# Outline:

I. Entropy and subregion dualities of T T bar deformed theories including the bulk dS case (w/Lewkowycz, Liu, Torroba). Gibbons-Hawking dS entropy from dS/dS, at the appropriate level of approximation. (works w/ Dong, Gorbenko, Torroba,...)

II. Time permitting: Quick overview of dS and large-field inflation in string theory (=quantum gravity), new class of compactifications w/Torroba (+ I. Agol)

# Main content based on:


A. Lewkowycz, J. Liu, E. Silverstein and G. Torroba, "$T\bar{T}$ and EE, with implications for (A)dS subregion encodings," arXiv:1909.13808 [hep-th].

V. Gorbenko, E. Silverstein and G. Torroba, "dS/dS and $T\bar{T}$," arXiv:1811.07965 [hep-th].

X. Dong, E. Silverstein and G. Torroba, "De Sitter Holography and Entanglement Entropy," JHEP **1807**, 050 (2018) doi:10.1007/JHEP07(2018)050 [arXiv:1804.08623 [hep-th]].

M. Alishahiha, A. Karch, E. Silverstein, and D. Tong, "The dS/dS correspondence," *AIP Conf. Proc.* **743** (2005) 393–409, arXiv:hep-th/0407125 [hep-th]. [,393(2004)].

M. Alishahiha, A. Karch, and E. Silverstein, "Hologravity," *JHEP* **06** (2005) 028, arXiv:hep-th/0504056 [hep-th].

X. Dong, B. Horn, E. Silverstein, and G. Torroba, "Micromanaging de Sitter holography," *Class. Quant. Grav.* **27** (2010) 245020, arXiv:1005.5403 [hep-th].

X. Dong, B. Horn, S. Matsuura, E. Silverstein, and G. Torroba, "FRW solutions and holography from uplifted AdS/CFT," *Phys. Rev.* **D85** (2012) 104035, arXiv:1108.5732 [hep-th].


# Other approaches (dS/CFT, FRW/CFT, cosmo within AdS,…) reviewed in e.g.

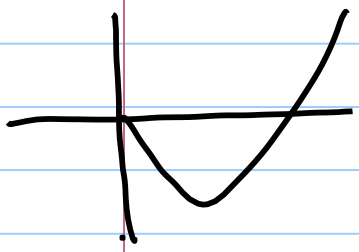

D. Anninos, "De Sitter Musings," *Int. J. Mod. Phys.* **A27** (2012) 1230013, arXiv:1205.3855 [hep-th].

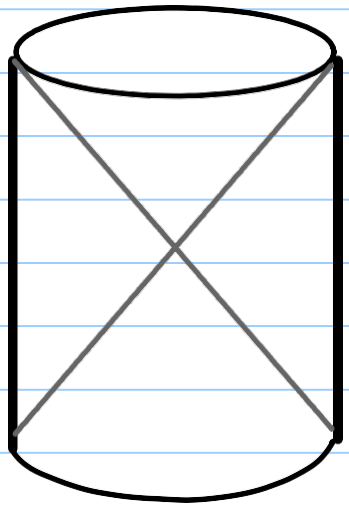
D. Anninos and D. M. Hofman, "Infrared Realization of $dS_2$ in $AdS_2$," *Class. Quant. Grav.* **35** no. 8, (2018) 085003, arXiv:1703.04622 [hep-th].

S. Cooper, M. Rozali, B. Swingle, M. Van Raamsdonk, C. Waddell, and D. Wakeham, "Black Hole Microstate Cosmology," *JHEP* **07** (2019) 065, arXiv:1810.10601 [hep-th].

**I. Holography**: We need a more complete QG framework for the V>0 landscape.

$\Lambda < 0$



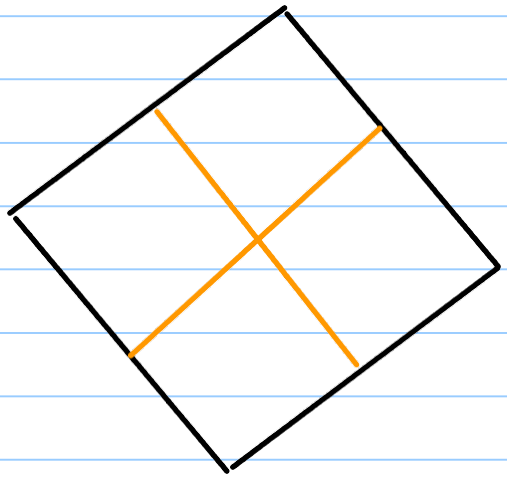


## AdS:

observables are conformal field theory (CFT) correlation functions

Timelike boundary at infinity pins fields.

$\Lambda = 0$

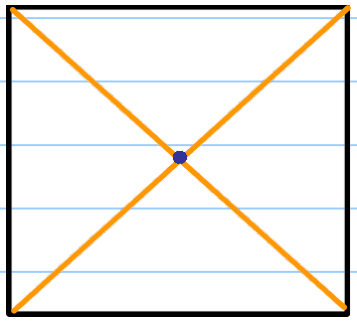


## Minkowski

Scattering matrix

Asymptotics also very special.

$\Lambda > 0$



## dS

no boundary analogous to the AdS one. (Finite, fluctuating space. Eventually decays.)

AdS/CFT: Low energy QFT = highly redshifted region of D-brane construction. The near-horizon AdS x S has a timelike boundary at spatial infinity. This, along with the negative cosmological constant, is very special and unrealistic.

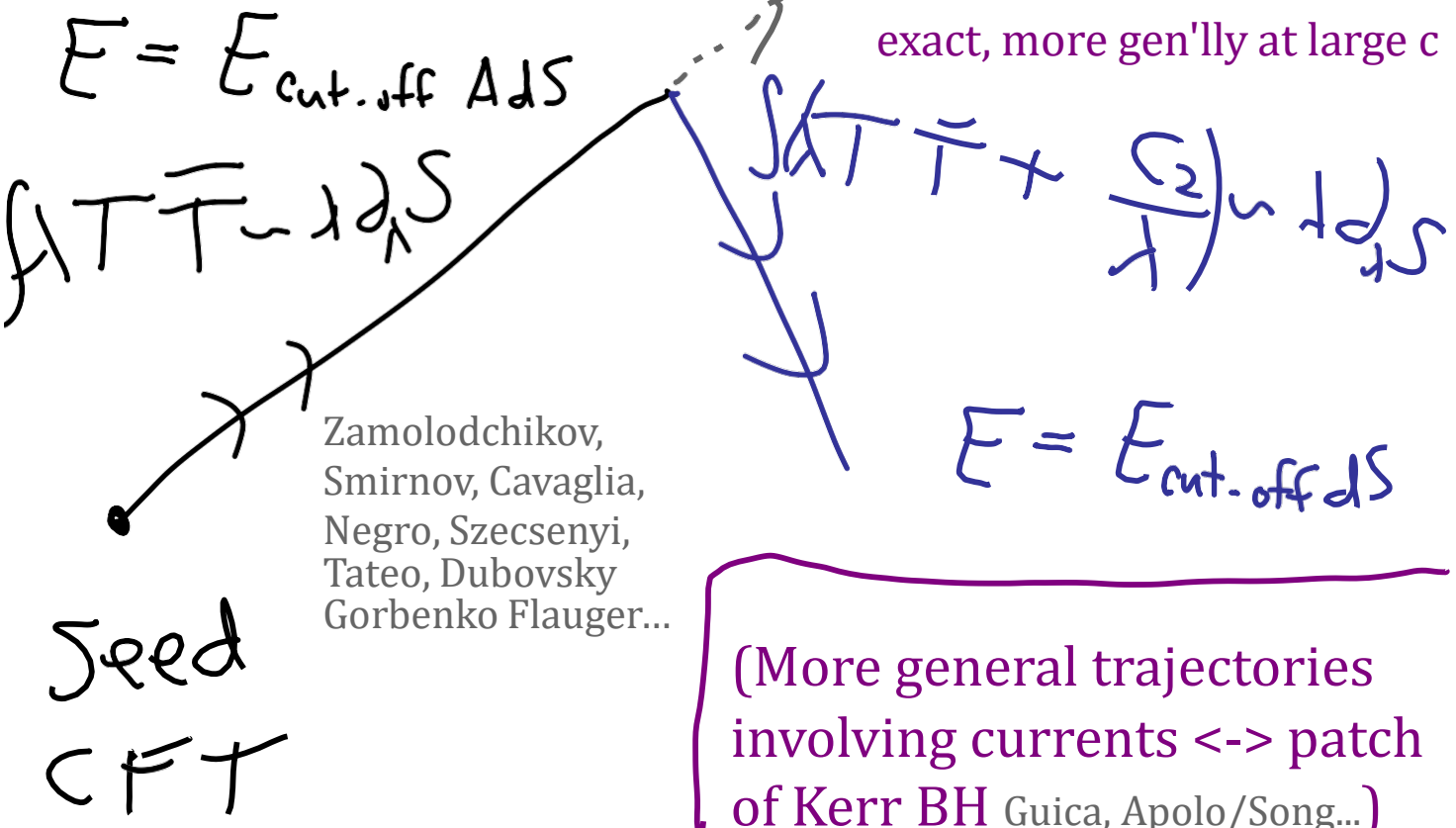
Within the near-horizon, AdS x S solution, there is a `scale-radius' duality.
To progress toward more realistic QG, want to determine the dual of a finite patch of this and other spacetimes (e.g. a radial cutoff).

Recently, McGough, Mezei, Verlinde and followups proposed a specific answer to this question, isolating a radially cutoff region of AdS as the dual of a tractable `irrelevant' deformation of the dual CFT known as T-Tbar. We then (w/Gorbenko, Torroba) generalize this deformation to capture finite patches of bulk **dS** instead. cf Miyaji Takayanagi Sato, Nomura Rath Salzetta...

# TTbar +... & Holography:

$$8\langle n|T\bar{T}|n\rangle = \langle n|T^{\alpha\beta}|n\rangle\langle n|T_{\alpha\beta}|n\rangle - \langle n|T^{\alpha}_{\alpha}|n\rangle^2$$

Factorization on cylinder
exact, more gen'lly at large c




Zamolodchikov, Smirnov, Cavaglia, Negro, Szecsenyi, Tateo, Dubovsky Gorbenko Flauger...

(More general trajectories involving currents <-> patch of Kerr BH Guica, Apolo/Song...)

Both universal, solvable deformations whose dressed energies match quasilocal Brown-York energy of a patch of (A)dS. The new extension enters nonlinearly in the integrated trajectory. There are generalizations with varying degrees of solvability, including for local bulk matter. Hartman Kruthoff Shaghoulian Tajdini, Gross Kruthoff Rolph Shaghoulian; Giveon Kutasov et al, ...

here we will focus on entropy and subregion dualities:

# $T\bar{T}$ and EE, with implications for (A)dS subregion encodings

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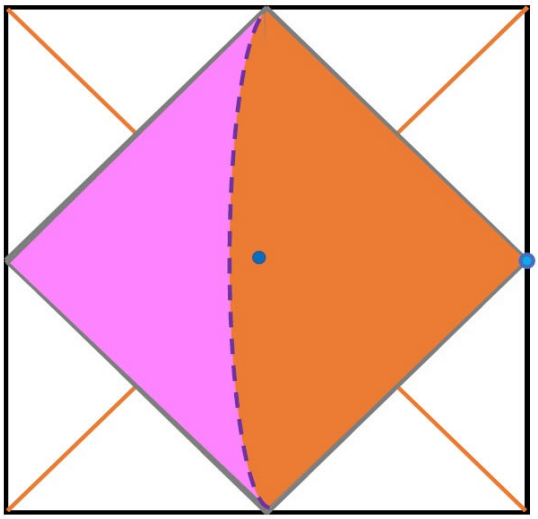
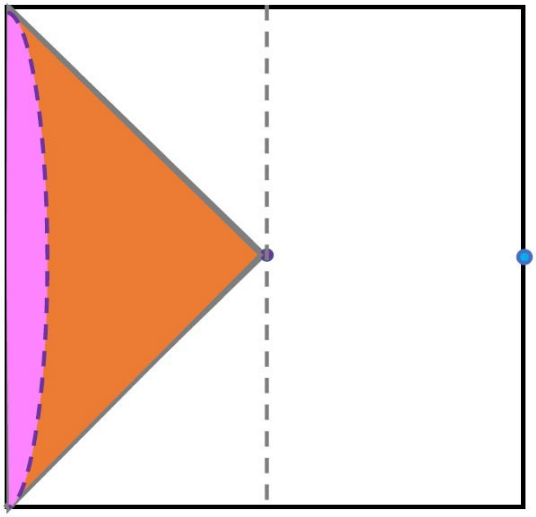
arXiv:1909.13808v1 [hep-th] 30 Sep 2019

