**

91 What is the value of z in the figure below?

- 7.37 feet
- 10.32 feet
- 10.99 feet
- 12.85 feet

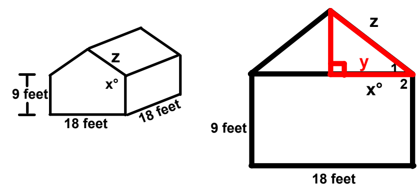


Front view of the shed

Question 1/11 Part A **Topic: Trigonometric Ratios**

92 What is the area of the roof?

- 98.91 ft^2
- 197.82 ft^2
- 296.73 ft^2
- 395.64 ft^2



Front view of the shed

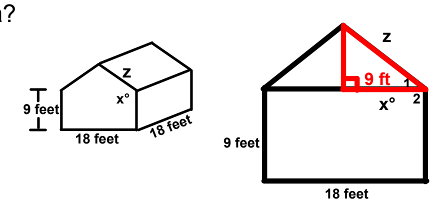
Question 1/11 Part B **Topic: Trigonometric Ratios**

Without changing the measurements of the base of the shed, the builder is also considering using a roof angle that will create a roof surface area that is 10% less than the area obtained in Part A. Less surface area will require less roofing shingles. Will such an angle meet the specified drainage requirements. Explain how you came to your conclusion.

Question 1/11 Part B **Topic: Trigonometric Ratios**

93 After finding the answer that the area of the roof was 395.64 ft^2, what would be the area of a roof that has 10% less area?

- 356.08 ft^2
- 316.52 ft^2
- 197.8 ft^2
- 39.56 ft^2

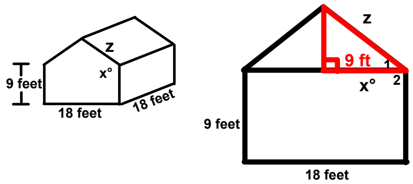


Front view of the shed

Question 1/11 Part B **Topic: Trigonometric Ratios**

94 Using the area that we found in the previous slide, what is the new value of z?

- 10.99 ft
- 17.58 ft
- 19.78 ft
- 9.89 ft

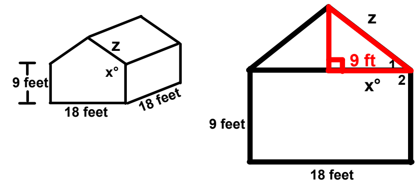


Front view of the shed

Question 1/11 Part B **Topic: Trigonometric Ratios**

95 Using the new value of z, what is the new m∠1?

- 65.51°
- 24.49°
- 42.30°
- 47.70°

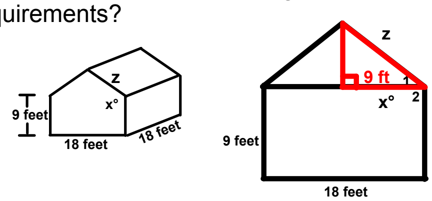


Front view of the shed

Question 1/11 Part B **Topic: Trigonometric Ratios**

96 Does the measurement of our new angle x meet the building requirements?

- Yes
- No



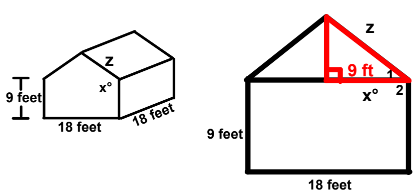
Front view of the shed

Question 1/11 Part C **Topic: Trigonometric Ratios**

Question 1/11 Part C **Topic: Trigonometric Ratios**

97 If the roofing shingles cost $27.75 for a bundle and his budget is $325, how many bundles of shingles can he buy?

- 10
- 11
- 11.71
- 12

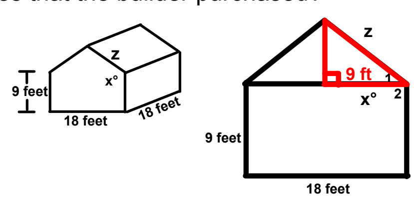


Front view of the shed

Question 1/11 Part C **Topic: Trigonometric Ratios**

98 If each bundle of shingles covers an area of 35 square feet, then what is the area is covered by the the amount of bundles that the builder purchased?

- 420 ft^2
- 409.85 ft^2
- 385 ft^2
- 350 ft^2



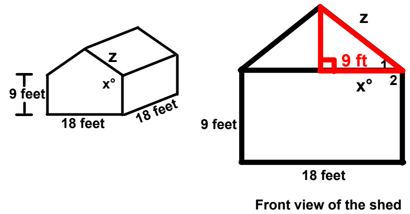
Front view of the shed

Slide 241 / 252

Question 1/11 Part C **Topic: Trigonometric Ratios**

99 Using the new area found in the last question, what is the value of z in the figures below?

- 10.69 ft
- 14.26 ft
- 16.04 ft
- 21.39 ft

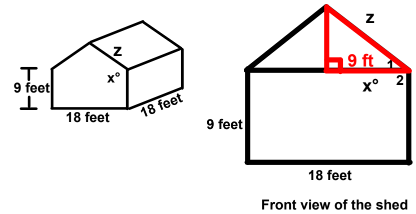


Slide 242 / 252

Question 1/11 Part C **Topic: Trigonometric Ratios**

100 Using the new value of z found in the last question, what is the new value of x in the figures below?

- 32.66°
- 57.34°
- 122.66°
- 147.34°



Slide 243 / 252

Released PARCC Exam Question

The following question from the released PARCC - PBA exam uses what we just learned and combines it with what we learned earlier to create an interesting question.