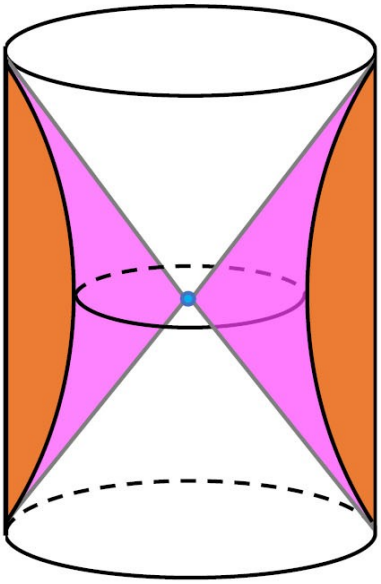
**Abstract**

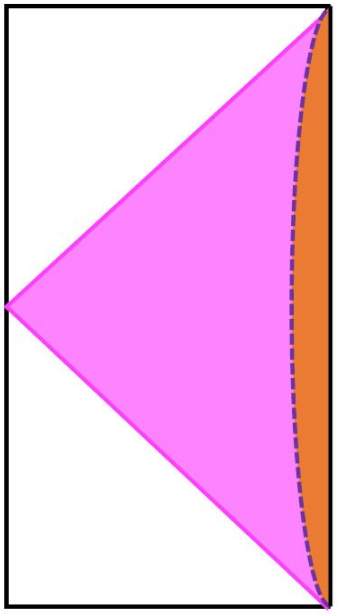
We initiate a study of subregion dualities, entropy, and redundant encoding of bulk points in holographic theories deformed by $T\bar{T}$ and its generalizations. This includes both cut off versions of Anti de Sitter spacetime, as well as the generalization to bulk de Sitter spacetime, for which we introduce two additional examples capturing different patches of the bulk and incorporating the second branch of the square root dressed energy formula. We provide new calculations of entanglement entropy (EE) for more general divisions of the system than the symmetric ones previously available. We find precise agreement between the gravity side and deformed-CFT side results to all orders in the deformation parameter at large central charge. An analysis of the fate of strong subadditivity for relatively boosted regions indicates nonlocality reminiscent of string theory. We introduce the structure of operator algebras in these systems. The causal and entanglement wedges generalize to appropriate deformed theories but exhibit qualitatively new behaviors, e.g. the causal wedge may exceed the entanglement wedge. This leads to subtleties which we express in terms of the Hamiltonian and modular Hamiltonian evolution. Finally, we exhibit redundant encoding of bulk points, including the cosmological case.

# TTbar +... isolates a radially cut off subregion of (A)dS geometries:









$$
\begin{aligned}
ds^2_{\text{(A)dS/dS}} &= dw^2 + \sin(h)^2 \frac{w}{\ell}\left(-d\tau^2 + \ell^2 \cosh^2 \frac{\tau}{\ell} d\phi^2\right) \quad w \le w_c \\
ds^2_{\text{(A)dS/cylinder}} &= -(1 \pm r^2/\ell^2)dt^2 + \frac{dr^2}{1 \pm r^2/\ell^2} + r^2 d\phi^2 \quad r \le r_c \\
ds^2_{\text{AdS/Poincare}} &= \ell^2 \frac{-dt^2 + dx^2 + dz^2}{z^2} \quad z \ge z_c
\end{aligned}
$$

We focus on the holographic patches closest to AdS/CFT:

- Radial direction emergent
- Boundary at the most UV radial slice
- Boundary causality (signals fastest along boundary)

In this framework, we find precise functional match between energies and entropies (beyond the symmetric cases), computed independently on the two sides.

Original AdS/cylinder case (more below):

$$
\begin{aligned}
E &= \frac{r_c}{4G}\left(1-\sqrt{1-\frac{8GM}{r_c^2}+\frac{16G^2J^2}{r_c^4}}\right) \\
r_c &= \frac{L_{proper}}{2\pi\ell_{AdS}}, \quad \lambda = 8G\ell, \quad c=\frac{3\ell}{2G} \\
M\ell &= \Delta+\bar{\Delta}-\frac{c}{12}, \quad J\ell = \Delta-\bar{\Delta} \\
E &= \frac{L}{\pi\lambda}\left(1-\sqrt{1-\frac{4\pi^2\lambda(\Delta+\bar{\Delta}-c/12)}{L^2}+\frac{4\pi^4\lambda^2(\Delta-\bar{\Delta})^2}{L^4}}\right)
\end{aligned}
$$

BH energies go complex above cutoff scale. They are absent in the unitary theory obtained by truncating to real levels => finite entropy. To match GR side, we should have emergent locality down to the bulk string scale.

**Various subtle aspects addressed in recent and ongoing work, including bulk matter beyond pure gravity, higher dimensions, classifying well-defined observables, ...** To binge-watch T-Tbar, see talks at Simons Center workshop April 2019...

**Many recent lessons in AdS/CFT, e.g.:**

*Causal wedge reconstruction, Hamiltonian evolution (HKLL) **U** Hamilton Kabat Lifshytz Lowe, Bousso et al, Morrison, ...

*Entanglement entropy (RT) Ryu-Takayanagi, HRT,...

*Algebraic QFT and EE Casini Huerta...

*E. wedge reconstruction and modular Hamiltonian evolution **ρ** Faulkner Lewkowyz Maldacena (but entanglement shadows, e.g. cf Freivogel Jefferson et al and islands, Almheiri et al, Pennington,...)

*Redundant encoding of bulk points (quantum error correction) Almheiri Dong Harlow...

**Goal**: fate of these properties in the cut off (A)dS cases, via T Tbar +.... trajectories.

**Summary**: Ryu-Takayanagi works for cutoff (A)dS, *new calculations: precise agreement with deformed-CFT side non-perturbatively in TTbar coupling at large c.* Technical reason: effect localizes to endpoints, order of limits crucial. Implications for subregion dualities: operator algebras and commutator structure reveal nature of non-locality, e.g. relative boost expands operator algebra. Some redundancy of bulk point encoding (cf quantum error correction).

Zamolodchikov, Smirnov, Cavaglia, Negro, Szecsenyi, Tateo

$$\frac{dS}{d\lambda} = 2\pi \int d^2x \sqrt{-g}\, T\bar{T}$$

## Apply step by step: generates a tractable trajectory in the space of QFTs

Recompute stress energy tensor at each step.

$$8\langle n|T\bar{T}|n\rangle = \langle n|T^{\alpha\beta}|n\rangle\langle n|T_{\alpha\beta}|n\rangle - \langle n|T^{\alpha}_{\alpha}|n\rangle^2$$

Factorization for 2d theory on flat spacetime for any theory, and any spacetime for large number of degrees of freedom.

# One scale S(λ) ~>

cf McGough Mezei Verlinde, Kraus Liu Marolf

$$T_i^i = -4\pi\lambda T\bar{T}$$

Factorization => for homogeneous states, algebraic equation for pressure in terms of energy density. e.g. zero momentum:

$$\begin{aligned}
\mathrm{Tr}\, T &= T_\tau^\tau + T_\phi^\phi = -4\pi\lambda T\bar{T} \Rightarrow \langle T_\phi^\phi\rangle = \frac{\langle T_\tau^\tau\rangle}{\pi\lambda\langle T_\tau^\tau\rangle - 1} \\
\langle T_\phi^\phi\rangle &= -dE/dL \\
\langle T_\tau^\tau\rangle &= -\frac{E}{L} = \frac{1}{\pi\lambda}\left(1 - \sqrt{1 + 2\pi\lambda\langle T_\tau^{\tau\,(0)}\rangle}\right)
\end{aligned}$$

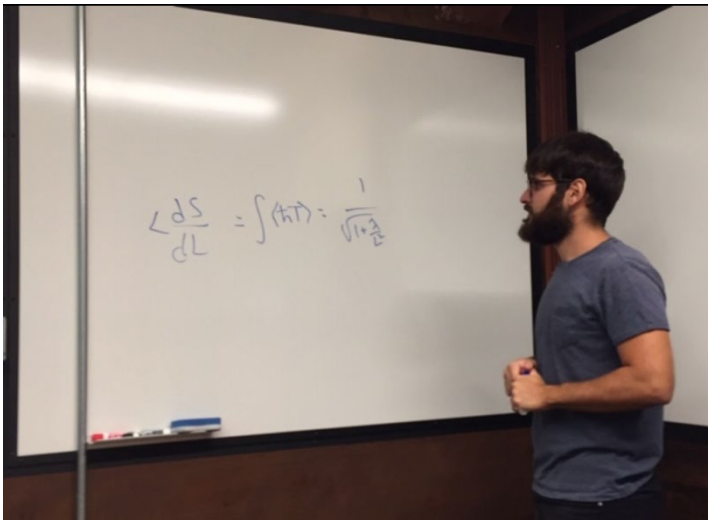
Result is exact nonlinear formula for`dressed' energies along trajectory.

Zamolodchikov, Smirnov, Cavaglia, Negro, Szecsenyi, Tateo, Cardy, Flauger, Dubovsky, Gorbenko, Mirbabayi, Hernandez-Chifflet, Aharony, Guica et al, Cotrell, Hashimoto, Giveon, Kutasov, Aharony, Datta, Giveon, Vaknin,...

(These same dressed levels arise if we couple the CFT to 2d Jackiw-Teitelboim', or Polyakov, gravity...dressing is like spacetime energy of the Polyakov string)

# T Tbar and dS/dS

## T Tbar, (A)dS/X, and EE



Repeat MMV et al calculation for (A)dS/dS. Solve for stress energy, matching AdS/dS and dS/dS near w=0.

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\,\sqrt{-g}\left(\mathcal{R}^{(3)} + \frac{2\eta}{\ell^2}\right) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\,\sqrt{-g}\left(K - \frac{b_{CT}}{\ell}\right)$$

Taking radial slices

$$ds_3^2 = dw^2 + g_{ij}\,dx^i dx^j\,,$$

the quasilocal stress-tensor is

$$T_{ij} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\text{on-shell}}}{\delta g^{ij}} = \frac{1}{8\pi G}\left(K_{ij} - K g_{ij} + \frac{b_{CT}}{\ell}g_{ij}\right)\,.$$

Result: 2 new terms in trace flow equation

$$\int d^2x\sqrt{-g}T^i_i \quad = \quad \int d^2x\sqrt{-g}\left(-4\pi\lambda T\bar{T} - \frac{c\mathcal{R}^{(2)}}{24\pi} + \frac{1-\eta}{\pi\lambda}\right)$$

e.g. in flat cylinder case exactly solvable for energy levels as in original T Tbar deformation. We use this equation and stress-energy conservation to compute entanglement entropy and properties of the density matrix on the 2d deformed-CFT side.

# Starting from a seed CFT, holographically reconstruct a patch of de Sitter spacetime.

# Beyond pure gravity, the matching the two parts of the trajectory for local bulk examples involves interpolation between the AdS and dS minima e.g. in the connected scalar sector of string theory.

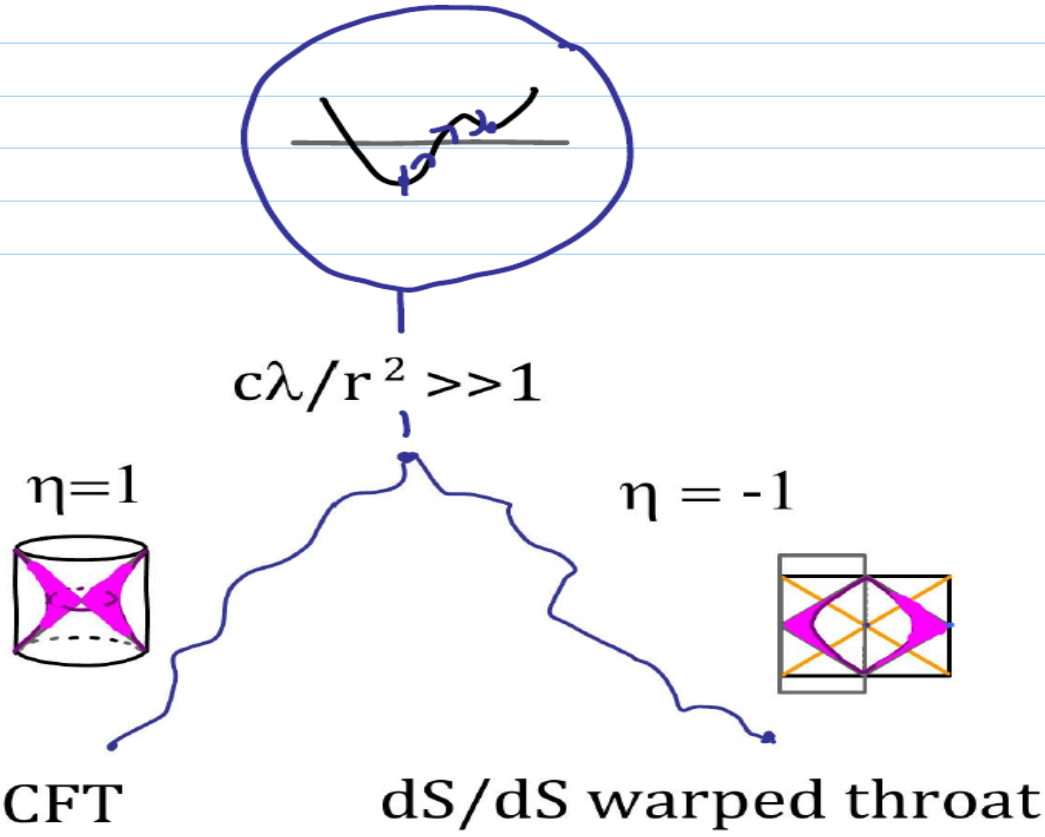
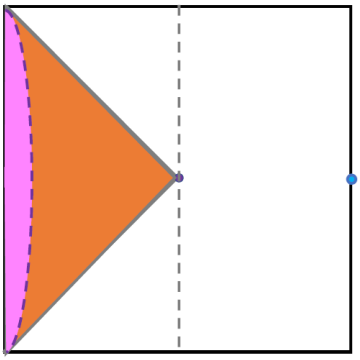


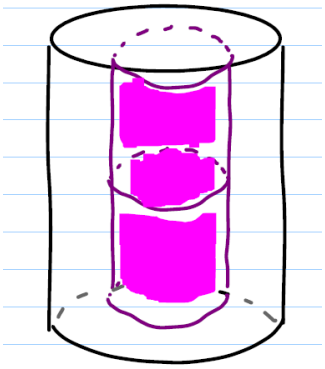

**Figure 1:** The reconstruction of the $dS/dS$ throat from a seed CFT proceeds via two joined trajectories as described in the text. The first trajectory (on the left) evolves the system from a pure CFT, via a sequence of cutoff AdS/dS systems, to the limit where this cutoff scale goes to zero, indicated by the point at the top of the figure. That point is the start of a new trajectory incorporating $\Lambda_2 \propto \eta - 1$, with increasing cutoff scale, culminating in the full $dS/dS$ warped throat.

# Dressed energies and additional dualities

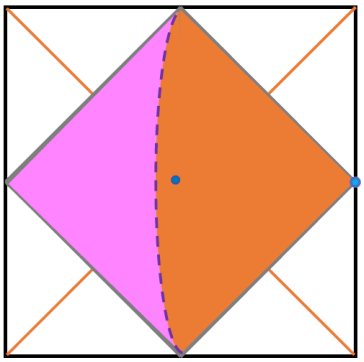
$$E = -2\pi L T_t^t = \frac{L}{\pi\lambda}\left(1 - \sqrt{\eta - \frac{4\pi^2\lambda(\Delta + \bar{\Delta} - c/12)}{L^2} + \frac{4\pi^4\lambda^2(\Delta - \bar{\Delta})^2}{L^4}}\right)$$



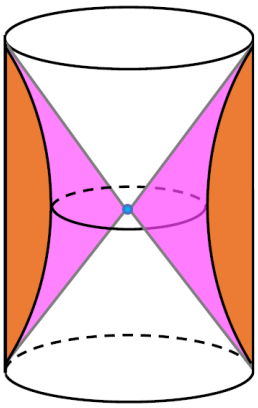
(A)dS/cylinder, formulates dS static patch



$$\langle T_\tau^\tau \rangle = \frac{1}{\pi\lambda}\left(1 \mp \sqrt{\eta + c\frac{\mathcal{R}^{(2)}\lambda}{24} - \frac{C_1\lambda}{L^2}}\right)$$



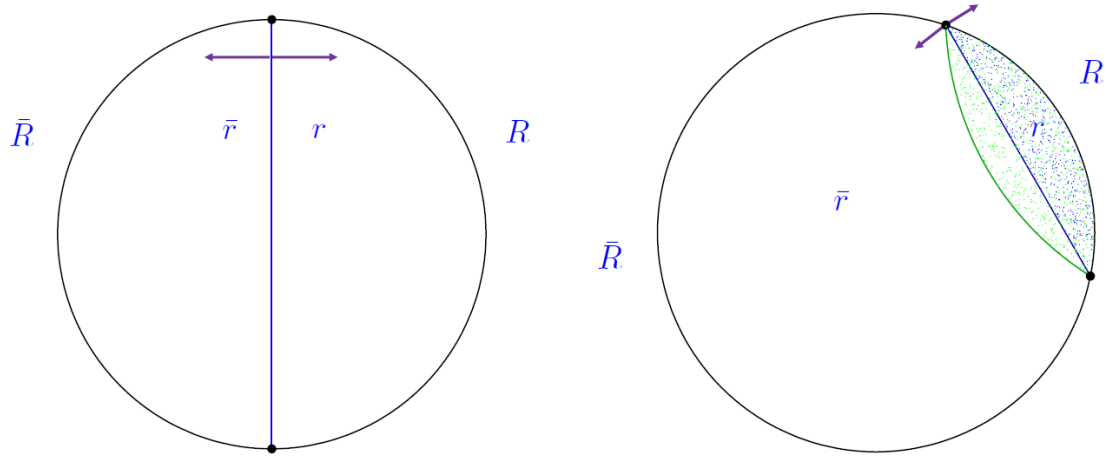
(A)dS/dS, including full dS/dS patch with 2nd sign of square root



# Energy and Entropy, U and ρ:

The deformed theories are non-local, so no *a priori* guarantee of a division of the system (i.e. its operator algebra) into subregions of the boundary spacetime. Also no guarantee a priori of causality in time evolution. But we have emergent bulk locality down to string scale, and causal signaling along bdry (as well as in bulk GR) for AdS/Poincare and (A)dS/dS, and extremal surfaces dividing the bulk system. Find Causal Wedge and Entanglement Wedge continue to exist, but CW can exceed EW unlike AdS/CFT.

$$[\mathcal{A}_{\bar{R}_s}, \mathcal{A}_{D[R]}] \neq 0\,.$$



$$\mathcal{A}_{\mathrm{CW}} = \mathcal{A}_{D[R]} = \{O(x),\ x \in D[R]\}\,.$$

$$\mathcal{A}_{\mathrm{EW}} = \mathcal{A}_{R_s} = \{\rho_R^{is}\, O(x)\, \rho_R^{-is},\ x \in R,\ s \in \mathbb{R}\}\,.$$

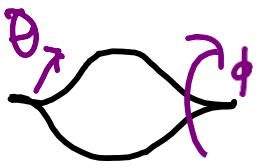
As in Faulkner-Lewkowycz

First, analyze entropy on deformed-CFT side. Equations for stress energy (quasilocal PDEs):

$$\nabla_a T^a_b = 0$$

$$T^a_a = -\frac{cR}{24\pi} + \frac{c_2}{\pi\lambda} - \frac{\pi\lambda}{2}(T^{ab}T_{ab} - (T^a_a)^2), \qquad c_2 = 1-\eta$$

with appropriate boundary conditions along with generalization for probe matter. Donelly and Shyam used these to compute R=1/2 stress tensor, here generalized with η:




$$T^\theta_\theta = \frac{1}{\pi\lambda}\left(1-\sqrt{\eta+\frac{c\lambda}{12r^2}\left[1+\left(\frac{1}{n^2}-1\right)\frac{1}{\sin^2\theta+n^2\delta^2}\right]}\right)$$

$$T^\phi_\phi = \frac{1}{\pi\lambda}\left(1-\frac{\eta+\frac{c\lambda}{12r^2}\left[1-\delta^2\frac{n^2-1}{(\sin^2\theta+\delta^2n^2)^2}\right]}{\sqrt{\eta+\frac{c\lambda}{12r^2}\left[1+\left(\frac{1}{n^2}-1\right)\frac{1}{\sin^2\theta+n^2\delta^2}\right]}}\right).$$

symmetry in $\phi$ ⇒ ODE

$$L\frac{d}{dL}\tilde{S}_n = -(1-n\partial_n)\int d^2x\sqrt{g}\,\mathrm{tr}\,T\,.$$

= RT result

Near one endpoint sin(θ)->0 :

$$\epsilon \equiv 1 - n \ll 1 ,$$

$$\partial_n \langle \mathrm{tr} T \rangle|_{n\to 1} = \epsilon \, \frac{c}{3} \, \frac{\lambda c}{24} \, \frac{1}{r^3 \sin^4\theta} \, \frac{1}{\left(1 + \frac{\lambda c}{12 r^2}\left(1 + \frac{2\epsilon}{\sin^2\theta}\right)\right)^{3/2}} .$$

Order of limits between n–>1 and λ–>0. Correct result from integrating –(1-$nd_n$)(Tr(T) first. Shift in denominator regulates endpoint singularity, no need for conical singularity cutoff: T Tb coupling regulates the endpoint itself.

Using the endpoint-dominant effect of λ enables exact (in λ at fixed c) calculation of entanglement entropy much more generally than R=1/2-space. We find agreement with RT.

(Probably not an accident: the Faulkner-Lewkowycz-Maldacena path integral derivations of RT + semiclassical corrections goes through if taken at face value. Still, helpful to have methods for computing entanglement in a greater variety of interacting theories using standard methods.)

# Recall: interval L in Poincare AdS conformally maps to half of dS neck, L=2$r_{dS}$

ref

$$ds^2 = -(dx^0)^2 + (dx^1)^2 = \Omega(\theta,\tau)^2\,(-\sin^2\theta\,d\tau^2 + r^2\,d\theta^2)$$

$$\Omega(\theta,\tau) = \frac{1}{1+\sin\theta\,\cosh(\tau/r)}$$

$$L = 2r$$

Non-CFT couplings are dimensionful; this map would appear not useful since the conformal factor multiplies the couplings, infecting the formulas.