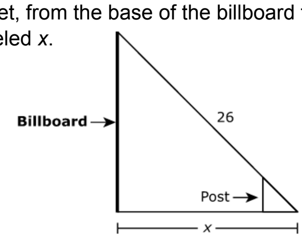
Please try it on your own.

Then we'll go through the processes that we can use to solve it.

Slide 244 / 252

Question 3/11 **Topic: Problem Solving w/ Similar Triangles**

A billboard at ground level has a support length of 26 feet that extends from the top of the billboard to the ground. A post that is 5 feet tall is attached to the support and is 4 feet from where the base of the support is attached to the ground. In the figure shown, the distance, in feet, from the base of the billboard to the base of the support is labeled *x*.



Create an equation that can be used to determine *x*. Discuss any assumptions that should be made concerning the equation. Use your equation to find the value of *x*. Show your work or explain your answer.

Slide 245 / 252

Question 3/11 **Topic: Problem Solving w/ Similar Triangles**

101 Is this problem solvable?

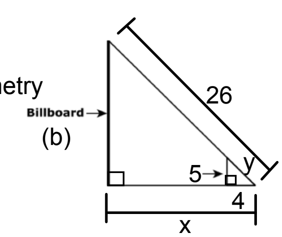
- Yes
- No

Slide 246 / 252

Question 3/11 **Topic: Problem Solving w/ Similar Triangles**

102 If we assume that both the billboard & the post are perpendicular with the ground, what concepts could we use to solve this problem?

- A Pythagorean Theorem
- B Right Triangle Trigonometry
- C Similar Triangles
- D All of the above



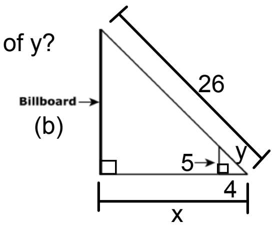
Question 3/11

Topic: Problem Solving w/ Similar Triangles

Let's use first, the combination of
A Pythagorean Theorem &
C Similar Triangles.

103 What would be the value of y?

- A 3
- B 9
- C $\sqrt{41}$
- D 41

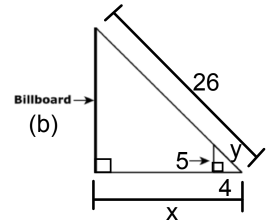


Question 3/11

Topic: Problem Solving w/ Similar Triangles

104 What proportion would we use to find the value of x?

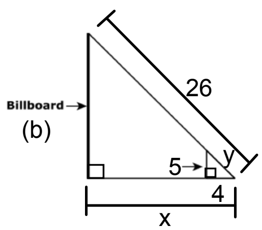
- A $\frac{5}{x} = \frac{\sqrt{41}}{26}$
- B $\frac{4}{x} = \frac{\sqrt{41}}{26}$
- C $\frac{5}{b} = \frac{\sqrt{41}}{26}$
- D $\frac{4}{x} = \frac{26}{\sqrt{41}}$



Question 3/11

Topic: Problem Solving w/ Similar Triangles

105 What is the value of x?



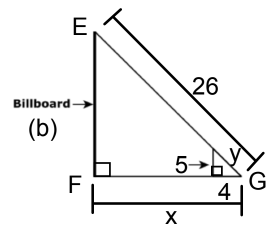
Question 3/11

Topic: Problem Solving w/ Similar Triangles

Now, let's use the combination of
B Right Triangle Trigonomety &
C Similar Triangles.

106 What would be the ratio that we would use to find the measurement of Angle G?

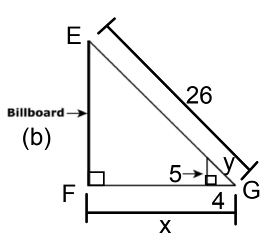
- A tan G = $\frac{5}{4}$
- B sin G = $\frac{5}{26}$
- C cos G = $\frac{4}{26}$
- D tan G = $\frac{4}{5}$



Question 3/11

Topic: Problem Solving w/ Similar Triangles

107 What is the measurement of angle G?



Question 3/11

Topic: Problem Solving w/ Similar Triangles

Since the two triangles are similar, the measurement of angle G is the same in both triangles.

108 Using the measurement of angle G, what is the value of x?