# AdS/Poincare case study

$$n\partial_n \langle \mathrm{tr}\, T\rangle|_{n\to 1} = \epsilon\, \frac{c}{3}\, \frac{\lambda c}{24\pi}\, \frac{C(\frac{\lambda}{L^2})^2}{\rho\left(\rho^2 + \epsilon \frac{\lambda c}{6} C(\frac{\lambda}{L^2})\right)^{3/2}}\,.$$

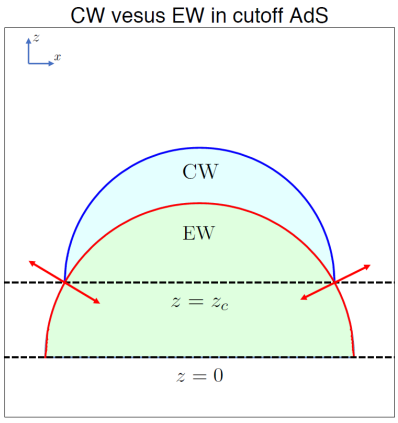
$$\begin{aligned} LS'(L) &= 2 \times \lim_{n\to 1}(2\pi n) \int_0^{\rho_0 \ll L} \rho d\rho\, n\partial_n \langle \mathrm{tr}\, T\rangle \\ &= \frac{c}{3} C(\frac{\lambda}{L^2}) \frac{1}{\sqrt{1+\epsilon \frac{\lambda c}{6\rho_0^2} C(\frac{\lambda}{L^2})}}\Big|_{\epsilon\to 0} \\ &= \frac{c}{3} C(\frac{\lambda}{L^2})\,. \end{aligned}$$

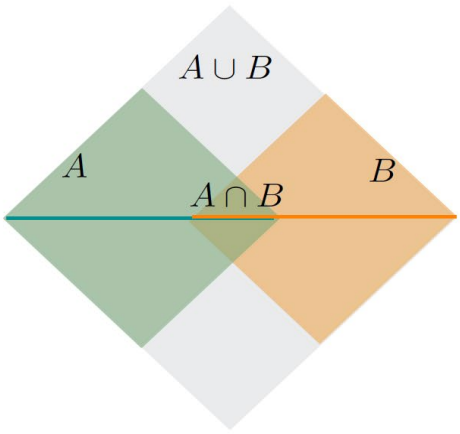
Finite interval in Poincare boundary can *still* be done via Weyl rescaling map to static patch in dS boundary cf Casini/Huerta/Myers because the Weyl factor Ω which normally complicates that map in a non-conformal theory reduces to 1 at the endpoints of the interval. This matches precisely with the Ryu-Takayanagi result.

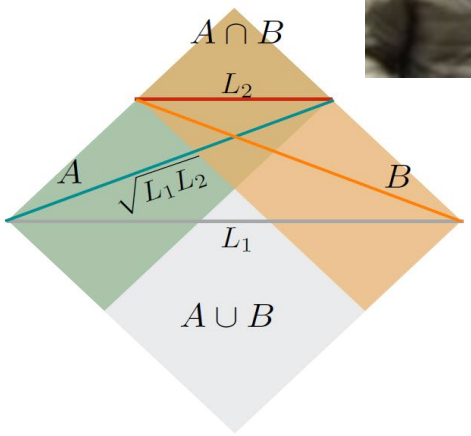
Leading (CFT) result from this order of limits; full non-perturbative (at large c) result from CHM Weyl map => precise match.

$$LS'(L) = \frac{c}{3} \frac{1}{\sqrt{1+\frac{\lambda c}{3L^2}}}$$

Note $O(\lambda/L^2)$ Contribution, cf Chakraborty et al





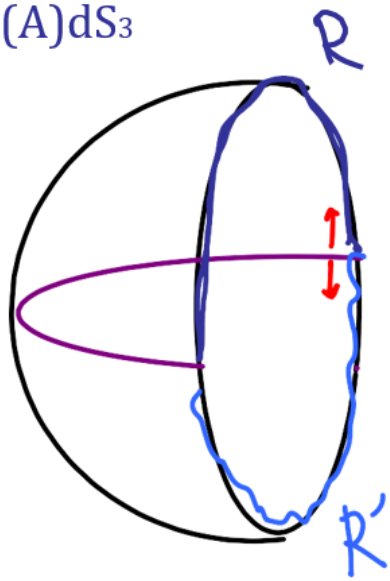




Boosted strong subadditivity of Casini And Huerta is violated. Interpretation: Relative boost => additional operators join the accessible operator algebra. The non-locality is similar to string theory

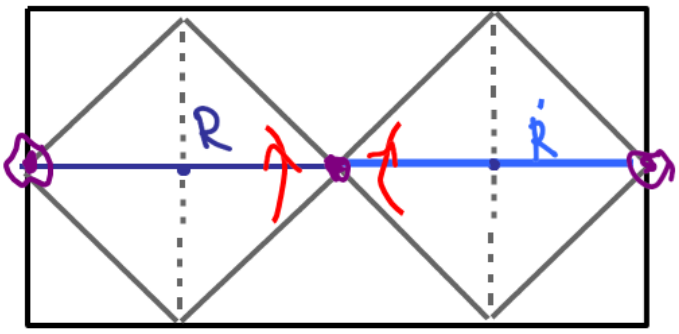
$$\mathcal{A}_{A_s} \not\subseteq \mathcal{A}_{(A\cup B)_s}$$

# dS/dS case study: again sharp match

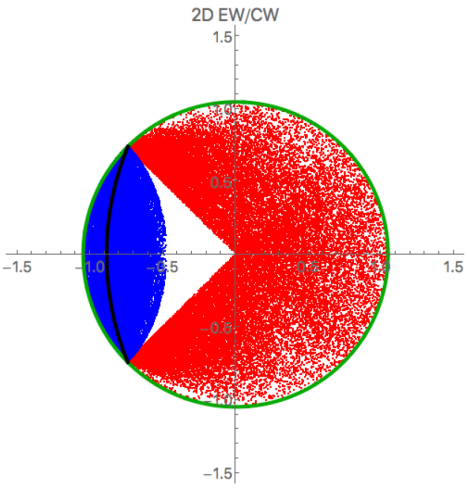
spatial slice of bulk (A)dS3

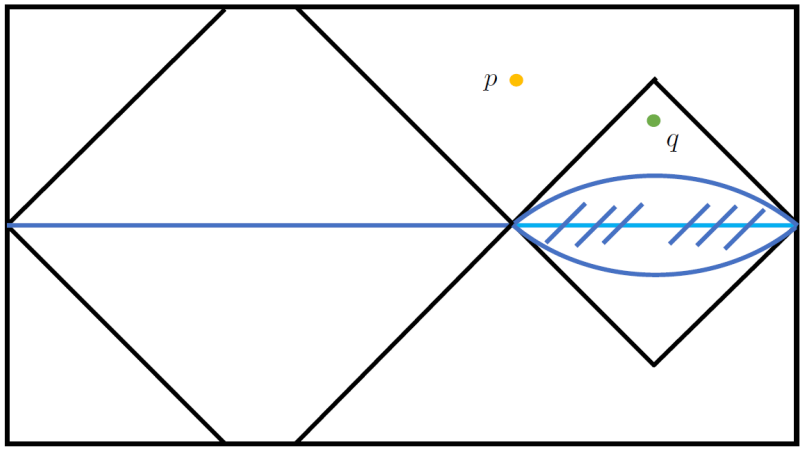



Boundary dS2




Modular Hamiltonian $K_R$ is local in this case (just Ttt) and modular evolution takes Ops(R)->Ops(D(R)).




$$\partial_r \log Z_n = -2\pi n r \int_0^\pi d\theta \, \sin\theta \, (T^\theta_\theta + T^\phi_\phi) \, .$$

maximal mixing:

$$S_0(r) = S_1(r) = \frac{\pi c}{6} \quad \text{for} \quad r = \sqrt{\frac{c\lambda}{12}} \, .$$

In gravity language, this corresponds to the central slice $w_c = \frac{\pi}{2}\ell$.

# All cases: modular evolution gets stalled for R<1/2

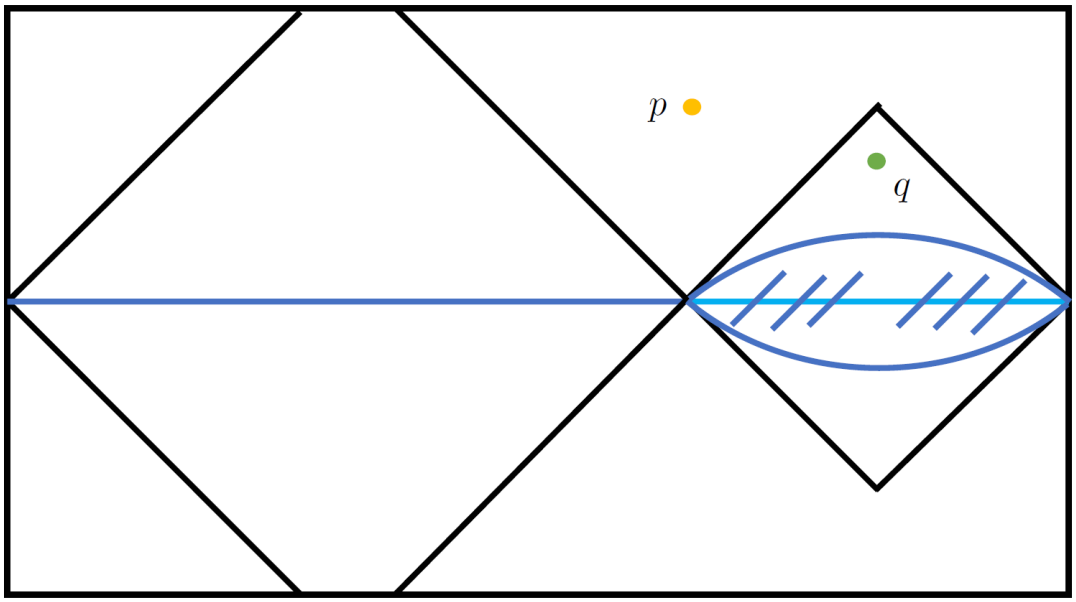
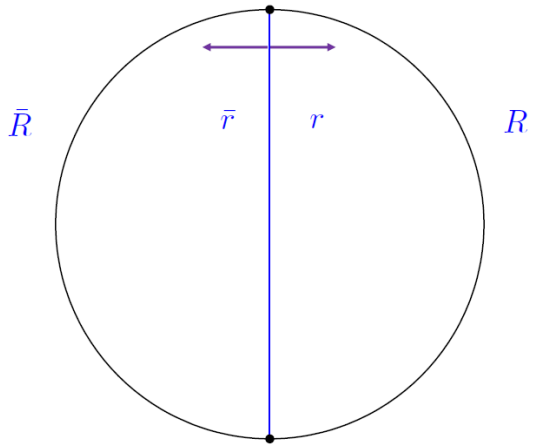
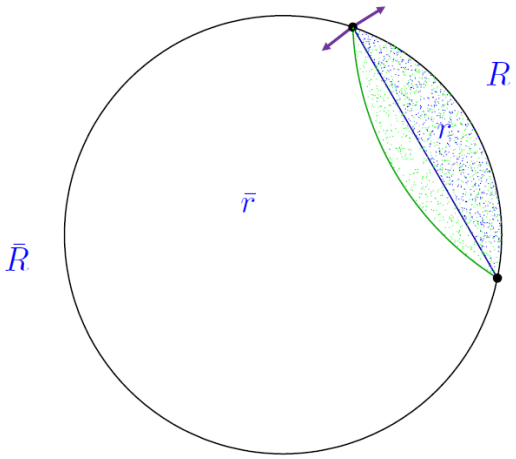



**Figure 4:** The regions $D[R]$ and $D[\bar{R}]$ for an asymmetric division of the system in the case of a boundary $dS_2$ spacetime, whose Penrose diagram we depict here. The region $E[R]$ for $R$ the smaller subregion is indicated by the dark blue hatchmarks. Operators at the point $p$ can be causally influenced from either region and are not in EW[R] or EW$[\bar{R}]$; this is familiar in asymptotic AdS/CFT. The point $q$ illustrates a point in $D[R]$ which is not in $E[R]$. Operators at that point do not commute with those in the algebra $\mathcal{A}_{\bar{R}_s}$ corresponding to the complementary subsystem to $R$. We note that this point $q$ on the boundary is also not part of the EW of either $R$ or $\bar{R}$, since it can be reached causally from both.

The full dS/dS warped throat is a simple extreme example: max mixing means $\rho \sim 1/\dim(H)$ so modular Hamiltonian acts as a constant (**no** evolution).




$$\begin{aligned}\Phi(X \in \mathrm{EW}[\bar{R}] \cap \mathrm{CW}[R]) &= \int_{\bar{R}} dx_{\bar{R}} \int ds\, f^{\bar{R}}_{\Delta,s}\left(X, x_{\bar{R}}\right) \rho_{\bar{R}}^{-is/2\pi} O\left(x_{\bar{R}}\right) \rho_{\bar{R}}^{is/2\pi} \\ &= \int_{D[R]} \tilde{f}\left(X, x_{D[R]}\right) U^{\dagger}\left(t_{D[R]}, t_R\right) O\left(x_R\right) U\left(t_{D[R]}, t_R\right)\end{aligned}$$

Consider formulating a bulk field $\phi$ in the overlap region using the top line, and its conjugate $\pi_\phi$ at the same point using the bottom line of (4.7). First, we use the fact that $[O(x_R), O(x_{\bar{R}})] = 0$. Thus in order to obtain the required nonzero canonical commutator in the bulk, $[\pi(X), \phi(X')] = i\delta(X - X')$, it must happen that

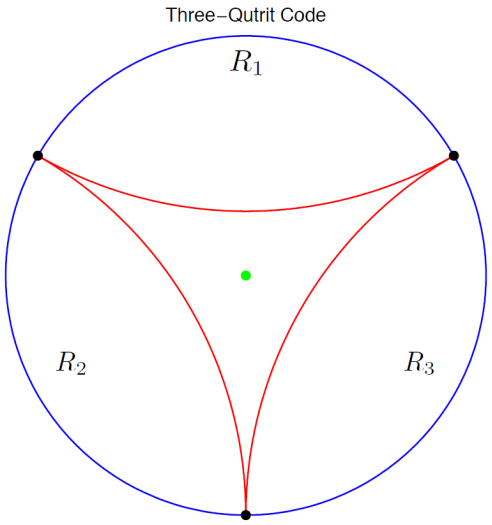
$$[\rho_{\bar{R}}, O(x_R)] \neq 0\,, \quad [U_{D[R]}, O(x_{\bar{R}})] \neq 0\,, \quad \text{or} \quad [\rho_{\bar{R}}, U_{D[R]}] \neq 0\,, \tag{4.8}$$

# Causal wedge > Entanglement wedge for R<1/2 in cut-off patches consistent with novel commutator structure.

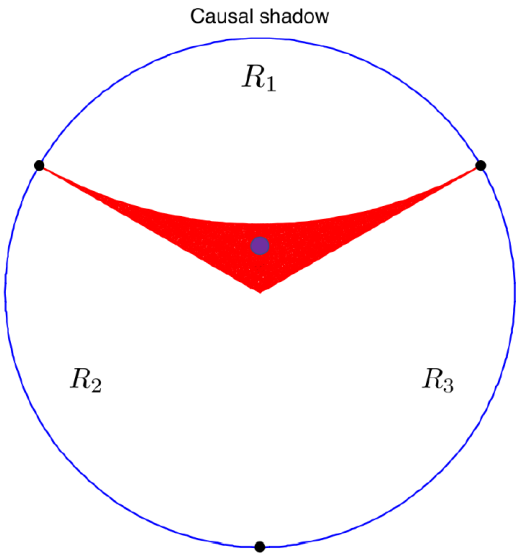
$$[\rho_{\bar{R}}, U_{D[R]}] \neq 0\,.$$

# Redundant encoding (QEC)

Almheiri Dong Harlow…The point in the middle is redundantly encoded, reconstructible by any of the complements of R1, R2, and R3.




For CW (HKLL) reconstruction, the causal shadow in our case limiting, but still exhibits Redundancy:

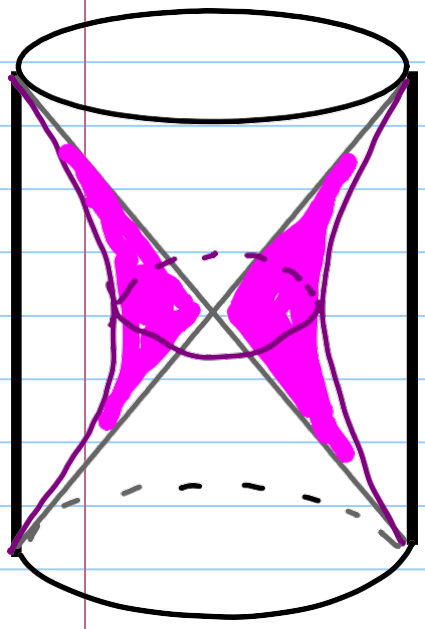



HKLL-Reconstructible via the complements of R2 and R3 (but not by the complement of R1).

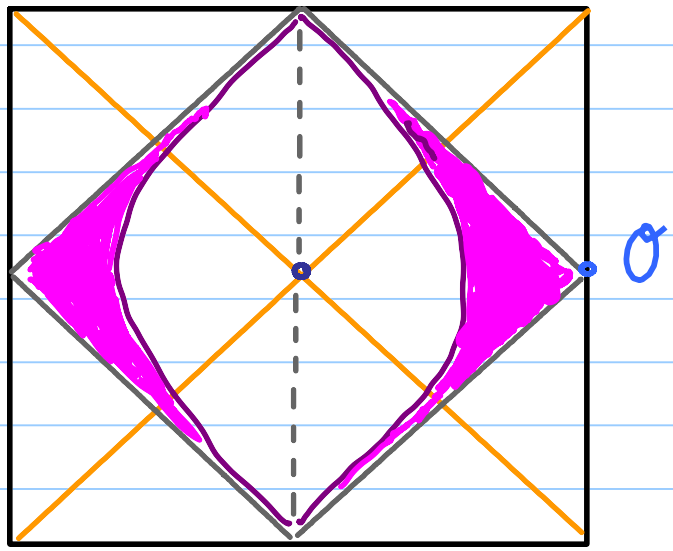
Full dS/dS patch involves two such coupled sectors of finite entropy. Macroscopically:

$$ds^2_{(A)dS_{d+1}} = dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})ds^2_{dS_d}$$

$$= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})\left[-d\tau^2 + \ell^2_{dS}\cosh^2\frac{\tau}{\ell_{dS}}d\Omega^2_{d-1}\right] .$$



AdS/dS

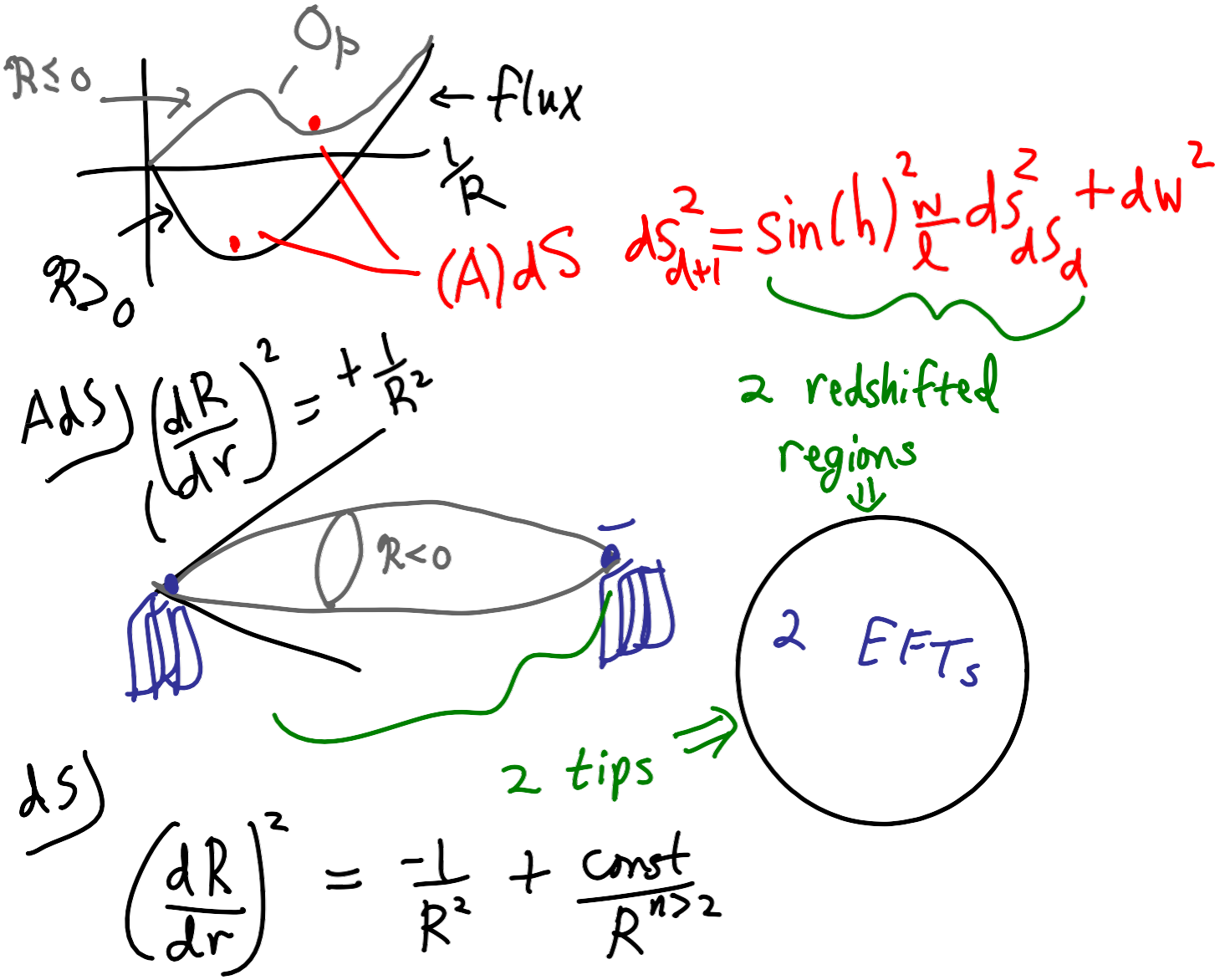



dS/dS (each point is (d-1)-sphere)

2 highly redshifted (IR) regions, each ~ IR region of AdS/dS

With T-Tbar+c2/ we first formulated each warped throat

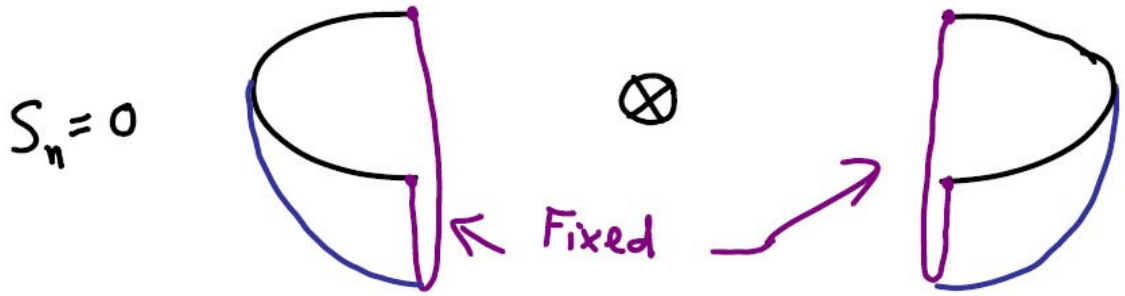
# Microscopic dS/dS: Uplifting AdS/CFT Dong Horn ES Torroba '10

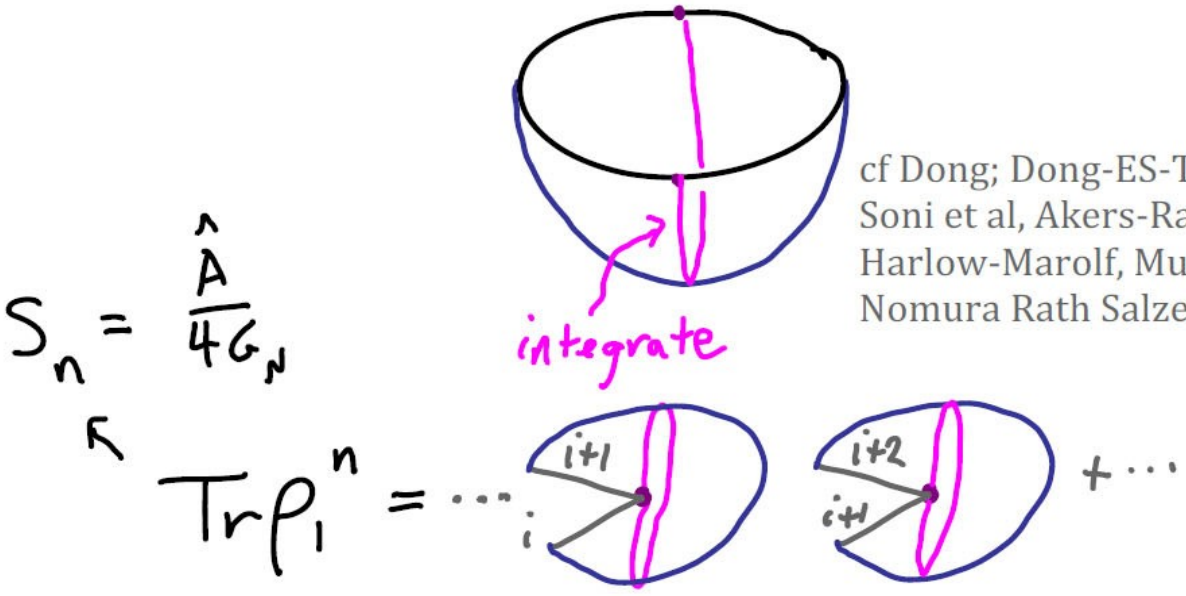



dS vs AdS brane construction: independent derivation of the two sectors because of metastability.

Also true in dS/CFT

**Joining throats:** Given the duality for each throat, generalized to any fixed boundary metric and fields, we can construct the ground state of the joined system and compute the Renyi entropies.



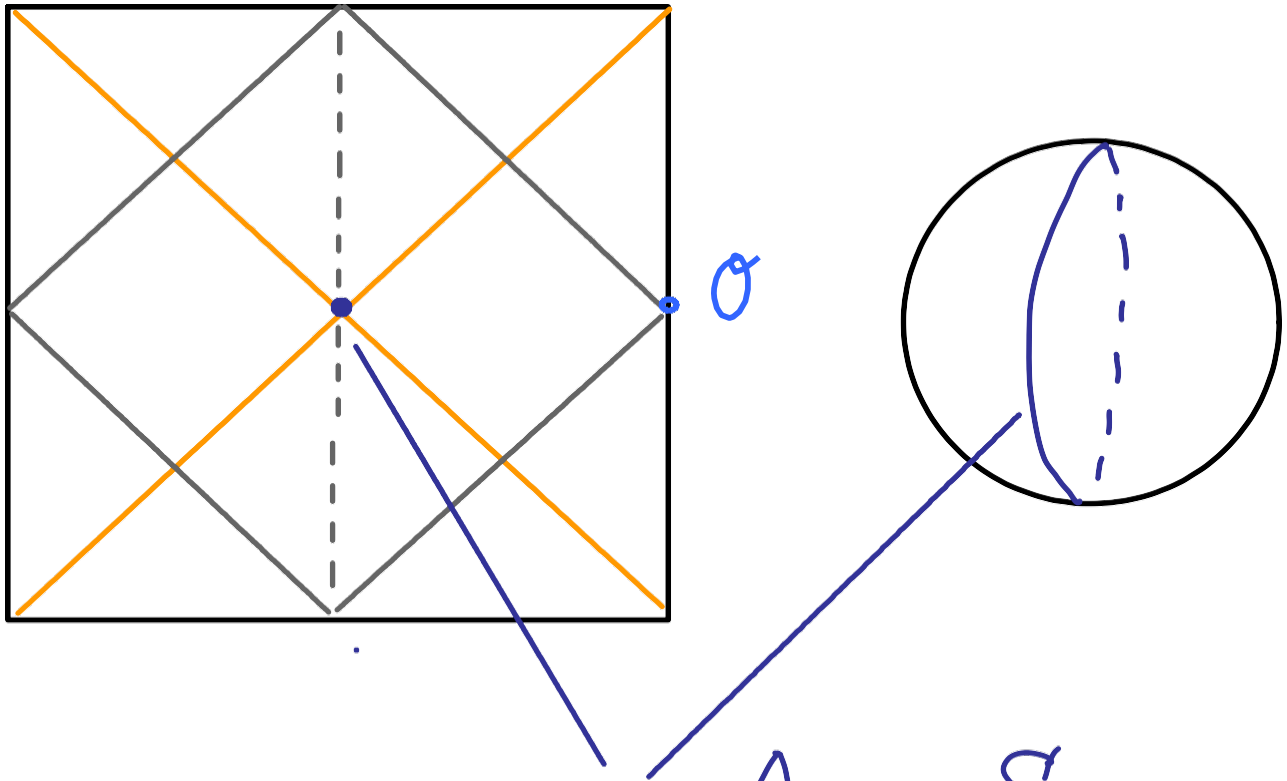




cf Dong; Dong-ES-Torroba; Soni et al, Akers-Rath/ Dong-Harlow-Marolf, Murdia Nomura Rath Salzetta, ...

In our case, the area of the shared locus has a saddle point independent of n, flat entanglement spectrum

Interactions -> highly mixed grnd state

# Interpretation of $S_{\text{Gibbons-Hawking}}$:




Ryu-Takayanagi Surface

$$\frac{A}{4G_N} = S_{RT}$$

$$= S_n = S_{vN} = \log \dim H = S^{(\vartheta)}_{G\text{-}H}$$

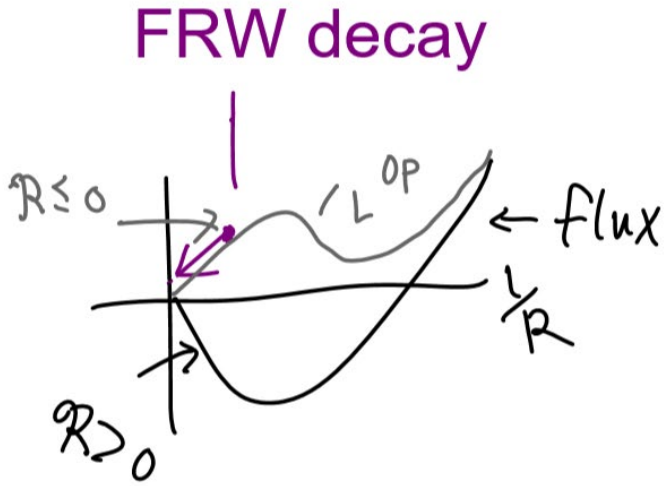
Observer O cannot interact with 2nd matter sector, traces over it.

Can generalize to more divisions, matter field profiles, etc. cf Geng Grieninger Karch

$S_{Gibbons-Hawking}$ is an approximate notion, obtained in a semiclassical (large c) limit.

This should not concern us any more than the count of protons in the periodic table does (the proton and neutron may decay, but we would not want to teach high school chemistry by working back from the ultimate decay products).

dS/dS part of a larger theory with asymptotic unbounded growth of entropy => potentially arbitrary precision then




Again 2-throated warped metric

$$ds^2 = C^2(\tau^{2/C} - w^2)^{C-1}dw^2 + (1 - \frac{w^2}{\tau^{2/C}})^{C-1}(-d\tau^2 + C^2\tau^2 dH_{d-1})$$

Previously found (Dong, Horn, ES, Matsuura, Torroba '12) entropy bound and d.o.f. count grows like $\tau$. $GR_d$ also decouples at late times.

# Future directions on part I

*T Tbar flow of the modular Hamiltonian from other points of view, e.g. generalization (Mazenc, Shyam, Soni) of the 2d gravity formulation w/Parrikar, Kruthoff,…

*Fully specify algebra of observables and its relation to EW and CW results? Cf Type … VN? cf Casini et al,…

*Entanglement shadows (cf Freivogel Jefferson et al) and islands (cf Pennington; Almheiri et al,…) in T T bar deformed theories including bulk dS.

*2d/1d case of dS/dS: e.g. integral over shared sources replaces SYK disorder average w/Kruthoff, Z. Yang, …

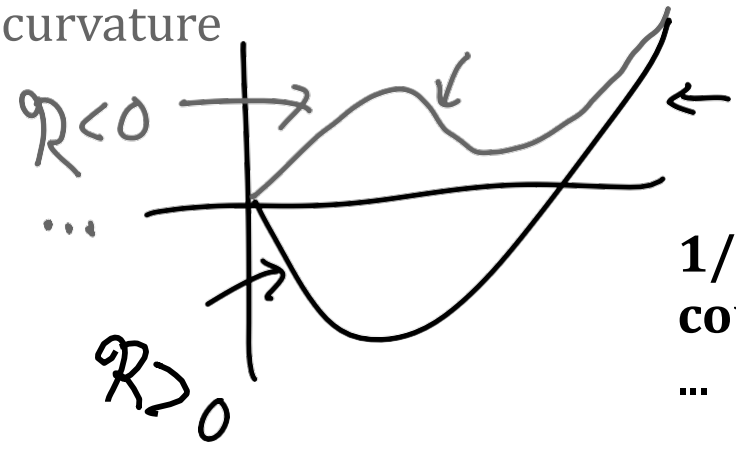
Generalize 2d gravity path integral formulations of T-Tbar and the single-trace version Giveon et al to include the $\Lambda_2$ deformation and curvature cf Mazenc, Shyam Soni. 2d side solvable, must have gravity-side realization. Analogy: double-trace deformations were novel on GR side since nonlocal on the internal sphere, but standard on QFT side.

In the single-trace version, the sign of $\lambda$ corresponding to the cutoff spacetime introduces singularities. Resolved in string theory, e.g. via enhancon mechanism? wip w/Anderson, Coleman, Mousatov

# Λ and String Theory: Potential energy $V(\phi_i)$ in 4d has mostly positive contributions, along with controlled negative sources

D-10
anti D3 brane
negative
curvature

Orientifolds
Quantum




Flux quanta
Bousso-
Polchinski

**1/Volume coupling**

**...**

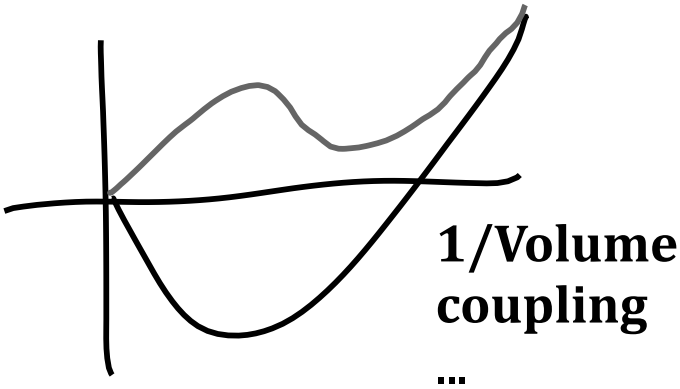
+other sources to fully stabilize

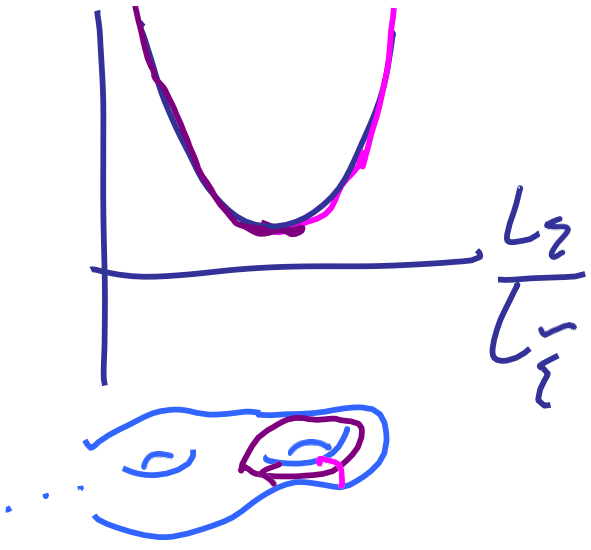
ES, +Maloney Strominger '01 Giddings/Kachru/Polchinski '01/KKLT'03 (studied in great detail), Large Volume, Riemann Surfaces, Flauger et al,….,Explicit example (2+1 bulk dimensions): Dong, Horn ES, Torroba 2010. Cordova/de Luca/Tomassiello '18, **...**

Fits with interpretation of small Λ as a selection effect.

**In effective field theory, we could simply write any constant Λ. In string theory, it eventually decays.**

Basic methods -- perturbation theory, numerics, mathematical theorems (e.g. existence of Calabi-Yau metric, structure of complex moduli of Calabi-Yau manifolds and Riemann Surfaces, exponential growth of the fundamental group and Mostow rigidity of hyperbolic manifolds, Selberg trace formula...); physical understanding grounded by controlled calculations. Effective field theory descending from string theory, analyzed at the appropriate level of approximation, also addressing situations where dangerous irrelevance enters (such as inflation) Sometimes worldsheet string theory is solvable classically (e.g. linear dilaton solution despite large string scale gradients); Fischler-Susskind. Integrability methods, holography, large Nf, large Nc,...



3-term structure, need large coefficients of subleading terms or small coeff of leading

(generalized) fluxes can stabilize dual cycles, e.g. complex moduli, in 2-term structure.

## Structure of the potential in perturbative string thy:

$$S = \frac{1}{2\alpha'^{\frac{D-2}{2}}} \int d^D x \sqrt{g}\, e^{-2\phi_s} \left( \mathcal{R} - \frac{2}{3}\frac{c - c_{\rm crit}}{\alpha'} + 4(\partial\phi_s)^2 \right) + S_{\rm matter}\,.$$

$$S_{matter} = \int d^D x \sqrt{-G}\{ \; - \sum_{n_B} \tau_{n_B} \frac{\delta^{(D-1-n_B)}(x_\perp)}{\sqrt{G_\perp}} + \sum_{n_O} \tau_{n_O} \frac{\delta^{(D-1-n_O)}(x_\perp)}{\sqrt{G_\perp}}$$
$$+ \; e^{-2\phi_s} |H_3|^2 + \sum_p |\tilde{F}_p|^2 + C.S. + h.d.\}$$

$$\tilde{F}_p = F_p + B \wedge F_{p-2} = dC_{p-1} + B \wedge dC_{p-3} \qquad \tilde{F}_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B \wedge dC_2$$

## Reduce to 4d, schematically

$$\frac{V}{M_P^4} = g_s^2 \left[ \frac{D-10}{L^{D-4}} + \frac{c_h}{L^{D-2}} + (\frac{N_3}{L^3})^2 \frac{1}{L^{D-4}} + (NS \text{ or } p-q \text{ branes}) \right]$$
$$+ \; g_s^3 \left[ - \sum_O \frac{\hat{\tau}_O L^{n_O}}{L^{2(D-4)}} + \sum_D \frac{\hat{\tau}_D L^{n_D} \sqrt{1 + b^2/L^{m_D}}}{L^{2(D-4)}} \right]$$
$$+ \; g_s^4 \left[ (\frac{Q_3 - C_0 N_3}{L^3})^2 \frac{1}{L^{D-4}} + (\frac{Q_5 - \frac{1}{2}c_2 N_3 + \frac{1}{2} b N_3}{L^3})^2 \frac{1}{L^{D-4}} + \dots \right]$$
$$+ \; h.d. + warping + quantum$$

1 term alone won't suffice, other limited no gos

**11d SUGRA (new) approach:** with G. Torroba; I. Agol

No dilaton. Consider hyperbolic space $H_7/\Gamma$, Casimir energy, and 7-form flux.

cf Kaloper et al, Saltman ES (string theory), Arkani-Hamed Dubovsky et al (Standard Model), Maldacena et al (SM wormholes)...

$$U(\ell) = M_4^4 \frac{1}{v_7 \hat{\ell}^7}\left(\frac{1}{\hat{\ell}^2} - \frac{\int_{H_7/\Gamma}\sqrt{-g}\rho_{Casimir}\ell_{11}^4}{v_7\hat{\ell}^7} + \frac{N_7^2}{v_7^2\hat{\ell}^{14}}\right)$$

$$= M_4^4 \frac{1}{v_7 \hat{\ell}^7}\left(\frac{1}{\hat{\ell}^2} - \frac{K}{\hat{\ell}^{11}} + \frac{N_7^2}{v_7^2\hat{\ell}^{14}}\right), \qquad M_4^2 \sim v_7\ell^7/\ell_{11}^9 = v_7\frac{\hat{\ell}^7}{\ell_{11}^2}$$

$$\hat{\ell} \equiv \ell/\ell_{11}$$

Can K, and all radii, be large? Yes given e.g. hyperbolic cusps with asymmetric cross section. cf discussions with I. Agol, M. Freedman

$$ds^2_{local} = dy^2 + e^{-2y/\ell}ds^2_{T^6} \qquad y_0 < y < y_c$$

$$\frac{\int\sqrt{-g}\rho_{Casimir}\ell_{11}^4}{v_7\hat{\ell}^7} \sim \pm n_c \frac{v_T\hat{\ell}^6}{v_7\hat{\ell}^7}\int_{\hat{y}_0}^{\hat{y}_c}\frac{d\hat{y}}{\hat{\lambda}_c^{11}}e^{-6\hat{y}/\hat{\ell}}e^{11\hat{y}/\hat{\ell}} \simeq \pm\frac{v_T n_c}{5v_7\hat{\lambda}_c^{11}}e^{5y_c/\ell} \sim \pm\frac{1}{\hat{\ell}^{11}}\left(\frac{\hat{\ell}}{\hat{\ell}_c}\right)^5\frac{Vol(T^6)}{\lambda_c^6}\frac{n_c}{v_7}$$

Mostow Rigidity => Hyperbolic metric is extremum of deformations of the geometry aside from overall curvature radius. To do: estimate shifts and warping effects from variation of Casimir energy within $H_7/\Gamma$.

# M. Douglas '09: Model-independent, well defined expression for effective 4d potential incorporating GR constraints

Let us recap the final expression for the effective potential. We have eliminated all dependence on $d$-dimensional physics except the number $d$, which we now set to $d = 4$. The metric, curvature, fluxes and warp factor $u$ are all defined on $X$, as is the additional "stringy source" $T^{(d)}_{string}$.

$$V_{eff} = \frac{1}{2}\int \sqrt{g}\left[u^2\left(-R + \frac{1}{2}\sum_p |F^{(p)}|^2 - \frac{1}{2}T^{(d)}_{string}\right) - 3(\nabla u)^2\right] + \frac{1}{2}C\left(\frac{1}{G_N} - \int \sqrt{g}\, u\right). \tag{3.1}$$

To use it, one enforces the warp factor constraint $\delta V_{eff}/\delta u = 0$, which in $d = 4$ is linear,

$$-\frac{1}{6}C = \left(-\nabla^2 + \frac{2}{3}U\right)u; \qquad U = \frac{1}{2}R - \frac{1}{4}\sum_p |F^{(p)}|^2 + \frac{1}{4}T^{(d)}_{string} \tag{3.2}$$

thus determining $C$ and $u$ up to an overall normalization, and the warped volume constraint

$$\frac{1}{G_N} = \int \sqrt{g}u,$$

thus determining the normalization. Due to the simple form of Eq. (3.1), its final value on substituting $u$ is simply $C/4G_N$, as in Eq. (2.41).

## Warp factor absorbs naïve instability for any manifold in the direction of the conformal factor of internal metric. For the hyperbolic case, no moduli (dim n>2) and other directions positive.

# What about SUSY?

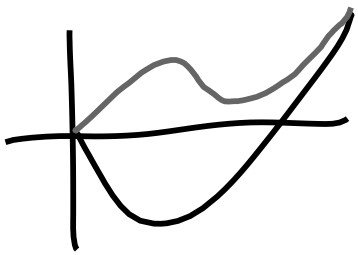
The classical de Sitter models and their later generalizations were initially not very popular in the field in part because they did not incorporate a symmetry known as supersymmetry, a hypothetical relation between particles of different spins. An extended version of this symmetry plays a crucial role in theoretically controlling an interpolation between systems which interact weakly and those which interact strongly, enabling beautiful calculations of black hole states and other strong-coupling theoretical physics [42].

However, such a symmetry is neither observed nor generically preserved among the energy sources in string theory. It is not required for controlled theoretical calculations in a regime with weak interactions (as well as appropriate numerical calculations beyond those regimes). Indeed, all of the empirically tested theoretical physics that has ever been done – including breathtaking phenomena such as relativity, quantum physics, development of the standard models of particle physics and cosmology, many body effects such as superconductivity, topological phases, and so on – does not rely on such a symmetry.

*There are trade-offs: Less SUSY enables more contributions to V, power law rather than exponential stabilization. Pert thy suffices.
*dS can help with mathematical rigor in some situations: e.g. Vasy's proof of stability of Kerr relies on it.

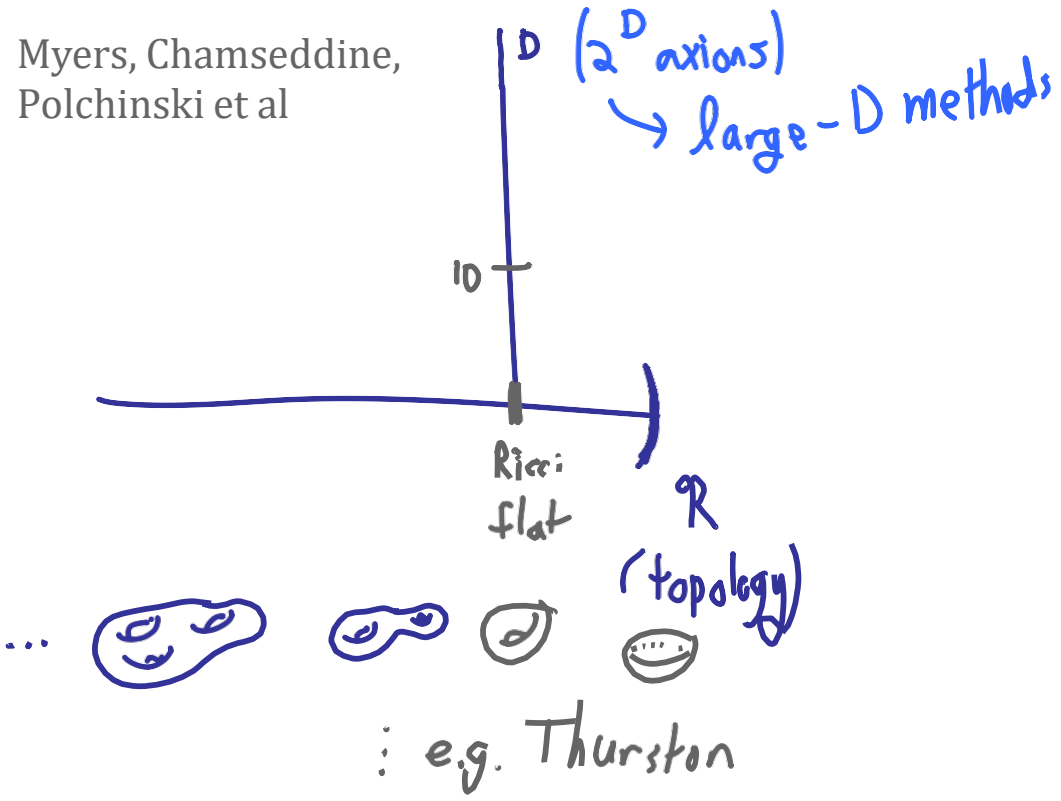
Note: Hyperbolic compactification better understood starting point than CY, in the sense:

1st positive term has known metric, then add 2 more sources. CY: classically zero potential (metric unknown), then add 3 sources in each such direction.



String theory configuration space has many directions (but lots of structure). Leading behavior:

Myers, Chamseddine, Polchinski et al




These different starting points are dynamically connected to each other Hellerman-Swanson, Green Lawrence McGreevy Morrison ES. e.g. Winding string condensation links different topologies Adams Liu McGreevy Saltman ES. T duality generalizes in a precise way to compact hyperbolic spaces: McGreevy ES Starr:

$$Z_{\rm IR} \equiv Z_1|_{\tau_2\to\infty} \sim e^{-\pi\tau_2\alpha' m_{\rm min}^2}. \qquad Z_{\rm UV} \equiv Z_1|_{\tau_2\to 0} \sim \sum_m \rho(m) e^{-\pi\tau_2\alpha' m^2} \sim e^{\frac{\pi c_{\rm eff}}{6\tau_2}}.$$

$$Z_{\rm IR} \sim e^{\pi\alpha'\tau_2(n-1)^2/(4t^2)}. \qquad \rho(l) \sim l^\alpha e^{l/l_0} \qquad Z_{\rm UV} \sim \exp\left(\frac{\pi\alpha'(n-1)^2}{4t^2\tau_2} + \frac{\pi c_{\rm eff}^{\rm crit}}{6\tau_2}\right)$$

D duality: Buscher derivation of T duality, as well as D-brane version, extends to hyperbolic spaces (eg Riemann surfaces, giving Jacobian torus of dimension 2 x genus as the dual). Green Lawrence McGreevy Morrison ES.

Jacobian=torus made of Wilson lines.

$$L = \int d^2\tilde{\theta}\frac{1}{2}\gamma_{ab}\left[\tilde{\bar{\Sigma}}^a\left(X^b - \int^Z \omega^b\right) - \bar{\Sigma}^a X^b\right] + \text{h.c.}$$
$$+ \int d^4\theta\gamma_{ab}\left[\frac{1}{2}(Y^a + \bar{Y}^a + V^a)(Y^b + \bar{Y}^b + V^b) - \frac{1}{2e^2}\tilde{\Sigma}^a\,{}^\dagger\tilde{\Sigma}^b\right]$$

For similar reasons, we expect that our derivation will generalize to yield a dual description of any space Mn in terms of a ˜ b1-dimensional torus, where ˜ b1 is the first Betti number of any finite cover of Mn (known as a virtual first betti number of Mn). For example, there exist many hyperbolic 3-manifolds which are homology spheres, i.e., H1 = {0} so that b1 = 0 [29]. Mathematicians have conjectured that these always have finite covers with nonzero ˜ b1 (see, for example, [30]); this would provide a way to extend D-duality to general 3-manifolds with ceff > ccrit. Conversely, it would be interesting to see if our physical methods can provide insight into the validity of the virtual first betti number conjecture.

The Thurston conjecture about finite covers with nonzero b1 was proved recently by I. Agol.

## Status of **Power Law Stabilization:**

* D>Dc: Maloney ES Strominger '01-'02, Dodelson et al '13, +Huajia Wang unpublished

$$U(\phi) = \left(a - be^{\phi} + \frac{b^2}{4a}(1+\delta)e^{2\phi}\right)e^{\frac{4\phi}{d-2}}$$

$b \sim 2^{D/4}$ Gives the needed large parameter for the middle term, RR flux term also large. Compete with (exactly solvable) stringy gradient in a term. Unpublished calculation by Wang indicates alpha' corrections from flux suppressed at large D. In general need more analysis of large D. Cf Emparan, Minwalla et al

* D=10 many authors (including collaborators Dong, Flauger, Horn, McAllister, Saltman, Torroba, ...) ... varying sets of ingredients and results

**explicit 3d model Dong Horn ES Torroba (numerically large radius, localized sources treated with consistent approximation as above).

$$\mathcal{U} \approx 16 M_3^3 k^3 \left\{ \left( 4\pi^2 - \frac{2\pi^2}{3\beta^2}\left[ 24 - n_\rho - \hat{n}_\rho \left( \left( \log \frac{L^2}{L_*^2} \right)^2 + \frac{(b_T - b_*)^2}{L^4} \right) \right] + \frac{\pi k n_{NS5}}{L^2 \beta^3} \right) \frac{\tilde{\eta}^4}{k} \right.$$
$$\left. - \left( 2\pi R^2 - \frac{n_{D7} R^4 \beta}{2k} \right) \frac{\tilde{\eta}^5}{\beta^3} + 4\pi^2 \left( N_{D5}^2 L^4 + \frac{(N_{D1} + b_T^2 N_{D5})^2}{L^4} + 2 b_T^2 N_{D5}^2 \right) \frac{k \tilde{\eta}^6}{\beta^4} \right\} \quad (4.39)$$

**~~Nil manifold example~~ ES '07 example **Incorrect**. Guy Gur-Ari '13 was **correct**. Introduced methods and sources that were useful later, stimulated monodromy.

## ...Power law stabilization status
## **Product of Riemann surfaces:

Saltman ES '04. Logical status similar to KKLT, though less extensively studied: full mechanism for volume, dilaton, complex moduli, but not explicit about 7-brane positions. Here also anti 7-branes but well separated so no annihilation instabilities.

$$A(\tau)=i\begin{pmatrix} 2\tau(\tau-\bar{\tau})^{-1}\bar{\tau} & -(\tau+\bar{\tau})(\tau-\bar{\tau})^{-1} \\ -(\tau-\bar{\tau})^{-1}(\tau+\bar{\tau}) & 2(\tau-\bar{\tau})^{-1}\end{pmatrix} \qquad U_{\text{flux}}\propto Q^{ikm}A_i^j(\tau_1)A_k^l(\tau_2)A_m^n(\tau_3)Q_{jln}$$

complex moduli stabilization by fluxes precistly as in Giddings-Kachru-Polchinski. Volumes, dilaton, complex moduli stabilized:

$$U=\frac{1}{l_4^4}\frac{g_s^4}{(V_1V_2V_3)^2}\left\{\sum_r\frac{1}{g_s^2}2(h_r+n_7-1)V_{r+1}V_{r+2}+\frac{1}{g_s^2}N^{i_1i_2i_3}A(\tau_1)_{i_1}^{j_1}A(\tau_2)_{i_2}^{j_2}A(\tau_3)_{i_3}^{j_3}N_{j_1j_2j_3}\right.$$
$$-\frac{1}{g_s}N_7+Q_3^{i_1i_2i_3}A(\tau_1)_{i_1}^{j_1}A(\tau_2)_{i_2}^{j_2}A(\tau_3)_{i_3}^{j_3}Q_{3\ j_1j_2j_3}$$
$$\left.+\sum_r\left[Q_1^{i_r}A(\tau_r)_{i_rj_r}Q_1^{j_r}V_{r+1}V_{r+2}+Q_5^{i_r}A(\tau_r)_{i_rj_r}Q_5^{j_r}\frac{1}{V_{r+1}V_{r+2}}\right]\right\}$$

## Large field inflation e.g. McAllister ES Westphal, Kaloper Lawrence, Flauger et al, D'Amico et al, Kim Nilles Peloso, Kachru et al... e.g. axion monodromy on Riemann surfaces:

$$\tilde{F}_p = F_p + B \wedge F_{p-2} = dC_{p-1} + B \wedge dC_{p-3}$$

## Axion potential can help stabilize other moduli, giving strong accelerated expansion (inflation):

cf McAllister ES Westphal Wrase '14, Dodelson Dong ES Torroba '13

$$\mathcal{U}|_{\sigma=\sigma_{min}} \sim M_P^4 \left\{ C_h^2 n_3^2 \frac{1}{\mathcal{V}^{2/3}} + C_h^4 (q_3^2 + q_1^2 b^2) \mathcal{V}^{2/3} + C_h^4 q_5^2 \right\}$$

$$\frac{\dot{b}^2}{\mathcal{V}^{2/3}} \propto \dot{b}^2 b \Rightarrow \phi_b \propto b^{3/2} \qquad V \approx \mu^{10/3} \phi^{2/3} .$$

## Back reaction from light degrees of freedom part of all original analyses of monodromy! and large field range:

$b \propto L^{-4}$ where L is the size of the compact space.

$b \propto \phi_{canonical}^{2/3}$

During inflation, $\phi_{canonical}$ evolves from $10M_p$ to $M_p$. Thus it rescales by a factor of $1/10$. So $b$ rescales by a factor of $1/10^{2/3}$. So $L$ rescales by a factor of $10^{1/6}$. Although $L$ changes, and $L$ is exponential in the canonical field, and it determines KK masses, this change over 10 Planck units of canonical field does not affect the KK spectrum much at all.

Note: As it stands, string theory has a nontrivial but model- dependent interaction with early universe data.

No model-independent prediction is at all justified for tensor mode amplitude or for the form of the non-Gaussianity of the distribution of primordial scalar perturbations, or for the form of dark energy.

Input from string theory models has helped systematize non-Gaussianity analysis (introducing mechanisms and related signatures that had not been considered although they can also be incorporated into EFT) and provides interesting targets for r but no universal prediction.

Further reading: Various reviews, some detailed and accurate (none fully comprehensive). Original papers.

## Somewhat persistent incorrect statements:

*`Supercritical has ghosts, KKLT antibrane fails to uplift.' No, must solve correct eom correctly.

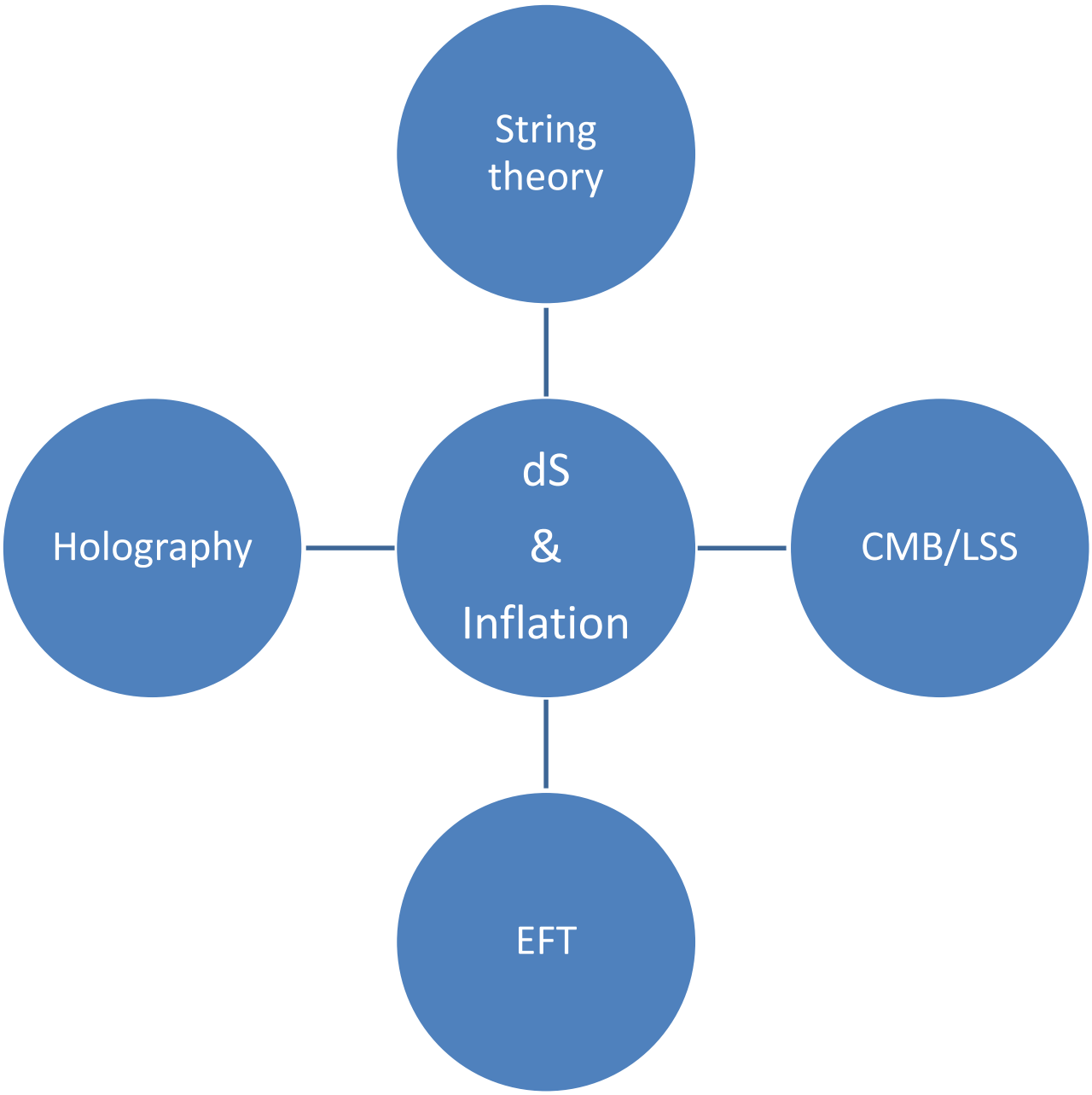
* Models with classical + quantum playoff problematic? No. The stability of matter depends on QM. Near- stability of white dwarfs and neutron stars depends on this playoff.

*More SUSY breaking means less control. No. There are clear tradeoffs mentioned above. Again, all physics that has ever been done has no SUSY and we can control that in string theory with same essential tools.

Another trade-off: large field inflation via symmetry can be simpler than accidental small field. The back reaction, which is included in the worked examples, leads to phenomenologically viable flattened potentials and no significant back reaction from light fields.

*No go theorems essentially exclude classical dS. No. They apply to 0% of models. (0 = finite/infinite).

*Landscape based on arguments like ``why not" or ``anything goes". No. Just, no.


String theory
Holography
dS & Inflation
CMB/LSS
EFT